

Lecture 17

on

Numerical Methods & Computer Programming

Topic: **Solution of systems of linear equations**

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Solution of Systems of Linear Equations

Systems of Equations

A set of equations is called a **system of equations**.

The **solutions** must satisfy each equation in the system.

If all equations in a system are linear, the system is a **system of linear equations**, or a **linear system**.

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Solution of Systems of Linear Equations

Systems of Linear Equations:

A solution to a system of equations is an **ordered pair** that satisfy all the equations in the system.

A system of linear equations can have:

1. Exactly one solution
2. No solutions
3. Infinitely many solutions



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Solution of Systems of Linear Equations

Systems of Linear Equations:

There are four ways to **solve systems of linear equations**:

1. By graphing
2. By substitution
3. By elimination
4. By multiplication (Matrices)



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Solution of Systems of Linear Equations

Solving Systems by Graphing:

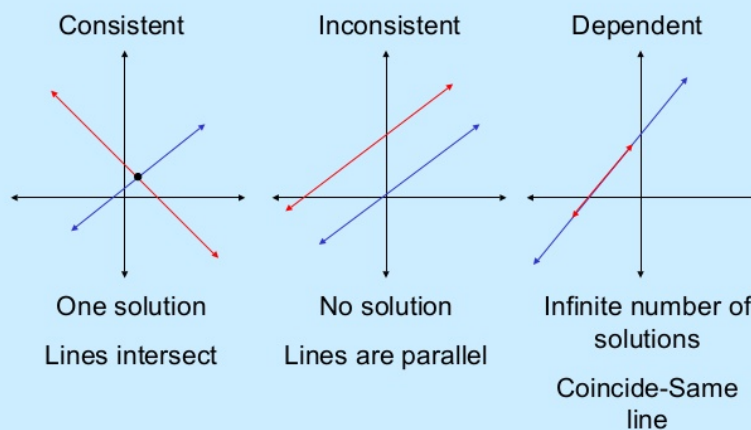
When solving a system by graphing:

1. Find ordered pairs that satisfy each of the equations.
2. Plot the ordered pairs and sketch the graphs of both equations on the same axis.
3. The coordinates of the point or points of intersection of the graphs are the solution or solutions to the system of equations.



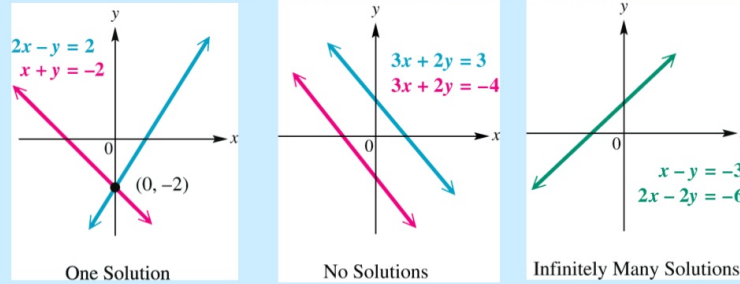
Solution of Systems of Linear Equations

Solving Systems by Graphing:



Solution of Systems of Linear Equations

Linear System in Two Variables



Three possible solutions to a linear system in two variables:

One solution: coordinates of a point

No solutions: **inconsistent** case

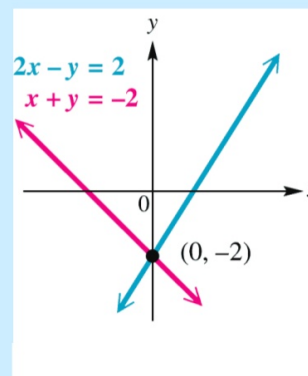
Infinitely many solutions: **dependent** case

Solution of Systems of Linear Equations

$$\begin{aligned} 2x - y &= 2 \\ x + y &= -2 \end{aligned}$$

$$\begin{aligned} 2x - y &= 2 \\ -y &= -2x + 2 \\ y &= 2x - 2 \end{aligned}$$

$$\begin{aligned} x + y &= -2 \\ y &= -x - 2 \end{aligned}$$



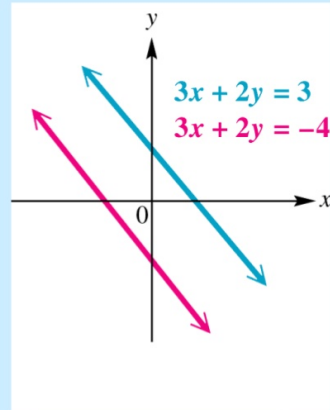
Different slope, different intercept!

Solution of Systems of Linear Equations

$$\begin{aligned}3x + 2y &= 3 \\3x + 2y &= -4\end{aligned}$$

$$\begin{aligned}3x + 2y &= 3 \\2y &= -3x + 3 \\y &= -\frac{3}{2}x + \frac{3}{2}\end{aligned}$$

$$\begin{aligned}3x + 2y &= -4 \\2y &= -3x - 4 \\y &= -\frac{3}{2}x - 2\end{aligned}$$



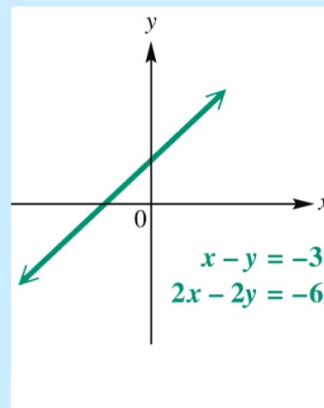
Same slope, different intercept!!

Solution of Systems of Linear Equations

$$\begin{aligned}x - y &= -3 \\2x - 2y &= -6\end{aligned}$$

$$\begin{aligned}x - y &= -3 \\-y &= -x - 3 \\y &= x + 3\end{aligned}$$

$$\begin{aligned}2x - 2y &= -6 \\-2y &= -2x - 6 \\y &= x + 3\end{aligned}$$



Same slope, same intercept!
Same equation!!

Solution of Systems of Linear Equations

Determine Without Graphing:

- There is a somewhat shortened way to determine *what type* (one solution, no solutions, infinitely many solutions) of solution exists within a system.
- Notice we are not finding *the* solution, just *what type* of solution.
- Write the equations in slope-intercept form: $y = mx + b$.
(i.e., solve the equations for y , remember that $m = \text{slope}$, $b = y - \text{intercept}$).

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Solution of Systems of Linear Equations

Determine Without Graphing:

Once the equations are in slope-intercept form, compare the slopes and intercepts.

One solution – the lines will have different slopes.

No solution – the lines will have the **same slope**, but different intercepts.

Infinitely many solutions – the lines will have the **same slope and the same intercept**.



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Solution of Systems of Linear Equations

Determine Without Graphing:

Given the following lines, determine what type of solution exists, **without graphing**.

Equation 1: $3x = 6y + 5$

Equation 2: $y = (1/2)x - 3$

Writing each in slope-intercept form (solve for y)

Equation 1: $y = (1/2)x - 5/6$

Equation 2: $y = (1/2)x - 3$

Since the lines have the same slope but different y-intercepts, there is no solution to the system of equations. The lines are parallel.

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Thank You

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