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পুরকৌশল বিভাগ / 060007

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THANKS TO ALL

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...CTOR INTEGRAL:

Let $f(t)$ and $G(t)$ be the two vector

$$\frac{dF(t)}{dt} = G(t)$$

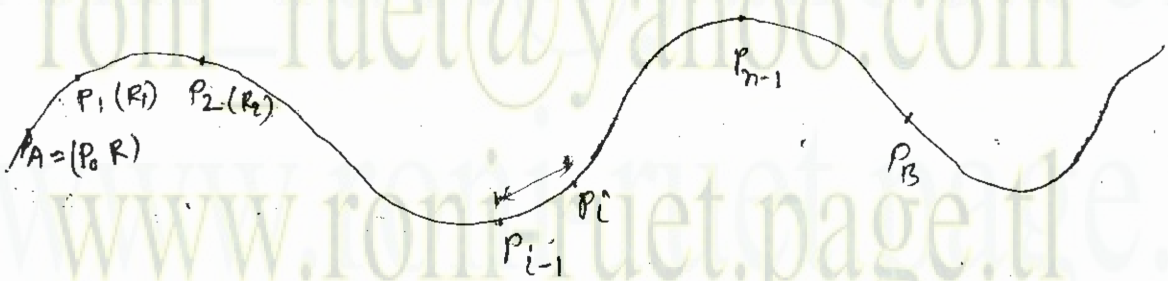
$$\Rightarrow dF(t) = G(t)\,dt$$

$$\Rightarrow \int G(t)\,dt = F(t) + C$$ indefinite integral

definite integral $$\int_a^b G(t)\,dt = [F(t)]_a^b = F(b) - F(a)$$

... INTEGRATION:

Consider the continuous vector function $F(R)$



and $R_1, R_2 \cdots R_n$ be there position vector, let v_i there position vector. of the are P_{i-1}, P_i. Then the vector point bunch at that segment $F(v_i)$

Considering to the sum $S = \sum_{i=0}^{n} F(v_i)\,\delta R_i$

where $\delta R_i = (R_{i-1} - R_i)$

If $n \to \infty$ then $|\delta R_0| = 0$

i.e $\int_c F(R)\, dr$ or $\int_c F\left(\frac{dR}{dt}\right) dt$

The line integration from A to B. we can write

$$= \int_A^B F(R)\, dR$$

If F represent a force action on a particle along the are AB. Then the work done $= \int_A^B F(r)(dr)$

$r = xi + jy + kz$

$dr = i dx + j dy + k dz$.

Ex: If a force $F = 2x^2y\,i + 3xy\,j$ displaces a particle in the plane xy from (0,0) to (1,4) along the curve $y = 4x^2$ find the work done

Sol^n: we know work done $= \int_A^B F(r)\, dr$

$$= \int_A^B (2x^2y\,i + 3xy\,j)(i\,dx + j\,dy)$$

$$= \int_c 2x^2y\, dx + \int_c 3xy\, dy$$

$$= \int_c (2x^2y\,dx + 3xy\,dy)$$

$y = 4x^2$
$dy = 8x\,dx$

$$= \int_0^1 2x^2(4x^2)\,dx + 3x(4x^2)\,\cancel{dy}\,8x\,dx$$ [∵ point (0,0) to (1,4)]

$$= \left[8\times\frac{x^5}{5}\right]_0^1 + 8\times 12\left[\frac{x^5}{5}\right]_0^1$$

$$= \frac{8}{5} + 12\times 8\times 1/5 = \frac{8}{5} + \frac{96}{5} = \frac{104}{5}$$ Ans

2) COMPUTE $\int_c F\cdot dr$ where $F = \dfrac{iy - jx}{x^2+y^2}$ and c is the cycle $x^2+y^2=1$ traversed counter clockwise.

Soln: $\bar{r} = ix + jy + kz$ and $dr = i\,dx + j\,dy + k\,dz$

$$\int_c F\,dr = \int_c \frac{iy - jx}{x^2+y^2}(i\,dx + j\,dy + k\,dz)$$

$$= \int_c \frac{y\,dx - x\,dy}{x^2+y^2}$$

$\left[x^2+y^2 = 1 \Rightarrow y = \sqrt{1-x^2} \right.$
$2x\,dx + 2y\,dy = 0$
$\left. dy = -\frac{x}{y}\,dx \right]$

$$= \int_c \frac{y\,dx - x(-x/y)\,dx}{1}$$

$$= \int_c \sqrt{1-x^2}\,dx + \frac{x^2}{y}\,dx.$$

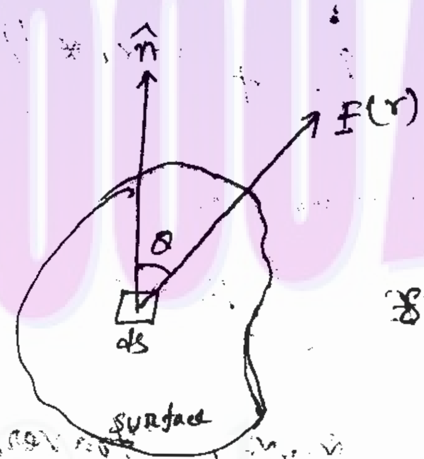
$$= \int \left(\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}} \right) dx$$

$$= \int \left(\frac{1-x^2+x^2}{\sqrt{1-x^2}} \right) dx$$

$$= \int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x \quad (\text{Ans:})$$

SURFACE INTEGRAL:

Let $\underline{F}$ be the vector function and S be the given surface



surface integral of $\underline{F}$ along the normal $= \underline{F}\cdot\vec{n}$

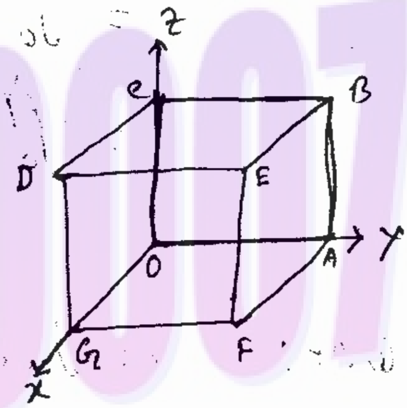
" " $\underline{F}$ over $S = \sum F\cdot\vec{n} = \iint_S F\cdot\vec{n}\, ds$

F represent the velocity of a fluid

If $\iint_S (F\cdot\vec{n})\, ds = 0$ then F said to be a solenodial vector

Evaluate $\iint_S \underline{v}\, \vec{n}\, ds$ where $\underline{v} = 4xz\,i - y^2 j + yz\,K$ and S is the surface of the cube bounded by the planes $x=0$, $x=1$, $y=0$, $y=1$ and $z=0$, $z=1$

Soln:



Face DEFG: $n = i$ $x=1$

$$\iint_{DEFG} \bar{v}\, \vec{n}\, ds =$$

$$= \int_0^1\int_0^1 (4xz\,i - y^2 j + yz\,k)\, i\, dy\, dz$$

$$= \int_0^1\int_0^1 4z\, dy\, dz = \int_0^1 [4zy]_0^1\, dz$$

$$= \int_0^1 4z\, dz = 4\cdot\left[\frac{z^2}{2}\right] = 4\times\frac{1}{2} = 2$$

Face OABC: $n = -i$ $x = 0$

$$\iint_{OABC} \bar{v}\, \vec{n}\, ds = \int_0^1\int_0^1 (4xz\,i - y^2 j + yz\,k)(-i)\, dy\, dz$$

$$= \int_0^1\int_0^1 (-y^2 j + yz\,k)(-i)\, dy\, dz$$

$$= 0$$

Face ABEF: $n = \vec{j}$ $y = 1$ then

$$\iint_{ABEF} \underline{v}\, \vec{n}\, ds = \int_0^1\int_0^1 (4xz\,i - j + z\,k)\, j \; dx\, dz$$

$$= \int_0^1\int_0^1 (-1)\, dx\, dz$$

$$= \int_0^1 [-x]_0^1\, dz = \int_0^1 -dz = [-z]_0^1 = -1$$

Face OGDE: $n = -\vec{j}$, $y = 0$

$$\iint_{OGDE} \underline{v}\, \vec{n}\, ds = \int_0^1\int_0^1 (4xz\,i)(-j)\, dx\, dz = 0$$

Face DEBC $n = k$ $z = 1$

$$\iint_{DEBC} \underline{v}\, \vec{n}\, ds = \int_0^1\int_0^1 (4x\,i - y^2 j + y\,k)\, k\, dx\, dy$$

$$= \int_0^1\int_0^1 y\, dx\, dy = \int_0^1 \left[\frac{y^2}{2}\right]_0^1 dx = \frac{1}{2}\int_0^1 dx$$

$$= \frac{1}{2} \times 1 = \frac{1}{2}$$

Face AOGF: $n = -k$ $z = 0$

$$\iint_{AOGF} \underline{v}\, \vec{n}\, ds = \int_0^1\int_0^1 (-y^2 j)(-k)\, dx\, dy = 0$$

Adding $\iint_S \underline{v}\, \vec{n}\, ds = 2 + 0 - 1 + 1/2 = 1 + \frac{1}{2} = 3/2$ Ans.

Evaluate $\iint_S \bar{A}\cdot\vec{n}\, ds$ where $\bar{A} = 18z\,i - 12j + 3y\,k$ and S is that part of the plane $2x+3y+6z=12$ which is located at the first order octal.

[straight line 2.0(28(2 x, y plane]

Sol^n:

$$\iint_S \bar{A}\cdot\vec{n}\, ds = \iint_R \bar{A}\,\vec{n}\,\frac{dy\cdot dx}{|\vec{n}\cdot\hat{k}|}$$

$$n = \frac{2i+3j+6k}{\sqrt{2^2+3^2+6^2}} = \frac{1}{7}(2i+3j+6k)$$

$$nk = \frac{1}{7}(2i+3j+6k)k = \frac{1}{7}*6 = 6/7$$

Also $\bar{A}\cdot\vec{n} = (18z\,i - 12j + 3y\,k)\,\frac{1}{7}(2i+3j+6k)$

$$= \frac{1}{7}(36z - 36 + 18y)$$

since $2x+3y+6z=12$

$$\therefore z = \frac{12-2x-3y}{6}$$

$$\bar{A}\,\vec{n} = \frac{1}{7}\left(36*\frac{12-2x-3y}{6} - 36 + 18y\right)$$

$$= \frac{1}{7}(36-12x)$$

$\therefore \iint_S \bar{A}\,\vec{n}\,ds =$

$\therefore \; y = \frac{12-2x}{3}.$

$$\therefore \iint_S \bar{A}\,\vec{n}\,ds = \iint_R \bar{A}\,\vec{n}\,\frac{dx\,dy}{|\hat{n}\,\hat{k}|} = \iint_R \frac{36-12x}{7}\cdot\frac{7}{6}\,dx\,dy$$

$$= \int_{x=0}^{6}\int_{y=0}^{\frac{12-2x}{3}} (6-2x)\,dx\,dy$$

$$= \int_{x=0}^{6}\int_{y=0}^{12-2x/3} (\cancel{6y-2xy})$$

$$= \int_{x=0}^{6} \Big[6y - 2xy\Big]_{y=0}^{12-2x/3} dx$$

$$= \int_{x=0}^{6} \left(24 - 12x + \frac{4x^2}{3}\right) dx$$

$$= \left[24x - 12x^2/2 + \frac{4x^3}{9}\right]_0^6$$

$$= 6\times 24 - \frac{12\times 6^2}{2} + \frac{4\times 6^3}{9} = 24 \;(\underline{\text{Ans:}})$$

NOTE: [$2x + 3y + 6z = 12$ For x co-ordinate $y=0$, $z=0$

$2x = 12 \Rightarrow x = 6$, $\longrightarrow \int_0^6$

$2x + 3y = 12$ y coordinate $z=0$

$y = \frac{12-2x}{3}$. $\int_0^{\frac{12-2x}{3}}$]

<u>Evaluate</u> $\iint_S \bar{A}\cdot\vec{n}\, ds$ where $A = zi + xj - 3y^2zk$ and is the surface of the $x^2+y^2=16$ include in the first <u>octant</u> between $z=0$ and $z=5$

*cylinder র ক্ষেত্রে x, z plane
*circle এর ক্ষেত্রে x, z plane

<u>soln:</u> $\iint_S \bar{A}\cdot\hat{n}\, ds = \iint_R \bar{A}\cdot\vec{n}\,\frac{dx\,dz}{|\hat{n}\cdot\hat{j}|}$

$$n = \frac{2xi + 2yj}{\sqrt{(2x)^2+(2y)^2}} = \frac{2(xi+jy)}{\sqrt{4(x^2+y^2)}} = \frac{xi+yj}{4}$$

$$nj = \frac{xi+jy}{4}\cdot j = y/4$$

$$\therefore \bar{A}\cdot\vec{n} = (zi + xj - 3y^2zk)\cdot\frac{1}{4}(xi+jy)$$

$$= \frac{1}{4}(xz + xy)$$

$$\therefore \iint_S \bar{A}\cdot\vec{n}\, ds = \iint_R \frac{1}{4}(xz+xy) \times \frac{4}{y} \times dx\,dz$$

$$= \int_{z=0}^{5}\int_{x=0}^{4} \frac{xz+xy}{y}\, dx\, dz$$

$$= \int_{z=0}^{5}\int_{x=0}^{4}\left(\frac{xz}{\sqrt{16-x^2}} + x\right) dx\, dz$$

$$\left[\int \frac{x}{\sqrt{16-x^2}} = \frac{x}{\sqrt{16-x^2}} = \left[\sqrt{16-x^2}\right]_4^0 = \sqrt{16} = 4\right]$$

$$= \int_{z=0}^{5} (4z+8)\,dz = \left[4 * z^2/2 + 8\right]_0^5$$

$$= \left[\frac{4*25}{2} + 8z\right] = \frac{100}{2} + 40$$

$$= 50 + 40 = 90 \text{ (Ans)}$$

VOLUAME INTEGRAL

(ii) Applying the divergance theorem to compute $\iint u.n\,ds$. s is the surface of the cysilinder $x^2+y^2=a^2$ bounded the planes $z=0$ $z=b$ and where $u = ix - jy + kz$

Soln: $\iint u.n\,ds = \iiint_V \nabla u.\,dv$ = voluame Integral

$$= \iiint_V \left(i\frac{\partial}{\partial x} + j\,\partial/\partial y + k\,\partial/\partial z\right)(ix - jy + kz)\,dv$$

$$= \iiint_V (1-1+1)\,dv$$

$$= \iiint_V dv = \iiint_V \cancel{dx\,dy\,dz}$$

$$= \int_{x=-a}^{a} \int_{y=-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_0^b (dx\,dy\,dz)$$

$$= \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dx\,dy * [z]_0^b = b\int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dx\,dy$$

$$= b\int_{-a}^{a} [y]_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dx = b\int_{-a}^{a} dx\,\left(2\sqrt{a^2-x^2}\right)$$

$$= b\,\cancel{2\sqrt{a^2-x^2}\,[x]_{-a}^{a}}$$

$$= b\int_{-a}^{a} \left(2\sqrt{a^2-x^2}\right) dx =$$

$$= 2b\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1} x/a\right]_{-a}^{a} = 2b\left[\frac{a^2}{2}\sin^{-1} a/a - \frac{a^2}{2}\sin^{-1}\left(\frac{-a}{a}\right)\right]$$

$$= 2b\left(a^2 \pi/2\right) = \pi a^2 b \text{ Ans}$$

VOLUME INTEGRAL:

If we consider a close surface in a space enclosing a volume v. Then the volume integral can be written

$$\iiint_V \underline{F}\, dv.$$

where $F = iF_x + jF_y + kF_z$

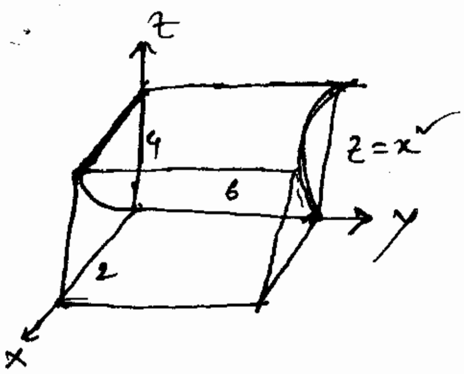
$dv = dx\,dy\,dz$.

$$\iiint_V (iF_x + jF_y + kF_z)\, dx\,dy\,dz.$$

$$= i\iiint_{xyz} F_x\, dx\,dy\,dz + j\iiint_{xyz} F_y\, dx\,dy\,dz$$

$$+ k\iiint_{xyz} F_z\, dx\,dy\,dz$$

1) Evaluate $\iiint_V \underline{F}\, dv$ where $\underline{F} = 2xzi - xj + y^2k$ and v is the region bounded by the surface $x=2$ $y=6$, $z=x^2$ and $z=4$



Soln:

$$\iiint_V \underline{F}\, dv = \iiint (2xzi - xj + y^2k)\, dx\,dy\,dz$$

$$= \int_0^2\int_0^6\int_{x^2}^4 (2xzi - xj + y^2k)\, dx\,dy\,dz$$

$$= i\int_0^2\int_0^6\int_{x^2}^4 2xz\,dx\,dy\,dz - j\int_0^2\int_0^6\int_{x^2}^4 x\,dx\,dy\,dz + k\int_0^2\int_0^6\int_{x^2}^4 y^2\,dx\,dy\,dz$$

$$= 128i - 24j + 384k$$

(Ans:)

$$\iint\left[2x\cdot\frac{z^2}{2}\right]_{x^2}^{4} = \iint(16x - x^5)\,dy\,dz$$

$$= \int\left[(16x-x^5)\,y\right]_0^6 = \int 6(16x-x^5)\,dx$$

$$= \left[6\times16x - \frac{x^6}{6}\times 6\right]_0^2$$

$$= 192 - 64$$

$$= 128$$

Q. Use divergence theorem to evaluate $\int_S \bar{A}\cdot ds$ wh… $A = x^3 i + y^3 j + z^3 k$ and S is the surface of s…

Soln:

$$\iint_S \bar{A}\cdot d\bar{s} = \iiint \text{div}\,A\cdot dv = \iiint \nabla A\cdot dv$$

$$= \iiint\left(i\,\partial/\partial x + j\,\partial/\partial y + k\,\partial/\partial z\right)(x^3 i + y^3 j + z^3 k)\,dv$$

$$= \iiint(3x^2 + 3y^2 + 3z^2)\,dv$$

$$= \iiint 3(a^2)\,dv = 3a^2\iiint dv = 3a^2\times V$$

$$= 3a^2\times\frac{4}{3}\pi\times a^3$$

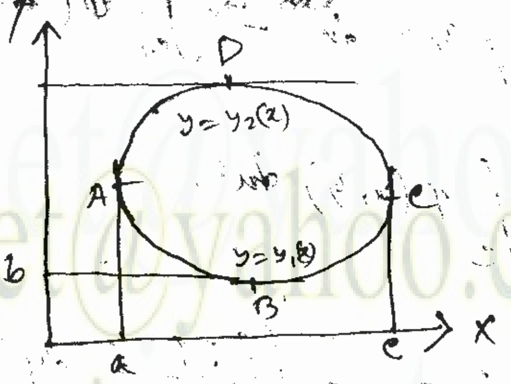
$$= 4\pi a^5 \text{ Ans}$$

GREEN'S THEORUM: (FOR A PLANE)

statement: If $\phi(x,y)$, $\psi(x,y)$ $\frac{\partial \psi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ be continuous function over a region R bounded by the sample close curve C in x-y plane then

$$\oint_C (\phi\, dx + \psi\, dy) = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dy\, dx.$$

$$\Rightarrow \oint_C (\phi\, dx + \psi\, dy) = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy$$



Proof:

let $C_1(ABC)$ be the equation $y = y_1(x)$

$C_2(CDA)$ " " " $y = y_2(x)$

let we take $\iint_R \frac{\partial \phi}{\partial y}\, dx\, dy$.

$$= \int_{x=a}^{x=c} \left[\int_{y=y_1(x)}^{y=y_2(x)} \frac{\partial \phi}{\partial y}\, dy\right] dx.$$

$$= \int_{x=a}^{x=c} \left[\phi(x,y)\right]_{y_1}^{y_2} dx$$

$$= \int_{x=a}^{x=c} \left[\phi(x,y_2) - \phi(x,y_1)\right] dx$$

$$= \int_{x=a}^{c} \phi(x,y_2)\, dx - \int_{x=a}^{c} \phi(x,y_1)\, dx$$

$$= -\int_{c}^{a} \phi(x,y_2)\, dx - \int_{a}^{c} \phi(x,y_1)\, dx$$

$$= -\int_{C_2} \phi(x,y)\, dx - \int_{C_1} \phi(x,y)\, dx$$

$$= -\int_{C} \phi(x,y)\, dx$$

$$\oint_C \phi(x,y)\, dx = -\iint_R \frac{\partial \phi}{\partial y}\, dx\, dy \longrightarrow (i)$$

similarly we get $\oint_C \psi(x,y)\, dy = \iint_R \frac{\partial \psi}{\partial x}\, dx\, dy \longrightarrow (ii)$

adding we get (i) and (ii)

$$\oint_C (\phi\, dx + \psi\, dy) = \iint_R \left[\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right] dy\, dx \quad \text{(proved)}$$

Use green's theorem to evaluate $\oint_C (x^2+xy)\,dx + (x^2+y^2)\,dy$ where c is the square formed by the lines. $y = \pm 1, x = \pm 1$

By Green's theorem $\oint_C (\phi\, dx + \psi\, dy) = \iint \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy$

$$= \int_{-1}^{1}\int_{-1}^{1}\left[\frac{\partial}{\partial x}(x^2+y^2) - \frac{\partial}{\partial y}(x^2+xy)\right] dx\, dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} (2x - x)\, dx\, dy$$

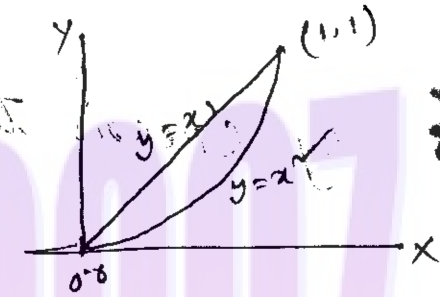
$$= \int_{-1}^{1}\int_{-1}^{1} x\, dx\, dy$$

$$= \int_{-1}^{1} x\, dx\, [y]_{-1}^{1} = \int_{-1}^{1} x\, dx * 2$$

$$= 2\left[\frac{x^2}{2}\right]_{-1}^{1} = [1-1]$$

$$= 0$$ Ans

verify Green's theorem in the plane for $\oint_C$ [redacted] where c is the closed curve of the region bounded by $y=x$, and $y=x^2$



Sol^n:

$y=x$ and $y=x^2$ intersect at $(0,0)$ and $(1,1)$. ~~The~~

Applying Greens theorem

$$\oint_C (\phi\,dx + \psi\,dy) = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dy\,dx$$

$$= \iint_R \left\{\frac{\partial}{\partial x}(x^2) - \frac{\partial}{\partial y}(xy+y^2)\right\} dy\,dx.$$

$$= \int_{x=0}^{1}\int_{y=x^2}^{x} (2x - x + 2y)\,dy\,dx.$$

$$= \int_{x=0}^{1}\int_{y=x^2}^{x} (x-2y)\,dy\,dx.$$

$$= \int_{x=0}^{1} \left[xy - y^2\right]_{x^2}^{x} dx \qquad = \int_0^x dx \left[x^2 - x^2 - x^3 + x^4\right]$$

$$= \int_0^1 (x^4 - x^3)\,dx$$

$$= \left[\frac{x^5}{5} - \frac{x^4}{4}\right]_0^1 = \frac{1}{5} - \frac{1}{4}$$

$$= \frac{4-5}{20} = -\frac{1}{20}$$

(Ans)

Along $y=x^2$, the line integration equal

$$\int_0^1 ((x)(x^2)+x^4)\,dx + x^2(2x)\,dx = \int_0^1 (3x^3+x^4)\,dx.$$

$$= \left[\frac{3x^4}{4}+\frac{x^5}{5}\right]_0^1 = \left[\frac{3}{4}+\frac{1}{5}\right] = \frac{19}{20}$$

along $y=x$ from $(1,1)$ ~~and~~ to $(0,0)$ the line integral equal

$$\int_1^0 \{x(x)+x^2\}\,dx + x^2\,dx = \int_1^0 3x^2\,dx$$

$$= \left[\frac{3x^3}{3}\right]_1^0 = (0-1) = -1$$

so the required integral $= \frac{19}{20} - 1 = \frac{19-20}{20} = -\frac{1}{20}$

(Ans)

GAUSS'S THEORUM OF DIVERGENCE: (relation between S.I and V.I)

statement: The surface integral of the normal component of a vector function $\bar{F}$ taken around a close surface S is equal to the integral of the divergence of F taken over the volume V enclosed by the surface S.

mathematically $\iint_S \bar{F}\cdot\hat{n}\cdot ds = \iiint_V div\,\bar{F}\, dv$

PROOF: - Let $F = iF_1 + jF_2 + kF_3$

putting the value F.n statement of the Divergence Theorem

$$\iint_S (iF_1+jF_2+kF_3)\,\hat{n}\cdot ds = \iiint_V \left(\frac{\partial}{\partial x}i + j\,\partial/\partial y + k\,\partial/\partial z\right)(iF_1+jF_2+kF_3)\,dx\,dy\,dz$$

$$= \iiint_V \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dx\,dy\,dz \longrightarrow (i)$$

~~Let~~

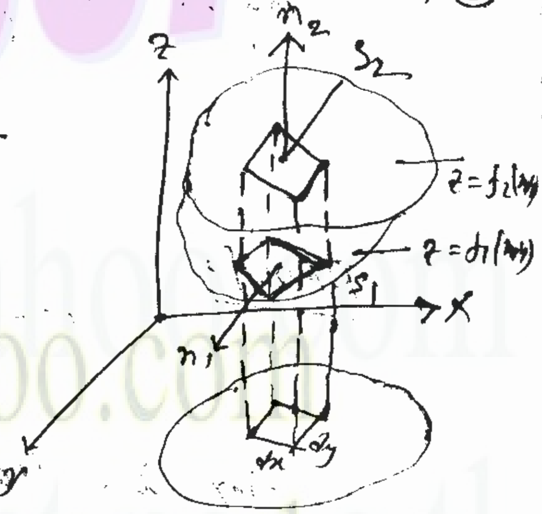
P.T.O

Let us evaluate first $\iiint_V \frac{\partial F_3}{\partial z} \, dx\,dy\,dz$.

$$\iiint_V \frac{\partial F_3}{\partial z} \, dx\,dy\,dz = \iint_R \left[\int_{z_1 = f_1(x,y)}^{f_2(x,y)} \frac{\partial F_3}{\partial z} \, dz \right] dx\,dy$$

$$= \iint_R \left[F_3(x,y,z) \right]_{f_1(x,y)}^{f_2(x,y)} dx\,dy$$

$$= \iint_R \left[F_3(x,y,f_2) - F_3(x,y,f_1) \right] dx\,dy \longrightarrow (2)$$



From upper side part the surface, i.e., S_2

we have $dx\,dy = dS_2 \cos\gamma_2 = n_2 \cdot k \, dS_2$

Again lower portion/part of surface S_1

we have

$$dy\,dx = -\cos\gamma_1 \, dS_1 = -n_1 \cdot k \, dS_1$$

$$\iint_R F_3(x,y,f_2) \, dx\,dy = \iint_{S_2} F_3 \, n_2 \cdot k \, dS_2$$

and $\iint_R F_3(x,y,f_1) \, dx\,dy = -\iint_{S_1} F_3 \, n_1 \cdot k \, dS_1$

putting the value in eqn ② we get

$$\iiint_V \frac{\partial F_3}{\partial z}\, dv = \iint_{S_2} F_3\, \vec{n}_2 \cdot k\, ds_2 + \iint_{S_1} F_3\, \vec{n}_1 \cdot k\, ds_1$$

$$= \iint_S F_3\, \vec{n} \cdot k\, ds \longrightarrow ③$$

Similarly it can be show that

$$\iiint_V \frac{\partial F_2}{\partial y}\, dv = \iint_S F_2\, \vec{n} \cdot j\, ds \longrightarrow ④$$

$$\iiint_V \frac{\partial F_1}{\partial x}\, dv = \iint_S F_1\, \vec{n} \cdot i\, ds \longrightarrow ⑤$$

Adding ③ ④ and ⑤ we get

$$\iiint_V \left(\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}\right) dv = \iint_S (F_1 i + F_2 j + F_3 k) \cdot \vec{n}\, ds$$

$$\Rightarrow \iiint_V \operatorname{div} (\nabla \cdot \vec{F})\, dv = \iint \vec{F} \cdot \vec{n}\, ds$$

$$\Rightarrow \iiint_V (\nabla \cdot \vec{F})\, dv = \iint_S \vec{F} \cdot \vec{n}\, ds$$

(proved)

Ex: If $\vec{F} = a x\hat{i} + b y\hat{j} + c z\hat{k}$, a, b, c constant,

then $\iint \vec{F} \cdot d\vec{s}$, where s is the surface of a unit sphere, is

(i) 0 (ii) $\frac{4}{3}\pi(a+b+c)$ (iii) $\frac{4}{3}\pi(a+b+c)^2$ (iv) none of these.

choose correct answer.

Soln

$$\operatorname{div} F = \nabla \cdot F$$
$$= \left(\hat{i}\,\partial/\partial x + \hat{j}\,\partial/\partial y + \hat{k}\,\partial/\partial z\right)(a x\hat{i} + b y\hat{j} + c z\hat{k})$$
$$= (a+b+c)$$

$$\therefore \iint F \cdot ds = \int_V \operatorname{div} F \, dV = a+b+c \int_V dV = (a+b+c) * V$$
$$= (a+b+c) * \frac{4}{3}\pi(1)^3$$
$$= \frac{4}{3}\pi(a+b+c) \quad \text{(Ans)}$$

PROB If $E = \operatorname{grad}\phi$ and $\nabla^2\phi = -4\pi\rho$ prove that

$$\iint_S \vec{E} \cdot \vec{n} \, ds = -4\pi \iiint_V \rho \, dV$$

Soln By Gauss D.T. $\iint_S E \cdot n \, ds = \iiint (\nabla E) \, dV$ BUT $E = \nabla\phi$

$$= \iiint_V \nabla(\nabla\phi) \, dV = \iiint \nabla^2\phi \, dV$$
$$= -4\pi \iiint_V \rho \, dV \quad \text{(proved)}$$

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THANKS TO ALL

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