Numerical Analysis and Computer Programming

Solutions of systems of linear equations

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The Inverse of a Matrix

❖ Find the Inverse of the Matrix A = $\begin{bmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{bmatrix}$

Sol^n:

We have $A^{-1} = \frac{1}{|A|}$ Adj $[A]$

$|A|$ = $\begin{vmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{vmatrix}$ = 5 (0 – 1) + 2 (0 – 4) + 4 (-2 -4) = -5 -8 -24 = -37

The Inverse of a Matrix

Transpose of A = $A' = \begin{bmatrix} 5 & -2 & 4 \\ -2 & 1 & 1 \\ 4 & 1 & 0 \end{bmatrix}$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Adj of A = $\begin{bmatrix} -1 & 4 & -6 \\ 4 & -16 & -13 \\ -6 & -13 & 1 \end{bmatrix}$

A^{-1} = $\frac{1}{-37}\begin{bmatrix} -1 & 4 & -6 \\ 4 & -16 & -13 \\ -6 & -13 & 1 \end{bmatrix}$

The Inverse of a Matrix

❖ Solve the equations, 3x + y + 2z = 3, 2x – 3y –z = -3, x + 2y + z = 4.

***Sol^n*:**

We can write AX = B , where

A = $\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

Therefore, X = A^{-1}B

$|A|$ = $\begin{vmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{vmatrix}$ = 8 and A'= $\begin{bmatrix} 3 & 2 & 1 \\ 1 & -3 & 2 \\ 2 & -1 & 1 \end{bmatrix}$; Adj of A = $\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$; $A^{-1} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$

The Inverse of a Matrix

$$X = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -3-9+20 \\ -9-3+28 \\ 21+15-44 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$; which gives x = 1, y =2, z = -1.

Rank of a Matrix

(a) A = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; r (A) = 0; since all the elements are zero.

(b) A = $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$; r (A) = 2, since $|A| \neq 0$.

(c) A = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$; $|A| = 0$; r(A) = 1.

(d) A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 1 \end{bmatrix}$; $|A| = 0$ and $\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \neq 0$, hence r(A) = 2.

(e) A = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 8 \\ 3 & 2 & 1 \end{bmatrix}$; $|A| \neq 0$; hence r(A) = 3.

Consistency of a Linear System of Equations

- If r (A) = r (A, b) ; then the equations are consistent.
- If r (A) ≠ r (A, b) ; then the equations are inconsistent.

Where,

r (A) = rank of A [Coefficient matrix]

r (A, b) = rank of (A, b) [Augmented matrix]

Consistency of a Linear System of Equations

❖ Find whether the following system is consistent

x – 4y + 5z = 8

3x + 7y –z = 3

x + 15y -11z = -14

***Sol^n*:**

A = $\begin{bmatrix} 1 & -4 & 5 \\ 3 & 7 & -1 \\ 1 & 15 & -11 \end{bmatrix}$; since $|A|$ = 0 and $\begin{vmatrix} 1 & -4 \\ 3 & 7 \end{vmatrix}$ ≠ 0 ; Therefore, r (A) = 2.

Consistency of a Linear System of Equations

The augmented matrix, (A, b) = $\begin{bmatrix} 1 & -4 & 5 & 8 \\ 3 & 7 & -1 & 3 \\ 1 & 15 & -11 & -14 \end{bmatrix}$

Since $\begin{vmatrix} -4 & 5 & 8 \\ 7 & -1 & 3 \\ 15 & -11 & -14 \end{vmatrix}$ = 31 ≠ 0 ; Therefore, r (A, b) = 3.

Since **r(A) ≠ r(A, b).** Therefore, the equations are inconsistent and there exists no solution.

Consistency of a Linear System of Equations

❖ **Exercise-1:** Examine for consistency the equations

$$2x - 3y + 5z = 1$$

$$3x + y - z = 2$$

$$x + 4y - 6z = 1$$

Gauss Elimination Method

❖ **Condition:**

i. x y z [No elimination]

ii. ~~x~~ y z [Eliminate **x**]

iii. ~~x~~ ~~y~~ z [Eliminate **x**, **y**]

Only row operation should be done.

Gauss Elimination Method

❖ Use Gauss elimination method to solve

$$x + y + z = 5$$
$$2x + 3y + 5z = 8$$
$$4x + 5z = 2$$

***Sol^n*:**

Augmented matrix = $\begin{bmatrix} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{bmatrix} \xrightarrow[R_{32}(-2)]{R_{21}(-2)} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & -6 & -5 & -14 \end{bmatrix} \xrightarrow{R_{32}(6)} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{bmatrix}$

From the matrix, we get the equations

$x + y + z = 5$ ....(i)

$y + 3z = -2$ ....(ii)

$13z = -26$ ....(iii)

From (iii), **z** = $-\frac{26}{13}$ = **- 2**

From (ii), **y** = -2 -3(-2) = -2 +6 = **4**

From (i), **x** = 5 – 4 + 2 = **3**

Therefore, x = 3, y = 4, z = -2. (Ans.)

Gauss Elimination Method

❖ The cost of Concrete at the ratio 1 : 3 : 6, 1: 2: 4 and 1: 2: 3 are 200 TK/ft^2, 225 TK/ft^2 and 240 TK/ft^2 respectively. Calculate cost per ft^2 cement, sand and coarse aggregate by using Gauss elimination method.

***Sol^n*:**

Let,

Cement = x TK/ft^2

Sand = y TK/ft^2

Coarse aggregate = z TK/ft^2

$$\text{Augmented matrix} = \begin{bmatrix} 1 & 3 & 6 & 200 \\ 1 & 2 & 4 & 225 \\ 1 & 2 & 3 & 240 \end{bmatrix} \xrightarrow[R_{32}(-1)]{R_{21}(-1)} \begin{bmatrix} 1 & 3 & 6 & 200 \\ 0 & -1 & -2 & 25 \\ 0 & 0 & -1 & 15 \end{bmatrix}$$

We get the equations,

x + 3y + 6z = 200 ....(i)

-y - 2z = 25 ....(ii)

-z = 15 ....(iii)

From (iii),

z = - 15

From (ii),

- y = 25 -30 = - 5

y = 5

From (i),

X = 200 – 3(5) - 6(-15)

X = 275

Cement = 275tk/ft^2, Sand = 5tk/ft^2, Coarse aggregate = 15 tk/ft^2

Gauss Elimination Method

- **Exercise-1:** Use Gauss elimination method to solve

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 4y + 9z = 16$$

- **Exercise-2:** Use Gauss elimination method to solve

$$x_1 + 7x_2 - 4x_3 = -51$$

$$4x_1 - 4x_2 + 9x_3 = 62$$

$$12x_1 - x_2 + 3x_3 = 8$$

Gauss- Jordan Method

❖ Condition:

i. **x** ~~y~~ ~~z~~ [Elimination **y, z**]

ii. ~~x~~ **y** ~~z~~ [Eliminate **x**, z]

iii. ~~x~~ ~~y~~ **z** [Eliminate **x**, **y**]

Only row operation should be done.

Gauss- Jordan Method

Solve the system

x + y + z = 5

2x + 3y + 5z = 8

4x + 5z = 2

By the Gauss- Jordan method.

$\boldsymbol{Sol^n}$:

Augmented matrix = $\begin{bmatrix} 1 & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{bmatrix} \xrightarrow[R_{32}(-2)]{R_{21}(-2)} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & -6 & -5 & -14 \end{bmatrix} \xrightarrow{R_{32}(6)} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{bmatrix}$

$\xrightarrow{R_3(\frac{1}{13})} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_{12}(-1)} \begin{bmatrix} 1 & 0 & -2 & 7 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_{23}(-3)} \begin{bmatrix} 1 & 0 & -2 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$\xrightarrow{R_{13}(2)} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

Therefore, x = 3, y = 4, z = -2. (Ans.)

Gauss- Jordan Method

❖ **Exercise-1:** Use Gauss-Jordan method to solve the following system

$$2x + y + z = 10$$

$$3x + 2y + 3z = 18$$

$$x + 4y + 9z = 16.$$

Thank You All