

# Probability

Mithun sin 20 Pdf  
Shamst n n n

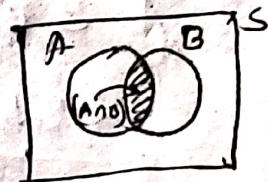
#  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

স্বতন্ত্র বস্তুসমূহের ক্ষেত্রে,  $\Rightarrow P(A \cup B) = P(A) + P(B)$

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} = P(A) + P(B)$$

→ অস্বতন্ত্র বস্তুসমূহের ক্ষেত্রে,  $\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = \frac{n(A)}{n(S)}$  ;  $P(B) = \frac{n(B)}{n(S)}$



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B) - n(A \cap B)}{n(S)}$

#  $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{n(A \cap B)/n(S)}{n(B)/n(S)}$

$= \frac{n(A \cap B)}{n(B)}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$

$= P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = \frac{n(A \cap B)}{n(S) \cdot n(S)} \cdot \frac{n(S) \cdot n(S)}{n(S) \cdot n(S)}$

$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)}$

$= P(A) \cdot P(B)$

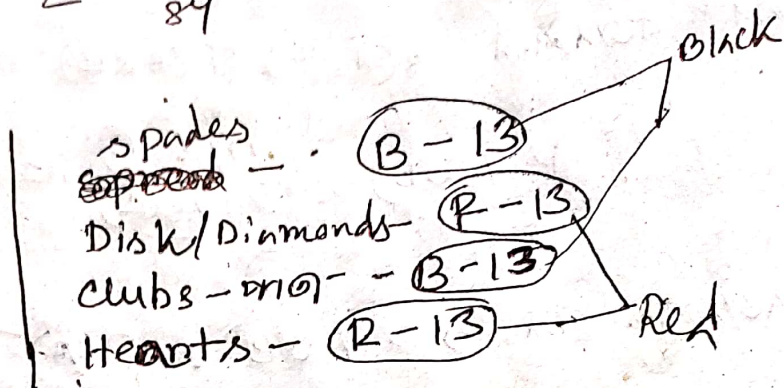
$$\begin{cases} W=4 \\ B=5 \end{cases}$$

$$W+B = 4+5 = 9$$

→ 301 - 220 (n) 23, 20 possibility

$$\frac{5C_3}{9C_3} = \frac{10}{84}$$

\* 52  
26  
26



\* A & B are independent events  $\frac{1}{3}$  &  $\frac{1}{4}$

Joint Probability?

$$P(A \cap B) = P(A) \times P(B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{1}{2}$$

\* Students = 40

$$F = 20$$

$$P \& C = 10$$

$$C = 25$$

$$P(F) = \frac{20}{40} = \frac{1}{2}; P(C) = \frac{25}{40} = \frac{5}{8}$$

$$P(F \cap C) = \frac{10}{40} = \frac{1}{4}$$

$$P(C/F) = \frac{P(C \cap F)}{P(F)}$$

$$= \frac{1/4}{1/2} = \frac{1}{2}$$

Probability of event (A, B) given event (C)?

\*  $\begin{matrix} |W-3| \\ |B-2| \\ T=5 \end{matrix} \quad \begin{matrix} |W-2| \\ |B-5| \\ T=7 \end{matrix} \rightarrow$  କାଳୀ 23ମା ମଧ୍ୟାହନ  $\frac{5}{7}$

→ ହୁଅନ୍ତି ସମ୍ପୂର୍ଣ୍ଣ ସମ୍ପୂର୍ଣ୍ଣ କଣ୍ଠ ଏକ ଦୁଇମା <sup>ସମ୍ପୂର୍ଣ୍ଣ</sup> ଅନୁକ୍ରମ <sup>କାଳୀ</sup> 23ମା ମଧ୍ୟାହନ ?

→ କାଳୀ-23ମା ମଧ୍ୟାହନ =  $\frac{2}{5}$

∴ ଉପ-ସମ୍ପୂର୍ଣ୍ଣ ସମ୍ପୂର୍ଣ୍ଣ କାଳୀ ଏକ 23ମା ମଧ୍ୟାହନ =  $\frac{5}{7} \times \frac{2}{5}$   
 $= \frac{2}{7}$

∴ ଅନ୍ତର ସମ୍ପୂର୍ଣ୍ଣ କାଳୀ 23ମା ମଧ୍ୟାହନ =  $1 - \frac{2}{7}$   
 $= \frac{5}{7}$

\*  $\begin{matrix} |W-4| \\ |R-3| \\ T=7 \end{matrix} \quad \begin{matrix} |W-3| \\ |R-7| \\ T=10 \end{matrix} \quad \begin{matrix} |W-6| \\ |R-7| \\ |B-9| \\ T=22 \end{matrix}$

\* କାଳୀ ~~କାଳୀ~~ ସମ୍ପୂର୍ଣ୍ଣ କଣ୍ଠ ତିନିଟି ଦୁଇମା କାଳୀ 23ମା ମଧ୍ୟାହନ =  $\frac{3}{7} \times \frac{7}{10} \times \frac{7}{22} = \frac{21}{220}$

\* ଉଦାହରଣ ଉପରେ ତିନିଟି ଏକ ଦୁଇମା (କାଳୀ) 23ମା ମଧ୍ୟାହନ ?  
 → କାଳୀ ଏକ  $6 + 7 + 9 = 22$

$P(RUW) = P(R) + P(W)$   
 $= \frac{7C3}{22C3} + \frac{6C3}{22C3} = \frac{35}{1540} + \frac{20}{1540}$   
 $= \frac{55}{1540}$

(c) एक 3 24 को एक ~~सिक्का~~ एक को एक सिक्का  
एक को एक सिक्का, एक को एक सिक्का,  
 $\Rightarrow$  एक को एक सिक्का को एक सिक्का को एक सिक्का =  $\frac{1}{2}$

$$\therefore \text{एक को एक 24 को एक सिक्का को एक सिक्का} = \frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$$

$$\therefore 24 \text{ को एक सिक्का को एक सिक्का} = \frac{1}{2} \times \frac{3}{10} = \frac{3}{20}$$

$$\therefore \text{एक को एक को एक सिक्का को एक सिक्का} = \frac{2}{7} + \frac{3}{20}$$

A को एक सिक्का को एक सिक्का	= $\frac{61}{140}$
B को एक सिक्का को एक सिक्का	
C को एक सिक्का को एक सिक्का	

A को एक  $P(A) = \frac{3}{5}$ , A को एक  $q(A) = \left(1 - \frac{3}{5}\right) = \frac{2}{5}$

B को एक  $P(B) = \frac{2}{6} = \frac{1}{3}$ , B को एक  $q(B) = 1 - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

C को एक  $P(C) = \frac{3}{7}$ ; C को एक  $q(C) = 1 - \frac{3}{7} = \frac{4}{7}$

\* 2 सिक्का को एक सिक्का (एक को एक सिक्का)

$$\begin{aligned} \Rightarrow P(2 \text{ सिक्का को एक सिक्का}) &= P(A)P(B)q(C) + P(A)q(B)P(C) + q(A)P(B)P(C) \\ &= \frac{3}{5} \times \frac{1}{3} \times \frac{4}{7} + \frac{3}{5} \times \frac{2}{3} \times \frac{3}{7} + \frac{2}{5} \times \frac{1}{3} \times \frac{3}{7} \\ &= \frac{12}{35} \text{ Ans.} \end{aligned}$$

(ii)  $P(\text{At least 2 shoots hit})$ ? (also 12/20/15, 12/20/15)

$$= P(A) \cdot P(B) \cdot P(C) + P(A) \cdot P(B) \cdot \bar{P}(C) + P(A) \cdot \bar{P}(B) \cdot P(C) + \bar{P}(A) \cdot P(B) \cdot P(C)$$

$$= \frac{3}{5} \times \frac{1}{3} \times \frac{3}{7} + \frac{3}{5} \times \frac{1}{3} \times \frac{4}{7} + \frac{3}{5} \times \frac{2}{5} \times \frac{3}{7} + \frac{2}{5} \times \frac{1}{3} \times \frac{3}{7}$$

$$= \frac{3}{7} \quad \underline{\text{Ans}}$$

\* (B)

W-3
B-5

T=8

(C)

W-6
B-8

T=14

~~B~~ A ball is transferred from B box to C.  
Then a ball draw from box C. Find the probability that it is B-white.

Case I: A white ball is transferred from B to C.

$$P(\text{white from B}) = \frac{3}{8}$$

$$P(\text{white in C}) = \frac{3}{8} \times \frac{7}{15} = \frac{7}{40}$$

Case II: A black ball is transferred from B to C.

$$P(\text{Black from B}) = \frac{5}{8}$$

$$P(\text{white ball from box C}) = \frac{5}{8} \times \frac{6}{15} = \frac{1}{4}$$

Probability of white ball from C,

$$= P(\text{case 1}) \text{ or } P(\text{case 2})$$

$$= \frac{7}{40} + \frac{1}{4} = \frac{17}{40}$$

Ans.

$$P(H \cap W) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35}$$

$$P(H \cup W) = \frac{1}{7} + \frac{1}{5} - \frac{1}{35} = \frac{11}{35}$$

Husband & wife  $\frac{1}{7}$  &  $\frac{1}{5}$  respectively are selected

apply  $\rightarrow$  Husband's selection probability =  $\frac{1}{7}$

wife " " " " =  $\frac{1}{5}$

find the probability of one of them being selected.

Sol<sup>n</sup>: Given,

$$P(H) = \frac{1}{7}; \text{ not selected } g(H) = \left(1 - \frac{1}{7}\right) = \frac{6}{7}$$

$$P(W) = \frac{1}{5}; \text{ " " } g(W) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}$$

$P(\text{one of them will be selected})$

$$= P(H) \cdot g(W) \text{ or } g(H) \cdot P(W)$$

$$= \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{6}{7} \times \frac{1}{5}\right)$$

$$= \frac{4}{35} + \frac{6}{35} = \frac{2}{7} \quad \underline{\text{Ans}}$$

# 1st student solved problem,  $P(A) = \frac{1}{2}$ ; Not solved  $g(A) = 1 - \frac{1}{2} = \frac{1}{2}$   
 2nd " " " " " " ,  $P(B) = \frac{1}{3}$ ; " " " "  $g(B) = \frac{2}{3}$   
 3rd " " " " " " ,  $P(C) = \frac{1}{4}$ ; " " " "  $g(C) = 1 - \frac{1}{4} = \frac{3}{4}$

$$P(\text{Problem will be solved}) = 1 - P(\text{fails to solve problem})$$

$$= 1 - g(A) \cdot g(B) \cdot g(C)$$

$$= 1 - \left( \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \right)$$

$$= \frac{3}{4}$$

Ans

# Total cards = 52

Cards drawn = 3

$\therefore$  sample space =  ${}^{52}C_3 = 22100$

Probability of a king =  ${}^4C_1 = 4$

" " " " a queen =  ${}^4C_1 = 4$

" " " " a ten =  ${}^4C_1 = 4$

$\therefore$  favourable event = ~~4~~  $4 \times 4 \times 4 = 64$

$\therefore$  Required probability =  $\frac{\text{favourable event}}{\text{sample space}}$

$$= \frac{64}{22100}$$

# Total cards = 52

drawn = 4

~~Sample~~ Sample space =  $52C_4$

(i) P(All cards are diamond)

$$= \frac{13C_4}{52C_4}$$

$$= \frac{11}{4165}$$

(ii) There is one card in each suit:

Probability of a diamond =  $13C_1$

" " a club =  $13C_1$

" " a Heart =  $13C_1$

" " a Spade =  $13C_1$

Favourable event =  $13C_1 \times 13C_1 \times 13C_1 \times 13C_1$

∴ Required probability =  $\frac{\text{favourable event}}{\text{sample space}}$

$$= \frac{13 \times 13 \times 13 \times 13}{52C_4} = \frac{28561}{270725}$$

#  $\left[ \begin{array}{l} R-5 \\ W-7 \\ B-9 \end{array} \right]$

$$T = 5 + 7 + 9 = 21$$

Drawn,  $D = 3$

(i) P(Three balls are different color) =  $\frac{5C_1 * 7C_1 * 9C_1}{21C_3}$

$$= \frac{9}{38} \text{ Ans}$$

(ii) P(two balls are same color) :

	S R	7W	9B	
$S_1 \rightarrow$	2	1	0	$\rightarrow 5C_2 * 7C_1 * 9C_0 = 70$
$S_2 \rightarrow$	2	0	1	$\rightarrow 5C_2 * 7C_0 * 9C_1 = 90$
$S_3 \rightarrow$	1	2	0	$\rightarrow 5C_1 * 7C_2 * 9C_0 = 105$
$S_4 \rightarrow$	1	0	2	$\rightarrow 5C_1 * 7C_0 * 9C_2 = 180$
$S_5 \rightarrow$	0	2	1	$\rightarrow 5C_0 * 7C_2 * 9C_1 = 189$
$S_6 \rightarrow$	0	1	2	$\rightarrow 5C_0 * 7C_1 * 9C_2 = 252$

Required probability =  $\frac{S_1 + S_2 + S_3 + S_4 + S_5 + S_6}{21C_3} = \frac{886}{1330}$

#  $\frac{443}{665}$

P(solved by at least two) :

$P(x) = \frac{1}{2}$  , not solved  $g(x) = \frac{1}{2}$

$P(y) = \frac{1}{3}$  , " "  $g(y) = \frac{2}{3}$

$P(z) = \frac{1}{4}$  , " "  $g(z) = \frac{3}{4}$

$\therefore P(\text{solved by at least two}) = P(x) \cdot P(y) \cdot g(z) + P(x) \cdot g(y) \cdot P(z) + g(x) \cdot P(y) \cdot P(z) + P(x) \cdot P(y) \cdot P(z)$   
 $= \frac{7}{24}$

#

$$\frac{400}{500} = \frac{1}{224}$$

52 weeks in a year  
 52 weeks in a year possibility

# 53 Sunday possibility

How?

Leap year = 366 days

$$52 \times 7 = 364 \text{ days}$$

$$366 - 364 = 2 \text{ days}$$

↓  
 52 weeks  
 Sunday 2 days  
 2 days = 2 Sundays

2 days = 2 Sundays

~~2 days = 2 Sundays~~ 23:10

- (i) Sun, Mon (ii) Mon, Tues (iii) Tues, Wed
- (iv) Wed, Thurs (v) Thurs, Fri (vi) Fri, Sat (vii) Sat, Sun

$$\therefore \text{Chance} = \frac{2}{7}$$

$$\frac{4 \times 4 \times 4}{500} = \frac{64}{22100} = \frac{16}{5525}$$

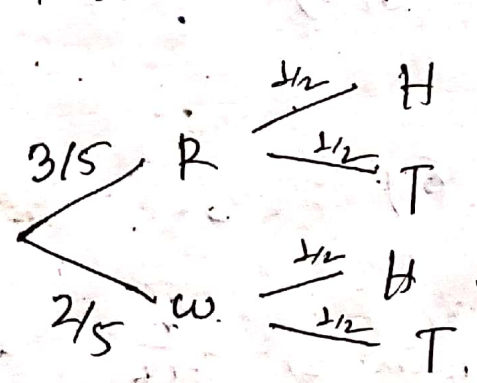


# 16 lettered cards  $\rightarrow$  4 Aces, 12 Face cards

# 36 Numbered  $\rightarrow$   $\boxed{2, 3, 4, 5, 6, 7, 8, 9, 10}$   
 $\rightarrow 9 \times 4 = 36$

#  $\begin{matrix} R=3 \\ W=2 \end{matrix}$  & coin

Probability tree:



$$P(RH) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} = 0.3$$

$$P(RT) = \frac{3}{5} \times \frac{1}{2} = \frac{3}{10} = 0.3$$

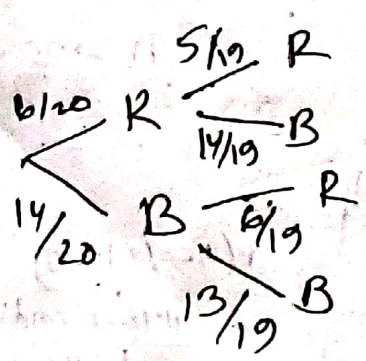
$$P(WH) = \frac{2}{5} \times \frac{1}{2} = \frac{2}{10} = 0.2$$

$$P(WT) = \frac{2}{5} \times \frac{1}{2} = 0.2$$

- $1.7 \times 10^{-3}$
- $17 \times 10^{-2}$
- $0.17 \times 10^{-1}$
- $0.017$
- $0.02$

# Tree Diagrams

Select 2 pen from 20 pens.  
 14 Blue, & 6 Red. Don't Replace.



$$\# P(A|D) = \frac{P(A \cap D)}{P(D)} \quad \therefore P(A \cap B) = P(A \text{ and } D)$$

$$\# P(\text{Ace/Black}) = \frac{P(\text{Ace and Black})}{P(\text{Ace})}$$

$$P(\text{Ace and Black}) = P(\text{Ace}) \cdot P(\text{Black/Ace})$$

# Sin 28 slide (Nazmul shouf)

Ex 30

W-5
R-6
B-4

$$T = 5 + 6 + 4 = 15$$

Drawn = 2

urn → 0/378

$$\textcircled{i} P(\text{both are red}) = \frac{{}^6C_2}{{}^{15}C_2} = \frac{1}{7}$$

$$\textcircled{ii} P(\text{one white and one black ball}) = \frac{{}^5C_1 \cdot {}^4C_1}{{}^{15}C_2} = \frac{4}{21}$$

Ex-5

$$T, \text{ Cards} = 52$$

drawn = 4

$$\textcircled{i} P(\text{All cards are spades}) = \frac{{}^{13}C_4}{{}^{52}C_4}$$

$$\textcircled{ii} P(\text{Two spades and two hearts}) = \frac{{}^{13}C_2 \cdot {}^{13}C_2}{{}^{52}C_4}$$

$$\textcircled{iii} P(\text{All cards are black}) = \frac{{}^{26}C_4}{{}^{52}C_4}$$

Ex: 1, 38

R-8
W-3
B-9

$$T = 8 + 3 + 9 = 20$$

$$a) \text{ all 3 are red} = \frac{{}^8C_3}{{}^{20}C_3}$$

$$b) \text{ " 3 " white} = \frac{{}^3C_3}{{}^{20}C_3}$$

$$\textcircled{c} P(\text{2 are red and 1 is white}) = \frac{{}^8C_2 \cdot {}^3C_1}{{}^{20}C_3}$$

$$\textcircled{d} P(\text{at least 1 is white}) = \frac{{}^3C_1 \cdot {}^3C_2 \cdot {}^3C_3}{{}^{20}C_3}$$

$$\textcircled{e} P(\text{1 of each color drawn}) = \frac{{}^8C_1 \times {}^3C_1 \times {}^9C_1}{{}^{20}C_3}$$

$$\textcircled{f} P(\text{The balls are drawn in the order red, white, blue})$$

$$= \frac{{}^8C_1}{{}^{20}C_1} \times \frac{{}^3C_1}{{}^{19}C_1} \times \frac{{}^9C_1}{{}^{18}C_1}$$

Ex-9 A bag contains 50 tickets from 1 to 50. 5 are drawn and arranged in ascending order ( $t_1 < t_2 < t_3 < t_4 < t_5$ ). Find the probability of  $t_4$  carrying the number 45.

$\Rightarrow t_1, t_2, t_3$  must be drawn out of tickets numbered from 1 to 44.  $\therefore$  favourable numbers  ${}^{44}C_3$ .

$t_4$  draw at 45  $\therefore$  " " "  ${}^1C_1$

$t_5$  " " at 46-50  $\therefore$  " " "  ${}^5C_1$

$$\therefore \text{Required probability} = \frac{{}^{44}C_3 \times {}^1C_1 \times {}^5C_1}{{}^{50}C_5}$$

$$= \underline{0.03}$$

# Find the probability of getting neither heart nor king when a card is drawn from a well shuffled of 52 cards.

$$\Rightarrow \text{Getting a card of heart, } P(A) = \frac{13}{52}$$

$$\text{Getting a card of king, } P(B) = \frac{4}{52}$$

$$A \cap B: \text{Getting a King of heart, } P(A \cap B) = \frac{1}{52}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Again probability of neither heart nor king is given

$$\text{by } P(A^c \cap B^c)$$

$$P(A^c \cap B^c) = P(A \cup B)^c = 1 - P(A \cup B)$$
$$= 1 - \frac{4}{13}$$

$$= \frac{9}{13}$$

Ex-15:  $P(A) = \frac{25}{100}, P(B) = \frac{30}{100}$

$$P(C) = \frac{20}{100}, P(A \cap B) = \frac{10}{100}$$

$$P(A \cap C) = \frac{5}{100}, P(B \cap C) = \frac{8}{100}, P(A \cap B \cap C) = \frac{3}{100}$$

→ [उपरोक्त सभी newspaper खरीदने वाले]

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$= \frac{25}{100} + \frac{30}{100} + \frac{20}{100} - \frac{10}{100} - \frac{8}{100} - \frac{5}{100} + \frac{3}{100}$$

$$= \frac{11}{20}$$

~~$P(A \cup B \cup C)$~~  → +

For a person chosen at random, find the probability that he reads none of the newspaper.

$$P(A \cup B \cup C)^c = 1 - P(A \cup B \cup C) = 1 - \frac{11}{20} = \frac{9}{20}$$

# for mutually exclusive events;

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) \leq 1$$

$$P(A \cap B) = P(A \cup B) = P(A) + P(B) \leq 1$$

# for 3 coins:

example = {HHH, HHT, HTH, HTT, TTT, TTH, THT, THT}

{HHH, HHT, HTH, HTT, TTT, TTH, THT, THT}

# ~~Q10~~ A & B are independent, will be independent if;

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex-17 ✓  
(42/6) 2(0)

# Ex-19 20031 ના 48 બોલોમાં 4 બોલો કાઢવામાં આવ્યા છે.  
drawn 42/6 2(0)

$$P(A) = \frac{{}^{48}C_n}{{}^{52}C_n} = \frac{48!}{n!(48-n)!} \cdot \frac{n!(52-n)!}{52!}$$

$$= \frac{(52-n)(51-n)(50-n)(49-n)}{52 \cdot 51 \cdot 50 \cdot 49}$$

$$\text{Also, } P(B|A) = \frac{{}^4C_1}{{}^{(52-n)}C_1} = \frac{4}{52-n}$$

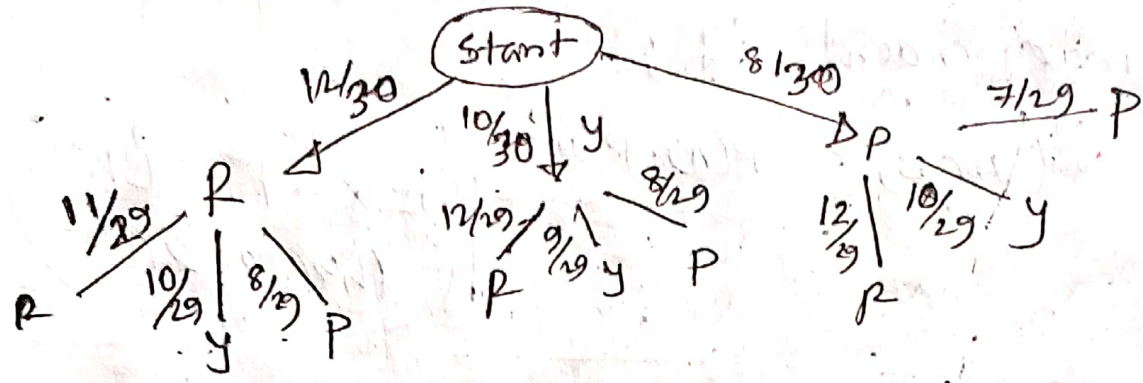
$$\therefore P(A \cap B) = P(A) \cdot P(B|A) = \frac{4(52-n)(50-n)(49-n)}{52 \cdot 51 \cdot 50 \cdot 49}$$

# Find the conditional probability

~~$P(A|B)$  or  $P(B|A)$~~

$P(A|B)$  or  $P(B|A)$  (or)  $P(A|B)$  or  $P(B|A)$

# A bag of 30 tulip bulbs was purchased from a nursery. The bag contains 12 red, 10 yellow & 8 purple tulip bulbs



#

	Factory ①	Factory ②	Total
defective			
good			
Total			

Numbers of Heads, $x_i$	0	1	2	3
Probability $P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

mean,  $\mu = \sum x_i \cdot P(x)$

Variance,  $\sigma^2 = \sum [x_i^2 \cdot P(x)] - \mu^2$

SD,  $\sigma = \sqrt{\sum [x_i^2 \cdot P(x)] - \mu^2}$

# mean  $\sum x \cdot P(x)$   
 # variance  $\sum x^2 \cdot P(x)$

# find the mean, variance and standard deviation of the probability function

$$f(x) = \frac{x}{10}; \text{ for } x = 1, 2, 3, 4$$

$$\Rightarrow \text{Mean, } E(x) = \sum_{i=1}^n x \cdot f(x)$$

$$= \sum_{i=1}^4 x \cdot \frac{x}{10}$$

$$= 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 3 \cdot \frac{3}{10} + 4 \cdot \frac{4}{10}$$

$$= \frac{30}{10} = 3$$

$$\Rightarrow \text{Variance, } E(x^2) = \sum_{i=1}^n x^2 \cdot f(x)$$

$$= \sum_{i=1}^4 x^2 \cdot \frac{x}{10}$$

$$= 1^2 \cdot \frac{1}{10} + 2^2 \cdot \frac{2}{10} + 3^2 \cdot \frac{3}{10} + 4^2 \cdot \frac{4}{10}$$

$$= 10 \quad \therefore \boxed{\text{Var}(x) = E(x^2) - (E(x))^2}$$

$$= 10 - 3^2 = 1$$

$$\therefore \text{S.D, } \sigma_x = \sqrt{\text{Var}(x)} = 1$$

Ans

$$\# f(x) = \begin{cases} \frac{3}{4}x(2-x), & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

$$E(x) = \int_0^2 x \cdot f(x) dx = \int_0^2 x \cdot \frac{3}{4}x(2-x) dx$$

$$E(x^2) = \int_0^2 x^2 f(x) dx$$

$$\text{Var}(x) = E(x^2) - E(x)^2$$

# Let  $x$  and  $y$  be random variables  $E(x) = 7$ ,  
 $E(y) = -5$ . Find  $E(4x - 2y + 6)$

$\Rightarrow$

Hint: Use the properties

i)  $E(a) = a$

(ii)  $E(bx) = bE(x)$

~~(iii)~~  $E(ax + b) = aE(x) + b$

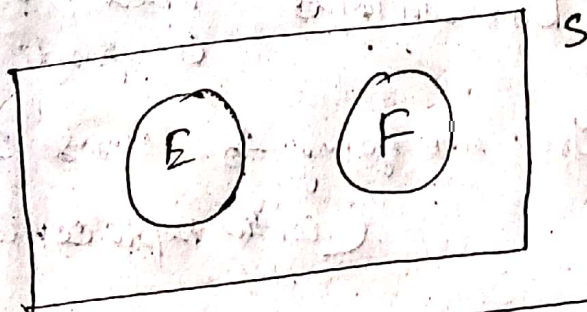
(iv)  $E(x + y) = E(x) + E(y)$

# # Properties of Variances, for any $a$ & $b$

- (i)  $\text{Var}(a) = 0$
- (ii)  $\text{Var}(bx) = b^2 \text{Var}(x)$
- (iii)  $\text{Var}(ax+tb) = a^2 \text{Var}(x)$

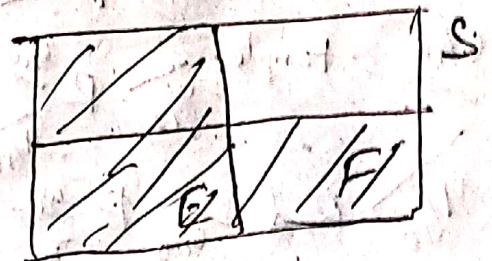
Mutually Exclusive

$$P(E \cup F) = P(E) + P(F)$$



Independent

$$P(E|F) = P(E) = \frac{P(E \cap F)}{P(F)}$$



(8-26 Jun lecture)

	factory 1	factory 2	total
defective	15	5	20
good	45	35	80
Total	60	40	100

make joint probability table

	factory 1	factory 2	total
defective	0.15	0.05	0.20
good	0.45	0.35	0.80
Total	0.60	0.40	1

①  $\rightarrow 0.20$

②  $\rightarrow 0.60$

③  $\rightarrow 0.80$

④  $\rightarrow 0.40$

⑤  $\rightarrow 0.15$

⑥  $\rightarrow P(D|F_1) = \frac{0.15}{0.60}$

⑦  $\rightarrow P(F_1|D) = \frac{0.15}{0.20}$

$\rightarrow$  (defective and made in factory one)

(defective given made in factory one)

(in Factory one given defective)

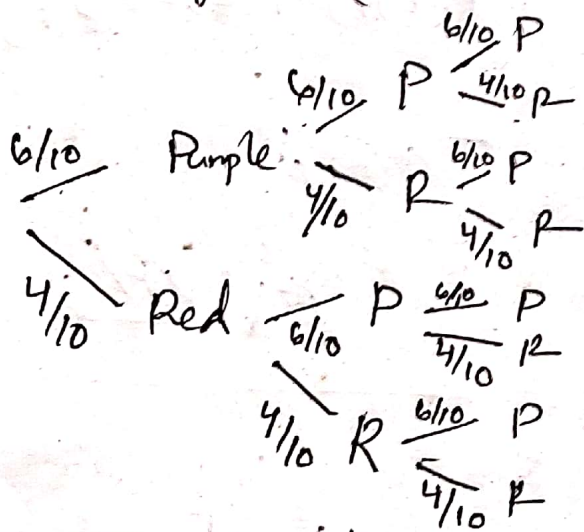
8. Are the events of selecting a defective chip and one made at factory one independent events?

$$P(D|Fact) = 0.25 \neq 0.20 = P(D)$$

9. Are the events of selecting a defective chip and one made at factory one mutually exclusive events?

$$P(D \cap Fact) = 0.15 \neq 0$$

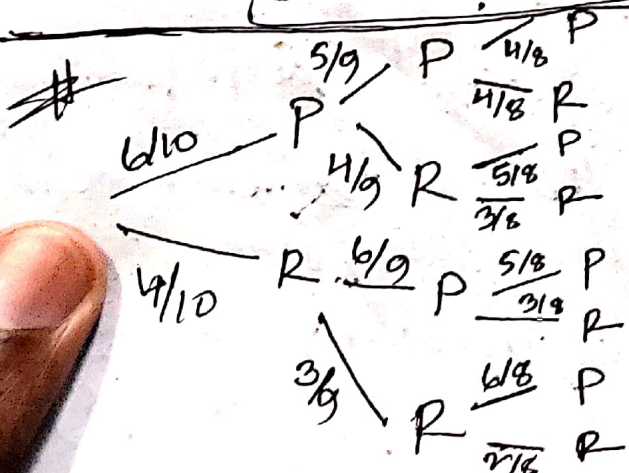
# Probability tree (with replacement)



A box contains 4 red marbles  
6 purple m.  
You are going to choose 3 marbles  
(with replacement)

①  $\rightarrow \frac{6}{10} \cdot \frac{6}{10} \cdot \frac{4}{10} = \frac{18}{125}$  (2 red, 1 purple)

~~Probability tree (without replacement)~~



(without replacement)

①  $\rightarrow \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8}$  (2P, 1R)

~~$\frac{18}{125}$~~   $= \frac{1}{6}$  (or 16.7%)