Lecture 2

On

Numerical Methods & Computer Programming

Dr Md Abu Sayeed
Associate Professor
Civil Engineering Department, RUET

Solution of Algebraic and Transcendental Equation

- ✓ Bisection Method
- ✓ Method of false position
- ✓ Iteration method
- ✓ Newton-Raphson Method
- ✓ Ramanujan's method
- ✓ The Secant Method

1. Bisection Method

- Generally, if *f(x)* is real and continuous in the interval x_l to x_u and $f(x_l).f(x_u)<0$, then there is <u>at least one real root</u> between x_l and x_u to this function.
- The interval at which the function changes sign is located. Then the interval is divided in half with the root lies in the midpoint of the subinterval. This process is repeated to obtained refined estimates.

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Bisection method

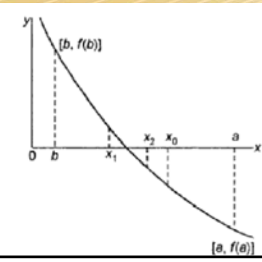
Choose two real numbers a and b such that $f(a)\,f(b)<0$.

Set $x_r=(a+b)/2$.

(a) If $f(a)\,f(x_r)<0$, the root lies in the interval (a, x_r). Then, set $b=x_r$ and go to step 2 above.

(b) If $f(a)\,f(x_r)>0$, the root lies in the interval (x_r, b). Then, set $a=x_r$ and go to step 2.

(c) If $f(a)\,f(x_r)=0$, it means that x_r is a root of the equation $f(x)=0$ and the computation may be terminated.



Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Example 2.1 Find a real root of the equation $f(x) = x^3 - x - 1 = 0$.

Since $f(1)$ is negative and $f(2)$ positive, a root lies between 1 and 2 and therefore we take $x_0 = 3/2$. Then

$$f(x_0) = \frac{27}{8} - \frac{3}{2} = \frac{15}{8}, \text{ which is positive.}$$

Hence the root lies between 1 and 1.5 and we obtain

$$x_1 = \frac{1 + 1.5}{2} = 1.25$$

We find $f(x_1) = -19/64$, which is negative. We therefore conclude that the root lies between 1.25 and 1.5. If follows that

$$x_2 = \frac{1.25 + 1.5}{2} = 1.375$$

The procedure is repeated and the successive approximations are

$$x_3 = 1.3125, \quad x_4 = 1.34375, \quad x_5 = 1.328125, \text{ etc.}$$

Find a real root of the equation $x^3 - 2x - 5 = 0$.

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Find a real root of the equation $x^3 - 2x - 5 = 0$.

Solution

Let $f(x) = x^3 - 2x - 5$. Then

$$f(2) = -1 \quad \text{and} \quad f(3) = 16.$$

Hence a root lies between 2 and 3 and we take

$$x_0 = \frac{2 + 3}{2} = 2.5$$

Since $f(x_0) = 5.6250$, we choose [2, 2.5] as the new interval. Then

$$x_1 = \frac{2 + 2.5}{2} = 2.25 \quad \text{and} \quad f(x_1) = 1.890625$$

Proceeding in this way, the following table is obtained.

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Find a real root of the equation $x^3 - 2x - 5 = 0$.

Solution

Proceeding in this way, the following table is obtained.

n	a	b	x	$f(x)$
1	2	3	2.5	5.6250
2	2	2.5	2.25	1.8906
3	2	2.25	2.125	0.3457
4	2	2.125	2.0625	−0.3513
5	2.0625	2.125	2.09375	−0.0089
6	2.09375	2.125	2.10938	0.1668
7	2.09375	2.10938	2.10156	0.07856
8	2.09375	2.10156	2.09766	0.03471
9	2.09375	2.09766	2.09570	0.01286
10	2.09375	2.09570	2.09473	0.00195
11	2.09375	2.09473	2.09424	−0.0035
12	2.09424	2.09473		

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Find a real root of the equation $x^3 - 2x - 5 = 0$.

Solution

Proceeding in this way, the following table is obtained.

n	a	b	x	$f(x)$
1	2	3	2.5	5.6250
2	2	2.5	2.25	1.8906
3	2	2.25	2.125	0.3457
4	2	2.125	2.0625	−0.3513
5	2.0625	2.125	2.09375	−0.0089
6	2.09375	2.125	2.10938	0.1668
7	2.09375	2.10938	2.10156	0.07856
8	2.09375	2.10156	2.09766	0.03471
9	2.09375	2.09766	2.09570	0.01286
10	2.09375	2.09570	2.09473	0.00195
11	2.09375	2.09473	2.09424	−0.0035
12	2.09424	2.09473		

Difference is less than .001

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Bisection Method - Termination Criteria

Approximate relative Error :

$$\varepsilon_a = \left| \frac{X_r^n - X_r^{n-1}}{X_r^n} \right| \times 100\%$$

$$\varepsilon_a = \left| \frac{X_u - X_l}{X_u + X_l} \right| \times 100\% \quad \text{(Bisection)}$$

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Homework

Find a positive root of the equation

$xe^x = 1$,

which lies between 0 to 1.

Dr. Md Abu Sayeed, Associate Professor, Dept of CE, RUET

Thank You

Dr. Md Abu Sayeed, Associate Prof., Dept of CE, RUET