Fundamentals of Hypothesis Testing

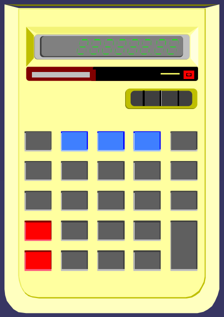
Hypothesis Testing Process

Assume the population mean TV sets is 3.

(Null Hypothesis)

Identify the Population

Take a Sample



Compute the Sample Mean to be 2.0

Do a statistical test and conclude

REJECT

Null Hypothesis

General Steps in Hypothesis Testing

1. State H_0

   State H_1

2. Choose α

3. Collect data

4. Compute test statistic
   (and/or the *p* value)

5. Make the decision

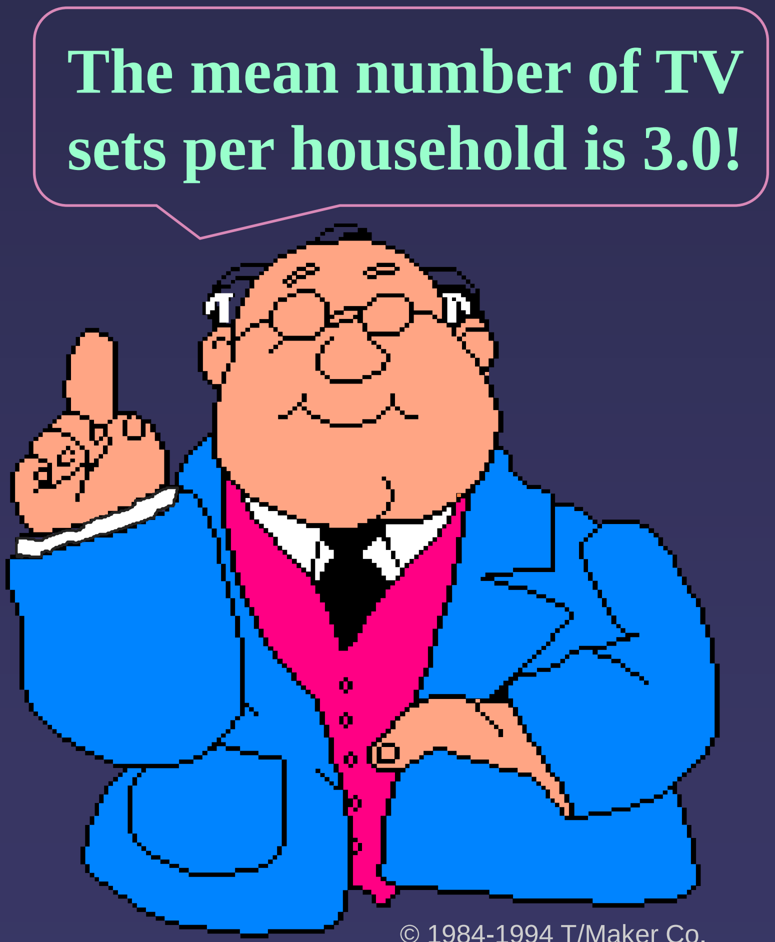
Step 1

Define the null and alternative hypotheses

What is a Hypothesis?

A hypothesis is a claim about the population parameter.

- Examples of a parameter are population mean or proportion



The Null Hypothesis, H_0

- States the assumption to be tested

  e.g. The mean number of TV sets is 3

  (H_0: $\mu = 3$)

- The null hypothesis is always about a population parameter (H_0: $\mu \underline{=} 3$), ***not*** about a sample statistic (H_0: $X = 3$)

The Null Hypothesis, H_0 *(continued)*

- **Begins with the assumption that the null hypothesis is *TRUE***

(Similar to the notion of innocent until proven guilty)

Always contains the " = " sign.

There may be enough evidence to reject the Null Hypothesis. Otherwise it is not rejected.

The Alternative Hypothesis, H_1

- Is the opposite of the null hypothesis

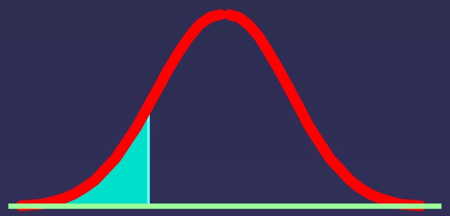
  e.g. The mean number of TV sets is not 3 (H_1: $\mu \neq 3$)

- Never contains an "=" sign.

One and Two Tail Hypotheses

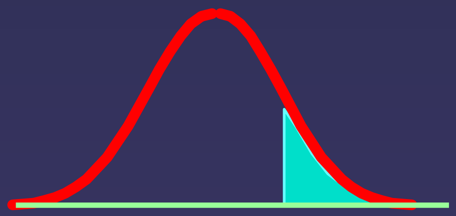
$H_0: \mu \geq 3$

$H_1: \mu < 3$



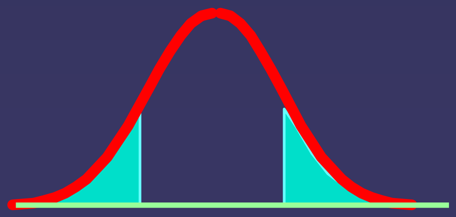
$H_0: \mu \leq 3$

$H_1: \mu > 3$



$H_0: \mu = 3$

$H_1: \mu \neq 3$



Step 2

Set the level of significance α

Result Possibilities

H_0: Innocent

Jury Trial

	The Truth	
Verdict	Innocent	Guilty
Innocent	**Correct**	**Error**
Guilty	**Error**	**Correct**

Hypothesis Test

	The Truth	
Decision	H_0 True	H_0 False
Do Not Reject **H_0**	**1 - α**	**Type II Error (β)**
Reject **H_0**	**Type I Error (α)**	**Power (1 - β)**

Level of Significance, α

- The α (level of significance) is selected by the researcher at the start of the research. Typical values are 0.01, 0.05, and 0.10
- The α defines unlikely values of sample statistic if null hypothesis is true. This is called rejection region.

Level of Significance, α and the Rejection Region

$H_0: \mu \geq 3$

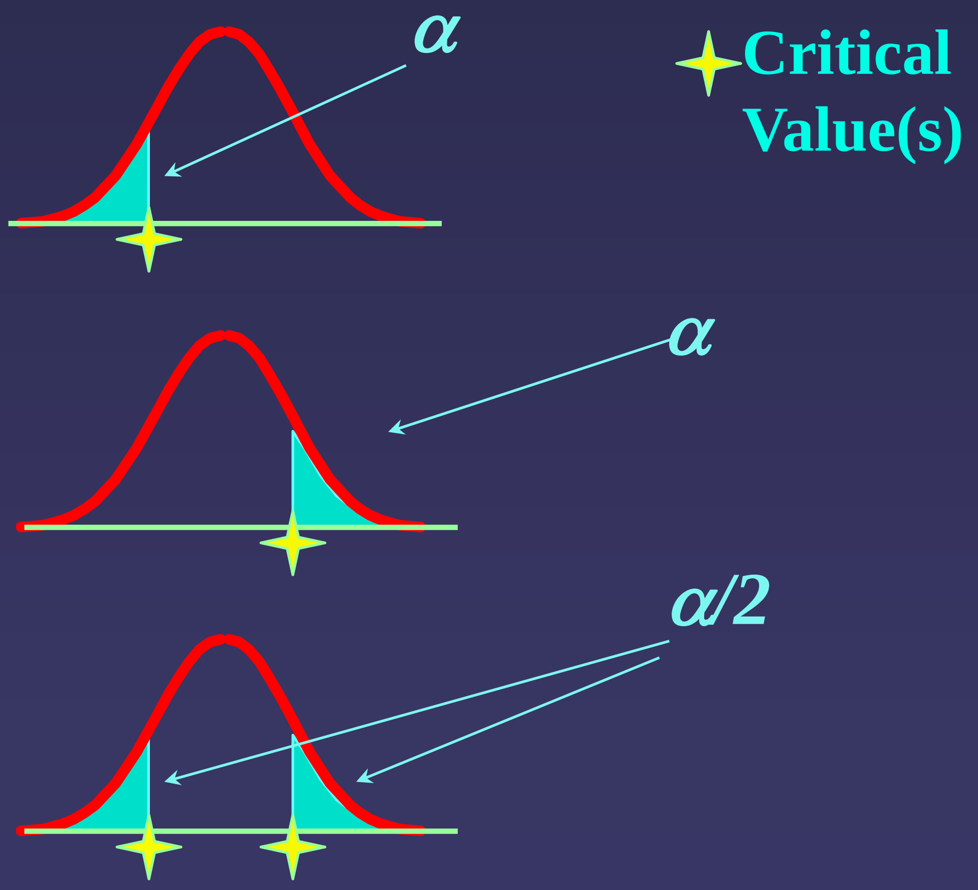
$H_1: \mu < 3$

$H_0: \mu \leq 3$

$H_1: \mu > 3$

$H_0: \mu = 3$

$H_1: \mu \neq 3$



Errors in Making Decisions

Type I error

- Is when you reject the null hypothesis when it is true.
- Probability of type I error is α and is set by the researcher.

Errors in Making Decisions

Type II Error

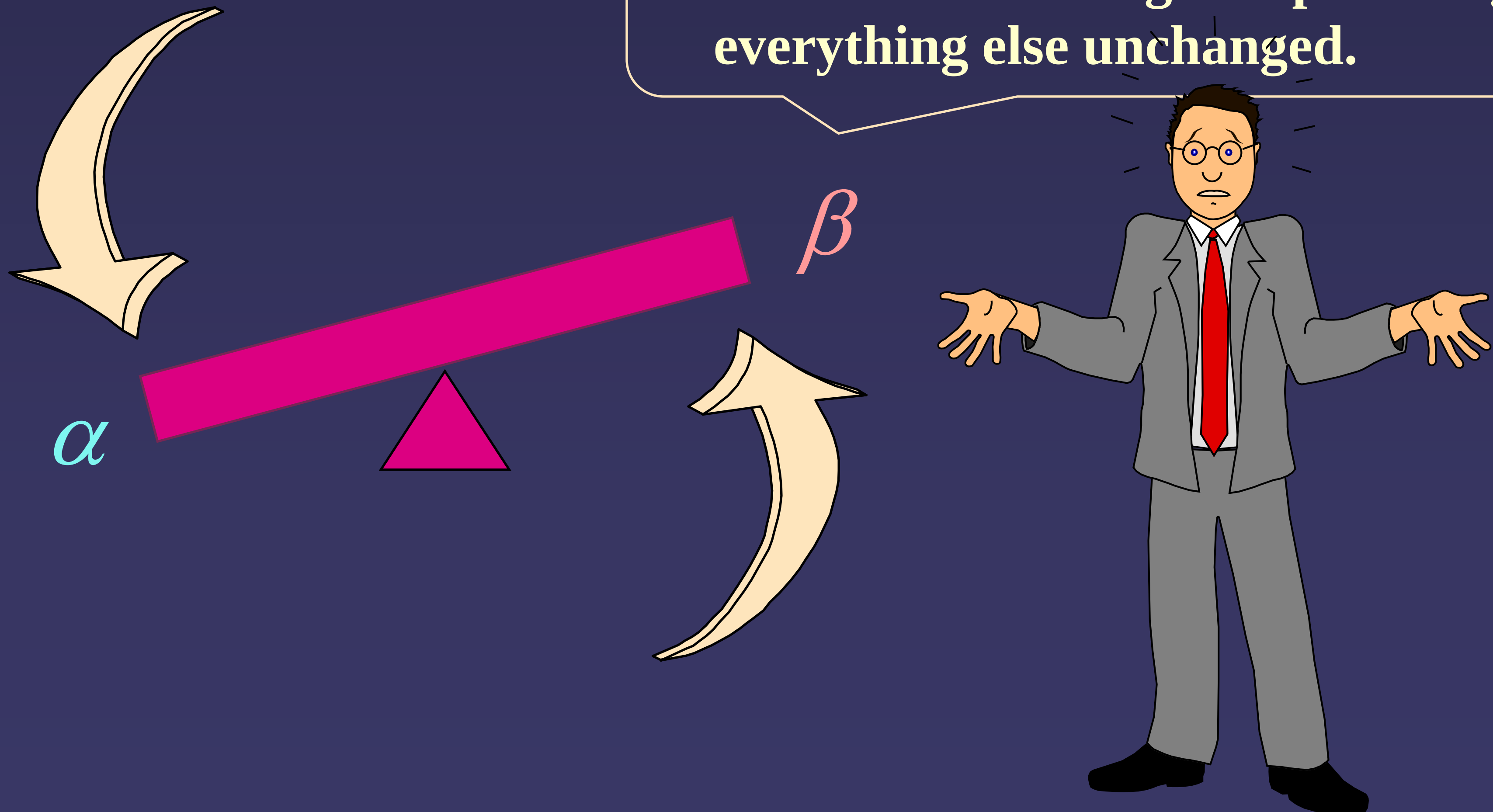
Is failing to Reject a False Null Hypothesis

Probability of Type II Error Is β (beta)

The *Power* of The Test Is *$(1-\beta)$*

α & β Have an Inverse Relationship

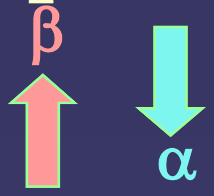
Reduce probability of one error and the other one goes up holding everything else unchanged.



Factors Affecting Type II Error β

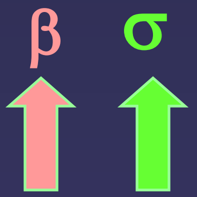
- β increases when difference between hypothesized parameter & its true value decreases.

- β increases when you are willing to take a bigger chance of a type I error (α decreases).

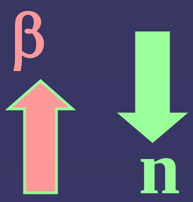


Factors Affecting Type II Error β *(continued)*

- β increases when population standard deviation σ increases



- β increases when sample size n decreases



How to Choose between Type I and Type II Errors

- Choice depends on the costs of the errors.
- Choose smaller type I error when the cost of rejecting the hypothesis is high.
  - At a criminal trial, the presumption is innocence. A type I error is convicting an innocent person.

Step 3

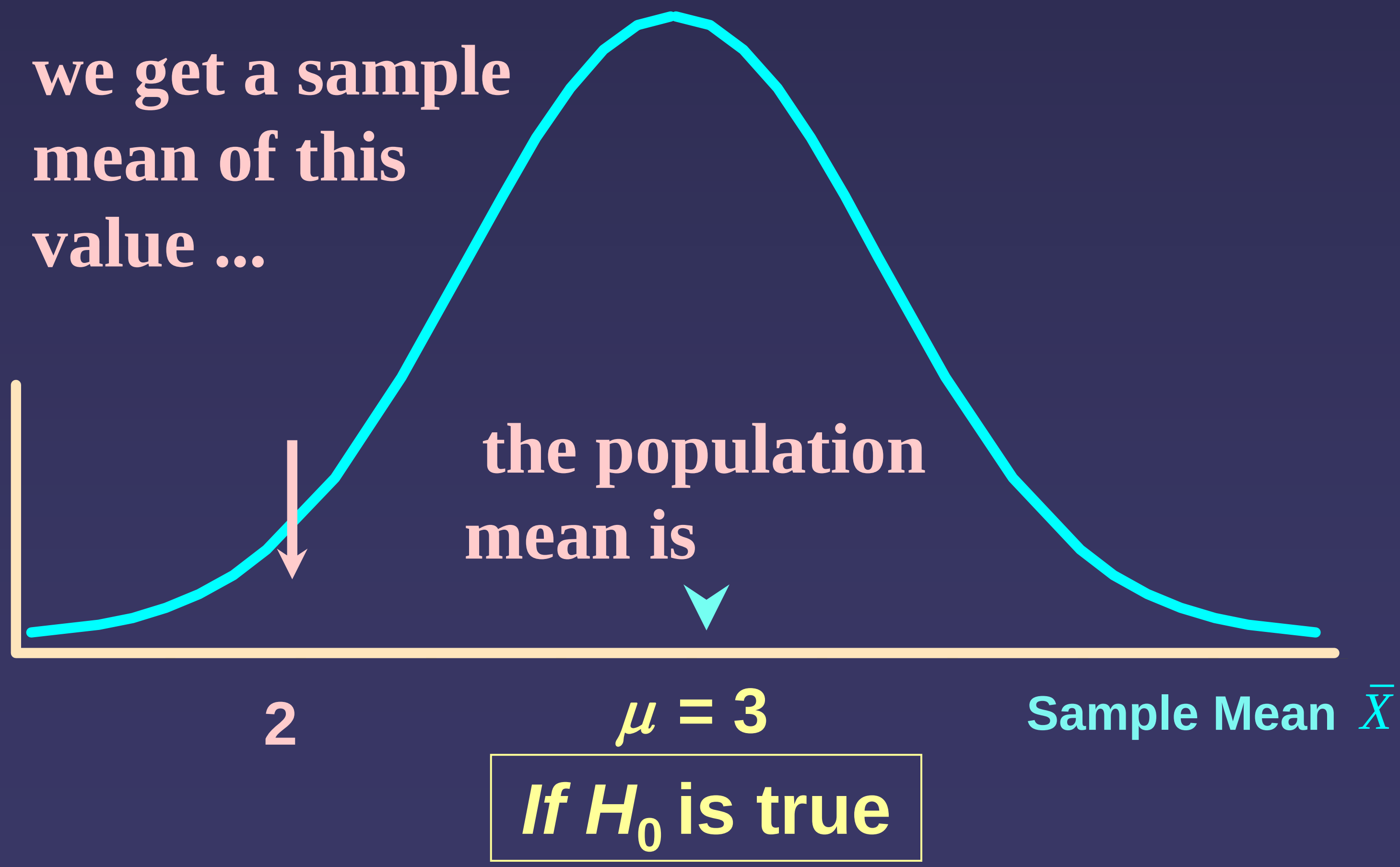
Gather the data

Step 4

Conduct a statistical test to measure the strength of the evidence

Weighting the evidence

Sampling Distribution of $\bar{X}$

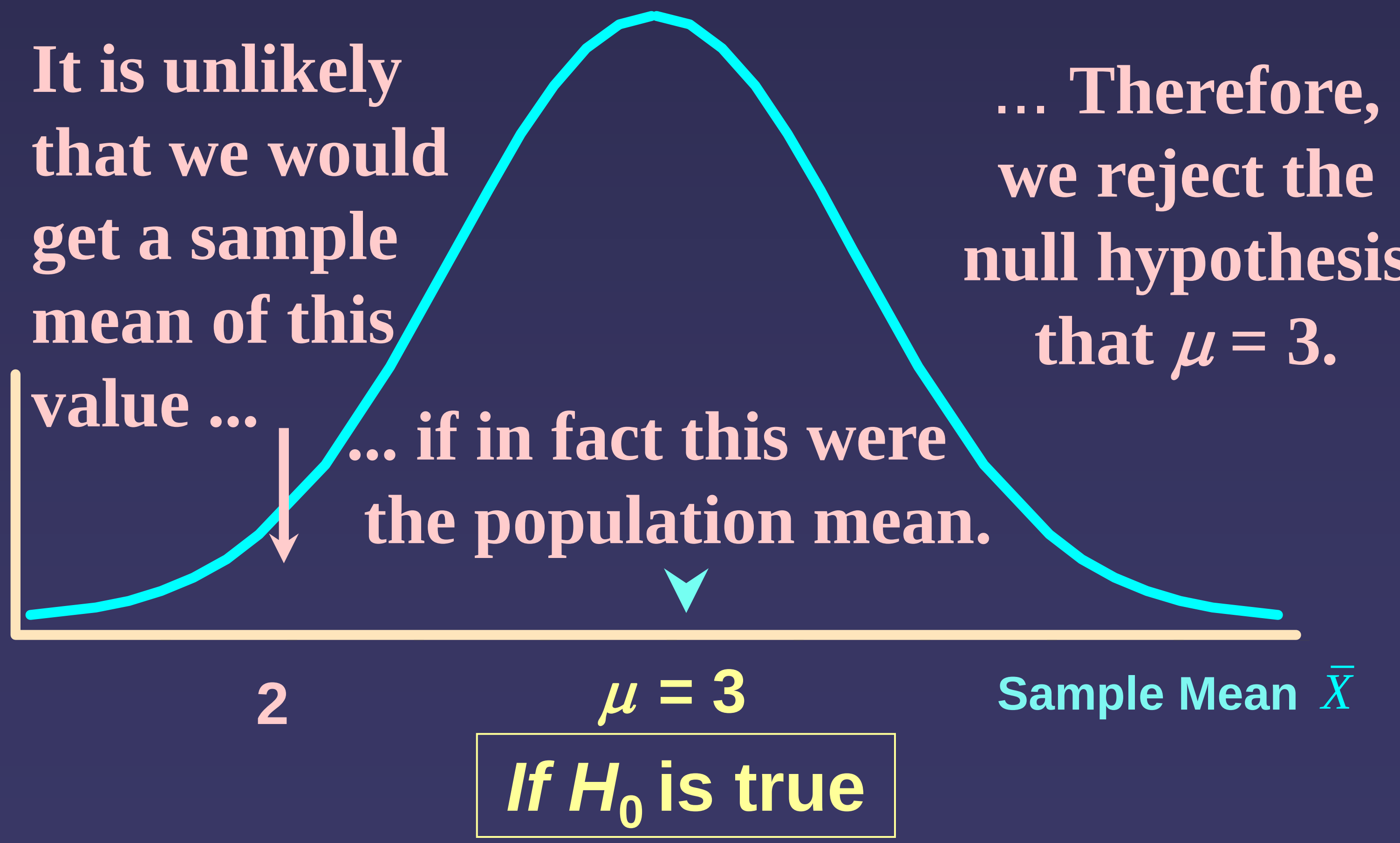


Step 5

Reach a conclusion

Rejecting H_0

Sampling Distribution of $\bar{X}$



An Example of the 5 Steps

For the two tail test on the mean where the population standard deviation σ is known.

Two-Tail Z Test for the Mean (σ Known)

Assumptions

Population is normally distributed.

If not normal, use large samples.

Z **test statistic:**

$$Z = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

Example

The daily yield for a chemical plant has averaged 880 tons for several years. The quality control manager wants to know if this average has changed. She randomly selects 50 days and records an average yield of 871 tons with a standard deviation of 21 tons.

$$H_0 : \mu = 880$$
$$H_a : \mu \neq 880$$

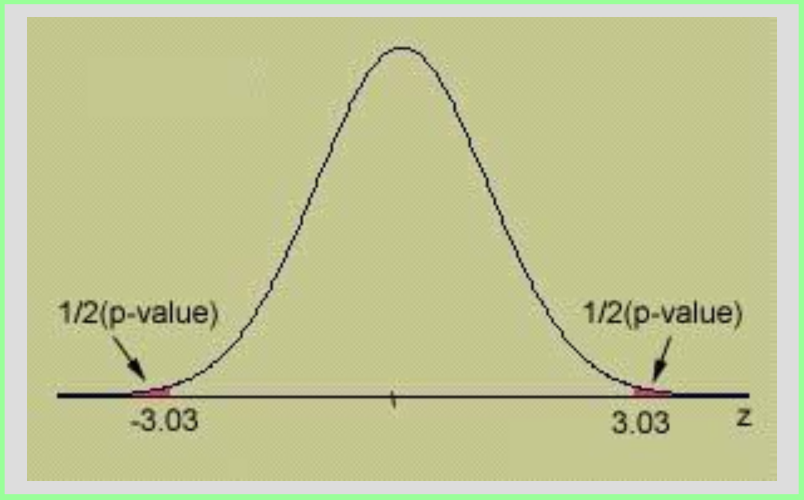
Test statistic :

$$z \approx \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{871 - 880}{21 / \sqrt{50}} = -3.03$$

Example

What is the probability that this test statistic or something even more extreme (far from what is expected if H_0 is true) could have happened *just by chance*?

$$p\text{-value}: P(z > 3.03) + P(z < -3.03)$$
$$= 2P(z < -3.03) = 2(.0012) = .0024$$



This is an unlikely occurrence, which happens about 2 times in 1000, assuming $\mu = 880$!

Example

To make our decision clear, we choose a significance level, say $\alpha = .01$.

If the *p*-value is less than α, H_0 is rejected as false. You report that the results are statistically significant at level α.

If the *p*-value is greater than a, H_0 is not rejected. You report that the results are not significant at level α.

Since our *p*-value =.0024 is less than, we reject H_0 and conclude that the average yield has changed.

Using a Rejection Region

If $\alpha = .01$, what would be the **critical value** that marks the "dividing line" between "not rejecting" and "rejecting" H_0?

If p-value < α, H_0 is rejected.

If p-value > α, H_0 is not rejected.

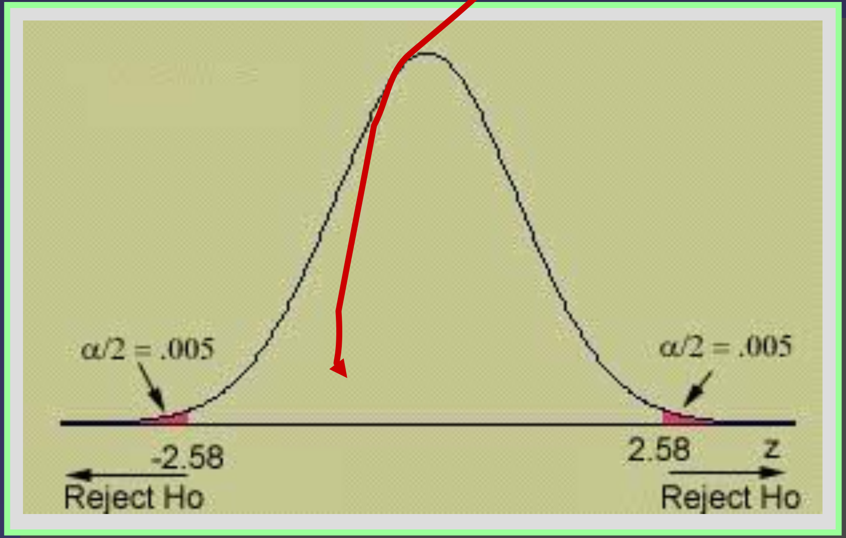
The dividing line occurs when p-value = α. This is called the **critical value** of the test statistic.

Test statistic > critical value implies p-value < α, H_0 is rejected.

Test statistic < critical value implies p-value > α, H_0 is not rejected.

Example

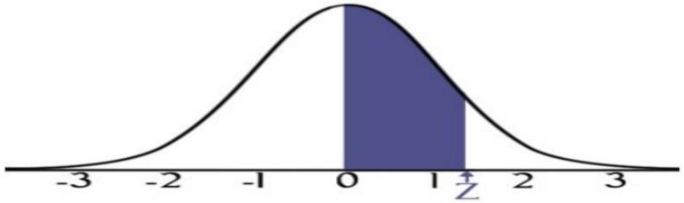
What is the critical value of z that cuts off exactly $\alpha/2 = .01/2 = .005$ in the tail of the z distribution?



For our example, $z = -3.03$ falls in the rejection region and H_0 is rejected at the 1% significance level.

Rejection Region: Reject H_0 if $z > 2.58$ or $z < -2.58$. If the test statistic falls in the rejection region, its *p*-value will be less than a = .01.

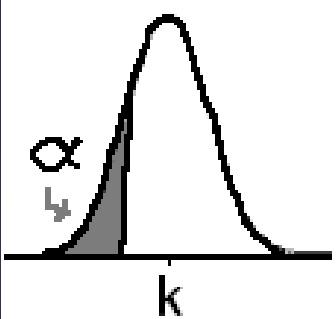
Figure 1.3: Standard Normal Table



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and *z* standard deviations above the mean. For example, for *z* = 1.25 the area under the curve between the mean (0) and *z* is 0.3944.

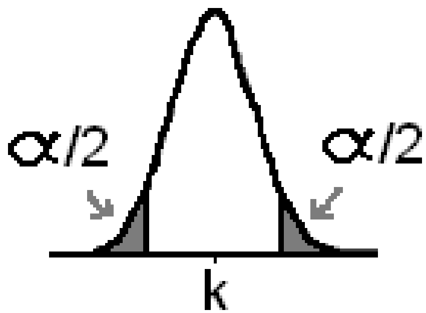
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998



$H_0: \mu = k$

$H_1: \mu < k$

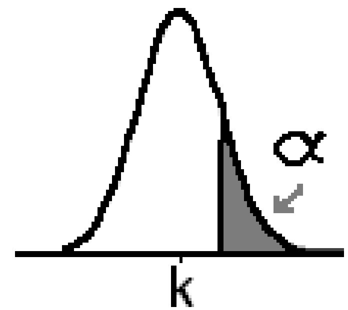
α	z critical
0.10	-1.28
0.05	-1.65
0.01	-2.33



$H_0: \mu = k$

$H_1: \mu \neq k$

α	z critical
0.10	± 1.65
0.05	± 1.96
0.01	± 2.58



$H_0: \mu = k$

$H_1: \mu > k$

α	z critical
0.10	1.28
0.05	1.65
0.01	2.33

Example: Two Tail Test

Does an average box of cereal contain 368 grams of cereal? A random sample of 36 boxes showed $\bar{X}$ = 372.5. The company has specified σ to be 15 grams. Test at the $\alpha = 0.05$ level

$H_0: \mu = 368$

$H_1: \mu \neq 368$

Example Solution: Two Tail

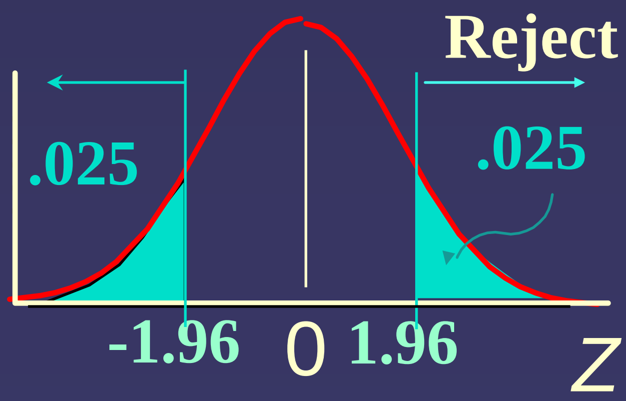
$H_0: \mu = 368$

$H_1: \mu \neq 368$

$\alpha = 0.05$

$n = 36$

Critical value: ±1.96



Test Statistic:

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{372.5 - 368}{15 / \sqrt{36}} = 1.8$$

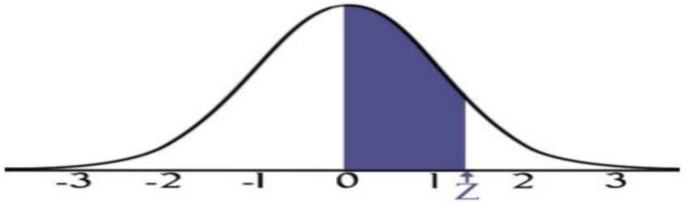
Decision:

Do Not Reject at $\alpha = .05$

Conclusion:

Cannot prove that the population mean is other than 368

Figure 1.3: Standard Normal Table



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0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
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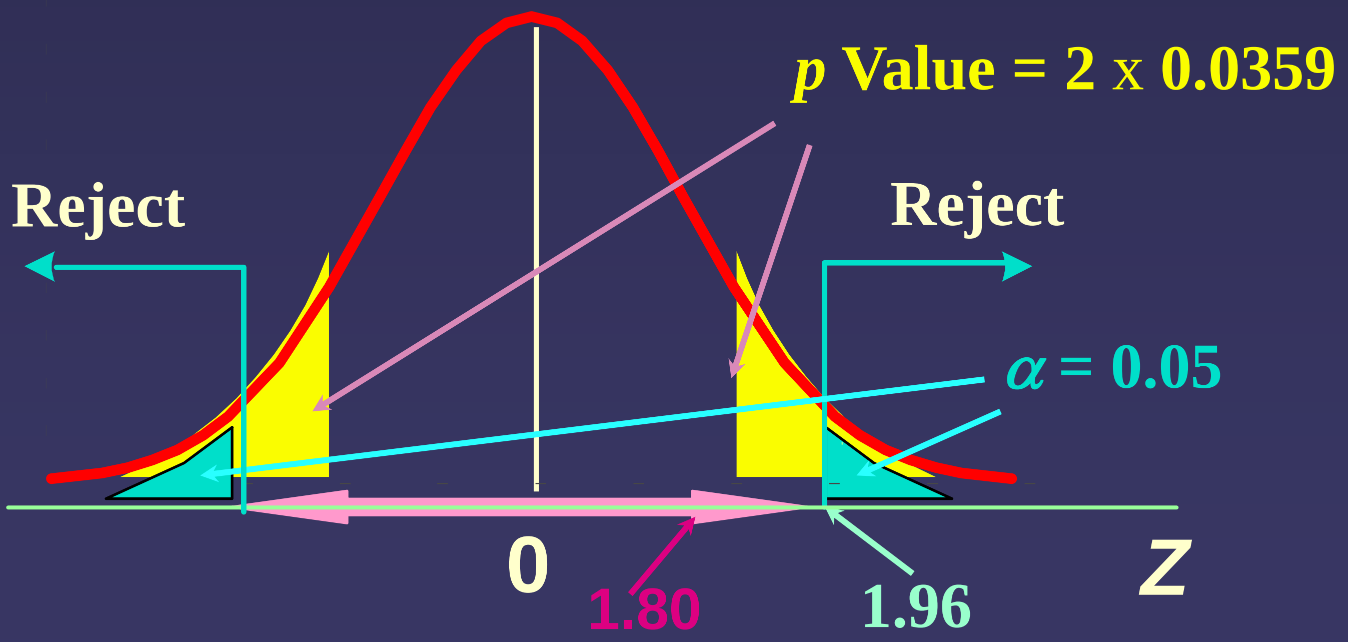
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

p Value Solution

(p Value = 0.0718) $\geq$ (α = 0.05)
Do Not Reject.



The Z Test and Confidence Interval

You will find both give the same conclusion but the test is easier to use and provides more information.

Connection to Confidence Intervals

For $\bar{X} = 372.5$ oz, $\sigma = 15$ and $n = 36$,

The 95% Confidence Interval is :

$$372.5 - (1.96)\,{}^{15}\!/\!_{\sqrt{36}} \text{ to } 372.5 + (1.96)\,{}^{15}\!/\!_{\sqrt{36}}$$

or

$$367.6 \le \mu \le 377.4$$

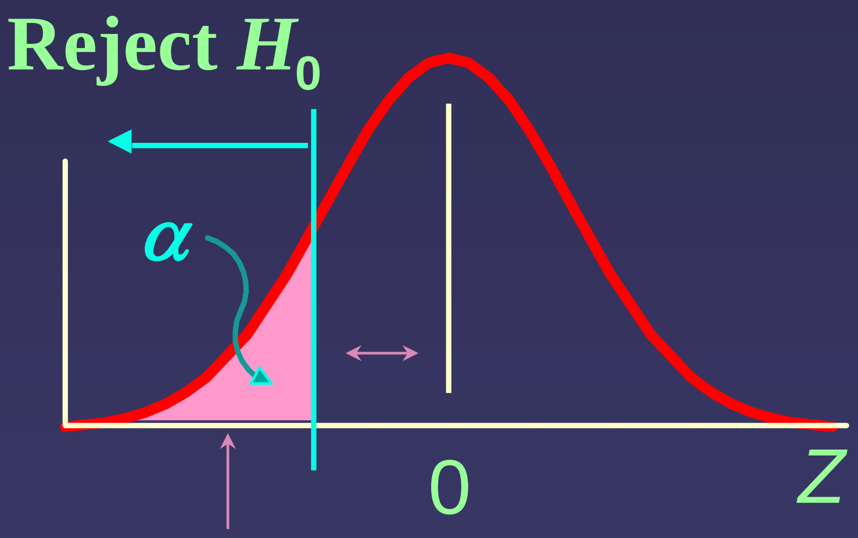
If this interval contains the hypothesized mean (368), we do not reject the null hypothesis.

An Example of the 5 Steps

For the one tail test on the mean where the population standard deviation σ is known.

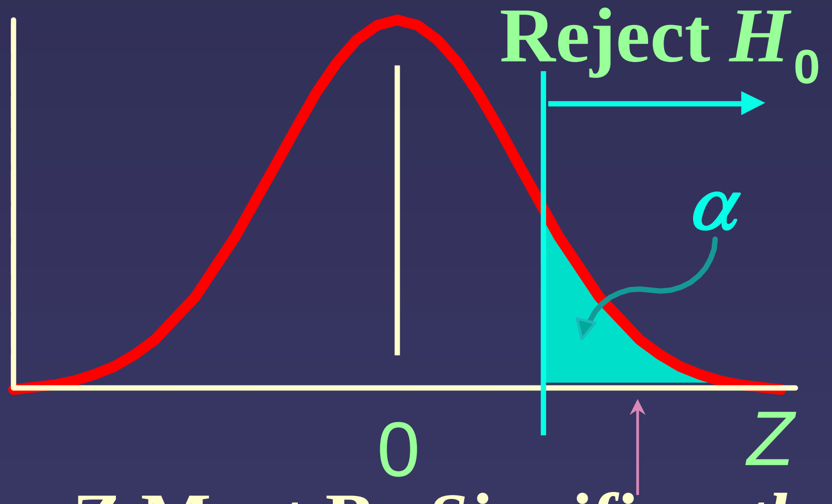
One Tail Tests

H_0: $\mu \geq \mu_0$
H_1: $\mu < \mu_0$



Z Must Be *Significantly* Below 0 to reject H_0

H_0: $\mu \leq \mu_0$
H_1: $\mu > \mu_0$



Z Must Be *Significantly* Above 0 to reject H_0

One-Tail Z Test for the Mean (σ Known)

Assumptions not changed:

Population is normally distributed.

If not normal, use large samples.

Z **test statistic not changed:**

$$Z = \frac{\overline{X} - \mu_{\overline{X}}}{\sigma_{\overline{X}}} = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

Example: One Tail Test

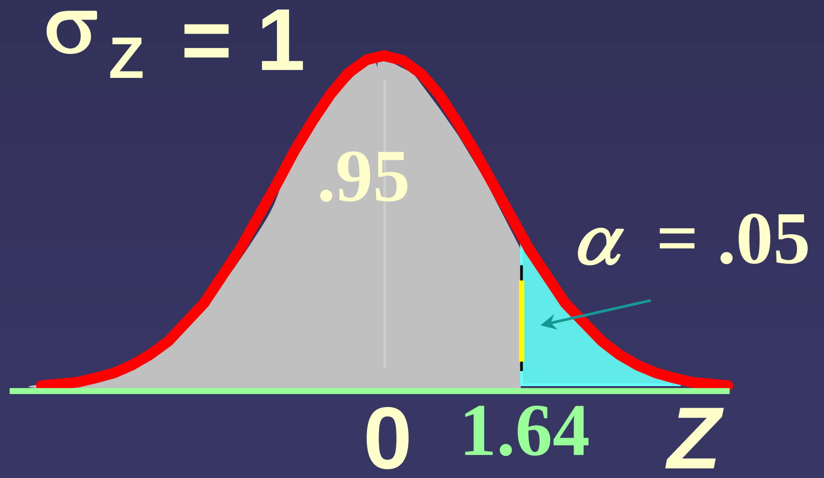
Does an average box of cereal contain more than 368 grams of cereal? A random sample of 36 boxes showed $\bar{X} = 372.5$. The company has specified σ to be 15 grams. Test at the $\alpha = 0.05$ level

H_0: $\mu \leq 368$

H_1: $\mu > 368$

Finding Critical Values: One Tail Tests

What is Z given $\alpha = 0.05$?



Critical Value = 1.64

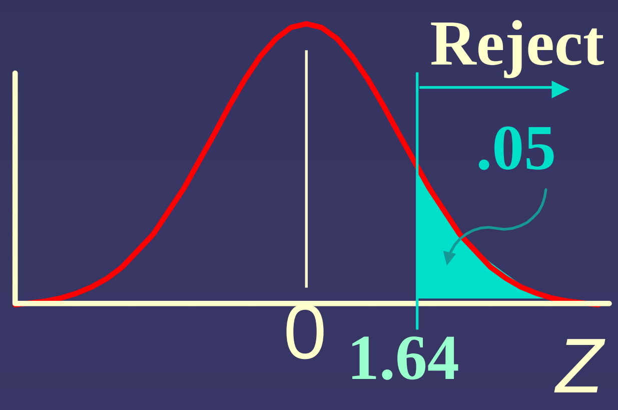
Example Solution: One Tail

H_0: $\mu \leq$ 368
H_1: $\mu >$ 368

$\alpha = 0.5$

$n = 36$

Critical value: 1.645



Test Statistic:

$$z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{372.5 - 368}{15 / \sqrt{36}} = 1.8$$

Decision:

Reject at $\alpha = .05$

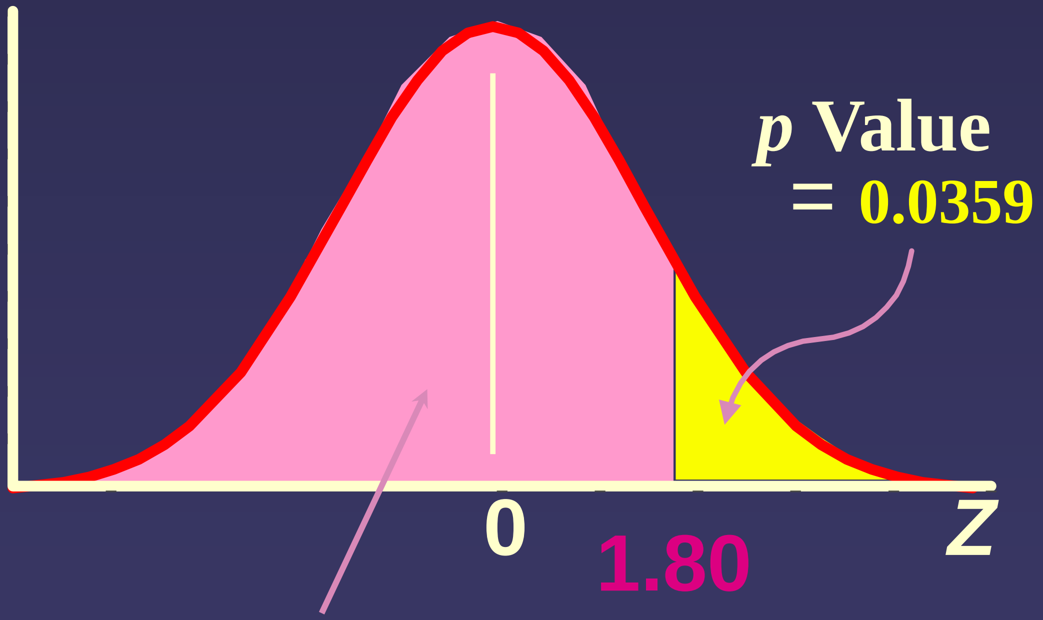
Conclusion:

The population mean is more than 368

p Value Solution

p Value is $P(Z \geq 1.80) = 0.0359$

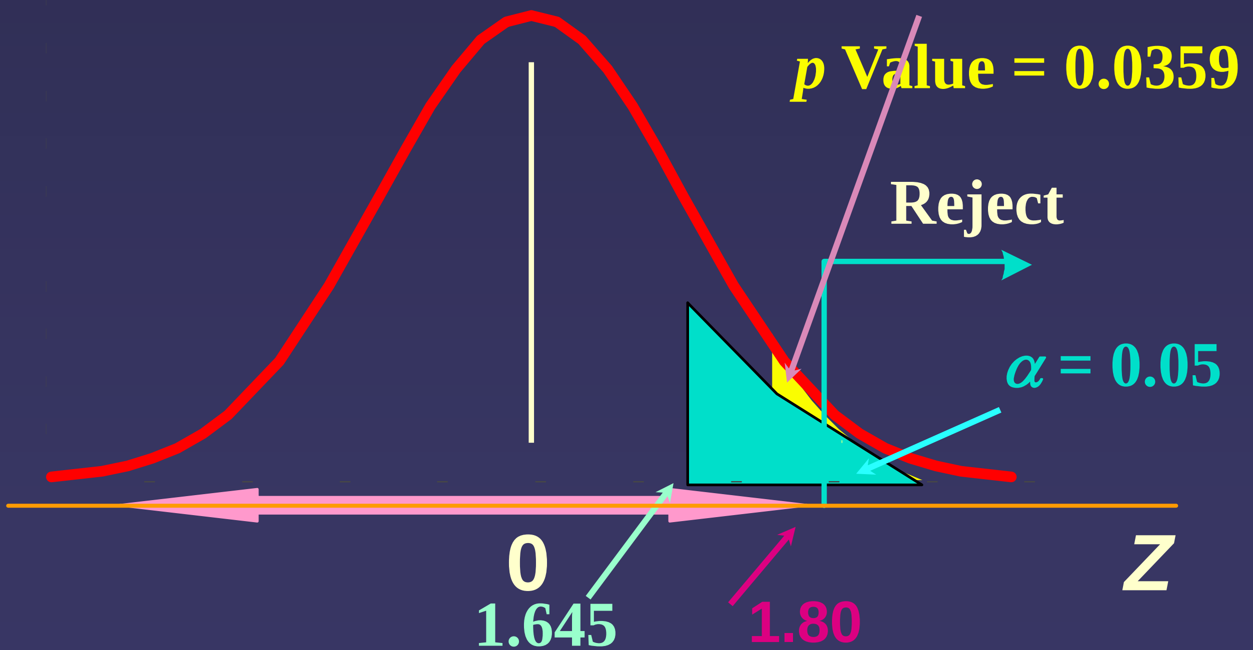
Use the alternative hypothesis to find the direction of the rejection region.



Z Value of Sample Statistic

p Value Solution

(p Value = 0.0359) ≤ (α = 0.05)
Reject.



Test Statistic 1.80 Is In the Reject Region

An Example of the 5 Steps

For the test on the mean where the population standard deviation σ is not known.

t Test for the Mean (σ Unknown)

Assumption:

The population is normally distributed or only slightly skewed & a large sample taken.

t test with n-1 degrees of freedom

$$t = \frac{\overline{X} - \mu}{S / \sqrt{n}}$$

Example: One Tail t-Test

Does an average box of cereal contain more than 368 grams of cereal? A random sample of 25 boxes showed $\bar{X}$ = 372.5, and s = 15. Test at the α = 0.01 level.

H_0: $\mu \le 368$
H_1: $\mu > 368$

σ is not given

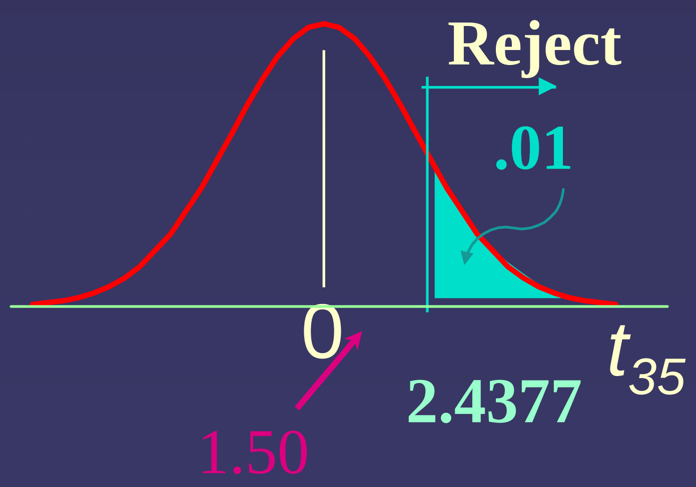
Example Solution: One Tail

H_0: $\mu \le 368$
H_1: $\mu > 368$

$\alpha = 0.01$

$n = 25$, df = 24

Critical value: 2.492



Test Statistic:

$$t = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{372.5 - 368}{15 / \sqrt{25}} = 1.5$$

Decision:

Do Not Reject at $\alpha = .01$

Conclusion:

Cannot prove that the population mean is more than 368

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646