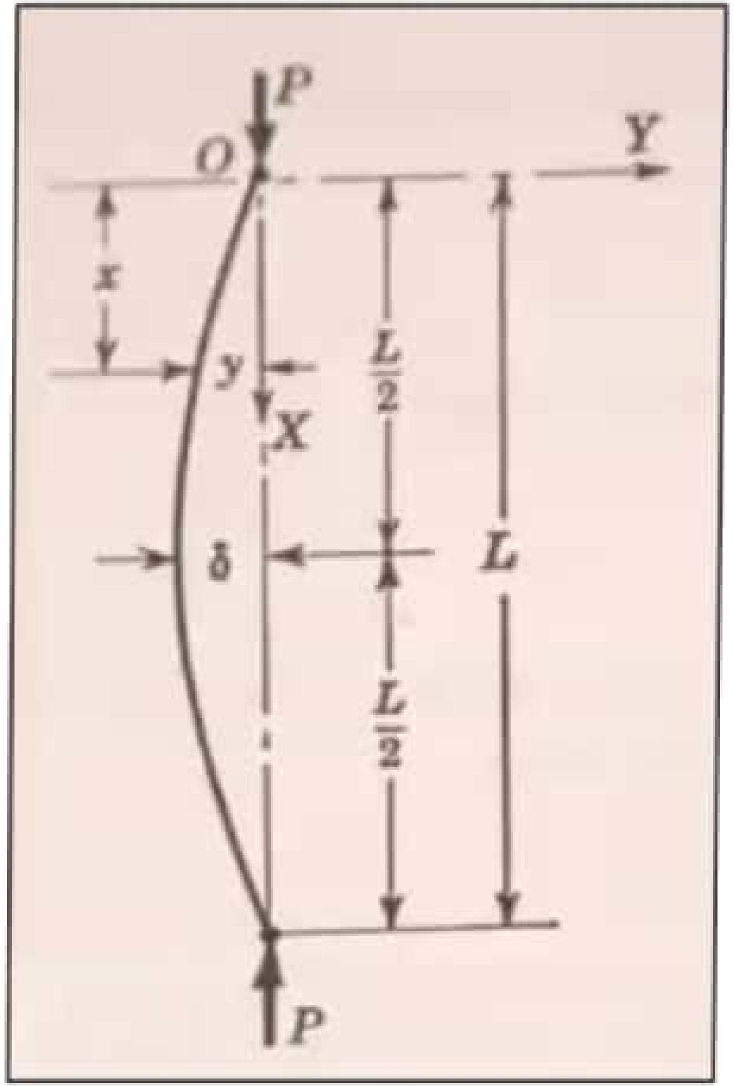
Euler's Formula for Long Column

- A Theoretical analysis of the critical load for long columns was made by the great Swiss mathematician Leonhard Euler in 1757. The illustration is followed below.



Euler's Formula for Long Column

- Fig. shows the centerline of a column in equilibrium under the action of its critical load P.
- The column is assumed to have hinged ends restrained against lateral movement.
- The maximum deflection 'δ' is so small that there is no appreciable difference between the original length of the column and its projection on vertical plane.
- Under these conditions the slope 'dy/dx' is so small that we may apply the approximate differential equation of the elastic curve of a beam –

Euler's Formula for Long Column

$$EI\frac{d^2y}{dx^2} = M = P(-y) = -Py$$

$$\Rightarrow EI\frac{d}{dx}\left(\frac{dy}{dx}\right) = -Py \quad \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

- Multiplying this by **2dy** to obtain perfect differentials, we get by integration –

$$2EI\int\left[\frac{d}{dx}\left(\frac{dy}{dx}\right)\right]dy = -2P\int y.dy$$

$$\Rightarrow EI\left(\frac{dy}{dx}\right)^2 = -Py^2 + C_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Euler's Formula for Long Column

- According to the Fig, at $y = \delta;\ \dfrac{dy}{dx} = 0$

From Eq^n. (ii); $0 = -P.\delta^2 + C_1 \Rightarrow C_1 = P.\delta^2$

From Eqn. (ii); $EI\left(\dfrac{dy}{dx}\right)^2 = P.\left(\delta^2 - y^2\right)$

$$\frac{dy}{dx} = \sqrt{\frac{P}{EI}\left(\delta^2 - y^2\right)} \Rightarrow \frac{dy}{\sqrt{\delta^2 - y^2}} = \sqrt{\frac{P}{EI}}\ dx$$

Euler's Formula for Long Column

$$\text{By Integrating: } \int \frac{dy}{\sqrt{\delta^2 - y^2}} = \int \sqrt{\frac{P}{EI}}\, dx$$

$$\Rightarrow \sin^{-1}\frac{y}{\delta} = x\sqrt{\frac{P}{EI}} + C_2$$

According to the Fig, at $x = 0; y = 0;$ Hence $C_2 = 0$

$$\Rightarrow \sin^{-1}\frac{y}{\delta} = x\sqrt{\frac{P}{EI}} \text{ or, } y = \delta \sin\left(x\sqrt{\frac{P}{EI}} \right)$$

Euler's Formula for Long Column

According to the Fig, at x = L; y = 0; *we get* –

$$\Rightarrow 0 = \delta \sin^{-1}\left(L\sqrt{\frac{P}{EI}}\right)$$

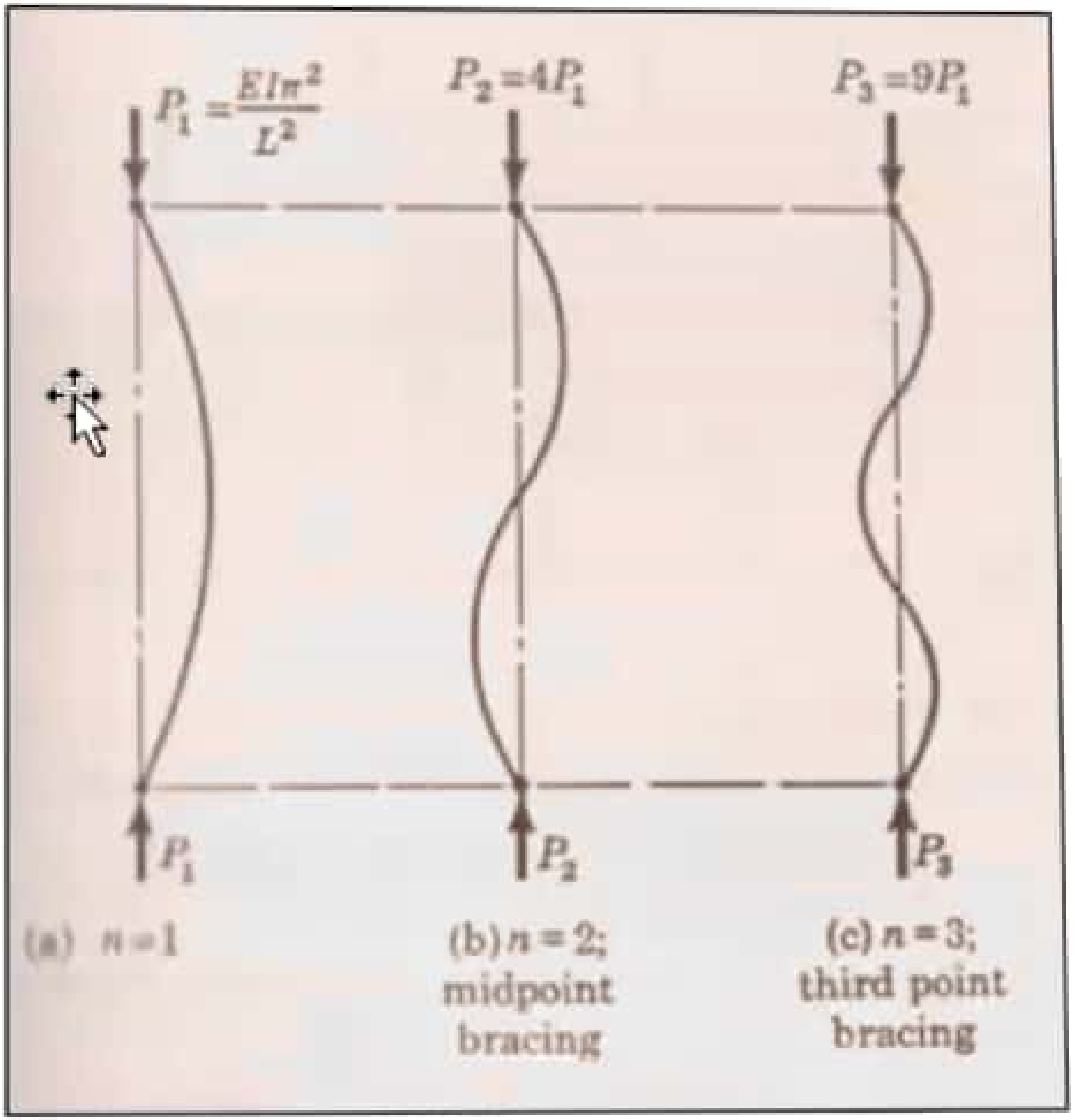
$$\Rightarrow L\sqrt{\frac{P}{EI}} = n\pi \ (n = 0, 1, 2, 3 \ldots\ldots)$$

From Which; $P = n^2 \dfrac{EI\pi^2}{L^2}$

Euler's Formula for Long Column

- The critical load for a hinged ended column is therefore –

$$P = n^2 \frac{EI\pi^2}{L^2}$$

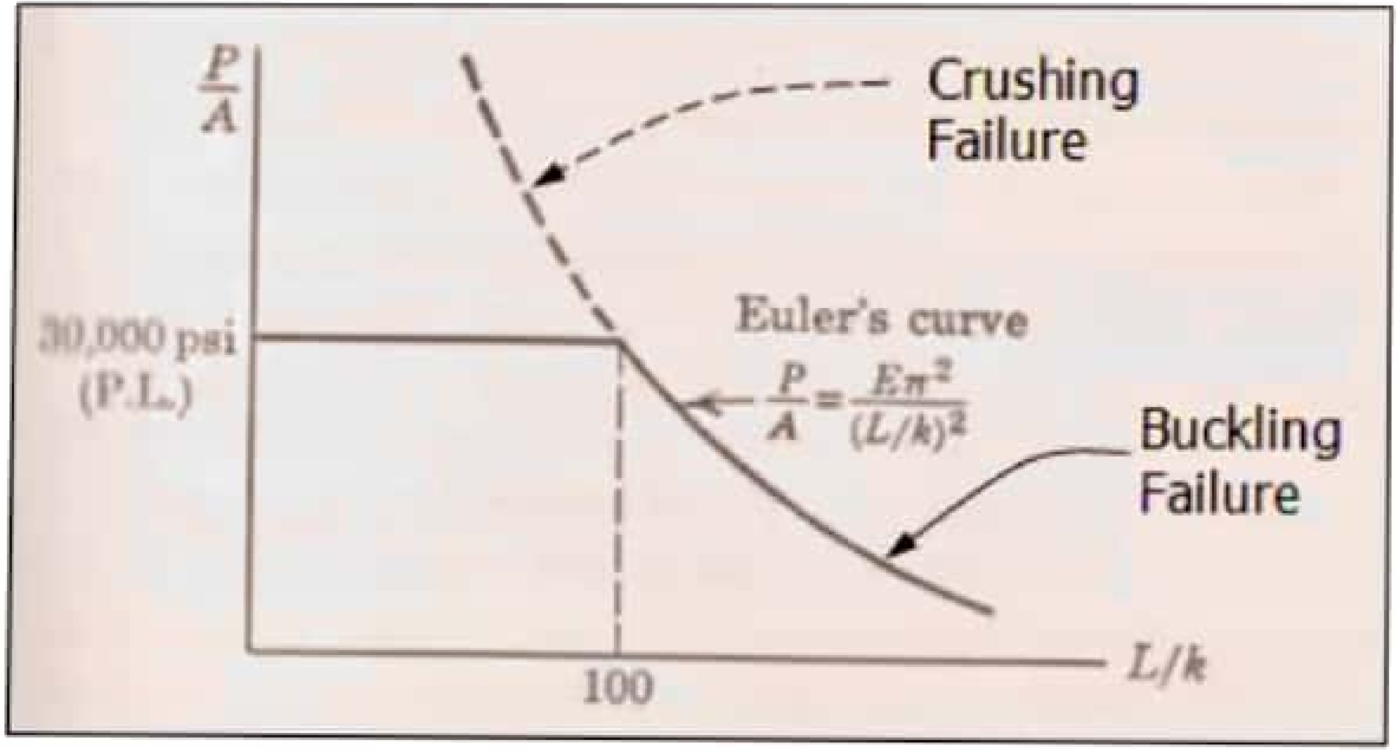


The Effect of End Condition on the Critical Load

End Condition	N = Number of Times Strength of Hinged Column	Le = Effective Length
Fixed Ends	4	½ L
One End Fixed, the Other Hinged	2	0.7 L
Both End Hinged	1	L
One End Fixed, the Other Free	1/4	2L

Euler's Curve

- Slenderness Ratio = L/K & K = √(I/A)
- For (L/K) < 100; Euler's Formula is not valid.



Problem - 1

- An Aluminum start 6 ft long has a rectangular section ¾ inch by 2 inch. A bolt through each end secures the start so that it acts as a hinged column about an axis perpendicular to the 2-inch dimension, and as a fixed-ended column about an axis perpendicular to the ¾-inch dimension. Determine the safe load, using a factor of safety 2 and $E = 10.3 \times 10^6$ psi.

Solution

- (1) The critical load for a hinged ended column is;

$$P = \frac{EI\pi^2}{L^2} \Rightarrow \frac{\pi^2 \times 10.3 \times 10^6 \times (3/4) \times 2^3}{72^2 \times 12} = 9804.87\ lb$$

- Using factor of safety = 2; P = 9,804.87÷2 = 4,902.43 lb
- (2) The critical load for a Fixed ended column is;

$$P = 4\frac{EI\pi^2}{L^2} \Rightarrow \frac{4 \times 10.3 \times 10^6 \times (3/4)^3 \times 2 \times \pi^2}{72^2 \times 12} = 5{,}515.24\ lb$$

- Using factor of safety = 2; P = 5515.24÷2 = 2,757.62 lb

(Ans)

Intermediate Column – Empirical Formula

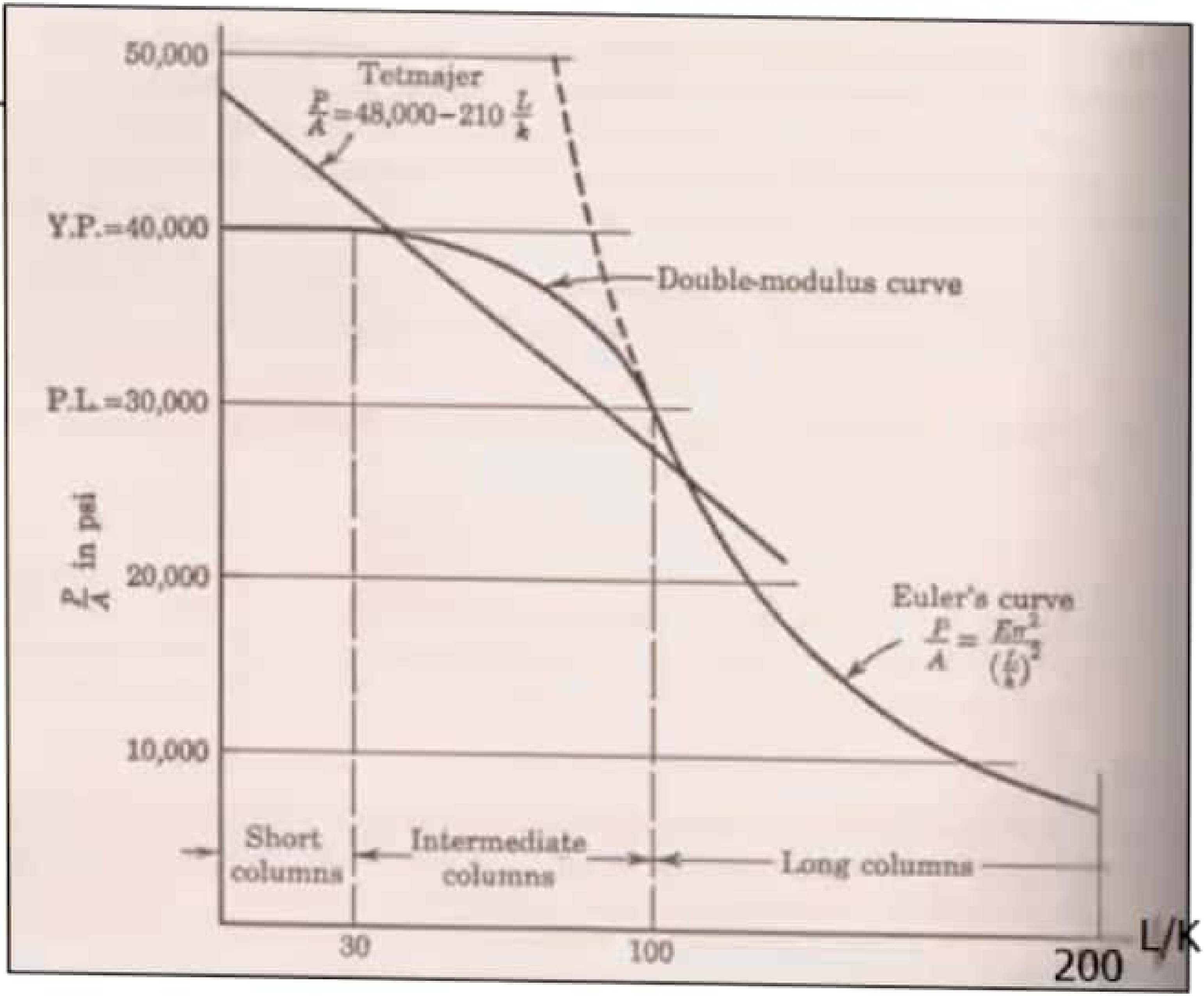
- According to AISC (American Institute of Steel Construction)

$$\frac{P}{A} = 17{,}000 - 0.485\left(\frac{L}{K}\right)^2$$

- According to Rankine – Gordon Formula

$$\frac{P}{A} = \frac{18000}{1 + \frac{1}{18000}\left(\frac{L}{K}\right)^2}$$

Intermediate Column – Empirical Formula



Problem - 2

- A 14WF87 is used as column 40 ft long with built-in ends.
  - (a) Using AISC formula compute the safe load that can be applied if the effective length is three quarter of the given length.
  - (b) What is the safe load if the column is also braced at its mid-point?

Solution

PART - A

- From Table B-2, Appendix B;
- Area = 25.26 in² & K = 3.70 (least)
- Effective Length, L_e = ¾ × 40 × 12 = 360 in
- So, (L/K) = 360 ÷ 3.70 = 97.297
- Applying the AISC formula -

$$\frac{P}{A} = 17{,}000 - 0.485(97.297)^2 \Rightarrow \frac{P}{A} = 12408.619\, psi$$

- Hence; P = 12,408.62 × 25.56 = 3,17,164.31 lb

(Ans)

Solution

PART - B

- Braced at Midpoint, the column is equivalent to one having a length of 20 ft.
- Hence the effective length, $L_e = 20 \times 12 = 240$ in
- So, $(L/K) = 240 \div 3.70 = 64.86$
- Applying the AISC formula -

$$\frac{P}{A} = 17{,}000 - 0.485(64.86)^2 \Rightarrow \frac{P}{A} = 14959.38\,psi$$

- Hence; $P = 14959.38 \times 25.56 = 3{,}82{,}361.91$ lb

(Ans)