Lecture 24

On

Numerical Methods & Computer Programming

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NUMERICAL METHODS

- The numerical methods for ODE include:
  - Picard
  - Taylor series
  - Euler series
  - Runge-kutta methods
  - Milne
  - Adams-Bashforth

Numerical Solution of Differential Equations

Taylor's Series Method:

- Consider the first order Differential Equation

$$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$$

- The Taylor's series is

$$y(x) = y(x_0) + \frac{(x - x_0)}{1!} y'(x_0) + \frac{(x - x_0)^2}{2!} y''(x_0) + \ldots$$

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Numerical Solution of Differential Equations

TAYLOR'S SERIES METHOD

Ex:1

Solve $y' = x + y$, $y(0) = 1$ by Taylor's series method. Hence find values of y at $x = 0.1$ and $x = 0.2$

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Numerical Solution of Differential Equations

Taylor's Series Method

Sol.: Here $y' = f(x,y) = x + y$, $x_0 = 0$ and $y_0 = 1$

$$y' = x + y \Rightarrow y'(0) = 1$$
$$y'' = 1 + y' \Rightarrow y''(0) = 1 + y'(0) = 1 + 1 = 2$$
$$y''' = y'' \Rightarrow y'''(0) = y''(0) = 2$$
$$y^{iv} = y''' \Rightarrow y^{iv}(0) = y'''(0) = 2 \ldots$$

Taylor's series is

$$y(x) = y(x_0) + \frac{(x-x_0)}{1!}y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \ldots$$

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Taylor's Series Method

$$\begin{aligned} y(x) &= 1 + xy'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0)\ldots \\ &= 1 + x(1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(2) + \frac{x^4}{24}(2) + \ldots \\ &= 1 + x + x^2 + \frac{x^3}{3} + \frac{x^4}{12} + \ldots \\ y(0.1) &= 1 + (0.1) + (0.1)^2 + \frac{(0.1)^3}{3} + \frac{(0.1)^4}{12} + \ldots \\ &= 1.1103 \\ y(0.2) &= 1 + (0.2) + (0.2)^2 + \frac{(0.2)^3}{3} + \frac{(0.2)^4}{12} + \ldots \\ &= 1.2428 \end{aligned}$$

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Taylor's Series

Ex Using Taylor's series method, obtain the solution of $\frac{dy}{dx} = 3x + y^2$, given that $y(0) = 1$. Find the value of y for $x = 0.1$

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Taylor's Series

Sol. Here, $y' = f(x,y) = 3x + y^2$, $x_0 = 0$ and $y_0 = 1$.

$$y' = 3x + y^2 \Rightarrow y'(x_0) = 3(x_0) + y_0^2 = 3(0) + 1 = 1$$
$$y'' = 3 + 2yy' \Rightarrow y''(x_0) = 3 + 2(1)(1) = 5$$
$$y''' = 2(y')^2 + 2yy'' \Rightarrow y'''(x_0) = 2(1)^2 + 2(5) = 12$$

By Taylor's series,

$$\begin{aligned} y(x) &= y_0 + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \ldots \\ &= 1 + x + \frac{x^2}{2!}(5) + \frac{x^3}{3!}(12) + \ldots \\ &= 1 + x + \frac{5x^2}{2!} + 2x^3 + \ldots \\ y(0.1) &= \end{aligned}$$

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Ex Using Taylor's series method, find the solution of $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$, at $x = 0.2$

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Sol. Here, $y' = f(x, y) = 2y + 3e^x$, $x_0 = 0$ and $y_0 = 0$.

$$y' = 2y + 3e^x \Rightarrow y'(x_0) = 2(y_0) + 3e^{x_0} = 2(0) + 3e^0 = 3$$

$$y'' = 2y' + 3e^x \Rightarrow y''(x_0) = 2(y_0') + 3e^{x_0} = 2(3) + 3e^0 = 9$$

$$y''' = 2y'' + 3e^x \Rightarrow y'''(x_0) = 2(y_0'') + 3e^{x_0} = 2(9) + 3e^0 = 21$$

$$y^{iv} = 2y''' + 3e^x \Rightarrow y^{iv}(x_0) = 45$$

By Taylor's series,

$$y(x) = y_0 + (x - x_0)y'(x_0) + \frac{(x - x_0)^2}{2!}y''(x_0) + \frac{(x - x_0)^3}{3!}y'''(x_0) + \ldots$$

$$= 0 + x(3) + \frac{x^2}{2!}(9) + \frac{x^3}{3!}(21) + \frac{x^4}{4!}(45) + \ldots$$

$$= 3x + \frac{9x^2}{2} + \frac{7x^3}{2} + \frac{15x^4}{8} + \ldots$$

$y(0.2) =$

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Thank You

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