

# Lecture 20

on

## Numerical Methods & Computer Programming

### Numerical Differentiation

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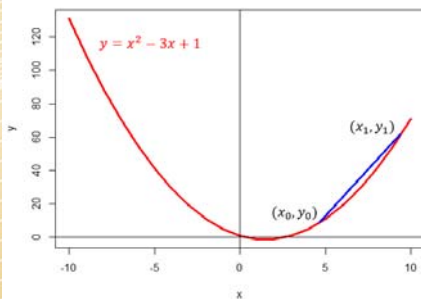
1

## Numerical Differentiation

Finding the derivative of a curve is known as differentiation.

But how can we measure the slope of the tangent at any point on the curve?

A **chord** is a straight line connecting two points on a curve. The slope of a chord is equal to the *average* slope of the curve between the two points. For example, in the plot below we have added a chord to the graph of  $y = x^2 - 3x + 1$  between the points  $(x_0, y_0)$  and  $(x_1, y_1)$ . The slope of this chord is equal to



$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{\Delta y}{\Delta x}$$

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## Numerical Differentiation

A curve is following the values of x and y as given in the Table; determine  $dy/dx$  and  $d^2y/dx^2$  when  $x = 1.2$ .

x	y
1.0	2.7183
1.2	3.3201
1.4	4.0552
1.6	4.9530
1.8	6.0496
2.0	7.3891
2.2	9.0250

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## Numerical Differentiation

If a derivative is required near the front of the table, one of the following formula may be used to get better accuracy

$$hy'_0 = \left( \Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \frac{1}{5}\Delta^5 - \frac{1}{6}\Delta^6 + \dots \right) y_0$$

$$h^2 y''_0 = \left( \Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \frac{5}{6}\Delta^5 + \frac{137}{180}\Delta^6 - \frac{7}{10}\Delta^7 + \frac{363}{560}\Delta^8 - \dots \right) y_0$$

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4



## Numerical Differentiation

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## Numerical Differentiation

From the following table of values of x and y; determine  $dy/dx$  and  $d^2y/dx^2$  when  $x = 1.2$ .

x	y	$\Delta$	$\Delta^2$	$\Delta^3$	$\Delta^4$	$\Delta^5$	$\Delta^6$
1.0	2.7183						
		0.6018					
1.2	3.3201		0.1333				
		0.7351		0.0294			
1.4	4.0552		0.1627		0.0067		
		0.8978		0.0361		0.0013	
1.6	4.9530		0.1988		0.0080		0.0001
		1.0966		0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891		0.2964				
		1.6359					
2.2	9.0250						

$$hy'_0 = \left( \Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \frac{1}{5}\Delta^5 - \frac{1}{6}\Delta^6 + \dots \right) y_0$$

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6

## Numerical Differentiation

From the following table of values of x and y; determine dy/dx and d<sup>2</sup>y/dx<sup>2</sup> when x = 1.2.

x	y	Δ	Δ <sup>2</sup>	Δ <sup>3</sup>	Δ <sup>4</sup>	Δ <sup>5</sup>	Δ <sup>6</sup>
1.0	2.7183						
		0.6018					
1.2	3.3201	0.7351	0.1333				
				0.0294			
1.4	4.0552	0.8978	0.1627	0.0361	0.0067		
						0.0013	
1.6	4.9530	1.0966	0.1988	0.0441	0.0080		0.0001
						0.0014	
1.8	6.0496	1.3395	0.2429	0.0535	0.0094		
2.0	7.3891	1.6359	0.2964				
2.2	9.0250						

$$hy'_0 = \left( \Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 - \frac{1}{4}\Delta^4 + \frac{1}{5}\Delta^5 - \frac{1}{6}\Delta^6 + \dots \right) y_0$$

Here  $x_0 = 1.2$ ,  $y_0 = 3.3201$  and  $h = 0.2$ . Hence (5.11) gives

$$\left[ \frac{dy}{dx} \right]_{x=1.2} = \frac{1}{0.2} \left[ 0.7351 - \frac{1}{2}(0.1627) + \frac{1}{3}(0.0361) - \frac{1}{4}(0.0080) + \frac{1}{5}(0.0014) \right]$$

$$= 3.3205.$$

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## Numerical Differentiation

From the following table of values of x and y; determine dy/dx and d<sup>2</sup>y/dx<sup>2</sup> when x = 1.2.

x	y	Δ	Δ <sup>2</sup>	Δ <sup>3</sup>	Δ <sup>4</sup>	Δ <sup>5</sup>	Δ <sup>6</sup>
1.0	2.7183						
		0.6018					
1.2	3.3201	0.7351	0.1333				
				0.0294			
1.4	4.0552	0.8978	0.1627	0.0361	0.0067		
						0.0013	
1.6	4.9530	1.0966	0.1988	0.0441	0.0080		0.0001
						0.0014	
1.8	6.0496	1.3395	0.2429	0.0535	0.0094		
2.0	7.3891	1.6359	0.2964				
2.2	9.0250						

$$h^2 y''_0 = \left( \Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \frac{5}{6}\Delta^5 + \frac{137}{180}\Delta^6 - \frac{7}{10}\Delta^7 + \frac{363}{560}\Delta^8 - \dots \right) y_0$$

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## Numerical Differentiation

From the following table of values of x and y; determine dy/dx and  $d^2y/dx^2$  when x = 1.2.

x	y	$\Delta$	$\Delta^2$	$\Delta^3$	$\Delta^4$	$\Delta^5$	$\Delta^6$
1.0	2.7183						
		0.6018					
1.2	3.3201		0.1333				
		0.7351		0.0294			
1.4	4.0552		0.1627		0.0067		
		0.8978		0.0361		0.0013	
1.6	4.9530		0.1988		0.0080		0.0001
		1.0966		0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891						
		1.6359					
2.2	9.0250						

$$h^2 y_0'' = \left( \Delta^2 - \Delta^3 + \frac{11}{12} \Delta^4 - \frac{5}{6} \Delta^5 + \frac{137}{180} \Delta^6 - \frac{7}{10} \Delta^7 + \frac{363}{560} \Delta^8 - \dots \right) y_0$$

Dr Md Abi  $\left[ \frac{d^2y}{dx^2} \right]_{x=1.2} = \frac{1}{0.04} \left[ 0.1627 - 0.0361 + \frac{11}{12}(0.0080) - \frac{5}{6}(0.0014) \right] = 3.318.$

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## Numerical Differentiation

A curve is following the values of x and y as given in the Table; determine dy/dx and  $d^2y/dx^2$  when x = 2.2.

x	y
1.0	2.7183
1.2	3.3201
1.4	4.0552
1.6	4.9530
1.8	6.0496
2.0	7.3891
2.2	9.0250

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10

## Numerical Differentiation

If a derivative is required near the end of the table, one of the following formula may be used to get better accuracy

$$hy'_n = \left( \nabla + \frac{1}{2}\nabla^2 + \frac{1}{3}\nabla^3 + \frac{1}{4}\nabla^4 + \frac{1}{5}\nabla^5 + \frac{1}{6}\nabla^6 + \frac{1}{7}\nabla^7 + \frac{1}{8}\nabla^8 + \dots \right) y_n$$

$$h^2 y''_n = \left( \nabla^2 + \nabla^3 + \frac{11}{12}\nabla^4 + \frac{5}{6}\nabla^5 + \frac{137}{180}\nabla^6 + \frac{7}{10}\nabla^7 + \frac{363}{560}\nabla^8 + \dots \right) y_n$$

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11

## Numerical Differentiation

From the following table of values of x and y; determine  $dy/dx$  and  $d^2y/dx^2$  when  $x = 2.2$ .

x	y	$\Delta$	$\Delta^2$	$\Delta^3$	$\Delta^4$	$\Delta^5$	$\Delta^6$
1.0	2.7183						
		0.6018					
1.2	3.3201		0.1333				
		0.7351		0.0294			
1.4	4.0552		0.1627		0.0067		
		0.8978		0.0361		0.0013	
1.6	4.9530		0.1988		0.0080		0.0001
		1.0966		0.0441		0.0014	
1.8	6.0496		0.2429		0.0094		
		1.3395		0.0535			
2.0	7.3891		0.2964				
		1.6359					
2.2	9.0250						

H.W.

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12

*Thank  
you*



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