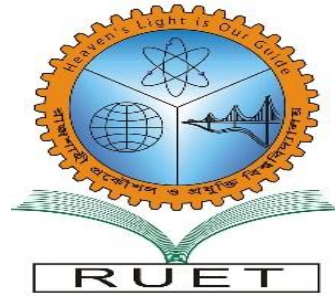


BECM 2203

Numerical Analysis and Computer Programming



Md. Ashrafur Islam
Lecturer

Dept. of Building Engineering & Construction Management
Rajshahi University of Engineering & Technology (RUET)

Syllabus

Introduction: Introduction to computer (fundamental terms and terminology, hardware, software and their applications) and operating systems fundamentals. **Solution of algebraic and transcendental equation: Bisection method, Regular false method, Newton-Raphson method, Iteration method; Rate of convergence; Order of errors; Interpolation: simple differences, difference tables, differences of a polynomial; Newton's formula for interpolation, central difference interpolation formula, divided differences, tables of divided differences, Newton's general interpolation formula, Lagrange's interpolation formula, inverse interpolation by Lagrange's formula and by successive approximation; Solutions of systems of linear equations: matrices, Gaussian elimination method, Gauss-Seidal integration method; Numerical differentiation and integration; Finite differences: Curve fitting by least squares; Solution of differential equations; Picard's method, Euler's method and Runge-Kutta method. Computer Programming: Introduction to programming language C and solution of numerical problems.**

Reference books

- ❖ Introductory Methods of Numerical Analysis by **S.S. Sastry**
- ❖ Numerical Mathematical Analysis by **James B. Scarborough**
- ❖ Numerical Methods for Engineers by **Steven C. Chapra and Raymond P. Canale**

Learning Objectives

After completing this course students will be able to:

- Understand the importance of numerical analysis;
- Learn the development of Mathematical Models related to Engineering and Science problems;
- Apply various methods in different fields of Engineering.

Introduction

In real world many physical problems are complex for which analytical solution may not be available or may be so complex that they are quite unsuitable for practical purposes. In this situation, the only alternate way is to approximate the problem. The approximate solution can be obtained through automatic computation to a repetition process of series of steps. Such process is known as numerical methods and the analysis of such methods is called Numerical Analysis.

Solution of Algebraic and Transcendental Equations

□ Introduction

A polynomial equation of the form

$$f(x) = p_n(x) = a_0x^{n-1} + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0 \dots(1)$$

is called an Algebraic equation.

For example, $x^4 - 4x^2 + 5 = 0$, $4x^2 - 5x + 7 = 0$; $2x^3 - 5x^2 + 7x + 5 = 0$ are algebraic equations.

An equation which contains polynomials, trigonometric functions, logarithmic functions, exponential functions etc., is called a Transcendental equation.

For example,

$$\tan x - e^x = 0; \sin x - xe^{2x} = 0; xe^x = \cos x$$
 are transcendental equations

Solution of Algebraic and Transcendental Equations

- ❖ The **Bisection** method
- ❖ The method of **False Position**
- ❖ The **Iteraion** method
- ❖ **Newton-Raphson** method

The Bisection Method

The bisection method, which is alternatively called **Binary search method**, **binary chopping**, **interval halving**, or **Bolzano's method**, is based on **Intermediate value theory (IVT)**.

Intermediate value theory (IVT):

If a function $f(x)$ is continuous between a and b , and $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one root between a and b .

The Bisection Method

Procedure:

1. Choose two real numbers a and b such that $f(a) f(b) < 0$.
2. Set $x_r = \frac{(a+b)}{2}$.
3. (a) If $f(a) f(x_r) < 0$, the root lies in the interval (a, x_r) . Then, set $b = x_r$ and go to step 2 above.
(b) If $f(a) f(x_r) > 0$, the root lies in the interval (x_r, b) . Then, set $a = x_r$ and go to step 2.
(c) If $f(a) f(x_r) = 0$, it means that x_r is a root of the equation $f(x)=0$ and the computation may be terminated.

Example 1

Find the real root of the equation $x^3 - x - 1 = 0$.

Solution:

Let,

$$f(x) = x^3 - x - 1 = 0$$

$$f(1) = 1^3 - 1 - 1 = -1 \text{ (negative)}$$

$$f(2) = 2^3 - 2 - 1 = +5 \text{ (Positive)}$$

Since $f(1)$ is negative and $f(2)$ is positive, a root lies between 1 and 2.

$$\text{Therefore, } x_0 = \frac{1+2}{2} = 1.5 \text{ and}$$

$$f(x_0) = f(1.5) = \frac{7}{8} \text{ (Positive)}$$

So root lies *betⁿ* 1 and 1.5 and

$$x_1 = \frac{1+1.5}{2} = 1.25$$

$$f(x_1) = f(1.25) = -\frac{19}{64} \text{ (negative)}$$

So root lies *betⁿ* 1.25 and 1.5 and

$$x_2 = \frac{1.25+1.5}{2} = 1.375 \text{ and } f(x_2) = 0.224 \text{ (possiive)}$$

Proceeding in this way, the following table is obtained (shown in the next slide)

Example 1 continue.....

n	a	b	x	f(x)
1	1	2	1.5	0.875
2	1	1.5	1.25	-0.2968
3	1.25	1.5	1.375	0.224
4	1.25	1.375	1.3125	-0.0515
5	1.3125	1.375	1.34375	0.0826
6	1.3125	1.34375	1.328125	0.01457
7	1.3125	1.328125	1.3203125	-0.01871
8	1.320312	1.328125	1.324218	-0.002127
9	1.32418	1.328125	1.32617	0.0062
10	1.32418	1.32617	1.325175	0.0019
11	1.32418	1.325175	1.32467	-0.000204
12	1.32467	1.325175	1.32492	0.00088
13	1.32467	1.32492	1.32479	0.000307

At $n=13$, it is seen that the difference between two successive iterates is 0.0005, which is less than 0.001. thus the root is 1.325 (upto 3 decimal).

Exercises

1. Find a real root of the equation $x^3 - 2x - 5 = 0$.
2. Use bisection method to determine the drag coefficient (c) needed for a parachutist of mass $m = 68.1$ kg to have a velocity of 40 ms^{-1} after free falling for time $t = 10$ s and $g = 9.81 \text{ ms}^{-2}$. The velocity V of a falling parachutist is given by

$$V = \frac{gm}{c} [1 - e^{-(c/m)t}]$$

The method of False Position

This is the **oldest method** for finding the real root of a nonlinear equation $f(x) = 0$ and also known as **regular falsi** or the **method of chords** or **linear interpolation method**. In this method, we choose two points a and b such that $f(a)$ and $f(b)$ are of opposite signs. Hence, a root must lie in between these points. The equation of the chord joining the two points, $[a, f(a)]$ and $[b, f(b)]$ is given by

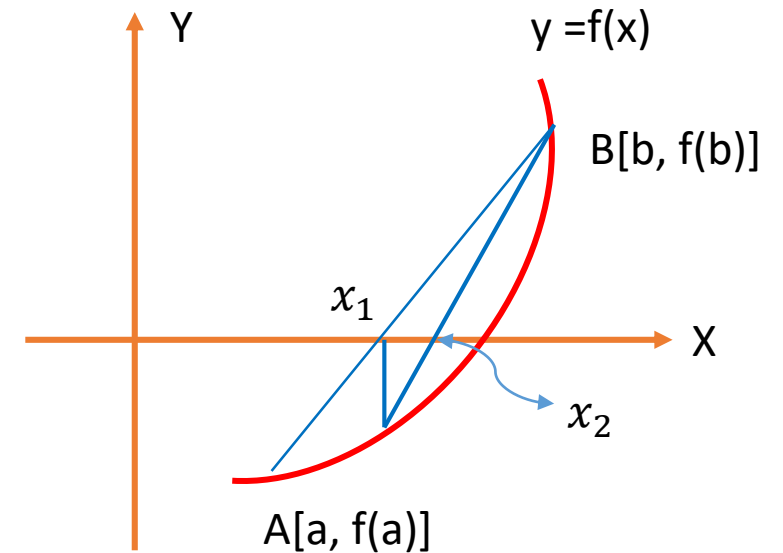
$$\frac{y-f(a)}{x-a} = \frac{f(b)-f(a)}{b-a}$$

$$\frac{0-f(a)}{x-a} = \frac{f(b)-f(a)}{b-a} \quad [\text{Putting } y=0]$$

$$\frac{x-a}{-f(a)} = \frac{(b-a)}{f(b)-f(a)}$$

$$x-a = -\frac{f(a)}{f(b)-f(a)} (b-a)$$

$$\mathbf{x = x_1 = a - \frac{f(a)}{f(b)-f(a)} (b-a)}$$



Example 2

Find a real root of the equation $x^3 - 2x - 5 = 0$.

Solution:

$$\text{Let, } f(x) = x^3 - 2x - 5 = 0$$

$$f(2) = -1$$

$$f(3) = +16$$

Since $f(2)$ is negative and $f(3)$ is positive, hence $a = 2$, $b = 3$, and a root lies between 2 and 3.

$$x_1 = a - \frac{f(a)}{f(b) - f(a)}(b - a) = 2.058824 \text{ and}$$

$f(x_1) = -0.3908$; hence the root lies *betⁿ* 2.058824 and 3.

$$x_2 = a - \frac{f(a)}{f(b)-f(a)}(b-a) = 2.081264 \text{ and}$$

$f(x_2) = -0.1472$; hence the root lies *betⁿ* 2.081264 and 3.

$$x_3 = a - \frac{f(a)}{f(b)-f(a)}(b-a) = 2.089639 \text{ and}$$

$f(x_3) = -0.0547$; hence the root lies *betⁿ* 2.089639 and 3.

$$x_4 = a - \frac{f(a)}{f(b)-f(a)}(b-a) = 2.092739 \text{ and}$$

$f(x_4) = -0.0202$; hence the root lies *betⁿ* 2.092739 and 3.

$$x_5 = a - \frac{f(a)}{f(b)-f(a)}(b-a) = 2.093883 \text{ and}$$

$f(x_5) = -0.0075$; hence the root lies *betⁿ* 2.093883 and 3.

$$x_6 = a - \frac{f(a)}{f(b) - f(a)}(b - a) = 2.094305 \text{ and}$$

$f(x_6) = -0.00275$; hence the root lies *betⁿ* 2.094305 and 3.

$$x_7 = a - \frac{f(a)}{f(b) - f(a)}(b - a) = 2.094460 \text{ and}$$

$f(x_7) = -0.001$; hence the root lies *betⁿ* 2.094460 and 3.

The correct value is 2.0945..., so that x_7 is correct to five significant figures.

Exercise

1. Determine the root of the equation $\cos x - xe^x = 0$ by the method of False position.

The Iteration Method

The iteration method is one such method, which **requires one starting value of x**.

We can use this method, if we can express $f(x) = 0$, as

$$x = \phi(x) \quad \dots\dots\dots (1)$$

We can express $f(x) = 0$, in the above form in more than one way also. For example, the equation $x^3 + x^2 - 1 = 0$ can be expressed in the following ways.

$$x = (1 + x)^{-1/2}$$

$$x = (1 - x^3)^{1/2}$$

$$x = (1 - x^2)^{1/3} \text{ and so on.}$$

Example 3

Find the root of the equation $2x = \cos x + 3$ correct to three decimal places.

Solⁿ:

We rewrite the equation in the form

$$x = \frac{1}{2}(\cos x + 3) \quad \dots\dots\dots(i)$$

We start with $x_0 = \frac{\pi}{2}$; so that

$$x_1 = \frac{1}{2}(\cos \frac{\pi}{2} + 3) = 1.5$$

$$x_2 = \frac{1}{2}(\cos(1.5) + 3) = 1.535$$

$$x_3 = \frac{1}{2}(\cos(1.535) + 3) = 1.518$$

$$x_4 = \frac{1}{2}(\cos(1.518) + 3) = 1.526$$

$$x_5 = \frac{1}{2}(\cos(1.526) + 3) = 1.522$$

.....the successive iterates are

$$x_6 = 1.524; x_7 = 1.523; x_8 = 1.524$$

Hence we take the solution as 1.524 correct to three decimal places.

Newton-Raphson Method

Let,

x_0 be an approximate root of $f(x) = 0$ and let $x_1 = x_0 + h$ be the correct root of $f(x_1) = 0$.

Therefore,

$$f(x_1) = 0$$

$$f(x_0 + h) = 0 \dots\dots\dots (i)$$

Expanding equation (i) by Taylor's series, we obtain

$$f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots\dots\dots = 0$$

Neglecting the second and higher-order derivatives, we have

$$f(x_0) + hf'(x_0) = 0$$

Newton-Raphson Method continue.....

$$h = -\frac{f(x_0)}{f'(x_0)}$$

$$x_1 - x_0 = -\frac{f(x_0)}{f'(x_0)} \quad [\text{since, } x_1 = x_0 + h ; \text{ hence, } h = x_1 - x_0]$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

.....

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad [\text{This is the Newton-Raphson formula}]$$

Example 4

Use the Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

Solⁿ:

Let,

$$f(x) = x^3 - 2x - 5$$

$$f'(x) = 3x^2 - 2$$

From Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \dots\dots\dots(i)$$

Putting $n = 0$ and choosing $x_0 = 2$ in equation (i),

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.1 ; x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.0945$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.0945$$

Therefore, the root is 2.0945.

Example 5

Find a root of the equation $x \sin x + \cos x = 0$

Solⁿ:

Let,

$$f(x) = x \sin x + \cos x$$

$$f'(x) = x \cos x + \cancel{\sin x} - \cancel{\sin x} = x \cos x$$

From Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \dots\dots\dots(i)$$

With $x_0 = \pi$, the successive iterates are given below

n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
0	π (3.1416)	-1.0	-3.1416	2.8233
1	2.8233	-0.0662	-2.6815	2.7986
2	2.7986	-0.0006	-2.6356	2.7984
3	2.7984	0	-2.6352	2.7984

Exercises

1. Find a root of the equation $3x - \cos x - 1 = 0$
2. Find a real root of the equation $x = e^{-x}$, using the newton- Raphson method.

Thank You All