Lecture 19

on

Numerical Methods & Computer Programming

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Basic Definitions of Matrices

m x n Matrix

A *matrix* is an array of *mn* elements arranged in *m* rows and *n* columns. Such a matrix *A* is usually denoted by

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}],$$

where $a_{11}, a_{12}, \ldots$ are called its *elements* and may be either real or complex. The matrix *A* is said to be of size $(m \times n)$.

Basic Definitions of Matrices

Square Matrix

If $m = n$, the matrix is said to be a *square* matrix of *order n*. Thus,

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Single row matrix or a row vector

is a square matrix of order 3. We may also have *single-row* or *single-column* matrices. These are called *vectors*. Thus, $[a_{11}, a_{12}, a_{13}, \ldots, a_{1n}]$ is a single-row matrix or a *row vector*, and

Single column matrix or a column vector

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{bmatrix}$$

is a single-column matrix or a *column vector*.

Principal diagonal or main diagonal

Trace of matrix *A*

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

The elements a_{ii} in a square matrix form the *principal diagonal* (or *main diagonal*). Their sum $a_{11} + a_{22} + \cdots + a_{nn}$ is called the *trace* of *A*.

A null matrix:

If all the elements of a square matrix are zero, then the matrix is said to be a null matrix.

Diagonal matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

On the other hand, if only the elements on the main diagonal are nonzero, then the matrix is said to be a *diagonal* matrix. For example,

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

is a diagonal matrix.

Unit matrix

In particular, the diagonal matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

in which all of the diagonal elements are equal to one, is called a *unit* matrix of order 3. Unit matrices are usually denoted by I.

Inverse of a matrix

Can you determine the Inverse of *matrix A*

$$A = \begin{bmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{bmatrix}.$$

Inverse of a matrix

$$A = \begin{bmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{bmatrix}.$$

We have $|A| = -37$, and

$$A' = \begin{bmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{bmatrix} = A.$$

Hence

$$A^{-1} = -\frac{1}{37}\begin{bmatrix} -1 & 4 & -6 \\ 4 & -16 & -13 \\ -6 & -13 & 1 \end{bmatrix}.$$

The reader should verify that $AA^{-1} = I$.

Solve the equations

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

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