Measures of Central Tendency and Location

Measures of Central Tendency

- ✓ a single value that represents a data set.
- ✓ Its purpose is to locate the center of a data set.
- ✓ commonly referred to as an **average.**

Properties of Mean

- ✓ A set of data has only one mean
- ✓ Applied for interval and ratio data
- ✓ All values in the data set are included
- ✓ Very useful in comparing two or more data sets.
- ✓ Affected by the extreme small or large values on a data set
- ✓ Cannot be computed for the data in a frequency distribution with an open-ended class

Arithmetic Mean (Mean)

- ✓ The only common measure in which all values plays an equal role meaning to determine its values you would need to consider all the values of any given data set.

$\bar{X}$ X bar (for sample)

μ mu (for population)

Mean for Ungrouped Data

$$\text{Mean} = \frac{\text{Sum of all values}}{\text{Number of values}}$$

Sample Mean: $$\overline{X} = \frac{\sum X}{n}$$

Population Mean: $$\mu = \frac{\sum X}{N}$$

Example 1: Mean for Ungrouped Data

The daily rates of a sample of eight employees at GMS Inc. are 550, 420, 560, 500, 700, 670, 860, 480. Find the mean daily rate of employee.

Solution:

$$\bar{X} = \frac{\sum X}{n} = \frac{X_1 + X_2 + X_3 + \cdots + X_n}{n}$$

$$= \frac{550 + 420 + 560 + 500 + 700 + 670 + 860 + 480}{8} = \frac{4{,}740}{8} = 592.50$$

The sample mean daily salary of employees is 592.50

Example 2: Mean for Ungrouped Data

Find the population mean of the ages of 9 middle-management employees of a certain company. The ages are 53, 45, 59, 48, 54, 46, 51, 58, and 55.

Solution:

$$\mu = \frac{\sum X}{N} = \frac{X_1 + X_2 + X_3 + \cdots + X_n}{N}$$

$$= \frac{53 + 45 + 59 + 48 + 54 + 46 + 51 + 58 + 55}{9} = \frac{469}{9} = 52.11$$

The mean population age of middle-management employee is 52.11.

Mean for Grouped Data

$$\text{Mean} = \frac{\text{Sum of all values}}{\text{Number of values}}$$

Sample Mean:

$$\bar{X} = \frac{\sum fX}{n}$$

Population Mean:

$$\mu = \frac{\sum fX}{N}$$

Example 3: Mean for Grouped Data

Using the example provided in on SJS Travel Agency. Determine the mean of the frequency distribution on the ages of 50 people taking travel tours.

Class Limits	Frequency
18 – 26	3
27 – 35	5
36 – 44	9
45 – 53	14
54 – 62	11
63 – 71	6
72 – 80	2

Solution for Example 3

Class Limits	f	X	fX
18 – 26	3	22	66
27 – 35	5	31	155
36 – 44	9	40	360
45 – 53	14	49	686
54 – 62	11	58	638
63 – 71	6	67	402
72 – 80	2	76	152
Total	50		2,459

Get the midpoint

Get the product of f and X

Get the sum

$$\bar{X} = \frac{\sum fX}{n} = \frac{2,459}{50} = 49.18$$

Thus, the mean age of people taking travel is 49.18.

When arithmetic mean should not be used

In the following cases arithmetic mean should not be used:

- In highly - skewed distributions.
- When the distribution is unevenly spread. Concentration being small or large at irregular points.
- When an average rate of growth or change over a period of time is required.
- When the observation are from geometric progression.
- When averaging rates (that is speed, fluctuations in the prices of articles, etc.)
- When there are very large and very small values of observations.

Other Types of Mean

- ✓ **Weighted Mean**
- ✓ **Geometric Mean**
- ✓ **Combined Mean**

Weighted Mean

- ✓ It is useful when various classes or groups contribute differently to the total.
- ✓ It is found by multiplying each value by its corresponding weight and dividing by the sum of the weights.

$$\bar{X}_w = \frac{\sum_{i=1}^{n} w_i X_i}{\sum_{i=1}^{n} w_i} = \frac{w_1 X_1 + w_2 X_2 + \cdots + w_n X_n}{w_1 + w_2 + \cdots + w_n}$$

w = weight of each entry
--weights may not sum to 100%
--if weights sum to 100%, then $\sum w = 1$

Example for Weighted Mean

Ex 1) A class is graded based on weighted mean as follows:

Homework: 20%
Quizzes: 35%
Tests: 45%

Let's say scores are 95 on homework, 82 on quizzes and 79 on tests. What is the weighted mean?

$$\bar{x} = \frac{\sum(x \bullet w)}{\sum w} = \frac{95(.2) + 82(.35) + 79(.45)}{1} = 83.5$$

What if there were no test scores yet?

$$\bar{x} = \frac{\sum(x \bullet w)}{\sum w} = \frac{95(.2) + 82(.35)}{.55} = 86.7$$

Example for Weighted Mean

Ex 2) Rachel is taking a class in which her grade is determined as follows:

50% from her test mean
15% from her midterm
20% from her final
15% from her homework

Her scores are 86 (tests), 96 (midterm), and 100 (homework). What does Rachel need to get on her final exam to receive a 90% in class?

$$\bar{x} = \frac{\sum(x \bullet w)}{\sum w} = \frac{86(.5) + 96(.15) + 100(.15) + x(.2)}{1} = 90$$

70.9 + .2x = 90
.2x = 19.1
X = 95.5

Rachel needs to get a 95.5% on the final to earn a 90% in the class.

Example for Weighted Mean

Ex 3) For the month of April, a checking account has a balance of $523 for 24 days, $2415 for 2 days and $250 for 4 days. What is the account's mean daily balance for April?

$$523\left(\frac{24}{30}\right)+2415\left(\frac{2}{30}\right)+250\left(\frac{4}{30}\right)=612.73$$

Geometric Mean

- To determine the average percents, indexes, and relatives;

$$GM = \sqrt[n]{(X_1)(X_2)(X_3)\cdots(X_n)}$$

- to establish the average percent increase in production, sales, or other business transaction or economic series from one period of time to another.

$$GM = \sqrt[n-1]{\frac{\text{value at the end of the period}}{\text{value at the start of the period}}} - 1$$

Example 1 for Geometric Mean

Suppose the profits earned by the MSS Construction Company on five projects were 5, 6, 4, 8, and 10 percent, respectively. What is the geometric mean profit?

Solution:

$$GM = \sqrt[n]{(X_1)(X_2)(X_3)(X_4)(X_5)}$$

$$= \sqrt[5]{(5)(6)(4)(8)(10)} = \sqrt[5]{9,600} = 6.26$$

The geometric mean profit is 6.26%.

Example 2 for Geometric Mean

Badminton as a sport grew rapidly in 2008. From January to December 2008 the number of badminton clubs in Metro Manila increased from 20 to 155. Compute the mean monthly percent increase in the number of badminton clubs.

Solution:

$$GM = \sqrt[n-1]{\frac{\text{value at the end of the period}}{\text{value at the start of the period}}} - 1$$

$$= \sqrt[12-1]{\frac{155}{20}} - 1 = \sqrt[11]{7.75} - 1 = 0.2046$$

Hence, badminton clubs are increasing at a rate of almost 0.2046 or 20.46% per month.

When geometric mean should be used

The geometric mean is often used for a set of numbers whose values are meant to be multiplied together or are exponential in nature, such as a set of growth figures: values of the human population or interest rates of a financial investment over time.