Lecture 18

on

Numerical Methods & Computer Programming

Topic: Solution of systems of linear equations

Dr. Md. Abu Sayeed
Associate Professor
Department of Civil Engineering, RUET

Solution of Systems of Linear Equations

Substitution Method:

Procedure for Substitution Method

1. Solve *one* of the equations for *one* of the variables.
2. Substitute the expression found in step 1 into the other equation.
3. Now solve for the remaining variable.
4. Substitute the value from step 2 into the equation written in step 1, and solve for the remaining variable.

Solution of Systems of Linear Equations

Substitution Method:

1. Solve the following system of equations by substitution.

$$y = x + 3$$
$$x + y = -5$$

Step 1 is already completed.

Step 2: Substitute x+3 into 2nd equation and solve.

$$x + (x+3) = -5$$
$$2x + 3 = -5$$
$$2x = -8$$
$$x = -4$$

Step 3: Substitute –4 into 1st equation and solve.

$$y = x + 3$$
$$y = -4 + 3$$
$$y = -1$$

The answer: (-4 , -1)

Solution of Systems of Linear Equations

1) Solve the system using substitution

$$x + y = 5$$
$$y = 3 + x$$

Step 1: Solve an equation for one variable. — The second equation is already solved for y!

Step 2: Substitute — $x + y = 5$; $x + (3 + x) = 5$

Step 3: Solve the equation. — $2x + 3 = 5$; $2x = 2$; $x = 1$

Solution of Systems of Linear Equations

1) Solve the system using substitution

$x + y = 5$
$y = 3 + x$

Step 4: Plug back in to find the other variable.

$x + y = 5$
$(1) + y = 5$
$y = 4$

Step 5: Check your solution.

(1, 4)
$(1) + (4) = 5$ ✓
$(4) = 3 + (1)$ ✓

The solution is (1, 4). What do you think the answer would be if you graphed the two equations?

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

2) Solve the system using substitution

$3y + x = 7$
$4x – 2y = 0$

Step 1: Solve an equation for one variable.

It is easiest to solve the first equation for x.

$3y + x = 7$
$-3y \quad -3y$
$x = -3y + 7$

Step 2: Substitute

$4x – 2y = 0$
$4(-3y + 7) – 2y = 0$

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

2) Solve the system using substitution

$3y + x = 7$
$4x – 2y = 0$

Step 3: Solve the equation.

$-12y + 28 – 2y = 0$
$-14y + 28 = 0$
$-14y = -28$
$y = 2$

Step 4: Plug back in to find the other variable.

$4x – 2y = 0$
$4x – 2(2) = 0$
$4x – 4 = 0$
$4x = 4$
$x = 1$

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

2) Solve the system using substitution

$3y + x = 7$
$4x – 2y = 0$

Step 5: Check your solution.

(1, 2)
$3(2) + (1) = 7$ ✓
$4(1) – 2(2) = 0$ ✓

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

Deciding whether an ordered pair is a solution of a linear system.

The solution set of a linear system of equations contains all ordered pairs that satisfy all the equations at the same time.

Example 1: Is the ordered pair a solution of the given system?

$$2x + y = -6$$
$$x + 3y = 2$$

Substitute the ordered pair into each equation.
Both equations must be satisfied.

A) (-4, 2)

$2(-4) + 2 = -6$
$(-4) + 3(2) = 2$
$-6 = -6$
$2 = 2$
$\therefore$ Yes

B) (3, -12)

$2(3) + (-12) = -6$
$(3) + 3(-12) = 2$
$-6 = -6$
$-33 \neq -6$
$\therefore$ No

Solution of Systems of Linear Equations

Substitution Method

Example Solve the system.

$$3x + 2y = 11 \quad \textbf{(1)}$$
$$-x + y = 3 \quad \textbf{(2)}$$

Solution

$y = x + 3$	Solve **(2)** for y.
$3x + 2(x+3) = 11$	Substitute $y = x + 3$ in **(1)**.
$3x + 2x + 6 = 11$	Solve for x.
$5x = 5$	
$x = 1$	
$y = 1 + 3$	Substitute $x = 1$ in $y = x + 3$.
$y = 4$	Solution set: $\{(1, 4)\}$

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

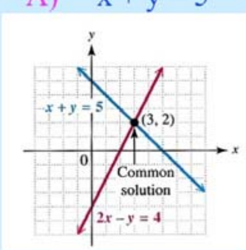
Systems of Linear Equations in Two Variables

Solving Linear Systems by Graphing.

One way to find the solution set of a linear system of equations is to graph each equation and find the point where the graphs intersect.

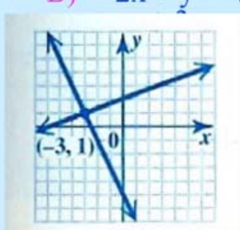
Example 1: Solve the system of equations by graphing.

A) $x + y = 5$



Solution: {(3,2)}

B) $2x + y = -5$



Solution: {(-3,1)}

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

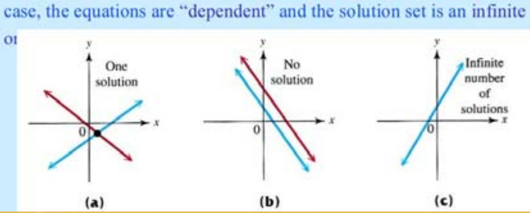
Solution of Systems of Linear Equations

Systems of Linear Equations in Two Variables

Solving Linear Systems by Graphing.

There are three possible solutions to a system of linear equations in two variables that have been graphed:

1) The two graphs intersect at a single point. The coordinates give the solution of the system. In this case, the solution is "consistent" and the equations are "independent".
2) The graphs are parallel lines. (Slopes are equal) In this case the system is "inconsistent" and the solution set is 0 or null.
3) The graphs are the same line. (Slopes and y-intercepts are the same) In this case, the equations are "dependent" and the solution set is an infinite set of or



Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

4-1 Systems of Linear Equations in Two Variables

Solving Linear Systems of two variables by Method of Substitution.

Step 1: Solve one of the equations for either variable

Step 2: Substitute for that variable in the other equation

(The result should be an equation with just one variable)

Step 3: Solve the equation from step 2

Step 4: Substitute the result of Step 3 into either of the original equations and solve for the other value.

Step 6: Check the solution and write the solution set.

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

4-1 Systems of Linear Equations in Two Variables

Solving Linear Systems of two variables by Method of Substitution.

Example 6: Solve the system : $4x + y = 5$, $2x - 3y = 13$

Step 1: Choose the variable y to solve for in the top equation:

$y = -4x + 5$

Step 2: Substitute this variable into the bottom equation

$2x - 3(-4x + 5) = 13 \longrightarrow 2x + 12x - 15 = 13$

Step 3: Solve the equation formed in step 2

$14x = 28$ $\quad x = 2$

Step 4: Substitute the result of Step 3 into either of the original equations and solve for the other value. $4(2) + y = 5$

$y = -3$

Solution Set: $\{(2,-3)\}$

Step 5: Check the solution and write the solution set.

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Solution of Systems of Linear Equations

Systems of Linear Equations in Two Variables

Solving Linear Systems of two variables by Method of Substitution.

Example 7:

Solve the system : $\frac{1}{2}x + \frac{1}{4}y = \frac{1}{2}$ rewrite as $\Rightarrow 4[\frac{1}{2}x + \frac{1}{4}y = \frac{1}{2}] \Rightarrow 2x + y = 2$

$-2x + 5y = 22$

Solve: $2x + y = 2$

$-2x + 5y = 22$

$y = -2x + 2$

$-2x + 5(-2x + 2) = 22 \longrightarrow -2x - 10x + 10 = 22$

$-12x = 12$

$x = -1$ $\quad 2(-1) + y = 2$

$y = 4$

Solution Set: $\{(-1,4)\}$

Dr. Md. Abu Sayeed, Assoc Prof., Dept of CE, RUET

Thank You

Dr. Md Abu Sayeed, Associate Prof., Dept of CE, RUET