Lecture 6

On

Numerical Methods & Computer Programming

Topic: The Secant Method

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The Secant Method

We have seen that the Newton–Raphson method requires the evaluation of derivatives of the function and this is not always possible, particularly in the case of functions arising in practical problems. In the secant method, the derivative at x_i is approximated by the formula

$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}},$$

which can be written as

$$f_i' = \frac{f_i - f_{i-1}}{x_i - x_{i-1}}, \qquad (2.36)$$

where $f_i = f(x_i)$. Hence, the Newton–Raphson formula becomes

$$x_{i+1} = x_i - \frac{f_i(x_i - x_{i-1})}{f_i - f_{i-1}} = \frac{x_{i-1}f_i - x_i f_{i-1}}{f_i - f_{i-1}}. \qquad (2.37)$$

It should be noted that this formula requires two initial approximations to the root.

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The Secant Method

Example 2.18 Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method.

Let the two initial approximations be given by

$$x_{-1} = 2 \quad \text{and} \quad x_0 = 3.$$

We have

$$f(x_{-1}) = f_1 = 8 - 9 = -1, \quad \text{and} \quad f(x_0) = f_0 = 27 - 11 = 16.$$

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Putting $i = 0$ in (2.37), we obtain

$$x_1 = \frac{2(16) - 3(-1)}{17} = \frac{35}{17} = 2.058823529.$$

Also,

$$f(x_1) = f_1 = -0.390799923.$$

Putting $i = 1$ in (2.37), we obtain

$$x_2 = \frac{x_0 f_1 - x_1 f_0}{f_1 - f_0} = \frac{3(-0.390799923) - 2.058823529(16)}{-16.390799923} = 2.08126366.$$

Again

$$f(x_2) = f_2 = -0.147204057.$$

Setting $i = 2$ in (2.37), and simplifying, we get $x_3 = 2.094824145$, which is correct to three significant figures.

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Revision

Example 2.7 Find the root of the equation $2x = \cos x + 3$ correct to three decimal places.

We rewrite the equation in the form

$$x = \frac{1}{2}(\cos x + 3) \qquad \text{(i)}$$

so that

$$\phi(x) = \frac{1}{2}(\cos x + 3),$$

and

$$|\phi'(x)| = \left|\frac{\sin x}{2}\right| < 1.$$

Hence the iteration method can be applied to the eq. (i) and we start with $x_0 = \pi/2$. The successive iterates are

$$x_1 = 1.5, \quad x_2 = 1.535, \quad x_3 = 1.518,$$
$$x_4 = 1.526, \quad x_5 = 1.522, \quad x_6 = 1.524,$$
$$x_7 = 1.523, \quad x_8 = 1.524.$$

Hence we take the solution as 1.524 correct to three decimal places.

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Home Work

Find a root of the equation $x \sin x + \cos x = 0$.

Find a real root of the equation $x = e^{-x}$,

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The Method of False Position

$$f(x) = x^3 - 2x - 5 = 0.$$

We find $f(2) = -1$ and $f(3) = 16$. Hence $a = 2$, $b = 3$, and a root lies between 2 and 3. Equation (2.7) gives

$$x_1 = \frac{2(16) - 3(-1)}{16 - (-1)} = \frac{35}{17} = 2.058823529.$$

Now, $f(x_1) = -0.390799917$ and hence the root lies between 2.058823529 and 3.0. Using formula (2.7), we obtain

$$x_2 = \frac{2.058823529(16) - 3(-0.390799917)}{16.390799917} = 2.08126366.$$

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The Method of False Position

Since $f(x_2) = -0.147204057$, it follows that the root lies between 2.08126366 and 3.0. Hence, we have

$$x_3 = \frac{2.08126366(16) - 3(-0.147204057)}{16.147204057} = 2.089639211.$$

Proceeding in this way, we obtain successively:

$$x_4 = 2.092739575, \qquad x_5 = 2.09388371,$$
$$x_6 = 2.094305452, \qquad x_7 = 2.094460846, \ldots$$

The correct value is 2.0945..., so that x_7 is correct to five significant figures.

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The Method of False Position

Example 2.5 Given that the equation $x^{2.2} = 69$ has a root between 5 and 8. Use the method of regula-falsi to determine it.

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The Method of False Position

Let $f(x) = x^{2.2} - 69$. We find

$$f(5) = -34.50675846 \quad \text{and} \quad f(8) = 28.00586026.$$

Hence

$$x_1 = \frac{5(28.00586026) - 8(-34.50675846)}{28.00586026 + 34.50675846} = 6.655990062.$$

Now, $f(x_1) = -4.275625415$ and therefore, the root lies between 6.655990062 and 8.0. We obtain

$$x_2 = 6.83400179, \qquad x_3 = 6.850669653.$$

The correct root is 6.8523651..., so that x_3 is correct to three significant figures.

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