Lecture 9

On

Numerical Methods & Computer Programming

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Backward Differences

The differences $y_1 - y_0, y_2 - y_1, \ldots, y_n - y_{n-1}$ are called *first backward differences* if they are denoted by $\nabla y_1, \nabla y_2, \ldots, \nabla y_n$ respectively, so that $\nabla y_1 = y_1 - y_0$, $\nabla y_2 = y_2 - y_1, \ldots, \nabla y_n = y_n - y_{n-1}$, where ∇ is called the *backward difference operator*. In a similar way, one can define backward differences of higher orders. Thus we obtain

$$\nabla^2 y_2 = \nabla y_2 - \nabla y_1 = y_2 - y_1 - (y_1 - y_0) = y_2 - 2y_1 + y_0,$$

$$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2 = y_3 - 3y_2 + 3y_1 - y_0, \text{ etc.}$$

With the same values of x and y as in Table 3.1, a backward difference table can be formed:

Backward Differences

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Table 3.2 Backward Difference Table

x	y	∇	∇^2	∇^3	∇^4	∇^5	∇^6
x_0	y_0						
x_1	y_1	∇y_1					
x_2	y_2	∇y_2	$\nabla^2 y_2$				
x_3	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$			
x_4	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$		
x_5	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$	
x_6	y_6	∇y_6	$\nabla^2 y_6$	$\nabla^3 y_6$	$\nabla^4 y_6$	$\nabla^5 y_6$	$\nabla^6 y_6$

Newton's Backward Formula for Interpolation

$$y_n(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots + \frac{p(p+1)\ldots(p+n-1)}{n!}\nabla^n y_n, \quad (3.14)$$

where $p = (x - x_n)/h$.

This is *Newton's backward difference interpolation formula* and it uses tabular values to the left of y_n. This formula is therefore useful for interpolation *near the end of* the tabular values.

It can be shown that the error in this formula may be written as

$$y(x) - y_n(x) = \frac{p(p+1)(p+2)\ldots(p+n)}{(n+1)!} h^{n+1} y^{(n+1)}(\xi), \quad (3.15)$$

where $x_0 < \xi < x_n$ and $x = x_n + ph$.

Newton's Backward Formula for Interpolation

Example 3.6 Values of x (in degrees) and sin x are given in the following table:

x (in degrees)	sin x
15	0.2588190
20	0.3420201
25	0.4226183
30	0.5
35	0.5735764
40	0.6427876

Determine the value of sin 38°.

Newton's Backward Formula for Interpolation

The difference table is

x	sin x	Δ	Δ^2	Δ^3	Δ^4	Δ^5
15	0.2588190					
		0.0832011				
20	0.3420201		−0.0026029			
		0.0805982		−0.0006136		
25	0.4226183		−0.0032165		0.0000248	
		0.0773817		−0.0005888		0.0000041
30	0.5		−0.0038053		0.0000289	
		0.0735764		−0.0005599		
35	0.5735764		−0.0043652			
		0.0692112				
40	0.6427876					

Newton's Backward Formula for Interpolation

To find sin 38°, we use Newton's backward difference formula with $x_n = 40$ and $x = 38$. This gives

$$p = \frac{x - x_n}{h} = \frac{38 - 40}{5} = -\frac{2}{5} = -0.4.$$

Hence, using formula (3.14), we obtain

$$y(38) = 0.6427876 - 0.4\,(0.0692112) + \frac{-0.4(-0.4-1)}{2}(-0.0043652)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)}{6}(-0.0005599)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{24}(0.0000289)$$

$$+ \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)(-0.4+4)}{120}(0.0000041)$$

$$= 0.6427876 - 0.02768448 + 0.00052382 + 0.00003583 - 0.00000120$$

$$= 0.6156614.$$