Lecture 5

On

Numerical Methods & Computer Programming

Topic: Newton Raphson Method

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NEWTON RAPHSON METHOD

This method is generally used to improve the result obtained by one of the previous methods. Let x_0 be an approximate root of $f(x)=0$ and let $x_1 = x_0 + h$ be the correct root so that $f(x_1)=0$. Expanding $f(x_0+h)$ by Taylor's series, we obtain

$$f(x_0)+hf'(x_0)+\frac{h^2}{2!}f''(x_0)+\cdots=0.$$

Neglecting the second- and higher-order derivatives, we have

$$f(x_0)+hf'(x_0)=0,$$

which gives

$$h=-\frac{f(x_0)}{f'(x_0)}.$$

NEWTON RAPHSON METHOD

A better approximation than x_0 is therefore given by x_1, where

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

Successive approximations are given by $x_2, x_3, \ldots, x_{n+1}$, where

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

which is the *Newton–Raphson formula.*

NEWTON RAPHSON METHOD

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

If we compare the Newton Raphson Formula with the Iterative method

$$x_{n+1} = \phi(x_n)$$

$$\phi(x) = x - \frac{f(x)}{f'(x)},$$

$$\phi'(x) = \frac{f(x)\,f''(x)}{[f'(x)]^2}.$$

$$|\phi'(x)| < 1$$

Newton Raphson Method

Find a root of the equation $x \sin x + \cos x = 0$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)},$$

Newton Raphson Method

Example 2.11 Find a root of the equation $x \sin x + \cos x = 0$.

We have

$$f(x) = x \sin x + \cos x \quad \text{and} \quad f'(x) = x \cos x.$$

The iteration formula is therefore

$$x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n}.$$

With $x_0 = \pi$, the successive iterates are given below

n	x_n	$f(x_n)$		x_{n+1}
0	3.1416	–1.0		2.8233
1	2.8233	–0.0662		2.7986
2	2.7986	–0.0006		2.7984
3	2.7984	0.0		2.7984

Newton Raphson Method

Example 2.12 Find a real root of the equation $x = e^{-x}$, using the Newton–Raphson method.

We write the equation in the form

$$f(x) = xe^x - 1 = 0 \qquad \text{(i)}$$

Let $x_0 = 1$. Then

$$x_1 = 1 - \frac{e-1}{2e} = \frac{1}{2}\left(1 + \frac{1}{e}\right) = 0.6839397$$

Now

$$f(x_1) = 0.3553424, \quad \text{and} \quad f'(x_1) = 3.337012,$$

Newton Raphson Method

so that

$$x_2 = 0.6839397 - \frac{0.3553424}{3.337012} = 0.5774545.$$

Proceeding in this way, we obtain

$$x_3 = 0.5672297 \quad \text{and} \quad x_4 = 0.5671433.$$


Thank you