

CHAPTER

RANDOM VARIABLES

RANDOM VARIABLES

Introduction

Discrete probability distribution

Continuous probability
distribution

Cumulative distribution function

Expected value, variance and
standard deviation

INTRODUCTION



RANDOM

- In an experiment of chance, outcomes occur randomly. We often summarize the outcome from a random experiment by a simple number.

Definition

- A ***variable*** is a symbol such as X , Y , Z , x or H , that assumes values for different elements. If the variable can assume only one value, it is called a constant.
- A ***random variable*** is a variable whose value is determined by the outcome of a random experiment.

Example

A balanced coin is tossed two times. List the elements of the sample space, the corresponding probabilities and the corresponding values X , where X is the number of getting head.

Solution

Elements of sample space	Probability	X
HH	$\frac{1}{4}$	2
HT	$\frac{1}{4}$	1
TH	$\frac{1}{4}$	1
TT	$\frac{1}{4}$	0

Exercise

- The time to recharge the flash is tested in three cell phone cameras. The probability a camera passes the test is 0.8 and the camera perform independently. List the elements of the sample space, the corresponding probabilities and the corresponding values X , where X denotes the number of camera passes the test.

Solution

X : the number of cameras that pass the test

Camera 1	Camera 2	Camera 3	Probability	X
Pass	Pass	Pass	0.512	3
Pass	Pass	Fail	0.128	2
Pass	Fail	Pass	0.128	2
Pass	Fail	Fail	0.032	1
Fail	Pass	Pass	0.128	2
Fail	Pass	Fail	0.032	1
Fail	Fail	Pass	0.032	1
Fail	Fail	Fail	0.008	0

TWO TYPES OF RANDOM VARIABLES

Discrete Random Variables

A random variable is **discrete** if its set of possible values consist of **discrete points** on the number line.

Continuous Random Variables

A random variable is **continuous** if its set of possible values consist of an **entire interval** on the number line.

EXAMPLES

Examples of discrete random variables:

-number of scratches on a surface

-number of defective parts among 1000 tested

-number of transmitted bits received error

Examples of continuous random variables:

-electrical current

-length

-time

DISCRETE PROBABILITY DISTRIBUTIONS

Definition:

- If X is a discrete random variable, the function given by $f(x)=P(X=x)$ for each x within the range of X is called the **probability distribution** of X .
- Requirements for a discrete probability distribution:

1) The probability of each value of the discrete random variable is between 0 and 1, inclusive. That is, $0 \leq P(X = x) \leq 1$

2) The sum of all the probabilities is 1. That is, $\sum_{x \in S} P(X = x) = 1$

Example

- Check whether the distribution is a probability distribution.

X	0	1	2	3	4
$P(X=x)$	0.125	0.375	0.025	0.375	0.125

Solution

$$\begin{aligned}\sum_0^4 P(X = x) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) \\ &= 0.125 + 0.375 + 0.025 + 0.375 + 0.125 \\ &= 1.025 \\ &\neq 1\end{aligned}$$

- **so the distribution is not a probability distribution.**

Example

- Check whether the function given by

$$f(x) = \frac{x+2}{25} \text{ for } x=1,2,3,4,5$$

can serve as the probability distribution of a discrete random variable.

Solution

Solution

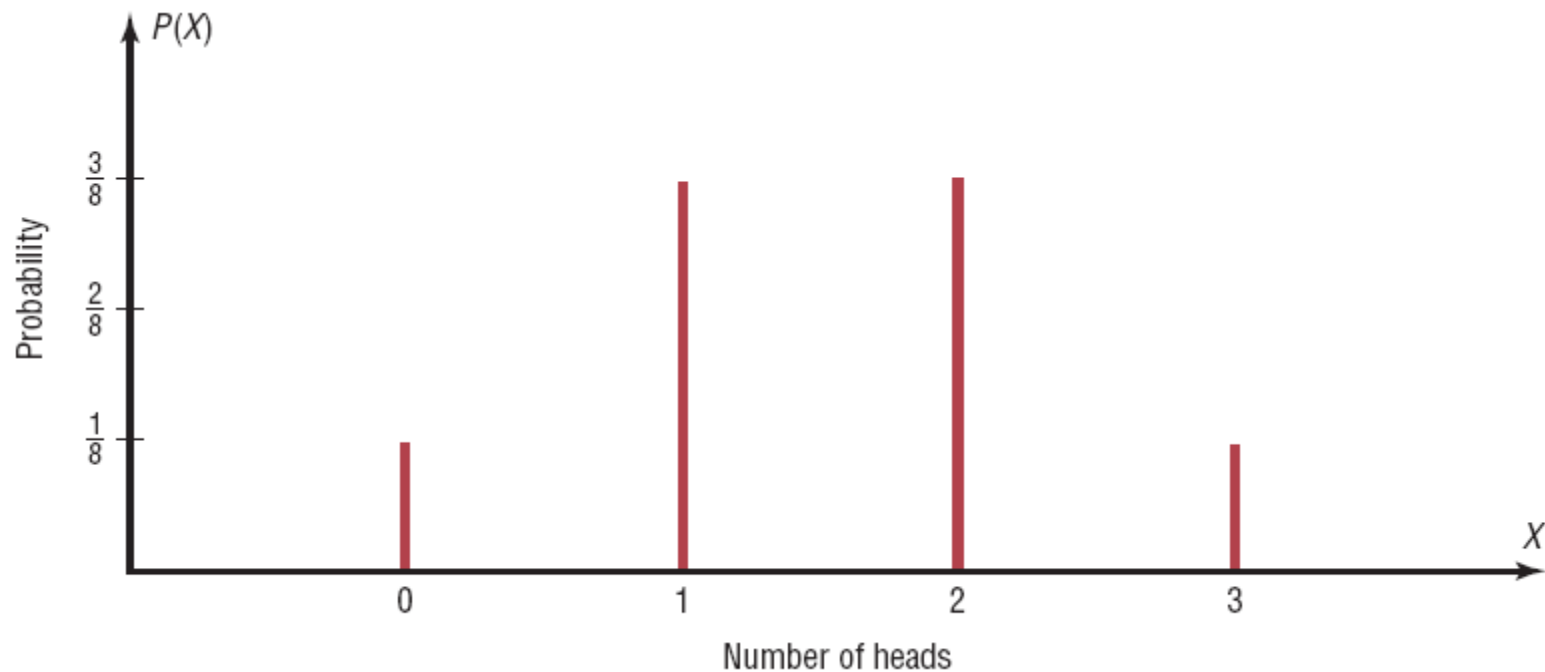
$$\begin{aligned}\sum_1^5 f(x) &= \sum_1^5 \frac{x+2}{25} \\ &= f(1) + f(2) + f(3) + f(4) + f(5) \\ &= \frac{1+2}{25} + \frac{2+2}{25} + \frac{3+2}{25} + \frac{4+2}{25} + \frac{5+2}{25} \\ &= \frac{3}{25} + \frac{4}{25} + \frac{5}{25} + \frac{6}{25} + \frac{7}{25} \\ &= \frac{25}{25} \\ &= 1\end{aligned}$$

so the given function is a probability distribution of a discrete random variable.

Example: Tossing Coins

Represent graphically the probability distribution for the sample space for tossing three coins.

• Number of heads X	0	1	2	3
Probability $P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



Mean, Variance, Standard Deviation, and Expectation

$$\text{MEAN: } \mu = \sum X \cdot P(X)$$

VARIANCE:

$$\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$

Example: Rolling a Die

Find the mean of the number of spots that appear when a die is tossed.

Outcome X	1	2	3	4	5	6
Probability $P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\mu = \sum X \cdot P(X)$$

$$= 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$$

$$= \frac{21}{6} = \boxed{3.5}$$

Example: Rolling a Die

Compute the variance and standard deviation for the probability distribution in Example 5–5.

Outcome X	1	2	3	4	5	6
Probability $P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$

$$\begin{aligned}\sigma^2 &= 1^2 \cdot \frac{1}{6} + 2^2 \cdot \frac{1}{6} + 3^2 \cdot \frac{1}{6} + 4^2 \cdot \frac{1}{6} \\ &\quad + 5^2 \cdot \frac{1}{6} + 6^2 \cdot \frac{1}{6} - (3.5)^2\end{aligned}$$

$$\sigma^2 = \boxed{2.9}, \quad \sigma = \boxed{1.7}$$

Example: On Hold for Talk Radio

A talk radio station has four telephone lines. If the host is unable to talk (i.e., during a commercial) or is talking to a person, the other callers are placed on hold. When all lines are in use, others who are trying to call in get a busy signal. The probability that 0, 1, 2, 3, or 4 people will get through is shown in the distribution. Find the variance and standard deviation for the distribution.

X	0	1	2	3	4
$P(X)$	0.18	0.34	0.23	0.21	0.04

Example: On Hold for Talk Radio

X	0	1	2	3	4
$P(X)$	0.18	0.34	0.23	0.21	0.04

$$\begin{aligned}\mu &= 0(0.18) + 1(0.34) + 2(0.23) \\ &\quad + 3(0.21) + 4(0.04) = 1.6\end{aligned}$$

$$\begin{aligned}\sigma^2 &= 0^2(0.18) + 1^2(0.34) + 2^2(0.23) \\ &\quad + 3^2(0.21) + 4^2(0.04) - (1.6)^2\end{aligned}$$

$$\sigma^2 = \boxed{1.2}, \quad \sigma = \boxed{1.1}$$

Expectation

- The **expected value**, or **expectation**, of a discrete random variable of a probability distribution is the theoretical average of the variable.
- The expected value is, by definition, the mean of the probability distribution.

$$E(X) = \mu = \sum X \cdot P(X)$$

Example: Winning Tickets

One thousand tickets are sold at \$1 each for four prizes of \$100, \$50, \$25, and \$10. After each prize drawing, the winning ticket is then returned to the pool of tickets. What is the expected value if you purchase two tickets?

Gain X	\$98	\$48	\$23	\$8	-\$2
Probability P(X)	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{992}{1000}$

$$\begin{aligned} E(X) &= \$98 \cdot \frac{2}{1000} + \$48 \cdot \frac{2}{1000} + \$23 \cdot \frac{2}{1000} \\ &\quad + \$8 \cdot \frac{2}{1000} + (-\$2) \cdot \frac{992}{1000} = \boxed{-\$1.63} \end{aligned}$$

Example: Winning Tickets

One thousand tickets are sold at \$1 each for four prizes of \$100, \$50, \$25, and \$10. After each prize drawing, the winning ticket is then returned to the pool of tickets. What is the expected value if you purchase two tickets?

Alternate Approach

Gain X	\$100	\$50	\$25	\$10	\$0
Probability P(X)	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{2}{1000}$	$\frac{992}{1000}$

$$\begin{aligned} E(X) &= \$100 \cdot \frac{2}{1000} + \$50 \cdot \frac{2}{1000} + \$25 \cdot \frac{2}{1000} \\ &\quad + \$10 \cdot \frac{2}{1000} + \$0 \cdot \frac{992}{1000} - \$2 = \boxed{-\$1.63} \end{aligned}$$

CONTINUOUS PROBABILITY DISTRIBUTIONS

Definition:

- A function with values $f(x)$, defined over the set of all numbers, is called a **probability density function** of the continuous random variable X if and only if

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

for any real constant a and b with $a \leq b$

Requirements for a probability density function of a continuous random variable X :

1) $f(x) \geq 0$ for $-\infty \leq x \leq \infty$

2) $\int_{-\infty}^{\infty} f(x) dx = 1$. That is the total area under graph is 1.

Example:

Consider the function

$$f(x) = \begin{cases} 6x(x-1) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find $P(X \leq -3)$
- (b) Find $P(0.5 \leq X \leq 0.8)$

Example

Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} c(2x^3 + 5) & , \quad -1 \leq x \leq 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

- 1) Evaluate c
- 2) Find $P(0 \leq X \leq 1)$

Solution

$$\begin{aligned} \text{a)} \quad P(-1 \leq X \leq 1) &= \int_{-1}^1 c(2x^3 + 5)dx \\ &= c \int_{-1}^1 (2x^3 + 5)dx \\ &= c \left(\frac{2x^4}{4} + 5x \right) \Big|_{-1}^1 \\ &= c \left[\left(\frac{2(1)^4}{4} + 5(1) \right) - \left(\frac{2(-1)^4}{4} + 5(-1) \right) \right] \\ &= c \left[\left(\frac{11}{2} \right) - \left(-\frac{9}{2} \right) \right] \\ &= c(10) \\ &= 1 \end{aligned}$$

$$\therefore c = \frac{1}{10}$$

b)

$$\begin{aligned} P(0 \leq X \leq 1) &= \int_0^1 \frac{1}{10} (2x^3 + 5) dx \\ &= \frac{1}{10} \left(\frac{2x^4}{4} + 5x \right) \Big|_0^1 \\ &= \frac{1}{10} \left[\left(\frac{2(1)^4}{4} + 5(1) \right) - \left(\frac{2(0)^4}{4} + 5(0) \right) \right] \\ &= \frac{1}{10} \left(\frac{11}{2} \right) \\ &= \frac{11}{20} \\ &= 0.55 \end{aligned}$$

Example

Let X be a continuous random variable with the following

$$f(x) = \begin{cases} \frac{3}{4}(x^2 + 1), & 0 \leq x \leq 1 \\ 0, & \textit{otherwise} \end{cases}$$

- a) Verify whether this distribution is a probability density function
- b) Find $P(0 \leq X \leq 0.5)$
- c) Find $P(0.5 \leq X \leq 2)$

Answer;

a) The distribution is probability density function if it fulfill the following requirements,

1) All $f(x) \geq 0$

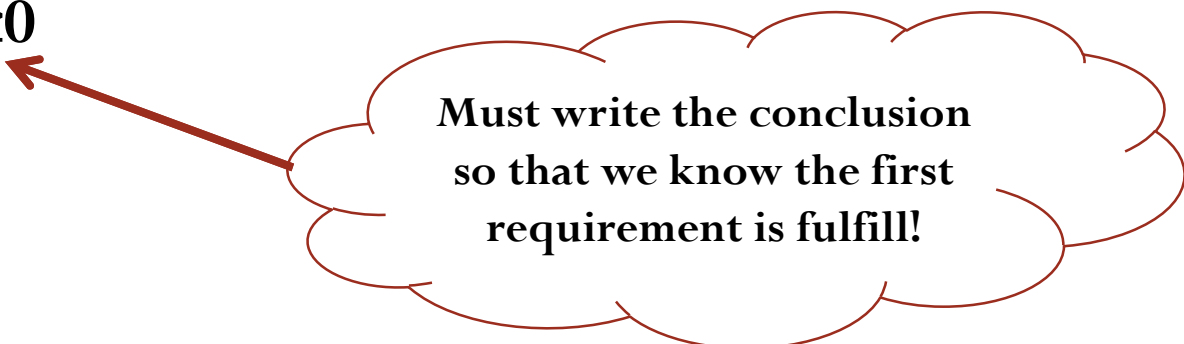
2) If $\int_{-\infty}^{\infty} f(x) dx = 1$

In this problem,

1) First requirement

$f(0) = 3/4 \geq 0$, $f(1) = 3/2 \geq 0$, $f(x) = 0$, otherwise

- **All $f(x) \geq 0$**



**Must write the conclusion
so that we know the first
requirement is fulfill!**

2) Second requirement

$$\int_{-\infty}^0 0 dx + \int_0^1 \frac{3}{4} (x^2 + 1) dx + \int_1^{\infty} 0 dx$$

$$= \frac{3}{4} \left[\frac{x^3}{3} + x \right]_0^1$$

$$= \frac{3}{4} \left[\frac{4}{3} \right]$$

$$= 1$$

$$- \int_{-\infty}^{\infty} f(x) dx = 1$$

**Must write the conclusion
so that we know the
second requirement is
fulfill!**

**Write last
conclusion to answer
the question!**

Since all requirements all fulfill, the distribution is probability density function.

$$b) P(0 \leq X \leq 0.5)$$

$$= \int_0^{0.5} \frac{3}{4} (x^2 + 1) dx$$

$$= \frac{3}{4} \int_0^{0.5} (x^2 + 1) dx$$

$$= \frac{3}{4} \left[\frac{x^3}{3} + x \right]_0^{0.5}$$

$$= \frac{3}{4} \left[\left(\frac{0.5^3}{3} + 0.5 \right) - 0 \right] = 0.40625$$

$$c) P(0.5 \leq X \leq 2)$$

$$= \int_{0.5}^1 \frac{3}{4} (x^2 + 1) dx + \int_1^2 0 dx$$

$$= \frac{3}{4} \int_{0.5}^1 (x^2 + 1) dx + 0$$

$$= \frac{3}{4} \left[\frac{x^3}{3} + x \right]_{0.5}^1$$

$$= \frac{3}{4} \left[\left(\frac{1^3}{3} + 1 \right) - \left(\frac{0.5^3}{3} + 0.5 \right) \right] = 0.59375$$

EXERCISE

1. A random variable x can assume 0,1,2,3,4. A probability distribution is shown here:

X	0	1	2	3	4
P(X)	0.1	0.3	0.3	?	0.1

- (b) Find $P(X = 3)$
- (c) Find $P(X \geq 2)$

2. Let $f(x) = \begin{cases} 12.5x - 1.25 & , 0.1 \leq x \leq 0.5 \\ 0 & , \text{otherwise} \end{cases}$

Find $P(0.2 \leq X \leq 0.3)$

3. Let $f(x) = \begin{cases} e^{-x+6} & , x > 6 \\ 0 & , \text{otherwise} \end{cases}$

(a) Find $P(X > 6)$

(b) Find $P(6 \leq X < 8)$

CUMULATIVE DISTRIBUTION FUNCTION

- The cumulative distribution function of a **discrete random variable X** , denoted as **$F(x)$** , is

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t) \text{ for } -\infty < x < \infty$$

- For a discrete random variable X , $F(x)$ satisfies the following properties:

$$1) 0 \leq F(x) \leq 1$$

$$2) \text{ If } x \leq y, \text{ then } F(x) \leq F(y)$$

- If the range of a random variable X consists of the values $x_1 < x_2 < x_3 < \dots < x_n$, then $f(x_1) = F(x_1)$ and $f(x_i) = F(x_i) - F(x_{i-1})$ for $i = 2, 3, \dots, n$

- The cumulative distribution function of a **continuous random variable X** is

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt \text{ for } -\infty < x < \infty$$

Let $F(x)$ be the distribution function for a continuous random

variable X . Then $f(x) = \frac{dF(x)}{dx} = F'(x)$

wherever the derivative exists.

Example

Given the probability function $f(x) = \frac{5-x}{10}$ for $x = 1, 2, 3, 4$,

find $F(x)$

Solution

x	1	2	3	4
$f(x)$	4/10	3/10	2/10	1/10
$F(x)$	4/10	7/10	9/10	1

Example

If X has the probability density

$$f(x) = \begin{cases} k \cdot e^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find

i) k

ii) $F(x)$

iii) $P(0.5 \leq x \leq 1)$

Solution

$$\text{i) } \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_0^{\infty} k \cdot e^{-3x} dx = k \left(\frac{e^{-3x}}{-3} \right) \Big|_0^{\infty}$$

$$= k \left(0 - \left(\frac{1}{-3} \right) \right)$$

$$= \frac{k}{3} = 1$$

$$\therefore k = 3$$

ii) for $x \leq 0$,

$$F(x) = \int_{-\infty}^x 0 dt = 0$$

for $x > 0$,

$$F(x) = \int_{-\infty}^0 0 dt + \int_0^x 3e^{-3t} dt$$

$$= 0 + 3 \int_0^x e^{-3t} dt$$

$$= 3 \left(\frac{e^{-3t}}{-3} \right) \Big|_0^x$$

$$= -1(e^{-3x} - e^0) = 1 - e^{-3x}$$

$$\therefore F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 - e^{-3x} & \text{for } x > 0 \end{cases}$$

Summary is
important!!!

$$\begin{aligned}\text{iii) } P(0.5 \leq X \leq 1) &= F(1) - F(0.5) \\ &= (1 - e^{-3(1)}) - (1 - e^{-3(0.5)}) \\ &= (1 - e^{-3}) - (1 - e^{-1.5}) \\ &= 0.173\end{aligned}$$

EXERCISE :

Given the probability density function of a random variable X as follows;

$$f(x) = \begin{cases} \frac{3}{8}x^2, & 0 \leq x \leq 2 \\ 0, & \textit{otherwise} \end{cases}$$

- i) Find the cumulative distribution function, $F(X)$
- ii) Find $P(1 \leq X \leq 2)$

How to change CDF to PDF

Given

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{8} & 0 \leq x < 2 \\ 1 & x > 2 \end{cases}$$

Find $f(x)$.

Solution:

$$\frac{d}{dx}(F(x)) = \frac{d}{dx}\left(\frac{x^3}{8}\right) = \frac{3x^2}{8}$$

$$f(x) = \frac{3x^2}{8}$$

EXPECTED VALUE, VARIANCE AND STANDARD DEVIATION

Expected Value

- The **mean** of a random variable X is also known as the **expected value** of X as $\mu = \mu_X = E(X)$

If X is a **discrete** random variable,

$$\mu_X = E(X) = \sum_{x \in S} xf(x) = \sum_{x \in S} xP(X = x)$$

If X is a **continuous** random variable,

$$\mu_X = E(X) = \int_{-\infty}^{\infty} xf(x)$$

Variance

$$\text{Var}(X) = \sigma^2 = \sigma_X^2 = E((X - \mu)^2), \text{ where}$$

$$\text{Var}(X) = \sum_{x \in S} (X - \mu)^2 P(X = x), \text{ in the discrete case,}$$

$$\text{Var}(X) = \int_{-\infty}^{\infty} (X - \mu)^2 f(x) dx, \text{ in the continuous case when it exists.}$$

$\text{Var}(X)$ exists if and only if $\mu = E(X)$ and $E(X^2)$ both exist, and then $\text{Var}(X) = E(X^2) - (E(X))^2$

Standard Deviation

The standard deviation is $\sigma = \sigma_X = \sqrt{\text{Var}(X)} = \sqrt{\sigma_X^2}$

Properties of Expected Values

- For any constant a and b ,

$$\text{i) } E(a) = a$$

$$\text{ii) } E(bX) = bE(X)$$

$$\text{iii) } E(aX + b) = aE(X) + b$$

$$\text{iv) } E(X + Y) = E(X) + E(Y)$$

Properties of Variances

For any constant a and b ,

$$\text{i) } \mathit{Var}(a) = 0$$

$$\text{ii) } \mathit{Var}(bX) = b^2 \mathit{Var}(X)$$

$$\text{iii) } \mathit{Var}(aX + b) = a^2 \mathit{Var}(X)$$

Example

Find the mean, variance and standard deviation of the probability function

$$f(x) = \frac{x}{10} \quad \text{for } x = 1, 2, 3, 4$$

Solution

Mean:

$$\begin{aligned} E(X) &= \sum_{i=1}^n x \cdot f(x) \\ &= \sum_{i=1}^4 x \cdot \frac{x}{10} \\ &= 1 \cdot \frac{1}{10} + 2 \cdot \frac{2}{10} + 3 \cdot \frac{3}{10} + 4 \cdot \frac{4}{10} \\ &= \frac{30}{10} = 3 \end{aligned}$$

Variance:

$$\begin{aligned} E(X^2) &= \sum_{i=1}^n x^2 \cdot f(x) \\ &= \sum_{i=1}^4 x^2 \cdot \frac{x}{10} \\ &= 1^2 \cdot \frac{1}{10} + 2^2 \cdot \frac{2}{10} + 3^2 \cdot \frac{3}{10} + 4^2 \cdot \frac{4}{10} \\ &= 10 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= 10 - 3^2 = 1 \end{aligned}$$

$$\sigma_X = \sqrt{\text{Var}(X)} = 1$$

Example

Let X be a continuous random variable with the following probability density function

$$f(x) = \begin{cases} \frac{3}{4}x(2-x), & 0 \leq x \leq 2 \\ 0 & , \text{ otherwise} \end{cases}$$

Find

a) $E(X)$

b) $Var(X)$

Solution

$$\begin{aligned} \text{a) } E(X) &= \int_{-\infty}^{\infty} x \cdot f(x) \, dx \\ &= \int_0^2 x \cdot \frac{3}{4} x (2 - x) \, dx \\ &= \frac{3}{4} \int_0^2 x^2 (2 - x) \, dx \\ &= \frac{3}{4} \int_0^2 (2x^2 - x^3) \, dx \\ &= \frac{3}{4} \left(\frac{2x^3}{3} - \frac{x^4}{4} \right) \Big|_0^2 \\ &= \frac{3}{4} \left[\left(\frac{2(2^3)}{3} - \frac{2^4}{4} \right) - 0 \right] \\ &= \frac{3}{4} \left(\frac{4}{3} \right) = 1 \end{aligned}$$

$$\begin{aligned}
\text{b) } E(X^2) &= \int_{-\infty}^{\infty} x^2 \cdot f(x) dx \\
&= \int_0^2 x^2 \cdot \frac{3}{4} x (2 - x) dx \\
&= \frac{3}{4} \int_0^2 x^3 (2 - x) dx \\
&= \frac{3}{4} \int_0^2 (2x^3 - x^4) dx \\
&= \frac{3}{4} \left(\frac{2x^4}{4} - \frac{x^5}{5} \right) \Big|_0^2 \\
&= \frac{3}{4} \left[\left(\frac{2(2^4)}{4} - \frac{2^5}{5} \right) - 0 \right] \\
&= \frac{3}{4} \left(\frac{8}{5} \right) = \frac{6}{5}
\end{aligned}$$

$$\begin{aligned}
\text{Var}(X) &= E(X^2) - (E(X))^2 \\
&= \frac{6}{5} - 1^2 = \frac{1}{5}
\end{aligned}$$

Example:

Let X and Y be random variables with $E(X) = 7, E(Y) = -5$.

Find $E(4X - 2Y + 6)$

Hint: Use this properties!

$$\text{i) } E(a) = a$$

$$\text{ii) } E(bX) = bE(X)$$

$$\text{iii) } E(aX + b) = aE(X) + b$$

$$\text{iv) } E(X + Y) = E(X) + E(Y)$$

Ans: 44

Exercise:

1. The number of holes, X that can be drilled per bit while drilling into limestone is given in table below:

X	1	2	3	4	5	6	7	8
P(X=x)	0.02	0.03	0.05	0.2	0.4	0.2	0.07	y

- (a) Find y . (Ans: 0.03)
- (b) Find $E[X], E[X^2]$. (Ans: 4.96, 26.34)
- (c) Find $Var(X), \sigma_x$. (Ans: 1.7384, 1.3185)
2. Let X and Y be random variables with
 $E(X) = 3, E(X^2) = 25, E(Y) = 10, E(Y^2) = 164$
- (a) Find $E(3X + Y - 8)$ (Ans: 11)
- (b) Find $Var(3X + Y - 8)$ (Ans: 208)

EXERCISE:

1. The table below represents the number of CDs sold for a certain month and their probability distribution. Find the value of A and B if expected value $E(X) = 104$. (Ans: 0.1,0.2)

X	80	90	100	110	120	130
P(X=x)	A	0.2	B	0.3	0.1	0.1

2. Given

$$f(x) = \begin{cases} \frac{3+7x}{3c} & x = 1, 2, 3 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Find the value c (Ans: 17)
- (b) Build a cumulative frequency distribution table.

3. The temperature readings from a thermocouple in a furnace fluctuate according to a cumulative distribution function.

$$F(x) = \begin{cases} 0 & x < 800^\circ C \\ 0.1x - 80 & 800^\circ C \leq x < 810^\circ C \\ 1 & x > 810^\circ C \end{cases}$$

Determine $P(X < 805)$, $P(800 < X \leq 805)$, $P(X > 808)$

(Ans: 0.5, 0.5, 0.2)