Chapter-01: Measures of Central Tendency and Location.

Arithmetic Mean

For ungrouped Data

sample mean
$$\boxed{\bar{x} = \frac{\sum x}{n}}$$

Population mean
$$\boxed{\mu = \frac{\sum x}{n}}$$

For grouped Data

$$\boxed{\bar{x} = \frac{\sum fx}{n}}$$

$$\boxed{\mu = \frac{\sum fx}{n}}$$

Math-01

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{550+420+560+500+700+670+860+480}{\cancel{n}\ \cancel{7}\ 8}$$

$$= 592.50$$

Math-02

$$\bar{\mu} = \frac{\sum x}{n}$$

$$= \frac{53+45+59+48+54+46+51+58+55}{9}$$

$$= 52.11$$

Math-03

Geo

Class limit	x	f	fx
18 — 26	22	3	66
27 — 35	31	5	155
36 — 44	40	9	360
45 — 53	49	14	686
54 — 62	58	11	638
63 — 71	67	6	402
72 — 80	76	2	152
		$n = 50$	$\Sigma fx = 2,459$

$$\therefore \bar{x} = \frac{\Sigma fx}{n}$$

$$= \frac{2,459}{50} = 49.18 \quad \text{Ans.}$$

Combined Mean

$$\boxed{\bar{X}_{cm} = \frac{\Sigma N \bar{X}}{\Sigma N}}$$

Math-07:

$$\bar{X}_{cm} = \frac{30400 \times 12 + 27,300 \times 18 + 42,500 \times 15}{45}$$

$$= 33,193.33 \text{ T.k}$$

Ans.

Geometric Mean

$$\boxed{GM = \sqrt[n]{(x_1)(x_2)(x_3)\cdots(x_n)}}$$

→ To determine the average percentage, indexes, and relatives.

$$\boxed{GM = \sqrt[n-1]{\frac{\text{value at the end of the period}}{\text{value at the start of the period}}} - 1}$$

→ To determine the average percents from one period of time to another

Math-04

$$GM = \sqrt[n]{x_1 x_2 x_3 \cdots x_n}$$

$$= \sqrt[5]{5 \times 6 \times 4 \times 8 \times 10}$$

$$= 6.26$$

Math-05

$$GM = \sqrt[12]{\frac{155}{20}} - 1$$

$$= 0.2046$$

$$= 20.46\%$$

→ per month.

Median

For ungrouped

$$\boxed{\text{Median} = \frac{n+1}{2}th}$$

→ for odd the middle ranked

→ for even, the median is the average of the two middle ranked values.

For grouped Data

$$\boxed{\text{Median} = LB + \left(\frac{\frac{N}{2} - cf}{f}\right) \times i}$$

Math-0B

45 46 48 51 53 54 55, 58, 59

$$\text{Median} = \frac{9+1}{2} = 5th$$

53 Ans.

Math-02

420 480 500 550 ↑ 670 700 860

$$\text{Median} = \frac{8+1}{2} th$$

$$= 4.5th$$

$$\therefore \text{Median} = \frac{550+670}{2}$$

$$= 555 th \quad \text{Ans}$$

Math-10

	Class Limit	f	cf	
	18 - 26	3	3	
9	27 - 35	5	8	
	36 - 44	9	17	cf
LB = 44.5	45 - 53	14 f	31	
	54 - 62	11	42	
	63 - 71	6	48	
	72 - 8	2	50	

$\frac{N}{2} = 25$

$n f = 50$

$$\text{Midian} = LB + \left(\frac{\frac{N}{2} - cf}{f}\right) * i$$

$$= 44.5 + \left(\frac{25 - 17}{14}\right) * 9$$

$$= 49.64$$ Ans.

Mode

Unimode – 1

Bimodal – 2

Multimodal – more than 2

No mode – without mode

Ungrouped

* যেইটা বেশি সংখ্যক বার.

For grouped

$$\text{Mode} = LB + \left(\frac{d_1}{d_1 + d_2}\right) * i$$

Math-14

Class Limit	Frequency
18 — 26	3
27 — 35	5
36 — 44	9
45 — 53	14
54 — 62	11
63 — 71	6
72 — 80	2

i = 9

LB = 44.5

f

$$\text{Mode} = 44.5 + \left(\frac{5}{5+3}\right) * 9$$

$$= 50.13 \text{ Ans.}$$

Quartiles, Deciles and Percentiles

	Ungrouped	Grouped
Quartiles	$Q_k = \frac{k(n+1)}{4}$	$Q_k = LB + \left(\frac{\frac{KN}{4} - cf}{f}\right) * i$
Deciles	$D_k = \frac{K(n+1)}{10}$	$D_k = LB + \left(\frac{\frac{KN}{10} - cf}{f}\right) * i$
Percentiles	$P_k = \frac{K(N+1)}{100}$	$P_k = LB + \left(\frac{\frac{KN}{100} - cf}{f}\right) * i$

Math

Class Limit	Frequency	cf
18 — 26	3	3
27 — 35	5	8
36 — 44	9	17
45 — 53	14	31
54 — 62	11	42
63 — 71	6	48
72 — 80	2	50
	$n = 50$	

$$Q_1 = 35.5 + \left(\frac{12.5 - 8}{9}\right) * 9$$
$$= 40$$

$$Q_2 = 44.5 + \left(\frac{25 - 17}{14}\right) * 9$$
$$= 49.64$$

$$D_7 = 53.5 + \left(\frac{35 - 31}{11}\right) * 9 \quad \left(\frac{7N}{10}\right)$$

$$P_{22} = 35.5 + \left(\frac{11 - 8}{9}\right) * 9 \quad \left(\frac{22N}{100}\right)$$
$$= 38.5 \text{ Ans}$$

Chapter-02: Measure of Dispersion

- Range
- Average Deviation
- Variance
- standard Deviation
- Quartile Deviation
- Co-efficient of variation

Rang: The difference of the highest value and the lowest value in the Data set.

$$\text{Rang} = HV - LV$$

Math-01

420, 480, 500, 550, 560, 670, 700, 860

$$\text{Rang} = HV - LV$$
$$= 860 - 420$$
$$= 440$$

Average Deviation:

For. Ungrouped

$$AD = \frac{\sum |x-\bar{x}|}{N}$$

$$AD = \frac{\sum |x-\mu|}{N}$$

For. grouped

$$AD = \frac{\sum f|x-\bar{x}|}{n}$$

$$AD = \frac{\sum f|x-\mu|}{n}$$

Math: The daily rates of a sample of eight employees at GMS Inc are ~~420~~, 550, 420, 560, 500, 700, 670, 860, 480. Find the average deviation

Soln

$$\bar{x} = \frac{550+420+560+500+700+670+860+480}{8}$$

$= 592.50$

Sample	$x-\bar{x}$	$\lvert x-\bar{x}\rvert$
550	−42.5	42.5
420	−172.5	172.5
560	−32.5	32.5
500	−92.5	92.5
700	107.5	107.5
670	77.5	77.5
860	267.5	267.5
480	−112.5	112.5

$\sum |x-\bar{x}| = 905$

$$\therefore AD = \frac{\sum |x-\bar{x}|}{N} = \frac{905}{8} = 113.225$$

Ans.

Example-02

Class limit	x	f	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
700 − 849	774.5	2	1549	322.5	645
850 − 999	924.5	9	8320.5	172.5	1552.5
1000 − 1,149	1074.5	15	16117.5	22.5	337.5
1,150 − 1299	1224.5	9	11020.5	127.5	1147.5
1300 − 1499	1374.5	5	6872.5	277.5	1387.5

$n = 40$ $\quad \sum fx = 43880$ $\quad \sum f\lvert x-\bar{x}\rvert = ??$

$$\bar{x} = \frac{\sum fx}{n} = \frac{43880}{40} = 1,097$$

$$A.D = \frac{\sum f\lvert x-\bar{x}\rvert}{n} = \frac{??}{40} = \boxed{??}$$ Ans.

Varience and Standard Deviation

Ungrouped

$s = \sigma$ when population

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

Varience

Grouped

$$s^2 = \frac{\sum f (x-\bar{x})^2}{n-1}$$

$$s^2 = \frac{\sum f x^2 - \frac{(\sum fx)^2}{n}}{n-1}$$

$\sigma = s$ population Deviation

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}}$$

$$s = \sqrt{\frac{\sum f (x-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{\sum f x^2 - \frac{(\sum fx)^2}{n}}{n-1}}$$

Example

x	x^2
550	30,2500
420	17,6400
560	31,3600
500	25,0000
700	49,0000
670	44,8900
860	73,9600
486	23,6400

$\Sigma x = 4740 \qquad \Sigma x^2 = 2951400$

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{2951400 - \frac{(4740)^2}{8}}{8-1}}$$

$$= 20421.43$$

Ans.

Example:

Class limit	x	f	fx	x^2	fx^2
18 — 26	22	3	66	484	1452
27 — 35	31	5	155	961	4805
36 — 44	40	9	360	1600	14400
45 — 53	49	14	686	2401	33614
54 — 62	58	11	638	3364	37004
63 — 71	67	6	462	4489	26934
72 — 80	76	2	152	5776	11552
		$n = 50$	$\Sigma fx = 2459$		$\Sigma fx^2 = 129761$

$$s^2 = \frac{\Sigma fx^2 - \frac{(\Sigma fx)^2}{n}}{n-1}$$

$$= \frac{129761 - \frac{(2459)^2}{50}}{49} = \boxed{}$$

$$s = \sqrt{s^2}$$

$$= \boxed{} \quad \text{Ans.}$$

Quartile Deviation

$$\boxed{Q.D = \frac{Q_3 - Q_1}{2}}$$

Co-efficient of variance. (C.V)

$$\boxed{\begin{aligned} CV &= \frac{S}{\bar{X}} \times 100\% \\ C.V &= \frac{S}{\mu} \times 100\% \end{aligned}}$$

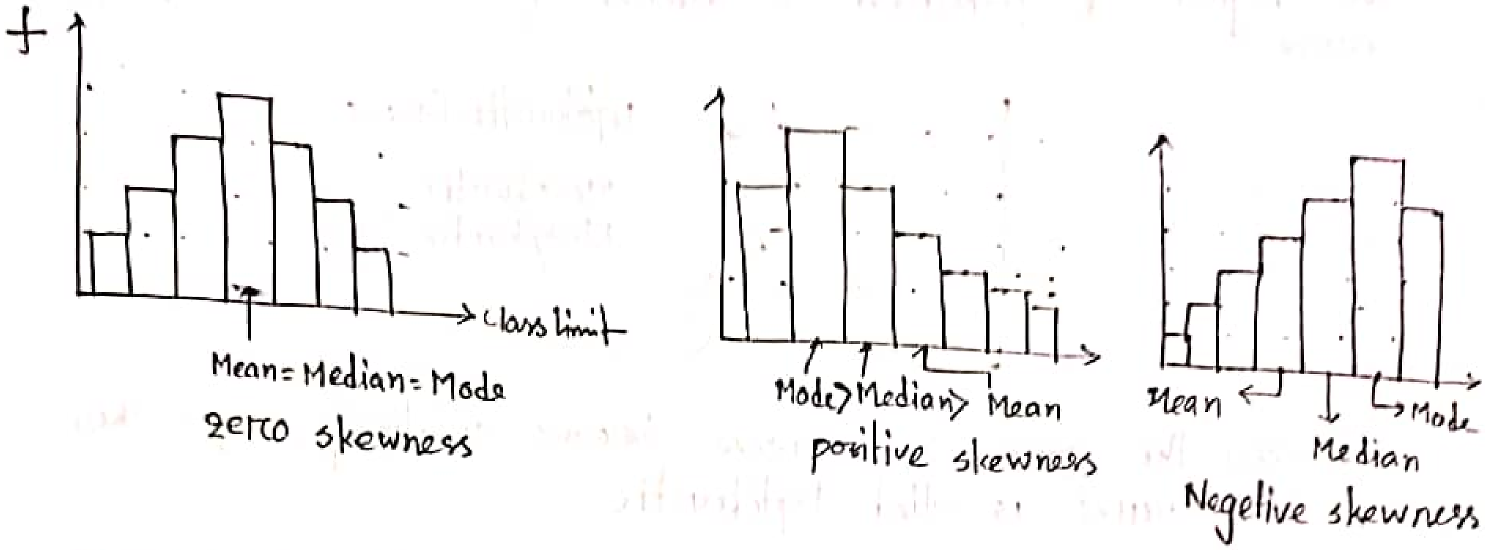
Math

$$C.V = \frac{3}{33} \times 100\% = 10\%$$

$$CV = \frac{3150}{45,000} \times 100\% = 7\%$$

Chapter-03 (Measure of skewness, kurtosis, moments)

Skewness



$$\boxed{Sk = \frac{\text{Mean} - \text{Mode}}{SD\,(s)}}$$ (Purchapes [illegible])

$$\boxed{Sk = \frac{3(\text{Median} - \text{Mean})}{SD\,(s)}}$$

it ranges b/w (-3 to 3)

$$\boxed{\begin{aligned} Sk &= \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1} \\ &= \frac{Q_1 - 2\,\text{Median} + Q_3}{Q_3 - Q_1} \end{aligned}}$$

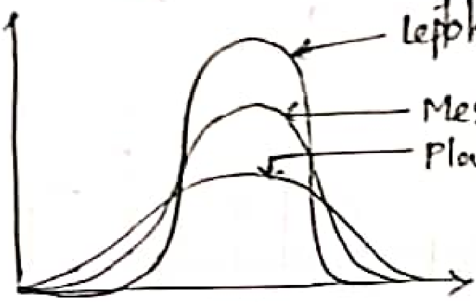
$$\boxed{Sk = \frac{m_3}{s^3} \text{ or, } \frac{\mu_3}{\sigma^3}}$$

$$\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$$

$$\boxed{\sigma^2 = \mu_2}$$

Kurtosis

The degree of peakedness or flatness of a unimodal frequency curve.



- When the peak of a curve becomes relatively high then the curve is called Leptokurtic
- When the curve is flat-topped, then it called Platykurtic
- Neither very peaked nor very flat topped; this called normal curve or mesokurtic.

$$\boxed{(\text{kurt})\ \beta_2 = \frac{\mu_4}{(\mu_2)^2} \quad \text{Or} \quad \frac{M_4}{M_2^2}}$$

For normal $\beta = 3$

leptocurtic $\beta > 3$

platycurtic $\beta < 3$

$$\boxed{\text{Kurt} = \frac{Q.D}{P_{90} - P_{10}}}$$

→ quartile deviation $\frac{Q_3 - Q_1}{2}$

→ 10th percentile.

90th Percentile

<u>Moments</u> (r moment designates the power to which deviations are raised before averaging them)

The rth moment about the mean

$$(m_r)\ \boxed{\mu_r = \frac{\sum f_i (x_i-\bar{x})^r}{N}}$$

→ for grouped data

$$(m_r)\ \boxed{\mu_r = \frac{\sum (x_i-\bar{x})^r}{N}}$$

→ for ungrouped data

when $r=0$

$$\mu_0 = \frac{\sum f_i (x_i-\bar{x})^0}{N} = \frac{\sum f_i}{N}$$

$$= \frac{N}{N}$$

$$= \underline{1}$$

when $r=1$

$$\mu_1 = \frac{\sum f_i (x_i-\bar{x})}{N}$$

$$= \frac{\sum f_i x_i}{N} - \frac{\bar{x}\sum f_i}{N}$$

$$= \bar{x} - \bar{x}\frac{N}{N}$$

$$= 0$$

when $r=2$

$$\mu_2 = \frac{\sum f_i (x_i-\bar{x})^2}{N} \longrightarrow \text{variance}$$

$$= \cancel{\sum f_i x_i^2 - (\sum f_i x_i)^2}$$

$$= \frac{\sum f_i (x_i^2 - 2x_i\bar{x} + \bar{x}^2)}{N}$$

$$= \frac{\sum f_i x_i^2}{N} - \frac{2\bar{x}\sum f_i x_i}{N} + \frac{\bar{x}^2 \sum f_i}{N}$$

$$= \frac{\sum f_i x_i^2}{N} - 2\bar{x}^2 + \bar{x}^2 = \frac{\sum f_i x_i^2}{N} - \bar{x}^2 = \sigma^2$$

The rth moment about an arbitrary value a

$m_r' / \boxed{\mu_r' = \frac{\sum f_i (x_i - a)^r}{N}}$ for grouped data

$m_r' / \boxed{\mu_r' = \frac{\sum (x_i - a)^r}{N}}$ for ungrouped data

when $r=0$

$$\mu_0' = \frac{\sum f_i (x_i - a)^0}{N}$$

$$= \frac{\sum f_i}{N} = \frac{\cancel{\Sigma} N}{N} = 1$$

when $r=1$

$$\mu_1' = \frac{\sum f_i (x_i - a)^1}{N}$$

$$= \frac{\sum f_i x_i}{N} - a \frac{\sum f_i}{N}$$

$$= \bar{x} - a$$

when $r=2$

$$\mu_2' = \frac{\sum f_i (x_i - a)^2}{N}$$

$= s^2$ (root mean square deviation)

~~when = a~~

when moment about the origin

the $a = 0$

$$\therefore \boxed{\mu_r' = \frac{\sum f_i x_i^r}{N}}$$

Relation between μ_r and μ_r':

$$\left\{\begin{aligned} \mu_2 &= \mu_2' - \mu_1'^2 \\ \mu_3 &= \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 \\ \mu_4 &= \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 \end{aligned}\right\}$$

We know,

$$\mu_r = \frac{1}{N}\sum_{i=1}^{k} f_i (x_i - \bar{x})^r$$

$$\therefore \mu_r = \frac{1}{N}\sum_{i=1}^{k} f_i (x_i - A + A - \bar{x})^r$$

$$= \frac{1}{N}\sum_{i=1}^{k} f_i \{(x_i - A) - (\bar{x} - A)\}^r$$

If $\boxed{d_i = x_i - A}$ then

$$x_i = A + d_i$$

$$\frac{1}{n}\sum x_i = A + \frac{1}{n}\sum d_i$$

$$\bar{x} = A + \mu_1'$$

$$\because \mu_1' = \bar{x} - A$$

$$\therefore \mu_r = \frac{1}{N}\sum_{i=1}^{k} f_i \{d_i - \mu_1'\}^r$$

$$= \frac{1}{N}\sum_{i=1}^{k} f_i \{d_i^r - {}^rC_1 d_i^{r-1}\mu_1' + {}^rC_2 d_i^{r-2}\mu_1'^2 - {}^rC_3 d_i^{r-3}\mu_1'^3 + \cdots + (-1)^r \mu_1'^r\}$$

$$= \frac{1}{n}\sum_{i=1}^{k} f_i d_i^r - {}^rC_1 \mu_1' \sum_{i=1}^{k} f_i d_i^{r-1} + \cdots\cdots$$

$$\therefore \boxed{\mu_r = \mu_r' - {}^rC_1 \mu_{r-1}'\mu_1' + {}^rC_2 \mu_{r-2}'\mu_1'^2 - {}^rC_3 \mu_{r-3}'\mu_1'^3 + \cdots}$$

Math-01 : The 1st three moments of a distribution about the value 2 of the variable are 1, 16, −40. show that the mean is 3 the variance is 15 and $\mu_3 = -86$. Also show that the 1st three moments about $x=0$ are 3, 24, 76

Soln

$\mu_1' = 1$

$\mu_2' = 16$

$\mu_3' = -40$

$a = 2$

(i) $\mu_1' = \bar{x} - a$

$1 = \bar{x} - 2$

$\therefore \bar{x} = 3$ Ans.

(ii) $\sigma^2 = \mu_2$

$= \mu_2' - \mu_1'^2$

$= 16 - 1^2$

$= 15$ Ans

(iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= -40 - 3(16\times1) + 2(1)^3$

$= -86$ Ans.

again

$\mu_1' = \bar{x} - a$

$= 3 - 0$

$= 3$

$\cancel{\mu_1}$

$\mu_2' = \mu_2 + \mu_1'^2$

$= 15 + 3^2$

$= 24$

$\mu_3' = \mu_3 + 3\mu_2'\mu_1' - 2\mu_1'^3$

$= -86 + 3\times(24)\times3 - 2\times(3^3)$

$= 76$ Ans.

Math-02:

The 1st ~~three~~ four moments of a distribution about the value ~~2~~ 5 of the variable are 2, 20, +40. and 50. show that mean is 7 and variance is 16. and also show that $\mu_3 = -64, \mu_4 = 162$ $\beta_1 = 1$, $\beta_2 = 0.63$.

Soln

Given that

$\mu_1' = 2$

$\mu_2' = 20$

$\mu_3' = 40$

$\mu_4' = 50$

$a = 5$

(i) $\mu_1' = \bar{x} - a$

$2 = \bar{x} - 5$

$\therefore \bar{x} = 7$ Ans.

(ii) $\sigma^2 = \mu_2$

$= \mu_2' - \mu_1'^2$

$= 20 - 2^2$

$= 16$ Ans

(iii) $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 40 - 3 \times (20) \times 2 + 2 \times 40^3$

$= -64$

(iv) $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 50 - 4 \times 40 \times 2 + 6 \times 20 \times 2^2 - 3 \times 2^4$

$= 162$ Ans.

(v) $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

$= \frac{(64)^2}{(16)^3}$

$= 1$

(vi) Kurt $\beta_2 = \frac{\mu_4}{\mu_2^2}$

$= \frac{162}{16^2}$

$= 0.63$

Math: Calculate the 1st four moment about the mean fore the following data. Also calculate β_1 and β_2 and comment about the shape of the curve

$X = 1 \quad 2 \quad 3 \quad 4 \quad 5, \quad 6 \quad 7 \quad 8 \quad 9$

$f = 1 \quad 6 \quad 13 \quad 25 \quad 30 \quad 22 \quad 9 \quad 5 \quad 2$

$h=1$

x	f	$d=\frac{x-a}{h}$	fd	fd^2	fd^3	fd^4
1	1	-4	-4	16	-64	256
2	6	-3	-18	54	-135	486
3	13	-2	-26	52	-104	208
4	25	-1	-25	25	-25	25
5 (a)	30	0	0	0	0	0
6	22	1	22	22	22	22
7	9	2	18	36	72	144
8	5	3	15	45	135	405
9	2	4	8	32	128	512
	$n=113$		$\Sigma fd = 10$	$\Sigma fd^2 = 282$	$\Sigma fd^3 = 29$	$\Sigma fd^4 = 2058$

$$\mu_1' = \frac{\Sigma fd}{n} \times h = \frac{-10}{113} \times 1 = -0.088$$

$$\mu_2' = \frac{\Sigma fd^2}{n} \times h^2 = 2.49$$

$$\mu_3' = \frac{\Sigma fd^3}{n} \times h^3 = 0.256$$

$$\mu_4' = \frac{\Sigma fd^4}{n} \times h^4 = 18.21$$

Moments about mean

$$\mu_1 = \frac{\sum f_i (x_i - \bar{x})}{N} = \mu_1' - {}^1C_1 \mu_0' \mu_1'$$

$$= 0 \qquad\qquad = \mu_1' - \mu_1'$$

$$\mu_2 = (\mu_2') - (\mu_1')^2$$

$$= 2.49 - (0.088)^2$$

$$= 2.48$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1 + \mu_1'^3$$

$$= 0.256 - 3.(2.49) \times (-0.088) + (-0.088)^3$$

$$\cancel{= 0.256}$$

$$= 0.91$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 18.21 - 4 \times (-0.088) \times (0.256) + 6(2.49) \times (-0.088)^2 - 3(-0.088)^4$$

$$= 18.41$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{0.91^3}{(2.48)^3}$$

$$= 0.049 \quad [\text{skewness}]$$

* Sheet এর ২ নং/ক্রমিক math করতে হবে।

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{18.41}{2.48^2}$$

$$= 2.993 < 3 \quad [\text{Kurtosis}]$$

↳ Platykurtic

Chapter - 04: Probability

* Probability $= \dfrac{\text{No. of favourable events}}{\text{Total number of outcomes in sample space}}$

* If A and B are two events

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If A and B are mutually exclusive events then $A \cap B = \phi$

$$\therefore P(A \cap B) = 0$$

Hence $\boxed{P(\underbrace{A \cup B}_{\text{or}}) = P(A) + P(B)}$

* Conditional probability

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

→ Probability of A when B is Already happend.

If A and B are independent event then $P(A/B) = P(A)$

$$\therefore P(A) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \boxed{P(A) . P(B) = P(A \overset{\text{and}}{\cap} B)}$$

Q-01: Randomly select one card from the deck. What is the probability that the selected card

1) Is a face card?

2) Is a club?

3) Is a heart and less than 8?

4) Is a black ace

5) Is red and higher than 10?

6) Is heart or less than 8

soln

1) $\frac{12}{52}$

2) $\frac{13}{52}$

~~3) $P(H \cup \text{less than } 8) = P(H) + P(\text{less than } 8) - P(H \cap \text{less than})$~~

~~$= \frac{13}{52} +$~~

3) $P(H \cap \text{less than } 8) = \frac{6}{52}$ // $P(H) * P(\text{less less } 8)$

$\frac{13}{52} * \frac{24}{52}$

$= \frac{6}{52}$

04) $\frac{2}{52}$

05) $\frac{8}{52}$

06) $P(H \cup \text{less than } 8) = P(H) + P(\text{less than } 8) - P(H \cap \text{less than } 8)$

$= \frac{13}{52} + \frac{24}{52} - \frac{6}{52}$

$= 5$

Event Probability Using Contingency table:

	B_1	B_2	Total
A_1	$P(A_1 \cap B_1)$	$P(A_1 \cap B_2)$	$P(A_1)$
A_2	$P(A_2 \cap B_1)$	$P(A_2 \cap B_2)$	$P(A_2)$
Total	$P(B_1)$	$P(B_2)$	1

Math-02

	Full prof.	Associ. Prof.	Assis. prof.	Instructor	Total
under 30 A_1	2	3	57	6	68
30-39 A_2	52	170	163	17	402
40-49 A_3	156	125	61	6	348
50-59 A_4	145	68	36	4	253
60 & or A_5	75	15	3	0	93
Total	430	381	320	33	1164

- suppose that an Asu faculty member is selected at random
  - Determine the probability that the faculty member selected is in his or her 50s.
  - Determine the probability that the faculty member selected is at least associate in his or her 40s.

Soln (a) $\frac{25}{1164} = 0.217$

(b) $\frac{156+125}{1164} = 0.241$

সাইট এর sheet এর problem

Example-01

১। $A = \{2\}$, $B = \{3\}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{4}{52} - 0$

$= \frac{2}{13}$

২। $C = \{4\}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \frac{4}{52} + \frac{4}{52} + \frac{4}{52}$

$= \frac{3}{13}$

৩। $D = \{club\}$

$P(A \cup D) = P(A) + P(D) - P(A \cap D)$

$= \frac{4}{52} + \frac{13}{12} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$

৪। $A = \{2\}$

$P(A) = \frac{4}{52}$

$P(A^c) = 1 - P(A)$

$= 1 - \frac{4}{52}$

$= \frac{48}{52}$ Ans.

Example-02:

1) $\frac{{}^{12}C_2}{{}^{30}C_2}$

$= 0.152$

2) $P(R_1 \text{ than } Y_2) = P(R_1 \cap Y_2) = P(R_1) * P(Y_2|R_1)$

$= \frac{12}{30} * \frac{10}{29}$

$= 0.138$

3) $P(Y_1 \text{ than } R_2) = P(Y_1 \cap R_2) = P(Y_1) * P(R_2|Y_1)$

$= \frac{10}{30} * \frac{12}{29} = 0.138$

4) $P(R \text{ and } Y) = P(R_1 \text{ than } Y_2)$ or $P(Y_1 \text{ than } R_2)$

$= 0.138 + 0.138$

$= 0.276$

Example-03

	fac 1	fac 2	Total
defect	0.15	0.05	0.20
good	0.45	0.35	0.80
Total	0.60	0.40	1.00

1) 0.20

2) 0.60

3) 0.80

4) 0.40

b) $P(\text{Fac1 than Def}) = P(\text{Fac 1} \cap \text{Def})$

$$= P(\overset{\text{fac1}}{\text{Def}}) \cdot P(\text{Def} \mid \text{fac 1})$$

$$= \cancel{0.20}$$

$$= 0.60 \times \frac{15}{60}$$

$$= 0.15 \quad \text{Ans.}$$

c) ~~P(Def than Fac1~~

$$P(\text{Def} \mid \text{fac 1}) = \frac{P(\text{Def} \cap \text{fac1})}{P(\text{fac1})}$$

$$= \frac{0.15}{0.60}$$

$$= 0.25 \quad \text{Ans.}$$

d) $$P(\text{fac1} \mid \text{Def}) = \frac{P(\text{fac1} \cap \text{Def})}{P(\text{Def})}$$

$$= \frac{0.15}{0.20}$$

$$= 0.75 \quad \text{Ans.}$$

e)

Ex-04: with replacement (combination এর rule apply করা যাবে না)

b) P(1st draw purple) * P(2nd draw purple) * P(3rd draw red)

$$\frac{6}{10} * \frac{6}{10} * \frac{4}{10}$$

without replacement

b) P(1st draw purple) * P(2nd draw purple) * P(3rd draw red)

$$\frac{6}{10} * \frac{5}{9} \times \frac{4}{8}$$

Or,

$$\frac{{}^6C_2 * {}^4C_1}{{}^{10}C_3} * \frac{1}{\frac{3!}{2!}}$$ [with flowing order or succession]

when flowing order or succession missing then Answer

$$\frac{{}^6C_2 * {}^4C_1}{{}^{10}C_3}$$

c) P(4 red than 6 purple) = $\frac{{}^4C_4 \times {}^6C_6}{{}^{10}C_{10}} \times \frac{1}{\frac{10!}{4!6!}}$ [with succession

$= 0.0048$ Ans.

Ex-05

$P(w,w)$ or $P(b,b)$

$$\frac{^6C_2}{^{10}C_2} + \frac{^4C_2}{^{10}C_2} = \boxed{?}$$

Ex-06

$$P(w \text{ than } B) = \frac{12}{30} * \frac{18}{29} = 0.248$$

$$\text{or, } \frac{^{12}C_1 * {}^{18}C_1}{^{30}C_2} * \frac{1}{2!}$$

$$= 0.248$$

Ex-07

$$\frac{^{10}C_1 * {}^{15}C_2}{^{25}C_3}$$

Ex-08

(i) $\frac{^6C_2}{^{15}C_2}$ Ans

(ii) $\frac{^5C_1 * {}^4C_1}{^{15}C_2}$ Ans.

Ex-09

(i) $\frac{^{13}C_4}{^{52}C_4}$

(ii) $\frac{^{13}C_2 * {}^{13}C_2}{^{15}C_4}$

(iii) $\frac{^{26}C_4}{^{52}C_4}$

Ex-10

(a) $\frac{8C_3}{20C_3}$

(b) $\frac{3C_3}{20C_3}$

(c) $\frac{8C_2 \times 9C_1}{20C_3}$

(d) P(1W, 2other) or P(2W, 1other) or P(3W)

$\Rightarrow \frac{3C_1 \times 17C_2}{20C_3} + \frac{3C_2 \times 17C_1}{20C_3} + \frac{3C_3}{20C_3}$

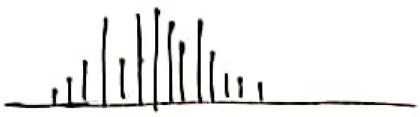
(e)

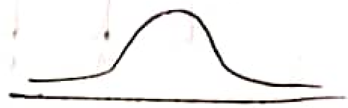
(f) $\frac{8C_1 \times 3C_1 \times 9C_1}{20C_3} \times \frac{1}{3!}$ Ans.

Chapter-05 (Random variable)/Probability Distribution

Random Variables

- Discrete Random Variable/Discrete probability distribution
  Or Probability mass function (PMF)
- Continuous Random Variable / Continuous Probability distribution.
  Or, Probability ~~desni~~ density function (Pdf)





In general $f(x)$ is a probability (density/mass) function if

$$\left\{\begin{array}{l} 1.\ f(x) \geq 0 \ ;\ f(x) \leq 1 \\ 2.\ \sum_{i=1}^{n} f(x_i) = 1 \end{array}\right.$$

This is for descrete random variable

and

$$\left\{\begin{array}{l} 1.\ f(x) \geq 0 \ ;\ f(x) \leq 1 \\ 2.\ \int_{-\infty}^{\infty} f(x)\, dx = 1 \end{array}\right.$$

This is for continuous random variable.

Example:

Expectation $E(x) = \mu$

$\boxed{\text{Mean: } \mu = \sum x . f(x)}$ for descrete $\boxed{n = \sum f(x) = 1}$

Probability mean

$\boxed{\text{Variance: } \sigma^2 = \sum (f(x) . x^2) - \mu^2}$

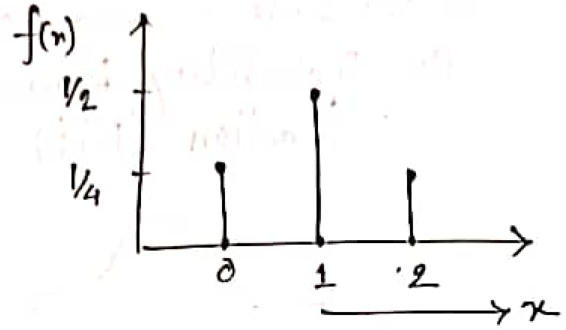
μ / $\boxed{E(x) = \int_{-\infty}^{\infty} x f(x)\, dx}$ for continuous

Probability distribution.

let x denotes head. In throwing two coins construct a probability table and graph distribution

{TT, HT, TH, HH}

x	0	1	2
$f(x)$	1/4	2/4	1/4

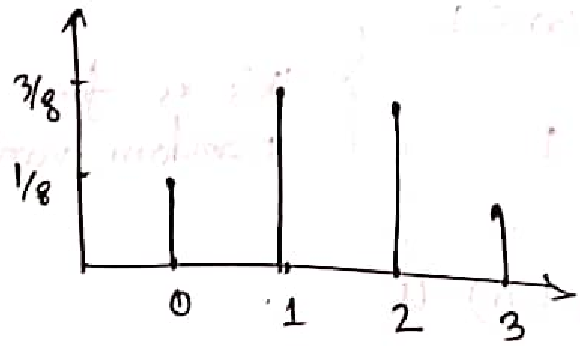


Graph distribution.

let x denotes the head. In throwing three coins construct a probability table and graph distribution

{HHH, HHT, HTT, HTH, THT, THH, TTH, TTT}

X	0	1	2	3
$f(x)$	1/8	3/8	3/8	1/8



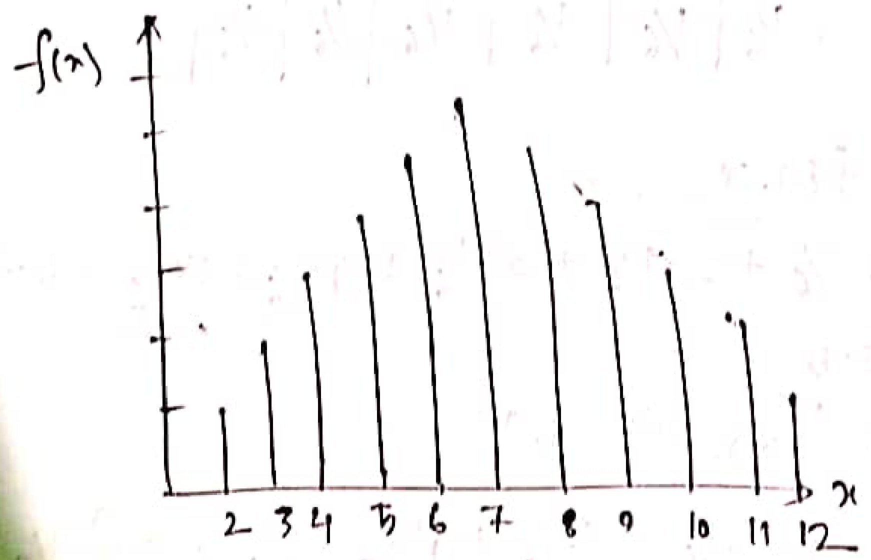
// Let x denotes the random variable & x represent the sum of numbers outcomes in throwing a pair of fair dice construct probability table and graph distribution.

Sample space

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Probability table

x	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$



Check whether the function given by

$$f(x) = \frac{x+2}{25} \quad \text{for } x = 1,2,3,4,5$$

can serve as the probability distribution of a discrete random variable.

Solution:

$$\sum_{n=1}^{5} f(n) = \sum_{1}^{5} \frac{x+2}{25}$$

$$\Rightarrow f(1)+f(2)+f(3)+f(4)+f(5) = \frac{3}{25} + \frac{4}{25} + \frac{5}{25} + \frac{6}{25} + \frac{7}{25}$$

$$= \frac{25}{25}$$

$= 1$ ∴ So the given function is a probability distribution of a discrete random variable.

Find the mean of the number of spots that appear when a die is tossed.

outcome X	1	2	3	4	5	6
$f(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

$$\mu = \sum f(x) \cdot X$$

$$= 1 \cdot 1/6 + 2 \times 1/6 + 3 \times 1/6 + 4 \times 1/6 + 6 \times 1/6 + 5 \times 1/6$$

$$= 3.5$$

$$\sigma^2 = \sum \left(f(x) \cdot x^2\right) - \bar{x}^2$$

$$= 1^2 \cdot 1/6 + 2^2 \cdot 1/6 + 3^2 \cdot 1/6 + 4^2 \cdot 1/6 + 6^2 \cdot 1/6 + 5^2 \cdot 1/6 - 3.5^2$$

$$= 2.9$$

∴ $\sigma = 1.7$ Ans.

One thousand tickets are sold at \$1 each for four prizes of \$100, \$50, \$25 and \$10. After each prize drawing the winning ticket is then returned to the pool of tickets. What is expected value if the you purchase two tickets?

Soln

Gain x	\$100	\$50	\$25	\$10	\$0
Probability $f(x)$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{992}{1000}$

$$E(x) = \$100 \times \frac{2}{1000} + \$50 \times \frac{2}{1000} + \$25 \times \frac{2}{1000} + \$10 \times \frac{2}{1000} + \$0 \cdot \frac{992}{1000} - \$2$$

$$= -\$\cdot 1.63$$

Let x be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} c(2x^3+5) , & -1<x<1 \\ 0 , & \text{otherwise} \end{cases}$$

1) Evaluate c
2) Find $P(0 \leq x \leq 1)$

1) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_{-\infty}^{-1} \cancelto{0}{f(x)\,dx} + \int_{-1}^{1} f(x)\,dx + \int_{1}^{\infty} \cancelto{0}{f(x)\,dx} = 1$

$\Rightarrow \int_{-1}^{1} c(2x^3+5) = 1$

$\Rightarrow c\left(\frac{2x^4}{4} + 5x\right)\Big|_{-1}^{1} = 1$

$\Rightarrow c\left(\frac{11}{2} + \frac{9}{2}\right) = 1$

$\Rightarrow c = \frac{1}{10}$

b)

$$p(0 \le x \le 1) = \int_0^1 \frac{1}{10}(2x^3 + 5)$$

$$= \frac{1}{10}\left[\frac{2x^4}{4} + 5x\right]_0^1$$

$$= 0.55 \text{ Ans}$$

A random variable x has the density function $f(x) = \frac{c}{x^2+1}$ where $-\infty < x < \infty$

1. Find the value of const c
2. Find the probability that x^2 lies between $\frac{1}{3}$ & 1

Soln

1. $$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{\infty}^{\infty} \frac{c}{x^2+1} = 1$$

$$c\left[\tan^{-1}x\right]_{-\infty}^{\infty} = 1$$

$$c\left[\tan^{-1}\infty - \tan^{-1}(-\infty)\right] = 1$$

$$c\left[\pi/2 - (-\pi/2)\right] = 1$$

$$c.\pi. = 1$$

$$c = 1/\pi$$

2. The range of x^2 $\quad 1/3 \le x^2 \le 1$

$$\pm \frac{1}{\sqrt{3}} < x < \pm 1$$

Now $P(1/\sqrt{3} \le x \le 1) = \int_{\frac{1}{\sqrt{3}}}^{1} f(x)\,dx$

$$= \int_{+1/\sqrt{3}}^{1} \frac{1/\pi}{1+x^2}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}x\right]_{1/\sqrt{3}}^{1}$$

$$= \frac{1}{\pi}\left[\pi/4 - \pi/6\right)$$

$$= \frac{1}{12}$$

and $P(-\frac{1}{\sqrt{3}} \le x \le 1) = \int_{-1}^{-1/\sqrt{3}} f(x)\,dx$

$$= \int_{-1}^{-1/\sqrt{3}} \frac{\frac{1}{\pi}}{1+x^2}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}x\right]_{-1}^{-1/\sqrt{3}}$$

$$= \frac{1}{\pi}\left[-\pi/6 - (-\pi/4)\right]$$

$$= \frac{1}{\pi}\cdot \pi/12$$

$$= \frac{1}{12}$$

$\therefore P(1/3 \le x^2 \le 1) = 1/12 + 1/12$

$= 1/6$ Ans.

The rth moment about the mean = μ_r

The rth moment about an arbitrary value $a = \mu_r'$

We know

$\mu_1' = \text{mean} - a$

$$\boxed{\text{mean} = \mu_1' + a}$$

The rth moment of x about the origin $(a=0)$ is defined as

$$\boxed{\mu_r' = E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\, dx}$$ as $r = 1, 2, \ldots$

$$\boxed{\text{mean} = \mu_1' = E(x)}$$

Variance $\mu_2 = \mu_2' - \mu_1'^2$

$$\therefore \boxed{\text{Variance} = E(x^2) - (E(x))^2}$$

Find the 1st four moments (a) about the origin
(b) about the mean
for a random variable x having density function

$$f(x) = \begin{cases} \frac{4x(9-x^2)}{81} & ; \quad 0 \le x \le 3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Solution:

(a) About the origin

$$\mu_1' = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \frac{4x(9-x^2)}{81}\, dx$$

$$= 0 + \int_{0}^{3} x \frac{4x(9-x^2)}{81}\, dx + 0$$

$$= 1.6 \quad [\text{By calculator}]$$

$$\mu_2' = E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^2 \frac{4x(9-x^2)}{81}\, dx$$

$$= \int_{0}^{3} x^2 \frac{4x(9-x^2)}{81}\, dx$$

$$= 3$$

$$\mu_3' = E(x^3) = \int_{-\infty}^{\infty} x^3 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x^3 \frac{4x(9-x^2)}{81}\, dx$$

$$= 6.17$$

$$\mu_4' = E(x^4) = \int_{-\infty}^{\infty} x^4 f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^4 \frac{4x(9-x^2)}{81}\,dx$$

$$= \frac{27}{2}$$

$$= 13.5$$

b. About the mean

1st moment $\mu_1 = 0$

2nd moment $\mu_2 = \mu_2' - \mu_1'^2$

$= 3 - 1.6^2$

$= 3 - 2.56$

$= 0.44$

3rd moment $\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= \boxed{?} \quad -0.038$

4th moment $\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= \boxed{?} \quad 0.4312$

skewness $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3} = \dfrac{(-0.038)^2}{(0.44)^3} = 0.01695$

Kurtosis $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

$= \dfrac{0.4312}{0.44^2}$

$= 2.23$

Probability Distributions

- Discrete Probability Distributions
  - Binomial
  - Poisson
- Continuous Probability Distribution
  - normal .

Chapter-06: Probability Distribution Function

"Binomial"

- There are n trails
- Each trail can result in a success or a failure.
- All the trails of the experiment are independent.

If P be the probability of fabourable events
q be the probability of failure events.
We evalute the number of r success in the n trails by the number of combinition.

$$\boxed{f(x) = P(x=r) = {}^{n}C_{r}\, p^{r}.\, q^{n-r}}$$

Mean of Binomial Distribution:

$$\text{Mean} = \sum_{r=0}^{n} r\,P(r)$$

$$= \sum_{r=0}^{n} r.\, {}^{n}C_{r}.\, p^{r}.\, q^{n-r}$$

$$= {}^{n}C_{1}\, q^{n-1} p^{1} + 2\, {}^{n}C_{2}\, q^{n-2}. p^{r} + 3\, {}^{n}C_{3}\, q^{n-3} p^{3} + \cdots\cdots n\, {}^{n}C_{n}\, q^{0} p^{n}$$

$$= n q^{n-1} p + \frac{2n(n-1)}{2!} q^{n-2} p^{r} + \frac{3(n)(n-1)(n-2)}{3!} q^{n-3} p^{3} + \cdots\cdots n p^{n}$$

$$= np\left[q^{n-1} + (n-1) q^{n-2} p + \frac{(n-1)(n-2)}{2!} q^{n-3} p^{r} + \cdots\cdots p^{n-1}\right]$$

$$= np\,(q+p)^{n-1}$$

$$q+P = 1$$

$$= np$$

$$\therefore \boxed{\text{Mean} = np}$$

Variance of Binomial Distribution:

$$\text{Variance} = \sum_{r=0}^{n} r^2 P(r) - \text{mean}^2$$

Now,
$$\sum_{r=0}^{n} r^2 P(r)$$
$$= \sum_{r=0}^{n} r^2 \, {}^nC_r \, q^{n-r} p^r$$
$$= {}^nC_1 q^{n-1} p^1 + 2^2 \, {}^nC_2 q^{n-2} p^2 + 3^2 \, {}^nC_3 q^{n-3} p^3 + \dots n^2 \, {}^nC_n p^n$$
$$= nq^{n-1}p + 4\frac{(n)(n-1)}{2!} q^{n-2} \cdot p^2 + \frac{9n(n-1)(n-2)}{3!} q^{n-3} p^3 + \dots n^2 p^n$$
$$= np\left[q^{n-1} + 2(n-1)q^{n-2} \cdot p + \frac{3(n-1)(n-2)}{2!} q^{n-3} p^2 + \dots np^{n-1}\right]$$
$$= np\left[q^{n-1} + (n-1)q^{n-2} \cdot p + \frac{(n-1)(n-2)}{2!} q^{n-3} p^2 + \dots + p^{n-1}\right] + np\left[(n-1)q^{n-2} p + (n-1)(n-2)q^{n-3} p^2 + \dots (n-1)p^{n-1}\right]$$
$$= np\left[(q+p)^{n-1} + (n-1)(q+p)^{n-2} \cdot p\right]$$
$$= np\left[1 + (n-1)p\right]$$

$$\text{Variance} = np\left[1 + (n-1)p\right] - n^2p^2$$
$$= np\left[1 + np - p\right] - n^2p^2$$
$$= np\left[q + np\right] - n^2p^2$$
$$= npq + n^2p^2 - n^2p^2$$

$$\boxed{\text{Variance} = npq}$$

Sir এর sheet এর math

Math-01 (i) No answer correct.

(i) $^{10}C_{10}\left(\frac{4}{5}\right)^{10}\cdot\left(\frac{1}{5}\right)^{0} = 0.1074$ Ans.

1st 2nd 10th

$\frac{1}{5}$ $\frac{4}{5}$

c 1

(ii) $^{10}C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8 = 0.3020$ Ans

(iii) $P(x\leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$$= {}^{10}C_0\cdot\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10} + {}^{10}C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^9 + {}^{10}C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8 + {}^{10}C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^7 + {}^{10}C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$$

$= 0.9672$ Ans.

Math-02

$p = \frac{1}{20}$; $q = \frac{19}{20}$

i.e. $P(x \geq 1) > \frac{1}{2}$

$\therefore 1 - P(x=0) > \frac{1}{2}$

$$1 - {}^{n}C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^n > \frac{1}{2}$$

$${}^{n}C_0\left(\frac{1}{20}\right)^0\left(\frac{19}{20}\right)^n < \frac{1}{2}$$

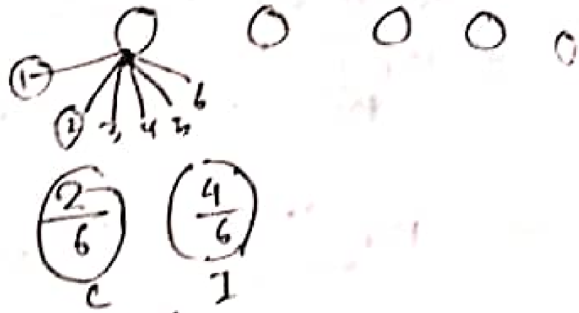
$$\left(\frac{19}{20}\right)^n < \frac{1}{2}$$

$$n\log_{10}\frac{19}{20} < \log_{10}\frac{1}{2}$$

$$n > \frac{\log_{10}\frac{1}{2}}{\log_{10}\frac{19}{20}}$$

$n = 13.5099$ $\therefore n = 14$ Ans

Example-03



$$P(x \geq 3) = 729 \left[{}^6C_3 \left(\frac{2}{6}\right)^3 \left(\frac{4}{6}\right)^3 + {}^6C_4 \left(\frac{2}{6}\right)^4 \left(\frac{4}{6}\right)^2 + {}^6C_5 \left(\frac{2}{6}\right)^5 \left(\frac{4}{6}\right)^1 + {}^6C_6 \left(\frac{2}{6}\right)^6 \right]$$

$$= \frac{729}{3^6} \left[160 + 60 + 12 + 1\right] = 233 \quad \text{Ans.}$$

Ex-04

$$P(7) + P(8) + P(9) + P(10)$$

$$= {}^{10}C_7 (0.65)^7 (0.35)^3 + {}^{10}C_8 (0.65)^8 (0.35)^2 + {}^{10}C_9 (0.65)^9 (0.35)^1 + {}^{10}C_{10} (0.65)^{10} (0.35)^0$$

$$= 0.5067 \quad \text{Ans.}$$

Ex-05

$np = 4$ —— (i)

$npq = 2$ —— (ii)

(i) ÷ (ii)

$q = \frac{1}{2}$

$\therefore p = \frac{1}{2}$

$\therefore n = 8$

(i) ${}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6$

(ii) $P(0) + P(1) + \cancel{P(2)}$

${}^8C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^8 + {}^8C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^7 + \cancel{{}^8C_2 \left(\frac{1}{2}\right)^2}$

(iii) $1 - P(0) - P(1)$

$= 1 - \{P(0) + P(1)\}$

$= 1 - \text{(ii)}$

$= \text{Ans.}$

Chapter-07: Distribution Function

"Poisson"

Poisson distribution with $P(r) = \frac{e^{-\lambda}\lambda^r}{r!}$ is a limiting case of Binomial distribution, under the condition:

i. $n \longrightarrow \infty$

ii $p \longrightarrow 0$

iii $np \longrightarrow \lambda$ is finit

<u>Proof</u>

$$P(r) = {}^nC_r\, q^{n-r} p^r$$

$$= {}^nC_r (1-p)^{n-r} \cdot p^r$$

$$= {}^nC_r \left(1-\frac{\lambda}{n}\right)^{n-r} \left(\frac{\lambda}{n}\right)^r$$

$$= \frac{n(n-1)(n-2)\ldots\ldots(n-(r-1))}{r!} \left(1-\frac{\lambda}{n}\right)^{n-r} \left(\frac{\lambda}{n}\right)^r$$

$$= \frac{1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\left(1-\frac{r-1}{n}\right)}{r!\left(1-\frac{\lambda}{n}\right)^r} \left(1-\frac{\lambda}{n}\right)^n \cdot \lambda^r$$

limit $n \longrightarrow \infty$

$$P(r) = \frac{\lambda^r}{r!} \operatorname{limit}_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^n$$

$$= +\frac{\lambda^r}{r!} \lim_{n\to\infty} \left[\left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right]^{-\lambda}$$

$$= \frac{+\lambda^r e^{-\lambda}}{r!}$$

$$\therefore \boxed{P(r) = \frac{e^{-\lambda}\lambda^r}{r!}}$$

$\boxed{\text{mean} = \lambda}$

$\boxed{\text{Variance} = \lambda}$

Sir এর math

Example-01

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \frac{e^{-1.5} \cdot 1.5^{0}}{0!}$$

$$= 0.2231$$

Example-02

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \frac{e^{-6} \cdot 6^{0}}{0!}$$

$$= 0.002479$$

$$P(x \leq 5) = P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$$

$$= \frac{e^{-6} \cdot 6^{0}}{0!} + \frac{e^{-6} \cdot 6^{1}}{1!} + \frac{e^{-6} \cdot 6^{2}}{2!} + \frac{e^{-6} \cdot 6^{3}}{3!}$$

$$+ \frac{e^{-6} \cdot 6^{4}}{4!} + \frac{e^{-6} \cdot 6^{5}}{5!}$$

$$= 0.4457$$ An.

Ex-03:

$P = 0.001$

$n = 2000$

$\lambda = Pn$

$= 0.001 \times 2000$

$= 2$

(a) $P(3) = \dfrac{e^{-2} \cdot 2^3}{3!}$

$= 0.18$

(b) $P(\text{more than two}) = 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left[\dfrac{e^{-2} \cdot 2^0}{0!} + \dfrac{e^{-2} 2^1}{1!} + \dfrac{e^{-2} \cdot 2^2}{2!}\right]$

$= 1 - e^{-2}[1 + 2 + 2]$

$= 0.325$ Ans.

(c) $P(0) = \dfrac{e^{-2} \cdot 2^0}{0!} = 0.135$

(d) $P(\text{more than } 1) = 1 - [P(0) + P(1)]$

$= 1 - \left[\dfrac{e^{-2} \cdot 2^0}{0!} + \dfrac{e^{-2} \cdot 2^1}{1!}\right]$

$= 0.595$ Ans.

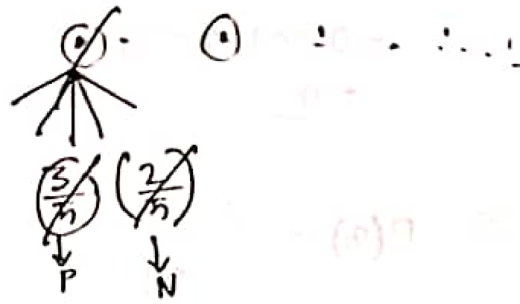
Problem 7.40 : Out of 800 families with 5 children each, how many would be expect to have (a) 3 boys (b) 5 girls and (c) either 2 or 3 boys? Assume equal probabilities for boys and girls

(a) $p = \frac{1}{2}$

$q = \frac{1}{2}$

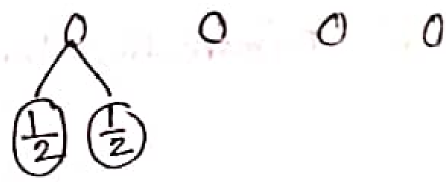
$n = 5$

$r = 3$



$$P(r=3) = {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3} \times 800$$

$$= 250$$



(b) $$P(r=5) = {}^5C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 \times 800$$

$$= 25$$

(c) $P(r=2)$ or $P(r=3)$

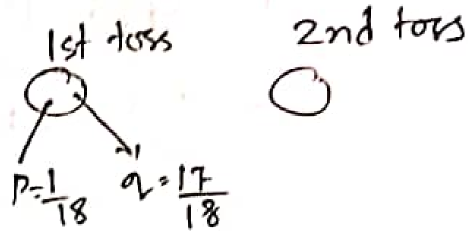
$$\left\{ {}^5C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 + {}^5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 \right\} \times 800$$

$$= 500$$

7.41 Find the probability of getting a total of 11 (a) once and (b) twice in two tosses of a pair of fair dice.

$P = \frac{2}{36} = \frac{1}{18}$

$q = \frac{17}{18}$



a) $n = 2$

$r = 1$

$^2C_1 \left(\frac{1}{18}\right)^1 \left(\frac{17}{18}\right)^1$

$= \frac{17}{162}$

b) $n = 2$

$r = 2$

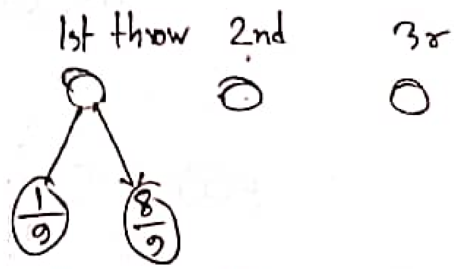
$^2C_2 \times \left(\frac{1}{18}\right)^2 \left(\frac{17}{18}\right)^0$

$= \frac{1}{324}$

7.42: What is the probability of getting a 9 exactly once in 3 throws with a pair of dice.

$P = \frac{4}{36} = \frac{1}{9}$

$q = \frac{8}{9}$



$^3C_1 \left(\frac{1}{9}\right)^1 \left(\frac{8}{9}\right)^2$

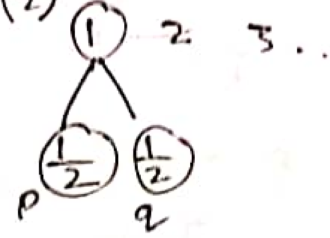
$= \frac{64}{243}$ Ans.

7.43 : Find the probability of guessing correctly at least 6 of the 10 answers on a true false examination

$P(x \geq 6) \; P(6) + P(7) + P(8) + P(9) + P(10)$

$= {}^{10}C_6 \left(\frac{1}{2}\right)^6 \cdot \left(\frac{1}{2}\right)^4 + {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2$

$+ {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0$



$= \frac{193}{512}$ Ans

7.67 :

7.69 :

7.70 :

7.71 :

[illegible]

7.70

$n = 20,00000$

$P = \frac{3}{1000000}$ or, $\cancel{A}$ .. $\frac{6}{2000000}$

$\lambda = np$

$= 2000000 \times \frac{3}{100,0000}$

$= 6$

$$P(r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

$$P(0) = \frac{e^{-6} \cdot 6^0}{0!} = 0.00248$$

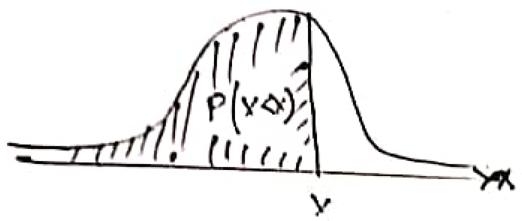
Chapter-08: "Normal" Distribution Function.

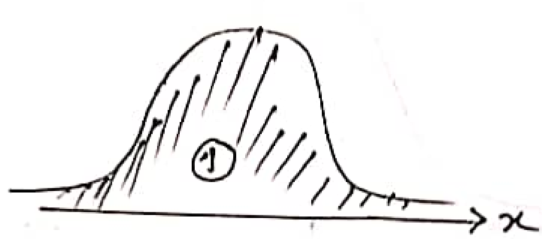
Normal distribution:

A continuous probability distribution for a given random variable x, that is completely defined by its mean and variance.

Properties of a normal Distributions -

1. A normal curve is symmetric
2. A normal curve is completely defined by its mean, μ, and variance σ^2
3. The total area under a curve equals 1





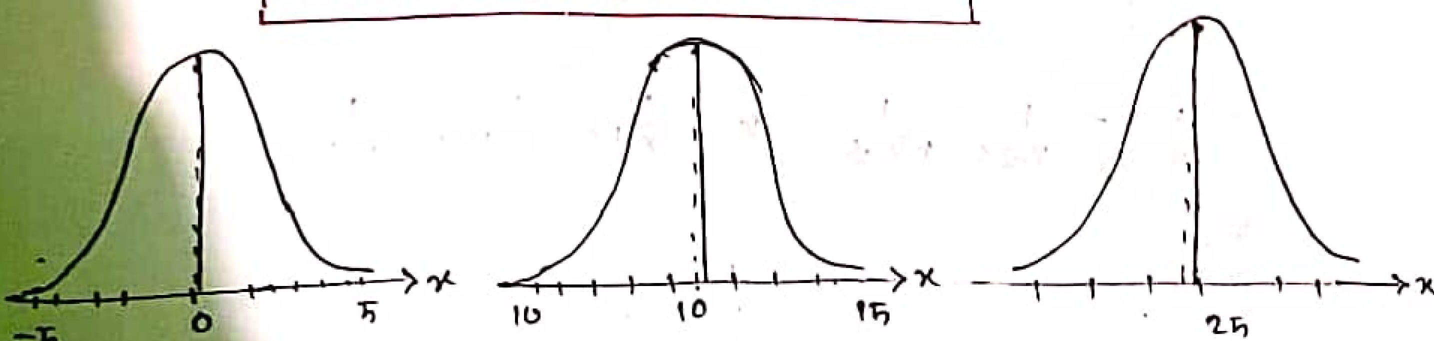
The normal distribution is given by the equation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}$$

→ mean

→ standard deviation.



$x = 0$
$\therefore \mu = 0$

$x = 10$
$\mu = 0$

$x = 25$
$\mu = 0$

So, we can say,

$$\therefore \frac{x-\mu}{\sigma} = Z = 0$$

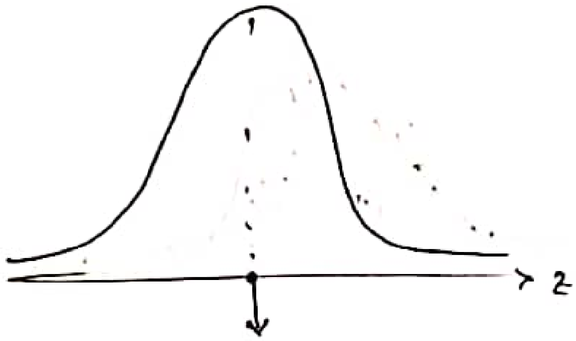
standard normal distribution

let $\frac{x-\mu}{\sigma} = z$

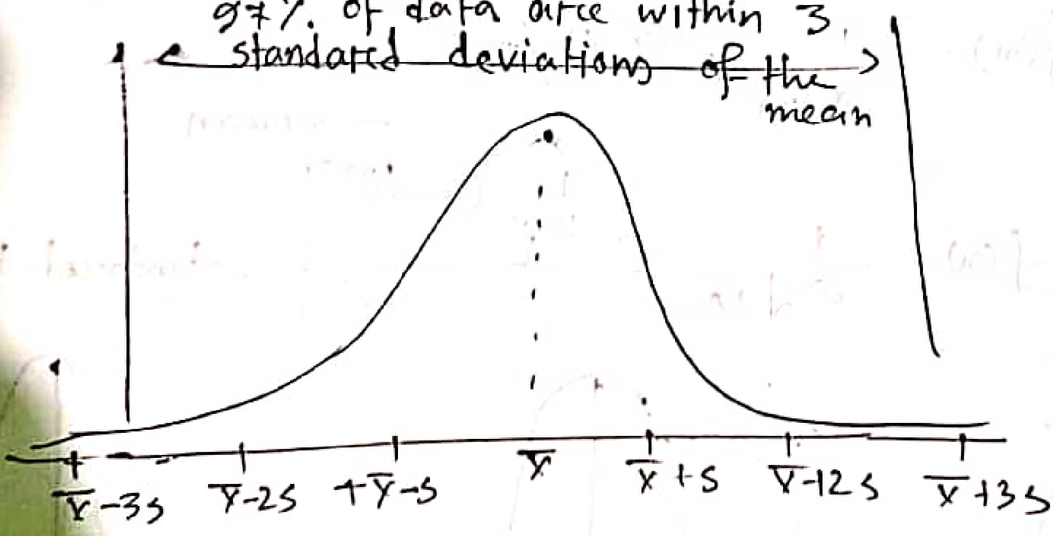
$dx = \sigma dz$

$$\therefore p(-\infty \le z \le \infty) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} \sigma . dz$$

$$\boxed{\therefore p(-\infty \le z \le \infty) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} dz}$$



97% of data are within 3 standard deviations of the mean



" Find the area between z_1 and z_2

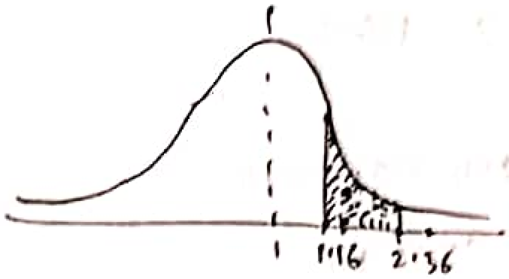
a. $z = 1.16$, $z_2 = 2.31$

<u>Soln</u>

~~= 0.3770~~

$= 0.4896 - 0.3770$

$= 0.1126$



<u>**Example**</u>: Calculate the probability that a normal random variable with a mean of 10 and standard deviation 20 will lie between 10 and 40.

<u>Soln</u>

$\bar{x} = 10$

$\sigma = 20$

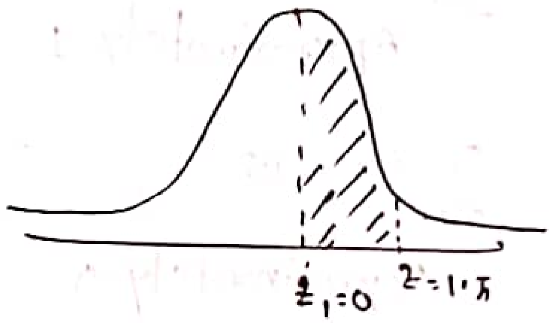
$x = 10$ and 40

$z_1 = 0$

$$z_2 = \frac{x - \bar{x}}{\sigma}$$

$$= \frac{40 - 10}{20}$$

$= 1.5$



$P(10 \leq x \leq 40) = 0.4332$ <u>Ans</u>.

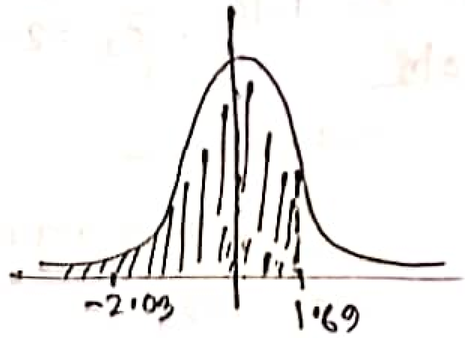
3. Find the area of the left of z

a. $z = 1.69$

Soln

$0.5 + 0.4545$

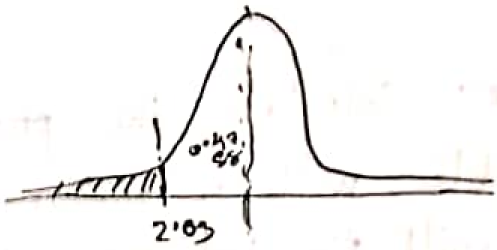
$= 0.9545$



b. $z = -2.03$

Soln $= 0.5 - 0.4788$

$= 0.0212$



c. $z = 0$

Soln

0.5000

d. $z = 4.2$

Soln

Approximately 1

e. $z = -4.2$

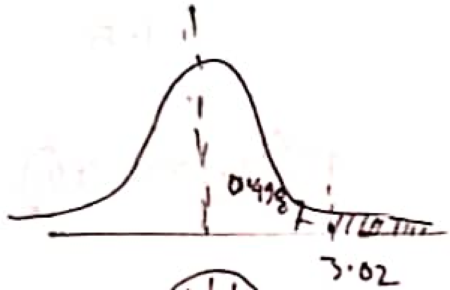
Soln

Approximately 0

4 Find the area of the right of z

a. $z = 3.02$

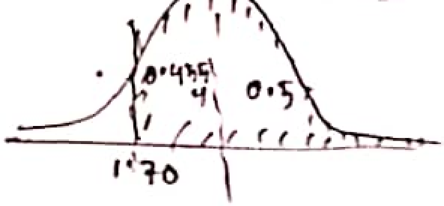
Soln $= 0.5 - 0.4987$



b $z = -1.70$

Soln P $= 0.4554 + 0.5$

$= 0.9554$



Example

Calculate the probability that a normal random variable with a mean of 10 and a standard deviation of 20 will be greater than 30

Soln

$\bar{x} = 10$

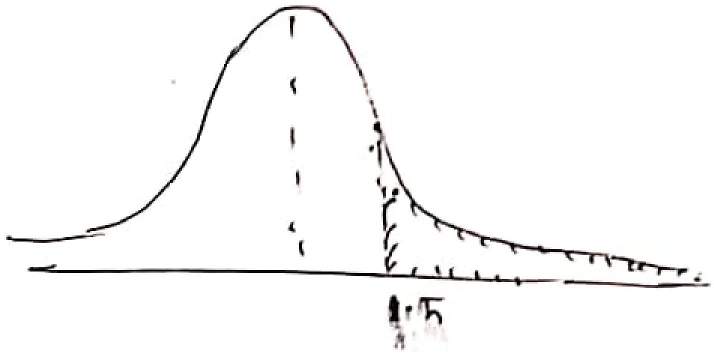
$\sigma = 20$

$x = 30$

$z = \frac{x - \bar{x}}{\sigma}$

$= \frac{30 - 10}{20}$

$= 1.5$



$P(x > 1.5) = 0.5 - 0.4332$