Lecture 4

On

Numerical Methods & Computer Programming

Topic: Iteration Method

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Iteration Method

INTRODUCTION

- ✓ In various applications of Engineering the Linear Algebraic Equations occurs quite often.
- ✓ The Solution of such system of the Equations can be obtained by Direct Methods or Iterative Method.
- ✓ Here we are Going to Talk about the ITERATIVE METHOD used in Engineering.

Iteration Method

ITERATIVE METHODS

- ✓ Whenever the Algebraic Equations contains large no of equations at that time DIRECT METHOD becomes lengthy and Time Consuming Headache.
- ✓ n this Case the Iteration methods plays a Vital Role by being the Substitute of the Method.
- ✓ In Iterative methods we start from an Initial Approximation to actual Solution & obtain the better Approximation after Repeating the ITERATION. This method gives Desired Accuracy.

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Iteration Method

2.4 THE ITERATION METHOD

We have so far discussed root-finding methods, which require the interval in which the root lies. We now describe methods which require one or more starting values of x. These values need not necessarily bracket the root. The first is the iteration method, which requires one starting value of x.

To describe this method for finding the roots of the equation

$$f(x) = 0, \qquad (2.1)$$

we rewrite this equation in the form

$$x = \phi(x). \qquad (2.8)$$

There are many ways of doing this. For example, the equation

$$x^3 + x^2 - 1 = 0$$

can be expressed as either of the forms:

$$x = (1+x)^{-1/2}, \qquad x = (1-x^3)^{1/2}, \qquad x = (1-x^2)^{1/3}, \ldots$$

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Example 2.6 Find a real root of the equation $x^3 + x^2 - 1 = 0$ on the interval [0, 1] with an accuracy of 10^{-4}.

To find this root, we rewrite the given equation in the form

$$x = \frac{1}{\sqrt{x+1}} \qquad \text{(i)}$$

Thus

$$\phi(x) = \frac{1}{\sqrt{x+1}}, \qquad \phi'(x) = -\frac{1}{2}\frac{1}{(x+1)^{3/2}}$$

and

$$\max_{[0,1]} |\phi'(x)| = \frac{1}{2\sqrt{8}} = k = 0.17678 < 0.2.$$

Since Equation (1) satisfy $\max_{[0,1]} |\phi'(x)|$ <1, so consider

$x_{n+1} = 1/(xn + 1)$ as iterative formula

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Iteration Method

Since Equation (1) satisfy $\max_{[0,1]} |\phi'(x)|$ <1, so consider

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Hence when the absolute value of the difference does not exceed 0.0004, the required accuracy will be achieved and then the iteration can be terminated.

Starting with $x_0 = 0.75$, we obtain the following table:

n	x_n	$\sqrt{x_n + 1}$	$x_{n+1} = 1/\sqrt{x_n + 1}$
0	0.75	1.3228756	0.7559289
1	0.7559289	1.3251146	0.7546517
2	0.7546517	1.3246326	0.7549263

At this stage, we find that

$$|x_{n+1} - x_n| = 0.7549263 - 0.7546517 = 0.0002746,$$

which is less than 0.0004. The iteration is therefore terminated and the root to the required accuracy is 0.7549.

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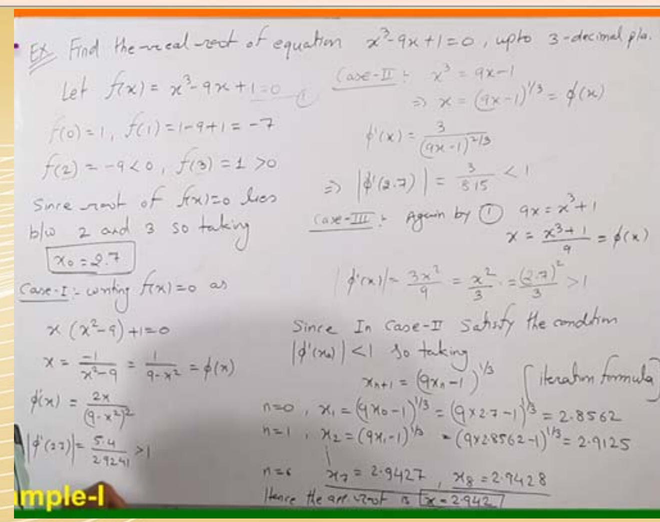

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APPLICATIONS

- Analysis of the Circuit containing the Invariant Elements.
- Determination of Output of Chemical Plants.
- **Study of the Civil Engineering Structures.**
- Problems of Layered Media in Siesmology etc.

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Home work

Find a positive root of the equation $xe^x = 1$, which lies between 0 to 1.

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Thank You

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