UNIT 3 MOMENTS

Structure

3.1 INTRODUCTION

Moments are popularly used to describe the characteristic of a distribution. They represent a convenient and unifying method for summarizing many of the most commonly used statistical measures such as measures of tendency, variation, skewness and kurtosis. Moments are statistical measures that give certain characteristics of the distribution. Moments can be raw moments, central moments and moments about any arbitrary point. For example, the first raw moment gives mean and the second central moment gives variance. Although direct formulae exist for central moments even then they can be easily calculated with the help of raw moments. The r^{th} central moment of the variable x is h^r times the r^{th} central moment of u where $u = (x – A)/h$ is a new variable obtained by subjecting x to a change of origin and scale. Since A does not come into the scene so there is no effect of change of origin on moments.

The basic concepts of moments are described in Section 3.2. In Section 3.3, the methods of calculation of different kinds of moments are explored. In this section the methods of calculation of raw moments, moments about zero and central moments are explained. In Section 3.4, the Pearson's Beta and Gamma coefficients of skewness and kurtosis are described. We shall discuss the skewness and kurtosis in details in Unit 4 of this block.

Objectives

After studying this unit, you would be able to

- define moments;
- explain different types of moments and their uses;
- derive the relation between raw and central moments;

- describe the effect of change of origin and scale on moments;
- calculate the raw and central moments for grouped and ungrouped frequency distributions;
- define the Shephard's correction for moments; and
- calculate the Beta and Gamma coefficients of skewness and kurtosis.

3.2 INTRODUCTION TO MOMENTS

Moment word is very popular in mechanical sciences. In science moment is a measure of energy which generates the frequency. In Statistics, moments are the arithmetic means of first, second, third and so on, i.e. r^{th} power of the deviation taken from either mean or an arbitrary point of a distribution. In other words, moments are statistical measures that give certain characteristics of the distribution. In statistics, some moments are very important. Generally, in any frequency distribution, four moments are obtained which are known as first, second, third and fourth moments. These four moments describe the information about mean, variance, skewness and kurtosis of a frequency distribution. Calculation of moments gives some features of a distribution which are of statistical importance. Moments can be classified in raw and central moment. Raw moments are measured about any arbitrary point A (say). If A is taken to be zero then raw moments are called moments about origin. When A is taken to be Arithmetic mean we get central moments. The first raw moment about origin is mean whereas the first central moment is zero. The second raw and central moments are mean square deviation and variance, respectively. The third and fourth moments are useful in measuring skewness and kurtosis.

3.3 METHODS OF CALCULATION OF MOMENTS

Three types of moments are:

1. Moments about arbitrary point,
2. Moments about mean, and
3. Moments about origin

3.3.1 Moments about Arbitrary Point

When actual mean is in fraction, moments are first calculated about an arbitrary point and then converted to moments about the actual mean. When deviations are taken from arbitrary point, the formula's are:

For Ungrouped Data

If $x_1, x_2, ..., x_n$ are the n observations of a variable X, then their moments about an arbitrary point A are

Zero order moment A $$\mu'_0 = \frac{\sum_{i=1}^{n}(x_i - A)^0}{n} = 1$$

First order moment $\mu_1' = \frac{\sum_{i=1}^{n}(x_i - A)^1}{n}$

Second order moment $\mu_2' = \frac{\sum_{i=1}^{n}(x_i - A)^2}{n}$

Third order moment $\mu_3' = \frac{\sum_{i=1}^{n}(x_i - A)^3}{n}$

and Fourth order moment $\mu_4' = \frac{\sum_{i=1}^{n}(x_i - A)^4}{n}$

In general the r^{th} order moment about arbitrary point A is given by

$$\mu_r' = \frac{\sum_{i=1}^{n}(x_i - A)^r}{n}; \quad \text{for } r = 1, 2, ---$$

For Grouped Data

If $x_1, x_2, ..., x_k$ are k values (or mid values in case of class intervals) of a variable X with their corresponding frequencies $f_1, f_2, ..., f_k$, then moments about an arbitrary point A are

Zero order moment $\mu_0' = \frac{\sum_{i=1}^{k} f_i (x_i - A)^0}{N} = 1$; where N = $\sum_{i=1}^{k} fi$

First order moment $\mu_1' = \frac{\sum_{i=1}^{k} f_i (x_i - A)^1}{N}$

Second order moment $\mu_2' = \frac{\sum_{i=1}^{k} f_i (x_i - A)^2}{N}$

Third order moment $\mu_3' = \frac{\sum_{i=1}^{k} f_i (x_i - A)^3}{N}$

Fourth order moment $\mu_4' = \frac{\sum_{i=1}^{k} f_i (x_i - A)^4}{N}$

In general, the r^{th} order moment about an arbitrary point A can be obtained as

$$\mu_r' = \frac{\sum_{i=1}^{k} f_i (x_i - A)^r}{N} \quad ; r = 1, 2, \ldots$$

In a frequency distribution, to simplify calculation we can use short-cut method.

If $d_i = \frac{(x_i - A)}{h}$ or $(x_i - A) = hd_i$ then, we get the moments about an arbitrary point A are

First order moment $\mu_1' = \frac{\sum_{i=1}^{k} f_i d_i^1}{N} \times h$

Second order moment $\mu_2' = \frac{\sum_{i=1}^{k} f_i d_i^2}{N} \times h^2$

Third order moment $\mu_3' = \frac{\sum_{i=1}^{k} f_i d_i^3}{N} \times h^3$

Fourth order moments $\mu_4' = \frac{\sum_{i=1}^{k} f_i d_i^4}{N} \times h^4$

Similarly, r^{th} order moment about A is given by

$$\mu_r' = \frac{\sum_{i=1}^{k} f_i d_i^r}{N} \times h^r; \quad \text{for } r = 1, 2, \ldots$$

3.3.2 Moments about Origin

In case, when we take an arbitrary point A = 0 then, we get the moments about origin.

For Ungrouped Data

The r^{th} order moment about origin is defined as:

r^{th} order moment $\mu_r' = \frac{\sum_{i=1}^{n} (x_i - 0)^r}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i^r$

First order moment $\mu_1' = \frac{\sum_{i=1}^{n} (x_i - 0)}{n} = \bar{x}$

Second order moment $\mu_2' = \frac{\sum_{i=1}^{n} (x_i - 0)^2}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i^2$

Third order moment $\mu_3' = \frac{\sum_{i=1}^{n} (x_i - 0)^3}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i^3$

Forth order moment $\mu_4' = \frac{\sum_{i=1}^{n} (x_i - 0)^4}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i^4$

For Grouped Data

r^{th} order moment $\quad \mu'_r = \dfrac{\sum_{i=1}^{k} f_i (x_i - 0)^r}{N} = \dfrac{1}{N}\sum_{i=1}^{k} f_i x_i^r$

First order moment $\quad \mu'_1 = \dfrac{\sum_{i=1}^{k} f_i (x_i - 0)}{N} = \dfrac{1}{N}\sum_{i=1}^{k} f_i x_i$

Second order moment $\quad \mu'_2 = \dfrac{\sum_{i=1}^{k} f_i (x_i - 0)^2}{N} = \dfrac{1}{N}\sum_{i=1}^{k} f_i x_i^2$

Third order moment $\quad \mu'_3 = \dfrac{\sum_{i=1}^{k} f_i (x_i - 0)^3}{N} = \dfrac{1}{N}\sum_{i=1}^{k} f_i x_i^3$

Fourth order moment $\quad \mu'_4 = \dfrac{\sum_{i=1}^{k} f_i (x_i - 0)^4}{N} = \dfrac{1}{N}\sum_{i=1}^{k} f_i x_i^4$

3.3.3 Moments about Mean

When we take the deviation from the actual mean and calculate the moments, these are known as moments about mean or central moments. The formulae are:

For Ungrouped Data

Zero order moment $\quad \mu_0 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^0}{n} = 1$

First order moment $\quad \mu_1 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^1}{n} = 0$

Thus first order moment about mean is zero, because the algebraic sum of the deviation from the mean is zero $\sum_{i=1}^{n} (x_i - \bar{x}) = 0$.

Second order moment $\quad \mu_2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n} = \sigma^2$ (variance)

Therefore, second order moment about mean is variance. These results, viz, $\mu_0 = 1$, $\mu_1 = 0$ and $\mu_2 = \sigma^2$ are found very important in statistical theory and practical.

Third order moment $\quad \mu_3 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^3}{n}$

and Fourth order moment $\mu_4 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^4}{n}$

In general, the r^{th} order moment of a variable about the mean is given by

$$\mu_r = \frac{\sum_{i=1}^{k} (x_i - \bar{x})^r}{N}; \quad \text{for } r = 0, 1, 2, \ldots$$

For Grouped Data

In case of frequency distribution the r^{th} order moment about mean is given by:

$$\mu_r = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^r}{N}; \quad \text{for } r = 0, 1, 2 \ldots$$

By substituting the different value of are we can gate different orders moment about mean as follows:

Zero order moment $\mu_0 = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^0}{N} = 1$ as $N = \sum_{i=1}^{k} f_i$

First order moment $\mu_1 = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^1}{N} = 0$

Because $\sum_{i=1}^{k} f_i \left(x_i - x\right) = 0$

Second order moment $\mu_2 = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^2}{N}$ = Variance (σ^2)

Third order moment $\mu_3 = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^3}{N}$

and Fourth order moment $\mu_4 = \frac{\sum_{i=1}^{k} f_i (x_i - \bar{x})^4}{N}$

3.3.4 Relation between Moments about Mean and Moments about Arbitrary Point

The r^{th} moment about mean is given by

$$u_r = \frac{1}{N} \sum_{i=1}^{k} f_i (x_i - \bar{x})^r \text{ ;where } r = 0, 1, \ldots$$

$$u_r = \frac{1}{N} \sum_{i=1}^{k} f_i (x_i - A + A - \bar{x})^r$$

$$u_r = \frac{1}{N}\sum_{i=1}^{k} f_i \{(x_i - A) - (\bar{x} - A)\}^r \qquad \ldots (1)$$

If $d_i = x_i - A$ then,

$$x_i = A + d_i$$

$$\frac{1}{n}\sum x_i = A + \frac{1}{n}\sum d_i$$

$$\bar{x} = (A + \mu_1')$$

$$\because \mu_1' = \frac{1}{n}\sum d_i$$

$$\mu_1' = \bar{x} - A$$

Therefore, we get from equation (1)

$$u_r = \frac{1}{N}\sum_{i=1}^{k} f_i (d_i - \mu_1')^r$$

$$\Rightarrow \frac{1}{N}\sum_{i=1}^{k} f_i \left\{d_i^r - {}^rC_1 d_1^{r-1}\mu_1' + {}^rC_2 d_i^{r-2}(\mu_1')^2 - {}^rC_3 d_i^{r-3}(\mu_1')^3 + \ldots + (-1)^r(\mu_1')^r\right\}$$

$$\Rightarrow \frac{1}{N}\sum_{i=1}^{k} f_i d_i^r - {}^rC_1\mu_1' \frac{1}{N}\sum_{i=1}^{k} f_i d_i^{r-1} + {}^rC_2\mu_1'^2 \frac{1}{N}\sum_{i=1}^{k} f_i d_i^{r-2} - {}^rC_3\mu_1'^3 \frac{1}{N}\sum_{i=1}^{k} f_i d_i^{r-3} \ldots + (-1)^r \mu_1'^r$$

Then,

$$\mu_r = \mu_r' - {}^rC_1\mu_{r-1}'\mu_1' + {}^rC_2\mu_{r-2}'\mu_1'^2 - {}^rC_3\mu_{r-3}'\mu_1'^3 + \ldots + (-1)^r \mu_1'^r \qquad \ldots (2)$$

In particular on putting r = 2, 3 and 4 in equation (2), we get

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

3.3.5 Effect of Change of Origin and Scale on Moments

Let $u_i = \frac{x_i - A}{h}$ so that $x_i = A + hu_i$ and $(x_i - \bar{x}) = h(u_i - \bar{u})$

Thus, r^{th} moment of x about arbitrary point x = A is given by

$$\mu_r'(x) = \frac{1}{N}\sum_{i=1}^{k} f_i (x_i - A)^r$$

$$= \frac{1}{N}\sum_{i=1}^{k} f_i (hu_i)^r$$

$$\mu_r'(x) = h^r \frac{1}{N}\sum_{i=1}^{k} f_i u_i^r = h^r \mu_r'(u)$$

and, r^{th} moment of x about mean is given by

$$\mu_r(x) = \frac{1}{N}\sum_{i=1}^{k} f_i\,(x_i - \bar{x})^r$$

$$= \frac{1}{N}\sum_{i=1}^{k} f_i\,\{h(u_i - \bar{u})\}^r$$

$$\mu_r(x) = h^r\,\frac{1}{N}\sum_{i=1}^{k} f_i\,(u_i - \bar{u})^r = h^r \mu_r(u)$$

Thus, the r^{th} moment of the variable x about mean is h^r times the r^{th} moment of the new variable u about mean after changing the origin and scale.

3.3.6 Sheppard's Corrections for Moments

The fundamental assumption that we make in farming class intervals is that the frequencies are uniformly distributed about the mid points of the class intervals. All the moment calculations in case of grouped frequency distributions rely on this assumption. The aggregate of the observations or their powers in a class is approximated by multiplying the class mid point or its power by the corresponding class frequency. For distributions that are either symmetrical or close to being symmetrical, this assumption is acceptable. But it is not acceptable for highly skewed distributions or when the class intervals exceed about $1/20^{th}$ of the range. In such situations, W. F. Sheppard suggested some corrections to be made to get rid of the so called "grouping errors" that enter into the calculation of moments.

Sheppard suggested the following corrections known as Sheppard's corrections in the calculation of central moments assuming continuous frequency distributions if the frequency tapers off to zero in both directions

$$\mu_2\,(\text{corrected}) = \mu_2 - \frac{h^2}{12}$$

$$\mu_3\,(\text{corrected}) = \mu_3$$

$$\mu_4\,(\text{corrected}) = \mu_4 - \frac{1}{2}h^2\,\mu_2 + \frac{7}{240}h^4$$

where, h is the width of class interval.

3.3.7 Charlier's Checks for Moments

The following identities

$$\sum f\,(x+1) = \sum fx + N$$

$$\sum f\,(x+1)^2 = \sum fx^2 + 2\sum fx + N$$

$$\sum f(x+1)^3 = \sum fx^3 + 3\sum fx^2 + 3\sum fx + N$$

$$\sum f(x+1)^4 = \sum fx^4 + 4\sum fx^3 + 6\sum fx^2 + 4\sum fx + N$$

are used to check the calculations done for finding moments.

3.4 PEARSON'S BETA AND GAMMA COEFFICIENTS

Karl Pearson defined the following four coefficients, based upon the first four central moments:

1. β_1 is defined as

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

It is used as measure of skewness. For a symmetrical distribution, β_1 shall be zero.

β_1 as a measure of skewness does not tell about the direction of skewness, i.e. positive or negative. Because μ_3 being the sum of cubes of the deviations from mean may be positive or negative but $\mu^2{}_3$ is always positive. Also μ_2 being the variance always positive. Hence β_1 would be always positive. This drawback is removed if we calculate Karl Pearson's coefficient of skewness γ_1 which is the square root of β_1 **,**i. e.

$$\gamma_1 = \pm\sqrt{\beta_1} = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{\mu 3}{\sigma^2}$$

Then the sign of skewness would depend upon the value of μ_3 whether it is positive or negative. It is advisable to use γ_1 as measure of skewness.

2. β_2 measures kurtosis and it is defined by

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

and similarly**,** coefficient of kurtosis γ_2 is defined as

$$\gamma_2 = \beta_2 - 3$$

Example 1: For the following distribution calculate first four moments about mean and also find β_1, β_2, γ_1 and γ_2:

Marks	5	10	15	20	25	30	35
Frequency	4	10	20	36	16	12	2

Solution: First we construct the following frequency distribution for calculation of moments:

Marks (x)	**f**	$\mathbf{d = \frac{(x-20)}{5}}$	**fd**	**fd^2**	**fd^3**	**fd^4**
5	4	-3	-12	36	-108	324
10	10	-2	-20	40	-80	160
15	20	-1	-20	20	-20	20
20	36	0	0	0	0	0
25	16	1	16	16	16	16
30	12	2	24	48	96	192
35	2	3	6	18	54	162
			$\sum fd = -6$	$\sum fd^2 = 178$	$\sum fd^3 = -42$	$\sum fd^4 = 874$

Then

$$\mu_1' = \frac{\sum fd}{N} \times h = \frac{-6}{100} \times 5 = -0.3$$

$$\mu_2' = \frac{\sum fd^2}{N} \times h^2 = \frac{178}{100} \times 25 = 44.5$$

$$\mu_3' = \frac{\sum fd^3}{N} \times h^3 = \frac{-42}{100} \times 125 = -52.5$$

$$\mu_4' = \frac{\sum fd^4}{N} \times h^4 = \frac{874}{100} \times 625 = 5462.5$$

Moments about mean

$$\mu_2 = \mu_2' - \mu_1'^2 = 44.5 - 0.09 = 44.41 = \sigma^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + \mu_1'^3$$

$$= -52.5 - 3 \times 44.5 \times -0.3 + 2(-0.3)^3$$

$$= -52.5 + 40.05 - 0.054 = -12.504$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 5462.5 - 4\,(-0.3 \times -52.5) + 6\,(44.5)\,(-0.3)^2 - 3\,(-0.3)^4$$

$$= 5462.5 - 63 + 24.03 - 0.0243$$

$$= 5423.5057$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(-12.504)^2}{(44.41)^3} = 0.001785$$

$$\gamma_1 = \frac{\mu_3}{\sigma^3} = \frac{-12.504}{(6.6641)^3} = -0.0422$$

$$\beta_2 = \frac{\mu_4}{\sigma_2^2} = \frac{5423.5057}{(44.41)^2} = 02.7499$$

$$\gamma_2 = \beta_2 - 3 = 2.7499 - 3 = -\ 0.2501$$

E1) The first four moments of a distribution about the value 5 of a variable are 1, 10, 20 and 25. Find the central moments, β_1 and β_2.

E2) For the following distribution, find central moments, β_1 and β_2:

Class	1.5-2.5	2.5-3.5	3.5-4.5	4.5-5.5	5.5-6.5
Frequency	1	3	7	3	1

E3) Wages of workers are given in the following table:

Weekly Wages	10-12	12-14	14-16	16-18	18-20	20-22	22-24
Frequency	1	3	7	12	12	4	3

Find the first four central moment and β_1 and β_2.

3.5 SUMMARY

In this unit, we have discussed:

1. What is moments;
2. The different types of moments and their uses;
3. The relation between raw and central moments;
4. The effect of change of origin and scale on moments;
5. How to calculate the raw and central moments for the given frequency distribution;
6. Shephard's Correction for Moments; and
7. The Beta and Gamma coefficients.

3.6 SOLUTIONS / ANSWERS

E1) We have given

$$\mu_1^{'} = 1,\ \mu_2^{'} = 10,\ \mu_3^{'} = 20 \text{ and } \mu_4^{'} = 25$$

Now we have to find out moments about mean

$$\mu_2 = \mu_2^{'} - \mu_1^{'2} = 10 - (1)^2 = 9$$

$$\mu_3 = \mu_3^{'} - 3\mu_2^{'}\mu_1^{1} + 2\mu_1^{'3} = 20 - 3 \times 10 \times 1 + 2(1)^3$$

$$= 20 - 30 + 2 = -8$$

$$\mu_4 = \mu_4^{'} - 4\mu_3^{'}\mu_1^{'} + 6\mu_2^{'}\mu_1^{'2} - 3\mu_1^{'4}$$

$$= 25 - 4 \times 20 \times 1 + 6 \times 10 \times 1^2 - 3 \times 1^4 = 2$$

So therefore, $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(-8)^2}{(9)^3} = \frac{64}{729} = 0.0877$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{2}{81} = 0.0247$$

E2) For calculation of moments

x	**f**	$\mathbf{d = (x - \bar{x})}$	**fd**	$\mathbf{fd^2}$	$\mathbf{fd^3}$	$\mathbf{fd^4}$
2	1	–2	–2	4	–8	16
3	3	–1	–3	3	–3	3
4	7	0	0	0	0	0
5	3	1	3	3	3	3
6	1	2	2	4	8	16
	N =15		$\sum fd = 0$	$\sum fd^2 = 14$	$\sum fd^3 = 0$	$\sum fd^4 = 38$

We therefore have,

$$\mu_1 = \frac{\sum fd}{N} = \frac{0}{15} = 0$$

$$\mu_2 = \frac{\sum fd^2}{N} = \frac{14}{150} = 0.933$$

$$\mu_3 = \frac{\sum fd^3}{N} = \frac{0}{15} = 0$$

$$\mu_4 = \frac{\sum fd^4}{N} = \frac{38}{15} = 2.533$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(0)}{(0.933)^2}$$

Since β_1=0 that means the distributions is symmetrical.

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{(2.53)}{(0.933)^2} = 2.91$$

As, $\beta_2 < 3$ that means curve is platykurtic.

E3) Calculation of first four moments

Wages	**f**	**x**	$\mathbf{d' = \frac{x-17}{2}}$	**fd'**	**fd'^{2}**	**fd'^{3}**	**fd'^{4}**
10-12	1	11	–3	–3	9	–27	81
12-14	3	13	–2	–6	12	–24	48
14-16	7	15	–1	–7	7	–7	7
16-18	20	17	0	0	0	0	0
18-20	12	19	1	12	12	12	12
20-22	4	21	2	8	16	32	64
22-24	3	23	3	9	27	81	243
				$\sum fd' = 13$	$\sum fd'^2 = 27$	$\sum fd'^3 = 67$	$\sum fd'^4 = 455$

Therefore, using formula

$$\mu_1' = \frac{\sum fd'}{N} \times h = \frac{13}{50} \times 2 = 0.52$$

$$\mu_2' = \frac{\sum fd'^2}{N} \times h^2 = \frac{27}{50} \times 4 = 2.16$$

$$\mu_3' = \frac{\sum fd'^3}{N} \times h^3 = \frac{67}{50} \times 8 = +10.72$$

$$\mu_4' = \frac{\sum fd'^4}{N} \times h^4 = \frac{455}{50} \times 16 = 145.6$$

So, we have

$$\mu_1 = 0,\ \mu_2 = \mu_2' - \mu_1'^2 = 2.16 - 0.2704 = 1.8896$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_3 = 10.72 - 3 \times 2.16 \times 0.52 + (0.52)^3$$
$$= 10.72 - 3.3696 + 0.1406$$

$$= 7.491$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\mu_4 = 145.6 - 4\times 0.52\times 10.72 + 6\times 2.56\times 0.2704$$
$$- 3\times 0.07312$$

$$= 145.6 - 22.2976 + 3.5043 - 0.2193$$

$$= 126.5874$$

Thus, the β coefficients

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(7.491)^2}{(1.8896)^3} = \frac{56.11508}{6.7469} = 8.317$$

and $$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{(126.5874)}{(1.8896)^2} = \frac{126.5874}{3.5706} = 35.4527$$