2018 SECTION-A

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$\boxed{Q.1(b)}$ Given that, $\phi = x^2yz^3$

$$\vec{\nabla}.\phi = \left(\frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{K}\right).(x^2yz^3)$$

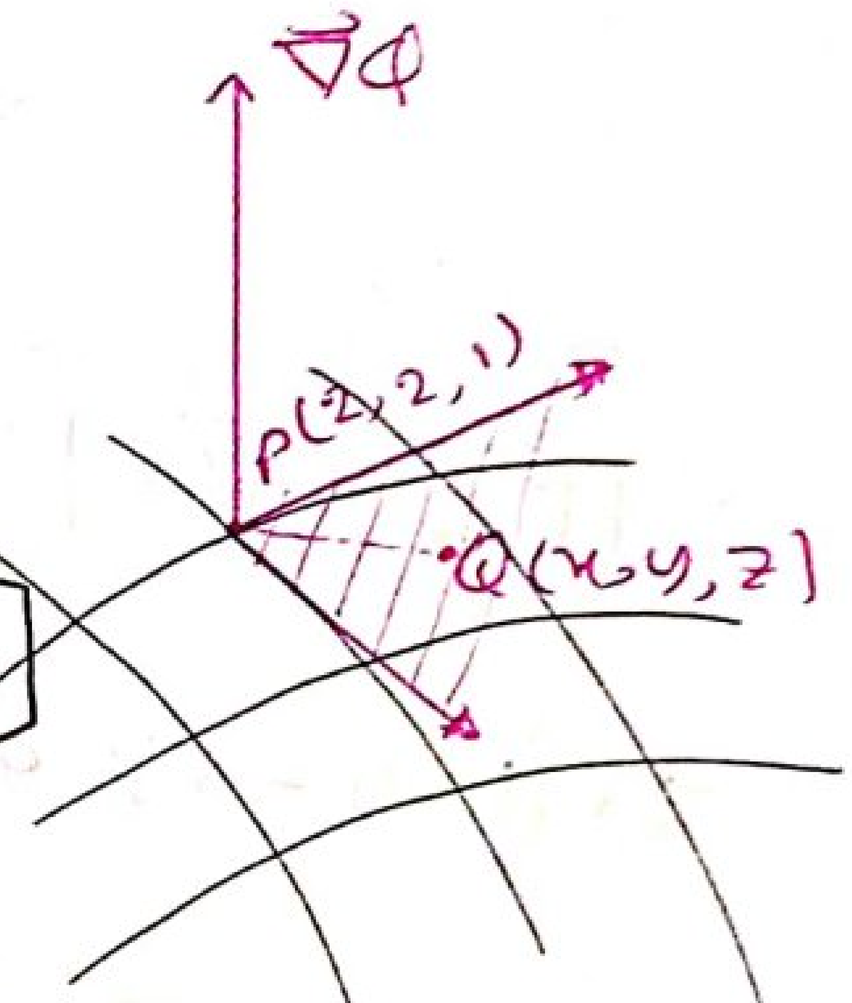
$$= 2xyz^3\underline{i} + x^2z^3\underline{j} + 3x^2yz^2\underline{K}$$

$$= -4i - 4\underline{j} + 12\underline{K} \quad [\text{at point } (2,1,-1)]$$

The directional derivative of ϕ is a maximum in the direction $\bar{\nabla}.\phi = -4\underline{i} - 4\underline{j} + 12\underline{K}$

The magnitude of this maximum is

$$= \sqrt{(-4)^2 + (-4)^2 + 12^2} = 4\sqrt{11}$$



$\boxed{Q.1(c)}$ $\phi = xyz^2 - 4$

$$\vec{\nabla}\phi = yz^2\underline{i} + xz^2\underline{j} + 2xyz\,\underline{K}$$

$$= 8\underline{i} + 8\underline{j} + 16\,\underline{K} \quad [\text{at } P(2,2,1)]$$

$$\bar{N} = 8\underline{i} + 8\underline{j} + 16\,\underline{K}$$

Let, a position vector r_0 at the tangent plane. Then the equation of the tangent plane is

$$\vec{PQ} . \vec{\nabla}\phi = 0$$

$$\Rightarrow [(x-2)i + (y-2)j + (z-1)\underline{K}].[8i + 8j + 16\underline{K}] = 0$$

$$\Rightarrow 8(x-2) + 8(y-2) + 16(z-1) = 0$$

$$\Rightarrow 8x - 16 + 8y - 16 + 16z - 16 = 0$$

$$\Rightarrow 8x + 8y + 16z = 48$$

$$\Rightarrow \therefore x + y + 2z = 6 \quad (\text{Ans})$$

The equations of the normal line to the surface is

$$|\vec{PQ} \times \vec{\nabla}Q| = 0$$

Now, $\vec{PQ} \times \vec{\nabla}Q = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x-2 & y-2 & z-1 \\ 8 & 8 & 16 \end{vmatrix}$

$$= \underline{i}(16y-32-8z+8) - \underline{j}(16x-32-8z+8) + \underline{k}(8x-16-8y+16)$$

$$= (16y-8z-24)\underline{i} + \underline{j}(8z-16x+24) + \underline{k}(8x-8y)$$

$$\therefore |\vec{PQ} \times \vec{\nabla}Q| = \sqrt{(16y-8z-24)^2+(8z-16x+24)^2+(8x-8y)^2}$$

$$\Rightarrow (16y-8z-24)^2+(8z-16x+24)^2+(8x-8y)^2 = 0$$

$\therefore 2y - z - 3 = 0$

$z - 2x + 3 = 0$

$x - y = 0$ (Ans)

$\boxed{\text{Q. 3(a)}}$

We know that, $L\{\sin bt\} = \dfrac{b}{s^2+b^2} = F(s)$

and first shifting property,

if $L\{f(t)\} = F(s)$

then $L\{e^{at} f(t)\} = F(s-a)$

$$\therefore L\{e^{at}\sin bt\} = \frac{b}{(s-a)^2+b^2} \quad \text{(proved)}$$

or,

$$L\{e^{at}\sin bt\} = \int_0^\infty e^{-st}\cdot e^{at}\sin bt\,dt$$

$$= \int_0^\infty e^{-t(s-a)}\sin bt\,dt$$

$$= \int_0^\infty e^{-tu}\sin bt\,dt \quad [\text{Let, } s-a=u]$$

$$= \cancel{F(u)}$$

$$= \frac{b}{u^2+b^2}$$

$$= \frac{b}{(s-a)^2+b^2} \quad \text{(proved)}$$

$\boxed{Q.3(a)}$ $L^{-1}\left\{\frac{2s+5}{(s^2+1)(s-4)}\right\}$

Let,

$$\frac{2s+5}{(s^2+1)(s-4)} = \frac{A}{(s-4)} + \frac{Bs+C}{(s^2+1)(s-4)}$$

$$\Rightarrow 2s+5 = A(s^2+1) + (Bs+c)(s-4)$$

$$\Rightarrow 2s+5 = As^2 + A + Bs^2 + Cs - 4Bs - 4c$$

$$\Rightarrow 2s+5 = s^2(A+B) + s(c-4B) + (A-4c)$$

Equating the coefficient of s^2, s and constant we get,

$A + B = 0$ —— (i)

$C - 4B = 2$ —— (ii)

$A - 4C = 5$ —— (iii)

Solving these three equation we get,

$A = \frac{13}{17}$, $B = -\frac{13}{17}$, $C = -\frac{18}{17}$

$$\therefore \frac{2s+5}{(s^2+1)(s-4)} = \frac{13}{17(s-4)} - \frac{13s+18}{17(s^2+1)}$$

$$= \frac{13}{17(s-4)} - \frac{13s}{17(s^2+1)} - \frac{18}{17(s^2+1)}$$

$$\therefore L^{-1}\left\{\frac{2s+5}{(s^2+1)(s-4)}\right\}$$

$$= L^{-1}\left\{\frac{13}{17(s-4)} - \frac{13s}{17(s^2+1)} - \frac{18}{17(s^2+1)}\right\}$$

$$= \frac{13}{17}e^{4t} - \frac{13}{17}\cos t - \frac{18}{17}\sin t$$

$\boxed{Q.4(a)}$ Given that,

$$t\frac{d^2y}{dt^2} + \frac{dy}{dt} + 4ty = 6 \quad \text{where } y'(0) = 0, y(0) = 3$$

$$\Rightarrow t y'' + y' + 4ty = 0 \text{ —— (i)}$$

Taking the laplace transform of both sides of (i) we get,

$L\{t y''\} + L\{y'\} + 4L\{t y\} = 0$

$\Rightarrow -\frac{d}{ds}\{s^2 Y(s) - sY(0) - Y'(0)\} + \{sY(s) - Y(0)\} - 4\frac{d}{ds}Y(s) = 0$

$\Rightarrow -\frac{d}{ds}\{s^2 Y(s) - 3s\} + sY(s) - 3 - 4\frac{dY}{ds} = 0$

$\Rightarrow -s^2\frac{dY}{ds} - 2sY + 3 + sY - 3 - 4\frac{dY}{ds} = 0$

$\Rightarrow -\frac{dY}{ds}(s^2+4) - sY = 0$

$\Rightarrow (s^2+4)\frac{dY}{ds} + sY = 0$

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$\Rightarrow \frac{dY}{ds} = -\frac{sY}{(s^2+4)}$

$\Rightarrow \frac{dY}{Y} + \frac{1}{2}\frac{2s}{s^2+4}\,ds = 0$

Integrating ~~bot~~ we get,

$\log Y + \frac{1}{2}\log(s^2+4) = \log c$

$\Rightarrow \log Y = \log\left(\frac{c}{\sqrt{s^2+4}}\right)$

$\Rightarrow Y(s) = \frac{c}{\sqrt{s^2+4}}$

Taking inverse laplace transform of both side we get,

$L^{-1}\{Y(s)\} = L^{-1}\left\{\frac{c}{\sqrt{s^2+4}}\right\}$

$\Rightarrow Y(t) = \cancel{\frac{c}{2}\sin 2t}\; c\,J_0(2t)$

~~Here, $y(0) = 3$~~

$\Rightarrow y'(t) = \frac{c}{2} \cdot 2 \cos 2t$

$\Rightarrow y'(0) = c \cos 0$

$\Rightarrow c = 0$

Here, $y(0) = c\, J_0(0)$

$\Rightarrow 3 = c \cdot 1$ $\quad [\because J_0(0) = 1]$

$\therefore c = 3$

$\therefore Y = 3 J_0(2t)$ (Ans)

Q.4(b) Given that,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{---} \text{ (1)}, \quad u(x,0) = 3\sin 2\pi x, \; u(0,t) = 0$$

and $u(1,t) = 0$ where $0 < x < 1$

$\Rightarrow$ Taking Laplace transform of the PDE (1), we have $\quad$ — $u = u(x,t)$

$$L\left\{\frac{\partial u}{\partial t}\right\} = L\left\{\frac{\partial^2 u}{\partial x^2}\right\}$$

$$\Rightarrow s u - u(x,0) = \frac{d^2 u}{dx^2} \quad [u = u(x,s)]$$

$$\Rightarrow \frac{d^2 u}{dx^2} - s u = u(x,0)$$

$$\Rightarrow \frac{d^2 u}{dx^2} - s u = -3\sin 2\pi x$$

$$\Rightarrow (D^2 - s) u = -3\sin 2\pi x$$

The auxiliary equation is: $m^2 - 5 = 0$

$\therefore m = \pm\sqrt{5}$

$\therefore CF = U_c = c_1 e^{\sqrt{5}x} + c_2 e^{-\sqrt{5}x}$

$$U_p = \frac{1}{D^2-5}\{-3\sin 2\pi x\}$$

$$= \frac{-3\sin 2\pi x}{-4\pi^2-5}$$

$$= \frac{3\sin 2\pi x}{4\pi^2+5}$$

$$\therefore U = U_c + U_p = c_1 e^{\sqrt{5}x} + c_2 e^{-\sqrt{5}x} + \frac{3\sin 2\pi x}{4\pi^2+5} \quad \text{(2)}$$

Taking Laplace transform on boundary conditions:

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$L\{u(0,t)\} = u(0,s) = 0$

$L\{u(1,t)\} = u(1,s) = 0$

Using these condition

$c_1 + c_2 = 0$ — (iii)

$c_1 e^{\sqrt{5}} + c_2 e^{-\sqrt{5}} = 0$ — (iv)

Solving (iii) and (iv) we get $c_1 = 0,\ c_2 = 0$

$$\therefore U = \frac{3\sin 2\pi x}{4\pi^2 + 5}$$

Taking inverse laplace transform on both side

$$u(x,t) = 3\sin 2\pi x\ L^{-1}\left\{\frac{1}{s+(2\pi)^2}\right\}$$

$\therefore u(x,t) = 3\sin 2\pi x \, e^{-4\pi^2 t}$

2017 SECTION-A

$\boxed{Q1(a)}$

Given that, $\vec{r_1} = 2\vec{a} - 3\vec{b} + \vec{c}$,

$\vec{r_2} = 3\vec{a} - 5\vec{b} + 2\vec{c}$

$\vec{r_3} = 4\vec{a} - 5\vec{b} + \vec{c}$

~~If $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ are~~

$[\vec{r_1}, \vec{r_2}. \vec{r_3}]$

$$= \begin{vmatrix} 2 & -3 & 1 \\ 3 & -5 & 2 \\ 4 & -5 & 1 \end{vmatrix}$$

$= 2(-5+10) + 3(3-8) + 1(-15+20)$

$= \cancel{10}\ 2\times 5 - 3\times 5 + 5$

$= 10 - 15 + 5 = 0$

$\therefore \vec{r_1}, \vec{r_2}, \vec{r_3}$ are linearly dependent.

$\boxed{Q.1(b)}$ Given that, $\vec{A} = 2\underline{i} + \underline{j} - 3\underline{k}$

$\vec{B} = \underline{i} - 2\underline{j} + \underline{k}$

Let a position vector, $\vec{r} = x\underline{i} + y\underline{j} + z\underline{k}$

$\vec{A}\cdot\vec{r} = 2x + y - 3z$

$\therefore 2x + y - 3z = 0$ ——— (i)

$\vec{B}\cdot\vec{r} = x - 2y + z$

$\therefore x - 2y + z = 0$ ——— (ii)

$r = \sqrt{x^2+y^2+z^2}$

$\Rightarrow x^2+y^2+z^2 = 5^2$ —— (iii)

(i) $\times 2$ + (ii)

$4x + 2y - 6z = 0$

$x - 2y + z = 0$

$5x - 5z = 0$

$\therefore x = z$

From (ii) $x - 2y + x = 0$

$\therefore x = y$

$\therefore x = y = z$

From (iii) $\Rightarrow 3x^2 = 5^2$

$\Rightarrow x = \sqrt{\frac{5^2}{3}}$

$\therefore x = y = z = \pm\frac{5}{\sqrt{3}}$

$\therefore \vec{r} = \pm\frac{5}{\sqrt{3}}(\underline{i}+\underline{j}+\underline{k})$ (Ans)

$\boxed{Q.2(a)}$ Given that, $\vec{F} = (\sin y + z)\,i + (x\cos y - z)\,\underline{j} + (x-y)\,\underline{k}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin y + z & x\cos y - z & x - y \end{vmatrix}$$

$= \underline{i}(-1+1) - \underline{j}(1-1) + \underline{k}(\cos y - \cos y)$

$= 0$

Hence $\vec{F}$ is irrotational.

Given that,

$$\vec{\nabla}\phi = \vec{F}$$

$$\Rightarrow \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k = (\sin y + z) i + (x\cos y - z) j + (x-y) k$$

$$\therefore \frac{\partial\phi}{\partial x} = \sin y + z \quad \text{—— (i)}$$

$$\frac{\partial\phi}{\partial y} = x\cancel{a}\cos y - z \quad \text{—— (ii)}$$

$$\frac{\partial\phi}{\partial z} = x - y \quad \text{—— (iii)}$$

Integrating (i) w.r.t x we get,

$$\phi = x\sin y + xz + f(y,z) \quad \text{—— (iv)}$$

partially differentiating eqn (iv) w.r.to y

$$\frac{\partial\phi}{\partial y} = x\cos y + \frac{df'(y,z)}{dy} \quad \text{—— (v)}$$

Comparing (v) and (ii) we get,

$$\cancel{f}\; x\cos y - z = x\cos y + f'(y,z)$$

$$\Rightarrow \frac{df^{\cancel{\prime}}(y,z)}{dy} = -z$$

$$\Rightarrow f(y,z) = -yz + g(z) \quad \text{—— (vi)} \quad \boxed{\text{By integrating}}$$

$$\therefore \phi = x\sin y + xy - yz + g(z) \quad \text{—— (vii)}$$

Partially differentiating (vii) w.r.to. z

$$\frac{\partial\phi}{\partial z} = -y + g'(z) \quad \text{—— (viii)}$$

comparing eqn (viii) and (viii)

$$\therefore \quad g'(z) = x$$

$\therefore \cancel{\phi = x}$

$\Rightarrow g(z) = zx + c$

$\therefore \phi = x \sin y + xy - yz + zx + c$

which is the required scalar function.

$\boxed{Q.3(b)}$

If $F(t)$ be a function of t defined for all positive value of t i.e $t > 0$, the the Laplace transform of $F(t)$ denoted by $L\{F(t)\}$ or $F(s)$ is defined by the expression

$$L\{F(t)\} = F(s) = \int_0^\infty e^{-st} F(t)\, dt$$

∂ $L\{9e^{-7t} - 3t^6 - 2\sin 5t - 5t^2\cos 2t\}$

$$= 9L\{e^{-7t}\} - 3L\{t^6\} - 2L\{\sin 5t\} - 5L\{t^2 \cos 2t\}$$

$$= 9\frac{1}{s+7} - 3\frac{6!}{s^7} - 2\frac{5}{s^2+5^2} - 5(-1)^2 \frac{d^2}{ds^2} \cdot \frac{s}{s^2+2^2}$$

$$= \frac{9}{s+7} - \frac{2160}{s^7} - \frac{10}{s^2+25} - 5\frac{d}{ds}\left\{\frac{(s^2+4)\cdot 1 - s \cdot 2s}{(s^2+4)^2}\right\}$$

$$= \frac{9}{s+7} - \frac{2160}{s^7} - \frac{10}{s^2+25} - 5\frac{d}{ds}\frac{4-s^2}{(s^2+4)^2}$$

$$= \frac{9}{s+7} - \frac{2160}{s^7} - \frac{10}{s^2+25} - 5\left\{\frac{(s^2+4)^2 \cdot (-2s) - (4-s^2)\cdot 2(s^2+4)\cdot 2s}{(s^2+4)^4}\right\}$$

$$= \frac{9}{s+7} - \frac{2160}{s^7} - \frac{10}{s^2+25} - 5\left\{\frac{2s(s^2+4)(-s^2-4-8+2s^2)}{(s^2+4)^4}\right\}$$

$$= \frac{9}{s+7} - \frac{2160}{s^7} - \frac{10}{s^2+25} - \frac{10s(2s^2-9)}{(s^2+4)^3}$$

(Ans)

$\boxed{Q.3(c)}$

By the theorem,

If $L^{-1}\{F(s)\} = F(t)$

$\therefore L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t F(u)\,du$

Here $F(s) = \frac{1}{s^2+1}$

$\therefore L^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t = F(t)$ $\therefore F(u) = \sin u$

$$\therefore L^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = \int_0^t \sin u\,du$$

$$= \left[-\cos u\right]_0^t$$

$$= -\cos t + 1$$

$$L^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\} = \int_0^t (1-\cos u)\,du$$

$$= \left[u - \sin u\right]_0^t$$

$$= t - \sin t$$

$$L^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\} = \int_0^t (u - \sin t)\,du$$

$$= \left[\frac{u^2}{2} + \cos t\right]_0^t$$

$$= \frac{t^2}{2} + \cos t - 1$$

$y(0) = y'(0) = 0$

Q.4(a)

Given that, $y'' + y = e^{-t}$ —— ①

Taking L.T of both side of ① we get

$L\{y''\} + L\{y\} = L\{e^{-t}\}$

$\Rightarrow s^2Y(s) - sY(0) - Y'(0) + Y(s) = \frac{1}{s+1}$

$\Rightarrow s^2Y(s) + Y(s) = \frac{1}{s+1}$

$\Rightarrow Y(s)(s^2+1) = \frac{1}{s+1}$

$\Rightarrow Y(s) = \frac{1}{(s^2+1)(s+1)}$

Now, $\frac{1}{(s^2+1)(s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+1}$

$\Rightarrow 1 = A(s^2+1) + (Bs+C)(s+1)$

$\Rightarrow 1 = As^2 + A + Bs^2 + Cs + Bs + C$

$\Rightarrow 1 = s^2(A+B) + (C+B)s + A + C$

Equating the co-efficient of s^2, s and constant

$A + B = 0$ —— (i)

$B + C = 0$ —— (ii)

$A + C = 1$ —— (iii)

From (i) $A = -B$ —— (iv)

From (iii), $-B + C = 1$

(iii) + (iv) $\Rightarrow 2C = 1$

$\therefore C = \frac{1}{2}$

$A = \frac{1}{2}$, $B = -\frac{1}{2}$

$\therefore \frac{1}{(s^2+1)(s+1)} = \frac{1}{2(s+1)} - \frac{s}{2(s^2+1)} + \frac{1}{2(s^2+1)}$

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$$\therefore Y(s) = \frac{1}{2(s+1)} - \frac{s}{2(s^2+1)} + \frac{1}{2(s^2+1)} \quad \text{——} (v)$$

Taking inverse Laplace transform of both side of eqn (v),

$$Y(t) = \frac{1}{2} L^{-1}\left\{\frac{1}{s+1}\right\} - \frac{1}{2} L^{-1}\left\{\frac{s}{s^2+1}\right\} + \frac{1}{2} L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$\therefore Y = \frac{e^{-t}}{2} - \frac{\cos t}{2} + \frac{\sin t}{2} \quad (Ans)$$

2015 SECTION-A

$\boxed{Q.1(a)}$ ~~Trible~~

Triple product of vectors is defined as the dot product of one vector with the cross product of the other two vectors. It is denoted by $\vec{A}.(\vec{B} \times \vec{C})$.

Given that,

$$\vec{A} = 2\underline{i} - \underline{j} + \underline{k}$$

$$\vec{B} = \underline{i} + 2\underline{j} - 3\underline{k}$$

$$\bar{C} = 3\underline{i} + \lambda\underline{j} + 5\underline{k}$$

If $\vec{A}, \vec{B}, \vec{C}$ are co-planar then $[\vec{A}, \vec{B}, \vec{C}] = 0$

that is

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2(10 + 3\lambda) + 1(5 + 6) + 1(\lambda - 6) = 0$$

$\Rightarrow 20 + 6\lambda + 11 + \lambda - 6 = 0$

$\Rightarrow 7\lambda + 25 = 0$

$\therefore \lambda = -\frac{25}{7}$ (Ans)

$\boxed{Q.2(b)}$ Given that,

$H = e^{-\lambda x}(c_1 \sin \lambda y + c_2 \cos \lambda y)$

$\Rightarrow \frac{\partial H}{\partial x} = -\lambda e^{-\lambda x}(c_1 \sin \lambda y + c_2 \cos \lambda y)$

$\Rightarrow \frac{\partial^2 H}{\partial x^2} = \lambda^2 e^{-\lambda x}(c_1 \sin \lambda y + c_2 \cos \lambda y)$ — (i)

Again

$\frac{\partial H}{\partial y} = e^{-\lambda x}(c_1 \lambda \cos \lambda y - c_2 \lambda \sin \lambda y)$

$\Rightarrow \frac{\partial^2 H}{\partial y^2} = e^{-\lambda x}(-c_1 \lambda^2 \sin \lambda y - c_2 \lambda^2 \cos \lambda y)$

$\Rightarrow \frac{\partial^2 H}{\partial y^2} = -\lambda^2 e^{-\lambda x}(c_1 \sin \lambda y + c_2 \cos \lambda y)$ — (ii)

The given partial differential equation

$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$

~~Equating~~ Adding (i) and (ii)

$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = \lambda^2 e^{-\lambda x}(c_1 \sin \lambda y + c_2 \cos \lambda y) - \lambda^2 e^{-\lambda x}(c_1 \sin \lambda y + c_2 \cos \lambda y)$

$= 0$

(shown)

$\boxed{Q.1(c)}$ Given that,

$P = 4e^{2x-y+z}$

and point, $M(1,1,-1)$, $N(-3,5,6)$

The position vector of point N,

$\therefore \vec{N} = -3\underline{i} + 5\underline{j} + 6\underline{k}$

$$\text{Now, } \vec{\nabla} P = \left(\frac{\partial}{\partial x}\underline{i} + \frac{\partial}{\partial y}\underline{j} + \frac{\partial}{\partial z}\underline{k}\right).(4e^{2x-y+z})$$

$$= 4(2e^{2x-y+z}\underline{i} - e^{2x-y+z}\underline{j} + e^{2x-y+z}\underline{k})$$

$$= (8e^{0}\underline{i} - 4e^{0}\underline{j} + 4e^{0}\underline{k})$$

$$= 8\underline{i} - 4\underline{j} + 4\underline{k}$$

The unit vector in the direction of $\vec{N}$,

$$\hat{n} = \frac{-3\underline{i} + 5\underline{j} + 6\underline{k}}{\sqrt{(-3)^2 + 5^2 + 6^2}}$$

$$= \frac{-3\underline{i} + 5\underline{j} + 6\underline{k}}{\sqrt{70}}$$

Then the required directional derivative is

$$\vec{\nabla} P . \hat{n} = (8\underline{i} - 4\underline{j} + 4\underline{k}).\left(\frac{-3\underline{i} + 5\underline{j} + 6\underline{k}}{\sqrt{70}}\right)$$

$$= -\frac{24}{\sqrt{70}} - \frac{20}{\sqrt{70}} + \frac{24}{\sqrt{70}}$$

$$= -\frac{2\sqrt{70}}{7} \quad (\text{Ans})$$

Since this is ~~posit~~ negative, P is decreasing in this direction.

Q. 2(b)

$L^{-1}\left\{\frac{s+1}{6s^2+7s+2}\right\}$

Now, $\frac{s+1}{6s^2+7s+2}$

$= \frac{s+1}{6s^2+4s+3s+2}$

$= \frac{s+1}{2s(3s+2)+1(3s+2)}$

$= \frac{s+1}{(3s+2)(2s+1)}$

Let, $\therefore \frac{s+1}{(2s+1)(3s+2)} = \frac{A}{(2s+1)} + \frac{B}{(3s+2)}$

we get,

$\therefore A = 1$ and $B = -1$

$\therefore \frac{s+1}{(2s+1)(3s+2)} = \frac{1}{2s+1} + -\frac{1}{(3s+2)}$

$\therefore L^{-1}\left\{\frac{s+1}{6s^2+7s+2}\right\}$

$= L^{-1}\left\{\frac{1}{2s+1}\right\} - L^{-1}\left\{\frac{1}{3s+2}\right\}$

$= L^{-1}\left\{\frac{1}{2(s+\frac{1}{2})}\right\} - L^{-1}\left\{\frac{1}{3(s+\frac{2}{3})}\right\}$

$= \frac{1}{2} L^{-1}\left\{\frac{1}{s+1/2}\right\} - \frac{1}{3} L^{-1}\left\{\frac{1}{s+2/3}\right\}$

$= \frac{1}{2} e^{-t/2} - \frac{1}{3} e^{-2t/3}$ (Ans)

Q.2 (c)

LINATE (d)

$$L\left\{\frac{d^2y}{dt^2}\right\} = L\{y''\}$$

$$\cancel{= L\{y''\}} = \int_0^\infty e^{-st}\, y''\, dt$$

$$= \left[e^{-st} y'\right]_0^\infty - \int_0^\infty (-s\, e^{-st} y')\, dt$$

$$= -y'(0) + s\int_0^\infty e^{-st} y'\, dt$$

$$= -y'(0) + s\left\{\left[e^{-st} y\right]_0^\infty - \int_0^\infty (-s\, e^{-st} y)\, dt\right.$$

$$= -y'(0) - s\,y(0) + s^2 \int_0^\infty e^{-st} y\, dt$$

$$= s^2 y - s\,y(0) - y'(0)$$

(shown)

Q.6 (i)

Given that,

$$Y'' + 2Y' + 5Y = e^{-t}\sin t \quad \text{—(1)} \qquad Y(0) = 0,\ Y'(0) = 0$$

Taking Laplace transform of both side of equ (1)

$$s^2 y - s\,Y(0) - Y'(0) + 2sy - 2Y(0) + 5y = \frac{1}{(s+1)^2+1}$$

$$\Rightarrow s^2 y - 0 - 1 + 2sy - 0 + 5y = \frac{1}{s^2+2s+2}$$

$$\Rightarrow y(s^2 + 2s + 5) = \frac{1}{s^2+2s+2} + 1$$

$$\Rightarrow Y = \frac{1}{(s^2+2s+2)(s^2+2s+5)} + \frac{1}{(s^2+2s+5)}$$

$$= \frac{1 + s^2 + 2s + 2}{(s^2+2s+2)\,(s^2+2s+5)}$$

$$= \frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$$

Let, $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)} = \frac{As+B}{(s^2+2s+5)} + \frac{Cs+D}{(s^2+2s+2)}$

$$\Rightarrow s^2+2s+3 = (As+B)(s^2+2s+2) + (Cs+D)(s^2+2s+5)$$

$$= As^3+Bs^2+2As^2+2Bs+2As+2B+Cs^3+Ds^2$$

$$+ 2Cs^2+2Ds+5Cs+5D$$

$$= s^3(A+C)+s^2(B+2A+D+2C)+(2B+5D)$$

$$+s(2B+2A+2D+5C)$$

$s^3 \longrightarrow 0 = A+C \quad \Rightarrow A=-C$ —— (i)

$s^2 \Longrightarrow 1 = B+2A+D+2C$

$\Rightarrow B+D-2C+2C = 1$

$\therefore B+D = 1$ —— (ii)

$s \longrightarrow 2 = 2B+2A+2D+5C$

$\Rightarrow 2(B+D) - 2C+5C = 2$

$\Rightarrow 2+3C = 2$

$\therefore C = 0$

$\therefore A = 0$

$s^0 \longrightarrow 2B+5D = 3$ —— (iii)

(iii) $-$ (ii) $\times 2$

$$\begin{array}{r} 2B+5D = 3 \\ -2B-2D = -2 \\ \hline 3D = 1 \end{array}$$

$\therefore D = \frac{1}{3}$

$B = 1-\frac{1}{3} = \frac{2}{3}$

$$y(s) = \frac{1}{3(s^2+2s+2)} + \frac{2}{3(s^2+2s+5)}$$

$$= \frac{1}{3\{(s^2+1)^2+1\}} + \left\{\frac{2}{3(s+1)^2+2^2}\right\}$$

Taking ~~to~~ Inverse laplace transform,

$$y(t) = \frac{1}{3}L^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} + \frac{2}{3}L^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}$$

$$= \frac{1}{3}e^{-t}\sin t + \frac{1}{3}e^{-t}\sin 2t \quad \text{(A)}$$

$\boxed{Q.d(ii)}$ Given that,

$Y'' - tY' + Y = 1$ — ①, $Y(0) = 1$, $Y'(0) = 2$

Taking Laplace transform of both side of ①

$$s^2y - sy(0) - y'(0) - (-1)^1\frac{d}{ds}\{y'\} + y = \frac{1}{s}$$

$$\Rightarrow s^2y - s - 2 + s\frac{dy}{ds} + y - 0 + y = \frac{1}{s}$$

$$\Rightarrow y(s^2+2) + s\frac{dy}{ds} = \frac{1}{s} + s + 2$$

$$\Rightarrow \frac{dy}{ds} + \left(s + \frac{2}{s}\right)y = \frac{1}{s^2} + 1 + \frac{2}{s}$$

$$IF = e^{\int\left(s+\frac{2}{s}\right)ds} = e^{\frac{s^2}{2}+2\ln s}$$

$$= e^{s^2/2}\cdot e^{\ln s^2}$$

$$= s^2e^{s^2/2}$$

$$\therefore y s^2 e^{s^2/2} = \cancel{\int (1+2\,\cdot +} \int \left(\frac{1}{s^2} + 1 + \frac{2}{s}\right) s^2 e^{s^2/2}\, ds$$

$$= \int (1 + 2s + s^2)\, e^{s^2/2}\, ds$$

$$\Rightarrow s^2 e^{s^2/2} y = \cancel{s e^{s^2/2} + 2 e^{s^2/2} +}$$

$$= 2 e^{s^2/2} + 2s \cdot \frac{e^{s^2/2}}{1/2} - 2 \times 2 \times e^{s^2/2}$$

$$+ s^2 \frac{e^{s^2/2}}{s} - \int 2s \frac{e^{s^2/2}}{1/2}\, ds$$

$$\Rightarrow s^2 e^{s^2/2} y = \quad - \quad - \quad -$$

$$\Rightarrow s^2 e^{s^2/2} y = s e^{s^2/2} + 2 e^{s^2/2} + c_1$$

$$\therefore y = \frac{1}{s} + \frac{2}{s^2} + \frac{c}{s^2} e^{s^2/2}$$

$c = 0$

$$\therefore y = \frac{1}{s} + \frac{2}{s^2}$$

Taking inverse laplace transform.

$$y(t) = 1 + 2t \quad (A)$$

2014 SECTION - B

$\boxed{Q.5(c)}$

Given that,

$$\phi = 2xz^2 - 3xy - 4x - 7$$

and point $P(1, -1, 2)$

$$\vec{\nabla}\phi = (2z^2 - 3y - 4)i + (-3x)j + (4xz)k$$

$$= (2z^2 - 3y - 4)i - 3xj + 4xzk$$

$$= 7i - 3j + 8k$$

Let a position vector on the tangent plane $\vec{r} = xi + yj + zk$

$$\therefore \vec{P} = i - j + 2k$$

$$\therefore \vec{Pr} = (x-1)i + (y+1)j + (z-2)k$$

The equation of the tangent plane is

$$\vec{Pr} \cdot \vec{\nabla}\phi = 0$$

$$\cancel{\Rightarrow \{(x-1)i + (y+1)j + (z-2)k\} \cdot \{(2z^2 - 3y - 4)i - 3xj + 4xzk\} = 0}$$

$$\cancel{\Rightarrow (x-1)(2z^2 - 3y - 4) - (y+1) \times 3x + (z-2) \times 4xz = 0}$$

$$\cancel{\Rightarrow 2xz^2 - 3xy - 2z^2 + 3y + 4 - 3xy - 3x + 4xz^2 - 8xz = 0}$$

$$\cancel{\Rightarrow -3x + 3y - 6xy + 6xz^2 - 2z^2 - 8xz + 4 = 0}$$

$$\Rightarrow \{(x-1)i + (y+1)j + (z-2)k\} \cdot (7i - 3j + 8k) = 0$$

$$\Rightarrow 7x - 7 - 3y - 3 + 8z - 16 = 0$$

$$\Rightarrow 7x - 3y + 8z = 26 \quad \text{(Ans)}$$

Q.6(b)

(i) $\vec{\nabla}\times\vec{\nabla}\phi = \nabla\times\left(\frac{\partial\phi}{\partial x}\underline{i}+\frac{\partial\phi}{\partial y}\underline{j}+\frac{\partial\phi}{\partial z}\underline{k}\right)$

$$=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix}$$

$$=\underline{i}\left(\frac{\partial}{\partial y}\frac{\partial\phi}{\partial z}-\frac{\partial}{\partial z}\frac{\partial\phi}{\partial y}\right)+\underline{j}\left(\frac{\partial}{\partial z}\frac{\partial\phi}{\partial x}-\frac{\partial}{\partial x}\frac{\partial\phi}{\partial z}\right)$$
$$+\underline{k}\left(\frac{\partial}{\partial x}\frac{\partial\phi}{\partial y}-\frac{\partial}{\partial y}\frac{\partial\phi}{\partial x}\right)$$

$$=\left(\frac{\partial^2\phi}{\partial y\partial z}-\frac{\partial^2\phi}{\partial z\partial y}\right)i+\left(\frac{\partial^2\phi}{\partial z\partial x}-\frac{\partial^2\phi}{\partial x\partial z}\right)\underline{j}$$
$$+\left(\frac{\partial^2\phi}{\partial x\partial y}-\frac{\partial^2\phi}{\partial y\partial x}\right)\underline{k}$$

$=0$ (shown)

(ii) $\vec{\nabla}\cdot\vec{\nabla}\times A=\vec{\nabla}\cdot\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$

$$=\vec{\nabla}\cdot\left[\underline{i}\left(\frac{\partial A_3}{\partial y}-\frac{\partial A_2}{\partial z}\right)+\underline{j}\left(\frac{\partial A_1}{\partial z}-\frac{\partial A_3}{\partial x}\right)+\underline{k}\left(\frac{\partial A_2}{\partial x}-\frac{\partial A_1}{\partial y}\right)\right]$$

$$=\frac{\partial}{\partial x}\left(\frac{\partial A_3}{\partial y}-\frac{\partial A_2}{\partial z}\right)+\frac{\partial}{\partial y}\left(\frac{\partial A_1}{\partial z}-\frac{\partial A_3}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\partial A_2}{\partial x}-\frac{\partial A_1}{\partial y}\right)$$

$$=\frac{\partial^2 A_3}{\partial x\partial y}-\frac{\partial^2 A_2}{\partial x\partial z}+\frac{\partial^2 A_1}{\partial y\partial z}-\frac{\partial^2 A_3}{\partial y\partial x}+\frac{\partial^2 A_2}{\partial z\partial x}-\frac{\partial^2 A_1}{\partial z\partial y}$$

$=0$ (shown)

$\boxed{Q.6(c)}$ If $\vec{v}(x,y,z)$ is a differentiable vector field then the curl of $\vec{v}$, written $\vec{\nabla} \cdot \vec{v}$. If the curl of the vector is zero then it is irrotational.

Given that,

$$\vec{A} = (x+2y+az)\underline{i} + (bx-3y-z)\underline{j} + (4x+cy+2z)\underline{k}$$

$$\therefore \vec{\nabla} \times \vec{A} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \partial/\partial y & \partial/\partial z \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix}$$

$$= \underline{i}(c+1) - \underline{j}(4-a) + \underline{k}(b-2)$$

For its irrotation

$c+1 = 0$

$\therefore c = -1$

$4-a = 0$

$\therefore a = 4$

and $b-2 = 0$

$\therefore b = 2$

$\therefore a = 4,\ b = 2,\ c = -1$ (A)

$\boxed{Q.7(a)}$

ii) $L\{t^3 e^{at}\}$

We know, $L\{e^{at}\} = \frac{1}{s-a}$

$\therefore L\{t^3 e^{at}\} = (-1)^3 \frac{d^3}{ds^3}\left(\frac{1}{s-a}\right)$

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$$= - \frac{d^2}{ds^2} \frac{(-1)}{(s-a)^2}$$

$$= - \frac{d}{ds} \frac{(-1)(-2)}{(s-a)^3}$$

$$= - \frac{(-1)(-2)(-3)}{(s-a)^4}$$

$$= \frac{6}{(s-a)^4}$$

$\boxed{Q.7.(b)(i)}$

$$L^{-1}\left\{\frac{6s-4}{s^2-4s+20}\right\}$$

$$= L^{-1}\left\{\frac{6s-4}{(s-2)^2+4^2}\right\}$$

$$= L^{-1}\left\{\frac{6s}{(s-2)^2+4^2}\right\} - L^{-1}\left\{\frac{4}{(s-2)^2+4^2}\right\}$$

$$= L^{-1}\left\{\frac{6(s-2)+12}{(s-2)^2+4^2}\right\} - 4e^{2t}\sin 4t$$

$$= 6 L^{-1}\left\{\frac{1}{(s-2)+4^2}\right\} + 3 L^{-1}\left\{\frac{4}{(s-2)^2+4^2}\right\} - 4e^{2t}\sin 4t$$

$$= 6 e^{2t} e^{-4^2} + 3e^{2t}\sin 4t - 4e^{2t}\sin 4t$$

$$= 6e^{2t}e^{-4^2} - e^{2t}\sin 4t \quad (\text{Ans})$$

Q.8 (i)

Given that,

$$Y'' - 3Y' + 2Y = 4e^{2t} \quad \text{---(1)}, \quad Y(0) = -3, \quad Y'(0) = 5$$

Taking Laplace transform of both side of (1)

$$\Rightarrow L\{Y''\} - 3L\{Y'\} + 2L\{Y\} = 4L\{e^{2t}\}$$

$$\Rightarrow s^2 y(s) - s\,y(0) - y'(0) - 3s\,y(s) + 3y(0) + 2y(s) = \frac{4}{s-2}$$

$$\Rightarrow s^2 y + 3s - 5 - 3sy - 9 + 2y = \frac{4}{s-2}$$

$$\Rightarrow y(s^2 - 3s + 2) = \frac{4}{s-2} - 3s + 14$$

$$\Rightarrow y(s^2 - 2s - s + 2) = \frac{4}{s-2} - 3s + 14$$

$$\Rightarrow y\{s(s-2) - 1(s-2)\} = \frac{4}{s-2} - 3s + 14$$

$$\Rightarrow y(s-2)(s-1) = \frac{4}{s-2} - 3s + 14$$

$$\cancel{\Rightarrow y = \frac{4}{(s-2)^2(s-1)} - \frac{3s}{(s-2)(s-1)}}$$

$$\Rightarrow y(s-2)(s-1)$$

$$\Rightarrow y(s^2 - 3s + 2) = \frac{4 - 3s^2 + 6s + 14s - 28}{s-2}$$

$$\Rightarrow y(s^2 - 3s + 2) = \frac{-3(s^2 - 3s + 2) + 11s - 18}{(s-2)}$$

$$\Rightarrow y = \frac{-3}{(s-2)} + \frac{11s - 18}{(s-2)^2(s-1)}$$

Now, $\dfrac{11s-18}{(s-2)^2(s-1)} = \dfrac{A}{s-1} + \dfrac{B}{s-2} + \dfrac{C}{(s-2)^2}$

$\Rightarrow 11s-18 = A(s-2)^2 + B(s-2)(s-1) + C(s-1)$

$= A(s^2-2s+4) + B(s^2-2s-s+2) + Cs - C$

$= As^2 - 2As + 4A + Bs^2 - 3Bs + 2B + Cs - C$

$= s^2(A+B) + s(-2A-3B+C) + (4A+2B-C)$

$s^2 \rightarrow A+B=0 \quad \therefore \cancel{A=-B}$ — (i)

$s \Rightarrow -2A-3B+C = 11$ — (ii)

$\Rightarrow 2B - 3B + C = 11$

$\Rightarrow -B + C = 1$ — (ii)

$4A+2B-C = -18$ — (iii)

$\Rightarrow -4B+2B-C = -18$

$\Rightarrow -2B - C = -18$ — (iii)

(ii) + (iii)

$-3B = -17$

$\therefore B =$

$A = -\dfrac{7}{3}, \quad B = \dfrac{7}{3}, \quad C = \dfrac{40}{3}$

$\dfrac{11s-18}{(s-2)^2(s-1)} = -\dfrac{7}{3(s-1)} + \dfrac{7}{3(s-2)} + \dfrac{40}{3(s-2)^2}$

$\therefore Y = -\dfrac{3}{s-2} - \dfrac{7}{3(s-1)} + \dfrac{7}{3(s-2)} + \dfrac{40}{3(s-2)^2}$

$y = -\dfrac{2}{3(s-2)} - \dfrac{7}{3(s-1)} + \dfrac{40}{3(s-2)^2}$

Taking inverse laplace transform,

$$Y(t) = -\frac{2e^{2t}}{3} - \frac{7e^{t}}{3} + \frac{40}{3} e^{2t}. t$$

(Ans)

2019 SECTION - A

$\boxed{Q.3(a)}$

$$L\{F(t)\} = \int_0^\infty e^{-st} F(t)\, dt$$

$$= \int_0^2 e^{-st} F(t) + \int_2^\infty e^{-st} F(t)\, dt$$

$$= \int_0^2 e^{-st} t^2\, dt + \int_2^\infty e^{-st} (t+2)^2\, dt$$

$u = t+2$

$$= \left(t^2 \frac{e^{-st}}{-s}\right)_0^2 + \frac{2}{s}\int_0^2 e^{-st} t\, dt + \int_0^\infty e^{-s(u-2)} u^2\, du$$

$$= -\frac{4e^{-2s}}{s} + \frac{2}{s}\left\{\left[t\frac{e^{-st}}{-s}\right]_0^2 + \frac{1}{s}\left[\frac{e^{-st}}{-s}\right]_0^2 \cancel{*}\right\} \cancel{\phi}$$

$$+ \int_0^\infty e^{-su+2s} u^2\, du$$

$$= -\frac{4e^{-2s}}{2} + \frac{2}{s}\left[-\frac{2e^{-2s}}{s} - \frac{e^{-2s}}{s^2}\right] + \cancel{\int_0} e^{2s} \int_0^\infty e^{-su} u^2 d$$

$$= -\frac{4e^{-2s}}{2} - \frac{4e^{-2s}}{s^2} - \frac{2e^{-2s}}{s^3} + e^{2s} \cdot \frac{2}{s^3}$$

$$= -2e^{-2s} - \frac{4e^{-2s}}{s^2} - \frac{2e^{-2s}}{s^3} + \frac{2e^{2s}}{s^3}$$

(Ans)

$\boxed{Q.4(c)}$

$4s(s+1)+1$

$4(s^2+s+\frac{1}{4})$

$(s+\frac{1}{2})^2$

$$L^{-1}\left\{\frac{s+3}{4s^2+4s+1}\right\}$$

$$\cancel{=L^{-1}\left\{\frac{s+3}{4s^2+3s+s+1}\right\}}$$

$$\cancel{=L^{-1}\left\{\frac{s+3}{s(4s+3)+1}\right\}}$$

$$=L^{-1}\left\{\frac{s+3}{4(s^2+s+1/4)}\right\}$$

$$=\frac{1}{4}L^{-1}\left\{\frac{s+3}{(s+1/2)^2}\right\}$$

$\cancel{\frac{1}{4}e^{-t/2}\cdot\frac{3}{4}}$

$\cancel{\frac{1}{4}e^{-t/2}\cdot(1/s)}$

$L^{-1}\{F(s-a)\} = e^{at}F(t)$

$$=\frac{1}{4}L^{-1}\left\{\frac{s}{(s+1/2)^2}\right\}+\frac{3}{4}L^{-1}\left\{\frac{1}{(s+1/2)^2}\right\}$$

$$=\frac{1}{4}L^{-1}\left\{\frac{(s+1/2)-\frac{1}{2}}{(s+1/2)^2}\right\}+\frac{3}{4}e^{-t/2}\cdot t$$

$$=\frac{1}{4}L^{-1}\left\{\frac{1}{s+1/2}\right\}-\frac{1}{8}L^{-1}\left\{\frac{1}{(s+1/2)^2}\right\}+\frac{3t\,e^{-t/2}}{4}$$

$$=\frac{1}{4}\cdot e^{-t/2}-\frac{t}{8}e^{-t/2}+\frac{3t\,e^{-t/2}}{4} \quad (*)$$

$\boxed{Q.2(a)}$

$$\text{Work done} = \int_{P_1}^{P_2}\vec{F}\cdot d\vec{r} = \int_{P_1}^{P_2}\vec{\nabla}\phi\cdot d\vec{r}$$

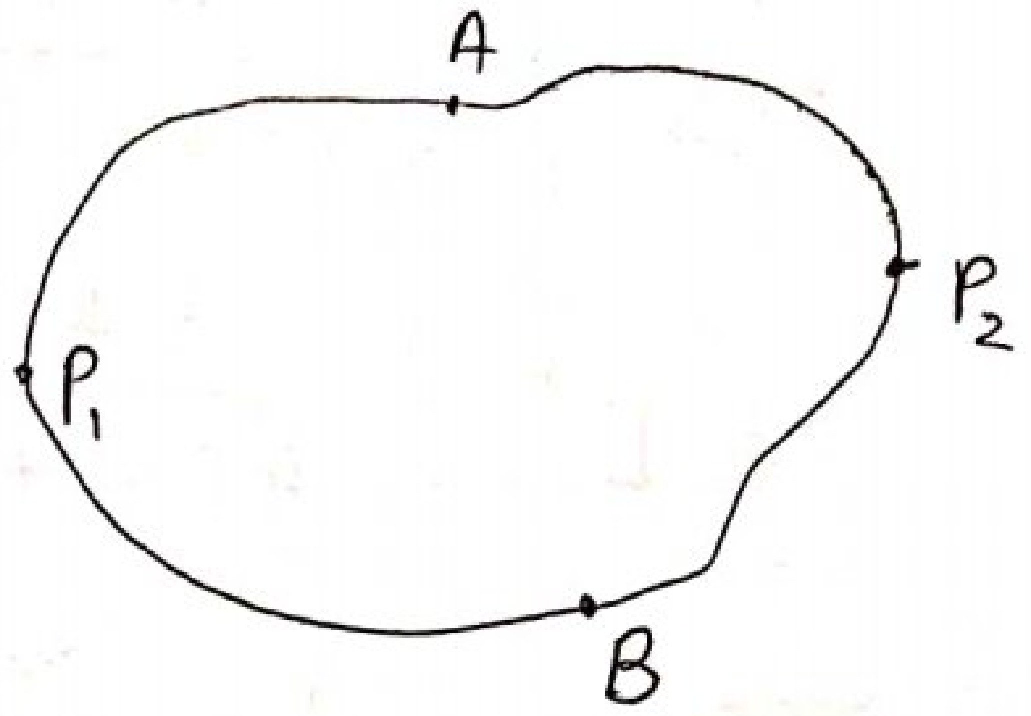
$$=\int_{P_1}^{P_2}\left(\frac{\partial\phi}{\partial x}\underline{i}+\frac{\partial\phi}{\partial y}\underline{j}+\frac{\partial\phi}{\partial z}\underline{k}\right)\cdot\left(dx\,\underline{i}+dy\,\underline{j}+dz\,\underline{k}\right)$$

$$=\int_{P_1}^{P_2}\left[\frac{\partial\phi}{\partial x}dx+\frac{\partial\phi}{\partial y}dy+\frac{\partial\phi}{\partial z}dz\right]$$

$$= \int_{P_1}^{P_2} d\phi = \phi(P_2) - \phi(P_1) = \phi(x_2, y_2, z_2) - \phi(x_1, y_1, z_1)$$

Then the integral depends only on points P_1 and P_2 and not on the path joining them.

Now, let $P_1AP_2BP_1$ be a closed curve. Then



$$\oint \vec{F}\cdot d\vec{r} = \int_{P_1AP_2BP_1} \vec{F}\cdot d\vec{r} = \int_{P_1AP_2} \vec{F}\cdot d\vec{r} + \int_{P_2BP_1} \vec{F}\cdot d\vec{r}$$

$$= \int_{P_1AP_2} \vec{F}\cdot d\vec{r} - \int_{P_1BP_2} \vec{F}\cdot d\vec{r}$$

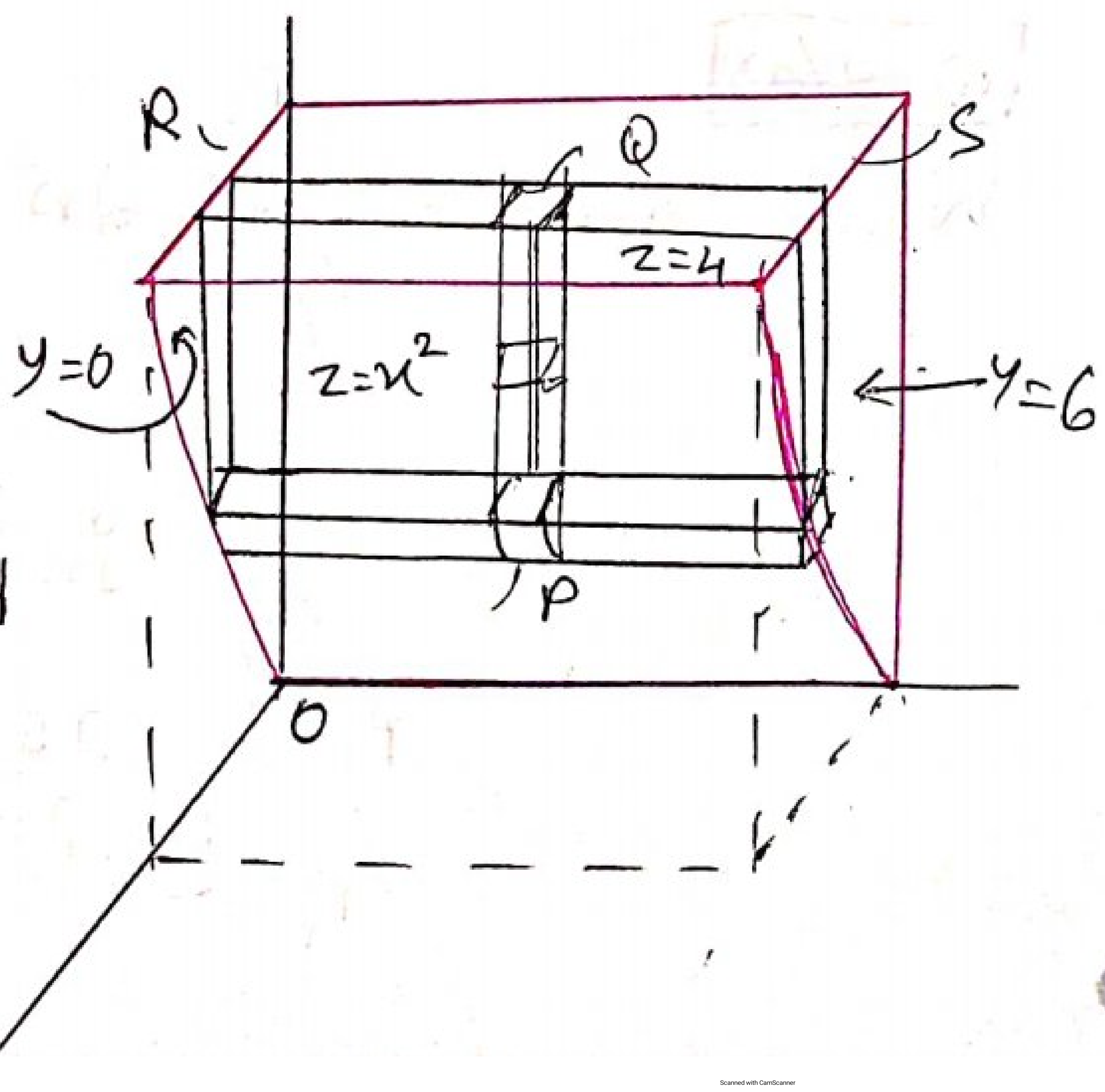
$$= 0$$

Since the integral from P_1 to P_2 along a path trough A is the same as that along a path through B, by hypothesis

conversely if $\oint \vec{F}\cdot d\vec{r} = 0$, then

$$\int_{P_1AP_2BP_1} \vec{F}\cdot d\vec{r} = \int_{P_1AP_2} \vec{F}\cdot d\vec{r} + \int_{P_2BP_1} \vec{F}\cdot d\vec{r} = \int_{P_1AP_2} \vec{F}\cdot d\vec{r} - \int_{P_1BP_2} \vec{F}\cdot d\vec{r} = 0$$

so that $\int_{P_1AP_2} \vec{F}\cdot d\vec{r} = \int_{P_1BP_2} \vec{F}\cdot d\vec{r}$



Q. 2 (b)

The region V is covered

a) by keeping x and y fixed and integrating from $z=x^2$ to $z=4$ (base to top of column PQ)

~~2018 (SECTION-B)~~

(b) then by keeping x fixed and integrating from $y=0$ to $y=6$ (R to S in the slab), (c) finally integrating from $x=0$ to $x=2$ (where $z=x^2$ meets $z=4$). Then the required integral is

$$\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4}(2xz\,\underline{i} - x\underline{j} + y^2\underline{k})\,dz\,dy\,dx$$

$$=2\underline{i}\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4} xz\,dz\,dy\,dx - \underline{j}\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4} x\,dz\,dy\,dx + \underline{k}\int_{x=0}^{2}\int_{y=0}^{6}\int_{z=x^2}^{4} y^2\,dz\,dy\,dx$$

$$=2\underline{i}\int_{x=0}^{2}\int_{y=0}^{6}\left[\frac{xz^2}{2}\right]_{x^2}^{4}dy\,dx - \underline{j}\int_{x=0}^{2}\int_{y=0}^{6}\left[xz\right]_{x^2}^{4}dy\,dx + \underline{k}\int_{x=0}^{2}\int_{y=0}^{6}\left[y^2z\right]_{x^2}^{4}dy\,dx$$

$$=\underline{i}\int_{x=0}^{2}\int_{y=0}^{6}(16x - x.x^4)\,dy\,dx - \underline{j}\int_{x=0}^{2}\int_{y=0}^{6}(4x - x.x^2)\,dy\,dx + \underline{k}\int_{x=0}^{2}\int_{y=0}^{6}(4y^2 - y^2.x^2)\,dy\,dx$$

$$=\underline{i}\int_{x=0}^{2}\left[16xy - x^5y\right]_0^6 dx - \underline{j}\int_{x=0}^{2}\left[4xy - x^3y\right]_0^6 + \underline{k}\int_{x=0}^{2}\left[\frac{4y^3}{3} - \frac{y^3}{3}x^2\right]_0^6$$

$$=\underline{i}\int_{x=0}^{2}(16\times 6x - 6x^5)\,dx - \underline{j}\int_{x=0}^{2}(4\times 6x - 6x^3)\,dx + \underline{k}\int_{x=0}^{2}\left(\frac{4\times 6^3}{3} - \frac{6^3}{3}x^2\right)$$

$$=\underline{i}\left[\frac{96x^2}{2} - \frac{6x^6}{6}\right]_0^2 - \underline{j}\left[\frac{24x^2}{2} - \frac{6x^4}{4}\right]_0^2 + \underline{k}\left[288x - \frac{72x^3}{3}\right]$$

$$=128\,\underline{i} - 24\,\underline{j} + 384\,\underline{k} \quad \text{(Ans)}$$

SECTION-B (2019)

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Q.5(a) <u>Skewness:</u> A distribution in which the values equidistant from the mean have equal frequencies and is called symmetric distribution. Any departure from symmetry is called skewness.

- In a perfect symmetric distribution,
  Mean = Median = Mode
- In positive skewness, Mean > Median > Mode
- In negative skewness, Mean < Median < Mode

<u>Kurtosis:</u> Kurtosis is a measure that describes the shape of a distribution's tails in relation to its overall shape.

- When the peak of a curve becomes relatively high then that curve is called Leptokurtic
- When the curve is flat-topped, then it is called Platykurtic
- Since normal curve is neither very peaked nor very flat-topped. and it is called Mesokurtic

Q.5(b) Given that,

$\mu_R = 56$ inches

$S.d_R = 2$ inches

$n_R = 64$

$n_R = 25$

$$\mu_R = \frac{\Sigma x}{n_R}$$

$$\Rightarrow \Sigma x = \mu_R \times n_R = 56 \times \overset{25}{\cancel{64}} = \cancel{3584}\ 1400,\ldots$$

and, $S.d_R = \sqrt{\cancel{\Sigma(x-\mu}}$

$$= \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n}}$$

$$\Rightarrow 2 = \sqrt{\frac{\Sigma x^2 - \frac{(3584)^2}{64}}{64}} \qquad \Rightarrow 2 = \sqrt{\frac{\Sigma x^2 - \frac{1400^2}{25}}{25}}$$

$$\Rightarrow \Sigma x^2 - \frac{(3584)^2}{64} = 2^2 \times 64 \qquad \Rightarrow \Sigma x^2 - \frac{1400^2}{25} = 2^2 \times 25$$

$$\Rightarrow \Sigma x^2 = 2^2 \times 64 + \frac{(3584)^2}{64} \qquad \Rightarrow \Sigma x^2 = 2^2 \times 25 + \frac{1400^2}{25}$$

$$\therefore \Sigma x^2 = 200960 \qquad \therefore \Sigma x^2 = 78500$$

$\therefore$ For correct mean and S.d

$$\mu = \frac{\Sigma x}{n} = \frac{3584}{25} = 143.36 \qquad \mu = \frac{\Sigma x}{n} = \frac{1400}{64} = 21.87$$

$$\text{and } S.d = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n}} \qquad S.d = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n}}$$

$$= \sqrt{\frac{200960 - \frac{(3584)^2}{25}}{25}} \qquad = \sqrt{\frac{78500 - \frac{1400^2}{64}}{64}}$$

$$= \qquad\qquad = 27.35\ (A)$$

Q.5(c)

For first samples:

$n = 100$

$\bar{x} = 15$

$S.d = 3$

$\therefore \bar{x} = \frac{\sum x_r}{n}$

$\Rightarrow \sum x_r = \bar{x} \times n = 15 \times 100 = 1500$

and $S.d = \sqrt{\frac{\sum x_r^2 - \frac{(\sum x_r)^2}{n}}{n-1}}$

ERA
1800051

$\Rightarrow 3 = \sqrt{\frac{\sum x_r^2 - \frac{(1500)^2}{100}}{100-1}}$

$\Rightarrow \sum x_r^2 - \frac{1500^2}{100} = 3^2 \times (100-1)$

$\Rightarrow \sum x_r^2 = 900 + \frac{1500^2}{100}$

$\therefore \sum x_r^2 = \cancel{23400}\ 23391$

<u>For whole group:</u>

$\bar{x} = 15.6$

$S.d = \sqrt{13.44}$

$n = 250$

$\therefore \bar{x} = \frac{\sum x}{N}$

$\Rightarrow \sum x = \mu \times N = 15.6 \times 250 = 3900$

and $S.d = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{N}}{n-1}}$

$\Rightarrow \sqrt{13.14} = \sqrt{\frac{\sum x^2 - \frac{3900^2}{250}}{250-1}}$

$\Rightarrow \sum x^2 - \frac{3900^2}{250} = 13.14 \times (250-1)$

$\Rightarrow \sum x^2 = \frac{3900^2}{250} + 13.14 \times (250-1)$

$\therefore \sum x^2 = \cancel{64125}\ 64111.86$

$\therefore \bar{x} = \dfrac{\sum x_f}{n}$

$\Rightarrow \sum x_f = \bar{x} \times n = 15 \times 100 = 1500$

and $S.d = \sqrt{\dfrac{\sum x_f^2 - \frac{(\sum x_f)^2}{n}}{n-1}}$

ERA
1800051

$\Rightarrow 3 = \sqrt{\dfrac{\sum x_f^2 - \frac{(1500)^2}{100}}{100-1}}$

$\Rightarrow \sum x_f^2 - \dfrac{1500^2}{100} = 3^2 \times (100-1)$

$\Rightarrow \sum x_f^2 = 900 + \dfrac{1500^2}{100}$

$\therefore \sum x_f^2 = \cancel{23400}\ 23391$

For whole group:

$\bar{x} = 15.6$

$S.d = \sqrt{13.44}$

$n = 250$

$\therefore \bar{x} = \dfrac{\sum x}{N}$

$\Rightarrow \sum x = \mu \times N = 15.6 \times 250 = 3900$

and $S.d = \sqrt{\dfrac{\sum x^2 - \frac{(\sum x)^2}{N}}{n-1}}$

$\Rightarrow \sqrt{13.14} = \sqrt{\dfrac{\sum x^2 - \frac{3900^2}{250}}{250-1}}$

$\Rightarrow \sum x^2 - \dfrac{3900^2}{250} = 13.14 \times (250-1)$

$\Rightarrow \sum x^2 = \dfrac{3900^2}{250} + 13.14 \times (250-1)$

$\therefore \sum x^2 = \cancel{64125}\ 64111.86$

For rest group:

$\Sigma X_r = 3900 - 1500 = 2400$

$\Sigma X_r^2 = 64111.86 - 23391 = 40720.86$

$n = 250 - 100 = 150$

$$\therefore \bar{x} = \frac{\Sigma X_r}{n} = \frac{2400}{150} = 16 \text{ (An)}$$

$$S.d = \sqrt{\frac{\Sigma x_r^2 - \frac{(\Sigma x_r)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{40720.86 - \frac{2400^2}{150}}{150-1}}$$

$= 3.93$ (An)

$\boxed{Q.6(a)}$ Given that,

$\Sigma X = 136$, $\Sigma Y = 243$, $\Sigma X^2 = 2278$, $\Sigma XY = 3476$,

$\Sigma Y^2 = 6129.$, $n = 10$

We know, the correlation coefficient,

$$r = \frac{n\Sigma XY - \Sigma X \Sigma Y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

$$= \frac{10 \times 3476 - 136 \times 243}{\sqrt{10 \times 2278 - 136^2} \cdot \sqrt{10 \times 6129 - 243^2}}$$

$= 0.55$ (An)

$\boxed{Q.6(b)}$

Age	f	x	fx	fx^2
20-25	170	22.5	3825	86062.5
25-30	110	27.5	3025	83187.5
30-35	80	32.5	2600	84500
35-40	~~40~~ 45	37.5	1687.5	63281.25
40-45	40	42.5	1700	72250
45-50	35	47.5	1662.5	~~1662.5~~ 78968.75
	$\Sigma f = 480$ $= n = 480$		$\Sigma fx = 14500$	$\Sigma fx^2 = \cancel{390943.75}$ 468250

$\therefore$

$$S.d = \sqrt{\frac{\Sigma fx^2 - \frac{(\Sigma fx)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{\cancel{390943.75}\ 468250 - \frac{(14500)^2}{480}}{480}}$$

$$= 7.936 \text{ (A)}$$

$\boxed{Q.7(a)}$ Given that,

$\mu_1' = -1.5$

$\mu_2' = 17$

$\mu_3' = -30$

$\mu_4' = 102$

We know,

$$\mu_r = \mu_r' - {}^rC_1 \mu_{r-1}' \mu_1' + {}^rC_2 \mu_{r-2}' \mu_1'^2 \cancel{{}^rC_3 \mu_{r-3}'} - \ldots\ldots (-1)^r \mu_1'^r$$

$\therefore \mu_2 = \mu_2' - 2\mu_1'^2 = 17 - 2\times(-1.5)^2 = 12.5$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 3\mu_1'^3 - \mu'^3$

$\mu_2 = \mu_2' - 2\mu_1'^2 + \mu_1'^2 = \mu_2' - \mu_1'^2 = 17 - (-1.5)^2 = 14.75$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 3\mu_1'^3 - \mu'^3$

$= \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = -30 - 3\times 17\times(-1.5) + 2\times(-1.5)^3$

$= 39.75$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 102 - 4\times(-30)\times(-1.5) + 6\times 17\times(-1.5)^2 - 3\times 102^4$

$= 136.31$

Now, $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3} = \dfrac{(39.75)^2}{(14.75)^3} = 0.49$ (Ans)

$\beta_2 = \dfrac{\mu_4}{\mu_2^2} = \dfrac{136.31}{(14.75)^2} = 0.63$ (Ans)

$\boxed{Q.7(b)}$

Given that,

$f(x) = Ce^{-kx}$

α কোথায় নাকি ∞ !!!

$\alpha = \infty$

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = C\int_{-\infty}^{\infty} e^{-kx}\,dx$

$= -C\left[\dfrac{e^{-kx}}{k}\right]_{-\infty}^{\infty}$

$= C\int_{-\infty}^{0} 0\cdot dx + C\int_{0}^{\alpha} e^{-kx}\,dx + C\int_{\alpha}^{\infty} 0\cdot dx$

$= -C\left[\dfrac{e^{-kx}}{k}\right]_0^{\alpha} = -\dfrac{C}{k}\left(e^{-k\alpha} - 1\right)$

$= \dfrac{C}{k}\left(1 - e^{-k\alpha}\right)$

$\int_{-\alpha}^{\alpha} f(x)\,dx = -C\left(1 - \dfrac{1}{k}\right)$

$\Rightarrow 1 = \dfrac{c}{k}$

$\therefore c = K$ ✓

$$\Rightarrow 1 = \frac{c}{k}\left(1 - e^{-k\alpha}\right)$$

$$\therefore c = \frac{k}{1 - e^{-k\alpha}} \quad \text{(Ans)}$$

$$\therefore f(x) = k e^{-kx}$$

$$\cancel{= \frac{k}{(1 - e^{-k\alpha}) \times e^{k\alpha}}}$$

$$\cancel{= \frac{k}{e^{k\alpha} - 1}}$$

$$\therefore \text{Mean}, \mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\infty}^{0} x f(x)\, dx + \int_{0}^{\alpha} x f(x)\, dx + \int_{\alpha}^{\infty} x f(x)\, dx$$

$$= 0 + \int_{0}^{\alpha} x f(x)\, dx + 0$$

$$= k \int_{0}^{\alpha} x e^{-kx}\, dx$$

$$= k \left[\left[\frac{x e^{-kx}}{-k} \right]_0^{\alpha} - \int_0^{\alpha} 1 \cdot \frac{e^{-kx}}{-k}\, dx \right]$$

$$= k \left[0 + \frac{1}{k} \left[\frac{e^{-kx}}{-k} \right]_0^{\alpha} \right]$$

$$= k \left[0 - \frac{1}{k^2} (0 - 1) \right]$$

$$= k \left[\frac{1}{k^2} \right]$$

$$= \frac{1}{K} \quad (A_2)$$

$$S.d = \sqrt{\Sigma x^2 f(x) - \mu^2}$$

Now $\int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^{\alpha} x^2 \cdot K e^{-Kx} \, dx$

$$= K\left[\left(\frac{x^2 e^{-Kx}}{-K}\right)_0^{\alpha} + \frac{2}{K}\int_0^{\alpha} x e^{-Kx}\,dx\right.$$

$$= K\left\{2 \times \frac{1}{K^3}\right\}$$

$$= \frac{2}{K^2}$$

$$= \frac{K}{1-e^{-K\alpha}}\left\{\int_0^{\alpha} x^2 e^{-Kx}\,dx\right\}$$

$$= \frac{K}{1-e^{-K\alpha}}\left\{\left[x^2 \frac{e^{-Kx}}{-K}\right]_0^{\alpha} + \frac{2}{K}\int_0^{\alpha} x e^{-Kx}\,dx\right\}$$

$$= \frac{K}{1-e^{-K\alpha}}\left\{-\frac{\alpha^2 e^{-K\alpha}}{K} + \frac{2}{K}\left[-\frac{\alpha e^{-K\alpha}}{K} + \frac{1}{K^2}(1-e^{-K\alpha})\right]\right\}$$

$$= -\frac{\alpha^2 e^{-K\alpha}}{1-e^{-K\alpha}} - \frac{2\alpha e^{-K\alpha}}{K(1-e^{-K\alpha})} + \frac{2}{K^2}$$

$$= -\frac{\alpha^2}{e^{K\alpha}-1} - \frac{2\alpha}{K(e^{K\alpha}-1)} + \frac{2}{K^2}$$

$$\therefore S.d = \sqrt{\frac{2}{K^2} - \frac{1}{K^2}}$$

$$= \frac{1}{K} \quad (A_2)$$

$\boxed{Q.7(c)}$

If σ is standard deviation and s is the root mean square deviation, then we can write,

$$s^2 = \sigma^2 + d^2$$

where d = deviation from assumed mean

$\boxed{Q.8(a)}$

$na_0 + a_1\Sigma x = \Sigma y$
$a_0\Sigma x + a_1\Sigma x^2 = \Sigma xy$

Let the required line,

$$y = a_0 + a_1 x$$

x	y	x^2	xy
16	192	256	3072
19	218	361	4142
25	210	625	5250
28	~~3~~232	784	6496
36	236	1296	8496
40	249	1600	9960
$\Sigma x = 164$	$\Sigma y = 1337$	$\Sigma x^2 = 4922$	$\Sigma xy = 37416$

Here, $n = 6$

$\therefore\ 6a_0 + 164a_1 = 1337$

and $164a_0 + 4922a_1 = 37416$

Solving these equation we get,

$a_0 = 168.62$
$a_1 = 1.98$

$\therefore y = 168.62 + 1.98x$ (Ans)

Regression: Regression is a statistical method that attempts to determine the strength and character of the relationship between one dependent variable and a series of other variables (known as independent variable)

$\boxed{Q.8(b)}$ We know,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\therefore$ Mean,

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z)\, e^{-z^2/2}\, \sigma\, dz$$

Let, $\frac{x-\mu}{\sigma} = z$

$\Rightarrow \frac{dx}{\sigma} = dz$

$\therefore dx = \sigma\, dz$

and, $x = \mu + \sigma z$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \mu e^{-z^2/2} dz + \int_{-\infty}^{\infty} \sigma z e^{-z^2/2} dz \right]$$

power 1 odd function $= 0$

$$= \frac{2\mu}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-z^2/2} dz$$

Let, $z^2/2 = \cancel{\sigma} p \Rightarrow \sqrt{2p}$

$\Rightarrow \frac{2z\,dz}{2} = dp$

$\therefore dz = \frac{dp}{z} = \frac{dp}{\sqrt{2p}}$

$$= \frac{2\mu}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-p} \frac{dp}{\sqrt{2p}}$$

$$\therefore E(x) = \frac{2\mu}{2\sqrt{\pi}} \int_0^\infty p^{-1/2} e^{-p}\, dp$$

$$= \frac{\mu}{\sqrt{\pi}} \Gamma\tfrac{1}{2} = \frac{\mu}{\sqrt{\pi}} \cdot \sqrt{\pi} \qquad \left[\because \int_0^\infty x^{n-1} e^{-x} dx = \Gamma n\right]$$

$$\therefore\ E(x) = \mu$$

Variance

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let,

$\frac{x-\mu}{\sigma} = z$

$\frac{dx}{\sigma} = z$

$dx = \sigma z$

$\therefore x = \sigma z + \mu$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu+\sigma z)^2 e^{-1/2 z^2} \sigma\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \mu^2 e^{-1/2 z^2} dz + \int_{-\infty}^{\infty} \underbrace{2\sigma\mu z e^{-1/2 z^2}}_{=0} dz + \int_{-\infty}^{\infty} \sigma^2 z^2 e^{-1/2 z^2} dz\right]$$

$$= \frac{2\mu^2}{\sqrt{2\pi}} \int_0^\infty e^{-z^2/2} dz + \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^\infty z^2 e^{-z^2/2} dz$$

Let,

$\frac{z^2}{2} = P$

$\Rightarrow z = \sqrt{2P}$

$\therefore \frac{2z\,dz}{2} = dP$

$\therefore dz = \frac{dP}{\sqrt{2P}}$

$$= \frac{2\mu^2}{\sqrt{2\pi}} \int_0^\infty e^{-P} \frac{dP}{\sqrt{2P}} + \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^\infty 2P\, e^{-P} \frac{dP}{\sqrt{2P}}$$

$$= \frac{2\mu^2}{2\sqrt{\pi}} \int_0^\infty e^{-1/2} e^{-P} dP + \frac{2\sigma^2}{\sqrt{\pi}} \int_0^\infty P^{1/2} e^{-P} dP$$

$$= \frac{\mu^2}{\sqrt{\pi}} \Gamma\tfrac{1}{2} + \frac{2\sigma^2}{\sqrt{\pi}} \Gamma\tfrac{3}{2}$$

$$= \frac{\mu^2}{\sqrt{\pi}}\sqrt{\pi} + \frac{2\sigma^2}{\sqrt{\pi}}\frac{1}{2}\sqrt{\pi}$$

$$\Gamma \tfrac{3}{2} = \Gamma\left(1+\tfrac{1}{2}\right) = \tfrac{1}{2}\Gamma\tfrac{1}{2} = \tfrac{1}{2}\sqrt{\pi}$$

$E(x^2) = \mu^2 + \sigma^2$

~~Var~~ Variance $(x) = E(x^2) - \{E(x)\}^2$

$= \mu^2 + \sigma^2 - \mu^2$

$= \sigma^2$ (Ans)

$\boxed{Q.8(C)}$ Given that,

$\mu = 68.22$ inches $x = 6$ feet $= 72$ inch

$\sigma^2 = 10.8$ inches

$\therefore \sigma = 3.286$

Now, $\mu = np = 68.22$

and $npq = 10.8$

$\therefore q = \frac{10.8}{68.22} = 0.158$ $\therefore$

$\therefore p = \frac{68.22}{1000} =$

$\therefore p = (1 - 0.158) = 0.842$

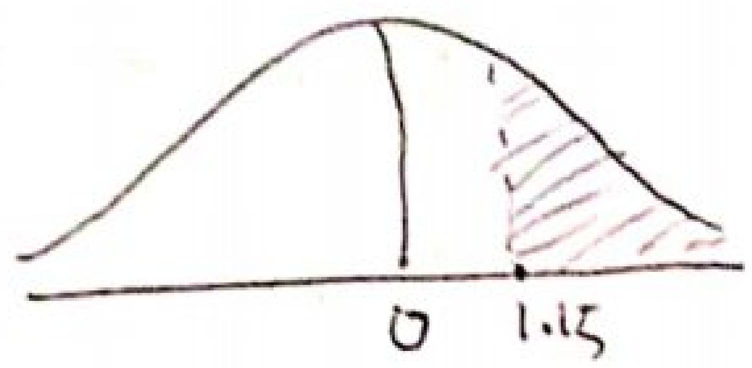
$\therefore n = \frac{68.22}{0.842} = 81$

$\therefore$ Mean to

$\therefore z = \frac{x-\mu}{\sigma}$

$= \frac{72 - 68.22}{3.286}$

$= 1.15$



$\therefore P(z > 1.15) = 0.1250$

SECTION-B (2018)

$\boxed{Q.5(a)}$ Given that,

$r = 0.60$, $\sigma_x = 1.50$, $\sigma_y = 2.0$, $\bar{X} = 10$ and $\bar{Y} = 2$

Y on X

$$\boxed{y - \bar{y} = b_{yx}(X - \bar{X})}$$

$\Rightarrow y - \bar{y} = r \frac{\sigma_y}{\sigma_x}(x - \bar{x})$ $\left[\because b_{yx} = r \frac{\sigma_y}{\sigma_x}\right]$

$\Rightarrow y - 20 = 0.60 \times \frac{2.0}{1.50}(x - 10)$

$\Rightarrow y - 20 = 0.8x - 8$

$\therefore y = 12 + 0.8x$ (Ans)

Extra

If $x = 7$, find the value of y.

Solve

$y = 12 + 0.8 \times 7 = 17.6$

Or,

If $y = 7$, find the value of x

Solve

$x = 1 + 0.45 \times 7 = 4.15$

X on Y

$x - \bar{x} = b_{xy}(y - \bar{y})$

$\Rightarrow x - \bar{x} = r \frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$\Rightarrow x - 10 = 0.6 \frac{1.5}{2}(y - 20)$

$\Rightarrow x - 10 = 0.45y - 9$

$\therefore x = 1 + 0.45y$ (Ans)

এখানে $d = \frac{x-35}{10}$ ধরে calculation করতে হবে

$\boxed{Q.5(b)}$ For calculation of moment

Expenditure	f	cf	$d = \frac{x-27}{10}$ (35)	fd	fd^2	fd^3	fd^4	x
10–20	14	14	−1.2	−16.8	20.16	−24.192	29.0304	15
20–30	25	39	−0.2	−5	1	−0.2	0.04	25
A → 30–40	27	66	0.8	21.6	17.28	13.824	11.0592	35
→ 40–50	26	92	1.8	46.8	84.24	151.632	272.9376	45
50–60	15	107	2.8	42	117.6	329.28	921.984	55

$\Sigma fd = 88.6$, $\Sigma fd^2 = 240.28$, $\Sigma fd^3 = 470.344$

$\Sigma fd^4 = 1235.05$

$N = 107$

Upper Quartiles = Size of $\left(\frac{3N}{4}\right)$th item $= \frac{3 \times 107}{4}$ th item

$= 80.25$ th item

$\therefore LB = 40$

$cf = 66$

$f = 26$

$h = 10$

$k = 3$

$$\therefore Q_3 = 40 + \left(\frac{\frac{3 \times 107}{4} - 66}{26} \right) \times 10$$

$$= 45.48 \quad (\text{Ans})$$

Now, calculation for moments

$$\mu_1' = \frac{\sum fd}{N} \times h = \frac{88.6}{107} \times 10 = 8.28$$

$$\mu_2' = \frac{\sum fd^2}{N} \times h^2 = \frac{240.28}{107} \times 10^2 = 224.56$$

$$\mu_3' = \frac{\sum fd^3}{N} \times h^3 = \frac{470.344}{107} \times 10^3 = 4395.74$$

$$\mu_4' = \frac{\sum fd^4}{N} \times h^4 = \frac{1235.05}{107} \times 10^4 = 115425.23$$

Moment about mean:

$$\mu_2 = \mu_2' - \mu_1'^2 = 224.56 - (8.28)^2 = 156.00$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = 1812.35$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 48110.40$$

$$\therefore \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(1812.35)^2}{(156)^3} = 0.865 \quad (\text{Ans})$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{48110.40}{(156)^2} = 1.977 \quad (\text{Ans})$$

$\boxed{Q.6(a)}$ Give

For sample 1

$n = 50, \ \bar{x} = 54.4, \ s = 8$

We know,

$$\bar{x} = \frac{\sum x_1}{n} \Rightarrow \sum x_1 = \bar{x} \times n = 54.4 \times 50 = 2720$$

and $s = \sqrt{\dfrac{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}{n-1}}$

$$\Rightarrow \Sigma x^2 - \frac{(\Sigma x)^2}{n} = s^2 \times (n-1)$$

$$\Rightarrow \Sigma x^2 - \frac{(2720)^2}{50} = 8^2 \times (50-1)$$

$$\Rightarrow \Sigma x_1^2 = 8^2 \times 49 + \frac{2720^2}{50} = 151104$$

For sample 2

$n = 100$, $\bar{x} = 50.3$, $S = 7$

$$\therefore \bar{x} = \frac{\Sigma x}{n}$$

$$\Rightarrow \Sigma x_2 = \bar{x} \times n = 50.3 \times 100 = 5030$$

and,

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}}$$

$$\Rightarrow \Sigma x_2^2 = \frac{(\Sigma x)^2}{n} + s^2(n-1)$$

$$= \frac{(5030)^2}{100} + 7^2 \times (100-1)$$

$$= 257860$$

For sample of size 150

$$\therefore \Sigma x = \Sigma x_1 + \Sigma x_2 = 2720 + 5030 = 7750$$

$$\Sigma x^2_{n=150} = \Sigma x_1^2 + \Sigma x_2^2 = 151104 + 257860 = 408964$$

$$\therefore \bar{x} = \frac{\Sigma x}{n} = \frac{7750}{150} = 51.67 \text{ (Ans)}$$

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{408964 - \frac{(7750)^2}{150}}{150-1}} = 7.57 \text{ (Ans)}$$

Q.6(h)

Variable	f	cf
10-20	12	12
20-30	30	42
30-40	A	42+A
40-50	65	107+A
50-60	B	107+A+B
60-70	25	132+A+B
70-80	18	150+A+B

n = 229

Here median (ranked value) $= \frac{N}{2}$th value $= \frac{229}{2} = 114.5$th

LB = 50

f = B , cf = 107+A , i = 10

Now, Median $= LB + \left(\frac{N/2 - cf}{f}\right) \times i$

$\Rightarrow 46 = 50 + \left(\frac{\frac{229}{2} - (107+A)}{65}\right) \times 10$

$\Rightarrow 46 - 50 = \frac{114.5 - 107 - A}{B} \times 10$

$\Rightarrow -\frac{4 \times B}{10} = \cancel{114.5 - 4}\ 7.5 - A$

$\therefore \cancel{A = 72.5 - \frac{6 \times 65}{10} = 33.5 \approx 34}$

$\therefore A - 0.4B = 7.5$ —— (i)

and $150 + A + B = 229$

$\cancel{\Rightarrow B = 229 - 150 - 34 = 45}$

$\therefore A + B = 79$ —— (ii)

(ii) − (i) $\Rightarrow 1.4B = 71.5$

$\therefore B = 51$

$\therefore A = 79 - 51 = 28$ (A)

$\boxed{Q.7(a)}$ (i) Number of purchase department member = 3

P(at leas one from the purchase department) = P

Total = 3+4+2+4 = 13

$$\therefore P = \frac{3C_1 \times {}^{10}C_3}{13C_4} + \frac{3C_2 \times {}^{10}C_2}{13C_4} + \frac{3C_3 \times {}^{10}C_1}{13C_4}$$

$$= 0.71 \text{ (A.)}$$

(ii) Number of chartered accountan member = 4

Total = 13

$$\therefore P = \frac{4C_1 \times {}^{9}C_3}{13C_4} + \frac{4C_2 \times {}^{9}C_2}{13C_4} + \frac{4C_3 \times {}^{9}C_1}{13C_4} + \frac{4C_4}{13C_4}$$

$$= 0.82 \text{ (A.)}$$

$\boxed{Q.7(b)}$ Given that,

$$f(x) = 3x^2 \quad 0 \le x \le 1$$

(i) $P(x \le a) = P(a > x)$

$$\Rightarrow \int_0^a 3x^2 dx = \int_a^1 3x^2 dx$$

$$\Rightarrow 3\left[\frac{x^3}{3}\right]_0^a = 3\left[\frac{x^3}{3}\right]_a^1$$

$$\Rightarrow a^3 = 1 - a^3$$

$$\Rightarrow 2a^3 = 1$$

$$\Rightarrow a^3 = 0.5$$

$$\therefore a = 0.79 \text{ (A.)}$$

(ii) $P(x > b) = 0.05$

$$\Rightarrow \int_b^1 3x^2 dx = 0.05$$

$$\Rightarrow 3\left[\frac{x^3}{3}\right]_b^1 = 0.05$$

$$\Rightarrow 1 - b^3 = 0.05$$

$$\Rightarrow b^3 = 1 - 0.05 = 0.95$$

$$\therefore b = 0.98 \text{ (A.)}$$

Q.7(a) (ii)

Probability of A machin will be performing in 5 years time = $P_A = \frac{1}{4}$

probability of

$\therefore$ A will not be operating, $q_A = 1 - P_A = 1 - \frac{1}{4} = \frac{3}{4}$

For B machine

$P_B = \frac{1}{3}$

$\therefore q_B = 1 - \frac{1}{3} = \frac{2}{3}$

(i) P(Both machines will be performing) $= \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$

(ii) P(Neither will be operating) $= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$

(iii) P(only machine B will be operating) $= \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

Q.8(a) Required table is given below:

Mistakes per day, x_i	No of days, f_i	$f_i x_i$
0	143	0
1	90	90
2	42	84
3	12	36
4	9	36
5	3	15
6	1	6
	$\Sigma f_i = 300$	$\Sigma f_i x_i = 267$

$\therefore \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{267}{300} = 0.89$

$\therefore \lambda = 0.89$

Poission distribution

$P(0) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.89} \times 0.89^0}{0!} = \cancel{0.365}\ 0.411$ $E_i(0) = 123.3$

$P(1) = \frac{e^{-.89} \times 0.89^1}{1!} = 0.365$ $E_i(1) = 300 \times 0.365 = 109.5$

$P(2) = \frac{e^{-0.89} \times (0.89)^2}{2!} = 0.163$ $E_i(2) = 48.9$

$P(3) = \frac{e^{-0.89} \times (0.89)^3}{3!} = 0.048$ $E_i(3) = 14.4$

$P(4) = \frac{e^{-0.89} \times (0.89)^4}{4!} = 0.011$ $E_i(4) = 3.3$

$P(5) = \frac{e^{-0.89} \times (0.89)^5}{5!} = 0.002$ $E_i(5) = 0.6$

$P(6) = \frac{e^{-0.89} \times (0.89)^5}{6!} = \cancel{0.000}0$ $E_i(0) = 0$

$\boxed{Q.8(b)}$ β_1, β_2

x	f	$d = \frac{x-5}{1}$	$f_i d$	$f_i d^2$	$f_i d^3$	$f_i d^4$
1	1	-4	-4	16	-64	256
2	6	-3	-18	54	-162	486
3	13	-2	-26	52	-104	208
4	25	-1	-25	25	-25	25
5	30	0	0	0	0	0
6	22	1	22	22	22	22
7	9	2	18	36	72	144
8	5	3	15	45	135	405
9	2	4	8	32	128	512

$n = 113$ $\Sigma f_i d = -92$ $\Sigma f_i d^2 = 282$ $\Sigma f_i d^3 = 2$ $\Sigma f_i d^4 = 2058$

$\mu_1' = \frac{\Sigma f_i d}{N} \times h = \frac{-92}{113} \times 1 = -0.81$

$\mu_2' =$ - - -

same process

SECTION-B (2017)

$\boxed{Q.5(a)}$ Dispersion is a statistical term that describes the size of the distribution of values expected for a particular variable and can be measured by several different statistics, such as range, variance and standard deviation. In other words, the difference between actual value and the average value is dispersion.

$\boxed{Q.5(b)}$ Given that,

$\mu_1' = 2\cancel{0}, \mu_2' = 20, \mu_3' = 40, \mu_4' = 50$

$\therefore \mu_2 = \mu_2' - \mu_1'^2 = 16$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = \cancel{\;\;} -64$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + \cancel{8}\, 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = -\cancel{0}\,194$

$\therefore$ skewness, $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(-64)^2}{16^3} = 1$

kurtosis, $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{-\cancel{0}194}{16^2} = \cancel{-0.875} \approx 0.76$

As kurtosis is less than 3. The curve is flat-topped and it is called platykurtic.

Q.5(c) The required table is given below.

For A, x	$(x-\bar{x})^2_A$	For B, x	$(x-\bar{x})^2_B$
15	1089	48	156.25
101	2809	10	650.25
8	1600	35	0.25
80	1024	22	182.25
10	1444	50	210.25
120	5184	60	600.25
35	169	25	110.25
15	1089	34	2.25

$\Sigma X_A = 384, \Sigma(x-\bar{x})^2_A = 14408 \quad \Sigma X_B = 284 \quad \Sigma(x-\bar{x})^2_B = 1912$

Here $n = 8$

Average

$\therefore \bar{x}_A = \frac{\Sigma x}{n} = \frac{384}{8} = 48$

$\bar{x}_B = \frac{284}{8} = 35.5$

Standard Deviation

$S_A = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{14408}{8-1}} = \sqrt{2058.29} = 45.37$

$S_B = \sqrt{\frac{1912}{8-1}} = \sqrt{273.14} = 16.53$

A is the better scorer as his average is greater than B.

And B is more consistent player as his standard deviation of score is less than A.

It means that dispersion from mean is of B is less than A. That's why B is more consistent player.

Q.6(a) Given that, a density function

$$f(x) = \begin{cases} \lambda e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$$

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$$\Rightarrow 1 = 0 + \int_0^{\infty} \lambda e^{-x}\,dx$$

$$\Rightarrow 1 = \lambda \left[\frac{e^{-x}}{-1}\right]_0^{\infty}$$

$$\Rightarrow 1 = -\lambda(0-1) \quad [\because e^{-\infty} = 0]$$

$$\therefore \lambda = 1$$

$$\therefore f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\therefore \mu_1' = \int_0^{\infty} x\,F(x)\,dx = \int_0^{\infty} x e^{-x}\,dx$$

$$= \left[\frac{xe^{-x}}{-1}\right]_0^{\infty} + \int_0^{\infty} 1.e^{-x}\,dx$$

$$= 0 + \left[\frac{e^{-x}}{-1}\right]_0^{\infty} = -1(0-1) = 1$$

$$\mu_2' = \int_0^{\infty} x^2 e^{-x}\,dx = -\left[x^2 e^{-x}\right]_0^{\infty} + 2\int_0^{\infty} x e^{-x}\,dx = 0 + 2\left\{-\left[xe^{-x}\right]_0^{\infty} + \int_0^{\infty} e^{-x}\,dx\right.$$

$$= 2\left\{-0 - \left[e^{-x}\right]_0^{\infty}\right\}$$

$$= 2\left\{-(0-1)\right\}$$

$$= 2$$

$$\mu_3' = \int_0^{\infty} x^3 e^{-x}\,dx = \left[x^3 e^{-x}\right]_0^{\infty} + 3\int_0^{\infty} x^2 e^{-x}\,dx = 3 \times 2 = 6$$

$$\cancel{= 0 + 3\left\{\left[x^2 e^{-x}\right]_0^{\infty} + 2\int_0^{\infty} x e^{-x}\,dx\right\}}$$

$$= 6\left\{-\left[x e^{-x}\right]_0^{\infty} + \int_0^{\infty} 1.e^{-x}dx\right\}$$

$$= 6\left\{-\left[e^{-x}\right]_0^{\infty}\right\} = 6\{-(0-1)\} = 6$$

$$\mu_4' = \int_0^{\infty} x^4 e^{-x} dx = \left[-\left[x^4 e^{-x}\right]_0^{\infty} + 4\int_0^{\infty} x^3 e^{-x} dx\right]$$

$$= 0 + 4 \times 6 = 24$$

$$\therefore \mu_2 = \mu_2' - \mu_1'^2 = 2 - 1^2 = 1$$

$$\mu_3 = \mu_3' - 3\mu_2\mu_1 + 2\mu_1^3 = 2$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1' - 3\mu_1'^4 = 9$$

$$\therefore \beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{2^2}{1^3} = 4$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9}{1} = 9$$

$\therefore$ coefficient of skewness, $\gamma_1 = \pm\sqrt{\beta_1} = \pm\sqrt{4} = \pm 2$

coefficient of kurtosis, $\gamma_2 = \beta_2 - 3 = 9 - 3 = 6$ (A)

Q.7(b)

Central Value: x	1	2	3	4	5	6	7	8	9	10	11	12
Frequency, y; f	2	10	19	25	40	44	41	28	25	15	5	1

The equation of the normal curve for n observation,

$$y = N \times \frac{1}{6\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{6}\right)^2} \quad \text{——— (1)}$$

Here $\mu = \bar{x}$

$f_i x$	2	20	57	100	200	264	287	224	225	150	55	12
$f_i x^2$	2	40	171	400	1000	1584	2009	1792	2025	1500	605	144

$\sum f_i x = 1596$

$\sum f_i x^2 = 11272$

$\therefore \mu = \frac{1596}{255} = 6.26 = \mu_1'$

$\mu_2' = \frac{\sum f_i x^2}{N} = \frac{11272}{255} = 44.20$

Standard deviation, $\sigma = \sqrt{\mu_2' - \mu_1'^2}$

$= \sqrt{\frac{11272}{255} - \left(\frac{1596}{255}\right)^2}$

$\therefore$ From ① $= 2.24$

$y = 255 \times \frac{1}{2.24 \times \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-6.26}{2.24}\right)^2}$

$= 45.42\, e^{-\frac{(x-6.26)^2}{10.035}}$

Therefore $(x - 6.26)$ considers as center of the normal curve.

Q.7(c)

Stratified sampling: The sample which is the aggregate of the sample unit of each of the stratum is called stratified sample and the technique of drawing this ~~sa~~ ~~so~~ smale is called stratified sampling.

(a)

0.

(b)

Q.8(a)

Weight, x	Height, y	x^2	y^2	xy
64	57	4096	3249	3648
71	59	5041	3480	4189
53	49	2809	2401	2597
67	62	4489	3844	4154
55	51	3025	2601	2805
58	50	3364	2500	2900
77	55	5926	3025	4235
57	48	3249	2304	2736
56	52	3136	2704	2912
51	42	2601	1764	2142

$\Sigma x = 609$ $\Sigma y = 525$ $\Sigma x^2 = 37736$, $\Sigma y^2 = 27873$ $\Sigma xy = 32318$

$n = 10$

$$\therefore r = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{10 \times 32318 - 609 \times 525}{\sqrt{10 \times 37736 - (609)^2}\sqrt{10 \times 27873 - (525)^2}} = 0.77$$

(A)

Q.8(b) for one die ant trail 1,

$P(4) = \frac{1}{6}$ $\therefore q = 1 - \frac{1}{6} = \frac{5}{6}$

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$n = 120$, $r = 14$

~~$\therefore P(x \leq 14) = {}^{120}C_{14}$~~

$\mu = \lambda = np = 120 \times \frac{1}{6} = 20$

$\sigma^2 = npq$

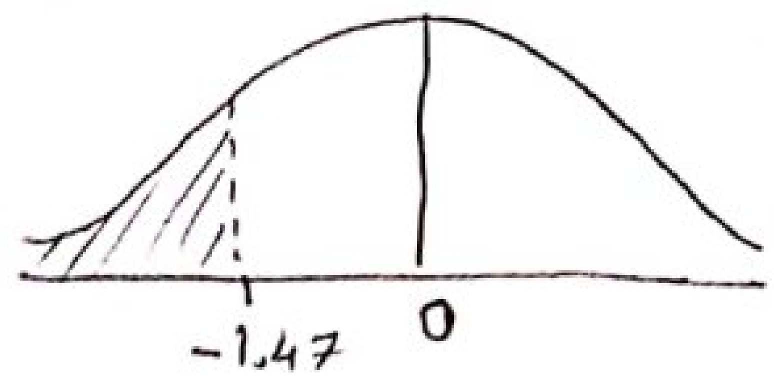
$\therefore \sigma = \sqrt{npq} = \sqrt{120 \times \frac{1}{6} \times \frac{5}{6}} = 4.08$

$\therefore z = \frac{14 - 20}{4.08}$ $\left[z = \frac{X - \mu}{\sigma}\right]$

$= -\frac{25}{17} = -1.47$

$\therefore P(z \le -1.47) = 0.07$ (Ans)

SECTION-B(2015)



Q.3(a) Given that, $P_h(s) = \frac{1}{7}$, $P_w(s) = \frac{1}{5}$

(i) $P(\text{both of them } \cancel{\text{selected}}) = \frac{1}{7} \times \frac{1}{5} = \frac{1}{35} = 0.029$

(ii) $\cancel{P}\ q_h = 1 - P_h(s) = 1 - \frac{1}{7} = \frac{6}{7}$

$q_w = 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore P(\text{only one of them}) = \frac{1}{7} \times \frac{4}{5} + \frac{1}{5} \times \frac{6}{7} = 0.286$

(iii) $P(\text{none of them}) = \frac{6}{7} \times \frac{4}{5} = 0.686$

Q.4(b) Different types of averages are given below:

(i) Harmonic mean: Harmonic Mean (HM) is defined as "A value that is the reciprocal of the mean of the reciprocals of a set of numbers".

merits: The Harmonic Mean is computed based on every observation in the data set.

demerits: The H.M. cannot be calculated if a data set values has negative or zero elemen

H.M of $x = \Sigma f / \Sigma (f/x)$

Geometric Mean: To determine the average percents, indexes and relatives:

$GM = \sqrt[n]{(x_1)(x_2)(x_3)\cdots(x_n)}$

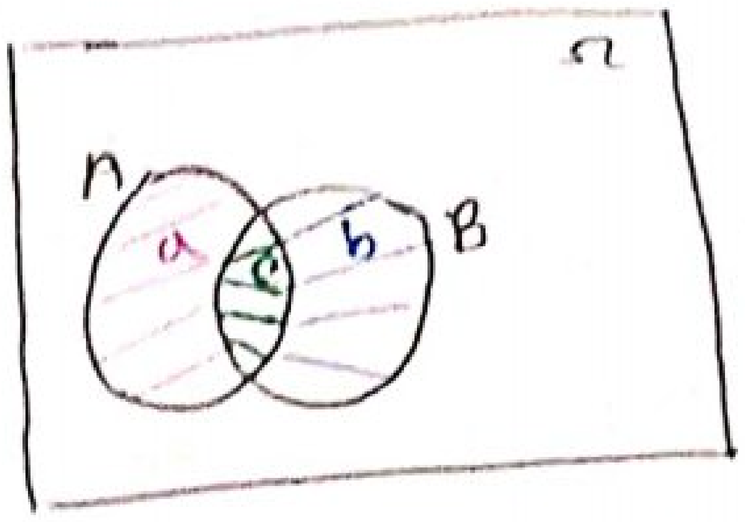
merit: We can easily find out average percents by using this method.

demerit: We, can not use GM always

$\boxed{Q.4(c)}$ $P(A\cup B) = P[(A\cap B^c)\cup(A\cap B)\cup(B\cap A^c)]$

$= P(A\cap B^c) + P(A\cap B) + P(B\cap A^c)$

$= a + c + b$



and, $P(A) + P(B) - P(A\cap B) = P(A\cap B^c) + P(A\cap B) + P(B\cap A^c) + P(A\cap B) - P(A\cap B)$

$= a + c + b$

$\therefore$ $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ (shown)

$\boxed{Q.5(a)}$ Given that, $\mu = 60\% = 0.6$ and $\sigma = 6.5, n = 200$

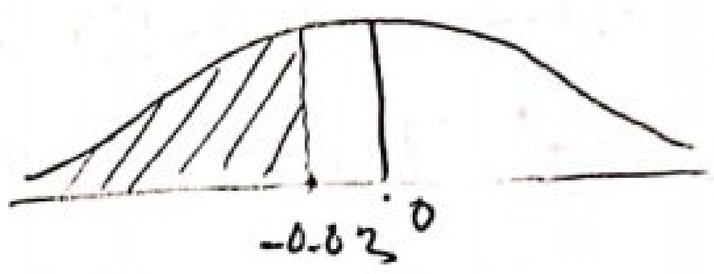
(i) $Z = \dfrac{0.8 - 0.6}{6.5}$ $\left[\because Z = \dfrac{X-\mu}{\sigma}\right]$

$= 0.03$

$\therefore P(Z \geq 0.03) = 0.48803$

$N_{80\%} = 0.48803 \times 200 = 98$

(ii) $Z = \dfrac{0.4 - 0.6}{6.5} = -0.03$



$P(Z \leq -0.03) = 0.48803$

$\therefore$ 48.803 % student got ~~less~~ ~~an~~ Less than 40% marks

$\therefore$ Numbers of student $N_{40\%} = 200 \times 0.48803 = 97.606$
$\simeq 98$

Now, we know the normal curve

$$y = N \times \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Here, $N = 200$, $\mu = 0.6$, $\sigma^2 = 42.25$, $\sigma = 6.5$

$$\therefore y = \frac{200}{6.5 \times \sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-.6)^2}{42.25}}$$

$$e^{-\frac{(x-.6)^2}{84.5}}$$

$$\therefore y = 12.28\, e$$

Let, $X_0 = 1$

$X_n = 200$

$$\text{Area} = \int_{x_0}^{x_n} y\,dx = 12.28\int_{1}^{200} e^{-\frac{(x-0.6)^2}{84.5}}\,dx$$

Let. $x-0.6=z$

$dx = dz$

$$= 12.28\int_{1}^{200} e^{-\frac{z^2}{84.5}}\,dz$$

$$= 12.28 \times (-84.5)\left[\frac{e^{-\frac{z^2}{84.5}}}{2z}\right]_{1}^{200}$$

$$= -\frac{1037.66}{2}(0 - 0.988)$$

$$= 512.60 \quad (\text{Ans})$$

$\boxed{Q.5(b)}$ Given that, $f(x) = \frac{c}{x^2+1}, \; -\alpha < x < \alpha$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\alpha}^{\alpha} f(x)\,dx = \int_{-\alpha}^{\alpha} \frac{c}{x^2+1}\,dx$$

$$\Rightarrow 1 = c\left[\tan^{-1} x\right]_{-\alpha}^{\alpha}$$

$$\Rightarrow 1 = c\left(\tan^{-1}\alpha - \tan^{-1}(-\alpha)\right)$$

$$\Rightarrow 1 = c\left(\tan^{-1}\alpha + \tan^{-1}\alpha\right)$$

$$\Rightarrow 1 = 2c\tan^{-1}\alpha$$

$$\Rightarrow 1 = 2c\cdot\frac{\pi}{2}$$

$$\cancel{\Rightarrow c = \frac{1}{2\tan^{-1}\alpha}}$$

$$\cancel{\therefore c = \frac{\tan\alpha}{2} \quad (\text{Ans})}$$

$$\therefore c = \frac{1}{\pi} \quad (\text{Ans})$$

$\boxed{Q.7\ (c)}$

Mistake per day, x_i	0	1	2	3	4	5	6
No of day, f_i	143	90	42	12	9	3	1
$f_i x_i$	0	90	84	36	36	15	6
$f_i x_i^2$	0	90	168	108	144	75	36

$\Sigma f_i x_i = 267$, $\Sigma f_i x_i^2 = 621$ $N = \Sigma f_i = 300$

mean. $\mu_1' = \mu = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{267}{300} = 0.89$

$\mu_2' = \frac{\Sigma f_i x_i^2}{\Sigma f_i} = \frac{621}{300} = 2.07$

$\therefore \sigma = \sqrt{\mu_2' - \mu_1'^2}$

$= \sqrt{2.07 - (0.89)^2} = 1.13$

The equation of normal curve distribution

$y = N \times \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$= 300 \times \frac{1}{1.13 \times \sqrt{2\pi}} e^{-\frac{(x-0.89)^2}{2\times(1.13)^2}}$

$\therefore y = 105.91\, e^{-\frac{(x-0.89)^2}{2.55}}$

Now. $(x - 0.89)$ take as the center of the normal curve

$x - 0.89 = 0 \quad \pm 1 \quad \pm 2 \quad \pm 3$

$y = 105.91 \quad 156.76 \quad 232.04 \quad 343.45$

Fitting these value we get the required curve.

$\boxed{\text{Q. 8(a)}}$ First let us rearrange the values of x and y ascending order and give the rank according to their value

x :	16	19	25	25	28	30	32	36
Rank of x ;	1	2	3.5	3.5	5	6	7	8

Y : 192 210 210 218 218 232 236 236

Rank of Y : 1 2.5 2.5 4.5 4.5 6 7.5 7.5

Now let us tabulate the values

x	Rank of x, R_1	y	Rank of y, R_2	$d = R_1 - R_2$	d^2
16	1	192	1	0	0
19	2	218	4.5	-2.5	6.25
25	3.5	210	2.5	1	1
28	5	232	6	-1	1
25	3.5	218	4.5	-1	1
30	6	236	7.5	-1.5	2.25
32	7	210	2.5	4.5	20.25
36	8	236	7.5	0.5	0.25

$\Sigma d^2 = 32$

$n = 8$

coefficient of Rank correlation,

$$r = 1 - \frac{6 \times \Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 32}{8 \times (8^2 - 1)} = 0.619 \text{ (Ans)}$$

Q.8(b) **Sampling:** Sampling is a process used in statistical analysis in which a predetermined number of observations are taken from a larger population.

Stratified sampling: আগে আছে

Sequential Sampling: Sequential sampling is a sampling technique that involves the evaluation of each sample taken from a population to see if it fits a desired conclusion.

Poition distribution

$\boxed{Q.1(b)}$ (I) ~~Lede~~, orange balls $= \cancel{9}7$ and otuers $= 8$
Total $= 15$

$\therefore P(\text{orange}) = \frac{7}{15}$

$q = 1 - \frac{7}{15} = \frac{8}{15}$

$n = 15$

$\therefore P(x) = {}^nC_x \, p^x \, q^{n-x} = \frac{15!}{x!(n-x)!}\left(\frac{7}{15}\right)^x\left(\frac{8}{15}\right)^{15-x}$

$\therefore P(2) = {}^{15}C_2 \left(\frac{7}{15}\right)^2 \left(\frac{8}{15}\right)^{15-2} = 6.46\times10^{-3}$

For white ball

$P = \frac{5}{15} = \frac{1}{3}$

$q = 1 - \frac{1}{3} = \frac{2}{3}$ Extra

$\therefore P(x) = \frac{15!}{x!(n-x)!}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{n-x}$

For black ball

$P = \frac{3}{15} = \frac{1}{5}$, $q = 1 - \frac{1}{5} = \frac{4}{5}$ Extra

$P(x) = \frac{15!}{x!(n-x)!}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{n-x}$

Normal distribution

orange ball

$n = 15$, $P = \frac{7}{15}$, $q = \frac{1}{2}$, $\lambda = nP = 7$

$P(x) = \frac{e^{-\lambda}.\lambda^x}{x!} = \frac{e^{-7}.7^x}{x!} = \frac{e^{-7}.7^2}{2!} = 0.02$

white ball

$\lambda = \frac{1}{3}\times15 = 5$ Extra

$\therefore P(x) = \frac{e^{-5}.5^x}{x!}$

Black ball

$\lambda = \frac{1}{5}\times15 = 3$ Extra

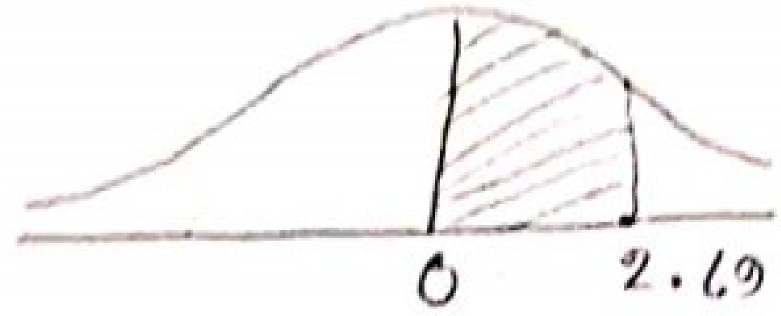
$P(x) = \frac{e^{-3}.3^x}{x!}$

Q.(a) We know, $z = \frac{x-\mu}{\sigma}$

Here, $\mu = 120$, $\sigma = 13$, $n = 500$

$\therefore \frac{120-120}{13} \le \frac{x-\mu}{\sigma} \le \frac{155-120}{13}$

$\Rightarrow 0 \le z < 2.69$



$\therefore P(0 \le z < 2.69) = 0.49643$

$\therefore$ 49.643% students are between 120 and 155

$\therefore$ Number of these student $= 500 \times \frac{49.643}{100} = 248.215$
≈ 248 (A)

Q.2(b)

Systematic sampling: Systematic sampling is a type of probability sampling method in which sample members from a larger population are selected according to a random starting point but with a fixed, periodic interval.

Q.3(b) Total = 10

(i) $P(\text{one from each category}) = \frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1 \times {}^1C_1}{{}^{10}C_4}$

$= \frac{4}{35} = 0.11$

(ii) purchase = 4

others = 6

$\therefore P(\text{at least one from purchase}) = 1 - \frac{{}^4C_0 \times {}^6C_4}{{}^{10}C_4}$

$= \frac{13}{14} = 0.929$ (A)

Q.3 (c) (i) $P(3) = \frac{1}{6}$, $n = 8$, $q = 1 - \frac{1}{6} = \frac{5}{6}$

$\therefore$ P(at least once) $= 1 - P(0)$ $\left[\because P(x) = {}^nC_x\, p^x q^{n-x}\right]$

$$= 1 - {}^8C_0 \times \left(\frac{1}{6}\right)^0 \times \left(\frac{5}{6}\right)^{8-0}$$

$$= 0.77$$

(ii) P(at most 7 time) $= 1 - P(8)$

$$= 1 - {}^8C_8 \times \left(\frac{1}{6}\right)^8 \times \left(\frac{5}{6}\right)^{8-8}$$

$$= 1 - 0.000000595$$

$$= 0.99999994 \text{ (A)}$$

Q.4(a)

Probability of being defective, $P = \frac{1}{500}$

$n = \cancel{8000}\ 10$, $N = 8000$ $\therefore \lambda = 10 \times \frac{1}{500} = \frac{1}{50}$

~~(i) P(no defective) =~~

$\cancel{\therefore P(r=0) = N \times \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-\frac{499}{50}}\left(\frac{499}{50}\right)^0}{0!} \times 8000 = 0.3}$ $\cancel{\lambda = 160}$

(1) $P(r=0) = \cancel{8000\times} \dfrac{e^{-\frac{1}{50}} \times \left(\frac{1}{50}\right)^0}{0!} = 0.98$

$\therefore$ No of lots without defective $= 0.98 \times 8000 = 7840$

(2) $P(r=2) = \dfrac{e^{-\frac{1}{50}}\left(\frac{1}{50}\right)^2}{2!} = 1.96 \times 10^{-4}$

$\therefore$ No of lots with 2 defective $= 8000 \times 1.96 \times 10^{-4}$

$= 1.6 \approx 2$

$\boxed{Q.4(b)}$ Given that, $f(x) = x(5-x^2),\quad 0 \le x \le 2$

$\therefore$ mean $= \mu_1' = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_0^2 x \cdot x(5-x^2)\,dx$

$= \int_0^2 5x^2\,dx - \int_0^2 x^4\,dx = 5\left[\frac{x^3}{3}\right]_0^2 - \left[\frac{x^5}{5}\right]_0^2$

$= \frac{5}{3} \times 2^3 - \frac{1}{5} \times 2^5$

$= \cancel{9.33}\ 6.93$

$\sigma^2 = \mu_2' - \mu_1'^2$

Here, $\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^2 x^3(5-x^2)\,dx$

$= 5\int_0^2 x^3\,dx - \int_0^2 x^5\,dx$

$= 5\left[\frac{x^4}{4}\right]_0^2 - \left[\frac{x^6}{6}\right]_0^2$

$= \frac{5}{4} 2^4 - \frac{1}{6} 2^6 = 9.33$

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$\therefore$ variance $= \sigma^2 = 9.33 - (6.93)^2 = -38.69$

$\boxed{Q.4(b)}$

Candidate	Rank by A, R_1	Rank by B, R_2	$d = R_1 - R_2$	d^2
A	4	1	3	9
B	3	6	-3	9
C	1	7	-6	36
D	2	5	-3	9
E	7	8	-1	1
F	9	10	-1	1
G	8	9	-1	1
H	10	3	7	49
I	5	2	3	9
J	6	4	2	4

$\Sigma d^2 = 119$

$n = 10$

$$\therefore r = 1 - \frac{6 \Sigma d_i^2}{n(n^2-1)} = 1 - \frac{6 \times 119}{10 \times (10^2-1)} = 0.279.$$