Lecture 25

On

Numerical Methods & Computer Programming

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Numerical Solution of Differential Equations

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Ex:

Use Taylor's series method to solve $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. Find $y(0.1)$ correct up to 4 decimal places.

Ex:

Use Taylor's series method to solve $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$. Find $y(0.03)$.

Numerical Solution of Differential Equations

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Picard's Method:

Consider the first order differential equation.

$$\frac{dy}{dx} = f(x, y) - - - (1)$$

subject to $y(x_0) = y_0$

The equation (1) can be written as

$$dy = f(x, y)dx$$

Integrating between the limits for x and y, we get

$$\int_{y_0}^{y} dy = \int_{x_0}^{x} f(x, y)dx$$

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Picard's Method

For 2^{nd} approximation y_2,
we replace y by y_1 in $f(x,y)$ in R.H.S of eq. (2),
we get

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)dx$$

In general,

$$y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)dx \text{ for } n = 0, 1, 2, \ldots$$

stop the process when the two consecutive values of y are same up to desired accuracy.

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Note:

This method is applicable to a limited class of equations in which the successive integration can be performed easily.

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Picard's Method

Ex Using Picard's method solve

$$\frac{dy}{dx} = 3 + 2xy \text{ where } y(0) = 1 \text{ for } x = 0.1.$$

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Picard's Method

Sol.:

By Picard's method $y_{n+1} = y_0 + \int_{x_0}^{x} f(x, y_n)\,dx$

Here $x_0 = 0, y_0 = 1, f(x,y) = 3 + 2xy$

- 1^{st} approximation:

put $n = 0$ and $y_0 = 1$ in $f(x,y)$

$$y_1 = 1 + \int_0^x (3 + 2x)\,dx$$

$$\therefore y_1 = 1 + 3x + x^2$$

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Picard's Method

- 2^{nd} approximation:

put $n = 1$ and $y_1 = 1 + 3x + x^2$ in $f(x, y)$

$$y_2 = 1 + \int_0^x \left[3 + 2x(1 + 3x + x^2)\right]\, dx$$

$$= 1 + \int_0^x \left[3 + 2x + 6x^2 + 2x^3)\right]\, dx$$

$$\therefore y_2 = 1 + 3x + x^2 + 2x^3 + \frac{x^4}{2}$$

which is approximate solution, putting $x = 0.1$

Numerical Solution of Differential Equations

Picard's Method

- 2^{nd} approximation:

put $n = 1$ and $y_1 = 1 + 3x + x^2$ in $f(x, y)$

$$y_2 = 1 + \int_0^x \left[3 + 2x(1 + 3x + x^2)\right]\, dx$$

$$= 1 + \int_0^x \left[3 + 2x + 6x^2 + 2x^3)\right]\, dx$$

$$\therefore y_2 = 1 + 3x + x^2 + 2x^3 + \frac{x^4}{2}$$

which is approximate solution, putting $x = 0.1$

$y(0.1) = 1.31205$

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Ex:

Using Picard's method, obtain a solution upto 4^{th} approx of the equation $\frac{dy}{dx} = y + x$, $y(0) = 1$.

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Thank You