Lecture 3

On

Numerical Methods & Computer Programming

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The Method of False Position

This is the oldest method for finding the real root of a nonlinear equation $f(x)=0$ and closely resembles the bisection method. In this method, also known as *regula falsi* or the *method of chords*, we choose two points a and b such that $f(a)$ and $f(b)$ are of opposite signs. Hence, a root must lie in between these points. Now, the equation of the chord joining the two points $[a, f(a)]$ and $[b, f(b)]$ is given by

$$\frac{y-f(a)}{x-a}=\frac{f(b)-f(a)}{b-a}. \qquad (2.6)$$

The method consists in replacing the part of the curve between the points $[a, f(a)]$ and $[b, f(b)]$ by means of the *chord* joining these points, and taking the point of intersection of the chord with the x-axis as an *approximation* to the root. The point of intersection in the present case is obtained by putting $y=0$ in (2.6). Thus, we obtain

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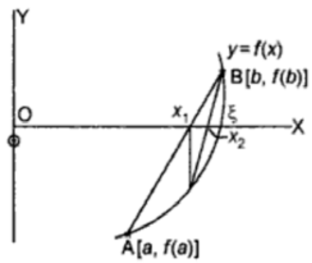
$$x_1 = a-\frac{f(a)}{f(b)-f(a)}(b-a)=\frac{af(b)-bf(a)}{f(b)-f(a)}, \qquad (2.7)$$

which is the *first approximation* to the root of $f(x)=0$. If now $f(x_1)$ and $f(a)$ are of opposite signs, then the root lies between a and x_1, and we replace b by x_1 in (2.7), and obtain the *next* approximation. Otherwise, we replace a by x_1 and generate the next approximation. The procedure is repeated till the root is obtained to the desired accuracy. Figure 2.2 gives

The Method of False Position

a graphical representation of the method. The error criterion (2.5) can be used in this case also.



The Method of False Position

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Find a real root of the equation :

$$f(x) = x^3 - 2x - 5 = 0.$$

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We find $f(2) = -1$ and $f(3) = 16$. Hence $a = 2$, $b = 3$, and a root lies between 2 and 3. Equation (2.7) gives

$$x_1 = \frac{2(16) - 3(-1)}{16 - (-1)} = \frac{35}{17} = 2.058823529.$$

Now, $f(x_1) = -0.390799917$ and hence the root lies between 2.058823529 and 3.0. Using formula (2.7), we obtain

$$x_2 = \frac{2.058823529(16) - 3(-0.390799917)}{16.390799917} = 2.08126366.$$

The Method of False Position

Since $f(x_2) = -0.147204057$, it follows that the root lies between 2.08126366 and 3.0. Hence, we have

$$x_3 = \frac{2.08126366(16) - 3(-0.147204057)}{16.147204057} = 2.089639211.$$

Proceeding in this way, we obtain successively:

$$x_4 = 2.092739575, \qquad x_5 = 2.09388371,$$

$$x_6 = 2.094305452, \qquad x_7 = 2.094460846, \ldots$$

The correct value is 2.0945..., so that x_7 is correct to five significant figures.

The Method of False Position

Example 2.5 Given that the equation $x^{2.2} = 69$ has a root between 5 and 8. Use the method of regula-falsi to determine it.

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The Method of False Position

Let $f(x) = x^{2.2} - 69$. We find

$$f(5) = -34.50675846 \quad \text{and} \quad f(8) = 28.00586026.$$

Hence

$$x_1 = \frac{5(28.00586026) - 8(-34.50675846)}{28.00586026 + 34.50675846} = 6.655990062.$$

Now, $f(x_1) = -4.275625415$ and therefore, the root lies between 6.655990062 and 8.0. We obtain

$$x_2 = 6.83400179, \qquad x_3 = 6.850669653.$$

The correct root is 6.8523651..., so that x_3 is correct to three significant figures.

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