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SOIL MECHANICS AND FOUNDATIONS

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SOIL MECHANICS AND FOUNDATIONS

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SOIL MECHANICS AND FOUNDATION

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Preface

PREFACE TO THE SECOND EDITION

Soil Engineering, soil mechanics or geotechnique is one of the youngest disciplines of civil engineering involving the study of soil, its behaviour and application as an engineering material. The term 'Soil Engineering' is currently used to cover a much wider scope implying that it is a practical science rather than a purely fundamental or mathematical one. Good soil engineering embodies the use of the best practices in exploration, testing, design and construction control, in addition to the basic idealised theories.

The field of Soil Mechanics is very vast. The civil engineer has many diverse and important encounters with soil. Apart from testing and classification of various types of soils in order to know its physical properties, the knowledge of soil mechanics is particularly helpful in the designs of foundations, rigid and flexible pavements, underground and earth retaining structures, embankments, and excavations, and earth dams.

This textbook is intended to present the currently accepted theories, design principles and practices of soil mechanics and foundation engineering. The assumptions and limitations used in developing a theory or a method are pointed out and sufficient number of examples and practice problems are included to illustrate the application of basic principles of practical problems. The text provides sufficient material, ranging from the simple to very complex, for the undergraduate and postgraduate courses in the subject of soil engineering in the engineering colleges, universities and professional examinations in India. The book should also prove a useful design aid for the practising engineer and a convenient reference source.

The book has been sub-divided into eight parts : (1) Elementary Properties, (2) Soil Hydraulics, (3) Elasticity Applied to Soils, (4) Compressibility, (5) Strength and Stability, (6) Foundation Engineering, (7) Pavement Design and (8) Miscellaneous Problems. More advanced topics have been indicated by an asterisk which may be omitted in the first reading or at the undergraduate level. In addition, *twenty-one* more common laboratory experiments have been included to illustrate the practice of soil testing. The test procedures conform to the latest Indian Standards issued to date. Metric Units have been used in the text. The author is highly thankful to Dr. Alam Singh for his kind permission to reproduce the matter of the first edition of the book published in his collaboration. Account has been taken throughout of the suggestions offered by the many users of the book and grateful acknowledgement is made to them. Further suggestions will be greatly appreciated.

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PREFACE TO THE THIRD EDITION

In the Third Edition, the subject-matter has been updated. In order to make the book more useful to the students appearing at the A.M.I.E. Examinations, questions from the examination papers of section B have been given in Appendix.

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PREFACE TO THE FOURTH EDITION

In the Fourth Edition of the book, the subject-matter has been thoroughly revised and enlarged to incorporate the latest developments. An article on 'dynamic analysis of block foundations' having six modes of vibrations has been added. The author is thankful to many readers of the book for useful suggestions.

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PREFACE TO THE FIFTH EDITION

In the Fifth Edition, the subject-matter has been revised, and an Appendix on SI units has been added at the end of the book.

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PREFACE TO THE SIXTH EDITION

In the Sixth Edition, the subject-matter has been revised and updated. Matter on under-reamed pile foundation has been added in Chapter 26.

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PREFACE TO THE EIGHTH EDITION

In the Eighth Edition, many new articles have been added and the subject-matter has been revised and updated.

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PREFACE TO THE NINTH EDITION

In the Ninth Edition, few new articles have been added. Notable amongst these are : bored compaction piles, field set-ups for plate load test, field set-up for pile load test and pneumatic caissions. The subject-matter has been revised and updated.

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PREFACE TO THE TENTH EDITION

In this edition, few misprints of the previous edition have been removed and the subject-matter has been updated.

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PREFACE TO THE ELEVENTH EDITION

In the Eleventh Edition, the book has been thoroughly revised and enlarged, using SI units. Care has been taken to distinguish between 'mass' and 'weight' and between 'density' and 'unit weight' and separate symbols have been assigned to these quantities to avoid confusion. Density (ρ), based on laboratory measurements has been expressed in the units of g/cm^3 (or kg/m^3) while the unit weight (γ) has been expressed in kN/m^3. The value of gravitational constant g has been taken as 981 cm/s^2. Thus, the density of water (ρ_w) has been taken as 1 g/cm^3 (or 1000 kg/m^3) while its unit weight has been taken as 9.81 kN/m^3.

A chapter on 'Advanced Measuring Instruments' has been added at the end. A large number of photographic plates, illustrating various testing equipment used in 'Soil Testing Laboratory' have been added. The authors are thankful to M/s Associated Instruments Manufactures, India (AIMIL), Delhi and M/s HEICO Instruments, Delhi for supplying photographs and illustrative catalogues of the instruments/ equipment manufactured by them.

It is hoped that this revised and updated edition will be useful to the students and practicing engineers. Further suggestions will be gratefully accepted.

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1-7-88 **A.K. JAIN**

PREFACE TO THE TWELFTH EDITION

In the Twelfth Edition, subject matter has been revised and enlarged. Chapter 24 on Bearing Capacity and chapter 25 on Shallow Foundations have been rewritten and many new articles and new examples have been added.

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PREFACE TO THE THIRTEENTH EDITION

In the Thirteenth Edition of the book, the subject matter has been thoroughly revised and updated. Many new articles and solved examples have been added. The entire book has been typeset using laser printer. The authors are thankful to Shri Mool Singh Gahlot and Shri Prem Singh Sankhla for the fine laser typesetting done by them.

JODHPUR
14th Nov. 1994

B.C. PUNMIA
ASHOK K. JAIN
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PREFACE TO THE SIXTEENTH EDITION

In the Sixteenth Edition, the subject matter has been thoroughly revised and updated, and rearranged. In each chapter, many new articles have been added. All the diagrams have been redrawn using computer graphics and the book has been computer type-set in a bigger format, keeping in pace with the modem trend. At the end of each chapter, problems appearing at various central competitive examinations (such as Civil Services, Engineering Services and Gate) have been solved. In addition to these, a new chapter on 'Geotextiles' have been added at the end of the book.

It is hoped, this thoroughly revised and updated edition will be useful to the students, teachers and practicing engineers. Further suggestions will be gratefully acknowledged. The authors are thankful to Shri R.K. Gupta of Laxmi Publications (P) Ltd. for good printing and excellent get-up of the book and that too in a record short time.

JODHPUR
1st March 2005

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Symbols

A = Pore pressure parameter ; area, constant

$\bar{A}$ = Pore pressure parameter

$\mathring{A}$ = Angstrom

A_f = Parameter A at failure

A_p = Sectional area of pile toe

A_v = Area of voids

A_s = Amplitude of vibrations

A_{sp} = Permissible amplitude

a = Area; constant ; radius

a_c = Air content

B = Pore pressure parameter ; constant ; width

$\bar{B}$ = Pore pressure parameter ;

B_p = Width of test plate

b = Constant ; width ; thickness of aquifer

C = Composite correction (hydrometer) ; constant ; total cohesion

C_e = Compression index ; coefficient of curvature

C_d = Dispersing agent correction

C_m = Meniscus correction

C_s = Expansion index ; shape constant

C_t = Temperature correction

C_u = Uniformity coefficient ; coefficient of elastic uniform compression of soil

C_v = Coefficient of volume compressibility

c = Unit cohesion, damping factor

c' = Effective unit cohesion

c_c = Critical damping factor

C_d' = Cohesion in drained test

c_e = Hvorslev true cohesion

c_m = Mobilised cohesion

c_u = Apparent cohesion

c_v = Coefficient of consolidation

c = Diameter ; depth ; distance

D_a = Diameter of average particle

D_{10} = Effective size; 10 per cent finer size

D_F = Depth factor

D_p = Depth of test plate

D_{min} = Minimum depth of foundation

d = Diameter; distance

E = Young's modulus of elasticity ; electric potential

e = Voids ratio; coefficient of restitution ; eccentricity; base of naturallogarithms (2.7183)

e_w = Water voids ratio

e_{max} = Maximum voids ratio

e_{min} = Minimum voids ratio

F = Force ; factor of safety ; deflection factor ; function of

F_c = Factor of safety with respect to cohesion

F_0 = Dynamic force

F_T = Transmitted Force

f = Silt factor ; stress ; function of; operating frequency

f_c = Unit skin friction of shaft of penetrometer

f_n = Natural frequency

f_s = Unit skin friction

G = Specific gravity of soil particles ; centre of gravity

G_k = Specific gravity of kerosene

G_m = Mass specific gravity of soil mass

g = Acceleration due to gravity

H = Height ; thickness ; depth total hydraulic head

H_a = Height of air in specimen

H_c = Critical height (depth) of slope or cut

H_s = Effective height in hydrometer analysis, sampling depth

H_s = Height of soil solids

H_w = Height of water

h = Hydraulic head ; height ; thickness

h_c = Capillary rise; Henry's constant of solubility of air in water

h_w = Piezometric head

I = Current; Index

I_D = Density index

I_f = Flow index ; Impact factor

I_L = Liquidity index

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I_T = Toughness index
i = Hydraulic gradient
i_c = Critical hydraulic gradient
i_e = Exit gradient
J = Seepage force
j = Seepage per unit volume
K = Coefficient; coefficient of earth pressure ; complete elliptic integral of the first kind with modulus m ; Influence factor
K' = Complete elliptic integral of the first kind with modulus m'
K_a = Coefficient of active earth pressure
K_B = Boussinesq influence factor for vertical pressure
K_0 = Coefficient of earth pressure at test
K_P = Coefficient of passive earth pressure
K_W = Westergaard's influence factor for vertical pressure
k = Coefficient of permeability; spring constant; modulus
k' = Effective permeability in a transformed flow net
k_p = Coefficient of percolation
k_s = Modulus of subgrade reaction
$k_{physical}$ = Physical permeability coefficient
L = Length; distance
$\bar{L}$ = Length of chord
L = Length of arc
l = Length; distance
M = Moment; factor
M_D = Driving moment
M_o = Overturning moment
M_B = Resisting moment
m = Mass; modulus ; ratio ; coefficient
m' = Complimentary modulus
m_s = Equivalent soil mass
m_c = Coefficient of volume change ; mass of vibrator
N = Normal force ; percent finer ; number of blows ; integer
N_c, N_q, N_γ = Bearing capacity factors (general shear failure)
N_c', N_q', N_γ' = Bearing capacity factors (local shear failure)
N_d = Number of potential drops
N_f = Number of flow channels
$N_{\gamma q}$ = Resultant bearing capacity factor (Meyerhof)
n = Porosity ; number ; exponent ; ratio
n_a = Percentage air voids
P = Force; load; weight
P_a = Resultant active pressure
P_0 = Equivalent wheel load
P_p = Resultant passive load
p = Pressure;reaction;percentage
p_a = Intensity of active earth pressure ; atmospheric pressure
p_p = Intensity of passive earth pressure
pF = Logarithm of soil suction
p_s = Seepage pressure
Q = Total quantity of flow or discharge load
Q_d = Allowable load
Q_{up} = Failure load ; ultimate bearing capacity of pile
Q_{ug} = Ultimate bearing capacity of pile group
q = Unit surcharge ; intensity of uniformly distributed load ; rate of flow or unit discharge
q_c = Static cone resistance, North Dakota cone bearing value
q_n = Net foundation pressure
q_f = Ultimate bearing capacity
q_u = Unconfined compressive strength
R = Corrected hydrometer reading ; radius; radius of zero drawdown ; reaction ; resistance ; dial reading ; amplitude ratio ; maximum scour depth
R_H = Hydraulic radius
R_h' = Hydrometer reading at top of meniscus
R_h = Hydrometer reading at bottom of meniscus, graduation reading on hydrometer stem
R_f = Total ultimate skin friction
R_L = Lacey's normal scour depth
R_p = Total ultimate point bearing resistance
R_T = Total tension (pull)
r = Radius; distance, frequency ratio
r_f = Unit skin friction
r_p = Unit point resistance
r_u = Pore-pressure ratio
S = Degree of saturation; storage coefficient ; shear force ; real set per blow
S_n = Stability number
S_R = Specific retention
S_s = Specific surface
S_Y = Specific yield
s = Soil suction; drawdown
T = Torque ; tangential component ; temperature coefficient of transmissibility

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T_s = Surface tension
T_v = Time factor
t = time; thickness
U = Degree of consolidation; total pore pressure ; Jacobi's symbol for the elliptic integral of the first kind
u = Pore pressure ; stream function
$\bar{u}$ = Excess hydrostatic pressure
V = Volume ; total volume of soil ; velocity of wave
V_a = Volume of air
V_d = Volume of dry soil
V_h = Volume of hydrometer
V_p = Volume of pipette
V_s = Volume of solids ; seepage velocity
V_p = Volume of voids
V_w = Volume of water
V_{wY} = Volume of water drained by gravity
V_{wR} = Volume of water retained
v = Velocity of flow ; superficial velocity ; pressure function
v_c = Critical velocity
v_s = Seepage velocity
W = Weight; total weight of soil
W_a = Weight of anvil
W_b = Weight of foundation block
W_d = Dry weight
W_D = Weight of soil solids per millilitre of suspension smaller than diameter D
W_f = Weight of frame
w = Water content; complex potential
w_f = Water content at failure
w_L = Liquid limit
w_P = Plastic limit
w_S = Shrinkage limit
w_{sat} = Water content at full saturation
x = Abscissa ; coordinate ; distance
$\bar{x}$ = Distance
y = Ordinate ; coordinate ; distance
$\bar{y}$ = Distance
Z = Depth ; thickness ; distance ; complex coordinate
$\bar{Z}$ = Distance
Z_c = Depth of tension crack
z = Coordinate ; variable ; depth ; thickness; height; position head; complex coordinate
α = Angle
α_A, α_B= Directional angles
β = Angle; angle of obliquity
γ = bulk density; variable
γ_d = Dry density
γ_{sat} = Saturated density
γ' = Submerged density
γ_s = Density of soil solids
γ_w = Density of water
Δ = Change or increment (or decrement); triangle
δ = Angle of wall friction : elastic deflection of machine foundation
s = Static deflection of spring
$\in$ = strain
ε = Base of natural logarithm (=2.7183)
η = Viscosity ; coordinate variable
η_b = Efficiency of blow
η_a = Efficiency of pile group
η_h = Efficiency of hammer
σ_1', σ_2', σ_3' = Effective principal stress
θ = Angle; inclination
θ_f = Inclination of failure plane
μ = Poisson's ratio ; micron ; magnification factor
π = 3.1416
$\Pi(U, a)$ = Jacobi's elliptic integral of third kind
$\Pi(n, \phi, k)$ = Legendre's elliptic integral of third kind
ρ = Settlement, deflection ; penetration ; resistivity horizontal displacement of well foundation
ρ_c = Corrected, final consolidation settlement
ρ_f = Final settlement
ρ_p = Settlement of test plate
Σ = Summation of
σ = Total pressure; normal stress
σ' = Effective pressure (stress)
σ_1, σ_2, σ_3 = Major, intermediate and minor principal stresses
ϕ_u = Apparent angle of shearing resistance
σ_d = Deviator stress
σ_c' = Consolidation pressure
σ_p' = Preconsolidation pressure
σ_x = Vertical pressure, normal
σ_x, σ_y, σ_z = Stress component in x, y, z directions
τ = Shear stress ; tangential stress
τ_f = Shear strength
Φ = Hydraulic potential ; Stress function
ϕ = Angle; angle of shearing resistance, velocity potential; a mu

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ψ' = Effective angle of shearing resistance

ϕ_d' = Angle of shearing resitance in drained test

ϕ_e' = Hvorslev true friction angle

ϕ_μ = Mobilised ϕ

ϕ_w = Weighted friction angle

ϕ_x = % pressure at any point

ψ = Angle; Stream function

ω = Angle ; operating frequency

ω_n = Natural frequency

∞ = Infinity

ζ = Complex coordinate

ξ = Coordinate variable ; damping ratio

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Part I
Elementary Properties

1. Introduction
2. Preliminary Definitions and Relationships
3. Determination of Index Properties
4. Classification of Soils
5. Soil Structure and Clay Mineralogy

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CHAPTER 1

Introduction

1.1 SOIL AND SOIL ENGINEERING

The term *'Soil'* has various meanings, depending upon the general professional field in which it is being considered. To an agriculturist, soil is the substance existing on the earth's surface, which grows and develops plant life. To the geologist also, soil is the material in the relatively thin surface zone within which roots occur, and all the rest of the crust is grouped under the term *rock* irrespective of its hardness. To an engineer, soil is the unaggregated or uncemented deposits of mineral and/or organic particles or fragments covering large portion of the earth's crust. It includes widely different materials like boulders, sands, gravels, clays and silts, and the range in the particle sizes in a soil may extend from grains only a fraction of a micron (10^{-4} cm) in diameter up to large size boulders.

Soil engineering, *Soil Mechanics* or Geotechnique is one of the youngest disciplines of civil engineering involving the study of soil, its behaviour and application as an engineering material. According to Terzaghi (1948) : 'Soil Mechanics is the application of laws of mechanics and hydraulics to engineering problems dealing with sediments and other unconsolidated accumulations of solid particles produced by the mechanical and chemical disintegration of rocks regardless of whether or not they contain an admixture of organic constituent'. The term *Soil Engineering* is currently used to cover a much wider scope implying that it is a practical science rather than a purely fundamental or mathematical one. The term *Foundation Engineering* is a branch of civil engineering, which is associated with the design, construction, maintenance, and renovation of footings, foundation walls, pile foundations, caissons, and all other structural members which form the foundations of buildings and other engineering structures (Taylor, 1948).

Soil is considered by the engineer as a complex material produced by the weathering of the solid rock. The formation of soil is as a result of the geologic cycle continually taking place on the face of the earth. The cycle consists of weathering or denudation, transportation, deposition and upheaval, again followed by weathering, and so on. *Weathering* is caused by the physical agencies such as periodical temperature changes, impact and splitting action of flowing water, ice and wind, and splitting actions of ice, plants and animals. Cohesionless soils are formed due to physical disintegration of rocks. Chemical weathering may

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be caused due to oxidation, hydration, carbonation and leaching by organic acids and water. Clay minerals are produced by chemical weathering. Soil obtained due to weathering may be residual or transported. *Residual* soils, which remain in place directly over the parent rock, are relatively shallow in depth. The deposits of the *transported soils* may be considerable in depth and their homogeneity or heterogeneity depends upon the manner of their transportation and deposition. The various agencies of transporting and redepositing soils are : water, ice, wind and gravity. Water-formed transported soils are termed as *alluvial, marine* or *lacustrine.* All the material, picked up, mixed, disintegrated, transported and redeposited by glaciers either by ice or by water issuing from melting of glaciers, is termed *glacial drift* or *simple drift.* The glacial deposits in general consists of a heterogeneous mixture of rock fragments and soils of varying sizes and proportions and, except the stratified drift deposited by glacial streams, are without any normal stratification. Dune sand and loss are the wind-blown *(aeoline)* deposits. *Loess* is the wind-blown silt or silty clay having little or no stratification. Soils transported by gravitational forces are termed *colluvial soils,* such as *talus.* The accumulation of decaying and chemically deposited vegetable matter under conditions of excessive moisture results in the formation of *cumulose* soils, such as *peat* and *muck.*

1.2 HISTORY OF DEVELOPMENT OF SOIL MECHANICS

The knowledge of the use of soil extends into prehistoric times, when man started constructing dwellings for living and roads for transportation. In the more primitive civilisations, soil was used by man as a construction material for foundations of structure and for the structures themselves. The knowledge of soils for the foundations, bunds and roads was gained by trial and error experiences. Through ancient times and even within the last few generations practically all improvement was the result of a continuously broadening by empirical knowledge.

The use of both timber and stone caissons of soft-ground shaft construction was known in Egypt in 2000 BC. The cutting edge was made of a round limestone block with a vertical hole bored into its middle. The outside surface of the caission was made smooth for reasons of reducing sinking resistance caused by friction. One of the greatest structures in ancient times was the famous 'hanging garden' built by the Babylonian King Nebuchadnezzar. The big retaining walls to support the terraces of the garden required some knowledge of earth pressures, even if the knowledge was empirical (Jumikis, 1962). The technical literature of the time during the Roman Empire supplies ample evidence that the Romans paid much attention to some properties of soils, and to the stability of foundations. The Romans built notable engineering structures, such as : harbours, moles, break-waters, aqueducts, bridges, large public buildings, sewage lines and a vast network of durable and excellent roads, requiring solutions of earth work and foundation design. The Roman engineer Vitruvious wrote his *Ten books on Architecture* sometimes in the first century BC discussing the stability of buildings, Vitruvious writes that "..... greatest care must be taken in the substructure, because, in these, immense damage is caused by the earth piled against them. For it cannot remain of the same weight as it usually has in the summer ; it swells in the winter by absorbing water from the rains. Consequently, by its weight and expansion it bursts and thrusts out the retaining wall" –(Jumikis, 1962). For foundations in loose or marshy land, he recommends the use of 'piles to be driven close together by machinery, and the intervals between them to be filled with charcoal.' In India also, Mansar, Mayamata, Visvakarma, Agastya, Santakumara, Mandana, Srikumara, etc. wrote books laying down rules of construction. Among these, Mansar's 'Silpa Sastra,' written sometimes in sixth or seventh century, became very popular. Mansar recommended compaction of soil by cows and oxen, and dewatering of foundations.

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Many structures were built during the medieval period (about 400 to 1400 AD). One of the main problems they had was about the compression of soil and the consequent settlement of buildings. During the past centuries, the compressible soil upon which heavy structures such as cathedrals etc. were built had enough time to consolidate, causing large settlements. The Leaning Tower of Pisa, constructed between 1174 to 1350 AD, is one such example. In India, Taj Mahal was constructed between 1632 to 1650 AD. It had unique foundation problems because of its proximity of the river Jamuna. The terrace and the mausoleum building, as well as the minarets, rest on one firm, compact bed of masonry, supported on masonry cylindrical wells sunk at close intervals. In the field of earth dams, the most notable example reported is that of Mudduk Masur dam in South India, of 33 m height, and built in 1500 AD (Oza, 1969).

In 1661, France undertook an extensive public works programme in improving the highways, and the building of canals. In the later part of the 17th century, French Military engineers contributed some empirical and analytical data pertaining to earth pressure on retaining walls for the design of revetments of fortifications. France established a Department of Roads and Bridges in 1715, and in 1747, the Famous *Ecole des ponts et chausses* was started. The first major contribution to the present scientific study of soil behaviour may be traced back towards the end of the eighteenth century, when Coulomb (1776), a Frenchman, published his wedge theory of earth pressure. Coulomb was the first to introduce the concept that shearing resistance of soil is composed of two components, namely, cohesion and friction. Poncelet (1788–1867), a famous geometer, extended Coulomb's theory, giving a graphical method of finding the magnitude of earth pressure on the wall, vertical as well as for inclined wall surface on the backfill side, and for arbitrary broken polygonal surfaces. K.Culmann (1866) gave the Coulomb-Poncelet theory a geometrical formulation. The earth pressure theory was elaborated by graphical analysis also by Rebhann (1871) and Weyrauch (1878). Two important laws—Darcy's law for flow of water through soils and Stoke's law for settlement of soild particles in liquid—were put forward in 1856. Even today, these laws play an important role in soil engineering. In 1857, Rankine presented his theory for calculating earth pressure and safe bearing capacity of foundation. Rankine and other workers of his time did not take cohesion of clay soil into calculations , although they knew its existence. Another important contribution in the nineteenth century was made by Boussinesq (1885) who gave his analysis for stress distribution in a semi-infinite, elastic medium under surface point loads. To test the earth pressure theories, Muller-Breslau (1906) performed some relatively extensive and elaborate experiments with a large scale model retaining wall.

In 1871, O. Mohr gave a graphical representation of stress at a point, popularly known as Mohr's stress circle. In soil mechanics, Mohr's stress circles are extensively used in the analysis of the shearing strength of soils. It is only in the beginning of the twentieth century that the basic physical properties of soil in general were understood, and the work of Atterberg, a Swedish soil scientist, and that of the Geotechnical Commission of the Swedish Government under the chairmanship of Dr. Fellenius, in this direction are remarkable. Atterberg was the first to propose in 1911 the different stages of consistency in which a clay soil may exist, depending upon its water content. To measure the shear strength of sand, shear box was probably first developed in France, by Leygue in about 1885. Later, it was improved by Krey (1918) in Germany, and Terzaghi and Casagrande in U.S.A. Resal (1910) and Bell (1915) are credited to have extended Rankine's analysis of earth pressure so as to include soil with both friction and cohesion. Bell also suggested a method of calculating the bearing capacity of cohesive soils. In 1916, Petterson and Hultin used the circular sliding theory with the so-called friction circle in stability calculations. This method was further developed by Fellenius in 1926, and is now known as the Swedish method of slope analysis. In 1913, the Swedish Geotechnical Commission was appointed, with Fellenius as its chairman.

In 1920, L. Praridtl gave his theory of plastic equilibrium, which forms the basis of various bearing capacity theories developed later. Dr. Terzaghi published his theory of consolidation in 1923 and the

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term *Soil Mechanics* was coined by him in 1925 when his book under the equivalent German title *Erdbaumechanik* was published. Dr. Terzaghi's contributions in the field of soil engineering have been immense and he is fittingly called the 'Father of Soil Mechanics'. Another important contribution made recently (1933) is that of Proctor on the principles of soil compaction. In 1922–23 Pavlovsky in Russia solved the complex problems of seepage below the hydraulic structures, and gave the electrical analogy method for the seepage computations. However, since his work was in Russian language, it remained unknown to the English literature till 1933, Weaver (1934) and Khosla (1936) solved some of the seepage problems independently. During World War II (1939–45) and after, a great impetus to the development of soil engineering has been made by various scientists and engineers of different countries of the World, and today it is recognised as a well-established branch of engineering. Several International conferences on Soil Mechanics and Foundation Engineering have been held till now under the auspices of International Society of Soil Mechanics and Foundation Engineering such as at Harvard (Massachusetts, U.S.A.) 1936, Rotterdam (Netherlands) 1948, Zurich (Switzerland) 1953, London (U.K.) 1957, Paris (France) 1961, Montreal (Canada) 1965, Mexico city (Mexico) 1969, Moscow (U.S.S.R.) 1973, Tokyo (Japan) 1977, Stockholm (Sweden) 1981, San Fransisco (U.S.A) 1985, Riode Jeneiro (Brazil) 1989, New Delhi (India) 1994, etc.

1.3 FIELD OF SOIL MECHANICS

The field of soil mechanics is very vast. The civil engineer has many diverse and important encounters with soil. Apart from the testing and classification of various types of soils in order to determine its physical properties, the knowledge of soil mechanics is particularly helpful in the following problems in civil engineering.

1. **Foundation design and construction.** Foundation is an important element of all civil engineering structures. Every structure — building, bridge, highway, tunnel, canal or dam — is founded in or on the surface of the earth. It is, therefore, necessary to know the bearing capacity of the soil, the pattern of stress distribution in the soil beneath the loaded area, the probable settlement of the foundation, effect of groundwater and the effect of vibrations, etc. The suitability of various types of foundations — *i.e.,* spread foundation, pile foundation, well foundation, etc. — depend upon the type of soil strata, the magnitude of loads and groundwater conditions. A knowledge of shrinkage and swelling characteristics of soil beneath the foundation is also very essential.
2. **Pavement design.** A pavement can either be flexible or rigid, and its performance depends upon the subsoil on which it rests. The thickness of a pavement and its component parts, depends upon some certain characteristics of the subsoil, which should be determined before the design is made. On busy pavements, where the intensity of traffic is very high, the effect of repetition of loading and the consequent fatigue failure has to be taken into account. Apart from these, other problems of pavement design are : frost, heave and thaw with their associated problems of frost damage to pavements ; frost penetration depth ; remedial measures to prevent frost damage ; problems of 'pumping' of clay subsoils and suitability of a soil as a construction material for building highways or railways, earth fills or cuts, etc. A knowledge of the techniques for the improvement of the soil properties such as strength and stability is very much helpful in constructing pavements on poor soils by stabilising them.
3. **Design of underground structures and earth retaining structures.** The design and construction of underground (subterranean) and earth retaining structures constitute an important phase of engineering. The examples of underground structures include tunnels, underground buildings,

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drainage structures and pipelines. The examples of earth retaining structures are : gravity retaining wall, anchored bulk heads and cofferdams. A knowledge of *soil structure interaction* is essential to design properly such structures subjected to soil loadings.

4. **Design of embankments and excavations.** When the surface of the soil structure is not horizontal, the component of gravity tends to move the soil downward, and may disturb the stability of the earth structure. A thorough knowledge of shear-strength and related properties of soil is essential to design the slope and height (or depth) of the embankment (or excavation). The possibility of the seeping groundwater reducing the soil strength while excavating must also be taken into account. It may sometimes be essential to drain the subsoil water, to increase the soil strength and to reduce the seepage forces. Deep excavations require lateral braces and sheet walls to prevent caving in.
5. **Design of earth dams.** The construction of an earth dam requires a very thorough knowledge of whole of the Soil Mechanics. Since soil is used as the only construction material in an earth dam, which may either be homogeneous or of composite section, its design involves the determination of the following physical properties of soil : index properties such as density, plasticity characteristics and specific gravity, particle size distribution and gradation of the soil; permeability, consolidation and compaction characteristics, and shear strength parameters under various drainage conditions. Since huge earth mass is involved in its construction, suitable soil survey to the nearby area may be essential for the borrow-pit area. The determination of the optimum water content at which maximum density will be obtained on compaction, is probably the most essential aspect of the design. Apart from the seepage, characteristics of the dam section must be thoroughly investigated since these have the greatest impact on the stability of the slopes as well as the foundations of the dam. The consolidation characteristics help in predicting the long range behaviour of the dam toward settlement and the consequent reduction in the pore pressure. Lastly, the possible effect of vibrations during an earthquake should also be taken into account while designing.

The performance of the soil in the designs cited above depends upon the characteristics of soil. Therefore, the testing of soil with relation to the determination of its physical properties, and the evaluation of effects of certain other factors such as seepage conditions, etc. forms the most essential part of the development of soil engineering. It is through research only that design and construction methods are modified to give maximum safety and/or economy, and new methods are evolved. The knowledge of theoretical soil mechanics, assuming the soil to be an ideal elastic isotropic and homogeneous material, helps in predicting the behaviour of the soil in the field.

1.4 SI UNITS

In this textbook, SI units have been used throughout. However, prior to this, metric units were in use, not only in India, but also in several parts of the world. Most of the equipment in these laboratories have the use of metric units. These equipment use grams and kilograms. Hence, *mass units of grams and kilograms* (formerly called 'weight') will continue to be used. *Also, the term 'mass'* and '*weight*' are *commonly used interchangeably*. This may cause some problem with 'unit weight' terms. In this text, '*mass*' will be represented in terms of grams and kilograms while '*weight*' will be represented in terms of Newtons and kilo Newtons. The term *density* is defined as mass per unit volume; hence it will be represented in terms of g/cm^3 or kg/m^3 units. On the contrary, the term '*unit weight*' is the weight per unit volume. Hence, it will be represented in terms of kN/m^3.

The value of gravitational constant g will be taken as 981 cm/s^2, for computational purposes.

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Thus, in order to convert the density (expressed in terms of g/cm^3) into unit weight, multiply the former by 9.81. For example, if a soil mass has a mass of 216 g and volume of 120 cm^3.

$$\text{Density} = \frac{\text{Mass}}{\text{Vol.}} = \frac{216}{120} = 1.8 \text{ g/cm}^3$$

and

$$\text{Unit weight} = \frac{\text{Weight}}{\text{Vol.}} = \frac{216}{120} \times 9.81 = 17.66 \text{ kN/m}^3$$

Pressure, which is defined to be a force per unit area, will be expressed in terms of kN/m^2 or in term of kilopascals (kPa).

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CHAPTER 2

Preliminary Definitions and Relationships

2.1 SOIL AS A THREE PHASE SYSTEM

A soil mass is a *three phase system* consisting of solid particles (called soil grains), water and air. The void space between the soil grains is filled partly with water and partly with air. However, if we take a dry soil mass, the voids are filled with air only. In case of a perfectly saturated soil, the voids are filled completely with water. In general, the soil mass has three constituents which do not occupy separate spaces but are blended together forming a complex material [Fig. 2.1(*a*)], the properties of which depend

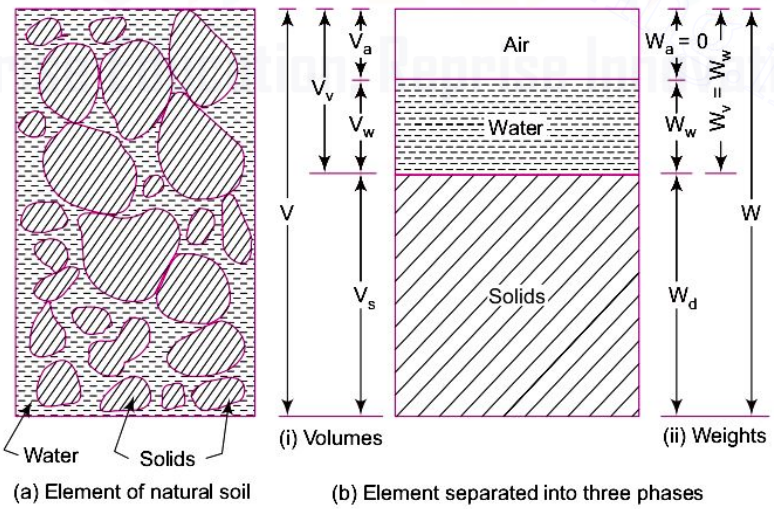

Fig. 2.1 Soil as a three phase system

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upon the relative percentages of these constituents, their arrangement and a variety of other factors. For calculation purposes, it is always more convenient to show these constituents occupying separate spaces, as shown in Fig. 2.1(*b*)(*i*) and Fig. 2.1(*b*)(*ii*).

As shown in Fig. 2.1(*b*) (*i*), the total volume V of the soil mass consists of (*i*) volume of air V_a, (*ii*) volume of water V_w and (*iii*) the volume of solids V_s. The volume of voids V_v, is, therefore, equal to volume of air plus the volume of water. Similarly, Fig. 2.1(*b*) (*ii*) shows the weights. The weight of air is considered to be negligible. Hence, the weight of total voids is equal to the weight of water W_w. The weight of solids is represented by W_d (or W_s) which is evidently equal to the dry weight of soil sample. The total weight W of the moist sample is, therefore, equal to $(W_w + W_d)$.

2.2 WATER CONTENT, DENSITY AND UNIT WEIGHTS

(*a*) Water content. The water content w, also called the *moisture content*, is defined as the ratio of weight of water W_w to the weight of solids (W_s or W_d) in a given mass of soil.

$$w = \frac{W_w}{W_d} \times 100 \qquad ...(2.1)$$

The water content is generally expressed as a percentage. However, when used in the formulae giving relationship between certain quantities, it may also be expressed as a fraction.

Rewriting equation 2.1, we have

$$w = \frac{W - W_d}{W_d} \times 100 = \left(\frac{W}{W_d} - 1\right) \times 100 \qquad ...(2.1a)$$

The usual procedure to find the natural water content is to take a mass of about 20 g to 30 g of soil sample in a container and determine its mass M very accurately. The soil sample is then kept in an oven (105°C–110°C) for about 24 hours so that it becomes perfectly dry. Its dry mass M_d is then determined and the water content is calculated from the relation,

$$w = \frac{M_w}{M_d} \times 100 = \frac{M - M_d}{M_d} \times 100 = \left(\frac{M}{M_d} - 1\right) \times 100 \qquad ...(2.1b)$$

In the field of geology and agriculture, water content (w') is defined as the ratio of the weight of water to the total weight of soil mass. Thus,

$$w' = \frac{W_w}{W} \times 100 = \frac{M_w}{M} \times 100 \qquad ...(2.1c)$$

In order to find relation between w and w', we have from equation 2.1(*c*)

$$w' = \frac{M_w}{M_w + M_d} \quad \text{or} \quad \frac{1}{w'} = \frac{M_w + M_d}{M_w} = 1 + \frac{1}{w}$$

From which we get $$w = \frac{w'}{1 - w'} \qquad ...(2.1d)$$

Expressed as percentage, $$w = \frac{w'}{100 - w'} \qquad ...(2.1e)$$

(*b*) Density of soil. The density of soil is defined as the mass of the soil per unit volume.

(*i*) Bulk density (ρ). The bulk density or *moist density* is the total mass M of the soil per unit of its total volume.

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Thus, $$\rho = \frac{M}{V} \quad ...(2.2)$$

It is expressed in terms of g/cm^3 or kg/m^3.

(ii) Dry density (ρ_d). The dry density is the mass of solids per unit of total volume (prior to drying) of the soil mass:

$$\rho_d = \frac{M_d}{V} \quad ...(2.2a)$$

(iii) Density of solids (ρ_s). The density of soil solids is the mass of soil solids (M_d) per unit of volume of solids (V_s):

$$\rho_s = \frac{M_d}{V_s} \quad ...(2.2b)$$

(iv) Saturated density (ρ_{sat}). When the soil mass is saturated, its bulk density is called saturated density. Thus, saturated density is the ratio of the total soil mass of saturated sample to its total volume.

(v) Submerged density (ρ'). The submerged density is the submerged mass of soil solids $(M_d)_{sub}$ per unit of total volume V of the soil mass:

$$\rho' = \frac{(M_d)_{sub}}{V} \quad ...(2.2c)$$

The *submerged density* or *buoyant density* is also expressed as:

$$\rho' = \rho_{sat} - \rho_w \quad ...(2.2d)$$

where ρ_w is the density of water which may be taken as 1 g/cm^3 of calculation purposes.

(c) Unit weight of soil mass. The unit weight of a soil mass is defined as its weight per unit volume.

(i) Bulk unit weight (γ). The bulk weight or *moist unit weight* is the total weight W of a soil mass per unit of its total volume V.

Thus $$\gamma = \frac{W}{V} \quad ...(2.3)$$

(ii) Dry unit weight (γ_d). The dry unit weight is the weight of solids per unit of its total volume (prior to drying) of the soil mass:

$$\gamma_d = \frac{W_d}{V} \quad ...(2.4)$$

(iii) Unit weight of solids (γ_s). The unit weight of soil solids is the weight of soil solids W_d per unit volume of solids (V_s):

$$\gamma_s = \frac{W_d}{V_s} \quad ...(2.5)$$

Thus, when the dry weight is reckoned with reference to the total original volume V, it is called the dry unit weight and when it is reckoned with reference to the volume of solids, we get unit weight of soil solids. Since the volume V_s of the solids does not alter, γ_s is a constant for a given soil, whereas γ_d is not a constant, being dependent upon the initial volume V of the soil mass.

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(iv) Saturated unit weight (γ_{sat}). When the soil mass is saturated, its bulk unit weight is called the saturated unit weight. Thus, saturated unit weight is the ratio of the total weight of a saturated soil sample to its total volume.

(v) Submerged unit weight (γ'). The submerged unit weight γ' is the submerged weight of soil solids $(W_d)_{sub}$ per unit of total volume V of the soil mass:

$$\gamma' = \frac{(W_d)_{sub}}{V} \qquad ...(2.6)$$

When the soil mass is submerged, the weight of soil solids is reduced due to buoyancy. The submerged weight $(W_d)_{sub}$ is, therefore, equal to the weight of soil solids in air minus the weight of water displaced by solids. Hence, the submerged unit weight or the *buoyant unit weight* is also expressed as:

$$\gamma' = \gamma_{sat} - \gamma_w \qquad ...(2.6a)$$

where, γ_w is the unit weight of water. For calculation purposes in SI units, γ_w may be taken as 9.81 kN/m^3.

(d) Inter-conversion between density and unit weight. In order to convert the density (expressed in terms of g/cm^3) into unit weight (kN/m^3) multiply the former by 9.81. This is so because

$$1\text{g/cm}^3 = \frac{9.81 \times 10^{-6}\text{ kN}}{1 \times 10^{-6}\text{ m}^3} = 9.81\text{ kN/m}^3$$

Hence, $\gamma \quad = 9.81 \times \rho$

(kN/m^3) (g/cm^3).

2.3 SPECIFIC GRAVITY

Specific gravity G is defined as the ratio of the weight of a given volume of soil solids at a given temperature to the weight of an equal volume of distilled water at that temperature, both weights being taken in air. In other words, it is the ratio of the unit weight of soil solids to that of water.

$$G = \frac{\gamma_s}{\gamma_w} \qquad ...(2.7)$$

The Indian Standard specifies 27°C as the *standard temperature* for reporting the specific gravity.

Some qualifying words like: *true, absolute, apparent, bulk* or *mass*, etc., are sometimes added to the term 'specific gravity'. These qualifying words modify the sense of specific gravity as to whether it refers to soil particles or to soil mass. The soil solids have permeable and impermeable voids inside them, the permeable voids being capable of getting filled with water. If all the internal voids of soil particles (permeable and impermeable) are excluded for determining the true volume of solids, the specific gravity obtained is called *absolute* or *true* specific gravity. The *apparent* or *mass* or bulk specific gravity G_m denotes the specific gravity of *soil mass* and is given by

$$G_m = \frac{\gamma}{\gamma_w}. \qquad ...(2.8)$$

Unless otherwise specified, we shall denote the Specific Gravity G (defined by equation 2.7) as the specific gravity of soil solids.

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Table 2.1 gives the values of specific gravity of some important soil constituents.

Table 2.1 Specific gravity of soil constituents

Soil Constituent	*Specific Gravity*	*Soil Constituent*	*Specific Gravity*
1. Aragonite	2.94	13. Kaolinite	2.64
2. Attapulgite	2.30	14. K. Feldspars	2.54–2.57
3. Augite	3.2–3.4	15. Limonite	3.8
4. Biotite	3.0–3.1	16. Magnetite	5.17
5. Calcite	2.72	17. Montmorillonite	2.75–2.78
6. Chlorite	2.6–2.9	18. Muscovite	2.7–3.1
7. Dolomite	2.85	19. Na-Ca-Feldspars	2.62–2.76
8. Gypsum	2.32	20. Orthoclase	2.56
9. Halloysite ($2H_2O$)	2.55	21. Pyrophyllite	2.84
10. Hematite	5.2	22. Quartz	2.65
11. Hornblende	3.2–3.5	23. Serpentine	2.2–2.7
12. Illite	2.6–2.86	24. Talc	2.7

2.4 VOIDS RATIO, POROSITY AND DEGREE OF SATURATION

Voids ratio. Voids ratio e of a given soil sample is the ratio of the volume of voids to the volume of soil solids in the given soil mass.

Thus, $$e = \frac{V_v}{V_s} \qquad ...(2.9)$$

Porosity. The porosity n of a given soil sample is the ratio of the volume of voids to the *total* volume of the given soil mass

$$n = \frac{V_v}{V} \qquad ...(2.10)$$

The voids ratio e is generally expressed as a fraction, while the porosity n is expressed as a percentage and is, therefore, also referred to as *percentage voids*.

Figure 2.2(a) shows the soil element in terms of voids ratio e. If the volume of voids is taken equal to e, the volume of solids, by definition (equation 2.9) would be equal to 1, and the total volume equal to $(1 + e)$. Similarly, if the volume of the voids is taken equal to n [Fig. 2.2(b)], the total volume of the element will be 1(unity) and hence the volume of solids would be equal to $(1 - n)$.

From Fig. 2.2(a), we have by definition of porosity,

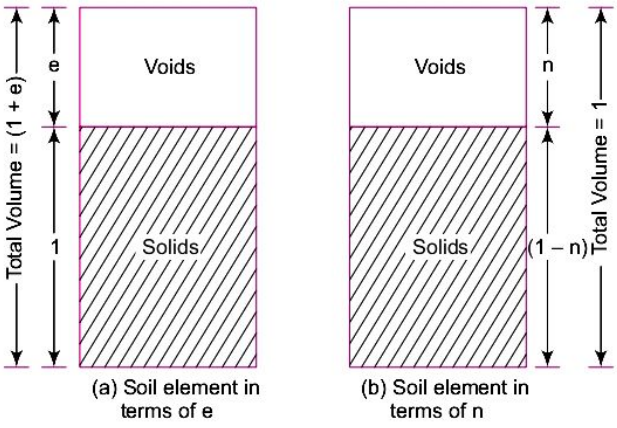

Fig. 2.2 Voids ratio and porosity

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$$n = \frac{V_v}{V} = \frac{e}{1+e} \qquad ...(2.11)$$

Similarly, from Fig. 2.2 (*b*), we get by definition of voids ratio,

$$e = \frac{V_v}{V_s} = \frac{n}{1-n} \qquad ...(2.12)$$

Equations 2.11 and 2.12 give two relations between n and e. Combining equations 2.11 and 2.12, we get

$$n = \frac{e}{1+e} = e(1-n)$$

or
$$(1-n) = \frac{1}{1+e} \qquad ...(2.12a)$$

Degree of saturation. In a given volume of voids of a sample, some space is occupied by water and the rest by air. In a *fully saturated* sample, the voids get completely filled with water. The *degree of saturation* S is defined as the ratio of the volume of water present in a given soil mass to the total volume of voids in it.

Thus,
$$S = \frac{V_w}{V_v} \qquad ...(2.13)$$

The degree of saturation is usually expressed as a percentage and is also known as *percent saturation*. For a fully saturated sample $V_w = V_v$ and hence $S = 1$. For a perfectly dry sample, V_w = zero, and hence $S = 0$.

Depending upon degree of saturation, a soil is generally described as dry, damp, moist, etc. Figure 2.3 shows such *descriptive terms* corresponding to various values of degree of saturation.

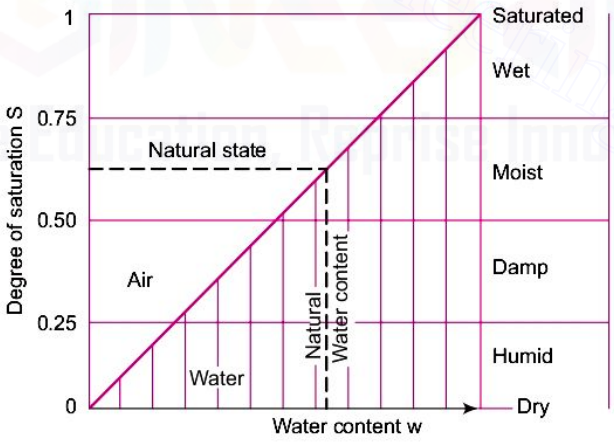

Fig. 2.3 Descriptive terms of soil state with various values of S

Percentage air voids. Percentage air voids n_a is defined as the ratio of the volume of air voids to the total volume of the soil mass and is expressed as percentage:

$$n_a = \frac{V_a}{V} \times 100 \qquad ...(2.14)$$

Air content. The air content a_c is defined as the ratio of volume of air voids to the volume of voids:

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$$a_c = \frac{V_a}{V_v} \qquad ...(2.15)$$

Since $V_a = V_v - V_w$, we get $a_c = 1 - \frac{V_w}{V_v} = 1 - S$ ...(2.16)

2.5 DENSITY INDEX AND RELATIVE COMPACTION

The term *density index* I_D or *relative density* or *degree of density* is used to express the relative compactness (or degree of compaction) of a natural cohesionless soil deposit. The density index is defined as the ratio of the difference between the voids ratio of the soil in its loosest state e_{max} and its natural voids ratio e to the difference between the voids ratios in the loosest and densest states:

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \qquad ...(2.17)$$

where e_{max} = voids ratio in the loosest state

e_{min} = voids ratio in the densest state

e = natural voids ratio of the deposit.

This term is used for cohesionless soil only. This term is not applicable to cohesive soil because of uncertaintities in the laboratory determination of the voids ratio in the loosest state of the soil (e_{max}). When the natural state of the cohesionless soil is in its loosest form, $e = e_{max}$ and hence $I_D = 0$. When the natural deposit is in its densest state, $e = e_{min}$ and hence $I_D = 1$. For any intermediate state, the density index varies between zero and one.

Equation 2.17, defining density index, can be easily derived by noting the fact that the density index is a function of voids ratio expressed by:

$$I_D = f(e) \qquad ...(1)$$

This relationship between I_D and e may be represented graphically, as shown in Fig. 2.4. The slope of the straight line *AB*, representing the relationship between I_D and e is given by

$$\tan\theta = \frac{1}{e_{max} - e_{min}}$$

or $$\cot\theta = (e_{max} - e_{min}) \qquad ...(2)$$

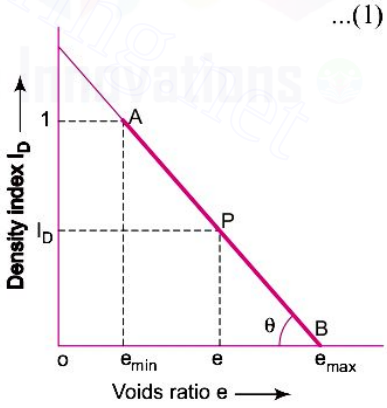

Fig. 2.4 $I_D - e$ Relationship

Now, for an intermediate value e

We have, $(e_{max} - e) = I_D \cot\theta$

or $$I_D = \frac{e_{max} - e}{\cot\theta} \qquad ...(3)$$

Substituting the value of $\cot\theta$ from equation 2, we get

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

From Fig. 2.4, we observe that when $e = e_{max}$, $I_D = 0$ and when $e = e_{min}$, $I_D = 1$.

Now from equation 2.24, we have

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$$e = \frac{G\gamma_w}{\gamma_d} - 1;\ e_{max} = \frac{G\gamma_w}{\gamma_{d,min}} - 1 \text{ and } e_{min} = \frac{G\gamma_w}{\gamma_{d,max}} - 1.$$

Substituting in eqn. 2.17, we get $I_D = \dfrac{\dfrac{G\gamma_w}{\gamma_{d,min}} - \dfrac{G\gamma_w}{\gamma_d}}{\dfrac{G\gamma_w}{\gamma_{d,min}} - \dfrac{G\gamma_w}{\gamma_{d,max}}}$

or
$$I_D = \frac{\gamma_d - \gamma_{d,min}}{\gamma_{d,max} - \gamma_{d,min}} \cdot \frac{\gamma_{d,max}}{\gamma_d} \quad ...(2.17a)$$

The above equation gives density index in terms of densities.

Density index is also expressed in terms of porosity as follows.

$$I_D = \frac{(n_{max} - n)(1 - n_{min})}{(n_{max} - n_{min})(1 - n)} \quad ...(2.17b)$$

where, γ_d = in-situ dry density; n = in-situ porosity.

$\gamma_{d,max}$ = maximum dry density or dry density corresponding to most compact state.

$\gamma_{d,min}$ = minimum dry density or dry density corresponding to most loosest state.

n_{max} = maximum porosity at loosest state; n_{min} = minimum porosity at densest state.

Table 2.2 gives the maximum and minimum voids ratio, porosity and dry unit weight of some typical granular soils. Table 2.3 gives the characteristics of density of granular soils on the basis of relative density.

Table 2.2 Characteristics of granular soils in dense and loose states

Description	*Voids ratio*		*Porosity*		*Dry unit weight (kN/m³)*	
	e_{max}	e_{min}	n_{max}	n_{min}	$(\gamma_d)_{min}$	$(\gamma_d)_{max}$
1. Uniform spheres	0.92	0.35	47.6	26.0	–	–
2. Clean uniform sand	1.00	0.40	50	29	13	18.5
3. Uniform inorganic silt	1.10	0.40	52	29	12.6	18.5
4. Silty sand	0.90	0.30	47	23	13.6	20.0
5. Fine to coarse sand	0.95	0.20	49	17	13.3	21.7
6. Micaceous sand	1.20	0.40	55	29	12.0	18.8
7.Silty sand and gravel	0.85	0.14	46	12	14.0	23.0

Table 2.3 Relative density

Relative density (%)	*Density description*
0–15	Very loose
15–35	Loose
35–65	Medium
65–85	Dense
85–100	Very dense

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Relative Compaction (R_C)

Degree of compaction is also sometimes expressed in terms of an *index* called *relative compaction* (R_C) defined as follows:

$$R_C = \frac{\gamma_d}{\gamma_{d,\max}} \qquad ...(2.18)$$

where, $\gamma_{d,\max}$ is the maximum dry density from compaction test.

In recent years, the use of the above index has become a generally accepted practice for judging the measure of compaction of both coarse-grained as well as cohesive soils. Since $\gamma_d = \gamma_s (1 + e)$, in general, we have

$$R_C = \frac{1 + e_{\min}}{1 + e} \qquad ...(2.18a)$$

Relative compaction (R_C) can also be expressed in terms of relative density (I_D) as follows:

$$R_C = \frac{R_0}{1 - I_D (1 - R_0)} \qquad ...(2.18b)$$

where, $R_0 = \gamma_{d,\min}/\gamma_{d,\max}$ and R_C and I_D are in fraction form.

Lee and Singh (1971) give the following approximate relation between R_C and I_D

$$R_C = 80 + 0.2\, I_D \qquad ...(2.18c)$$

(where both R_C and I_D are in percent form)

When the soil is in looset form, $I_D = 0$, which gives minimum value of R_C as 80% from equation 2.18(c). When the soil is in densest form, $I_D = 100\%$ corresponding to which $R_C = 100\%$ from equation 2.18(c). Thus, relative compaction varies from 80% to 100% according to equation 2.18(c).

2.6 FUNCTIONAL RELATIONSHIPS

(*i*) Relation between *e*, *G*, *w* and *S*:

In Fig. 2.5, e_w represents volume of water, e represents the volume of voids, and the volume of solids is equal to unity.

Now, $$S = \frac{V_w}{V_v} = \frac{e_w}{e}$$

Hence, $$e_w = eS \qquad ...(2.19a)$$

The term e_w is known as the *water voids ratio*. For a fully saturated sample, $e_w = e$.

Now, $$w = \frac{W_w}{W_d} = \frac{e_w \gamma_w}{\gamma_s \cdot 1}$$

But, $$G = \frac{\gamma_s}{\gamma_w} \quad \text{or} \quad \gamma_s = G \gamma_w$$

$$w = \frac{e_w \gamma_w}{G \gamma_w} = \frac{e_w}{G}$$

or $$e_w = w\, G \qquad ...(2.19b)$$

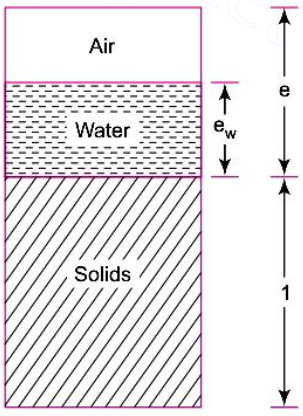

Fig. 2.5 Soil element in terms of e_w and e

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Equating Eqs. 2.19(a) and 2.19(b), we get

$$e = \frac{w\,G}{S} \qquad ...(2.20)$$

For a fully saturated soil, $S = 1$ and $w = w_{sat}$

$$e = w_{sat}\,G \qquad ...(2.20a)$$

(*ii*) Relation between e, S and n_a:

$$n_a = \frac{V_a}{V}$$

From Fig. 2.5, $V_a = V_v - V_w = e - e_w$ and $V = V_s + V_v = 1 + e$

$$\therefore \quad n_a = \frac{e - e_w}{1 + e} \quad \text{But } e_w = e\,S \text{ (Eq. 2.18)}$$

$$\therefore \quad n_a = \frac{e(1 - S)}{1 + e} \qquad ...(2.21)$$

(*iii*) Relation between n_a, a_c and n:

$$a_c = \frac{V_a}{V_v}, \quad n = \frac{V_v}{V}$$

$$\therefore \quad n_a = \frac{V_a}{V} = n \cdot a_c \qquad ...(2.22)$$

(*iv*) Relation between γ_d . G and e (or n):

$$\gamma_d = \frac{W_d}{V} \qquad \gamma_d = \frac{\gamma_s \cdot V_s}{V} \qquad ...(i)$$

Now from Fig. 2.2(a),

$$V_s = 1 \text{ and } V = (1 + e)$$

$$\therefore \quad \gamma_d = \frac{\gamma_s \cdot 1}{1 + e} \quad \text{But} \quad \gamma_s = G\,\gamma_w$$

$$\therefore \quad \gamma_d = \frac{G\,\gamma_w}{1 + e} \qquad ...(2.23)$$

A convenient expression for calculating the voids ratio of soil mass is obtained from equation 2.23, as:

$$e = \frac{G\,\gamma_w}{\gamma_d} - 1 \qquad ...(2.24)$$

Again, from Fig. 2.2(b), $V_s = (1 - n)$ and $V = 1$

Substituting in (i) and taking $\gamma_s = G\,\gamma_w$, we get

$$\gamma_d = \frac{G\,\gamma_w(1 - n)}{1} = (1 - n)\,G\,\gamma_w \qquad ...(2.25)$$

(*v*) Relation between γ_{sat}, G and e (or n):

$$\gamma_{sat} = \frac{\text{Total weight of saturated soil}}{\text{Total volume of soil}} = \frac{W_{sat}}{V} = \frac{W_d + W_w}{V} = \frac{\gamma_s\,V_s + \gamma_w\,V_w}{V} \qquad ...(i)$$

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From Fig. 2.2(a), $V_s = 1$, $V_w = e$ and $V = 1 + e$

$$\therefore \quad \gamma_{sat} = \frac{\gamma_s \cdot 1 + \gamma_w \cdot e}{1+e} = \frac{G\gamma_w + \gamma_w \cdot e}{1+e}$$

or
$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} \qquad ...(2.26)$$

Similarly, from Fig. 2.2(b), $V_s = 1 - n$; $V_w = n$; $V = 1$

$$\therefore \quad \gamma_{sat} = \frac{\gamma_s (1-n) + \gamma_w n}{1}$$

or
$$\gamma_{sat} = G\gamma_w (1-n) + \gamma_w n \qquad ...(2.27)$$

(*vi*) Relation between γ, G, e, and S:

We have, by definition, $\gamma = \dfrac{W}{V} = \dfrac{\gamma_s V_s + \gamma_w V_w}{V}$

From Fig. 2.4, illustrating a partially saturated sample, we have

$$V_s = 1 \; ; \; V_w = e_w \text{ and } V = (1+e)$$

$$\therefore \quad \gamma = \frac{\gamma_s \cdot 1 + \gamma_w \cdot e_w}{1+e} \quad \text{But} \quad \gamma_s = G\gamma_w \text{ and } e_w = eS$$

$$\therefore \quad \gamma = \frac{G\gamma_w + \gamma_w \cdot eS}{1+e} = \frac{(G+eS)\gamma_w}{1+e} \qquad ...(2.28)$$

This is a general relationship from which equations 2.23 and 2.26 can be derived. For example, when the soil is perfectly dry, $S = 0$ and hence, γ reduces to

$$\gamma_d = \frac{G\gamma_w}{1+e} \quad \text{Eq. (2.23)}$$

Similarly, when $S = 1$, γ becomes

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} \quad \text{Eq. (2.26)}$$

(*vii*) Relation between γ', G and e:

$$\gamma' = \gamma_{sat} - \gamma_w \quad \frac{(G+e)\gamma_w}{1+e} - \gamma_w \quad \text{or} \quad \gamma' = \frac{(G-1)\gamma_w}{1+e} \qquad ...(2.29)$$

(*viii*) Relation between γ_d, γ and w:

Water content, $w = \dfrac{W_w}{W_d}$ Hence $1 + w = \dfrac{W_w + W_d}{W_d} = \dfrac{W}{W_d}$

$$\therefore \quad W_d = \frac{W}{1+w} \qquad ...(2.30)$$

Now, $\gamma_d = \dfrac{W_d}{V} = \dfrac{W}{(1+w)V} \quad \therefore \quad \gamma_d = \dfrac{\gamma}{1+w} \qquad ...(2.31)$

(*ix*) Relation between γ', γ_d and n:

From Fig. 2.3, $(W_d)_{sub} = 1 \cdot \gamma_s - 1 \cdot \gamma_w = G\gamma_w - \gamma_w$

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$$(W_d)_{sub} = (G-1)\gamma_w \quad \text{and} \quad V = 1+e$$

or

$$\therefore \quad \gamma' = \frac{(W_d)_{sub}}{V} = \frac{(G-1)\gamma_w}{1+e} \text{ (Eq. 2.29) or } \gamma' = \frac{G\gamma_w}{1+e} - \frac{\gamma_w}{1+e}$$

But $\frac{G\gamma_w}{1+e} = \gamma_d$ (Eq. 2.23) and $\frac{1}{1+e} = 1-n$ [Eq. 2.12(*a*)]

$$\gamma' = \gamma_d - (1-n)\gamma_w \qquad ...(2.32)$$

(*x*) Relation between γ_{sat}, γ, γ_d and S: We have already proved in Eq. 2.28,

that $$\gamma = \frac{(G+eS)\gamma_w}{1+e} \quad \text{or} \quad \gamma = \frac{G\gamma_w}{1+e} + S\frac{e\gamma_w}{1+e}$$

or $$\gamma = \gamma_d + S\left[\frac{(G+e)\gamma_w}{1+e} - \frac{G\gamma_w}{1+e}\right]$$

Hence, $$\gamma = \gamma_d + S[\gamma_{sat} - \gamma_d] \qquad ...(2.33)$$

(*xi*) Relation between γ_d, G, w and S:

From Eq. 2.23, $$\gamma_d = \frac{G\gamma_w}{1+e} \text{ But } e = \frac{wG}{S} \qquad \text{(From Eq. 2.20)}$$

$$\therefore \quad \gamma_d = \frac{G\gamma_w}{1+\frac{w}{S}} \qquad ...(2.34)$$

When $$S = 1,\ \gamma_d = \frac{G\gamma_w}{1+w_{sat}G} \qquad ...(2.34a)$$

(*xii*) Relation between γ_d, G, w and n_a:

From Fig. 2.1, $V = V_a + V_w + V_s$ or $V = V_a + \frac{W_w}{\gamma_w} + \frac{W_d}{\gamma_s}$

$$\therefore \quad 1 = \frac{V_a}{V} + \frac{wW_d}{\gamma_w V} + \frac{W_d}{\gamma_s V} = \frac{V_a}{V} + \frac{w\gamma_d}{\gamma_w} + \frac{\gamma_d}{\gamma_s}$$

$$\therefore \quad \left(1 - \frac{V_a}{V}\right) = \frac{w\gamma_d}{\gamma_w} + \frac{\gamma_d}{G\gamma_w} \quad \text{or} \quad (1-n_a) = \frac{\gamma_d}{\gamma_w}\left(w + \frac{1}{G}\right)$$

$$\therefore \quad \gamma_d = \frac{(1-n_a)\gamma_w}{w+\frac{1}{G}} \quad \text{or} \quad \gamma_d = \frac{(1-n_a)G\gamma_w}{1+wG} \qquad ...(2.35)$$

Inter-relationships in terms of densities: Similar inter-relationships can be obtained in terms of densities (ρ). Thus, we have the following inter-relationships.

$$\rho_d = \frac{G\cdot\rho_w}{1+e} \qquad ...(2.23a)\ ; \qquad e = \frac{G\cdot\rho_w}{\rho_d} - 1 \qquad ...(2.24a)$$

$$\rho_d = (1-n)\,G\,\rho_w \qquad ...(2.25a)\ ; \qquad \rho_{sat} = \frac{(G+e)\rho_w}{1+e} \qquad ...(2.26a)$$

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$$\rho_{sat} = G\cdot\rho_w (1-n) + \rho_w\cdot n \quad ...(2.27a)\,; \qquad \rho = \frac{(G+eS)\rho_w}{1+e} \quad ...(2.28a)$$

$$\rho' = \frac{(G-1)\rho_w}{1+e} \quad ...(2.29a)\,; \qquad \rho_d = \frac{\rho}{1+w} \quad ...(2.31a)$$

$$\rho' = \rho_d - (1-n)\rho_w \quad ...(2.32a)\,; \qquad \rho = \rho_d + S(\rho_{sat} - \rho_d) \quad ...(2.33a)$$

$$\rho_d = \frac{G\rho_w}{1+\frac{wG}{S}} \quad ...(2.34a)\,; \text{ and } \quad \rho_d = \frac{(1-n_a)G\rho_w}{1+wG} \quad ...(2.35a)$$

SOLVED EXAMPLES

Example 2.1. *A soil sample has a porosity of 40%. The specific gravity of solids is 2.70. Calculate (a) voids ratio, (b) dry density, (c) unit weight if the soil is 50% saturated, and (d) unit weight if the soil is completely saturated.*

Solution. Given, $n = 40\% = 0.4$; $G = 2.70$

(*a*) We have $e = \dfrac{n}{1-n} = \dfrac{0.4}{1-0.4} = \mathbf{0.667}$

(*b*) $\gamma_d = \dfrac{G\gamma_w}{1+e} = \dfrac{2.7\times 9.81}{1+0.667} = \mathbf{15.89\ kN/m^3}$ (Taking $\gamma_w = 9.81\ kN/m^3$)

(*c*) $e = \dfrac{wG}{S}$ or $w = \dfrac{eS}{G} = \dfrac{0.667\times 0.5}{2.70} = 0.124$

$\gamma_d = 15.89\ kN/m^3$ (as before)

$\therefore \quad \gamma = \gamma_d (1+w) = 15.89 \times 1.124 = \mathbf{17.86\ kN/m^3}$

***Check*:** $\gamma = \dfrac{\gamma_w (G+eS)}{1+e} = \dfrac{9.81(2.70 + 0.667\times 0.5)}{1+0.667} = \mathbf{17.86\ kN/m^3}$

(*d*) When the soil is fully saturated, $e = w_{sat}\cdot G$

$\therefore \quad w_{sat} = \dfrac{e}{G} = \dfrac{0.667}{2.70} = 0.247$

$\therefore \quad \gamma_{sat} = \gamma_d (1 + w_{sat}) = 15.89 \times 1.247 = \mathbf{19.81\ kN/m^3}$

Alternatively, $\gamma_{sat} = G\gamma_w (1-n) + \gamma_w n = 2.7 \times 9.81\ (1 - 0.4) + 9.81 \times 0.4$

$= 15.89 + 3.92 = \mathbf{19.81\ kN/m^3}$.

Example 2.2. *An undisturbed sample of soil has a volume of 100 cm^3 and mass of 190 g. On oven drying for 24 hours, the mass is reduced to 160 g. If the specific gravity of grains is 2.68, determine the water content, voids ratio and degree of saturation of the soil.*

Solution. $M_w = 190 - 160 = 30$ g ; $M_d = 160$ g

$\therefore \quad w = \dfrac{M_w}{M_d} = \dfrac{30}{160} = 0.188 = \mathbf{18.8\%}$

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Mass of moist soil = $M = 190$ g $\quad \therefore \quad$ Bulk density, $\rho = \dfrac{M}{V} = \dfrac{190}{100} = 1.9$ g/cm^3

Hence, $\gamma = 9.81 \times \rho = 9.81 \times 1.9 = 18.64$ kN/m^3

Alternatively, $\gamma = \dfrac{W}{V} = \dfrac{190(\text{g})}{100(\text{cm}^3)} \times 9.81 = 18.64$ kN/m^3

(Since 1 g/cm^3 = 9.81 kN/m^3)

$\therefore \quad \gamma_d = \dfrac{\gamma}{1+w} = \dfrac{18.64}{1+0.188} = 15.69$ kN/m^3

Alternatively, $\gamma_d = \dfrac{W_d}{V} = \dfrac{160}{100} \times 9.81 = 15.69$ kN/m^3

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.68 \times 9.81}{15.69} - 1 = \mathbf{0.67}$$

$$S = \frac{wG}{e} = \frac{0.188 \times 2.68}{0.67} = 0.744 = \mathbf{74.4\%}.$$

Example 2.3. *The in situ-density of an embankment, compacted at a water content of 12% was determined with the help of a core cutter. The empty mass of the cutter was 1286 g and the cutter full of soil has a mass of 3195 g, the volume of the cutter being 1000 cm^3. Determine the bulk density, dry density and the degree of saturation of the embankment.*

If the embankment becomes fully saturated during rains, what would be its water content and saturated unit weight? Assume no volume changes in soil on saturation. Take the specific gravity of the soil as 2.70.

Solution. Mass of soil in cutter, $M = 3195 - 1286 = 1909$ g

$\therefore$ Bulk density, $\rho = \dfrac{M}{V} = \dfrac{1909}{1000} = 1909$ g/cm^3

$\therefore$ Bulk unit weight, $\gamma = 9.81\ \rho = 9.81 \times 1.909 = 18.73$ kN/m^3

$$\gamma_d = \frac{\gamma}{1+w} = \frac{18.73}{1+0.12} = \mathbf{16.72\ kN/m^3}$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{16.72} - 1 = 0.584$$

and $$S = \frac{wG}{e} = \frac{0.12 \times 2.70}{0.584} = 0.555 = \mathbf{55.5\%}.$$

At Saturation. Since the volume remains the same, the voids ratio also remains unchanged. Now $e = w_{sat} \cdot G$

$\therefore$

$$w_{sat} = \frac{e}{G} = \frac{0.584}{2.70} = 0.216 = \mathbf{21.6\%}$$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \frac{2.7+0.584}{1+0.584} \times 9.81 = \mathbf{20.34\ kN/m^3}.$$

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Example 2.4. *The in situ-percentage voids of a sand deposit is 34%. For determining the density index, dried sand from the stratum was first filled loosely in a 1000 cm^3 mould and was then vibrated to give a maximum density. The loose dry mass in the mould was 1610 g and the dense dry mass at maximum compaction was found to be 1980 g. Determine the density index if the specific gravity of the sand particles is 2.67.*

Solution. $n = 34\% = 0.34$

$$e = \frac{n}{1-n} = \frac{0.34}{1-0.34} = \frac{0.34}{0.66} = 0.515\,; \qquad \gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.67 \times 9.81}{1+0.515} = 17.289$$

$$(\gamma_d)_{max} = \frac{1980}{1000} \times 9.81 = 19.42\,\text{kN/m}^3\,; \qquad (\gamma_d)_{min} = \frac{1610}{1000} \times 9.81 = 15.79\,\text{kN/m}^3$$

$$e_{min} = \frac{G\gamma_w}{(\gamma_d)_{max}} - 1 = \frac{2.67 \times 9.81}{19.42} - 1 = 0.349\,; \quad e_{max} = \frac{G\gamma_w}{(\gamma_d)_{min}} - 1 = \frac{2.67 \times 9.81}{15.79} - 1 = 0.659$$

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.659 - 0.515}{0.659 - 0.349} = 0.465 = \mathbf{46.5\%}$$

Example 2.5. *The mass specific gravity (apparent specific gravity) of a soil equals 1.64. The specific gravity of solids is 2.70. Determine the voids ratio under the assumption that the soil is perfectly dry. What would be the voids ratio, if the sample is assumed to have a water content of 8%?*

Solution. When the sample is dry, $G_m = \frac{\gamma_d}{\gamma_w} = 1.64$ (given)

$\therefore$ $\gamma_d = 1.64\,\gamma_w = 1.64 \times 9.81 = 16.09\,\text{kN/m}^3$

Now, $e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{16.09} - 1 = \mathbf{0.646.}$

When the sample has the water content $w = 8\%$, $G_m = \frac{\gamma}{\gamma_w} = 1.64$

$\therefore$ $\gamma = 1.64\,\gamma_w = 1.64 \times 9.81 = 16.09\,\text{kN/m}^3$

$\therefore$ $\gamma_d = \frac{\gamma}{1+w} = \frac{16.09}{1+0.08} = 14.9\,\text{kN/m}^3$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{14.9} - 1 = \mathbf{0.78.}$$

Example 2.6. *A natural soil deposit has a bulk unit weight of 18.44 kN/m^3 and water content of 5%. Calculate the amount of water required to be added to 1 cubic metre of soil to raise the water content to 15%. Assume the voids ratio to remain constant. What will then be the degree of saturation? Assume $G = 2.67$.*

Solution. Given : $\gamma = 18.44\,\text{kN/m}^3$ and $w = 5\%$

$\therefore$ $\gamma_d = \frac{\gamma}{1+w} = \frac{18.44}{1+0.05} = 17.56\,\text{kN/m}^3$

Earlier, when $w = 5\%$, $w = 0.05 = \frac{W_w}{W_d}$. For one cubic metre of soil $V = 1\ \text{m}^3$

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$\therefore \quad W_d = \gamma_d \cdot V = 17.56 \times 1 = 17.56 \text{ kN}$

$\therefore \quad W_w = 0.05 \times W_d = 0.05 \times 17.56 = 0.878 \text{ kN}$

$\therefore \quad V_w = \dfrac{W_w}{\gamma_w} = \dfrac{0.878}{9.81} = 0.0895 \text{ m}^3$

Later, when $w = 15\%$; $\quad W_w = w\, W_d = 0.15 \times 17.56 = 2.634 \text{ kN}$

$\therefore \quad V_w = \dfrac{W_w}{\gamma_w} = \dfrac{2.634}{9.81} = 0.2685 \text{ m}^3$

Hence, additional water required to raise the water content from 5% to 15% = 0.2685 – 0.0895 = 0.179 m^3 = **179 litres**.

Voids ratio, $\quad e = \dfrac{G\gamma_w}{\gamma_d} - 1 = \dfrac{2.67 \times 9.81}{17.56} - 1 = 0.49$

After the water has been added, e remains the same.

$$S = \frac{wG}{e} = \frac{0.15 \times 2.67}{0.49} = 0.817 = \mathbf{81.7\%}.$$

Example 2.7. *Calculate the unit weights and specific gravities of solids of (a) a soil composed of pure quartz and (b) a soil composed of 60% quartz, 25% mica and 15% iron oxide. Assume that both soils are saturated and have voids ratio of 0.63. Take average G for quartz = 2.66, for mica = 3.0 and for iron oxide = 3.8.*

Solution.

(*a*) For the soil composed of pure quartz, G for quartz = 2.66

$\therefore \quad \gamma_{sat} = \dfrac{G+e}{1+e}\gamma_w = \dfrac{2.66+0.63}{1+0.63} \times 9.81 = \mathbf{19.8 \text{ kN/m}^3}$

(*b*) For the composite soil,

$$G_{average} = (2.66 \times 0.6) + (3.0 \times 0.25) + (3.8 \times 0.15) = 1.60 + 0.75 + 0.57 = 2.92$$

$$\gamma_{sat} = \frac{2.92 + 0.63}{1 + 0.63} \times 9.81 = \mathbf{21.36 \text{ kN/m}^3}.$$

Example 2.8. *A soil has a bulk unit weight of 20.11 kN/m^3 and water content of 15%. Calculate the water content if the soil partially dries to a unit weight of 19.42 kN/m^3 and the voids ratio remains unchanged.*

Solution. Before drying, $\gamma = 20.11 \text{ kN/m}^3$

$\therefore \quad \gamma_d = \dfrac{20.11}{1+0.15} = \mathbf{17.49 \text{ kN/m}^3}$

Since, after drying, e does not change, V and γ_d are the same,

$\therefore \quad \gamma = \gamma_d(1 + w)$ or $1 + w = \dfrac{\gamma}{\gamma_d} = \dfrac{19.42}{17.49} = 1.11$

$\therefore \quad w = 1.11 - 1 = 0.11 = \mathbf{11\%}.$

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Example 2.9. *A cube of dried clay having sides 4 cm long has a mass of 110 g. The same cube of soil, when saturated at unchanged volume, has mass of 135 g. Draw the soil element showing the volumes and weights of the constituents, and then determine the specific gravity of soil solids and the voids ratio.*

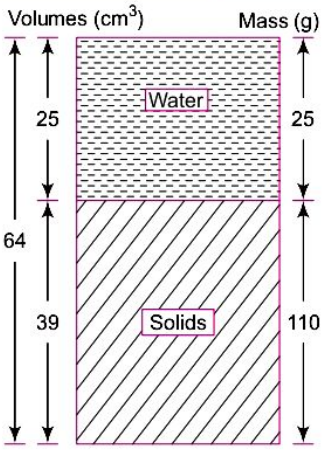

Fig. 2.6

Solution. Volume of soil = 4^3 = 64 cm^3 (Before and after saturation).

Mass of water after saturation = 135 – 110 = 25 g

∴ Volume of voids, $V_v = 25$ cm^3

Volume of solids, $V_s = 64 - 25 = 39$ cm^3

Total mass of dry solids, $M_s = 110$ g

The mass and volume has been marked in Fig. 2.6, from which:

$$G = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w} = \frac{M_s}{V_s \times 1} = \frac{110}{39 \times 1}$$

$$= \mathbf{2.82}$$

$$e = \frac{V_v}{V_s} = \frac{25}{39} = \mathbf{0.64.}$$

Example 2.10. *It is required to prepare a compacted cylindrical specimen of 40 mm dia. and 80 mm length from oven dry soil. The specimen is required to have water content of 10% and percent air voids of 18%. Taking G = 2.70, determine the mass of soil and mass of water, required for the preparation of the above specimen.*

Solution.

Total volume of compacted sample, $V = \frac{\pi}{4}(0.04)^2 \times 0.08 = 1.0053 \times 10^{-4}\,\text{m}^3$ ...(1)

Let the mass of solids (*i.e.* dry weight of soil) = M_d kg.

∴ Mass of water, $M_w = wM_d = 0.16\,M_d$

Volume of solids, $V_s = \frac{M_d}{G\gamma_w} = \frac{M_d}{2.7 \times 1000} = \frac{M_d}{2700}\,\text{m}^3$ ...(2)

Volume of water, $V_w = \frac{M_w}{\rho_w} = \frac{0.16 M_d}{1000} = 1.6 \times 10^{-4} M_d\,\text{m}^3$ ...(3)

Volume of air, $V_a = n_a \cdot V = 0.18 \times 1.0053 \times 10^{-4} = 1.8095 \times 10^{-5}\,\text{m}^3$ ...(4)

Now, total volume, $V = V_s + V_w + V_a$

∴ $$1.0053 \times 10^{-4} = \frac{M_d}{2700} + 1.6 \times 10^{-4} M_d + 1.8095 \times 10^{-5}$$

From which, we get $M_d = 0.1554$ kg = **155.4 g**

Mass of water, $M_w = 0.16\,M_d = 0.16 \times 0.1554 =$ **0.0249 kg = 24.9 g**

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Example 2.11. *An embankment, having a total volume of 5000 m^3 has a water content of 16% and dry density of 1.75 g/cm^3. If it was constructed from a borrow pit where the undisturbed soil has a water content of 13% and voids ratio of 0.6, calculate the quantity of soil which was excavated for the construction of the above embankment. Take specific gravity of soil solids as 2.68.*

Solution. Total volume of embankment, $V = 5 \times 10^3$ m^3

Now dry density, $$\rho_d = \frac{M_d}{V} = 1.75 \text{ g/cm}^3 = 1.75 \text{ t/m}^3$$

$\therefore$ $$M_d = \rho_d \,.\, V = 1.75 \times 5 \times 10^3 = 8750 \text{ t}$$

In the borrow pit, $$(\rho_d)_b = \frac{G\rho_w}{1+e} = \frac{2.68 \times 1}{1+0.6} = 1.675 \text{ t/m}^3$$

Now, volume of soil from borrow pit, $V_b = \left(\frac{M_d}{\rho_d}\right)_b = \frac{8750}{1.675} = \mathbf{5223.8\ m^3}$

Example 2.12. *An embankment, having total volume of 2000 m^3 is to be constructed having a bulk density of 1.98 g/cm^3 and a placement water content of 18%. The soil is to be obtained either from borrow area A or borrow area B, which have voids ratio of 0.78 and 0.69, respectively and water content of 16% and 12%, respectively. Taking G = 2.66, for both the soils, determine the volume of soil required to be excavated from each of the areas. If the cost of excavation is ₹ 35 per m^3 in each area, but cost of transportation is ₹ 32 and 36 per m^3 from areas A and B respectively, which of the borrow areas is more economical?*

Solution. ***For the embankment,*** $\rho_d = \frac{\rho}{1+w} = \frac{1.98}{1+0.18} = 1.678$ g/cm^3 = 1.678 t/m^3

$\therefore$ $$M_d = \rho_d \cdot V = 1.678 \times 2000 = 3356 \text{ t}$$

(a) Borrow area A: $\rho_d = \frac{G\rho_w}{1+e} = \frac{2.66 \times 1}{1+0.78} = 1.494$ g /cm^3 = 1.494 t/m^3

$\therefore$ $$V = \frac{M_d}{\rho_d} = \frac{3356}{1.494} = \mathbf{2246\ m^3}$$

Cost of excavation and transportation = (35 + 32) 2246 = ₹ 150482

(b) Borrow area B: $\rho_d = \frac{G\gamma_w}{1+e} = \frac{2.66 \times 1}{1+0.69} = 1.574$ g/cm^3 = 1.574 t/m^3

$\therefore$ $$V = \frac{M_d}{\rho_d} = \frac{3356}{1.574} = \mathbf{2132\ m^3}$$

Cost of excavation and transportation = (35 + 36) 2132 = ₹ 151372

Hence borrow area *A* is more economical.

Example 2.13. *Sandy soil in a borrow pit has unit weight of solids as 26.3 kN/m^3, water content equal to 11% and bulk unit weight equal to 16.4 kN/m^3. How many cubic metre of compacted fill could be constructed of 3500 m^3 of sand excavated from the borrow pit, if the required value of porosity in the compacted fill is 30%? Also compute the change in degree of saturation.*

Solution. Let us use suffix 1 for the borrow pit soil and 2 for the compacted soil. Assuming that weight and water content do not change during construction, the change in the volume can be calculated from the change in the unit weight.

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i.e.,
$$\frac{V_1}{V_2} = \frac{W/\gamma_1}{W/\gamma_2} = \frac{\gamma_2}{\gamma_1} = \frac{1+e_1}{1+e_2}$$

Hence,
$$V_2 = V_1\left(\frac{1+e_2}{1+e_1}\right) = V_1\left(\frac{1-n_1}{1-n_2}\right) \qquad ...(1)$$

From equation 2.24,
$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{\gamma_s}{\gamma/(1+w)} - 1 = \frac{\gamma_s(1+w)}{\gamma} - 1$$

Hence,
$$e_1 = \frac{\gamma_s(1+w)}{\gamma_1} - 1 = \frac{26.3(1+0.11)}{16.4} - 1 = 0.780$$

Also,
$$e_2 = \frac{n_2}{1-n_2} = \frac{0.3}{1-0.3} = 0.429$$

Hence,
$$V_2 = V_1\left(\frac{1+e_2}{1+e_1}\right) = 3500\left(\frac{1+0.429}{1+0.780}\right) = \mathbf{2810\ m^3}$$

Now,
$$S_1 = \frac{w\gamma_s}{e_1\gamma_w} = \frac{0.11\times 26.3}{0.78\times 9.81} = \mathbf{0.378}$$
$$S_2 = \frac{w\gamma_s}{e_2\gamma_w} = \frac{0.11\times 26.3}{0.429\times 9.81} = \mathbf{0.687.}$$

Thus, the degree of saturation will be considerably increased in the compacted fill.

Example 2.14. *A sand deposit has following properties in its natural state:*

A(i) bulk unit weight: 18.6 kN/m^3, (ii) unit weight of soil solids: 26.3 kN/m^3, (iii) water content: 10%.

The same sand, when dumped loosely will experience a volume change of 18% due to bulking. Determine the porosity of the sand in both the states.

Solution. Given: (*A*)

We have,
$$\gamma_1 = 18.6\ \text{kN/m}^3$$
$$\gamma_s = 26.3\ \text{kN/m}^3$$
$$w = 10 = .1$$

Now,
$$\gamma_1 = \gamma_s\frac{1+w}{1+e_1} = \gamma_s(1+w)(1-n_1)$$

$\therefore$
$$1-n = \frac{\gamma_1}{\gamma_s(1+w)} = \frac{18.6}{26.3(1+0.1)} = 0.643$$

or
$$n_1 = 1 - 0.643 = 0.357 = \mathbf{35.7\%}$$

Let us now assume that weight and water content of the material do not change during dumping.

$\therefore$
$$\frac{V_1}{V_2} = \frac{W/\gamma_1}{W/\gamma_2} = \frac{\gamma_2}{\gamma_1} = \frac{1+e_1}{1+e_2} = \frac{1-n_2}{1-n_1}$$

or
$$1-n_2 = \frac{V_1}{V_2}(1-n_1)$$

or
$$n_2 = 1 - \frac{V_1}{V_2}(1-n_1) = 1 - \frac{1}{1.18}(1-0.357) = 0.455 = \mathbf{45.5\%}$$

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2.7 PACKING OF UNIFORM SPHERES

Packing of *uniform spheres* is the only granular system, the voids ratio and porosity of which can be determined mathematically. According to Slichter (1899) the loosest stable arrangement of equal sized spheres is obtained when the sphere centres form a rectangular space lattice. Such a packing, shown in Fig. 2.7(*a*) is known as *cubic packing*, wherein each sphere is in contact with six neighbouring spheres. The porosity for such a packing is $n = 47.64\%$ (see example 2.15).

Similarly, the densest state of packing is obtained (Fig. 2.7(*b*)) when sphere centres form a rhombohedral array ($\alpha = 60°$) wherein each sphere is in contact with 12 neighbouring spheres. The porosity for such a packing is $n = 25.95\%$ (see example 2.16).

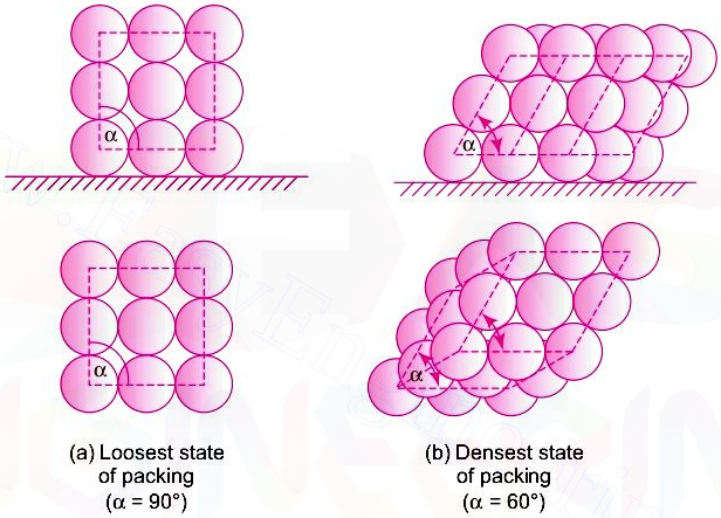

Fig. 2.7 Packing of uniform spheres

For an *intermediate packing* (*i.e.*, neither loosest packing nor densest packing), the centres of any 8 spheres, *originally arranged in cubic packing* form the corners of a rhombohedron, with acute face angle α, known as angle of *orientation of the rhombohedron*. Corresponding to original unit cube, the total volume of the rhombohedron, or volume of *unit cell* is given by

$$V = \sqrt{1 + 2\cos\alpha}\,(1 - \cos\alpha) \qquad ...(2.36)$$

The volume of solids in this rhombohedron is evidently equal to original volume $\pi/6$, which is constant and which does not change with the angle α. The porosity for such an intermediate packing is given by the following formula by Slichter:

$$n = 1 - \frac{\pi}{6(1 - \cos\alpha)\sqrt{1 + 2\cos\alpha}} \qquad ...(2.37)$$

Example 2.15. *Determine the maximum possible voids ratio for a uniformly graded sand of perfectly spherical grains.*

Solution. The soil will have maximum possible voids when its grains are arranged in a cubical array of spheres. Consider a *unit cube* of soil having spherical particles of diameter *d*.

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Volume of each spherical particle = $\frac{\pi}{6}d^3$

Total volume of container = $1 \times 1 \times 1 = 1$

$\therefore$ No. of solids in the container = $\frac{1}{d}.\frac{1}{d}.\frac{1}{d} = \frac{1}{d^3}$

$\therefore$ Volume of solids, $(V_s) = \left(\frac{\pi}{6}d^3\right) \times \frac{1}{d^3} = \frac{\pi}{6}$

$\therefore$ Volume of voids, $(V_v) = 1 - \frac{\pi}{6}$

$\therefore$ Voids ratio, $e = \frac{1-\frac{\pi}{6}}{\frac{\pi}{6}} = 0.9099$

Porosity, $n = \frac{e}{1+e} = \frac{0.9099}{1+0.9099} = 0.4764 = \mathbf{47.64\%}.$

Example 2.16. *Determine the minimum possible voids ratio of a uniformly graded sand of perfectly spherical grains.*

Solution. Consider a *unit cube* of soil having spherical particles of diameter d. The soil will have maximum possible voids in this packing wherein the angle $\alpha = 90°$ (Fig. 2.7*a*). Now rearrange the spheres so that the sphere centres form a rhombohedral array with angle $\alpha = 60°$. The soil will attain densest state of packing in this arrangement. By such an rearrangement the volume of solids will remain the same, while the volume of voids will change, thus changing the total volume V.

If d is the diameter of each sphere, each of volume $\pi\, d^3/6$, the number of solids in the original unit cube = $\frac{1}{d}.\frac{1}{d}.\frac{1}{d} = \frac{1}{d^3}$

$\therefore$ Volume of solids, $V_s = \frac{\pi}{6}d^3 \times \frac{1}{d^3} = \frac{\pi}{6}$

Now the changed volume of rhombohedron is given by equation 2.36

$$V = \sqrt{1+2\cos\alpha}\,(1-\cos\alpha)$$

Here, for densest state of packing, $\alpha = 60°$

$\therefore$ $$V = \sqrt{1+2\cos 60°}\,(1-\cos 60°) = 0.7071$$

Thus, the *original unit volume* is changed to 0.7071 due to densest packing. Since the volume of solids remain the same, we have $V_s = \frac{\pi}{6} = 0.5236$

$\therefore$ Volume of voids, $V_v = V - V_s = 0.7071 - 0.5236 = 0.1835$

$\therefore$ Voids ratio, $e = \frac{V_v}{v_s} = \frac{0.1835}{0.5236} = 0.35$

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Hence porosity $$n = \frac{e}{1+e} = \frac{0.35}{1+0.35} = 0.2595 = \mathbf{25.95\%}$$

Example 2.17. *Determine the voids ratio and porosity of a uniformly graded sand of perfectly spherical grains arranged to form a rhombohedral arry with angle of orientation of rhombohedron equal to 75°.*

Solution. Refer Fig. 2.7(*b*), where α is equal to 75° (instead of 60°).

The volume of original unit cube will change to new volume V due to this arrangement, without changing the initial volume of solids, V_s (= $\pi/6$).

New volume is given by equation 2.36.

$$V = \sqrt{1+2\cos\alpha}\,(1-\cos\alpha) = \sqrt{1+2\cos 75°}\,(1-\cos 75°) = 0.9131$$

$$V_s = \pi/6 = 0.5236.$$

Hence, $$V_v = V - V_s = 0.9131 - 0.5236 = 0.3895$$

Hence voids ratio, $$e = \frac{V_v}{V_s} = \frac{0.3895}{0.5236} = \mathbf{0.7439}$$

and porosity, $$n = \frac{e}{1+e} = \frac{0.7439}{1+0.7439} = 0.4266 = \mathbf{42.66\%.}$$

2.8 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 2.18. *The soil fill for a road embankment is to be compacted in place to a void ratio of 0.7. If the void ratio of the borrow pit soil is 1.2, how many cubic metres of compacted fill can be placed in the embankment per 1000 cubic metres of borrow materials? If no water is either added or lost during the placement of the fill, what would be the percentage change in the degree of saturation of the soil? The compacted fill is not fully saturated.* **(Civil Services Exam. 1982)**

Solution. Use suffix 1 for borrow pit soil and 2 for embankment fill soil.

$$e = \frac{V_v}{V_s};\ 1+e = 1+\frac{V_v}{V_s} = \frac{V_s+V_v}{V_s} = \frac{V}{V_s}.\ \text{Hence } V = V_s\,(1+e)$$

Since V_s remains constant, $$\frac{V_2}{V_1} = \frac{1+e_2}{1+e_1} \quad \therefore \quad V_2 = V_1\left(\frac{1+e_2}{1+e_1}\right) = \frac{1000(1+0.7)}{(1+1.2)} = \mathbf{772.73\ m^3}$$

Now, $w = M_w/M_d$. Since neither M_d change nor M_w changes, the water content also remains the same *i.e.*, $w_1 = w_2$. Only, volume changes from V_1 to V_2 due to expulsion of air voids.

$$\therefore \qquad S_1 = \frac{wG}{e_1} \text{ and } S_2 = \frac{wG}{e_2} \quad \therefore \quad \frac{S_1}{S_2} = \frac{e_2}{e_1}$$

$$\therefore \qquad \%\text{ change is } S = \left(\frac{S_2 - S_1}{S_1}\right)100 = \left(\frac{S_2}{S_1} - 1\right)100 = \left(\frac{e_1}{e_2} - 1\right)100 = \left(\frac{1.2}{0.7} - 1\right)100$$

$$= \mathbf{71.43\%.}$$

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Example 2.19. *Test on a fill reveal that one cubic metre of soil in the fill weighs 1624 kg and after being dried, 1.40 tonnes. If the specific gravity of solids is 2.65, determine the water content, void ratio, porosity and degree of saturation of the soil mass in moist state.* **(Civil Services Exam. 1983)**

Solution. $M_w = 1624 - 1400 = 224$ kg ; $V_w = 224/1000 = 0.224$ m^3

$$M_s = M_d = 1400 \text{ kg} ; V_s = \frac{M_d}{G\rho_w} = \frac{1400 \times 1000}{2.65 \times 1} = 528302 \text{ cm}^3 = 0.5283 \text{ m}^3$$

$$\therefore \quad V_a = 1 - V_w - V_s = 1 - 0.224 - 0.5283 = 0.2477 \text{ m}^3 ; V_v = V_a + V_w$$

$$= 0.2477 + 0.2240 = 0.4717 \text{ m}^3$$

(*i*) Moisture content, $w = M_w/M_d = 224/1400 = 0.16 =$ **16%**

(*ii*) Voids ratio, $e = V_v/V_s = 0.4717/0.5283 =$ **0.893**

(*iii*) Degree of saturation, $S = \frac{G \,.\, w}{e} = \frac{2.65 \times 0.16}{0.893} = 0.4749 =$ **47.49%**

(*iv*) Porosity, $n = \frac{e}{1+e} = \frac{0.893}{1+0.893} =$ **0.472**

Example 2.20. *A soil sample is partially saturated. Its natural moisture content was found to be 22% and bulk density 2 g/cm^3. If the specific gravity of solid particles is 2.65 and the density of water be taken as 1 g/cm^3, find out the degree of saturation and the void ratio.* **(Civil Services Exam. 1984)**

Solution. Given $w = 0.22$; $\rho = 2$ g/cm^3 ; $G = 2.65$; $\rho_w = 1$ g/cm^3

From equation 2.28(*a*), $$\rho = \frac{(G + e S_r)\rho_w}{1+e} = \frac{G(1+w)\rho_w}{1+e}$$

From which, $$e = \frac{G\rho_w(1+w)}{\rho} - 1 = \frac{2.65 \times 1(1+0.22)}{2} - 1 =$$ **0.6165**

Also from equation 2.20, $$S = \frac{wG}{e} = \frac{0.22 \times 2.65}{0.6165} = 0.9457 =$$ **94.57%.**

Example 2.21. *A natural soil sample has a bulk density of 2 g/cm^3 with 6% water content. Calculate the amount of water required to be added to one cubic metre of soil to raise the water content to 15% while the voids ratio remains constant. What is then the degree of saturation? G = 2.67.*

(Civil Services Exam. 1986)

Solution. Given $\rho = 2$ g/cm^3, $w = 0.06$; $G = 2.67$

$$\rho_d = \frac{\rho}{1+w} = \frac{2}{1+0.06} = 1.887 \text{ g/cm}^3$$

First stage: When $w = 6\% = 0.06 = \frac{M_w}{M_d}$. Consider a unit volume, $V = 1 \text{ m}^3 = 10^6 \text{ cm}^3$

Now, $M_d = \rho_d \,.\, V = 1.887 \times 10^6$ g $\quad \therefore \quad M_w = w\, M_d = 0.06 \times 1.887 \times 10^6$

$= 113220$ g

$$V_w = \frac{M_w}{\rho_w} = \frac{113220}{1} = 11.3220 \times 10^4 \text{ cm}^3$$

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Second stage: When $w = 15\% = 0.15$ $\therefore$ $M_w = w\,M_d = 0.15 \times 1.887 \times 10^6$ g $= 28.305 \times 10^4$ g

$$\therefore \quad V_w = \frac{M_w}{\rho_w} = \frac{28.305 \times 10^4}{1} = 28.305 \times 10^4 \text{ cm}^3.$$

$\therefore$ Additional volume of water required to raise w from 6% to 15%.

$$= 28.305 \times 10^4 - 11.3220 \times 10^4 = 16.983 \times 10^4 \text{ cm}^3 = \textbf{169.83 litres}$$

$$\text{Now voids ratio} = e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.67 \times 1}{1.887} - 1 = 0.415$$

After adding water, e remains the same. Hence, $S = \dfrac{wG}{e} = \dfrac{0.15 \times 2.67}{0.415} = 0.9652 = \textbf{96.52\%}$

Example 2.22. *A mass of soil coated with thin layer of paraffin wax weighs 690.6 g and the soil alone weighs 683 g. When the sample is immersed in water, it displaces 350 ml of water. The specific gravity of the soil is 2.73 and that of wax is 0.89. Find out void ratio and degree of saturation if it has got water content of 17%. Unit weight of water is 1000 kg/cu.m.* **(Civil Services Exam. 1990)**

Solution. Given: $M_1 = 683$ g ; $M_2 = 690.6$ g ; $G_p = 0.89$; $G = 2.73$

$$\therefore \quad M_p = M_2 - M_1 = 690.6 - 683 = 7.6 \text{ g} \quad \therefore \quad V_p = M_p/G_p = 7.6/0.89 = 8.539 \text{ cm}^3$$

$$V_w = 350 \text{ cm}^3 \text{ (gives)} \; ; \; V = V_w - V_p = 350 - 8.539 = 341.461 \text{ cm}^3$$

$$\therefore \quad \rho = M_1/V = 683/341.461 = 2 \text{ g/cm}^3 \text{ or } w = 0.17.$$

Hence, $\rho_d = \rho/(1 + w) = 2/(1 + 0.17) = 1.709$ g/cm^3

Now, $\rho_w = 1000$ kg/m^3 $= 1$ g/cm^3 $\therefore$ $e = \dfrac{G\rho_w}{\rho_d} - 1 = \dfrac{2.73 \times 1}{1.709} - 1 = \textbf{0.597}$

and $S = \dfrac{wG}{e} = \dfrac{0.17 \times 2.73}{0.597} = 0.7773 = \textbf{77.73\%}$

Example 2.23. *A compacted cylindrical specimen 50 mm diameter and 100 mm long is to be prepared from dry soil. If the specimen is required to have a water content of 15% and the percentage of air voids is 20, calculate the weight of soil and water required in the preparation of the soil whose specific gravity is 2.69.*

(Civil Services Exam, 1991)

Solution. Given: $w = 15\%$; $a_c = 20\%$; $G = 2.69$

$$V = \frac{\pi}{4}(50)^2 \times 10 = 196.35 \text{ cm}^3 \; ; \; M_w = 0.15\,M_d \; ; \; V_a = 0.20\,V$$

Now, $$V_w = \frac{M_w}{\rho_w} = \frac{0.15 M_d}{\rho_w} = \frac{0.15(V_s \cdot G\rho_w)}{\rho_w} = 0.15 \times 2.69\,V_s = 0.4035\,V_s$$

Now, $$V = V_a + V_w + V_s = 0.20\,V + 0.4035\,V_s + V_s$$

or $0.8\,V = 1.4035\,V_s$, from which $V_s = 0.57\,V = 0.57 \times 196.35 = 111.92$ cm^3

$\therefore$ $M_d = V_s\,G\,\rho_w = 111.92 \times 2.69 \times 1 = \textbf{301.1 g}$

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Also, $M_w = 0.15\ M_d = 0.15 \times 301.1 = \mathbf{45.2\ g.}$

Example 2.24 *A clayey soil has saturated moisture content of 15.8%. The specific gravity is 2.72. Its saturation percentage is 70.8%. The soil is allowed to absorb water. After sometime, the saturation increased to 90.8%. Find the water content of the soil in the latter case.* **(Civil Services Exam. 1992)**

Solution. Given: $w = 15.8\%$; $G = 2.72$; $S = 70.8\%$

Initially, $$e = \frac{wG}{S} = \frac{0.158 \times 2.72}{0.708} = 0.607$$

When the soil is allowed to absorb water, the air voids are replaced partially (or fully) by water and hence the voids ratio remains constant.

Now, after absorption,

$$e = \frac{wG}{S} = 0.607$$

$\therefore$ $$w = \frac{0.607 \times S}{G} = \frac{0.607 \times 0.908}{2.72} = 0.2026 = \mathbf{20.26\%}$$

Example 2.25. *A core cutter 12.6 cm in height and 10.2 cm in diameter weighs 1071 g when empty. It is used to determine the in-situ unit weight of an embankment. The weight of core cutter full of soil of 2970 g. If the water content is 6%, what are in situ-dry weight and porosity? (ii) If the embankment gets fully saturated due to heavy rains what will be the increase in water content and bulk unit weight, if no volume change occurs? The specific gravity of soil solids is 2.69.* **(Civil Services Exam. 1997)**

Solution. $$V = \frac{\pi}{4} D^2 H = \frac{\pi}{4}(10.2)^2 \times 12.6 = 1029.58\ \text{cm}^3$$

Mass of soil sample, $M = 2970 - 1071 = 1899\ \text{cm}^3$

$\therefore$ $\rho = M/V = 1899/1029.58 = 1.844\ \text{g/cm}^3$; $\gamma = 1.844 \times 9.81 = 18.094\ \text{kN/m}^3$

$$\rho_d = \frac{\rho}{1+w} = \frac{1.844}{1+0.06} = 1.74\ \text{g/cm}^3 \quad \therefore\ \gamma_d = 1.74 \times 9.81 \approx \mathbf{17.07\ kN/m^3}$$

$$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.69 \times 9.81}{17.07} - 1 = \mathbf{0.546}$$

(*ii*) When the embankment gets saturated, e remain the same, since water outer by expelling air from the voids.

Now, $$e = w_{sat} \cdot G \quad \therefore \quad w_{sat} = \frac{e}{G} = \frac{0.546}{2.69} = 0.203 = 20.3\%$$

$\therefore$ Increase in water content $= 0.203 - 0.6 = 0.143 = \mathbf{14.3\%}$

Also, $\gamma_{sat} = \gamma_d\ (1 + w_{sat}) = 17.07\ (1 + 0.203) = 20.535\ \text{kN/m}^3$

$\therefore$ Increase is bulk unit weight $= 20.535 - 18.094 = \mathbf{2.441\ kN/m^3}$.

Example 2.26. *A highly sensitive volcanic clay was investigated in the laboratory and found to have the following properties:*

(*i*) $\gamma_{wet} = 12.50\ kN/m^3$ (*ii*) $G = 2.75$ (*iii*) $e = 9.0$ (*iv*) $w = 311\%$

In rechecking the above values, one was found to be inconsistent with the rest. Find the inconsistent value and report it correctly. **(Civil services Exam. 2001)**

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Solution. The solution will be done by trial.

***Assumption* (*i*):** Let γ_{wet} be the inconsistent value, and the values of G, e and w be correct.

Then $$S = \frac{wG}{e} = \frac{3.11 \times 2.75}{9.0} = 0.95 < 1 \text{ (Hence OK)}$$

From equation 2.28, $$\gamma = \frac{(G + eS)\gamma_w}{1 + e} = \frac{(2.75 + 9.0 \times 0.95) \times 9.81}{1 + 9.0} = 11.09 \text{ kN/m}^3$$

Since this computed value of γ does not match with the given value of 12.56 kN/m^3, the given value is inconsistent. In order to show that γ_{wet} is the only inconsistent value, let us check back by assuming any of the remaining value to be inconsistent.

***Assumption* (*ii*):** Let value of G be in consistent, and let γ_{wet} ; e and w be correct.

$$\therefore \quad \gamma_d = \frac{\gamma_{wet}}{1 + w} = \frac{12.56}{1 + 3.11} = 3.06 \text{ kN/m}^3$$

Hence, $$G = \frac{\gamma_d (1 + e)}{\gamma_w} = \frac{3.06(1 + 9.0)}{9.81} = 3.115 \text{ and } S = \frac{wG}{e} = \frac{3.11 \times 3.115}{9.0} = 1.076 > 1$$

Since S is more than 1, the above assumption is not feasible, *i.e.*, G is not inconsistent.

***Assumption* (*iii*)**: Let the value of e be inconsistent and let γ_{wet}, G and w be correct.

Then, $$\gamma_d = \frac{\gamma_{wet}}{1 + w} = \frac{12.53}{1 + 3.11} = 3.06 \text{ kN/m}^3$$

$$\therefore \quad e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.75 \times 9.81}{3.06} - 1 = 7.828 \text{ and } S = \frac{wG}{e} = \frac{3.11 \times 2.75}{7.828} = 1.093 > 1.$$

Since S is more than 1, the above assumption is not feasible, *i.e.*, the value of e (= 9.0) is not inconsistent.

***Assumption* (*iv*):** Lastly, let w be incorrect and γ_{wet}, G and e be correct.

Now, $$\gamma_d = \frac{G\gamma_w}{1 + e} = \frac{2.75 \times 9.81}{1 + 9.0} = 2.698 \text{ kN/m}^3$$

$$\therefore \quad w = \frac{\gamma_{wet}}{\gamma_d} - 1 = \frac{12.56}{2.698} - 1 = 3.655$$

and $$S = \frac{wG}{e} = \frac{3.655 \times 2.75}{9.0} = 1.117 > 1$$

Since S is more than 1, the above assumption is incorrect, *i.e.*, the value of w is not inconsistent.

Final result: Hence, only assumption (*i*) is correct, *i.e.*, the value of γ_{wet} in inconsistent. The correct value of γ_{wet} = **11.09 kN/m^3**.

Example 2.27. *The unit weight of a sand backfill was determined by field measurements to be 1746 kg per cu. m. The water content at the time of test was 8.6% and the unit weight of the soil constituents was 2.6 gram per cu cm. In the laboratory, the void ratios in the loosest and densest states were found to be 0.642 and 0.462 respectively. What was the relative density of the fill? Write the importance of this term.* **(U.P.S.C. Engg. Services Exam. 1994)**

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Solution. Given $\rho = 1746$ kg/m^3 $= 1.746$ g/cm^3 ; $w = 0.086$; $\gamma_s = 2.6$ g/cm^3.

$$\therefore \quad G = \frac{\gamma_s}{\gamma_w} = \frac{2.6}{1} = 2.6; \quad \text{Now} \quad \rho = \frac{G\rho_w(1+w)}{1+e}$$

From which $$e = \frac{G\rho_w(1+w)}{\rho} - 1 = \frac{2.6 \times 1(1+0.086)}{1.746} - 1 = 0.617$$

Also, given $e_{max} = 0.642$ and $e_{min} = 0.462$

$$\therefore \quad I_D = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.642 - 0.617}{0.642 - 0.462} = 0.1379 = \mathbf{13.79\%}.$$

Such a low value of I_D indicates that the soil is very loose.

Example 2.28. *Soil is to be excavated from a borrow pit which has a density of 1.75 g/cc and water content of 12%. The specific gravity of soil particles is 2.7. The soil is compacted so that water content is 18% and dry density is 1.65 g/cc. For 1000 cu.m of soil in fill, estimate (i) the quantity of soil to be excavated from the pit in cu.m; and (ii) the amount of water to be added. Also determine the void ratios of the soil in borrow pit and fill.* **(Engg. Service Exam., 1995)**

Solution. Given: $\gamma = 1.75$ g/cm^3; $w = 12\%$; $G = 2.7$. After compaction; $\gamma_d = 1.65$ g/cm^3 ; $w = 18\%$

Let us use suffix 1 for borrow pit and 2 for the fill.

For the Borrow pit: $$\rho = \frac{G\rho_w(1+w)}{1+e}$$

or $$e_1 = \frac{G\rho_w(1+w)}{\rho} - 1 = \frac{2.7 \times (1+0.12)}{1.75} - 1 = 0.728$$

For the fill: $$\rho_d = \frac{G\rho_w}{1+e}$$

or $$e_2 = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.7 \times 1}{1.65} - 1 = 0.6364$$

Since the volume of solids remains constant,

$$V_s = \frac{V_1}{1+e_1} = \frac{V_2}{1+e_2} = \frac{1000\,\text{m}^3}{1+0.6364} \approx 611.1\,\text{m}^3 \quad \therefore \quad V_1 = 611.1\,(1+0.728) \approx \mathbf{1056\ m^3}.$$

Again, in general, $w = M_w/M_d$

But $$M_d = M_s = \text{mass of solids} = V_s\, G\, \rho_w = 611.1 \times 2.7 \times 1 = 1649.97\ t$$

Now, $$M_{w1} = M_d \cdot w_1 = 1649.97 \times 0.12 = 197.996\ \text{m}^3\ ;$$

$$M_{w2} = M_d \cdot w_2 = 1649.97 \times 0.18 = 296.995\ \text{m}^3$$

$\therefore$ Water to be added $= M_{w2} - M_{w1} = 296.995 - 197.996 \approx \mathbf{99\ m^3}$

Example 2.29. *A sampler with a volume of 45 cm^3 is filled with a soil sample. When the soil is poured into a graduated cylinder, if displaces 25 cm^3 of water. What is the porosity and wind ratio of the soil?* **(Engg. Services Exam., 1998)**

Solution. Given: $V = 45$ cm^3; Volume of solids, V_s = Volume of water displaced = 25 cm^3

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$$V_v = V - V_s = 45 - 25 = 20 \text{ cm}^3 \quad \therefore \quad e = V_v/V_s = 20/25 = \mathbf{0.8}$$

$$n = V_v/V = 20/45 = \mathbf{0.444.}$$

Example 2.30. *The void ratio and specific gravity of a sample of clay are 0.73 and 2.7, respectively. If the voids are 92% saturated, find the bulk density, dry density and the water content. What would be the water content for complete saturation, the void ratio remaining the same?* **(Engg. Services Exam., 1998)**

Solution. Given : $e = 0.73$; $G = 2.7$; $S = 92\%$

$$w = \frac{eS}{G} = \frac{0.73 \times 0.92}{2.7} = 0.2487 = \mathbf{24.87\%;}$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.73} = \mathbf{15.31 \text{ kN/m}^3}$$

$$\gamma = \gamma_d (1 + w) = 15.31\ (1 + 0.2487) = \mathbf{19.118 \text{ kN/m}^3;}$$

$$w_{sat} = \frac{e}{G} = \frac{0.72}{2.7} = 0.2703 = \mathbf{27.03\%}$$

Example 2.31. *A sample of sand above water table was found to have a natural moisture content of 15% and a unit weight of 18.84 kN/m^3. Laboratory tests on a dried sample indicated values of 0.5 and 0.85 for minimum and maximum void ratios, respectively for densest and loosest states. Calculate the degree of saturation and the relative density. Assure $G = 2.65$.* **(Engg. Services Exam. 2000)**

Solution. Given: $w = 15\%$, $\gamma = 18.84$ kN/m^3 ; $e_{min} = 0.5$; $e_{max} = 0.85$

From equations 2.24 and 2.31, $e = \dfrac{G\gamma_w(1+w)}{\gamma} - 1 = \dfrac{2.65 \times 9.81(1+0.15)}{18.84} - 1 = \mathbf{0.587}$

From equation 220, $S = \dfrac{wG}{e} = \dfrac{0.15 \times 2.65}{0.587} = 0.6774 = \mathbf{67.74\%}$

Also, $I_D = \dfrac{e_{max} - e}{e_{max} - e_{min}} = \dfrac{0.85 - 0.587}{0.85 - 0.5} = 0.7514 = \mathbf{75.14\%}$

Example 2.32. *Cohesive soil yields a maximum dry density of 1.8 g/cc at on OMC of 16% during a standard Proctor test. If the value of G is 2.65, what is the degree of saturation? What is the maximum dry density it can be further compacted to?* **(Gate Exam. 1992)**

Solution. Given: $\rho_d = 1.8$ g/cm^3; $w = 16\%$; $G = 2.65$

$$e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.65 \times 1}{1.8} - 1 = 0.472 \quad \therefore \quad S = \frac{wG}{e} = \frac{0.16 \times 2.65}{0.472} = 0.8979 = \mathbf{89.79\%}$$

Theoretically $\gamma_{d,\,max}$ is obtained at zero air voids, *i.e.*, when $S = 1$.

$$\therefore \quad \rho_{d\,,\,max} = \frac{G\rho_w}{1+wG} = \frac{2.65 \times 1}{1+0.16 \times 2.65} = \mathbf{1.861 \text{ g/cm}^3}$$

Example 2.33. *The total unit weight (γ_t) of soil is 18.8 kN/m^3 the specific gravity (G) of the solid particles of soil is 2.67 and the water content (w) of the soil is 12%. Calculate the dry unit weight (γ_d), the void ratio (e) and the degree of saturation.* **(Gate Exam. 1994)**

Solution. $\gamma_t\ (= \gamma) = 18.8$ kN/m^3 ; $G = 2.67$; $w = 12\%$;

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Now, $$\gamma_d = \frac{\gamma_t}{1+w} = \frac{18.8}{1+0.12} = \mathbf{16.786\ kN/m^3}$$

$\therefore$ $$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.67 \times 9.81}{16.786} - 1 = \mathbf{0.56}$$

and $$S = \frac{wG}{e} = \frac{0.12 \times 2.67}{0.56} = 0.5721 = \mathbf{57.21\%}$$

Example 2.34. *The total unit weight of the glacial outwash soil is 16 kN/m^3. The specific gravity of soil particles of the soil is 2.67. The water content of the soil is 17%. Calculate:*

(a) Dry unit weight *(b) Porosity*
(c) Void ratio *(d) Degree of saturation*

(Gate Exam. 1998)

Solution. $\gamma_t (= \gamma) = 16$ kN/m^3 ; $G = 2.67$; $w = 17\%$

(a) $\gamma_d = \frac{\gamma}{1+w} = \frac{16}{1+0.17} = \mathbf{13.675\ kN/m^3}$;

(b) $e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.67 \times 9.81}{13.675} - 1 = \mathbf{0.9153}$

(c) $n = \frac{e}{1+e} = \frac{0.9153}{1+0.9153} = 0.4779 = \mathbf{47.79\%;}$

(d) $S = \frac{wG}{e} = \frac{0.17 \times 2.67}{0.9153} = 0.4959 = \mathbf{49.59\%.}$

Example 2.35. *Soil has been compacted in an embankment at a bulk density of 2.15 mg/m^3 and water content of 12%. The value of specific gravity of soil solids is 2.65. The water table is well below the foundation level. Estimate the dry density, void ratio, degree of saturation and air constent of compacted soil.*

(Gate Exam. 2002)

Solution. Given: $\rho = 2.15$ mg/m^3 $= 2.15$ g/cm^3 ; $w = 12\%$; $G = 2.65$

$$\rho_d = \frac{\rho}{1+w} = \frac{2.15}{1+0.12} = \mathbf{1.92\ g/cm^3;}$$

$$e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.65 \times 1}{1.92} - 1 = \mathbf{0.38}$$

$$S = \frac{wG}{e} = \frac{0.12 \times 2.65}{0.38} = 0.8368 = \mathbf{83.68\%\ ;}$$

$$a_c = 1 - S = 1 - 0.8368 = 0.1632 = \mathbf{16.32\%}$$

PROBLEMS

1. The natural bulk unit weight of a sandy stratum is 18.54 kN/m^3 and it has a water content of 8%. For determining the density index, dried sand from the stratum was first filled loosely in a 300 cm^3 mould and then vibrated to give a maximum density. The loose dry mass in the mould was 480 g and the dense dry mass at the maximum compaction was 570 g. If the specific gravity of solids is 2.66, find the density index of the sand in the stratum. [**Ans.** $I_D = 0.54$]

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2. A saturated specimen of undisturbed inorganic clay has a volume of 19.2 cm^3 and mass 32.5 g. After oven-drying at 105°C for 24 hours, the mass reduces to 20.9 g. For the soil in the natural state, find (*i*) w, (*ii*) G, (*iii*) e, (*iv*) γ_{sat} and (*v*) γ_d. [**Ans.** (*i*) 55.4%, (*ii*) 2.76, (*iii*) 1.53, (*iv*) 16.68 kN/m^3, (*v*) 10.69 kN/m^3]
3. In a Jodhpur-Mini-Compactor test, 612 g of wet soil occupies a volume of 300 cm^3 at a moisture content of 12.6%. Determine (*i*) γ, (*ii*) γ_d, (*iii*) e, (*iv*) n and (*v*) S in the compacted soil if the specific gravity of soil solids is 2.68. [**Ans.** (*i*) 20.01 kN/m^3, (*ii*) 17.76 kN/m^3, (*iii*) 0.48, (*iv*) 32.4% and (*v*) 70.3%]
4. A moist sample of soil has a mass of 633 g and a volume of 300 cm^3 at a water content of 11%. Taking $G = 2.68$, determine e, S and n_a. Also determine the water content at which the soil gets fully saturated without any increase in the volume. What will be the unit weight at saturation?
[**Ans.** $e = 0.41$; $S = 71.9\%$; $n_a = 8.2\%$; $w_{sat} = 15.3\%$; $\gamma_{sat} = 12.48$ kN/m^3]
5. A compacted sample of soil with a bulk unit weight of 19.62 kN/m^3 has a water content of 15%. What are its dry density, degree of saturation and air-content? Assume $G = 2.65$.
[**Ans.** $\gamma_d = 17.07$ kN/m^3 , $S = 76\%$; $a_c = 24\%$]
6. A saturated sample of soil has a water content of 35%. Adopting $G = 2.70$, calculate γ_d, γ_{sat} and γ'.
[**Ans.** $\gamma_d = 13.64$ kN/m^3, $\gamma_{sat} = 18.44$ kN/m^3, $\gamma' = 8.63$ kN/m^3]
7. A fully saturated clay sample has mass of 101.5 g and a volume of 50 cm^3. After oven drying, the clay has mass of 84.5 g. Assuming that the volume does not change during drying, determine (*i*) G, (*ii*) e, (*iii*) n and (*iv*) γ_d. [**Ans.** (*i*) $G = 2.55$, (*ii*) $e = 0.51$, (*iii*) $n = 33.8\%$, (*iv*) $\gamma_d = 16.58$ kN/m^3]
8. By three phase soil system show that the degree of saturation S (as ratio) in terms of mass unit weight γ, water content w (as ratio), specific gravity of soil grains G, and unit weight of water γ_w, is given by the expression

$$S = \frac{w}{\frac{\gamma_w}{\gamma}(1+w) - \frac{1}{G}}.$$

9. An imaginary soil mass is contained in a container measuring 10 cm × 10 cm × 10 cm. The soil consists of spherical grains of size 1 cm in diameter. Determine the maximum possible voids ratio, porosity and per cent solids. [**Ans.** 0.91 ; 47.6% ; 52.4%]

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CHAPTER 3

Determination of Index Properties

3.1 GENERAL

In this chapter, we shall describe the methods of determining those properties of soils which are used in their identification and classification. These include the determination of : (*i*) water content, (*ii*) specific gravity, (*iii*) particle size distribution, (*iv*) consistency limits, (*v*) in situ-density, and (*vi*) density index. These properties are known as *index properties*.

3.2 WATER CONTENT

The water content of a soil sample can be determined by the following methods:

1. Oven drying method
2. Sand bath method
3. Alcohol method
4. Calcium carbide method
5. Pycnometer method
6. Radiation method
7. Torsion balance method.

1. Oven drying method. This is the most accurate method of determining the water content, and is, therefore, used in the laboratory. A specimen of soil sample is kept in a clean container and put in a thermostatically controlled oven with interior of non-corroding material to maintain the temperature between 105°C to 110°C. For complete drying, sandy soils take about four hours and fat clays take about 14 to 16 hours. Usually the sample is kept for about 24 hours in the oven so that complete drying is assured. A temperature higher than 110°C may break the crystalline structure of clay particles resulting in the loss of chemically bound structural water. For highly organic soils, such as peat, a lower temperature

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of about 60°C is preferable to prevent the oxidation of the organic matter. Certain soils contain gypsum which on heating looses its water of crystallisation. If it is suspected that gypsum is present in the soil, the sample is dried at not more than 80°C but for a longer time (IS : 2720 Part II-1969).

A clean non-corrodible container is taken and its mass is found with its lid, on a balance accurate to 0.01 g. A specimen of the moist soil is placed in the container and the lid is replaced. The mass of the container and the contents is determined. With the lid removed, the container is then placed in the oven for drying. After drying, the container is removed from the oven and allowed to cool in a desiccator. The lid is then replaced, and the mass of container and the dry soil is found. The water content is calculated from the following expression:

$$w = \frac{M_2 - M_3}{M_3 - M_1} \times 100 \text{ (percent)} \qquad ...(3.1)$$

where, M_1 = mass of container with lid

M_2 = mass of container with lid and wet soil

M_3 = mass of container with lid and dry soil.

The observations are tabulated as illustrated in Table 3.7, Experiment 1.

2. Sand bath method. This is a field method of determining rough value of the water content, where the facility of an oven is not available. The container with the soil is placed on a sand bath. The sand bath is heated over a kerosene stove. The soil becomes dry within 1/2 to 1 hour. The water content is then determined from equation 3.1. However, higher temperature may break the crystalline structure of soils. This method should not be used for organic soils, or for soils having higher percentage of gypsum (B.S. 1377 : 1961).

3. Alcohol method. This is also a crude field method. The wet soil sample is kept in a evaporating dish and mixed with sufficient quantity of methylated spirit. The dish is then properly covered and the mixture is ignited. The mixture is kept stirred by a wire during ignition. Since there is no control over the temperature it should not be used for soils containing large percentage of organic matter or gypsum. The water content is determined from the expression:

$$w = \frac{M_2 - M_3}{M_3 - M_1} \times 100 \text{ where, } M_1 = \text{mass of empty dish}$$

M_2 = mass of dish + wet soil; M_3 = mass of dish + dry soil.

4. Calcium carbide method. In this method, 6 g of wet soil sample is placed in an airtight container (called moisture tester) and is mixed with sufficient quantity of fresh calcium carbide powder. The mixture is shaken vigorously. The acetylene gas, produced by the reaction of the moisture of the soil and the calcium carbide, exerts pressure on a sensitive diaphragm placed at the end of the container. The dial gauge located at the diaphragm reads the water content directly. However, the calibration of the dial gauge is such that is gives the water content (w') based on the wet weight of the sample. Knowing the water content w' based on wet weight, the water content (w) based on dry weight can be found from equation 2.1(d):

$$w = \frac{w'}{1 - w'}$$

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The method is very quick—the result can be obtained in 5 to 10 minutes. The field kit contains the moisture tester, a small single-pan weighing balance, a bottle containing calcium carbide and a brush. A big container permits the use of 26 g sample (Blysten : 1961). In order that balls, having wet soil inside and dry soil outside, may not form during the reaction with calcium carbide, the soil may be mixed with perfectly dry sand.

In the larger container, two 30 mm diameter balls are placed for proper pulverisation of clay soils. This method is specially suited to a circumstance where water content is to be quickly determined for the purpose of proper field control, such as in the compaction of an embankment.

5. Pycnometer method. This is also a quick method of determining the water content of those soils whose specific gravity G is accurately known. Pycnometer (Fig. 3.1a) is a large size density bottle of about 900 mL capacity. A conical brass cap, having a 6 mm diameter hole at its top is screwed to the open end of the pycnometer. A rubber washer is placed between conical cap and the rim of the bottle so that there is no leakage of water.

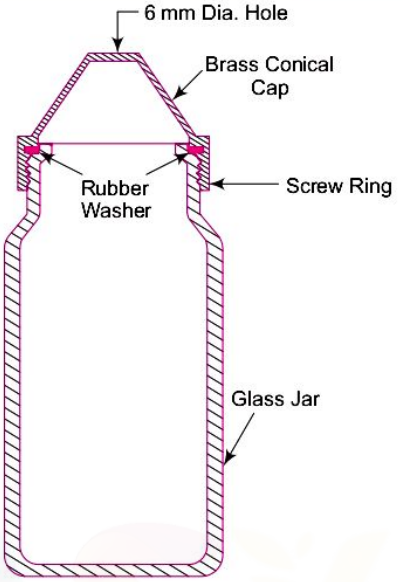

Fig. 3.1 (a) Pycnometer

Test procedure:

1. Take a clean, dry pycnometer, and find its mass (M_1) with its cap and washer.
2. Put about 200 g to 400 g of wet soil sample in the pycnometer and find its mass (M_2) with its cap and washer.
3. Fill the pycnometer to half its height and mix it thoroughly with the glass rod. Add more water, and stir it. Replace the screw top and fill the pycnometer flush with the hole in the conical cap. Dry the pycnometer from outside, and find its mass (M_3).
4. Empty the pycnometer, clean it thoroughly, and fill it with clean water to the hole of the conical cap, and find its mass (M_4).

The water content is then calculated from the following expression:

$$w = \left[\left(\frac{M_2 - M_1}{M_3 - M_4}\right)\left(\frac{G-1}{G}\right) - 1\right] \times 100 \qquad ...(3.2)$$

The above expression can be derived with reference to Fig. 3.2.

In Fig. 3.1(b) (iii), if M_d is the mass of soil particles, the volume of solid particles will be equal to M_d/G. Thus, if the solids from Fig. 3.1(b) (iii) are replaced with water of mass M_d/G, we get the mass M_4 indicated in Fig. 3.1(b) (iv). Thus,

$$M_4 = M_3 - M_d + \frac{M_d}{G} \quad \text{or} \quad M_d\left(\frac{G-1}{G}\right) = M_3 - M_4$$

From which,
$$M_d = (M_3 - M_4)\,\frac{G}{G-1} \qquad ...(i)$$

Now mass of water M_w in the wet soil sample = $(M_2 - M_1) - M_d$

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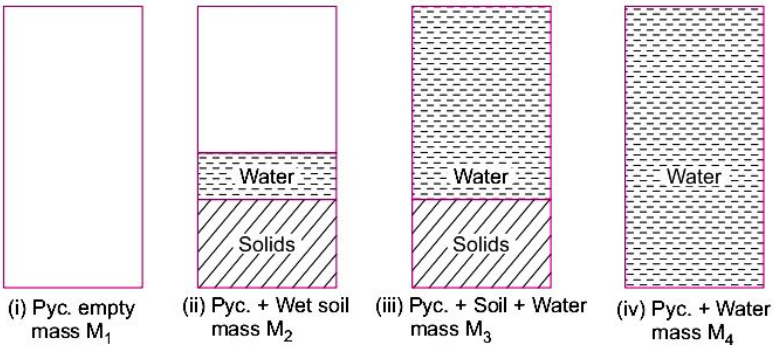

Fig. 3.1 (b) Water content determination

$$\therefore \quad w = \frac{M_w}{M_d} \times 100 = \frac{M_2 - M_1 - M_d}{M_d} \times 100$$

$$= \left(\frac{M_2 - M_1}{M_d} - 1\right) 100 = \left[\left(\frac{M_2 - M_1}{M_3 - M_4}\right)\left(\frac{G-1}{G}\right) - 1\right] \times 100 \quad ...(3.2a)$$

This method is suitable for coarse grained soils only.

6. Radiation method. This method is extremely useful for the determination of water content of soil deposit in the in situ condition. It uses two steel casings—casing *A* and casing *B* which are placed in two bore holes at some distance apart, in the soil deposit the field moisture content of which is to be determined. A device containing some radioactive isotope material (such as cobalt – 60) is placed in a capsule which in turn is lowered into casing *A*.

Similarly, a detector unit is lowered in steel casing *B*. Small openings are made in both casings *A* and *B*, facing each other. When the radioactive device is activated, it emits neutrons. When these neutrons strike with the hydrogen atoms of water in the sub-soil, they loose energy. The loss of energy is evidently equal to water content in the soil. The detector device is calibrated to given directly the water content of the subsoil, at that level of emission. However, proper shielding precautions should be taken to avoid radiation problems.

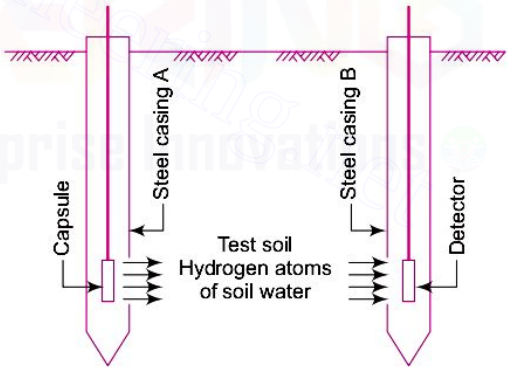

Fig. 3.2 Radiation method

7. Torsion balance method (IS : 2720, Part II–1973). The equipment has two main parts: (*i*) *infrared* lamp, and (*ii*) torsion balance. The infrared radiation is provided by 250 watt lamp built in the balance for use with alternating current 220–230 V, 50 cycle, single phase mains supply. The weighing mechanism, a torsion balance, has a built-in magnetic damper to reduce pan vibrations during quick drying. The balance scale is divided in terms of water percentages from 1 to 100 water content in 0.2 percent division. The moisture meter is generally calibrated to use 25 g of soil, and hence the maximum size of particle present in the specimen should be less than 2 mm.

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The test specimen is kept in a suitable container so that the water content to be determined is not affected by ambient conditions. Torque is applied to one end of the torsion wire by means of a calibrated drum to balance the loss of weight of the sample as it dries out under infrared lamp. To determine the percent reduction of mass at any instant, rotate the drum scale by turning the drum drive knob until the point returns to the index. The percent is read directly from the scale. However, this per cent (w') is the percent of water based upon the initial mass (*i.e.*, wet mass) of the sample. The water content (w) based on the dry mass is computed from equation 2.1 (*d*).

$$w = \frac{w'}{1 - w'}$$

Provision is made to adjust the input voltage to the infrared lamp to control the heat for drying the specimen. A suitable thermometer graduated from 40°C to 150°C is provided for ascertaining the temperature of drying in the pan housing. Normally, the temperature is kept between 110°C ± 5°C. The time required for the test depends upon the type of soil and the quantity of water present; the normal time varies between 15 to 30 minutes. Drying and weighing occur simultaneously, and hence the method is specially suitable for those soils which re-absorb moisture quickly on drying. The criterion for taking the *final reading* is that the pointer should remain steady on index mark which shows that the sample has dried to constant mass.

3.3 SPECIFIC GRAVITY

The specific gravity of soil solids is determined by: (*i*) a 50 mL density bottle, or (*ii*) a 500 mL flask, or (*iii*) a pycnometer. The density bottle method is the most accurate, and is suitable for all types of soils. The flask or pycnometer is used only for coarse grained soils. The density bottle method is the standard method used in the laboratory.

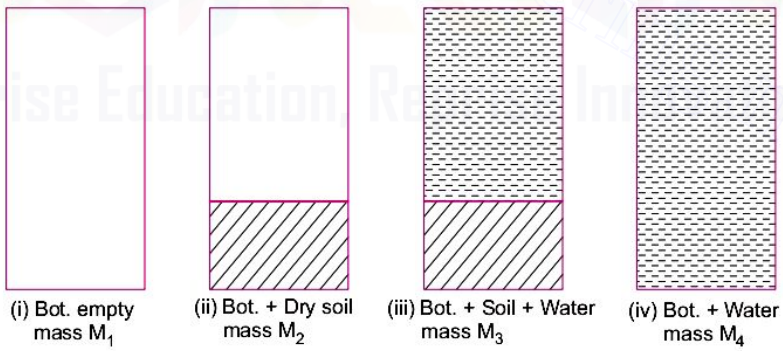

Fig. 3.3 Specific gravity computation

However, in all the three methods, the sequence of observations is the same. The mass M_1 of the empty, dry, bottle (or flask or pycnometer) is first taken. A sample of oven-dried soil, cooled in a desiccator, is put in the bottle, and the mass M_2 is taken. The bottle is then filled with distilled water (or kerosene) gradually, removing the entrapped air either by applying vacuum or by shaking the bottle. The mass M_3 of the bottle, soil and water (full up to the top) is taken. Finally, the bottle is emptied completely and thoroughly washed, and clean water (or kerosene) is filled to the top, and the mass M_4 is taken. Based on these four observations, the specific gravity can be computed as follows (Fig. 3.3).

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From Figs. 3.3 (*i*) and (*ii*), dry mass M_d of the soil is $M_d = M_2 - M_1$ ...(*a*)

Mass of water in Fig. 3.3 (*iii*) = $M_3 - M_2$; Mass of water in Fig. 3.3 (*iv*) = $M_4 - M_1$

Hence, mass of water having the same volume as that of soil solids is

$$= (M_4 - M_1) - (M_3 - M_2)$$

Now,
$$G = \frac{\text{Dry mass of soil}}{\text{Mass of water of equal volume}} = \frac{M_2 - M_1}{(M_4 - M_1) - (M_3 - M_2)}$$

or
$$G = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)} = \frac{M_d}{M_d - (M_3 - M_4)} \quad ...(3.3)$$

Alternative Proof. If the soil solids are removed from M_3 [Fig. 3.3 (*iii*)] and replaced by water of equal volume, M_4 is obtained.

Volume of solids in Fig. 3.3 (*iii*) = $\frac{M_d}{G}$

$\therefore$
$$M_4 = M_3 - M_d + \frac{M_d}{G}$$

or
$$G = \frac{M_d}{M_d - (M_3 - M_4)} = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

When a 500 mL flask is used, mass is taken to an accuracy of 0.1 g. In case of pycnometer, mass measurements are taken to 1 g accuracy. In both the cases, the entrapped air is expelled by stirring and distilled water is used. However, in the density bottle method, mass measurements are taken to an accuracy of 0.001 to 0.005 g and kerosene is used since it is a better wetting agent. In this case, specific gravity of soil solids is given by

$$G = \frac{M_d \cdot G_k}{M_d - (M_3 - M_4)} \quad ...(3.3a)$$

where G_k is the specific gravity of kerosene at the test temperature T_t°C.

When specific gravity of a cohesive soil is determined, the density bottle with the dry soil and kerosene covering the soil is placed in a vacuum desiccator, and vacuum is gradually applied. A vacuum of 65 cm is maintained, usually for 16 to 24 hours, till no more air is released from the soil. As far as possible, de-aired kerosene should be used. The bottle is then removed from the desiccator and filled with de-aired kerosene and the stopper is inserted. The bottle is then inserted in a constant temperature (27°C) water bath for about half an hour. When the bottle has reached the constant temperature, a few drops of kerosene are added to the bottle (if the room temperature is more than 27°C) or the extraneous kerosene exuded from the capillary tube is mopped up using a filter paper (if the room temperature is less than 27°C) as the case may be. The bottle is then wiped dry from outside and weighed. Similarly, before taking mass M_4, the bottle full of kerosene is kept in the constant temperature bath and then weighed. For accurate determination, three sets of observations are taken and the mean taken to get the final value of G.

If, however, the test is conducted on any other temperature T_t°C (*i.e.*, room temperature in the case of flask or pycnometer), the specific gravity, to be reported at 27°C, is found as under.

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$$G \text{ (at } 27°C) = G \text{ (at } T_t°C) \frac{\text{Sp. gr. of water at } T_t°C}{\text{Sp. gr. of water at } 27°C} \quad ...(3.4)$$

Distilled water is used in the flask or pycnometer test, and kerosene is used for the density bottle method. When kerosene is used, its specific gravity is determined accurately by a separate test. The specific gravity of each consignment of kerosene is determined and is repeated monthly or biweekly if the same consignment is still in use.

Table 3.8 shows the observation sheet for the specific gravity determination (See Experiment 2).

SOLVED EXAMPLES

Example 3.1. *In order to determine the water content, 370 g of a wet sandy sample was placed in a pycnometer. The mass of the pycnometer, sand and water full to the top of the conical cap was found to be 2148 g. The mass of pycnometer full of clean water was 1932 g. Taking G = 2.65, determine the water content of the sample.*

Solution.

From Eq. 3.2, $$w = \left[\left(\frac{M_2 - M_1}{M_3 - M_4}\right)\left(\frac{G-1}{G}\right) - 1\right] \times 100 = \left[\left(\frac{M}{M_3 - M_4}\right)\left(\frac{G-1}{G}\right) - 1\right] \times 100$$

where M = wet mass of soil = 370 g; M_3 = 2148 g; M_4 = 1932 g

$$\therefore \quad w = \left[\left(\frac{370}{2148 - 1932}\right) \times \left(\frac{2.65 - 1}{2.65}\right) - 1\right] \times 100 = \mathbf{6.5\%}.$$

Example 3.2. *An oven dried soil having a mass of 200 g is placed in a pycnometer which is then completely filled with water. The total mass of the pycnometer with water and soil inside is 1605 g. The pycnometer filled with water alone has a mass of 1480 g. Calculate the specific gravity of the soil.*

Solution. Given: M_d = 200 g; M_3 = 1605 g; M_4 = 1480 g

$$\therefore \quad G = \frac{M_d}{M_d - (M_3 - M_4)} = \frac{200}{200 - (1605 - 1480)} = \mathbf{2.67}.$$

Example 3.3. *The specific gravity of a soil sample was found with the help of a 50 mL density bottle using kerosene. The mass of empty bottle was found to be 62.12 g. A sample of oven dried clay was placed in the bottle and the total mass of bottle and clay was found to be 83.49 g. The mass of bottle, clay and kerosene filled up to the top of the capillary tube of the stopper was found to be 264.41 g. Finally, the bottle full of clean kerosene only has mass of 249.24 g. Every time the temperature of the contents of the bottle was maintained at 27°C by placing the bottle in a thermostatically controlled water bath, before weighing. In a separate test, the specific gravity of kerosene was found to be 0.773 at 27°C. What is the specific gravity of soil? If the specific gravity is to be reported at 4°C, what will be its value?*

Solution.

$$G = \frac{M_d \times G_k}{M_d - (M_3 - M_4)}$$

Here, M_d = 83.49 – 62.12 = 21.37 g; M_3 = 264.41 g; M_4 = 249.24 g

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$$M_3 - M_4 = 264.41 - 249.24 = 15.17 \text{ g}$$

$$\therefore \quad G = \frac{21.37 \times 0.773}{21.37 - 15.17} = \frac{21.37 \times 0.773}{6.2} = \mathbf{2.66}$$

Now, $$G \text{ at } 4°\text{C} = G \text{ at } 27°\text{C} \frac{\text{Sp. gr. of water at } 27°\text{C}}{\text{Sp. gr. of water at } 4°\text{C}} = 2.66 \times \frac{0.9965}{1.000} = \mathbf{2.65.}$$

3.4 PARTICLE SIZE DISTRIBUTION

The percentage of various sizes of particles in a given dry soil sample is found by a particle size analysis or *mechanical analysis*. By mechanical analysis is meant the separation of a soil into its different size fractions. The mechanical analysis is performed in two stages: (*i*) sieve analysis, and (*ii*) sedimentation analysis or wet mechanical analysis.

The first stage is meant for coarse-grained soils only, while the second stage is performed for fine-grained soils. In general, a soil sample may contain both coarse-grained particles as well as fine particles, and hence both the stages of the mechanical analysis may be necessary. The sieve analysis is, however, the true representative of grain size distribution, since the test is not affected by temperature, etc.

3.5 SIEVE ANALYSIS

In the BS and ASTM standards, the sieve sizes are given in terms of the number of openings per inch. The number of openings per square inch is equal to the square of the number of the sieve. In the Indian Standard (IS : 460–1962), the sieves are designated by the size of the aperture in mm. Table 3.1 gives a list of sieves and their openings, for IS, BS and ASTM specifications.

The complete sieve analysis can be divided into two parts—the *coarse analysis* and *fine analysis*. An oven dried sample of soil is separated into two fractions by sieving it through a 4.75 mm IS sieve. The portion retained on it (+ 4.75 mm size) is termed as the *gravel fraction* and is kept for the coarse analysis, while the portion passing through it (– 4.75 mm size) is subjected to fine sieve analysis. The following set of sieves are used for coarse sieve analysis: IS : 100, 63, 20, 10 and 4.75 mm. The sieves used for fine sieve analysis are : 2 mm, 1.0 mm, 600, 425, 300, 212, 150 and 75 micron IS sieves.

Sieving is performed by arranging the various sieves one over the other in the order of their mesh openings—the largest aperture sieve being kept at the top and the smallest aperture sieve at the bottom. A receiver is kept at the bottom and a cover is kept at the top of the whole assembly. The soil sample is put on the top sieve, and the whole assembly is fitted on a sieve shaking machine. The amount of shaking depends upon the shape and the number of particles. At least 10 minutes of shaking is desirable for soils with small particles. The portion of the soil sample retained on each sieve is weighed. The percentage of soil retained on each sieve is calculated on the basis of the total mass of soil sample taken and from these results, percentage passing through each sieve is calculated. Table 3.14 shows the specimen observation and calculation sheet.

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Table 3.1 Sieves designation and their sizes

IS Sieves IS: 460–1962		*BS Sieves BS: 410–1962*		*ASTM Sieves ASTM E 11–1961*	
Designation	*Aperture (mm)*	*Designation*	*Aperture (mm)*	*Designation*	*Aperture (mm)*
50 mm	50.00	2 in.	50.80	2 in.	50.80
40 mm	40.00	$1\frac{1}{2}$ in	38.11	$1\frac{1}{2}$ in	38.10
20 mm	20.00	3/4 in	19.05	3/4 in	19.00
10 mm	10.00	3/8 in	9.52	3/8 in	9.51
*5.60 mm	5.60	–	–	$3\frac{1}{2}$	5.66
4.75 mm	4.75	3/16-in	4.76	4	4.76
*4.00 mm	4.00	–	–	5	4.00
*2.80 mm	2.80	6	2.80	7	2.83
2.36 mm	2.36	7	2.40	8	2.38
*2.00 mm	2.00	8	2.00	10	2.00
*1.40 mm	1.40	12	1.40	14	1.41
1.18 mm	1.18	14	1.20	16	1.19
*1.00 mm	1.00	16	1.00	18	1.00
710 micron	0.710	22	0.710	25	0.707
600 micron	0.600	25	0.600	30	0.595
*500 micron	0.500	30	0.500	35	0.500
425 micron	0.425	36	0.420	40	0.420
*355 micron	0.355	44	0.355	45	0.354
300 micron	0.300	52	0.300	50	0.297
250 micron	0.250	60	0.250	60	0.250
212 micron	0.212	72	0.210	70	0.210
*180 micron	0.180	85	0.180	80	0.177
150 micron	0.150	100	0.150	100	0.149
*125 micron	0.125	120	0.125	120	0.125
*90 micron	0.090	170	0.090	170	0.088
75 micron	0.075	200	0.075	200	0.074
*63 micron	0.063	240	0.063	230	0.063
*45 micron	0.045	350	0.045	325	0.044

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*Sieves marked with *have been proposed as an International (ISO) Standard.
It is recommended to include, if possible, these sieves in all sieve analysis data or reports.

It is advisable to wash the soil portion passing through 4.75 mm sieve over 75 micron sieve so that silt and clay particles sticking to the sand particles may be dislodged. Two grams of sodium hexametaphosphate is added per litre of water used. Washing should be continued until the water passing through 75 micron

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sieve is substantially clean. The fraction retained on the 75 micron sieve is dried in the oven. The dried portion is then re-sieved through 2 mm, 1 mm, 600, 425, 300, 212, 150 and 75 micron IS sieves. The portion passing 75 micron sieve (while washing) is also dried separately and its mass determined to get % finer than 75 micron size. If the portion passing 75 micron size is substantial, wet analysis is done for further sub-division of particle size distribution. [See Experiment 8].

3.6 SEDIMENTATION ANALYSIS: THEORY

In the wet mechanical analysis, or sedimentation analysis, the soil fraction, finer than 75 micron size is kept in suspension in a liquid (usually, water) medium. The analysis is based on Stokes' law, according to which the velocity at which grains settle out of suspension, all other factors being equal, is dependent upon the shape, weight and size of the grain. However, in the usual analysis it is assumed that the soil particles are spherical and have the same specific gravity (the average specific gravity). With this assumption, the coarser particles settle more quickly than the finer ones. If v is the terminal velocity of sinking of a spherical particle, it is given by

$$v = \frac{2}{9} r^2 \frac{\gamma_s - \gamma_w}{\eta} \qquad ...(3.5)$$

or

$$v = \frac{1}{18} D^2 \frac{\gamma_s - \gamma_w}{\eta} \qquad ...(3.5a)$$

where, r = radius of the spherical particle (m)

D = diameter of the spherical particle (m)

v = terminal velocity (m/sec)

γ_s = unit weight of particles (kN/m^3)

γ_w = unit weight of water/liquid (kN/m^3)

η = viscosity of water/liquid (kN-s/m^2) = $\frac{\mu}{g}$

μ = viscosity in absolute units of poise

g = acceleration due to gravity

If water is used as the medium for suspension, γ_w is equal to 9.81 kN/m^3. Similarly, $\gamma_s = G\,\gamma_w$. Substituting this, we get

$$v = \frac{1}{18} D^2 \frac{(G-1)\gamma_w}{\eta} \qquad ...(3.5b)$$

The above formula should be expressed in the consistent units of metres, seconds and kilo-newton. If the diameter (D) of the particles is in mm, we have

$$v = \frac{1}{18}\left(\frac{D}{1000}\right)^2 \cdot \frac{(G-1)\gamma_w}{\eta} = \frac{D^2 \cdot \gamma_w (G-1)}{18 \times 10^6\, \eta} \qquad ...(3.5c)$$

Taking $\gamma_w = 9.81$ kN/m^3, we get $v = \dfrac{D^2 (G-1)}{1.835 \times 10^6\, \eta}$ ...(3.6)

$$\therefore \quad D = \sqrt{\frac{18 \times 10^6\, \eta\, v}{(G-1)\gamma_w}} \text{ mm} \qquad ...(3.7)$$

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or $$D = 1355\sqrt{\frac{\eta v}{G-1}} \text{ mm} \qquad ...(3.7a)$$

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It should be noted that 1 poise is equivalent to 0.1 N – s/m^2 or to 10^{-4} kN - s/m^2.

If a particle of diameter D mm falls through a height H_e cm in t minutes,

$$v = \frac{H_e}{60t} \text{ cm/sec} = \frac{H_e}{6000t} \text{ m/sec}$$

Substituting in Eq. 3.7, we get

Table 3.2 Values of factor F
(After calculations by the author)

Temp °C	m (Poise)	Factor F					
		G = 2.50	G = 2.60	G = 2.65	G = 2.70	G = 2.75	G = 2.80
15	0.01145	1528	1479	1458	1435	1414	1395
16	0.01116	1508	1460	1438	1417	1396	1377
17	0.01088	1490	1442	1420	1399	1379	1360
18	0.01060	1470	1423	1402	1381	1361	1342
19	0.01034	1452	1406	1384	1364	1344	1325
20	0.01009	1434	1389	1367	1347	1328	1309
21	0.00984	1417	1372	1351	1331	1311	1294
22	0.00961	1400	1355	1335	1315	1296	1278
23	0.00938	1383	1339	1318	1291	1280	1262
24	0.00916	1367	1323	1305	1284	1265	1248
25	0.00896	1351	1308	1288	1269	1251	1233
26	0.00875	1335	1293	1273	1254	1236	1219
27	0.00855	1321	1279	1259	1241	1224	1206
28	0.00836	1305	1264	1244	1226	1208	1191
29	0.00818	1291	1250	1231	1213	1195	1178
30	0.00800	1277	1236	1217	1199	1182	1165
31	0.00783	1264	1224	1205	1187	1170	1154
32	0.00767	1251	1211	1193	1175	1158	1142
33	0.00751	1238	1199	1180	1163	1146	1130
34	0.00736	1225	1186	1168	1151	1134	1118
35	0.00721	1212	1174	1156	1139	1120	1107
36	0.00706	1199	1161	1144	1127	1110	1095
37	0.00692	1188	1150	1133	1116	1100	1084
38	0.00679	1176	1139	1122	1105	1089	1074
39	0.00666	1165	1128	1110	1094	1079	1064
40	0.00654	1155	1118	1101	1084	1069	1055

Note. For intermediate values, interpolation may be done.

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$$D = \sqrt{\frac{18\times10^{6}\,\eta}{(G-1)\gamma_w}\times\frac{H_e}{6000t}} = \sqrt{\frac{3000\,\eta}{(G-1)\gamma_w}}\cdot\sqrt{\frac{H_e}{t}} \qquad ...(3.8)$$

or

$$D = 10^{-5}\,F\sqrt{\frac{H_e}{t}} \qquad ...(3.9)$$

where $F = 10^{5}\sqrt{\dfrac{3000\,\eta}{(G-1)\gamma_w}}$ is a constant factor for given values of η and G.

However, η depends upon the temperature (see Table 3.2) and hence the factor F varies with the temperature which changes during the total time of testing. Table 3.2 gives the values of factor F for various values of G and temperature. Also, Fig. 3.4 gives the curves for factor F.

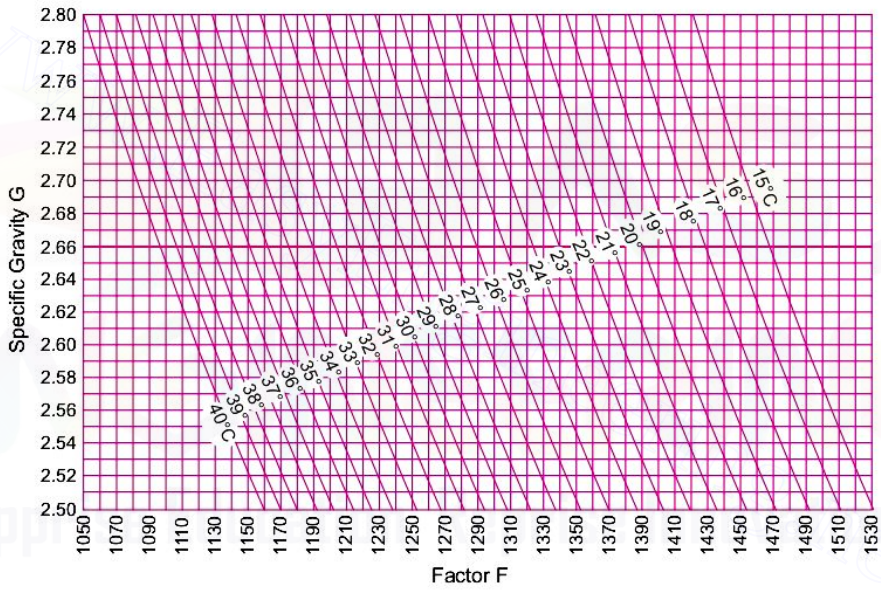

Fig. 3.4 Values of factor F (after calculation by the author)

At 27 °C, the viscosity μ of distilled water is approximately 0.00855 poise. Since 1 poise is equivalent to 10^{-4} kN-s/m^2, we have

$$\eta = 0.00855\times10^{-4}\ \text{kN-s/m}^2$$

Taking an average value of $G = 2.68$, we get from Eq. 3.5 (c), we get

$$v = \frac{D^2\,(9.81)(2.68-1)}{18\times10^{6}\times0.0085\times10^{-4}} \approx 1.077\,D^2\ (\text{m/sec}) \qquad ...(3.10)$$

Equation 3.10 is an approximate version of Stoke's law and can be easily remembered for rough determinations. Based on this, the time of settlement of particles of various diameters, through a height of 10 cm are as under:

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Diameter (mm)	*Time*
0.06	25.8 s
0.02	3 m 52 s
0.01	15 m 28 s
0.006	42 m 59 s
0.004	1 h 36 m 43 s
0.002	6 h 26 m 53 s
0.001	25 h 47 m 31 s

The sedimentation analysis is done either with the help of a hydrometer or a pipette. In both the methods, a suitable amount of oven-dried soil sample, finer than 75 micron size, is mixed with a given volume V of distilled water. The mixture is shaken thoroughly and the test is started by keeping the jar, containing soil water mixture, vertical. At the commencement of sedimentation test, soil particles are assumed to be uniformly distributed throughout the suspension. After any time interval t, if a sample of soil suspension is taken from a height H_e (measured from the top level of suspension), only those particles will remain in the suspension which have not settled during this time interval. The diameter of those particles, which are finer than those which have already settled, can be found from Eq. 3.9. The greater the time interval t allowed for suspension to settle, the finer are the particles sizes retained at this depth H_e. Hence sampling at different time intervals, at this sampling depth H_e, would give the content of particles of different sizes.

If, at any time interval t, M_D is the mass, per mL, of all particles smaller than the diameter D (determined from equation 3.9) still in suspension at the depth H_e the percentage finer than D is given by

$$N = \frac{M_D}{M_d / V} \times 1000 \qquad ...(3.11)$$

where, N = percentage finer than the diameter D

M_d = total dry mass of all particles put in the suspension

V = volume of suspension.

Thus, with the help of equations 3.9 and 3.11, we can get various diameter D and the percentage of particles finer (N%) than this diameter.

Limitations of sedimentation analysis. The analysis is based on the assumptions that (*i*) soil particles are spherical, (*ii*) particles settle independent of other particles and the neighbouring particles do not have any effect on its velocity of settlement, and (*iii*) the walls of jar, in which the suspension is kept, also do not affect the settlement. In actual practice, the fine particles of soil, for which this analysis is primarily meant, are not truly spherical. The particles of fine grained soils are thin platelets which do not settle out of suspension in the same manner and at the same rate as smooth spheres. Thus, the sedimentation analysis gives the particles size *equivalent diameter*. The upper limit of particle size for the validity of the law is about 0.2 mm (Taylor : 1948) beyond which the liquid tends to develop a turbulent motion at the boundaries of the particles. The lower limit of particle size is about 0.0002 mm. For particle smaller than 0.0002 mm equivalent diameter, Brownian movement affects their settlement, and Stoke's law no longer remains valid. Also it is assumed that the soil has an average specific gravity, the value of which is used in computing the diameter D. Actually, different particles may have different specific gravity, depending upon their mineral constituents. The settlement of the particles is influenced by the surrounding particles as the liquid is not of infinite extent. The particles falling near the wall of the jar are also affected.

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3.7 PIPETTE METHOD

The pipette method is the standard sedimentation method used in the laboratory. The equipment consists of a pipette, a jar and a number of sampling bottles. Generally, a boiling tube of 500 mL capacity is used in place of a jar. Figure 3.5 shows a pipette for extracting samples from the jar (or tube) from a desired depth (H_e). The pipette consists of (*i*) a 125 mL bulb with stopcock, for keeping distilled water, (*ii*) a three way stopcock, (*iii*) suction and waste water outlets, and (*iv*) sampling pipette of 10 mL capacity (including the capacity of the cock). The method consists in drawing off samples of soil suspension, 10 mL in volume, by means of this pipette from a depth of 10 cm (H_e) at various time intervals after the commencement of the sedimentation. The recommended time intervals are: 1/2, 1, 2, 4, 8, 15 and 30 minutes, and 1, 2, 4, 8, 16 and 24 hours, reckoned from the commencement of the test. The pipette should be inserted in the boiling tube about 25 seconds before the selected time interval and the time taken for sucking the sample should not be more than 10 to 20 seconds. Each sample, so taken, is transferred into suitable sampling bottles and dried in an oven. The mass M_D of solids per mL of suspension is thus found by taking the dry mass and dividing it by 10.

Method of preparing soil suspension. In the sedimentation analysis, only those particles which are finer than 75 micron size are included. Hence, the soil sample is washed through a 75 micron sieve. About 12 to 30 g of oven-drived sample (depending upon the type of soil) is accurately weighed and mixed with distilled water in a dish or beaker to form a smooth thin paste. To have proper dispersion of soil, a dispersing agent (deflocculating agent) is added to the soil. Some of the common dispersing agents are: sodium oxalate, sodium silicate and sodium polyphosphate compounds, such as tetra sodium pyrophosphate, sodium hexametaphosphate (*calgon*) and sodium tripolyphosphate. IS : 2720 (Part IV)-1965 recommends the use of dispersing solution containing 33 g of the sodium hexametaphosphate and 7 g of sodium carbonate in distilled water to make one litre of solution. 25 mL of this solution is added to the dish (containing the soil and distilled water) and the mixture is warmed gently for about 10 minutes. The contents are then transferred to the cup of a mechanical mixer, using a jet of distilled water to wash all traces of the soil out of the evaporating dish. The soil suspension is then stirred well for 15 minutes or longer in the case of highly clayey soils. The suspension is then washed through 75 micron sieve, using jet of distilled water and the suspension, which has passed through the sieve, is transferred to the 500 mL capacity boiling tube (sedimentation tube). Care should be taken that all the particles finer than 75 micron in size are transferred to the tube. The tube is then filled to the 500 mL mark, by adding distilled water. The tube is then put in a constant temperature water bath [Fig. 3.5 (*b*)], where used. When the temperature in the tube has been stabilised to the temperature of the bath, the soil suspension is thoroughly shaken by inverting the tube several times, and then replaced in the bath. The stopwatch is then started, and soil samples are collected at various time intervals, with the help of pipette.

Those soils, which contain organic matter and calcium compounds, are *pretreated* before the dispersing agent is mixed (as explained above) since these contents act as cementing agents and cause the particles to settle as aggregations of particles instead of as individuals. The process of removal of organic matter and calcium compounds is known as *pretreatment*.

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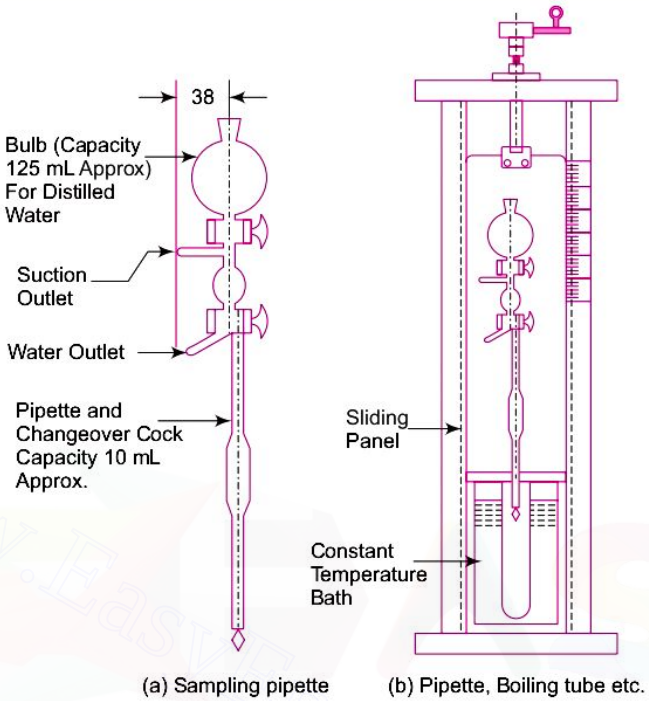

Fig. 3.5 Pipette analysis apparatus

The soil is first treated with hydrogen peroxide solution to remove the organic matter by oxidation. The mixture of soil and hydrogen peroxide is kept warm at a temperature not exceeding 60°C, till no further evolution of the gas takes place. The remaining hydrogen peroxide in the solution is then decomposed by boiling the solution. To remove the calcium compound, the cooled mixture of soil is then treated with 0.2N hydrochloric acid. When the reaction is complete, the mixture is filtered and the filtrate is washed with distilled water until it becomes free from the acid. The filtrate is then dried in the oven, to know the loss of mass due to pretreatment.

Calculation of *D* and *N*

10 mL samples are collected from the soil suspension (sedimentation tube) from a depth of 10 cm, with the help of the pipette, at various time intervals. The samples are collected into the weighing bottles (sampling bottles), and kept in the oven for drying. The mass M_D, per mL of suspension so collected is calculated as under:

$$M_D = \frac{\text{Dry mass of sample in the weighing bottle}}{V_p} \qquad ...(3.12)$$

where V_p = volume of the pipette
= volume of sample collected in the weighing bottle = 10 mL.

The percentage finer is calculated from the following expression based on equation 3.11.

$$N' = \frac{M_D - \frac{m}{V}}{M_d/V} \times 100 \qquad ...(3.13)$$

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where m = mass of dispersing agent present in the total suspension of volume V

V = volume of suspension = 500 mL

N' = percentage finer, based on M_d

If 25 mL of dispersing agent solution, containing 33 g of sodium hexametaphosphate and 7 g of sodium carbonate per litre is used, we get

$$m = \frac{33+7}{1000} \times 25 = 1 \text{ g}$$

The corresponding diameter D of the particle, to which the above percentage of soil is finer, is calculated from equation 3.9. The observation may be recorded as shown in Table 3.16. The pipette method, though very simple, requires more time, and is not suitable for routine control tests. The apparatus is very sensitive, and very accurate weighings are required. Due to these reasons, sometimes the hydrometer method of sedimentation analysis is preferred.

3.8 HYDROMETER METHOD

The hydrometer method of sedimentation analysis differs from the pipette analysis in the method of taking the observations—the principles of the test being the same in both the cases. In the pipette analysis, the mass M_D per mL of suspension is found directly by collecting a 10 mL sample of soil suspension from the sampling depth H_e. However, in the hydrometer analysis, M_D is computed indirectly by reading the density of the soil suspension at a depth H_e at various time intervals. In the pipette test, the sampling depth H_e is kept constant (= 10 cm) while in the hydrometer test, the sampling depth H_e (also known as the effective depth) goes on increasing as the particles settle with the increase in the time intervals. It is, therefore, necessary to calibrate the hydrometer and the sedimentation jar before the start of the sedimentation test, to find a relation between H_e and the density readings of the hydrometer. If the same set of hydrometer and sedimentation jar are used for a number of tests, one calibration chart will serve the purpose of the tests.

Calibration of hydrometer. Figure 3.6(a) shows the hydrometer. The readings on the hydrometer stem give the density of the soil suspension situated at the centre of the bulb at any time. For convenience, the hydrometer readings are recorded after subtracting 1 and multiplying the remaining digits by 1000. Such a *reduced reading* is designated as R_h. For example, if the density reading at the intersection of horizontal surface of soil suspension with the stem, is 1.010, it is recorded as 10 (*i.e.*, $R_h = 10$). Similarly, a density reading of 0.995 is recorded as $R_h = -5$. As indicated in Fig. 3.6(a) the hydrometer readings R_h (and also the density readings) increase in the downward direction towards the hydrometer bulb. Let H be the height, in cm, between any hydrometer reading R_h and the neck, and h the height of the bulb.

Figure 3.6(b) shows the jar, containing the soil suspension. When the hydrometer is immersed in the jar [Fig. 3.6(c)], the water level aa rises to $a_1 a_1$, the rise being equal to the volume V_h of the hydrometer divided by the internal area of cross-section A of the jar. Similarly, the level bb rises to $b_1 b_1$, where bb is the level, situated at a depth H_e below the top level aa, at which the density measurements of the soil suspension are being taken. The rise between bb and $b_1 b_1$ will be approximately equal to $V_h/2A$. The level $b_1 b_1$ is now corresponding to the centre of the bulb, but the soil particles at $b_1 b_1$ are of the same concentration as they were at bb. Therefore, we have

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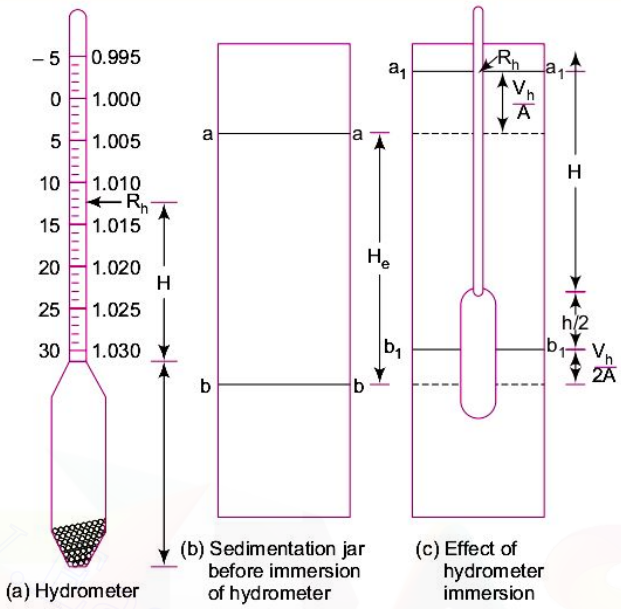

Fig. 3.6 Hydrometer analysis

$$H_e = \left(H + \frac{h}{2} + \frac{V_h}{2A}\right) - \frac{V_h}{A} = H + \frac{1}{2}\left(h - \frac{V_h}{A}\right) \qquad ...(3.14)$$

In the above expression, there are two variables: the effective depth H_e and the depth H which depends upon the hydrometer reading R_h. Therefore, by selecting various hydrometer readings R_h, the depth H can be measured with the help of an accurate scale, and the corresponding depth H_e can be found. The height h of the bulb is constant. Similarly, V_h and A are constant. To find the volume of the hydrometer, it is weighed accurately. The mass of the hydrometer in grams gives the volume of the hydrometer in millilitres. The observations are tabulated as shown in Table 3.3.

Table 3.3 Calibration of hydrometer

1. Hydrometer No. 25.
2. Volume of Hydrometer $V_h = 72$ cm^3.
3. Height of bulb (h) = 16.6 cm.

1. Sedimentation Jar No. 5.
2. Sectional Area of Jar $A = 30$ cm^2.
3. Constant $\frac{1}{2}\left(h - \frac{V_h}{A}\right) = 7.1$ cm.

Hydrometer reading R_h	*H (cm)*	*Effective depth H_e (cm)*
30	1.8	8.9
25	3.6	10.7
20	5.4	12.5
15	7.3	14.4
10	9.2	16.3
5	11.1	18.2
0	13.0	20.1
– 5	14.9	22.0

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Based on the observations of Table 3.3, a calibration curve (straight line) has been shown in Fig. 3.7. The curve is useful in finding the value of H_e for any value of hydrometer reading R_h.

Test procedure. The method of preparation of soil suspension is the same as indicated in the pipette test. However, the volume of suspension is 1000 mL in this case and hence, double the quantity of dry soil and dispersing agent is taken. The sedimentation jar (cylinder) is shaken vigorously and is then kept vertical over a solid base. The stopwatch is simultaneously started. The hydrometer is slowly inserted in the jar and readings are taken at 1/2, 1 and 2 minutes time intervals. The hydrometer is then taken out. More readings are then taken at the following time intervals: 4, 8, 15, 30 minutes and 1, 2, 4 hours etc. To take the reading, the hydrometer is inserted about 30 seconds before the given time interval, so that it is stable at the time when the reading is to be taken. Since the soil suspension is opaque, the reading is taken corresponding to the upper level of the meniscus. A suitable meniscus correction is then applied to the hydrometer readings.

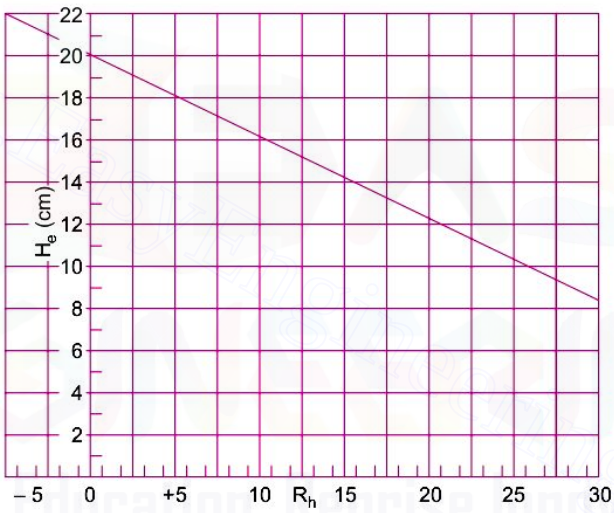

Fig. 3.7 Calibration curve for the hydrometer

Corrections to the hydrometer readings. The hydrometers are generally calibrated at 27°C. If the temperature of the soil suspension is not 27°C, a *temperature correction* C_t should be applied to the observed hydrometer reading. If the test temperature is more than 27°C, the hydrometer readings will naturally be less than what they should be, and hence the temperature correction will be *positive*. Similarly, if the test temperature is lower than 27°C, the temperature correction will be negative. In addition to this, two more corrections are to be applied to the hydrometer readings: the *meniscus correction* and the *dispersing agent correction*. Since the soil suspension is opaque, the hydrometer reading is taken at the top of the meniscus. Actual reading, to be taken at the water level, will be *more* since the readings increase in the downward direction. Hence the meniscus correction C_m is always *positive*. Its magnitude can be found by immersing hydrometer in a jar containing clear water, and finding the difference between the reading corresponding to the top and bottom of the meniscus. Similarly, the addition of dispersing agent in water increase its density, and hence the *dispersing agent correction* C_d is always *negative*. Thus, the corrected hydrometer reading R *is given by*

$$R = R_h' + C_m \pm C_t - C_d \qquad ...(3.15)$$

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where, R_h' = observed hydrometer reading at the top of the meniscus.

The three corrections can be combined into one correction, known as the *composite correction* $\pm C$ and equation 3.15 can be represented as

$$R = R_h' \pm C \qquad ...(3.16)$$

Similarly, the hydrometer reading, corrected for meniscus above, is given by

$$R_h = R_h' + C_m \qquad ...(3.17)$$

Equation 3.17 is useful for finding the effective height H_e from the calibration chart (curve) corresponding to the observed reading R_h'.

In order to find the composite correction C, an identical cylinder with 100 mL capacity is taken, and filled with distilled water and the same quantity of dispersing agent is used in the test cylinder. The temperature of both the cylinders should be the same. The hydrometer is immersed in this comparison cylinders containing distilled water and dispersing agent, and reading is taken at the *top* of the meniscus. The *negative* of the hydrometer reading so obtained gives the composite correction. For example, if the hydrometer reading in the comparison cylinder is +2, the composite correction $C = -2$. Similarly, if the reading is –1, the composite correction will be $C = +1$. The composite correction is found before the start of the test, and also, at every time interval exceeding 30 minutes.

Computation of *D* and *N*. The particle size D is calculated from Eq. 3.9:

$$D = 10^{-5} F \sqrt{\frac{H_e}{t}} \qquad ...(3.9)$$

For various time intervals, R_h is found from equation 3.7 and corresponding values of H_e from Fig. 3.7. Substituting the values of H_e (cm) and t (minutes) in equation 3.9, the diameter D (mm) is computed. To compute the percentage of the soil finer than this diameter, the mass M_D per mL of suspension at effective depth H_e is first computed as under. Since the hydrometer readings have been recorded by subtracting 1 from the density (ρ) readings and multiplying them by 1000, we have

$$R = (\rho - 1)\,1000 \quad \text{or} \quad \rho = 1 + \frac{R}{1000} \qquad ...(i)$$

where ρ is the density reading actually marked on the hydrometer, and R is the hydrometer reading corrected for the composite correction.

Now let us consider 1 mL of soil suspension, at a time interval t, at the effective depth H_e. If M_D is the mass of solids in this 1 mL suspension, the mass of water in it will be $= 1 - \frac{M_D}{G}$

Total mass of 1 mL suspension $= 1 - \frac{M_D}{G} + M_D$

Hence density of the suspension $= 1 - \frac{M_D}{G} + M_D$ ...(*ii*)

Equating (*i*) and (*ii*), we get $1 + \frac{R}{1000} = 1 - \frac{M_D}{G} + M_D$

or
$$M_D = \frac{R}{1000}\left(\frac{G}{G-1}\right) \qquad ...(3.18)$$

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where, G = specific gravity of soil solids (average)

Substituting this values of M_D in equation 3.11, we get

$$N' = \frac{\frac{R}{1000}\left(\frac{G}{G-1}\right)}{M_d/V} \times 100 \text{ ; Taking } V = 1000 \text{ mL, we get}$$

$$N' = \frac{100G}{M_d\,(G-1)}R \qquad ...(3.19)$$

where, N' = percentage finer with respect to M_d

Thus, for various values of R, N' can be computed. For a combined sieve and sedimentation analysis, if M is the total dry mass of soil originally taken (before sieving it over 2 mm sieve), the overall percentage finer N is given by

$$N = N' \times \frac{M'}{M} \qquad ...(3.20)$$

where M' = cumulative mass passing 2 mm sieve (out of which the soil having mass M_d was taken for the wet analysis)

M = total dry mass of soil sample

If the soil sample does not contain particles coarser than 2 mm size, N and N' will be equal. Table 3.15 gives the observation sheet for hydrometer analysis [See Experiment 9].

3.9 PARTICLE SIZE DISTRIBUTION CURVE

The results of the mechanical analysis are plotted to get a particle size distribution curve with the percentage finer N as the ordinate and the particle diameter as the abscissa, the diameter being plotted on a logarithmic scale. Figure 3.8 shows some typical curves for various soils. A particle size distribution curve gives us an idea about the type and *gradation* of the soil. A curve situated higher up or to the left represents a relatively fine grained soil while a curve situated to the right represents a coarse grained soil.

A soil sample may be either *well graded* or *poorly graded* (*uniformly graded*). A soil is said to be well graded when it has good representation of particles of all sizes. On the other hand, a soil is said to be poorly graded if it has an excess of certain particles and deficiency of other, or if it has most of the particles of about the same size ; in the latter case it is known as a uniformly graded soil. Thus, soil *A* (Fig. 3.8) is well graded while soil *B* is uniformly graded. A curve with a flat portion represents a soil in which some intermediate size particles are missing (soil E in Fig. 3.8). Such a soil is known as *gap graded* or *skip graded*.

For coarse grained soil, certain particle sizes such as D_{10}, D_{30} and D_{60} are important. The D_{10} represents a size, in mm such that 10% of the particles are finer than this size. Similarly, the soil particles finer than D_{60} size are 60 per cent of the total mass of the sample. The size D_{10} is sometimes called the *effective size* or *effective diameter*. The *uniformity coefficient* C_U (or coefficient of uniformity) is a measure of particle size range and is given by the ratio of D_{60} and D_{10} sizes:

$$C_U = \frac{D_{60}}{D_{10}} \qquad ...(3.21)$$

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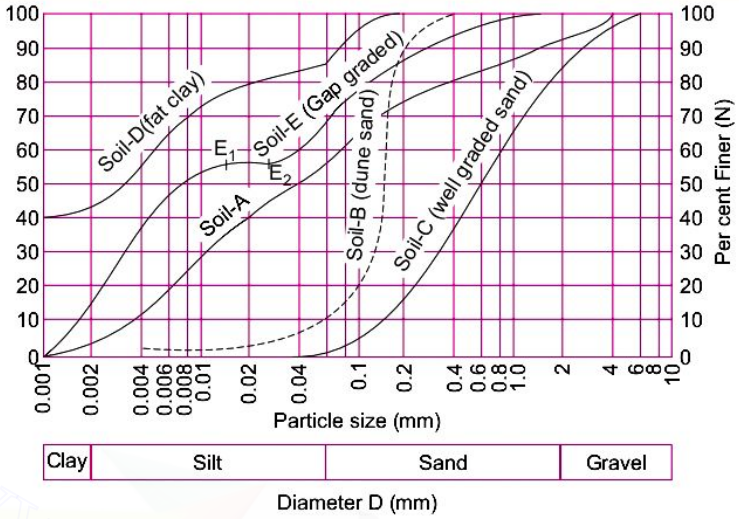

Fig. 3.8 Particle size distribution curve

Similarly, the shape of the particle size curve is represented by the *co-efficient of the curvature* C_C, given by

$$C_C = \frac{(D_{30})^2}{D_{10} \times D_{60}} \qquad ...(3.22)$$

For a uniformly graded soil, C_U is nearly unity. For a well graded soil, C_C must be between 1 to 3 and in addition C_U must be greater than 4 for gravels and 6 for sands (USBR : 1960).

Example 3.4. *A soil sample, consisting of particles of size ranging from 0.5 mm to 0.01 mm, is put on the surface of still water tank 5 metres deep. Calculate the time of settlement of the coarsest and the finest particles of the sample, to the bottom of the tank. Assume average specific gravity of soil particles as 2.66 and viscosity of water as 0.01 poise.*

Solution.

$$v = \frac{D^2 \gamma_w (G-1)}{18 \times 10^6 \eta} = \frac{D^2 (G-1)}{1.835 \times 10^6 \eta} \qquad ...(3.6)$$

Here, $G = 2.66$ and $\eta = 0.01 \times 10^{-4} = 10^{-6}$ kN-s/m^2

$$\therefore \quad v = \frac{D^2}{1.835} \times \frac{2.66-1}{10^6 (10^{-6})} \approx 0.905 D^2$$

where v is in m/sec and D is in mm.

For coarsest particle, $D = 0.5$ mm ; $v = 0.905\ (0.5)^2 = 0.2263$ m/sec

$$\therefore \quad t = \frac{h}{v} = \frac{5}{0.2263} = \textbf{22.1 seconds}$$

For the finest particle, $D = 0.01$ mm ; $v = 0.905\ (0.01)^2 = 9.05 \times 10^{-5}$ m/sec

$$\therefore \quad t = \frac{5}{9.05 \times 10^{-5}} = 55249 \text{ sec} = \textbf{15 hours 20 min 49 sec.}$$

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Example 3.5 *50 grams of oven dried soil sample is taken for sedimentation analysis. The hydrometer reading in a 100 mL soil suspension 30 minutes after the commencement of sedimentation test is 24.5. The effective depth for R_h = 25, found from the calibration curve is 10.7 cm. The meniscus correction is found to be +0.5 and the composite correction as –2.50 at the test temperature of 30°C. Taking the specific gravity of particles as 2.75 and viscosity of water as 0.008 poise, calculate the smallest particle size which would have settled during this interval of 30 minutes and the percentage of particles finer than this size.*

Solution.

$$R_h' = 24.5; \quad \therefore \quad R_h = 24.5 + 0.5 = 25; \quad R = 24.5 - 2.50 = 22$$

From Eq. 3.8, $$D = \sqrt{\frac{3000\eta}{(G-1)\gamma_w}} \cdot \sqrt{\frac{H_e}{t}}$$

where D is in mm, H_e is in cm and t is in min.

For the present case, $\eta = 0.008 \times 10^{-4}$ kN-s/m^2, H_e = 10.7 cm G = 2.75 and γ_w = 9.81 kN/m^3; t = 30 min.

$$\therefore \quad D = \sqrt{\frac{3000 \times 0.008 \times 10^{-4}}{(2.75-1)9.81}} \times \sqrt{\frac{H_e}{t}} = 0.01182\sqrt{\frac{H_e}{t}}$$

or $$D = 0.01182\sqrt{\frac{10.7}{30}} = 7.06 \times 10^{-3} \text{ mm} = \mathbf{0.00706 \text{ mm}}$$

Alternatively, the diameter could be found from the expression $D = 10^{-5} F \sqrt{\frac{H_e}{t}}$

From Table 3.2, for G = 2.75 and temperature = 30°C, we get F = 1182

$$\therefore \quad D = 10^{-5} \times 1182 \sqrt{\frac{10.7}{30}} = \mathbf{0.00706 \text{ mm}}$$

The percentage finer is given by

$$N = \frac{100\,G}{M_d\,(G-1)} R \text{ , where, } M_d = \text{mass of dry soil} = 50 \text{ g}$$

$$\therefore \quad N = \frac{100 \times 2.75}{50(2.75-1)} \times 22 = \mathbf{69.1\%}.$$

3.10 CONSISTENCY OF SOILS

By *consistency* is meant the relative ease with which soil can be deformed. This term is mostly used for fine grained soils for which the consistency is related to a large extent to water content. Consistency denotes degree of firmness of the soil which may be termed as *soft*, *firm*, *stiff* or *hard*. Fine grained soil may be mixed with water to form a plastic paste which can be moulded into any form by pressure. The addition of water reduces the cohesion making the soil still easier to mould. Further addition of water reduces the cohesion until the material no longer retains its shape under its own weight, but flows as a liquid. Enough water may be added until the soil grains are dispersed in a suspension. If water is evaporated from such a soil suspension, the soil passes through various stages or *states of consistency*. In 1911, the Swedish agriculturist Atterberg divided the entire range from liquid to solid state into four stages: (*i*) the liquid state, (*ii*) the plastic state, (*iii*) the semi-solid state, and (*iv*) the solid state. He set arbitrary limits,

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known as *consistency limits* or *Atterberg limits*, for these divisions in terms of water content. Thus, the consistency limits are the water contents at which the soil mass passes from one state to the next. Figure 3.9 shows the four states of consistency, with the appropriate consistency limits. The Atterberg limits which are most useful for engineering purposes are: *liquid limit*, *plastic limit* and *shrinkage limit*. These limits are expressed as per cent water content.

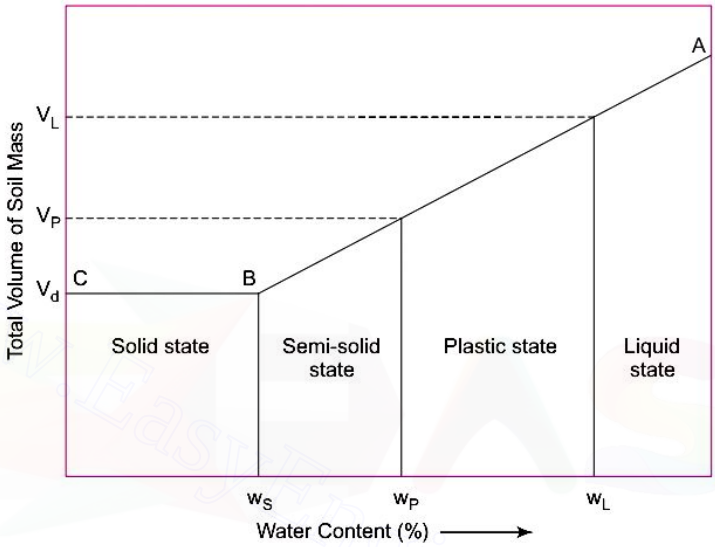

Fig. 3.9 Consistency limits

Liquid limit (w_L). Liquid limit is the water content corresponding to the arbitrary limit between liquid and plastic state of consistency of a soil. It is defined as the minimum water content at which the soil is still in the liquid state, but has a small shearing strength against flowing which can be measured by standard available means. With reference to the standard liquid limit device, it is defined as the minimum water content at which a part of soil cut by a groove of standard dimensions, will flow together for a distance of 12 mm $\left(\frac{1}{2}\text{ inch}\right)$ under an impact of 25 blows in the device.

Plastic limit (w_P). Plastic limit is the water content corresponding to an arbitrary limit between the plastic and the semi-solid states of consistency of a soil. It is defined as the minimum water content at which a soil will just begin to crumble when rolled into a thread approximately 3 mm in diameter.

Shrinkage limit (w_S). Shrinkage limit is defined as the maximum water content at which a reduction in water content will not cause a decrease in the volume of a soil mass. It is the lowest water content at which a soil can still be completely saturated.

Plasticity index (I_P). The range of consistency within which a soil exhibits plastic properties is called *plastic range* and is indicated by *plasticity index*. The *plasticity index* is defined as the numerical difference between the liquid limit and the plastic limit of a soil:

$$I_P = w_L - w_P \qquad ...(3.23)$$

In the case of sandy soils, plastic limit should be determined first. When plastic limit cannot be determined, the plasticity index is reported as *NP* (non-plastic). When the plastic limit is equal to or greater than the liquid limit, the plasticity index is reported as zero.

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Plasticity. Plasticity is defined as that property of a soil which allows it to be deformed rapidly, without rupture, without elastic rebound, and without volume change. According to Goldschmidt theory, the plasticity is due to the presence of thin scale like particles which carry on their surfaces' electromagnetic charges. Water molecules are bi-polar and orient themselves like tiny magnets in the magnetic field next to the surface of the soil particles. Water becomes highly viscous near the particles, but as the distance increases, the viscosity of water decreases until at some distance ordinary water exists. When enough water is present (corresponding to the plastic state of consistency), the particles are separated by molasses-like water which allows particles to slip past each other to new positions without any tendency to return to their former positions without change in volume of voids, and without impairing the cohesion. The correctness of Goldschmidt's theory for the cause of plasticity is evidenced by the fact that clay does not become plastic when mixed with a liquid of non-polarizing molecules like kerosene.

Consistency index (I_C). The *consistency index* or the *relative consistency* is defined as the ratio of the liquid limit minus the natural water content to the plasticity index of a soil:

$$I_C = \frac{w_L - w}{I_P} \qquad ...(3.24)$$

where w is the natural water content of the soil.

Consistency index is useful in the study of the field behaviour of saturated fine grained soils. Thus, if the consistency index of a soil is equal of unity, it is at the plastic limit. Similarly, a soil with I_C equal to zero is at its liquid limit. If I_C exceeds unity, the soil is in a semi-solid state and will be stiff. A negative consistency index indicates that the soil has natural water content greater than the liquid limit and hence behaves just like a liquid.

Liquidity index (I_L). The *liquidity index* or *water-plasticity ratio* is the ratio, expressed as a percentage, of the natural water content of a soil minus its plastic limit, to its plasticity index:

$$I_L = \frac{w - w_P}{I_P} \qquad ...(3.25)$$

where w is the natural water content of the soil.

3.11 DETERMINATION OF LIQUID AND PLASTIC LIMITS

The liquid limit is determined in the laboratory with the help of the standard liquid limit apparatus designed by Casagrande. The apparatus (Fig. 3.10) consists of a hard rubber base of B.S. hardness 21–25, over which a brass cup drops through a desired height. The brass cup can be raised and lowered to fall on the rubber base with the help of a cam operated by a handle. The height of fall of the cup can be adjusted with the help of adjusting screws. Before starting the test, the height of fall of the cup is adjusted to 1 cm. Two types of grooving tools are used: (*i*) the Casagrande (BS) tool and (*ii*) ASTM tool. The IS: 9259–1979 designates these tools as grooving tool (*a*) and *grooving tool* (*b*) respectively. The Casagrande tool cuts a groove of size 2 mm wide at the bottom, 11.0 mm wide at the top and 8 mm high while the ASTM tool cuts a groove 2 mm wide at the bottom, 13.6 mm at the top and 10 mm deep. The ASTM tool is used only for more sandy soils where the Casagrande tool tends to tear the sides of the groove.

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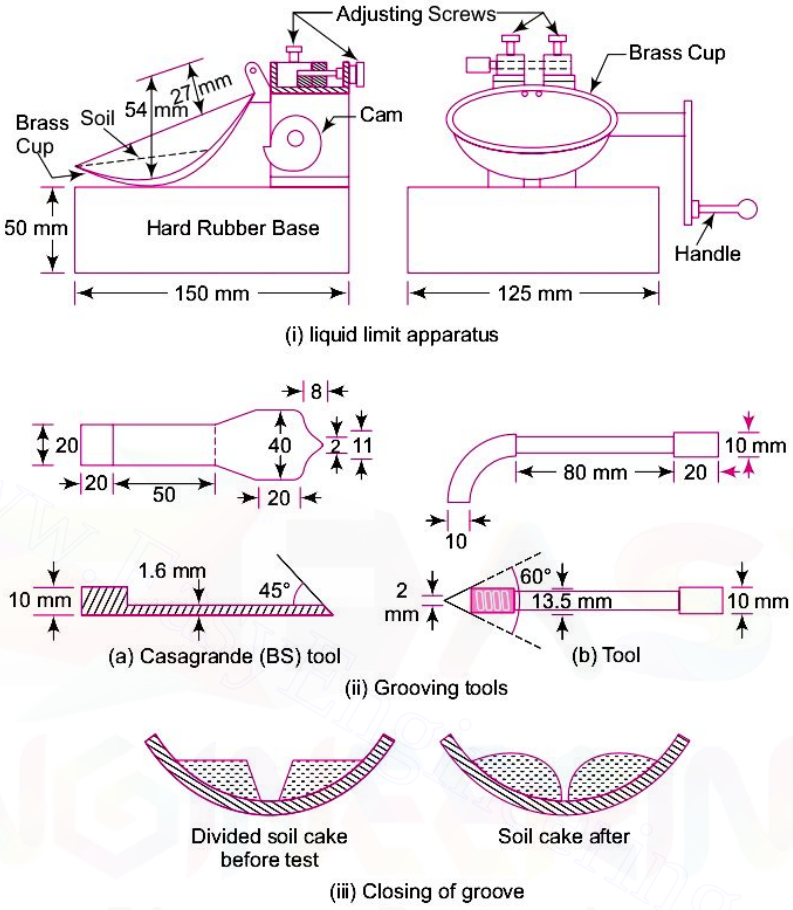

Fig. 3.10 Liquid limit apparatus

About 120 g of the specimen passing through 425 micron sieve is mixed thoroughly with distilled water in the evaporation dish or on a marble plate to form a uniform paste. A portion of the paste is placed in the cup over the spot where the cup rests on the base, squeezed down and spread into position and the groove is cut in the soil pat [Fig. 3.10 (*iii*)]. The handle is rotated at a rate about 2 revolutions per second, and the number of blows are counted until the two parts of the soil sample come into contact at the bottom of the groove along a distance of 10 mm. Some soils tend to slide on the surface of the cup instead of the flowing. If this occurs, the result should be discarded and the test repeated until flowing does not occur. After recording the number of blows, approximately 10 gram of soil from near the closed groove is taken for water content determination. Since it is difficult to adjust the water content precisely equal to the liquid limit when the groove should close in 25 blows, the liquid limit is determined by plotting a graph between number of blows as abscissa on a logarithmic scale and the corresponding water content as ordinate. Experience shows that such a graph, known as the *flow curve*, is a straight line having the following equation:

$$w_1 - w_2 = I_f \log_{10} \frac{n_2}{n_1} \qquad ...(3.26)$$

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where w_1 = water content corresponding to blows n_1

w_2 = water content corresponding to blows n_2

I_f = slope of the curve, known as the *flow index*.

For plotting the flow curve (Fig. 3.11), at least four to five sets of reading in the range of 10 to 50 blows should be taken. The water content corresponding to 25 blows is taken as the liquid limit.

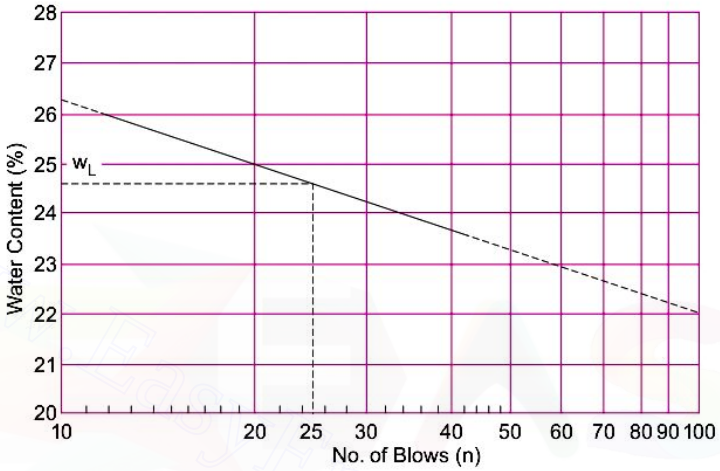

Fig. 3.11 Flow curve

Flow index. The flow index or the slope of the curve can be determined from the relation:

$$I_f = \frac{w_1 - w_2}{\log_{10} \frac{n_2}{n_1}} \qquad ...(3.26a)$$

Selecting the values of n_2 and n_1 corresponding to the number of blows over one log-cycle difference, $\log_{10} n_2/n_1$ becomes equal to unity, and hence I_f becomes equal to the difference between the corresponding water contents. Thus, if the flow curve is extended at either end so as to intersect the ordinates corresponding to 10 and 100 blows the numerical difference in water contents at 10 and 100 blows gives directly the flow index.

Table 3.17 gives the observation sheet for the liquid limit test (See Experiment 11).

One-point method. Attempts have been made to determine the liquid limit by taking only one reading of water content and its corresponding number of blows. The liquid limit is then estimated from the following equation:

$$w_L = w_n \left(\frac{n}{25} \right)^e \qquad ...(3.27a)$$

where, w_n = water content corresponding to n number of blows

w_L = water content at liquid limit

e = index, the value of which varies from 0.068 to 0.121 (Mohan: 1959).

If the value of n is taken between 20 to 30 blows the index e can be taken to be 0.1, and resulting value of w_L is fairly accurate. However, this method is used only for getting rough value of the liquid limit.

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Alternatively, for the range of blows between 15 and 35, the liquid limit water content (w_L) is computed employing the following formula (IS : 2720–1985).

$$w_L = \frac{w_n}{1.3213 - 0.23 \log_{10} n} \quad ...(3.27)$$

Static cone penetration method. The Soviet liquid limit device is based on the principle of static penetration. A 30° cone of stainless steel is made to penetrate the soil pat, under a mass of 75 grams inclusive of the mass of the cone. If the cone penetrates through a depth of 1 cm in 5 seconds, the soil pat is at the liquid limit.

IS : 2720 (Part V)–1985 Specifies a similar penetrometer (Fig. 3.12) for the determination of liquid limit. The cone has a central angle of 31° and total sliding mass of 80 g. The soil pat is kept in a cylindrical trough, 5 cm in diameter and 5 cm high, below the cone. The liquid limit of the soil corresponds to the water content of a paste which would give 20 mm penetration of the soil.

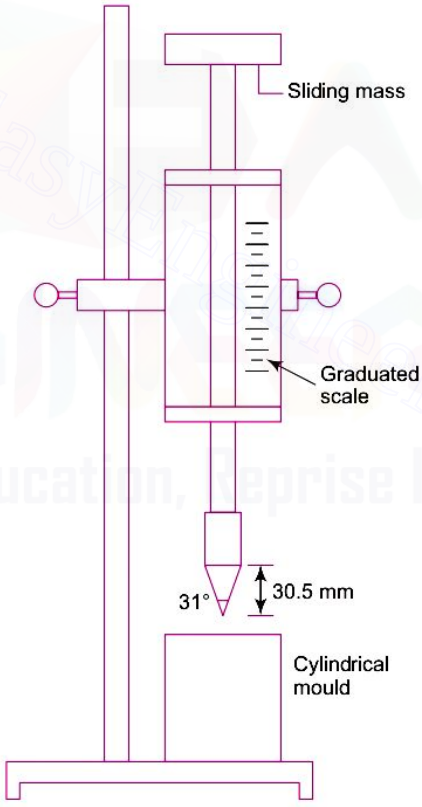

Fig. 3.12 Cone penetration IS : 2720 (Part V)–1965

Soil pats are prepared at various water contents and depth of penetration (x) for each pat is noted. A graph is plotted representing water content (w) on the y-axis and cone penetration (x) on the x-axis. The best fitting straight line is then drawn. The water content corresponding to a cone penetration of 20 mm is then taken as the liquid limit. The sets of values used for the graphs should be such that the values of penetration are in the range of 14 to 28 mm.

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Alternatively, w_L can be computed by *one-point method* using cone penetrometer, from any one of the following relationships:

$$w_L = \frac{w}{0.77 \log_{10} x} \qquad ...(3.28a)$$

or

$$w_L = \frac{w}{0.65 + 0.0175x} \qquad ...(3.28b)$$

The above expression is applicable only if the depth of penetration is between 20 to 30 mm. The expression is based on the assumption that at the liquid limit the shear strength of the soil is about 1.76 kN/m^2 (17.6 g/cm^2) which the penetrometer gives for a depth of 25 mm under a total sliding mass of 148 g.

Plastic Limit Determination. To determine the plastic limit, the soil specimen, passing 425 micron sieve, is mixed thoroughly with distilled water until the soil mass becomes plastic enough to be easily moulded with fingers. The plastic soil mass should be left for enough time to allow water to permeate through the soil mass. A ball is formed with about 8 g of this plastic soil mass and rolled between the fingers and a glass plate (or marble plate) with just sufficient pressure to roll the mass into a thread of uniform diameter throughout its length. When a diameter of 3 mm is reached, the soil is remoulded again into a ball. This process of rolling and remoulding is repeated until the thread starts just crumbling at a diameter of 3 mm. The crumbled threads are kept for water content determination. The test is repeated twice more with fresh samples. The plastic limit w_p is then taken as the average of three water contents.

The *plasticity index* is calculated from the relation:

$$I_P = w_L - w_P$$

Toughness Index (I_T). The toughness index is defined as the ratio of the plasticity index to the flow index:

$$I_T = I_P / I_f \qquad ...(3.29)$$

Knowing I_P and I_f, I_T can be computed. Similarly, if the natural water content of a soil is known, its consistency index, and liquidity index can be calculated using equations 3.24 and 3.25.

3.12 SHRINKAGE LIMIT

An expression for the shrinkage limit of a soil can be derived with reference to Fig. 3.13. If a saturated soil sample is taken (having water content more than the shrinkage limit) and allowed to dry up gradually, its volume will go on reducing till a stage will come after which the reduction in the soil water will not result in further reduction in the total volume of the soil sample. The water content corresponding to that stage is known as the shrinkage limit. Figure 3.13(a) shows such a soil sample of volume V_1 and mass M_1 while Fig. 3.13 (b) is the stage of the sample at its shrinkage limit. Figure 3.13(c) is the oven dried sample having volume V_d and mass M_d. Evidently, according to the definition, volumes V_2 and V_d are equal.

Now, mass of water in (a) $= M_1 - M_d$

Loss of water from (a) to (b) $= (V_1 - V_2)\rho_w$

$\therefore$ Mass of water in (b) $= (M_1 - M_d) - (V_1 - V_2)\rho_w$

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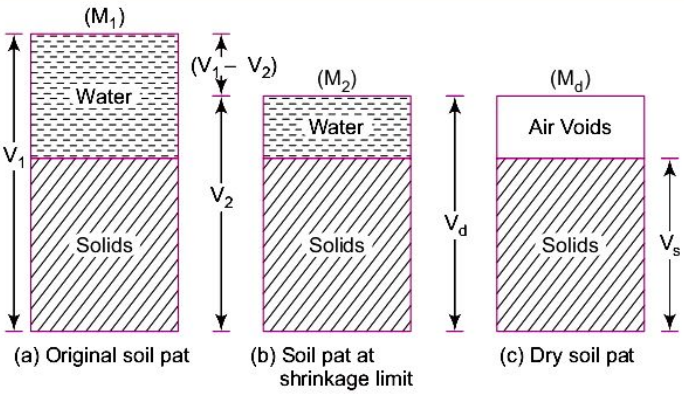

Fig. 3.13 Shrinkage limit determination

$$\therefore \quad w_s = \frac{(M_1 - M_d) - (V_1 - V_2)\rho_w}{M_d} \times 100 \qquad ...(3.30\ a)$$

$$\text{or} \quad w_s = \left[w_1 - \frac{(V_1 - V_d)\rho_w}{M_d} \right] \times 100 \qquad ...(3.30)$$

$$\text{or} \quad w_s = \left[w_1 - \frac{(V_1 - V_d)\gamma_w}{W_d} \right] \times 100 \qquad ...(3.30\ b)$$

where, w_1 = water content of the original saturated sample of volume V_1

V_d = dry volume of the soil sample ; W_d = dry weight of the soil sample

M_d = dry mass of the soil sample.

The form of equation 3.30 immediately suggests the method of determining the shrinkage limit in the laboratory. The equipment for the determination consists of (*i*) a porcelain evaporating dish, about 12 cm in diameter with flat bottom, (*ii*) a stainless steel shrinkage dish, 45 mm in diameter and 15 mm in height, with flat bottom, (*iii*) two glass plates, each 75 × 75 mm, one of plain glass and the other having three metal prongs, and (*iv*) a glass cup 50 mm in diameter and 25 mm in height, with its top rim ground smooth and level (Fig. 3.14).

The volume V_1 of the shrinkage dish is first determined by filling it to overflow with mercury, removing the excess by pressing a flat glass plate over its top and then taking the mass of the dish filled with mercury. The mass of the mercury contained in the dish, divided by its density (13.6 g/cm^3) gives the volume of the dish. About 50 g of soil passing 425 micron IS sieve is mixed with distilled water sufficient to fill the voids completely and to make the soil pasty enough to be readily worked into the shrinkage dish without the inclusion of air bubbles. The inside of the shrinkage dish is coated with a thin layer of vaseline. A volume of wet soil of about one-third the volume of dish is put in its centre and the soil is caused to flow to the edges by tapping it gently on a hard surface. The dish is gradually filled by adding more soil in instalment followed by gently tappings to exclude the inclusion of air. The excess soil is struck off with straight edge and all soil adhering to the outside of the dish is wiped off. The dish filled with soil is then immediately weighed. The mass M_1 of the wet soil pat, of volume V_1, is thus known by subtracting the mass of the empty dish from the mass of the wet soil plus the dish taken above. The dish is then placed

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in the oven. The soil pat will have volumetric shrinkage on drying, as shown in Fig. 3.14 (*b*). The mass M_d of the dry soil pat is found. To find the volume V_d of the dry soil pat, the glass cup is first filled with mercury and the excess mercury is removed by pressing the glass plate with three prongs firmly over the top of the cup. The cup is wiped off any mercury which may be adhering to its outside surface, and is placed in the evaporating dish. The dry soil pat is placed on the surface of the mercury of the cup and is carefully forced down by means of glass with prongs. The mass of the mercury so displaced divided by its density gives the volume V_d of the dry soil pat. The shrinkage limit is then calculated from Eq. 3.30. Table 3.20 gives the observation sheet for the shrinkage limit determination (See Experiment 13).

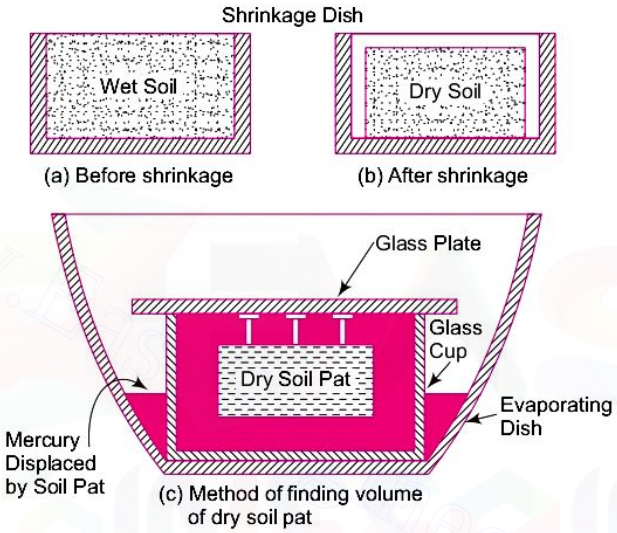

Fig. 3.14 Apparatus of shrinkage limit determination

Alternative method. Alternatively shrinkage limit can be found if the specific gravity G of the soil grains is known. If V_s is the volume of solids, and V_d is the dry volume, we have

$$w_s = \frac{(V_d - V_s)\rho_w}{M_d} \times 100 = \left(V_d - \frac{M_d}{\rho_s}\right)\frac{\rho_w}{M_d} \times 100 \qquad ...(3.31)$$

where ρ_s = density of soil solids

$$\therefore \quad w_s = \left(\frac{V_d}{M_d}\rho_w - \frac{1}{G}\right)100 = \left(\frac{V_d}{M_d} - \frac{1}{G}\right)100 \qquad ...(3.31a)$$

Since $\rho_w = 1$ (g/cm^3)

Equation 3.31 may also be written as

$$w_s = \left(\frac{\rho_w}{\rho_d} - \frac{1}{G}\right)100 = \left(\frac{\gamma_w}{\gamma_d} - \frac{1}{G}\right)100 \qquad ...(3.31b)$$

where, ρ_d = dry density of soil based on its minimum (dry) volume

γ_d = dry unit weight of soil sample.

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Substituting, $\gamma_d = \dfrac{G\gamma_w}{1+e}$ in equation 3.31(b), we get

$$w_s = \frac{e}{G} \qquad ...(3.32)$$

where e = voids ratio of the sample at its minimum (dry) volume.

Approximate value of G from shrinkage limit test. The observation of a shrinkage limit test can also be used to determine the approximate value of G as under:

$$\gamma_s = G\gamma_w = \frac{M_d \times 9.81}{V_s} \text{ Hence } G = \frac{M_d}{V_s} \text{ (Since } \gamma_w = 9.81 \text{ kN/m}^3)$$

But $V_s = V_1 - \dfrac{M_1 - M_d}{\rho_w}$ [From Fig. 3.13 (a)]

$$\therefore \quad G = \frac{M_d\,\rho_w}{V_1\,\rho_w - (M_1 - M_d)} \text{ (where } \rho_w = 1 \text{ g/cm}^3) \qquad ...(3.33\ a)$$

$$\therefore \quad G = \frac{M_d}{V_1 - (M_1 - M_d)} \qquad ...(3.33)$$

Also, if the shrinkage limit is known, we get from equation 3.31 (b)

$$G = \frac{1}{\dfrac{\gamma_w}{\gamma_d} - \dfrac{w_s}{100}} = \frac{1}{\dfrac{\rho_w}{\rho_d} - \dfrac{w_s}{100}} \qquad ...(3.34)$$

where, ρ_d = dry density of soil (g/cm^3) ; ρ_w = density of water (= 1 g/cm^3).

Shrinkage ratio (*SR*). We have seen that if a soil slurry is allowed to dry up, the reduction in water content of the saturated soil mass is accompanied by a change in the volume of the soil mass, the total change in the volume of the soil mass is equal to the volume of water evaporated. This reduction in volume takes place up to the shrinkage limit. The reduction in the volume is represented by the straight line *AB* in Fig. 3.9. *Shrinkage ratio* is defined as the ratio of a given volume change expressed as a percentage of dry volume, to the corresponding change in water content above the shrinkage limit expressed as a percentage of the weight of the oven dried soil:

$$SR = \frac{\dfrac{V_1 - V_2}{V_d} \times 100}{w_1 - w_2} \qquad ...(3.35)$$

where, V_1 = volume of soil mass at water content w_1

V_2 = volume of soil mass at water content w_2

V_d = volume of dry soil mass

w_1, w_2 = water content, expressed as percentage.

At the shrinkage limit [Fig. 3.13 (b)], $V_2 = V_d$ and $w_2 = w_s$; hence

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$$SR = \frac{\left(\frac{V_1 - V_d}{V_d}\right)100}{w_1 - w_s} \qquad ...(3.36)$$

Also, in equation 3.35, the change in the water contents $(w_1 - w_2)$ is given by

$$w_1 - w_2 = \frac{(V_1 - V_2)\rho_w}{M_d} \times 100$$

Hence,
$$SR = \frac{M_d}{V_d \rho_w} = \frac{\rho_d}{\rho_w} = \frac{\gamma_d}{\gamma_w} \qquad ...(3.37)$$

Thus, the shrinkage ratio of a soil is equal to the mass specific gravity of the soil in its dry state. The shrinkage limit test data can be substituted in equation 3.36 to determine the shrinkage ratio of the soil.

Volumetric shrinkage (*VS*). The volumetric shrinkage or *volumetric change* is defined as the decrease in the volume of a soil mass, expressed as a percentage of the dry volume of the soil mass, when the water content is reduced from a given percentage to the shrinkage limit:

$$VS = \frac{V_1 - V_d}{V_d} \times 100 \qquad ...(3.38a)$$

But
$$\frac{V_1 - V_d}{V_d} \times 100 = (w_1 - w_s)\, SR \quad \text{Equation (3.36)}$$

$\therefore$
$$VS = (w_1 - w_s)\, SR \qquad ...(3.38)$$

where V_1 is the volume of soil mass at any water content w_1.

Linear shrinkage (L_S). It is defined as the decrease in one dimension of a soil mass expressed as a percentage of the original dimension, when the water content is reduced from a given value to the shrinkage limit. It is calculated from the following formula:

$$L_S = 100\left[1 - \left(\frac{100}{VS + 100}\right)^{\frac{1}{3}}\right] \qquad ...(3.39)$$

Example 3.6. *An undisturbed saturated specimen of clay has a volume of 18.9 cm^3 and a mass of 30.2 g. On oven drying, the mass reduces to 18.0 g. The volume of dry specimen as determined by displacement of mercury is 9.9 cm^3. Determine shrinkage limit, specific gravity, shrinkage ratio and volumetric shrinkage.*

Solution. Given:
$$M_1 = 30.2 \text{ g} \; ; \; M_d = 18.0 \text{ g} \; ; \; \rho_w = 1 \text{ g/cm}^3$$
$$V_1 = 18.9 \text{ cm}^3 \; ; \; V_2 = 9.9 \text{ cm}^3$$

(*i*)
$$w_s = \left[\frac{M_1 - M_d}{M_d} - \frac{(V_1 - V_2)\rho_w}{M_d}\right] \times 100 \qquad ...(3.30a)$$
$$= \left[\frac{30.2 - 18.0}{18.0} - \frac{(18.9 - 9.9)1}{18.0}\right] \times 100 = \mathbf{17.8\%}.$$

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(*ii*) $$G = \frac{M_d}{V_1 - (M_1 - M_d)} = \frac{18.0}{18.9 - (30.2 - 18.0)} = \mathbf{2.69.} \qquad ...(3.33)$$

Alternatively, from equation 3.34; $G = \dfrac{1}{\left(\dfrac{\rho_w}{\rho_d} - \dfrac{w_s}{100}\right)}$

where, ρ_w = density of water = 1 g/cm^3

ρ_d = dry density of soil specimen = $\dfrac{18.0}{9.9} = 1.818$ g/cm^3

$\therefore$ $$G = \frac{1}{\left(\dfrac{1}{1.818} - \dfrac{17.8}{100}\right)} = \mathbf{2.69}.$$

(*iii*) Shrinkage ratio, $$SR = \frac{\gamma_d}{\gamma_w} = \frac{\rho_d}{\rho_w} = \frac{1.818}{1} \approx \mathbf{1.82}.$$

(*iv*) Volumetric shrinkage, $$VS = \frac{(V_1 - V_d)100}{V_d} = \frac{18.9 - 9.9}{9.9} \times 100 = \mathbf{91\%}$$

Alternatively, $$VS = (w_1 - w_s)\, SR = (67.8 - 17.8) \times 1.82 = \mathbf{91\%}$$

Example 3.7. *The mass specific gravity of a fully saturated specimen of clay having a water content of 36% is 1.86. On oven drying, the mass specific gravity drops to 1.72. Calculate the specific gravity of clay and its shrinkage limit.*

Solution.

$$e = w_{sat} \,.\, G = 0.36\, G$$

Mass specific gravity, $$G_m = \left[\frac{(G + e)\gamma_w}{1 + e}\right]\frac{1}{\gamma_w}$$

$\therefore$ $$1.86 = \frac{G + 0.36G}{1 + 0.36G} = \frac{1.36G}{1 + 0.36G}$$

From which, $$G = \mathbf{2.69}$$

Now, $$w_s = \frac{\gamma_w}{\gamma_d} - \frac{1}{G} \text{ where } \frac{\gamma_d}{\gamma_w} = \text{mass specific gravity of dry soil} = 1.72$$

$\therefore$ $$w_s = \frac{1}{1.72} - \frac{1}{2.69} = 0.21 = \mathbf{21\%}$$

Alternatively, $$(e)_{dry} = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.69 \times 1}{1.72} - 1 = 0.565$$

$$w_s = \frac{e}{G} = \frac{0.565}{2.69} = 0.21 = \mathbf{21\%}.$$

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Example 3.8. *The Atterberg limits of a clay soil are : liquid limit 52%, plastic limit 30% and shrinkage limit 18%. If the specimen of this soil shrinks from a volume of 39.5 cm^3 at the liquid limit to a volume of 24.2 cm^3 at the shrinkage limit, calculate the true specific gravity.*

Solution. Figure 3.15 (*a*, *b*, *c*) shows that the states of the specimen at liquid limit, shrinkage limit and dry condition, respectively.

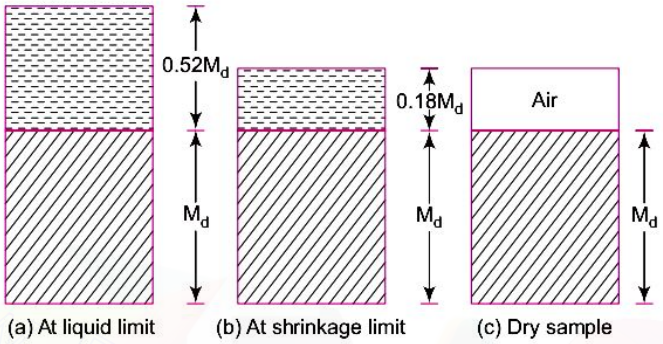

Fig. 3.15

Difference of volume of water in (*a*) and (*b*)

$$= 39.5 - 24.2 = 15.3 \text{ cm}^3$$

Difference of mass of water in (*a*) and (*b*) = 15.3 g.

But from Fig. 3.15 (*a*), (*b*), this difference is equal to $(0.52 - 0.18)\, M_d$

$$\therefore \quad (0.52 - 0.18)\, M_d = 15.3 \text{ or } M_d = \frac{15.3}{0.34} = 45 \text{ g}$$

Mass of water in (*b*) $= 0.18\, M_d = 0.18 \times 45 = 8.1$ g

Volume of water in (*b*) $= 8.1 \text{ cm}^3$

$\therefore$ Volume of solids, V_s in (*b*) $= 24.2 - 8.1 = 16.1 \text{ cm}^3$

Hence, $$\rho_s = \text{density of solids} = \frac{M_d}{V_s} = \frac{45}{16.1} = 2.8 \text{ g/cm}^3$$

$$\therefore \quad G_S = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w} = \frac{2.8}{1} = \mathbf{2.8}.$$

Example 3.9 *The plastic limit of a soil is 25% and its plasticity index is 8%. When the soil is dried from its state at plastic limit, the volume change is 25% of its volume at plastic limit. Similarly, the corresponding volume change from the liquid limit to the dry state is 34% of its volume at liquid limit. Determine the shrinkage limit and the shrinkage ratio.*

Solution.

$$w_P = 25\% \; ; \; I_P = 8\%$$

$$\therefore \quad w_L = 25 + 8 = 33\%$$

Volume change at liquid limit $= 34\%$

$\therefore$ Dry volume, $V_d = V_L - 0.34\, V_L = 0.66\, V_L$

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where, V_L = volume at liquid limit ...(*i*)

Similarly, volume change at plastic limit = 25%

$\therefore$ $V_d = V_P - 0.25\ V_P = 0.75\ V_P$ ...(*ii*)

where, V_P = volume at plastic limit

Equating (*i*) and (*ii*), we get $V_P = \dfrac{0.66}{0.75} V_L = 0.88 V_L$

Figure 3.16 shows the consistency limits. From the diagram, it is clear that when the soil passes from liquid limit to plastic limit, there is a change of (1 – 0.88) V_L in volume and 8% change in water content.

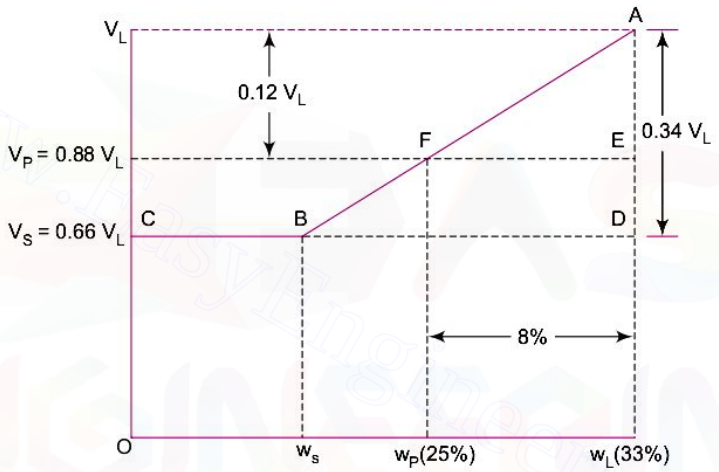

Fig. 3.16

$\therefore$ $\dfrac{BD}{AD} = \dfrac{FE}{AE} = \dfrac{8}{0.12 V_L}$; $\therefore$ $BD = \dfrac{8}{0.12 V_L} \times 0.34\ V_L = 22.6\%$

$\therefore$ $w_s = w_L - 22.6\% = 33 - 22.6 = \mathbf{10.4\%}$

$$SR = \frac{\dfrac{V_1 - V_2}{V}}{w_1 - w_2} = \frac{(V_L - V_P) \times 100}{V_d\,(w_L - w_P)} = \frac{V_L (1 - 0.88) 100}{0.66\, V_L (33 - 25)} = \frac{0.12 \times 100}{0.66 \times 8}$$

$$= \mathbf{2.27}$$

Example 3.10. *A saturated soil sample has a volume of 25 cm^3 at the liquid limit. If the soil has liquid limit and shrinkage limit of 42% and 20%, respectively, determine the minimum volume which can be attained by the soil specimen. Take G = 2.72.*

Solution. The soil specimen will attain minimum volume at shrinkage limit. Figure 3.17 (*a*) and (*b*) show the states of the specimen at liquid limit and shrinkage limit respectively.

If M_d is the mass of solids in *g*, volume of water at liquid limit is

$$V_L = 0.42\ M_d\ \text{cm}^3$$

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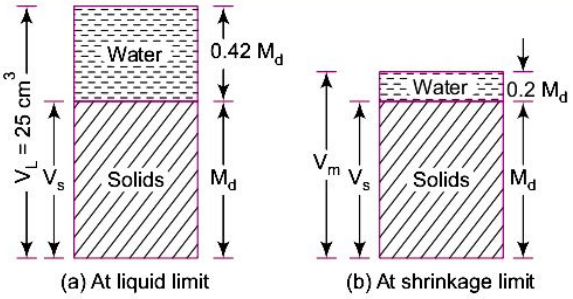

Fig. 3.17

Volume of solids,

$$V_s = \frac{M_d}{G\rho_w} = \frac{M_d}{2.72 \times 1} = \frac{M_d}{2.72} \text{ cm}^3$$
$$= 0.368\ M_d \text{ cm}^3$$

$\therefore$ Total volume $= 0.42\ M_d + 0.368\ M_d$. But this is equal to 25 cm^3

$\therefore$ $0.42\ M_d + 0.368\ M_d = 25$, from which $M_d = 31.74$ g

At the shrinkage limit, soil attains its minimum volume V_m

$\therefore$ $V_m = V_s + 0.2\ M_d = 0.368\ M_d + 0.2\ M_d = 0.568 \times 31.74 = \mathbf{18.03\ cm^3}$

Example 3.11. *An oven dried sample of soil has a volume of 265 cm^3 and a mass of 456 g. Taking G = 2.71, determine the voids ratio and shrinkage limit. What will be the, water content which will fully saturate the soil sample and also cause an increase in volume equal to 10% of the original dry volume?*

Solution.

$$\text{Dry density, } \rho_d = \frac{M_d}{V} = \frac{456}{265} = 1.721 \text{ g/cm}^3$$

$$\text{But } \rho_d = \frac{G\rho_w}{1+e}$$

$\therefore$ $$1.721 = \frac{2.7 \times 1}{1+e} \text{ From which}$$

$$e = \frac{2.71}{1.721} - 1 = \mathbf{0.575}$$

$$\text{Shrinkage limit} = \frac{e}{G} = \frac{0.575}{2.71} = 0.212$$
$$= \mathbf{21.2\%}$$

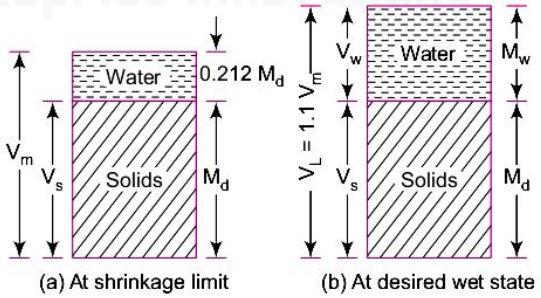

Fig. 3.18

Figures 3.18 (*a*) and (*b*) show the states of the specimen at shrinkage limit and desired final state, respectively.

Now, $V_m = 265$ cm^3 (given). Also, $V_s = \frac{M_d}{G} = \frac{456}{2.71} = 168.27$ cm^3

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At the final desired state, $V = 1.1\ V_m = 1.1 \times 265 = 291.5\ \text{cm}^3$ ...(1)

But from Fig. 3.18 (*a*), $V = V_w + V_s = V_w + 168.27$ ...(2)

From (1) and (2), we get $V_w = 291.5 - 168.27 = 123.23\ \text{cm}^3$

$\therefore$ Mass of water in final state, $M_w = 123.23$ g

Hence, water content in final state $= \dfrac{M_w}{M_d} = \dfrac{123.23}{456} = 0.27 = \mathbf{27\%}$.

3.13 DETERMINATION OF IN-SITU DENSITY, VOIDS RATIO AND DENSITY INDEX

The field density of a natural soil deposit or of a compacted soil can be determined by the following methods:

1. Sand replacement method
2. Core cutter method
3. Water displacement method
4. Submerged mass density method
5. Rubber balloon method.

1. Sand Replacement method. The equipment in the sand replacement method consists of (*i*) sand pouring cylinder mounted above a pouring cone and separated by a valve or shutter, (*ii*) calibrating container, (*iii*) tray with central circular hole, and (*iv*) chisel, scoop, balance, etc. The procedure consists of (*a*) calibration of the cylinder, (*b*) measurement of a soil density, and (*c*) determination of water content and dry density.

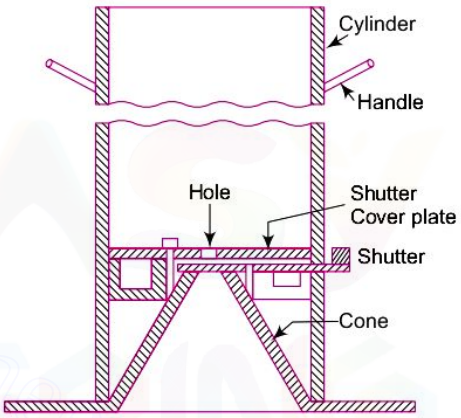

Fig. 3.19 Sand replacement cylinder

(a) Calibration of the cylinder. This consists of the determination of the weight of sand required to fill the pouring cone of the cylinder, and the determination of the bulk density of sand. Uniformly graded, dry, clean sand preferably passing a 600 micron sieve and retained on 300 micron IS sieve is used in the cylinder. The cylinder is filled up to a height of 1 cm below the top, and its initial mass M_1 is taken. The sand is run out of cylinder, equal in volume to that of the calibrating container. The cyclinder is then placed over a plane surface and the sand is allowed to run out to fill the cone below.

When no further sand runs out, the valve is closed. The sand filled in the cone is collected, and its mass M_2 is found. All the sand is then refilled in the cylinder so that the total mass of sand and cylinder is equal to the original mass M_1. The cylinder is then put centrally above the calibrating container, and the sand is allowed to run into the calibrating container. The valve is closed when there is no further movement of sand. The mass of cylinder with sand is found (M_3). The mass M' of the sand required to fill the calibrating container will be equal to $M_1 - M_3 - M_2$. The mass M' divided by the volume of the calibrating container gives the bulk density of the sand. All the sand is then refilled in the cylinder.

(b) Measurement of soil density. The site is cleaned and levelled, and the tray placed over it. A test hole, approximately of a depth equal to that of the calibrating container is excavated in the ground, and the soil is collected in the tray. The mass M of the excavated soil is found. The cylinder is centrally placed over the hole, and the sand is allowed to run into it. The valve is closed when no further movement of

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sand takes place. The mass M_4 of the cylinder and the remaining sand in it is measured. The mass M'' run into the hole, up to level ground surface, will evidently be equal to $M_1 - M_2 - M_4$. Dividing M'' by the bulk density of sand, the volume of the hole and hence the volume V of the excavated soil is known. Dividing the mass M by the volume V, the bulk density ρ of the soil excavated is known.

(c) Dry density. A suitable sample of the excavated soil is kept for water content (w) determination. The dry density of the soil will be equal to the bulk density divided by $(1 + w)$. Table 3.10 gives the observation sheet for the test. The bulk unit weight $\gamma = \rho\, g = 9.81\rho$ (kN/m^3) and the dry unit weight $\gamma_d = \rho_d \cdot g = 9.81\rho_d$ (kN/m^3) [See Experiment 7].

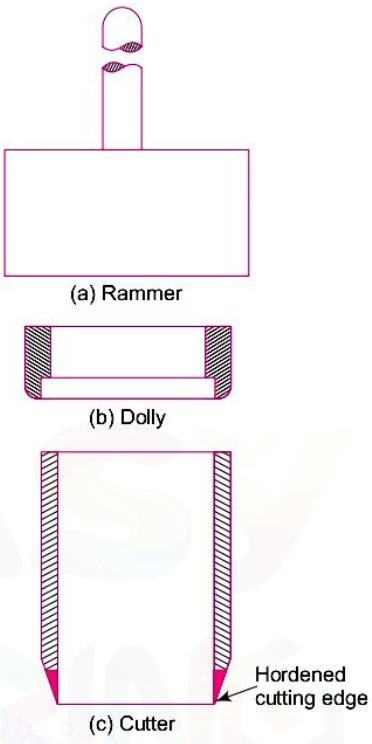

Fig. 3.20 Core cutter

2. Core cutter method. A core cutter, consisting of a steel cutter, 10 cm in diameter and about 13 cm high, and a 2.5 cm high dolly is driven in the cleaned surface with the help of a suitable rammer, till about 1 cm of the dolly protrudes above the surface. The cutter, containing the soil, is dug out of the ground, the dolly is removed and the excess soil is trimmed off. The mass of the soil in the cutter is found. By dividing it by the volume of the cutter the bulk density is determined. The water content of the excavated soil is found in the laboratory, and the dry density is computed, [See Experiment 6].

3. Water displacement method. This method is suitable only for cohesive soil samples brought from the field. A small specimen is trimmed to a more or less regular shape, from a larger sample, and its mass M_1 is found. The specimen is covered with a thin layer of paraffin wax and the mass M_2 of the coated specimen is taken. A metal container is filled above the overflow level, and excess water is allowed to run off through the overflow outlet. The coated specimen is then slowly immersed in the container, and the overflow water is collected in a measuring jar. The volume V_w of the displaced water is thus known. The volume V of the uncoated specimen is then calculated from the following relation.

$$V = V_w - \frac{M_2 - M_1}{G_P} \qquad ...(3.40)$$

where G_P = density of paraffin wax (g/mL)

In the absence of any other test, G_P may be taken as 0.908 g/mL.

The bulk density and the dry density of the specimen are determined from the relations

$$\rho = \frac{M_1}{V} \quad \text{and} \quad \rho_d = \frac{\rho}{1+w}$$

The bulk unit weight, $\gamma = \rho \cdot g = \frac{M_1}{V} \times 9.81$ (kN/m^3) and dry unit weight, $\gamma_d = \frac{\gamma}{1+w}$

where w is the water content of the specimen, to be determined by oven drying a small specimen. [See Experiment 5].

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4. Submerged mass density method. This method is used to determine the volume (V_w) of the wax coated specimen, and is based on Archimedes' principle that when a body is submerged in water, the reduction in its mass is equal to mass of volume of water displaced. The soil sample is trimmed and its mass M_1 is found. It is then covered with a thin layer of paraffin wax and the mass M_2 of wax coated specimen is found. The wax coated specimen is then placed in the cradle of special type balance. The cradle is then dipped in water contained in the bucket placed just below, and the apparent mass (M_3) of waxed specimen is found.

From Archimedes' principle, $M_3 = M_2 - V_w \rho_w$

or
$$V_w = \frac{M_2 - M_3}{\rho_w}$$

Substituting the value of V_w in Eq. 3.40, we get.

$$V = \frac{M_2 - M_3}{\rho_w} - \frac{M_2 - M_1}{G_P} \qquad ...(3.40\ a)$$

Thus, the volume of the specimen is known. The bulk density is then found from the relation

$$\rho = M_1/V.$$

5. Rubber balloon method. In this method, the volume of the excavated hole is measured with the help of an inflated rubber balloon. The apparatus consists of (*i*) a **graduated** glass or lucite cylinder enclosed in an airtight aluminium case, with an opening in the bottom, and (*ii*) a tray with central circular hole of 10 cm diameter (Anon : 1941). The cylinder is partially filled with water. Pressure or Vacuum can be applied to the bottom of the cylinder with the help of a double acting rubber bulb.

The ground surface, where the density is to be determined is cleaned and levelled, and tray is placed over it. The cylinder is then placed centrally over the tray. The air valve is opened and air is pumped into the cylinder until the balloon is completely inflated against the surface of the soil in the opening of the tray. The water level is read in the cylinder. The cylinder is then removed, and a hole is excavated in the ground. The excavated soil is weighed, and a sample is kept for water content determination. The cylinder is then placed over the opening in the tray, air valve is opened and air is forced in the cylinder to inflate the bottom, until the base of the instrument is raised off the tray at least by 1 cm. The air valve is closed and both feet are placed firmly on the base plate so that the balloon is forced into any irregularities in the hole. The water level is read in the cylinder. The volume of the hole is found from the difference between the initial and final water level, in the glass cylinder. Knowing the mass, volume and water content, the bulk density and dry density can be computed.

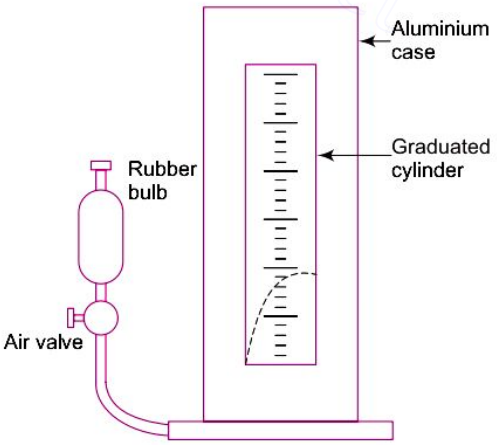

Fig. 3.21 Rubber balloon cylinder

Voids ratio and density index. The voids ratio of the soil in situ can be computed from the following expression if G and γ_d are known:

$$e = \frac{G\gamma_w}{\gamma_d} - 1$$

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The density index can then be calculated from the expression.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

To find e_{max}, the minimum density of the soil, corresponding to its loosest state, is found by pouring soil in a cylinder gently with the help of a funnel. Similarly, e_{min} is found by compacting the soil by vibrating it to get its maximum density.

3.14 ACTIVITY OF CLAYS

The properties of clays and their behaviour is influenced by presence of certain clay minerals (see chapter 5) even in small quantities. The thickness of the oriented water around a clay particle is dependant on type of clay mineral. Thus, the plasticity of a clay depends upon (*i*) the nature of clay mineral present, and (*ii*) amount of clay mineral present. On the basis of lab tests, Skempton (1953) observed that *for a given soil* the plasticity index is directly proportional to the per cent of clay-size fraction (*i.e.*, per cent by weight finer than 0.002 mm in size). He introduced the concept of *activity*, by relating the plasticity to the quantity of clay-size particles, and defined the *activity* (A_c) *as the ratio of plasticity index to the per cent by weight of soil particles of diameter smaller than two microns present in the soil.* Thus,

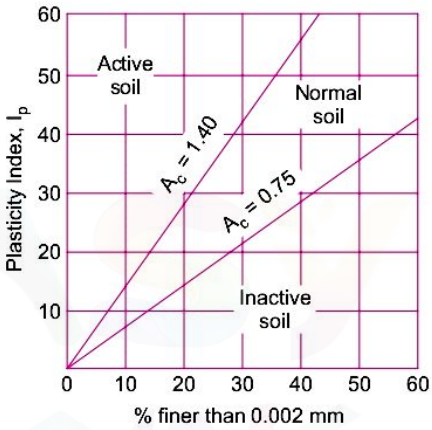

Fig. 3.22

$$A_c = \frac{I_p}{C_w} \qquad ...(3.41)$$

where, I_p = Plasticity index

C_w = percentage by weight of 'clay sizes', *i.e.*, of particles of size less than two microns.

Activity can be determined from the results of usual laboratory tests such as wet analysis, liquid limit and plastic limit.

Based on activity number, clays can be classified *qualitatively* into three categories as given in the Table 3.4 (also see Fig. 3.22).

Table 3.4 Classification based on activity

Activity	*Classification*
< 0.75	Inactive
0.75 – 1.40	Normal
> 1.40	Active

It should be noted that the activity of a given soil will be a function of type of clay mineral present in it. Clays containing kaolinite will have relatively low activity while those having montmorillonite will have high activity. Based on equation 3.41, typical activity values are as follows:

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Kaolinite	0.4 – 0.5
Illite	0.5 – 1.0
Montmorillonite	1.0 – 7.0

3.15 SENSITIVITY OF CLAYS

The consistency of an undisturbed sample of clay is altered, even at the same water content, if it is remoulded. It is because the original structure of clay is altered by reworking or remoulding. Since the strength of a clay soil is related, to its structure, remoulding results in decrease of its strength. The degree of disturbance of undisturbed clay sample due to remoulding is expressed by *sensitivity* (S_t) which is defined as the ratio of its unconfined compression strength in the natural or undisturbed state to that in the remoulded state, without change in the water content.

$$S_t = \frac{q_u\,(\text{undisturbed})}{q_u\,(\text{remoulded})} \qquad ...(3.42)$$

The sensitivity of most clays generally falls in a range of 1 to 8. Table 3.5 below gives the classification of clays based on the sensitivity. Clays having sensitivity greater than 16 are known as *quick* clays. Highly overconsolidated clays tend to be insensitive.

Table 3.5 Sensitivity classification

Sensitivity	*Classification*	*Structure*
1	Insensitive	–
2 to 4	Normal or less sensitive	Honeycomb structure
4 to 8	Sensitive	Honeycomb or Flocculent structure
8 to 16	Extra sensitive	Flocculent structure
> 16	Quick	Unstable

3.16 THIXOTROPY OF CLAYS

When sensitive clays are used in construction, they loose strength due to remoulding during construction operations. However, with passage of time, the strength again increases, though not to the same original level. This phenomenon of 'strength loss-strength gain' with no change in volume or water content is called '*thixotropy*' (from the Greek *thixis*, meaning 'touch' and *tropein*, meaning 'to change'). Thus, *thixotropy* is defined as an isothermal, reversible, time dependent process which occurs under constant composition and volume, thereby a material softens, as a result of remoulding, and then gradually returns to its original strength when allowed to rest. This is shown in Fig. 3.23. The larger the sensitivity, larger the *thixotropic hardening*.

The loss of strength due to remoulding is partly due to (*i*) permanent destruction of the structure due to in situ layers, and (*ii*) reorientation of the molecules in the adsorbed layers. *The gain is strength is due to rehabilitation of the molecular structure of the soil*, and is due to its thixotropic property. The loss of strength due to *destruction* of structure cannot be recouped with time. The regaining of a part of the

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strength after remoulding has important application in connection with pile driving operations, and other types of construction in which disturbance of natural clay formations is inevitable.

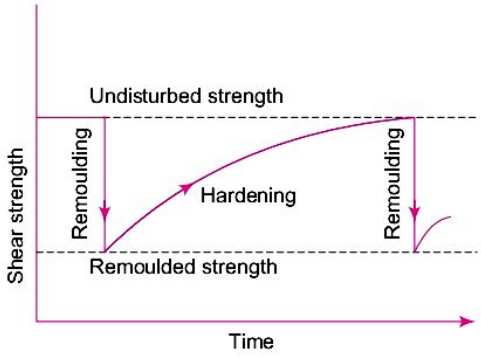

Fig. 3.23 Thixotropy of clays

3.17 COLLAPSIBLE SOILS

Collapsing or *collapsible soils* are those soils which undergo large decrease in volume due to increase in moisture content, even without increase in external loads. Examples of soils exhibiting this behaviour are loess, weakly cemented sands and silts where cementing agent is soluble (*e.g.*, soluble gypsum, halite, etc.) and certain granite residual soils. The Authors have observed some dune sands exhibiting such behaviour. A common feature of collapsible soils are the loose bulky grains held together by capillary stresses. Deposits of collapsible soils are usually associated with regions of moisture deficiency, such as those in arid and semi-arid regions.

Potentially collapsible soils could be identified and classified by detailed geologic studies. Figure 3.24 (Holtz and Gibbs, 1967) provides guidance for identifying the potential for collapse for clayey sands and sandy clays found in western U.S.

The potential for collapse is expressed by the term *collapse potential* (CP) or *coefficient of structural collapse* defined by

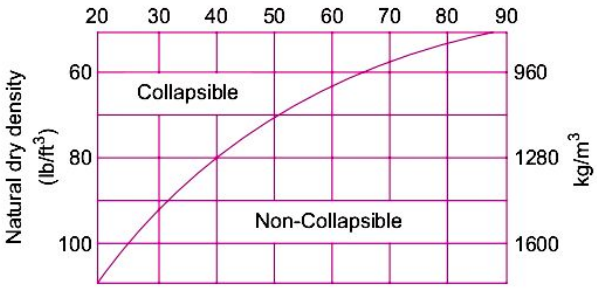

Fig. 3.24 Criterion for collapse potential

$$CP = \frac{\Delta e_c}{1 + e_1} \quad ...(3.43) \quad \text{or} \quad CP = \frac{\Delta H_c}{H_1} \quad ...(3.43a)$$

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where Δe_c = change in void ratio upon wetting

e_1 = voids ratio, before saturating or wetting the soil

ΔH_c = change in height upon wetting; H_1 = initial height.

The sudden volume decrease due to collapse of structure can be easily studied in the laboratory by consolidometer test conducted on special oedometer devised by Vasiliev (1949). During the test, the natural specimen is first consolidated under loads increased in steps, and a normal *p-e* curve is obtained (Fig. 3.25). At some pre-fixed load intensity, after the consolidation is complete, the specimen is wetted through porous stones with enough water to become fully saturated. This will result in a sudden compression (Δe_c) which can be observed.

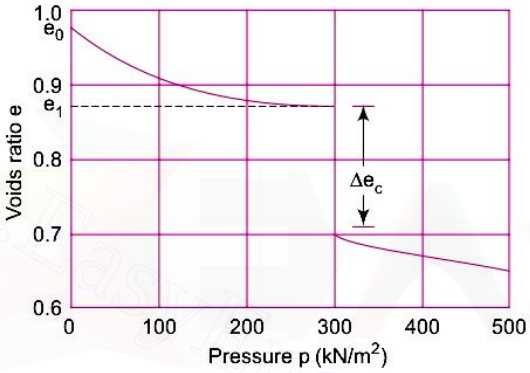

Fig. 3.25 Collapse potential

At given over-burden pressure (p_o), the collapse potential depends upon the degree of saturation (S) as shown in Fig. 3.26. The bulk of structural collapse usually occurs upon the first saturation, and a repeated submersion produces only a very small further decrease in volume. The collapse potential (CP) varies with the pressure as shown in Fig. 3.27. The reduction in volume is a maximum at a pressure which is usually in the vicinity 300 kN/m^2.

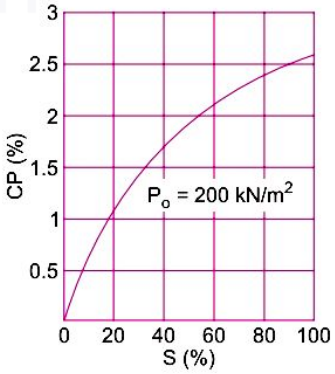

Fig. 3.26 Effect of S on CP

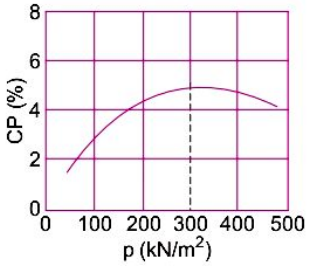

Fig. 3.27 Effect of *p* on CP

Denisov (1946) used the ratio e_L/e_o to characterise the collapsibility of a soil, where e_L is the voids ratio corresponding to liquid limit and e is the natural voids ratio. If this ratio is greater than unity, the soil is described as 'collapsible'.

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The potential for collapse can also be evaluated in the field by performing standard plate load tests under varied moisture environments.

Table 3.6 below gives the effect of collapse potential values on the severity of problem.

Table 3.6 Collapse potential values

CP	*Severity of Problem*
0–1%	No problem
1–5%	Moderate trouble
5–10%	Trouble
10–20%	Severe trouble
> 20%	Very severe trouble

Example 3.12. *A clay sample has liquid limit and plastic limit of 96% and 24% respectively. Sedimentation analysis reveals that the clay soil has 50% of the particles smaller that 0.002 mm. Indicate the activity classification of the clay soil and the probable type of clay mineral.*

Solution. We have $w_L = 96\%$ and $w_P = 24\%$

Hence plasticity index, $I_P = w_L - w_P = 96 - 24 = 72\%$

Now, from equation 3.41, activity $A_c = \dfrac{I_P}{C_w} = \dfrac{72}{50} = 1.44$

Since the activity No. is greater than 1.4, clay may be classified as being active. Also, the probable clay mineral is *montmorillonite*.

Example 3.13. *A clay specimen has unconfined compressive strength of 240 kN/m^2 in undisturbed state. Later, on remoulding, the unconfined compressive strength is found to be 54 kN/m^2. Classify the clay soil on the basis of sensitivity and indicate the probable structure of clay soil.*

Solution. From equation 3.42. Sensitivity, $S_t = \dfrac{q_u\,(\text{undisturbed})}{q_u\,(\text{disturbed})} = \dfrac{240}{54} = 4.44$

Since S_t is between 4 and 8, the given clay is classified as 'sensitive. The possible structure of clay soil may be *honey-comb* or *flocculent*.

3.18 AVERAGE DIAMETER OF A GROUP OF PARTICLES

A given mass of granular soil has particles of different sizes. Sometimes, it may be necessary to state the *average diameter* of the group of particles present in the soil mass. Following are the most common ways to interpret average diameter (D_{av}), along with corresponding expressions used for the determination.

(a) Arithmetic mean method: $D_a = \dfrac{\sum_{i=1}^{J} n_i D_i}{\sum_{i=1}^{J} n_i}$ ...(3.44*a*)

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(b) *Geometric mean method:* $D_g = (D_1\ D_2\ D_3\ \ldots\ D_n)^{1/J}$ ...(3.44*b*)

(c) *Harmonic mean method:* $D_h = \dfrac{1}{\sum_{i=1}^{J} n_i} \sum_{i=1}^{J} \dfrac{n_i}{D_i}$ ...(3.44*c*)

where, D_a = average diameter by arithmetic mean method
D_g = average diameter by geometric mean method
D_h = average diameter by harmonic mean method.

Example 3.13 illustrates the method of computing average diameter of a group of particles in a hypothetical soil mass, using the above equations.

Out of the three mean diameters (D_a, D_g and D_h) the average diameter by harmonic method (*i.e.*, D_h) is quite important since it is sometimes used to compute the *specific surface*. The harmonic mean diameter (D_h) can be obtained from the following expression assuming a continuous distribution [$P = f(D)$]:

$$\frac{1}{D_h} = \int_0^1 \frac{dP}{D} \qquad \ldots(3.45)$$

Kozney (1931) developed a simple graphical method for determining D_h, by plotting two sets of particle size distribution curves: (*i*) curve between percentage finer N and log D, and (*ii*) curve between percent finer N and $1/D$. The method of finding D_h from these two set of curves has been explained numerically in Example 3.15.

3.19 SPECIFIC SURFACE

Specific surface is defined as the total surface area of the soil particles in a unit mass. In order to understand and appreciate the physical significance of this term, let us consider a cube of soil particle having 10 mm side. The surface area of each face of the cube is 10×10 sq. mm and hence the total surface area of the cube $= 6 \times 10 \times 10 = 600$ sq. mm. Let us now subdivide this cube into 8 cubes, each with 5 mm side. The total surface area of the same original cube, subdivided into eight smaller cubes will be $= 8 \times 6 \times 5 \times 5 = 1200$ sq. mm. If the same cube is further subdivided into smaller cubes each with sides of 1 µm, then the number of small cubes will be 10^{12} and their total surface area will be 10 m^2 (10^7 mm^2). Thus the subdivision of the same mass yields greater surface area. *Hence specific surface increases very rapidly as the grain size decreases*. Microscopic particles have very large specific surface.

Another factor which is very important for the magnitude of specific surface is the *grain shape*. Let us consider soil solids of various shapes, all having a constant volume $V = 1$ cm^3 (1000 mm^3), and hence of constant mass. Their specific surfaces will be as under:

S.No.	*Particle shape*	*Specific surface (per cm^3)*
1.	Spherical	483 mm^2
2.	Cubical	600 mm^2
3.	Disc, 1 mm thick	2110 mm^2
4.	Plate, 1 µm thick	2×10^6 mm^2

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The specific surface per unit mass is of particular importance in the study of fine grained soils, since the electrical forces responsible for inter-particle effects are proportional to the specific surface (see chapter 5). The chemical activity of the clay minerals depends upon the surface area, because the electrical charge of clay minerals is high per unit mass. Permeability of soil greatly depends on the specific surface of the soil because of the viscous effects associated with surface characteristics of the grains. Specific surface is also related to many other physical properties of soils.

Expression for specific surface for soil having spherical particles: Let us consider unit mass of soil having spherical particles of various sizes. The *specific surface* for such a soil can be determined in terms of its *harmonic mean diameter* (D_h) defined by equation 3.44 (*c*). If there are n number of particles of mean diameter D_h in that unit soil mass, having mass density ρ_s of the soils, we have

$$n\left(\frac{\pi}{6} D_h^3 \rho_s\right) = \text{unit mass} = 1. \qquad ...(1)$$

Also, if F is the *specific surface* (*i.e.*, the total surface area of the particles on a unit mass), we have

$$F = n\,(\pi d) \qquad ...(2)$$

Substituting the value of n from (1) into (2), we get

$$F = \frac{6}{\rho_s} \cdot \frac{1}{D_h} \qquad ...(3.45a)$$

The harmonic mean diameter D_h can be found by Kozney's method explained in § 3.19 and illustrated in Example 3.14.

Specific surface (S_s) is also expressed as surface area of the particle per unit volume of the particle, *i.e.*,

$$\text{Specific surface,} \quad S_s = \frac{\text{Surface area of particle}}{\text{Volume of particle}} = \frac{\pi D^2}{\frac{1}{6}\pi D^3} = \frac{6}{D} \qquad ...(3.45)$$

For spheres uniformly distributed in size between the mesh sizes x and y of the adjacent sieves is given by

$$S_s = \frac{6}{\sqrt{x \cdot y}} \qquad ...(3.45b)$$

The above equation gives results accurate to 2% if x/y is not greater than 2 and the size distribution by mass is logarithmically uniform. The following table gives the specific surface of spheres lying between the BS sieves/equivalent IS sieves.

Equivalent IS sieves	*Specific surface (cm^2/cm^3)*
2.36–1.18 mm	35.2
1.18 mm–600 (microns)	70.6
600–300 (microns)	143.0
300–150 (microns)	284.0
150–75 (microns)	558.0

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Example 3.14. *Columns 1 and 2 of the table below give the limits of the particle size and the number of the particles respectively of a given soil mass. Determine arithmetic mean diameter, geometric mean diameter and harmonic mean diameter of the group of particles. Taking density of soil solids as 2.72 g/cm^3, determine the specific surface of the group of particles.*

Solution.

1	*2*	*3*	*4*	*5*
Limits of the particles size (μ)	*No. of particles (n)*	*Average size D, (μ)*	*nD*	*n/D*
0–6	3	3	9	1
6–12	12	9	108	1.33
12–18	60	15	900	4.0
18–24	85	21	1785	4.05
24–30	30	27	810	1.11
30–36	20	33	660	0.61
36–42	16	39	624	0.41
42–48	8	45	360	0.18
48–54	4	51	204	0.08
54–60	2	57	114	0.04
Sum	240		5574	12.81

Hence, J = No. of groups = 10

Now, $$D_a = \frac{\sum_{i=1}^{J} n_i D_i}{\sum_{i=1}^{J} n_i} = \frac{5574}{240} = 23.23\ \mu = \mathbf{23.23 \times 10^{-3}\ mm}$$

$$D_g = (D_1 D_2 D_3 \ldots D_n)^{1/J} = (3 \times 9 \times 15 \times 21 \times 27 \times 33 \times 39 \times 45 \times 51 \times 57)^{1/10}$$
$$= 22.84\ \mu = \mathbf{22.84 \times 10^{-3}\ mm}$$

$$\frac{1}{D_h} = \frac{1}{\sum_{i=1}^{J} n_i} \frac{n_i}{D_i} = \frac{1}{240}(12.01) \quad \text{or} \quad D_h = \frac{240}{12.01} = 19.98\ \mu = \mathbf{19.98 \times 10^{-3}\ mm.}$$

Example 3.15. *The particle size analysis of a soil sample gave the following results.*

Particle size (mm)	*% finer (N)*	*Particle size (mm)*	*% finer (N)*
0.2	*100*	*0.01*	*19*
0.1	*95*	*0.005*	*9*
0.05	*89*	*0.002*	*4*
0.02	*40*	*0.001*	*2*

Using Kozney's graphical method, determine the harmonic mean diameter, D_h.

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Solution.

***Step 1*:** Plot the usual particle size distribution curve between N and $\log_{10} D$. (curve I of Fig. 3.28).

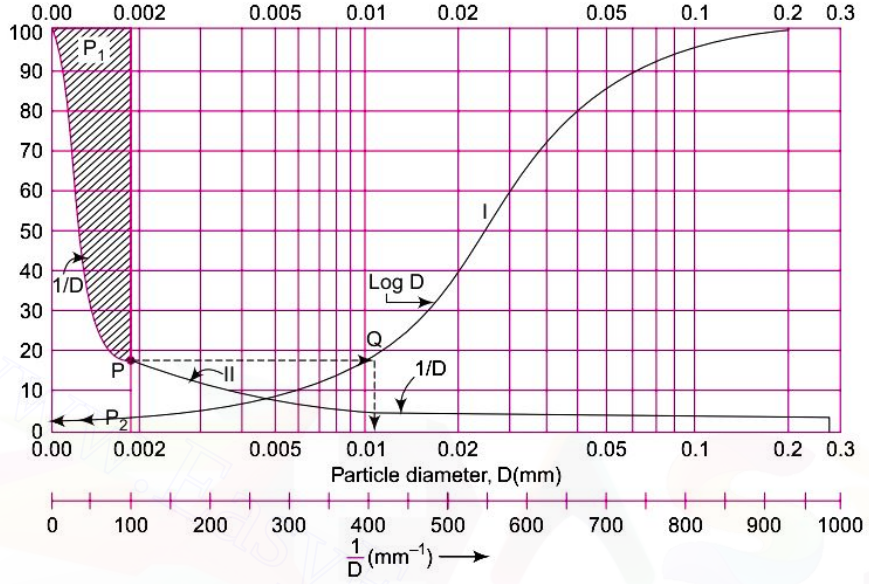

Fig. 3.28

***Step 2*:** From the given data, derive a table between 1/D and N, as follows.

1/D mm^{-1}	*N(%)*	*1/D mm*$^{-1}$	*N%*
5	100	100	19
10	95	200	9
20	89	500	4
50	40	1000	2

***Step 3*:** On the same sheet, plot a curve between *1/D* and N (curve II of Fig. 3.28).

***Step 4*:** Convert the area below curve II into a rectangle of height $N = 100\%$. This means that a vertical line $P_1 P_2$ is drawn in such a way that the two shaded areas, shown in Fig. 3.28 are equal. This is done by trial.

***Step 5*:** The vertical line $P_1 P_2$ will cut the curve II in point P which corresponds to the harmonic mean diameter D_h. Draw horizontal line PQ to intersect curve I in the point Q. Project point Q downwards on the log scale to get the required harmonic mean diameter D_h.

Thus, from Fig. 3.28, we get D_h = **0.01 mm.**

3.20 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 3.16. *The plastic limit of a soil is 25% and its plasticity index is 8%. When the soil is dried from its state of plastic limit, the volume change is 25% of its volume at plastic limit. Similarly, the corresponding volume change from liquid limit to dry state is 34% of its volume at liquid limit. Determine the shrinkage limit and shrinkage ratio.* **(Civil Services Exam. 1989)**

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Solution. This question was set from Author's present book. See example 3.9 for detailed solution.

Answers: $w_s = 10.4\%$ and $SR = 2.27$

Example 3.17. *The plastic limit of a soil is 24% and its plasticity index is 80%. When the soil is dried from its state at plastic limit, the volume change is 26% of its volume of plastic limit. The corresponding volume change from liquid limit to dry state is 35% of its volume at liquid limit. Determine the shrinkage limit and the shrinkage ratio.* **(Civil Services Exam. 1993)**

Solution. This question was set on the basis of example 3.9 of this book, simply by reducing all the volumes by 1%. Otherwise the question is exactly the same. Hence refer example 3.9 for the solution of this question.

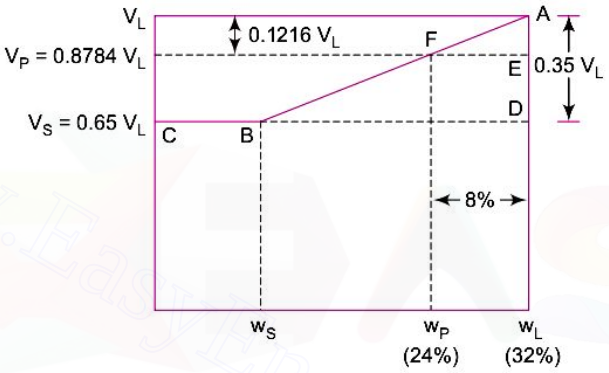

Fig. 3.29

Given: $w_P = 24\%$, $I_P = 8\%$;

$\therefore$ $w_L = 24 + 8 = 32\%$

Volume change at liquid limit = 35%; Volume change at plastic limit = 26%

$\therefore$ Dry volume, $V_d = V_L - 0.35\ V_L = 0.65\ V_L$ ...(*i*)

Also, $V_d = V_P - 0.26\ V_P = 0.74\ V_P$ ...(*ii*)

$\therefore$ From (*i*) and (*ii*), $V_P = \dfrac{0.65}{0.74} V_L$

$= 0.8784\ V_L$. From Fig. 3.29,

$\therefore$ $\dfrac{BD}{AD} = \dfrac{FE}{AE} = \dfrac{8}{0.1216 V_L}$ $\therefore$ $BD = \dfrac{8}{0.1216 V_L} \times 0.35\, V_L = 23.02\%$

$w_S = w_L - 23.02 = 32 - 2.02 = \mathbf{8.98\%}$

$$SR = \frac{(V_1 - V_2)/V_d}{w_1 - w_2} = \frac{(V_L - V_P)100}{V_d (w_L - w_P)} = \frac{V_L (1 - 0.8784)}{0.65\, V_L (32 - 24)} = \mathbf{2.338.}$$

Example 3.18 *The plastic limit and liquid limit of a soil are 30% and 42% respectively. The percentage volume change from the liquid limit to dry state is 35% of the dry volume. Similarly, the percentage volume change from the plastic limit to the dry state is 22% of the dry volume. Determine the shrinkage limit and the shrinkage ratio.* **(Civil Services Exam. 1997)**

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Solution. This question is similar to example 3.9 except that volume changes are corresponding to dry weights. Given: $w_L = 42\%$; $w_P = 30\%$; $V_L - V_d = 0.35\ V_d$ or $V_L = 1.35\ V_d$

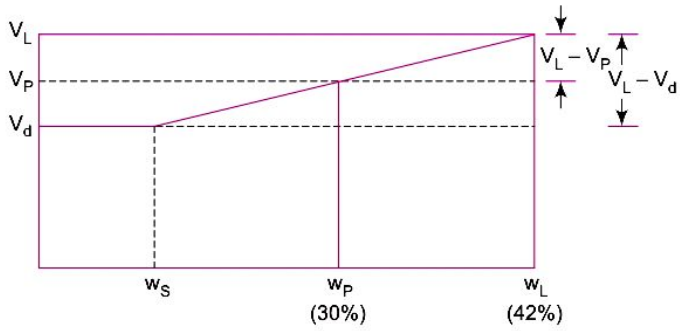

Fig. 3.30

Also, $V_P - V_d = 0.22\ V_d$

or $V_P = 1.22\ V_d$

From the geometry of Fig. 3.30

$$\frac{V_L - V_P}{w_L - w_P} = \frac{V_L - V_d}{w_L - w_S}$$

or $$\frac{1.35V_d - 1.22V_d}{0.42 - 0.30} = \frac{1.35V_d - V_d}{0.42 - w_S}$$

From which, $$w_S = 0.42 - \frac{1.35 - 1}{1.35 - 1.22} \times 0.12$$

$$= 0.0969 = 9.69\%$$

Also, $$SR = \frac{(V_1 - V_2)/V_d}{w_1 - w_2} = \frac{V_L - V_P}{V_d (w_L - w_P)} = \frac{1.35 - 1.22}{0.42 - 0.30} = \mathbf{1.083}$$

Example 3.19. *The mass specific gravity of a fully saturated specimen of clay having a water content of 40% is 1.88. On oven drying the mass specific gravity drops to 1.74. Calculate the specific gravity of clay and its shrinkage limit.* **(Engg. services Exam. 1992)**

Solution. The above problem is based on Example 3.7

Given, $G_{m,\ sat} = 1.88$; $w = 40\%$, $G_{m,\ dry} = 1.74$

For saturated soil, $$G_m = \frac{\gamma_{sat}}{\gamma_w} \quad \text{or} \quad \gamma_{sat} = G_m \cdot \gamma_w = 1.88 \times 9.81 = 18.443\ \text{kN/m}^3$$

Now, $$\gamma_{sat} = \frac{(G + e)\gamma_w}{1 + e}, \text{ where } e = e_{sat} = w_{sat} = G = 0.4\ G$$

$\therefore$ $$\gamma_{sat} = 18.443 = \frac{(G + 0.4G)}{1 + 0.4G} \cdot \gamma_w = \frac{1.4G}{1 + 0.4G} \times 9.81$$

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or $$\frac{1.4G}{1+0.4G} = \frac{18.443}{9.81} = 1.88, \quad \text{or} \quad G = 2.901$$

$$G_{m,\,dry} = 1.74 = \frac{\gamma_d}{\gamma_w}; \quad \text{Now } w_S = \frac{\gamma_w}{\gamma_d} - \frac{1}{G} = \frac{1}{1.74} - \frac{1}{2.901} = 0.23 = \mathbf{23\%}.$$

Example 3.20. *The values of liquid limit, plastic limit and shrinkage limit of a soil were reported as: $w_L = 60\%$, $w_P = 30\%$, $w_S = 20\%$*

If a sample of this soil at liquid limit has a volume of 40 cc and its volume measured at shrinkage limit was 23.5 cc, determine the specific gravity of the solids. What is the shrinkage ratio and volumetric shrinkage? **(Engg. Services Exam, 1996)**

Solution. This problem is exactly similar to example 3.8.

ΔM_w between (a) and $(b) = 0.6\,M_d - 0.2\,M_d = 0.4\,M_d$ ...(i)

Now, $V_1 - V_2 = 40 - 23.5 = 16.5$ cm^3. Hence $\Delta M_w = 16.5 \times \rho_w = 16.5$ g ...(ii)

From (i) and (ii), we get $M_d = 16.5/0.4 = 41.25$ g

$\therefore$ M_w in $(a) = 0.2\,M_d = 0.2 \times 41.25 = 8.25$ g

$\therefore$ V_w in $(b) = M_w/\rho_w = 8.25/1 = 8.25$ cm^3

$$V_s = V_2 - V_w = 23.5 - 8.25 = 15.25 \text{ cm}^3$$

Hence, $$\rho_s = \frac{M_d}{V_s} = \frac{41.25}{15.25} = 2.705; \quad \text{Hence } G_s = G = \frac{\rho_s}{\rho_w} = \frac{2.705}{1} = \mathbf{2.705}$$

Shrinkage ratio, $$SR = \frac{\rho_d}{\rho_w} = \frac{M_d/V_d}{1} = \frac{41.25}{23.5} = \mathbf{1.755.}$$

Also, volumetric shrinkage, $$VS = \frac{V_1 - V_d}{V_d} = \frac{40 - 23.5}{23.5} = 0.7021 = \mathbf{70.21\%}$$

Check: $VS = (w_1 - w_2)\,SR = (0.6 - 0.2) \times 1.755 = 0.702$

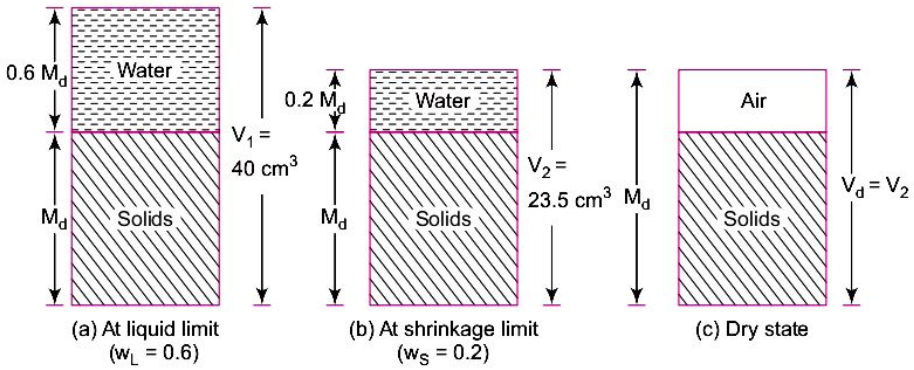

Fig. 3.31

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Example 3.21. *A soil deposit has a void ratio of 0.9. If the void ratio is reduced to 0.6 by compaction, find the percentage volume loss due to compaction.* **(Gate Exam. 1988)**

Solution.

Given: $e_1 = 0.9$; $e_2 = 0.6$. Let us use symbol 1 for initial condition and 2 for finally compacted condition.

Initially $\frac{V_{v_1}}{V_s} = e_1 = 0.9$; Hence $V_{v_1} = 0.9\ V_s \quad \therefore \quad V_1 = V_{v_1} + V_s = 0.9\ V_s + V_s = 1.9 V_s$ ...(1)

Finally $\frac{V_{v2}}{V_s} = e_2 = 0.6$; Hence $V_{v2} = 0.6\ V_s \quad \therefore \quad V_2 = V_{v2} + V_s = 0.6\ V_s + V_s = 1.6\ V_s$ ...(2)

$$\therefore \quad \% \text{ loss of volume} = \frac{1.9V_s - 1.6V_s}{1.9V_s} \times 100 = 15.79\%$$

3.21 LABORATORY EXPERIMENTS

Experiment 1: Determination of Water Content by Oven-Drying Method

Object and scope. The object of this test is to determine the water content of a soil sample in the Laboratory by oven-drying method. This experiment forms an essential part of many other laboratory experiments.

Material and equipment. (*i*) Non-corrodible air-tight containers, (*ii*) Balance of sufficient sensitivity to weigh soil samples to an accuracy of 0.04 per cent of the mass of the soil taken for the test. For fine grained soils the balance should have an accuracy of 0.01 g and for coarse-grained soil, it should have an accuracy of 0.1 g, (*iii*) Desiccator with any suitable desiccating agent, (*iv*) Thermostatically controlled oven, with interior of non-corroding material, to maintain temperature between 105°C to 110°C.

Test procedure

1. Clean the container and find its mass with lid (M_1).
2. Put the required quantity of the moist soil sample in the container and replace the lid. Take the mass M_2. The quantity of the soil sample to be taken depends upon its gradation and maximum size of particles. The following quantities are recommended for laboratory use:

Size of particles more than 90% passing	*Minimum quantity of soil specimen to be taken for testing (g)*
425-micron IS sieve	25
2 mm IS sieve	50
4.75 mm IS sieve	200
9.5 mm IS sieve	300
19 mm IS sieve	500
37.5 mm IS sieve	1000

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3. Keep the container in the oven with lid removed and maintain the temperature of the oven between 105°C to 110°C, for about 16 to 24 hours.
4. Take out the container, replace the lid and cool it in the desiccator. Find the mass M_3 of the container with lid and dried soil sample.

Tabulation of observations. The results of the test are recorded as illustrated in Table 3.7.

Table 3.7 Data and observation sheet for water content determination [IS: 2720 (Part II)—1973]

Sample No.... *Tested by.....*

1.	Container No.		42	
2.	Mass of container and wet soil	M_2 g	50.21	
3.	Mass of container and dry soil	M_3 g	48.05	
4.	Mass of container	M_1 g	20.42	
5.	Mass of dry soil	$(M_3 - M_1)$ g	27.63	
6.	Mass of water	$(M_2 - M_3)$ g	2.16	
7.	Water content $w = \frac{M_2 - M_3}{M_3 - M_1} \times 100\%$		7.8	

Calculations. The water content of the sample is calculated from the following expression (Eq. 3.1)

$$w = \frac{M_2 - M_3}{M_3 - M_1} \times 100\%$$

Precautions and discussions. (*i*) Oven-drying at 105°C to 110°C does not result in reliable water content values for soil containing gypsum or other minerals having loosely bound water of hydration or for soil containing significant amounts of organic material. Reliable water content values for these soils can be obtained by drying in oven at approximately 60°C to 80°C, (*ii*) The specimen should be dried in the oven to constant mass indicated by the difference between two consecutive mass of the container with the dried specimen, taken at suitable intervals after initial drying, being a maximum of 0.1% of the original mass of the soil specimen.

Reference to Indian Standard. IS: 2720 (Part II)–1973, Method of Test for Soils: Part II: Determination of Moisture Content.

Experiment 2: Determination of Specific Gravity of Soil by Density Bottle

Object and scope. The object of the test is to determine the specific gravity of soil fraction passing 4.75 mm sieve by density bottle.

Materials and equipment. (*i*) Density bottle of 50 mL or 100 mL capacity, with stopper having capillary hole at its centre, (*ii*) Constant temperature water bath maintaining a constant temperature of 27°C, (*iii*) Balance sensitive to 0.001 g, (*iv*) Vacuum source, (*v*) Wash bottle filled with deaired distilled water, (*vi*) Alcohol, and (*vii*) Ether.

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Test procedure

1. To clean and dry the density bottle, wash it thoroughly with distilled water and allow it to drain. Rinse the bottle with alcohol to remove the water and drain the alcohol. Then rinse the bottle with ether to remove the alcohol and drain the either. Turn the bottle upside down for a few minutes to permit the ether vapour to come out.
2. Find the mass of the empty cleaned bottle (M_1) accurate to 0.001 g with its stopper.
3. Take about 10 to 20 g of oven-dried soil sample, cooled in the desiccator and transfer it carefully to the density bottle. Find the mass (M_2) of the bottle and the soil, with the stopper.
4. Put about 10 mL deaired distilled water in the bottle, so that the soil is fully soaked. Leave it for a period of 2 to 10 hours.
5. Add more distilled water so that the bottle is about half full. Remove the entrapped air by subjecting the contents to a partial vacuum (air pressure not exceeding 100 mm of mercury).
6. Fill in the bottle completely, put the stopper on and place the bottle on the stand fitted in the constant temperature water bath. Keep it there for about one hour so that the temperature of soil and water in the bottle also reaches 27°C.
7. Take out the bottle and wipe it clean and dry from outside. Fill the capillary of the stopper with drops of distilled water, in case it is not full. Determine the mass of the bottle and its contents (M_3).
8. Empty the bottle and clean it thoroughly. Fill it with distilled water, put on the stopper and wipe the bottle dry from outside. Find the mass (M_4).
9. Repeat steps 3 to 8 and take two more determinations.

Tabulation of observations. The observations are tabulated as shown in Table 3.8

Table 3.8 Data and observation sheet for specific gravity determination by density bottle

Sample No. A–13

S. No.	*Determination*		*1*	*2*	*3*
1.	Density bottle No.		1	2	3
2.	Mass of density bottle	(M_1) g	31.352	32.560	32.125
3.	Mass of bottle + dry soil	(M_2) g	44.873	44.615	45.213
4.	Mass of bottle + soil + water	(M_3) g	140.563	140.468	140.260
5.	Mass of bottle + water	(M_4) g	132.052	132.875	132.024
	Specific gravity (G) Average specific gravity (at 27°C) = **2.70**		2.705	2.700	2.695

Calculations. The specific gravity of the soil is calculated from equation 3.3:

$$G = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

Experiment 3: Determination of Specific Gravity by Pycnometer

Object and scope. The object of the test is to determine the specific gravity of soil fraction passing 4.75 mm IS sieve by pycnometer.

Materials and equipment. (*i*) Pycnometer of about 900 mL capacity, with a conical brass cap and screwed at its top, (*ii*) Balance sensitive to 1 g, (*iii*) Glass rod, (*iv*) Deaired, distilled water.

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Test procedure

1. Clean the pycnometer and dry it. Find the mass (M_1) of the pycnometer, brass cap and washer, accurate to 1 g.
2. Take about 200 to 400 g of oven-dried soil and put it in the pycnometer. Find the mass of the pycnometer plus soil etc. (M_2).
3. Fill the pycnometer to half its height with distilled water and mix it thoroughly with glass rod. Add more water and stir it. Replace the screw top and fill the pycnometer flush with hole in the conical cap. Dry the pycnometer from outside, and find the mass (M_3).
4. Empty the pycnometer; clean it thoroughly and fill it with distilled water to the hole of the conical cap and find the mass (M_4).
5. Repeat steps 2 to 4 for two more determinations of specific gravity.

Tabulation of observations. See Table 3.9.

Table 3.9 Data and observation sheet for determination of specific gravity by pycnometer

S. No.	*Determination Number*		*1*	*2*	*3*
1.	Pycnometer No.		5	5	5
2.	Mass of Pycnometer	(M_1) g	652	652	652
3.	Mass of Pyc. + soil	(M_2) g	908	950	929
4.	Mass of Pyc. + soil + water	(M_3) g	1630	1656	1643
5.	Mass of Pyc. + water	(M_4) g	1470	1470	1470
6.	Specific Gravity Av. Specific Gravity: **2.66**		2.67	2.66	2.66

Calculations. The Specific Gravity is calculated from equation 3.3:

$$G = \frac{M_2 - M_1}{(M_2 - M_1) - (M_3 - M_4)}$$

Experiment 4: Determination of Water Content by Pycnometer

Object and scope. The object of the test is to determine the water content of a moist soil sample by pycnometer. For this determination, the specific gravity (G) of soil solids must be known.

Material and equipment. (*i*) Pycnometer of about 900 mL capacity, with a conical brass cap screwed at its top, (*ii*) Balance sensitive to 1 g, (*iii*) Glass rod.

Test procedure

1. Clean the pycnometer and dry it. Find the mass (M_1) of the pycnometer, brass cap and the washer, accurate to 1 g.
2. Put about 200 g to 400 g of wet soil sample in the pycnometer and find its mass with its cap and washer (M_2).
3. Fill the pycnometer to half its height and mix it thoroughly with the glass rod. Add more water and stir it. Replace the screw top and fill the pycnometer flush with the hole in the conical cap. Dry the pycnometer from outside and find its mass (M_3).

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4. Empty the pycnometer, clean it thoroughly, and fill it with clean water, to the hole of the conical cap and find its mass (M_4).

Tabulation of observations. See Table 3.10

Table 3.10 Data and observation sheet for water content determination by pycnometer

Specific Gravity = 2.66

1.	Mass of Pycnometer	(M_1) g	652
2.	Mass of Pyc. + moist soil	(M_2) g	891
3.	Mass of Pyc. + soil + water	(M_3) g	1608
4.	Mass of Pyc. + water	(M_4) g	1470
5.	Water content	(w) (%)	8

Calculations. Water content is calculated from equation 3.2.

$$w = \left[\frac{M_2 - M_1}{M_3 - M_4}\left(\frac{G-1}{G}\right) - 1\right] \times 100$$

Experiment 5: Determination of Dry Density and Dry Unit Weight by Water Displacement Method

Object and scope. The object of the test is to determine the dry density, dry unit weight and voids ratio of cohesive soil sample received from the field.

Materials and equipment. (*i*) A container about 30 cm diameter and 45 cm high, having an overflow outlet tube fitted in the upper half, (*ii*) A rubber tube with spring clip, fitted to the overflow outlet tube, (*iii*) Balance of 5 kg capacity, accurate to 1 g, (*iv*) Paraffin wax, of known specific gravity, (*v*) Container for water content determination, (*vi*) Thermostatically controlled oven, 105°C to 110°C, (*vii*) Glass beaker, 500 mL capacity, (*viii*) Cutting knife, (*ix*) Heater for melting wax and a brush.

Test procedure

1. With the help of knife, trim the sample to a more or less regular shape, avoiding re-entrant angles. Find the mass of the wax coated sample (M_1).
2. With the help of the brush, apply one coat of melted wax. When the coat is hardened, apply the second coat. Find the mass of the wax coated sample (M_2).
3. Fill fresh water in the container to overflow through the tubing. When the overflow stops, clamp the clip of the tubing.
4. Immerse the wax coated sample slowly into the container, taking care that it sinks completely. Put the beaker below the rubber tubing and open the clip. Collect all the overflow water. Find the mass of water so collected, accurate to 1 g.
5. Take out the wax coated sample, dry it from outside and remove the paraffin wax skin. Cut the sample into two pieces and keep a representative sample for water content determination.

Tabulation of observations. The test observations are tabulated as illustrated in Table 3.11.

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Table 3.11 Data and observation sheet for dry density determination by water displacement method

1.	Mass of the specimen	(M_1) (g)	768
2.	Mass of waxed specimen	(M_2) (g)	795
3.	Mass of wax coated	$(M_P = M_2 - M_1)$ (g)	27
4.	Density of paraffin wax	(GP) (g/mL)	0.908
5.	Volume of wax coated	$V_P = \frac{M_p}{G_p}$ (mL)	30
6.	Volume of water displaced	(V_w) (mL)	424
7.	Volume of specimen	$V = V_w - V_p$ (mL)	395
8.	Bulk density	$\rho = \frac{M_1}{V}$ (g/cm^3)	1.95
9.	Bulk unit weight	$\gamma = 9.81\ \rho$ (kN/m^3)	19.13
10.	Container No.		42
11.	Mass of container and wet soil	(g)	50.35
12.	Mass of container and dry soil	(g)	46.87
13.	Mass of container	(g)	20.42
14.	Mass of dry soil	(g)	26.45
15.	Mass of water	(g)	3.48
16.	Water content (w)	(Ratio)	0.13
17.	Dry density of specimen	$\rho_d = \frac{\rho}{1+w}$ (g/cm^3)	1.73
18.	Dry unit weight of specimen	$\gamma_d = 9.81\ \rho_d$ (kN/m^3)	16.97
19.	Specific gravity of soil particles	(Assumed)	2.70
20.	Voids ratio	$e = \frac{G\gamma_w}{\gamma_d} - 1$ (Ratio)	0.56

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Experiment 6: Determination of Field Density and Dry Unit Weight by Core Cutter Method

Object and scope. The object of the test is to determine the dry density and dry unit weight of soil in-place by the core cutter.

Materials and equipment. (*i*) Cylindrical core cutter of steel, 130 mm long and 10 cm internal diameter, with a wall thickness of 3 mm, bevelled at one end, (*ii*) Steel dolly, 2.5 cm high and 10 cm internal diameter, with wall thickness 7.5 mm fitted with a lip to enable it to be fitted on top of the core cutter, (*iii*) Steel rammer, having mass of 9 kg, (*iv*) Palette knife, (*v*) Steel rule, (*vi*) Spade or pickaxe or grafting tool, (*vii*) Straight edge, (*viii*) Balance accurate to 1 g, (*ix*) Container for water content determination.

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Test procedure

1. Measure the inside dimensions (accurate to 0.25 mm) of the core cutter and calculate its volume. Find the mass of the core cutter (without dolly), accurate to 1 g.

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2. Expose the small area, about 30 cm square, to be tested and level it. Put the dolly on the top of the core cutter and drive the assembly into the soil with the help of the rammer until the top of the dolly protrudes about 1.5 cm above the surface.
3. Dig out the container from the surrounding soil, and allow some soil to project from the lower end of the cutter. With the help of the straight edge, trim flat the end of the cutter. Take out the dolly and also trim flat the other end of the cutter.
4. Find the mass of cutter full of soil.
5. Keep some representative specimen of soil for water content determination.
6. Repeat the test at two or three locations nearby and get the average dry density.

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Tabulation of observations. See Table 3.12.

Table 3.12 Data and observation sheet for dry density determination by core cutter method

S. No.	Determination Number		1	2	3
1.	Mass of core cutter + wet soil	(g)	3027		
2.	Mass of core cutter	(g)	1058		
3.	Mass of wet soil	(g)	1969		
4.	Volume of core cutter	(mL)	1021		
5.	Bulk density $\rho = \frac{(3)}{(4)}$	(g/cm^3)	1.93		
6.	Bulk unit weight	$\gamma = 9.81\ \rho\ (kN/m^3)$	18.92		
7.	Container No.		8		
8.	Mass of container + wet soil	(g)	61.24		
9.	Mass of container + dry soil	(g)	56.36		
10.	Mass of container	(g)	20.76		
11.	Mass of dry soil	(g)	35.60		
12.	Mass of water	(g)	4.98		
13.	Water content	(Ratio)	0.14		
14.	Dry density	$\rho_d = \frac{\rho}{1+w}\left(g/cm^3\right)$	1.69		
15.	Dry unit weigh	$\gamma_d = \frac{\gamma}{1+w}\left(kN/m^3\right)$	16.60		

Reference to Indian Standard. IS: 2720–1975/88 (Part XXIX): Determination of dry density of soil in-place by the core cutter method.

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Experiment 7: Determination of Field Density and Dry Unit Weight by Sand Replacement Method

Object and scope. The object of the test is to determine the dry density of natural or compact soil, in-place, by the sand replacement method.

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Materials and equipment. (*i*) Sand pouring cylinder of about 3 litres capacity, mounted above a pouring cone and separated by a shutter cover plate and a shutter, (*ii*) Cylindrical calibrating container, 10 cm internal diameter and 15 cm internal depth, fitted with flange approximately 5 cm wide and about 5 mm thick, (*iii*) Glass plate, about 45 cm square and 1 cm thick, (*iv*) Metal tray with a central circular hole of diameter equal to the diameter of the pouring cone, (*v*) Tools for excavating hole, (*vi*) Balance accurate to 1 g, (*vii*) Container for water content determination, (*viii*) Clean, closely graded natural sand passing the 600-micron IS sieve and retained on the 300-micron IS sieve.

Test procedure

(A) *Determination of mass of sand filling the cone*

1. Fill the clean closely graded sand in the sand pouring cylinder up to a height 1 cm below the top. Determine the total initial mass of the cylinder plus sand (M_1). This total initial mass should be maintained constant throughout the tests for which the calibration is used.
2. Allow the sand of volume equivalent to that of the excavated hole in the soil (or equal to that of the calibrating container), to run out of cylinder by opening the shutter. Close the shutter and place the cylinder on glass plate.
3. Open the shutter and allow the sand to run out. Close the valve when no further movement of sand is observed. Remove the cylinder carefully. Weigh the sand collected on the glass surface. Its mass (M_2) will give the mass of sand filling the pouring cone. Repeat this step at least three times and take the mean mass (M_2). Put the sand back into the cylinder, to have the same constant mass (M_1).

(B) *Determination of bulk density of sand*

4. Determine the volume (V) of the calibrating container by filling it with water full to the brim and finding the mass of water. This volume should be checked by calculating it from the measured internal dimensions of the container.
5. Place the sand pouring cylinder concentrically on the top of the calibrating container, after being filled to constant mass (M_1). Open the shutter and permit the sand to run into the container. When no further movement of sand is seen, close the shutter. Remove the pouring cylinder and find its mass (M_3) to nearest gram.
6. Repeat step (5) at least thrice and find the mean mass M_3. Put the sand into the sand pouring cylinder.

(C) *Determination of dry density of soil in-place*

7. Expose about 45 cm square area of the soil to be tested and trim it down to level surface. Keep the tray on the level surface and excavate a circular hole of approximately 10 cm diameter and 15 cm deep and collect all the excavated soil in the tray. Find the mass (M) of the excavated soil.
8. Remove the tray, and place the sand pouring cylinder, so that the base of the cylinder concentrically covers the hole. The cylinder should have its constant mass M_1. Open the shutter and permit the sand to run into the hole. Close the shutter when no further movement of the sand is seen. Remove the cylinder and determine its mass (M_4).
9. Keep a representative sample of the excavated soil for water content determination.

Tabulation of observations. The observations are tabulated as illustrated in Table 3.13.

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Table 3.13 Data and observation sheet for determination of dry density by sand replacement method

(a) Determination of Mass of sand in the cone		
1. Mass of sand (+cylinder) before pouring	(M_1) (g)	10550
2. Mean mass of sand in cone	(M_2) (g)	445
(b) Determination of bulk density of sand		
3. Volume of calibrating container	(V) (mL)	1000
4. Mean mass of sand (+cylinder) after pouring	(M_3) (g)	8655
5. Mass of sand filling calibrating container $M' = M_1 - M_3 - M_2$	(g)	1450
6. Bulk density of sand	$\rho_s = \frac{(5)}{(3)}$ (g/cm^3)	1.45
(c) Bulk density and unit weight of soil		
7. Mass of wet soil from the hole	(M) (g)	2234
8. Mass of sand (+cylinder) after pouring in the hole	(M_4) (g)	8512
9. Mass of sand in the hole $M'' = M_1 - M_4 - M_2$	(g)	1593
10. Bulk density of soil	$\rho = \frac{M}{M''} \times \rho_s$ (g/cm^3)	2.03
11. Bulk unit weight of soil	$\gamma = 9.81\ \rho$ (kN/m^3)	19.91
(d) Water content determination		11
12. Container No.	(g)	62.48
13. Mass of container + wet soil	(g)	57.76
14. Mass of container + dry soil	(g)	21.43
15. Mass of container	(g)	36.33
16. Mass of dry soil	(g)	4.72
17. Mass of water	(g)	0.13
18. Water content (w)	(Ratio)	
19. Dry density	$\rho_d = \frac{\rho}{1+w}$ (g/cm^3)	1.8
20. Dry unit weight	$\gamma_d = \frac{\gamma}{1+w}$ (kN/m^3)	17.62

Reference to Indian standard: IS: 2720–1974/88 (Part XXVIII): Determination of dry density by sand eplacement method.

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Experiment 8: Determination of Grain-Size Distribution by Sieving

Object and scope. The object of this experiment is to determine grain-size distribution of coarse grained oil by sieving. The test covers both coarse sieve analysis (for gravel fraction) as well as fine sieve nalysis (for sand fraction).

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Material and equipment. (*i*) Balances accurate to 1 g and 0.1 g, (*ii*) Set of IS sieves: 100 mm, 75 mm, 22 mm, 10 mm, 4.75 mm, 2 mm, 1 mm, 600 microns, 425 microns, 300 microns, 212 microns, 150 microns and 75 microns size, (*iii*) Thermostatically controlled oven, (*iv*) Two or more large metal or plastic water tight trays, (*v*) Sieve brushes and a wire brush, (*vi*) Mortar with a rubber covered pestle, (*vii*) Mechanical sieve-shaker, and (*viii*) Riffler.

Test procedure

1. Using a riffler, take a representative sample of soil received from the field and dry it in the oven.
2. Weigh the required quantity of dried soil, keep it in a tray and soak it with water. Depending on the maximum size of material present in substantial quantities in the soil, the mass of soil sample taken for analysis may be as follows [IS : 2720 (Part IV)–1985].

Maximum size of material present in substantial quantities (mm)	*Mass to be taken for test (kg)*
75	60
40	25
25	13
19	6.5
12.5	3.5
10	1.5
6.5	0.75
4.75	0.40

3. Puddle the sample thoroughly in water and transfer the slurry to the 4.75 mm sieve which divides the gravel fraction from the sand fraction. Wash the slurry with jet of water. Collect the materials retained on 4.75 mm sieve and the material passing through it in separate containers. Keep the material retained on 4.75 mm sieve in the oven.
4. Wash the material passing through the 4.75 mm sieve through a 75-micron sieve so that silt and clay particles are separated from the sand fraction. Collect the material passing through 75-micron sieve and the material retained on it in separate containers, and keep them in the oven.
5. Sieve the dried material, retained on 4.75 mm sieve (step 3), through the following set of sieves: 100 mm, 19 mm, 10 mm and 4.75 mm by hand sieving. While sieving through each sieve, the sieve shall be agitated so that the sample rolls in irregular motion over the sieve. The material from the sieve may be rubbed, if necessary, with the rubber pestle in the mortar taking care to see that individual soil particles are not broken and re-sieved to make sure that only individual particles are retained. The mass of material retained on each sieve should be recorded.
6. Sieve the dried material, retained on 75-micron sieve (step 4), through the following set of sieves: 2 mm, 1 mm, 600 microns, 425 microns, 300 microns, 212 microns, 150 microns and 75 microns size. The set of sieves should be arranged one above the other and fitted to a mechanical sieve shaker such that the 2 mm sieve is at the top and the 75 microns sieve is at the bottom. A cover should be placed on the top of the 2 mm sieve, and a receiver should be placed below the 75 microns sieve. A minimum of 10 minutes sieving should be used. The soil fraction retained on

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each sieve should be carefully collected in containers and the mass of each fraction determined and recorded.

Alternatively, the material retained on 75 microns sieve (step 4), may not be dried, but be washed through a nest of sieves specified in step 6, nested in order of their fineness with the finest (75 micron) at the bottom. Washing should be continued until the water passing through each sieve is substantially clean. The fraction retained on each sieve should be emptied carefully without loss of material in separate containers and oven-dried. The oven-dried fraction should be weighed separate and their mass should be recorded.

7. The material passing 75 micron-sieve (step 4) may be used for sedimentation analysis. (See Experiments 9 or 10).

Tabulation of observations. The test observations and results are recorded as illustrated in Table 3.14.

Calculations. The percentage of soil retained on each sieve is calculated on the basis of total mass of soil sample taken and from these results the percent passing through each of the sieve is calculated, as illustrated in Table 3.14.

Table 3.14 Data and observation sheet for sieve analysis

Sample No. 108 *Mass retained on 4.75 mm sieve : 167 g*
Mass of dry soil sample: 1000 g Mass passing through 75 micron size : 77 g

S. No.	*IS Sieve*	*Particle size D (mm)*	*Mass retained (g)*	*% retained*	*Cumulative % retained*	*Cumulative % finer (N)*
1.	100 mm	100 mm	–	–	–	100
2.	75 mm	75 mm	–	–	–	100
3.	19 mm	19 mm	33	3.3	3.3	96.7
4.	10 mm	10 mm	49	4.9	8.2	91.8
5.	4.75 mm	4.75 mm	85	8.5	16.7	83.3
6.	2 mm	2 mm	140	14.0	30.7	69.3
7.	1 mm	1 mm	160	16.0	46.7	53.3
8.	600 micron	0.6 mm	142	14.2	60.9	39.1
9.	425 micron	0.425 mm	118	11.8	72.7	27.3
10.	300 micron	0.300 mm	82	8.2	80.9	19.1
11.	212 micron	0.212 mm	56	5.6	86.5	13.5
12.	150 micron	0.150 mm	35	3.5	90.0	10.0
13.	75 micron	0.075 mm	23	2.3	92.3	7.7

Note 1. Dry Sieve Analysis. If the soil sample contains little or no fines (passing 75 micron sieve), dry sieve analysis may be carried out. The gravel fraction and sand fraction are first separated by dry sieving through 4.75 mm sieve. The material retained on 4.75 mm size is further sieved through the following set of sieves: 100 mm, 75 mm, 19 mm, 10 mm and 4.75 mm sieves. The material passing 4.75 mm sieve is sieved through the following set of sieves: 2 mm, 1 mm, 600 micron, 425 micron, 300 micron, 212 micron, 150 micron and 75 micron sizes.

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Note 2. The permissible maximum mass of sample on the 200 mm diameter sieves should be as follows:

IS Sieve Designation	*Maximum mass of sample (g)*
600 micron	160
200 micron	55
75 micron	25

Note 3. If the soil contains greater amount of fines (finer than 75 micron size), the dried soil fraction passing through 4.75 mm sieve (step 6) should be soaked in water containing two grams of sodium hexametaphosphate (or one gram of sodium hydroxide and one gram of sodium carbonate) per litre of water. The soaked specimen should then be washed thoroughly over the nest of sieves specified in step 6.

Experiment 9: Determination of Grain Size Distribution by Hydrometer

Object and scope. The object of this experiment is to determine the distribution of particle size, finer than 75 micron sieve, by sedimentation analysis, using a density hydrometer, and then to plot the grain-size distribution curve.

Materials and equipment. (*i*) Density hydrometer conforming to IS: 3104–1965, (*ii*) Two glass measuring cylinders of 1000 mL capacity with ground glass or rubber stoppers about 7 cm diameter and 33 cm high marked at 1000 mL volume, (*iii*) Thermometer to cover the range 0 to 50°C, accurate to 0.50°C, (*iv*) Water bath or constant temperature room (optional), (*v*) Stirring apparatus, (*vi*) 75 micron sieve, (*vii*) Balance accurate to 0.01 g, (*viii*) Stop watch, (*ix*) Wash bottles containing distilled water, (*x*) Glass rod, about 15 to 20 cm long and 4 to 5 mm in diameter, (*xi*) Reagents: Hydrogen peroxide, Hydrochloric acid N solution and Sodium hexametaphosphate, (*xii*) Conical flask of 1000 mL capacity, (*xiii*) Buchner or Hirch funnel, (*xiv*) Filter flask, (*xv*) Measuring cylinder of 100 mL capacity, (*xvi*) Filter paper and blue litmus paper.

Test procedure

(A) *Calibration of hydrometer*

1. *Determination of volume of the hydrometer bulb* (V_h). Pour about 800 mL of water in the 1000 mL measuring cylinder and note the reading at the water level. Immerse the hydrometer in water and note the water reading. The difference between the two readings is recorded as the volume of the hydrometer bulb plus the volume of that part of the stem which is submerged. For practical purposes, the error due to the inclusion of this stem volume may be neglected.

   Alternatively, weigh the hydrometer to the nearest 0.2 g. This mass in grams is recorded as the volume of the hydrometer in mL. This includes the volume of the bulb plus the volume of the stem. For practical purposes the error due to the inclusion of the stem may be neglected.
2. In order to find the area of cross-section (A) of the measuring cylinder in which the hydrometer is to be used, measure the distance, in cm, between two graduations of the cylinder. The cross-sectional area (A) is then equal to the volume included between the two graduations divided by the distance between them.
3. Measure the distance (h) from the neck to the bottom of the bulb, and record it as the height of the bulb.

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about 15 minutes. The sodium hexametaphosphate solution is prepared by dissolving 33 g of sodium hexametaphosphate and 7 g of sodium carbonate in distilled water to make one litre of solution. This solution is unstable and should be freshly prepared approximately once in a month.

2. Transfer the soil suspension to the 75 micron IS sieve placed on a receiver and wash the soil on this sieve using jet of distilled water from a wash bottle. The amount of distilled water used during this operation may be about 500 mL.
3. Transfer the soil suspension passing the 75 micron IS sieve to the 1000 mL measuring cylinder, and add more distilled water to make the volume to exactly 1000 mL in the cylinder.
4. Collect the material retained on 75 micron sieve and put it in the oven for drying. Determine the dry mass of soil retained on 75 micron sieve.

(D) *Sedimentation test with hydrometer*

1. Insert a rubber bung or any other suitable cover on the top of the 1000 mL measuring cylinder containing the soil suspension (obtained in step 3 above) and shake it vigorously end over end. Stop shaking and allow it to stand. Immediately, start the stop watch, and remove the top cover from the cylinder.
2. Immerse the hydrometer gently to a depth slightly below its floating position and then allow it to float freely. Take the hydrometer readings after periods of 1/2, 1, 2 and 4 minutes. Take out the hydrometer, rinse it with distilled water and allow it to stand in a jar containing distilled water at the same temperature as that of the test cylinder.
3. The hydrometer is re-inserted in the suspension and readings are taken after periods of 8, 15 and 30 minutes; 1, 2 and 4 hours after shaking. The hydrometer should be removed, rinsed and placed in the distilled water after each reading. After the end of 4 hours, readings should be taken once or twice within 24 hours.
4. ***Composite correction.*** In order to determine the composite correction, put 100 mL of dispersing agent solution in another 1000 mL measuring cylinder and make it to 1000 mL by adding distilled water. The cylinder should be maintained at the same temperature as that of the test cylinder containing soil specimen. Insert the hydrometer in this comparison cylinder containing distilled water and the dispersing agent and take the reading corresponding to the *top* of the meniscus. The *negative* of the hydrometer reading so obtained gives the composite correction (C). The composite correction is found before the start of the test, and also at every time intervals of 30 minutes, 1 hour, 2 hours and 4 hours after the beginning of the test, and afterwards, just after each hydrometer reading is taken in test cylinder.
5. The temperature of the suspension should be observed and recorded once during the first 15 minutes and then after every subsequent reading.

Tabulation of observations. The test observations and results are recorded as illustrated in Table 3.15. The observations for the calibration of the hydrometer have been recorded in Table 3.3.

Particle size distribution curve. The results of the above table are plotted to get a particle size distribution curve with percentage finer N as the ordinate and the particle diameter (D) on logarithmic scale as abscissa.

Calculations.

(1) The loss in mass in pre-treatment of the soil in percentage is calculated from the following expression:

$$P = \left(1 - \frac{M_b}{M_d}\right) 100$$

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where, P = loss in mass in percentage.
M_d = mass of dry soil sample taken from the soil passing 2 mm sieve.
M_b = mass of the soil after pre-treatment.

(2) The diameter of the particles in suspension at any sampling time t is calculated from equation. 3.9:

$D = 10^{-5} F \sqrt{\frac{H_e}{t}}$, where factor F is taken either from Table 3.2 or from Fig. 3.4.

(3) The percentage finer N' based on the mass M_d is calculated from equation 3.19

$$N' = \frac{100\,G}{M_d\,(G-1)} R\,.$$

(4) The percentage finer N based on total mass of dry soil sample (M) is obtained from the relation:

$$N = N' \times \frac{M'}{M}$$

where, M' = cumulative mass passing 2 mm sieve
(out of which the soil weighing M_d was taken for the hydrometer analysis).

Table 3.15 Data and observation sheet for hydrometer analysis

1. Sample No. A—102.
2. Mass of dry soil sample (M) = 500 g.
3. Mass of fraction passing 2 mm sieve (M') = 500 g.
4. Mass of dry sample taken from minus 2 mm sieve (M_d) = 50 g.
5. Specific gravity of soil particles of minus 75 micron: G = 2.67.
6. Hydrometer No. 12: Sedimentation jar No. 36; Meniscus correction: C_m = + 0.5.

Date	*Time*	*Elapsed time t (min)*	*Hydro-meter Reading Rh′*	*Temp °C*	*Correc-tion C*	*$R_h = R_h' + C_m$*	*Eff. depth H_e (cm)*	*Factor F*	*Particle size D (mm)*	*$R = R_h' + C$*	*% finer (N′) based on M_d*	*% finer (N) based on whole $N = N' \times \frac{M'}{M}$*
12-4-76	08-27	1/2	20.25	26.5	– 0.5	20.75	12.2	1254	0.061	19.75	63	58.0
		1	19.00	26.5	– 0.5	19.50	12.7	1254	0.045	18.50	59	54.3
		2	17.50	26.5	– 0.5	18.00	13.3	1254	0.033	17.00	54.4	50.0
		4	15.50	26.5	– 0.5	16.00	14.0	1254	0.023	15.00	48	44.1
		8	15.50	26.5	– 0.5	16.00	14.0	1254	0.0165	15.00	48	44.1
		15	11.50	27.0	– 0.5	12.00	15.5	1252	0.0125	11.00	35	32.2
	08-57	30	9.50	27.5	– 0.5	10.00	16.3	1245	0.0092	9.00	29	26.7
	09-30	63	7.25	29	0	7.75	17.1	1224	0.0062	7.25	23	21.2
	10-20	113	6.00	30	+ 0.5	6.50	17.6	1210	0.0047	6.50	21	19.3
	12-27	240	4.00	31.5	+ 1	4.50	18.3	1190	0.0033	5.00	16	14.7
	17-00	513	2.25	31.5	+ 2	2.75	19.0	1190	0.0023	3.25	10.4	9.6
	07-55	1408	2.25	29	0	2.75	19.0	1224	0.0015	2.25	6.5	5.8

(5) The uniformity coefficient C_u and coefficient of curvature C_c are calculated respectively from equations 3.21 and 3.22.

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Note 1. If the soil sample does not contain calcium compounds or soluble salts, pre-treatment should be omitted.

Note 2. Sodium hexametaphosphate has been found to be ineffective when dealing with certain highly flocculated soils. In such cases dispersion may be carried out by adding *N*-sodium hydroxide solution at the rate of 4 mL per 10 g of soil.

Note 3. Asymmetrical heating of the suspension cause convection currents which affect the sedimentation process. The suspension should therefore, be kept out of direct sunlight and away from any local source of heat. Evaporation should be retarded by keeping a cover on the measuring cylinder between the readings.

Note 4. The specific gravity G used in the above expression should be determined for the fraction of the sample passing 75 micron sieve.

Experiment 10: Determination of Grain Size Distribution by Pipette

Object and scope. The object of the experiment is to determine the distribution of particle size, finer than 75 micron sieve, by sedimentation analysis, using a sampling pipette, and then to plot the grain size distribution curve.

Materials and equipment. (*i*) A sampling pipette fitted with a pressure and suction inlet having a capacity of approximately 10 mL fitted on a sliding panel, (*ii*) Glass sedimentation tubes (2 Nos.), 50 mm diameter and approx. 350 mm long marked at 500 mL volume, with rubber bungs to fit, (*iii*) Weighing bottles with round stopper, (*iv*) Constant temperature bath, capable of being maintained at 27°C ± 0.1°C, (*v*) Stirring apparatus, (*vi*) 75 micron sieve, (*vii*) Balance to weigh up to 0.001 g, (*viii*) Stop watch, (*ix*) Desiccator, (*x*) Evaporating dish, (*xi*) Conical beaker of capacity 650 mL or 1 litre with a cover glass to fit, (*xii*) Funnel and filter flask, (*xiii*) Measuring cylinder of 1000 mL capacity, (*xiv*) Pipette of 25 mL capacity, (*xv*) Glass filter funnel of 10 cm diameter, (*xvi*) Glass rod, (*xvii*) Filter papers, Blue litmus paper, (*xviii*) Thermometer to cover the range 0°C – 50°C, (*xix*) Reagents: Hydrogen peroxide, Hydrochloric acid *N* solution and Sodium hexametaphosphate, (*xx*) Wash bottles containing distilled water.

Test procedure

(A) *Pre-treatment of soil*

(1) Weigh accurately to 0.001 g, a quantity M_d of oven-dried soil sample passing the 2 mm IS sieve. The quantity M_d varies from 30 g for sandy soil to about 12 g with clay soil.

(2) Place the soil in a 650 mL conical beaker and add 55 mL of distilled water to it. Boil the suspension gently until the volume is reduced to 40 mL. After cooling, add 75 mL of hydrogen peroxide and allow to stand overnight.

(3) Next morning, gently heat the suspension, avoiding frothing over and agitating the contents frequently. As soon as vigorous frothing has subsided and when there is no further reaction by the addition to fresh hydrogen peroxide, the volume is reduced to about 30 mL by boiling.

(4) If the soil contains insoluble calcium compounds, add about 10 mL of hydrochloric acid to the cooled mixture. The solution is stirred with a glass rod for a few minutes and allowed to stand for one hour or longer periods if necessary. The treatment should be continued till the solution gives acid reaction to litmus. More acid may be required for soil containing considerable amount of calcium salts.

(5) Filter the mixture using the Buchner funnel and wash it with warm water until the filtrate shows no acid reaction to litmus. Transfer the damp soil on the filter paper and funnel to the

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evaporating dish using a jet of distilled water. Place the dish and its contents to the oven. Find the mass M_b, of the oven-dried soil remaining after pre-treament, accurate to 0.001 g.

Note. In case of soils containing no calcium compounds or soluble salts, and having a low organic content (less than 2%) the pre-treatment prescribed above may be omitted and the dispersing agent is added direct to the soil taken for analysis.

(B) *Dispersion of soil*

1. To the oven-dried soil in the evaporating dish (step 5 above) add 25 mL of sodium hexametaphosphate solution, together with about 25 mL of distilled water, and the soil is brought into suspension by stirring with a glass rod. Warm the mixture gently for about 10 minutes and transfer it to the cup of the mechanical mixer using a jet of distilled water and stir it well for about 15 minutes. The sodium hexametaphosphate solution is prepared by dissolving 33 g of sodium hexametaphosphate and 7 g of sodium carbonate in distilled water to make one litre solution.
2. Transfer the soil suspension to the 75 micron IS sieve placed on a receiver and wash the soil on this sieve using jet of distilled water from a wash bottle. The amount of distilled water used during this operation should not exceed 150 mL.
3. Transfer the soil suspension passing 75 micron sieve to the 500 mL sedimentation tube, and add more distilled water to make the volume to exactly 200 mL. Keep the tube in the constant temperature bath.
4. Collect the material retained on 75 micron sieve and put it in the oven for drying. Determine the dried mass accurate to 0.001 gram.

(C) *Sedimentation test with pipette*

1. ***Calibration of sampling pipette.*** Clean the sampling pipette and dry it. Immerse the nozzle in distilled water and close the cock of the bulb. Open the three way cock to connect the pipette to the suction outlet. With the help of rubber tubing attached to the suction outlet, suck up the water to rise above the three way cock. Close three way cock and remove the pipette from water. Remove the surplus water in the small bulb by connecting it to the water outlet through the three way cock. Now the water contained in the pipette and the three way cock is discharged into a glass weighing bottle. Determine the mass of water in grams, which will give the volume of the pipette.
2. The sedimentation tube containing the soil suspension, and with rubber bung inserted at its top, is shaken end over end. The tube is then replaced in the bath, and stop watch is simultaneously started. The rubber bung is carefully removed.
3. (*a*) Fill the upper bulb with distilled water after closing the cock below it. To collect the sample at any instant t after the start of the test, lower the pipette into the suspension until its end is 10 cm below the top of suspension, just 15 seconds before the sample is due to be taken. Connect the three way cock to the suction tube and suck the sample slowly till the sample comes just above the three way cock. This operation should take about 10 seconds.

   (*b*) Take out the pipette from the suspension. Drain out the excess sample collected in the small bulb by connecting it to the water outlet through the three ways cock. Open the cock below the bulb to allow the distilled water to run from the bulb to the water outlet until no solution remains in the system.

   (*c*) To collect the soil suspension contained in the pipette, insert glass weighing bottle below the nozzle of the pipette and open the three way stop cock so that the contents are drained into the bottle. Any suspension left on the inner walls of the pipette is washed into

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the weighing bottle by allowing distilled water from the upper bulb to run into the pipette through the intermediate small bulb.

4. Samples are drawn, as explained in step 3 above, after 0.5, 2, 4, 8, 15 and 30 minutes, and 1, 2, 4, 8, 16 and 24 hours, reckoned from the commencement of the test.
5. The weighing bottles, into which soil suspension are collected at various time intervals, are placed into the oven and the samples are evaporated to dryness. After cooling in a desiccator the weighing bottles, the mass of contents are found to the nearest 0.001 g.

Tabulation of observations. The test observations and results are recorded as illustrated in Table 3.16.

Table 3.16 Data and observation sheet for pipette analysis

Soil No. B/31
Specific gravity: 2.70
Volume of suspension: 500 mL
Mass of dispersing agent in the suspension: 1 g

Dry mass of soil taken: 20 g
Mass retained on 75 micron sieve: 3.96 g
% finer than 75 micron: 80%

Date and Time	*Elapsed time (min.)*	*Temp. (°C)*	*Factor F*	*D (mm)*	*Bottle No.*	*Bottle + Dry mass*	*Mass of Bottle*	*Dry mass of soil*	M_D	*N%*
12.1.73	$\frac{1}{2}$	20	1347	0.0604	2	22.942	22.610	0.332	0.0332	78.0
11.32										
	2	20	1347	0.0300	4	22.224	21.914	0.310	0.0310	72.5
	4	20	1347	0.0213	5	22.681	22.397	0.284	0.0284	66.0
	8	20	1347	0.0151	6	22.679	22.418	0.261	0.0261	60.3
	15	20	1347	0.0110	7	22.804	25.571	0.233	0.0233	53.3
12.02	30	20	1347	0.0078	8	22.987	22.773	0.214	0.0214	48.5
	60	20	1347	0.0055	9	22.610	22.414	0.196	0.0196	44.0
	120	21	1331	0.0039	10	22.315	22.142	0.173	0.0173	38.3
	240	21	1331	0.0034	11	22.416	22.269	0.147	0.0147	33.7
13.1.73	1440	20	1347	0.0012	12	22.570	22.483	0.087	0.0087	16.8
11.32										

Particle size distribution curve. The results of Table 3.16 are plotted to get a particle size distribution curve with N as ordinate and D, on logarithmic scale, as abscissa.

Calculations.

1. The loss of mass if pre-treatment of the soil in percentage is calculated from the following expression:

$$P = \left(1 - \frac{M_b}{M_d}\right)100$$

where, P = loss in mass in percentage.
M_d = mass of dry soil sample taken from the soil passing 2 mm sieve.
M_b = mass of the soil after pre-treatment.

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2. The diameter of the particle in suspension at any sampling time t is calculated from equation 3.9.

$$D = 10^{-5} F \sqrt{\frac{H_e}{t}} = 10^{-5} F \sqrt{\frac{10}{t}}$$

where factor F is taken either from Table 3.2 or from Fig. 3.4.

3. The percentage finer N' based on M_d is calculated from equaiton 3.13.

$$N = \frac{M_D - \frac{m}{V}}{M_d / V} \times 100$$

where, V = Total volume of suspension = 500 mL

m = Mass of dispersing agent present in the total suspension of volume

$$V = \frac{33+7}{1000} \times 25 = 1 \text{ g}$$

M_d = Mass of dry soil sample taken from the soil passing 2 mm sieve

M_D = Dry mass of the sample in the weighing bottle per mL of the suspension

$$= \frac{\text{Dry mass of sample in the weighing bottle}}{\text{Volume } (V_p) \text{ of the pipette}}$$

4. The percentage finer N based on total mass of dry soil sample (W) is obtained from the relation:

$$N = N' \times \frac{M'}{M}$$

where, M' = Cumulative mass passing 2 mm sieve (out of which the mass M_d was taken for the hydrometer analysis).

5. The uniformity coefficient C_u and the coefficient of curvature C_c are calculated from equations 3.21 and 3.22 respectively.

Experiment 11: Determination of Liquid Limit of Soil

Object and scope. The object of the test is to determine the liquid limit of the soil sample using Casagrande type mechanical liquid limit apparatus.

Material and equipment. (*i*) Mechanical liquid limit device (Casagrande type) consisting of a brass cup and carriage, mounted on base Micarta Number 221 A, (*ii*) Grooving tool '*a*' (Casagrande or B.S. tool) and grooving tool '*b*' (ASTM tool), (*iii*) Porcelain evaporating dish, about 12 cm in diameter or marble plate 30 cm square, (*iv*) Flexible spatula, with blade about 8 cm along and 2 cm wide, (*v*) Balance to weigh to 0.01 g, (*vi*) Airtight containers to determine water content, (*vii*) Thermostatically controlled oven to maintain temperature between 105°C to 110°C, (*viii*) Wash bottle containing distilled water, (*ix*) 425 micron sieve, (*x*) Desiccator.

Test procedure

1. By means of the gauge on the handle of the grooving tool and the adjustment plate, adjust the height through which the cup is lifted and dropped so that the point on the cup which comes in contact with the base falls through exactly one centimetre when the handle is rotated by one revolution. When the adjustment is complete, secure the adjustment plate by tightening its screws.

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2. Take about 120 g of the specimen, passing through the 425 micron sieve, and mix it thoroughly with distilled water in the evaporating dish or on the marble plate so that uniform paste is formed. Leave the soil for sufficient time so that water may permeate throughout the soil mass. In the case of fat clays, this maturing time may be up to 24 hours. For an average soil, thorough mixing for about 15 to 30 minutes may be sufficient. The amount of water to be added depends on the type of soil, and is a matter of experience.
3. Take a portion of the paste with the spatula and place it in the centre of the cup so that it is almost half filled. Level off the top of the wet soil symmetrically with the spatula, so that it is parallel to the rubber base and the maximum depth of the soil is 1 cm.
4. With the help of grooving tool '*a*', the paste in the cup is divided along the cup diameter (through the centre line of the cam follower), by holding the tool normal to the surface of the cup and drawing it firmly across. Thus, a V-shaped gap, 2 mm wide at the bottom and 11 mm at the top and 8 mm deep will be formed. However, in the case of sandy soils tool 'a' does not form a neat groove and hence tool '*b*' is used.
5. Turn the handle of the apparatus at the rate of 2 revolutions per seconds, until the two parts of the soil come in contact with the bottom of the groove along a distance of 10 mm. Record the number of blows required to cause the groove close for approximate length of 10 mm.
6. Collect a representative slice of soil by moving the spatula widthwise from one edge to the other edge of the soil cake at right angles to the groove, including the portion of the groove in which the soil flowed together, and put it in an airtight container. Its water content is later determined by method of Expt. 1.
7. Remove the remaining soil from the cup and mix it with the soil left earlier on the marble plate (or evaporating dish). Change the consistency of the mix by either adding more water or leaving the soil paste to dry, as the case may be, and repeat steps 3, 4, 5 and 6. Note the number of revolutions to close the groove and keep the soil for water content determination. These operations are repeated for 3 or 4 more additional trials. The soil paste in these operations should be of such consistency that number of revolutions or drops to close the groove is 25 ± 10. The test should always proceed from the dryer (more blows) to the wetter (less blows) condition of the soil.

Tabulation of observations. The observations are tabulated as illustrated in Table 3.17.

Table 3.17 Data and observation sheet for liquid limit determination

Soil No. C-108 *Date*

Determination No.		***1***	***2***	***3***	***4***
Number of blows		34	23	18	12
Container number		130	123	128	132
Mass of container +wet soil	(g)	38.86	46.63	60.36	43.43
Mass of container +dry soil	(g)	34.91	39.59	49.02	37.22
Mass of water	(g)	3.95	7.04	11.34	6.21
Mass of container	(g)	26.08	25.30	26.95	66.06
Mass of oven dry soil	(g)	8.83	14.29	22.07	11.16
Water content	(%)	44.6	49.4	51.4	55.6
Liquid limit (from graph) = 485.5%. Flow index (from graph) = 227					

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Calculations and results. Plot the flow curve (Fig. 3.11) with water content as the ordinate and log of number of blows as abscissa. The water content corresponding to 25 blows is taken as the liquid limit of the soil.

From the graph, find the difference between the water contents for the blows differing by one log cycle. To do this, extend the flow curve at either ends so as to intersect the ordinates corresponding to 10 and 100 blows, and note the numerical difference in water contents at 10 and 100 blows.

Note 1. Some soils tend to slide on the surface of the cup instead of 'flowing'. In such a case, the test should be repeated. If slipping still occurs, the test is not applicable, and a note to this effect should be made.

Note 2. It has been observed that heating a soil alters its liquid limit. Generally, liquid limit of natural soil is required. Drying may alter soil by causing the particles to subdivide or agglomerate by driving off absorbed water which is not completely regained on rewetting, or by effecting a chemical change in any organic matter in the soil. *Hence the soil should not be oven-dried before the commencement of the test.*

Note 3. Distilled water should be used in order to minimise the possibility of ion exchange between the soil and any impurities in the water.

Note 4. It is better if the test is performed in a humid room. Humid atmosphere minimizes surface drying of the soil while it is being tested.

Reference to Indian Standard. IS: 2720 (Part V)–1985. Determination of liquid and plastic limits.

Experiment 12: Determination of Plastic Limit of Soil

Object and scope. The object of the test is to determine the plastic limit of soil sample and then to calculate plasticity index, toughness index, liquidity index and consistency index of the soil.

Materials and equipment. The materials and equipment of experiment 11 plus a rod of 3 mm diameter.

Test procedure

1. Take about 20 g of air dried soil from the thoroughly mixed portion of the material passing 420 micron IS sieve. Mix it on the marble plate with sufficient distilled water to make it plastic enough to be shaped into a ball. Leave the plastic soil mass for some time to mature. In some fat clays, the plastic soil mass may be left to stand for 24 hours to allow water to permeate throughout the soil mass.
2. Take about 8 g of the plastic soil, make a ball of it, and roll it on the marble (or glass) plate with the hand with just sufficient pressure to roll the mass into a thread of uniform diameter throughout its length. When the diameter of the thread has decreased to 3 mm, the specimen is kneaded together and rolled out again. Continue the process until thread just crumbles at 3 mm diameter.
3. Collect the crumbled soil thread in the airtight container and keep it for water content determination. The test is repeated twice more. Thus three readings are obtained for the determination.
4. Also, determine the natural water content of the soil sample obtained from the field.

Tabulation of observations. The observations are tabulated as illustrated in Table 3.18.

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Table 3.18 Data and observation sheet for plastic limit determination

Soil No. C-108

Determination Number		*1*	*2*	*3*
Container Number		33	19	22
Mass of container + wet soil	(g)	31.29	30.39	30.87
Mass of container +oven dry soil	(g)	29.75	28.75	29.27
Mass of water	(g)	1.54	1.64	1.60
Mass of container	(g)	24.01	22.79	23.42
Mass of dry soil	(g)	5.74	5.96	5.85
Water content	(%)	26.8	27.5	27.3
Plastic limit = 27.2%		Natural water content of field soil = 32.8%		

Calculations

1. Plastic limit = 27.2
2. Plasticity index = 48.5 – 27.2 = 21.3 (since $w_L = 48.5\%$)
3. Liquidity index, $I_L = \dfrac{w - w_P}{I_P} = \dfrac{32.8 - 27.2}{21.3} = 0.263$
4. Consistency index, $I_C = \dfrac{w_L - w}{I_P} = \dfrac{48.5 - 32.2}{21.3} = 0.737$
5. Flow Index, $I_f = 22.7$ (from Experiment 11)
6. Toughness Index, $I_T = \dfrac{I_P}{I_f} = \dfrac{21.3}{22.7} = 0.94$

Summary of results. Given in Table 3.19.

Table 3.19

Liquid Limit w_L	*Flow Index* I_f	*Plastic Limit* w_P	*Plasticity Index* I_P	*Liquidity Index* I_L	*Consistency Index* I_C	*Toughness Index* I_T
48.5	22.7	27.2	21.3	0.263	0.734	0.94

Note 1. Undue pressure or oblique rolling, which might result in mechanical breaking of the soil thread, should not be used. The soil thread must crumble due to its decrease in water content only. The rate of rolling should be between 80 and 90 strokes per minute counting a stroke as one complete motion of the hand forward and back to the starting position again.

Note 2. In the case of sandy soils plastic limit should be determined first. When the plastic limit cannot be determined, the plasticity index should be reported as NP (non-plastic).

Note 3. When the plastic limit is equal to or greater than the liquid limit, the plasticity index should be reported as zero.

Reference to Indian Standard. IS : 2720 (Part V)–1985. Determination of liquid and plastic limits.

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Experiment 13: Determination of Shrinkage Factors of Soil

Object and scope. The object of the experiment is to determine shrinkage limit, shrinkage ratio and volumetric shrinkage.

Materials and equipment. (*i*) Evaporating dish (2 Nos.) of procelain, about 12 cm in diameter with flat bottom, (*ii*) Shrinkage dish, of non-corroding metal, having flat bottom and 45 mm in diameter and 15 mm in height internally (3 Nos.), (*iii*) Glass cup, 50 to 55 mm in diameter and 25 mm in height, the top rim of which is ground smooth and level, (*iv*) Glass plates, (2 Nos.), each 75 × 75 mm. One plate should be of plain glass and the other should have three metal prongs, (*v*) Spatula, (*vi*) Straight edge, (*vii*) 425 micron IS sieve, (*viii*) Balances, sensitive to 0.1 g and 0.01 g, (*ix*) Oven, thermostatically controlled (105°C–110°C), (*x*) Mercury, (*xi*) Desiccator, (*xii*) Wash bottle containing distilled water.

Test procedure (for Remoulded sample)

1. ***Preparation of soil paste.*** Take about 100 g of soil sample from a thoroughly mixed portion of the material passing 425 mircon IS sieve.
2. Place about 30 g of the above sample in evaporating dish and mix it thoroughly with distilled water. Water added should be sufficient to fill the voids in the soil completely and make the soil pasty enough to be readily worked into the shrinkage dish without entrapping air bubbles. In the case of plastic soils, the water content of the paste may exceed its liquid limit by as much as 10%, while for friable soil the amount of water required to obtain the desired consistency may be equal to or slightly greater than the liquid limit.
3. ***Determination of mass and volume of the shrinkage dish.*** Clean a shrinkage dish and determine its mass accurate to 0.1 g. To determine its volume, place the shrinkage dish in the evaporating dish and fill it to overflowing the mercury. Remove die excess mercury by pressing the plain glass plate firmly on its top, taking care that no air is entrapped. Wipe off, carefully, any mercury which may be adhering to the outside of the shrinkage dish. Carefully transfer the mercury of the shrinkage dish to die other evaporating dish, and then determine the mass of mercury, accurate to 0.1 g. The mass of mercury divided by its density would give the volume of the shrinkage dish, which is also the volume of wet soil pat.
4. ***Filling the shrinkage dish soil pat.*** Coat the inside of the shrinkage dish with a thin layer of silicon grease or vaseline. In the centre of dish, place the soil paste, about one-third of the volume of die dish, with the help of spatula. Tap the dish gently on a firm surface, cushioned with layers of blotting paper or rubber sheet and allow the paste to flow towards the edges. Place another equal instalment of the paste in the dish and make it flow toward the edges by tapping. Tapping should be continued till the paste is compacted and all the entrapped air is brought to the surface. Repeat the process till the dish is completely filled and the excess soil overflows. Strike off the excess soil paste with a straight edge. Wipe off the soil adhering to the outside of the dish.
5. ***Determination of wet and dry mass of soil pat.*** Weigh immediately the shrinkage dish plus the wet soil pat, accurate to 0.1 g. Keep the shrinkage dish open to air until the colour of pat turns from dark to light. Keep the shrinkage dish in the oven and thus dry the pat to constant mass at 105° to 110°C. Cool the dish in a desiccator and weigh immediately.

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6. ***Determination of volume of dry soil pat.*** To determine the volume of the dry soil pat, keep the glass cup in the evaporating dish. Fill the cup to overflowing with mercury. Remove the excees mercury by pressing the glass plate with the three prongs firmly over the top of the cup. Transfer the cup carefully to another evaporating dish, carefully wiping off any mercury which may be adhering to the outside of the cup. Place the oven-dried soil pat on the surface of mercury in the cup and carefully force the pat into the mercury by pressing it by the same glass plate containing three prongs. Press the plate firmly on the top of the cup. Collect carefully the displaced mercury and find its mass to an accuracry of 0.01 g. The volume of the dry soil pat is then determined by dividing this mass by the density of mercury.

Alternative Procedure (for undisturbed soil)

(*i*) Undisturbed soil sample obtained from the field is trimmed approximately 45 mm in diameter and 15 mm in height. The edges of the pat are rounded off so that no air is entrapped during mercury displacement.

(*ii*) Keep the pat in a suitable container and air dry it for some time so that its colour changes dark to light. Keep the container in the oven and dry it at 105°C to 110°C. Take out the pat and smoothen the edges by sand paper.

(*iii*) Place the pat again in the oven for some time and dry it to constant mass. Cool the even dried part in the desiccator, and then determine its dry mass.

(*v*) Determine the volume of the dry soil pat, as explained in step (5) above.

Tabulation of observations. See Tables 3.20 and 3.21.

Table 3.20 Data and observation sheet for shrinkage factors (Remoulded sample)

Determination Number		*1*	*2*	*3*
(*a*) ***Water content of wet soil pat***				
1. Shrinkage dish No.		1	2	3
2. Mass of shrinkage dish		23.0	22.2	21.7
3. Mass of shinkage dish + wet soil pat	(g)	63.5	61.3	62.0
4. Mass of shrinkage dish + dry soil pat	(g)	52.6	50.9	51.3
5. Mass of dry soil pat (M_d)	(g)	29.6	28.7	29.6
6. Mass of water	(g)	10.9	10.4	10.7
7. Water content of soil pat (w)	(Ratio)	36.8	36.3	36.2
(*b*) ***Volume of wet soil pat***				
8. Evaporating dish No.		1	1	1
9. Mass of mercury filling shrinkage dish + mass of evaporating dish	(g)	350.3	345.4	348.5
10. Mass of evaporating dish	(g)	76.2	76.2	76.2
11. Mass of mercury filling shrinkage dish	(g)	274.1	269.2	272.3
12. Vol. of wet soil pat $V = \frac{(11)}{13.6}$	(cm^3)	20.2	19.8	20.2

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(c) *Volume of dry soil pat*				
13. Evaporating dish No.		1	1	1
14. Mass of mercury displaced by dry soil pat + mass of evaporating dish	(g)	304.3	300.6	302.7
15. Mass of evaporating dish	(g)	76.2	76.2	76.2
16. Mass of mercury displaced by dry soil pat	(g)	228.1	224.4	226.5
17. Vol. of dry soil pat $V_d = \frac{(16)}{13.6}$	(cm^3)	16.8	16.5	16.7
(d) *Calculations*				
18. Shrinkage limit $w_s = \left[w - \frac{V - V_d}{M_d}\right]100$	(%)	25.3	24.8	24.4
19. Shrinkage ratio $SR = \frac{M_d}{V_d}$		1.76	1.74	1.77
20. Volumetric shrinkage $VS = (w - w_s)\,SR$		20.2%	20.0%	20.9%

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Table 3.21 Data and observation sheet for shrinkage limit (Undisturbed sample)

S. No.	*Determination Number*		*1*	*2*	*3*
1.	Dish No.		1	2	3
2.	Mass of dish + dry soil pat	(g)	52.1	50.7	50.6
3.	Mass of dish	(g)	23.1	22.2	21.7
4.	Mass of dry soil pat (M_d)	(g)	29.0	28.5	38.9
5.	Evaporating dish No.	(g)	1	1	1
6.	Mass of mercury displaced by dry pat + Mass of evaporating	(g)	308.6	306.2	310.2
7.	Mass of evaporating dish	(g)	76.2	76.2	76.2
8.	Mass of mercury displaced by dry soil pat	(g)	232.4	230.0	234.0
9.	Volume of dry soil pat $V_d = \frac{(8)}{13.6}$	(cm^3)	17.1	16.9	17.2
10.	Specific gravity of soil (G)		2.70	2.70	2.70
11.	Shrinkage limit $w_s = \left(\frac{V_d}{M_d} - \frac{1}{G}\right)100$	(%)	22.0	22.3	22.5

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Calculations. The various factors are calculated as under:

1. Shrinkage limit (remoulded sample) $w_s = \left[w - \frac{V - V_d}{M_d}\right] \times 100$ (Since $\rho_w = 1$ g/cm^3)

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2. Shrinkage ratio: $SR = \dfrac{M_d}{V_d \rho_w} = \dfrac{M_d}{V_d}$
3. Volumetric shrinkage: $VS = (w - w_s)\, SR$
4. Shrinkage limit (undisturbed sample): $w_s = \left(\dfrac{V_d}{M_d} - \dfrac{1}{G}\right) \times 100$

Results

1. Shrinkage Limit (Remoulded Sample) : 24.8%
2. Shrinkage Ratio : 1.76
3. Volumetric Shrinkage : 20.4%
4. Shrinkage Limit (Undisturbed Sample) : 22.3%

Reference to Indian Standard. IS : 2720–1972/78 (Part VI): Determination of Shrinkage Factors.

PROBLEMS

1. During a sedimentation test for grain size analysis, the corrected hydrometer reading in a 1000 mL uniform soil suspension at the instant of commencement of sedimentation is 1.030. After 30 minutes, the corrected hydrometer reading is observed as 1.015, and the corresponding effective depth is 10 cm. If the specific gravity of soil solids is 2.68 and viscosity of water is 0.01 Poise, find (*i*) the total mass of soil solids placed in the 1000 mL suspension, (*ii*) the effective diameter corresponding to the 30 minutes reading, and (*iii*) the percentage finer than this diameter. [**Ans.** (*i*) 47.9 g, (*ii*) 0.0078 mm, (*iii*) 50%]

2. To determine the in-situ density of a compacted embankment, a small hole is dug into the embankment and the excavated soil is weighed. Uniformly graded dry sand is then poured into the hole from a weighed sand pouring cylinder fitted with pouring cone at its bottom. The cylinder after filling the hole is weighed. The mass of sand filling the pouring cone is determined afterwards. The density of sand used in the cylinder is determined by filling it in a 1000 cm^3 calibrating can whose empty mass is 944 g. If the water content of the embankment soil is 8% and other test data are as given below, determine the in-situ dry density:

(*i*) Mass of excavated soil = 925 g
(*ii*) Mass of cylinder + sand before test = 5332 g
(*iii*) Mass of cylinder + sand after filling the hole = 4152 g
(*iv*) Mass of sand filling the pouring cone = 432 g
(*v*) Mass of calibrating can + sand = 2483 g. [**Ans.** 1.76 g/cm^3; 17.27 kN/m^3]

3. The Atterberg limits of a soil sample are $w_L = 50\%$, $w_P = 30\%$ and $w_S = 15\%$. If the specimen of this soil shrinks from a volume of 10 cm^3 at liquid limit to 5.94 cm^3 when it is oven-dried, calculate (*i*) shrinkage ratio, and (*ii*) specific gravity of the soil solids. [**Ans.** (*i*) 1.96, (*ii*) 2.78]

4. A saturated soil sample has a volume of 20 cm^3 at its liquid limit. Given $w_S = 42\%$, $w_L = 17\%$ and $G = 2.74$, find the minimum volume which the soil can attain. [**Ans.** 14.2 cm^3]

5. The oven-dry mass of pat of clay is 10.8 g and the mass of mercury displaced on immersion is 84.2 g. Taking the specific gravity of solids as 2.72, determine the shrinkage limit and shrinkage ratio.

[**Ans.** 20.4%; 1.745].

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6. A clay sample has a voids ratio of 0.53 in dry state. What will be its shrinkage limit if $G = 2.70$?

[**Ans.** 19.6%]

7. Define liquid limit, liquidity index and consistency index. Determine the value of the liquid limit of a soil from the following test data:

Number of blows	*Water content (%)*
38	16
34	17
20	20
12	22

[**Ans.** 18.7%]

8. An oven-dry sample of volume 220 cm^3 has a mass of 400 g. Determine its voids ratio and shrinkage limit, if the specific gravity of soil particles is assumed as 2.76. If the sample is allowed to swell and get fully saturated on contact with water, what will be the water content which will fully saturate the sample and also cause an increase in volume equal to 10% of the original dry volume?

[**Ans.** $e = 0.535$; $w_S = 19.4\%$; $w = 24.2\%$]

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CHAPTER 4

Classification of Soils

4.1 GENERAL

The purpose of soil classification is to arrange various types of soils into groups according to their engineering or agricultural properties and various other characteristics. Soil possessing similar characteristics can be placed in the same group. Soil survey and soil classification are carried out by several agencies for different purposes. For example, the agriculture departments undertake soil investigations from the point of view of the suitability, or otherwise, of the soil for crops and its fertility. However, from engineering point of view, the classification may be done with the objective of finding the suitability of the soil for construction of dams, highways or foundations, etc. For general engineering purposes, soils may be classified by the following systems:

1. Particle size classification
2. Textural classification
3. Highway Research Board (HRB) classification
4. Unified soil classification and IS classification system

4.2 PARTICLE SIZE CLASSIFICATION

In this system, soils are arranged according to the grain size. Terms such as gravel, sand, silt and clay are used to indicate grain sizes. These terms are used only as *designation of particles size*, and do not signify the naturally occurring soil types, which are mixtures of particles of different sizes and exhibit definite characteristics. It is preferable to use the word '*silt size*' and '*clay size*' in place of simply '*silt*' or '*clay*' in this system. There are various grain size classifications in use, but the more commonly used systems are:

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(*i*) U.S. Bureau of Soil and Public Road Administration (PRA) System of United States, (*ii*) International soil classification, proposed at the International Soil Congress at Washington, D.C. in 1927, and (*iii*) The M.I.T. classification proposed by Prof. Gilboy of Massachusetts Institute of Technology as a simplification of the Bureau of Soils Classification, and (*iv*) Indian Standard Classification (IS : 1948–1970) based on the M.I.T. system. These four systems are shown in Fig. 4.1.

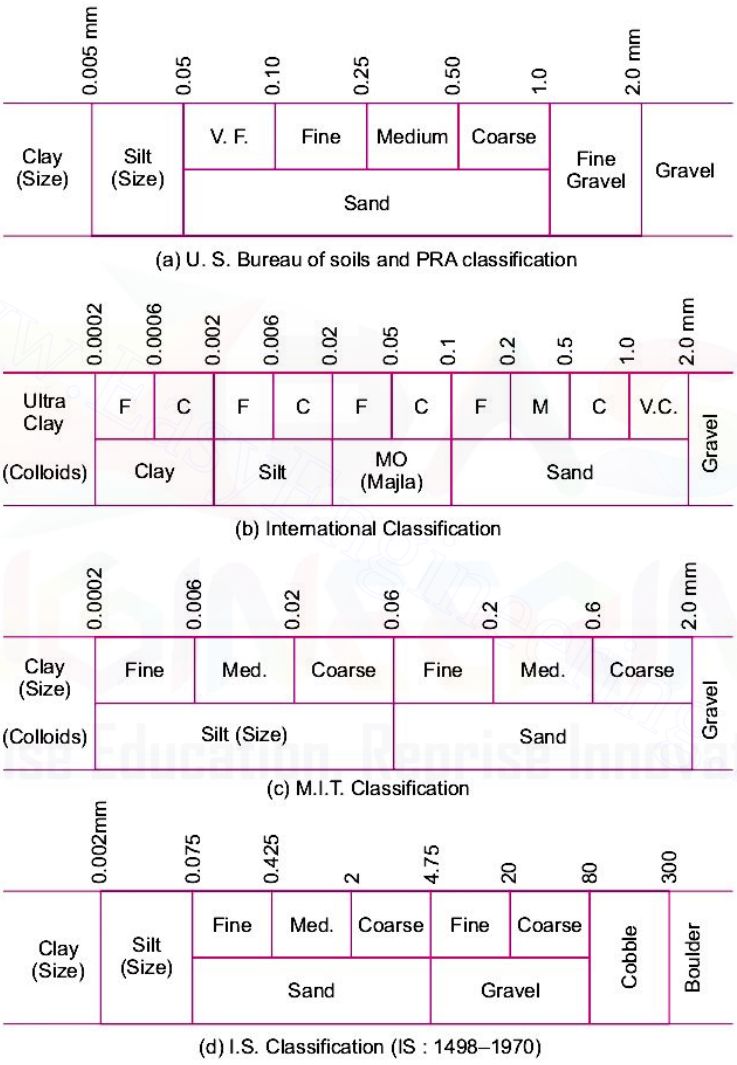

Fig. 4.1 Grain-size Classification Scales

4.3 TEXTURAL CLASSIFICATION

Soils occurring in nature are composed of different percentage of sand, silt and clay size particles. Soil classification of composite soils exclusively based on the particle size distribution is known as textural

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classification. Probably the best known of these textural classifications is the triangular classification of U.S. Public Roads Administraion, shown in Fig. 4.2. The classification is based on the percentages of sand, silt and clay sizes making up the soil. Such a classification is more suitable for describing coarse grained soils rather than clay soils whose properties are less dependent on the particle size distribution.

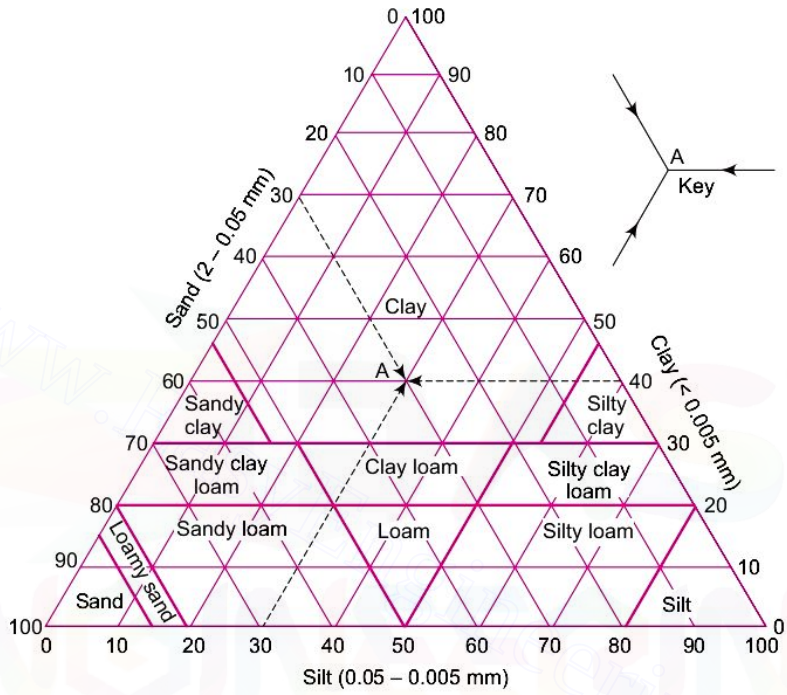

Fig. 4.2 Textural Classification Chart (Adapted from U.S. Public Roads Administration)

To use the chart, for the given percentages of the three constituents forming a soil, lines are drawn parallel to the three sides of the equilateral triangle, as shown by arrows in the 'key' of Fig. 4.2. For example, if a soil is composed of 30% sand, 30% silt sizes and 40% clay sizes, the three lines so drawn intersect at the point *A* situated in the sector designated as 'clay'. Such a soil will be termed as 'clay'.

4.4 HIGHWAY RESEARCH BOARD (HRB) CLASSIFICATION

The Highway Research Board (HRB) classification system, also known as Public Road Administration (PRA) classification system, is based on both the particle-size composition as well as the plasticity characteristics. The system is mostly used for pavement construction. Soils are divided into 7 primary groups, designated as A–1, A–2…A–7 as shown in Table 4.1. Group A–1 is divided into two sub-groups and group A–2 into four sub-groups. A characteristic *group index* is used to describe the performance of the soils when used for pavement construction. Group index is *not* used to place a soil in particular group; it is actually a means of rating the value of a soil as a sub-grade material within its own group. The higher the value of the index, the poorer is the quality of the material.

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The group index of a soil depends upon (*i*) the amount of material passing the 75-micron IS sieve, (*ii*) the liquid limit, and (*iii*) the Plastic limit, and is given by the following equation:

$$\text{Group index} = 0.2\,a + 0.005\,ac + 0.01\,bd \qquad \ldots(4.1)$$

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where, a = that portion of percentage passing 75 micron sieve greater than 35 and not exceeding 75 expressed as a whole number (0 to 40).

b = that portion of percentage passing 75 micron sieve greater than 15 and not exceeding 55 expressed as a whole number (0 to 40).

c = that portion of the numerical liquid limit greater than 40 and not exceeding 60 expressed as positive whole number (0 to 20).

and d = that portion of the numerical plasticity index greater than 10 and not exceeding 30 expressed as a positive whole number (0 to 20).

Table 4.1 HRB-Classification of soils and soil-aggregate mixtures

General Description	*Granular materials (35% or less passing 75 micron IS sieve)*							*Silt clay materials (more than 35 % passing 75 micron IS sieve)*			
Group Classification	*A–1*		*A–3*	*A–2*				*A–4*	*A–5*	*A–6*	*A–7*
	A–1–a	*A–1–b*		*A–2–4*	*A–2–5*	*A–2–6*	*A–2–7*				*A–7–5 A–7–6*
Sieve analysis, percent passing											
2.0 mim IS sieve	50 max										
425 micron sieve	30 max	50 max	51 min								
75 micron sieve	15 max	25 max	10 max	35 max	35 max	35 max	35 max	36 min	36 min	36 min	36 min
Characteristics of fraction passing											
425 micron sieve											
Liquid Limit				40 max	41 min	40 max	41 min	40 max	41 min	40 max	41 max
Plasticity Index	6 max		NP	10 max	10 max	11 min	11 max	10 max	10 max	11 min	11 min
Group Index	Zero					4 max		8 max	12 max	16 max	20 max
Usual type of significant constituent materials	Stone fragments gravel and sand		Fine sand	Silty or clayey gravel and sand				Silty soils		Clayey soils	
General rating as subgrade	Excellent to good			Fair to poor							

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Classification procedure. With required test data available, proceed from left to right on the chart (Table 4.1) and correct group will be found by process of elimination. The first group from left into which the test data will fit is the correct classification. The plasticity index of A –7 –5 sub-group is equal to or less than liquid limit minus 30. Plasticity index of A –7 –6 sub-group is greater than liquid limit minus 30.

The group index should be rounded off to the nearest whole number and placed in parenthesis, such as A –2 –2(6) or A –6(6). See example 4.3.

Classification of Indian Black Cotton soils. The divisions of A –7 group on the basis of the demarcation line ($I_P = w_L - 30$) into A –7 –5 and A –7 –6 sub-groups does not appear to divide the Indian Black Cotton soils into two distinct groups having maximum value of group index as 20 only. Based on investigations carried out at the Central Road Research Laboratory, New Delhi (1953), a classification of black cotton soil into narrow sub-groups (Table 4.2) has been suggested extending the maximum value of group index from 20 to 50. Accordingly the factors for the calculation of group index (Eq. 4.1) have the following maximum values: $a = 65$; $b = 65$; $c = 45$; and $d = 34$.

Table 4.2 Extension of HRB classification for indian black cotton soils

Group	*A–7*	*A–7 a*	*A–7 b*	*A–7 c*
Per cent passing 75 micron sieve	up to 75	55–95	80–95	85–100
Characteristics of fraction passing 425 micron sieve				
Liquid limit	below 55	below 65	above 65	above 65
Plasticity index	below 25	below 42	below 42	above 42
Group index	20 max	20–30	30–40	40–50
Usual type of significant constituent material	Clayey soils			
General rating as sub-grade	Fair to Poor	Poor		

4.5 UNIFIED SOIL CLASSIFICATION SYSTEM (USCS)

Originally developed by Casagrande (1940), the Unified Soil Classification System (USCS) was used for air field construction during World War II. It was later (1952) modified slightly by the Bureau of Reclamation and Corps of Engineers of USA, to make it applicable to other constructions like foundations, earth dams, earth canals, earth slopes, etc. The system has also been adopted by American Society of Testing Materials (ASTM) and later by Bureau of Indian Standards (1970). According to the USCS, the coarse grained soils are classified on the basis of their grain size distribution while the fine-grained soils, whose behaviour is controlled by plasticity, are classified on the basis of their plasticity.

Various soils are classified into four major groups: (*i*) Coarse grained, (*ii*) Fine grained, (*iii*) Organic soils, and (*iv*) Peat. Table 4.3 gives the group symbols consisting of a prefix and a suffix. Thus, there are in all 15 groups of soils – 8 groups of coarse grained soils, 6 groups of fine grained soils (including organic soils), and 1 group for peat. The soils are first classified into two categories: (*i*) coarse grained soils, and (*ii*) fine grained soils.

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Table 4.3 Prefixes and suffixes of USCS

Soil Type	*Prefix*	*Sub Group*	*Suffix*
1. Gravel	G	(*i*) Well graded	W
2. Sand	S	(*ii*) Poorly graded	P
3. Silt	M	(*iii*) Silty	M
4. Clay	C	(*iv*) Clayey	C
5. Organic	O	(*v*) $w_L < 50\%$	L
6. Peat	Pt	(*vi*) $w_L > 50$	H

1. **Coarse grained soils.** If more than 50% of the soil is retained on No. 200 US sieve (0.075 mm), it is designated as coarse grained soil. A coarse grained soil is designated as gravel (G) if 50% or more of the coarse fraction (plus 0.075 mm) is retained on No. 4 (4.75 mm) US sieve; otherwise it is termed as sand (S). Coarse grained soils, containing less than 5% fines, are designated by symbols GW and SW if they are well graded and by symbol GP and SP if they are poorly graded. If however the percentage of fines in more than 12%, the coarse grained soils are designated by symbols GM, GC, SM or SC, as per criterion laid down in Table 4.4. Similarly, if the percentage of fines lie between 5 to 12%, coarse grained soils are designated by dual symbols GW-GM or SP-SM.

2. **Fine grained soils.** A soil is termed as fine grained if more than 50% of the soil sample passes No. 200 US sieve. Fine grained soil are subdivided into silt (M) and clay (C), based on their liquid limit (w_L) and plasticity index (I_P). Organic soils (O) are also included in this group. Figure 4.3 shows the plasticity chart devised by Casagrande (1948), and used for the USCS system. The A – line in the chart has the equation $I_P = 0.73\ (w_L - 20)$. This A – line generally separates the more clay like materials from those that are silty, and also the organic soils from inorganic soils. The fine grained soils (*i.e.*, silt, clay and organic fractions) are further subdivided into soil possessing low (L) or high (H) plasticity when the liquid limit is less than 50% or more than 50%, respectively. When the plasticity index and liquid limit plot in the hatched portion of the plasticity chart, the soil is given dual symbol CL – ML. Soils possessing

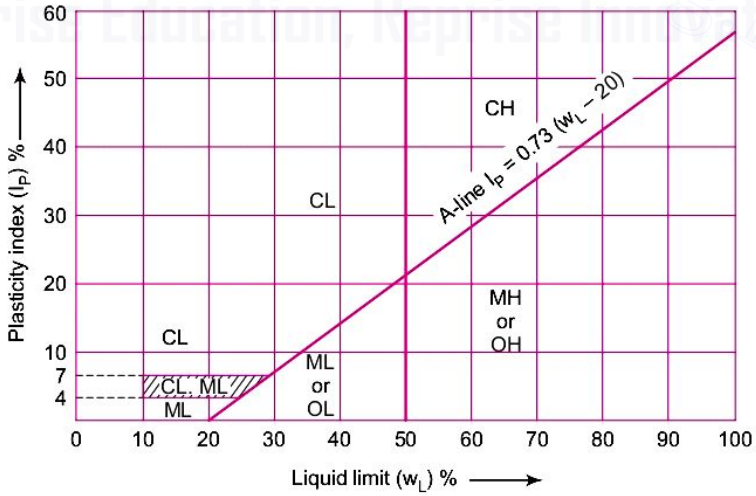

Fig. 4.3 Casagrande's plasticity chart (USCS)

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Table 4.4 Unified soil classification system

Major Division			*Group symbols*	*Typical names*	*Classification criteria*	
Coarse Grained Soils. (More than 50% retained on No. 200 sieve (0.075 mm)	Gravel [50% or more of coarse fraction retained on No. 4 sieve (4.75 mm)	Clean Gravels	GW	Well graded gravels	Percentage of fines (a) Less than 5% passing No. 200: GW; GP; SW; SP (b) More than 12% passing No. 200: GM; GC; SM; SC (c) 5 to 12% passing No. 200: Use of dual symbols as GW-GM; SP-SC	$C_u > 4$, C_c = 1 to 3
			GP	Poorly graded gravels		Not meeting both criteria for GW
		Gravels with fines	GM	Silty gravels	Atterberg Limits below A-line or plasticity index less than 4	Atterberg Limits in hatched area GM-GC
			GC	Clayey gravels	Atterberg Limits above A-line and plasticity index greater than 7	
	Sand [more than 50% of coarse fraction passing No. 4 sieve (4.75 mm)]	Clean Sands	SW	Well-graded sands	$C_u > 6$, C_c = 1 to 3,	
			SP	Poorly graded sands	Not meeting both criteria for SW	
		Sands with fines	SM	Silty sands	Atterberg Limits below A-line or plasticity index less than 4	Atterberg Limits in hatched area SM-SC
			SC	Clayey sands	Atterberg Limits above A-line and plasticity index greater than 7	
Fine grained soils [50% or more passing No. 200 sieve (0.075 mm)]	Silts and clays: Liquid Limit 50% or less		ML	Inorganic silts of low plasticity	Plasticity Chart (see Fig. 4.3)	
			CL	Inorganic clays of low to medium plasticity		
			OL	Organic silts of low plasticity		
	Silts and clays: Liquid Limit greater than 50%		MH	Inorganic silts of high plasticity		
			CH	Inorganic clays of high plasticity		
			OH	Organic clays of medium of high plasticity		
Highly Organic Soils			Pt	Peat; muck and other highly organic soil	Visual-manual identification	

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Table 4.5 Chart for engineering use of soils (Wagner, 1957)

Typical Names of Soil Groups	Groups Symbols	Important Properties				Relative Desirability for Various Uses									
						Rolled Earth Dams			Canal Sections		Foundations		Roadways		
		Permeability when Compacted	Shearing Strength when Compacted and Saturated	Compressibility when Compacted and Saturated	Workability as a Construction Material	Homogeneous Embankment	Core	Shell	Erosion Resistance	Compacted Earth Lining	Seepage Important	Seepage not Important	Fills		Surfacing
													Frost Heave not Possible	Frost Heave Possible	
Well graded gravels, gravel-sand mixtures, little or no fines	GW	pervious	excellent	negligible	excellent	-	-	1	1	-	-	1	1	1	3
Poorly graded gravels, gravel sand mixtures, little or no fines	GP	very pervious	good	negligible	good	-	-	2	2	-	-	3	3	3	-
Silty gravels, poorly graded gravel-sand-silt mixtures	GM	semipervious to impervious	good	negligible	good	2	4	-	4	4	1	4	4	9	5
Clayey gravels, poorly graded gravel-sand-clay mixtures	GC	impervious	good to fair	very low	good	1	1	-	3	1	2	6	5	5	1
Well graded sand, gravelly sand, little or no fines	SW	pervious	excellent	negligible	excellent	-	-	3 if gravelly	6	-	-	2	2	2	4
Poorly graded sands, gravelly sands, little or no fines	SP	pervious	good	very low	fair	-	-	4 if gravelly	7 if gravelly	-	-	5	6	4	
Silty sands poorly graded sand-silt mixtures	SM	semipervious to impervious	good	low	fair	4	5	-	8 if gravelly	5 erosion critical	3	7	8	10	6
Clayey sands, poorly graded sand-clay mixtures	SC	impervious	fair	low	good	3	2	-	5	2	4	8	7	6	2

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Typical Names of Soil Groups	Groups Symbols	Important Properties				Relative Desirability for Various Uses									
						Rolled Earth Dams			Canal Sections		Foundations		Roadways		
		Permeability when Compacted	Shearing Strength when Compacted and Saturated	Compressibility when Compacted and Saturated	Workability as a Construction Material	Homogeneous Embankment	Core	Shell	Erosion Resistance	Compacted Earth Lining	Seepage Important	Seepage not Important	Fills		Surfacing
													Frost Heave not Possible	Frost Heave Possible	
Inorganic silts and very find sands, rock flour, silty or clayey fine sands with slight plasticity	ML	semipervious to impervious	fair	medium	fair	6	6	-	-	6 erosion critical	6	9	10	11	-
Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays	CL	impervious	fair	medium	good to fair	5	3	-	9	3	5	10	9	7	7
Organic silts and organic silt clays of low plasticity	OL	semipervious to impervious	poor	medium	fair	8	8	-	-	7 erosion critcal	7	11	11	12	-
Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	MH	semipervious to impervious	fair to poor	high	poor	9	9	-	-	-	8	12	12	13	-
Inorganic clays of high plasticity, fat clays	CH	impervious	poor	high	poor	7	7	-	10	8 volume change critical	9	13	13	8	-
Organic clays of medium to high plasticity	OH	impervious	poor	high	poor	10	10	-	-	-	10	14	14	14	-
Peat and other highly	Pt	-	-	-	-	-	-	--	-	-	-	-	-	-	-

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the characteristics of more than one group are termed as boundary soils, and are designated by dual group symbols. For example, symbols GW – GC means that the soil is well graded gravel with some clay fines. Also, the inorganic soils (ML or MH) and organic soils (OL or OH) plot on the same zone of the plasticity chart. Oven drying method is used to distinguish between organic and inorganic soils. If the liquid limit of the soil decreases by 30% or more, it is classified as organic (OL or OH): otherwise it is classified as inorganic (ML or MH). *Highly organic soils*, fibrous in nature and having high compressibility, usually peat and swelling soils, are not further subdivided, but are put into one group only with group symbol *Pt*.t

After a soil has been classified according to unified soil classification system (USCS), its important properties and relative desirability for various uses can be identified from the *engineering use chart* given by Wagner (1957) and reproduced in Table 4.5. The numbers in the chart denote *relative suitability* with No. 1 as the best and No. 10 as the worst.

4.6 INDIAN STANDARD CLASSIFICATION SYSTEM, (ISCS) (IS : 1498–1970)

The Indian Standard Soil Classification System (ISCS), first developed in 1959, was revised in 1970. This revised version is essentially based on USCS with the *modification* that the fine grained soils have been subdivided into three groups (low, medium and high plasticity) as against only two groups (low and high) in the USCS. The ISCS classifies the soils into 18 groups (Tables 4.6 and 4.7) as against 15 groups of USCS.

Divisions. Soils are broadly divided into *three* divisions:

1. ***Coarse grained soil.***
   In these soils, more than half the total material by mass is larger than 75 micron IS sieve size.
2. ***Fine grained soils.***
   In these soils, more than half the material by mass is smaller than 75 micron IS sieve size.
3. ***Highly organic soils and other miscellaneous soil materials.***

These soil contain large percentages of fibrous organic matter, such as peat, and the particles of decomposed vegetation. In addition, certain soils containing shells, concretions, cinders and other non-soil materials in sufficient quantities are also grouped in this division.

1. **Coarse grained soils.**

Coarse grained soils are further divided into two sub-divisions:

(*a*) ***Gravels (G).***

In these soils, more than half the coarse fraction (+75 micron) is larger than 4.75 mm IS sieve size. This sub-division includes gravels and gravelly soil, and is designated by symbol *G*.

(*b*) ***Sands (S).***

In these soils more than half the coarse fraction (+75 micron) is smaller than 4.75 mm IS sieve size. This sub-division includes sands and sandy soils.

Each of the above sub-divisions are further sub-divided into four groups depending upon grading and inclusion of other materials:

W	:	Well graded, clean
C	:	Well graded with excellent clay binder
P	:	Poorly graded, fairly clean
M	:	Containing fine materials not covered in other groups.

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These symbols used in combination designate the type of coarse grained soils. For example, *GC* means clayey gravels. Table 4.6 gives the symbols and description for basic soil components. Table 4.7 gives the field identification, group symbols and typical names for all the soils. Table 4.8 gives the laboratory classification criteria for coarse grained soils. Table 4.9 gives the characteristics pertinent to embankments and foundations while Table 4.10 gives the characteristics pertinent to roads and airfields.

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Table 4.6 Basic soil components (IS classification) (IS : 1498–1970)

Soil	*Soil Component*	*Symbol*	*Particle size range and description*
Coarse-grained components	Boulder	None	Round to angular, bulky hard, rock particle, average diameter more than 30 cm.
	Cobble	None	Round to angular, bulky hard, rock particle, average diameter smaller than 30 cm but retained on 80 mm sieve.
	Gravel	G	Rounded to angular, bulk, hard, rock particle, passing 80 mm sieve but retained on 4.75 mm sieve. *Coarse* : 80 mm to 20 mm sieve. *Fine* : 20 mm to 4.75 mm sieve.
			Rounded to angular bulky, hard, rocky particle, passing 4.75 mm sieve retained on 75 micron sieve. *Coarse* : 4.75 mm to 2.0 mm sieve. *Medium* : 2.0 mm to 425 micron sieve. *Fine* : 425 micron to 75 micron sieve.
	Sand	S	
Fine-grained Components	Silt	M	Particles smaller than 75-micron sieve indentified by behaviour, that it is slightly plastic or non-plastic regardless of moisture and exhibits little or no strength when air dried.
	Clay	C	Particles smaller than 75-micron sieve identified by behaviour, that is, it can be made to exhibit plastic properties within a certain range of moisture and exhibits considerable strength when air dried.
	Organic matter	O	Organic matter in various sizes and stages of decomposition.

2. **Fine grained soils.** Fine grained soils are further divided into three sub-divisions:

(*a*) Inorganic silts and very fine sands: *M*
(*b*) Inorganic clays: *C*
(*c*) Organic silts and clays and organic matter: *O*.

The fine grained soils are further divided into the following groups on the basis of the following arbitrarily selected values of liquid limit which is a good index of compressibility:

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(*i*) *Silts and clays of low compressibility*, having a liquid less than 35, and represented by symbol *L*.
(*ii*) *Silts and clays of medium compressibility*, having a liquid limit greater than 35 and less than 50, and represented by symbol *I*.
(*iii*) *Silts and clays of high compressibility*, having liquid limit greater than 50, and represented by a symbol *H*.

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Major Divisions			Group Symbol	Typical Names	Field identification procedures (excluding particles larger than 80 mm and basing fractions on estimated weights)	Information required for describing soils
1	2		3	4	5	6
COARSE-GRAINED SOILS More than half of material is larger than 75-micron sieve size The 75-micron sieve size is about the smallest particle visible to the naked eye	GRAVELS More than half of coarse fraction is larger than 4.75 mm sieve size	*Clean gravels (Little or no fines)*	GW	Well-graded gravel, gravel-sand mixtures, little or no fines	Wide range in grain size and substantial amounts of all intermediate particle sizes	For undisturbed soils and information on stratification, degree of compactness, cementation, moisture conditions and drainage characteristics.
			GP	Poorly graded gravels or graveisand mixtures, little or no fines	Predominantly one size or a range of sizes with some intermediate sizes missing	
		Gravel with fines (Appreciable amount of fines)	GM	Silty gravels, poorly graded gravel-sand-silt mixtures	Non-plastic fines or fines with low plasticity (for identification procedures see ML and MI below)	Give typical name, indicate approximate percentages of sand gravel, maximum size, angularity, surface condition and hardness of the coarse grains, local or geologic name and other pertinent descriptive information and symbol in parentheses.
			GC	Clayey gravels, poorly graded gravel-sand-silt mixtures	Plastic fine (for identification procedures see CL and CI below)	
	SANDS More than half of coarse fraction is smaller than 4.75 mm sieve size	*Clean sands (Little or no fines)*	SW	Well graded sands, gravelly sands, little or no fines	Wide range in grain size and substantial amounts of all intermediate particle sizes	Example:
			SP	Poorly graded sands or gravelly sands, little or no fines	Predominantly one size of a range of sizes with some intermediate sizes missing	Silty sand gravelly, about 20% hard angular gravels 10 mm maximum size, rounded and subcircular sand grains, about 15%.
		Sand with fines (Appreciable amount of fines)	SM	Silty sands, poorly graded sand-silt mixtures	Non-plastic fines or fines with low plasticity (for intermediate procedures see MI and ML below)	
			SC	Clayey sands, poorly graded sand-clay mixtures	Plastic fines (for identification procedures see CL and CI below)	Non-plastic fines with low dry strength, well compacted and moist in place, alluvial sand, (SM).

(For visual classification, the 5 mm size may be used as equivalent to the 4.75 mm sieve size)

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				Identification procedures (on fraction smaller than 425 μ. sieve size)			
				Dry strength	*Dilatancy*	*Toughness*	
FINE-GRAINED SOILS **More than half of material is smaller than 75-micron sieve size** **The 75-micron sieve size is about the smallest particle visible to the naked eye**	SILTS AND CLAYS With low compressibility; liquid limit is less than 35	ML	Inorganic silts and very fine sands, rock flour, silty or clayey fine sand or clayey silts with none to low plasticity	None to low	Quick	None	For undisturbed soils, add information on structure, stratification, consistency in undisturbed and remoulded states, moisture and drainage conditions. Give typical names, indicate degree and character of plasticity, amount and maximum size of coarse grains, colour in wet condition, odour, if any, local or geologic name and other pertinent descriptive information, and symbol in parentheses.
		CL	Inorganic clays, gravelly clays, sandy clays, silty clays, lean clays of low plasticity	Medium	None to very slow	Medium	
		OL	Organic silts of low plasticity	Low	Slow	Low	
	SILTS AND CLAYS With medium compressibility; liquid limit is greater than 35 and less than 50	MI	Inorganic silts, silty or clayey fine sand or clayey silts of medium plasticity	Low	Quick to slow	None	
		CI	Inorganic clays, gravelly clays, sandy clays, silty clays, lean clays of medium plasticity	Medium to high	None	Medium	
		OI	Organic silts and organic silty clays of medium plasticity	Low to medium	Slow	Low	
	SILTS AND CLAYS With high compressibility liquid limit is greater than 50	MH	Inorganic silts of high compressibility, micaceous or diatomaceous fine sandy or silty soils, elastic silts	Low to medium	Slow to none	Low to medium	Example: Clayey silt, brown, slightly plastic, small percentage of fine sand, numerous vertical root holes, firm and dry in place ; losses, (ML).
		CH	Inorganic clays of high plasticity, fat clays	High to very high	None	High	
		OH	Organic clays of medium to high plasticity	Medium to high	Slow to very slow	Low to medium	
High organic soil		Pt	Peat and other highly organic soils with very high compressibility	Readily identified by colour, odour, spongy feel and frequently by fibrous texture			

Note. Boundary classification. Soil possessing characteristics of two groups are designated by combination of group symbols

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Combination of these symbols indicates the type of fine grained soil. For example, *ML* means inorganic silt with low to medium compressibility. Table 4.7 gives the field identification, group symbols and typical names. Laboratory classification of fine grained soil is done with the help of plasticity chart shown in Fig. 4.4. The *A-line*, dividing inorganic clay from silt and organic soil has the following equation:

$$I_P = 0.73\ (w_L - 20)$$

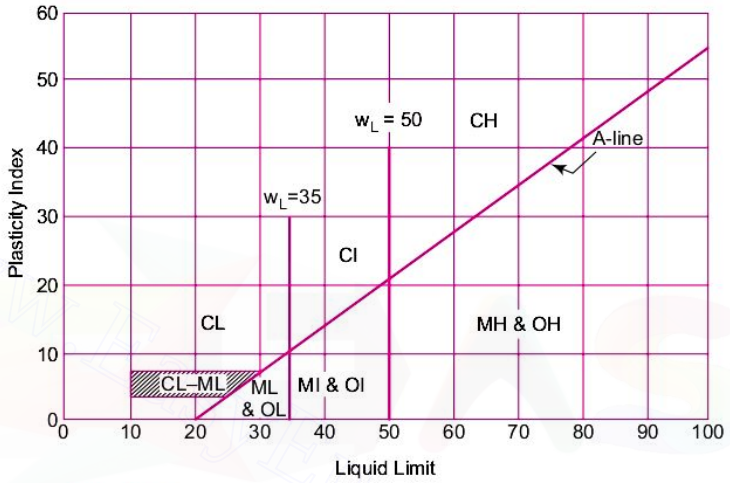

Fig. 4.4 Plasticity chart (IS soil classification system)

Table 4.8 IS soil classification: laboratory classification criteria for coarse grained soils (IS : 1498–1970)

Groups Symbols	*Laboratory Classification Criteria*		
GW	C_u Greater than 4 C_c Between 1 and 3		Determine percentages of gravel and sand from grain sizes curve depending on percentage of fines (fraction smaller than No. 75 micron sieve size); coarse–grained soils are classified as follows: Less than 5% : *GW*, *GP*, *SW*, *SP* More than 12%: *GM*, *GC*, *SM*, *SC* 5% to 12% : *Border line* cases requiring use of dual symbols $C_U = \frac{D_{60}}{D_{10}}$ (uniformity coefficient) $C_C = \frac{(D_{30})^2}{D_{10} \times D_{60}}$ (coefficient of curvature)
GP	Not meeting all gradation requirements for GW		
GM	Atterberg limits below "A" line or I_P less than 4	Above "A" line with I_P between 4 and 7 are *border line* cases requiring use of dual symbols.	
GC	Atterberg limit above "A" line with I_P greater than 7		
SW	C_U Greater than 6 C_C Between 1 and 3		
SP	Not meeting all gradation requirements for SW		
SM	Atterberg limits below "A" line or I_P less than 4	Limits plotting above "A" line with I_P between 4 and 7 are *border line* cases requiring use of dual symbols	
SC	Atterberg limit above "A" line with I_P greater than 7		

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Majority of Indian Black cotton soils lie along a band above the A-line. The plot of some of the black cotton soils is also found to lie below the A-line. Care should be taken in classifying such soils. Some other inorganic clays, such as kaolin, behave as inorganic silts and usually lie below A-line and should be classified as such (*ML*, *MI*, *MH*), although they are clays from mineralogical stand point.

3. **Boundary classification.** There are no rigid boundaries between soil groups, and the boundary cases can be conveniently designated by dual symbols, such as *GW-SW* or *CL-ML*. The common boundary classifications for coarse-grained soil are: *GW-GP*; *GM-GC*; *GW-GM*; *GW-GC*; *SW-SP*; *SM-SC*; *SW-SM*; *SW-SC*; *GW-SW*; *GP-SP*; *GM-SM*; and *GC-SC*. The common boundary classifications for the fine grained soils are : *ML-MI*; *CL-CI*; *OL-OI*; *MI-MH*; *CI-CH*; *OI-OH*; *CL-ML*; *ML-OL*; *CL-OL*; *CI-MI*; *MI-OI*; *CI-OI*; *MH-CH*; *MH-OH*; and *CH-OH*. The boundary classifications between coarse-grained and fine-grained soils are: *SM-ML* and *SC-CL*. It is possible in coarse grained soil with fines between 5 to 10 percent and I_P between 4 to 7 to have a soil which can be represented by both *GM* and *GC* or *SM* and *SC*. In such cases the non-plastic classification is favoured (Earth Manual: 1960).

SOLVED EXAMPLES

Example 4.1. *Sketch the plasticity chart used for classifying fine grained soil in the IS Soil Classification system.*

Give the group symbols for the following soils:

(i) Liquid limit = 40% ; Plastic limit = 22%
(ii) Liquid limit = 20% ; Plastic limit = 14%
(iii) Passing 4.75 mm sieve = 70%; Passing 75 micron sieve = 8%
Uniformity coefficient = 7; Coefficient of curvature = 3; Plasticity index = 3.

Solution. Fig. 4.4. shows the plasticity chart of the IS system.

(*i*) $w_L = 40\%$; $w_P = 22\%$: $I_P = 40 - 22 = 18\%$
Plotting the point for $I_P = 18\%$ and $w_L = 40\%$ on the plasticity chart, group symbol for the soil will be *CI*.

(*ii*) $w_L = 20\%$; $w_L = 14\%$ $\therefore$ $I_P = 6\%$
Plotting the point for $I_P = 60\%$ and $w_L = 20\%$, the soil falls in the *CL-ML* sector.

(*iii*) Since more than half the portion (70%) of the soil passes through 4.75 mm sieve, the soil is essentially sandy (*S*). Referring to Table 4.8, since $C_u = 7$ (greater than 6) and $C_c = 3$, the soil is of *SW* group. However, since percentage passing 75-micron size is 8% (between 5 and 12%), it is a border-line case. Also, since $I_P = 3$ (less than 4), it satisfies the requirement of *SM*. Hence the soil may be designated as *SW-SM*.

Example 4.2. *The following data on consistency limits are available for two soils A and B.*

	Soil A	*Soil B*
1. *Plastic limit*	*16%*	*19%*
2. *Liquid limit*	*30%*	*52%*
3. *Flow index*	*11*	*6*
4. *Natural water content*	*32%*	*40%*

Find which soil is (a) more plastic, (b) better foundation material on remoulding (c) better shear strength as a function of water content, (d) better shear strength at plastic limit. Classify the soil as per ISCS. Do these soils have organic matter?

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Solution.

(*a*) Plasticity index I_P for soil $A = 30 - 16 = 14$
Plasticity index I_P for soils $B = 52 - 19 = 33$
Since plasticity index of soil B is greater, soil B is *more plastic*.

(*b*) Consistency index I_c for soil $A = \dfrac{w_L - w}{I_p} = \dfrac{30 - 32}{14} = -0.142$

Consistency index I_c for soil $B = \dfrac{w_L - w}{I_P} = \dfrac{52 - 40}{33} = 0.3636$

The consistency index for soil A is negative. Hence it will turn into slurry when remoulded. Hence soil A is not suitable for foundations. However, soil B will be suitable.

(*c*) Flow index I_f for soil $A = 11$; Flow index I_f for soil $B = 6$
Since the flow index of soil B is lesser than of soil A, soil B has better shear strength as a function of water content.

(*d*) Toughness index I_T for soil $A = I_P / I_f = 14/11 = 1.27$
Toughness index I_T for soil $B = 33/6 = 5.5$
Since the toughness index for soil B is greater than that of A, soil B has better shear strength at plastic limit.

Classification of the soil as per ISCS: When I_P and w_L are marked on the plasticity chart, soils A and B fall in the zones of CL and CH respectively. Thus soil A is inorganic clay for low plasticity while soil B is inorganic clay of high plasticity. Hence these soils do not have organic matter.

Example 4.3. *Laboratony tests on a soil sample obtained from a highway project site reveal that 56 percent of the soil passes 200 No. US sieve and the liquid and plastic limits of the soil are 36 percent and 23 percent respectively. Determine the group index of the soil and classify it as per HRB/PRA classification system.*

Solution. Refer Table 4.1.

As more than 35% of the soil passes through 200 No. US sieve (or 75 micron IS sieve) the soil falls in the group between A –4 to A –7. Since the liquid limit is less than 40, the soil falls either in A –4 group or in A –6 group. Further, since the plastic limit is more than 21, it falls in A –6 group.

$$\text{Group index} = 0.2\,a + 0.005\,ac + 0.01\,bd \qquad ...(4.1)$$

where a = that portion of the percent passing 200 No. sieve (75 micron) greater than 35 and not exceeding $75 = 56 - 35 = 21$.

b = that portion of the percent passing 75 micron sieve, greater than 15 and not exceeding $75 = 56 - 15 = 41$, but not exceeding 40. Hence $b = 40$

d = that portion of the numerical plasticity index greater than 10 and not exceeding 30 = $(36 - 23) - 10 = 3$

$\therefore$ Group index $= 0.2 \times 21 + 0.005 \times 21 \times 0 + 0.01 \times 40 \times 3 = 5.4 \approx 5$

Hence the soil belongs to group A –6 (5).

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Example 4.4. *Laboratory tests on a soil sample obtained from a foundation site reveal the following data:*

Total mass of soil sieved = 200 g.

Cumulative mass retained on 4 mm sieve = 30 g.

Cumulative mass retained on 75 micron sieve = 150 g.

$D_{10} = 0.07$ *mm;* $D_{30} = 0.12$ *mm;* $D_{60} = 1.95$ *mm.*

Liquid limit = 38%; Plastic limit = 28%

Classify the soil according to USCS. It the soil suitable for foundations?

Solution. Percentage of soil finer than 75 micron size $= \dfrac{200 - 150}{200} \times 100 = 25\%$

Since more than half the soil particles are coarser than 0.075 mm size, the soil belongs to the group of '*coarse grained soils*.' Again, since the percentage of particles finer than 0.075 mm is more than 12%, the soil belongs to any one of the following groups: GM, GC, SM, SC.

In order to further narrow down the group, let us find relative percentage of gravel and sand, out of the total coarse fraction.

Mass of total coarse fraction, larger than 0.075 mm = 150 g.

$\therefore$ Percentage of fraction coarser than 4 mm $= \dfrac{30}{150} \times 100 = 20\%$ (*i.e.*, percent gravel size).

Percentage of fraction finer than 4 mm = 100 – 20 = 80% (*i.e.*, % sand size).

Since more than 50% of the coarse fraction is finer than 4 mm, the soil belongs to the category of 'sands', and the group symbols narrow down to SM, SC.

For the determination of final group, let us use the plasticity characteristics of soil. The soil has w_L = 38% and $I_P = 38 - 28 = 10$.

Hence, in the plasticity chart (Fig. 4.3), the point falls below A-line. Hence the soil falls to SM group.

In order to assess its suitabilty as a foundation material, refer Table 4.5 by Wegner. We find that soil with SM group has a rating of 3 (*i.e.*, good) if seepage is important. However, if seepage is not an important consideration, the soil has a rating of 7 (average) on a scale of 14.

Note: The students are advised to re-classify the above soil as per ISCS.

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Table 4.9 Characteristics pertinent to embankments and foundations

Soil group (1)	*Value of Embankment (2)*	*Permeability cm per sec. (3)*	*Compaction characteristics (4)*	*Unit dry density g/cm³ (5)*	*Value for foundation (6)*	*Requirements for seepage control (7)*
GW	Very stable, pervious shells of dikes and dams	$k > 10^{-2}$	Good; tractor, rubber-tyred, street-wheeled roller	2.00 – 2.16	Good bearing value	Positive cut-off
GP	Reasonably stable, pervious shells of dikes and dams	$k > 10^{-2}$	Good; tractor, rubber-tyred, street-wheeled roller	1.84 – 2.00	Good bearing value	Positive cut-off
GM	Reasonably stable, not particularly suited to shells, but may be used for impervious cores or blankets	$k = 10^{-3}$ to 10^{-8}	Good, with close control; rubber tyred, sheepsfoot roller	1.92 – 2.16	Good bearing value	Toe trench to none
GC	Fairly stable, may be used for impervious core	$k = 10^{-6}$ to 10^{-8}	Fair; rubber-tyred, sheepsfoot roller	1.84 – 2.08	Good bearing value	None
SW	Very stable, pervious sections, slope protection required	$k > 10^{-3}$	Good; tractor	1.76 – 2.08	Good bearing value	Upstream blanket and toe drainage or wells
SP	Reasonably stable, may be used in dike section with flat slopes	$k > 10^{-3}$	Good; tractor	1.60 – 1.92	Good to poor bearing value depending on density	Upstream blanket and toe drainage or wells
SM	Fairly stable, not particularly suited to shells, may be used for impervious cores or dikes	$k = 10^{-3}$ to 10^{-6}	Good, with close control; rubber tyred, sheepsfoot roller	1.76 – 2.00	Good to poor bearing value depending on density	Upstream blanket and toe drainage or wells
SC	Fairly stable, used for impervious core for flood control structures	$k = 10^{-6}$ to 10^{-8}	Fair; sheepsfoot roller, rubber-tyred	1.68 – 2.00	Good to poor bearing value	None
ML, MI	Poor stability, may be used for embankments with proper control	$k = 10^{-3}$ to 10^{-6}	Good to poor, close control essential; rubber-tyred roller, sheepsfoot roller	1.52 – 1.92	Very Poor; susceptible to liquefaction	Toe trench to none
CI, CL	Stable impervious cores and blankets	$k = 10^{-6}$ to 10^{-8}	Fair to good; sheepsfoot roller, rubber tyred	1.52 – 1.92	Good to poor bearing	None

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Soil group (1)	Value of Embankment (2)	Permeability cm per sec. (3)	Compaction characteristics (4)	Unit dry density g/cm^3 (5)	Value for foundation (6)	Requirements for seepage control (7)
OL, O1	Note suitable for embankments	$k = 10^{-4}$ to 10^{-6}	Fair to poor, sheepsfoot roller	1.28-1.60	Fair to poor bearing, may have excessive settlements	None
MH	Poor stability, cores of hydraulic fill dam, not desirable in rolled fill construction	$k = 10^{-4}$ to 10^{-6}	Poor to very poor, sheepsfoot roller	1.12-1.52	Poor bearing	None
CH	Fair stability with flat slope, thin cores, blanket and dike sections	$k = 10^{-6}$ to 10^{-8}	Fair to poor, sheepsfoot roller	1.20-1.68	Fair to poor bearing	None
OH	Not suitable for embankments	$k = 10^{-6}$ to 10^{-8}	Poor to very poor, sheepsfoot roller	1.04-1.60	Very poor bearing	None
Pt	Not used for construction		Compaction not practical		Remove from foundation	

Note

1. Values in columns 2 and 6 are for guidance only. Design should be based on test results.
2. In column 4, the equipment listed will usually produce densities with a reasonable number of passes when moisture conditions and thickness of lift are properly controlled.
3. Column 5, unit dry weights are compacted soil at optimum water content for IS light compaction effort.

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Soil group	Value as sub grade when not subject to frost Action	Value as sub-base when not subject to frost Action	Value as base when not subject to frost Action	Potential frost Action	Conipress-ibility and Expansion	Drainage Characteristics	Compaction Equipment	Unit Dry density g/cm^3	CBR values	Subgrade modulus (K) kg/cm^2
1	2	3	4	5	6	7	8	9	10	11
GW	Excellent	Excellent	Good	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tyre roller, steel-wheeled roller	2.00-2.24	40-80	8.3-13.84
GP	Good to excellent	Good	Fair to good	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tyre roller, close control of moisture	1.76-2.24	30-60	8.3-13.84
GM d	Good to excellent	Good	Fair to good	Slight to medium	Very slight	Fair to poor	Rubber-tyred roller, sheepsfoot roller, close control of moisture	2.00-2.32	40-80	8.3-13.84
u	Good	Fair	Poor to not suitable	Slight to medium	Slight	Poor to practi-cally impervious	Rubber-tyred roller, sheepsfoot roller	1.84-2.16	20-30	5.53-8.3
GC	Good	Fair	Poor to not suitable	Slight to medium	Slight	Poor to practi-cally impervious	Rubber-tyred roller, sheepsfoot roller	2.08-2.32	20-40	5.53-8.3
SW	Good	Fair to good	Poor	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tyred roller	1.76-2.08	20-40	5.53-11.07
SP	Fair to good	Fair to good	Poor to not suitable	None to very slight	Almost none	Excellent	Crawler-type tractor, rubber-tyred roller	1.68-2.16	10-40	4.65-11.07
SM d	Fair to good	Fair to good	Poor	Slight to high	Very slight	Fair to poor	Rubber-tyred roller, sheepsfoot roller, close control of moisture	1.92-2.16	15-40	4.65-11.07
u	Fair	Poor to fair	Not suit-able	Slight to high	Slight to medium	Poor to practi-cally impervious	Rubber-tyred roller, sheepsfoot roller	1.60-2.08	10-20	2.77-8.3
SC	Poor to Fair	Poor	Not suit-able	Slight to high	Slight to medium	Poor to practi-cally impervious	Rubber-tyred roller, sheepsfoot roller	1.60-2.16	5-20	2.77-8.3
ML, Ml	Poor to Fair	Not suit-able	Not suit-able	Medium to very high	Slight to medium	Fair to poor	Rubber-tyred roller, sheepsfoot roller close control of moisture	1.44-2.08	15 or less	2.77-5.3

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Soil group	Value as sub-grade when not subject to frost Action	Value as sub-base when not subject to frost Action	Value as base when not subject to frost Action	Potential frost Action	Compress-ibility and Expansion	Drainage Char-acteristics	Compaction Equipment	Unit Dry density g/cm^3	CBR values	Subgrade modulus (K) kg/cm^2
1	2	3	4	5	6	7	8	9	10	11
CL, CI	Poor to Fair	Not suit-able	Not suit-able	Medium to high	Medium	Practically impervious	Rubber-tyred roller, sheepsfoot roller	1.44-2.08	15 or less	1.38-4.15
OL, OI	Poor	Not suit-able	Not suit-able	Medium to high	Medium to high	Poor	Rubber-tyred roller, sheepsfoot roller	1.44-1.68	5 or less	1.38-2.77
MH	Poor	Not suit-able	Not suit-able	Medium to very high	High	Fair to poor	Sheepsfoot roller, rubber-tyred roller	1.28-1.68	10 or less	1.38-2.77
CH	Poor to very poor	Not suit-able	Not suit-able	Medium	High	Practically impervious	Sheepsfoot roller, rubber-tyred roller	1.44-1.84	15 or less	1.38-4.15
OH	Poor to very poor	Not suit-able	Not suit-able	Medium	High	Practically impervious	Sheepsfoot roller, rubber-tyred roller	1.28-1.76	5 or less	0.69-2.77
Pt	Not suit-able	Not suit-able	Not suit-able	Slight	Very high	Fair to poor	Compaction not practical	–	–	–

Note: 1. Column 1 Division of GM and SM groups into Sub-division of "*d*" and "*u*" are for roads and airfields only. Sub-division is on basis of Atterberg limits. Suffix *d* (*i.e.*, GM_d) will be used when the liquid limit is 25 or less and the plasticity index is 5 or less; the suffix "*u*" will be used otherwise.

2. In column 8, the equipment listed will usually produce the required densities with a reasonable number of passes when moisture condition and thickness of lift are properly controlled. In some instances, several types of equipment are listed because variable soil characteristics within which a given soil group may require different equipment. In some instances, a combination of two types may be necessary.

(*a*) *Processed base materials and other angular materials*. Steel-wheeled and rubber-tyred rollers are recommended for hard, angular materials with limited fines or screening. Rubber-tyred equipment is recommended for softer materials subjected to degradation.

(*b*) *Finishing*. Rubber-tyred equipment is recommended for rolling during final shaping operations for moist soils and processed materials.

(*c*) *Equipment size*. The following sizes of equipment are necessary to assure the high densities required for airfield construction.

Crawler-type tractor-total weight in excess of 136 KN. Rubber-tyred equipment-wheel load in excess of 68 KN, wheel loads as high as 181 kN may be necessary to obtain the required densities for some materials (based on contact pressure of approximately 457 to 1050 kN/m^2).

Sheepsfoot roller-unit pressure (on 38.7 to 77.4 cm^2) to be in excess of 1750 kN/m^2 and unit pressure as high as 4600 kN/m^2 may be necessary to obtain the required densities for some materials. The areas of the feet should be at least 5 percent of the total peripheral area of the drum

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4.7 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 4.5. *The result of laboratory tests conducted on two soils A and B are as follows*

Characteristic	*percent passing*		D_{10}^*	D_{30}	D_{60}	*Liquid limit*	*Plastic limit*
	0.075 mm sieve	*4.75 mm sieve*					
Soil A	*14*	*92*	*0.14*	*0.33*	*1.0*	*16*	*8*
Soil B	*75*	*100*	-	-	-	*58*	*14*

**D is in mm.*

Classify the soil as per Indian standards. **(Civil Services Exam., 2002)**

Solution.

(*i*) ***Soil A*:** Percent of soil between 4.75 mm and 0.075 mm = 92 – 14 = 78%. Hence the soil is *sandy*, with a symbol *S*.
Plasticity index = 16 – 8 = 8 > 7. Hence it is clayey sand, SC.

(*ii*) ***Soil B:*** Since more than half is passing 75 micron sieve, it is a *fine grained soil*. Also, $I_p = 58 - 14 = 44$.
Plotting the point w_L. = 58% and I_p = 44%, we find that the soil is of CH group. Hence soil *B* is clay of high compressibility.

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CHAPTER 5

Soil Structure and Clay Mineralogy

5.1 SOIL STRUCTURE

Soil Structure is usually defined as the arrangement and state of aggregation of soil particles in a soil mass. This term includes, in larger sense, consideration of the mineralogical composition, electrical properties, shape and orientation of solid particles; the nature and properties of soil water and its ionic composition; and the interaction forces between soil particles, soil water, and their adsorption complexes (Leonard: 1962). As far as structure is concerned, *soil particles* refer not only to the individual mechanical elements, such as sand, silt and clay, but also to the aggregates or structural elements which are formed by the aggregation of smaller mechanical fractions. Soil structure is an important factor which influences many soil properties, such as permeability, compressibility and shear strength etc. The following types of soil structure are generally recognised:

1. ***Single grained.*** An arrangement composed of individual soil particles.
2. ***Honeycomb.*** An arrangement of soil particles having a comparatively loose, stable structure resembling a honeycomb. The soil mass is composed of loosely arranged bundles of particles, irespective of the arrangement of the particles within the bundles (Terzaghi and Peck: 1967).
3. ***Flocculent.*** An arrangement composed of 'flocs' of soil particles instead of individual soil particles. The particles are oriented 'edge-to-edge' or 'edge-to-face' with respect; to one another.
4. ***Dispersed.*** An arrangement composed of particles having a 'face-to-face' or parallel orientation.
5. ***Coarse-grained skeleton.*** An arrangement of coarse grains forming a skeleton with its interstices partly filled by a relatively loose aggregation of the finest soil grains.
6. ***Cohesive matrix.*** An arrangement in which a particle-to-particle contact of coarse fraction is not possible. The coarse grains remain embedded in a large mass of cohesive fine grains.

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The single grained structure is characteristic of coarse-grained soils. The honeycomb, flocculent and dispersed structures are found in fine-grained soils. The skeleton and matrix structures represent composite soils. Before going into the characteristics of these structures, we shall briefly describe the various electrical forces which participate in the building of a soil structure.

5.2 SOLID PARTICLES IN SOILS

The particles of coarse-grained soils are composed of primary minerals *(i.e.,* they are the same as existing in present-day rocks). These particles are sometimes termed as *bulky* particles, and can be thought of rough-edged shapes approaching spheres. These particles do not possess the property of plasticity and cohesion and their behaviour is governed primarily by gravitational forces or mass energy rather than colloidal forces.

Many investigations have shown (Grim, 1959) that fine-grained soils are composed predominantly of crystalline minerals and the amorphous materials that may be present (such as allophane) have little, if any, effect on soil behaviour. These minerals, which have low surface activities and do not contribute appreciable plasticity or cohesion are referred to as *non-clay minerals.* The crystalline minerals whose surface activity is such that they develop cohesion and plasticity are called *clay minerals.* About 15 minerals are classed as clay minerals and these belong to four main groups : kaolin, montmorillonite, illite and playgorskite. Chemically, the clay minerals are silicates of aluminium and/or iron and magnesium. Some of them also contain alkalies and/or alkaline earths, as essential components. Most of the clay minerals have sheet or layered structures. Some of the clay minerals have elongate tubular or fibrous structures. Clays can be considered as essentially made up of extremely small particles, each one of which is either a book of sheet-like units or a bundle of tubes or fibres. Individual soils or clays may contain more than one kind of book-like units or a mixture of books and bundles of tubes or fibres (Grim, 1959). Clay particles behave like *colloids.* A colloid is a particle whose specific surface (surface area per unit mass or volume) is so high that its behaviour is controlled by surface energy rather than mass energy. The smaller is any given shaped particle, the larger is the surface area per unit volume. If we consider a cube of sides 1 cm long, the ratio of surface area to the volume is 6 per centimetre. Suppose the cube is sub-divided into smaller cubes whose sides are 1 μ ($= 10^{-3}$ mm) in length, there will be 10^{12} such cubes in a total volume of 1 cm^3 and a total surface area of 6×10^4 sq. cm increasing the ratio of surface area to volume by ten thousand times. Since the clay particles are plate, needle or rod shaped, they have still higher specific surfaces than a cube of equal volume. A montmorillonite platelet 0.1 μ × 0.002 μ has specific surface/volume equal to 1000/μ, which is five times that of an equivolume cube. The upper size limit of a colloid has arbitrary been set at approximately 0.2 to 1 μ. Nearly, all clay particles are colloidal even though the maximum particles dimension of several of clay minerals (such as kaolinite, dickite, attapulgite, etc.) is greater than 1 μ.

5.3 ATOMIC AND MOLECULAR BONDS

The nature of surface bonding forces is not completely understood. Possibly, one of the simplest general classification system for these bonding forces is: (1) Electrostatic or Primary Valence Bond, (2) Hydrogen Bond, and (3) Secondary Valence Bond. Lambe (1953) recognises the hydrogen bond as a special type of secondary valence bond. Atoms bonding to atoms forming molecules (intramolecular bond) are called *primary valence* bonds. Atoms in one molecule bonding to atoms in another molecule (intermolecular bonds)

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are called *secondary valence* bonds. The primary valence bonds are sufficiently strong so that they are seldom broken in engineering works. However, a secondary valence bond is the bonding force, relatively weaker, in which a student of Soil Mechanics is most interested.

Electrostatic or primary valence bonds. There are five types of primary valence bonds: the *covalent bond,* the *heterpolar bond,* the *ionic-bond,* the *co-ordinate bond* and the *metallic bond.* Some scientists, however, recognise only two types of bonds: ionic bond (sometimes called electrovalent), and covalent bond. Since the transition from one type of bond to another can be gradual, the classification of certain forces is arbitrary. The ionic bond is the simplest and strongest of the bonds that holds atoms together. This bond is made by the exchange of electrons in the union of ions. In the combination of simple elements, an element having an excess of electrons in a ring or shell can exchange the excess electron with an element lacking electrons to complete its outer ring or shell. The exchange of electrons ties the atoms together with an exceptionally strong electrostatic bond. An example of such a bond is illustrated by the union of Al and O. Aluminium has an excess of 3 electrons in its outer ring and oxygen lacks 2 electrons in its outer ring. The linking of the ions of these two elements can be indicated as shown in Fig. 5.1 in which each of the joining lines can be considered as a unit electrostatic force bonding the aluminium and oxygen ions into a molecule of aluminium oxide. In the covalent bond, elements are bonded by sharing of a common electron by two atoms. However, since the primary valence bonds in cohesive soils are seldom broken, they are of minor concern to the engineer.

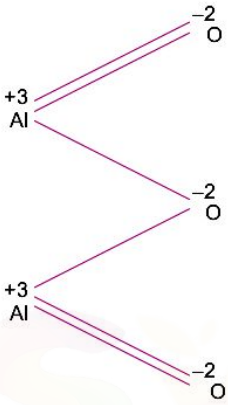
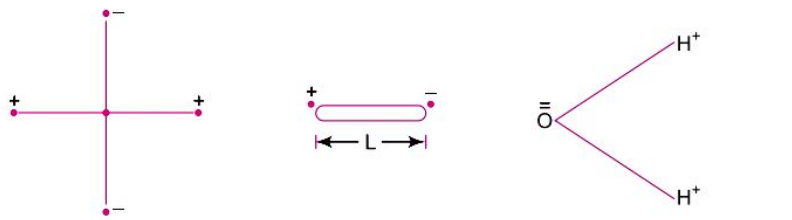


Fig. 5.1 Ionic bond (aluminium oxide)

Secondary valence forces. Secondary valence forces are also sometimes called residual valence forces, van der Waal's forces, van der Waals-London forces, intermolecular forces of attraction, and intermolecular cohesive forces. Lambe (1953) recognised two types of secondary valence forces: the *van der Waal forces* and the *hydrogen bonds.* Secondary valence forces acting between molecules are attributed to the presence of electric moments in the individual molecules. If, in an electrical system the centre of action of positive charge coincides with the centre of action of negative charges, the system has no dipole moment, and is termed *non-polar* [Fig. 5.2 (*a*)]. Although a molecule is electrically neutral, the centre of gravity of the positive and negative charges may not coincide. An electric moment is thus developed, and the system is referred to as being *polar.* System of Fig. 5.2 (*b*) has a *dipole mement* of $e \times L$. The atoms in the water molecule are held together by a heterpolar bond and the resulting molecule is not electrically symmetrical; the hydrogen-oxygen bonds are at an angle of 105° [Fig. 5.2 (*c*)]. The water molecule is therefore a dipole.



(a) Non-polar system (b) Polar system (dipole) (c) Water molecule (dipole)

Fig. 5.2 Polar and non-polar system (the solid lines represent primary valence bonds)

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Similarly, the unsymmetrical distribution of electrons in the silicate crystals (the most widespread and abundant constituent of clay particles) makes them polar.

van der Waals (1873) postulated the existence of a common attractive force acting between all atoms and molecules of matter. Later, Keeson proposed that this force is caused by two oriented dipoles and calculated the average attractive force between such oriented dipoles. Many types of orientations are possible with unsymmetrical water molecules (Fig. 5.3).

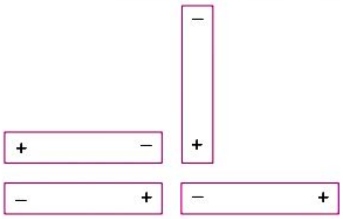

Fig. 5.3 Oriented dipole

Orientation is one of the three effects contributing to secondary valence forces. The secondary valence force caused by orientation effect is commonly known as the van der Waal force. Since thermal agitation tends to upset the alignment of dipoles, the *orientation effect* is highly dependent upon temperature. The other two effects are: the *induction effect* and the *dispersion effect*. Many normally non-polar molecules become polar when placed in an electric field, since it causes a slight displacement of the electrons and nuclei. Since the moment is induced, the phenomenon is called the *induced effect*. Debye suggested and wrote equations for the attraction between molecules that are permanent dipoles and the magnetic moments induced in the adjacent molecules that are not permanent dipoles. The extent to which the induced effect occurs in a molecule is called the *polarisability* of the molecules. The induction effect is only slightly affected by temperature. In 1930, London conceived another type of energy holding molecules together which is known as *dispersion energy* (dispersion effect). This type of attractive energy exists between atoms and molecules that are not normally dipolar. Since all electrons vibrate constantly, there occur in all molecules relative displacements between the electrons and nuclei (Fig. 5.4). As the electrons spin in their orbits, at one instant the negative electrons may be near the side of their orbits farthest away from the positive nucleus, which produces in effect a *statistical dipole*. This statistical dipole can induce magnetic moments in adjacent molecules and thus produce the type of attraction known as the *London van der Waal forces*. The dispersion effect, so produced, occurs in all molecules and is independent, within the normal temperature ranges, of temperature. The relative magnitude of the attractive forces resulting from the three effects may be assessed from the following breakdown of energies in the secondary valence forces between water molecules:

Fig. 5.4 Statistical dipole

Orientation, 77%; Dispersion, 19%; Induction, 4%

Thus, the *orientation* of water molecules has a dominating influence on the van der Waals attractive forces.

The hydrogen bond. The hydrogen bond occurs when an atom of hydrogen is rather strongly attracted by two other atoms. The hydrogen cannot decide to which atom to bond, and oscillates between them. The best example of the hydrogen bond is the bond between water molecules (Fig. 5.5). The hydrogen bond is normally considered a secondary valence bond. It is, however, stronger than the usual secondary valence bond, and is somewhat similar in character to the co-ordinate bond, a primary valence bond. It can be considered a strong secondary or a weak primary valence bond, or a unique bond between the secondary and primary bonds.

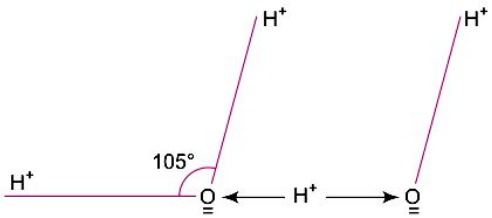

Fig. 5.5 Hydrogen bond

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An indication of the relative strength of the various bonds and the distance between bonded atoms can be obtained from the typical values given below (Lambe: 1953):

Type of bond	*Strength in kcal per g-mole*	*Interatomic or Intermolecular distance in Å*
1. Primary valence	20–200	1–2
2. Hydrogen	5–10	2–3
3. Secondary valence	0.5–5	> 5

Both the primary valence and hydrogen bonds are too strong to be broken under the stresses normally applied in engineering to a soil system. On the other hand, van der Waals forces are much weaker than the other two, and are greatly influenced by applied stresses and by the changes in nature of the soil water system (Lambe, 1958). These forces have their unique importance as they contribute to clay strength and cause soil to hold water.

Diffue double layer. Experiments have shown that a soil colloid suspended in water carries nearly always a net negative charge. The acidic nature of soil minerals suggests that hydrogen atoms of the hydroxyl groups come off in the presence of water and thereby give the minerals a net negative charge. Since the net electrical charge of the entire soil water suspension must be zero, the charge on each colloid must be neutralised by ions from the water which swarm around each colloid. These ions are called *counter ions or exchangeable ions,* since they can be replaced. If there were no thermal activity possessed by these ions, and if there were no attraction exerted on them by other ions and colloids, these counter ions would all swarm to the surface of the particles to neutralise the surface charge of the particle. Thus, their positions are compromises between the particle charge which pulls them in and their thermal activities plus the attraction by other bodies, which keeps them away. The counter ions thus constitute the *diffuse double layer* (Fig. 5.6) of the colloid; the surface charge of the colloid is the other layer of the double layer. The force fields that develop between the charged soil particles, the surrounding water, and the associated ions have a controlling influence on the soil properties which can be varied within wide limits by changing such factors as the types of ions, and their concentration, temperature, and the nature and amount of pore fluid.

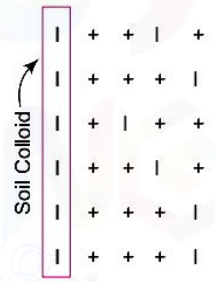

Fig. 5.6 Diffuse double layer

Colloid repulsion and attraction. The electric potential ψ responsible for the formation of the diffuse double layer, reduces with distance [Fig. 5.7 (*a*)], till, at some distance away from the colloid, free water exists. When two colloids in a suspension approach each other, they will reach an interparticle distance where their double layers interact. The most important result of interaction between double layers is the decrease effected in the charge of each double layer. A decrease in charge results in an increase of the free energy of the double layer. Since systems tend to exist in a state of minimum free energy, the colloids *repulse* each other when their double layer interact (Lambe, 1953). However, there exist long-range attractive forces between colloids in suspension. The attractive forces acting between colloids which are far apart are 'van der Waal forces' or 'van der Waal-London Forces'. Debye showed that two dipolar molecules would mutually influence their spatial orientations in such a way that on the average, attraction will occur. The repulsive and attractive energies between particles is a function of the distance between them. Let us now consider the total repulsive plus attractive energy between two particles. If the total

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potential energy between two particles decreases as they approach each other, there is attraction between the particles since the force between the particles is the negative derivative of the potential. The two particles will, therefore, approach each other until they touch or stick, or *flocculate*. If, however, the nearing particles increase the potential energy, repulsion or dispersion takes place. As thermal energy brings the particles towards each other, they will repulse and stay in suspension. The various factors in a soil-water system affecting flocculation or dispersion are (Lambe, 1958): electrolyte concentration, ion valence, dielectric constant, temperature, size of hydrated ion, relative acidity (or *p*H value), and anion absorption. Flocculation is caused by increasing electrolyte concentration, ion valence, and temperature, or by decreasing dielectric constant, size of the hydrated ion, *p*H and anion absorption. A decrease in the thickness of double layer reduces the electric repulsion and causes a tendency towards flocculation.

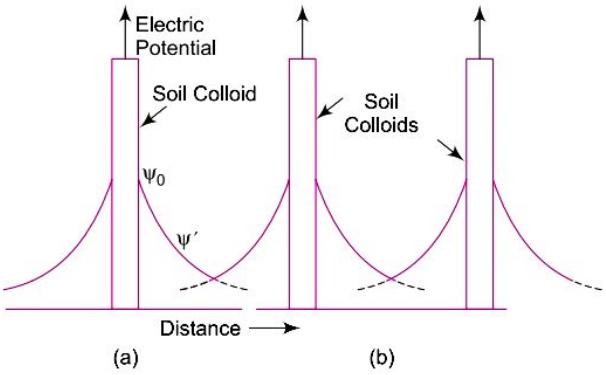

Fig. 5.7 Colloid potential

5.4 INTERPARTICLE FORCES IN A SOIL MASS

In the previous articles, we have considered the force between the particles which are at some significant distance apart in a suspension. In situ, however, the soil particles are very close, and their distance is further decreased by self-weight, capillary forces, seepage force and other causes. The forces between the particles may be of two types: *gravitational forces* and *surface forces*. The gravitational forces are proportional to the mass, and are important for coarse-grained soils only. In the fine-grained soils, the specific surface of the colloids is very large, and the surface forces dominate the gravitational forces. We shall consider the surface forces in detail.

The surface forces may be broadly classified under two heads: (1) attractive forces, and (2) repulsive forces.

(1) Attractive forces (Lambe, 1953). There are six possible types of attractive forces: (*i*) van der Waals London forces, *(ii)* hydrogen bond, *(iii)* cation linkage, *(iv)* dipole-cation linkage, (*v*) water dipole linkage, and *(vi)* ionic bond. The *van der Waals London* force is the universal attractive force and may be the only one effective in some soils. It gets much stronger as the particles approach. Just as the *hydrogen bond* can be effected between oxygen and hydroxyl groups within a particle, it can act between discrete particles. For example, in kaolinitic soil, particles may come together (Fig. 5.8 (*a*) and 5.16) and permit a hydrogen bonding between silica of one particle and gibbsite of another. Hydrogen bond is one of the strongest interparticle bond. A *cation linkage* acts as a bridge between two adjacent particles carrying negative charges [Fig. 5.8 (*b*)]. The cation linkage is similar to the intersheet bonds of illite (5.18). Particles can be linked together through water-cation-water mechanism [Fig. 5.8 (*c*)]. This linkage would be a weak one, but could exist at relatively large distances. Figure 5.8 *(d)* shows a possible dipolar linkage by water. It is possible for two particles to bond ionically to each other just as the atoms in a crystal bond. For example, the edges of two kaolinite crystals can be joined by the cations (aluminium and silicon) of one particle joining with the oxygen of the other particle. This type of joining amounts to crystal growth.

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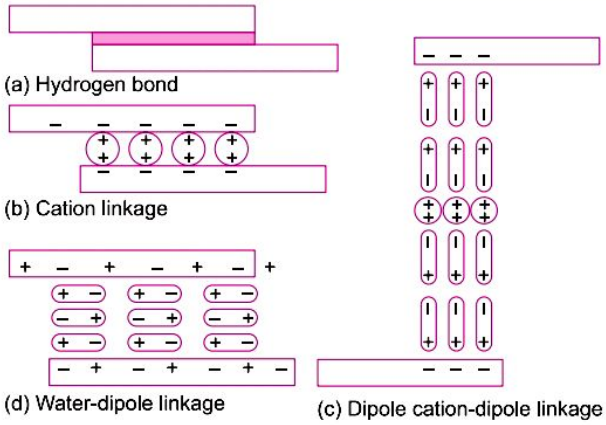

Fig. 5.8 Interparticle attractive forces (Lambe, 1953)

(2) Repulsive forces (Lambe, 1953). There are two types of repulsive forces: (*i*) Particle charge, and (*ii*) cation-cation repulsion. Since the particles are similarly charged (*i.e.*, carrying a residual negative charge), they repel each other [Fig. 5.9 (*a*)]. As explained in the previous article (colloid repulsion), the repulsive forces between two soil colloids become effective when they approach each other close enough for the double layers to overlap and interact. Similarly, when two particles with exchangeable cations are brought close to each other, the cations tend to repel each other, since they carry like charges [Fig. 5.9(*b*)]. However, this mechanism does not constitute a strong repulsive force between particles, since the mobile cations will move along the particle surface of positions not opposite other cation [Fig. 5.9(*c*)]. The cation charge may also be entirely balanced by the particle charge, so that there is no cation-cation repulsion.

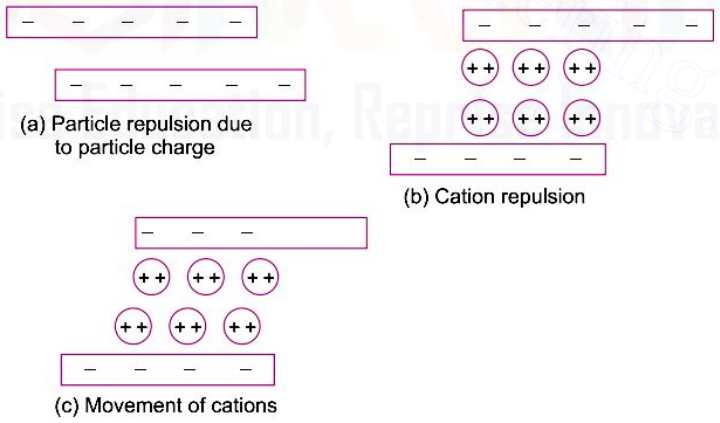

Fig. 5.9 Interparticle repulsive forces (Lambe, 1953)

5.5 SINGLE GRAINED STRUCTURE

Coarse-grained soils (diameter >0.02 mm) settle out of suspension in water as individual grains independently of other grains. The major force causing their deposition is gravitational and the surface forces are too small to be of practical importance. The weight of the grains causes them to settle, and get particle-

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to-particle contact on deposition. They may be deposited in a loose state having a high voids ratio or in a dense state having a low voids ratio.

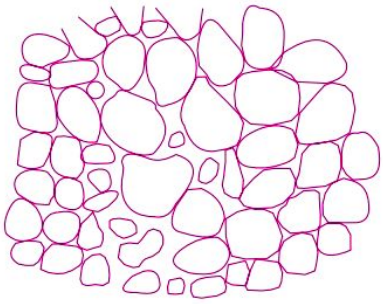

Fig. 5.10 Single grained structure

5.6 HONEYCOMB STRUCTURE

Such a structure exists in grains of silts or rock flour smaller than 0.02 mm diameter and larger than 0.0002 mm. When such grains settle under gravity, the surface forces also play an equally important role. The surface forces at the contact areas, as the grains come in contact at the bottom of suspension, are large enough compared to the submerged weight to prevent the grains from rolling down immediately into positions of equilibrium among the grains already deposited. The grains coming in contact are held until miniature arches are formed, bridging over relatively large void spacing and forming a *honeycomb structure* (Fig. 5.11). Each cell in the honeycomb structure is supposed to be made up of numerous single mineral grains. Comparatively large amount of water is enclosed within voids built up of aggregates of minerals glued to each other by the adhesion forces (Rosenqvist: 1959). The structure so formed has high voids ratio and is capable of carrying relatively heavy loads without excessive volume change. However, the structures might be broken down with a resulting volume decrease, by driving piles into a deposit of silt having honeycomb structure.

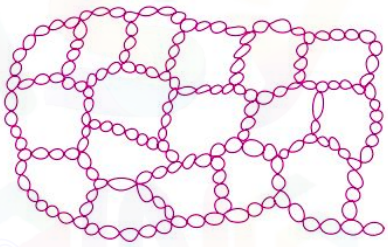

Fig. 5.11 Honeycomb structure

5.7 FLOCCULENT AND DISPERSED STRUCTURES

Clay, consisting of very fine particles or colloids, may have two types of structures: *flocculated* or *dispersed*. V.M.Goldschmidt (1926), known mostly because of his geological and geochemical work, expressed the opinion that the flaky minerals in the highly sensitive clays are arranged in unstable card house structures. Goldschmidt maintained that the surplus water was enclosed in the space between a few mineral flakes leaning upon each other and the difference between the clays of high and low sensitivity was due to a denser arrangement of the minerals in the clays of low sensitivity. Lambe (1953) explained a similar structure of clay from colloidal chemical viewpoint. According to him a *flocculated structure* of clay platelets is formed when there are edge-to-edge contacts between the platelets [Fig. 5.12 (*a*)]. Such a structure is formed if the net electrical forces between adjacent soil particles at the time of

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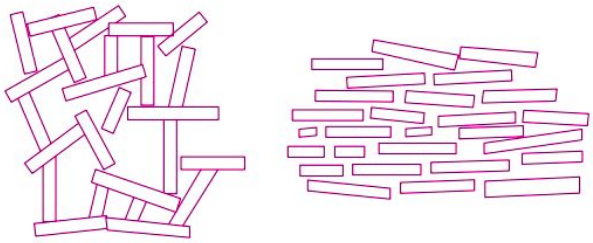
deposition are attraction forces. If there is a concentration of dissolved minerals in the water (*e.g.*, marine water) the tendency of flocculation is increased. On the other hand, a *dispersed* or *oriented* structure is formed when the platelets have face-to-face contact in more or less parallel array [Fig. 5.12 (*b*)]. Such a structure is formed if the net electrical forces between adjacent soil particles at the time of deposition are repulsion.



(a) Flocculated structure (b) Dispersed structure

Fig. 5.12 Goldschmidt-Lambe concept of cardhouse structure

A clay having flocculent structure has a high voids ratio. When pressure is applied to the deposit, high concentration of stress exists at the points of contact causing the particles to be bent and to slip along the contact surfaces to positions of greater stability and producing a denser arrangement of the particles with a resulting decrease in volume. Remoulding and compacting clays tend to produce a dispersed structure. Similarly, consolidation tends to orient particles into the dispersed arrangement. Pressure applied to a dispersed structure forces some of the double layer water out from between the flakes until they are separated by water with high enough viscosity to support the applied load. Montmorillonite crystals under some conditions may be only 10 Å (Å = 10^{-7} mm) thick with 10 Å of adsorbed water next to the surface and 200 Å of double layer water. Montmorillonite having a dispersed structure under this condition is 97% water and 3% solids by volume.

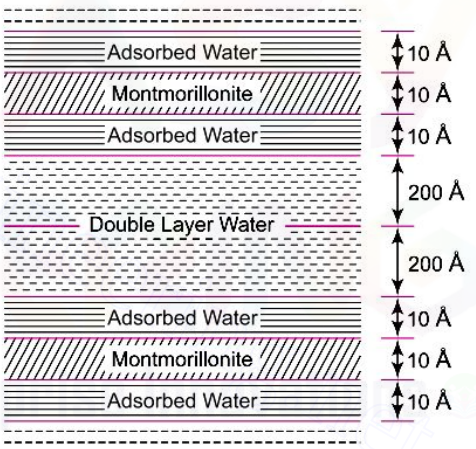


Fig. 5.13 Montmorillonite crystals with attached water (dispersed structure)

5.8 STRUCTURE OF COMPOSITE SOILS

Depending upon the relative proportion of coarse grained and fine grained particles, two types of structures of composite soils can be possible: the *coarse grained skeleton* structure and *cohesive matrix*. In the coarse-grained skeleton, the voids space of the single grained structure is filled with clay particles. The bulky particles form a continuous relatively incompressible frame-work. In the cohesive matrix, the clay content is more so that the bulky particles are not in a position to have particle-to-particle contact. The clay constitutes the load bearing member, and hence the soil formation is relatively more compressible.

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5.9 CLAY MINERALS

There are two fundamental *building blocks* for the clay mineral structures. One is a *silica tetrahedral unit* [Fig. 5.14 (*a*)] in which four oxygen or hydroxyls having the configuration of a tetrahedron enclose a silicon atom. The tetrahedra are combined in a sheet structure so the oxygens of the bases of all the tetrahedra are in a common plane, and each oxygen belongs to two tetrahedra [Fig. 5.14 (*b*)]. The silica tetrahedral sheet alone may be reviewed as a layer of silicon atom between a layer of oxygens and a layer of hydroxyls (tips of the tetrahedra). The silicon sheet is represented by the symbol, representing the oxygen basal layer and the hydroxyl apex layer.

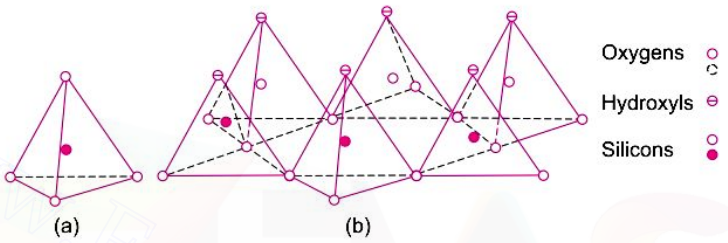


Fig. 5.14 Basic structural units in the silica sheet (Grim, 1959)

The second building block is an octahedral unit in which an aluminium, iron or magnesium atom is enclosed in six hydroxyls having the configuration of an octahedron Fig. 5.15 (*a*)]. The octahedral units are put together into a sheet structure [Fig. 5.15 (*b*)] which may be viewed as two layers of densely packed hydroxyls with cation between the sheets in octahedral co-ordination (Grim, 1959). This unit is symbolised by and is known as *gibbsite*.

About 15 minerals are ordinarily classified as clay minerals, and these belong to four main groups; kaolin, montmorillonite, illite and palygorskite. Out of these, the first three groups are the most common, and will be described here.

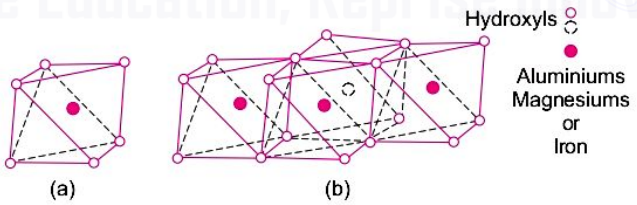
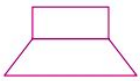


Fig. 5.15 Basic structural units in the octahedral sheet (Grim, 1959)

Kaolinite: Kaolinite is the most common mineral of the kaolin group. The kaolinite structural unit is made up of gibbsite sheets (with aluminium atoms at their centres) joined to silica sheets through the unbalanced oxygen atoms at the apexes of the silicas, (*i.e.,* the apexes of the silica layer and one of the gibbsite form a combined layer). This structural unit is symbolised by  which is about 7 Å (one angstrom, Å = 10^{-7} mm = 10^{-10} m) thick.

The *Kaolinite mineral* or *crystal,* is stacking of such 7 Å thick sheets symbolised as shown in Fig. 5.16 (*a*). The structure is like that of a book with each leaf of the book 7 Å thick. Successive 7 Å layers are held together with *hydrogen bonds* (Fig. 5.16 *b*). A kaolinite crystal may be made up of often 100 or

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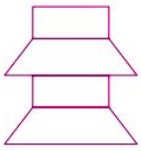

Fig. 5.16 (a)

more such stackings. The kaolinite particles occur in clay as platelets from 1000 Å to 20,000 Å wide by 100 Å to 1000 Å thick. Since the hydrogen bond is fairly strong, it is extremely difficult to separate the layers, and as a result kaolinite is relatively stable and water is unable to penetrate between the layers, Kaolinite consequently shows relatively little *swell* on wetting. The platelets carry negative electromagnetic charges on their flat surface which attract thick layers of adsorbed water thereby producing plasticity when the kaolinite is mixed with water. China clay is almost pure kaolinite.

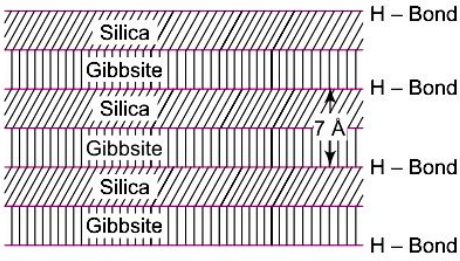

Fig. 5.16 (b) Structure of kaolinite

Montmorillonite. This is the most common of all the clay minerals in expansive clay soils. The mineral is made up of sheet like units. The basic structure of each unit is made up of gibbsite sheet (*i.e.,* the octahedral sheet) sandwiched between two silica sheets, and is symbolised as shown in figure. The thickness of each unit (or sheet) is about 10 Å and the dimensions in the other two directions are indefinite. The gibbsite layer may include atoms of aluminium, iron, magnesium or a combination of these. In addition, the silicon atoms of tetrahedra may interchange with aluminium atoms. These structural changes are called *amorphous changes* and result in a net negative charge on the clay mineral. Cations which are in soil water (*i.e.*, Na^+, Ca^{++}, K^+ etc.) are attracted to the negatively charged clay plates, and exist in a continuous state of interchange.

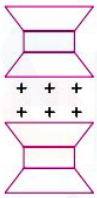

Fig. 5.17 (a)

The basic 10 Å thick units are stacked one above the other like the leaves of a book and symbolised as shown in Fig. 5.17 *(a)*. There is very weak bonding between the successive sheets and water may enter between the sheets causing the minerals to swell (Fig. 5.17 *b*). The spacing between the elemental silica-gibbsite-silica sheets depends upon the amount of available water to occupy the space. For this reason, montmorillonite is said to have an expanding lattice. Each thin platelet has a power to attract to each flat surface a layer of adsorbed water approximately 200 Å thick, thus separating platelets a distance of 400 Å under zero pressure. In the presence of abundance of water, the mineral can, in some cases, split up into about an individual unit layers of 10 Å thick. Soils containing montmorillonite minerals exhibit high shrinkage and swelling charactersitics, depending upon the nature of exchangeable cations present.

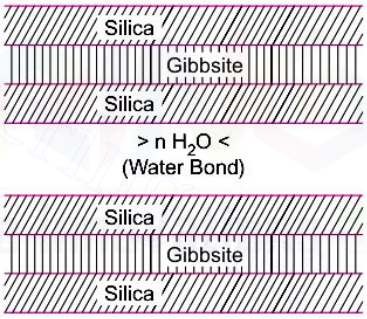

Fig. 5.17 (b) Structure of montmorillonite

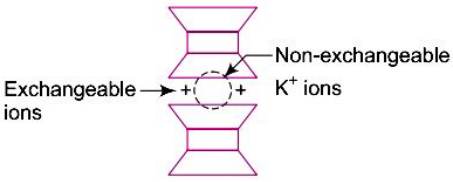

Fig. 5.18 (a)

Illite. The structure of illite is similar to that of montmorillonite except that there is always substantial (20% ∓) replacement of silicons by aluminium in the tetrahedral layers and potassiums are between the layers serving to balance the charge resulting from the replacement and to tie the sheet units together. The basic unit is symbolically represented as shown in Fig. 5.18 (*a*). The cation bond of illite is weaker than the hydrogen bond of kaolinite, but is stronger than the water

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bond of montmorillonite. Due to this, the illite crystal (Fig. 5.18 *b)* has a greater tendency to split into ultimate platelets consisting of gibbsite layer between two silica layers, than that in kaolinite. However, illite structure does not swell because of movement of water between the sheets, as in the case of montmorillonite. Illite clay particle may be 50 Å to 500 Å thick and 1000 Å to 5000 Å in lateral dimensions.

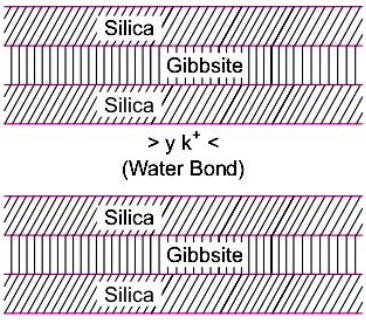

Fig. 5.18 (b) Structure of illite

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Part II
Soil Hydraulics

6. Soil Water: Effective and Neutral Stresses
7. Permeability
8. Well Hydraulics
9. Seepage Analysis
10. Seepage Below Hydraulic Structures
11. Drainage and Dewatering

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CHAPTER 6

Soil Water: Effective and Neutral Stresses

6.1 MODES OF OCCURRENCE OF WATER IN SOIL

Water present in the voids of soil mass is called *soil water.* It can be classified in several ways given below:

(a) Broad classification

1. Free water or gravitational water
2. Held water
   - (*i*) Structural water
   - (*ii*) Adsorbed water
   - (*iii*) Capillary water

(b) Classification on phenomenological basis

- (*i*) Ground water
- (*ii*) Capillary water
- (*iii*) Adsorbed water
- (*iv*) Infiltered water

(c) Classification on structural aspect **(Fig. 6.1)**

- (*i*) Pore water
- (*ii*) Solvate water
- (*iii*) Adsorbed water
- (*iv*) Structural water

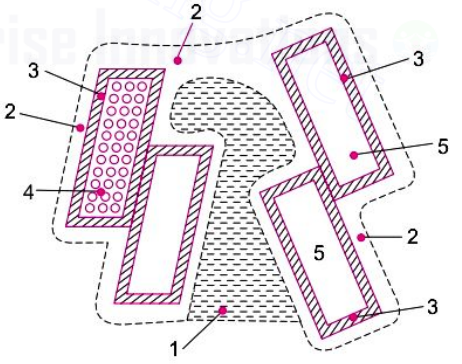

Fig. 6.1 Structural classification of water in soil

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Ground water: It is subsurface water that fills the voids continuously and is subjected to no forces other than gravity. Hence this water is also known as *gravitational water* or *free water.* Such a water has a free surface which can be easily observed in wells and bore holes. Ground water fills up the voids in the soil up to the *ground water table* and translocates through them. It fills coherently and completely all voids–making the soil saturated. Ground water obeys laws of hydraulics and is capable of moving under hydrodynamic forces.

Capillary water: It is that water which is lifted up by surface tension above the free ground water surface. This water is in *suspended condition* within the interstices and pores of capillary size of the soil. The capillary water fills all the pores in the soil, to a certain distance above the water table–this distance being known as *zone of capillary saturation.* The water saturated voids are bounded by menisci, in accordance with the capillary theory (see §6.3).

Adsorbed water: Adsorbed water comprises of (*i*) hygroscopic water, and (*ii*) film water. *Hygroscopic water* or *contact moisture* or *surface bound moisture* is that water which the soil particles freely adsorb from atmosphere by the *physical forces of attraction* and is held by the forces of *adhesion.* This form of moisture is in a dense state and surrounds the surfaces of individual soil grains as a very thin film. Coarse grained soils have relatively low hygroscopic moisture because of their low specific surface. The average *hygroscopicity* of sands, silts and clays is 1%, 6% and 16% respectively. *Hygroscopic moisture is affected neither by gravity nor by capillary forces*. Also, hygroscopic water has greater density, greater viscosity, higher boiling point and lower freezing point as compared to ordinary water. *Film moisture* is attached to the surface of soil particles as a film on the layer of hygroscopic film. This film forms because of condensation of aqueous vapour. Film moisture is also held by molecular forces of high intensity but not as high as in the case of hygroscopic film. *Film moisture* can migrate on the application of external energy potential (*i.e.*, thermal potential or electric potential). The greater the specific surface of soil, the more is the film moisture that can be contained.

Infiltered water: It is that portion of *surface precipitation* which soaks into ground, moving downwards through air-containing zones. It is subject to capillary forces.

Pore water (Fig. 6.1): From the point of view of inter-particle forces, soil water can also be divided into two heads : the *adsorbed water* which is attracted by forces within the soil strong enough to influence its behaviour, and *pore water* (Fig. 6.2) which is essentially free of strong soil attractive forces (Lambe, 1953). The capillary water and the gravitational water may be considered as the two types of pore water. It exhibits the physical and chemical properties of ordinary liquid water. It is capable of moving under hydrodynamic forces unless restricted in its free movement, such as when entrapped between air bubbles or by retention due to capillary forces that in fine pores may overcome the hydrodynamic forces.

Solvate water: It is that water which forms a hydration shell (presumably not more than 200 molecules thick) around soil grains. It is subject to polar, electrostatic and ionic binding forces. It remains mobile under hydrodynamic forces, though its density and viscosity are greater than those of ordinary water.

Structural water: It is the water chemically combined in the crystal structure of the soil mineral. It refers to *hydroxyl groups* that constitute parts of crystal lattice. Under loading encountered in soil engineering, the structural water cannot be separated or removed and is, therefore, unimportant. It can also not removed by oven drying at 105°C–110°C. However, it can only be driven off at such high temperatures as would cause the destruction of the crystal structure. We will therefore, consider the structural water as part and parcel of the soil particle.

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6.2 ADSORBED WATER

We have studied that the soil particles carry a net negative charge. Due to this charge, they attract water. The attractive forces (surface potential) is a function of the specific surface of the particles. The attractive forces between the particle and the water so adsorbed are polar bonds and soil-water forces *via* the exchangeable cations [Fig. 6.2 (*a*)]. The cations of the double layer also attract the dipolar molecules of water. Water in the vicinity of soil particle is thus subjected to an attractive force which basically consists of two components (Lambe, 1960) : (*i*) attraction of the dipolar water to the electrical charged soil, and (*ii*) attraction of dipolar water to the cation in the double layer, the cations in turn being attracted to the particle.

As the adsorbed water is under the influence of soil attractive forces, its behaviour is different from that of the pore water [Fig. 6.2. (*b*)]. The part of the adsorbed water immediately adjacent to the soil is held by forces that are so strong that the water is immobile to normal hydrodynamic forces. The density, freezing point and other properties of this tightly bound water are different from those of ordinary liquid water. It can be considered as bound water which is a part of the effective soil particle. However, the outer portion of the adsorbed water is partially immobilised and can be considered as viscous water. The forces between this water and the soil are of the same nature as those of the inner layer, but are weaker (Lambe, 1953).

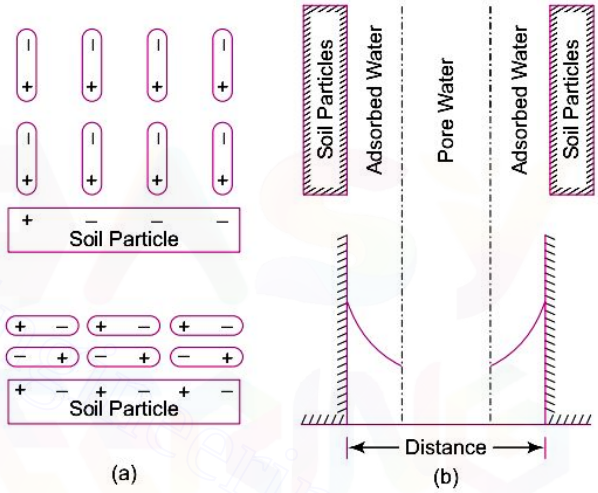

Fig. 6.2 Adsorbed water and pore water (Lambe, 1953)

If an oven-dried soil sample is placed in moist air, the sample adsorbs moisture, till its water content reaches some constant value corresponding to its maximum *hygroscopic capacity.* The quantity of hygroscopic water or adsorbed water for a given soil varies with the temperature and the relative humidity of the air, and the characteristics of the soil particles themselves. Coarse-grained soils have relatively low hygroscopic moisture due to their limited specific surface. The maximum hygroscopic capacity of various soils, *i.e.,* the ratio of water adsorbed by a dry soil in saturated atmosphere at a given temperature to the weight of oven-dried soil, has approximately the following average values: sands 1%, silts 7% and clays 17%. The thickness of adsorbed layer varies from about 200 *angstroms* (1 angstrom $=10^{-7}$ mm $= 10^{-10}$ metre) for silts to 30 angstroms for clay colloids. Adsorbed water has a very definite effect on the cohesion and plasticity characteristics for fine grained soils.

The water in the soil system that is not under significant forces of attraction from the soil particle is pore water. Pore water follows hydrodynamic laws and in general behaves as normal liquid water. Essentially all of the water in sandy soil is pore water; less than half the water in a saturated cohesive soil may be pore water or free water.

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6.3 CAPILLARY WATER

Capillary water is the soil moisture located within the interstices and voids of capillary size of the soil. Capillary water is held in the interstices of soil due to capillary forces. *Capillary action* or *capillarity* is the phenomenon of movement of water in the interstices of a soil due to capillary forces. The minute pores of soil serve as capillary tubes through which the moisture rises above the ground water table. The capillary forces depend upon various factors such as surface tension of water, pressure in water in relation to atmospheric pressure, and the size and conformation of soil pores.

Surface Tension. Surface tension of water is the property which exists in the surface film of water tending to contract the contained volume into a form having a minimum superficial area possible. The molecules on surface of a liquid are attracted by other molecules on the surface and inside the body of the liquid. Because there is no pull from outside, the surface molecules are pulled towards the inside of the liquid mass tending to reduce the surface to a minimum. Thus, in the case of molecules in a drop of water the drops tend to assume a spherical shape. The surface tension T_s (sometimes designated as the *coefficient of surface tension*) is approximately equal to 72.8 dynes per cm or 0.728×10^{-6} kN/cm at 20°C. The coefficient depends upon the chemical nature of liquids. The surface tension for other common liquids (such as acetone, benzene, petrol, etc.) at room temperature (20°C) are approximately of the order of 0.29×10^{-6} kN/cm. Thus, the surface tension for water is more than double the surface tension for other common liquids. However, the surface tension for mercury is as high 2.45×10^{-6} kN/cm. The formation of curved menicus around other material inserted in water is due to the surface tension. When a solid or hollow tube, wet with water, is partly inserted vertically in water, the molecules, due to attraction between the molecules of water and the material, climb the solid surface forming a curved meniscus adjacent to the walls of the tube or rod. The tensile strength, due to the attraction of the surface molecules for each other and for the rube wall, supports the water in the meniscus above the horizontal surface (Fig. 6.3).

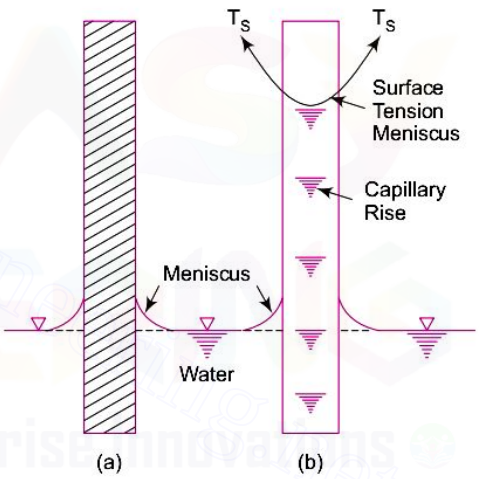

Fig. 6.3 Surface tension and formation of meniscus

Capillary Rise. The interstices or the pores of soil mass may be looked upon as a series of capillary tubes, extending vertically above water table. The rise of water in the capillary tubes, or the fine pores of the soil, is due to the existence of surface tension which pulls the water up against the gravitational force. The height of capillary rise, above the *ground water* (or free water) surface depends upon the diameter of the capillary tube (or fineness of the pores) and the value of the surface tension. The water in the capillary tube hangs in tension, and is supported from the side of the tube around the edge of the meniscus. Figure 6.4 shows an enlarged view of a capillary tube inserted in water, and the consequent capillary rise.

The formation of a concave meniscus will take place only if the inner walls of the tube are initially wet. If the walls are dry before insertion, a convex meniscus depressed below the water surface is formed. However since the soil pores always carry adsorbed water, the meniscus formation in soil will always be concave as shown in Fig. 6.4. The vertical component $T_s \cos \alpha$ of the surface tension force depends upon the angle of incidence α between the meniscus and the tube.

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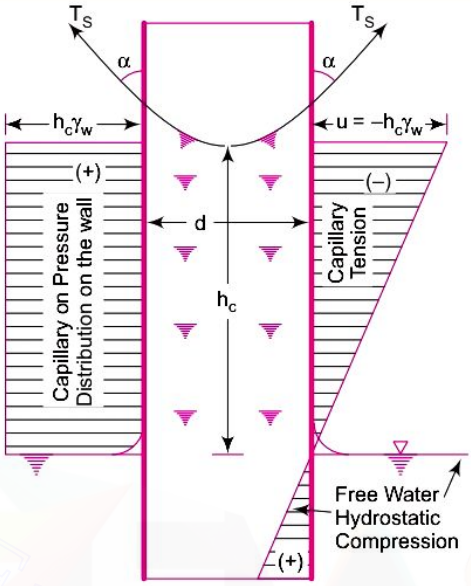

Fig. 6.4 Capillary rise

Let d = inner diameter of the tube and h_c = height of capillary rise.

When the capillary tube is inserted in water, the rise of water will take place. When equilibrium has reached, water will stop moving further. At this equilibrium position, when the height of rise is h_c, the weight of column of water is equal to $\frac{\pi d^2}{4} h_c \gamma_w$.

Table 6.1 Values of unit weight, dynamic viscosity and surface tension for water

Temp °C	*Unit weight (kN/m³)*	η *(dyne-s/cm²)*	*Surface tension (dynes/cm)*
4	9.8070	0.01567	75.6
16	9.7969	0.01111	73.4
18	9.7935	0.01056	73.1
20	9.7896	0.01005	72.8
22	9.7854	0.00958	72.4
24	9.7808	0.00914	72.2
26	9.7758	0.00874	71.8
28	9.7704	0.00836	71.4
30	9.7646	0.00801	71.2

Note. 1 kN = 10^8 dynes; 1 g = 981 dynes.

The vertical component of the reaction of meniscus against the inside circumference of the tube, supporting the above weight of water column, is equal to $\pi d T_S \cos \alpha$. Equating these two quantities at the equilibrium, we get

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$$\frac{\pi d^2}{4} h_c \gamma_w = \pi d\, T_S \cos\alpha \text{ from which, } h_c = \frac{4T_S \cos\alpha}{\gamma_w d} \quad ...(6.1)$$

The value of α will depend upon the initial conditions of the inner walls of the tube. If the tube is perfectly clean and wet, a semi-spherical meniscus will be formed. In that case, α will be zero, and maximum capillary rise will take place:

$$(h_c)_{max} = \frac{4T_S}{\gamma_w d} \quad ...(6.2)$$

For water at 4°C, $T_S = 75.6$ dynes/cm $= 75.6 \times 10^{-8}$ kN/cm and $\gamma_w = 9.807$ kN/cm^3.

If d is expressed in cm, the above expression reduces to

$$(h_c)_{max} = \frac{4 \times 75.6 \times 10^{-8}}{(9.807)(10^{-6})d} = \frac{0.3084}{d} \text{ cm (Since 1 m}^3 = 10^6 \text{ cm}^3) \quad ...(6.3a)$$

At 20°C, $T_S = 72.8$ dynes/cm $= 72.8 \times 10^{-8}$ kN/cm and $\gamma_w = 9.7876$ kN/m^3

$$(h_c)_{max} = \frac{4 \times 72.8 \times 10^{-8}}{(9.7896 \times 10^{-6})d} = \frac{0.2975}{d} \text{ cm} \quad ...(6.3b)$$

Thus, we see that height of capillary rise *decreases* with *increase* in temperature.

Retention of water in capillary tube, when lifted: It is interesting to note that if a capillary tube of uniform diameter is lifted, no water will be retained in the tube. This is because the surface tension at the two ends of the vertical water column will balance each other leaving no resultant force to oppose gravity. Thus in Fig. 6.5 (*a*) the upwards forces (F_u) due to surface tension are balanced by downward forces (F_d) at the lower end. However, if the tube is of non-uniform section, such as a *necked tube* with smaller diameter at the top, (Fig. 6.5 *b*) some water will be retained in the tube when lifted. This is so because the upward component of surface tension (*i.e.*, force F_u) is greater than the down ward component of surface tension (*i.e.*, force F_d).

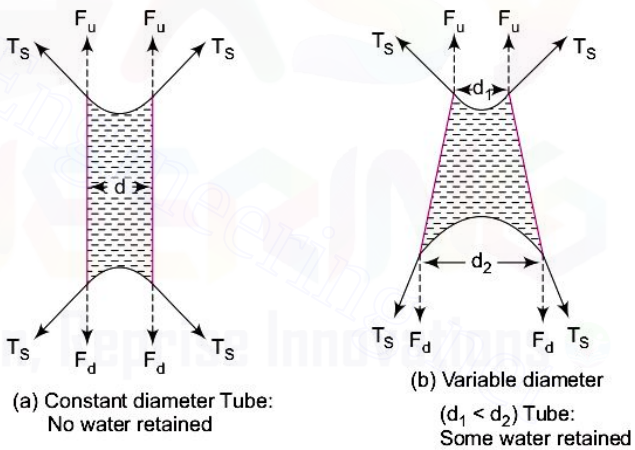

Fig. 6.5

Capillary rise in tubes of non-uniform diameter: If a capillary tube is of non-uniform diameter (*i.e.* of diameters d_1 and d_2) as shown in Fig. 6.6, the height of capillary rise will depend upon the *direction of flow* of water in the tube. When such as tube is dipped in water, water is lifted in the tube up to a height h_{c1} only.

It will not rise past the bulb because water cannot maintain equilibrium at the larger diameter d_2. In contrast to this if the tube is filled by pouring water from the top, or if the tube is lowered below the water table and then raised, equilibrium is maintained at a greater height h_{c2} (Fig. 6.6 *b*). Thus, the capillary height in capillary tubes of non-uniform diameter is more if the flow is downward than when the flow is upward.

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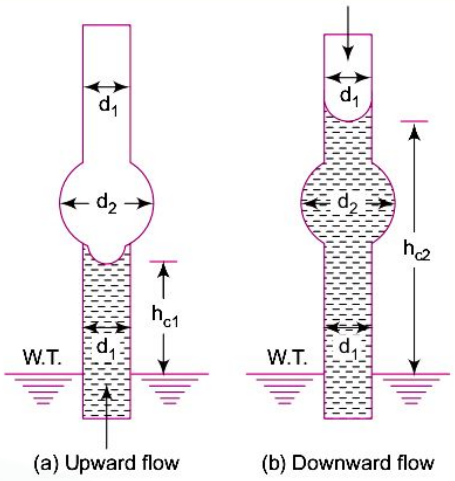

Fig. 6.6 Effect of direction of flow on capillary rise

6.4 CAPILLARY TENSION, CAPILLARY POTENTIAL AND SOIL SUCTION

It is to be noted that the water in the capillary tube, above the level of the ground water or free water surface, will be in a state of tension. However, the water in the tube, below the free surface, will be in hydrostatic compression as usual. The stress distribution is also shown in Fig. 6.4. At any height h *above* the water table, the stress u in water will be $-h\gamma_w$ (minus sign for tension). The maximum magnitude of the stress u will depend upon the radius R of the meniscus (Fig. 6.7). The relation between the diameter d and the radius R is:

$$\frac{d}{2} = R\cos\alpha \quad \text{or} \quad d = 2R\cos\alpha$$

Substituting this value of d in Eq. 6.1, we get

$$h_c = \frac{4T_S\cos\alpha}{\gamma_w\, 2R\cos\alpha} = \frac{4T_S}{\gamma_w\, 2R}$$

$\therefore$ u_c = maximum tension at the level of meniscus

$$= \gamma_w h_c = \gamma_w \frac{4T_S}{\gamma_w\, 2R} = \frac{2T_S}{R} \qquad ...(6.4)$$

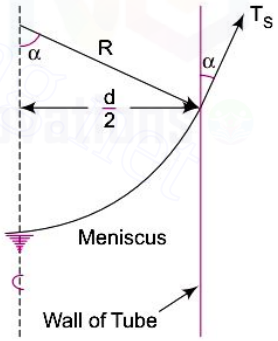

Fig. 6.7 Relation between R and α

Thus, the maximum tensile stress is inversely proportional to the radius of meniscus.

When $\alpha = 0, R = d/2$, we have $(u_c)_{max} = \dfrac{4T_S}{d}$ ...(6.4*a*)

This could also be obtained from Eq. 6.2 : $(u_c)_{max} = (h_c)_{max}\,\gamma_w = \dfrac{4T_S}{d}$

The tensile stress caused in water is called the *capillary tension* or the *capillary potential.* The capillary tension or capillary potential is the *pressure deficiency*, pressure reduction or negative pressure in

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the pore water (or the pressure below atmospheric) by which water is retained in a soil mass. It decreases linearly from a maximum value of $h_c \gamma_w$ at the level of the meniscus to zero value at the free water surface. The pressure deficiency in the held water is also termed as *soil suction* or *suction pressure.* Soil suction is measured by the height h_c in centimetres to which a water column could be drawn by suction in a soil mass free from external stress. The common logarithm of this height (cm) or pressure (g/cm^2) is known as the pF value (Schofield, 1935):

$$pF = \log_{10}(h_c) \qquad ...(6.5)$$

Thus, a pF value of 2 represents a soil suction of 100 cm of water or suction pressure and capillarity of 100 g/cm^2.

Factors affecting soil suction

Following are some of the factors affecting soil suction:

1. *Particle size of soil:* Smaller the size of the particles, smaller will be the pore size with small radii of menisci, resulting in greater capillary rise and hence greater suction. This is evident from Table 6.1.

2. *Water content:* Smaller the water content, greater will be the soil suction. Soil suction will attain its maximum value when the soil is dry (see Figs. 6.8 and 6.9)

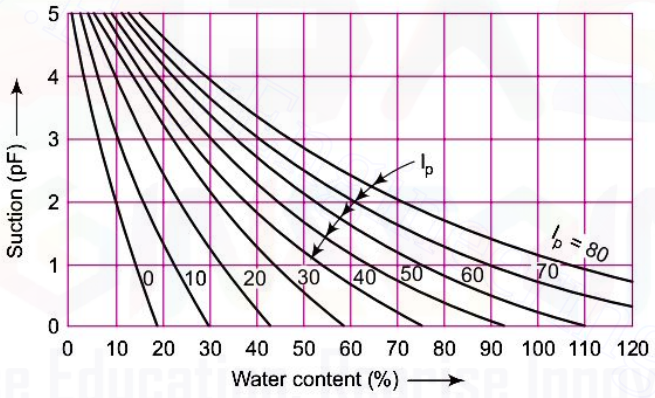

Fig. 6.8 Relation between soil suction and water content at various I_p

3. *Plasticity index of soil:* For a given water content, soil suction will be greater in a soil which has greater plasticity index than in the one which has lower plasticity index. Empirical relationship between the suction of remoulded soil and water content for a range of plasticity indices has been established at the Road Research Laboratory England, as shown in Fig. 6.8.

4. *History of drying and wetting:* For the same soil, suction is greater during drying cycle than during wetting cycle (see Fig. 6.9).

5. *Soil structure*: The size of interstices in a soil depends upon the structure of the soil. Change in the structure of a soil result in the change in the size of the interstices and hence change in soil suction.

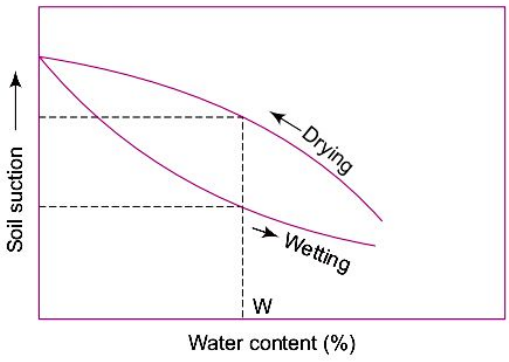

Fig. 6.9 Variation of soil suction during drying and wetting cycles

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6. *Temperature:* *Rise* in temperature results in decrease of surface tension (T_S) and hence *decrease* in soil suction. Similarly, fall in temperature results in increase of soil suction.

7. *Denseness of soil*: Increase in denseness of soil results in decrease in the size of the pores of the soil and hence increase in soil suction. At low density, the soil will be relatively loose, with larger size pores, resulting in decrease in soil suction.

8. *Angle of contact:* The mineralogical composition of soil governs the angle of contact between the soil particles and water. Soil suction decreases with increase in the angle of contact (α). As is evident from Eq. 6.1, the capillary height (h_c) and hence the soil suction is maximum when $\alpha = 0$.

9. *Dissolved salts in pore water:* Impurities, such as dissolved salts, etc. increase the surface tension, resulting in increase in soil suction.

Variation of capillary tension (or soil suction) during drying and wetting of soils

Up to this stage, we have discussed the development of capillary tension in soil zone just above the water table. It is interesting to note that capillary tension also develops in a saturated soil, when its water content is reduced by drying. This is because during the drying process, the water recedes into the interstices of soil, thus forming menisci. On further reduction in water content, the menisci recede, resulting in reduction of radii of curvature and corresponding increase in capillary tension or soil suction. The soil suction is maximum when the soil is oven dry. On the contrary, when the moisture content of an oven dried soil is increased, the soil suction is decreased. When a dry soil is submerged under water, meniscus is destroyed resulting in reduction of soil suction to zero value. Figure 6.9 shows the variation of soil suction during drying and wetting cycles. It is interesting to note that the suction at a particular water content (w) is more when the soil is drying than when the soil is wetting, resulting in the formation of a hysterisis loop. The process of drying is analogous to the flow of water in the downward direction (Fig. 6.6 *b*) resulting in greater capillary tension.

Capillary pressure. Capillary pressure and capillary tension are numerically equal at the level of meniscus. The meniscus transfers its surface tension force from liquid circumferentially around the wall of the capillary tube (or the solid particles forming the voids) causing the capillary pressure (↓). The pressure induces on the soil particle a compressive stress equal to the weight of the water column. The capillary pressure distribution is rectangular, unlike the triangular distribution of capillary tension, as shown in Fig. 6.3 (left-side). The magnitude of the pressure is the same at all heights above the free water surface. The capillary pressure, transferred from grain to grain may also be called *intergranular* or *contact* or *effective pressure* (see § 6.4 also). The intergranular pressure tends to force the solid particles together with a pressure equal and opposite to the tension through the water.

Non-uniform meniscus. If the meniscus is not of uniform curvature, but R_1 and R_2 are the radii of curvature in two orthogonal planes, the height of capillary rise is given by

$$h_c = \frac{T_S}{\gamma_w}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad ...(6.6)$$

Contact moisture. Water can also be held by surface tension round the point of contact of two particles (spheres). Capillary water in this form is known as contact moisture or *contact capillary water.* Because of the tension in the contact capillary water, the two particles tend to press against each other giving rise to a force known as *contact pressure*. Various factors affecting the contact pressure are (*i*) particle size, (*ii*) density of packing, (*iii*) angle of contact, and (*iv*) water content. *Increase* in water content results

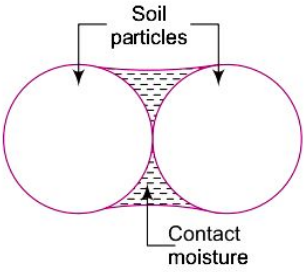

Fig. 6.10 Contact moisture

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in increase in the radius of meniscus, which in turn results in *decrease* of contact pressure. When the soil becomes saturated, the contact pressure reduces to zero. The contact pressure gives rise to the phenomenon called *apparent cohesion* due to which sands may acquire compressive and tensile strengths.

6.5 CAPILLARY PHENOMENON IN SOILS : CAPILLARY ZONES

Capillary rise in soil depends upon the size and grading of the particles and the state of their packing. Though the phenomenon of capillary rise in soils is similar to capillary rise in a capillary tube, the pores in soils are irregularly shaped and are interconnected not only in vertical direction but in horizontal directions also. Due to these limitations, a purely theoretical analysis, which may give satisfactory results, is impossible.

In order to apply Eq. 6.2 to compute $(h_c)_{max}$ it is essential to estimate the diameter d of the pore space. Two methods are available for assessing *effective pore diameter d.*

1st method: Take the diameter of pore size equal to 20% of the *effective grain size* (D_{10}),

i.e., $$d = 0.2\, D_{10} \qquad ...(6.7\ a)$$

Substituting this value of d in Eq. 6.2, $h_{c_{max}}$ can be calculated.

2nd method: Hazen has given the following equation for computing $h_{c_{max}}$

$$h_{c_{max}} = \frac{C}{e\, D_{10}} \qquad ...(6.7\ b)$$

where e = voids ratio and the value of C (an empirical constant) is between 0.1 to 0.5 sq. cm when $h_{c_{max}}$ and D_{10} are taken in cm.

The above expression shows that the decrease in effective grain size causes a decrease in voids ratio and an increase in the capillary rise in soils.

The average values of height of capillary rise in soils are given in Table 6.2.

Since the effect of molecular attraction is more near the water table, all the pores will be filled, irrespective of the pore space, and the soil will be fully saturated. However, at far off distance from the water table, only smaller voids will be filled, leaving the soil there *partially saturated.* In soil near the ground surface, there may be only contact moisture, without having any continuity. This soil above the water table may be divided following *three zones* (Fig. 6.11)

Table 6.2 Capillary heights of soil

Soil	*Fraction (mm)*	*Capillary height (cm)*
Fine gravel	2 to 1	2 to 10
Coarse sand	1 to 0.5	10 to 15
Medium sand	0.5 to 0.25	15 to 30
Fine sand	0.25 to 0.05	30 to 100
Silt	0.05 to 0.005	100 to 1000
Clay	0.005 to 0.0005	1000 to 3000
Colloids	< 0.0005	3000 and more

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(*i*) Zone I: Zone of capillary saturation
(*ii*) Zone II: Zone of partial saturation and
(*iii*) Zone III: Zone of contact water.

The zone of *capillary saturation* is the zone which has soil with 100% saturation. In the second zone, *i.e.* the zone of *partial saturation,* only small pores are filled with water while the large voids are filled with air, and hence the degree of saturation is less than 100%. The third zone near the ground surface contain water surrounding the particles at contact points, without having any continuity. Water in the third zone (top zone) may also reach there from ground surface by percolation/infiltration and may be held in suspension by capillary forces.

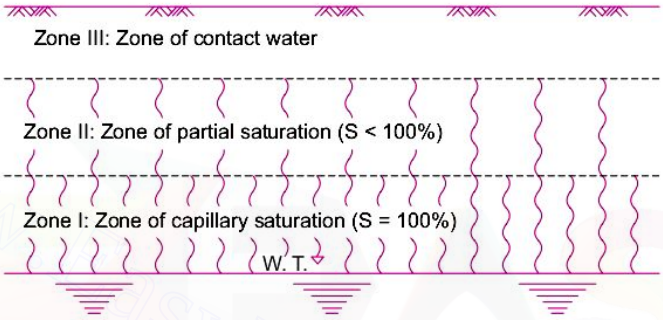

Fig. 6.11 Capillary zones in soil above water table

6.6 SHRINKAGE AND SWELLING OF SOILS

Soils undergo a volume change when the water content is changed: decrease in water content cause shrinkage while increase of water content cause swelling. Large volume changes of soils, specially clayey soils have resulted in extensive structural damages. However, coarse grained soils have very little shrinkage and swelling. For clayey soils, the degree of change in volume depends upon factors such as (*i*) type and amount of clay minerals present in the soil (*ii*) specific surface area of clay, (*iii*) structure of the soil, (*iv*) pore water salt concentration, (*v*) valence of the exchangeable cation etc.

Shrinkage of soils: Shrinkage takes place due to decrease in water content in the soils. When a saturated soils is allowed to dry, a meniscus develops in each void at the soil surface. Formation of such a meniscus causes tension in the soil water leading to a compression in the soil structure and consequent reduction in the volume. Figure 6.12 shows a soil mass consisting of spherical soils particles. Initially, when the soil is fully saturated, the capillary spaces between the particles are completely filled with water leading to the formation of meniscus in the form of plane *a – a.* Tension in water is zero at this stage. When the water evaporates, the meniscus retreats to positions *b – b, c – c,* etc. causing tension in water and consequent compression in solid grains. The magnitude of tension so developed depends upon the radius of meniscus at that stage. Finally, when the meniscus (*f – f,* say) attains its minimum value, compressive forces are maximum and shrinkage is also maximum. Soil at this stage has minimum volume and the water content corresponds to the shrinkage limit for the soil. Further recession of meniscus does not increase the compressive forces, as pores of smaller radius are not available at the that stage.

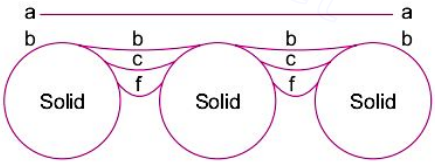

Fig. 6.12 Retreating of meniscus

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The *degree of shrinkage* depends upon several factors such as initial water content, type and amount of clay content, mode and environment, of geological deposition. Shrinkage occurs horizontally as well as vertically, causing vertical shrinkage cracks. Shrinkage is more prominent in clay soils. However, presence of sand and silt-size particles in a clay deposit reduce the total shrinkage.

Swelling of soil: When water is added to a soil which has shrunk, menisci are destroyed resulting in tension in the pore water and consequent reduction in compressive stresses in solid grains. This results in elastic expansion of the soil mass, causing *swelling*. However, in clay soils, swelling is caused mainly due to repulsive forces which separate the clay particles, causing volume change. The mechanism of swelling is much more complex than that of shrinkage. Swelling depends upon several factors such as (*i*) type and amount of clay mineral present in the soil, (*ii*) affinity of clay minerals for water, (*iii*) cation exchange capacity and electrical repulsive forces, (*iv*) expansion of entrapped air, (*v*) specific surface (*vi*) structure of soil, (*vii*) elastic rebound of soil grains, etc.

As per IS: 2720 - Part 40, 1977, *free swell* of a soil is defined as increase in the volume of a soil without any constraints, on submergence in water. Generally free swell ceases when the water content reaches the plastic limit. When the montmorillonite clay mineral content is very high in a soil, it shows an almost *reversible* swelling and shrinkage on rewetting and redrying. However, when illite and kaolinite clay minerals are present in a clay soil, the soil exhibit large initial volume decrease on drying with only a limited swelling on rewetting.

6.7 SLAKING OF CLAY

Consider a mass of clayey soil which has been dried well *below the shrinkage limit,* thus, attaining the minimum volume. When this mass of soil is suddenly immersed in water, it will cause its *slaking,* resulting in its disintegration into a soft wet mass. Such *slaking* is due to the entry of air into the void space during the drying of the soil below the shrinkage limit. When water enters these voids, due to immersion, menisci will be formed, causing high pressures and subsequent explosion of the voids leading to the disintegration of the soil structure.

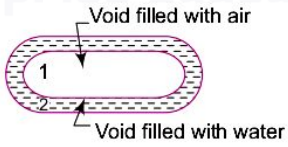

Fig. 6.13 Slaking of clay

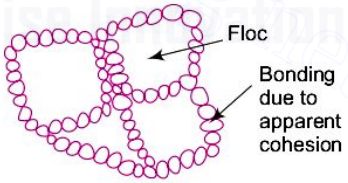

Fig. 6.14 Bulking of moist sand

6.8 BULKING OF SAND

If a dry mass of sand is moistened slightly and then shovelled or dumped loose into a heap, its volume will increase considerably relative to dry state. This phenomena is called *bulking* of sands. Apparent cohesion offers an explanation for the bulking of sands. As a result of inter-particle adhesion due to capillary action of water, apparent cohesion is developed. This apparent cohesion holds the particles together in clusters enclosing honey combs especially when the moist soil mass is loosely dumped or poured. The finer the grains, the greater is the increase in volume. The volume change depends also upon the water content, its maximum value being when $w = 5\%$ to 6%. When water content is slightly below

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complete saturation, no such bulking occurs. Saturating the soil mass would result in the destruction of the menisci and an appreciable decrease in volume.

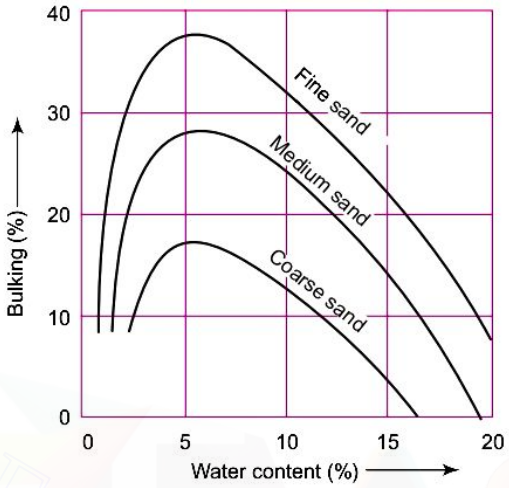

Fig. 6.15 Bulking of sands

6.9 FROST ACTION

The freezing and thawing of water present in soil and the resultant effects on soil and on structures founded in this soil is known as *frost action.* Frost action consists of two processes: (1) Frost heave, and (2) Frost boil.

1. Frost heave: *Frost heave* is defined as rise of ground water surface due to frost action. When the atmospheric temperature falls to the freezing point, the water present in the capillary fringe may freeze, leading to the formation of ice. When water is converted into ice, there is an increase of about 9% in the volume. Therefore, in a saturated soil, the void volume above the level of freezing will increase by the same amount, representing an overall increase in the volume of soil of 2.5% to 5%, depending upon the voids ratio. For example, if the voids ratio is 40%, the expansion of the soil if saturated, will be $0.09 \times 40 = 3.6\%$, which means that there will be a heave of 3.6 cm in every 1 m thickness of soil strata in the capillary fringe. In certain circumstances, a much greater increase in volume can occur due to formation of ice lenses within the soil (Fig. 6.16).

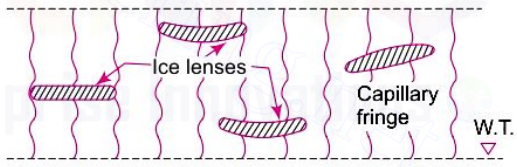

Fig. 6.16 Formation of ICE lenses due to frost

The temperature at which water freezes in the pores of the soil depends upon the pore size–smaller the pore size, lower the freezing temperature. Initially, water freezes in larger pores, remaining unfrozen in the smaller pores. As the temperature falls below zero, higher *soil suction* develops and water migrates towards the ice in the larger voids where it freezes and adds to the volume of ice. *Continued migration gradually results in the formation of ice lenses* (Fig. 6.16) *and a rise in the ground surface.* Only silts and fine sands are prone to frost action because these soils have large capillary rise due to relatively fine particles. Also because of relatively fair permeability, water can easily flow in these soils. In contrast to

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this, coarse grained soils have very little capillary rise and hence frost heave in these soils is very limited. Similarly, in clayey soils, though their capillary rise is very high, their permeability is very low resulting in limited water migration and hence limited frost heave. However, the presence of fissures can result in an increase in the rate of migration. A well graded soil is reckoned to be frost susceptible if more than 3% of the particles are smaller than 0.02 mm. A poorly graded soil is susceptible if more than 10% of the particles are smaller than 0.02 mm.

The depth or boundary below the ground surface up to which water may freeze is called *frost line.* Following are the conditions for the formation of ice lens and marked frost heave.

1. Soil to be saturated in the beginning and during the freezing period
2. Proximity of ground water reservoir so that water may be pulled up in the zone of freezing.
3. High capillarity of soil so that the soil possesses high capacity to pull water.
4. Moderate permeability of the soil, so as to allow unobstructed flow of water through it.
5. Gradual dropping of seasonal temperature, so that temperature of soil drops below freezing point, and this reduced temperature to persist for a longer duration. Also, there should be gradual rate of change of temperature with depth.

2. Frost boil: If the temperature rises in the soil in which frost heave has occured, the frozen soil mass *thaws* and free water is liberated. When thawing takes place, the soil previously frozen will contain an excess of water with the result that it will become soft and its strength is reduced. These effects are known as *frost boil.*

Silty soils are more susceptible to frost boil than sands and clayey soils. In sands, the released water due to thawing drains out quickly. However, silty soils have more plasticity index compared to clays and hence the softening effect due to excess water will be more severe in silts than in clays.

Effects of frost boil

1. Settlement of structures resting on the ground surface.
2. Settlement of highway pavements.
3. Formation of pot holes due to extrusion of soft soil and water under the dynamic action of wheel loads.
4. Eventual breaking of pavement and ejection of subgrade soil in a soft and soapy condition, under moving wheel loads.

3. Preventive measures: Ill effects of frost can be prevented or minimised by adopting the following preventive measures:

1. Founding buildings below the frost depth to avoid possible frost heave.
2. Placing insulating blankets of 15 to 30 cm thick layers of sand and gravel on the ground surface above the frost susceptible soil strata, to prevent deep frost penetration.
3. Interposing a pervious gravel blanket between the frost line and highest water table, to prevent considerably the capillary saturation of frost zone.
4. Providing proper drainage to lower the water table and to allow quick escape of water during thawing.
5. Removing the frost susceptible soil up to the frost depth and substituting by less susceptible soil.

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SOLVED EXAMPLES

Example 6.1. *Compute the maximum capillary tension for a tube 0.05 mm in diameter.*

Solution. The maximum capillary height at 4 °C is given by Eq. 6.3.

$$(h_c)_{max} = \frac{0.3084}{d} = \frac{0.3084}{0.005} = 61.7 \text{ cm} = 0.617 \text{ m}$$

$\therefore$ Capillary tension $= (h_c)_{max}\, \gamma_w = 0.617 \times 9.81 = \mathbf{6.05\ kN/m^3}$.

Example 6.2. *Compute the height of capillary rise in a soil whose D_{10} is 0.1 mm and voids ratio is 0.60.*

Solution. Let the average size of the void be d mm. Volume of each sphere of solids may be assumed proportional to D_{10}^3. Since the voids ratio is 0.6, the volume of void space, corresponding to the unit of volume of solids, will be proportional to 0.60 D_{10}^3.

But volume of each void space is also proportional to d^3.

Hence, $d^3 = 0.60\ D_{10}^3$

$$d = (0.60)^{1/3}\, D_{10} = 0.845 \times D_{10} = 0.845 \times 0.1 = 0.0845 \text{ mm} = 0.00845 \text{ cm}$$

Now, $$h_c = \frac{0.3084}{d} \text{ cm (at 4°C)} = \frac{0.3084}{0.00845} = \mathbf{36.5\ cm}.$$

Example 6.3. *When water at 20 °C is added to a fine sand and to a silt, a difference in capillary rise of 25 cm is observed between the two soils. If the capillary rise in fine sand is 25 cm, calculate the difference in the size of the voids of the two soils.*

Solution. Using suffix 1 for sand and 2 for silt,

$$h_{c1} = 25 \text{ cm}; \quad \therefore \quad h_{c2} = 25 + 25 = 50 \text{ cm}$$

Now, $$d_1 = \frac{0.2975}{h_{c1}} = \frac{0.2975}{25} = 0.0119 \text{ cm; and}$$

$$d_2 = \frac{0.2975}{h_{c2}} = \frac{0.2975}{50} = 0.00595 \text{ cm}$$

$\therefore$ Difference in the size of the voids is $= d_1 - d_2 = 0.0119 - 0.00595 = \mathbf{0.00595\ cm.}$

Example 6.4. *The Capillary rise in soil A with $D_{10}^3 = 0.06$ mm is 60 cm. Estimate the capillary rise in soil B with $D_{10}^3 = 0.1$ mm, assuming the same voids ratio in both the soils.*

Solution. Let the size of voids be d.

Now $$V_s \propto D_{10}^3 \quad \text{and} \quad V_v = eV_s$$

But $$V_v \propto d^3 \quad \therefore \quad e = \left(\frac{d}{D_{10}}\right)^3 \qquad ...(i)$$

For soil A, $$d = \frac{0.3084}{h_c} = \frac{0.3084}{60} = 5.14 \times 10^{-3} \text{ mm} = 5.14 \times 10^{-2} \text{ mm}$$

Substituting it in (i), we get

$$e = \left(\frac{d}{D_{10}}\right)^3 = \left(\frac{5.14 \times 10^{-2}}{0.06}\right)^3 = 0.629$$

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Now for soil B, $d = (e)^{1/3} D_{10} = (0.629)^{1/3} \times 0.1 = 0.857 \times 0.1 = 0.0857$ mm $= 0.00857$ cm.

Alternatively, since voids ratio for both soils is equal, we get from (*i*)

$$\left(\frac{d}{D_{10}}\right)_A = \left(\frac{d}{D_{10}}\right)_B$$

$$\therefore \quad (d)_B = \left(\frac{d}{D_{10}}\right)_A \times (D_{10})_B = \frac{5.14 \times 10^{-2}}{0.06} \times 0.1 = 0.0857 \text{ mm} = 0.00857 \text{ cm}$$

Hence, $$(h_c)_B = \frac{0.3084}{0.00857} = \mathbf{36\ cm.}$$

Example 6.5. *A glass vessel is filled with water, as shown in Fig. 6.17. A fully developed meniscus has formed in a small hole of diameter* $d_1 = 0.02$ *cm in the wall of the vessel. Another hole of diameter* d_2 *exists in the lower wall. What is the greatest value which* d_2 *may have? If* $d_2 = d_1$, *find the contact angle in the lower hole.*

Solution.

(*a*) As is evident from Fig. 6.17, the capillary heights supported by both the holes will be in opposite directions. However, the algebraic sum of the heights of water supported by them is equal to 9 cm.

i.e. $$h_{c1} - h_{c2} = 9 \text{ or } h_{C2} = h_{c1} - 9 \qquad ...(i)$$

Now $$h_{c1} = \frac{0.3084}{0.02} = 15.42 \text{ cm (when } \alpha = 0)$$

Thus, the upper hole can support a height of 15.42 cm

$$h_{C2} = 15.42 - 9 = 6.42 \text{ cm}$$

For full meniscus to be developed in the lower hole.

$$d_2 = \frac{0.3084}{6.42} = \mathbf{0.048\ cm.}$$

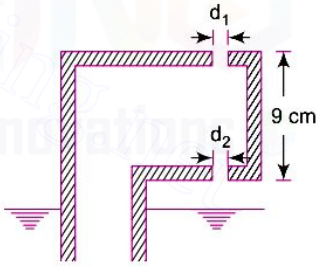

Fig. 6.17

This is the maximum value of d_2. If d_2 is increased, h_{c2} will be decreased, the corresponding value of h_{c1} ($= 9 + h_{c2}$) will also be decreased and the full meniscus will not be formed in the upper hole.

(*b*) If $d_1 = d_2 = 0.02$ cm, then for full meniscus to be developed in the upper hole, $h_{c1} = 15.42$ cm and $h_{c2} = 15.42 - 9 = 6.42$ cm

$$\therefore \quad h_{c2} = 6.2 = \frac{0.3084 \cos \alpha_2}{d_2}$$

$$\therefore \quad \cos \alpha_2 = \frac{h_{c2} \times d_2}{0.3084} = \frac{6.42 \times 0.02}{0.3084} = 0.416 \quad \text{or} \quad \alpha = \mathbf{65.4°}.$$

Note. The diameter of the lower hole can still be reduced. As reduction in diameter takes place, α_2 will go on increasing. When α_2 tends to reach 90° value, d_2 tends to reach a zero value, making the hole infinitely small.

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6.10 STRESS CONDITIONS IN SOIL: EFFECTIVE AND NEUTRAL PRESSURES

At any plane in a soil mass, the total stress or unit pressure σ is the total load per unit area. This pressure may be due to (*i*) self-weight of soil (saturated weight, if the soil is saturated), and (*ii*) over-burden on the soil. The total pressure consist of two distinct components : *intergranular pressure* or *effective pressure* and the *neutral pressure* or *pore pressure*. Effective pressure σ' is the pressure transmitted from particle through their point of contact through the soil mass above the plane. Such a pressure, also termed as *intergranular pressure,* is *effective* in decreasing the voids ratio of the soil mass and in mobilising its shear strength. The neutral pressure or the pore water pressure or pore pressure is the pressure transmitted through the pore fluid. This pressure, equal to water load per unit area above the plane, does not have any measurable influence on the voids ratio or any other mechanical property of the soil, such as the shearing resistance. Therefore, this pressure is also called *neutral pressure u.* Since the total vertical pressure at any plane is equal to the sum of the effective pressure and the pore pressure, we have

$$\sigma = \sigma' + u \qquad ...(6.8)$$

At any plane, the pore pressure is equal to piezometric head h_w times the unit weight of water, *i.e.*

$$u = h_w \gamma_w \qquad ...(6.8\ a)$$

To find the value of effective pressure, we shall consider different conditions of soil water system.

1. Submerged soil mass: Figure 6.18 shows a saturated soil mass of depth Z, submerged under water of height Z_1 above its top level. If a piezometric tube is inserted at level *AA*, water will rise in it up to level *CC*. Now, total pressure at *AA* is given by

$$\sigma = Z\gamma_{sat} + Z_1 \gamma_w \qquad ...(i)$$

Also, pore pressure $u = \gamma_w h_w$ ...(*ii*)

$$\therefore \quad \sigma' = \sigma - u = Z\gamma_{sat} + Z_1 \gamma_w - h_w \gamma_w$$

$$= Z\gamma_{sat} + Z_1 \gamma_w - (Z + Z_1)\gamma_w$$

or

$$\sigma' = Z(\gamma_{sat} - \gamma_w) = Z\gamma' \qquad ...(6.9)$$

Hence the effective pressure is equal to the thickness of the soil multiplied by the submerged weight of soil. It does not depend upon the height Z_1 of the water column. Even if Z_1 reduces to zero, σ' will remain equal to $Z\gamma'$ so long as the soil mass above *AA* remains fully saturated. At *BB*, the total pressure is equal to the water pressure $Z_1 \gamma_w$, and hence the effective pressure is zero. Thus, the effective pressure varies linearly, as shown in Fig. 6.18.

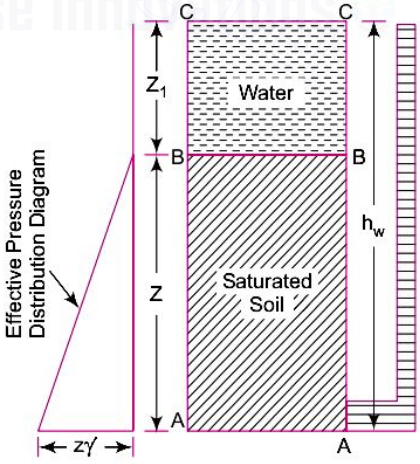

Fig. 6.18

2. Soil mass with surcharge: Let us now consider a moist soil mass of height Z_1 above a saturated mass of height *Z*. Soil mass supports a surcharge pressure of intensity *q* per unit area.

At the level *AA*, the pressure are:

$$\sigma = q + Z_1 \gamma + Z \gamma_{sat}$$

$$u = h_w \gamma_w = Z \gamma_w$$

$$\therefore \quad \sigma' = \sigma - u = q + Z_1 \gamma + Z \gamma_{sat} - Z \gamma_w$$

$$= q + Z_1 \gamma + Z\gamma' \qquad ...(6.10)$$

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At the plane BB

$$\sigma = q + Z_1 \gamma$$

$$u = h_w \gamma_w = 0$$

$$\therefore \quad \sigma' = \sigma = q + Z_1 \gamma$$

At the plane *CC*, effective pressure = total pressure = q. The effective pressure distribution diagram is also shown in Fig. 6.19.

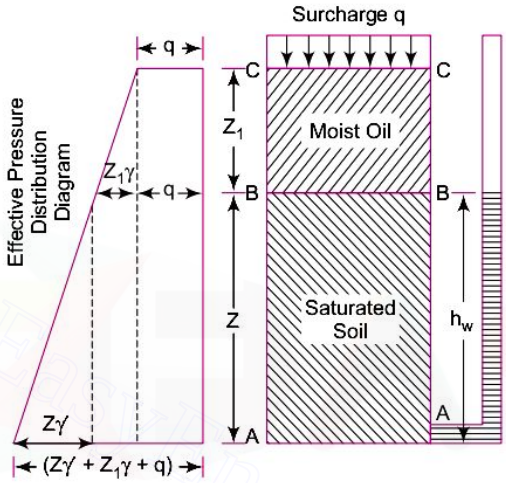

Fig. 6.19

3. Saturated soil with capillary fringe: Figure 6.20 shows a saturated soil mass of height Z. Above this, there is a soil mass of height Z_1 saturated by capillary water. If we insert a piezometric tube at *AA*, water will rise to a height corresponding to the free water level *BB*.

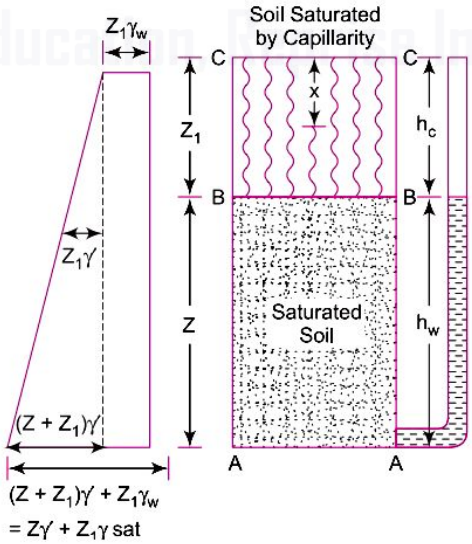

Fig. 6.20

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We have seen in the previous article that capillary induces a capillary pressure or compressive pressure on the soil grains. This pressure is also *intergranular* and is *effective* in reducing the voids ratio of the soil mass. This compressive pressure is equal to $h_c \gamma_w = Z_1 \gamma_w$ in the present case.

Hence, at the level *AA*,

$$\sigma' = (Z + Z_1)\gamma' \times Z_1 \gamma_w$$
$$= Z\gamma' + Z_1 \gamma' + Z_1 \gamma_w = Z\gamma' + Z_1 \gamma_{sat} \quad ...(6.11)$$

Alternatively, at AA, $\sigma = Z \gamma_{sat} + Z_1 \gamma_{sat}$

$u = Z \gamma_w$

$\therefore$ $\sigma' = \sigma - u = Z \gamma_{sat} + Z_1 \gamma_{sal} - Z \gamma_w = Z \gamma' + Z_1 \gamma_{sat}$

Similarly, at the level *BB*, $\sigma' = Z_1 \gamma' + Z_1 \gamma_w = Z_1 \gamma_{sat}$ ...(6.12)

This gives an interesting result. Ordinarily, in a saturated soil, σ' at depth Z_1 is $Z_1 \gamma'$, while in this case, when the soil is saturated by capillarity, this has been increased to $Z_1 \gamma_{sat}$. Finally, at *CC*, σ' = capillary pressure = $Z_1 \gamma_w$.

The effective pressure distribution diagram is shown in Fig. 6.20. *The effect of capillarity of height* Z_1, *is analogous to a surcharge* $q = Z_1 \gamma_w$ *placed on the saturated soil mass.*

At any depth x below the level *CC*, $\sigma' = x\gamma' + Z_1 \gamma_w$

Alternatively, $\sigma = x \cdot \gamma_{sat}$

u = – [pressure due to weight of water hanging below that level] = $-(Z_1 - x)\gamma_w$

$\therefore$ $\sigma' = \sigma - u = x \gamma_{sat} + (Z_1 - x)\gamma_w$

$= x(\gamma_{sat} - \gamma_w) + Z_1 \gamma_w$

$= x \cdot \gamma' + Z_1 \gamma_w$

The stress conditions in soil due to flowing water have been discussed in chapter 8.

4. Partially saturated soil: In a *partially saturated soil,* a part of void space is occupied by air. Hence, in addition to pore water pressure (u_w), *pore air pressure* (u_a) will also to there. Bishop (1959) based on his intuition, gave the following expression for the effective stress.

$$\sigma' = \sigma - u_a + \chi (u_a - u_w) \quad ...(6.13)$$

where u_a = pore air pressure

u_w = pore water pressure

χ = factor of unit cross-section area occupied by water = A_w/A

A_w = area of water;

A = area of cross-section of soil.

Figure 6.21 gives the variation of factor χ with degree of saturation. Apart from degree of saturation, factor χ also depends upon several other factors such as soil structure, process by which the soil reached the present degree of saturation (S) and stress changes. For degree

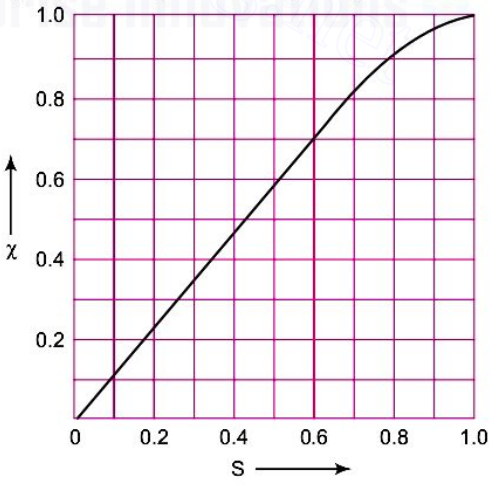

Fig. 6.21 Variation of χ with S (Bishop)

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of saturation near unity ($S \geq 90\%$), it is recommended to take χ as unity (*i.e.*, 1). In that case, the above equation reduces to the form.

$$\sigma' = \sigma - u_a + 1\,(u_a - u_w)$$
$$= \sigma - u_w = \sigma - u \qquad ...(6.14)$$

Example 6.6. *The water table in a certain area is at a depth of 4 m below the ground surface. To a depth of 12 m, the soil consists of very fine sand having an average voids ratio of 0.7. Above the water table the sand has an average degree of saturation of 50%. Calculate the effective pressure on a horizontal plane at a depth 10 metres below the ground surface. What will be the increase in the effective pressure if the soil gets saturated by capillarity up to a height of 1 m above the water table? Assume G = 2.65.*

Solution. Height of sand layer above water table = $Z_1 = 4$ m

Height of saturated layer = $12 - 4 = 8$ m

Depth of point X, where pressure is to be computed = 10 m

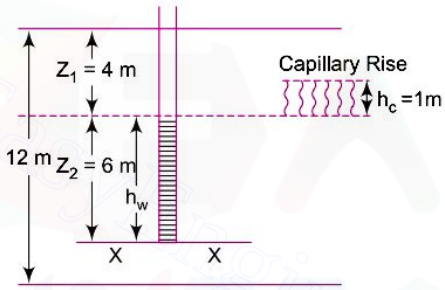

Fig. 6.22

Height of saturated layer above $X = Z_2 = 10 - 4 = 6$ m

Now, $$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.7}$$
$$= 15.29 \text{ kN/m}^3$$

(*i*) For sand above water table,

$$e = \frac{wG}{S_r} \quad \text{Hence,} \quad w = \frac{e\,S_r}{G} = \frac{0.7 \times 0.5}{2.65} = 0.132$$
$$\gamma_1 = \gamma_d\,(1 + w) = 15.29 \times 1.132 = 17.31 \text{ kN/m}^3.$$

(*ii*) For saturated sand below water table

$$w_{sat} = \frac{e}{G} = \frac{0.7}{2.65} = 0.264$$
$$\gamma_2 = \gamma_d\,(1 + w_{sat}) = 15.29 \times 1.264 = 19.33 \text{ kN/m}^3$$
$$\gamma_2' = 19.33 - 9.81 = 9.52 \text{ kN/m}^3.$$

Effective pressure at X

$$\sigma = Z_1\,\gamma_1 + Z_2\,\gamma_2 = 4 \times 17.31 + 6 \times 19.33 = 185.22 \text{ kN/m}^2$$

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$$u = h_w \gamma_w = 6 \times 9.81 = 58.86 \text{ kN/m}^2$$

$\therefore$ $\sigma' = \sigma - u = 185.22 - 58.86 = \mathbf{126.36\ kN/m^2}$.

Alternatively, $\sigma' = Z_1 \gamma_1 + Z_2 \gamma_2' = (4 \times 17.31) + (6 \times 9.52) = \mathbf{126.36\ kN/m^2}$.

Effective stress at X after capillary rise

$$\sigma' = 3 \gamma_1 + (6 + 1) \gamma_2' + h_c \gamma_w = (3 \times 17.31) + (7 \times 9.52) + (1 \times 9.81)$$

$$= 128.38 \text{ kN/m}^2$$

Increase in pressure= 128.38 – 126.36 = **2.02 kN/m^2**.

Example 6.7. *A 10 m thick bed of sand is underlain by a layer of clay of 6 m thickness. The water table which was originally at the ground surface is lowered by drainage to a depth of 4 m, whereupon the degree of saturation above the lowered water table reduces to 20%. Determine the increase in the magnitude of the vertical effective pressure at the middle of the clay layer due to lowering of water table. The saturated unit weights of sand and clay are respectively 20.6 kN/m^3 and 17.6 kN/m^3, and the dry unit weight of sand is 16.7 kN/m^3.*

Solution.

(*i*) Before lowering the water table, the pressures at the middle of the clay layer are

$$\sigma = (10 \times 20.6) + (3 \times 17.6) = 258.8 \text{ kN/m}^2$$

$$u = 13 \times 9.81 = 127.53 \text{ kN/m}^2 = 258.8 - 127.53 = \mathbf{131.27\ kN/m^2}.$$

(*ii*) After lowering the water table, the unit weight of sand is given by

$$\gamma = \gamma_d + S(\gamma_{sat} - \gamma_d) = 16.7 + 0.2\,(20.6 - 16.7) = 17.48 \text{ kN/m}^3$$

$$\sigma = (4 \times 17.48) + (6 \times 20.6) + (3 \times 17.6) = 246.32 \text{ kN/m}^2$$

$$u = 9 \times 9.81 = 88.29 \text{ kN/m}^2$$

$\therefore$ $\sigma' = 246.32 - 88.29 = \mathbf{158.03\ kN/m^2}$

$\therefore$ Increase in effective stress= 158.03 – 131.27 = **26.76 kN/m^2.**

Example 6.8. *The water table in a deposit of sand 8 m thick, is at a depth of 3 m below the surface. Above the water table, the sand is saturated with capillary water. The bulk density of sand is 19.62 kN/m^3. Calculate the effective pressure at 1 m, 3 m and 8 m below the surface. Hence plot the variation of total pressure, neutral pressure and effective pressure over the depth of 8 m.*

Solution.

(a) Stresses at D, 8 m below ground

If we insert a piezometric tube at *D*, water will rise through a height $h_w = 5$ m in it.

$$\sigma = (3 + 5) \gamma_{sat} = 8 \times 19.62 = 156.96 \text{ kN/m}^2$$

$$u = h_w \gamma_w = 5 \times 9.81 = 49.05 \text{ kN/m}^2$$

$$\sigma' = \sigma - u = 156.96 - 49.05 = \mathbf{107.91\ kN/m^2}.$$

Alternatively, $\sigma' = 5\gamma' + 3 \gamma_{sat} = 5 \times 9.81 + 3 \times 19.62 = 107.91 \text{ kN/m}^2$

or $\sigma' = 8 \gamma' + h_c \gamma_w = 8\,(19.62 - 9.81) + 3\,(9.81) = \mathbf{107.91\ kN/m^2}$.

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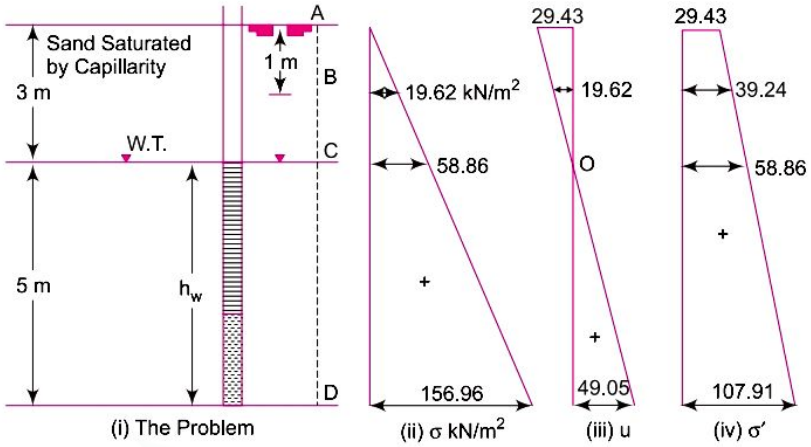

Fig. 6.23

(b) Stresses at C, 3 m below ground level

$$\sigma = 3\,\gamma_{sat} = 3 \times 19.62 = 58.86 \text{ kN/m}^2 \; ; \; u = \text{zero}$$

$$\sigma' = \mathbf{58.86 \text{ kN/m}^2}.$$

Alternatively, $\sigma' = h \times \gamma_{sat} = 3 \times 19.62 = 58.86$

or $\sigma' = 3 \times \gamma' + h_c\,\gamma_w = 3\,(19.62 - 9.81) + 3\,(9.81) = \mathbf{58.86 \text{ kN/m}^2}.$

(c) Stresses at A, at ground level

$$\sigma = 0$$

$$u = -\,h_c\,\gamma_w = -\,3\,(9.81) = -\,29.43 \text{ kN/m}^2$$

$$\sigma' = \sigma - u = \mathbf{29.43 \text{ kN/m}^2}.$$

(d) Stresses at B, 1 m below ground level

$$\sigma = 1\,\gamma_{sat} = 1 \times 19.62 = 19.62 \text{ kN/m}^2$$

$$u = -\,2\,\gamma_w = -\,2 \times 9.81 = -19.62 \text{ kN/m}^2$$

(*i.e.,* pressure due to weight of water hanging below that level)

$\therefore$ $\sigma' = (\sigma - u) = 19.62 - (-\,19.62) = 19.62 + 19.62 = \mathbf{39.24 \text{ kN/m}^2}.$

Alternatively, $\sigma' = 1\gamma' + h_c\,\gamma_w = 1\,(19.62 - 9.81) + 3 \times 9.81 = \mathbf{39.24 \text{ kN/m}^2}.$

Note. While finding the effective stress at any point within the capillary-saturated soil, it is always convenient to do the calculations by considering the soil to be freely saturated (so that $\sigma' = Z\gamma'$) and then adding the surcharge effect due to capillarity. The increase in σ' due to capillarity is always equal to $h_c\,\gamma_w$. The total stress, effective stress and pore pressure distributions are shown in Fig. 6.23.

6.11 CAPILLARY SIPHONING

We have seen that capillary forces can raise the water in the capillary tube against force of gravity. Same phenomenon takes place in the voids of dry soil. In case of earth dams, this may create serious problems.

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Figure 6.24 shows a composite earth dam with central impervious core provided to check the seepage of water through the body of the dam. The outer shells of the dam are made of highly pervious, granular soil. When the water level in the reservoir is corresponding to the flood level (H.F.L.), the portion to the u/s of the dam will be saturated. The water level in the u/s pervious shell will be practically the same as the H.F.L. Due to capillarity, water will rise through a height h_c. If the top of the core is situated at a height $y < h_c$ above the H.F.L., the capillary forces will pull the water in the descending part of the earth dam, and will slowly empty it. The process, known as *capillary siphoning*, may continue for a very long duration. This will result not only in the water loss but also in possible damage to the downstream slope. While designing the level of impervious core, therefore, this point should be kept in mind.

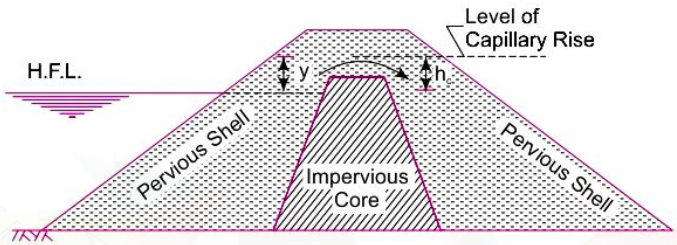

Fig. 6.24 Capillary siphoning

6.12 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 6.9. *A granular soil deposit is 7 m deep over an impermeable layer. The ground water table is 4 m below the ground surface. The deposit has a zone of capillary rise, of 1.2 m with a saturation of 50%. Plot the variation of total stress, pore water pressure and effective stress with the depth of deposit, $e = 0.6$ and $G_s = 2.65$.*

Solution.

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.6} = 16.248 \text{ kN/m}^3$$

Since e does not change with saturation,

$$w_{sat} = \frac{e\,s}{G} = \frac{0.6 \times 1}{2.65} = 0.2264 = 22.64\%$$

$$\gamma_{sat} = \gamma_d (1 + w_{sat}) = 16.248\ (1 + 0.2264) = 19.927 \text{ kN/m}^3$$

In the capillary zone, $S = 50\% = 0.5$ $\quad\therefore\quad w_{wet} = \dfrac{e\,s}{G} = \dfrac{0.6 \times 0.5}{2.65} = 0.1132$

and $$\gamma_{wet} = \gamma_d (1 + w_{wet}) = 16.248\ (1 + 0.1132) = 18.087 \text{ kN/m}^3$$

(a) *Total stress distribution:* Fig. 6.25 *(b)*

At A, $\sigma = 0$

At B, $\sigma = \gamma_{dz} = 16.248 \times 2.8 = 45.49 \text{ kN/m}^2$

At C, $\sigma = 16.248 \times 2.8 + 18.087 \times 1.2 = 67.20 \text{ kN/m}^2$

At D, $\sigma = 16.248 \times 2.8 + 18.087 \times 1.2 + 3 \times 19.927 = 126.28 \text{ kN/m}^3$

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(b) *Pore pressure distribution:* Fig. 6.25 (*c*)

At *A*, $u = 0$; At *B*, $u = -H_c \gamma_w = -1.2 \times 9.81 = -11.77$ kN/m^2

At *C*, $u = 0$; At *D*, $u = 3 \times 9.81 = 29.43$ kN/m^2

(c) *Effective pressure distribution:* Fig. 6.25 (*d*): $\sigma' = \sigma - u$

At A, $\sigma' = 0$; At *B*, $\sigma' = 45.49$ and $\sigma' = 45.49 - (-11.77) = 57.26$ kN/m^2

At *C*, $\sigma' = 67.20 + 0 = 67.20$ kN/m^2; At *D*, $\sigma' = 126.28 - 29.43 = 96.85$ kN/m^2

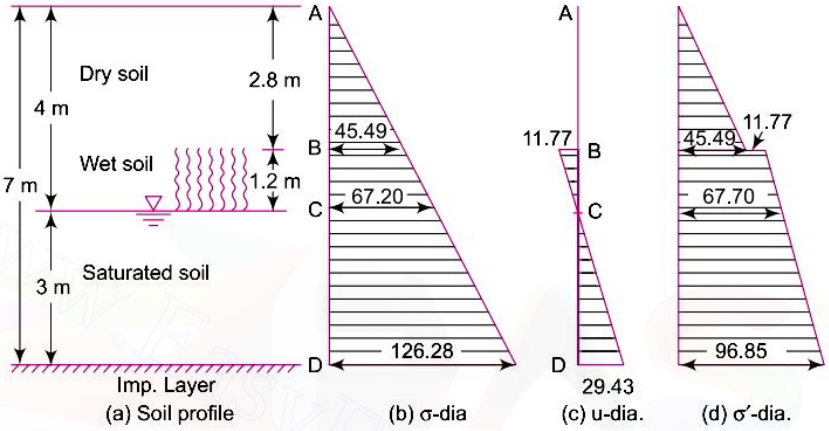

Fig. 6.25

Example 6.10. *For the subsoil conditions shown in Fig. 6.26 (a) below, draw the total, neutral and effective stress diagrams up to a depth of 9 m, neglecting capillary flow.* **(Civil Services Exam. 1997)**

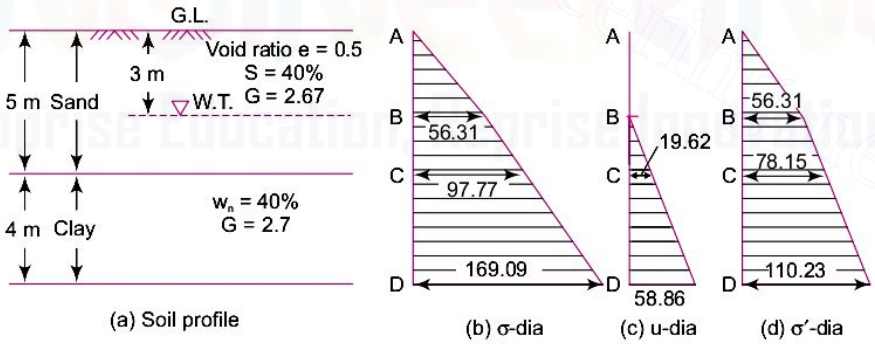

Fig. 6.26

Solution.

(a) *For sand:*

$$\gamma_d = \frac{G.\gamma_w}{1+e} = \frac{2.67 \times 9.81}{1+0.5} = 17.462 \text{ kN/m}^3$$

$$w_{wet} = \frac{e.S}{G} = \frac{0.5 \times 0.4}{2.657} = 0.0749 \; ; \gamma_{wet} = 17.462\,(1 + 0.0749) = 18.77 \text{ kN/m}^3$$

$$w_{sat} = \frac{0.5 \times 1}{2.67} = 0.1873 \; ; \gamma_{sat} = 17.462\,(1 + 0.1873) = 20.732 \text{ kN/m}^3$$

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(b) For clay: $$e = \frac{wG}{S} = \frac{0.4 \times 2.70}{1} = 1.08$$

$$\gamma_{sat} = \frac{G\gamma_w}{1+e}(1 + w_{sat}) = \frac{2.7 \times 9.81}{1+1.08}(1+0.4) = 17.828 \text{ kN/m}^3$$

The computations for total stress, pore pressure and effective stress are shown in tabular form below. The distribution of these are shown in Fig. 6.26 (*b*), (*c*) and (*d*) respectively.

Point	*Level Below G.L. (m)*	*Total stress σ (kN/m²)*	*Pore pressure u (kN/m²)*	*Effective stress σ′ = σ – u*
A	0	0	0	0
B	3	18.77 × 3 = 56.31	0	56.31
C	5	56.31 + 20.732 × 2 = 97.77	9.81 × 2 = 19.62	78.15
D	9	97.77 + 17.828 × 4 = 169.09	9.81 × 6 = 58.86	110.23

Example 6.11. *Compute the total, effective and pore pressure at a depth of 20 m below the bottom of a lake 6 m deep. The bottom of lake consists of soft clay with a thickness of more than 20 m. The average water content of the clay is 35% and the specific gravity of the soil may be assumed to be 2.65.*

(Civil Services Exam. 1998)

Solution.

$$e = w_{sat} G = 0.35 \times 2.65 = 0.9275$$

$$\gamma_{sat} = \frac{\gamma_w (G+e)}{1+e} = \frac{9.81(2.65+0.9275)}{1+0.9275} = 18.208 \text{ kN/m}^3$$

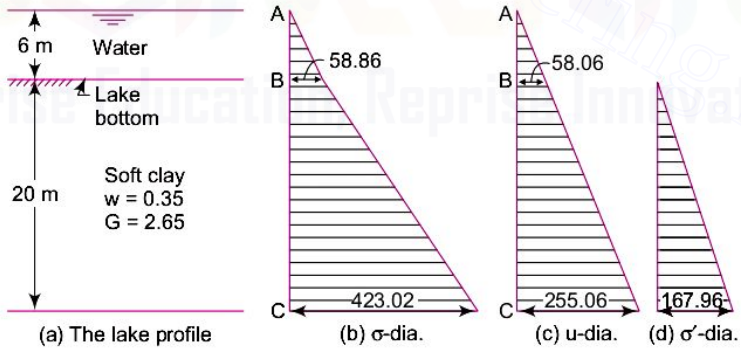

Fig. 6.27

(*i*) ***σ-diagram:***

At *A*, $\sigma = 0$; At *B*, $\sigma = 9.81 \times 6 = 58.86 \text{ kN/m}^2$;

At *C*, $\sigma = 58.86 + 18.208 \times 20 = 423.02 \text{ kN/m}^2$

(*ii*) ***u-diagram:***

At *A*, $u = 0$; at *B*, $u = 9.81 \times 6 = 58.86 \text{ kN/m}^2$;

At *C*, $u = 9.81 \times 26 = 255.06$

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(*iii*) *σ′-diagram* ($\sigma' = \sigma - u$)

At A, $\sigma' = 0$; At B, $\sigma' = 58 \cdot 86 - 58.86 = 0$

At C, $\sigma' = 423.02 - 255.06 = 167.96$ kN/m^2.

Example 6.12. *Compute the total, effective and pore pressure at a depth of 12 m below the bottom of a lake 6 m deep. The bottom of the lack consists of soft clay with a thickness of more than 15 m. The average water content for the clay is 40% and the specific gravity of soil may be assumed 2.65. Assume that the lake is filled up with water up to the top.* **(Civil Services Exam. 1999)**

Solution. This example is practically similar to example 6.11.

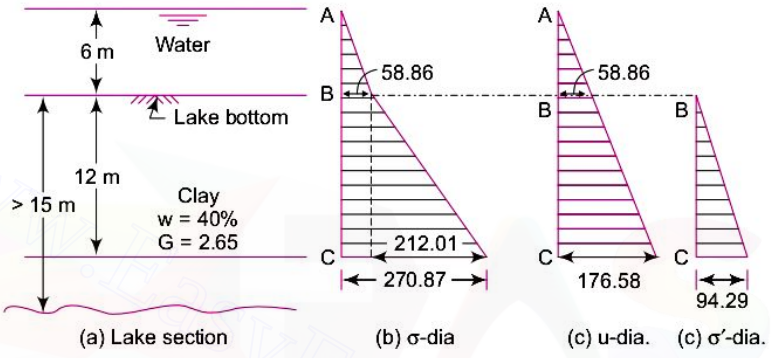

Fig. 6.28

$$e = w_{sat}\, G = 0.4 \times 2.65 = 1.06;\ \gamma_{sat} = \frac{\gamma_w (G+e)}{1+e} = \frac{9.81(2.65+1.06)}{1+1.06} = 17.667 \text{ kN/cm}^3$$

The pressure distributions are shown in Fig. 6.28.

Example 6.13. *At a construction site, a 3 m thick clay layer is followed by a 4 m thick gravel layer which is resting on impervious rock. A load of 25 kN/m^2 is applied suddenly at the surface. The saturated unit weight of the soils are 19 kN/m^3 and 20 kN/m^3 for the clay and gravel layers, respectively. The water table is at the surface. Draw diagrams showing variation with depth of total, neutral and effective stress in the layers.* **(Gate Exam. 1991)**

Solution.

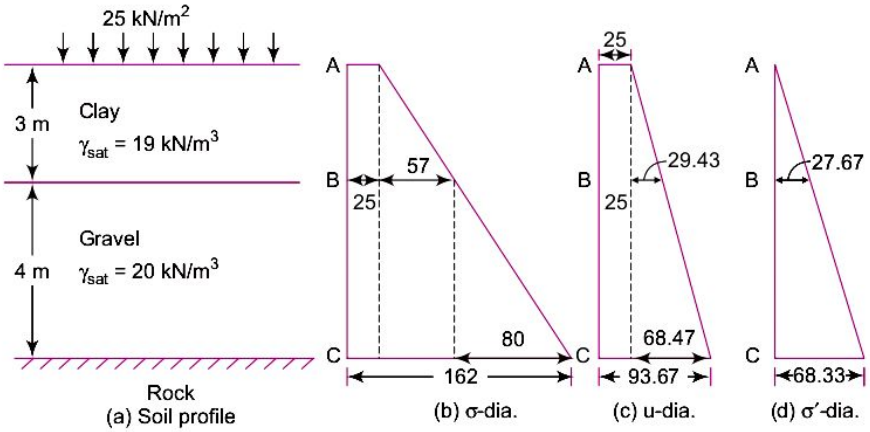

Fig. 6.29

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(*a*) *σ-diagram* (Fig. 6.29 *b*)

At A, $\sigma = 25$ kN/m^2; At B, $\sigma = 25 + 3 \times 19 = 25 + 57 = 82$ kN/m^2

At C, $\sigma = 25 + 57 + 4 \times 20 = 25 + 57 + 80 = 162$ kN/m^2

(*b*) *μ-diagram* (Fig. 6.29 *c*)

At A: Since load is applied suddenly, the entire load is taken by pore water

$\therefore$ $u_A = 25$ kN/m^2

At B, $u_B = 25 + 9.81 \times 3 = 25 + 29.43 = 54.43$ kN/m^2

At C, $u_C = 25 + 9.81 \times 7 = 25 + 68.67 = 93.67$ kN/m^2

(*c*) *σ′-diagram* (Fig. 6.29 *d*)

At A, $\sigma' = 25 - 25 = 0$

At B, $\sigma' = 82 - 54.43 = 27.57$ kN/m^2

At C, $\sigma' = 162 - 93.67 = 68.33$ kN/m^2.

Example 6.14. *The soil profile at the bottom of the valley comprises 2 m of sand overlying 10 m of clay. The clay layer itself is resting on highly permeable weathered sand stone. The unit weight of sand above the water table, which is 1 m below the ground level, is 16 kN/m^3, and below the water table, it is 20 kN/m^3. The saturated unit weight of clay is 22 kN/m^3. If the water table in the sand stone layer is under artesian condition, corresponding to a stand pipe level at 5 m above the ground level, plot the variation of (i) total stress, (ii) pore water pressure, and (iii) effective stress, with depth.*

(Gate Exam. 1993)

Solution.

(*i*) *Total pressure (σ) distributions*: Fig. 6.30 (*b*)

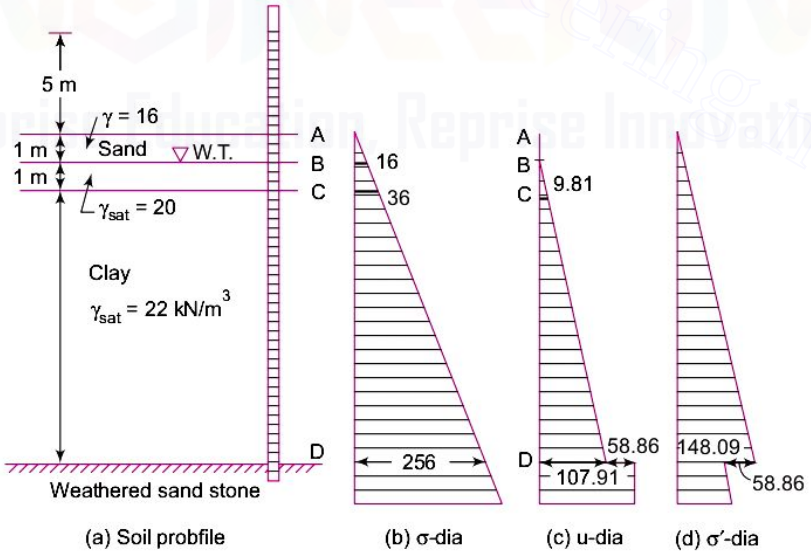

Fig. 6.30

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At A, $\sigma = 0$; At B, $\sigma = 16 \times 1 = 16$ kN/m^2

At C, $\sigma = 16 + 20 \times 1 = 36$ kN/m^2; At D, $\sigma = 36 + 22 \times 10 = 256$ kN/m^2

Below D, $\sigma = 256 + \gamma \cdot z$ where γ and z refer to the weathered sand stone

(ii) ***Pore pressure (u) distribution*:** Fig. 6.30 (*c*)

At A, $u = 0$, At B, $u = 0$

At C, $u = 9.81 \times 1 = 9.81$ kN/m^2 ; At D, $u = 9.81 \times 11 = 107.91$ kN/m^2

Below D, $u = 107.91 + 6 \times 9.81 = 107.91 + 58.86 = 166.77$ kN/m^2

Thus, below D, the pore water pressure suddenly increases by $6 \times 9.81 = 58.86$ due to artesian condition in weathered sand stone. The pore pressure remains constant throughout the depth of sold stone.

(iii) ***Effective stress distribution*** ($\sigma' = \sigma - u$)**:** Fig. 6.30 *(d)*

At A, $\sigma' = 0$; At B, $\sigma' = 16 - 0 = 16$ kN/m^2

At C, $\sigma' = 36 - 9.81 = 26.19$ kN/m^2;

At D, $\sigma' = 256 - 107.91 = 148.09$ kN/m^2

Below D, $\sigma' = 148.09 - 6 \times 9.81 = 148.09 - 58.86 = 89.23$ kN/m^2

Thus, below D, σ' suddenly decreases by the amount equal to excess artesian pressure ($= 6 \times \gamma_w$).

Example 6.15. *For the subsoil condition shown in Fig. 6.31 (a) calculate the total, neutral and effective stress at 1 m, 3 m and 6 m below the ground level. Assuming $\gamma_w = 10$ kN/m^3.*

(Gate Exam, 1997)

Solution. Above point A, assume soil as dry sand.

***For sand*:**

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 10}{1+0.4} = 18.929 \text{kN/m}^3$$

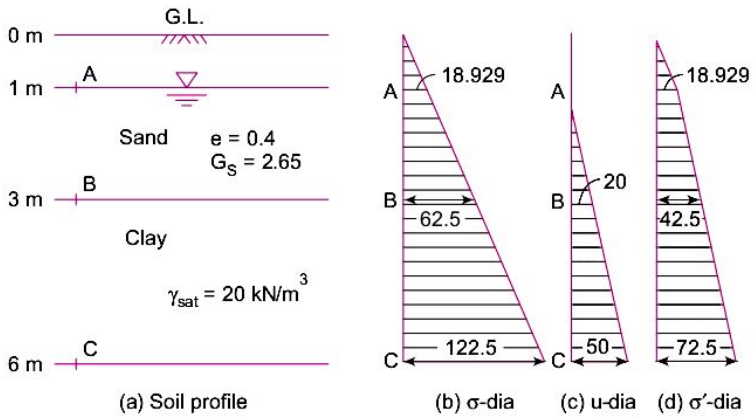

Fig. 6.31

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$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.4)}{1+0.4} \times 10 = 21.786 \text{ kN/m}^3$$

$$\gamma' = 21.786 - 10 = 11.786 \text{ kN/m}^3$$

For clay: $\gamma_{sat} = 20$ kN/m^3 (Given)

At *A*, $\sigma = 18.929 \times 1 = 18.929$ kN/m^2; $u = 0$; σ' 18.929 – 0 = 18.929 kN/m^2

At *B*, $\sigma = 18.929 + 21.786 \times 2 = 62.5$ kN/m^2

$u = 10 \times 2 = 20$ kN/m^2; $\sigma' = 62.5 - 20 = 42.5$ kN/m^2

At *C*, $\sigma = 62.5 + 20 \times 3 = 122.5$ kN/m^2

$u = 10 \times 5 = 50$ kN/m^2; $\sigma' = 122.5 - 50 = 72.5$ kN/m^2.

Example 6.16. *A layer of saturated clay 5 m thick is overtain by sand 4.0 m deep. The water-table is 3 m below the top surface. The saturated weights of clay and sand are 18 kN/m^3 and 20 kN/m^3, respectively. Above the water table, the unit weight of sand is 17 kN/m^3. Calculate the effective pressure on a horizontal plane at a depth of 9 m below the ground surface. What will be the increase in the effective pressure at 9 m, if the soil gets saturated by capillary, up to height of 1 m above the water table? γ_w = 9.81 kN/m^3.* **(Gate Exam. 1999)**

Solution. Refer Example 6.6.

$$\sigma' \text{ at } A = 3\gamma_{sand} + 1\gamma_{sat.,\,sand} + 5\gamma'_{clay} = 3 \times 17 + 1\,(20 - 9.81) + 5(18 - 9.81)$$
$$= 51 + 10.19 + 40.95 = \mathbf{102.14 \text{ kN/m}^2}$$

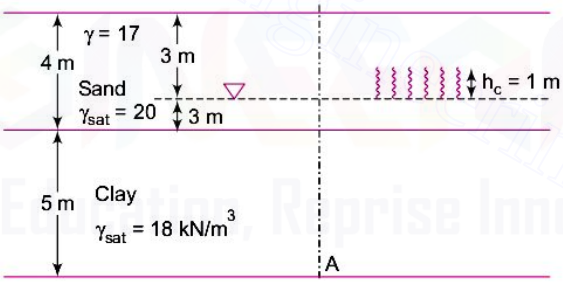

Fig. 6.32

If the soil gets saturated by capillarity, then

$$\sigma' \text{ at } A = 2\gamma_{sand} + 1\gamma_{sat.,\,sand} + \gamma'_{sand} + 5\gamma'_{clay} = 2 \times 17 + 1\,(20) + 1\,(20 - 9.81) + 5\,(18 - 9.81)$$
$$= 34 + 20 + 10.19 + 40.95 = 105.14 \text{ kN/m}^2$$

Alternatively: $\sigma'_A = 2 \times 17 + (1 + 1)\,(20 - 9.81) + 1 \times 9.81 + 5\,(18 - 9.81)$

$= 34 + 20.38 + 9.81 + 40.95 = 105.14$ kN/m^2

$\therefore$ Increase in pressure $= 105.14 - 102.14 = 3$ kN/m^2.

Example 6.17. *A 5 m thick clay layer lies between two layers of sand each 4 m thick, the top upper layer being at ground level. The water table is 2 m below the ground level but the lower layer of sand is under artesian pressure, the piezometric surface being 4 m above ground level. The saturated unit weight of clay is 20 kN/m^3 and that of sand is 19 kN/m^3. Above the water table, unit weight of sand is*

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16.5 kN/m³. Calculate the effective vertical stress at the top and bottom of the clay layer, (b) Also draw the total vertical stress diagram in the given soil layers. **(Gate Exam. 2000)**

Solution.

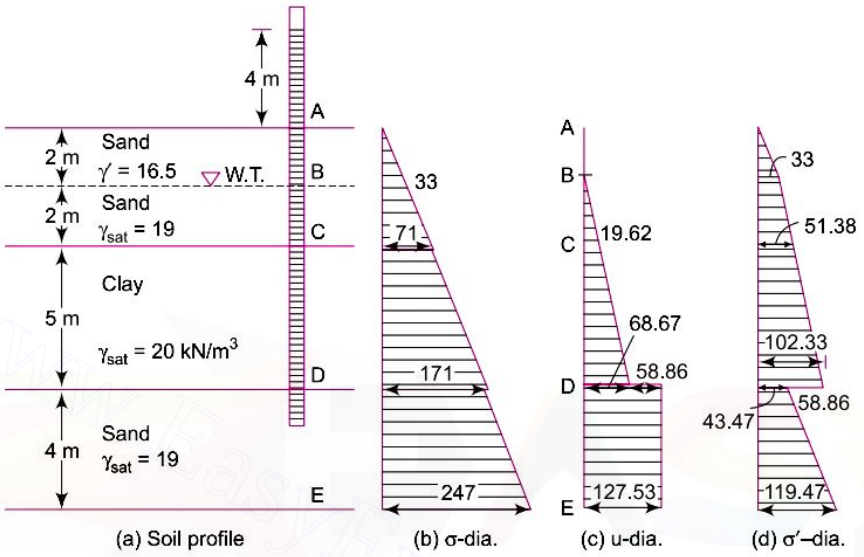

Fig. 6.33

***(i) σ′dia. (6.33 b)*:** The artesian pressure in the lower sand layer will not affect the total stress at any point.

At A, $\sigma = 0$, At B, $\sigma = 2 \times 16.5 = 33$ kN/m², At C, $\sigma = 33 + 2 \times 19 = 71$ kN/m²

At D, $\sigma = 71 + 5 \times 20 = 171$ kN/m², At E, $\sigma = 171 + 4 \times 19 = 247$ kN/m²

***(ii) u-variation (Fig. 6.33 c)*:**

At A, $u = 0$; At B, $u = 0$; At C, $u = 9.81 \times 2 = 19.62$ kN/m²

At D, $u = 9.81 \times 7 = 68.67$

Below level D, there is sudden increase in u due to excess artesian pressure head of $4 + 2 = 6$ m, the value of increase being equal to $6 \times 9.81 = 58.86$ kN/m². Thus total u below $D = 68.67 + 58.86 = 127.53$. This pore pressure will remain constant throughout the depth DE'

***(iii) Variation of σ′ (Fig. 6.33 d)*:**

At A, $\sigma' = 0$; At B, $\sigma' = 33 - 0 = 33$ kN/m²

At D, $\sigma' = 71 - 19.62 = 51.38$ kN/m²; At D, $\sigma' = 171 - 68.67 = 102.33$ kN/m²

$= 34 + 20.38 + 9.81 + 40.95 = 105.14$ kN/m²

Increase in pressure $= 105.14 - 102.14 = 3$ kN/m²

Just below D, σ' suddenly decreases further by an amount of 58.86 (being equal to excess artesian pressure). Thus, just below D, net $\sigma' = 102.33 - 58.86 = 43.47$ kN/m²

At E, $\sigma' = \sigma - u = 247 - 127.53 = 119.47$ kN/m².

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PROBLEMS

1. What is the absolute pressure (in kN/m^2) in the water just below the meniscus in a capillary tube whose diameter is 0.50 mm? Take the wetting angle as 10°, and atmospheric pressure 100 kN/m^2.
[Ans. 94.2 kN/m^2]
2. The radii of curvature of an irregular capillary are 0.5 mm and 0.01 mm. Calculate the surface tension across the meniscus formed by the double curvature if the height of capillary rise is 72 cm.
[Ans. 0.692×10^{-6} kN/cm]
3. Define capillary potential. The pF of soil is 2.8. Calculate the capillary height and capillary potential for the soil. **[Ans.** 6.31 m; 61.9 kN/m^2]
4. For the given soil profile (Fig. 6.34), compute the effective pressure at a depth of 12 m.
[Ans. 118.7 kN/m^2]

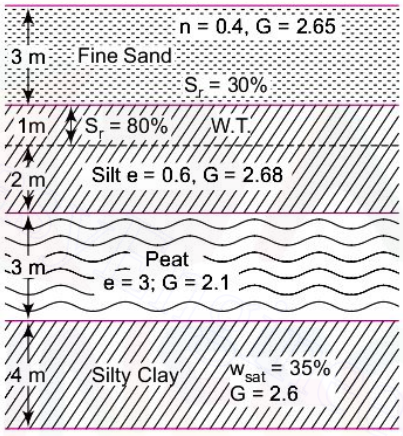

Fig. 6.34

5. The surface of a saturated clay deposit is located permanently below a body of water. Laboratory tests have indicated that the average natural water content of clay is 50% and the specific gravity of solid matter is 2.72. Calculate the inter-granular pressure at a depth of 8 m below the surface of clay layer. If the water level remains unchanged and excavation is made by dredging, how many metres of clay must be removed to reduce the inter-granular pressure at 8 m by 24.53 kN/m^2. **[Ans.** 57.29 kN/m^2]

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CHAPTER 7

Permeability

7.1 INTRODUCTION

Permeability is defined as the property of a porous material which permits the passage or seepage of water (or other fluids) through its interconnecting voids. A material having continuous voids is called *permeable*. Gravels are highly permeable while stiff clay is the least permeable, and hence such a clay may be termed *impermeable* for all practical purposes.

The flow of water through soils may either be a *laminar* flow or a *turbulent* flow. In laminar flow, each fluid particle travels along a definite path which never crosses the path of any other particle. In turbulent flow, the paths are irregular and twisting, crossing and recrossing at random (Taylor, 1948). In most of the practical flow problems in soil mechanics, the flow is laminar.

The study of seepage of water through soil is important for the following engineering problems:

1. Determination of rate of settlement of a saturated compressible soil layer.
2. Calculation of seepage through the body of earth dams, and stability of slopes.
3. Calculation of uplift pressure under hydraulic structures and their safety against piping.
4. Ground water flow towards wells and drainage of soil.

In this chapter, we shall discuss the methods of determining the permeability of soils, and the study of various factors affecting it. The ground water flow towards the wells has been discussed in Chapter 8, while the unconfined and confined flow problems of earth structures have been discussed in Chapters 9 and 10, respectively.

7.2 DARCY'S LAW

The law of flow of water through soil was first studied by Darcy (1856) who demonstrated experimentally that for laminer flow conditions in a saturated soil, the rate of flow or the discharge per unit time is proportional to the *hydraulic gradient.*

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$$q = kiA \qquad ...(7.1)$$

or
$$v = \frac{q}{A} = k\,i \qquad ...(7.2)$$

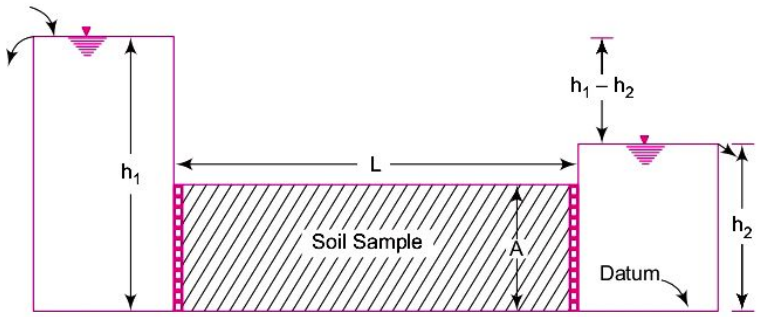

Fig. 7.1 Flow of water through soil

where, q = discharge per unit time

A = total cross-sectional area of soil mass, perpendicular to the direction of flow

i = hydraulic gradient

k = Darcy's coefficient of permeability

v = velocity of flow, or average *discharge velocity.*

If a soil sample of length L and cross-sectional area A, is subjected to differential head of water, $h_1 - h_2$, the hydraulic gradient i will be equal to $\frac{h_1 - h_2}{L}$ and, we have

$$q = k\,\frac{h_1 - h_2}{L}\,A \qquad ...(7.1\,a)$$

From Eq. 7.2, when hydraulic gradient is unity, k is equal to v. Thus, the *coefficient of permeability,* or simply *permeability,* is defined as the average velocity of flow that will occur through the total cross-sectional area of soil under unit hydraulic gradient. The dimensions of the coefficient of permeability k are the same as those of velocity. It is usually expressed as cm/sec or m/day or feet/day.

Table 7.1 gives some typical values of coefficient of permeability of various soils.

Table 7.1 Typical values of k

Soil type	*Coefficient of permeability cm/sec*
Clean gravel	1.0 and greater
Clean sand (coarse)	$1.0 - 1 \times 10^{-2}$
Sand (mixture)	$1 \times 10^{-2} - 5 \times 10^{-2}$
Fine sand	$5 \times 10^{-2} - 1 \times 10^{-3}$
Silty sand	$2 \times 10^{-3} - 1 \times 10^{-4}$
Silt	$5 \times 10^{-4} - 1 \times 10^{-5}$
Clay	1×10^{-6} and smaller

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7.3 DISCHARGE VELOCITY AND SEEPAGE VELOCITY

The Darcy's law (Eqs. 7.1, 7.2) in no way describes the state of flow within individual pores. Darcy's law represents the *statistical macroscopic equivalent* of the Navier-Stokes equations of motion for the viscous flow of ground water. The velocity of flow v is the rate of discharge of water per unit of *total* cross-sectional area A of soil. This total area of cross-section is composed of the area of solids A_s and area of voids A_v. Since the flow takes through the voids, the *actual* or *true* velocity of flow will be more than the discharge velocity. This actual velocity is called the *seepage velocity* v_s, and is defined as the rate of discharge of percolating water per unit cross-sectional area of voids perpendicular to the direction of flow.

From the definitions of the discharge velocity and seepage velocity, we have

$$q = vA = v_s A_v$$

$$\therefore \quad v_s = v\frac{A}{A_v}; \quad \text{But} \quad \frac{A_v}{A} = \frac{V_v}{v} = n$$

or
$$v_s = v\frac{A}{A_v} = v.\frac{1}{n} = \frac{v}{n} = \frac{1+e}{e}v \qquad ...(7.3)$$

The seepage velocity v_s is also proportional to the hydraulic gradient:

$$v_s = k_p\, i \text{ (where } k_p = \textit{coefficient of percolation)} \qquad ...(7.4)$$

From Darcy's law, $v = ki \quad \therefore \quad \frac{v_s}{v} = \frac{k_p}{k} = \frac{1}{n}$

or
$$k_p = \frac{k}{n}. \qquad ...(7.5)$$

7.4 VALIDITY OF DARCY'S LAW

Darcy's law of linear dependency between velocity of flow v and hydraulic gradient i is valid only for laminer flow conditions in the soil. From the experiments on flow through pipes, Reynolds found that the flow is laminar so long as the velocity of flow is less than a lower critical velocity v_c expressed in terms of Reynolds number as follows:

$$\frac{v_c\, d\, \rho_w}{\eta\, g} = 2000 \qquad ...(7.6)$$

where v_c = lower critical velocity in the pipe (cm/sec); d = diameter of pipe (cm)

ρ_w = density of water (g/mL); η = viscosity of water (g sec/cm^2).

and g = acceleration due to gravity (cm/sec^2).

Based on this analogy, the flow through soils may be assumed to depend upon the dimensions of the pore spaces. In coarse grained soils, where the pore dimensions are larger, there will be greater possibility of flow becoming turbulent. Francher, Lewis and Barnes (1933) demonstrated experimentally that flow

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through sands remain laminer and the Darcy's law valid so long as the Reynolds number, expressed in the form below, is equal to or less than unity:

$$\frac{v\, D_a\, \rho_w}{\eta\, g} \leq 1 \qquad ...(7.7)$$

where v = velocity of flow (cm/sec); D_a = diameter of average particle (cm).

Substituting the numerical values of physical constants and other parameters as follows:

$$\eta = 1 \times 10^{-5}\ \text{g} = \text{sec/cm}^2; \quad g = 981\ \text{cm/sec}^2$$

$$D_a = D_{10} = D\ \text{(uniform soil)}; \quad v = k\, i,\ i = 1$$

$$k = 100\ D_m^2 = 100\ D_{10}^2$$

we get $$D \approx 0.05\ \text{cm} = 0.5\ \text{mm}$$

The above criterion gives conservative results. Scheidegger's (1957) collected data show that critical Reynold's number may vary from 0.1 to 75, for Darcy's law to be valid. Such a wide variation is partly due to the different interpretations to the characteristic diameter used in the equation for Reynold's number. Allen Hazen (1892) concluded that linear dependency of the velocity and hydraulic gradient existed if the effective size of soil did not exceed 3 mm. Based on Piefke's experimental results, Prinz concluded that the law of proportionality between velocity and hydraulic gradient deviates faster in very fine sand than in coaser soils. The critical velocities of flow for fine sands and fine gravels were found to be 0.97×10^{-4} cm/sec and 5.56×10^{-4} cm/sec respectively. For the ground water flow occurring in nature and normally encountered in soil engineering problems, the Darcy's law is generally within its validity limits.

7.5 POISEUILLE'S LAW OF FLOW THROUGH CAPILLARY TUBE*

The relationship governing the laminer flow of water through capillary tube is known as *Poiseuille's law.* In the laminar flow, each particle of fluid flows in a direct line. At the surface of the tube, which will be wet with water, the molecular attraction holds a very thin layer of water immobile, so that the velocity at the surface of the tube is zero. The velocity increases to a maximum at the centre of the tube. The variations in velocity from point to the point are accompanied by small friction losses.

Figure 7.2 (*a*) shows a capillary tube, of length L and radius R. The velocity distribution is shown in Fig. 7.2 (*c*). At any radial distance r from the centre, the velocity is v and the velocity gradient (*i.e.,* space rate of change of velocity) is $-\frac{dv}{dr}$. The unit shear at the top and bottom of the cylinder of water (Fig. 7.2 *d*) of radius r is given by

$$\tau = -\eta \frac{dv}{dr} \qquad ...(7.8)$$

where η is the coefficient of viscosity, the approximate value of which may be taken about 10^{-5} gram sec per sq. cm. If the tube is subjected to a head of water h_1, at one end, and h_2 ($h_1 > h_2$) at the other end,

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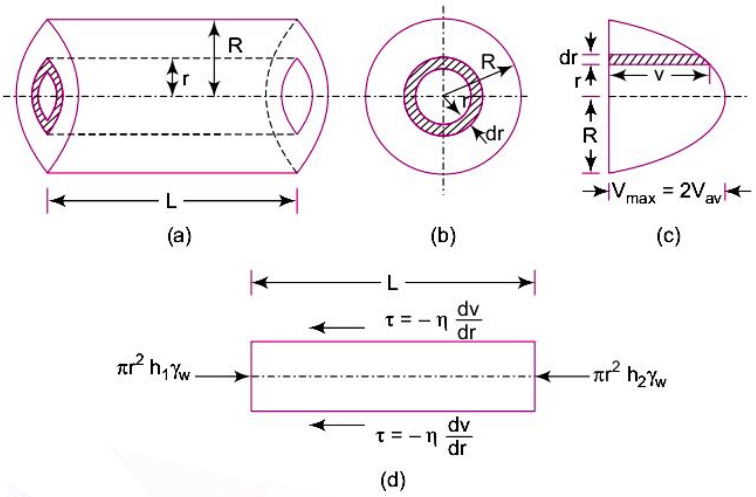

Fig. 7.2 Flow through capillary tube

flow will take place and various forces acting on the cylinder of water at any radius r will be as shown in Fig. 7.2 *d*. Since the tube is in equilibrium, the sum of all forces acting on the cylinder must be zero.

Hence, $h_1 \gamma_w (\pi r^2) - h_2 \gamma_w (\pi r^2) - \tau(2\pi r L) = 0$ or $\tau (2\pi r L) = (h_1 - h_2) \pi r^2 \gamma_w$

$$\therefore \quad 2\pi r L \left(-\eta \frac{dv}{dr}\right) = (h_1 - h_2) \pi r^2 \gamma_w \quad \text{or} \quad dv = -\frac{h_1 - h_2}{L} \cdot \frac{\gamma_w}{2\eta} r dr$$

Replacing $h_1 - h_2$ by h (net head causing flow) and intergrating, we get

$$v = -\frac{h\gamma_w}{4\eta L} r^2 + C$$

At $r = R, v = 0 \quad \therefore \quad C = \frac{h\gamma_w}{4\eta L} R^2$

Hence, $$v = \frac{h\gamma_w}{4\eta L}\left(R^2 - r^2\right) \qquad ...(7.9)$$

This is the law of variation of velocity.

The quantity of water flowing in the thin cylindrical sheet dr thick, is given by

$$dq = (2\pi r dr)\, v = \frac{h\gamma_w}{4\eta L}\left(R^2 - r^2\right) 2\pi r\, dr$$

$\therefore$ Total quantity of water flowing in the capillary tube, per unit time, is

$$q = \frac{h\gamma_w}{4\eta L} 2\pi \int_0^R \left(R^2 - r^2\right) r dr = \frac{h\gamma_w}{8\eta L} \pi R^4$$

Replacing $\frac{h}{L} = i$ = hydraulic gradient

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$$q = \frac{\gamma_w \pi R^4}{8 \eta} i \qquad ...(7.10)$$

If a is the area of the tube, average velocity is given by

$$v_{av} = \frac{q}{a} = \frac{q}{\pi R^2} = \frac{\gamma_w R^2}{8 \eta} i \qquad ...(7.10a)$$

or

$$v_{av} = \frac{\gamma_w}{8 \eta \pi} ia \qquad ...(7.11)$$

Equation 7.11 is the Poiseuille's law in which the velocity of flow during laminar flow varies as the first power of the hydraulic gradient.

Effect of shape of the capillary tube. Equations 7.10 and 7.11 are valid for *circular* capillary tube only. In engineering hydraulics, the velocity is generally designated in terms of the *hydraulic radius* R_H, which is defined as the ratio of area to the wetted perimeter.

For circular tube, $R_H = \frac{\pi R^2}{2 \pi R} = \frac{R}{2}$ or $R = 2 R_H$

Substituting this in equation 7.10, we get

$$q_{cir} = \frac{1}{2} \frac{\gamma_w R_H^2}{\eta} ia \qquad ...(7.12)$$

Similarly, the rate of flow between two flat, closely spaced, parallel plates, can be found to be given by

$$q_{pl} = \frac{1}{3} \frac{\gamma_w R_H^2}{\eta} ia \qquad ...(7.13)$$

Hence, we can conclude that for the capillary tube of any other given geometrical shape of cross-section, the flow may be expressed as

$$q = C_s \frac{\gamma_w R_H^2}{\eta} ia \qquad ...(7.14)$$

where C_s = shape constant.

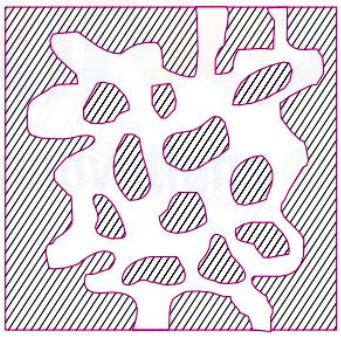

Fig. 7.3 Tube of irregular shape

Figure 7.3 shows the shape of a irregular capillary voids. Let the total cross-sectional area of the soil mass be A, and n be the porosity.

$\therefore$ a = area of flow passage = nA

$\therefore$ $$q = \left(C_s \frac{\gamma_w R_H^2}{\eta} n \right) iA = \left(C_s \frac{\gamma_w R_H^2}{\eta} \frac{e}{1+e} \right) iA \qquad ...(7.15)$$

Hydraulic mean radius in soil pores. Kozeny's (1927) concept of hydraulic mean radius in soils leads to the following expression:

$$R_H = \frac{A}{P} = \frac{AL}{PL}$$

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If V_s is the volume of solids in a soil mass having voids ratio e, the volume of the flow channel ($= AL$) will be eV_s. The total surface area of flow channel ($= PL$) is equal to the total *surface area* A_s of the soil grains.

$$\therefore \quad R_H = \frac{AL}{PL} = \frac{eV_s}{A_s} \qquad ...(7.16)$$

Let D_s = diameter of the spherical grain which has the same ratio of volume to surface area as holds collectively for the grains in a given soil.

$$\therefore \quad \frac{V_s}{A_s} = \frac{\frac{1}{6}\pi D_s^3}{\pi D_s^2} = \frac{D_s}{6}; \qquad \therefore \quad R_H = \frac{D_s}{6}e \qquad ...(7.17\ a)$$

Substituting this in Eq. 7.15,

$$q = \left(C_s \cdot \frac{\gamma_w}{\eta} \cdot \frac{e^3}{1+e} \frac{D_s^2}{36}\right) iA = \left(C \cdot \frac{\gamma_w}{\eta} \cdot \frac{e^3}{1+e} D_s^2\right) iA \qquad ...(7.17)$$

where C is a new shape constant.

7.6 FACTORS AFFECTING PERMEABILITY

Figure 7.17 is the Poiseuille's law adapted for the flow through the soil pores. Comparing it with the Darcy's law: $q = kiA$, we get

$$k = D_s^2 \cdot \frac{\gamma_w}{\eta} \cdot \frac{e^3}{1+e} \cdot C \qquad ...(7.18)$$

Thus, the factors affecting permeability are:

1. Grain size
2. Properties of the pore fluid
3. Voids ratio of the soil
4. Structural arrangement of the soil particles
5. Entrapped air and foreign-matter and
6. Adsorbed water in clayey soils.

1. Effect of size and shape of particles. Permeability varies approximately as the square of the grain size. Since soils consist of many different-sized grains, some specific grain size has to be used for comparison. Allen Hazen (1892), based on his experiments on filter sands of particle size between 0.1 and 3 mm, found that the permeability can be expressed as

$$k = CD_{10}^2 \qquad ...(7.19)$$

where k = coefficient of permeability (cm/sec); D_{10} = effective diameter (cm)

C = constant, approximately equal to 100 when D_{10} is expressed in centimetre.

Attempt have been made to correlate the permeability with specific surface of the soil particles. One such relationship is given by Kozeny (1907).

$$k = \frac{1}{K_k \eta S_s^2} \times \frac{n^3}{1-n^2} \qquad ...(7.20)$$

where k = coefficient of permeability (cm/sec per unit hydraulic gradient)

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n = porosity

S_s = specific surface of particles (cm^2/cm^3)

η = viscosity (g-sec/cm^2)

K_k = constant, equal to 5 for spherical particles

On the basis of his experiments, Loudon (1952–53) developed the following empirical formula:

$$\log_{10}\left(k\, S_s^2\right) = a + bn \qquad ...(7.21)$$

where *a, b* are constants, the values of which are 1.365 and 5.15 respectively for permeability at 10°C. As demonstrated by Loudon's experiments, the permeability of coarse grained soils is inversely proportional to the specific surface, *at a given porosity.*

2. Effect of properties of pore fluid. Equation 7.18 indicates that the permeability is directly proportional to the unit weight of water and inversely proportional to its viscosity. Though the unit weight of water does not change much with the change in temperature, there is great variation in viscosity with temperature. Hence, when other factors remain constant, the effect of the property of water on the values of permeability can be expressed as

as
$$\frac{k_1}{k_2} = \frac{\eta_2}{\eta_1} \qquad ...(7.22\ a)$$

It is usual to convert the permeability results to a standard temperature (27°C) for comparison purposes, by expression:

$$k_{27} = k\,\frac{\eta}{\eta_{27}} \qquad ...(7.22\ b)$$

However, if change in the unit weight of water due to temperature is also taken into account, we have the more general equation.

$$\frac{k_1}{k_2} = \frac{\eta_2}{\eta_1}\cdot\frac{\gamma_{w1}}{\gamma_{w2}} = \frac{\eta_2}{\eta_1}\cdot\frac{\rho_{w1}}{\rho_{w2}} \qquad ...(7.22)$$

where k_{27} = permeability at 27°C; η_{27} = viscosity at 27°C

k = permeability at test temperature; η = viscosity at test temperature.

Muskat (1937) pointed out that a more general coefficient of permeability, called the *physical permeability* k_p is related to the Darcy's coefficient of permeability k as follows:

$$k_p = k\,\frac{\eta}{\gamma_w} \qquad ...(7.22\ c)$$

In any soil, k_p has the same value for all fluids and all temperatures as long as the voids ratio and the structure of the soil skeleton are not changed.

3. Effect of voids ratio. Equation 7.18 indicates that the effect of voids ratio on the values of permeability can be expressed as

$$\frac{k_1}{k_2} = \left[\frac{C_1\, e_1^3}{1+e_1}\right] \div \left[\frac{C_2\, e_2^3}{1+e_2}\right] \qquad ...(7.23)$$

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Laboratory experiments have shown that the factor C changes very little with the change in the voids ratio of unstratified sand samples. However, for clays, it varies appreciably. Thus, for coarse graind soils, equation 7.24 reduces to

$$\frac{k_1}{k_2} = \frac{e_1^3}{1+e_1} \div \frac{e_2^3}{1+e_2} = \frac{e_1^3}{1+e_1} \cdot \frac{1+e_2}{e_2^3} \qquad ...(7.23\ a)$$

Based on another concept of mean hydraulic radius for the soils, the following relationship is obtained:

$$\frac{k_1}{k_2} = \frac{e_1^2}{e_2^2} \qquad ...(7.23\ b)$$

It has been found that a semi-logarithmic plot of voids ratio versus permeability is approximately a straight line for both coarse grained as well as fine grained soils (Fig. 7.4).

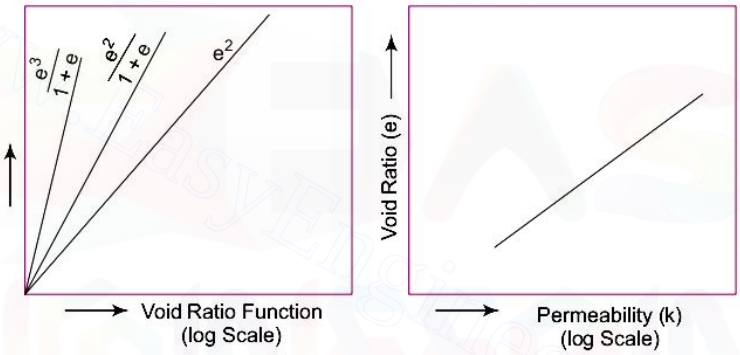

Fig. 7.4 Variation of k with e

4. **Effect of structural arrangement of particles and stratification.** The structural arrangement of the particle may vary, at the same voids ratio, depending upon the method of deposition or compacting the soil mass. The structure may be entirely different for a disturbed sample as compared to an undisturbed sample which may possess stratification. The effect of structural disturbance on permeability is much pronounced in fine-grained soils. Stratified soil masses have marked variations in their permeabilities in direction parallel and perpendicular to stratification, the permeability parallel to the stratification being always greater (§ 7.13). When flow through natural soil deposits is under consideration, permeability should be determined on undisturbed soil as its natural structural arrangement.

5. **Effect of degree of saturation and other foreign matter.** The permeability is greatly reduced if air is entrapped in the voids thus reducing its degree of saturation. The dissolved air in the pore fluid (water) may get liberated, thus changing the permeability. Ideal condition of test are when air-free distilled water is used and the soil is completely saturated by vacuum saturation, for measuring the permeability. However, since the percolating water in the field may have some gas content, it may appear more realistic to use the actual field water for testing in the laboratory. Organic foreign matter also has the tendency to move towards crictical flow channels and choke them up, thus decreasing the permeability.

6. **Effect of adsorbed water.** The adsorbed water surrounding the fine-soil particles is not free to move, and reduces the effective pore space available for the passage of water. According to a crude

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approximation after Casagrande, 0.1 may be taken as the voids ratio occupied by adsorbed water, and the permeability may be roughly assumed to be proportional to the square of the net voids ratio of $(e - 0.1)$.

7.7 COEFFICIENT OF ABSOLUTE PERMEABILITY

In the previous article, we have discussed various factors on which the coefficient of permeability (k) depends. Thus, the coefficient of permeability (k) depends not only on the properties of the soil mass (such as size, shape, specific surface, structural arrangement, stratification, voids ratio etc.) but also on the properties of the *permeant* (*i.e.*, water) which flows through it. Let us now introduce a coefficient which does not depend upon the properties of permeant. Such a coefficient, known as *coefficient of absolute permeability* (K) is defined by the expression

$$K = k\left(\frac{\eta}{\gamma_w}\right) \qquad ...(7.24\ a)$$

Substituting the value of $k\,\frac{\eta}{\gamma_w}$ from equation 7.18, we get

$$K = C\left(\frac{e^3}{1+e}\right)D^2 \qquad ...(7.24)$$

The above equation indicates that the coefficient of absolute permeability is *independent* of the properties of permeant (*i.e.*, water) and it depends solely on the properties of soil mass.

Dimensions of K: From equation 7.24 (a)

$$[K] = \left[\frac{L}{T}\right]\left[\frac{FT}{L^2}\right]\left[\frac{L^3}{F}\right] = \left[L^2\right]$$

Hence K has the dimensions of area. The units of K are: mm^2, cm^2, m^2 or darcy.

$$1 \text{ darcy} = 0.987 \times 10^{-8} \text{ cm}^2.$$

It is interesting to note that for a given voids ratio and structural arrangement of particles, the coefficient of absolute permeability (K) is constant, irrespective of type/properties of fluid.

7.8 DETERMINATION OF COEFFICIENT OF PERMEABILITY

The coefficient of permeability can be determined by the following methods:

(a) *Laboratory methods*
- (1) Constant head permeability test.
- (2) Falling head permeability test.

(b) *Field methods*
- (1) Pumping-out tests.
- (2) Pumping-in tests.

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(*c*) *Indirect methods*

(1) Computation from grain size or specific surface.
(2) Horizontal capillarity test.
(3) Consolidation test data.

Permeability can be determined in the laboratory by direct measurement with the help of *permeameters,* by allowing the water to flow through soil sample either under *constant head* or under *variable head.* Permeability can also be determined directly by field test, described in chapter 8 (well hydraulics). The indirect method of computing the permeability from consolidation test data has been explained in chapter 15.

Empirical formulae for determination of coefficient of permeability (*k*) Permeability can also be computed from several empirical formulae given below.

1. *Jaky's formula*

Jaky (1944) found that a fair estimate of the order of magnitude of k can be obtained for all soils from the formula

$$k = 100\, D_m^2 \qquad ...(7.25\ a)$$

where D_m denotes grain size (in cm) that occurs with the greatest frequency.

2. *Allen Hazen's formula*

$$k = C\, D_{10}^2 \qquad ...(7.25\ b)$$

where C is a constant, which is taken approximately equal to 100 when D_{10} is expressed in cm.

3. *Terzaghi's formula*: Terzaghi (1955) developed the following formula for fairly uniform sands, which reflects the effect of grain size and void ratio:

$$k = 200\, D_e^2\, e^2 \qquad ...(7.25\ c)$$

where D_e = *effective grain size* (*i.e.*, the diameter of the sphere for which the ratio of its volume to its surface area is the same as the similar ratio for a given assemblage of soil particles.

4. *Kozney's formula*

$$k = \frac{1}{K_k\, \eta\, S_s^2} \times \frac{n^3}{1-n^2} \qquad ...(7.25\ d)$$

where S_s = specific surface of particles (cm^2/cm^3)

η = viscosity ($g = sec/cm^2$)

K_k = constant, equal to 5 for spherical particles

5. *Louden's formula*

$$Log_{10}\left(k\, S_s^2\right) = a + bn$$

where a, b are constants, the value of which are 1.365 and 5.15 respectively for permeability at 10°C.

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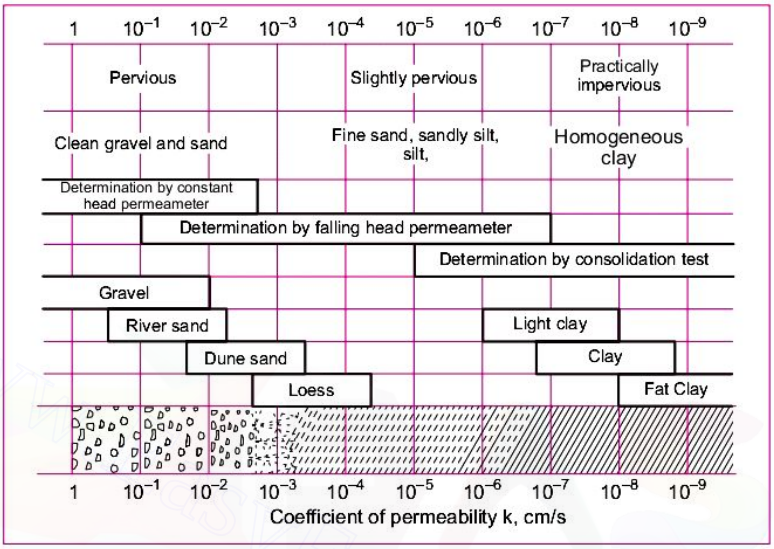
Soil type, numerical values of *k* and methods of its determination

Table 7.2 gives the soil types, numerical values of k and methods of its determination.

Table 7.2



7.9 CONSTANT HEAD PERMEABILITY TEST

Figure 7.5 shows the diagrammatical representation of constant head test.

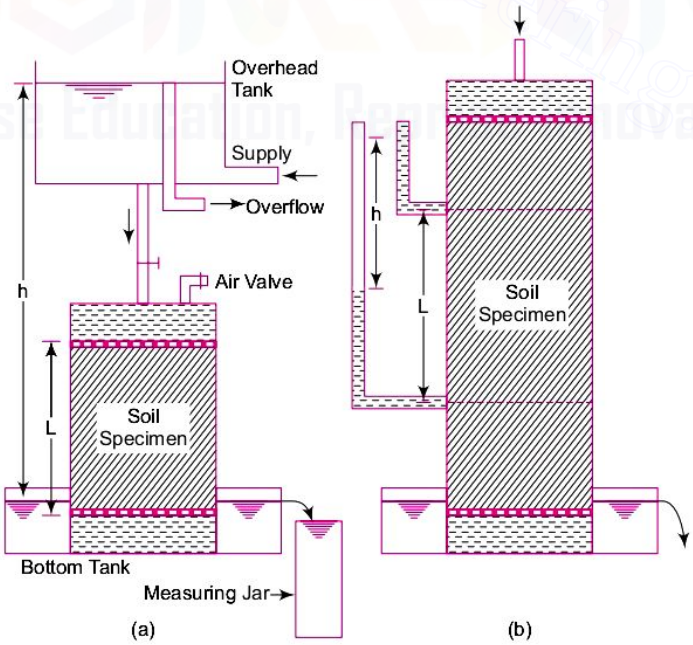


Fig. 7.5 Constant head test

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Water flows from the overhead tank consisting of three tubes: the inlet tube, the overflow tube and the outlet tube. The constant hydraulic gradient i causing the flow is the head h (*i.e.*, difference in the water levels of the overhead and bottom tanks) divided by the length L of the sample. If the length of the sample is large, the head lost over a length of specimen is measured by inserting piezometric tubes, as shown in Fig. 7.5 (*b*).

If Q is the total quantity of flow in a time interval t, we have from Darcy's law,

$$q = \frac{Q}{t} = k\,i\,A$$

$$\therefore \qquad k = \frac{Q}{t}\frac{1}{i\,A} = \frac{Q}{t}\frac{L}{h}\frac{1}{A} \qquad \text{...(7.26)}$$

where A = total cross-sectional area of sample.

When steady state of flow is reached, the total quantity of water Q in time t collected in a measuring jar. The observations are recorded as shown in Table 7.3 [See Experiment 14].

7.10 FALLING HEAD PERMEABILITY TEST

The constant head permeability test is used for coarse-grained soil only where a reasonable discharge can be collected in a given time. However, the falling head test is used for relatively less permeable soils where the discharge is small. Figure 7.6 shows the diagrammatical representation of a falling head test arrangement.

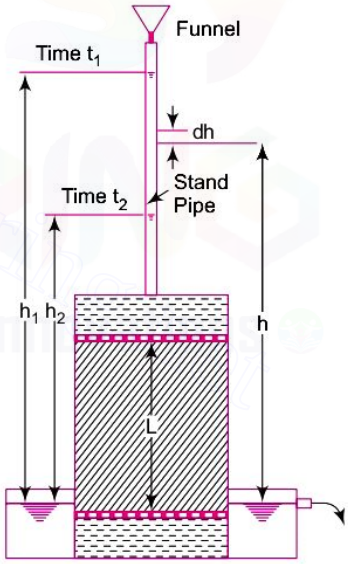

Fig. 7.6 Falling head test

A stand pipe of known cross-sectional area a is fitted over the permeameter, and water is allowed to run down. The water level in the stand pipe constantly falls as water flows. Observations are started after steady state of flow has reached. The head at any time instant t is equal to the difference in the water level in the stand pipe and the bottom tank. Let h_1 and h_2 be heads at time intervals t_1 and t_2 ($t_2 > t_1$) respectively. Let h be the head at any intermediate time interval t, and $-dh$ be the change in the head in a smaller time interval dt (minus sign has been used since h decreases as t increases). Hence, from Darcy's law, the rate of flow q is given by

$$q = \frac{(-dh \cdot a)}{dt} = kiA$$

where i = hydraulic gradient at time $t = \dfrac{h}{L}$

$$\therefore \qquad \frac{k\,h}{L}A = -\frac{dh}{dt}a \text{ or } \frac{Ak}{aL}dt = -\frac{dh}{h}$$

Integrating between two time limits, we get

$$\frac{AK}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2}\frac{dh}{h} = \int_{h_2}^{h_1}\frac{dh}{h}$$

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or $$\frac{Ak}{aL}(t_2 - t_1) = \log_e \frac{h_1}{h_2}$$

Denoting $t_2 - t_1 = t$, we get

$$k = \frac{aL}{At} \log_e \frac{h_1}{h_2} = 2.3 \frac{aL}{At} \log_{10} \frac{h_1}{h_2} \quad ...(7.27)$$

The laboratory observations consist of measurement of the heads h_1 and h_2 at two chosen time intervals t_1 and t_2. The observations are recorded as shown in Table 7.4.

7.11 THE JODHPUR PERMEAMETER

The *Jodhpur Permeameter* was designed and developed by Dr. Alam Singh (1958) at the Soil Engineering Laboratory of M.B.M.Engineering College, Jodhpur. The apparatus is meant for studying the permeability characteristics of all types of soil samples under different conditions of laboratory as well as in the field. Both falling head and constant head test can be performed on remoulded as well as undisturbed specimens. Remoulded specimens can be prepared either by static or by dynamic compaction method.

The Jodhpur permeameter (Model III) comprises the following:

1. Permeameter mould
2. Top cap fitted with water inlet nozzle and air release valve
3. Dynamic compaction base plate
4. Perforated base plate
5. Perforated top plate
6. Static compaction flanged end-plugs, 2 Nos., 3 cm and 2.5 cm high
7. Compaction collars, 2 Nos. 3 cm and 2.5 cm high
8. Split collar
9. 2.5 kg Dynamic Ramming Tool (DRT)
10. Rod temper
11. Bottom tank
12. Constant head tank fitted with air intake tube
13. Spare brass tube and rubber stopper for vacuum saturation
14. Set of three stand pipes, fixed to the back of constant head tank
15. Core cutter
16. Dolly for core cutter
17. Centering ring for cutter
18. Wire gauge, pad of filter paper, copper, wool pad, bolts, nuts, spanner, funnel, pinch cocks and flexible tubing.

The permeameter mould which is a cylinder of internal capacity 300 mL, 50 cm^2 cross-sectional area (79.8 mm diameter) and 6 cm effective height, has two studs fixed to the side lugs which aid in assembling the mould, the top cap and the perforated base plate (or the collars and the compacting base plate as the case may be). A rubber gasket (washer) under the top cap ensures water tightness. The permeameter assembly is placed in the bottom tank having a water outlet which permits accurate control for water level for falling heads tests. The bolt head under the perforated base plate keeps the permeameter mould assembly raised a little in the bottom tank thus allowing free flow of water through the base perforations.

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To test undisturbed specimen, the 0.3 litre core cutter (50 cm^2 in cross-section × 6 cm high) with dolly attached to top is pushed into the undisturbed block of soil. The soil surrounding the outside of the cutter dolly is cut, and the cutter and dolly full of undisturbed soil is removed. The soil is cut flush with top and bottom ends of the cutter, after removing the dolly. The centering ring for the cutter is placed over the perforated base plate and the core with the undisturbed specimen is placed centrally over the perforated base plate with the cutting edge downwards. The top is then tightened over the cutter.

The remoulded specimen can be prepared either by static compaction, or by dynamic compaction, at any desired density. The weight of the wet soil, to compact it at a given density and water content, is first calculated. To compact it by *static compaction,* the 3 cm collar is attached to the bottom end of the 0.3 litre mould and 2.5 cm collar to the top end. The split collar is placed around the 2.5 cm flanged end plug. The mould assembly is supported over the 2.5 cm end plug with the 2.5 cm collar resting on the split collar. The calculated weight of the wet soil is put into the mould and the top plug is inserted. The entire assembly is kept in a press and the sample is compacted. After compaction, the 3 cm plug and 3 cm collar are removed and, after putting the fine mesh gauge etc., perforated base plate is fixed over it. The mould is turned upside down, the plug and the collar are removed, and the top perforated plate and top cap are fixed.

To compact the specimen dynamically, using the rod temper, the mould is fixed upside down on the dynamic compaction plate, and the collar is fixed to its other end. The wet soil of pre-calculated quantity is then compacted into the mould by means of the rod temper, in two or three layers. After compaction, the collar is removed and, after placing the fine mesh gauge, the perforated base plate is fixed. The mould assembly is then turned upside down, the compaction base plate is detached, and the top cap is fixed.

Alternatively, if permeability at Proctor's maximum dry density and at a moulding water content equal to the optimum value is required, first the maximum dry density and optimum water content is determined. (This can be done by Jodhpur Mini compactor test, Proctor test or by compaction in permeameter mould itself). The soil is then compacted at the optimum water content in two layers in the 0.3 litre permeameter mould (mould assembly as described in the above para) with 15 blows of 2.5 kg DRT given to each layer. After the compaction, the compaction collar is removed, the excess soil is trimmed off, and the perforated base plate is fixed, as described in the above para.

SOLVED EXAMPLES

Example 7.1. *Calculate the coefficient of permeability of a soil sample, 6 cm in height and 50 cm^2 in cross-sectional area, if a quantity of water equal to 430 ml passed down in 10 minutes, under an effective constant head of 40 cm.*

On oven-drying, the test specimen has mass of 498 g. Taking the specific gravity of soil solids as 2.65, calculate the seepage velocity of water during the test.

Solution. Given: $Q = 430$ mL; $t = 10 \times 60 = 600$ seconds

$A = 50\ cm^2$; $L = 6$ cm; $h = 40$ cm

From equation 7.26, $k = \frac{Q}{t}\ \frac{L}{h}\ \frac{1}{A} = \frac{430}{600} \times \frac{6}{40} \times \frac{1}{50} = 2.15 \times 10^{-3}$ cm/sec

$= 2.15 \times 10^{-3} \times 864 = 1.86$ m/day (Since 1 cm/sec = 864 m/day)

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Now, $$v = \frac{q}{A} = \frac{430}{600 \times 50} = 1.435 \times 10^{-2} \text{ cm/sec}$$

Alternatively, $$v = ki = 2.15 \times 10^{-3} \times \frac{40}{6} = 1.435 \times 10^{-2} \text{ cm/sec}$$

Now, $$\rho_d = \frac{M_d}{V} = \frac{498}{50 \times 6} = 1.66 \text{ g/cm}^3 \quad \therefore \quad e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.65 \times 1}{1.66} - 1 = 0.595$$

$$\therefore \quad n = \frac{e}{1+e} = \frac{0.595}{1.595} = 0.373$$

$$\therefore \quad v_s = \frac{v}{n} = \frac{1.435 \times 10^{-2}}{0.373} = \mathbf{3.85 \times 10^{-2}\ cm/sec}.$$

Example 7.2. *In a falling head permeameter test, the initial head (t = 0) is 40 cm. The head drops by 5 cm in 10 minutes. Calculate the time required to run the test for the final head to be at 20 cm. If the sample is 6 cm is height and 50 cm^2 in cross-sectional area, calculate the coefficient of permeability, taking area of stand pipe = 0.5 cm^2.*

Solution. In a time interval t = 10 minutes, the head drops from initial value of h_1 = 40 to $h_2 = 40 - 5 = 35$ cm.

From equation 7.26, we have $k = 2.3 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}$

or $$t = \frac{2.3\, aL}{Ak} \log_{10} \frac{h_1}{h_2} = m \log_{10} \frac{h_1}{h_2}$$

where $$m = \frac{2.3\, aL}{Ak} = \text{constant for the set up}$$

$$\therefore \quad 10 = m \log_{10} \frac{40}{35} \quad \text{or} \quad m = \frac{10}{\log_{10} \frac{40}{35}} = \frac{10}{0.058} = 172.5 \text{ units}$$

$$\therefore \quad t = m \log_{10} \frac{h_1}{h_2} = 172.5 \log_{10} \frac{h_1}{h_2}$$

Now, let the time interval required for the head to drop from initial value of $h_1 = 40$ cm to a final value of $h_2 = 20$ cm, be t minutes.

$$t = 172.5 \log_{10} \frac{40}{20} = 172.5 \times 0.301 = \mathbf{51.9} \text{ minutes}$$

Again, $$m = \frac{2.3\, aL}{Ak} = 172.5 \text{ units} \quad \therefore \quad k = \frac{2.3\, aL}{A \times 172.5} \text{ cm/minute}$$

(Since t used to compute m was in minutes)

$$k = \frac{2.3 \times 0.5 \times 6}{50 \times 172.5 \times 60} \text{ cm/sec} = \mathbf{1.335 \times 10^{-5}} \text{ cm/sec}.$$

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Alternatively, $$k = 2.3 \frac{aL}{At} \log_{10} \frac{h_1}{h_2} = \frac{2.3 \times 0.5 \times 6}{50 \times 10 \times 60} \log_{10} \frac{40}{35} = \mathbf{1.335 \times 10^{-5}\ cm/sec}$$

Example 7.3. *Due to a rise of temperature, the viscosity and unit weight of the percolating fluid are reduced to 75% and 97% respectively. Other things being constant, calculate the percentage change in co-efficient of permeability.*

Solution. Let k_1, γ_{w1} and η_1, represent the coefficient of permeability, weight and viscosity at the increased temperature. Dropping the suffix 1 to represent these quantities at the standard (or original) temperature, we have

$$k = A \frac{\gamma_w}{\eta} \quad \text{and} \quad k_1 = A \frac{\gamma_{w1}}{\eta_1}, \quad \text{where} \quad A = \text{constant}$$

$$\therefore \quad \frac{k_1}{k} = \frac{\gamma_{w1}}{\eta_1} \times \frac{\eta}{\gamma_w} \quad \text{or} \quad k_1 = k \frac{\gamma_{w1}}{\gamma_w} \frac{\eta}{\eta_1}$$

Now, $$\gamma_{w_1} = 0.97\, \gamma_w \quad \text{and} \quad \eta_1 = 0.75\, \eta$$

$$\therefore \quad k_1 = k \frac{(0.97)}{0.75} = 1.295\, k$$

$\therefore$ Increase in $k = \mathbf{29.5\%}$.

Example 7.4. *A constant head permeability test was run on a sand sample 16 cm in length and 60 cm^2 in cross-sectional area. Porosity was $n_1 = 40\%$. Under a constant head of 30 cm, the discharge was found to be 45 cm^3 in 18 seconds. Calculate the co-efficient of permeability. Also, determine the discharge velocity and seepage velocity during the test. Estimate the permeability of the sand for a porosity of' $n_2 = 35\%$.*

Solution.

From equation 7.26, $$k = \frac{Q}{t} \cdot \frac{L}{h} \cdot \frac{1}{A} = \frac{45}{18} \times \frac{16}{30} \times \frac{1}{60} = \mathbf{2.22 \times 10^{-2}\ cm/s}$$

Discharge velocity, $$v = k\, i = k \frac{h}{l} = 2.22 \times 10^{-2} \times \frac{30}{16} = \mathbf{4.17 \times 10^{-2}\ cm/s}$$

Seepage velocity, $$v_s = \frac{v}{n} = \frac{4.7 \times 10^{-2}}{0.4} = \mathbf{10.42 \times 10^{-2}\ cm/s}$$

Again, from equation 7.23 (a)

$$\frac{k_1}{k_2} = \frac{e_1^3}{1 + e_1} \cdot \frac{1 + e_2}{e_2^3} = \frac{n_1^3}{(1 - n_1)^2} \Big/ \frac{n_2^3}{(1 - n_2)^2}$$

or $$k_2 = k_1 \frac{\dfrac{n_2^3}{(1 - n_2)^2}}{\dfrac{n_1^3}{(1 - n_1)^2}} = 2.22 \times 10^{-2} \times \frac{\dfrac{0.35^3}{(1 - 0.35)^2}}{\dfrac{0.4^3}{(1 - 0.4)^2}} = \mathbf{1.26 \times 10^{-2}\ cm/s}$$

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Example 7.5. *Permeability tests were performed on a soil sample, under different voids ratio and different temperatures and the following results were obtained.*

Test No.	*Voids ratio (e)*	*Temperature °C*	*k(cm/s)*
1	*0.65*	*25*	0.4×10^{-4}
2	*1.02*	*40*	1.9×10^{-4}

Estimate the co-efficient of permeability at a temperature of 20°C for a voids ratio of 0.80.

Given the following physical properties of water:

At 20°C, $\eta = 10.09 \times 10^{-4}$ *and* $\rho_w = 0.998\ g/cm^3$

At 25°C, $\eta = 8.95 \times 10^{-4}\ g\ sec/cm^2$ *and* $\rho_w = 0.997\ g/cm^3$

At 40°C, $\eta = 6.54 \times 10^{-4}\ g\ sec/cm^2$ *and* $\rho_w = 0.992\ g/cm^2$

Solution.

Step 1: Let us first convert both the test results to a temperature of 20°C.

From equation 7.22, $\dfrac{k_1}{k_2} = \dfrac{\eta_2}{\eta_1} \cdot \dfrac{\rho_{w1}}{\rho_{w2}}$

For first test, $(k_1)_{20} = 0.4 \times 10^{-4} \times \dfrac{8.95 \times 10^{-4}}{10.09 \times 10^{-4}} \times \dfrac{0.998}{0.997} = 0.355 \times 10^{-4}$ cm/s

For second test, $(k_2)_{20} = 1.9 \times 10^{-4} \times \dfrac{6.54 \times 10^{-4}}{10.09 \times 10^{-4}} \times \dfrac{0.998}{0.992} = 1.239 \times 10^{-4}$ cm/s

Step 2: Now convert these values for a void ratio of 0.8. Using equation 7.23 (*a*).

$$\left(\frac{e^3}{1+e}\right)_{0.8} = \frac{(0.8)^3}{1+0.8} = 0.284\ ; \left(\frac{e^3}{1+e}\right)_{0.65} = \frac{(0.65)^3}{1+0.65} = 0.166\ ; \left(\frac{e^3}{1+e}\right)_{1.02} = \frac{1.02^3}{1+1.02} = 0.525$$

For first test, $(k_1)_{0.80} = 0.355 \times 10^{-4} \times \dfrac{0.284}{0.166} = 0.607 \times 10^{-4}$ cm/sec.

For second test, $(k_2)_{0.8} = 1.239 \times 10^{-4} \times \dfrac{0.284}{0.525} = 0.670 \times 10^{-4}$ cm/sec.

Taking the average, the probable value is, $\boldsymbol{k = 6.4 \times 10^{-5}}$ **cm/sec.**

7.12 THE CAPILLARITY-PERMEABILITY TEST

The capillarity-permeability test or the *horizontal capillarity test* is used to determine the coefficient of permeability k as well as the capillary height h_c of the soil sample. Figure 7.7 shows the set up for the test.

Dry soil sample is placed in a transparent lucite or glass tube, about 4 cm in diameter and 35 cm long, at a desired density. Water is allowed to flow from one end, under a constant head h_0 and the other end is kept open to atmosphere through air vent tube. At any time interval t, after the commencement of the

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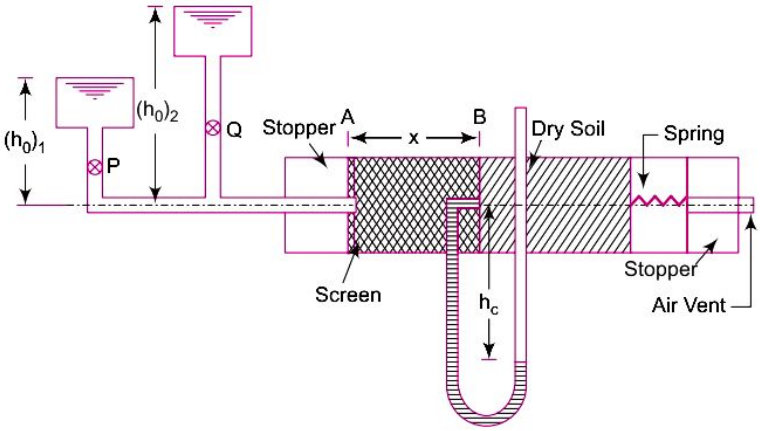

Fig. 7.7 Capillarity-permeability test

test, let the capillary water travel through a distance x, from point A to B. At point A, there is a pressure head h_0 while at the point B, there is a pressure deficiency (*i.e.*, a negative head) equal to h_c of water.

$\therefore$ Hydraulic head lost in causing the flow from A to $B = h_0 - (-h_c) = h_0 + h_c$

$\therefore$ Hydraulic gradient $= \dfrac{h_0 + h_c}{x}$

From Darcy's law, $v = ki$ or $nv_s = ki$ (assuming 100% saturation)

where v_s = seepage velocity, parallel to the direction of $x = \dfrac{dx}{dt}$

If the co-efficient of permeability is designated as k_u at a partial saturation S the above expression may be rewritten as

$$S\,n\,v_s = k_u\,i \quad \text{or} \quad S\,n\frac{dx}{dt} = k_u\frac{h_0 + h_c}{x}$$

$\therefore$ $$x\,dx = \frac{k_u}{S\,n}(h_0 + h_c)\,dt$$

Integrating between the limits x_1 and x_2 for x, with the corresponding values of t_1 and t_2, we get

$$\int_{x_2}^{x_1} x dx = \frac{k_u}{S\,n}(h_0 + h_c)\int_{t_1}^{t_2} dt$$

$\therefore$ $$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2\,k_u}{S\,n}(h_0 + h_c) \qquad ...(7.28\ a)$$

If $S = 100\%$, the above expression reduces to

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2\,k}{n}(h_0 + h_c) \qquad ...(7.28)$$

In the above equation, there are two unknowns: k and h_c. The first set of observations (up to about first half length of the tube) are taken under a head $(h_0)_1$. As the capillary saturation proceeds, the values of x

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are recorded at various time intervals t. A plot of x^2 with t gives a straight line, the slope of which gives the value of $\frac{x_2^2 - x_1^2}{t_2 - t_1}$ ($= m_1$ say). The second set of observations (for the next half of the tube) are taken under an increased head and the plotting of observed values of x^2 against t gives the value of the quantity $\frac{x_2^2 - x_1^2}{t_2 - t_1}$ ($= m_2$ say). Knowing the two slopes, the values of k and h_c can be found by the simultaneous solution of the following two equation:

$$\left(\frac{x_2^2 - x_1^2}{t_2 - t_1}\right)_1 = m_1 = \frac{2k}{n}(h_{01} + h_c) \quad \text{and} \quad \left(\frac{x_2^2 - x_1^2}{t_2 - t_1}\right)_2 = m_2 = \frac{2k}{n}(h_{02} + h_c)$$

The degree of saturation can be found by taking the wet mass of the soil sample at the end of the test. The porosity is computed from the known dry mass, volume and specific gravity.

7.13 PERMEABILITY OF STRATIFIED SOIL DEPOSITS

In nature, soil mass may consist of several layers deposited one above the other. Their bedding planes may be horizontal, inclined or vertical. Each layer, assumed to be homogeneous and isotropic, has its own value of coefficient of permeability. The *average permeability* of the whole deposit will depend upon the direction of flow with relation to the direction of the bedding planes. We shall consider both the cases of flow: (*i*) parallel to the bedding planes, and (*ii*) perpendicular to the bedding planes.

1. Average permeability parallel to the bedding planes.

Let $Z_1, Z_2, \ldots, Z_n$ = thickness of layers and $k_1, k_2, \ldots, k_n$ = permeabilities of the layers. For flow to be parallel to the bedding planes, the hydraulic gradient i will be the same for all the layers. However, since $v = ki$ and since k is different, the velocity of flow will be different in different layers.

Let k_x = average permeability of the soil deposit parallel to the bedding plane.

Total discharge through the soil deposit = Sum of discharge throught the individual layers

$$\therefore \qquad q = q_1 + q_2 + \ldots q_n$$

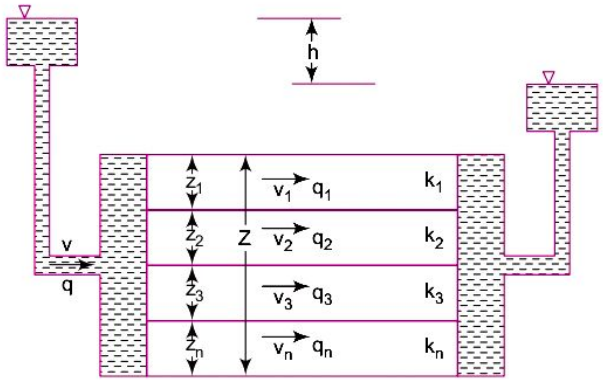

Fig. 7.8 Flow parallel to bedding plane

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or $$q = k_x i Z - k_1 i Z_1 + k_2 i Z_2 + \ldots k_n i Z_n$$

or $$k_x = \frac{k_1 Z_1 + k_2 Z_2 + \ldots + k_n Z_n}{Z} \quad (\text{where } Z = Z_1 + Z_2 + \ldots Z_n) \qquad \ldots(7.29)$$

2. Average permeability perpendicular to the bedding planes.

In this case, the velocity of flow, and hence the unit discharge, will be the same through each layer. However, the hydraulic gradient, and hence the head loss through each layer will be different. Denoting the head loss through the layers by $h_1, h_2, \ldots, h_n$ and the total head loss as h, we have

$$h = h_1 + h_2 \ldots, h_n$$

But $$h_1 = i_1 Z_1; h_2 = i_2 Z_2 \ldots; h_n = i_n Z_n$$

$\therefore$ $$h = i_1 Z_1 + i_2 Z_2 + \ldots i_n Z_n \qquad \ldots(i)$$

Now, if k_z = average permeability perpendicular to the bedding plane, we have

$$v = k_z i = k_z \frac{h}{Z}, \quad \text{or} \quad h = \frac{vZ}{k_z}$$

Also, $$i_1 = \frac{v}{k_1}; \quad i_2 = \frac{v}{k_2}; \quad i_n = \frac{v}{k_n}$$

Substituting these values in (i), we get

$$\frac{vZ}{k_z} = \frac{vZ_1}{k_1} + \frac{vZ_2}{k_2} + \ldots \frac{vZ_n}{k_n}$$

or $$k_z = \frac{Z}{\frac{Z_1}{k_1} + \frac{Z_2}{k_2} + \ldots \frac{Z_n}{k_n}} \qquad \ldots(7.30)$$

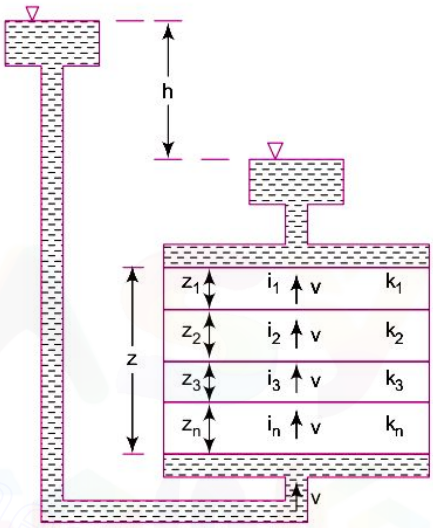

Fig. 7.9 Flow perpendicular to bedding plane

It can be shown that for any stratified soil mass k_x is always greater than k_z. For **example, consider** a three-layer system, having $k_1 = 2$, $k_2 = 1$, $k_3 = 4$ units.

Let $$Z_1 = 4, Z_2 = 1 \quad \text{and} \quad Z_3 = 2 \text{ units. } Z = 4 + 1 + 2 = 7 \text{ units}$$

$$k_x = \frac{(2 \times 4) + (1 \times 1) + (2 \times 4)}{7} = \frac{17}{7} = 2.43; \quad k_z = \frac{7}{\frac{4}{3} + \frac{1}{1} + \frac{2}{4}} = \frac{7}{3.5} = 2$$

$\therefore$ $$k_x > k_z$$

Example 7.6. *A stratified soil deposit consists of four layers of equal thickness. The coefficient of permeability of the second, third and fourth layers are respectively* $\frac{1}{3}$ *rd,* $\frac{1}{2}$ *and twice of the coefficient of permeability of the top layer. Compute the average permeabilities of the deposit, parallel and perpendicular to the direction of the stratification in terms of the permeability of the top layer.*

Solution. Let the thickness of the top layer be Z and its permeability be k.

$\therefore$ Total thickness of deposit = 4 Z

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Now, $$k_1 = k; k_2 = \frac{1}{3}k; \quad k_3 = \frac{k}{2}; \quad k_4 = 2k$$

$\therefore$ $$k_x = \frac{Z.k + \frac{Zk}{3} + \frac{Zk}{2} + Z.2k}{4Z} = \frac{\mathbf{23}}{\mathbf{24}}\boldsymbol{k}$$

$$k_z = \frac{4Z}{\frac{Z}{k} + \frac{3}{k}Z + \frac{2}{k}Z + \frac{Z}{2k}} = \frac{\mathbf{8}}{\mathbf{13}}\boldsymbol{k}\,.$$

Example 7.7. *Figure 7.10 shows an aquifer inclined at 12° to the horizontal. Two observation wells, dug upto the aquifer, at a horizontal distance of 80 m show a difference of 6 m in the water levels. Taking co-efficient of permeability of aquifer soil as 1.2 mm/sec, determine the discharge through the aquifer, per unit width. The thickness of aquifer normal to the direction of flow is 3.2 m.*

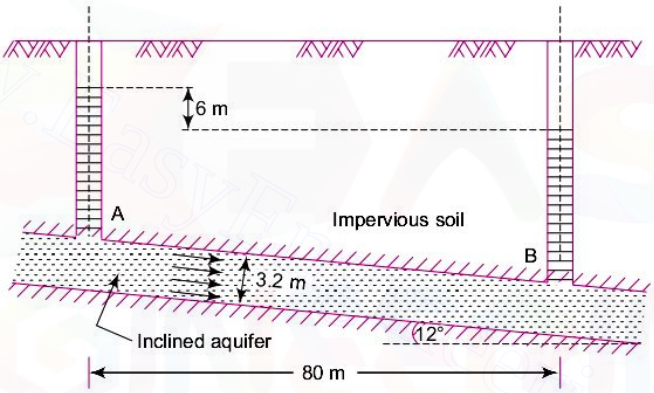

Fig. 7.10

Solution. Length of travel (L) between wells A and B

$$= \frac{80}{\cos 12°} = 81.787 \text{ m}$$

$\therefore$ $$i = \frac{h}{L} = \frac{6}{81.787} = 0.0734$$

Now, from Darcy law, flow per unit width of aquifer is given by

$$q = k\,i\,A = 1.2 \times 10^{-3}\,(0.0734)\,(3.2 \times 1) = 0.288 \times 10^{-3} \text{ m}^3/\text{sec}$$

$$= \mathbf{0.288 \text{ lit/sec.}}$$

Example 7.8. *A capillarity permeability test was performed in two stages. In the first stage, the wetted surface advanced from its initial position of 3 cm to 10 cm in 8 minutes, under a head of 60 cm at the entry of water. In the second stage, the wetted surface advanced from 10 cm to 23 cm in 24 minutes under a head of 230 cm. At the end of test, the degree of saturation was found to be 92% and the porosity was 32%. Determine the capillarity head and the co-efficient of permeability.*

Solution. For the first stage of test, we have, from equation 7.28 (a)

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$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2k_u}{S.n}(h_{01} + h_c) \text{ or } \frac{10^2 - 3^2}{8 \times 60} = \frac{2\,k_u}{0.92 \times 0.32}(60 + h_c)$$

or $$k_u\,(60 + h_c) = 0.0279 \qquad ...(i)$$

Modifying equation 7.28 (a) for the second stage of test. We get

$$\frac{x_3^2 - x_2^2}{t_3 - t_2} = \frac{2k_u}{S.n}(h_{02} + h_c) \quad \text{or} \quad \frac{23^2 - 10^2}{24 \times 60} = \frac{2\,k_u}{0.92 \times 0.32}(230 + h_c)$$

or $$k_u\,(230 + h_c) = 0.0439 \qquad ...(ii)$$

From (1) and (2), we have $\dfrac{230 + h_c}{60 + h_c} = \dfrac{0.0439}{0.0279} = 1.572$

From which, we get $h_c = \mathbf{237.2\ cm}$

Hence from (1), $k_u = \dfrac{0.0279}{60 + 237.2} = \mathbf{0.939 \times 10^{-4}\ cm/sec.}$

7.14 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 7.9. *Calculate the coefficient of permeability of a soil sample 6 cm is height and 50 cm^2 in cross-sectional area, if a quantity of water equal to 450 mL passed down in 10 minutes under an effective constant head of 40 cm. On oven drying, the test specimen weighs 495 g. Taking the specific gravity of soil solids as 2.65, calculate the seepage velocity of water during the test.* **(Civil Services Exam. 1989)**

Solution. Given: $L = 6$ cm; $A = 50$ cm^2; $Q = 450$ mL, $t = 10$ minutes; $h = 40$ cm

This question was set from example 7.1 of this book. All the data are exactly the same except that Q has been changed from 430 mL to 450 mL and mass has been changed from 498 g to 495 g.

Equation 7.26m $k = \dfrac{Q}{t} \cdot \dfrac{L}{h} \cdot \dfrac{1}{A} = \dfrac{450}{10 \times 60} \times \dfrac{6}{40} \times \dfrac{1}{50} = \mathbf{2.25 \times 10^{-3}\ cm/sec = 1.944\ m/day}$

$$v = \frac{Q}{tA} = \frac{40}{600 \times 50} = 1.5 \times 10^{-2}\ \text{cm/sec}; \quad \rho_d = \frac{M_d}{V} = \frac{495}{50 \times 6} = 1.65\ \text{g/cm}^3$$

$$e = \frac{G\,\rho_w}{\rho_d} - 1 = \frac{2.65 \times 1}{1.65} - 1 = 0.606; \quad n = \frac{e}{1+e} = \frac{0.606}{1 + 0.622} = 0.377$$

$\therefore$ $$v_s = \frac{v}{n} = \frac{1.5 \times 10^{-2}}{0.377} = \mathbf{3.975 \times 10^{-2}\ cm/sec.}$$

Example 7.10. *In a capillary permeability test conducted in two stages under a head of 50 cm and 200 cm, in the first stage the wetted surface rose from 20 mm to 80 mm in 6 minutes. In the second stage it rose from 80 mm to 200 mm in 20 minutes. If the degree of saturation is 90% and porosity is 30%, determine the capillary head and the co-efficient of permeability.* **(Civil Services Exam. 1991)**

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Solution.

For first stage, $\left(\frac{x_2^2 - x_1^2}{t_2 - t_1}\right)_1 = m_1 = \frac{2k}{S.n}(h_{01} + h_c)$; or $\frac{8^2 - 2^2}{6} = \frac{2k}{0.9 \times 0.3}(50 + h_c)$

or $h\,(50 + h_c) = 1.35$ ...(*i*)

For second stage, $\frac{20^2 - 8^2}{20} = \frac{2k}{0.9 \times 0.3}(200 + h_c)$

or $k\,(200 + h_c) = 2.268$ ...(*ii*)

Solving (*i*) and (*ii*), we get h_c = **170.59 cm**

and $k = 6.12 \times 10^{-3}$ cm/min = $\mathbf{1.02 \times 10^{-4}}$ **cm/sec.**

Example 7.11. *To determine the capillary head and the permeability of soil, a tube containing a soil with a void ratio of 0.6 was kept horizontally in a trough filled with water with its centre at a depth of 75 cm from the water level. Water was found to advance from 1.5 cm to 12 cm in 8.5 minutes. In another test with the same soil kept at a depth of 22.5 cm below the water level, water was found to advance from 13 cm to 21 cm in 10 minutes. Determine the capillary head and the co-efficient of permeability of the soil.*

(Civil Services Exam. 1993)

Solution.

$$e = 0.6; \quad \text{Hence } n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.375$$

Assume the soil to be fully saturated.

For first stage, $\frac{12^2 - 1.5^2}{8.5} = \frac{2k}{0.375}\ (7.5 + h_c)$ or $k\,(7.5 + h_c) = 3.1268$ ...(*i*)

For second stage, $\frac{21^2 - 13^2}{10} = \frac{2k}{0.375}\ (22.5 + h_c)$ or $h\,(22.5 + h_c) = 5.1$ ...(*ii*)

From (*i*) and (*ii*), h_c = **16.27 cm**

and $k = 0.1315$ cm/min = $\mathbf{2.19 \times 10^{-3}}$ **cm/sec.**

Example 7.12. *A horizontal stratified soil deposit consists of three uniform layers of thickness 6, 4 and 12 m respectively.*

The permeabilities of these layers are 8 $\times$ 10^{-4} cm/s, 52 $\times$ 10^{-4} cm/s and 6 $\times$ 10^{-4} cm/s, find the effective average permeability of the deposit in the horizontal and vertical direction.

(Civil Services Exam. 2001)

Solution. Refer example 7.4

From Eq. 7.29, $k_x = \frac{6 \times 8 \times 10^{-4} + 4 \times 52 \times 10^{-4} + 12 \times 6 \times 10^{-4}}{6 + 4 + 12} = \mathbf{14.909 \times 10^{-4}}$ **cm/s**

From Eq. 7.30, $k_z = \frac{(6 + 4 + 12)10^{-4}}{6/8 + 4/52 + 12/6} = \mathbf{7.782 \times 10^{-4}}$ **cm/s.**

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Example 7.13. *A capillary permeability test was conducted in two stages under a head of 60 cm and 180 cm respectively at the entry end. In the first stage, the wetted surface moved from 1.5 cm to 7 cm in 7 minutes. In the second stage, it advanced from 7 cm to 18.5 cm in 24 minutes. The degree of saturation at the end of the test was 85% and the porosity was 35%. Determine the capillary head and the co-efficient of permeability.* **(Engg. Services Exam. 1990)**

Solution. Refer equation 7.28 (*a*)

First stage, $$\frac{7^2 - 1.5^2}{7} = \frac{2\,k_u}{0.85 \times 0.35}(60 + h_c)$$

or $$k_u\,(60 + h_c) = 0.9934 \qquad ...(i)$$

Second stage, $$\frac{18.5^2 - T^2}{24} = \frac{2\,k_u}{0.85 \times 0.35}(180 + h_c)$$

or $$k_u\,(180 + h_c) = 1.8175 \qquad ...(ii)$$

From (*i*) and (*ii*) $$\frac{180 + h_c}{60 + h_c} = \frac{1.8175}{0.9934} = 1.8296,$$ from which h_c = **84.65 cm**

Hence from (*i*), $k_u = 6.87 \times 10^{-3}$ cm/min = $\mathbf{1.145 \times 10^{-4}}$ **cm/sec.**

Example 7.14. *What will be the ratio of average permeability in horizontal direction to that in the vertical direction for a soil deposit consisting of three horizontal layers, if the thickness and permeability of the second layer are twice of those of the first and those of the third layer twice those of second?* **(Engg. Services Exam. 2002)**

Solution. Refer Example 7.4. Let k_1 and z_1 stand for first layer.

For second layer, $k_2 = 2k_1$ and $z_2 = 2z_1$

For third layer, $k_3 = 2\,k_2 = 4\,k_1$ and $z_3 = 2\,z_2 = 4\,z_1$

Total $z = z_1 + 2\,z_1 + 4\,z_2 = 7\,z_1$

From equation 7.29, $$k_H = \frac{k_1\,z_1 + (2\,k_1)(2\,z_1) + (4\,k_1)(4\,z_1)}{7\,z_1} = \frac{21}{7}\,k_1$$

From equation 7.30, $$k_v = \frac{7\,z_1}{\frac{z_1}{k_1} + \frac{2\,z_1}{2\,k_1} + \frac{4\,z_1}{4\,k_1}} = \frac{7}{3}\,k_1$$

$\therefore$ $$\frac{k_H}{k_v} = \frac{21}{7} \times \frac{3}{7} = \frac{\mathbf{9}}{\mathbf{7}}.$$

Example 7.15. *In a falling head permeameter test on a silty clay sample, the following results were obtained: sample length 12 mm; sample diameter 80 mm; initial head 1200 mm; final head 400 mm; time for fall in head 6 minutes; stand pipe diameter 4 mm. Find the co-efficient of permeability of the soil in mm/sec.* **(Gate Exam. 1998)**

Solution. Refer Example 7.2

Given: $L = 12$ cm; $A = \frac{\pi}{4}(8)^2 = 50.265$ cm^2; $a = \frac{\pi}{4}(0.4)^2 = 0.1257$ cm^2

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$t = 6$ min $= 360$ sec. $h_1 = 120$ cm; $h_2 = 40$ cm

From equation 7.27, $k = 2.303 \dfrac{aL}{At} \log_{10} \dfrac{h_1}{h_2} = 2.303 \dfrac{0.1257 \times 12}{50.265 \times 30} \log_{10} \dfrac{120}{40} = \mathbf{9.159 \times 10^{-5}}$ **cm/sec.**

Example 7.16. *Estimate the flow quantity (in litres per second) through the soil in the pipe shown below. The pressure heads at two locations are shown in Fig.7.11. The internal diameter of the pipe is 1 m and the efficient of permeability of soil is 1×10^{-5} m/sec.* **(Gate Exam. 2001)**

Solution.

$$q = k\,i\,A = k \frac{\Delta h}{L} A = 1 \times 10^{-5} \times \frac{2 - 1.5}{10} \times \frac{\pi}{4} (1)^2 = 0.0393 \times 10^{-5} \text{ m}^3/\text{sec}$$

$$= \mathbf{3.93 \times 10^{-4}\ L/s}$$

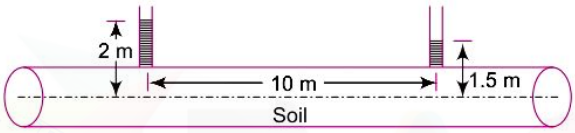

Fig. 7.11

7.15 LABORATORY EXPERIMENTS

Experiment 14: Determination of Permeability by Constant Head Test

Object and scope. The object of the experiment is to determine the co-efficient of permeability of soil in the laboratory by constant head test using Jodhpur Permeameter.

Materials and equipment. (*i*) Jodhpur permeameter complete with all accessories, (*ii*) De-aired water, (*iii*) Balance to weigh to 1 g, (*iv*) 4.75 mm and 2 mm IS sieves, (*v*) Mixing pan or basin, (*vi*) Stop watch, (*vii*) Graduated measuring cylinder, (*viii*) Metre scale, (*ix*) Beaker, (*x*) Thermometer, (*xi*) Containers for water content determination, (*xiii*) Straight edge or trimming knife.

Test procedure

(a) Preparation of statically compacted remoulded specimen

1. Take 800 to 1000 g of representative specimen of soil and mix water to it so that its water content is raised to the optimum water content for the soil determined by Proctor's test. If permeability is to be determined at any other water content, raise the water content of the soil to the desired value. Leave the soil mix for some time in air-tight container.
2. For the given volume (V) of the mould, calculate the mass (M) of the soil mix so as to give the desired dry density (ρ_d), using the following expression:

$$M = \rho_d (1 + w) V$$

   Take the mass of the above soil accuate to 1 g.
3. Assemble permeameter for static compaction. For this, attach the 3 cm collar to the bottom end of the 0.3 litre mould and 2.5 cm collar to its top end. Support the mould assembly over the 2.5 cm end plug with the 2.5 cm collar resting on the split collar kept around the 2.5 cm end plug. The 0.3 litre mould should be lightly greased from inside.

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4. Put the weighed quantity of soil (step 2) into the mould assembly. Insert the top 3 cm end plug into the top collar. The soil may be tamped with hand while being poured into the mould. Keep the entire assembly into a compression machine and remove the split collar. Apply compressive force on the assembly till the flanges of both the end plugs touch the corresponding collars.
5. Maintain the load for about 1 minute and then release it. Remove the top 3 cm plug and collar. Place a filter paper or fine wire gauze on the top of the specimen and fix the perforated base plate on it.
6. Turn the mould assembly upside down and remove the 2.5 cm end plug and collar. Place the top perforated plate on the top of the soil specimen and fix the top can on to it, after inserting the sealing gasket.

The specimen is now ready for the permeability test.

(b) Preparation of dynamically compacted remoulded specimen

1. Take 800 to 1000 g of representative specimen of soil and raise its water content to the optimum water content. Leave the soil mix in an air-tight container for some time.
2. Assemble the permeameter for dynamic compaction. For this, grease the mould lightly from inside and place it upside down on the dynamic compaction base. Find the mass of the assembly accurate to 1 g. Put the 3 cm collar to the other end.
3. Compact the wet soil mix in two layers, with 15 blows of the 2.5 kg dynamic ramming tool, given to each layer. Remove the collar and trim off the excess soil. Find the mass of mould assembly with soil. The difference of the, two masses taken in steps (2) and (3) would give the mass (M) of the soil compacted.
4. Place filter paper or fine wire mesh on the top of soil specimen and fix the perforated base plate on to it.
5. Turn the assembly upside down and remove the compaction plate. Place the top perforated plate on the top of the soil specimen and fix the top cap on to it, after inserting the sealing gasket.

The specimen is now ready for the permeability test.

(c) Saturation of compacted specimen

To saturate the compacted specimen, place the permeameter mould in the vacuum desiccator and open air release valve. Fill the desiccator with de-aired water till the water level reaches well above the top cap and the water inlet nozzle is submerged. Apply vacuum of about 5 to 10 cm of mercury and maintain it for some time. Increase this vacuum slowly in steps, to about 70 cm of mercury. In every increment, sufficient time should be given so that the air bubbles come out without vibrating the specimen. Take out specimen when the saturation is complete.

(d) Constant head test

1. Place the mould assembly in the bottom tank and fill the bottom tank with water upto its outlet.
2. Connect the outlet tube of the constant head tank to the inlet nozzle of the permeameter, after removing the air in the flexible rubber tubing connecting the tube. Adjust the hydraulic head by either adjusting the relative heights of the permeameter mould and the constant head tank, or by raising or lowering the air intake tube within the head tank.
3. Start the stop watch, and at the same time put a beaker under the outlet of the bottom tank. Run the test for some convenient time interval. Measure the quantity of water collected in the beaker during that time.
4. Repeat the test twice more, under the same head and for the same time interval.

Tabulation of observations. Test observations are tabulated as illustrated in Table 7.3.

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Table 7.3 Data and observation sheet for constant head permeability test

Sample No. A/46
Moulding water content : 14%
Dry density : 1.72 g/cm^3
Specific gravity : 2.68
Voids ratio : 0.56

1.	Hydraulic head h	(cm)	6
2.	Length of the sample (L)	(cm)	6
3.	Hydraulic gradient		1
4.	Cross-sectional area of sample	(cm^2)	50
5.	Time interval (t)	(sec)	600
6.	Quantity of flow (Q) : (i) I test	(mL)	860
	(ii) II test	(mL)	855
	(iii) III test	(mL)	862
	Average	(mL)	859
7.	Co-efficient of permeability	(cm/sec)	2.86×10^{-2}
8.	Test temperature	(°C)	32
9.	Permeability at 27°C	(cm/sec)	2.57×10^{-2}

Calculations. The co-efficient of permeability is calculated from Eq. 7.26.

$$k = \frac{Q}{t} \frac{L}{h} \frac{1}{A}$$

Experiment 15: Determination of Permeability by Falling Head Test

Object and scope. The object of the experiment is to determine the co-efficient of permeability of soil in the laboratory by falling head test using Jodhpur Permeameter.

Materials and equipment. Same as in Experiment 14.

Test procedure

1. Prepare the remoulded soil specimen in the permeameter and saturate it as explained in Experiment 14.
2. Keep the permeameter mould assembly in the bottom tank and fill the bottom tank with water upon its outlet.
3. Connect the water inlet nozzle of the mould to the stand pipe filled with water. Permit water to flow for some time till steady state of flow is reached.
4. With the help of the stop watch, note the time interval required for the water level in the stand pipe to fall from some convenient initial value to some final value.
5. Repeat step (4) at least twice and determine the time for the water level in the stand pipe to drop from the same initial head to the same final value.
6. In order to determine the inside area of cross-section of the stand pipe, collect the quantity of water contained in between two graduations of known distance apart. Find the mass of this water accurate to 0.1 g. The mass in grams divided by the distance, in cm, between the two graduations will give the inside area of cross-section of the stand pipe.

Tabulation of observations. The test observations are tabulated as illustrated in Table 7.4.

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Table 7.4 Data and observation sheet for falling head permeability test

Sample No. A/107 — Specific gravity : 2.68
Moulding water content: 12% — Void ratio : 0.60
Dry density: 1.67 g/cm^3

1.	Area of stand pipe (a)	(cm^2)	0.785
2.	Cross-sectional area of soil sample (A)	(cm^2)	50
3.	Length of the sample (L)		6
4.	Initial head (h_1)	(cm)	40
5.	Final head (h_2)	(cm)	20
6.	Time interval: (i) I test	(sec)	56
	(ii) II test	(sec)	57
	(iii) III test	(sec)	55
	Average	(sec)	56
7.	Coefficient of permeability at test temperature	(cm/sec)	1.17×10^{-3}
8.	Test temperature	(°C)	32
9.	Coefficient of permeability at 27°C	(cm/sec)	1.05×10^{-3}

Calculations. The coefficient of permeability is calculated from equation 7.27

$$k = 2.3 \frac{aL}{At} \log_{10} \frac{h_1}{h_2}.$$

PROBLEMS

1. Find the average horizontal and vertical permeabilities of a soil mass made up of three horizontal layers. The first and second layers have the same thickness of 0.5 metre each. The third layer is one metre thick. The co-efficients of permeability of the first, second the third layers are respectively 1×10^{-3} cm/sec, 2×10^{-2} cm/sec and 5×10^{-4} cm/sec. [**Ans.** $k_H = 5.5 \times 10^{-3}$ cm/sec; $k_v = 0.79 \times 10^{-3}$ cm/sec]
2. The co-efficient of permeability of a soil sample is found to be 1×10^{-3} cm/sec at a voids ratio of 0.4. Estimate its permeability at a voids ratio of 0.6. [**Ans.** 2.95×10^{-3} cm/sec, or 2.25×10^{-3} cm/sec]
3. A soil sample of height 6 cm and area of cross-section of 100 cm^2 was subjected to falling head permeability test. In a time interval of five minutes, the head dropped from 60 cm to 20 cm. If the cross-sectional area of the stand pipe is 2 cm^2, compute the co-efficient of permeability of the soil sample. If the same sample is subjected to a constant head of 18 cm, calculate the total quantity of water that will be collected after flowing through the sample. [**Ans.** 4.4×10^{-4} cm/sec; 475 mL]
4. A glass cylinder 50 cm^2 in inside cross-sectional area and 40 cm high is provided with a screen at the bottom and is open at the top. Saturated sand is filled in the cylinder up to a height of 10 cm above the screen. The cylinder is then filled with water up to its top. Determine the co-efficient of permeability in cm/sec if the water level drops from the top of the cylinder through a distance of 20 cm in half an hour. [**Ans.** 3.83×10^{-3} cm/sec]
5. If during a permeability test on a soil sample with a falling head permeameter, equal time intervals are noted for drops of head from h_1 to h_2 and again from h_2 to h_3, find a relationship between h_1, h_2 and h_3. [**Ans.** $h_2 = \sqrt{h_1 h_3}$]
6. A falling head permeameter accommodates a soil sample 6 cm high and 50 cm^2 in cross-sectional area. The permeability of the sample is expected to be 1×10^{-4} cm/sec. If it is desired that the head in the stand pipe should fall from 30 cm to 10 cm in 40 minutes, determine the size of the stand pipe which should be used. [**Ans.** 1.53 cm dia.]

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CHAPTER 8

Well Hydraulics

8.1 INTRODUCTION

The largest available source of fresh water lies underground. The total ground water potential is estimated to be one-third the capacity of oceans. The ground water reservoir consists of water held in voids within a geologic stratum. Water bearing formations of the earth's crust act as conduits for transmission and as reservoirs for storage of ground water. The discharge from ground water occurs in two ways: (1) natural ways, (2) artificial ways. The natural discharge occurs as flow in lakes, reservoirs, rivers, oceans and springs. Pumpage from wells constitutes the major artificial discharge of ground water. The discharge so obtained primarily depends upon the permeability of the soil strata in which well has been sunk. Hence pumping out tests are frequently adopted for determining soil permeability of soil formations.

8.2 SOME DEFINITIONS

1. Aquifer. Aquifers are permeable formations having structures which permit appreciable quantity of water to move through them under ordinary field conditions. These are the geologic formations in which ground water occurs. Aquifers are mainly of two types (*i*) unconfined aquifer; and (*ii*) confined aquifer. *Unconfined aquifer* or *water table aquifer* is the one in which a water table serves as the upper surface of zone of saturation. It is also sometimes known as *free, phreatic* or *non-artesian* aquifer. *Confined aquifer* or *artesian aquifer* is the one in which ground water is confined under pressure greater than atmospheric by overlying relatively impermeable strata. In a well penetrating such an aquifer, the water level will rise to the level of the local static pressure or *artesian head.*

2. Aquiclude and aquifuge. *Aquicludes* are the impermeable formations which contain water but are not capable of transmitting or supplying a significant quantity, *Aquifuge* is an impermeable formation which neither contains water nor transmits any water.

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3. Specific yield and specific retention. The capacity of a formation to contain water is measured by porosity. However, a high porosity does not indicate that an aquifer will yield large volumes of water to a well. The only water which can be obtained from the aquifer is that which will flow by gravity. The *specific yield* S_Y of an aquifer is defined as the ratio, expressed as percentage of the volume of water which, after being saturated, can be drained by gravity of the total volume of the aquifer.

$$S_Y = \text{Specific yield} = \frac{\text{Volume of water drained by gravity}}{\text{Total volume}} = \frac{V_{WY}}{V} \times 100 \qquad ...(8.1)$$

Specific yield is an indication of the water yielding capacity of an unconfined aquifer. Specific yield is always less than porosity since some water will be retained in the aquifer by molecular and surface tension forces. The *specific retention* S_R of an aquifer is the ratio expressed as a percentage of the volume of water it will retain after saturation against the force of gravity to its own volume.

$$S_R = \text{Specific retention}$$

$$= \frac{V_{wR}}{V} \times 100$$

where V_{wR} = Volume of water retained. ...(8.2)

Now, in a saturated soil, $n = \frac{V_w}{V} \times 100$; where V_w = Volume of water = $V_{wY} + V_{wR}$.

Hence, $$n = S_Y + S_R \qquad ...(8.3)$$

The value of specific yield depends upon grain size, shape and distribution of pores and compaction of stratum. Figure 8.1 shows the variation of porosity, specific yield and specific retention of soil with grain size.

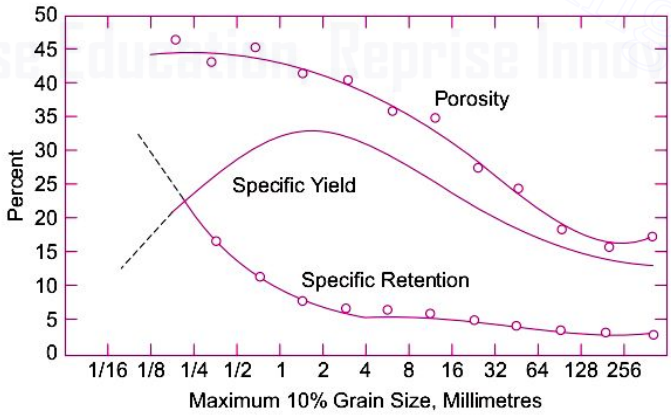

Fig. 8.1 Variation of specific yield etc. with grain size

4. Storage coefficient. The water yielding capacity of a confined aquifer can be expressed in terms of its *storage coefficient*. Storage coefficient is defined as the volume of water that an aquifer releases per unit surface area of aquifer per unit change in the component of head normal to that surface. In most of

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the confined aquifers, the value of storage coefficient ranges between 0.00005 to 0.005. Its value can be determined from pumping tests on wells penetrating fully into the confined aquifer. The storage coefficient for an unconfined aquifer corresponds to its specific yield.

5. **Coefficient of permeability and transmissibility.** The *coefficient of permeability k* is defined as the velocity of flow which will occur through the total cross-sectional area of the soil (or aquifer) under a unit hydraulic gradient. The *coefficient of transmissibility T* is defined as the rate of flow of water (in m^3/day) through a vertical strip of aquifer of unit width (1 m) and extending the full saturation height under unit hydraulic gradient. Thus, the coefficient of transmissibility T equals the field coefficient of permeability multiplied by the aquifer thickness b:

$$T = bk. \qquad ...(8.4)$$

8.3 STEADY RADIAL FLOW TO A WELL : DUPUIT'S THEORY

When a well is penetrated into an extensive homogeneous aquifer, the water table initially remains horizontal in the well. When the well is pumped, water is removed from the aquifer and the water table or the piezometric surface, depending upon the type of the aquifer, is lowered resulting in a parabolic depression in the water table or the piezometric surface. This depression is called the *cone of depression* or the *drawdown curve*. At any point, away from the well, the drawdown s is the vertical distance by which the water table or the piezometric surface is lowered. The analysis of such a radial flow towards a well was originally proposed by Dupuit in 1868 and later modified by Thiem (1906).

For the sake of analysis, we shall take two cases: (*i*) well fully penetrating an unconfined aquifer, (*ii*) well fully penetrating a confined aquifer.

1. Unconfined aquifer. Figure 8.2 shows a well penetrating an unconfined or free aquifer to its full depth.

Let r = radius of the well

H = thickness of the aquifer, measured from the impermeable layer to the initial level of water table

s = drawdown at the well

h = depth of water in the well measured above the impermeable layer

Considering the origin of coordinates at a point O at the centre of the well as its bottom, let the co-ordinates of any point P on the drawdown curve be (x, y).

Then from Darcy's law, Discharge $q = k A_x i_x$

where A_x = area of cross-section of the saturated part of the aquifer at P

$= (2\pi x)y = 2\pi x y$

i_x = hydraulic gradient at $P = \dfrac{dy}{dx}$

Hence, $q = k (2\pi x y) \dfrac{dy}{dx}$ or $q \dfrac{dx}{x} = 2 \pi ky\, dy$

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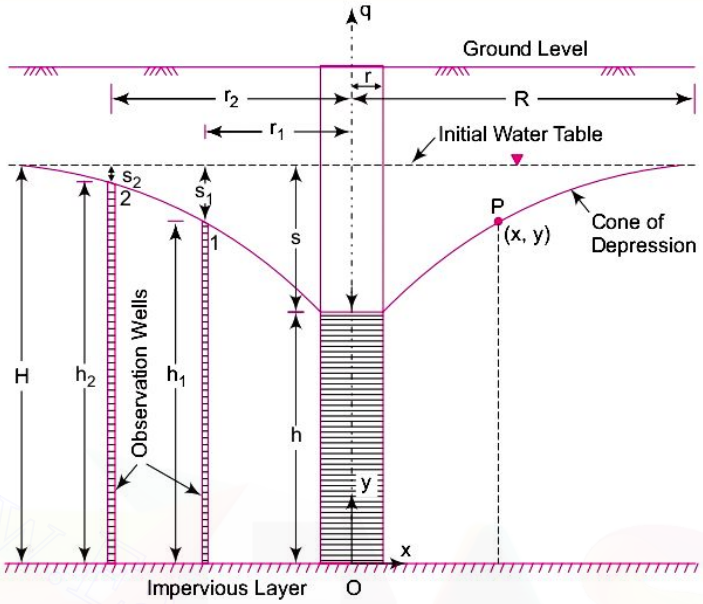

Fig. 8.2 Unconfined aquifer

Integrating between the limits (R, r) for x and (H, h) for y, we get

$$q\int_r^R \frac{dx}{x} = 2\pi k\int_h^H y\,dy \quad \therefore \quad q\,(\log_e x)_r^R = 2\pi k\left(\frac{y^2}{2}\right)_h^H$$

From which,

$$q = \frac{\pi k(H^2 - h^2)}{\log_e \frac{R}{r}} = \frac{1.36k\,(H^2 - h^2)}{\log_{10} \frac{R}{r}} \qquad ...(8.5)$$

If k is expressed in cubic metre per day per square metre (m^3/day/m^2) of the area of subsoil, the above expression for discharge will directly be in cubic metre per day units. In the above expression, R commonly known as *radius of zero drawdown or maximum radius of influence,* is the radius, measured from the centre of the well, where the drawdown curve meets the original water table tangentially. In practice, the selection of the radius of influence R is approximate and arbitrary, but the variation, in Q is small for a wide range of R. Suggested values of R fall in the range of 150 to 300 m.

Alternatively, R may be computed from the following approximate expression given by Sichardt:

$$R = 3000\,s\sqrt{k} \qquad ...(8.6)$$

where R and s are in metres, and k is in m/sec.

If there are two *observation wells* at radial distance r_1, and $r_2 (r_2 > r_1)$ and if the depths of water in them are h_1 and h_2 respectively. Equation (8.5) can also be expressed in the following form

$$q = \frac{\pi k(h_2^2 - h_1^2)}{\log_e \frac{r_2}{r_1}} = \frac{1.36k(h_2^2 - h_1^2)}{\log_{10} \frac{r_2}{r_1}} \qquad ...(8.7)$$

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If the drawdown s is measured at the well, we have

$$s = H - h \quad \text{and} \quad H = s + h \quad \text{or} \quad H + h = s + 2h$$

Then, from equation 8.5, we get

$$q = \frac{\pi k(H-h)(H+h)}{\log_e \frac{R}{r}} = \frac{\pi k s(s+2h)}{\log_e \frac{R}{r}} = \frac{\pi k s(s+2L)}{\log_e \frac{R}{r}} = \frac{1.36 k s(s+2L)}{\log_e \frac{R}{r}} \quad ...(8.8)$$

where L = effective length of the strainer = h.

Assumptions and limitations of Dupuit's theory. Dupuit's theory of flow is based on the following assumptions:

1. The velocity of flow is proportional to the tangent of the hydraulic gradient instead of its sine.
2. The flow is horizontal and uniform everywhere in the vertical section.
3. Aquifer is homogeneous, isotropic and of infinite aerial extent.
4. The well penetrates and receives water from the entire thickness of the aquifer.
5. The co-efficient of transmissibility is constant at all places and at all times.
6. Natural ground water regime affecting an aquifer remains constant with time.
7. Flow is laminar and Darcy's law is valid.

Out of these, assumptions 1, 2 and 7 are of particular importance. The flow is not horizontal, especially near the well. Also, the piezometric surface attains greater slope as it approaches the well boundary, with the result that assumption 1 is an approximation. Due to these reasons, the parabolic form of piezometric surface computed from the Dupuit's theory deviates from the observed surface. This deviation is large, resulting in the formation of a *seepage face*. In addition to these, the velocity near the well increases, and the flow no longer remains laminar. Thus, Darcy's law equation is not valid near the well face.

2. Confined aquifer. Figure 8.3 shows a well fully penetrating a confined or artesian aquifer. Let (x, y) be the coordinates of any point P on the drawdown curve, measured with respect to the origin O. Then, from Darcy's law, flow crossing a vertical plane through P is given by $q = k\, i_x\, A_x$.

where A_x = cross-sectional area of flow, measured at $P = 2\pi\, x\, b$

b = thickness of confined aquifer; i_x = hydraulic gradient at $P = \frac{dy}{dx}$

$$\therefore \quad q = k\left(\frac{dy}{dx}\right)(2\pi\, xb) \quad \text{or} \quad q\frac{dx}{x} = 2\pi k\, b dy$$

Integrating between the limits (R, r) for x and (H, h) for y, we get

$$q\int_r^R \frac{dx}{x} = 2x\, kb \int_h^H dy \quad \text{or} \quad q\,[\log_e x]_r^R = 2\pi kb\,[y]_h^H$$

From which

$$q = \frac{2\pi kb(H-h)}{\log_e \frac{R}{r}} = \frac{2.72 bk(H-h)}{\log_{10} \frac{R}{r}} \quad ...(8.9)$$

or

$$q = \frac{2\pi T\, s}{\log_e \frac{R}{r}} = \frac{2.72\, T\, s}{\log_{10} \frac{R}{r}} \quad ...(8.10)$$

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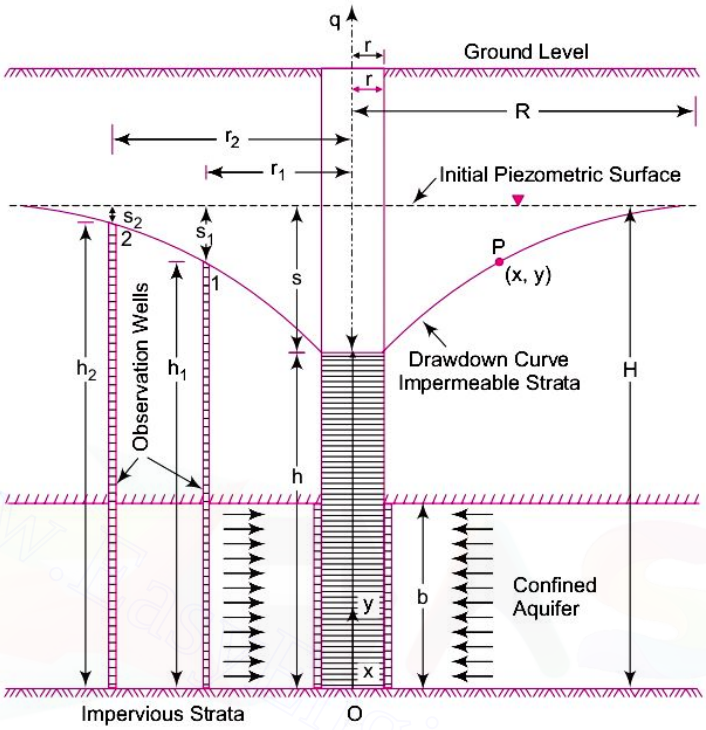

Fig. 8.3 Confined aquifer

where T = coefficient of transmissibility = bk and s = drawdown at the well

Equation 8.9 is known as the *equilibrium equation* or the *Thiem equation.*

If h_1 and h_2 are the measured depths of water in two observation wells situated radially at distance r_1 and r_2 respectively, we get

$$q = \frac{2.72\,bk\,(h_2 - h_1)}{\log_{10}\frac{r_2}{r_1}} = \frac{2.72T(h_2 - h_1)}{\log_{10}\frac{r_2}{r_1}} \qquad ...(8.11)$$

8.4 FIELD DETERMINATION OF *k* AND *T*: PUMPING OUT TESTS

Compared to laboratory tests, field permeability tests are more reliable. They give the in-situ value of permeability with minimum disturbance. The value of coefficient so obtained is an overall average for a large area. The permeability can be determined by either a pumping-out test or a pumping-in test. In the pumping-out test, drawdowns, corresponding to a steady discharge q, are observed at a number of observation wells. Pumping must continue at a uniform rate for a sufficient time to approach a *steady state* condition. Steady state condition is the one in which the drawdown changes negligibly with time.

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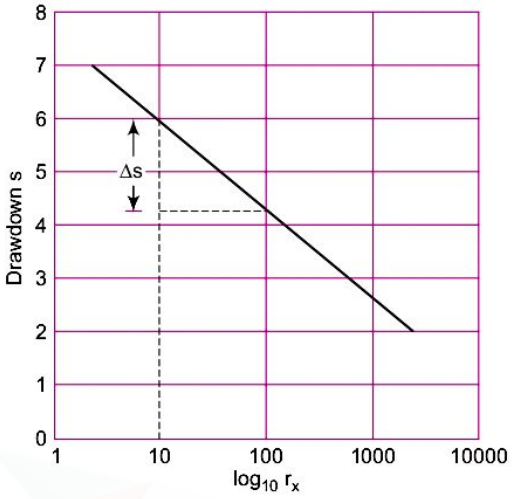

Fig. 8.4 Determination of T

(*a*) *Pumping-out test in unconfined aquifer:* From equations 8.5 and 8.7, we get

$$k = \frac{q}{\pi(H^2 - h^2)} \log_e \frac{R}{r} = \frac{q}{1.36\,(H^2 - h^2)} \log_{10} \frac{R}{r} \qquad ...(8.12)$$

and

$$k = \frac{q}{\pi(h_2^2 - h_1^2)} \log_e \frac{r_2}{r_1} = \frac{q}{1.36(h_2^2 - h_1^2)} \log_{10} \frac{r_2}{r_1} \qquad ...(8.13)$$

(*b*) *Pumping-out test in unconfined aquifer:* From equations 8.9 and 8.11, we get

$$k = \frac{q}{2\pi b(H - h)} \log_e \frac{R}{r} = \frac{q}{2.72b(H - h)} \log_{10} \frac{R}{r} \qquad ...(8.14)$$

and

$$k = \frac{q}{2\pi b(h_2 - h_1)} \log_e \frac{r_2}{r_1} = \frac{q}{2.72b(h_2 - h_1)} \log_{10} \frac{r_2}{r_1} \qquad ...(8.15)$$

From ground water point of view, however, the field practice is to determine the coefficient of transmissibility T. The drawdowns are observed at various observation wells. Referring to Fig. 8.3 (confined aquifer):

Let s_1 = drawdown in observation well 1 = $(H - h_1)$

s_2 = drawdown in observation well 2 = $(H - h_2)$

$\therefore$ $h_2 - h_1 = (H - s_2) - (H - s_1) = s_1 - s_2$

Then, from equation 8.11,

$$q = \frac{2.72T(h_2 - h_1)}{\log_{10} \frac{r_2}{r_1}} = \frac{2.72T(s_1 - s_2)}{\log_{10} \frac{r_2}{r_1}} \quad \text{Hence, } T = \frac{q}{2.72\,(s_1 - s_2)} \log_{10} \frac{r_2}{r_1} \qquad ...(8.16)$$

Choosing $r_2 = 10\,r_1$, we find $\log_{10} \frac{r_2}{r_1} = 1$. Hence, $T = \frac{q}{2.72\,(s_1 - s_2)} = \frac{q}{2.72\,\Delta s}$ ...(8.17)

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where, Δs = difference in drawdown at the two wells so selected that $r_2 - 10\ r_1$

The method, therefore, consists in observing drawdown $s_1, s_2, \ldots, s_n$ at certain observation wells distant $r_1, r_2, \ldots, r_n$ etc. and plotting a graph between s_x as ordinate and $\log_{10} r_x$ as abscissa, thus getting a straight line, as shown in Fig. 8.4. From the graph, Δs can be obtained for one log cycle of distance, and can be substituted in equation 8.17 to get T. Knowning T, $k = T/b$ can be computed.

SOLVED EXAMPLES

Example 8.1. *In order to determine the field permeability of a free aquifer, pumping out test was performed and following observations were made:*

Diameter of well = 20 cm. Discharge from the well = 240 m^3/hour R.L. of original water surface, before pumping started =240.5 m R.L. of water in the well at constant pumping =235.6 m R.L. of impervious layer = 210 m. R.L. of water in observation well = 239.8 m

Radial distance of observation well from the tube well = 50 m. Calculate k. Also calculate: (i) the error in k if observations are not taken in the observation well, (ii) the radius of influence is assumed to be 300 m, and (iii) actual radius of influence based on the observations of observation well.

Solution. (*i*) (Refer Fig. 8.2) (*a*) From Eq. 8.7, we have $q = \dfrac{1.36\,k(h_2^2 - h_1^2)}{\log_{10}\dfrac{r_2}{r_1}}$

Using suffix 1 for the well face and 2 for the observation well, we have

$$r_1 = 10 \text{ cm} = 0.1; \quad r_2 = 50 \text{ m}$$

$$h_1 = 235.6 - 210 = 25.6 \text{ m}; \quad h_2 = 239.8 - 210 = 29.8 \text{ m}$$

$$\therefore \quad (h_2 + h_1) = 29.8 + 25.6 = 55.4 \text{ m}; \quad (h_2 - h_1) = 29.8 - 25.6 = 4.2 \text{ m}$$

$$q = 24 \text{ m}^3\text{/hour} \qquad \therefore \quad 240 = \frac{1.36\,k(55.4) \times 4.2}{\log_{10}\dfrac{50}{0.1}}$$

$$\therefore \quad k = \frac{240}{1.36 \times 55.4 \times 4.2} \log_{10} 500 = 2.045 \text{ m/hour} = \mathbf{49.15\ m/day}$$

(*ii*) If k is computed on the basis of assumed radius of influence, we have, from equation 8.5

$$q = \frac{1.36\,k\,(H^2 - h^2)}{\log_{10}\dfrac{R}{r}}$$

Here $\dfrac{R}{r} = \dfrac{300}{0.1} = 3000$; $H = 240.5 - 210 = 30.5$ m; $h = 235.6 - 210 = 25.6$ m

$$\therefore \quad H + h = 30.5 + 25.6 = 56.1 \text{ m}; \; H - h = 30.5 - 25.6 = 4.9 \text{ m}$$

$$\therefore \quad k = \frac{240}{13.6 \times 56.1 \times 4.9} \log_{10} 3000 = 2.23 \text{ m/hour} = 53.4 \text{ m/day}$$

$$\therefore \quad \% \text{ error} = \frac{53.4 - 49.15}{49.15} \times 100 = \mathbf{8.6\%}$$

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(*iii*) Calculation of actual radius of influence

$$\log_{10}\frac{R}{r} = \frac{1.36\,k\,(H^2 - h^2)}{q}, \qquad \text{where} \quad k = 2.045 \text{ m/hour}$$

$$H + h = 56.1 \text{ m}; \; H - h = 4.9 \text{ m}; \; q = 240 \text{ m}^3\text{/hour}$$

$$\therefore \quad \log_{10}\frac{R}{r} = \frac{1.36 \times 2.045 \times 56.1 \times 4.9}{240} = 3.185$$

$$\therefore \quad \frac{R}{r} = 1531 \quad \text{or} \quad R = 1531 \times r = 1531 \times 0.1 = \mathbf{153} \text{ m}$$

Example 8.2. *A tube-well of 20 cm diameter penetrates fully an artesian aquifer of 30 m thickness. Compute the permeability of the aquifer, if a steady discharge of 40 litres/sec is obtained from the well under a drawdown of 4 m at the well face. Take radius of influence equal to 245 m.*

Solution.
$$k = \frac{q}{2.72\,b(H-h)}\log_{10}\frac{R}{r} = \frac{q}{2.72\,bs}\log_{10}\frac{R}{r} \qquad ...(8.14)$$

Here $b = 30$ m; $s = 4$ m; $r = 0.1$ m; $R = 245$ m; $q = 40$ litres/sec $= 0.04$ m^3/sec

$$\therefore \quad k = \frac{0.04}{2.72 \times 30 \times 4}\log_{10}\frac{245}{0.1} \text{ m/sec} = \mathbf{36 \text{ m/day}}$$

Example 8.3. *A steady ground water flow is occurring through an unconfined aquifer of coarse sand underlain by a horizontal impervious formation, as shown in Fig. 8.5. The depths of water table below the ground surface in the two observation wells, fixed at a spacing of 290 m along the direction of flow, are recorded as 5.0 m and 5.6 m. The sand layer is 20 m thick and its co-efficient of permeability is 4×10^{-3} cm/sec. Determine the rate of flow in m^3/day/m width of the aquifer.*

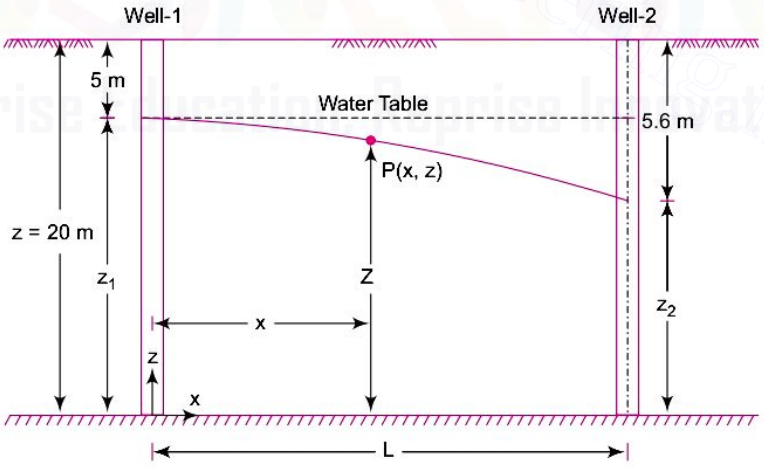

Fig. 8.5

Solution. The slope of the free water table represents the hydraulic gradient under which flow occurs. Let the co-ordinates of any point on the water table be measured with respect to the origin situated at the bottom of observation well No. 1. The co-ordinates of any point P are therefore (x, z).

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$$i = -\frac{dz}{dx} \text{ (Dupuit's assumption)}$$

$\therefore$ $q = kiA = -k\frac{dz}{dx}(z)$ per unit width of the aquifer

Intergrating, $q\int_0^L dx = -k\int_{z_1}^{z_2} z\,dz$ $\therefore$ $q = k\frac{z_1^2 - z_2^2}{2L} = k\left(\frac{z_1 - z_2}{L}\right)\left(\frac{z_1 + z_2}{2}\right)$

Now, $z_1 = 20 - 5 = 15$; $z_2 = 20 - 5.6 = 14.4$ m; $k = 4 \times 10^{-3}$ cm/sec $= 4 \times 10^{-5}$ m/sec

$\therefore$ $q = 4 \times 10^{-5}\left(\frac{15 - 14.4}{290}\right) \times \left(\frac{15 + 14.4}{2}\right) = 1.216 \times 10^{-6}$ m^3/sec/m width

$= 1.216 \times 10^{-6} \times 24 \times 60 \times 60 \times$ m^3/day/m width $=$ **0.105 m^3/day/m.**

8.5 PUMPING IN TESTS

The U.S. Bureau of Reclamation (Earth Manual, 1960) has devised two types of pumping in tests: (*i*) *Open-end tests* and (*ii*) *Packer tests.*

(*i*) Open-end tests. An open-end pipe is sunk in the strata and the soil is taken out of the pipe just to the bottom. Clean water, having temperature slightly higher than the ground water, is added through a metering system to maintain *gravity flow* under constant head. Water may also be allowed to enter the hole under some pressure head.

The permeability is calculated from the following expression determined from the electrical analogy experiments,

$$k = \frac{q}{5.5rh} \quad ...(8.18)$$

where h = differential head of water (gravity plus pressure, if any)

r = radius of casing ; q = constant rate of flow.

(*ii*) Packer tests. An uncased portion of the drill hole or a perforated portion of the casing is used for performing the test. In case the test is performed during drilling, a top packer is placed just inside or below the casing. Water is pumped in the lower portion of the hole. To perform the test after completion of the hole, which can stand without casing, two packers are set on a pipe or drill stem keeping the perforated portion of the pipe between the plugs. The bottom of the pipe is plugged. The length of packer on expansion should be five times the diameter of the hole. Testing is started from the bottom of the hole and continued upwards.

The coefficient of permeability is determined from the following expression:

$$k = \frac{q}{2\pi Lh}\log_{10}\frac{L}{r}\,;\; L \geq 10\,r \quad ...(8.19)$$

$$k = \frac{q}{2\pi Lh}\sinh^{-1}\frac{L}{2r}\,;\; 10\,r > L \geq r \quad ...(8.20)$$

where L = length of portion of the hole tested.

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8.6 INTERFERENCE AMONG WELLS

When two wells, situated near each other, are discharging, their drawdown curves intersect within their radii of zero drawdown. Thus, though the total discharge is increased, the discharge in individual well is decreased, due to interference.

Figure 8.6 shows interference between two wells. If the two wells are at distance B apart, and have the same diameter and drawdown and discharge over the same period of time, it can be shown with the help of method of complex variables, that the discharge through each well is given by

$$q_1 = q_2 = \frac{2\pi kb(H-h)}{\log_e \frac{R^2}{rB}} \qquad ...(8.21)$$

where R is the radius of area of influence ($R >> B$).

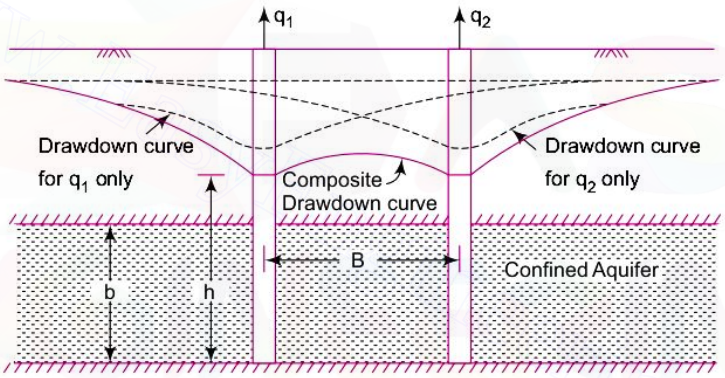

Fig. 8.6 Interference between two wells

If there were only one well, then the discharge under the same drawdown would be (Eq. 8.9)

$$q = \frac{2\pi kb\,(H-h)}{\log_e \frac{R}{r}} \qquad ...(8.9)$$

Since, $R >> B$, $\frac{R^2}{rB} > \frac{R}{r}$ Hence, $q > q_1$.

Thus, discharge in each well is decreased due to interference. Similarly, if there are three wells forming an equilateral triangle at a distance B on a side, and if all the three wells have the same characteristics,

$$q_1 = q_2 = q_3 = \frac{2\pi kb\,(H-h)}{\log_e \frac{R^3}{rB^2}} \qquad ...(8.22)$$

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8.7 FULLY PENETRATING ARTESIAN GRAVITY WELL

Sometimes, in an artesian well (*i.e.,* well in confined aquifer) high pumping rates may lower the water at the well face to a level below the top of the confined aquifer, as shown in Fig. 8.7. In such a case, the flow pattern close to the well is similar to that for a gravity well (*i.e.,* a well in unconfined aquifer) whereas at distances farther from the well, the flow is artesian. This type of well is known as a combined *artesian gravity well.* The flow from such a well can be computed from the following expression developed by Muskat (1937):

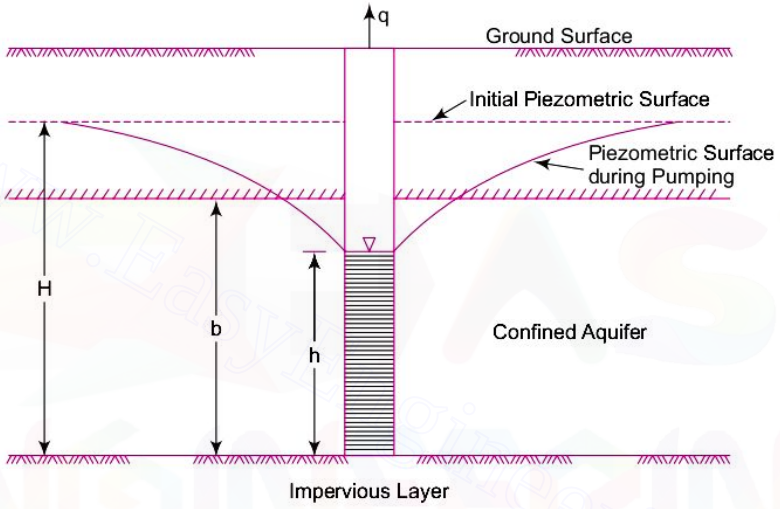

Fig. 8.7 Artesian-gravity well

$$q = \frac{\pi k (2bH - b^2 - h^2)}{\log_e \frac{R}{r}} \quad ...(8.23)$$

8.8 PARTIALLY PENETRATING ARTESIAN WELL

A partially penetrating artesian well is the one in which the well screen does not penetrate to the full depth of the confined aquifer. The pattern of flow in the aquifer in the vicinity of such a well deviates from that for a fully penetrating well. In practice, we often encounter such wells that extend only partly through the water bearing stratum.

Figure 8.8 shows a partially penetrating artesian well in which the strainer length b_1 is less than the aquifer thickness b. The discharge q_p from such a well can be computed from the following equation:

$$q_p = \frac{2\pi k b (H - h)}{\log_e \frac{R}{r}} G = q\,G \quad ...(8.24)$$

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where q_p = discharge for the partially penetrating well

q = discharge for a fully penetrating well for the same drawdown $(H - h)$

G = correction factor for partial penetration = q_p / q

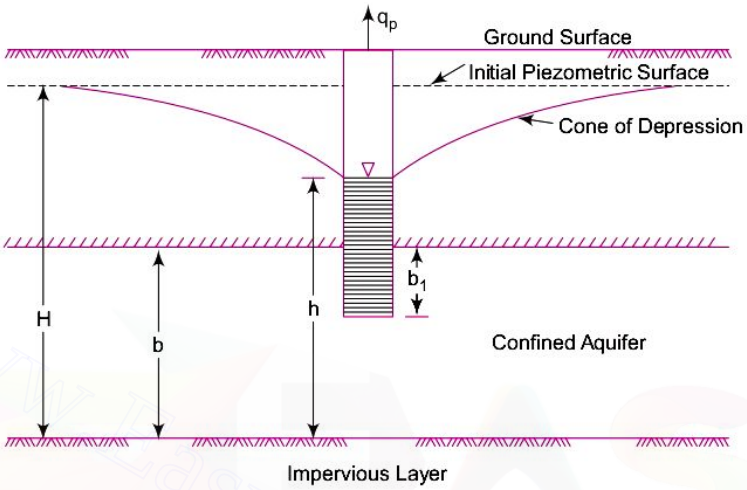

Fig. 8.8 Partially penetrating artesian well

A reasonable estimation of the correction factor G can be obtained from the following expression developed by Kozeny:

$$G = \frac{b_1}{b}\left(1 + 7\sqrt{\frac{r}{2b_1}}\cos\frac{\pi b_1}{2b}\right) \qquad ...(8.25)$$

8.9 SPHERICAL FLOW IN A WELL*

Figure 8.9 shows a special case of a partially penetrating well where the well just penetrates the top surface of a semi-infinite porous medium. Here $b_1 = 0$ and equation 8.24 does not apply because the flow towards the well is purely spherical. The discharge q_s from such a well can be computed from the following expression:

$$q_s = 2\pi kr\,(H - h) \qquad ...(8.26)$$

For the case of simple radial flow in fully penetrating well, the discharge q is given by equation 8.9:

$$q = \frac{2\pi kb(H - h)}{\log_e R/r} \qquad ...(8.9)$$

$$\therefore \quad \frac{q_s}{q} = \frac{r}{b}\log_e\frac{R}{r} = 2.3\frac{r}{b}\log_{10}\frac{R}{r} \qquad ...(8.27)$$

As a numerical example, let $r = 8$ cm $= 0.08$ m;

$$\frac{R}{r} = 1000; \quad b = 16 \text{ m} \qquad \therefore \quad \frac{q_s}{q} = 2.3 \times \frac{0.08}{16}\log_{10} 1000 \approx \frac{1}{30}$$

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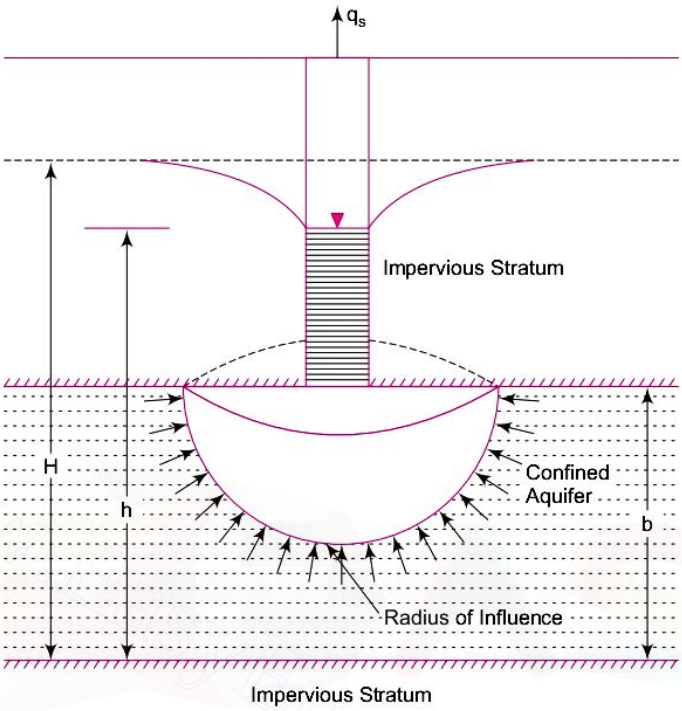

Fig. 8.9 Spherical flow in well

This shows that spherical flow is much less efficient than the radial flow.

8.10 FLOW TOWARDS OPEN WELL : RECUPERATION TEST

The hydraulics of flow towards open well is slightly different. In the case of tube well, radial flow takes place all around the well and there is no flow from the bottom of the well, while in the case of an open well, all the flow is essentially from the bottom. An open well has relatively larger diameter at its base, and its sides are mostly lined with brick. The discharge of an open well can be determined with the help of a *recuperation test*. In the recuperation test, water level is depressed to any level below the normal level, and the pumping is stopped. The time taken for the water to recuperate to the normal level is noted. From the data, the discharge from the well can be calculated as under:

Let aa = static water level in the well, before the pumping started

bb = water level in the well when pumping stopped

h_1 = depression head when the pumping stopped (metres)

cc = water level in the well at a time T after pumping stopped

h_2 = depression head in the well at a time T after pumping stopped (metres)

h = depression head in the well at a time t after the pumping stopped (metres)

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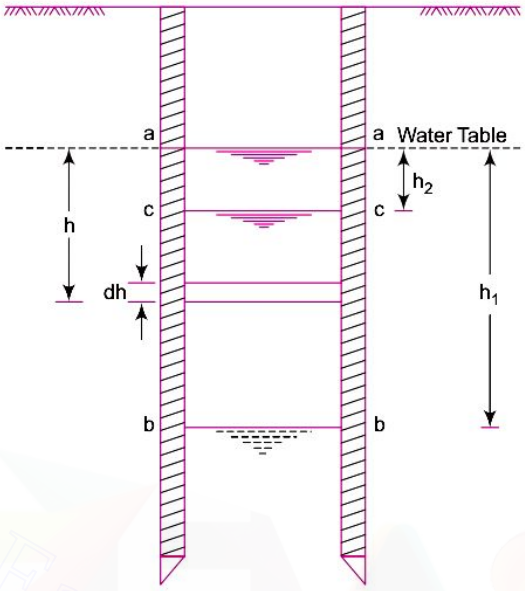

Fig. 8.10 Recuperation test

dh = decrease in depression head in a time dt

t, T = time in hours

Thus, in a time t, reckoned from the instant of stopping the pumping, the water level recuperates by $(h_1 - h)$ metres. In a time dt after this, the head recuperates by a value dh metres.

$\therefore$ Volume of water entering the well, when the head recuperates by dh is

$$dV = A\, dh \qquad ...(1)$$

where A = cross-sectional area of the well at its bottom.

Again, if q is the rate of discharge in the well at the time t, under the depression head h, the volume of water entering the well in a time dt hours is given by $dV = q\, dt$; But $q \propto h$ or $q = Kh$

$$\therefore \qquad dV = Khdt \qquad ...(2)$$

where K is constant depending upon the soil at the base of the well through which water enters. Equating (1) and (2), we get $Khdt = -Adh$ ...(3)

The minus sign indicates that h decreases as time t increase. Integrating the above between the limits $t = 0$ when $h = h_1$; to $t = T$ when $h = h_2$

we get $$\frac{K}{A}\int_0^T dt = -\int_{h_1}^{h_2} \frac{dh}{h}\ ;\ \text{From which,}\ \frac{K}{A}T = \left[\log_e h\right]_{h_2}^{h_1}$$

or $$\frac{K}{A} = \frac{1}{T}\log_e \frac{h_1}{h_2} = \frac{2.3}{T}\log_{10}\frac{h_1}{h_2} \qquad ...(8.28)$$

Thus, knowing value of h_1, h_2 and T from a recuperation test, the quantity K/A can be calculated. K/A is known as the *specific yield* or *specific capacity* of an open well, in cubic metre per hour per square

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metre of the area through which water percolates under one metre depression head. In the absence of the recuperation test, the following rough values of K/A, specified by Marrio, can be adopted.

Type of soil	*K/A (Cubic metre per hour, per sq m of area under 1 m depression head)*
Clay	0.25
Fine sand	0.50
Coarse sand	1.00

Knowing the value of K/A by observations, the discharge q from a well under a constant depression head H can be calculated as under:

$$q = KH = \left(\frac{K}{A}\right) AH = \frac{2.3}{T}\left(\log_{10}\frac{h_1}{h_2}\right) AH \text{ m}^3/\text{hour} \qquad ...(8.29)$$

Example 8.4. *During a recuperation test, the water in an open well was depressed, by pumping, by 2.5 metres and it recuperated 1.8 metres in 80 minutes. Find (a) yield from a well of 4 m diameter under a depression head of 3 metres, (b) the diameter of a well to yield 8 litres/second under a depression head of 2 metres.*

Solution. The specific yield is given by

$$\frac{K}{A} = \frac{2.3}{T}\log_{10}\frac{h_1}{h_2}, \qquad \text{where } T = \text{time in hours} = \frac{4}{3} \text{ hours}$$

$h_1 = 2.5$ m ; $h_2 = 2.5 - 1.8 = 0.7$ m $\quad \therefore \quad \frac{K}{A} = 0.955$ m^3/m^2/m/hour.

(a) Yield from the well of diameter 4 m:

$$A = \frac{\pi}{4}(4)^2 = 12.56 \text{ m}^2; \; q = \left(\frac{K}{A}\right) AH = 0.955 \times 12.56 \times 3 = 36 \text{ m}^3/\text{hour} = \textbf{10 litres/sec.}$$

(b) Yield = 8 L/sec = 28.8 m^3/hour (Since 1 L/sec = 3.6 m^3/hour)

$$\therefore \qquad q = \left(\frac{K}{A}\right) AH \quad \text{or} \quad A = \frac{q}{H} \times \left(\frac{A}{K}\right) = \frac{28.8}{2} \times \frac{1}{0.955} = 15.07 \text{ m}^2$$

$$\therefore \qquad d = \sqrt{\frac{4 \times 15.07}{\pi}} = 4.38 \text{ m} \approx \textbf{4.4 m} \text{ (say).}$$

Example 8.5. *Two tube wells, each of 20 cm diameter are spaced at 100 m distance. Both the wells penetrate fully a confined aquifer of 12 m thickness. Calculate the discharge if only one well is discharging under a depression head of 3 m. What will be the percentage decrease in the discharge of the well if both the wells are discharging under the depression head of 3 m. Take radius of influence for each well equal to 250 metres, and coefficient of permeability of aquifer as 60 metres/day.*

Solution. (Refer Fig. 8.6). In the first case, when only one well is discharging, the discharge is given by equation 8.9:

$$Q = \frac{2.72\, bk\, s}{\log_{10}\frac{R}{r}}$$

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where $b = 12$ m; $s = 3$ m; $k = 60$ m/day

$$\log_{10}\frac{R}{r} = \log_{10}\frac{250}{0.1} \approx 3.398$$

$$\therefore \quad Q = \frac{2.72 \times 12 \times 60 \times 3}{3.398}\text{ m}^3/\text{day} = 1729\text{ m}^3/\text{day} = \mathbf{72.04\ m^3/hour}$$

When both the wells are discharging, the discharge from each well is given by equation 8.21.

$$Q_1 = Q_2 = \frac{2\pi\, kb(H-h)}{\log_e \frac{R^2}{rB}} = \frac{2.72\, k\, b\, s}{\log_{10}\frac{R^2}{rB}} = \frac{2.72 \times 60 \times 12 \times 3}{\log_{10}\left(\frac{250 \times 250}{0.1 \times 100}\right)} = \frac{2.72 \times 60 \times 12 \times 3}{3.796}$$

$$= 1545\text{ m}^3/\text{day} = \mathbf{64.45\ m^3/hour.}$$

$\therefore$ Percentage decrease in the discharge $= \frac{72.04 - 64.45}{72.04} \times 100 = 10.53\%$

Example 8.6. *A gravity well has a diameter of 60 cm. The depth of water in the well is 40 metres before pumping is started. When pumping is being done at the rate of 2000 litres per minute, the drawdown in a well 10 metres away is 4 metres and in another well 20 metres away is 2 metres. Determine (a) radius of zero drawdown, (b) coefficient of permeability, (c) drawdown in the well, (d) specific capacity of the well, (e) maximum rate at which water can be pumped from the well.*

Solution.

(*a*) For a tube well in unconfined aquifer, we have from Eq. 8.5,

$$Q = \frac{1.36k\,(H^2 - h^2)}{\log_{10}\frac{R}{r}} \qquad \text{Here, we have } H = 40\ ;$$

At $r = 10$ m, $h = 40 - 4 = 36$ m. At $r = 20$ m, $h = 40 - 3 = 38$ m.

Applying the above equation at these two locations, we have

$$\frac{(40^2 - 36^2)}{\log_{10}\frac{R}{10}} = \frac{(40^2 - 38^2)}{\log_{10}\frac{R}{20}}$$

or $$\log_{10}\frac{R}{20} = \frac{(40^2 - 38^2)}{(40^2 - 36^2)} \quad \log_{10}\frac{R}{10} = 0.5132\ \log_{10}\frac{R}{10}$$

or $$\log_{10}\frac{R}{20} = \log_{10}\left(\frac{R}{10}\right)^{0.5132} \qquad \text{or} \qquad \frac{R}{20} = \frac{R^{0.5132}}{3.2599}$$

$$\therefore \quad R = (6.1352)^{\frac{1}{0.4868}} = (6.1352)^{2.0542} = \mathbf{41.52\ m.}$$

(*b*) $Q = 2000$ litres/min $= 2$ m^3/min (since 1 litres/min $= 0.001$/m^3/min).

$\therefore$ $R = 41.52$ m ; $H = 40$ m ; $h = 36$ m ; $r = 10$ m

$\therefore$ $$Q = 2 = \frac{1.36\, k\,(40^2 - 36^2)}{\log_{10}\frac{41.52}{10}}$$

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$$\therefore \qquad k = \frac{2}{1.36\,(40^2 - 36^2)} \log_{10} \frac{41.52}{10} = 0.003 \text{ m/min} = \textbf{4.31 m/day.}$$

(*c*) Depth of water in the well is given by

$$2 = \frac{1.36 \times 0.003\,(40^2 - H_0^2)}{\log_{10} \dfrac{41.52}{0.30}}$$

$$\therefore \qquad 1600 - H_0^2 = \frac{2}{1.36 \times 0.003} \times \log_{10} \frac{41.52}{0.30} = 1049.58.$$

$$\therefore \qquad H_0 = 23.46 \text{ m.} \quad \text{Hence drawdown at the well} = 40 - 23.46 = \textbf{16.54 m.}$$

(*d*) **Specific Capacity.** It is defined as the discharge per unit drawdown. Let it be designated by S_c. It is not contant, but decreases as the discharge increases. Let us assume that the yield is directly proportional to the drawdown or to the radius of zero-drawdown

$$\therefore \qquad Q \propto R \quad \text{or} \quad Q = CR \text{ where } C \text{ is a constant}$$

For the given data, $\quad C = \dfrac{Q}{R} = \dfrac{2}{41.52} = 0.04817 \qquad \therefore \quad R = \dfrac{Q}{C} = \dfrac{Q}{0.04817}$ in general

Now corresponding to drawdown of 1 m, discharge $Q = S_c$. Hence radius of zero drawdown

$$= \frac{S_c}{C} = \frac{S}{0.04817}$$

Hence, $$Q = S_c = \frac{1.36k\,(40^2 - 39^2)}{\log_{10} \dfrac{R}{r}} = \frac{1.36 \times 0.003\,(42^2 - 39^2)}{\log_{10} \left(\dfrac{S_c}{0.04817 \times 0.3} \right)}$$

or $$S_c \log_{10} (61.2\, S_c) = 0.3223.$$

Solving this by trial and error, we get $S_c = 0.266$ m^2/min.

Hence, specific capacity of the well is **0.266 m^3/min/m** depression head.

(*e*) Maximum rate of discharge Q_m will be obtained when drawdown in the well is equal to H, *i.e.*, when H_0 = zero.

$$Q_m = \frac{1.36 \times 0.003\,(40^2 - 0^2)}{\log_{10} \dfrac{Q_m}{0.04817 \times 0.3}}$$

or $$Q_m \log_{10} 69.2\, Q_m = 6.528.$$

Solving this by trial and error, we get $Q_m = \textbf{2.85 m}^3\textbf{/min.}$

8.11 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 8.7. *A 30 cm well completely penetrates an unconfined aquifer of depth 40 m. After a long period of pumping at a steady rate of 1500 lpm, the drawdown in two observation wells 25 m and 75 m from the pumping well were found to be 3.5 m and 2.0 m respectively. Determine the transmissibility of the aquifer. What is the drawn down at the pumping well ?* **(Civil Services Exam. 1994)**

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Solution

$$q = 1500 \text{ litres/min} = 1.5 \text{ m}^3/\text{min} = 0.025 \text{ m}^3/\text{sec}$$

$$h_2 = H - s_2 = 40 - 2 = 38 \text{ m}; \quad h_1 = H - s_1 = 40 - 3.5 = 36.5 \text{ m}$$

From Eq. 8.7, $q = \dfrac{1.36k\,(h_2^2 - h_1^2)}{\log_{10} r_2/r_1}$ or $0.025 = \dfrac{1.36k\,(38^2 - 36.5^2)}{\log_{10} 75/25}$

From which, $k = 7.848 \times 10^{-5}$ m/sec

$\therefore$ $T = Hk = 40 \times 7.848 \times 10^{-5} = \mathbf{3.14 \times 10^{-3}\ m^2/sec}$

Also, at the well face, $q = \dfrac{1.36k\,(h_1^2 - h_w^2)}{\log_{10}(r_1/r_w)}$ $\therefore$ $0.025 = \dfrac{1.36 \times 7.848 \times 10^{-5}(36.5^2 - h_w^2)}{\log_{10}(25/0.15)}$

or $h_w^2 = 36.5^2 - 520.42 = 811.83$ from which $h_w = 28.49$ m

$\therefore$ Drawdown at well face, $s_w = H - h_w = 40 - 28.49 = \mathbf{11.51\ m.}$

Example 8.8. *During a recuperation test, the water in an open well was depressed by pumping by 2.5 m and it recuperated 1.8 m in 80 minutes. Calculate the yield from a well 4 m diameter under a depression head of 3 m.* **(Engg. Services Exam. 1993)**

Solution. ***This problem was set from example 8.4 of author's book. For solution, see example 8.4.***

Example 8.9. *An aquifer of 20 m average thickness is overlain by an impermeable layer of 30 m thickness. A test well of 0.5 m diameter and two observation wells at distances of 10 m and 60 m from the test well are drilled through the aquifer. After pumping at a rate of 0.1 m^3/sec for a long time, the following drawdowns are stabilized in these wells: First observation well, 4 m; second observation well, 3 m. Show the arrangement in a diagram. Determine the coefficient of permeability and drawdown in the test well.* **(Engg. Services Exam. 1995)**

Solution. Refer Eq. 8.11. Here $h_2 = (20 + 30) - 3 = 47$ m; $h_1 = (20 + 30) - 4 = 46$ m

$r_2 = 60$ m; $r_1 = 10$ m; $b = 20$ m; $q = 0.1$ m^3/sec.

$$k = \frac{q \log_{10} r_2/r_1}{2.72\,b\,(h_2 - h_1)} = \frac{0.1 \log_{10}(60/10)}{2.72 \times 20(47 - 46)} = \mathbf{1.43 \times 10^{-3}\ m/sec.}$$

Let s be the drawdown at well face. Hence $h_w = (20 + 30) - s = 50 - s$

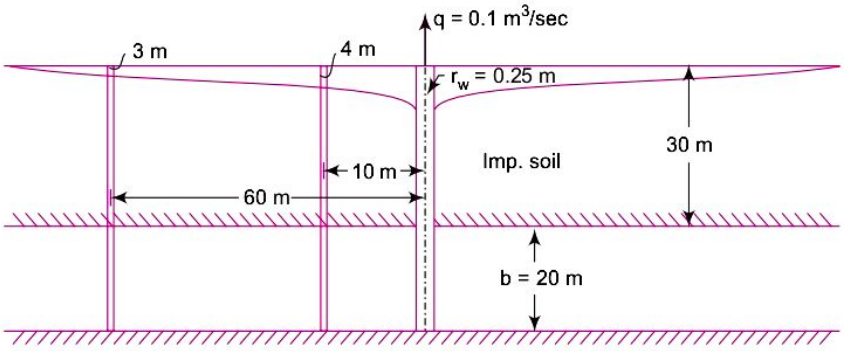

Fig. 8.11

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Hence, from Eq. 8.11, $(h_2 - h_w) = \dfrac{q \log_{10} r_2 / r_w}{2.72\, b\, k} = \dfrac{0.1 \log_{10}(60/0.25)}{2.72 \times 20 \times 1.43 \times 10^{-3}} = 3.0588$

$\therefore \quad h_w = h_2 - 3.0588 = 47 - 3.0588 = 43.94$ m. Hence $s = 50 - h_w = 50 - 43.94 = 6.06$ m.

Example 8.10. *A trench is to be excavated 240 m away from a river and in a confined aquifer 4.5 m thick. The trench has to run parallel to the river and is to be 300 m long. Water level in the trench is to be maintained 6 m above the lower confining layer and 3 m below the water level in the river. Determine the rate at which water should be pumped from the trench if the hydraulic conductivity of the aquifer is 4.5 m/day. Assume that there is no contribution to flow from the land side of the trench. State the condition when the assumption can be valid.* **(Engg. Services Exam. 1999)**

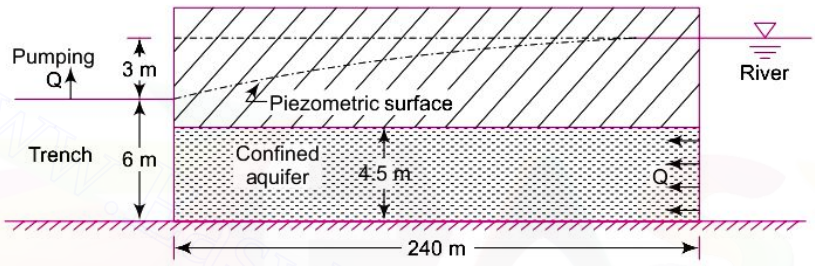

Fig. 8.12

Solution. Rate of pumping from the trench is equal to the discharge Q from river to the trench, flowing through the confined aquifer. From Darcy law, $Q = k \cdot i \cdot A$

where i = average hydraulic gradient = $h/L = 3/240 = 1/80$

A = area of flow = $4.5 \times 300 = 1350$ m^2; $k = 4.5$ m/day

$\therefore \quad Q = 4.5 \times \dfrac{1}{80} \times 1350 = \mathbf{75.94\ m^3/day} = 0.879$ litre/sec.

PROBLEMS

1. A coarse sand deposit 12.50 m thick overlies a stratum of clay. The ground water table is 2.0 metre below the ground surface. During the steady state of flow in a pumping-out test for finding k of sand in-situ, the quantity of water collected from the test-well is 550 litres per minute. The depths of the lowered water table, below the ground surface, in the two observation wells fixed radially at 20 metres and 40 metres respectively from the centre of the test well, are observed as 2.3 and 2.1 m respectively. Determine the coefficient of permeability of the sand deposit. [**Ans.** 4.9×10^{-2} cm/sec]
2. A field pumping-out test has the following data:
(*i*) Diameter of the well = 20 cm, (*ii*) Thickness of confined aquifer = 30 m, (*iii*) Radius of circle of influence = 350 m, (*iv*) Drawdown during the test = 5 m, (*v*) Pump discharge = 0.08 cumecs
Compute the permeability of aquifer in metres/day. [**Ans.** 60 m/day]
3. Two tube-wells, each of 20 cm diameter are spaced at 100 m distance. But the wells penetrate fully a confined aquifer of 12 m thickness. Calculate the discharge if only one well is discharging under a depression head of 3 m. What will be the percentage decrease in the discharge of this well if both the wells are discharging under the depression head of 3 m? Take the radius of influence of each well equal to 250 metres, and the coefficient of permeability of the aquifer as 60 metres/day. [**Ans.** 72 m^3/hour; 10.6%]

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CHAPTER 9

Seepage Analysis

9.1 HEAD, GRADIENT AND POTENTIAL

When water flows through a saturated soil mass, the *total head* at any point in the soil mass consists of (*i*) *piezometric head* or pressure head, (*ii*) the *velocity head,* and (*iii*) the *position head.* Figure 9.1 represents the flow of water through a saturated soil sample, of length L, due to the difference in elevation of free water surface at A and B.

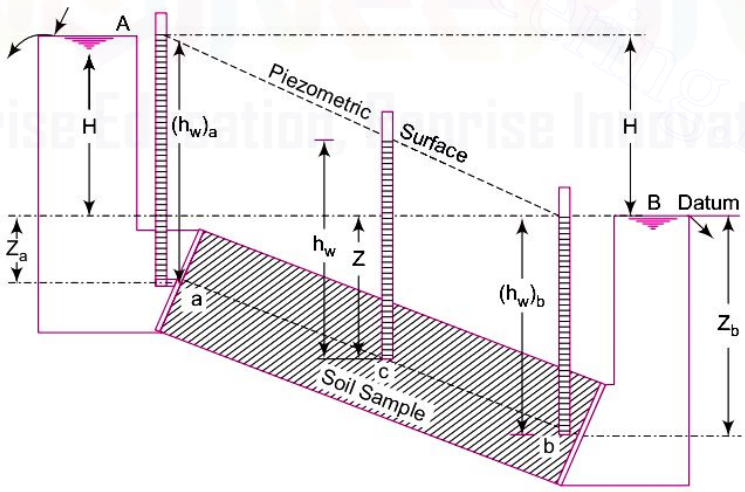

Fig. 9.1 Types of head

At the upper point a of the soil specimen, *piezometric head* is $(h_w)_a$. At the lower point b, the piezometric head is $(h_w)_b$. At any intermediate point c, the piezometric head h_w is equal to the height through which the water rises in a piezometric tube inserted at that point. The piezometric head is also called the *pressure*

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head. A *piezometric surface* is the line joining the water levels in the piezometres. The vertical distance between the piezometric levels at point a and b is called the initial *hydraulic head* H under which the flow takes place. The *position* or *elevation head* at any point is the elevation of that point with respect to any arbitrary datum. The position head Z is taken positive if it is situated above the datum and negative if below the datum. Thus, if the datum is taken at the *d/s* water level (tail water level), the position head at a is $-Z_a$ and at b is $-Z_b$.

The *total head* at any point may be regarded as the potential energy per unit weight of water measured with respect to the datum. The velocity head $V^2/2g$ is negligibly small for flow of water through the soil, and is usually neglected. Hence the total head at any point is equal to the algebraic sum of the piezometric head and the position head. Flow occurs between two points only when there is a difference in the total heads or potential energies or simply *potential* at those points.

In Fig. 9.1, the potential at b is $(h_w)_b - Z_b = 0$, and the potential at a is $(h_w)_a - Z_a = H$ = initial hydraulic head. The hydraulic head may be designated as *hydraulic potential* (h) also. The hydraulic potential at any point within the soil mass is given by

$$h = h_w \pm Z \qquad ...(9.1)$$

When *d/s* water level is the datum, the total head and the hydraulic potential at any point are equal.

A symbol ϕ is sometimes used in place of h to represent the hydraulic potential or the *potential function.* However, when ϕ represents a product of k and h, it is known as the *velocity potential* (See § 9.4). The loss of head or the dissipation of the hydraulic head per unit distance of flow through the soil is called the *hydraulic gradient:*

$$i = h/L \qquad ...(9.2)$$

9.2 SEEPAGE PRESSURE

By virtue of the viscous friction exerted on water flowing through soil pores, an energy transfer is effected between the water and the soil. The force corresponding to this energy transfer is called the *seepage force* or *seepage pressure.* Thus, *seepage pressure* is the pressure exerted by water on the soil through which it percolates. It is this seepage pressure that is responsible for the phenomenon known as *quick sand* and is of vital importance in the stability analysis of earth structures subjected to the action of seepage. The first rational approach to the problem was presented by Terzaghi in 1922 and forms the basis of the subsequent studies.

If h is the hydraulic head or the head lost due to frictional drag of water flowing through a soil mass of thickness z, the seepage pressure p_s is given by

$$p_s = h\gamma_w \qquad ...(9.3)$$

or

$$p_s = \frac{h}{z} z\gamma_w = iz\gamma_w \qquad ...(9.4)$$

where z = length over, which the head h is lost ; i = hydraulic gradient.

The *seepage force* J transmitted to the soil mass of total cross-sectional area A is:

$$J = p_s A = iz\gamma_w A \qquad ...(9.5)$$

The seepage force per unit volume is given by $J = \frac{iz\gamma_w A}{z A} = i\gamma_w$ ...(9.6)

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The seepage pressure always acts in the direction of flow. The vertical effective pressure may be decreased or increased, due to the seepage pressure depending upon the direction of flow. Thus, the effective pressure in a soil mass subjected to seepage pressure is given by

$$\sigma' = z\gamma' \pm p_s = z\gamma' \pm iz\gamma_w \qquad ...(9.7)$$

If the flow occurs in the *downward* direction, the effective pressure is *increased* and hence + sign is used. If, however, flow occurs in *upward* direction, the effective pressure is *decreased* and hence – sign is used in Eq. 9.7.

9.3 UPWARD FLOW : QUICK CONDITION

When flow takes place in an upward direction, the seepage pressure also acts in the upward direction and the effective pressure is reduced. If the seepage pressure becomes equal to the pressure due to submerged weight of the soil, the effective pressure is reduced to zero. In such a case, a cohesionless soil loses all its shear strength, and the soil particles have a tendency to move up in the direction of flow. This phenomenon of lifting of soil particles is called *quick condition, boiling condition* or *quick sand.* Thus, during the quick condition,

$$\sigma' = z\gamma' - p_s = 0$$

or $$p_s = z\gamma' \quad \text{or} \quad iz\gamma_w = z\gamma'$$

from which, $$i = i_c = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e} \qquad ...(9.8)$$

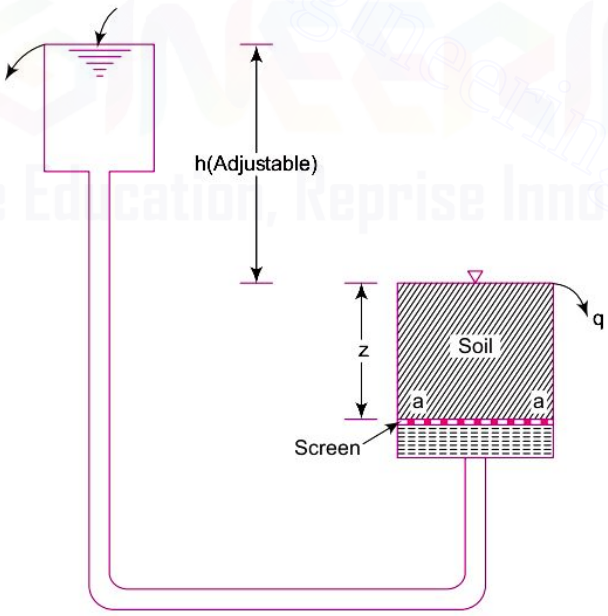

Fig. 9.2 Quick sand condition

The hydraulic gradient at such a critical state is called the *critical hydraulic gradient.* For loose deposits of sand or silt, if voids ratio e is taken as 0.67 and G as 2.67, the critical hydraulic gradient works out to

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be *unity*. It should be noted that *quick sand* is not a type of sand but a *flow condition* occurring within a cohesionless soil when its effective pressure is reduced to zero due to upward flow of water.

Figure 9.2 shows a set-up to demonstrate the phenomenon of quick sand. Water flows in an upward direction through a saturated soil sample of thickness z under a hydraulic head h. This head can be increased or decreased by moving the supply tank in the upward or downward direction. When the soil particles are in the state of critical equilibrium, the total upward force at the bottom of the soil becomes equal to the total weight of all the materials above the surface considered.

Equating the upward and downward forces at the level *a-a*, we have

$$(h+z)\,\gamma_w A = z\gamma_{sat} A \quad \therefore \quad h\gamma_w = z(\gamma_{sat} - \gamma_w) = z\gamma'$$

$$\frac{h}{z} = i_c = \frac{\gamma'}{\gamma_w} = \frac{G-1}{1+e}.$$

SOLVED EXAMPLES

Example 9.1. *A coarse-grained soil has a voids ratio of 0.78 and specific gravity as 2.67. Calculate the critical gradient at which quick sand condition will occur.*

Solution. Equation 9.8: $i_c = \dfrac{\gamma'}{\gamma_w} = \dfrac{G-1}{1+e} = \dfrac{2.67-1}{1+0.78} = \mathbf{0.94}$.

Example 9.2. *A large open excavation was made in a stratum of stiff clay with a saturated unit weight of 18.6 kN/m^3. When the depth of excavation reached 7 m, the bottom rose gradually, cracked and was flooded from below by a mixture of sand and water. Subsequent borings showed that the clay was underlain by a bed of sand with its surface at a depth of 12 m. Compute the elevation to which the water would have risen above the stratum into a drill hole before the excavation was started.*

Solution. (Fig. 9.3). Let the water in the drill hole rise to a value h_w metres above the sand strata, before the excavation was started. At the point A, the effective pressure is reduced when the excavation of the top soil is Started. When the soil is excavated by 7 m, the soil is lifted up. At that time, the effective pressure at A evidently becomes zero.

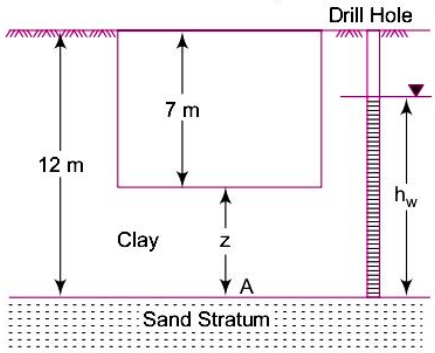

Fig. 9.3

$$\sigma' = z\gamma_{sat} - h_w\gamma_w = 0$$

$$\therefore \quad h_w = \frac{z\gamma_{sat}}{\gamma_w} = \frac{(12-7)\times 18.6}{9.81} = \mathbf{9.48\ m.}$$

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Example 9.3. *Water is flowing at the rate of 0.05 mL/sec in an upward direction through a fine sand sample whose co-efficient of permeability is 2×10^{-3} cm/sec. The sample thickness is 12 cm and cross-sectional area is 50 cm^2. Find the effective pressure at the middle and bottom sections of the sample, if the saturated unit weight of sand is 19.4 kN/m^3.*

Solution.

$$q = 0.05 \text{ cm}^3/\text{sec}; k = 2 \times 10^{-3} \text{ cm/sec} ; z = 12 \text{ cm} ; A = 50 \text{ cm}^2$$

$$\gamma' = 19.4 - 9.81 = 9.59 \text{ kN/m}^3$$

Now, $$i = \frac{q}{kA} = \frac{0.05}{2 \times 10^{-3} \times 50} = 0.5$$

For upward flow of water, the effective pressure is given by $\sigma' = z\gamma' - iz\gamma_w$

For the bottom section of the sample, $z = 12$ cm $= 0.12$ m measured from the top.

$\therefore$ $$\sigma' = (0.12 \times 9.59) - (0.5 \times 0.12 \times 9.81) = \mathbf{0.562 \text{ kN/m}^2}$$

At the middle section of the sample, $z = 6$ cm $= 0.06$ m measured from the top.

$\therefore$ $$\sigma' = (0.06 \times 9.59) - (0.5 \times 0.06 \times 9.81) = \mathbf{0.281 \text{ kN/m}^2.}$$

Example 9.4. *In the test set-up shown in Fig. 9.4, two different granular soils are placed in permeameter and flow is allowed to take place under a constant total head of 30 cm. (a) Determine the total head and pressure head at point A, (b) If 30% of the total head is lost as water flows upward through lower soil layer, what is the total head and pressure head at B?, (c) If the permeability of layer is 3×10^{-2} cm/sec, calculate the quantity of water per second flowing through unit area of the soil, (d) What is the co-efficient of permeability of the upper soil layer?*

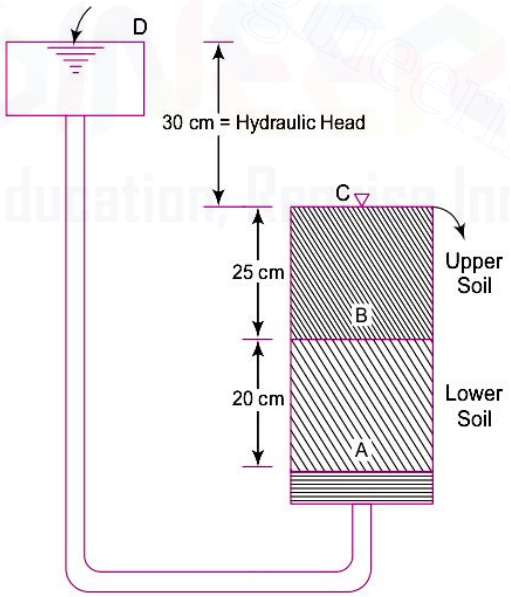

Fig. 9.4

Solution. Let the water level at C be the datum. The hydraulic head $h = 30$ cm.

(*a*) Total head at $D = h_w + z$ *where* h_w = *piezometric head or pressure head at* $D = 0$, z = *position head at* $D = +30$ cm

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$\therefore$ Total head at $D = 0 + 30 = 30$ cm

Total head at $A = h_w + z$

where h_w = piezometric or pressure head at A

$= 75$ cm

z = position head at $A = -45$ cm

Total head at $A = 75 - 45 = \mathbf{30\ cm} = 100\ \%\ h$.

(*b*) Loss of head from A to $B = 30\%$ of $h = 0.3 \times 30 = 9$ cm

$\therefore$ Total head at B = total head at A – head lost in $AB = 30 - 9 = \mathbf{21\ cm}$

But total head at $B = h_w + z$, where $z = -25$ cm

$\therefore$ $21 = h_w - 25$

or $h_w = 21 + 25 = \mathbf{46\ cm}$ = pressure head at B.

(*c*) Head lost between A and $B = 9$ cm

Now, $q = k\,i\,A = k\,.\,\dfrac{h}{z}\,A$

Taking $A = 1\ \text{cm}^2$; $h = 9$ cm ; $z = 20$ cm, we get

$$q = 3 \times 10^{-2} \times \frac{9}{20} \times 1 = \mathbf{1.35 \times 10^{-2}\ cm^3/sec.}$$

(*d*) Same flow takes place through the upper soil $q = 1.35 \times 10^{-2} = kiA$

Now, total head at $B = 21$ cm

Total head at $C = 0$

$\therefore$ Head lost between B and $C = 21$ cm

(Alternatively, head lost in upper soil = 70% of $h = 0.7 \times 30 = 21$ cm)

$\therefore$ i for the upper soil $= 21/25$

$\therefore$
$$k = \frac{q}{iA} = \frac{1.35 \times 10^{-2}}{\frac{21}{25} \times 1} \approx \mathbf{1.6 \times 10^{-2}\ cm/sec.}$$

Example 9.5. *In example 9.4, the voids ratio and specific gravities of the two soils are as follows:*

	Voids ratio	*Specific gravity*
Upper soil	*0.68*	*2.70*
Lower soil	*0.52*	*2.66*

(a) Determine the discharge velocity and seepage velocity through each soil.

(b) If the total head is increased, determine at what value of head will either soil be moved out of the container (become quick)?

Solution.

(*a*) Discharge velocity $= v = \dfrac{q}{A} = \dfrac{1.35 \times 10^{-2}}{1} = \mathbf{1.35 \times 10^{-2}\ cm/sec}$

This is evidently the same for both the soils.

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The seepage velocity (percolation velocity) v_p is given by $v_p = \dfrac{1+e}{e} v$

For the upper soil, $v_p = \dfrac{1+0.68}{0.68} \times v = \mathbf{3.34 \times 10^{-2}}$ **cm/sec.**

For the lower soil, $v_p = \dfrac{1+0.52}{0.52} \times v = \mathbf{3.95 \times 10^{-2}}$ **cm/sec.**

(*b*) The critical gradient i_c is given by $i_c = \dfrac{G-1}{1+e}$

For the upper soil, $i_c = \dfrac{2.70-1}{1+0.68} = 1.01$; For the lower soil, $i_c = \dfrac{2.66-1}{1+0.52} = 1.09$

The head lost in the upper soil is equal to 70% of the hydraulic head, while that lost in the lower soil is only 30%. Thus, for a given head the greatest hydraulic gradient occurs in the upper soil, and therefore, instability will occur in this material before it occurs in the lower soil.

Let the total head (hydraulic head) at the quick condition be h. Head lost in upper layer $= i_c \times z$ $= 1.01 \times 25$. But this is equal to 70% of $h = 0.7\,h$

$$0.7\,h = 1.01 \times 25 \text{ or } h = \frac{1.01 \times 25}{0.7} = \mathbf{36\ cm.}$$

Example 9.6. *Figure 9.5 shows the section of a water bearing sand stratum located between impervious clay stratums at top and bottom. There is a hydraulic gradient in the pervious layer causing a strata horizontal flow of water with a seepage velocity $v_s = 5 \times 10^{-4}$ cm/s. Two piezomtric tubes are inserted at points A and B, 12 m apart. If the water rises to a height of $h_A = 4.5$ m at A, what will be the height of rise in the piezometer at B? Laboratory test results for the sand show that it has $k = 1.24 \times 10^{-3}$ cm/s, $n = 36\%$ and $G = 2.66$. Determine the resultant body force acting on a soil element with unit volume in the vertical section C located midway between A and B.*

Also, draw the distribution of total and effective stresses along the vertical sections through A and B, taking the unit weight of overlying clay stratum as 19.8 kN/m^3.

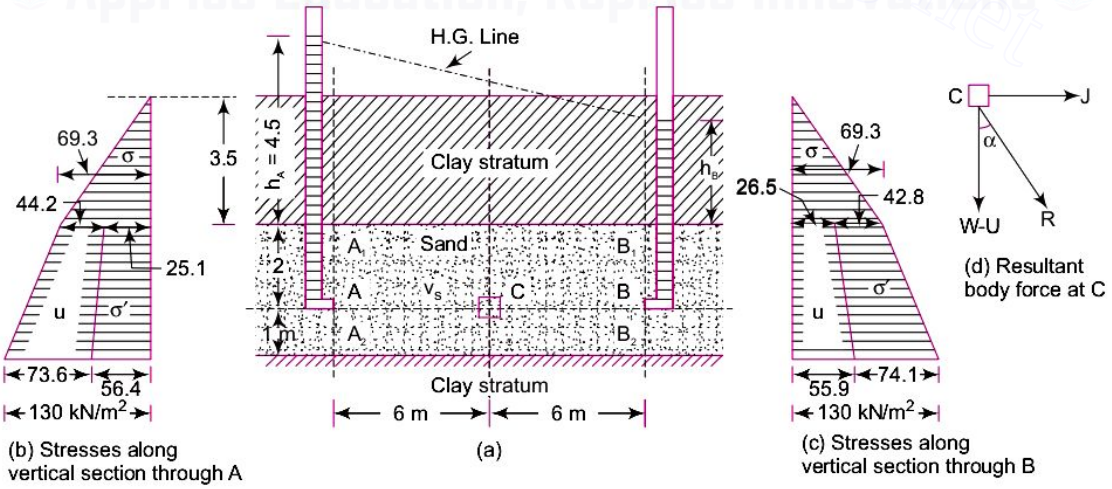

Fig. 9.5

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Solution.

(*i*) *Determination of h_B :*

The discharge velocity, $v = n\, V_s = 0.36 \times 5 \times 10^{-4} = 1.8 \times 10^{-4}$ cm/s

Saturated unit weight $\gamma_{sat} = G\gamma_w (1 - n) + \gamma_w n = 2.66 \times 9.81\ (1 - 0.36) + 9.81 \times 0.36$

$= \mathbf{20.23\ kN/m^3}$

Hydraulic gradient, $i = \dfrac{v}{k} = \dfrac{1.8 \times 10^{-4}}{1.2 \times 10^{-3}} = 0.15$

Neglecting the velocity head, the pressure head at B is given by

$$h_B = h_A - i\,L = 4.5 - 12 \times 0.15 = \mathbf{2.7\ m.}$$

(*ii*) *Resultant body force at C (Fig. 9.5 d):* Consider an element of volume 1 m^3 at C. Downward saturated weight of soil in the element,

$$W = 1\ m^3 \times 20.23\ kN/m^3 = 20.23\ kN$$

Buoyancy force, $F = 1\ m^3 \times \gamma_w = 1 \times 9.81 = 9.81$ kN

Horizontal seepage force, $J = V \cdot i\,\gamma_w = 1 \times 0.15 \times 9.81 = 1.47$ kN

$\therefore$ Resultant body force, $R = \sqrt{(W - F)^2 + J^2} = \sqrt{(19.6 - 9.81)^2 + 1.47^2} = \mathbf{9.9\ kN.}$

The inclination of R to the vertical, $\alpha = \tan^{-1}\left(\dfrac{1.47}{9.9}\right) = \mathbf{8.45°}$.

(*iii*) *Stresses along vertical section through A:* Consider point A_1 *at the to of sand stratum and point* A_2 at its bottom. The total vertical pressure at point A_1 will be equal to the weight of the overlying clay stratum.

$\therefore$ $\sigma_{A1} = 19.8 \times 3.5 = 69.3\ kN/m^2$; $\sigma_{A2} = 69.3 + 3 \times 20.23 \approx 130\ kN/m^2$

Also, $u_{A1} = h_A \cdot \gamma_w = 4.5 \times 9.81 \approx 44.2\ kN/m^2$ and $u_{A2} = (4.5 + 3)\ 9.81 = 73.6\ kN/m^2$

$\therefore$ $\sigma_{A1}' = \sigma_{A1} - u_{A1} = 69.3 - 44.2 = 25.1\ kN/m^2$; $\sigma_{A2}' = \sigma_{A2} - u_{A2} = 130 - 73.6 = 56.4\ kN/m^2$

The variations of pressure are shown in Fig. 9.5 (*b*).

(*iv*) *Stresses along vertical section through B:* Consider two points B_1, and B_2 at the top and bottom of sand stratum, respectively.

$\sigma_{B1} = 19.8 \times 3.5 = 69.3\ kN/m^2$; $\sigma_{B2} = 69.3 + 3 \times 20.23 = 130\ kN/m^2$

$u_{B1} = h_B \gamma_w = 2.7 \times 9.81 = 26.5\ kN/m^2$; $u_{B2} = (2.7 + 3) \times 9.81 = 55.9\ kN/m^2$

$\therefore$ $\sigma_{B1}' = \sigma_{B1} - u_{B1} = 69.3 - 26.5 = 42.8\ kN/m^2$; $\sigma_{B2}' = \sigma_{B2} - u_{B2} = 130 - 55.9 = 74.1\ kN/m^2$

The pressure variations are shown in Fig. 9.5(*c*).

9.4 TWO DIMENSIONAL FLOW : LAPLACE EQUATION

The quantity of water flowing through a saturated soil mass, as well as the distribution of water pressure can be estimated by the theory of flow of fluids through porous medium. While computing these quantities with the help of theoretical analysis that follow, the following assumptions are made:

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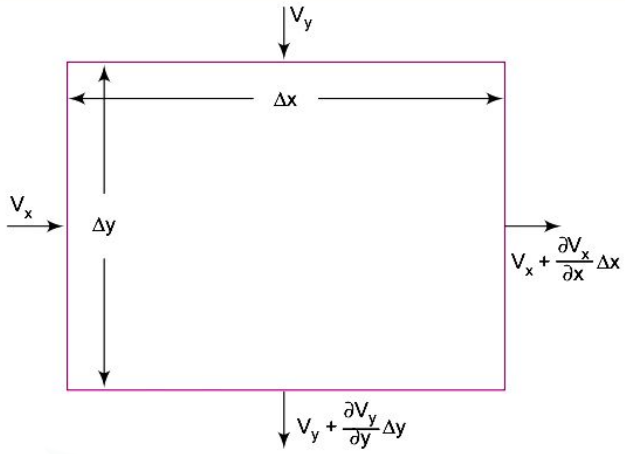

Fig. 9.6 Two dimensional flow

1. The saturated porous medium is *incompressible*. The size of the pore spaces does not change with time, regardless of water pressure.
2. The seeping water flows under a hydraulic gradient which is due only to gravity head loss, or Darcy's law for flow through porous medium is valid.
3. There is no change in the degree of saturation in the zone of soil through which water seeps and the quantity of water flowing into any element of volume is equal to the quantity which flows out in the same length of time.
4. The hydraulic boundary conditions at entry and exit are known.
5. Water is incompressible.

We shall first derive the Laplace equation for two dimensional flow.

Consider an element of soil of size Δx, Δy and of unit thickness perpendicular to the plane of the paper. Let v_x and v_y be the entry velocity components in x and y directions. Then $\left(v_x + \frac{\partial v_x}{\partial x}\Delta x\right)$ and $\left(v_y + \frac{\partial v_y}{\partial y}\Delta y\right)$ will be the corresponding velocity components at the exit of the element.

According to assumption 3 stated above, the quantity of water entering the element is equal to the quantity of water leaving it.

$$v_x(\Delta y \cdot 1) + v_y(\Delta x \cdot 1) = \left(v_x + \frac{\partial v_x}{\partial x}.\Delta x\right)(\Delta y \cdot 1) + \left(v_y + \frac{\partial v_y}{dy}\Delta y\right)(\Delta x \cdot 1)$$

From which $$\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 0 \qquad \text{...(9.9). This is the } \textit{continuity equation.}$$

According to assumption 2, $v_x = k_x \cdot i_x = k_x \cdot \frac{\partial h}{\partial x}$ and $v_y = k_y \cdot i_y = k_y \frac{\partial h}{\partial y}$

where h = hydraulic head under which water flows

k_x and k_y = coefficients of permeability in x and y directions.

Substituting these in Eq. 9.9, we get $$\frac{\partial^2(k_x h)}{\partial x^2} + \frac{\partial^2(k_y h)}{\partial y^2} = 0 \qquad \text{...(9.10)}$$

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For an isotropic soil, $k_x = k_y = k$ (say) $\dfrac{\partial^2 h}{\partial x^2} + \dfrac{\partial^2 h}{\partial y^2} = 0$ ...(9.11)

Substituting $\phi = kh$ = *velocity potential*, we get $\dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} = 0$ ...(9.12)

This is the Laplace equation of flow in two dimensions.

Velocity potential (ϕ). The velocity potential ϕ may be defined as a scalar function of space and time such that its derivative with respect to any direction gives the fluid velocity in that direction. This is evident, since, we have $\phi = kh$

$$\therefore \quad \frac{\partial \phi}{\partial x} = K \frac{\partial h}{\partial x} = k i_x = v_x \qquad ...(9.13(a))$$

Similarly, $$\frac{\partial \phi}{\partial y} = k \frac{\partial h}{\partial y} = k\, i_y = v_y \qquad ...(9.13(b))$$

The solution of Eq. 9.12 can be obtained by

(*i*) analytical methods (*ii*) graphical method (*iii*) experimental methods

The solution gives two sets of curves known as *'equipotential lines'* and *'stream lines'* (or flow lines), mutually orthogonal to each other, as shown in Fig. 9.7. The equipotential lines represent contours of equal head (potential). The direction of seepage is always perpendicular to the equipotential lines. The path along which the individual particles of water seep through the soil are called stream lines or flow lines.

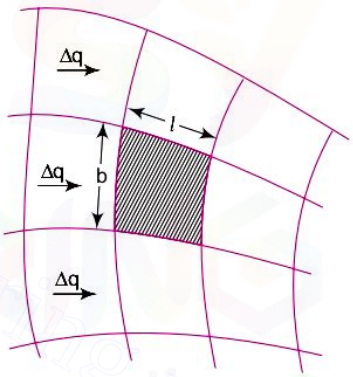

Fig. 9.7 Portion of a flow net

Stream function. It can be shown that Laplace's equation is satisfied by the conjugate harmonic function ϕ and ψ and that the curve 'ϕ (x, y) = constant' are the orthogonal trajectories of the curve 'ψ (x, y) = constant'. In ground water literature, the function ψ (x, y) or simply ψ, is called the *stream function*. It is defined as a scalar function of space and time such that a partial derivative of this function with respect any direction gives the velocity component in a direction + 90° (clockwise) to the original direction.

Thus, $$\frac{\partial \psi}{\partial y} = v_x \qquad ...(9.14(a))$$

and $$\frac{\partial \psi}{\partial x} = -v_y \qquad ...(9.14(b))$$

From equations 9.13 and 9.14, we find that

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \text{ and } \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{dx} \qquad ...(9.15)$$

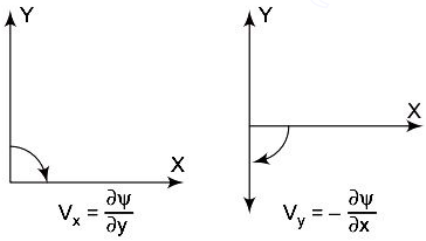

Fig. 9.8 Stream function

Equation 9.15 is known as Cauchy-Riemann equations.

Substituting the values of v_x and v_y in terms of ψ, in the continuity equation, we get

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = 0 \quad \text{or} \quad \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

Thus, the stream function satisfies the continuity equation.

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Similarly, substituting the values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$, in term of ψ, in Laplace equation,

We get $\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right)=0$ or $\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right)=0$

$\therefore \quad \frac{\partial^2 \psi}{\partial x \partial y}-\frac{\partial^2 \psi}{\partial y \partial x}=0$

Thus, the stream function also satisfies the Laplace equation.

Considering total differential, we have $d\psi = \frac{\partial \psi}{\partial x}dx+\frac{\partial \psi}{\partial y}dy$

and combining this with the Cauchy-Riemann equation (Eq. 9.15), we get

$$\psi = \int\left(\frac{\partial \phi}{\partial x}dx-\frac{\partial \phi}{\partial y}dx\right) \qquad ...(9.16(a))$$

Similarly, $$\phi = \int\left(\frac{\partial \psi}{\partial y}dy-\frac{\partial \psi}{\partial x}dy\right) \qquad ...(9.16(b))$$

In solving ground water or seepage problems, we need concern ourselves only with the determination of one of the two functions ϕ and ψ. The other functions will then follow from the relationships of Eq. 9.16.

Complex potential (w). A combination of function ϕ and ψ is called the *complex potential,* and is defined by

$$w = \phi + i\psi \qquad ...(9.17)$$

9.5 GRAPHICAL METHOD OF FLOW NET CONSTRUCTION

The graphical method of flow net construction, first given by Forchheimer (1930), is based on trial sketching. The hydraulic boundary conditions have a great effect on the general shape of the flow net, and hence must be examined before sketching is started. The flow net can be plotted by trial and error by observing the following properties of flow net and by following the practical suggestions given by *A*. Casagrande.

Properties of flow net

1. The flow lines and equipotential lines meet at right angles to one another.
2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.
3. The quantity of water flowing through each flow channel is the same. Similarly, the same potential drop occurs between two successive equipotential lines.
4. Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.
5. In a homogeneous soil, every transition in the shape of the curves is smooth, being either elliptical or parabolic in shape.

Arthur Casagrande (1937) gave the following excellent hints for the beginner in flow net sketching:

1. Use every opportunity to study the appearance of well constructed flow nets. When the picture is sufficiently absorbed in your mind, try to draw the same flow net without looking at the available solution; repeat this until you are able to sketch this flow net in a satisfactory manner.

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2. Four or five flow channels are usually sufficient for the first attempts; the use of too many flow channels may distract the attention from essential features.
3. Always watch the appearance of the entire flow net. Do not try to adjust details before the entire flow net is approximately correct.
4. The beginner usually makes the mistake of drawing too sharp transitions between straight and curved sections of flow lines or equipotential lines. Keep in mind that all transitions are smooth, of elliptical or parabolic shape. The size of the squares in each channel will change gradually.

9.6 FLOW NET BY ELECTRICAL ANALOGY

The Darcy's law governing the flow of water through soil is analogous to Ohm's law govering the flow of electric current through conductors. The corresponding analogous quantitites are shown in the table below:

Darcy' law of water seepage	*Ohm's law of electric flow*
$q = k\frac{h}{L}A$	$I = C\frac{E}{l}a$
q = quantity of seepage	I = rate of flow of electricity
k = coefficient of permeability	C = electric conductivity coefficient
A = cross-sectional area	a = cross-sectional area
h = hydraulic head	E = electric potential
L = length of seepage	l = length of path of electric current

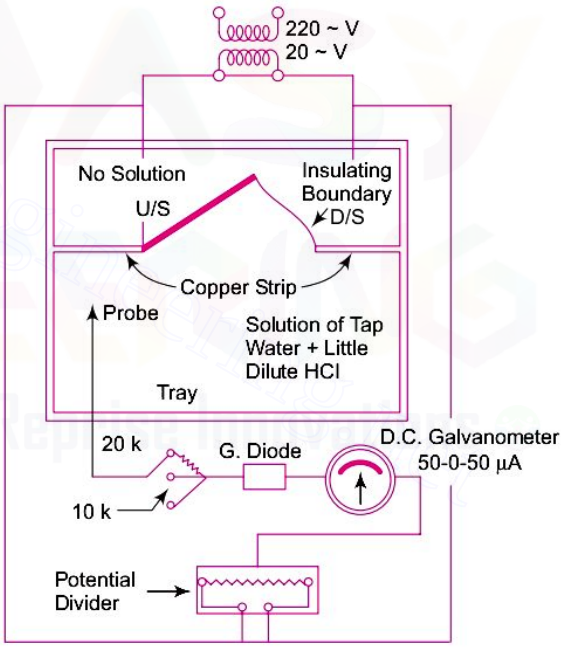

Fig. 9.9 Circuit diagram for electric analogy tray (After alam singh and B.C. punmia, 1962)

Thus, the solutions to seepage problems can be obtained with electric models which have the same geometric shape as the soil through which the water flows. The seepage medium is replaced by an electric conductor consisting of water with some salt or dilute hydrochloric acid. The boundary equipotential lines are made of copper. The boundary flow lines are simulated by non-conducting strips such as ebonite or perspex etc. An alternating voltage (generally of value 5 to 20 volts) is applied across the boundary equipotential strips. A potential divider is connected in parallel with the alternating current source. Figure 9.9 shows the electric analogy tray, the model and the complete circuit diagram, for study of seepage through an earth dam.

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To determine a line of contour of equal potential, the potentiometer is adjusted to a percentage of the total voltage drop, and the probe of a galvanometer is used to find the corresponding balance points (null points) on the model. Changes in the coefficient of permeability in soil zones in the seepage analogue are simulated by changes in the electric conductivity coefficient in the model. When once the equipotential lines are obtained, orthogonal flow lines conforming to the boundary conditions are then drawn as in the graphical method. Figure 9.10 shows a typical flow net for steady seepage case for an earth dam having the same foundation material as that of the body of the dam. Figure 9.11 shows the net for the sudden drawdown case for the same dam.

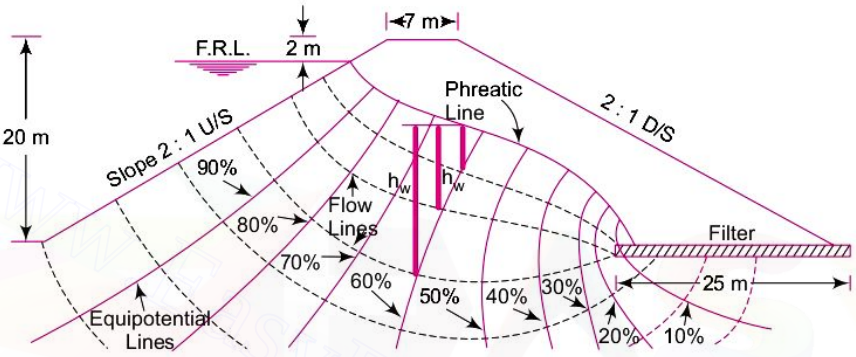

Fig. 9.10 Flow net for steady seepage (After alam singh and B.C. punmia, 1962)

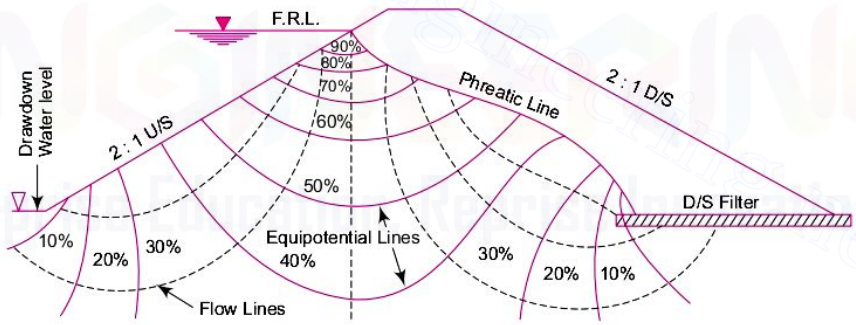

Fig. 9.11 Flow net for sudden drawdown (After alam singh and B.C. punmia, 1962)

9.7 APPLICATIONS OF FLOW NET

A flow net can be utilised for the following purposes:

(*i*) Determination of seepage,
(*ii*) Determination of hydrostatic pressure,
(*iii*) Determination of seepage pressure,
(*iv*) Determination of exit gradient.

(*i*) Determination of seepage. Figure 9.7 shows a portion of a flow net. The portion between any two successive flow lines is known as at *flow channel.* The portion enclosed between two successive equipotential lines and successive flow lines is known as *field* such as that shown hatched in Fig. 9.7.

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Let b and l be the width and length of the field,

Δh = head drop through the field;

Δq = discharge passing through the flow channel;

H = total hydraulic head causing flow

= difference between upstream and downstream heads.

Then, from Darcy's law of flow through soils:

$$\Delta q = k \cdot \frac{\Delta h}{l}(b \times 1) \text{ (Considering unit thickness)} \quad ...(i)$$

If N_d = total number of potential drops in the complete flow net then, $\Delta h = \frac{H}{N_d}$

Hence, $$\Delta q = k \cdot \frac{H}{N_d}\left(\frac{b}{l}\right) \quad ...(ii)$$

The total discharge through the complete flow net is given by

$$q = \Sigma \Delta q = k \cdot \frac{H}{N_d}\left(\frac{b}{l}\right) \cdot N_f = kH \frac{N_f}{N_d}\frac{b}{l} \quad ...(iii)$$

where N_f = total number of flow channels in the net.

The field is square; hence $b = l$

Thus, $$b = kH \frac{N_f}{N_d} \quad ...(9.18)$$

This is the required expression for the discharge passing through a flow net and is valid for isotropic soils in which $k_x = k_y = k$.

(ii) Determination of hydrostatic pressure. The hydrostatic pressure at any point within the soil mass is given by

$$u = h_w \gamma_w \quad ...(9.19)$$

where u = hydrostatic pressure ; h_w = piezometric head

The hydrostatic pressure in terms of piezometric head h_w is calculated from the following relation

$$h_w = h - Z \quad ...(9.20)$$

where h = hydraulic potential at the point under consideration

Z = position head of the point above datum, considered positive upwards.

All the three quantities h_w, h and Z can be expressed as the percentage of the total hydraulic head H. Equation 9.20 can be used to plot a *pressure net* representing lines of equal water pressure (piezometric head), within the saturated soil mass.

For example, if we want to plot the line of equal pressure corresponding to $h_w = 20\%$ (say), we have $h_w = 20 = h - Z$

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To find a point corresponding to $h_w = 20\%$ on $h = 30\%$ H, we get $Z = 30 - 20 = 10\%$ of H. Thus, the point is located. Similary, to get another point of $h_w = 20\%$ on $h = 40\%$, $Z = 40 - 20 = 20\%$ of H, and so on. The various points so obtained may be joined by a smooth curve to get a contour for $h_w = 20\%$. This way, a pressure net may be prepared. Figure 9.12 shows the pressure net for the sudden drawdown case of an earthen dam corresponding to the flow net shown in Fig. 9.11.

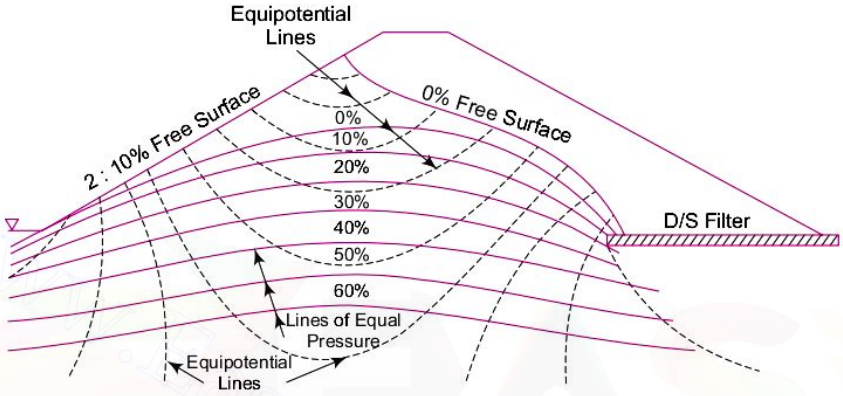

Fig. 9.12 Pressure net of sudden drawdown case (After alam singh and B.C. punmia, 1962)

***(iii)* Determination of seepage pressure.** The hydraulic potential h at any point located after n potential drops, each of value Δh is given by

$$h = H - n\Delta h \qquad ...(9.21)$$

The seepage pressure at any point equals the hydraulic potential or the balance hydraulic head multiplied by the unit weight of water (Eq. 9.3) and hence, is given by

$$p_s = h\,\gamma_w = (H - n\Delta h)\,\gamma_w \qquad ...(9.22)$$

The pressure acts in the direction of flow.

***(iv)* Determination of exit gradient.** The *exit gradient* is the hydraulic gradient at the downstream end of the flow line where the percolating water leaves the soil mass and emerges into the free water at the downstream. The exit gradient can be calculated from the following expression, in which Δh represents the potential drop and l the average length of last field in the flow net, at exit end:

$$i_e = \frac{\Delta h}{l} \qquad ...(9.23)$$

The importance of the exit gradient for the stability of granular solids has been discussed at length in Chapter 10.

9.8 SEEPAGE THROUGH ANISOTROPIC SOIL

Let us now consider the case of an anisotropic flow medium in which $k_x \neq k_y$. For such a case, the flow Eq. 9.10 becomes

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$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0$$

which is not a Laplacian equation. Hence flow net cannot be directly drawn Rewriting it, we get

$$\frac{k_x}{k_y} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Let us put $$x_n = x \sqrt{\frac{k_y}{k_x}}$$

where x_n is the new co-ordinate variable in the x-direction. Then the above equation becomes

$$\frac{\partial^2 h}{\partial x_n^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad ...(9.24)$$

This is in Laplacian form.

To plot the flow net for such a case, the cross-section through anisotropic soil is plotted to a natural scale in the y-direction, but to a transformed scale in the x-direction, all dimensions parallel to x-axis being redued by multiplying by the factor $\sqrt{k_y/k_x}$. The flow net for this *tranformed section* will now be constructed in the normal manner as if the soil were isotropic. The *actual flow net* is then obtained by re-transforming the cross-section including the flow net, back to the natural scale by multiplying by a factor $\sqrt{k_x/k_y}$. The actual flow net, thus, will not have orthogonal set of curves.

As shown in Fig. 9.13, the field of transformed section will be a square one, while the field of the actual section (re-transformed) will be rectangular one having its length in x-direction equal to $\sqrt{\frac{k_x}{k_y}}$ times the width in the y-direction.

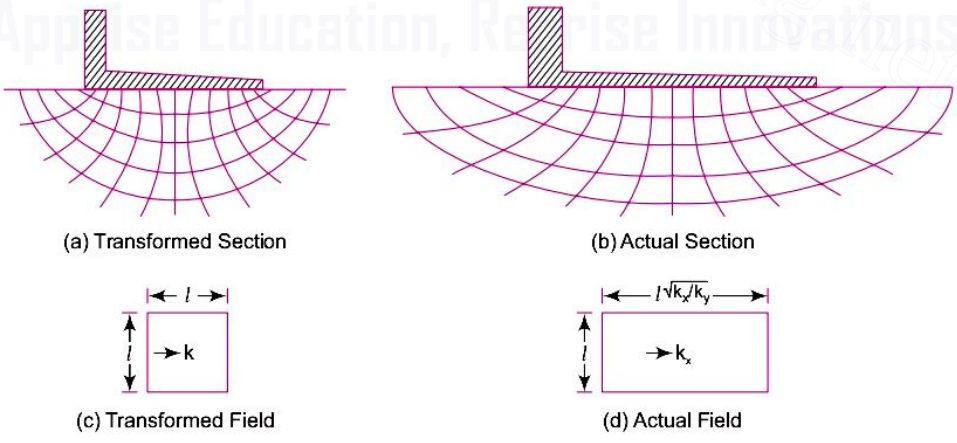

Fig. 9.13 Anisotropic flow

Let k_x = permeability coefficient in x-direction of the actual anisotropic soil field

k' = equivalent permeability of the transformed field.

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Then, for the transformed section,

$$\Delta q = k' \frac{\Delta h}{l}(l \times 1)$$

For the actual field, $\Delta q = k_x \dfrac{\Delta h}{l\sqrt{k_x/k_y}}(l \times 1)$

Since the quantity of flow is the same

$$k' \frac{\Delta h}{l}(l) = k_x \frac{\Delta h}{l\sqrt{k_x/k_y}}(l) \quad \therefore \quad k' = k_x \sqrt{\frac{k_y}{k_x}} = \sqrt{k_x k_y} \qquad ...(9.25)$$

Hence the discharge is given by

$$q = k'H \frac{N_f}{N_d} = \sqrt{k_x \cdot k_y}\, H \frac{N_f}{N_d} \qquad ...(9.26)$$

9.9 DEFLECTION OF FLOW LINES AT INTERFACE OF DISSIMILAR SOILS

The flow lines get deflected at the interface between two dissimilar soils when they pass from one soil to the other. Figure 9.14 shows two flow lines ψ_1, (AB) and ψ_2 (CD) being deflected to BA_1 and DC_1 respectively at the interface of two soils having permeability coefficients k_1, k_2. Let ϕ_1 and ϕ_2 be two equipotential lines, having a drop of potential Δh between them. The equipotential lines also get deflected at the interface. Let θ_1 and θ_2 be the angles that the flow line ψ_1 makes with the normal to the interface before and after deflection respectively.

The rate of flow Δq between the flow channel bounded by ψ_1, ψ_2 is given by

$$\Delta q = k_1 i_1 (ED) = k_2 i_2 (BF)$$

where $$i_1 = \frac{\Delta h}{EB} \text{ and } i_2 = \frac{\Delta h}{DF}$$

$$\therefore \quad \Delta q = k_1 \frac{\Delta h}{EB} ED = k_2 \frac{\Delta h}{DF} BF$$

or $$\frac{k_1}{k_2} = \frac{EB}{ED}\frac{BF}{DF}\text{; But } \frac{EB}{ED} = \tan\theta_1$$

and $$\frac{DF}{BF} = \tan\theta_2$$

$$\therefore \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{k_1}{k_2} \qquad ...(9.27)$$

Equation 9.27 defines the law of deflection of flow line at the interface of two dissimilar soils.

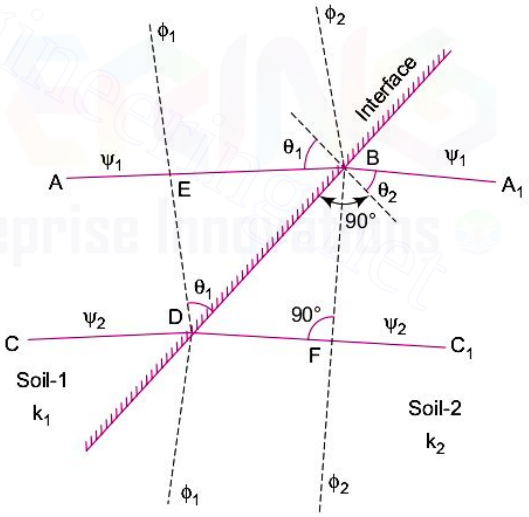

Fig. 9.14 Deflection of flow lines

9.10 PHREATIC LINE OF AN EARTH DAM

Let us now consider the case of a homogeneous earth dam with a horizontal filter, as shown in Fig. 9.15(*b*). In order to draw the flow net, it is first essential to find the location and shape of the *phreatic line* or

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the *top flow line* separating the saturated and unsaturated zones. The phreatic line or seepage line is defined as the line within a dam section below which there are positive hydrostatic pressures in the dam. The hydrostatic pressure on the phreatic line itself is atmospheric. The phreatic line can be located by (*i*) analytical method, (*ii*) graphical method, and (*iii*) experimental method. We shall first discuss the graphical method of determination of phreatic line suggested by *Casagrande.*

Casagrande Method. An analytically derived flow net has been given by Kozeny (1931) for the case of water flowing above an impervious, infinite, horizontal plane which at a certain point *A* becomes permeable. Kozeny's solution consists of a family of confocal parabolas representing the flow lines and the equipotential lines. The point *A* where the floor becomes permeable represents the *focus* for the parabolas. Casagrande (1940) established that the actual topflow line or the *phreatic line* in a homogeneous earth dam corresponds very closely to the solution of Kozeny, except at the entrance and exit ends, where it has to suit the boundary conditions. Kozeny's top flow line is called the *basic* or *base parabola.* Casagrande assumed the starting point *F* of the filter *FE* (Fig. 9.15 *b*) as the focus. The following is the procedure for locating the phreatic line graphically:

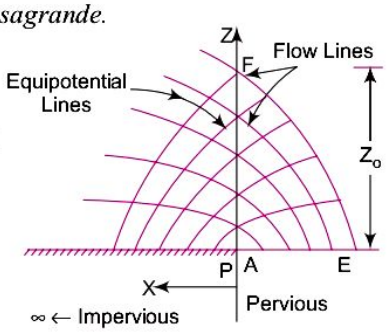

Fig. 9.15 (a) Parabolic flow (After Kozney)

1. *AB* is the upstream face. Let its horizontal projection be *L*. On the water surface, measure a distance *BC* = 0.3 *L*. Then the point *C* is the starting point of the base parabola.

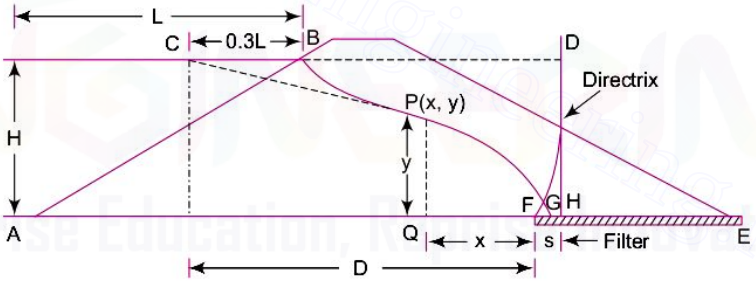

Fig. 9.15 (b) Casagrande's method of determining phreatic line in a dam with horizontal drainage filter

2. To locate the directrix of the parabola, we utilise the principle that any point on the parabola is equidistant from the focus as well as from the directrix. Hence with point *C* as the centre and *CF* as the radius, draw an arc to cut the horizontal line through *CB* in *D*. Draw a vertical tangent to the curve *FD* at *D*. Evidently, *CD* = *CF*. Hence the vertical line *DH* is the directrix.
3. The last point *G* on the parabola will lie midway between *F* and *H*.
4. In order to locate the intermediate points on the parabola we use the principle that its distances from the focus and directrix must be equal. For example, to locate any point *P*, draw vertical line *QP* at any distance *x* from *F*. Measure *QH*. With *F* as the centre and *QH* as the radius, draw an arc to cut vertical line through *Q* in point *P*.
5. Join all these points to get the base parabola. However, some correction is to be made at the entry point. The phreatic line must start from *B* and not from *C*. Also, the phreatic line is a flow line, and must start perpendicularly to the *u/s* face *AB* which is a 100% equipotential line. Hence the

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portion of the phreatic line at B is sketched free hand in such a way that it starts perpendicularly to AB, and meets the rest of the parabola tangentially without any kink. The base parabola should also meet the d/s filter perpendicularly (*i.e.*, vertically) at G.

Analytical method. In order to find the equation of the base parabola, consider any point P on it, with co-ordinates (x, y) with respect to the focus F as origin.

From the property of the parabola, we have $PF = QH$

$$\therefore \quad \sqrt{x^2 + y^2} = QF + FH = x + s \text{ (where } s = FH = \text{focal distance)} \qquad ...(i)$$

From (i), $$x^2 + y^2 = x^2 + s^2 + 2xs \quad \text{or} \quad x = \frac{y^2 - s^2}{2s} \qquad ...(9.28(a))$$

and $$y^2 = 2xs + s^2 \qquad ...(9.28(b))$$

This is the equation of the parabola. Taking various values of x, the values of y can be calculated from equation 9.28 (b) and the parabola can be plotted. The distance s can either be determined graphically (Fig. 9.15 a) or can be calculated analytically by considering the co-ordinates of point C, as follows :

From (i), $$s = \left(\sqrt{x^2 + y^2} - x\right)$$

At C, $$x = D \quad \text{and} \quad y = H \text{ (Fig. 9.15 } b\text{)}$$

$$s = \left(\sqrt{D^2 + H^2} - D\right) \qquad ...(9.29)$$

Discharge through the body of the dam. In order to get an expression for the discharge q through the body of the dam for present case of horizontal filter, we observe that through the vertical section PQ

$$q = kiA = k \frac{dy}{dx} y \qquad ...(ii)$$

But from (ii), $$y = (2xs + s^2)^{1/2} \quad \therefore \quad \frac{dy}{dx} = \frac{s}{\left(2xs + s^2\right)^{1/2}} \qquad ...(iii)$$

Substituting in (iii), we get

$$q = k \frac{s}{\left(2xs + s^2\right)^{1/2}} \times \left(2xs + s^2\right)^{1/2} \quad \text{or} \quad q = ks \qquad ...(9.30)$$

This is a very simple expression for discharge q in terms of focal distance s.

9.11 PHREATIC LINE IN AN EARTH DAM WITH NO FILTER

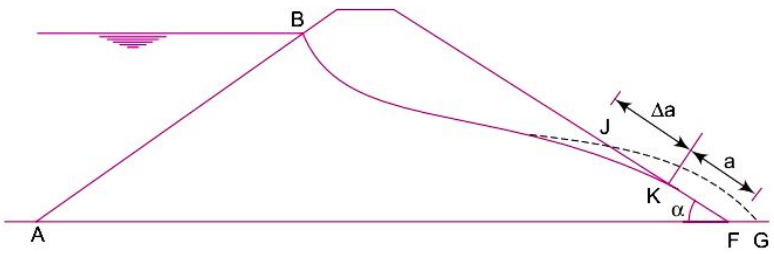

Fig. 9.16 Dam with no drainage filter

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Figure 9.16 shows a homogeneous earth dam with no horizontal drainage filter at the *d/s* side. The focus in this case will be lowest point *F* of the *d/s* slope, and the base parabola *BJG* will evidently cut the *d/s* slope at *J* and extend beyond the limit of the dam, as shown by dotted line. However, according to exit conditions

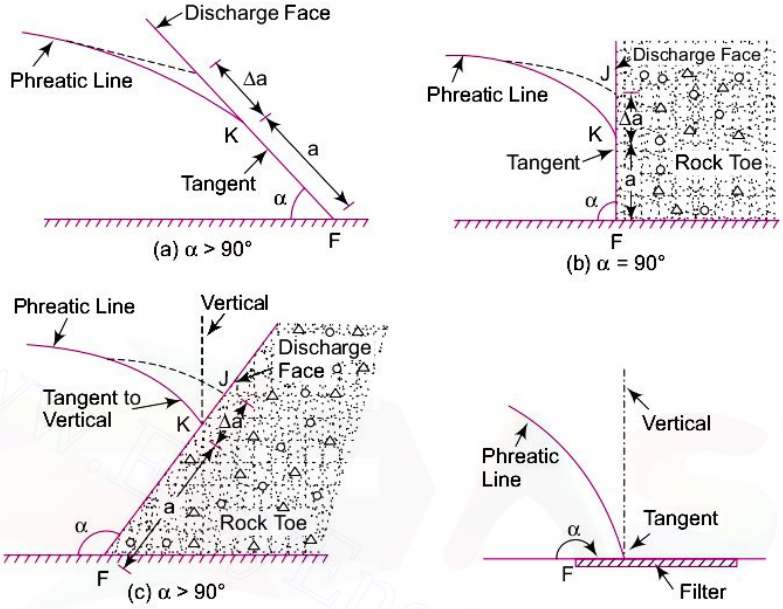
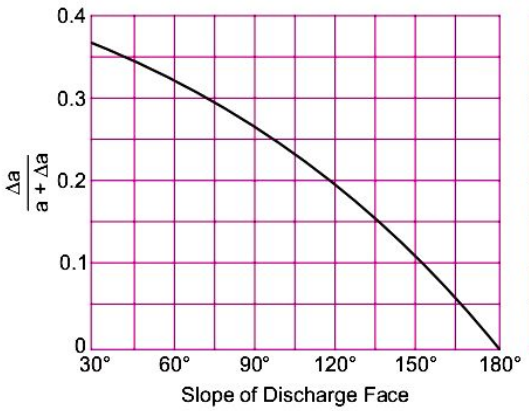


Fig. 9.17 Exit conditions for various slopes of the discharge face

(Fig. 9.17), the phreatic line must emerge out at some point *K* meeting the *d/s* face tangentially there. The portion *KF* is then known as the *discharge face*, and always remains wet. The correction Δa, by which the parabola is to be shifted downwards, is found by the values of $\frac{\Delta a}{a + \Delta a}$ given by Casagrande (Fig. 9.18) for various values of slope α of the discharge face. The slope α can even exceed the value of 90°, specially in the case of rockfill toe, shown in Fig. 9.17 (*c*).



α	$\frac{\Delta a}{a + \Delta a}$
30°	0.36
60°	0.32
90°	0.26
120°	0.18
135°	0.14
150°	0.1
180°	0

Fig. 9.18 Relation between α and $\frac{\Delta a}{a + \Delta a}$

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Thus, we observe that $\dfrac{\Delta a}{a+\Delta a}$ = value found from Fig. 9.18 ...(*i*)

and $a + \Delta a = JF$ = value found from Fig. 9.16 ...(*ii*)

Solving (*i*) and (*ii*), the value a of Δa can be found.

Analytical Solution of Schaffernak and Van Iterson for $\alpha < 30°$. In order to find the value of a analytically, Schaffernak and Van Iterson (1917) assumed that the energy gradient $i = \tan\alpha = dy/dx$. This means that the gradient is equal to the slope of the line of seepage, which is approximately true so long as the the slope is gentle (*i.e.*, $\alpha < 30°$).

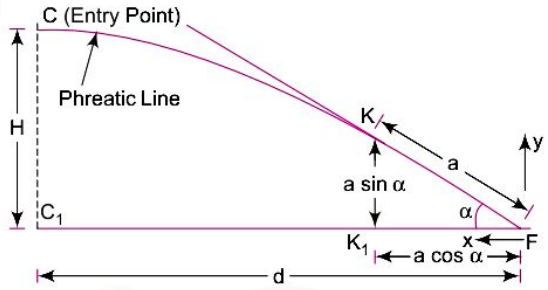

Fig. 9.19 Schaffernak's method

For vertical section KK_1,

$$q = k\frac{dy}{dx}y \qquad ...(i)$$

But $\dfrac{dy}{dx} = i = \tan\alpha$

and $y = KK_1 = a\sin\alpha$

Substituting in (*i*), we get $q = ka\sin\alpha\tan\alpha$ ...(9.31)

This is the expression for the discharge.

Again, $q = k\dfrac{dy}{dx}y = ka\sin\alpha\tan\alpha$

$\therefore$ $a\sin\alpha\tan\alpha\,dx = y\,dy$. Integrating between the limits $x = a\cos\alpha$ to $x = d$

and $y = a\sin\alpha$ to $y = H$, we get

$$a\sin\alpha\tan\alpha\int_{a\cos\alpha}^{d} dx = \int_{a\sin\alpha}^{H} y\,dy$$

$$\therefore\ a\sin\alpha\tan\alpha\,(d - a\cos\alpha) = \frac{H^2 - a^2\sin^2\alpha}{2}$$

From which, we obtain, after simplification

$$a = \frac{d}{\cos\alpha} - \sqrt{\frac{d^2}{\cos^2\alpha} - \frac{H^2}{\sin^2\alpha}} \qquad ...(9.32)$$

Analytical Solution of Leo Casagrande for $30° < \alpha < 60°$. It will be observed that the previous solution gives satisfactory results for slope < 30° . For steeper slopes, the deviation from correct values increases rapidly beyond tolerable limits. Leo Casagrande (1932) suggested the use of sin α instead of tan α.

Thus, $q = kiA = k\dfrac{dy}{ds}A$ ...(*i*)

At K, $s = a$ and $y = a\sin\alpha = A$; $\dfrac{dy}{ds} = \sin\alpha$

(where s = distance measured along the curve)

Substituting in (*i*), we get $q = k\sin\alpha\, a\sin\alpha = ka\sin^2\alpha$ ...(9.33)

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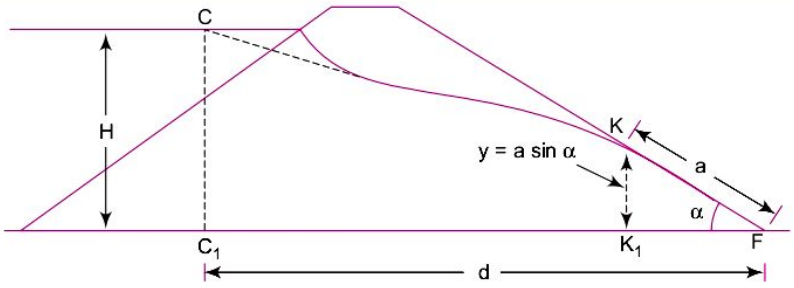

Fig. 9.20 Casagrande's method

which is the expression for discharge.

Again, $$q = k\frac{dy}{ds}\,y = ka\sin^2\alpha \quad \therefore \quad a\sin^2\alpha\,ds = y\,dy$$

Integrating between the limits $s = a$ to $s = S$ (where S = total length of the parabola) and $y = a\sin\alpha$ to $y = H$, we get

$$a\sin^2\alpha\int_a^S ds = \int_{a\sin\alpha}^{H} y\,dy$$

$$\therefore \quad a\sin^2\alpha\,(S-a) = \frac{H^2 - a^2\sin^2\alpha}{2} \quad \text{or} \quad a^2 - 2aS + \frac{H^2}{\sin^2\alpha} = 0$$

From which $$a = S - \sqrt{S^2 - \frac{H^2}{\sin^2\alpha}} \qquad ...(9.34)$$

Taking $S \approx (H^2 + d^2)^{1/2}$, we get

$$a = \sqrt{H^2 + d^2} - \sqrt{H^2 + d^2 - \frac{H^2}{\sin^2\alpha}} = \sqrt{H^2 + d^2} - \sqrt{d^2 - H^2\cot^2\alpha} \qquad ...(9.35)$$

Example 9.7. *For the earth dam of homogeneous section with a horizontal filter as shown in Fig. 9.21, draw the phreatic line. If the coefficient of permeability of the soil material used in the dam is 5×10^{-4} cm/sec, find the seepage flow per unit length of the dam.*

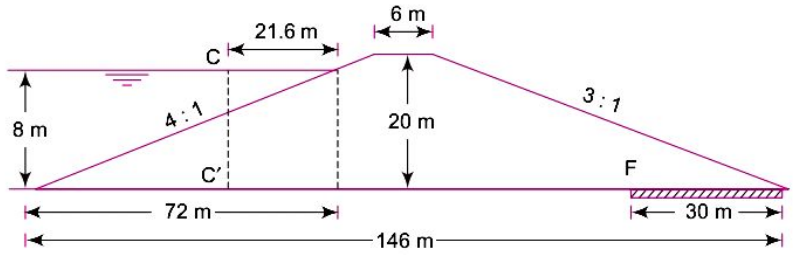

Fig. 9.21

Solution. Taking the focus as the origin, the equation of the parabola is

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$$\sqrt{x^2 + y^2} = x + s$$

The parabola cuts the reservoir water surface at C, such that

$$FC' = (146 - 30 - 72) + 21.6 = 65.6 \text{ m and } CC' = 18 \text{ m}$$

$$\therefore \quad s = \sqrt{(65.6)^2 + 18^2} - 65.6 = \mathbf{2.42\ m}$$

The equation of the parabola is, therefore,

$$y = \sqrt{2sx + s^2} = \sqrt{4.84\,x + 5.85}$$

The co-ordinates of the parabola are tabulated below:

x	0	10	20	30	40	50	60	65.6
y	2.42	7.36	10.11	12.30	14.1	15.7	17.25	18

The top flow line, therefore, can be drawn. A correction at the entry should be made as explained earlier.

Example 9.8. *For a homogeneous earth dam 52 m high and 2 m free board, a flow net was constructed and following results were obtained: Number of equipotential drops = 25; Number of flow channels = 4. The dam has a horizontal filter of 40 m length at its downstream end. Calculate the discharge per metre length of the dam if the co-efficient of permeability of the dam material is 3×10^{-3} cm/sec.*

Solution. The discharge per unit length is given by

$$q = kH \frac{N_f}{N_d} \text{ ; Here } H = \text{water depth} = 52 - 2 = 50 \text{ m}$$

$$k = 3 \times 10^{-3} \text{ cm/sec} = 3 \times 10^{-5} \text{ m/sec ; } N_f = 4 \text{ ; } N_d = 25$$

$$\therefore \quad q = 3 \times 10^{-5} \times 50 \times \frac{4}{25} = 24 \times 10^{-5} \text{ m}^3\text{/sec/m} = \mathbf{0.00024\ cumecs/metre\ length}.$$

9.12 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 9.9. *A 3 m thick soil stratum has coefficient of permeability of 3×10^{-7} m /sec. A separate test gave porosity 40% and bulk unit weight 21 kN/m^3 at a moisture content of 31%. Determine the head at which upward seepage will cause quick sand condition. What is the flow required to maintain critical condition?* **(Civil Services Exam. 1990)**

Solution.

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$$

$$G = \frac{\gamma_d}{\gamma_w}(1+e) = \frac{\gamma}{(1+w)\,\gamma_w}(1+e) = \frac{21}{(1+0.31)\,9.81} \times (1+0.667) = 2.724$$

$$i_c = \frac{G-1}{1+e} = \frac{2.724-1}{1+0.667} = 1.0342 = \frac{H_c}{L}$$

$$\therefore \quad H_c = i_c \cdot L = 1.0342 \times 3 = \mathbf{3.103\ m}$$

Now, $$Q_c = k\, i_c\, A = 3 \times 10^{-7} \times 1.0342\, A = 3.103 \times 10^{-7}\, A \text{ m}^3\text{/sec}$$

Flow = $\mathbf{3.103 \times 10^{-7}\ m^3/sec/m^2}$ area of soil stratum.

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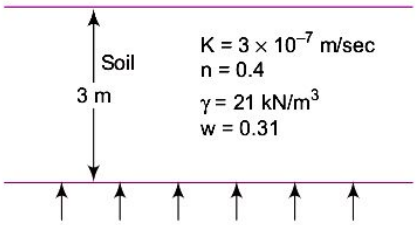

Fig. 9.22

Example 9.10. *Explain the phenomenon of quick sand. Determine the value of critical hydraulic gradient for a loose deposit of sand having void ratio of 0.67 and specific gravity of 2.67*

(Engg. Services Exam. 1991)

Solution. Refer Example 9.1. $i_c = \dfrac{G-1}{1+e} = \dfrac{2.67-1}{1+0.67} = 1.$

Example 9.11. *Calculate the seepage through an earth dam resting on an impervious foundation. The relevant data are given below : Height of a dam = 60.0 m*

Upstream slope = 2.75 : 1 (H : V); Downstream slope = 2.50 : 1 (H : V)

Free board =2.5 m; Crest width = 8.0 m ; Length of drainage blanket = 120.0 m

Co-efficient of permeability of the embankment material in

x-direction = 8×10^{-7} m/s; y-direction = 2×10^{-7} m/s. **(Engg. Services Exam. 1996)**

Solution. Discharges $q = k \cdot s$

Here $k = \sqrt{k_x \cdot k_y} = \sqrt{\left(8 \times 10^{-7}\right)\left(2 \times 10^{-7}\right)} = 4 \times 10^{-7}$ m/s.

s = focal distance $= \left(\sqrt{D^2 + H^2} - D\right)$

$H = 60 - 2.5 = 57.5$ m ; $L = 2.75 \times 57.5 = 158.13$ m

$0.3\,L = 0.3 \times 158.13 = 47.44$ m;

$D = 47.44 + (2.75 \times 2.5) + 8 + (2.5 \times 60) - 120 = 92.32$ m

$\therefore \quad s = \sqrt{(92.32)^2 + (57.5)^2} - 92.32 = 16.44$ m

$\therefore \quad q = 4 \times 10^{-7} \times 16.44 = \mathbf{65.77 \times 10^{-7}\ m^3/sec/m.}$

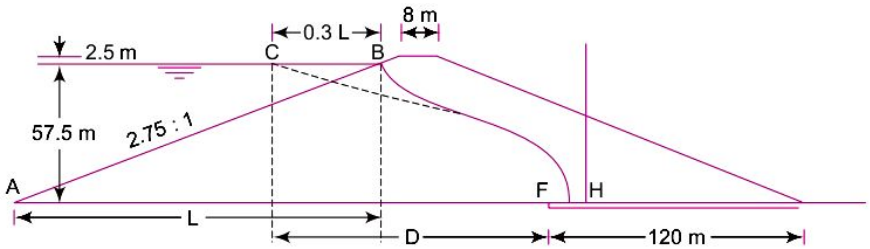

Fig. 9.23

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Example 9.12. *A homogeneous anisotropic earth dam, which is 20 m high is constructed on an impermeable foundation. The coefficients of permeability of soil used for the construction of the dam, in the horizontal and vertical direction are 4.8×10^{-8} m/s and 1.6×10^{-8} m/s respectively. The water level on the reservoir side is 18 m from the base of the dam: downstream side is dry. It is seen that there are 4 flow channels and 18 equipotential drops is a square flownet drawn in the transformed dam section. Estimate the quantity of seepage per unit length in m^3/s through the dam.* **(Gate Exam. 1994)**

Solution. $$q = k \cdot H \cdot \frac{N_f}{N_d}$$

Here, $$k = \sqrt{k_H \cdot k_v} = \sqrt{4.8 \times 1.6} \times 10^{-8} = 2.7713 \times 10^{-8} \text{ m/s}$$

$\therefore$ $$q = 2.7713 \times 10^{-8} \times 18 \times \frac{4}{18} = \mathbf{11.085 \times 10^{-8}\ m^3/s/m\ run.}$$

Example 9.13. *A layer of clay of thickness 12.5 m is underlain by sand. The saturated unit weight of the clay is 18.5 kN/m^3. When the depth of an open trench excavated in the clay reached a depth of 8 m, the bottom cracked and the water started entering the trench from below. Determine the height to which water would have risen from the top of sand in a bore hole if it were drilled into sand prior to the excavation. Take γ_w = 10 kN/m^3.* **(Gate Exam. 1995)**

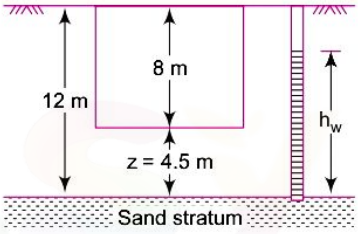

Fig. 9.24

Solution. This question is based on example 9.2.

At, A, $$\sigma' = z\,\gamma_{sat} - h_w\,\gamma_w = 0$$

$\therefore$ $$h_w = \frac{z\,\gamma_{sat}}{r_w} = \frac{(12.5 - 8) \times 18.5}{10} = \mathbf{8.325\ m.}$$

Example 9.14. *A 1.25 m layer of soil (n = 0.35; G = 2.65) is subjected to an upward seepage head of 1.85 m. What depth of coarse sand would be required above the existing soil to provide a factor of safety of 2 against piping? Assume that coarse sand has the same porosity and specific gravity as the soil and that there is negligible head loss in sand γ_w = 9.81 kN/m^3.* **(Gate Exam. 1999)**

Solution. Let the depth of coarse sand D m.

$\therefore$ $$e = \frac{n}{1-n} = \frac{0.35}{1-0.35} = 0.5385$$

$\therefore$ $$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \frac{2.65 + 0.5385}{1 + 0.5385} \times 9.81$$

$$= 20.331 \text{ kN/m}^3$$

$\therefore$ $$\gamma' = 20.331 - 9.81 = 10.52 \text{ kN/m}^3$$

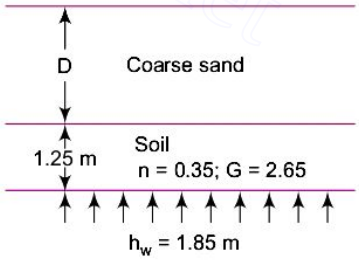

Fig. 9.25

Now upward force $$= h_w\,\gamma_w = 1.85 \times 9.81$$

$$= 18.1485 \text{ kN/m}^2 \quad \ldots (i)$$

Downward force $$= (D + 1.25)\,\gamma' = (D + 1.25) \times 10.521 \quad \ldots (ii)$$

In order to have a factor of safety of 2, the downward force should be equal to twice the upward force

$\therefore$ $$(D + 1.25) \times 10.521 = 2 \times 18.1485$$

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$$\therefore \quad D = \frac{2 \times 18.1485}{10.521} - 1.25 \approx \mathbf{2.2\ m}.$$

Example 9.15. *At given location, 8 m thick saturated clay (natural water content = 30%, G_s = 2.7) is undertain by sand. The sand layer is under artesian pressure equivalent to 3 m of water head. It is proposed to make an open excavation in the clay. How deep can this excavation be made before the bottom heaves ?* **(Civil Services Exam. 2000)**

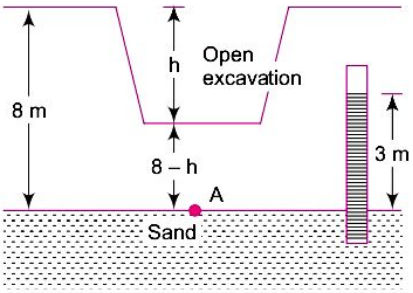

Fig. 9.26

Solution Refer Example 9.2.

Let h = depth of excavation

$\therefore$ Thickness of clay below excavation = $8 - h$

For stability at point A,

Upward hydrostatic pressure = effective pressure due to weight of clay

$$\therefore \quad 3\,\gamma_w = (8 - h)\,\gamma_{sat}$$

Now, $$\gamma_{sat} = \frac{\gamma_w (G + e)}{1 + e}$$

where $$e = w_{sat}\, G = 0.3 \times 2.7 = 0.81$$

$$\therefore \quad \gamma_{sat} = \frac{9.81 \times (2.7 + 0.81)}{1 + 0.81} = 19.024\ \text{kN/m}^3$$

$$\therefore \quad 3 \times 9.81 = (8 - h) \times 19.024 \text{ from which } h = \mathbf{6.453\ m.}$$

Example 9.16. *Given the flow situation shown in Fig. 9.27 below.*

(i) Is this a case of confined or unconfined flow?

(ii) What are the boundary condition for the flow situation depicted in the figure above.

(iii) The flow net drawn for this condition gives $n_f = 3$; $n_d = 30$. What is the quantity of flow per metre run occurring under the weir? Where n_f = No. of flow channels and n_d = number of equipotential drops.

(iv) The elementary square at the toe of the weir has dimensions of 0.6 m. What is the exit gradient?

(v) For what reservoir height would the exit gradient be equal to 1? What is the implication of condition to the safety of the weir ? **(Civil Engg. Services 1997)**

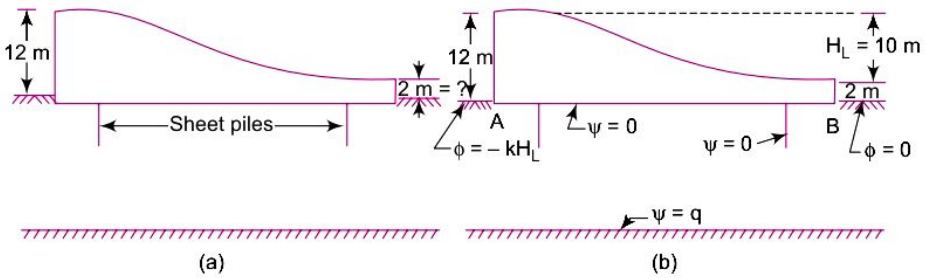

Fig. 9.27

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Solution

(*i*) This is a case of confined flow.

(*ii*) The boundaries are depicted in Fig. 9.27 (*a*)

(*iii*) $q = k\, H_L\, n_f/n_d$ where $k = 10^{-4}$ cm/s $= 10^{-6}$ m/sec ; $n_f = 3$; $n_d = 30$;

$$H_L = 12 - 2 = 10 \text{ m}$$

$$\therefore \quad q = 10^{-6} \times 10 \times 3/30 = \mathbf{1.0 \times 10^{-6}\ m^3/s/m\ width}$$

(*iv*) Size of elementary square $= b \times l = 0.6 \times 0.6 = 0.36\ m^2$

$$\Delta h = H_L/n_d = 10/30 = 1/3$$

$$\therefore \quad i_c = \Delta h/l = \frac{1}{3 \times 0.6} = \mathbf{0.556}$$

(*v*) Let the critical height be H_c

$$\therefore \quad i_c = 1 = \frac{H_c}{30} \times \frac{1}{l} = \frac{H_c}{30 \times 0.6}$$

$$\therefore \quad H_c = 30 \times 0.6 = \mathbf{18\ m}$$

$$\therefore \quad \text{Water depth} = 18 + 2 = \mathbf{20\ m}$$

PROBLEMS

1. A stratum, consisting of fine sand, is 2 metres thick. Under what head of water, flowing in an upward direction, will a quick condition develop? Take $G = 2.68$ and $e = 0.6$. [**Ans.** 2.1 m]
2. A foundation trench is to be excavated in a stratum of stiff clay, 8 metres thick, underlain by a bed of sand. In a trial bore hole, the ground water is observed to rise to an elevation 2 m below the ground surface. Find the depth to which the excavation can be safely carried out without the danger of the bottom becoming unstable under uplift pressure of ground water. The specific gravity of clay particles is 2.72 and voids ratio 0.72. If excavation is to be carried safely to a depth of 6 m, how much should the water be lowered in the vicinity of trench? [**Ans.** 5 m ; 2 m]
3. An experimental set up shown in Fig. 9.28 has a discharge of 20 mL per minute under a hydraulic head H cm. The coefficient of permeability for the sand is 0.02 cm/sec. Taking $G = 2.65$ and $\gamma_{sat} = 21$ kN/m^2 find,
   (*i*) Hydraulic head H
   (*ii*) Hydraulic gradients at sections A and B
   (*iii*) σ' at A and B under conditions of normal flow
   (*iv*) Critical hydraulic gradient at which quick sand will develop.
   [**Ans.** (*i*) $h_1 = 8.08$ cm, (*ii*) $i_A = 0.34$; $i_B = 0.85$, (*iii*) $\sigma_B' = 100$ N/m^2 ; $\sigma_A' = 860$ N/m^2, (*iv*) $i_c = 1.1$]
4. An earth dam is built on an impervious foundation with a horizontal filter under the d/s slope. The horizontal and vertical permeabilities of the soil material in the dam are respectively 4×10^{-3} and 1×10^{-3} cm/sec. The full reservoir level is 15 metres above the d/s filter. A flow net, constructed for the transformed section of the dam, consists of 4 flow channels and 15 equipotential drops. Estimate the seepage loss per metre length of the dam. [**Ans.** 80 mL/sec/m]

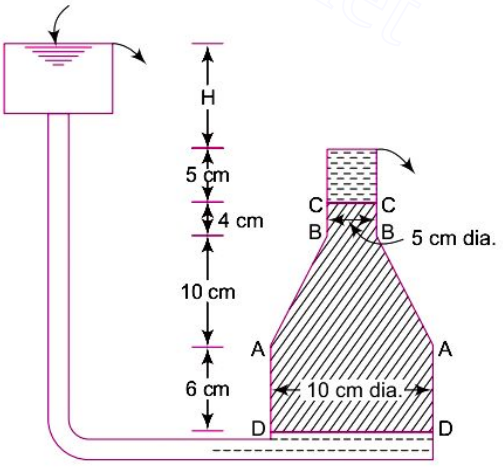

Fig. 9.28

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CHAPTER 10

Seepage Below Hydraulic Structures

10.1 INTRODUCTION

If a hydraulic structure is built on pervious soil, the higher water level at the upstream of the structure will cause seepage through the permeable soil. The design of the apron or the floor of the structure is intimately connected with the possibility of this seepage. Two sources of weakness are to be guarded against: (1) Percolation may cause undermining of the pervious, granular foundation which would necessarily be followed by the collapse of the whole structure, (2) the floor or the apron may be forced upwards, owing to the upward pressure of water filtering through the pervious soil under the structure. The failure of the old Manufia Regulator in Egypt was due to undermining, whereas the failure of Narora Weir in India was due to excessive water pressure (*uplift pressure*) causing the floor of the weir to be blown upwards (Leliavsky, 1965).

The law of flow of water through soil was first studied by Darcy (1856). The *hydraulic gradient theory* of weir design was developed as a result of experiments of Col. Clibborn and Beresford (1902). Later, in 1910, Bligh enunciated his *creep theory.* Bligh assumes as an approximation that the hydraulic slope or gradient is constant throughout the length of percolation. The safety of hydraulic structure depends on the length of the percolation path, generally known as the *creep length,* which is along the line of contact of the structure and its foundation. He stated that the length of path of flow has the same effectiveness, length for length, in reducing uplift pressure, whether it was along the horizontal or the vertical. Later, Lane (1932) gave his *weighted creep theory* according to which he recommended a weight of three to the vertical creep. However, both the theories are incorrect. The very experiments of Col. Clibborn which were supposed to form the basis of the creep theory, go to disprove it.

A general theory, and a large number of individual solutions of the conformal transformation problem as applied to weir-foundation design were published by Prof. N. N. Pavlovsky (1922, 33). He also evolved the electrical analogy method for the seepage problems. Khosla, Bose and Taylor (1936) solved the problems

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of seepage under the aprons founded on the pervious medium of infinite depth, both mathematically as well as experimentally with the help of hydraulic and electrical models. The charts given by them still form the basis of scientific design of weir aprons in most of the countries.

10.2 PIPING : EXIT GRADIENT

The undermining of the subsoil starts from the tail end of the work. It begins at the surface due to the residual forces of seepage water at this end being in excess of the restraining forces of the subsoil which tend to hold the latter in position. Once the surface is disturbed, the dislocation of subsoil particles works further down, and, if progressive, leads to the formation of cavities below the floor in which the latter may collapse. According to the commonly accepted ideas, this undermining is supposed to result from what is known as *piping,* that is, the erosion of the subsoil by the high velocities of flow of water through it when such velocities exceed a certain limit. But this concept of undermining is incomplete. Water has a certain residual force at each point along its flow through the sub-soil which acts in the direction of flow and is proportional to the pressure gradient at that point. At the tail end, this force is directed upward and will tend to lift up the soil particle if it is more than the submerged weight of the latter. Once the soil particles are disturbed, the resistance against upward pressure of water will be further reduced, tending to progressive disruption of the subsoil. The flow gathers into a series of pipes in the subsoil and dislocation of particles is accelerated. The subsoil is thus progressively undermined. This conception of undermining by 'foundation' was put forward by Prof. Terzaghi (1925) and independently arrived at by F. F. Haigh (1930). The pressure gradient at the exit at which the upward force is just equal to the submerged weight of the soil is called the *floatation gradient* (Terzaghi), or *bursting gradient* (Haigh) or the *critical gradient* (Khosla).

10.3 KHOSLA'S THEORY

Khosla and his associates took into account the flow pattern below the impermeable base of hydraulic structures, to calculate the *uplift pressure* and the gradient at the exit, called the *exit gradient.* Various cases were analysed mathematically. In order to appreciate the essential difference between Khosla's theory and Bligh's theory, we will start with a simple case of horizontal floor with negligible small thickness as shown in Fig. 10.1.

For the case of two dimensional flow occurring under a straight floor, it can be shown that the stream lines are confocal ellipses given by the equation:

$$\frac{x^2}{\left(\frac{b}{2}\cosh u\right)^2} + \frac{y^2}{\left(\frac{b}{2}\sinh u\right)^2} = 1 \qquad ...(10.1)$$

where u = stream line function given by

$$\cosh u = \frac{1}{2}\left(e^u + e^{-u}\right) \text{ and } \sinh u = \frac{1}{2}\left(e^u - e^{-u}\right) \qquad ...(10.2)$$

From equation 10.1, any particular stream line can be determined by giving a suitable value to u. To calculate the gradient diagram, we must consider the first stream line AB which hugs to the outline of the floor. Since the discharge between this line and the solid floor is zero, the stream line function $u = 0$,

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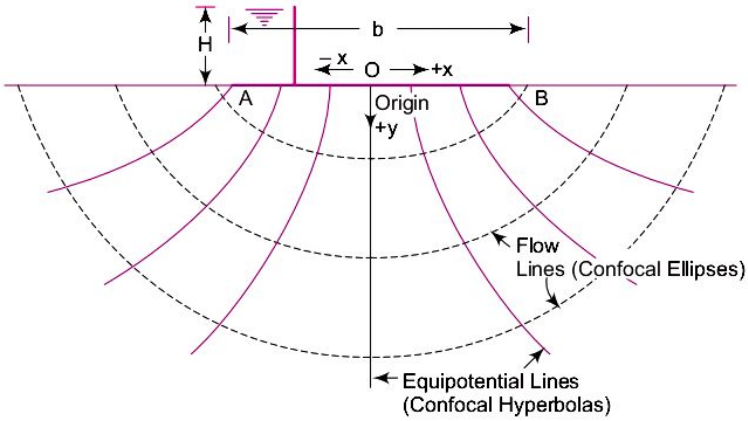

Fig. 10.1 Flow under horizontal floor

$$\cosh u = \cosh 0 = 1;\ \sinh u = \sinh 0 = 0$$

Substituting in equation 10.1, we get $\dfrac{x^2}{b^2/4} + \dfrac{y^2}{0} = 1$

which is true only when $y = 0$. This gives the equation of the straight line AB.

Again, the equipotential lines consist of a set of confocal hyperbolas given by

$$\frac{x^2}{\left(\frac{b}{2}\cos v\right)^2} - \frac{y^2}{\left(\frac{b}{2}\sin v\right)^2} = 1 \qquad ...(10.3)$$

where v = pressure function, given by $v = \pi\,\phi = \pi\,\dfrac{P}{H}$ ...(10.4)

where P = pressure head at any point ; H = total head causing seepage

In order to solve equations 10.1 and 10.3 for x for and y, let the complex co-ordinates z of any point be represented by

$$z = \frac{b}{2}\cosh w$$

where $z = x + iy$ and $w = u + iv$

$$\therefore \quad x + iy = \frac{b}{2}\cosh(u + iv)$$

$$= \frac{b}{2}\cosh u \cdot \cos v + i \sinh u \cdot \sin v$$

Separating real and imaginary parts, and equating them, we get

$$x = \frac{b}{2}\cosh u \cos v \quad \text{and}$$

$$y = \frac{b}{2}\sinh u \sin v \qquad ...(10.5)$$

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These values of x and y will be acceptable only if they satisfy both Eqs. 10.1 and 10.3. Thus, substituting these values of x and y in Eq. 10.1.

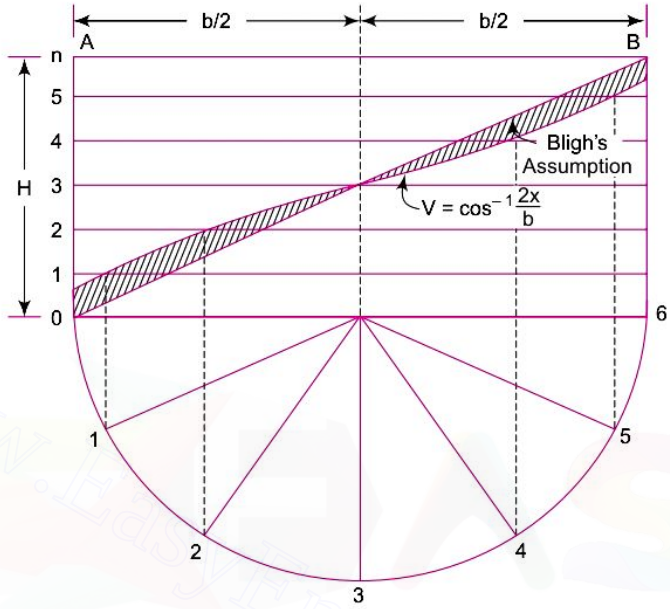

Fig. 10.2 Pressure distribution

We get, $\cos^2 v + \sin^2 v = 1$ (satisfied).

Similarly, from Eq. 10.3, $\cosh^2 u - \sinh^2 u = 1$ (satisfied)

Thus, the values of x and y are given by Eq. 10.5.

Hence $x = \frac{b}{2} \cosh u \cos v$. In the limiting case, for AB, $u = 0$

$$\therefore \quad x = \frac{b}{2} \cosh 0 \cdot \cos v = \frac{b}{2} \cos v \quad \text{or} \quad v = \cos^{-1}\left(\frac{2x}{b}\right) \qquad ...(10.6)$$

But $$v = \pi \phi = \pi \frac{P}{H} \quad \therefore \quad \phi = \frac{1}{\pi} \cos^{-1}\left(\frac{2x}{b}\right) \qquad ...(10.7)$$

or $$P = \frac{H}{\pi} \cos^{-1}\left(\frac{2x}{b}\right) \qquad ...(10.8)$$

Thus, for the first flow line AB, the pressure can be obtained by giving x different values in Eq. 10.8. Figure 10.2 shows the pressure distribution diagram both by Eq. 10.8 as well as by Bligh's method.

10.4 COMPOSITE PROFILES : SCHWARZ-CHRISTOFFEL TRANSFORMATION

Figure 10.3 (a) shows a composite profile, the most general case analysed by A. N. Khosla, Dr. N. K. Bose and Dr. E. M. Taylor (1936) by the method of Schwarz-Christoffel transformation. The flow lines and equipotential lines of the actual section are no longer ellipses and hyperbolas. By the mathematical process of transformation, the distorted flow net can be brought back to the normal elliptical and hyperbolic curves.

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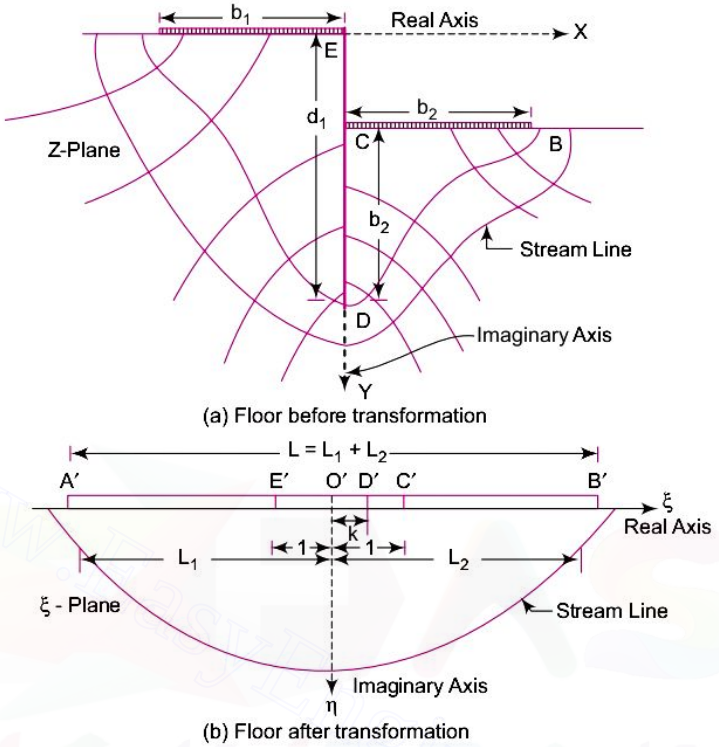

Fig. 10.3 Khosla's case of composite profile

If a polygon is located in the z-plane, then the Schwarz-Christoffel equation, that maps it *conformally* on to ζ, plane, is given by

$$z = A \int \frac{d\zeta}{(\zeta - \zeta_1)^{\lambda_1} (\zeta - \zeta_2)^{\lambda_2} (\zeta - \zeta_3)^{\lambda_3}} \quad ...(10.9)$$

where $\pi\lambda_1$, $\pi\lambda_2$, $\pi\lambda_3$ ... = changes in the angles of the polygon which are required to transform into a straight line.

$\zeta_1, \zeta_2, \zeta_3$ = co-ordinates of its vertices and A = complex constant.

Step 1. *Transformation Equation $z = f(\zeta)$*: Let us now apply this equation to transform the composite profile of Fig. 10.3 (a) into the straight line profile of Fig. 10.3 (b). The key points A, E, D, C and B of Fig. 10.3 (a) fall into points A', E', D', C', and B' respectively, in Fig. 10.3 (b). It will be seen that the angles at vertices E, D and C are respectively $\pi/2$, 2π and $\pi/2$, and these have been changed to π each at E', D', and C' on transformation. The Schwarz-Christoffel formula is:

$$z = A \int \frac{d\zeta}{(\zeta - \zeta_1)^{\lambda_1} (\zeta - \zeta_2)^{\lambda_2} (\zeta - \zeta_3)^{\lambda_3}} \quad ...(i)$$

where $\pi\lambda_1$ = change in the angle at $C = \pi - \pi/2 = + \pi/2$

$\pi\lambda_2$ = change in the angle at $D = \pi - 2\pi = - \pi$

$\pi\lambda_3$ = change in the angle at $E = \pi - \pi/2 = + \pi/2$

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Hence, $\lambda_1 = \frac{1}{2}$; $\lambda_2 = -1$ and $\lambda_3 = \frac{1}{2}$

Also, selecting the origin O midway between C' and E', OE' and OC' will be equal, and can be selected to be equal to unity. Let $OD = E$, where K is an absolute number.

Then ζ_1 = coordinate of $C = +1$; ζ_2 = coordinate of $D = K$

ζ_3 = coordinate of $E = -1$

Substituting these in (*i*), we get

$$z = A\int \frac{d\zeta}{(\zeta-1)^{1/2}(\zeta-K)^{-1}(\zeta+1)^{1/2}} = A\int \frac{(\zeta-K)\,d\zeta}{\sqrt{(\zeta^2-1)}} \qquad ...(ii)$$

Integrating the above, we get

$$z = A\sqrt{(\zeta^2-1)} - AK\log_e\left(\zeta+\sqrt{(\zeta^2-1)}\right) + B \qquad ...(iii)$$

Step 2. ***Determination of constants A and B*:** In order to find the constant B, let us compare the co-ordinates of point C and C' in the two planes. Evidently, the z co-ordinates of $C = i(d_1 - d_2)$ while that of C', $\zeta = 1$

Substituting these in (*ii*), we get $B = i(d_1 - d_2)$

Similarly, the coordinates of E and E' are $z = 0$ and $\zeta, = -1$.

Substituting these in (*iii*), we get

$$0 = B - AK\log_e(-1)$$

or $$AK\pi i = B = i(d_1 - d_2) \qquad [\text{since } \log_e(-1) = \pi i]$$

$$\therefore \quad AK = \frac{d_1 - d_2}{\pi}. \qquad ...(iv)$$

Step 3. ***Determination of parameter K:*** Comparing the coordinates of point D and D', we have $z = id_1$ and $\zeta = K$ Substituting these in (*iii*), we get

$$id_1 = A\sqrt{K^2-1} - AK\log_e\left(K+\sqrt{K^2-1}\right) + i(d_1 - d_2)$$

or $$id_1 = Ai\sqrt{1-K^2} - AK\log_e\left(K + i\sqrt{1-K^2}\right) + i(d_1 - d_2)$$

Putting $K = \cos\theta$, we get

$$id_1 = Ai\sin\theta - AK\log_e(\cos\theta + i\sin\theta) + i(d_1 - d_2)$$

or $$id_1 = Ai\sin\theta - AK\,i\theta + i(d_1 - d_2) \quad \therefore\ d_2 = A\sin\theta - AK\theta$$

Substituting the values of A and AK from (*iv*), we get

or $$d_2 = \frac{d_1 - d_2}{\pi K}\sin\theta - \frac{d_1 - d_2}{\pi}\theta = \frac{d_1 - d_2}{\pi}\tan\theta - \frac{d_1 - d_2}{\pi}\theta$$

Hence, $$\frac{d_2}{d_1 - d_2}\pi = \tan\theta - \theta$$

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or $$\pi\,\delta = \tan\theta - \theta = \frac{\sqrt{1-K^2}}{K} - \cos^{-1} K \quad \left(\text{where } \delta = \frac{d_2}{d_1 - d_2}\right) \qquad ...(v)$$

Thus, from (v) the values of K for various values of δ can be determined.

Step 4. ***Final transformation equation:*** Substituting the values of A and AK in *(iii)*, we get the final transformation formula.

$$z = \frac{d_1 - d_2}{\pi K}\sqrt{\zeta^2 - 1} - \frac{d_1 - d_2}{\pi}\log_e\left(\zeta + \sqrt{\zeta^2 - 1}\right) + i(d_1 - d_2) \qquad ...(10.10)$$

where K is now a known parameter.

Step 5. ***Determination of co-ordinates A′ and B′:*** Equation 10.10 is applicable to all points on the profile of the floor. Let the ζ, coordinates of points A' and B' be $-L_1$ and $+L_2$ respectively. The z-coordinates of the corresponding points A and B are $-b_1$ and $\{b_2 + i(d_1 - d_2)\}$ respectively. Substituting these coordinates, separately in Eq. 10.10,

we get $$-b_1 = \frac{d_1 - d_2}{\pi K}\sqrt{L_1^2 - 1} - \frac{d_1 - d_2}{\pi}\log_e\left(-L_1 + \sqrt{L_1^2 - 1}\right) + i(d_1 - d_2)$$

or $$-b_1 = \frac{d_1 - d_2}{\pi K}\sqrt{L_1^2 - 1} - \frac{d_1 - d_2}{\pi}\log_e\left(L_1^2 - \sqrt{L_1^2 - 1}\right) \qquad ...(vi)$$

and $$b_2 + i(d_1 - d_2) = \frac{d_1 - d_2}{\pi K}\sqrt{L_2^2 - 1} - \frac{d_1 - d_2}{\pi}\log_e\left(L_2 + \sqrt{L_2^2 - 1}\right) + i(d_1 - d_2)$$

or $$b_2 = \frac{d_1 - d_2}{\pi K}\sqrt{L_2^2 - 1} - \frac{d_1 - d_2}{\pi}\log_e\left(L_2 + \sqrt{L_2^2 - 1}\right) \qquad ...(vii)$$

Assuming $L_1 = \cosh\gamma_1$ and $L_2 = \cosh\gamma_2$, and substituting these in the above, we get

$$-b_1 = \frac{d_1 - d_2}{\pi K}\sinh\gamma_1 + \frac{d_1 - d_2}{\pi}\gamma_1 \text{ or } \sinh\gamma_1 + K\gamma_1 = \frac{\pi K b_1}{d_1 - d_2}$$

Similarly, $$\sinh\gamma_2 - K\gamma_2 = \frac{\pi K b_2}{d_1 - d_2}$$

Again, let $\delta_1 = \dfrac{b_1}{d_1 - d_2}$ and $\delta_2 = \dfrac{b_2}{d_1 - d_2}$. Substituting in the above, we get

$$\sinh\gamma_1 + K\gamma_1 = -\pi K\delta_1 \quad ...(viii) \quad \text{and} \quad \sinh\gamma_2 - K\gamma_2 = \pi K\delta_2 \qquad ...(ix)$$

Thus, from Eqs. *(viii)* and *(ix)*, γ_1 and γ_2 can be calculated. Hence $L_1 = \cosh\gamma_1$ and $L_2 = \cosh\gamma_2$ can be known. Thus, corresponding to given set of values of floor dimensions b_1 b_2, d_1 and d_2 we can first get K, and then L_1 and L_2.

Step 6. ***Pressure distribution below the floor:*** Now, in order get the pressure distribution under the floor, let us consider Eq. 10.10. In § 10.3, we have seen that for the case of a horizontal floor (Figs 10.1 and 10.2)

$$x + iy = z = \frac{b}{2}\cosh w = \frac{b}{2}\cosh(u + iv)$$

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Applying this to the *transformed floor* of Fig. 10.3 (*b*), where $b/2$ = half the width of foundation = $\frac{1}{2}(L_1 + L_2)$ and $z = \zeta + \frac{1}{2}(L_1 - L_2)$, we get

$$\zeta + \frac{1}{2}(L_1 - L_2) = \frac{1}{2}(L_1 + L_2)\cosh w$$

Putting $\frac{L_1 + L_2}{2} = \lambda$ and $\frac{L_1 - L_2}{2} = \lambda_1$, we get

$$\zeta = \lambda \cosh w - \lambda_1 \qquad ...(10.11)$$

Substituting this in Eq. 10.11, we get a general equation for the flow net diagram as under;

$$z = \frac{d_1 - d_2}{\pi K}\sqrt{(\lambda \cosh w - \lambda_1)^2 - 1}$$

$$- \frac{d_1 - d_2}{\pi}\log_e\left\{(\lambda \cosh w - \lambda_1) + \sqrt{(\lambda \cosh w - \lambda_1)^2 - 1}\right\} + i(d_1 - d_2)$$

For the first stream line which hugs to base, we have $u = 0$;

Hence $w = u + iv = iv$. Also $z = x + iy$.

Hence,
$$x + iy = \frac{d_1 - d_2}{\pi K}\sqrt{(\lambda \cos v - \lambda_1)^2 - 1}$$

$$- \frac{d_1 - d_2}{\pi}\log_e\left\{(\lambda \cos v - \lambda_1) + \sqrt{(\lambda \cos v - \lambda_1)^2 - 1}\right\} + i(d_1 - d_2) \qquad ...(10.12)$$

From Eq. 10.12, we can get the pressure function v for any point whose coordinate is $(x + iy)$. We shall apply this equation to different regions of the floor of Fig. 10.3 (*a*).

Step 7. ***Pressure distribution for length AE:*** In this length, $y = 0$. Also, introducing a new variable β'_1 defined by the equation,

$$\lambda \cos v - \lambda_1 = -\cosh \beta'_1 \qquad ...(x)$$

We get, from Eq. 10.12

$$x = \frac{d_1 - d_2}{\pi K}\sinh \beta'_1 - \frac{d_1 - d_2}{\pi}\log_e\left[-\cosh \beta'_1 + \sinh \beta'_1\right] + i(d_1 + d_2)$$

or
$$x = \frac{d_1 - d_2}{\pi K}\sinh \beta'_1 + \frac{d_1 - d_2}{\pi}\beta'_1 - \frac{d_1 - d_2}{\pi}\log_e(-1) + i(d_1 - d_2)$$

$$x = \frac{d_1 - d_2}{\pi K}\sinh \beta'_1 + \frac{d_1 - d_2}{\pi}\beta'_1$$

Hence,
$$\frac{x}{d_1 - d_2}\pi K = \delta_x \pi K = \sinh \beta_1' + K\beta_1' \qquad ...(xi)$$

From this equation, β_1' can be calculated. Hence from Eq. (*x*), the pressure function v is found to be

$$v = \cos^{-1}\left(\frac{\lambda_1 - \cosh \beta'_1}{\lambda}\right) = \pi \frac{P}{H}$$

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$$\therefore \quad P = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1 - \cosh\beta_1'}{\lambda}\right) \qquad ...(10.13a)$$

or
$$\phi = \frac{P}{H} = \frac{1}{\pi}\cos^{-1}\left(\frac{\lambda_1 - \cosh\beta_1'}{\lambda}\right) \qquad ...(10.13b)$$

This gives pressure distribution under *AE*.

Step 8. ***Pressure distribution for length CB*:** Introducing a new variable β'_2 defined by the equation $\lambda \cos v - \lambda_1 = \cosh \beta'_2$, we get from equation 10.12,

$$\sin\beta_2' - K\beta_2' = \pi K\frac{x}{d_1 - d_2} = \delta_x \pi K \qquad ...(xii)$$

From this equation, β'_2 can be known. Hence the pressure distribution is given by

$$v = \cos^{-1}\left(\frac{\lambda_1 + \cosh\beta_2'}{\lambda}\right) = \pi\frac{P}{H}$$

Hence,
$$P = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1 + \cosh\beta_2'}{\lambda}\right) \qquad ...(10.14a)$$

or
$$\phi = \frac{1}{\pi}\cos^{-1}\left(\frac{\lambda_1 + \cosh\beta_2'}{\lambda}\right) \qquad ...(10.14b)$$

Combining equations 10.13 and 10.14 in one common form, we get

$$P = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1 \mp \cosh\beta_{1,2}'}{\lambda}\right) \qquad ...(10.15)$$

where – sign is used for upstream and + sign for the downstream part of the floor.

Step 9. ***Pressure distribution along ED and DC*:** For the pressure distribution along the *u/s* and *d/s* faces of the pile, we have $x = 0$. Hence from Eq. (10.12),

$$iy = \frac{d_1 - d_2}{\pi K}\sqrt{(\lambda\cos v - \lambda_1)^2 - 1} - \frac{d_1 - d_2}{\pi}\log_e$$
$$[\lambda\cos v - \lambda_1) + \sqrt{(\lambda\cos v - \lambda_1)^2 - 1}] + i(d_1 - d_2)$$

Introducing a new variable β defined by the equation $\lambda \cos v - \lambda_1 = \cos \beta$

we get
$$iy = \frac{d_1 - d_2}{\pi K} i\sin\beta - \frac{d_1 - d_2}{\pi} i\beta + i(d_1 - d_2)$$

or $\left(\frac{y_1}{d_1 - d_2} - 1\right)\pi K = \sin\beta - K\beta$, from which β can be calculated.

Hence,
$$v = \cos^{-1}\left(\frac{\lambda_1 + \cos\beta}{\lambda}\right) = \pi\frac{P}{H} \quad \therefore \quad P = \frac{H}{\pi}\cos^{-1}\left(\frac{\lambda_1 + \cos\beta}{\lambda}\right) \qquad ...(10.16)$$

Step 10. ***Pressure at key points***

(i) Pressure at E

For $E'A'$, $\zeta = \lambda\cos v - \lambda_1$ (From equation 10.11) ; At E', $\zeta = -1$

Hence, $-1 = \lambda\cos v - \lambda_1$

or
$$v = \pi\frac{P}{H} = \cos^{-1}\frac{\lambda_1 - 1}{\lambda} \text{ or } P_E = \frac{H}{\pi}\cos^{-1}\frac{\lambda_1 - 1}{\lambda} \qquad ...(10.17)$$

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(ii) Pressure at C

For $C'B'\zeta = \lambda \cos v - \lambda_1$ (From Eq. 10.11) At $C', \zeta = +1$

$$\therefore \quad +1 = \lambda \cos v - \lambda_1 \quad \text{or} \quad v = \pi \frac{P}{H} = \cos^{-1} \frac{\lambda_1 + 1}{\lambda}$$

$$\therefore \quad P_C = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 + 1}{\lambda} \qquad ...(10.18)$$

(iii) Pressure at D

At point D', $\zeta = \lambda \cos v - \lambda_1 = K$

$$\therefore \quad P_D = \frac{H}{\pi} \cos^{-1} \frac{\lambda_1 + K}{\lambda}. \qquad ...(10.19)$$

Specific Cases

We shall now consider the following specific cases of the general form shown in Fig. 10.3.

(*i*) Straight horizontal floor of negligible thickness with pile either at the u/s end or the d/s end (Fig. 10.4 *a,b*).

(*ii*) Straight horizontal floor of negligible thickness with piles at some intermediate point (Fig. 10.4*c*).

These cases were derived from the general case discussed above. Expressions for the pressure at key-point *E, D* and *C* were found by substituting the values of *z*-co-ordinates of these points in the general equation for the pressure distribution. Figure 10.5 gives values of pressure at the key points *C, D* and *E,* for the case of a floor with pile at some intermediate point. Figure 10.6 gives the pressure at the key points and the values of exit gradient when the pile is at the *d/s* end of the floor. The formulae for the pressure at these points are given on the figures. The exit gradient for the case of pile at *d/s* end is given by

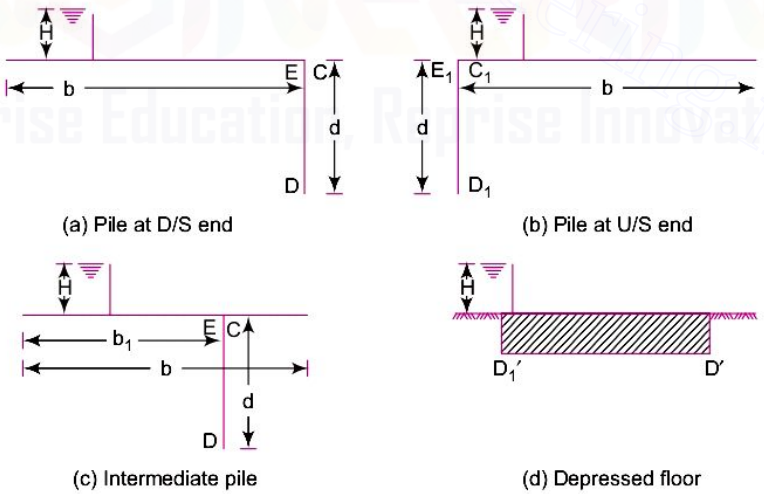

Fig. 10.4 Specific cases

$$G_E = \frac{H}{d} \frac{1}{\pi\sqrt{\lambda}} \quad \left(\text{where } \lambda = \frac{1 + \sqrt{1 + \alpha^2}}{2}\right) \qquad ...(10.20)$$

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$$\alpha = \frac{b}{d}\ ;\ b = \text{length of impervious floor}\ ;\ d = \text{depth of d/s sheet pile.}$$

The method of reading the graphs is explained on the figure itself. For example, let us take the case of a horizontal floor of length 20 m, with a pile of 4 m at the d/s end.

Then $$\frac{1}{\alpha} = \frac{d}{b} = \frac{4}{20} = \frac{1}{5} = 0.2$$

Hence from curves of Fig. 10.6, when $\frac{1}{\alpha} = 0.2$

we get $\phi_D = 27\%$ or $P_D = 0.27\,H$ and $\phi_E \approx 40\%$ or $P_E = 0.40\,H$

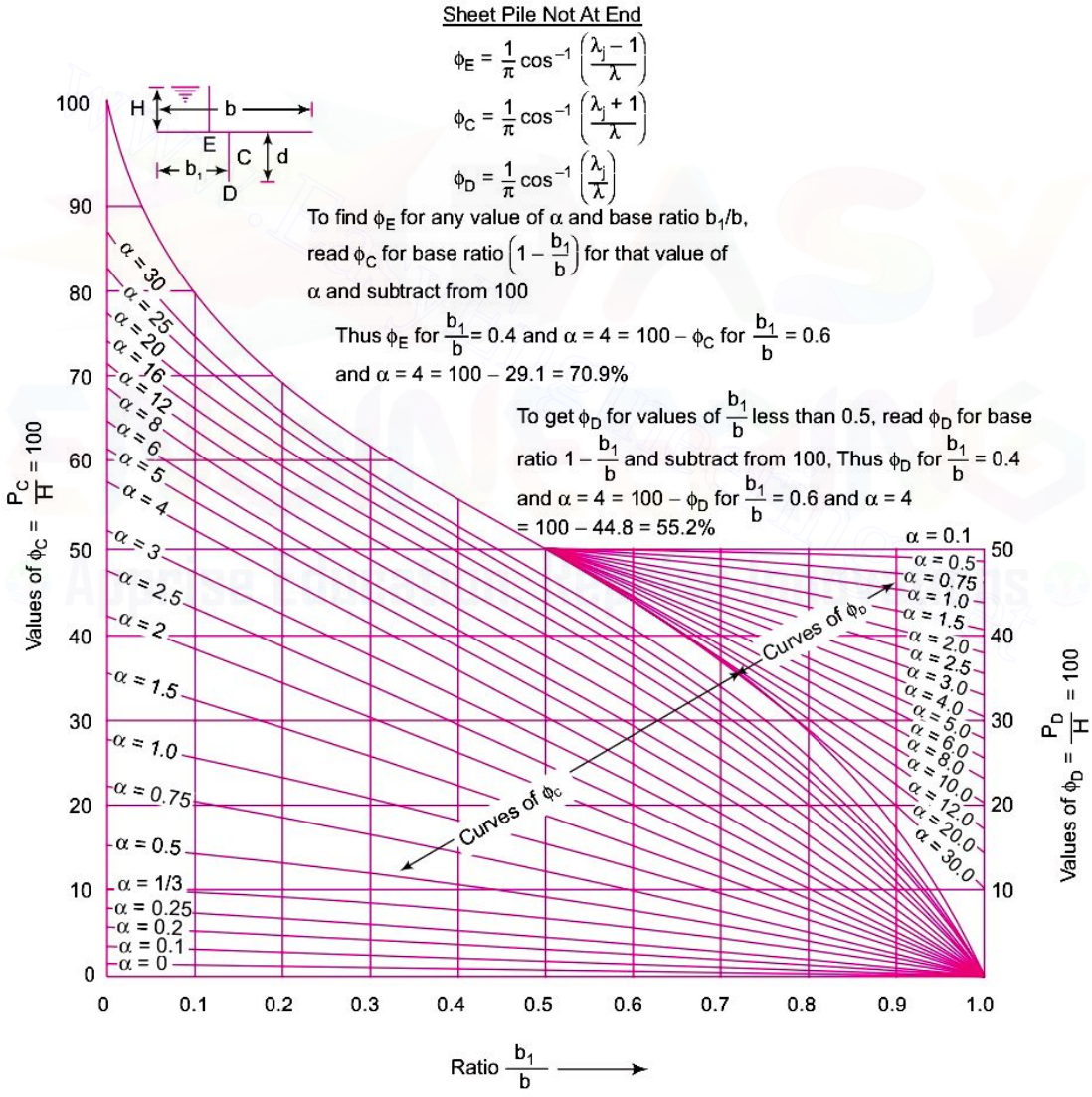

Fig. 10.5 Khosla curves

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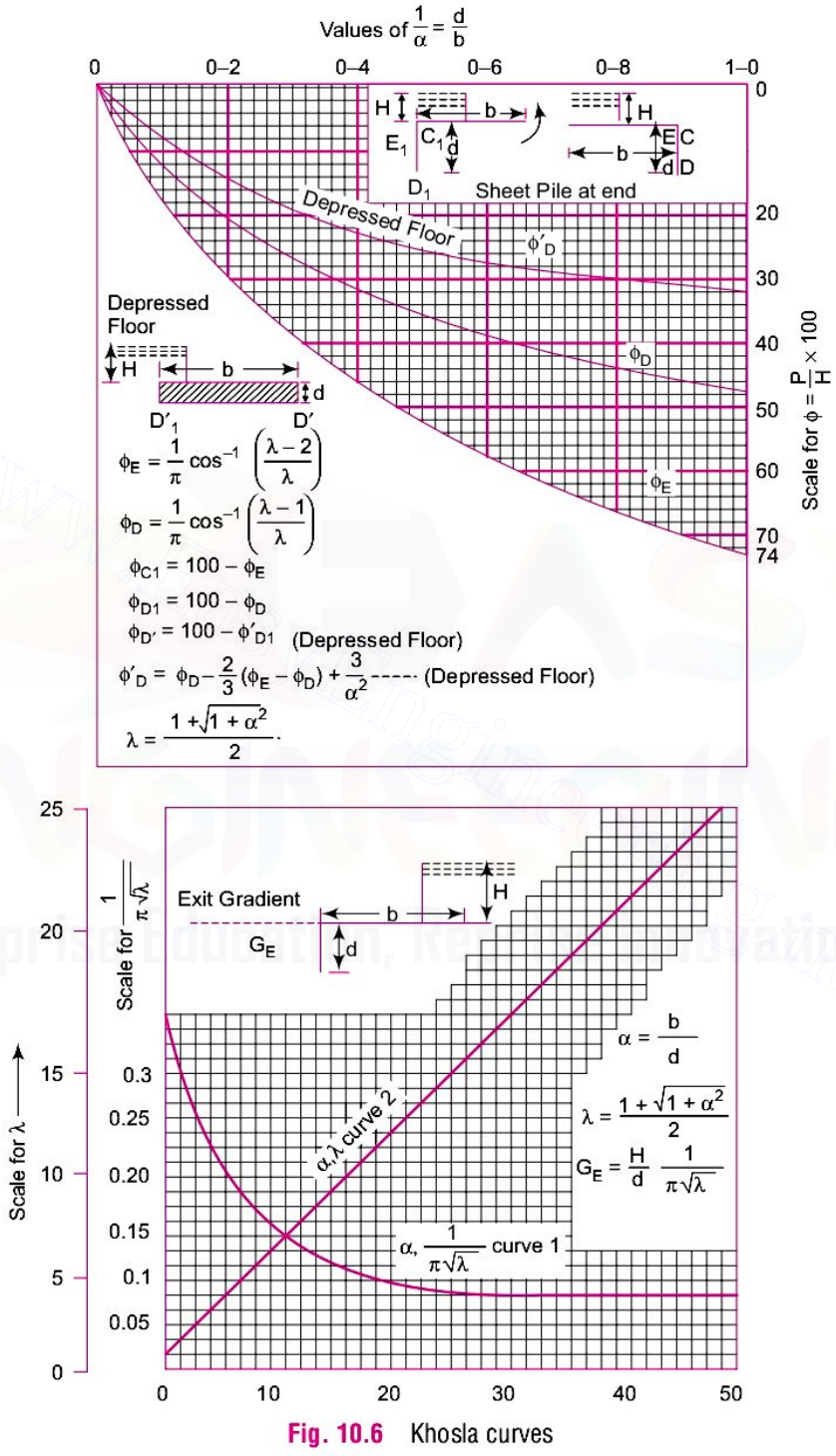

Fig. 10.6 Khosla curves

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If, however, the pile is provided at the upstream end (and not at the downstream), the pressure at the key points C_1 and D_1 can be calculated by first finding ϕ_D and ϕ_E (assuming as if the pile is at the d/s end) and then by the relation:

$$\phi_{C1} = 100 - \phi_E ; \quad \phi_{D1} = 100 - \phi_D$$

These relations are obtained by considering the principle of reversibility of flow. For the present case, when $\phi_D = 27\%$ and $\phi_E = 40\%$, we get

$$\phi_{C1} = 100 - \phi_E = 100 - 40 = 60\% \text{ and } \phi_{E1} = 100 - \phi_D = 100 - 27 = 73\%$$

To calculate the exit gradient for the case of pile at the d/s end, let us take $b = 20$ m, $d = 4$ m and $H = 5$ m.

$$\therefore \quad \alpha = \frac{b}{d} = \frac{20}{4} = 5$$

From the exit gradient curve, for $\alpha = 5$, we get $\dfrac{1}{\pi\sqrt{\lambda}} = 0.185$

$$\therefore \quad G_E = \frac{H}{d}\frac{1}{\pi\sqrt{\lambda}} = \frac{5}{4} \times 0.185 = \mathbf{0.231}$$

For alluvial soils, the critical hydraulic gradient may be approximately equal to 1. The permissible exit gradient can be found by adopting a suitable factor of safety of 6 to 7. The permissible exit gradients of three soils are given below:

Type of soil	*Exit gradient* (G_E)
1. Fine sand	$\frac{1}{6}$ to $\frac{1}{7}$
2. Coarse sand	$\frac{1}{5}$ to $\frac{1}{6}$
3. Shingle	$\frac{1}{4}$ to $\frac{1}{5}$

10.5 PAVLOVSKY'S METHOD : FINITE DEPTH PROBLEMS

Pavlovsky analysed various cases of seepage under the hydraulic structures in 1922, but as the text was in Russian, his work remained almost unknown. Apart from a purely mathematical analysis, Pavlovsky's investigations also comprised model-tank tests as well as the electrical analogy models. His analysis includes the problems of seepage under the aprons founded both finite as well as infinite depth. The transformations are performed into two operations : (*i*) in the first operation the field of the true profile (z-plane) is transformed on to the auxiliary semi-infinite plane (ζ-plane), and (*ii*) in the second operation, a rectangular field of known flow characteristics (Z-plane) is transformed on the same semi-infinite plane (ζ-plane). Thus, from the first operation, we get a transformation equation between z and ζ, while in the second operation, we get a transformation equation between Z and ζ. Finally, combining the two operations, we get a transformation equation between z and Z, which is the main purpose of the two stage transformation.

To understand the procedure, we will take first the case of a horizontal floor founded on a pervious medium of *finite depth*. Figure 10.7 (*a*) shows the actual profile in the z-plane, while Fig. 10.7 (*b*) shows the transformation of ζ-plane. Figure 10.7 (*c*) shows a known rectangular field in Z-plane.

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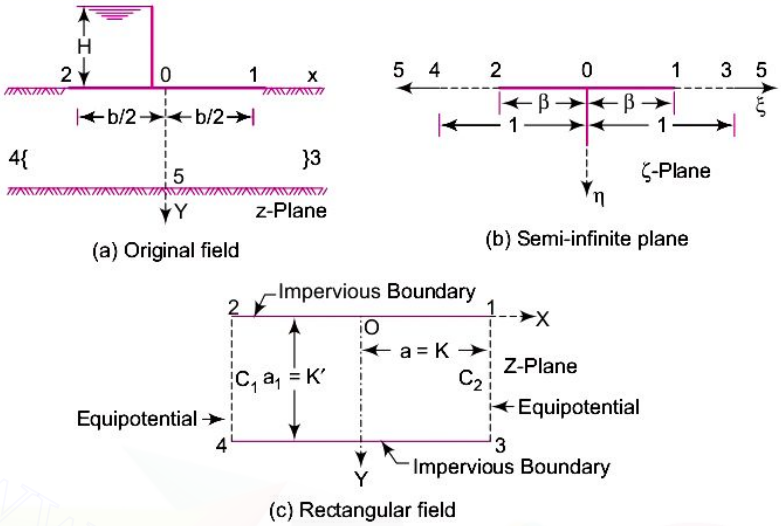

Fig. 10.7 Pavlovsky's transformations

In the first operation, the original field is transformed to the semi-infinite plane by Schwarz-Christoffel method. The original field is cut at point 5 and is opended out. Points 3 and 4 correspond to infinite distance. Comparing various points of Fig. 10.7 (*a*) and (*b*), it will be seen that there is no change in the angles at vertices 0, 1 and 2. The Schwarz-Christoffel equation therefore becomes

$$z = A_1 \int_0^{\zeta} \frac{d\zeta}{(\zeta - \zeta_3)^{\lambda_3} (\zeta - \zeta_4)^{\lambda_4}} \qquad ...(i)$$

The co-ordinates of corresponding pairs of points are chosen as follows :

$z_0 = 0$ and $\zeta_0 = 0$; $z_3 = +\infty$ and $\zeta_3 = 1$

$z_4 = -\infty$ and $\zeta_4 = -1$; $z_1 = +b/2$ and $\zeta_1 = +\beta$

$z_2 = -b/2$ and $\zeta_2 = -\beta$; $\therefore$ $\zeta - \zeta_3 = \zeta - 1$ and $\zeta - \zeta_4 = \zeta + 1$

Also, $\pi\lambda_3 = +\pi$ (since the angle subtended by points 1 and 5 at point 3 in z-plane is zero while that in the ζ-plane is π)

$\therefore$ $\lambda_3 = +1$; and $\pi\lambda_4 = \pi$ $\therefore$ $\lambda_4 = +1$

Substituting these in (*i*), we get $z = A_1 \int_0^{\zeta} \frac{d\zeta}{(\zeta - 1)(\zeta + 1)} = A \int_0^{\zeta} \frac{d\zeta}{1 - \zeta^2}$ ...(*ii*)

Integrating the above along the real axis,

$$z = A \int_0^{\infty} \frac{d\zeta}{1 - \zeta^2} = \frac{A\pi i}{2}$$

The right hand side contains no real value, and hence the corresponding point for which the above value is applicable must be situated on the imaginary axis in the z-plane. Evidently, this is applicable for point 5 for which $z_5 = iD$

$\therefore$ $iD = \frac{A\pi i}{2}$; From which $A = \frac{2D}{\pi}$

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Hence, $$z = \frac{2D}{\pi}\int_0^{\zeta}\frac{d\zeta}{1-\zeta^2} = \frac{2D}{\pi}\tanh^{-1}\zeta \qquad ...(iii)$$

Selecting $D = \frac{\pi}{2}$ = *main dimension*, we get the transformation equation between z and ζ-planes as follows:

$$z = \tanh^{-1}\zeta \text{ and } \zeta = \tanh z \qquad ...(10.21)$$

In the *second operation,* the rectangular field is transformed on to the semi-infinite plane, by the Schwarz-Christoffel equation,

$$Z = A\int_0^{\lambda}\frac{d\zeta}{(\zeta-\zeta_1)^{\lambda_1}(\zeta-\zeta_2)^{\lambda_2}(\zeta-\zeta_3)^{\lambda_3}(\zeta-\zeta_4)^{\lambda_4}} \qquad ...(iv)$$

The co-ordinates of the corresponding pairs of the points in the two planes are as follows :

$Z_1 = a$ and $\zeta_1 = \beta$; $Z_2 = -a$ and $\zeta_2 = -\beta$

$Z_3 = a + a_1 i$ and $\zeta_3 = 1$; $Z_4 = -a + a_1 i$ and $\zeta_4 = -1$

$$\pi\lambda_1 = \pi\lambda_2 = \pi\lambda_3 = \pi\lambda_4 = +\frac{\pi}{2}$$

(Since angles at all the points were equal to $\pi/2$ before transformation, while they were increased to π after the transformation)

Hence, $$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \frac{1}{2}$$

Substituting these in (*iv*), we get

$$Z = A\int_0^{\zeta}\frac{d\zeta}{(\zeta-\beta)^{1/2}(\zeta+\beta)^{1/2}(\zeta-1)^{1/2}(\zeta+1)^{1/2}} = A\int_0^{\zeta}\frac{d\zeta}{\sqrt{(\beta^2-\zeta^2)(1-\zeta^2)}} \qquad ...(v)$$

Let $\zeta = \beta \cdot \tau$, where τ is a new variable.

Assuming $A = 1$, we get

$$Z = \int_0^{\tau}\frac{d\tau}{\sqrt{(1-\tau^2)(1-\beta^2\tau^2)}} = F(\tau, m) = \text{Sn}^{-1}(\tau, m) \qquad ...(10.22)$$

where F = elliptic integral of the first kind ; m = modulus = $\beta \tanh b$

$\text{Sn}^{-1}(\tau, m)$ = function inverse to Jacobi's 'Sinus amplitudinis'.

Hence the transformation equation between Z and ζ planes are :

$$Z = \text{Sn}^{-1}\left(\frac{\zeta}{m}, m\right) \text{ and } \zeta = m\,\text{Sn}(Z, m) \qquad ...(10.22)$$

Combining Eqs. 10.21 and 10.22, we get the following transformation equation between z and Z :

$$Z = \text{Sn}^{-1}\left(\frac{\tanh z}{m}, m\right) \qquad ...(10.23)$$

Now, $Z = X + i\,Y$. For the under surface of the apron, $Y = y = 0$.

Hence, $$Z = \text{Sn}^{-1}\left(\frac{\tanh x}{m}, m\right) \qquad ...(10.24)$$

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Other cases: Pavlovsky analysed a number of other cases. We shall mention here only two cases shown in Figs. 10.8 and 10.9 .

(*a*) Apron with asymmetrical cutoff, resting on pervious medium of finite depth:

The transformation equations are

$$\zeta = \pm \cos d \sqrt{\tanh^2 z + \tan^2 d} \qquad ...(10.27)$$

and

$$Z = \mathrm{Sn}^{-1} \sqrt{\frac{(\zeta_4 - \zeta_7)(\zeta_6 - \zeta)}{(\zeta_6 - \zeta_7)(\zeta_4 - \zeta)}} \qquad ...(10.28)$$

From Eq. 10.27, we have

$$\zeta_4 = \cos d \sqrt{\tanh^2 b_2 + \tan^2 d} \text{ and } \zeta_5 = -\cos d \sqrt{\tanh^2 b_1 + \tan^2 d}$$

Also, $\zeta_6 = +1$ and $\zeta_7 = -1$

For any point under the foundation, $Z = X$

$$\therefore \quad X = \mathrm{Sn}^{-1} \sqrt{\frac{(\zeta_4 + 1)(1 - \zeta)}{2(\zeta_4 - \zeta)}}$$

where ζ is defined by Eq. 10.27 in which $z = x$.

Hence the pressure distribution is given by

$$\phi_x = \frac{H}{K} \mathrm{Sn}^{-1} \left[\sqrt{\frac{(\zeta_4 + 1)(1 - \zeta)}{2(\zeta_1 - \zeta)}}, m \right] \qquad ...(10.29)$$

where, the modulus m is given by

$$m = \sqrt{\frac{(\zeta_4 - \zeta_5)(\zeta_6 - \zeta_7)}{(\zeta_6 - \zeta_5)(\zeta_4 - \zeta_7)}} = \sqrt{\frac{2(\zeta_4 - \zeta_5)}{(1 - \zeta_5)(\zeta_4 + 1)}} \qquad ...(10.30)$$

and K = complete elliptic integral of the first kind.

For any point under the apron, $z = x$ and hence ζ, is known from Eq. 10.27. Knowing ζ, ϕ_x can be computed from Eq. 10.29.

(*b*) Depressed floor resting on pervious medium of finite depth:

The transformation equation for the case of depressed floor (a rectangular floor) are as follows:

$$z = \frac{\alpha}{\delta} \left[\cup - \delta \, \Pi (\cup, a) \right] \qquad ...(10.31)$$

where $\cup$ = Jacobi's symbol for the elliptic integral of the first kind

$\Pi(\cup, a)$ = Jacobi's elliptic integral of the third kind given by

$$\Pi (n, \phi, k) = \cup + \frac{\mathrm{tn}\, a}{\mathrm{dn}\, a} \Pi (\cup, a)$$

$\Pi(n, \phi, k)$ = Legendre's ellipitic integral of third kind with modulus $k = \alpha/\beta$

n = parameter $= -\alpha^2 = -k^2 \, \mathrm{Sn}\, a$

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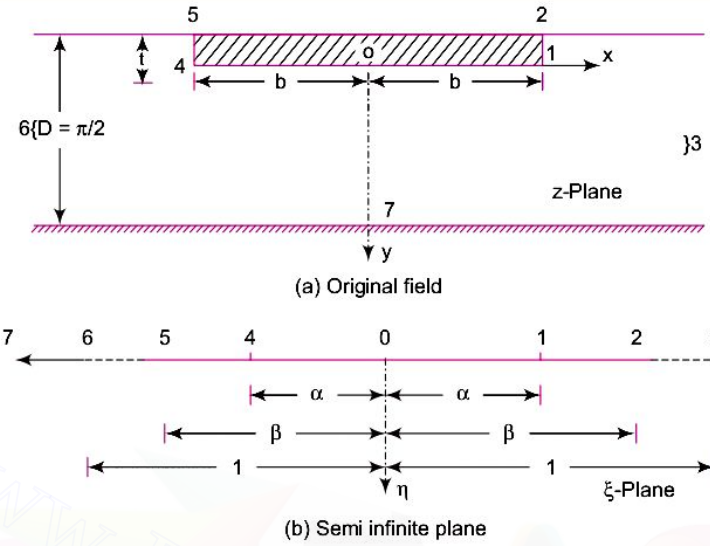

Fig 10.9 Depressed floor

a = Jacobi's factor, connected to n and k as shown above

$$\phi = \text{am} \cup = \sin^{-1}\frac{\zeta}{\alpha}\ ;\ \delta = \frac{\beta}{\alpha^2}\sqrt{\frac{1-\alpha^2}{1-\beta^2}}$$

$$\text{Sn}\ a = \beta = \text{sinus amplitudinis}$$

$$\text{Cn}\ a = \sqrt{1-\beta^2} = \text{consinus amplitudinis}$$

$$\text{tn}\ a = \frac{\beta}{\sqrt{(1-\beta^2)}} = \text{tangent amplitudinis}\ ;\ \text{dn}\ a = \sqrt{1-\alpha^2} = \text{delta amplitudinis}$$

The other transformation equation [Eq. 10.22 (*a*)] is

$$Z = \text{Sn}^{-1}\left(\frac{\zeta}{m}, m\right) \text{ where } m = \text{modulus} = \beta \qquad ...(10.32)$$

The pressure distribution is then given by

$$\phi_x = \frac{H}{2}\left[1 \pm \frac{X}{K}\right] \qquad ...(10.33)$$

For the points under the apron, $Z = X$ and $z = x$.

Thus, for any value of $z\ (=x)$, ζ, can be calculated from Eq. 10.31. Knowing ζ,, $Z(=X)$ can be calculated from Eq. 10.32 and hence pressure can be computed from Eq. 10.33.

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CHAPTER 11

Drainage and Dewatering

11.1 INTRODUCTION

The removal of excess water from the saturated soil mass is termed *drainage* or *dewatering*. In many civil engineering problems, such as excavations for basements and foundations of buildings, foundations of dams, or laying sewer lines, the excavations are often carried below water table. Such excavations require lowering of water table below the bottom of excavation to prevent revelling or sloughing of the sides and to get dry working conditions for construction purposes. Drainage is also required for increasing the stability of soil by reducing seepage and pore water pressures and for reducing the danger of frost action. Drainage reduces the natural stresses in cohesionless soil and thereby increases the effective stress and strength. Drainage may also be essential to lower the water table of a waterlogged area to make it more suitable for cultivation purposes. Other problems associated with ground water are drainage of water behind retaining structures, basement walls, and earth dams and embankment to prevent build-up of hydrostatic pressure.

The ground water table may be lowered by the following methods

1. Ditches and sumps
2. Well point system
3. Shallow well system
4. Deep well drainage system
5. Vacuum method
6. Electro-osmosis method.

11.2 DITCHES AND SUMPS

This is the simplest form of dewatering used in the shallow, excavations in coarse grained soils whose permeability is greater than 10^{-3} cm/sec. Shallow pits, called *sumps* are dug along the periphery of

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the area-drainage ditches. The water from the slopes or sides flows under gravity and is collected in the sumps from which it is pumped out. If the seepage is significant, it may cause softening and revelling or sloughing of the lower part of the slope. There is also possibility of piping in the sump bottom. In such circumstances, the sump can be *weighted down* with an inverted filter conisting of layers of successively coarser material from the bottom of the sump-pit upwards. The filter criteria have been discussed in §11.8.

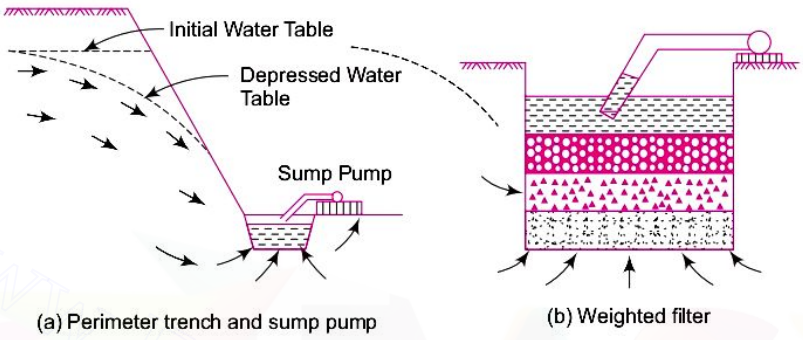

Fig. 11.1 Excavation drainage with sump

11.3 WELL POINT SYSTEM

A more complicated dewatering system based on gravity flow is the installation of *well points*. A well point is a perforated pipe about 0.5 to 1 m long and 5 to 8 cm in diameter, covered by cylindrical wire-gauge screen. A conical steel drive point is attached to the lower end of the pipe, with a ball valve fitted in the point to allow jetting of water to pass through it for driving it. The points are placed in a row or ring, and the riser pipes are attached through a common manifold or header pipe to a special well point pump. For inserting the well point into ground by jetting, water is pumped down the well point under pressure from where it emerges with a great velocity through the tip of the drive point. The emerging jet-stream dislodges the surrounding soil and the well point can be lowered to the desired depth. A further advantage

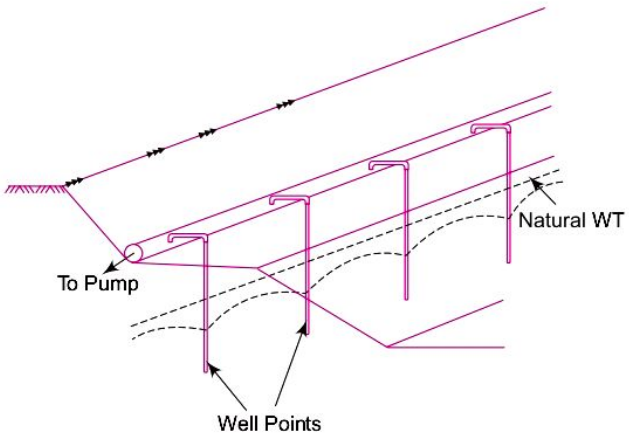

Fig. 11.2 Lowering water table by well point system

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of jetting is that water under pressure washes away soil fines from around the well point leaving a relatively coarser material to settle and form a natural filter around the well point. The hole formed around the riser pipe and the well point by jetting water is filled with coarse sand. The sand also helps in directing drainage to the well point. When suction commences, the ball valve closes and the water is drawn in the well point only through the surrounding screen. Suction at the very bottom of the well point permits lowering of the ground water table with maximum drawing in of air.

The suction pump used in the well point system has a capacity of bringing water to the surface from a minimum depth of about 6 m. The well points are generally spaced between 1 to 2 m. For dewatering excavations which are more than 6 m below the water table, a *multiplestage well point system* (Fig. 11.3) is used. Excavations exceeding 16 m depth are preferably drained by the *deep well* system. In the multiplestage well point system, the ground is first stripped to the natural water level where the first stage of well points is installed. After excavating about 5 m, second stage is installed to further lower the water table for advancing excavations.

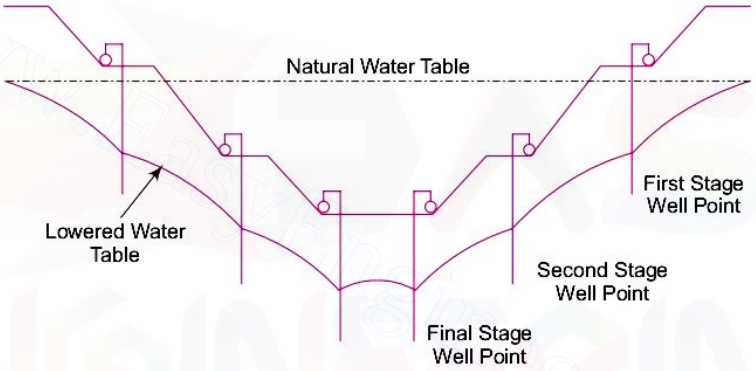

Fig.11.3 Multistage well point system

The other stages are put successively, up to a maximum depth of 16 m is reached. In the well point system, a round the clock pumping schedule is essential, as the interruption in pumping can have catastrophic consequences. Hence, one auxiliary pump for each two pumps in the use should always be available.

11.4 SHALLOW WELL SYSTEM

In this system, a hole of 30 cm in diameter or more is bored into the ground to a depth not exceeding 10 m below the axis of the pump. A strainer tube of 15 cm diameter is lowered in the bore hole having a casing tube. A gravel filter is formed around the stainer tube by gradually removing the casing tube and simultaneously pouring filter material, such as gravel, etc., in the annular space. A suction pipe is lowered into filter well so formed. The suction pipes from a number of such wells may be connected to one common header leading to the pumping unit.

11.5 DEEP WELL DRAINAGE

When the depth of excavation is more than about 16 m below the water table, deep well drainage system may be used with advantage. The system is also useful where artesian water is present. A 15 to 60 cm diameter hole is bored and a casing with a long screen (5 to 25 cm) is provided. A submersible pump

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with a capacity to push the water up to a height of 30 m or more is installed near the bottom of the well. Each well has its own pump. Along with the deep well arranged on the outer side of the area under excavation, a row of well points is frequently installed at the toe of the side slopes of the deep excavation [Fig. 11.4(*a*)].

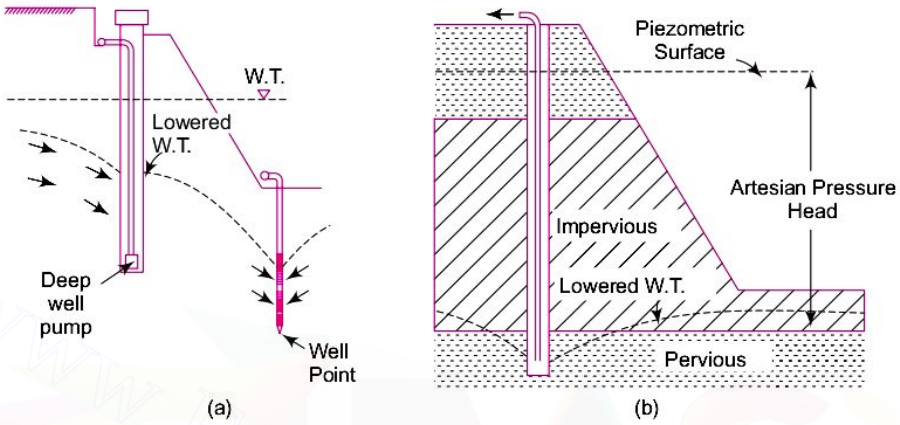

Fig. 11.4 Deep well system

Deep well system is also very helpful where high artesian pressure exists. Pumping lowers the piezometric surface and thereby reduces the artesian pressure head.

11.6 VACUUM METHOD: FORCED FLOW

For fine-grained soils, the well point system can be extended by the vacuum method. For successful dewatering in the fine, noncohesive soils, such as silty sands and other fine sands whose average effective particles size D_{10} is smaller than about 0.05 mm and coefficient of permeability between 10^{-3} and 10^{-5} cm/sec, it is necessary to apply a suction head in excess of the capillary head to the dewatering system. Both the well point system and the deep well system can be adopted for dewatering such soils by maintaining a vacuum in the well with the use of airtight seals for all points. A hole of about 25 cm diameter is created around the well point and the rise pipe by jetting water under sufficient pressure. While the jetting water is still flowing, medium to coarse sand is rapidly shovelled into the hole to fill it up to about 0.75 or 1 m from the top. The top portion of the hole is then sealed up by tamping bentonite, soil cement or clay. Vacuum pumps are used to create a vacuum in the sand filling. When the vacuum is drawn on the well point, the ground surface is subjected to unbalanced atmospheric pressure. Although the quantity of water drawn out does not increase much, the unbalanced atmospheric pressure acting on the ground surface consolidates the subsoil which becomes stiff enough for carrying out excavations.

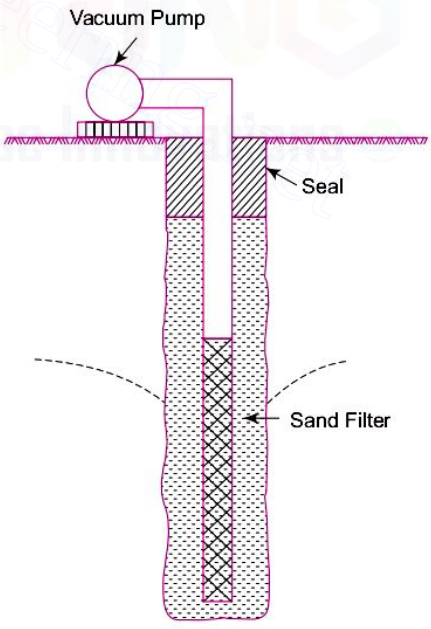

Fig. 11.5 Vacuum method

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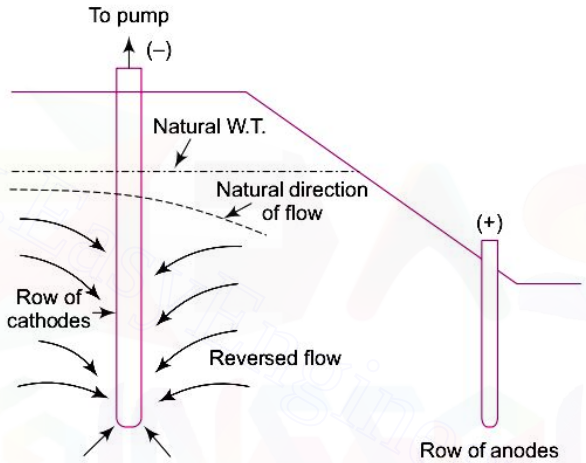
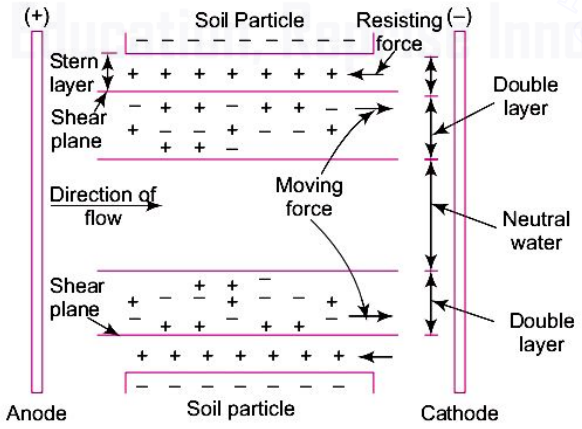
11.7 ELECTRO-OSMOSIS METHOD

None of the above methods are suitable for fine grained cohesive soil. Such soils can be drained or *stabilised* by electro-osmosis. The application of electro-osmosis to dewatering of soil was largely developed by L, Casagrande (1952). If direct current is passed between two electrodes into saturated soil mass, the soil water will travel from the positive electrode (anode) to the negative electrode (cathode). The cathode is made in the form of a well point or a metal tube for pumping out the seeping water. A steel rod, a pipe or sheet piling of excavation can serve as the anode. The arrangement of the electrodes is done in such a way that the natural direction of flow of water is reversed away from the excavation, thereby increasing the shear strength of the soil and stability of the slope (Fig. 11.6).



(a) The Arrangement



(b) The Principle

Fig. 11.6 Electro-osmotic drainage

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The velocity of flow towards the cathode can be expressed as follows (Scott, 1968):

$$v_e = k_e - i_e$$

where k_e = electro-osmotic coefficient of permeability

i_e = electric gradient, or the electric potential divided by the distance between the electrodes.

The electro-osmotic permeability k_e varies with the porosity of soils and the electrolytic and viscous properties of the fluid, but is independent of the *size* of the soil pores *(cf.* Darcy's permeability). Since the range of porosity variations for soils is not large and the electrolytic properties of pore water are also relatively constant, k_e may be taken to be roughly independent of the soil type. For practical purposes, k_e may be assumed as 0.5×10^{-4} cm/sec for most soils for an electric gradient of 1 volt/cm. The potentials generally used are from 40 to 180 volts with electrode spacing 4 to 5 metres.

The principle of electro-osmosis can be explained with respect to the *electric double layer* on the fine grained soil particles. The outer part of this layer carries, a net positive charge and it starts moving towards the cathode, as soon as an electric potential is applied to a saturated soil. The thin inner layer remains attached to the particle. The neutral water filling the pores is also dragged towards the cathode by moving outer laters of the electrical double layer, from where it may be pumped out.

Electro-osmosis is very successful in draining fine-grained soils, but is quite costly. It is mainly used for fine grained soils where main purpose of dewatering is to increase consolidation and shear strength of the soil.

11.8 SEEPAGE ANALYSIS

A complete design of dewatering system consists of the diameter and spacing of well points or wells, penetration of wells and the pumping capacities. These factors depend upon the rate or quantity of water to be drained out and the corresponding drawdown. The wells may be either *artesian* or *gravity* wells, depending upon conditions of flow. Where the dewatering system consists of a number of closely spaced well points or wells, a simplified solution can be obtained by considering the line of wells equivalent to a linear drainage slot. The seepage analysis is done on the assumption that the flow is laminar and Darcy's law is valid. It is further assumed that the soil to be dewatered is homogeneous and isotropic. We shall consider the following cases of flow towards a continuously discharging line slot: (*i*) fully penetrating slot : unconfined flow, (*ii*) fully penetrating slot : confined flow, (*iii*) fully penetrating slot : combined gravity-artesian flow, (*iv*) partially penetrating slot, (*v*) flow to a slot from two line sources.

1. Fully penetrating slot: unconfined flow. Figure 11.7 shows an infinitely long slot, with seepage from a line source parallel to the slot. It is assumed that the flow originates on one side of the slot. Consider a length *a* of the slot. The discharge is given on the assumption that on any vertical line below the drawdown curve, the hydraulic gradient is constant and is equal to the slope of the drawdown curve at the point where the vertical line intersects the drawdown curve *(Dupuit-Forchheimer assumption)*. For any point *P* (*x, y*), the discharge crossing the vertical plane through

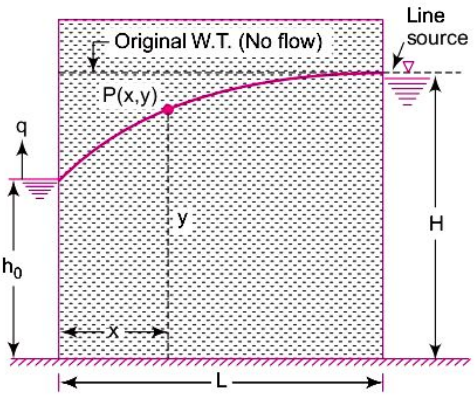

Fig. 11.7 Unconfined flow towards a slot: One line source

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P, per unit length a of the slot is given by

$$q = k\,i\,A,$$

where $$i = \frac{dy}{dx} \text{ and } A = ya$$

$$\therefore \quad q = k\,\frac{dy}{dx}\,ya \quad \text{or} \quad \frac{q}{ka}\,dx = y\,dy \qquad ...(11.1)$$

Integrating between the limits

$y = h_0$ to $y = \text{H}$, we get

$$\frac{q}{ka}\int_0^L dx = \int_{h_0}^{H} y\,dy$$

$$\therefore \quad \frac{qL}{ka} = \frac{1}{2}\left(H^2 - h_0^2\right)$$

or $$q = \frac{ka}{2L}\left(H^2 - h_0^2\right) \qquad ...(11.2)$$

2. Fully penetrating slot: artesian flow.

$$q = k\,i\,A$$

where $$i = \frac{dy}{dx};\ A = ab$$

$$\therefore \quad q = k\,\frac{dy}{dx}\,ab$$

$$\therefore \quad \frac{q}{kab}\int_0^L dx = \int_h^H dy$$

$$\therefore \quad \frac{q\,L}{k\,a\,b} = (\text{H} - h_0)$$

or $$q = \frac{k\,a\,b}{L}\left(H - h_0\right) \qquad ...(11.3)$$

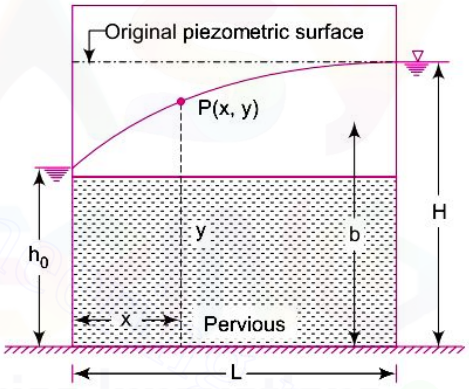

Fig. 11.8 Confined flow: One line source

3. Fully penetrating slot: combined gravity-artesian flow.

In this case, the rate of withdrawal is such that the draw-down curve goes below the impervious layer, at the slot. The analysis can be done by using the discharge equation for gravity flow for length L_G (*i.e.*, using L_G in place of L and b in place of H in equations 11.2) and the discharge equation for artesian flow for the length $L - L_G$ (*i.e.*, using $L - L_G$ in place of L and b in place of h_0). If q_1 and q_2 are the corresponding discharge, we have

$$q_1 = \frac{ka}{2\,L_G}\left(b^2 - h_0^2\right) \qquad ...(11.4)$$

$$q_2 = \frac{kab}{L - L_G}\left(H - b\right) \qquad ...(11.5)$$

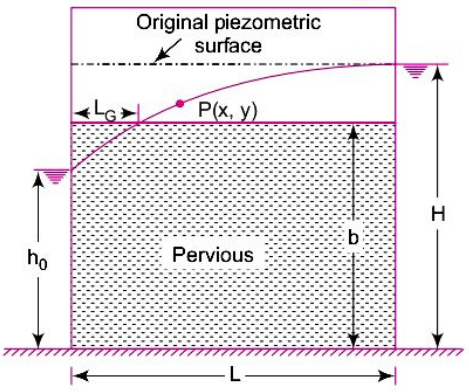

Fig. 11.9 Combined artesian-gravity flow towards a line slot

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Since both the discharges should be equal at the plane passing through the point P, where the flow changes from artesian to gravity conditions, we have

$$\frac{ka}{2L_G}\left(b^2 - h_0^2\right) = \frac{kab}{L - L_G}(H - b)$$

From which,
$$L_G = \frac{L\left(b^2 - h_0^2\right)}{2bH - b^2 - h_0^2} \qquad ...(11.6)$$

Substituting this in equation 11.4, we have

$$q = \frac{ka\left(2bH - b^2 - h_0^2\right)}{2L} \qquad ...(11.7)$$

4. Partially penetrating slot. The discharge q_p from a partially penetrating slot in an unconfined aquifer can be found from the following expression developed by Chapman (1956) by model studies:

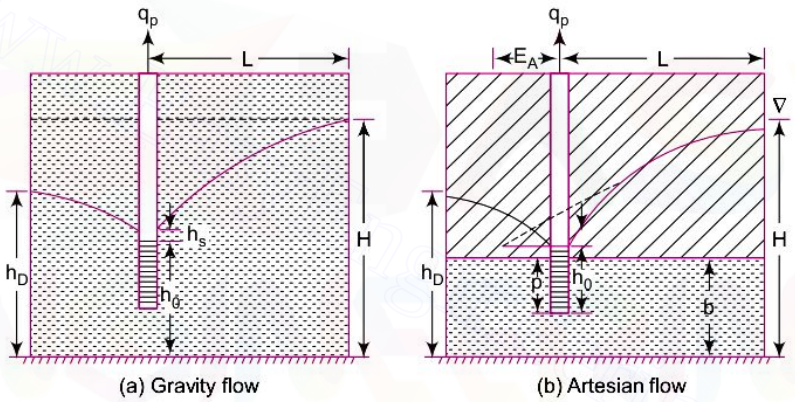

Fig. 11.10 Partially penetrating slots

$$q_p = \left(0.73 + 0.27\frac{H - h_0}{H}\right)\frac{ka}{2L}\left(H^2 - h_0^2\right) \qquad ...(11.8)$$

The symbols are shown in Fig. 11.10. The maximum residual head h downstream from the slot is given by

$$h_D = h_0\left[\frac{1.48}{L}(H - h_0) + 1\right] \qquad ...(11.9)$$

For the case of a slot partially penetrating an artesian aquifer, the discharge q_p per unit length a is given by

$$q_p = \frac{kba\,(H - h_0)}{L + E_A} \qquad ...(11.10)$$

where E_A = extra length factor which is a function of the ratio of slot penetration p to the thickness of pervious stratum b. It can be determined from Fig. 11.11 developed by Baron (1952).

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5. Flow to a slot from two line sources. In most of the practical problems, the flow towards a slot originates both sides of the slot instead of only one side, as discussed in the above cases. For fully penetrating slots in confined as well as unconfined flows, the discharge will be double of the discharge given by equations 11.2 and 11.3 respectively. For *gravity flow* conditions of a partially penetrating slot, with two parallel line sources situated equidistant on either side, the discharge can be estimated from the following equation developed by Chapman (1956) by model studies

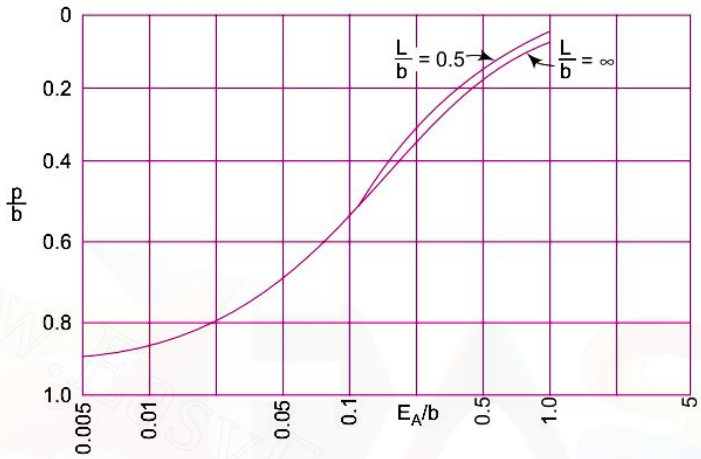

Fig. 11.11 Curves for extra length factor E_A

$$q_p = \left[0.73 - 0.27\frac{H - h_0}{H}\right]\frac{ka}{L}\left(H^2 - h_0^2\right) \qquad ...(11.11)$$

where L = distance from slot to either of two line sources.

For *artesian flow* to a partially penetrating slot midway between two line sources, the discharge per unit length a is given by

where $\lambda = a$ factor which depends upon p/b ratio, as given by the curve of Fig. 11.12.

$$q_p = \frac{2kba\left(H - h_0\right)}{L + \lambda b}$$

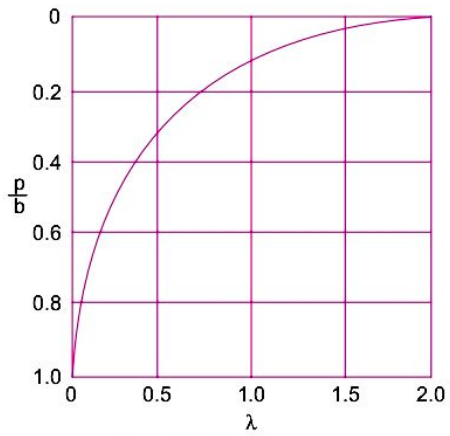

Fig. 11.12 Factor λ

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11.9 PROTECTIVE FILTERS

Downstream drainage filters and the drains in earth dams, and *'weighting filters'* at the bottom of drainage sumps and trenches arrest the percolating water at the exit and allow its safe exit without inducing piping. With a proper drainage filter under the d/s slope or a rockfill toe, the phreatic line in an earth dam can be kept well within the body of the dam and the danger of d/s slope is removed. Horizontal filters incorporated in the d/s toe of a dam are used to cover d/s area adjoining the toe add to the downwards effective pressure of the underneath protected soil and assist in balancing the upward seepage forces. Similarly, the d/s surface area adjoining a sheet-pile wall can be loaded with filters to increase the factor of safety against piping.

A protective filter consists of a combination of layers of pervious materials and is designed in such a manner as to provide quick drainage, yet prevent the movement of soil particles due to flowing water. In such a filter, each subsequent layer becomes increasingly coarser than the previous one, and hence, it is also called *reverse filter* or *inverted filter*. The soil to be protected (*i.e.*, the embankment or foundation material surrounding the filter) is known as the *base material.* The four main requirements to be satisfied by filter material are as follows (U.S.B.R. Earth Manual):

1. The filter material should be sufficiently fine and so graded that the voids of the filter are small enough to prevent base material particles from penetrating and clogging the filter.
2. The filter material should be sufficiently coarse and pervious compared to the base material so that the incoming water is rapidly removed without any appreciable build up of seepage forces within the filter.
3. The filter material should be coarse enough not to be carried away through the drainage pipe openings. The drainage pipe should be provided with sufficiently small openings or perforations, or additional coarser layer should be used if necessary.
4. The filter layer should be sufficiently thick to provide good distribution of all particle sizes throughout the filter and to be able to carry the seepage discharge. The filter thickness should ensure an adequate safety against piping and proper insulation for frost-susceptible base material, as the case may be.

According to Terzaghi, the filter material should fulfil the following two criteria:

1. The D_{15} size of filter material must not be more than 4 to 5 times D_{85} size of the base material. This prevents the foundation material from passing through the pores of the filter material.
2. The D_{15} size of filter material must be at least 4 to 5 times the D_{15} size of the base material. This keeps the seepage forces within the filter to permissible small magnitude.

The above criteria may be expressed as:

$$\frac{D_{15}\text{ of filter}}{D_{85}\text{ of base material}} < 4 \text{ to } 5 < \frac{D_{15}\text{ of filter}}{D_{15}\text{ of base material}}$$

The requirements cited above must be satisfied between any two adjacent layers of the filter. The criteria given by Terzaghi have been further modified as follows (U.S.B.R.: Design of small dams):

(*a*) $\dfrac{D_{15}\text{ of filter}}{D_{15}\text{ of base material}} = 5$ to 40, provided that the filter does not contain more than 5 per cent of material finer than 0.075 mm (no. 200 sieve).

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(*b*) $\dfrac{D_{15} \text{ of filter}}{D_{85} \text{ of base material}} = 5$ or less.

(*c*) $\dfrac{D_{85} \text{ of filter}}{\text{Maximum opening of pipe of drains}} = 2$ or more.

(*d*) The grain size curve of the filter should be roughly parallel to that of the base material.

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Part III
Elasticity Applied to Soils

12. Elements of Elasticity
13. Stress Distribution : I
14. Stress Distribution : II

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CHAPTER 12

Elements of Elasticity

12.1 STATE OF STRESS AT A POINT

Force systems acting on an elastic body in equilibrium are of two kinds: *body forces* and *surface forces*. Forces distributed over the surface of the body, such as the pressure of one body on another, hydrostatic pressure etc., are called *surface forces*. Such forces are applied externally at the boundaries of the body, and dimensionally, a surface force is defined as a force per unit area. Forces distributed over the volume of a body, such as gravitational forces, magnetic forces, seepage forces or in the case of a body in motion, inertia forces, are called *body forces*. Dimensionally, a body force is taken as a force per unit volume.

The total stress field on any three-dimensional element is determined by the following stresses:

$$\begin{pmatrix} \sigma_{XX} & \tau_{XY} & \tau_{XZ} \\ \tau_{YX} & \sigma_{YY} & \tau_{YZ} \\ \tau_{ZX} & \tau_{ZY} & \sigma_{ZZ} \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} \sigma_{X} & \tau_{XY} & \tau_{ZX} \\ \tau_{XY} & \sigma_{Y} & \tau_{ZY} \\ \tau_{XZ} & \tau_{YZ} & \sigma_{Z} \end{pmatrix}$$

These nine stress components, as given by this group of square matrix of stresses, are the components of a mathematical entity called the *stress tensor,* with a symmetrical matrix relative to its main diagonal (upper left to lower right). The main diagonal elements of the stress tensor are, the normal stress components (compression), and the off diagonal elements are shear stresses.

Each stress component in it is represented by its magnitude, direction as well as the position of the plane on which it is acting. For example, σ_{XX} (or simply σ_X) signifies the normal stress acting on the face of the element that is perpendicular to x-axis, and the stress is acting in the x-direction. Similarly, the shearing stress τ_{XY} denotes a stress acting in the face of an element that is perpendicular to x-axis, the stress acting in the direction of y-axis. The stress τ_{YZ} denotes a stress on the face of an element that is perpendicular to y-axis, the stress acting in the direction of z-axis. Thus, at a point, there are three normal stresses: σ_X (or σ_{XX}), σ_Y (or σ_{YY}) and σ_Z (or σ_{ZZ}) and six shearing stresses: τ_{XY}, τ_{YX}; τ_{YZ} ; τ_{ZY} ; τ_{ZX} and τ_{XZ} as shown in Fig. 12.1(a).

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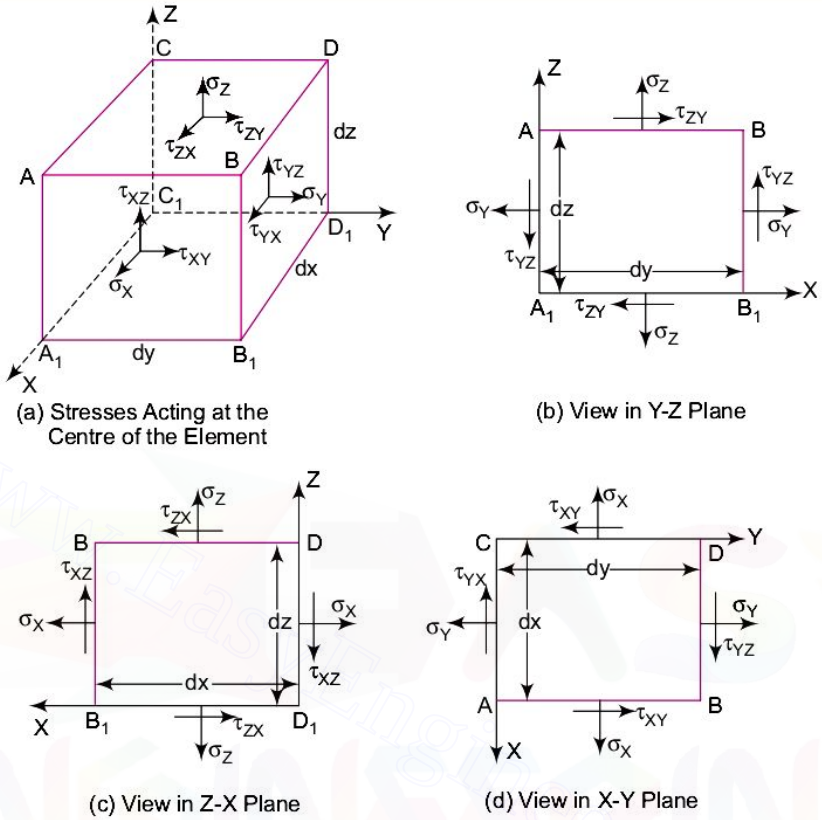

Fig. 12.1 Three dimensional field

Figure 12.1 (*a*) shows the stresses acting at the centre of an elemental volume of size *dx, dy* and *dz*. Figure 12.1 (*b*),(*c*) and (*d*) show the views in *y-z, z-x* and *x-y* planes, respectively. Considering the equilibrium of the elemental volume, and applying equation $\Sigma M_z = 0$ (where M_z represents the moment about *z*-axis), we get, from Fig. 12.1(*d*).

$$\therefore \qquad (\tau_{YX} \cdot dx\, dz)\, dy = (\tau_{XY}\, dy\, dz)\, dx$$

$$\tau_{XY} = \tau_{YX} \qquad ...(12.1)$$

Similarly, from Figs. 12.1 (*b*) and (*c*), we get:

$$\tau_{YZ} = \tau_{ZY} \text{ and } \tau_{ZX} = \tau_{XZ} \qquad ...(12.2)$$

Thus, out of six shearing stresses, there are only three independent shearing stresses and total independent stresses are therefore six only (*i.e.*, three normal stresses and three shearing stresses).

12.2 EQUILIBRIUM EQUATIONS

Figure 12.2 shows an elemental volume of size *dx, dy* and *dz* with the nine stress components acting at the centre of the element. The stress on each face will be equal to the stress at the centre increased or reduced by the distance from the centre to the face times the spatial derivative of the stress. For example, normal

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stress component acting at the centre will be increased to $\left(\sigma_Y + \frac{\partial \sigma_Y}{\partial y}\frac{dy}{2}\right)$ to the face $B\,D\,D_1\,B_1$ and decreased to $\left(\sigma_Y - \frac{\partial \sigma_Y}{\partial y}\frac{dy}{2}\right)$ at the face $ACC_1\,A_1$.

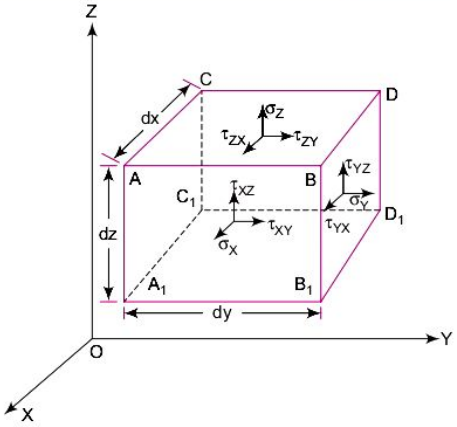

Fig. 12.2

If X, Y and Z denote the components of body forces per unit volume, in the three corresponding directions, then the equation of equilibrium obtained by summing all the forces acting on the element in the x-direction is:

$$\left\{\left(\sigma_X + \frac{\partial \sigma_X}{\partial x}\frac{dx}{2}\right)dy\cdot dz - \left(\sigma_X - \frac{\partial \sigma_X}{\partial x}\frac{dx}{2}\right)dy\cdot dz\right\}$$

$$+\left\{\left(\tau_{YX} + \frac{\partial \tau_{YX}}{\partial y}\frac{dx}{2}\right)dx\cdot dz - \left(\tau_{YX} - \frac{\partial \tau_{YX}}{\partial y}\frac{dy}{2}\right)dx\cdot dz\right\}$$

$$+\left\{\left(\tau_{ZX} + \frac{\partial \tau_{ZX}}{\partial z}\frac{dz}{2}\right)dx\cdot dy - \left(\tau_{ZX} - \frac{\partial \tau_{ZX}}{\partial z}\frac{dz}{2}\right)dx\cdot dy\right\} + X dx\cdot dy\cdot dz = 0$$

Noting that $\tau_{XY} = \tau_{YX}$ and $\tau_{XZ} = \tau_{ZX}$, and dividing all the terms by $dx.\ dy.\ dz$, we get

$$\frac{\partial \sigma_X}{\partial x} + \frac{\partial \tau_{YX}}{\partial z} + \frac{\partial \tau_{ZX}}{\partial z} + X = 0 \quad ...(I) \qquad ...(12.3\ a)$$

Similarly, in the y-direction, the balance of forces requires that

$$\left\{\left(\sigma_Y + \frac{\partial \sigma_Y}{\partial y}\frac{dy}{2}\right)dx\cdot dz - \left(\sigma_Y - \frac{\partial \sigma_Y}{dy}\frac{dy}{2}\right)dx\cdot dz\right\}$$

$$+\left\{\left(\tau_{ZY} + \frac{\partial \tau_{ZY}}{dz}\frac{dz}{2}\right)dy\cdot dx - \left(\tau_{ZY} - \frac{\partial \tau_{ZY}}{\partial z}\frac{dz}{2}\right)dy\cdot dx\right\}$$

$$+\left\{\left(\tau_{XY} + \frac{\partial \tau_{XY}}{\partial x}\frac{dx}{2}\right)dy\cdot dz - \left(\tau_{XY} - \frac{\partial \tau_{XY}}{\partial x}\frac{dx}{2}\right)dy\cdot dz\right\} + Y\,dx\,dy\,dz = 0$$

which on simplification reduces to: $\frac{\partial \sigma_Y}{\partial y} + \frac{\partial \tau_{ZY}}{\partial z} + \frac{\partial \tau_{XY}}{\partial x} + Y = 0$ ...(II) ...(12.3 b)

Lastly in the z-direction, we have

$$\left\{\left(\sigma_Z - \frac{\partial \sigma_Z}{\partial z}\frac{dz}{2}\right)dx\cdot dy - \left(\sigma_Z - \frac{\partial \sigma_Z}{\partial z}\frac{dz}{2}\right)dx\cdot dy\right\}$$

$$+\left\{\left(\tau_{XZ} + \frac{\partial \tau_{XZ}}{\partial x}\frac{dx}{2}\right)dz\cdot dy - \left(\tau_{XZ} - \frac{\partial \tau_{XZ}}{\partial x}\frac{dx}{2}\right)dz\cdot dy\right\}$$

$$+\left\{\left(\tau_{YZ} + \frac{\partial \tau_{YZ}}{\partial y}\frac{dy}{2}\right)dz\cdot dx - \left(\tau_{YZ} - \frac{\partial \tau_{YZ}}{\partial y}\frac{dy}{2}\right)dz\cdot dx\right\} + Z\cdot dx\cdot dy\cdot dz = 0$$

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which on simplification reduces to: $\frac{\partial \sigma_Z}{\partial z} + \frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \sigma_{YZ}}{\partial y} + Z = 0$ ...(III) ...(12.3 *c*)

Equation 12.3 *a*, *b*, *c* are three equilibrium equations which are also sometimes written in the following order:

$$\frac{\partial \sigma_X}{\partial x} + \frac{\partial \tau_{YX}}{\partial y} + \frac{\partial \tau_{ZX}}{\partial z} + X = 0 \quad \text{...(I)}$$

$$\frac{\partial \tau_{XY}}{\partial x} + \frac{\partial \sigma_Y}{\partial y} + \frac{\partial \tau_{ZY}}{\partial z} + Y = 0 \quad \text{...(IV)} \quad \text{...(12.3)}$$

$$\frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \tau_{YZ}}{\partial y} + \frac{\partial \sigma_Z}{\partial z} + Z = 0 \quad \text{...(III)}$$

However, we have seen in §12.1 that there are six independent stress components acting at a point and the complete solution of the problem requires the determination of the these six stress components. Thus there are six unknowns, and only three *equations of equilibrium* are available. Thus, the problem of elasticity is strictly of indeterminate nature. These equations of static equilibrium must be supplemented with equations of compatibility (§12.5) of deformations to get the complete solution. In addition to this, the final solution should satisfy the boundary conditions (§ 12.6).

12.3 EQUILIBRIUM EQUATIONS FOR SATURATED SOIL BODY

The equilibrium equations derived in the previous article are satisfied both in terms of the total stresses and the effective stresses acting on a saturated soil element in equilibrium.

(*a*) Total stresses. If the equilibrium equations are formulated in terms of total stresses which include the water pressure or seepage force, the body forces will be equal to those due to gravity in the respective directions. If *z*-axis is directed downwards, we have: $X = 0$, $Y = 0$ and $Z = \gamma$. Thus, the equilibrium equations in terms of total stresses can be written as:

$$\frac{\partial \sigma_X}{\partial x} + \frac{\partial \tau_{YX}}{\partial y} + \frac{\partial \tau_{ZX}}{\partial z} = 0 \quad \text{...(12.4 } a\text{)}$$

$$\frac{\partial \tau_{XY}}{\partial x} + \frac{\partial \sigma_Y}{\partial y} + \frac{\partial \tau_{ZY}}{\partial z} = 0 \quad \text{...(12.4 } b\text{)}$$

and $$\frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \tau_{YZ}}{\partial y} + \frac{\partial \sigma_Z}{\partial z} + \gamma = 0 \quad \text{...(12.4 } c\text{)}$$

where σ_X, σ_Y and σ_Z are the total stresses, which include the seepage forces. Thus,

Equation 12.4 are the equilibrium equations in terms of the total stresses.

Now, $$\sigma_X - \sigma_X' + u = \sigma_X' + \gamma_w (h - h_e)$$

u = pore pressure = $\gamma_w h_w = \gamma_w (h - h_e)$; h = total head at the point

h_e = elevation head, which is a function of z

Differentiating (*i*) partially which respect to *x*, we get

$$\therefore \quad \frac{\partial \sigma_X}{\partial x} = \frac{\partial \sigma_{X'}}{\partial x} + \gamma_w \frac{\partial h}{\partial x} \qquad \left[\text{since } \frac{\partial h_e}{\partial x} = 0\right]$$

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Similarly, $\frac{\partial \sigma_Y}{\partial y} = \frac{\partial \sigma_{Y'}}{\partial y} + \gamma_w \frac{\partial h}{\partial y}$ and $\frac{\partial \sigma_Z}{\partial z} = \frac{\partial \sigma_{Z'}}{\partial z} + \gamma_w \frac{\partial h}{\partial z} - \gamma_w \left[\text{since } \frac{\partial h_e}{\partial x} = 1\right]$

Substituting these in equation 12.4, we get the following equilibrium equation in terms of effective stresses:

$$\frac{\partial \sigma_{X'}}{\partial x} + \frac{\partial \tau_{YX}}{\partial y} + \frac{\partial \tau_{ZX}}{\partial z} + \gamma_w \frac{\partial h}{\partial x} = 0 \qquad ...(12.5\ a)$$

$$\frac{\partial \tau_{XY}}{\partial x} + \frac{\partial \sigma_{Y'}}{\partial y} + \frac{\partial \tau_{ZY}}{\partial z} + \gamma_w \frac{\partial h}{\partial y} = 0 \qquad ...(12.5\ b)$$

and $$\frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \tau_{YZ}}{\partial y} + \frac{\partial \sigma_{Z'}}{\partial z} + (\gamma - \gamma_w) + \gamma_w \frac{\partial h}{\partial z} = 0$$

or $$\frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \tau_{YZ}}{\partial y} + \frac{\partial \sigma_{Z'}}{\partial z} + \gamma_w \frac{\partial h}{\partial z} + \gamma' = 0 \qquad ...(12.5\ c)$$

(*b*) Effective stresses. The above equilbrium equations (equation 12.5) in terms of effective stresses can also be derived directly from the original equilibrium equations (equation 12.3), by considering σ_X, σ_Y and σ_Z terms in equation 12.3 as the effective stresses, and *X, Y* and *Z* or the body forces *inclusive* of seepage forces. Thus, we have, from Eqs. 12.3,

$$\frac{\partial \sigma_{X'}}{\partial x} + \frac{\partial \tau_{YX}}{\partial y} + \frac{\partial \tau_{ZX}}{\partial z} + X = 0$$

$$\frac{\partial \tau_{XY}}{\partial x} + \frac{\partial \sigma_{Y'}}{\partial y} + \frac{\partial \tau_{ZY}}{\partial z} + Y = 0 \qquad ...(12.6)$$

or $$\frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \tau_{YZ}}{\partial y} + \frac{\partial \sigma_{Z'}}{\partial z} + Z = 0$$

Equation 12.6 are equilibrium equations in terms of effective stresses, in which the body forces are given by

$$X = i_x \cdot \gamma_w = \gamma_w \frac{\partial h}{\partial x}$$

(Since, from equation 9.6 seepage force per unit volume is given by $J = i\ \gamma_w$)

$$Y = i_Y \cdot \gamma_w = \gamma_w \frac{\partial h}{\partial y} \text{ and } Z = \gamma' + i_Z \cdot \gamma_w = \gamma' + \gamma_w \frac{\partial h}{\partial z}$$

where γ' = effective unit weight.

Substituting these values of body forces, in equation 12.6, we get

$$\frac{\partial \sigma_{X'}}{\partial x} + \frac{\partial \tau_{YX}}{\partial y} + \frac{\partial \tau_{ZX}}{\partial z} + \gamma_w \frac{\partial h}{\partial x} = 0$$

$$\frac{\partial \tau_{XY}}{\partial x} + \frac{\partial \sigma_{Y'}}{\partial y} + \frac{\partial \tau_{ZY}}{\partial z} + \gamma_w \frac{\partial h}{\partial y} = 0$$

$$\frac{\partial \tau_{XZ}}{\partial x} + \frac{\partial \tau_{YZ}}{\partial y} + \frac{\partial \sigma_{Z'}}{\partial z} + \gamma_w \frac{\partial h}{\partial z} + \gamma' = 0$$

The above set of equations is the same as equation 12.5 derived earlier from the equilibrium in terms of total stresses. Hence we conclude that the general form of equilibrium equations (Equation 12.3) can be represented both in terms of total stresses as well as effective stresses, provided appropriate values of

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the body forces are substituted in each case. If σ_X, σ_Y and σ_Z of Eqs. 12.3 are designated as total stresses, then corresponding body forces will be $X = 0$, $Y = 0$ and $Z = \gamma$. If however, σ_X, σ_Y and σ_Z of Eqs. 12.3. are considered as the effective stresses, the corresponding body forces will be

$$X = \gamma_w \frac{\partial h}{\partial x},\ Z = \gamma_w \frac{\partial h}{\partial y} \text{ and } Z = \gamma' + \gamma_w \frac{\partial h}{\partial z}.$$

12.4 STRAIN COMPONENTS : STRAIN TENSOR

Let u, v and w be the displacements in x, y and z directions respectively. For a three dimensional case, there are six strain components: ε_X, ε_Y, ε_Z, Υ_{XY}, Υ_{YZ} and Υ_{ZX}. The three linear strain components are defined by:

$$\varepsilon_X = \frac{\partial u}{\partial x} \qquad ...(12.7\ a)$$

$$\varepsilon_Y = \frac{\partial v}{\partial y} \qquad ...(12.7\ b)$$

$$\varepsilon_Z = \frac{\partial w}{\partial z} \qquad ...(12.7\ c)$$

In order to find the other three strain components (called the shearing strain components), consider a plane lamina of size dx, dy in the x, y plane. The lines OA and OB originally orthogonal to each other, are displaced to the positions $O'A'$ and $O'B'$ respectively. The shearing strain is equal to the change in angle at O

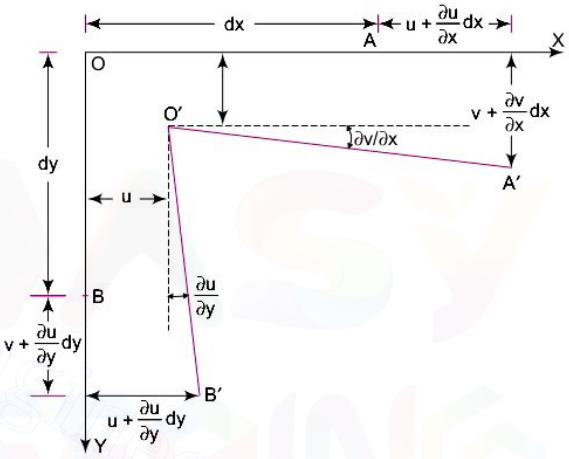

Fig. 12.3 Shear strains

Displacement of A in x-direction $= u + \dfrac{\partial u}{\partial x} dx$

Displacement of B in y-direction $= v + \dfrac{\partial v}{\partial y} dy$

Displacement of A in y-direction $= v + \dfrac{\partial v}{\partial x} dx$

Displacement of B in x-direction $= u + \dfrac{\partial u}{\partial y} dy$

$\therefore$ Total change in the angle at $O = \Upsilon_{XY} = \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}$ ...(12.8 a)

Similarly, $\Upsilon_{YZ} = \dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}$ ...(12.8 b) ; and $\Upsilon_{ZX} = \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}$ ...(12.8 c)

It can be shown that linear strain of a diagonal is equal to half the shearing strain. Thus, if, ε_{XY}, ε_{YZ} and ε_{ZX} represent the linear strains of the diagonals of a plane lamina,

We have $\varepsilon_{XY} = \dfrac{1}{2}\Upsilon_{XY}$; $\varepsilon_{YZ} = \dfrac{1}{2}\Upsilon_{YZ}$; $\varepsilon_{ZX} = \dfrac{1}{2}\Upsilon_{ZX}$ ...(12.9)

Therefore, the *strain tensor* consisting of nine strain components; can be represented as under:

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$$\begin{pmatrix} \varepsilon_{XX} & \varepsilon_{XY} & \varepsilon_{XZ} \\ \varepsilon_{YX} & \varepsilon_{YY} & \varepsilon_{YZ} \\ \varepsilon_{ZX} & \varepsilon_{ZY} & \varepsilon_{ZZ} \end{pmatrix} \text{ or } \begin{pmatrix} \varepsilon_X & \frac{1}{2}\Upsilon_{XY} & \frac{1}{2}\Upsilon_{XZ} \\ \frac{1}{2}\Upsilon_{YX} & \varepsilon_Y & \frac{1}{2}\Upsilon_{YZ} \\ \frac{1}{2}\Upsilon_{ZX} & \frac{1}{2}\Upsilon_{ZY} & \varepsilon_Z \end{pmatrix}$$

12.5 COMPATIBILITY EQUATIONS

The equations resulting from the application of the strain equations are known as the compatibility equations, or the Saint-Venant's equations.

Differentiating Eq. 12.7 (*a*) twice with respect to *y*, Eq. 12.7 (*b*) twice with respect to *x* and Eq. 12.8 (*a*) once with respect to *x* and then with respect to *y*, we get

$$\frac{\partial^2 \varepsilon_X}{\partial y^2} = \frac{\partial^3 u}{dx\,\partial y^2} \quad ...(i) \qquad \frac{\partial^2 \varepsilon_Y}{\partial x^2} = \frac{\partial^3 v}{\partial y\,\partial x^2} \quad ...(ii)$$

and $$\frac{\partial^2 \Upsilon_{XY}}{\partial x\,\partial y} = \frac{\partial^3 u}{\partial x\,\partial y^2} + \frac{\partial^3 v}{\partial x^2\,\partial y} \quad ...(iii)$$

By inspection from (*i*), (*ii*) and (*iii*), we get $\frac{\partial^2 \varepsilon_X}{\partial y^2} + \frac{\partial^2 \varepsilon_Y}{\partial x^2} = \frac{\partial^2 \Upsilon_{XY}}{\partial x\,\partial y}$ ...(12.10 *a*)

Similarly, $\frac{\partial^2 \varepsilon_Y}{\partial z^2} + \frac{\partial^2 \varepsilon_Z}{\partial y^2} = \frac{\partial^2 \Upsilon_{YZ}}{\partial y\,\partial z}$ ...(12.10 *b*) and $\frac{\partial^2 \varepsilon_Z}{\partial x^2} + \frac{\partial^2 \varepsilon_X}{\partial z^2} = \frac{\partial^2 \Upsilon_{ZX}}{\partial z\,\partial x}$ ...(12.10 *c*)

Again, from Eq. 12.7 (*a*), $\frac{\partial^2 \varepsilon_X}{\partial y\,\partial x} = \frac{\partial^3 u}{\partial x\,\partial y\,\partial z}$ ...(*iv*)

From Eq. 12.8 (*a*), $\frac{\partial^2 \Upsilon_{XY}}{\partial z\,\partial x} = \frac{\partial^3 v}{\partial x^2\,\partial z} + \frac{\partial^3 u}{\partial y\,\partial z\,\partial x}$ ...(*v*)

From Eq. 12.8 (*b*), $\frac{\partial^2 \Upsilon_{YZ}}{\partial x^2} = \frac{\partial^3 w}{\partial x^2\,\partial y} + \frac{\partial^3 v}{\partial x^2\,\partial z}$ ...(*vi*)

From Eq. 12.8 (*c*), $\frac{\partial^2 \Upsilon_{ZX}}{\partial y\,\partial x} = \frac{\partial^3 w}{\partial x^2\,\partial y} + \frac{\partial^3 u}{\partial y\,\partial z\,\partial x}$ ...(*vii*)

From (*iv*), (*v*), (*vi*) and (*vii*), we have

$$2\frac{\partial^2 \varepsilon_X}{\partial y\,\partial z} = \frac{\partial^2 \Upsilon_{XZ}}{\partial z\,\partial x} - \frac{\partial^2 \Upsilon_{YZ}}{\partial x^2} + \frac{\partial^2 \Upsilon_{ZX}}{\partial y\,\partial x}$$

or $$2\frac{\partial^2 \varepsilon_X}{\partial y\,\partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \Upsilon_{YZ}}{\partial x} + \frac{\partial \Upsilon_{ZX}}{\partial y} + \frac{\partial \Upsilon_{XY}}{\partial z}\right) \quad ...(12.10\ d)$$

Similarly, $$2\frac{\partial^2 \varepsilon_Y}{\partial z\,\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial \Upsilon_{YZ}}{\partial x} - \frac{\partial \Upsilon_{ZX}}{\partial y} + \frac{\partial \Upsilon_{XY}}{\partial z}\right) \quad ...(12.10\ e)$$

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and $$2\frac{\partial^2 \varepsilon_Z}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \Upsilon_{YZ}}{\partial x} + \frac{\partial \Upsilon_{ZX}}{\partial y} - \frac{\partial \Upsilon_{XY}}{\partial z}\right) \quad ...(12.10\,f)$$

Equation 12.10 are the compatibility equations.

12.6 BOUNDARY CONDITION EQUATIONS

The complete solution of an elasticity problem is obtained by the solution of equilibrium and compatibility equations, but the final solution must also satisfy the boundary condition equations. In order to derive the boundary condition equations, consider a boundary plane *ABC* (Fig. 12.4 *a*) with *l*, *m* and *n* as the direction consines of the external normal to its surface at any point. Let $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ be the components of surface forces per unit area on the elementary area *ABC*. Figure 12.4 *a* shows the nine stress components on the faces *OBC*, *OAC* and *OAB*. If the element volume is considered to be shrunk to a point, these nine stress components are assumed to act at the point.

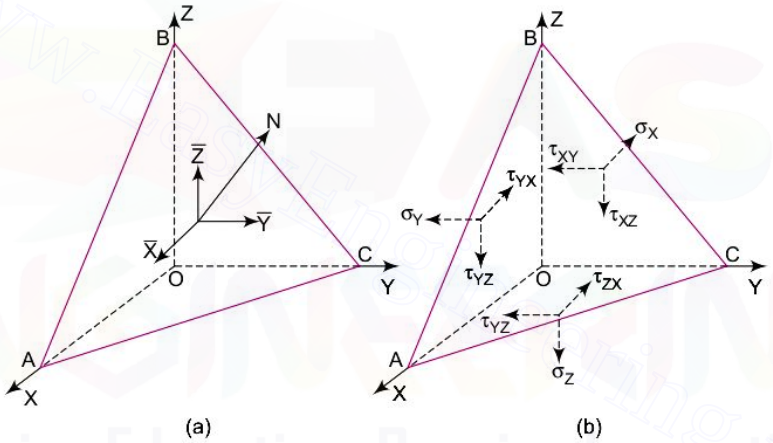

Fig. 12.4 Boundary conditions

Let area $ABC = ds \quad \therefore \quad$ Area $OBC = ds \cos(N, x) = ds \cdot l$

$OAB = ds \cos(N, y) = ds \cdot m$; $OAC = ds \cos(N, z) = ds \cdot n$

Resolving all the forces in *x*-direction, and equating them to zero, we get

$$\Sigma X = 0 = \bar{X} ds - \sigma_X ds\, l - \tau_{YX} ds\, m - \tau_{ZX} ds\, n$$

or $$\bar{X} = \sigma_X l + \tau_{YX} m + \tau_{ZX} n \quad ...(12.11\,a)$$

Similarly, $$\bar{Y} = \sigma_Y m + \tau_{ZY} n + \tau_{XY} l \quad ...(12.11\,b)$$

and $$\bar{Z} = \sigma_Z n + \tau_{XZ} l + \tau_{YZ} m \quad ...(12.11\,c)$$

These are the *boundary condition equations,* which are also sometimes written in the matrix form:

$$\begin{bmatrix} \bar{X} \\ \bar{Y} \\ \bar{Z} \end{bmatrix} = \begin{bmatrix} \sigma_X & \tau_{YX} & \tau_{ZX} \\ \tau_{XY} & \sigma_Y & \tau_{ZY} \\ \tau_{XZ} & \tau_{YZ} & \sigma_Z \end{bmatrix} \begin{bmatrix} l \\ m \\ n \end{bmatrix} \quad ...(12.11)$$

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12.7 GENERALISED HOOKE'S LAW : HOMOGENEITY AND ISOTROPY

In its simplest form, Hooke's law states that, within elastic limits, stress is proportional to strain. Following are the generalised Hooke's law equations (due to Navier-Stockes) connecting the strain components to stress components:

$$\varepsilon_X = C_{11}\sigma_X + C_{12}\sigma_Y + C_{13}\sigma_Z + C_{14}\tau_{XY} + C_{15}\tau_{YZ} + C_{16}\tau_{ZX} \quad ...(12.12\ a)$$

$$\varepsilon_Y = C_{21}\sigma_X + C_{22}\sigma_Y + C_{23}\sigma_Z + C_{24}\tau_{XY} + C_{25}\tau_{YZ} + C_{26}\tau_{ZX} \quad ...(12.12\ b)$$

$$\varepsilon_Z = C_{31}\sigma_X + C_{32}\sigma_v + C_{33}\sigma_Z + C_{34}\tau_{XY} + C_{35}\tau_{YZ} + C_{36}\tau_{ZX} \quad ...(12.12\ c)$$

$$\Upsilon_{XY} = C_{41}\sigma_X + C_{42}\sigma_Y + C_{43}\sigma_Z + C_{44}\tau_{XY} + C_{45}\tau_{YZ} + C_{46}\tau_{ZX} \quad ...(12.12\ d)$$

$$\Upsilon_{YZ} = C_{51}\sigma_X + C_{52}\sigma_Y + C_{53}\sigma_Z + C_{54}\tau_{XY} + C_{55}\tau_{YZ} + C_{56}\tau_{ZX} \quad ...(12.12\ e)$$

$$\Upsilon_{ZX} = C_{61}\sigma_X + C_{62}\sigma_Y + C_{63}\sigma_Z + C_{64}\tau_{XY} + C_{65}\tau_{YZ} + C_{66}\tau_{ZX} \quad ...(12.12\ f)$$

These equations contain 36 *elastic constants*. These elastic constants are independent of the stress components at a point. An elastic body is said to be *homogeneous* if there are only 36 elastic constants, and they are the same at all points within a region. *A* point in a soil body is called *isotropic* if its elastic constants are the *same* in all *directions* at the point. By a series of rotations of axes, it can be shown (Harr. 1966) that the number of independent elastic constants for a homogeneous, isotropic body are only 2. The considerations of homogeneity restrict the total elastic constants within a region to a finite value of 36, while the considerations of isotropy further reduces these constants to only 2. The generalised Hooke's law equations (equation 12.12) then reduce to:

$$\varepsilon_X = C_{11}\sigma_X + C_{12}(\sigma_Y + \sigma_Z);\quad \varepsilon_Y = C_{11}\sigma_Y + C_{12}(\sigma_Z + \sigma_X);\quad \varepsilon_Z = C_{11}\sigma_Z + C_{12}(\sigma_X + \sigma_Y) \quad ...(12.13)$$

$$\Upsilon_{XY} = 2(C_{11} - C_{12})\tau_{XY};\qquad \Upsilon_{YZ} = 2(C_{11} - C_{12})\tau_{YZ};\qquad \Upsilon_{ZX} = 2(C_{11} - C_{12})\tau_{ZX} \quad ...(12.14)$$

In order to evaluate the values of the two constants C_{11} and C_{12} let us take the case of uniaxial stress (such as in unconfined compression tests) in the x-direction, with $\sigma_Y = \sigma_Z = 0$

$\therefore \quad \varepsilon_X = C_{11}\sigma_X$

From which $\quad C_{11} = \dfrac{\varepsilon_X}{\sigma_X} = \dfrac{1}{E}$, where E = Young's modulus of elasticity.

Also, $\quad \varepsilon_Y = \varepsilon_Z = C_{12}\sigma_X = C_{12}E\,.\,\varepsilon_X$

$\therefore \quad C_{12} = \dfrac{1}{E}\dfrac{\varepsilon_Y}{\varepsilon_X} = -\dfrac{\mu}{E}$, where μ = Poisson's ratio = $\dfrac{1}{m}$

Substituting these in Eqs. 12.13, we get final equations as under:

$$\varepsilon_X = \frac{1}{E}[\sigma_X - \mu(\sigma_Y + \sigma_Z)] \quad ...(12.14\ a) \qquad \varepsilon_Y = \frac{1}{E}[\sigma_Y - \mu(\sigma_Z + \sigma_X)] \quad ...(12.14\ b)$$

$$\varepsilon_Z = \frac{1}{E}[\sigma_Z - \mu(\sigma_Z + \sigma_{YZ})] \quad ...(12.14\ c) \qquad \Upsilon_{XY} = \frac{2(1+\mu)}{E}\tau_{XY} \quad ...(12.14\ d)$$

$$\Upsilon_{YZ} = \frac{2(1+\mu)}{E}\tau_{YZ} \quad ...(12.14\ e) \qquad \Upsilon_{ZX} = \frac{2(1+\mu)}{E}\tau_{ZX} \quad ...(12.14\ f)$$

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12.8 TYPICAL VALUES OF MODULUS OF ELASTICITY AND POISSON'S RATIO

Table 12.1 and 12.2 show respectively the typical ranges of values of modulus of elasticity E and Poisson's ratio μ (Bowles, 1988).

Table 12.1 Values of E

	Type of soil	*Range of values of E (MP a or kN/m² × 1000)*
1	Clay	
	(*i*) Very soft	2–15
	(*ii*) Soft	5–25
	(*iii*) Medium	15–20
	(*iv*) Hard	50–100
	(*v*) Sandy	25–250
2	Loess	15–60
3.	Sand	
	(*i*) Silty	5–20
	(*ii*) Loose	10–25
	(*iii*) Dense	50–81
4.	Sand and Gravel	
	(*i*) Loose	50–150
	(*ii*) Dense	100–200
5	Silt	2–20

Table 12.2 Values of μ

	Type of soil	*Range of values of* μ
1.	Clay	
	(*i*) Saturated	0.4 – 0.5
	(*ii*) Unsaturated	0.1 – 0.3
2	Sandy clay	0.2 – 0.3
3	Silt	0.3 – 0.35
4	Sand	0.15 – 0.4
5	Loess	0.1 – 0.3

12.9 TWO DIMENSIONAL PROBLEMS

(*a*) Plane stress. If a *thin* plate or layer of soils is uniformly loaded by forces applied at the boundary, parallel to the plane (say xz-plane) of the plate, the stress components $\sigma_Y = \tau_{XY} = \tau_{YZ} = 0$ on both the face of the plate. Such a state of stresses is called the *plane stress*. The components of stress are σ_X, σ_Z,

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and τ_{XZ} ; these components are independent of y (*i.e.*, they do not vary with y). The Hooke's law equation for plane stress case are:

$$\varepsilon_X = \frac{1}{E}(\sigma_X - \mu\sigma_Z);\ \varepsilon_Z = \frac{1}{E}(\sigma_Z - \mu\sigma_X);\ \varepsilon_Y = \frac{\mu}{E}(\sigma_X + \sigma_Z) \qquad ...(12.15)$$

$$\Upsilon_{XZ} = \frac{2(1+\mu)}{E}\tau_{XZ} \text{ and } \Upsilon_{XY} = \Upsilon_{YZ} = 0 \qquad (\because \tau_{XY} = \tau_{YZ} = 0)$$

(*b*) Plain strain. There are many problems in soil mechanics—a retaining wall with lateral pressure, a continuous footing, a continuous slope or a tunnel—in which one dimension (say, in y-direction) is very large in comparison to the other two. If such bodies are loaded by forces which are perpendicular to the longitudinal elements (in the long direction y) and do not vary along the length, all cross-sections will be in the same direction. The state of affairs existing in the xz plane through a point holds for all planes parallel to it. Such a case is known as *plane strain* case. The strain components ε_Y, Υ_{XY} and Υ_{ZY} will each be zero. The other strain components ε_X, ε_Y and Υ_{XZ} are given by the following Hooke's law equations:

$$\varepsilon_Y = 0 = \frac{1}{E}[\sigma_Y - \mu(\sigma_Z + \sigma_X)] \text{ or } \sigma_Y = \mu(\sigma_Z + \sigma_X) \qquad ...(12.16)$$

$$\therefore \quad \varepsilon_X = \frac{1}{E}[\sigma_X - \mu\sigma_Y - \mu\sigma_Z] = \frac{1}{E}[\sigma_X - \mu\sigma_Z - \mu^2(\sigma_X + \sigma_Z)]$$

or
$$\varepsilon_X = \frac{1-\mu^2}{E}\left[\sigma_X - \frac{\mu}{1-\mu}\sigma_Z\right] \qquad ...(12.17\ a)$$

Similarly
$$\varepsilon_Z = \frac{1-\mu^2}{E}\left[\sigma_Z - \frac{\mu}{1-\mu}\sigma_X\right] \qquad ...(12.17\ b)$$

and
$$\Upsilon_{XZ} = \frac{2(1+\mu)}{E}\tau_{XZ} \qquad ...(12.17\ c)$$

$$\Upsilon_{XY} = \Upsilon_{XZ} = 0 \text{ [Hence } \tau_{XY} = \tau_{XZ} = 0]$$

Equilibrium equations. For both plane stress as well as plane strain case, the equilibrium equations are as follows:

$$\frac{\partial\sigma_X}{\partial x} + \frac{\partial\tau_{XZ}}{\partial z} + X = 0;\ \frac{\partial\tau_{XZ}}{\partial x} + \frac{\partial\sigma_Z}{\partial z} + Z = 0 \qquad ...(12.18)$$

12.10 COMPATIBILITY EQUATION IN TWO DIMENSIONAL CASE

For the two dimensional case, the six compatibility equations (Eqs. 12.10) evidently reduce to one single equation:

$$\frac{\partial^2\varepsilon_X}{\partial z^2} + \frac{\partial^2\varepsilon_Z}{\partial x^2} = \frac{\partial^2\Upsilon_{XZ}}{\partial x\,\partial z} \qquad ...(12.19)$$

Compatibility equation in terms of stress. The above compatibility equation in terms of strain can be converted into the compatibility equation in terms of stress. We shall consider both the cases-plane stress case and plane strain case.

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(1) Plane stress case

Substituting the value of ε_X, ε_Z and Υ_{XZ} from the Hooke's law equation, Eq.12.15 to Eq.12.19, we get

$$\frac{\partial^2}{\partial z^2}(\sigma_X - \mu\sigma_Z) + \frac{\partial^2}{\partial x^2}(\sigma_Z - \mu\sigma_X) = 2(1+\mu)\frac{\partial^2 \tau_{XZ}}{\partial x\,\partial z} \quad ...(i)$$

Again, differentiating first of the equilibrium equations, (Eq. 12.18 *a*), with respect to x and second (Eq. 12.18 *b*) with respect to z and adding them together, we get

$$2\frac{\partial^2 \tau_{XZ}}{\partial x\,\partial z} = -\left(\frac{\partial^2 \sigma_X}{\partial x^2} + \frac{\partial^2 \sigma_Z}{\partial z^2}\right) - \left(\frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z}\right) \quad ...(ii)$$

Substituting the value of $\frac{2\,\partial^2 \tau_{XZ}}{\partial x\,\partial z}$ in (*i*), we get

$$\frac{\partial^2}{\partial z^2}(\sigma_X - \mu\sigma_Z) + \frac{\partial^2}{\partial x^2}(\sigma_Z - \mu\sigma_X) + (1+\mu)\left(\frac{\partial^2 \sigma_X}{\partial x^2} + \frac{\partial^2 \sigma_Z}{\partial x^2}\right) + (1+\mu)\left(\frac{\partial X}{\partial x^2} + \frac{\partial Z}{\partial z}\right) = 0$$

This, on simplification, gives

$$\frac{\partial^2 \sigma_X}{\partial z^2} + \frac{\partial^2 \sigma_Z}{\partial x^2} + \frac{\partial^2 \sigma_X}{\partial x^2} + \frac{\partial^2 \sigma_Z}{\partial z^2} = -(1+\mu)\left(\frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z}\right)$$

or

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)(\sigma_X + \sigma_Z) = -(1+\mu)\left(\frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z}\right) \quad ...(12.20)$$

If body forces are absent, or constant, we have

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)(\sigma_X + \sigma_Z) = 0 \quad ...(12.21)$$

which is the required compatibility equation in terms of stress, for the plane stress case.

(2) Plane strain case

Substituting the values of ε_X, ε_Y and Υ_{XZ} (Eq.12.17), we get from Eq.12.19.

$$(1-\mu^2)\frac{\partial^2 \sigma_X}{\partial z^2} - \mu(1+\mu)\frac{\partial^2 \sigma_Z}{\partial z^2} + (1-\mu^2)\frac{\partial^2 \sigma_Z}{\partial x^2} - \mu(1+\mu)\frac{\sigma^2 \sigma_X}{\partial x^2} = 2(1+\mu)\frac{\partial^2 \tau_{XZ}}{\partial x\,\partial z} \quad ...(iii)$$

Substituting the value of $\frac{2\,\partial^2 \tau_{XZ}}{\partial x\,\partial z}$ as found in (*ii*), we get

$$(1-\mu^2)\frac{\partial^2 \sigma_X}{\partial z^2} - \mu(1+\mu)\frac{\partial^2 \sigma_X}{\partial z^2} + (1-\mu^2)\frac{\partial^2 \sigma_Z}{\partial x^2} - \mu(1+\mu)\frac{\sigma^2\,\sigma_X}{\partial x^2} + (1+\mu)\left[\frac{\partial^2 \sigma_X}{\partial x^2} + \frac{\partial^2 \sigma_Z}{\partial z^2} + \frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z}\right] = 0$$

which on simplification, reduces to,

$$\frac{\partial^2 \sigma_X}{\partial z^2} + \frac{\partial^2 \sigma_Z}{\partial z^2} + \frac{\partial^2 \sigma_Z}{\partial x^2} + \frac{\partial^2 \sigma_X}{\partial x^2} = -\frac{1}{1-\mu}\left(\frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z}\right)$$

or

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)(\sigma_X + \sigma_Z) = -\frac{1}{1-\mu}\left(\frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z}\right) \quad ...(12.22)$$

If the body forces are absent, or constant, we have

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$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)(\sigma_X+\sigma_Z)=0 \qquad ...(12.23)$$

which is the same as equation 12.21 found for the plane stress case. Thus, in case of constant body forces (or no body forces), same compatibility equation holds both for the case of plane stress and for case of plane strain. Hence the stress distribution is the same in both the cases, provided the shape of the boundary and the external forces are the same. Also, the stress distribution is the same for all the isotropic materials, since Eq. 12.21 or Eq. 12.23 do not contain any elastic constant. The photo-elastic method of determination of stress distribution is based on this conclusion.

12.11 STRESS FUNCTION

The solution of a two dimensional problem of elasticity reduces to the integration of the differential equation of equilibrium together with the compatibility equation and the boundary conditions. The equations are:

$$\frac{\partial\sigma_X}{\partial x}+\frac{\partial\tau_{XZ}}{\partial z}=0 \quad ...(12.24\,a) \qquad \frac{\partial\tau_{XZ}}{\partial x}+\frac{\partial\sigma_Z}{\partial z}+\gamma=0 \quad ...(12.24\,b)$$

and $$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)(\sigma_X+\sigma_Z)=0 \qquad ...(12.24\,c)$$

It is usual to reduce three above the equations into one single equation in terms of the so-called '*stress junction*' Φ defined by

$$\sigma_X=\frac{\partial^2\Phi}{\partial z^2};\ \sigma_Z=\frac{\partial^2\Phi}{\partial x^2};\ \text{and}\ \tau_{XZ}=-\frac{\partial^2\Phi}{\partial x\,\partial z}-\gamma.x \qquad ...(12.25)$$

The above function Φ is called the Airy *stress function,* and was introduced by G.B. Airy in 1862. It can be easily seen that the stress function Φ defined by Eq. 12.25, satisfies equilibrium equations (Eqs. 12.24 *a*, *b*). Performing differentiation on Eqs. 12.25,

we get $$\frac{\partial\sigma_X}{\partial x}=\frac{\partial^3\Phi}{\partial x\,\partial z^2};\ \frac{\partial\tau_{XZ}}{\partial z}=-\frac{\partial^3\Phi}{\partial x\,\partial z^2}$$

$$\frac{\partial\sigma_Z}{\partial z}=\frac{\partial^3\Phi}{\partial z\,\partial x^2};\ \frac{\partial\tau_{XZ}}{\partial x}=-\frac{\partial^3\Phi}{\partial x^2\,\partial z}-\gamma$$

$\therefore$ $$\frac{\partial\sigma_X}{\partial x}+\frac{\partial\tau_{XZ}}{\partial z}=\frac{\partial^3\Phi}{\partial x\,\partial z^2}-\frac{\partial^3\Phi}{\partial x\,\partial z^2}=0\ \text{(satisfied)}$$

$$\frac{\partial\tau_{XZ}}{\partial x}+\frac{\partial\sigma_Z}{\partial z}+\gamma=-\frac{\partial^3\Phi}{\partial x^2\,\partial z}-\gamma+\frac{\partial^3\Phi}{\partial z\,\partial x^2}+\gamma=0\ \text{(satisfied)}$$

Again, substituting the proper derivatives of stress components as function of Airy stress function in the compatibility equation Eq. 12.24 *c*, we get

$$\frac{\partial^4\Phi}{\partial x^4}+2\frac{\partial^4\Phi}{\partial x^2\,\partial z^2}+\frac{\partial^4\Phi}{\partial z^4}=0 \text{ or } \nabla^2\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2}\right)=0 \quad \text{or} \quad \nabla^4\Phi=0 \qquad ...(12.26)$$

This is a biharmonic differential equation of fourth order, and is known as the compatibility equation in terms of stress function. The solution of Eq. 12.26 must also satisfy the boundary conditions. In soil

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mechanics problems, it is usual to find the stresses due to the body forces (*i.e.*, self-weight and seepage forces, etc.) and those due to boundary forces separately.

In that case, Eq. 12.25 *c* is modified to $\tau_{XZ} = -\dfrac{\partial^2 \Phi}{\partial x\, \partial z}$

The solution of two dimensional problems is thus reduced to the integration of the differential equation 12.26, having regard to the boundary conditions. In case of long rectangular strips, the solution of the biharmonic equation is done in the form of polynomials of various degrees.

12.12 EQUILIBRIUM EQUATIONS IN POLAR CO-ORDINATES

In most of the soil engineering problems, it is advantageous to use polar co-ordinates.

An infinitesimal element of soil in the polar co-ordinate system is shown in Fig. 12.5.

Let σ_r = normal stress component in radial direction

σ_θ = normal stress component in circumferential direction.

$\tau_{r\theta}$ = shear stress component acting tangentially to the four faces.

Let the stresses on the four faces be further defined by suffixes 1, 2, 3, and 4 respectively.

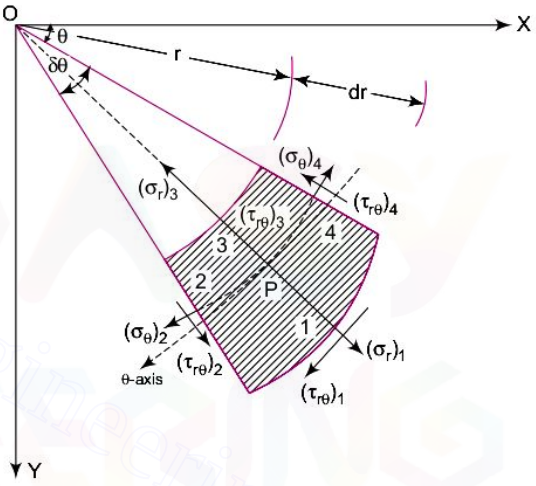

Fig. 12.5 Stresses in polar co-ordinates

Resolving the forces in θ direction, and noting that

$$\sin\frac{d\theta}{2} \approx \frac{d\theta}{2};\ \cos\frac{d\theta}{2} \approx 1$$

and assuming body forces in θ direction to be zero, we get

$$\{(\sigma_\theta)_2 - (\sigma_\theta)_4\}\, dr \cos\frac{d\theta}{2} + \{(\tau_{r\theta})_2 + (\tau_{r\theta})_4\}\, dr \sin\frac{d\theta}{2} + \{(\tau_{r\theta})_1 - (\tau_{r\theta})_3\}\, r\, d\theta = 0$$

or $$\{(\sigma_\theta)_2 - (\sigma_\theta)_4\}\, dr + \{(\tau_{r\theta})_2 + (\tau_{r\theta})_4\}\, dr\, \frac{d\theta}{2} + \{(\tau_{r\theta})_1 - (\tau_{r\theta})_3\}\, r\, d\theta = 0$$

Dividing by $dr\, d\theta$, we get

$$\frac{(\sigma_\theta)_2 - (\sigma_\theta)_4}{d\theta} + \frac{(\tau_{r\theta})_2 + (\tau_{r\theta})_4}{2} + \frac{\{(\tau_{r\theta})_1 - (\tau_{r\theta})_3\}\, r}{dr} = 0 \qquad ...(i)$$

Assuming the element to be shrunk to a point, we have

$$\frac{(\sigma_\theta)_2 - (\sigma_\theta)_4}{d\theta} = \frac{\partial \sigma_\theta}{d\theta};\ \frac{(\tau_{r\theta})_2 + (\tau_{r\theta})_4}{2} = \tau_{r\theta}$$

and $$\frac{\{(\tau_{r\theta})_1 - (\tau_{r\theta})_3\}\, r}{dr} = \frac{\partial (\tau_{r\theta} \cdot r)}{\partial r} = \tau_{r\theta} + r\frac{\partial \tau_{r\theta}}{\partial r}$$

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Substituting in (*i*), we get $\frac{\partial \sigma_\theta}{\partial \theta} + \tau_{r\theta} + \left(\tau_{r\theta} + r \frac{\partial \tau_{r\theta}}{\partial r} \right) = 0$

which gives $\frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = 0$ ...(I) ...(12.27 *a*)

which is the equilibrium equation in θ-direction.

Similarly, the resolving in *r*-direction, we get

$$\{(\sigma_r)_1 - (\sigma_r)_3\}\, r d\theta - \{(\sigma_\theta)_2 + (\sigma_\theta)_4\}\, dr \sin \frac{d\theta}{2} + \{(\tau_{r\theta})_2 - (\tau_{r\theta})_4\}\, dr \cos \frac{d\theta}{2} = 0 \quad ...(ii)$$

or $\{(\sigma_r)_1 - (\sigma_r)_3\}\, r d\theta - \{(\sigma_\theta)_2 + (\sigma_\theta)_4\}\, dr \frac{d\theta}{2} + \{(\tau_{r\theta})_2 - (\tau_{r\theta})_4\}\, dr = 0$ ...(*ii*)

Dividing by $dr\, d\theta$, we get $\frac{\{(\sigma_r)_1 + (\sigma_r)_3\} r}{dr} - \frac{(\sigma_\theta)_2 + (\sigma_\theta)_4}{2} + \frac{(\tau_{r\theta})_2 - (\tau_{r\theta})_4}{d\theta} = 0$

or $\frac{\partial (\sigma_r) r}{dr} - \sigma_\theta + \frac{\partial \tau_{r\theta}}{\partial \theta} = 0$

or $r \frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta + \frac{\partial \sigma_{r\theta}}{\partial \theta} = 0$ which may be re-written as

$$\frac{\partial \sigma_r}{\partial r} + \left(\frac{\sigma_r - \sigma_\theta}{r} \right) + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} = 0 \quad ...(II) \quad ...(12.27\ b)$$

which is the second equilibrium equation. If, however, R is the body force per unit volume in the r-direction, one more term $R \cdot r d\theta \cdot dr$ will added to the LHS in equation (*ii*) and equation II will be modified as

$$\frac{\partial \sigma_r}{\partial r} + \left(\frac{\sigma_r - \sigma_\theta}{r} \right) + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + R = \mathbf{0}. \quad ...(II\ a) \quad ...(12.27\ c)$$

12.13 COMPATIBILITY EQUATIONS AND STRESS FUNCTION IN POLAR CO-ORDINATES

It can be shown that the compatibility equation in terms of stress components, in polar co-ordinates, is given by

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) (\sigma_r + \sigma_\theta) = 0 \quad ...(12.28)$$

The stress function Φ is defined in terms of components in polar co-ordinates as follows:

$$\sigma_r = \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2}; \quad \sigma_\theta = \frac{\partial^2 \Phi}{\partial r^2} \text{ and } \tau_{r\theta} = \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial r\, \partial \theta} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \Phi}{\partial \theta} \right) \quad ...(12.29)$$

To yield a possible stress distribution, this stress function must ensure that the condition of compatibility (Eq. 12.28) is satisfied. In cartesian co-ordinates, this condition is $\nabla^4 \Phi =$ (Eq. 12.26). A corresponding condition in polar co-ordinates can be obtained by transforming Eq. 12.26 in polar co-ordinates by the substitutions: $r^2 = x^2 + z^2$ and $\theta = \tan^{-1} \left(\frac{z}{x} \right)$. Thus, the compatibility equation, in terms of Φ in polar co-ordinates becomes:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right) \left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \right) = 0 \text{ or } \nabla_r^2 \left(\nabla_r^2 \Phi \right) = 0 \quad ...(12.30)$$

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Axis symmetric case. If the stress distribution is symmetrical with respect to the axis through O perpendicular to the xy-plane, the stress components are functions of r only, and do not vary with θ. The two equilibrium equations then reduce to one single equation:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad ...(12.31)$$

The compatibility equation reduces to

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)\left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r}\right) = 0$$

$$\text{or } \frac{\partial^4 \Phi}{dr^4} + \frac{2}{r}\frac{d^3\Phi}{\partial r^3} + \frac{1}{r^2}\frac{d^2\Phi}{dr^2} + \frac{1}{r^3}\frac{d\Phi}{dr} = 0. \qquad ...(12.32)$$

12.14 CYLINDRICAL CO-ORDINATES

For many stress problems in soil mechanics, such as the stress distribution due to single concentrated load, the solution requires the use of cylinderical co-ordinates — a three dimensional case. The three axes of reference are: r , θ and z. The normal stress components are σ_θ, σ_r and σ_z, the shear stress components are $\tau_{r\theta}$, $\tau_{\theta z}$, and $\tau_r z$.

Equilibrium equations. They are as follows:

$$\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} + \frac{\partial \tau_{\theta z}}{\partial z} = 0 \qquad ...(12.33\ a)$$

$$\frac{\partial \sigma_r}{\partial z} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{(\sigma_r - \sigma_\theta)}{r} + \frac{\partial \tau_{rz}}{\partial z} = 0 \qquad ...(12.33\ b)$$

$$\text{or} \quad \frac{\partial \sigma_r}{\partial z} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0 \qquad ...(12.33\ c)$$

If, however, we take an *axis symmetric* case, in which the stresses do not vary with θ, and in which the tangential stresses $\tau_{r\theta}$ and $\tau_{\theta z}$ are zero, the above equilibrium equations reduce to the following two equations.

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0; \text{ and } \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} = 0. \qquad ...(12.34)$$

Love's cylindrical stress components. Love (1984) has indicated that, for the plane stress and plane strain case *(i.e.,* axis symmetric case) *in cylindrical co-ordinates* the four stresses σ_r, σ_θ, σ_z and as functions of Airy's stress function Φ can be determined by the third derivative:

$$\sigma_r = \frac{\partial}{\partial z}\left(\mu \nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial r^2}\right) \quad ...(12.35\ a); \quad \sigma_\theta = \frac{\partial}{\partial z}\left(\mu \nabla^2 \Phi - \frac{1}{r}\frac{\partial \Phi}{\partial r}\right) \qquad ...(12.35\ b)$$

$$\sigma_z = \frac{\partial}{\partial z}\left[(2-\mu)\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial r^2}\right] \quad ...(12.35\ c); \text{ and } \tau_{rz} = \frac{\partial}{\partial r}\left[(1-\mu)\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial r^2}\right] \qquad ...(12.35\ d)$$

The corresponding compatibility equation is

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial z^2}\right) = 0 \qquad ...(12.36)$$

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CHAPTER 13

Stress Distribution : I

13.1 INTRODUCTION : STRESSES DUE TO SELF WEIGHT

In this chapter, we shall discuss the stresses and displacements in soil mass, due to various types of surface loading. The derivation of various formulae used for stress distribution in this chapter have been given in chapter 14, wherein some advanced problems of loading have been discussed.

Before proceeding further, we shall consider the stresses within a soil mass due to its own weight. Stresses due to self weight are sometimes known as *geostatic stresses.* Let us take the soil mass to be bounded by the horizontal plane (ground surface) *xy,* and the *z*-axis be directed downwards. Under this condition, the soil mass is said to be *semi-infinite.* Where there is no external loading, the ground plane becomes a principal plane since it is devoid of any shear loading. From the symmetry and the orthogonality of principal planes, one can conclude that all the horizontal and vertical planes will be devoid of shear stresses, so that within the soil mass, $\tau_{XY} = \tau_{XZ} = \tau_{YZ} = 0$. Substituting this in the equilibrium equations (Eq. 12.4), we get

$$\sigma_Z = \gamma z \qquad ...(13.1)$$

where γ = unit weight of soil and σ_Z = vertical stress at a point within and soil mass, situated at a depth z below the ground surface.

Similarly, from compatibility equations in terms of stresses (for a three dimentional case), one obtains

$$\sigma_X = \sigma_Y = \frac{\mu}{1-\mu}\,\gamma z = K_0\,\gamma z \qquad ...(13.2)$$

where μ = Poisson's ratio (Table 12.2) and

$K_0 = \dfrac{\mu}{1-\mu}$ = coefficient of lateral pressure at rest.

Thus, Eqs. 13.1 and 13.2 give stress components at a point situated at depth z below the ground surface, due to self-weight of the soil mass above it.

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At a certain point within the soil mass, the stresses are caused due to both surface loadings as well as due to self-weight of soil above it. The stress components due to both these loading (*i.e.,* self weight and surface loads) can be found separately, and then added algebraically to get then final stresses at the points. In the following articles, we shall, therefore, discuss the stress distribution due to surface loading alone.

Symbols. The total stress field at a point within a soil mass loaded at its boundary, consists of nine stress components given below:

$$\begin{pmatrix} \sigma_X & \tau_{YX} & \tau_{ZX} \\ \tau_{XY} & \sigma_Y & \tau_{ZY} \\ \tau_{XZ} & \tau_{YZ} & \sigma_Z \end{pmatrix}$$

These nine stress components, as given by this group of square matrix of stresses, are the components of a mathematical entity called the *stress tensor,* with a symmetrical matrix relative to its main diagonal (upper left to lower right). The main diagonal elements of the stress tensor are the normal stress components, and the off-diagonal elements are shear stress. Out of the nine stress components indicated above, there are three independent shear components, making the total unknowns to be equal to six (see § 12.1 also). The corresponding nine strain components are given by the following strain tensor :

$$\begin{pmatrix} \varepsilon_X & \frac{1}{2}\Upsilon_{XY} & \frac{1}{2}\Upsilon_{XZ} \\ \frac{1}{2}\Upsilon_{YX} & \varepsilon_Y & \frac{1}{2}\Upsilon_{YZ} \\ \frac{1}{2}\Upsilon_{ZX} & \frac{1}{2}\Upsilon_{ZY} & \varepsilon_Z \end{pmatrix}$$

, where ε denotes the linear or direct strain and γ denotes the shearing strain (see § 12.4).

13.2 CONCENTRATED FORCE : BOUSSINESQ EQUATIONS

Boussinesq (1885) solved the problem of stress distribution in soils due to a concentrated load acting at the ground surface, by assuming a suitable stress function (§14.4). The following assumptions are made in the solutions by the theory of elasticity :

1. The soil mass is an elastic medium, for which the modulus of elasticity E is constant.
2. The soil mass is *homogeneous,* that is, all its constituent parts or elements are similar and it has identical properties at every point in it in identical directions.
3. The soil mass is *isotropic,* that is, it has identical elastic properties in all directions through any point of it.
4. The soil mass is *semi-infinite,* that is, it extends infinitely in all directions below a level surface.

Let a point load Q (single concentrated vertical load) act at the ground surface, at a point O which may be taken as the origin of the x, y and z-axes as shown. Let us find the stress components at a point P in the soil mass, having coordinates x, y and z, or having a radial horizontal distance r and vertical distance z from the point O.

Using the logarithmic stress function (see § 14.4), Boussinesq showed that the polar radial stress may be expressed as

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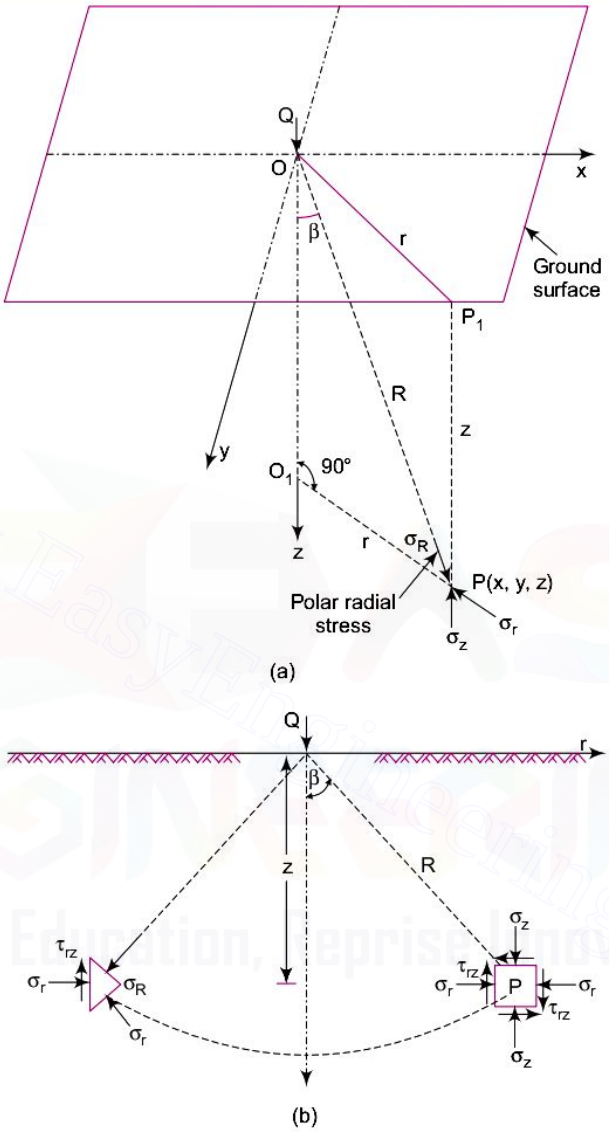

Fig. 13.1 Concentrated load: Boussinesq analysis

$$\sigma_R = \frac{3}{2}\frac{Q}{\pi}\frac{\cos\beta}{R^2} \qquad ...(13.1\ a)$$

where R = polar radial co-ordinate of point $P = \sqrt{r^2 + z^2} = (x^2 + y^2 + z^2)^{1/2}$ and $\cos\beta = \frac{z}{R}$

In the cylindrical co-ordinates the corresponding vertical stress σ_z and tangential stress $\tau_{r,z}$ are given by :

$$\sigma_Z = \sigma_R \cos^2\beta = \frac{3}{2}\frac{Q}{\pi}\frac{\cos^3\beta}{R^2} = \frac{3}{2}\frac{Q}{\pi}\frac{z^3}{R^5} \qquad ...(13.2\ a)$$

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or
$$\sigma_Z = \frac{3Q}{2\pi}\frac{z^3}{\left(r^2+z^2\right)^{5/2}} = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \quad ...(13.2\ b)$$

and
$$\tau_{r\,z} = \frac{1}{2}\sigma_R \sin 2\beta$$
$$= \frac{3}{2}\frac{Q}{\pi}\frac{\cos^2\beta \sin\beta}{R^3} = \frac{3}{2}\frac{Q}{\pi}\frac{z^2 r}{R^5} \quad ...(13.3\ a)$$
$$= \frac{3Q}{2\pi}\frac{r\,z^2}{\left(r^2+z^2\right)^{5/2}} = \frac{3Q\,r}{2\pi z^3}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \quad ...(13.3)$$

It should be emphasized that although both the vertical normal stress and shearing stress are independent of the elastic constants (E and μ) they are very much dependent on the assumptions of linear elasticity.

Eq. 13.2 may be re-written as
$$\sigma_Z = K_B \frac{Q}{z^2} \quad ...(13.4)$$

K_B = Boussinesq *influence factor*
$$= \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \quad ...(13.5)$$

The influence factor is a function of *r/z* ratio which is dimensionless. Table 13.1 gives values of influence factors (Gilboy, 1953). The influence factors for shear stress (Eq.13.3) can also be found by multiplying the factor K_B (Table 13.1) by the ratio *r/z* . Thus, if it is required to find the vertical stress at a point situated at a radial distance *r* and depth *z* below the loaded point, find the influence factor for *r/z* ratio and then multiply it by Q/z^2 to get the vertical stress.

Table 13.1 Boussinesq influence factor for vertical pressure caused by a surface point load

r/z	K_B	*r/z*	K_B	*r/z*	K_B	*r/z*	K_B	*r/z*	K_B	*r/z*	K_B
0.00	0.4775	0.50	0.2733	1.00	0.0844	1.50	0.0251	2.00	0.0085	2.50	0.0034
0.01	0.4773	0.51	0.2679	1.01	0.0823	1.51	0.0245	2.01	0.0084	2.51	0.0033
0.02	0.4770	0.52	0.2625	1.02	0.0803	1.52	0.0240	2.02	0.0082	2.52	0.0033
0.03	0.4764	0.53	0.2571	1.03	0.0783	1.53	0.0234	2.03	0.0081	2.53	0.0032
0.04	0.4755	0.54	0.2518	1.04	0.0764	1.54	0.0229	2.04	0.0079	2.54	0.0032
0.05	0.4745	0.55	0.2466	1.05	0.0744	1.55	0.0224	2.05	0.0078	2.55	0.0031
0.06	0.4723	0.56	0.2414	1.06	0.0727	1.56	0.0219	2.06	0.0076	2.56	0.0031
0.07	0.4717	0.57	0.2363	1.07	0.0709	1.57	0.0214	2.07	0.0075	2.57	0.0030
0.08	0.4699	0.58	0.2813	1.08	0.0691	1.58	0.0209	2.08	0.0073	2.58	0.0030
0.09	0.4679	0.59	0.2263	1.09	0.0674	1.59	0.0204	2.09	0.0072	2.59	0.0029
0.10	0.4657	0.60	0.2214	1.10	0.0658	1.60	0.0200	2.10	0.0070	2.60	0.0029

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0.11	0.4633	0.61	0.2165	1.11	0.0641	1.61	0.0195	2.11	0.0069	2.61	0.0028
0.12	0.4607	0.62	0.2117	1.12	0.0626	1.62	0.0191	2.12	0.0068	2.62	0.0028
0.13	0.4579	0.63	0.2070	1.13	0.0610	1.63	0.0187	2.13	0.0066	2.63	0.0027
0.14	0.4548	0.64	0.2040	1.14	0.0595	1.64	0.0183	2.14	0.0065	2.64	0.0027
0.15	0.4516	0.65	0.1978	1.15	0.0581	1.65	0.0179	2.15	0.0064	2.65	0.0026
0.16	0.4482	0.66	0.1934	1.16	0.0567	1.66	0.0175	2.16	0.0063	2.66	0.0026
0.17	0.4446	0.67	0.1889	1.17	0.0553	1.67	0.0171	2.17	0.0062	2.67	0.0025
0.18	0.4409	0.68	0.1846	1.18	0.0539	1.68	0.0167	2.18	0.0060	2.68	0.0025
0.19	0.4370	0.69	0.1804	1.19	0.0526	1.69	0.0163	2.19	0.0059	2.69	0.0025
0.20	0.4329	0.70	0.1762	1.20	0.0513	1.70	0.0160	2.20	0.0058	2.70	0.0024
0.21	0.4286	0.71	0.1721	1.21	0.0501	1.71	0.0157	2.21	0.0057	2.72	0.0023
0.22	0.4242	0.72	0.1681	1.22	0.0489	1.72	0.0153	2.22	0.0056	2.74	0.0022
0.23	0.4197	0.73	0.1641	1.23	0.0477	1.73	0.0150	2.23	0.0055	2.76	0.0022
0.24	0.4151	0.74	0.1603	1.24	0.0466	1.74	0.0147	2.24	0.0054	2.78	0.0021
0.25	0.4103	0.75	0.1565	1.25	0.0454	1.75	0.0144	2.25	0.0053	3.80	0.0021
0.26	0.4054	0.76	0.1527	1.26	0.0443	1.76	0.0141	2.26	0.0052	2.84	0.0019
0.27	0.4004	0.77	0.1491	1.27	0.0433	1.77	0.0138	2.27	0.0051	2.91	0.0017
0.28	0.3954	0.78	0.1455	1.28	0.0422	1.78	0.0135	2.28	0.0050	2.99	0.0015
0.29	0.3902	0.79	0.1420	1.29	0.0412	1.79	0.0132	2.29	0.0049	3.08	0.0013
0.30	0.3840	0.80	0.1386	1.30	0.0402	1.80	0.0129	2.30	0.0048	3.19	0.0011
0.31	0.3796	0.81	0.1353	1.31	0.0393	1.81	0.0126	2.31	0.0047	3.31	0.0009
0.32	0.3742	0.82	0.1320	1.32	0.0384	1.82	0.0124	2.32	0.0047	3.50	0.0007
0.33	0.3687	0.83	0.1288	1.33	0.0374	1.83	0.0121	2.33	0.0046	3.75	0.0005
0.34	0.3632	0.84	0.1257	1.34	0.0365	1.84	0.0119	2.34	0.0045	4.13	0.0003
0.35	0.3577	0.85	0.1226	1.35	0.0357	1.85	0.0116	2.35	0.0044	4.91	0.0001
0.36	0.3521	0.86	0.1196	1.36	0.0348	1.86	0.0114	2.36	0.0043	6.15	0.0001
0.37	0.3465	0.87	0.1166	1.37	0.0340	1.87	0.0112	2.37	0.0043		
0.38	0.3408	0.88	0.1138	1.38	0.0332	1.88	0.0109	2.38	0.0042		
0.39	0.3351	0.89	0.1110	1.39	0.0324	1.89	0.0107	2.39	0.0041		
0.40	0.3294	0.90	0.1083	1.40	0.0317	1.90	0.0105	2.40	0.0040		
0.41	0.3238	0.91	0.1057	1.41	0.0309	1.91	0.0103	2.41	0.0040		
0.42	0.3181	0.92	0.1031	1.42	0.0302	1.92	0.0101	2.42	0.0039		
0.43	0.3124	0.93	0.1005	1.43	0.0295	1.93	0.0099	2.43	0.0038		
0.44	0.3068	0.94	0.0981	1.44	0.0288	1.94	0.0097	2.44	0.0038		
0.45	0.3011	0.95	0.0956	1.45	0.0282	1.95	0.0095	2.45	0.0037		
0.46	0.2955	0.96	0.0933	1.46	0.0275	1.96	0.0093	2.46	0.0036		
0.47	0.2899	0.97	0.0910	1.47	0.0269	1.97	0.0091	2.47	0.0036		
0.48	0.2843	0.98	0.0887	1.48	0.0263	1.98	0.0089	2.48	0.0035		
0.49	0.2788	0.99	0.0865	1.49	0.0257	1.99	0.0087	2.49	0.0034		

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The intensities of vertical pressure, directly below the point load (where $r = 0$), on its axis of loading is given by

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$$\sigma_Z = \frac{0.4775\,Q}{z^2} \qquad ...(13.6)$$

Further, the horizontal radial stress component σ_r, and the displacement u, v *and* w in x, y and z-directions are given by the following expressions:

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3zr^2}{R^5} - \frac{1-2\mu}{R(R+z)}\right] \qquad ...(13.7)$$

$$u = \frac{Q(1+\mu)}{2\pi E}\left[\frac{xz}{R^3} - \frac{(1-2\mu)x}{R(R+z)}\right] \qquad ...(13.8)$$

$$v = \frac{Q(1+\mu)}{2\pi E}\left[\frac{yz}{R^3} - \frac{(1-2\mu)y}{R(R+z)}\right] \qquad ...(13.9)$$ and

$$w = \frac{Q(1+\mu)}{2\pi E}\left[\frac{z^2}{R^3} + \frac{2(1-\mu)}{R}\right] \qquad ...(13.10)$$

13.3 PRESSURE DISTRIBUTION DIAGRAMS

By means of Boussinesq's stress distribution theory, the following vertical pressure distribution diagrams can be prepared :

1. Stress isobar or isobar diagram
2. Vertical pressure distribution on a horizontal plane
3. Vertical pressure distribution on a vertical line.

Isobars. An isobar is a curve or contour connecting all points below the ground surface of equal vertical pressure. An isobar is a spatial, curved surface of the shape of a bulb, because the vertical pressure on a given horizontal plane is the same in all directions at points located at equal radial distances around the axis of loading. The zone in a loaded soil mass bounded by an isobar of given vertical pressure intensity is called a *pressure bulb.* The vertical pressure at every point on the surface of pressure bulb is the same.

Suppose an isobar of value $\sigma_Z = 0.25\,Q$ (or 25% of Q) per unit area is to be plotted. Then, from equation 13.4,

$$K_B = \frac{\sigma_Z \times z^2}{Q} = \frac{0.25\,Q\,z^2}{Q} = 0.25\,z^2 \qquad ...(i)$$

A number of numerical values of z are selected, and the values of K_B are calculated from (i). Corresponding to these values of K_B, r/z are found from Table 13.1 and hence corresponding values r are computed. Thus, we get the coordinates (r, z) of a number of points, where $\sigma_Z = 0.25\,Q$. The calculations are better performed in a tabular form shown below :

Table 13.2 Isobar data ($\sigma_Z = 0.25\,Q$/unit area)

Z (units)	K_B	*r/z*	*r (units)*
0.2	0.0100	1.92	0.38
0.4	0.0400	1.30	0.52
0.6	0.0900	0.97	0.58
0.8	0.1600	0.74	0.59
1.0	0.2500	0.54	0.54
1.2	0.3600	0.34	0.41
1.38	0.4775	0.00	0.00

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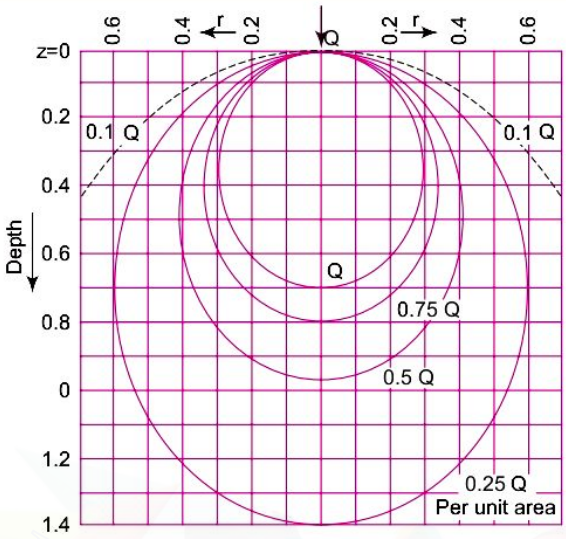

Fig. 13.2 Isobar diagram

On any horizontal plane at a depth of z, σ_Z is the same for the same horizontal distance r on either side of axis of loading. This makes the isobar symmetrical about the axis of loading. The depth at which the isobar $\sigma_Z = 0.25\ Q$ crosses the axis of loading is calculated by first finding K_B when $r = 0$. Thus,

When $r = 0, K_B = 0.4775$

$$\therefore \quad z = \sqrt{\frac{0.4775}{0.25}} = 1.38 \text{ units}$$

For any given load system, a number of isobars corresponding to various intensities of vertical pressure are drawn. Thus, an *isobar diagram,* in Fig. 13.2, consists of a family of isobars of various intensities.

Vertical pressure distribution on a horizontal plane. The vertical pressure distribution on any horizontal plane at a depth z below the ground surface, due to a concentrated load, is given by

$$\sigma_Z = K_B \frac{Q}{z^2}$$

Depth z is a known depth. Selecting different values of horizontal distance r, K_B can be found from Table 13.1, and hence σ_Z can be computed. Below the load, the vertical pressure will be equal to $0.4775\ Q/z^2$, and it decreases very rapidly with the increase in the value of r, as is evident from Table 13.3.

Table 13.3 Variation of σ_Z with r at constant depth

r/z	K_B	σ_Z
0.0	0.4775	Maximum
0.5	0.2733	57% of the maximum
1.0	0.0844	17.7% of the maximum
2.0	0.0085	1.8% of the maximum

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From the above table, it can be concluded that at a given depth, when horizontal radial distance is equal to twice the depth, the vertical pressure due to single concentrated load can be considered negligible.

Figure 13.3 shows a vertical stress distribution diagram, due to a concentrated load, at a depth z. If such a diagram is plotted for unit load ($Q = 1$), it is called the *influence diagram* for point A below the axis. Such a diagram is helpful in computing the vertical stress σ_z at A due to a number of concentrated loads $Q_1, Q_2, ..., Q_n$ etc. situated at radial distances $r_1, r_2, ..., r_n$ from the vertical axis through point A. The vertical stresss is then given by

$$\sigma_Z = \Sigma Q \cdot O$$
$$= Q_1O_1 + Q_2O_2 + ... Q_n \cdot O_n \quad ...(13.11)$$

where $O, O_1, O_2, ..., O_n$ are the ordinates of the influence diagram plotted for σ_Z at A.

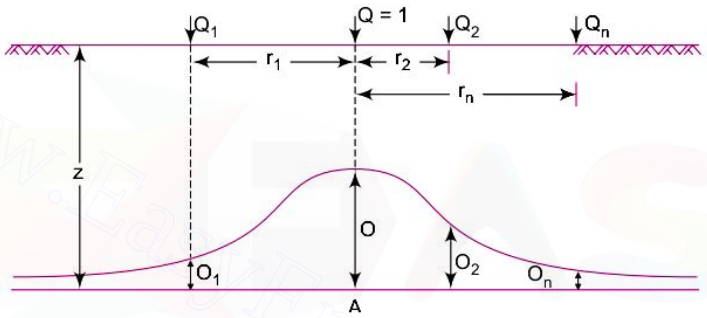

Fig. 13.3 Vertical stress distribution on a horizontal plane (Influence diagram for σ_Z at A)

The influence diagram can be used to find σ_Z at any point on a horizontal plane, by orienting the diagram on the plane in such a way that vertical axis through that point coincides with the maximum ordinate (O) of the influence diagram. When once this is done, the ordinates $O_1, O_2, ..., O_n$ due to any given system of loads can be found and σ_Z can be computed from Eq. 13.11.

Vertical pressure distribution on vertical line. From eq. 13.4, it is clear that σ_Z also decreases with increase in the depth z. On any vertical line distant r from the axis of the load, the variation of σ_Z can be plotted from the relation :

$$\sigma_Z = K_B\, Q/z^2$$

In the above expression, the radial distance r associated with K_B is constant. Hence various values of z and r/z can be selected, and K_B can be found. Then σ_Z can be computed, which will be proportional to K_B/z^2. A table can be prepared as under :

Table 13.4 Variation of σ_Z with Z at constant r (r = 1 unit)

z (units)	$\frac{r}{z}$	K_B	$\frac{K_B}{z^2}$	σ_Z
0	∞	–	–	Intermediate
0.2	5	0.0001	0.0025	0.0025 Q
0.5	2	0.0085	0.0340	0.0340 Q
1	1	0.0844	0.0844	0.0844 Q

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1.225	0.817	0.1332	0.0888	0.0888 Q (Max. value)
2	0.5	0.2733	0.0683	0.0683 Q
4	0.25	0.4103	0.0256	0.0256 Q
5	0.2	0.4329	0.0172	0.0172 Q
10	0.1	0.4657	0.0046	0.0046 Q

Figure 13.4 shows the vertical stress distribution on a vertical line at distance r from the axis of loading. The vertical stress first increases, attains a maximum value, and then decreases. It can be shown (see example 13.2) that the maximum value of σ_Z on a vertical line is obtained at the point of intersection of the vertical plane with a radial line at $\beta = 39° 15'$ through the point load, as shown in Fig. 13.4. The corresponding value of $\frac{r}{z} = 0.817$

or $$z = \frac{\pi}{0.817} = \frac{1}{0.817} = 1.225$$

for which $$K_B = 0.1332$$

Hence, $$(\sigma_Z)_{max} = \frac{0.1332\,Q}{(1.225)^2} = \mathbf{0.0888\ Q.}$$

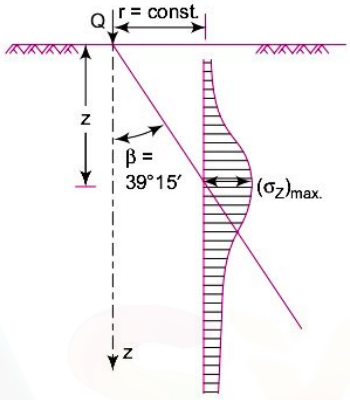

Fig. 13.4 σ_Z Distribution on vertical line

SOLVED EXAMPLES

Example 13.1. *Find the intensity of vertical pressure and horizontal shear stress at a point 4 m directly below a 20 kN point load acting at a horizontal ground surface. What will be vertical pressure and shear stress at a point 2 m horizontally away from the axis of loading but at the same depth of 4 m?*

Solution. (*a*) $r = 0$; $z = 4$ m ; $Q = 20$ kN.

From Eq. 13.2, $$\sigma_Z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = \frac{3\times 20}{2\times\pi\times(4)^2} = \mathbf{0.597\ kN/m^2}$$

From Eq. 13.3, $$\tau_{rZ} = \frac{3Q}{2\pi}\frac{r\,z^2}{\left(r^2+z^2\right)^{5/2}} = \frac{3Qr}{2\pi z^3}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = 0 \qquad \text{(Since } r = 0\text{)}$$

Alternatively, From Table 13.1, $K_B\left(\text{for } \frac{r}{z} = 0\right) = 0.4775$

$$\therefore \quad \sigma_Z = K_B\frac{Q}{z^2} = \frac{0.4775\times 20}{(4)^2} = 0.597\ \text{kN/m}^2.$$

(*b*) $r = 2$ m ; $z = 4$ m $\therefore$ $\frac{r}{z} = 0.5$

$$\therefore \quad \sigma_Z = \frac{3\times 20}{2\pi(4)^2}\left[\frac{1}{1+(0.5)^2}\right]^{5/2} = \mathbf{0.342\ kN/m^2}$$

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$$\tau_{rZ} = \frac{3 \times 2 \times 20}{2\pi (4)^2}\left[\frac{1}{1+(0.5)^2}\right]^{5/2} = \mathbf{0.171\ kN/m^2}$$

Alternatively, $K_B \left(\text{for } \frac{r}{z} = 0.5\right) = 0.2733$

$$\therefore \quad \sigma_Z = K_B \frac{Q}{z^2} = \frac{0.2733 \times 20}{(4)^2} = 0.342\ \text{kN/m}^2$$

and $$\tau_{rZ} = \sigma_Z . \frac{r}{z} = 0.342 \times \frac{2}{4} = \mathbf{0.171\ kN/m^2}.$$

Example 13.2. *Prove that the maximum vertical stress on a vertical line at a constant radial distance r from the axis of a vertical load is induced at the point of intersection of the vertical line with a radial line at ($\beta = 39° 15'$ from the point of application of concentrated load. What will be the value of shear stress at the point? Hence, or otherwise, find the maximum vertical stress on a line situated at r = 2 m from the axis of a concentrated load of value 20 kN.*

Solution. (Refer Fig. 13.4)

We have $$\sigma_Z = \frac{3Q}{2\pi}\frac{z^3}{\left(x^2+r^2\right)^{5/2}} = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \qquad ...(13.2)$$

For the maximum value of σ_Z (where r is constant), differentiate Eq. 13.2 with respect to z and equate it to zero.

$$\frac{d\sigma_Z}{dz} = \frac{3Q}{2\pi}\left[\frac{3z^2\left(z^2+r^2\right)^{5/2} - z^3 \times \frac{5}{2}\left(z^2+r^2\right)^{3/2} 2z}{\left(z^2+r^2\right)^{5/2}}\right] = 0$$

$\therefore$ $3z^2(r^2+z^2) - 5z^4 = 0$, from which $z = \left(\sqrt{3/2}\right) r = 1.225\ r$ ...(13.12)

$$\therefore \quad \frac{r}{z} = \sqrt{\frac{2}{3}} = \frac{1}{1.225} = 0.817 = \tan\beta \quad \therefore \quad \beta = \mathbf{39° 15'}$$

Substituting the value of $\frac{r}{z} = \sqrt{\frac{2}{3}}$ and $z = \sqrt{\frac{3}{2}}\, r$ in Eq. 13.12, we get

$$(\sigma_Z)_{\max} = \frac{3Q}{2\pi}\frac{1}{\left(\sqrt{\frac{3}{2}}\right)^2}\left[\frac{1}{1+\left(\frac{2}{3}\right)}\right]^{5/2} = \frac{Q}{\pi r^2} \times \frac{1}{\left(1+\frac{2}{3}\right)^{5/2}} = \mathbf{0.0888}\,\frac{Q}{r^2} \qquad ...(13.13)$$

$$\tau_{rz} = \frac{3Q}{2\pi}\frac{r\,z^2}{\left(r^2+z^2\right)^{5/2}} = \frac{3Q}{2\pi}\frac{r}{z^3}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$= (\sigma_Z)_{\max}\frac{r}{z} = \left(0.0888\,\frac{Q}{r^2}\right) \times 0.817 = \mathbf{0.0725}\,\frac{Q}{r^2}. \qquad ...(13.14)$$

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When $r = 2$ m and $Q = 20$ kN, $(\sigma_Z)_{max} = 0.0888 \times \dfrac{20}{4} = \mathbf{0.444\ kN/m^2}$

which occurs at $z = 1.225$, $r = \mathbf{2.45\ m}$

Also, $\tau_{rz} = 0.0725 \dfrac{Q}{r^2} = \dfrac{0.0725 \times 20}{4} = \mathbf{0.362\ kN/m^2}.$

13.4 VERTICAL PRESSURE UNDER A UNIFORMLY LOADED CIRCULAR AREA

The Boussinesq equation for the vertical stress due to a single concentrated load can now be extended to find the vertical pressure on any point on the vertical axis passing through the centre of a uniformly loaded circular area. Figure 13.5 shows a uniformly loaded circular area of radius a and load intensity q per unit area. Assume the soil as an elastic, isotropic, semi-infinite mass.

Consider an elementary ring of radius r and width δr on the loaded area. If the elementary ring is further divided into small parts, each of area δA, the load on each elementary area will be $q\ \delta A$. This load may be considered as a point load. Hence the vertical pressure at point P, situated at depth z on the vertical axis through the centre of the area, is evidently given by Eq. 13.2 .

$$\delta\sigma_Z = \frac{3(q \cdot \delta A)}{2\pi} \frac{z^3}{\left(r^2 + z^2\right)^{5/2}}$$

Integrating over the entire ring of radius r, the vertical stress $\Delta\sigma_Z$ is given by

$$\Delta\sigma_Z = \frac{3q}{2\pi}(\Sigma\delta A) \frac{z^3}{\left(r^2 + z^2\right)^{5/2}} = \frac{3q\,(2\pi r\,\delta r)}{2\pi} \frac{z^3}{\left(r^2 + z^2\right)^{5/2}} = 3qr\,\delta r \frac{z^3}{\left(r^2 + z^2\right)^{5/2}}$$

The total vertical pressure σ_Z due to the entire loaded area is given by integrating the above expression between the limits $r = 0$ to $r = a$.

$$\therefore \quad \sigma_Z = 3q z^3 \int_0^a \frac{r\,dr}{\left(r^2 + z^2\right)^{5/2}}$$

Put $r^2 + z^2 = n^2$, so that $r\,dr = n\,dn$

Limits : $\begin{cases} \text{when } r = 0,\ n = z \\ \text{when } r = a,\ n = \left(a^2 + z^2\right)^{1/2} \end{cases}$

$$\therefore \quad \sigma_Z = 3q z^3 \int_z^{\left(a^2 + z^2\right)^{1/2}} \frac{dn}{n^4}$$

$$= q z^3 \left[\frac{1}{z^3} - \frac{1}{\left(a^2 + z^2\right)^{3/2}}\right]$$

$$\therefore \quad \sigma_Z = q \left[1 - \left\{\frac{1}{1 + \left(\frac{a}{z}\right)^2}\right\}^{3/2}\right] \qquad ...(13.15)$$

or $$\sigma_Z = K_B \cdot q \qquad ...(13.16)$$

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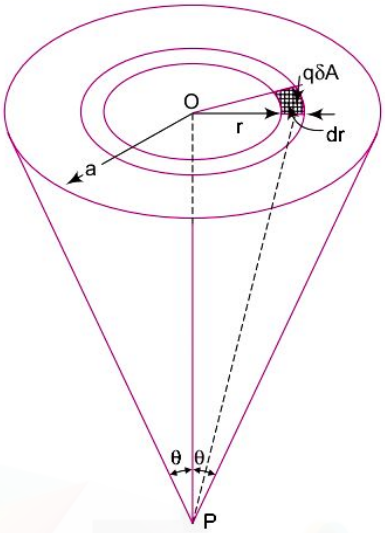

Fig. 13.5 Uniformly distributed load over circular area

where K_B = Boussinesq influence factor for uniformly distributed circular load

$$= 1 - \left\{ \frac{1}{1 + \left(\frac{a}{z}\right)^2} \right\}^{3/2} \quad ...(13.16\ a)$$

Table 13.5 given the value of the influence factors for various values of a/z. The vertical pressure at a given depth on the vertical axis through the centre of the circular loaded area can be found by multiplying the influence factor by the load intensity q. For the vertical pressure at any other point not situated under the centre of the circular load, reference is drawn to article 14.5 of chapter 14.

If θ is the angle which the line joining the point P makes with the outer edge of the loading, Eq. 13.15 reduces to

$$\sigma_Z = q[1 - \cos^3 \theta] \quad ...(13.17)$$

Table 13.5 Influence factors for vertical pressure under centre of uniformly loaded circular area

$\frac{a}{z}$	K_B	$\frac{a}{z}$	K_B	$\frac{a}{z}$	K_B
0.00	0.0000	1.00	0.6465	2.0	0.9106
0.05	0.0037	1.05	0.6720	2.5	0.9488
0.10	0.0148	1.10	0.6956	3.0	0.9684
0.15	0.0328	1.15	0.7175	3.5	0.9793
0.20	0.0571	1.20	0.7376	4.0	0.9857
0.25	0.0869	1.25	0.7562	4.5	0.9898
0.30	0.1213	1.30	0.7733	5.0	0.9925

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0.35	0.1592	1.35	0.7891	5.5	0.9943
0.40	0.1996	1.40	0.8036	6.0	0.9956
0.45	0.2417	1.45	0.8170	6.5	0.9965
0.50	0.2845	1.50	0.8293	7.0	0.9972
0.55	0.3273	1.55	0.8407	8.0	0.9981
0.60	0.3695	1.60	0.8511	9.0	0.9987
0.65	0.4106	1.65	0.8608	10	0.9990
0.70	0.4052	1.70	0.8697	12	0.9994
0.75	0.4880	1.75	0.8779	14	0.9996
0.80	0.5239	1.80	0.8855	16	0.9998
0.85	0.5577	1.85	0.8925	20	0.9999
0.90	0.5893	1.90	0.8990	100	1.0000
0.95	0.6189	1.95	0.9050	∞	1.0000

Figure 13.6 shows a family of isobars under a uniformly loaded circular area, first presented by Jurgenson (1934).

With the help of this diagram, the vertical pressure of various points below a circular loaded area can be conveniently determined.

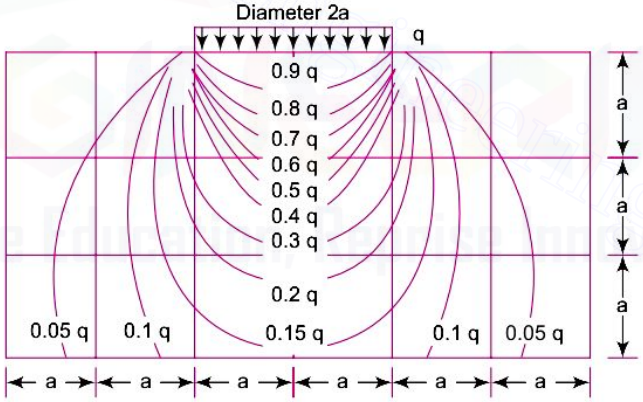

Fig. 13.6 Isobars under a uniformly loaded circular area

13.5 VERTICAL PRESSURE DUE TO A LINE LOAD

Let us consider an infinitely long line load of intensity q' per unit length, acting on the surface of a semi-infinite elastic medium. Let the y-axis be directed along the direction of the line load, as shown in Fig. 13.7. Let us find the expression for the vertical stress at any point P having co-ordinates (x, y, z).

The radial distance of point $P = r = (x^2 + y^2)^{1/2}$

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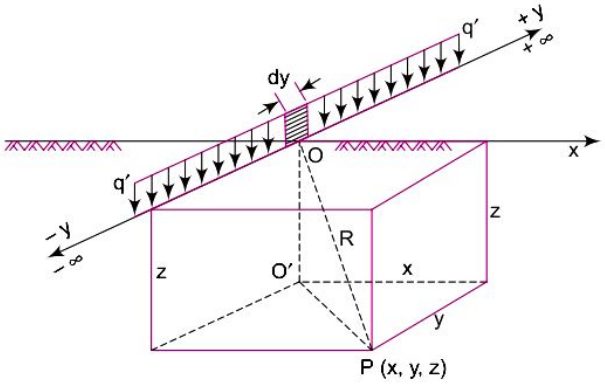

Fig. 13.7 Line load

The polar distance of point $P = R = (r^2 + z^2)^{1/2} = (x^2 + y^2 + z^2)^{1/2}$.

Consider a small length δy along the line load. The elementary load in this length will be equal to $q'\ \delta y$, which can be considered to be a concentrated load. Hence the vertical stress $\Delta\sigma_Z$ due to this elementary load is given by

$$\Delta\sigma_Z = \frac{3(q'\delta y)z^3}{2\pi R^5} = \frac{3q'\delta y z^3}{2\pi\left(x^2 + y^2 + z^2\right)^{5/2}}$$

$$\therefore \quad \sigma_Z = \int_{-\infty}^{+\infty} \frac{3q'z^3\,dy}{2\pi\left(x^2 + y^2 + z^2\right)^{5/2}} = 2\int_0^{\infty} \frac{3q'z^3\,dy}{2\pi\left(x^2 + y^2 + z^2\right)^{5/2}}$$

or
$$\sigma_Z = \frac{2q'z^2}{\pi\left(x^2 + z^2\right)^2} = \frac{2q'}{\pi z}\frac{1}{\left[1 + \left(\frac{x}{z}\right)^2\right]^2} \qquad ...(13.18)$$

In the above expression x and z are constants for a given position of a point P and the only variable is y. Also, x is the horizontal distance of point P from the line load, in a direction perpendicular to the line load. When the point P is situated vertically below the line load, at a depth z, we have $x = 0$, and hence the vertical stress is given by

$$\sigma_Z = \frac{2q'}{\pi z}. \qquad ...(13.19)$$

13.6 VERTICAL PRESSURE UNDER STRIP LOAD

Figure 13.8 shows an infinite strip of width B, loaded with uniformly distributed load intensity q per unit area. Let us find the vertical pressure at a point P situated below a depth z, on a vertical axis passing through the centre of the strip.

Consider a strip load of width dx, at distance x from the centre. The elementary line load intensity along this elementary strip of width dx will be $q \cdot dx$. The vertical pressure at P due to this elementary line load is given by Eq. 13.18.

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$$\Delta\sigma_Z = \frac{2(q\,dx)}{\pi z}\,\frac{1}{\left[1+\left(\frac{x}{z}\right)^2\right]^2}$$

Total vertical pressure due to the whole strip load is given by

$$\sigma_Z = \frac{2q}{\pi z}\int_{-B/2}^{+B/2}\frac{dx}{\left[1+(x/z)^2\right]^2} = \frac{4q}{\pi z}\int_0^{B/2}\frac{dx}{\left[1+(x/z)^2\right]^2}$$

Put $\frac{x}{z} = \tan\beta, \quad \therefore \quad dz = z\sec^2\beta$

$$\therefore \quad \sigma_Z = \frac{4q}{\pi}\int_\theta^{\theta/2}\frac{\sec^2\beta\,d\beta}{\left(1+\tan^2\beta\right)^2}$$

$$= \frac{4q}{\pi}\int_0^{\theta/2}\cos^2\beta\,d\beta = \frac{q}{\pi}(\theta+\sin\theta) \qquad ...(13.20)$$

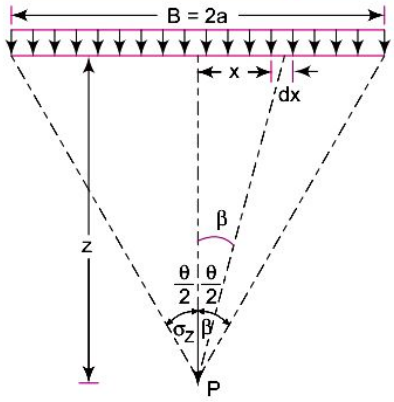

Fig. 13.8 Strip load

Table 13.6 gives vertical pressure at different depths below the centre of a uniform load of intensity q and width B.

Table 13.6 Vertical pressure under centre of strip load

z/B Ratio	*Vertical pressure σ_Z as per cent of q*
0.1	99.7
0.2	97.7
0.5	81.7
1.0	55.0
2.0	30.6
5.0	12.7
10.0	6.4

13.7 VERTICAL PRESSURE UNDER A UNIFORMLY LOADED RECTANGULAR AREA

Let us take the case of a rectangular load area of length $2a$ and width $2b$, and let the reference axes pass through the centre of the area, as shown in Fig. 13.9. Let the point P, where vertical stress has to be found, have the co-ordinates (x, y, z).

Consider an elementary area $\delta A = \delta\xi\delta\eta$, and let the x and y co-ordinates of the centre of this elementary area be ξ and η respectively. Evidently, the x and y co-ordinates of point P, with respect to the elementary area, will be $(x - \xi)$ and $(y - \eta)$ respectively. Hence the polar distance R between the elementary load and point P is given by

$$R = [(x-\xi)^2 + (y-\eta)^2 + z^2]^{1/2}$$

The vertical stress $\Delta\sigma_Z$ at P due to this elementary load $(\delta\xi \cdot \delta\eta \cdot q)$ is given by

Eq. 13.2 : $$\Delta\sigma_Z = \frac{3}{2\pi}(\delta\xi\,\delta\eta\,q)\frac{r^3}{R^5}$$

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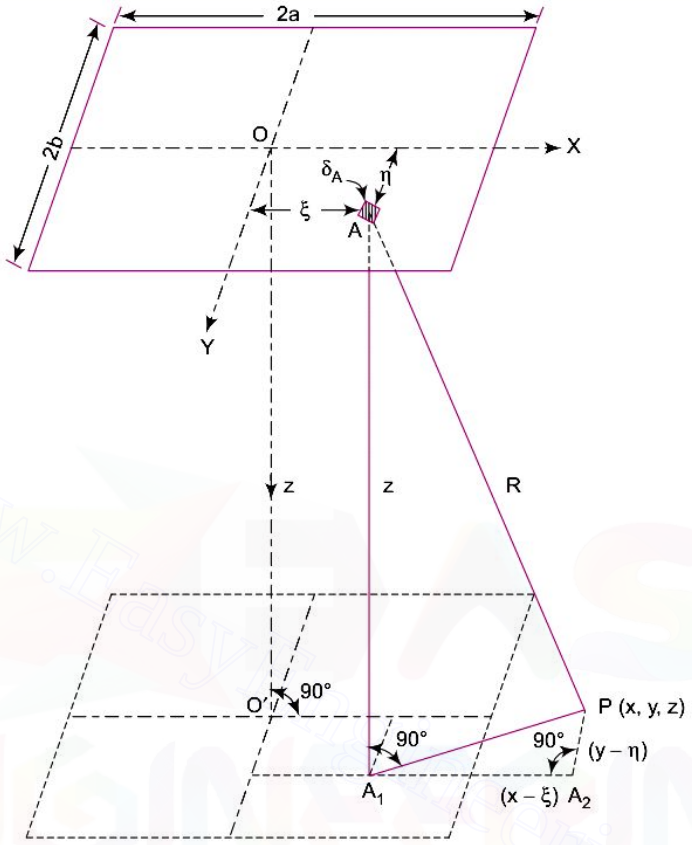

Fig. 13.9 Rectangular loaded area

Hence the vertical stress at *P*, due to the entire loaded area is given by

$$\sigma_Z = \frac{3qz^3}{2\pi}\int_{-a}^{a}\int_{-b}^{b}\frac{d\xi\, d\eta}{\left[(x-\xi)^2+(y-\eta)^2+z^2\right]^{5/2}}$$

Florin (1959, 61) obtained the above integral. However, the integral is far too lengthy to be of practical value. A more practical case is the vertical stress (σ_Z) under the centre of the rectangle ($x = y = 0$)

$$\therefore \qquad (\sigma_Z)_0 = \frac{3qz^3}{2\pi}\int_{-a}^{a}\int_{-b}^{b}\frac{d\xi\, d\eta}{(\xi^2+\eta^2+z^2)^{5/2}}$$

or
$$(\sigma_Z)_0 = \frac{2q}{\pi}\left[\frac{abz\left(a^2+b^2+2z^2\right)}{\left(a^2+z^2\right)\left(b^2+z^2\right)\sqrt{a^2+b^2+z^2}}+\sin^{-1}\frac{ab}{\sqrt{a^2+z^2}\,.\sqrt{b^2+z^2}}\right] \qquad ...(13.21)$$

The above expression can now be utilized to find the vertical stress under the corner of a rectangular area of the size *a, b*. From principle of superposition, the vertical stress under the coner of the rectangle of size *a, b* will be one quarter of the above expression :

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$$(\sigma_Z)_c = \frac{q}{2\pi}\left[\frac{m'n'}{\sqrt{1+m'^2+n'^2}}\,\frac{1+m'^2+2n'^2}{\left(1+n'^2\right)\times\left(m'^2+n'^2\right)} + \sin^{-1}\frac{m'}{\sqrt{m'^2+n'^2}\,\sqrt{\left(1+n'^2\right)}}\right] \quad ...(13.22)$$

$$\left(\text{where } m' = \frac{a}{b};\ n' = \frac{z}{b}\right)$$

or $$(\sigma_Z)_c = q\,K_S \quad ...(13.23)$$

where K_S = Steinbrenner (1936) influence factor, given by curves of Fig. 13.10.

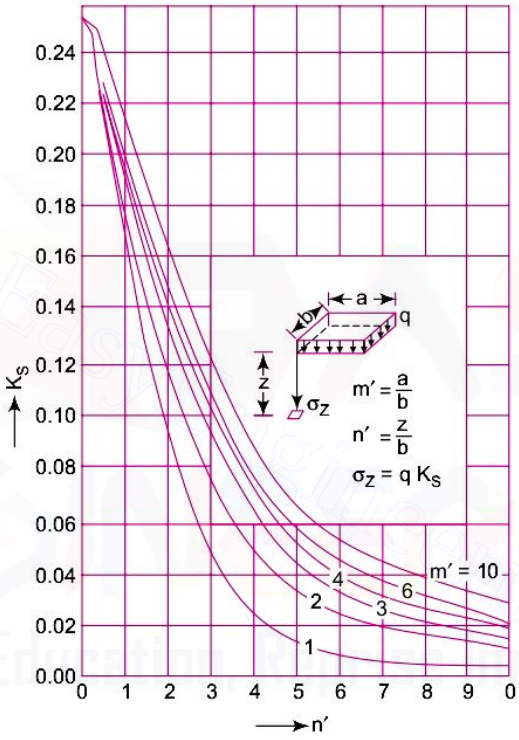

Fig. 13.10 Influence factor for rectangular area (After Steinbrenner, 1936)

A more common form of the vertical stress under the corner of a rectangular area of size a, b is as follows :

$$(\sigma_Z)_c = \frac{q}{4\pi}\left[\frac{2\,m\,n\left(m^2+n^2+1\right)^{1/2}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\frac{2\,m\,n\left(m^2+n^2+1\right)^{1/2}}{m^2+n^2-m^2n^2+1}\right] \quad ...(13.24)$$

or $$(\sigma_Z)_c = K\,q \quad ...(13.24)\quad \left(\text{where } m = \frac{a}{z},\ n = \frac{b}{z}\right)$$

where K = Influence factor. Table 13.7 gives the values of influence factor K. In the above formulae a and b or m and n are *interchangeable*. The above form of solution is after Newmark (1935).

Equation 13.24 can also be utilised for vertical stress at a point P not situated at the corner of the rectangle, but below some other point A either inside or outside the rectangle as shown in Figs. 13.11 and 13.12. When the point A is *inside* the rectangle, the vertical stress at point P, vertically below A at depth z is given by

$$\sigma_Z = q(K_1 + K_2 + K_3 + K_4) \quad ...(13.25)$$

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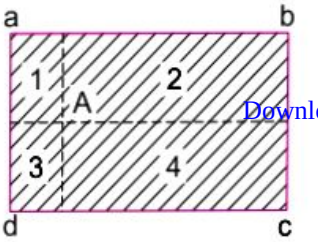

Fig. 13.11 Point *A* inside the rectangle

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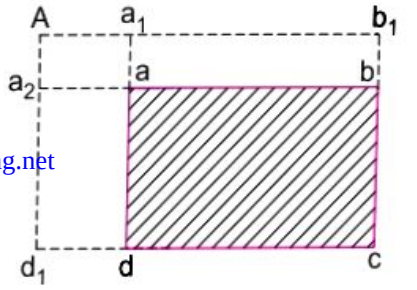

Fig. 13.12 Point *A* outside the rectangle

where K_1, K_2, K_3 and K_4 are the influence factors for the four rectangles 1, 2, 3 and 4. Similarly, if the point *A* is *outside* the loaded rectangle, construct the four rectangles as shown in Fig. 13.12. The shaded area in the loaded rectangle may be considered to be the algebraic sum of the four rectangles, each with the corner at *A*;

$$\text{Area } abcd = Ab_1cd_1 - Ab_1ba_2 - Aa_1dd_1 + Aa_1aa_2$$

$$\therefore \quad \sigma_Z \text{ at } A = q(K_1 - K_2 - K_3 + K_4) \qquad ...(13.26)$$

where K_1 = influence factor for area Ab_1cd_1 ; K_2 = influence factor for area Ab_1ba_2

K_3 = influence factor for area Aa_1dd_1 ; K_4 = influence factor for area Aa_1aa_2

Table 13.7 Influence factors for vertical pressure under a corner of uniformly loaded rectangular area (After newmark, 1935)

m	*n*									
	0.1	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*	*1.0*
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1391
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1168	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1648	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2046
10.0	0.0316	0.0620	0.0902	0.1154	0.1375	0.1562	0.1720	0.1850	0.1958	0.2044
∞	0.0316	0.0620	0.0902	0.1154	0.1375	0.1562	0.1720	0.1850	0.1958	0.2046

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Table 13.7 (Contd.) Influence factors for vertical pressure under a corner of uniformly loaded rectangular area (After newmark, 1935)

m	*n*									
	1.2	*1.4*	*1.6*	*1.8*	*2.0*	*2.5*	*3.0*	*5.0*	*10.0*	∞
0.1	0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.2	0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0620	0.0620	0.0620
0.3	0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0902	0.0902	0.0902
0.4	0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1154	0.1154	0.1154
0.5	0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1374	0.1375	0.1375
0.6	0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1561	0.1562	0.1562
0.7	0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1719	0.1720	0.1720
0.8	0.1648	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1849	0.1850	0.1850
0.9	0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1956	0.1958	0.1958
1.0	0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2044	0.2046	0.2046
1.2	0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2175	0.2177	0.2177
1.4	0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2263	0.2265	0.2266
1.6	0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2324	0.2326	0.2326
1.8	0.2103	0.2184	0.2237	0.2274	0.2299	0.2333	0.2350	0.2366	0.2368	0.2369
2.0	0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2395	0.2399	0.2399
2.5	0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2439	0.2443	0.2443
3.0	0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2461	0.2465	0.2465
5.0	0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2461	0.2486	0.2491	0.2492
10.0	0.2177	0.2265	0.2368	0.2368	0.2399	0.2443	0.2465	0.2491	0.2498	0.2499
∞	0.2177	0.2266	0.2326	0.2369	0.2399	0.2443	0.2465	0.2492	0.2499	0.2500

13.8 EQUIVALENT POINT LOAD METHOD

This is an approximate method of calculating the vertical stress at any point due to any loaded area. The entire area is divided into a number of small *area units* and the total distributed load over a unit area is replaced by a point load of the same magnitude acting at the centroid of the area unit. Thus, the distributed loads over the whole area is replaced by a number of point loads situated at the centroids of the various area units. The influence factors for each of these load positions can be found with respect to the point *P*, where σ_Z is to be determined. The vertical stress is then given by

$$\sigma_Z = \frac{1}{z^2}\left[Q_1 K_{B1} + Q_2 K_{B2} \ldots Q_n K_{Bn}\right] \quad \ldots(13.27)$$

If all the point loads are of equal magnitude Q'

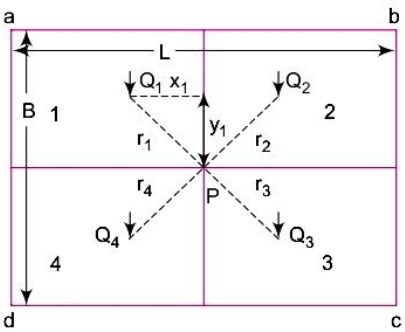

Fig. 13.13 Equivalent point load

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$$\sigma_Z = \frac{Q'}{z^2} \Sigma K_B \qquad ...(13.28)$$

where ΣK_B = sum of the individual influence factors for the various area units.

The accuracy of the result will depend upon the size of the area unit chosen. If the length of the side of the small area unit is less than one-third of the depth at which vertical pressure is required, the error involved in the result is within 3 per cent.

13.9 NEWMARK'S INFLUENCE CHART

A more accurate method of determining the vertical stress at any point under a uniformly loaded area of *any shape* is with the help of *influence chart* or *influence diagram* originally suggested by Newmark (1942). A chart, consisting of number of circles and radiating lines, is so prepared that the influence of each *area unit* (formed in the shape of a sector between two concentric circles and two adjacent radial lines) is the same at the centre of the circles, *i.e.*, each area unit causes the equal vertical stress at the centre of the diagram.

Let a uniformly loaded circular area of radius r_1 cm be divided into 20 sectors (area units) as shown in Fig. 13.14. If q is the intensity of loading, and σ_Z is the vertical pressure at a depth z below the centre of the area, each unit such as $OA_1 B_1$ exerts a pressure equal to $\sigma_Z/20$ at the centre.

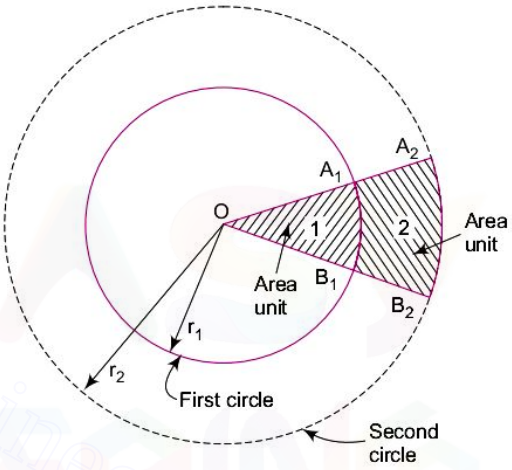

Fig. 13.14 Preparation of influence chart

Hence, from Eq. 13.15

$$\frac{\sigma_Z}{20} = \frac{q}{20}\left[1 - \left\{\frac{1}{1+\left(\frac{r_1}{z}\right)^2}\right\}^{3/2}\right] = i_f\, q \qquad ...(13.29)$$

where

$$i_f = \text{influence value} = \frac{1}{20}\left[\left\{1 - \frac{1}{1+\left(\frac{r_1}{z}\right)^2}\right\}^{3/2}\right]$$

If i_f be made equal to an arbitrarily fixed value, say 0.005, we have

$$\frac{q}{20}\left[1 - \left\{\frac{1}{1+\left(\frac{r_1}{z}\right)^2}\right\}^{3/2}\right] = 0.005\, q \qquad ...(13.30)$$

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Selecting the value of $z = 5$ cm (say), the value of r_1 solved from Eq. 13.30 comes out be 1.35 cm. Hence if a circle is drawn with radius $r_1 = 1.35$ cm and divided into 20 equal area units, each area unit will exert a pressure equal to 0.005 q intensity at a depth of 5 cm.

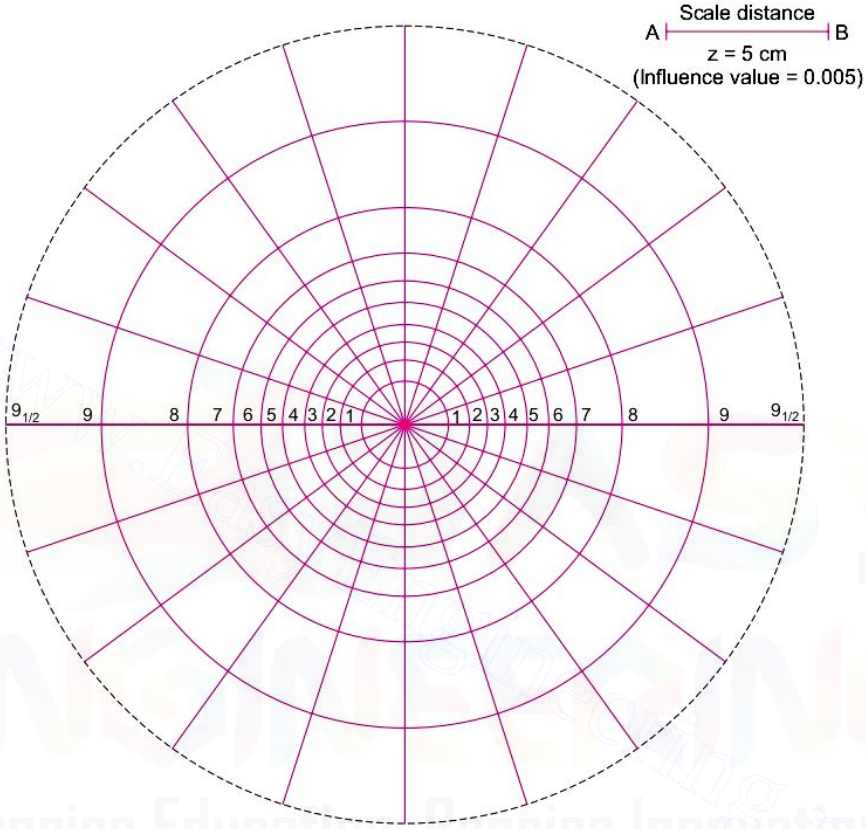

Fig. 13.15 Newmark's influence chart

Let the radius of second concentric circle be equal to r_2 cm. By extending the twenty radial lines, the space between the two concentric circles is again divided into 20 equal area units ; $A_1A_2B_2B_1$ is one such area unit. The vertical pressure, at the centre, due to each of these area units is to be of intensity 0.005 q. Therefore, the total pressure due to area units $O A_1 B_1$ and $A_1A_2B_2B_1$ at depth $z = 5$ cm below the centre is $2 \times 0.005\, q$.

Hence from Eq. 13.15 :

$$\text{Vertical pressure due to } OA_2B_2 = \frac{q}{20}\left[1 - \left\{\frac{1}{1+\left(\frac{r_2}{z}\right)^2}\right\}^{3/2}\right] = 2 \times 0.005\, q$$

Substituting $z = 5$ cm, we get $r_2 = 2.00$ cm from the above relation. Similarly, the radii of 3rd, 4th, 5th, 6th, 7th, 8th, 9th circles can be calculated, as tabulated in Table 13.8. The radius of 10th circle is given by the following governing equation :

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$$\frac{q}{20}\left[1-\left\{\frac{1}{1+\left(\frac{r_{10}}{z}\right)^2}\right\}^{3/2}\right] = 10 \times 0.005\, q = \frac{q}{20}$$

From the above r_{10} = infinity.

Table 13.8 Radii of concentric circles for influence chart (z = 5 cm; i_f = 0.005; each circle divided into 20 parts)

Number of circles	1	2	3	4	5	6	7	8	9	$9\frac{1}{2}$	10
Radius (cm)	1.35	2.00	2.59	3.18	3.83	4.59	5.54	6.94	9.54	12.62	∞

Figure 13.15 shows the influence chart drawn on the basis of Table 13.8.

To use the chart for determining the vertical stress at any point under the loaded area, the plan of the loaded area is first drawn on a tracing paper to such a scale that the length AB (= 5 cm) drawn on the chart represents the depth to the point at which pressure is required. For example, if the pressure is to be found at a depth of 5 m, the scale of plan will be 5 cm = 5 m, or 1 cm = 1 m. The plan of the loaded area is then so placed over the chart that the point below which pressure is required coincides with the centre of the chart. The point below which pressure is required may lie within or outside the loaded area. The total number of area units (including the fractions) covered by the plan of the loaded area is counted. The vertical pressure is then calculated from the relation:

$$\sigma_A = 0.005\, q \times N_A \text{ (where } N_A = \text{number of area units under the loaded area).} \qquad ...(13.31)$$

Example 13.3. *A rectangular area 2 m × 4 m carries a uniform load of 80 kN/m² at the ground surface. Find the vertical pressures at 5 m below the centre and corner of the loaded area.*

Solution. (*a*) For the point under the centre of the area, there will be influence of four rectangles of size 1 m × 2 m, having a common corner at the centre of the loaded rectangle.

Hence, $a = 1 \text{ m}\ ;\ b = 2 \text{ m}\ ;\ m = \frac{a}{z} = \frac{1}{5} = 0.2\ ;\ n = \frac{b}{z} = \frac{2}{5} = 0.4$

$\therefore$ K_{B1} (for one quadrant) = 0.0328 $\quad\therefore\quad \sigma_Z = 4\, qK_{B1} = 4 \times 80 \times 0.0328 = \mathbf{10.5\ kN/m^2}$

(*b*) For the point under the corner of rectangle

$$a = 2 \text{ m}\ ;\ b = 4 \text{ m}\ \therefore\quad m = \frac{2}{5} = 0.4\ ;\ n = \frac{4}{5} = 0.8$$

$\therefore$ $K_B = 0.0931 \quad\therefore\quad \sigma_Z = qK_B = 80 \times 0.0931 = \mathbf{7.45\ kN/m^2}.$

Example 13.4. *Solve example 13.3 by the equivalent load method.*

Solution. Divide loaded area into four equal rectangles of size 1 m × 2 m. Each area will represent a point load $Q' = 1 \times 2 \times 80 = 160$ kN acting at its centroid.

(a) For the point under the centre **:** The influence of each area unit will be equal

$$r' = \sqrt{1+(0.5)^2} = 1.118$$

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$$\therefore \quad \frac{r'}{z} = \frac{1.118}{5} = 0.223 \quad \therefore \quad K_B = 0.4247$$

$$\therefore \quad \sigma_Z = \frac{Q'}{z^2} \Sigma K_B = \frac{160 \times 4 \times 0.4247}{5 \times 5} = \mathbf{10.87\ kN/m^2}$$

By exact method (example 13.3), $\sigma_Z = 10.5$ kN/m^2

$$\therefore \text{ \% error } = \frac{10.87 - 10.5}{10.5} = 3.5\%.$$

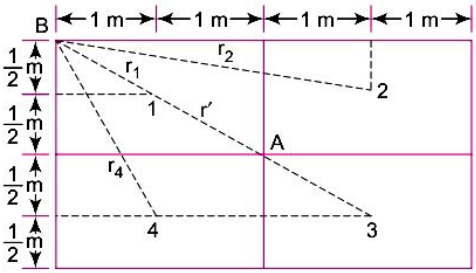

Fig. 13.16

(b) For the point under corner B: The influence of each area unit will be different. Let r_1, r_2, r_3, r_4, be the radial distance of centroids of each unit from *B*.

The corresponding values of *r/z* and K_B are as under :

Area unit	*r*	*r/z*	K_B
1	1.118	0.223	0.4247
2	3.040	0.608	0.2174
3	3.360	0.672	0.1880
4	1.800	0.360	0.3521
			$\Sigma K_B = 1.1822$

$$\therefore \quad \sigma_Z = \frac{Q'}{z^2} \Sigma K_B = \frac{160 \times 1.1822}{25} = \mathbf{7.57\ kN/m^2}$$

But by exact method (example 13.3), $\sigma_Z = 7.45$ kN/m^2

$$\therefore \quad \text{\% error} = \frac{7.57 - 7.45}{7.45} = 1.56\%.$$

Example 13.5. *Solve example 13.3 using Newmark's influence chart.*

Solution. $z = 5$ m. Hence the scale of the plan will be

$$AB(= 5 \text{ cm}) = 5 \text{ m or } 1 \text{ cm} = 1 \text{ m}$$

(*a*) The plan of the rectangular area is drawn to the scale of 1 cm = 1 m, and oriented on the chart in such a way that its centroid is over the centre of the diagram.

Number of area units under the rectangle = $N_A = 25.5$ units

$\therefore$ σ_Z under centre of area = $0.005 \times q\ N_A = 0.005 \times 80 \times 25.5 = \mathbf{10.2\ kN/m^2}$.

(*b*) The plan of the rectangular area is then oriented in such a way that one of its corner is above the centre of chart. Then $N_A = 18.5$ units σ_Z under corner of area = $0.005 \times 80 \times 18.5 = \mathbf{7.4\ kN/m^2}$.

13.10 WESTERGAARD'S ANALYSIS

Westergaard (1938) also solved the problem of pressure distribution in soil under point load, assuming the soil to be an elastic medium of semi-infinite extent but containing numerous, closely spaced, horizontal sheets of negligible thickness of an infinite rigid material which permits only downward deformation on the mass as a whole without allowing it to undergo any lateral strain.

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The assumption of no lateral displacement implies that : $u = v = \varepsilon_X = \varepsilon_Y = \gamma_{XY} = 0$ and volumedilatation $= \varepsilon = \varepsilon_Z = \dfrac{\partial w}{\partial z}$ where u, v and w are displacement in x, y and z-directions.

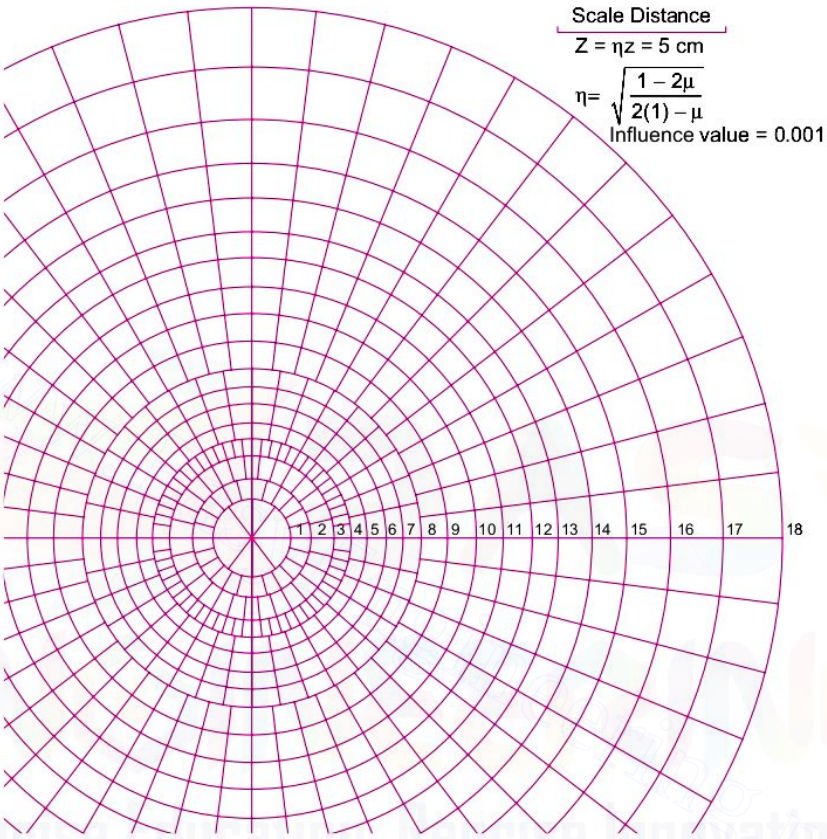

Fig. 13.17 Influence chart for σ_Z based on Westergaard analysis

The stresses on a horizontal plane, which are now simply function of vertical displacement are given by:

$$\sigma_Z = \frac{2(1-\mu)G}{(1-2\mu)}\frac{\partial w}{\partial x} \quad ...(13.31\ a) \qquad \text{where } G = \text{shear modulus} = \frac{E}{2(1+\mu)}$$

$$\tau_{XZ} = G\left(\frac{\partial w}{\partial x}\right) \quad ...(13.31\ b) \qquad \text{and} \qquad \tau_{ZY} = G\left(\frac{\partial w}{\partial y}\right) \quad ...(13.31\ c)$$

Substituting these into equilibrium equations in the vertical direction (Eq. 12.3 c), we get

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial Z^2} = \nabla^2 w = 0 \quad ...(13.32)$$

where $Z = \eta z$, and $\eta = \sqrt{\dfrac{1-2\mu}{2(1-\mu)}}$ ...(13.33)

For the point load Q applied at the origin of the co-ordinates, Westergaard obtained

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$$w = \eta \frac{Q}{2\pi RG} \qquad ...(13.34)$$

where $R^2 = r^2 + Z^2 = x^2 + y^2 + \eta^2 z^2$. Substituting in Eq. 13.21*a*, we get

$$\sigma_Z = \frac{QZ}{2\pi R^3} = \frac{Q}{2\pi\eta^2 z^2 \left[1 + \left(\frac{r}{\eta z}\right)^2\right]^{3/2}} \qquad ...(13.35)$$

The value of μ varies from 0 to 0.5 for elastic materials. For a case of large lateral restraint, the lateral strain is very small and μ may be assumed as zero.

Eq. 13.35 then reduces to $$\sigma_Z = \frac{1}{\pi\left[1 + 2\left(\frac{r}{z}\right)^2\right]^{3/2}} \frac{Q}{z^2} = K_w \frac{Q}{z^2} \qquad ...(13.36)$$

where K_w = Westergaard *influence factor* = $$\frac{1}{\pi\left[1 + 2\left(\frac{r}{z}\right)^2\right]^{3/2}} \qquad ...(13.37)$$

Value of K_w as function of $\frac{r}{z}$ are given in Table 13.9

Table 13.9 Westergaard's influence factor for vertical pressure caused by a surface point load

r/z	K_w	r/z	K_w	r/z	K_w	r/z	K_w	r/z	K_w
0.00	0.3183	0.44	0.1949	0.88	0.0783	1.32	0.0335	1.76	0.0165
0.02	0.3178	0.46	0.1875	0.90	0.0751	1.34	0.0324	1.78	0.0160
0.04	0.3168	0.48	0.1803	0.92	0.0721	1.36	0.0312	1.80	0.0156
0.06	0.3149	0.50	0.1733	0.94	0.0692	1.38	0.0302	1.82	0.0151
0.08	0.3123	0.52	0.1664	0.96	0.0664	1.40	0.0292	1.84	0.0147
0.10	0.3090	0.54	0.1598	0.98	0.0638	1.42	0.0282	1.86	0.0143
0.12	0.3050	0.56	0.1534	1.00	0.0613	1.44	0.0273	1.88	0.0139
0.14	0.3005	0.58	0.1471	1.02	0.0589	1.46	0.0264	1.90	0.0135
0.16	0.2953	0.60	0.1411	1.04	0.0566	1.48	0.0255	1.92	0.0131
0.18	0.2897	0.62	0.1353	1.06	0.0544	1.50	0.0247	1.94	0.0128
0.20	0.2836	0.64	0.1298	1.08	0.0523	1.52	0.0239	1.96	0.0124
0.22	0.2771	0.66	0.1244	1.10	0.0503	1.54	0.0231	1.98	0.0121
0.24	0.2703	0.68	0.1192	1.12	0.0484	1.56	0.0224	2.00	0.0118
0.26	0.2632	0.70	0.1142	1.14	0.0466	1.58	0.0217	2.10	0.0103
0.28	0.2558	0.72	0.1095	1.16	0.0499	1.60	0.0210	2.20	0.0091
0.30	0.2483	0.74	0.1050	1.18	0.0432	1.62	0.0204	2.30	0.0081
0.32	0.2407	0.76	0.1006	1.20	0.0416	1.64	0.0198	2.40	0.0072

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0.34	0.2331	0.78	0.0964	1.22	0.0401	1.66	0.0192	2.50	0.0064
0.36	0.2254	0.80	0.0925	1.24	0.0386	1.68	0.0186	2.60	0.0058
0.38	0.2175	0.82	0.0887	1.26	0.0373	1.70	0.0180	2.70	0.0052
0.40	0.2099	0.84	0.0850	1.28	0.0300	1.72	0.0175	2.80	0.0047
0.42	0.2023	0.86	0.0815	1.30	0.0347	1.74	0.0170	3.00	0.0038

Uniformly loaded circular area : Equation 13.36 can be integrated to obtain vertical pressure under a uniform circular load of intensity q and radius a. The vertical pressure under the centre of circular load is given by

$$\sigma_Z = q\left[1-\left\{\frac{1}{1+\left(\frac{a}{\eta z}\right)^2}\right\}^{1/2}\right] \qquad ...(13.38)$$

Fenske (1951) obtained influence chart for the Westergaard case, using Newmark's method. Table 13.10 gives the data for influence chart (Fig. 13.7) with the influence value of 0.001 and the *modified depth* $Z = \eta z = 4$ cm.

Table 13.10 Data for influence chart based on Westergaard analysis

$\eta z = 5$ cm; Influence value = 0.001

Circle No.	*Radius (cm)*	*Radial division (degrees)*	*Circle No.*	*Radius (cm)*	*Radial division (degrees)*
1	0.64	45.00	11	5.88	7.50
2	1.02	30.00	12	6.66	7.50
3	1.46	18.00	13	7.56	7.50
4	1.88	15.00	14	8.56	7.50
5	2.36	11.25	15	9.76	7.50
6	2.80	11.25	16	11.18	7.50
7	3.32	9.00	17	12.96	7.50
8	3.86	9.00	18	15.22	7.50
9	4.50	7.50	19	17.68	6.00
10	5.16	7.50	20	20.96	6.00

The method of using the chart is the same as the Newmark's chart except that the scale for plotting the loaded area is obtained by equating the modified depth ηz of given problem to 5 cm (scale distance) of the chart. When $\mu = 0$, $\eta = 0.707$.

Uniformly loaded rectangular area

σ_Z at the corner of a rectangular area with sides mz and nz is given by

$$\sigma_Z = \frac{q}{2\pi}\cot^{-1}\sqrt{\frac{1-2\mu}{2-2\mu}\left(\frac{1}{m^2}+\frac{1}{n^2}\right)+\left(\frac{1-2\mu}{2-2\mu}\right)^2\frac{1}{m^2\,n^2}} \qquad ...(13.39)$$

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Taking $\mu = 0$, the above equation takes the form

$$(\sigma_Z)_{\mu=0} = \frac{q}{2\pi} \cot^{-1} \sqrt{\frac{1}{2m^2} + \frac{1}{2n^2} + \frac{1}{4m^2 n^2}} \qquad ...(13.40)$$

Comparison between Boussinesq's and Westergaard's solutions.

For a point load, we have, in general $\sigma_Z = K \frac{Q}{z^2}$

where

$$K = K_B \text{ in Boussinesq's analysis} = \frac{3/2\pi}{\left[1+(r/z)^2\right]^{5/2}}$$

$$K = K_w \text{ in Westergaard's analysis} = \frac{1/\pi}{\left[1+2(r/z)^2\right]^{3/2}}$$

Table 13.11 gives the value K_B and K_w for some salient values of r/z, for comparison. Similarly, Fig. 13.18 gives the variation of K_B and K_w for various values of r/z, for comparison.

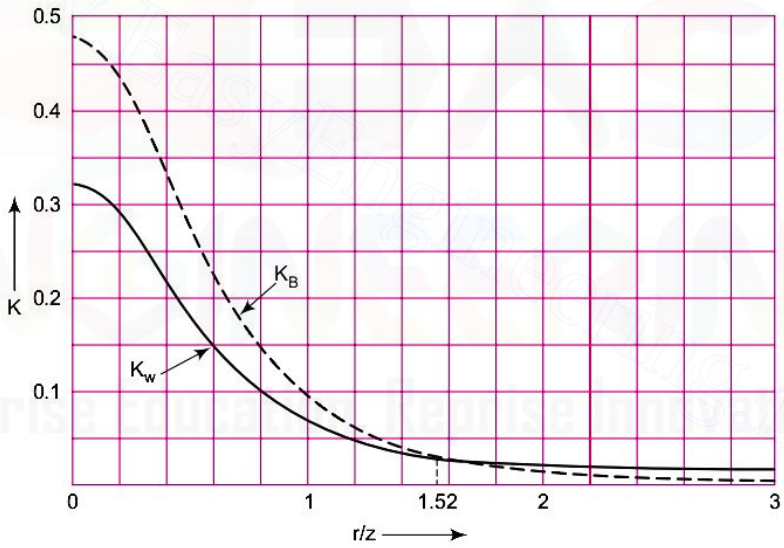

Fig. 3.18 Variation of K_B and K_w with r/z

Table 13.11 Comparison between K_B and K_w

r/z	K_B	K_w	r/z	K_B	K_w
0.00	0.4775	0.3183	1.40	0.0317	0.0292
0.20	0.4329	0.2836	1.60	0.0200	0.0210
0.40	0.3294	0.2099	1.80	0.0129	0.0156
0.60	0.2214	0.1411	2.00	0.0085	0.0118
0.80	0.1386	0.0925	2.5	0.0034	0.0064
1.00	0.0844	0.0613	3.0	0.0014	0.0038
1.20	0.0513	0.0416			

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From both Table 13.11 as well as Fig. 13.18, it is clear that K_w is smaller than K_B for $r/z < 1.52$ and is greater than K_B for $r/z > 1.52$ approximately. This fact is also reflected in the comparison of vertical stress distribution on a horizontal plane at a specified depth, from both the solutions, as shown in Fig. 13.19.

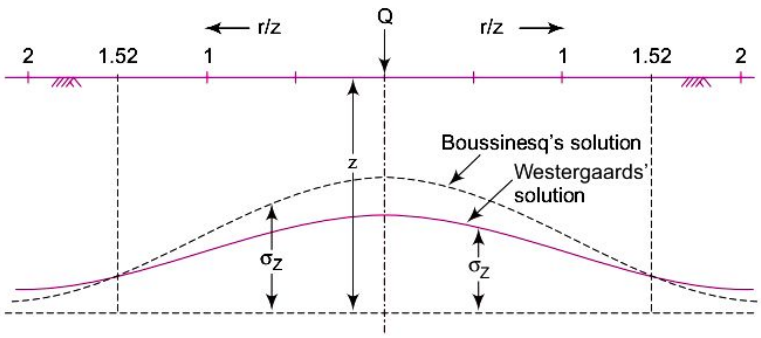

Fig. 13.19 σ_Z Distribution on a horizontal plane

Example 13.6 *Solve example 13.3 by Westergaard's analysis, taking* $\mu = 0$

Solution.

(a) Vertical pressure below the corner P_1 of the loaded area

Here $z = 5$ m ; $m = 2/5 = 0.4$; $n = 4/5 = 0.8$

Eq. 13.40:
$$(\sigma_Z)_{P_1} = \frac{80}{2\pi}\cot^{-1}\sqrt{\frac{1}{2(0.4)^2}+\frac{1}{2(0.8)^2}+\frac{1}{4}\frac{1}{(0.4\times 0.8)^2}}$$

$$= \frac{80}{2\pi}\cot^{-1} 2.519 = \frac{80}{2\pi}(0.3779)^* = 4.81 \text{ kN/m}^2$$

(against 7.45 kN/m^2 by Boussinesq's analysis)

(b) Vertical pressure below the centre P of the loaded area

Subdivide the rectangle into four smaller rectangles of size 1 m × 2 m so that point P becomes the corner of each of the four rectangles. Evidently, σ_Z at P will be equal to four times σ_Z at P due to each small rectangular area load.

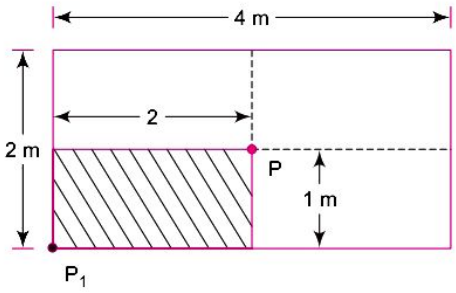

Fig. 13.20

*Let $\cot^{-1} 2.519 = \theta$; or $\cot\theta = 2.519$

$\therefore \quad \tan\theta = \dfrac{1}{2.519} = 0.397$ or $\theta = \tan^{-1} 0.397 = 21.65° = 21.65 \times \dfrac{\pi}{180} = 0.3779$ rad.

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For each subdivided rectangle, $m = 1/5 = 0.2$ and $n = 2/5 = 0.4$

$$\therefore \quad (\sigma_Z)_P = 4\left[\frac{80}{2\pi}\cot^{-1}\sqrt{\frac{1}{2(0.2)^2}+\frac{1}{2(0.4)^2}+\frac{1}{4(0.2\times 0.4)^2}}\right]$$

$$= \frac{160}{\pi}\cot^{-1} 8.0429 = \frac{160}{\pi}(0.1237) = \mathbf{6.3\ kN/m^2}$$

(against 10.5 kN/m^2 found by Boussinesq's analysis).

Example 13.7. *A water tank is supported by a ring foundation having outer diameter of 10 m and inner diameter of 7.5 m. The ring foundation transmits uniform load intensity of 160 kN/m^2. Compute the vertical stress induced at a depth of 4 m, below the centre of ring foundation, using (i) Boussinesq analysis, and (ii) Westergaard's analysis, taking* $\mu = 0$.

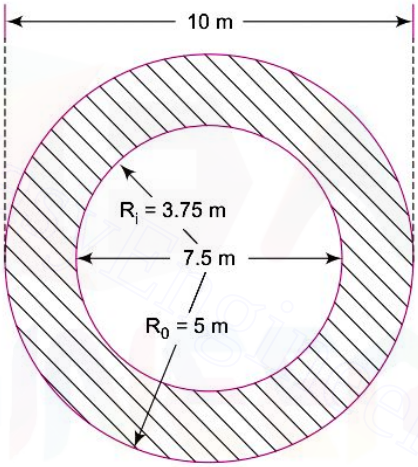

Fig. 13.21

Solution.

(i) Boussinesq Analysis

Inner radius, $R_i = 7.5/2 = 3.75$ m

Outer radius, $R_0 = 10/2 = 5$ m

$R_i/z = 3.75/4 = 0.9375$;

$R_0/z = 5/4 = 1.25$

$$\sigma_Z = 160\left[1-\left\{\frac{1}{1+(1.25)^2}\right\}^{3/2}\right] - 160\left[1-\left\{\frac{1}{1+(0.9375)^2}\right\}^{3/2}\right]$$

$$= 120.99 - 97.88 = \mathbf{23.11\ kN/m^2}.$$

(ii) Westergaard's Analysis : Use Eq. 13.38

Here $a = R_0$ or R_i for outer and inner radii respectively.

$\therefore \quad R_i/z = 3.75/4 = 0.9375\ R_0/z = 5/4 = 1.25$

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$$\therefore \quad \sigma_Z = 160\left[1-\left\{\frac{1}{1+2(1.25)^2}\right\}^{1/2}\right]-160\left[1-\left\{\frac{1}{1+2(0.9375)^2}\right\}^{1/2}\right]$$

$$= 81.22 - 63.65 = \mathbf{17.57\ kN/m^2.}$$

Example 13.8. *The base of a tower consists of a equilateral triangular frame, on the corners of which the three legs of the tower is supported. The total weight of the tower is 600 kN, which is equally carried by all the three legs. Compute the increase in the vertical stress in the soil caused at a point 5 m below one of the legs.*

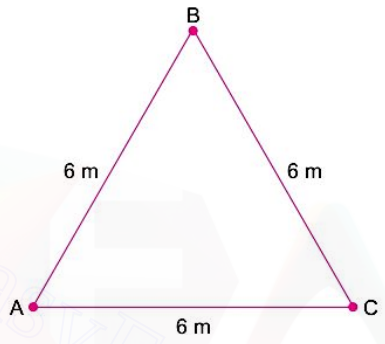

Fig. 13.22

Solution.

Let us find σ_Z at a point P vertically below A. The stresses at P will be caused due to (i) point load of leg at A, for which $r = r_A = 0$, (ii) point load of leg at B, for which $r = r_B = 6$ m, and (iii) point load of leg at C for which $r = r_C = 6$ m.

$$\therefore \quad (\sigma_Z)_{P,A} = \frac{3Q}{2\pi z^2}\left[\left\{\frac{1}{1+(r_A/z)^2}\right\}^{5/2}+2\left\{\frac{1}{1+(r_{B,C}/z)^2}\right\}^{5/2}\right]$$

where $\quad Q = 600/3 = 200$ kN

$$\therefore \quad (\sigma_Z)_{P,A} = \frac{3\times 200}{2\pi(5)^2}\left[\left\{\frac{1}{1+(0)^2}\right\}^{5/2}+2\left\{\frac{1}{1+(6/5)^2}\right\}^{5/2}\right]$$

$$= 3.8197\,[1 + 2\times 0.1075] = \mathbf{4.641\ kN/m^2.}$$

13.11 CONTACT PRESSURE

Contact pressure is defined as the vertical pressure acting at the surface of contact between the base of a footing and the underlying soil mass. To simplify design, the computation of the bending moments etc. in the footings is commonly based on the assumption that the footings rest on a uniformaly spaced bed of springs so that the distribution of contact pressure is uniform. The actual contact pressure distribution, however, depends upon the flexural rigidity of the footing and the elastic properties of the subgrade.

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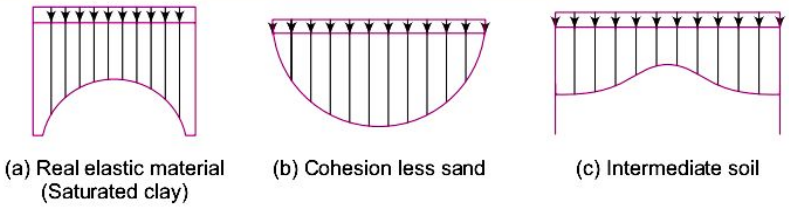

Fig. 13.23 Contact pressure distribution under rigid footings

If the footing in *flexible,* the distribution of contact pressure is uniform irrespective of the type of the subgrade or under-soil material. If the footing is perfectly rigid, the contact pressure distribution depends upon the type of the subgrade. Figure 13.23 shows the pressure distribution under rigid footing resting over (*a*) real, elastic material (such as saturated clay), and (*b*) cohesionless sand, and (*c*) soil having intermediate characteristics.

In the case of a real elastic material, theoretical intensity of contact pressure at the centre is $q/2$, and infinite at the outer edges. However, local yielding causes redistribution of pressure, making it finite at the edges. When the loading approaches a value sufficient to cause failure of soil, the contact pressure distribution may probably be very nearly uniform. In the case of sand, no resistance to deformation is offered at the outer edges of the footing, making the contact pressure zero there. The pressure distribution is parabolic with maximum value at the centre, though it tends to become more uniform with increasing footing width. When a footing is neither perfectly flexible, nor perfectly rigid, and the underlying soil possesses both cohesion and friction, the contact pressure lies between the extreme conditions for uniform and non uniform distribution for flexible and rigid footings.

13.12 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 13.9. *Calculate the intensity of vertical stress induced at a point 2 m vertically below the corner of a square footing of 2 m side transmitting a bearing pressure of 12 t/m² to the surface of the soil deposit. Use the Boussinesq equation given below*

$$\sigma_{Z,r} = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

where $\sigma_{z,r}$ is the intensity of vertical pressure at a depth z below and horizontal distance r away from the point of application on the soil surface of the point load Q. **(Civil Services Exam. 1982)**

Solution. For a point load, $\sigma_{Z,r} = \dfrac{3Q}{2\pi z^2}\left[\dfrac{1}{1+(r/z)^2}\right]^{5/2} = \dfrac{3}{2}\dfrac{Q}{\pi}\dfrac{z^3}{R^5}$

Following the procedure outlined in §13.7, one can obtain the following expression for σ_z under the corner of a rectangular area.

$$\sigma_Z = \frac{q}{4\pi}\left[\frac{2mn\left(m^2+n^2+1\right)^{1/2}}{m^2+n^2+m^2n^2+1}\times\frac{m^2+n^2+2}{m^2+n^2+1}+\tan^{-1}\frac{2mn\left(m^2+n^2+1\right)^{1/2}}{m^2+n^2-m^2n^2+1}\right]$$

where $m = a/z = 2/2 = 1$ and $n = b/z = 2/2 = 1$

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$$\therefore \quad \sigma_Z = \frac{12}{4\pi}\left[\frac{2\times1\times1\left(1+1+1\right)^{1/2}}{1+1+1\times1+1}\times\frac{1+1+2}{1+1+1}+\tan^{-1}\frac{2\times1\times1\left(1+1+1\right)^{1/2}}{1+1-1\times1+1}\right]$$

$$= \mathbf{2.1027\ t/m^2.}$$

PROBLEMS

1. A circular area of 7.5 metres in diameter on the ground surface carries a uniformly distributed load 3 kN/m^2. Find the intensity of vertical pressure below the centre of the loaded area at a depth of 6 metres below the ground surface. Use Boussinesq analysis. [**Ans.** 1.17 kN/m^2]
2. A circular area is loaded with a uniform load intensity of 100 kN/m^2 at ground surface. Calculate the vertical pressure at a point *P* so situated on the vertical line through the centre of loaded area that the area subtends an angle 90° at *P*. Use the Boussinesq analysis. [**Ans.** 64.6 kN/m^2]
3. A concentrated point load of 200 kN acts at the ground surface. Find the intensity of vertical pressure at a depth of 10 metres below the ground surface, and situated on the axis of the loading. What will be the vertical pressure at a point at a depth of 5 m and at a distance of 2 m from the axis of loading? Use Boussinesq analysis. [**Ans.** 0.955 kN/m^2; 2.64 kN/m^2]
4. Solve problem 3 by Westergaard analysis, taking $\mu = 0$ [**Ans.** 0.64 kN/m^2; 1.68 kN/m^2]
5. A rectangular area 3 × 1 m is uniformly loaded with load intensity 100 kN/m^2 at the ground surface. Calculate the vertical pressure at a point 4 m below one of its corners. [**Ans.** 6 kN/m^2]
6. Solve problem 5 by Newmark's chart. [**Ans.** $N_A = 12$; $\sigma_Z = 6$ kN/m^2]
7. The uniform intensity of loading at the foundation level of a building is 10 m in width and very great extent in length, with the intensity of loading of 100 kN/m^2. Using Newmark's chart, find the vertical stress at the depth of 1 m under the centre line and the edge of the building. Check your answers by analytical solution. [**Ans.** (*i*) Under centre line $N_A = 110$ units; $\sigma_Z = 55$ kN/m^2]
   (*ii*) Under the edge $N_A = 80$ units; $\sigma_Z = 40$ kN/m^2]
8. Using Boussinesq analysis, compute analytically the vertical pressure under the centre line of footing of problem 7, at a depth of 10 m. [**Ans.** 54.7 kN/m^2]
9. A point load of 100 kN acts on the ground surface. Using Boussinesq analysis, find the maximum vertical pressure on a vertical plane distant 2 metres from the loading. [**Ans.** 2.22 kN/m^2 at $z = 2.45$ m]

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CHAPTER 14

Stress Distribution : II*

14.1 VERTICAL LINE LOAD

In article 13.5, we have considered the case of line load acting at the horizontal ground surface. Let us now take the case of a vertical line load acting on an inclined surface in the form of a wedge or cone of an angle 2α at its apex, as shown in Fig. 14.1.

Let the intensity of line load, directed in the y-axis, be Q_1 per unit length. The x-axis is in the direction perpendicular to the direction of loading and z-axis is directed downwards. At any radial distance r and polar angle θ, Mitchell found that the three stress components (two dimensional case) are given by

$$\sigma_r = K \frac{Q_1 \cos\theta}{r} \qquad ...(14.1)$$

$$\sigma_\theta = 0 \; ; \; \tau_{r\theta} = 0$$

where K is constant, to be determined by considering the boundary conditions. The distribution of stress in this case is called a *simple radial distribution*. The above form of solution for the three stress components will be valid only if it satisfies the equilibrium equation (Eq. 12.27) and compatibility equation (Eq. 12.28) in terms of stress components.

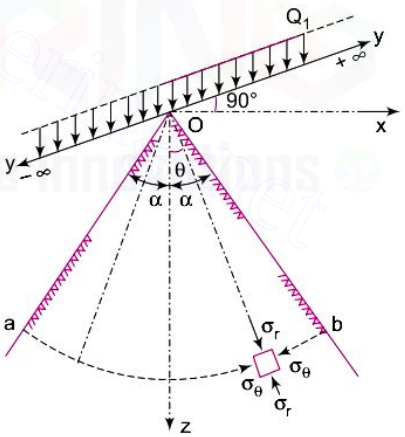

Fig. 14.1

For example, the equilibrium equations in polar co-ordinates are

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad ...(12.27\ a)$$

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and $$\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = 0 \qquad ...(12.27\ b)$$

Now, $$\frac{\partial \sigma_r}{\partial r} = -\frac{KQ_1 \cos\theta}{r^2} \qquad \text{(from Eq. 14.1)}$$

Substituting the values in Eq. 12.27. (*b*), we get

$$-\frac{KQ_1 \cos\theta}{r^2} + 0 + \left(\frac{KQ_1 \cos\theta}{r}\right)\frac{1}{r} = 0$$

which is satisfied. Similarly, Eq. 12.27 (*a*) is also satisfied.

The compatibility equation is $$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)(\sigma_r + \sigma_\theta) = 0$$

For the present case, where $\sigma_\theta = 0$, the above equation takes the form:

$$\left(\frac{\partial^2 \sigma_r}{\partial r^2} + \frac{1}{r}\frac{\partial \sigma_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \sigma_r}{\partial \theta^2}\right) = 0$$

where $\frac{\partial \sigma_r}{\partial r} = -\frac{KQ_1 \cos\theta}{r^2}$; $\frac{\partial^2 \sigma_r}{\partial r^2} = +\frac{2KQ_1 \cos\theta}{r^3}$ and $\frac{\partial^2 \sigma_r}{\partial \theta^2} = -\frac{KQ_1 \cos\theta}{r}$

Substituting these in the compatibility equation, we see that it is satisfied.

Hence Eq. 14.1 gives expression for the stress components, in which the constant K is still to be determined.

Considering the equilibrium of the sector of the wedge (free body) *acb*, we have

$$2\int_0^\alpha \sigma_r\, r\, d\theta \cos\theta = Q_1$$

or $$2\int_0^\alpha \left(\frac{KQ_1 \cos\theta}{r}\right)(r\, d\theta)\cos\theta = Q_1 \text{ or } KQ_1\left(\alpha + \frac{1}{2}\sin 2\alpha\right) = Q_1 \quad \text{or} \quad K = \frac{2}{2\alpha + \sin 2\alpha} \qquad ...(14.2)$$

Substituting this in Eq. 14.1, we get

$$\sigma_r = \frac{2Q_1 \cos\theta}{r(2\alpha + \sin 2\alpha)} \qquad ...(14.3)$$

Taking this special case, when ground is horizontal, $\alpha = \pi/2$. Hence

$$\sigma_r = \frac{2Q_1 \cos\theta}{\pi r} \qquad ...(14.4)$$

On any horizontal plane, relations between the stress components in Cartesian co-ordinates and polar co-ordinates are as follows:

$$\sigma_z = \sigma_r \cos^2\theta\ ;\ \sigma_x = \sigma_r \sin^2\theta\ ;\ \tau_{xz} = \sigma_r \sin\theta \cos\theta$$

Hence the stress components in Cartesian system are

$$\sigma_z = \frac{2Q_1 \cos^3\theta}{\pi r} = \frac{2Q_1 z^3}{\pi r^4} = \frac{2Q_1 z^3}{\pi\left(x^2 + z^2\right)^2} \qquad ...(14.5)$$

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This is the same as Eq. 13.18. The other two stress components are

$$\sigma_x = \frac{2Q_1 x^2 z}{\pi\left(x^2 + z^2\right)^2} \qquad ...(14.6)$$

and

$$\tau_{xz} = \frac{2Q_1 xz^2}{\pi\left(x^2 + z^2\right)^2} \qquad ...(14.7)$$

Figure 14.2 shows the variation of the stress σ_z, σ_x and τ_{xz} on a horizontal plane at the depth z. If the line load intensity Q_1 is equated to unity, these curves are called *influence curves* for the stresses at point A situated below the loading. The influence curves can be used to find the stress components at A due to a number of line loads P_1, P_2 etc. situated at horizontal distance x_1, x_2 etc. and due to a strip load $q(x)$ also. For example σ_z at A is given by

$$(\sigma_z)_A = P_1 (O_1) + P_2 (O_2) + Q(O) + ...q(x)\, a_x \qquad ...(14.8)$$

where O_1, O_2, ... etc. are ordinates of the influence diagram (σ_z) under line loads of intensity P_1, P_2, ... etc. and a_x = shaded area of the influence diagram under the strip load.

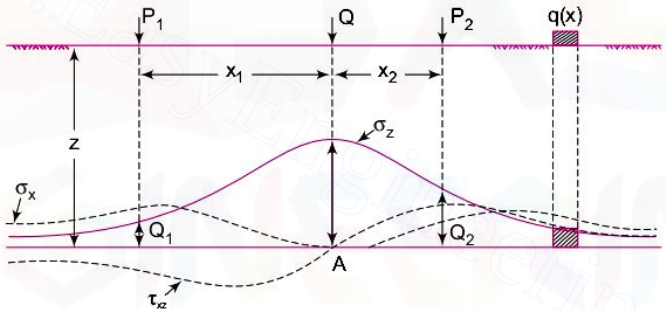

Fig. 14.2 Influence curve

14.2 HORIZONTAL LINE LOAD

Figure 14.3 (*a*) shows a horizontal line load of infinite extent, acting on a wedge-shaped earth mass. Let the intensity of loading be Q_2 per unit length. The stress components in polar co-ordinates at any point (r, θ) are given by

$$\sigma_r = \frac{kQ_2 \sin\theta}{r}\ ;\ \sigma_\theta = 0\ ;\ \tau_{r\theta} = 0 \qquad ...(14.9)$$

It can be shown that the above stress components satisfy the equilibrium equations as well as the compatibility equations. Again considering any wedge-shaped sector *aob* as free body, the following expressions are obtained:

$$\sigma_r = \frac{2Q_2 \sin\theta}{r\left(2\alpha - \sin 2\alpha\right)} \qquad ...(14.10)$$

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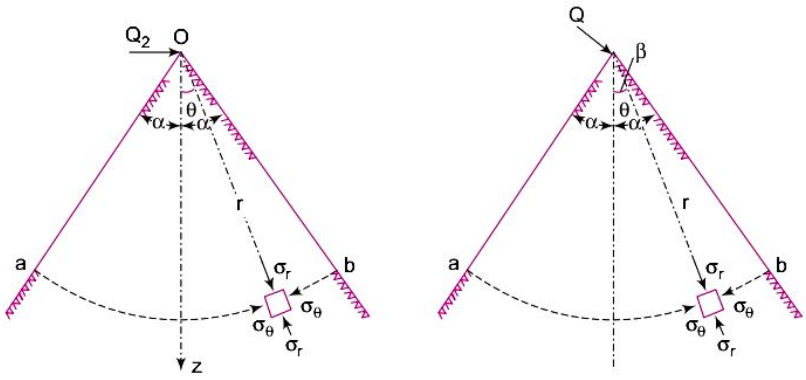

Fig. 14.3 Horizontal and inclined line loads

For a horizontal ground surface, with horizontal line load of intensity Q_2 per unit length, we have

$$\sigma_r = \frac{2Q_2 \sin\theta}{\pi r} \qquad ...(14.11)$$

Hence the stress components in Cartesian system are given by

$$\sigma_z = \sigma_r \cos^2\theta = \frac{2Q_2 \sin\theta \cos^2\theta}{\pi r} = \frac{2Q_2 xz^2}{\pi\left(x^2+z^2\right)^2} \qquad ...(14.12)$$

$$\sigma_x = \sigma_r \sin^2\theta = \frac{2Q_2 \sin^3\theta}{\pi r} = \frac{2Q_2 x^3}{\pi\left(x^2+z^2\right)^2} \qquad ...(14.13)$$

$$\tau_{xz} = \sigma_r \sin\theta \cos\theta = \frac{2Q_2 \sin^2\theta \cos\theta}{\pi r} = \frac{2Q_2 x^2 z}{\pi\left(x^2+z^2\right)^2} \qquad ...(14.14)$$

Inclined line load. The stresses due to an inclined line load (Fig. 14.2. *b*) of intensity Q per unit length can be found by resolving the inclined load into horizontal and vertical components, and then applying Eqs. 14.3 and 14.11.

Thus,
$$\sigma_r = \frac{2Q}{r}\left(\frac{\cos\beta\cos\theta}{2\alpha+\sin 2\alpha} + \frac{\sin\beta\sin\theta}{2\alpha-\sin 2\alpha}\right);\ \sigma_\theta = 0\ ;\ \tau_{r\theta} = 0 \qquad ...(14.15)$$

where β is the angle which the inclined load makes with the vertical.

When the ground surface is horizontal, $\alpha = \frac{\pi}{2}$. Hence stress components due to an inclined line load acting at the horizontal ground surface is given by

$$\sigma_r = \frac{2Q}{\pi r}\cos(\theta-\beta) \qquad ...(14.16)$$

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14.3 UNIFORM VERTICAL LOAD OVER A STRIP

Figure 14.4 (*a*) shows a uniform load of intensity q per unit area distributed over a flexible strip of width $2a$ and of infinite length. Consider any point P with coordinates x, z. If we take an elementary strip of width $d\xi$, distant ξ from the centre of the strip, it may be considered to be a line load of intensity $q\,d\xi$ per unit length. The stress at P, due to this elementary strip can be found from Eqs. 14.6 and 14.7.

Thus,
$$\Delta\sigma_z = \frac{2qz^3}{\pi}\frac{d\xi}{\left[(x-\xi)^2+z^2\right]^2}\;;\quad \Delta\sigma_x = \frac{2q(x-\xi)^2 z\,d\xi}{\pi\left[(x-\xi)^2+z^2\right]^2}$$

And
$$\Delta\tau_{xz} = \frac{q(x-\xi)z^2 d\xi}{\pi\left[(x-\xi)^2+z^2\right]^2}$$

Integrating the above between the limits $\xi = +a$ to $\xi = -a$, we get the final expression for the stress components at P:

$$\sigma_z = \frac{2q}{\pi}\int_{-a}^{a}\frac{z^3 d\xi}{\left[(x-\xi)^2+z^2\right]^2} = \frac{q}{\pi}\left[\tan^{-1}\frac{z}{x-a} - \tan^{-1}\frac{z}{x+a} - \frac{2az\left(x^2-z^2-a^2\right)}{\left(x^2+z^2-a^2\right)^2+4a^2z^2}\right] \quad ...(14.17)$$

Similarly,
$$\sigma_x = \frac{q}{\pi}\left[\tan^{-1}\frac{z}{x-a} - \tan^{-1}\frac{z}{x+a} + \frac{2az\left(x^2-z^2-a^2\right)}{\left(x^2+z^2-a^2\right)^2+4a^2z^2}\right] \quad ...(14.18)$$

and
$$\tau_{xz} = \frac{4aq\,xz^2}{\pi\left[\left(x^2+z^2-a^2\right)^2+4a^2z^2\right]} \quad ...(14.19)$$

In another system [Fig. 14.4(*b*)], the stress components are expressed in terms of angles θ, β_1 and β_2 as under:

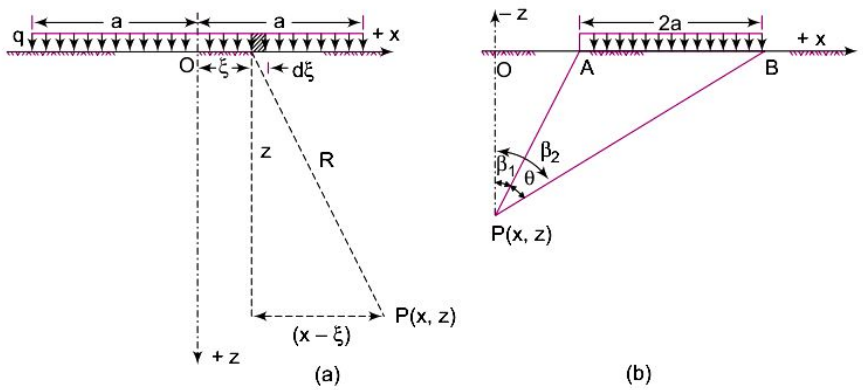

Fig. 14.4 Strip load

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$$\sigma_z = \frac{q}{\pi}\left[\theta + \sin\theta \cos\left(\beta_1 + \beta_2\right)\right] \qquad ...(14.17\ a)$$

$$\sigma_z = \frac{q}{\pi}\left[\theta - \sin\theta \cos\left(\beta_1 + \beta_2\right)\right] \qquad ...(14.18\ a)$$

$$\tau_{xz} = \frac{q}{\pi}\left[\sin\theta \sin\left(\beta_1 + \beta_2\right)\right] \qquad ...(14.19\ a)$$

If the point load P is situated under the centre of the strip we have

$$\beta_1 = -\frac{\theta}{2} \quad \text{and} \quad \beta_2 = +\frac{\theta}{2} \text{ or } \beta_1 + \beta_2 = 0$$

$\therefore$ $$\sigma_z = \frac{q}{\pi}\left[\theta + \sin\theta\right], \text{ which is same as Eq. 13.20.}$$

14.4 CONCENTRATED FORCE: BOUSSINESQ PROBLEM

Let us take the three dimensional case of a concentrated force (Boussinesq problem). Similar to the two dimensional case of vertical line load, the polar stress due to the concentrated force may be expressed as

$$\sigma_R = 1\frac{kQ\cos\beta}{R^2} \qquad ...(14.20)$$

where k is a constant, which is to be determined.

The stress components σ_z and τ_{rz} can be expressed in terms of σ_R as under:

$$\sigma_x = \sigma_R \cos^2\beta;$$

$$\tau_{rz} = \sigma_R \sin\beta \cos\beta$$

Noting that $\cos\beta = \frac{z}{R}$; $\sin\beta = \frac{r}{R}$;

$$R^2 = r^2 + z^2 \text{ and } R\,dR = r\,dr,$$

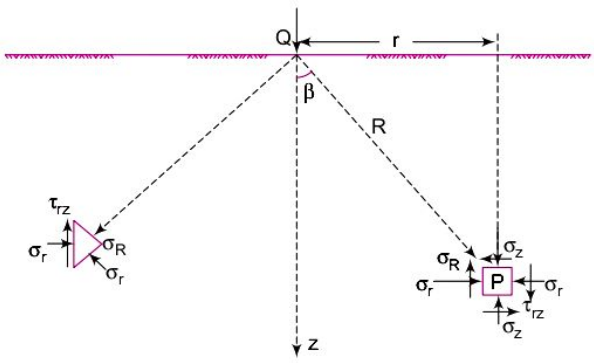

Fig. 14.5 Concentrated force

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we get, $$\sigma_z = \frac{kQ\cos^3\beta}{R^2} = \frac{kQz^3}{R^5}$$

and $$\tau_{rz} = \frac{kQ\cos^3\beta\sin\beta}{R^2} = \frac{kQz^2r}{R^5}$$

From the consideration of equilibrium of forces in vertical direction, the integration of σ_z on any horizontal plane, at depth z, over $r = 0$ to $r = \infty$ must be equal to the applied force Q. Thus,

$$\int_0^\infty \sigma z(2\pi r)dr = Q$$

Putting $R.\,dR$ for rdr, and noting that $R = z$ when $r = 0$ and $R = \infty$ when $r = \infty$ we get,

$$2\pi kQz^3 \int_z^\infty \frac{dR}{R^4} = Q \qquad ...(14.21)$$

or $$2\pi kQz^3\left[-\frac{1}{3R^2}\right]_z^\infty = Q$$

from which $$k = \frac{3}{2\pi}$$

Substituting this in Eq. 14.21, we have

$$\sigma_z = \frac{3Q}{2\pi}\frac{z^3}{R^5} = \frac{3Q}{2\pi z^2}\frac{1}{\left[1+(r/z)^2\right]^{5/2}} \quad \text{(which is the same as Eq. 13.2.)} \qquad ...(14.22)$$

and $$\tau_{rz} = \frac{3P}{2\pi}\frac{z^2r}{R^5} \qquad ...(14.23)$$

Boussinesq originally solved this problem by assuming stress function:

$$\phi = c_1 z\log_e r + c_2(r^2+z^2)^{1/2} + c_3 z\log_e\left(\frac{\sqrt{r^2+z^2}-z}{\sqrt{r^2+z^2}+z}\right) \qquad ...(14.24)$$

With the help of above stress function, one can find all the stress components, as well as displacement components u, v and w. The simpler proof given above does not enable us to find the displacement components. Melan (1919) used the following trigonometric stress function for the solution of this problem.

$$\Phi = \Sigma(A\cosh\alpha x + B\sinh\alpha x)\cos\alpha y \qquad ...(14.25)$$

in which A and B are constants and x represents the various values over which to extend the summation (Σ) of the stress function.

14.5 NORMAL LOAD OVER A CIRCULAR AREA

Figure 14.6 shows a uniformly loaded circular area of radius a and load intensity q per unit area. Let the point P be at a radial distance b from the centre of the area, and at a depth z below ground surface. On an elementary ring of radius r and thickness dr, if we take an elementary load $q\,rd\theta\,dr$, this load can be treated as a point load. The polar distance R of point P from the elementary load is given by:

$$R = (AA'^2 + A'P^2)^{1/2} \text{ But } AA' = z, \text{ and } A'P = r^2 + b^2 - 2br\cos\theta$$

$$\therefore \quad R = (r^2 + b^2 + z^2 - 2br\cos\theta)^{1/2}$$

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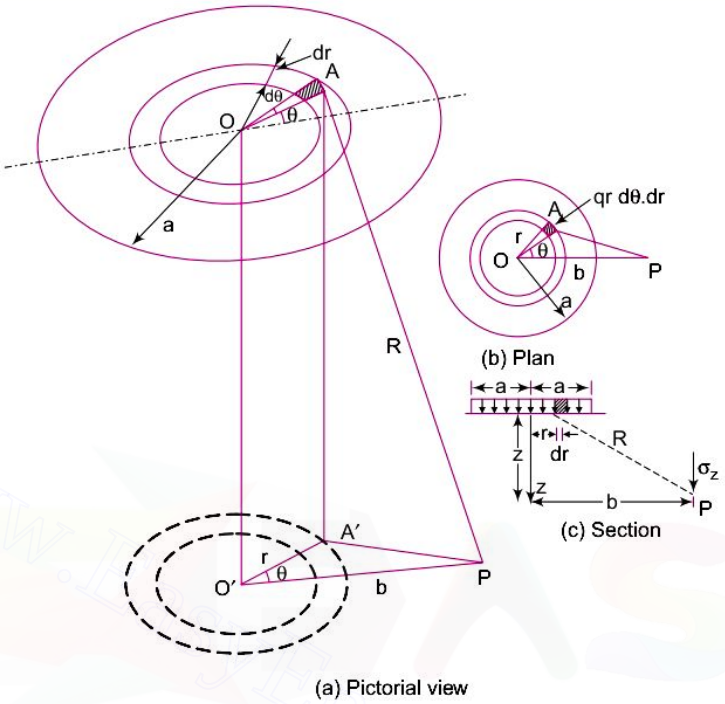

Fig. 14.6 Uniform load over circular area

Hence
$$\delta\sigma_z = \frac{3z^3 (q\, r dr\, d\theta)}{2\pi \left(r^2 + b^2 + z^2 - 2br\cos\theta\right)^{5/2}}$$

The total vertical stress at P, due to the entire circular area is given by

$$\sigma_x = \frac{3z^3 q}{2\pi} \int_0^a \int_0^{2\pi} \frac{r dr\, d\theta}{\left(r^2 + b^2 + z^2 - 2br\cos\theta\right)^{5/2}}$$

Egorov (1958) performed the above integration and found the expression for σ_z. as under

$$\sigma_z = q\left[A - \frac{n}{\pi\sqrt{n^2 + (1+t)^2}} \left\{ \frac{n^2 - 1 + t^2}{n^2 + (1-t)^2} E(k) + \frac{1-t}{1+t} \Pi_0(k, p) \right\}\right] \quad ...(14.26)$$

where $E(k)$ and $\Pi_0(k, p)$ are complete elliptic integrals of second and third kind respectively, of modulus k and parameter p.

$$t = \frac{b}{a}; n = \frac{z}{a};$$

$$k^2 = \frac{4t}{n^2 + (t+1)^2};$$

$$p = -\frac{4t}{(t+1)^2},$$

and
$$A = 1 \text{ if } b < a;$$

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$$A = \frac{1}{2} \text{ if } b = a;$$

$$A = 0 \text{ if } b > a$$

Figure 14.7 shows the curves for determination of σ_z. If the point P lies over the centre line through the circular area, the above equation takes the form

$$\sigma_z = q\left[1 - \frac{1}{\left[1+\left(\frac{a}{z}\right)^2\right]^{3/2}}\right] \qquad ...(14.27)$$

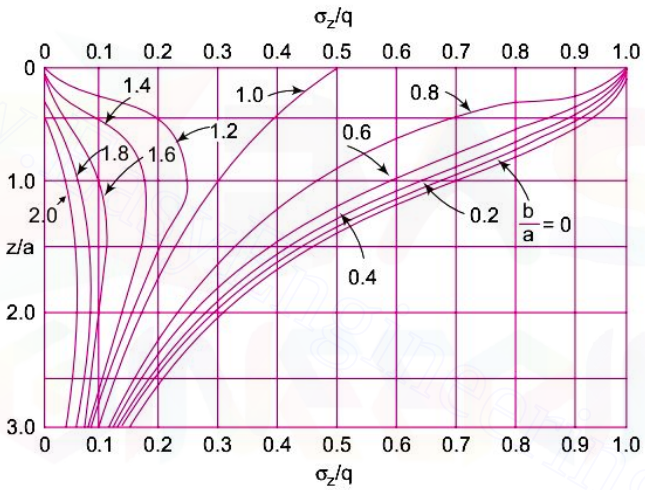

Fig. 14.7 Curves for σ_z (Egorov, 1958)

14.6 TRIANGULAR AND OTHER LOADINGS

We shall now consider various cases of triangular and other loadings, in which the load is not uniformly distributed, but the rate of loading increases linearly from zero at one end.

1. Triangularly distributed vertical load. (Fig. 14.8). For any position of point P, substending an α angle with AB, the vertical stress is given by

$$\sigma_z = \frac{q}{a\pi}\left[x\alpha - \frac{az}{(x-\alpha)^2 + z^2}(x-a)\right] \qquad ...(14.28)$$

If the point P happens to be vertically below A (*i.e.*, point P_A) we have

$$(\sigma_z)_A = \frac{q}{\pi}\frac{\sin 2\alpha_A}{2} = \frac{q}{\pi}\frac{az}{\left(a^2+z^2\right)} \qquad ...(14.29)$$

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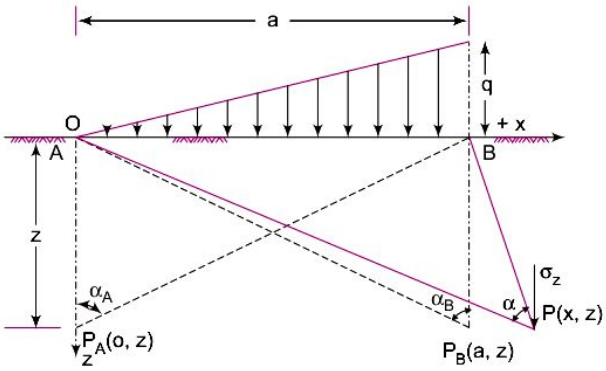

Fig. 14.8

Similarly, for point P_B vertically below B, we have

$$(\sigma_z)_B = \frac{q}{\pi}\alpha_B \qquad ...(14.30)$$

2. Linearly variable infinite load (long uniform slope). Let the load intensity be q at a horizontal distance a from the origin so that load intensity at x is $q_x = q\dfrac{x}{a}$.

For any point P (x, y), such that PO makes an angle α with the horizontal through P as shown in Fig. 14.9, we have

$$\sigma_z = \frac{q}{\pi a}(x\alpha + z) \qquad ...(14.31)$$

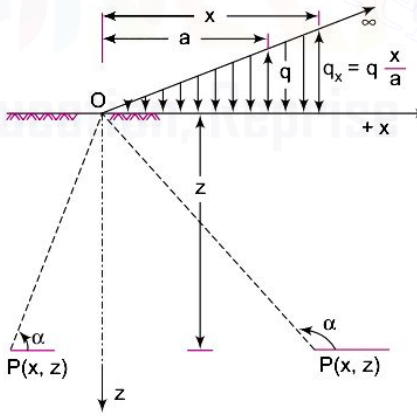

Fig. 14.9 Long uniform slope

3. Symmetrically distributed triangular load. (Fig. 14.10). Let q be the maximum intensity of load at its centre. The stress components at any point $P(x, z)$ subtending angles α_1 and α_2 with AO and OB, as shown, are given by

$$\sigma_x = \frac{q}{\pi a}\left[a(\alpha_1 + \alpha_2) + x(\alpha_1 - \alpha_2)\right] \qquad ...(14.32)$$

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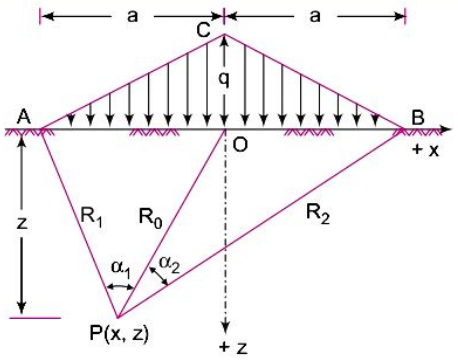

Fig. 14.10 Symmetrical distributed triangular load

$$\sigma_z = \frac{q}{\pi a}\left[a(\alpha_1 + \alpha_2) + x(\alpha_1 - \alpha_2) - 2z \log_e \frac{R_1 R_2}{R_0^2}\right] \quad ...(14.33)$$

where R_1, R_2 and R_0 are polar distances.

$$\sigma_y = \frac{2q}{\pi a m}\left[a(\alpha_1 + \alpha_2) + x(\alpha_1 - \alpha_2) - z \log_e \frac{R_1 R_2}{R_0^2}\right] \quad ...(14.34)$$

(where m = Poisson's coefficient)

$$\tau_{xz} = -\frac{qz}{\pi a}(\alpha_1 - \alpha_2) \quad ...(14.35)$$

Under the centre of the triangular load, the vertical stress at any point at depth z is given by

$$\sigma_z = \frac{q}{\pi}(\alpha_1 + \alpha_2) \quad ...(14.36)$$

The equation could also be got from Eq. 14.30, by considering two triangular loads of AOC and OCB separately. Figure 14.11 shows a family of isobars for σ_z under a strip with triangular load.

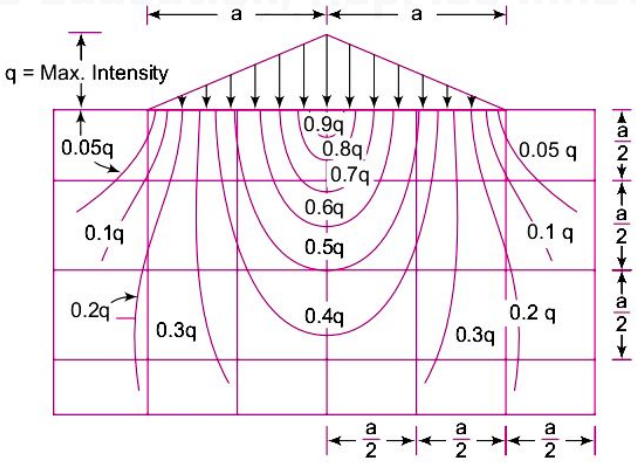

Fig. 14.11 Family of isobars under a strip with triangular load

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4. Triangular and uniformly distributed semi-infinite loads.

$$\sigma_z = \frac{q}{\pi a}(a\beta + x\alpha) \qquad ...(14.37)$$

$$\sigma_x = \frac{q}{\pi a}\left(a\beta + x\alpha + 2z \log_e \frac{R_1}{R_2}\right) \qquad ...(14.38)$$

$$\tau_{xz} = -\frac{q}{\pi a} z\alpha \qquad ...(14.39)$$

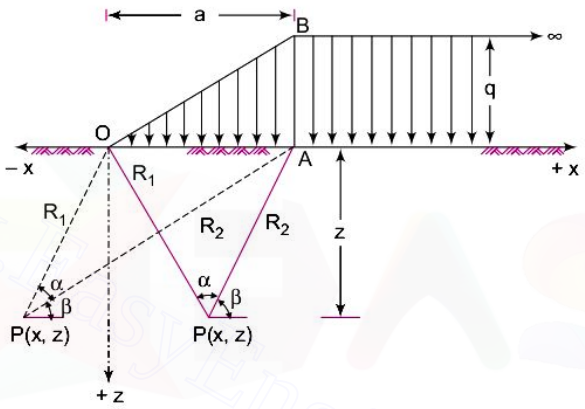

Fig. 14.12

5. Triangular and uniformly distributed strip load of finite width.

$$\sigma_z = \frac{q}{\pi a}\left[a\beta + x\alpha - \frac{az}{R_2^2} z(x-b)\right] \qquad ...(14.40)$$

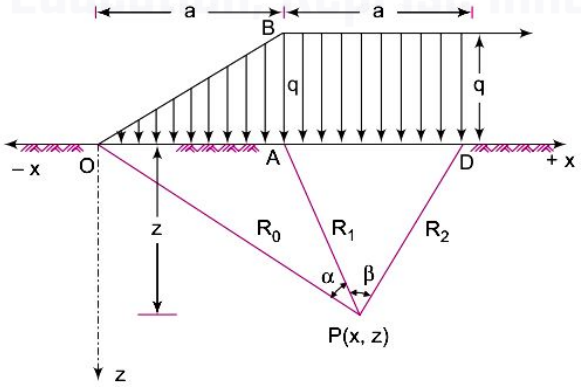

Fig. 14.13

$$\sigma_x = \frac{q}{\pi a}\left[(a\beta + x\alpha) + \frac{az}{R_2^2}(x-b) + 2z \log_e \frac{R_1}{R_0}\right] \qquad ...(14.41)$$

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$$\tau_{xz} = -\frac{q}{\pi a}\left(z\alpha - a\frac{z^2}{R_2^2}\right) \qquad ...(14.42)$$

6. Trapezoidal loading. Such loadings are quite commonly encountered in earth fills, embankments, highways, rail roads, earth dams etc. A trapezoidal loading can be split into a number of basic loadings as under:

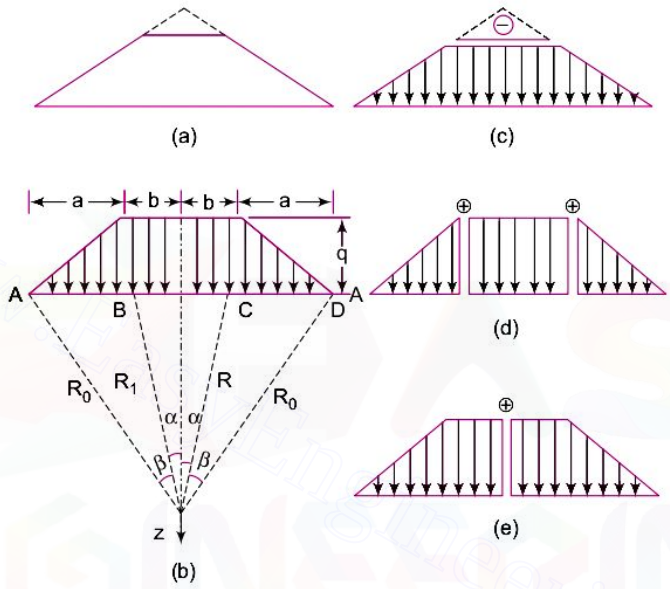

Fig. 14.14 Trapezoidal loading.

(1) A triangular loading minus another triangular loading of smaller magnitude (Fig. 14.14 *c*).
(2) A triangular loading plus a strip loading (Fig. 14.14 *d*).
(3) A triangular finite-width strip plus another triangular finite width strip loads (Fig. 14.14 *e*)

The vertical stress under the centre of the loading is given by

$$\sigma_z = \frac{2q}{\pi a}\left[a(\alpha+\beta)+b\beta\right] \qquad ...(14.43)$$

14.7 PRINCIPAL STRESSES AND MAXIMUM SHEAR

For two dimensional problems, the principal stress σ_1 and σ_2 can be expressed in terms of stress components σ_z, σ_x and τ_{xz} as under:

$$\sigma_1 = \frac{\sigma_z+\sigma_x}{2} + \sqrt{\frac{(\sigma_z-\sigma_x)^2}{4}+\tau_{xz}^2} \quad \text{(major)} \qquad ...(14.44\ a)$$

$$\sigma_3 = \frac{\sigma_z+\sigma_x}{2} - \sqrt{\frac{(\sigma_z-\sigma_x)^2}{4}+\tau_{xz}^2} \quad \text{(minor)} \qquad ...(14.44\ b)$$

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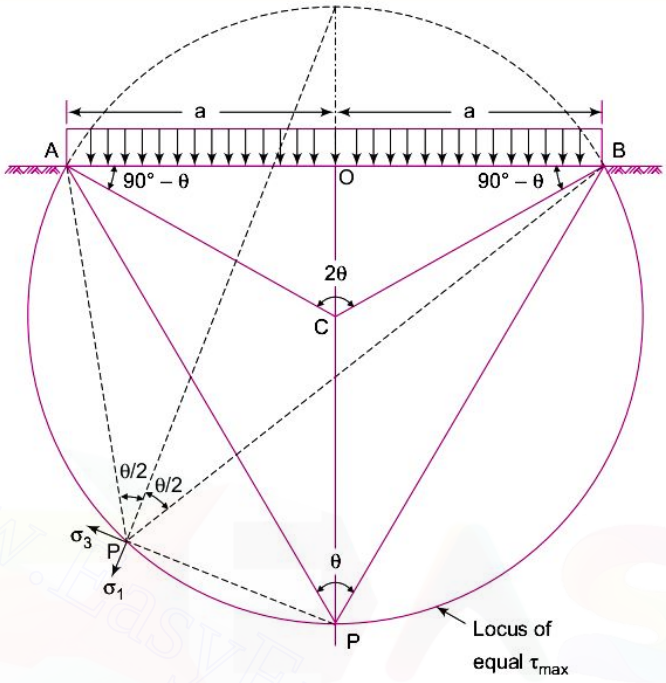

Fig. 14.15 Orientation of principal stresses and locus of point of equal maximum shear under a strip load

If we take the case of strip loading, which is a more common case, and substitute the stress components (Eq. 14.17. (*a*), 14.18, 14.19 (*a*) in Eq. 14.44, we get the following expressions for the principal stresses:

$$\text{Major principal stress, } \sigma_1 = \frac{q}{\pi}\,[\theta + \sin\theta] \qquad ...(14.45)$$

$$\text{Minor principal stress, } \sigma_3 = \frac{q}{\pi}\,[\theta - \sin\theta] \qquad ...(14.46)$$

The *maximum shear stress* τ_{max} is half the difference of the major and minor principal stresses:

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{q}{\pi}\sin\theta = \frac{q}{\pi}\,\frac{2\,\frac{a}{z}}{1+\left(\frac{a}{z}\right)^2} \qquad ...(14.47)$$

τ_{max} is constant so long as θ is constant. Thus, if a circular arc is drawn with centre given by the intersection of lines forming an angle $(90° - \theta)$ from each end of the strip load, a locus of points of equal τ_{max} is obtained. The *greatest* value of τ_{max} occurs when $\theta = \frac{\pi}{2}$.

Hence greatest $\tau_{max} = \frac{q}{\pi}$

The locus of the points having greatest τ_{max} is a semi-circle whose diameter is the width of the loaded strip.

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Part IV
Compressibility

15. One Dimensional Consolidation
16. Three Dimensional Consolidation
17. Compaction

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CHAPTER 15

One Dimensional Consolidation

15.1 INTRODUCTION

When a compressive load is applied to soil mass, a decrease in its volume takes place. The decrease in the volume of soil mass under stress is known as *compression* and the property of soil mass pertaining to its susceptibility to decrease in volume under pressure is known as *compressibility*. A solid crystalline material, like steel, is deformed, under stress, by a relative distortion of the position of atoms in the molecular structure. However, soils are composed of small solid particles not bonded together, except by the small van der Waals forces and adsorbed double layer water. When a stress is applied to it, the elastic deformation of solid particles is negligibly small compared to the deformation caused by change in relative position of the discrete particles and the resulting decrease in the volume of voids. When the voids are filled with air alone, compression of soil occurs rapidly, because air is compressible and can escape easily from the voids. In a saturated soil mass having its voids filled with incompressible water, decrease in volume or compression can take place when water is expelled out of the voids. Such a compression resulting from a long term static load and the consequent escape of pore water is termed as *consolidation.* According to Terzaghi: *"every process involving a decrease in the water content of a saturated soil without replacement of the water by air is called a process of consolidation.* The opposite process is called a process of *swelling,* which involves an increase in the water content due to an increase in the volume of voids." Compression of soil also takes place by expulsion of air from the voids, under short duration moving or vibratory loads. Such a compression is usually known as *compaction.* The compression of partly saturated soils is accompanied by expulsion and compression of air and its partial dissolution in water. Depending upon degree of saturation, water may also be expelled out along with air.

When a compressive load is applied to a laterally confined layer of sand, *rapid* vertical deformation occurs. The rate at which this deformation can take place depends upon the permeability of soil and upon the distance the water must travel to reach a drainage surface. The compressibility of clays may also be caused by three factors: (*i*) the expulsion of double layer water from between the grains (*ii*) slipping of the particles to new positions of greater density, and *(iii)* bending of particles as elastic sheets. The permeability of clay being very small, time is an important factor in the consolidation of clays.

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15.2 THE CONSOLIDATION PROCESS: SPRING ANALOGY

The mechanics of consolidation was demonstrated by Terzaghi, by means of the *piston and spring analogy.*

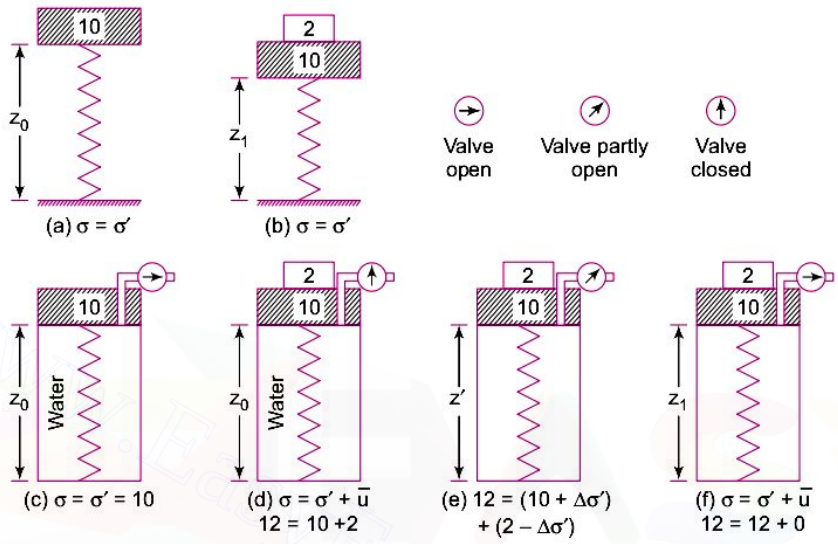

Fig. 15.1 The spring analogy

Figure 15.1 shows a spring with a piston on its top. Let the length of the spring be z_0 under a pressure of 10 units. If 12 units of pressure are added to its top, the spring will be compressed immediately to a length z_1 . A further application of load will result in further decrease in the length of the spring. Within elastic limit, the load deflection curve may be assumed to be straight. If this spring and piston is placed in a cylinder containing water upto the bottom of the piston, and a valve at its bottom, water will be free of stress since the whole load is carried by the spring alone. If the pressure on the piston is increased to 12 units, and the valve is closed, the spring *cannot* deform since water is incompressible. Hence the additional pressure of 2 units is entirely borne by water. If σ denotes the total pressure, σ the pressure in spring and $\bar{u}$ as the pressure in water (*i.e.*, pore pressure), the governing equation in Fig. 15.1 (*d*) is given by

$$12 = 10 + 2 \text{ or } \sigma = \sigma' + \bar{u} \qquad ...(15.1)$$

Now, let the valve be opened slightly so that some water escapes and then valve is closed. Due to escape of some water, the piston moves down, the spring is compressed and hence some pressure, out of pressure of 2 units entirely borne by water, is now transferred to the spring. Thus, at any intermediate stage, the pressure equation becomes:

$$12 = (10 + \Delta\sigma') + (2 - \Delta\sigma') \qquad ...(15.2)$$

where $\Delta\sigma'$ is the transfer of pressure from water to the spring corresponding to a given amount of expulsion of water. If the valve is fully opened, sufficient water will escape till the length of spring is reduced to a height of z_1 . Thus, the whole of 2 units of pressure is transferred from water to the spring, the water becomes free of pressure and the spring carries the whole of pressure. The pressure equation at this stage becomes:

$$12 = 12 + 0 \text{ or } \sigma = \sigma' + \bar{u} \qquad ...(15.3)$$

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Thus, we see that when there is a pressure increment, the whole of pressure is first taken by water. As the water escapes out of the system, the load transfer takes place from water to the spring till the spring is deformed by the full amount corresponding to the applied stress increment. This analogy can be applied to the consolidation process of a soil mass consisting of soil water system. The grain structure represents the spring while the voids filled with water represent the cylinder. The valve opening is represented by the permeability of the soil mass, and the rate of load transfer from water to soil depends upon the permeability and the boundary conditions *(i.e.* the drainage faces available). The pressure that builds up in pore water due to load increment on the soil is termed *excess pore pressure* or *excess hydrostatic pressure* or *hydrodynamic pressure* $\bar{u}$, because it is in excess of the initial pressure in water under static condition. The excess hydrostatic pressure forces the water to drain out of the voids. As water starts escaping from the voids, the excess hydrostatic pressure in water gets gradually dissipated and the pressure increment is shifted as an increase in effective pressure on the soil solids and the soil mass decreases in volume. When the whole of the pressure increment or the *consolidation pressure* is carried as an increase in the effective pressure on the solids, no more water escapes from the voids and a condition of equilibrium is attained. Under different applied pressures, soil attains different *equilibrium* or *final voids ratio,* and under each equilibrium condition the whole of the applied pressure is carried by the solids as an effective pressure. The delay caused in consolidation by the slow drainage of water out of a saturated soil mass is called *hydrodynamic lag.* The reduction in volume of soil which is due principally to a squeezing out of water from the voids is termed *primary consolidation, primary compression* or *primary time effect.* Even after the reduction of all excess hydrostatic pressure to zero, some compression of soil takes place at a very slow rate. This is known as *secondary consolidation, secondary compression* or *secondary time effect.* During the secondary compression, some of the highly viscous water between the points of contact is forced out from between the particles.

15.3 CONSOLIDATION OF LATERALLY CONFINED SOIL

If a remoulded soil is laterally confined in a consolidometer, consisting of a metal ring, and porous stones are placed both at its top and bottom faces, the compression or consolidation of soil sample takes place under a vertical pressure applied on the top of porous stones. The porous stones provide free drainage of water and air from or into the soil sample. Under a given applied pressure, a final settlement and equilibrium voids ratio is attained after certain time. At the equilibrium stage, the applied pressure naturally becomes the effective pressure σ' on the soil. The pressure can then be increased and a new equilibrium voids ratio is attained. Thus, a relationship can be obtained between the effective pressure σ' and the equilibrium voids ratio e (generally called 'voids ratio' only), in the form of curve shown in Fig. 15.2. If, at any intermediate stage at point B, when $\sigma' = 300$ kN/m^2 (say), the pressure is completely removed, the sample expands, as represented by the expansion curve BC. During expansion, the sample never attains the original voids ratio, because of some permanent compression mainly due to some irreversible orientation undergone by the soil particles under compression. If the soil is again put under compression, a re-compression curve, such as CD is obtained, the voids ratio at D being always less than that at B at the same pressure of 300 kN/m^2 . On further pressure increments, the curve DE is obtained. The portion AB of the curve represents the compression of a soil which has not been subjected in the past to pressures greater than those which are being applied for the present compression, Such a curve is called the *virgin compression curve.* The curve DE is also the virgin curve.

If the pressure voids ratio curve is plotted on a semi-logarithmic plot, with σ' as abscissa on logarithmic scale and voids ratio as ordinate on an arithmetic scale the virgin compression curve, becomes a straight line as shown in Fig. 15.3.

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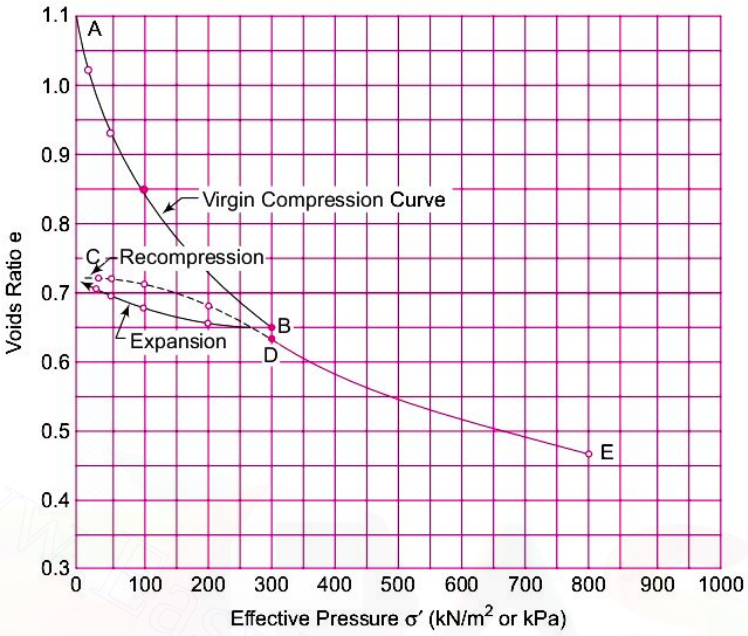

Fig. 15.2 Pressure voids ratio curve for remoulded specimen

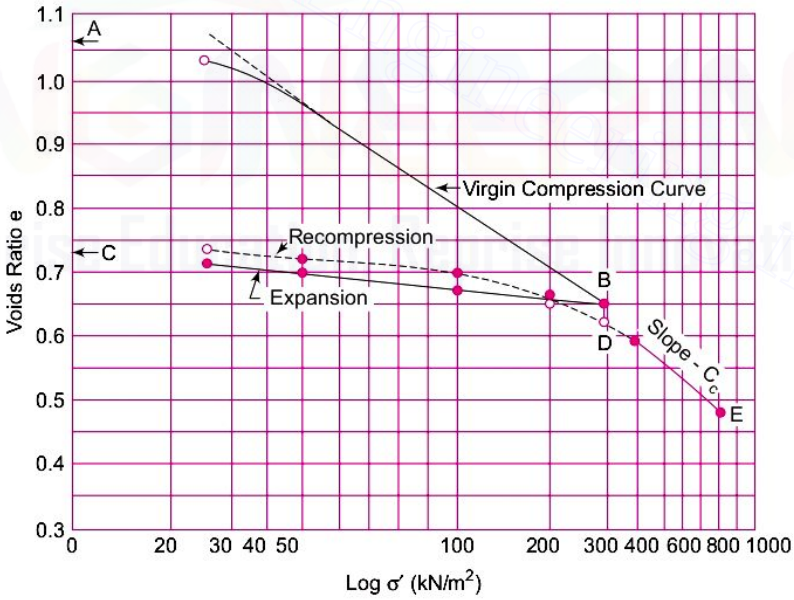

Fig. 15.3 Semi-log plot of pressure-voids ratio relationship

The straight line portion of the virgin compression curve can be expressed by the following empirical relationship given by Terzaghi:

$$e = e_0 - C_c \log_{10} \frac{\sigma'}{\sigma'_0} \qquad ...(15.4)$$

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where e_0 = initial voids ratio corresponding to the initial pressure σ_0'

e = voids ratio at increased pressure σ'

C_c = compression index (dimensionless)

The compression index represents the slope of the linear portion of the pressure-voids ratio curve, and remains constant within a fairly large range of pressure.

Thus, from Eq. 15.4, $$C_c = \frac{e_0 - e}{\log_{10} \frac{\sigma'}{\sigma_0'}} = \frac{\Delta e}{\Delta \log_{10} \sigma'} \quad ...(15.5)$$

Eq. 15.5 may be written as $\Delta_e = C_c \log_{10} \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}$ ...(15.6)

The expansion curve is also a fairly straight line on the semilog plot, and is expressed by

$$e_0 = e + C_s \log_{10} \frac{\sigma'}{\sigma_0'} \quad ...(15.7)$$

where C_s = *expansion* or *swelling index*. It is a measure of the volume increase due to the removal of pressure.

Skempton (1944) conducted consolidation tests on a number of clays from different parts of the World, and gave the following equation for the compression index for a remoulded sample:

$$C_c = 0.007\ (w_L - 10\%) \quad ...(15.8\ a)$$

For an ordinary clay of medium to low sensitivity, the value of C_c *corresponding to the field consolidation* line is roughly equal to 1.3 times the value of C_c corresponding to remoulded sample.

Hence, $$C_c = 0.009\ (w_L - 10\%) \quad ...(15.8\ b)$$

Hough (1957) gave the following equation for *precompressed* soils:

$$C_c = 0.3\ (e_0 - 0.27) \quad ...(15.9)$$

where e_0 = *in-situ* void ratio.

Coefficient of compressibility a_v. The coefficient of compressibility is defined as the decrease in voids ratio per unit increase of pressure:

$$a_v = \frac{-\Delta e}{\Delta \sigma'} = \frac{e_0 - e}{\sigma' - \sigma_0'} \quad ...(15.10)$$

For a given difference in pressure, the value of coefficient of compressibility decreases as the pressure increases.

Coefficient of volume change m_v. The coefficient of volume change or the coefficient of volume compressibility is defined as the change in volume of a soil per unit of initial volume due to a given unit increase in the pressure:

$$m_v = -\frac{\Delta e}{1 + e_0} \frac{1}{\Delta\sigma'} \quad ...(15.11)$$

Substituting $-\frac{\Delta e}{\Delta \sigma'} = a_v$, we get $m_v = \frac{a_v}{1 + e_0}$ ...(15.12)

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When the soil is laterally confined, the change in the volume is proportional to change in the thickness ΔH and the initial volume is proportional to the initial thickness H_0

Hence, $$m_v = -\frac{\Delta H}{H_0}\frac{1}{\Delta\sigma'} \quad ...(15.13)$$

Thus the change in the thickness, ΔH due to pressure increment is given by

$$\Delta H = -m_v H_0 \Delta\sigma' \quad ...(15.13\ a)$$

(The minus sign in the above equation simply denotes that the voids ratio or thickness decreases with the increase in the pressure.)

Consolidation settlement

Consolidations settlement can be computed by two methods:

Method (*a*): Using coefficient of volume change (m_v)

Method (*b*): Using voids ratio.

(*a*) Final settlement using coefficient of volume change (m_v)

The consolidation settlement ρ_f when the soil stratum of thickness H has fully consolidated under a pressure increment $\Delta\sigma'$, is given by Eq. 15.13 *a*:

$$\rho_f = m_v H \Delta\sigma' \quad ...(15.14)$$

This is on the assumption that the pressure increment is transmitted uniformly over the thickness H. However, in practical cases under a finite surface loading, the intensity of $\Delta\sigma'$ *decreases* with the depth of layer in a non-linear manner. In such circumstances, the consolidation settlement $\Delta\rho_f$ of an element of thickness dz is calculated under an average effective pressure increment $\Delta\sigma'$ by Eq. 15.14:

$$\Delta\rho_f = m_v \Delta\sigma'\, dz$$

Integrating for the total thickness H of the layer:

$$\rho_f = \int_0^H m_v \Delta\sigma'\, dz \quad ...(15.14\ a)$$

in which both m_v and $\Delta\sigma'$ are variables. The integration may be performed numerically or graphically.

The numerical integration may be performed by dividing the total thickness H into a number of thin layers, and $\Delta\sigma'$ at the mid-height of each layer may be considered to represent a *constant average pressure increment* for the layer. The settlement for each layer can then be calculated from Eq. 15.14. The total settlement of the layer of thickness H will then be equal to the sum of individual settlements of the various thin layers.

(*b*) Final settlement by voids ratio: The final settlement ρ_f can also be computed from the following relationship:

$$\frac{\Delta H}{H} = \frac{e_0 - e}{1 + e_0} \text{ or } \rho_f = \Delta H = \frac{e_0 - e}{1 + e_0} H \quad ...(15.15\ a)$$

(*i*) Normally consolidated soils: Compression index for normally consolidated soil is constant. Hence substituting the value of $e_0 - e$ in terms of C_c

$$\rho_f = H \frac{C_c}{1 + e_0} \log_{10} \frac{\sigma'}{\sigma_0'} \quad ...(15.15)$$

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where $\sigma' = \sigma_0' + \Delta\sigma'$.

(ii) Preconsolidated soils: Final settlement is small in case of preconsolidated soils as the recompression index or the swelling index C_s is very small in comparison to C_c.

Now from Eq. 15.7, $e_0 - e = C_s \log_{10} \dfrac{\sigma'}{\sigma_0'}$ where $\sigma' = \sigma_0' + \Delta\sigma$

Hence from Eq. 15.15 (*a*), $\rho_f = \dfrac{C_s}{1+e_0} H \log_{10} \dfrac{\sigma'}{\sigma_0'}$ ...(15.15 *b*)

The above equation is applicable when σ' is smaller than the preconsolidation pressure σ_p'. If the preconsolidation pressure σ_p' is greater than σ_0' but *smaller* than σ', then final settlement is computed in two parts:

(*i*) settlement due to pressure σ_0' to σ_p', using C_s and
(*ii*) settlement due to pressure σ_p' to σ', using C_c

Thus, $\rho_f = \dfrac{C_s}{1+e_0} H \log_{10} \dfrac{\sigma_p'}{\sigma_0'} + \dfrac{C_c}{1+e_0} H \log_{10} \dfrac{\sigma'}{\sigma_p'}$ ...(5.15 *c*)

See example 15.12 for illustration.

15.4 CONSOLIDATION OF UNDISTURBED SPECIMEN

Soil deposits may be divided into three classes as regards to the consolidation history: pre-consolidated, normally consolidated and under consolidated. A clay is said to be *pre-compressed, pre-consolidated* or *overconsolidated* if it has ever been subjected to a pressure in excess of its present overburden pressure. The temporary overburden pressure to which a soil has been subjected and under which it got consolidated is known as *pre-consolidation* pressure. A soil may have been pre-consolidated during the geologic past by the weight of an ice sheet or glacier which has melted away, or by other geologic overburden and structural load which no longer exist now. A *normally consolidated* soil is one which has never been subjected to an effective pressure greater than the existing overburden pressure and which is also completely consolidated by the existing overburden. A soil which is not fully consolidated under the existing overburden pressure is called an *under-consolidated soil.*

Determination of pre-consolidation pressure. To find the preconsolidation pressure, an undisturbed sample of clay is consolidated in the laboratory and the pressure voids-ratio relationship is plotted on a semilog plot, as shown in Fig. 15.4.

The initial portion of the curve is flat and resembles the recompression curve of a remoulded specimen. The lower portion of the curve, which is a straight line, is the laboratory virgin curve. The approximate value of the pre-consolidation pressure σ_p' may be determined by the following empirical method of *A*. Casagrande (1936).

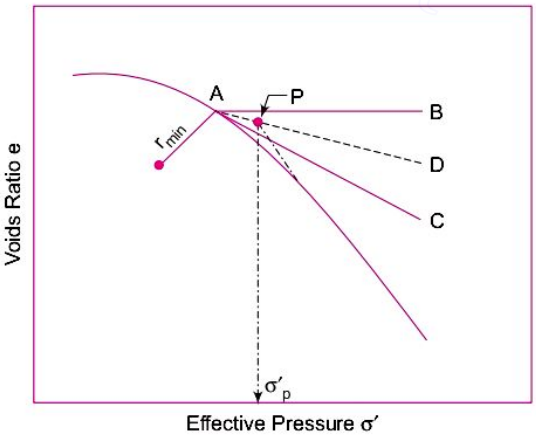

Fig. 15.4 Pre-consolidation pressure

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The point A of maximum curvature (minimum radius) is selected and horizontal line AB is drawn. A tangent AC is drawn to the curve and the bisector AD, bisecting angle BAC is drawn. The straight portion of the virgin curve is extended back to meet the bisector AD in P. The point P corresponds to the pre-consolidation pressure σ_p'.

15.5 TERZAGHI'S THEORY OF ONE DIMENSIONAL CONSOLIDATION

The theoretical concept of the consolidation process was developed by Terzaghi (1923). In the development of the mathematical statement of the consolidation process, the following simplifying assumptions are made: (*i*) The soil is homogeneous and fully saturated, (*ii*) Soil particles and water are incompressible, (*iii*) The deformation of the soil is due entirely to change in volume, (*iv*) Darcy's law for the velocity of flow of water through soil is perfectly valid, (*v*) Coefficient of permeability is constant during consolidation, (*vi*) Load is applied in one direction only and deformation occurs only in the direction of the load application, *i.e.* the soil is restrained against lateral deformation, (*vii*) Excess pore water drains out only in the vertical direction, (*viii*) The boundary is a free surface offering no resistance to the flow of water from the soil, (*ix*) The change in thickness of the layer during consolidation is insignificant, (*x*) The time lag in consolidation is due entirely to the permeability of soil, and thus, the secondary consolidation is disregarded.

Figure 15.5 (*a*) shows a clay layer, of thickness H, sandwiched between two layers of sand which serves as drainage faces. When the layer is subjected to a pressure increment $\Delta\sigma$, excess hydrostatic pressure is set up in the clay layer. At the time t_0, the instant of pressure application, whole of the consolidating pressure $\Delta\sigma$ is carried by the pore water so that the initial excess hydrostatic pressure $\bar{u}_0$ is equal to $\Delta\sigma$, and is represented by a straight line $\bar{u} = \Delta\sigma$ on the pressure distribution diagram. The straight line CED joining the water levels in the piezometric tubes represent this distribution. As water starts escaping into the sand, the excess hydrostatic pressure at the pervious boundaries drops to zero and remains so at all times. After a very great time t_f, the whole of the excess hydrostatic pressure is dissipated so that $\bar{u} = 0$,

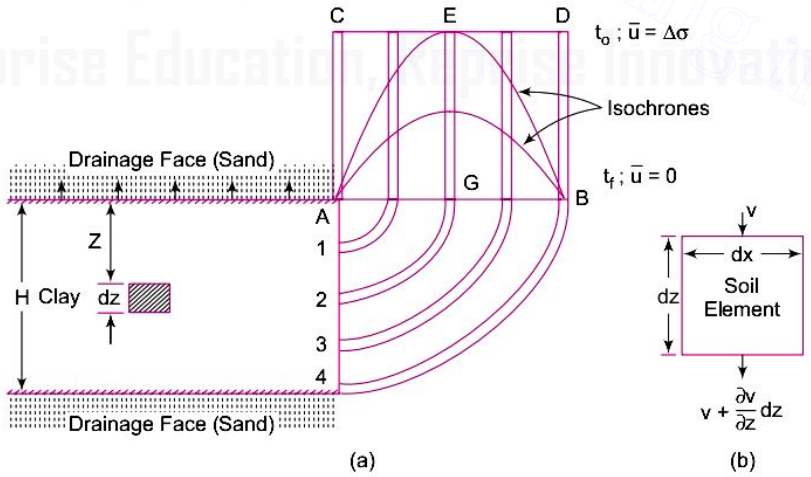

Fig. 15.5 One dimensional consolidation

represented by line AGB. At an intermediate time t, the consolidating pressure $\Delta\sigma$ is partly carried by water and partly by soil, and the following relationship is obtained: $\Delta\sigma = \Delta\sigma' + \bar{u}$. The distribution of

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excess hydrostatic pressure $\bar{u}$ at any time t is indicated by the curve *AFB*, joining water levels, in the piezometric tubes; this curve is known as *isochrone*, and number of such isochrones can be drawn at various time intervals t_1, t_2, t_3 etc. The slope of isochrones at any point at a given time indicates the rate of change of $\bar{u}$ with depth.

At any times t, the hydraulic head h corresponding to the excess hydrostatic pressure is given by

$$h = \frac{\bar{u}}{\gamma \omega} \qquad ...(i)$$

Hence the hydraulic gradient i is given by

$$i = \frac{\partial h}{\partial z} = \frac{1}{\gamma \omega} \frac{\partial \bar{u}}{\partial z} \qquad ...(ii)$$

Thus, the rate of change of $\bar{u}$ along the depth of the layer represents the hydraulic gradient. The velocity with which the excess pore water flows at the depth z is given by Darcy's law

$$v = ki = \frac{k}{\gamma \omega} \frac{\partial \bar{u}}{\partial z} \qquad ...(iii)$$

The rate of change of velocity along the depth of the layer is then given by

$$\frac{\partial v}{\partial z} = \frac{k}{\gamma \omega} \frac{\partial^2 \bar{u}}{\partial z^2} \qquad ...(iv)$$

Consider a small soil element of size dx, dz, and of width dy perpendicular to the xz plane. If v is the velocity of water at the entry into the elements, the velocity at the exit will be equal to $v + \frac{\partial v}{\partial z} dz$

The quantity of water entering the soil element = $v\, dxdy$

The quantity of water leaving the soil element = $\left(v + \frac{\partial v}{\partial z} dz \right) dxdy.$

Hence the net quantity of water dq squeezed out of the soil element per unit time is given by

$$\Delta q = \frac{\partial v}{\partial z}\, dx\, dy\, dz \qquad ...(v)$$

The decrease in the volume of soil is equal to the volume of water squeezed out.

However, from Eq. 15.4. $\Delta V = - m_v V_0 \Delta\sigma'$ ...(vi)

where V_0 = volume of soil element at time $t_0 = dx\, dy\, dz$.

$\therefore$ Change of volume per unit time is given by

$$\frac{\partial (\Delta V)}{\partial t} = -m_v\, dx\, dy\, dz\, \frac{\partial (\Delta\sigma')}{\partial t} \qquad ...(vii)$$

Equating (v) and (vii), we get $\frac{\partial v}{\partial z} = -m_v \frac{\partial (\Delta\sigma')}{\partial t}$ ...(viii)

Now, $\Delta\sigma = \Delta\sigma' + \bar{u}$, where $\Delta\sigma$ is constant.

$\therefore$ $$\frac{\partial (\Delta\sigma')}{\partial t} = - \frac{\partial \bar{u}}{\partial t} \qquad ...(ix)$$

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Hence, from (*viii*) and (*ix*), $\frac{\partial v}{\partial z} = m_v \frac{\partial \bar{u}}{\partial t}$ ...(*x*)

Combining Eqs. (*iv*) and (*x*), we get

$$\frac{\partial \bar{u}}{\partial t} = \frac{k}{m_v \gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} \quad \text{...(15.16)}$$

or $$\frac{\partial \bar{u}}{\partial t} = c_v \frac{\partial^2 \bar{u}}{\partial z^2} \quad \text{...(15.17)}$$

where $$c_v = \textit{coefficient of consolidation} = \frac{k}{m_v \gamma_w} \quad \text{...(15.18)}$$

$$= \frac{k(1+e_0)}{a_v \gamma_w} \quad \text{...(15.19)}$$

Equation 15.17 is the basic differential equation of consolidation which relates the rate of change of excess hydrostatic pressure to the rate of expulsion of excess pore water from a unit volume of soil during the same time interval. The term *coefficient of consolidation* c_v used in the equation is adopted to indicate the combined effects of permeability and compressibility of soil on the rate of volume change. The units of c_v are cm^2/sec.

15.6 SOLUTION OF THE CONSOLIDATION EQUATION*

The solution of the differential equation of consolidation is obtained by means of the Fourier series. The solution must satisfy the following hydraulic boundary conditions [Fig. 15.5 *(a)*] :

(*i*) At $t = 0$, at any distance z, $\bar{u} = \bar{u}_0 = \Delta\sigma$ (constant)

(*ii*) At $t = \infty$, at any distance z, $\bar{u}_0 = 0$

(*iii*) At $t = t$, at $z = 0$, $\bar{u} = 0$ and at $z = H$, $\bar{u} = 0$ ...(15.20)

If $\bar{u}$ is assumed to be a product of some function of z and t, it may be represented by the following expression : $\bar{u} = f_1(z) \cdot f_2(t)$ ...(1)

Eq. 15.17 may therefore be written as

$$f_1(z) \cdot \frac{\partial}{\partial t}\left[f_2(t)\right] = c_v f_2(t) \cdot \frac{\partial^2}{\partial z^2}\left[f_1(z)\right] \text{ or } \frac{\frac{\partial^2}{\partial z^2}\left[f_1(z)\right]}{f_1(z)} = \frac{\frac{\partial}{\partial t}\left[f_1(t)\right]}{c_v f_2(t)} \quad \text{...(2)}$$

The left hand term does not contain t and hence is constant if t is considered variable. Similarly, the right hand term is constant when z is considered variable. Hence each term must be equal to a constant (say $-A^2$) and they may be represented by the following relations.

$$\frac{\partial^2}{\partial z^2}\left[f_1(z)\right] = -A^2 f_1(z) \quad \text{...(3)}$$

and $$\frac{\partial^2}{\partial t^2}\left[f_2(t)\right] = -A^2 c_v f_2(t) \quad \text{...(4)}$$

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Eqs. (3) and (4) are satisfied respectively by the following expression.

$$f_1(z) = C_1 \cos Az + C_2 \sin Az \quad ...(5)$$

and $$f_2(t) = C_3\, \varepsilon^{-A^2 c_v . t} \quad ...(6)$$

where C_1, C_2, C_3 = arbitrary constants;

ε = base of hyperbolic or Napierian logarithm = 2.718

Substituting the above values, Eq. (1) becomes

$$\bar{u} = (C_4 \cos Az + C_5 \sin Az)\ \varepsilon^{-A^2 c_v . t} \quad ...(7)$$

The solution of Eq. (7) must satisfy the boundary conditions stated in Eq. 15.20.

Thus, at time t when $z = 0,\ \bar{u} = 0$

$\therefore$ $C_4 = 0$

Hence Eq. (7) reduces to $\bar{u} = C_5 (\sin Az)\, \varepsilon^{-A^2 c_v . t}$ ...(8)

$\therefore$ Also, at time t, at $z = H,\ \bar{u} = 0 = C_5 (\sin AH) \varepsilon^{-A^2 c_v . t}$ ...(9)

Eq. (9) is satisfied when $AH = n\pi$, where n is any integer.

Hence $$\bar{u} = C_5 \left(\sin \frac{n\pi z}{H} \right) \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v . t} \quad ...(15.21)$$

The above expression may be written in the following form:

$$\bar{u} = B_1 \left(\sin \frac{\pi z}{H} \right) \varepsilon^{\frac{-\pi^2}{H^2} c_v . t} + B_2 \left(\sin \frac{2\pi z}{H} \right) \varepsilon^{\frac{-4\pi^2}{H^2} c_v . t} + \ldots B_n \left(\sin \frac{n\pi z}{H} \right) \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v . t} + \ldots$$

or $$\bar{u} = \sum_{n=1}^{n=\infty} B_n \left(\sin \frac{n\pi z}{H} \right) \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v . t} \quad ...(15.22)$$

When $t = 0,\ \varepsilon^{\frac{-n^2\pi^2}{H^2} c_v . t} = 1$ and $\bar{u} = \bar{u}_0$

$\therefore$ $$\bar{u}_0 = \sum_{n=1}^{n=\infty} B_n \left(\sin \frac{n\pi z}{H} \right) \quad ...(15.23)$$

The following relationships are used to determine the value of B_n in the Fourier expansion (Eq. 15.23):

$$\int_0^{\pi} \sin mx \sin nx\, dx = 0 \quad ...(10)$$

and $$\int_0^{\pi} \sin^2 nx\, dx = \frac{\pi}{2} \quad ...(11)$$

where m and n are unequal integers.

In the above expression (Eqs. 10 and 11), if variable x is changed to $\frac{\pi z}{H}$, dx changes to $\frac{\pi}{H} dz$, and the limits of integration become 0 to H. Hence Eqs. 10 and 11 become:

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$$\int_0^H \sin\frac{m\pi z}{H}\sin\frac{n\pi z}{H}\,dz = 0 \qquad ...(12)$$

and
$$\int_0^H \sin^2\frac{n\pi z}{H}\,dz = \frac{H}{2} \qquad ...(13)$$

Multiplying both sides of Eq. 15.23 by $\sin\frac{n\pi z}{H}$ and integrating between the limits 0 to H:

$$\int_0^H \bar{u}_0 \sin\frac{n\pi z}{H}\,dz = \sum_{\substack{m=1\\ m\neq n}}^{m=\infty} B_m \int_0^H \sin\frac{m\pi z}{H}\sin\frac{n\pi z}{H}\,dz + B_n\int_0^H \sin^2\frac{n\pi z}{H}\,dz \qquad ...(14)$$

On multiplication by $\sin\frac{n\pi z}{H}$, the right hand side of Eq. 15.23 is split into two parts: the nth term which is of the form of Eq. 13, and a series of all terms, except nth term, which being of the form of Eq. 12 vanishes.

Hence, $$\int_0^H \bar{u}_0 \sin\frac{n\pi z}{H}\,dz = B_n\frac{H}{2} \text{ or } B_n = \frac{2}{H}\int_0^H \bar{u}_0 \sin\frac{n\pi z}{H}\,dx \qquad ...(15)$$

Substituting this is Eq. 15.22, we get

$$\bar{u} = \sum_{n=1}^{n=\infty}\left[\frac{2}{H}\int_0^H \bar{u}_0 \sin\frac{n\pi z}{H}\,dz\right]\left(\sin\frac{n\pi z}{H}\right)\varepsilon^{\frac{-n^2\pi^2}{H^2}c_v\,.\,t} \qquad ...(16)$$

or
$$\bar{u} = \sum_{n=1}^{n=\infty}\frac{2\Delta\sigma}{n\pi}(1-\cos n\pi)\left(\sin\frac{n\pi z}{H}\right)\varepsilon^{\frac{-n^2\pi^2}{H^2}c_v\,.\,t} \qquad ...(17)$$

when n = even, $1 - \cos n\pi = 0$; n = odd, $1 - \cos n\pi = 2$

Substituting $n = 2N+1$, when N is an integer, the above equation becomes:

$$\bar{u} = \frac{4}{n}\Delta\sigma\sum_{N=1}^{N=\infty}\frac{1}{(2N+1)}\left[\sin\frac{(2N+1)\pi z}{H}\right]\varepsilon^{-\frac{(2N+1)^2\pi^2}{H^2}c_v\,.\,t} \qquad ...(15.24)$$

Eq. 15.24 gives the variation of excess hydrostatic pressure $\bar{u}$ with depth z at any time in terms of applied consolidating pressure $\Delta\sigma$.

The *consolidation settlement* $\Delta\rho$ or the downward movement of the surface of a consolidating layer, at any time t during the process of consolidation can be found from

Eqs. 15.14 and 15.24. $\Delta\rho = m_v\,\Delta\sigma' dz$

where $\Delta\rho'$ = effective pressure increment at time $t = \Delta\sigma - \bar{u}$

$\therefore$ $\Delta\rho = m_v(\Delta\sigma - \bar{u})\,dz$

Integrating between the limits 0 to H, the settlement ρ of the fully thickness of the clay at times t is given by

$$\rho = \int_0^H m_v(\Delta\sigma - \bar{u})\,dz = m_v\left[\Delta\sigma\, H - \int_0^H \bar{u}\,dz\right]$$

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Substituting $\bar{u}$ from Eq., 15.24 and integrating,

$$\rho = m_v \, \Delta\sigma H \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} \varepsilon^{\frac{-(2N+1)^2 \pi^2}{H^2} c_v . t} \right] \quad ...(15.25)$$

At $t = \infty$, when the process of consolidation is complete, the *ultimate* or final settlement ρ_f is given by

$$\rho_f = m_v \Delta\sigma \, H. \quad ...(15.26)$$

The ratio ρ to ρ_f, expressed as a percentage, is termed the *degree of consolidation U:*

$$U\,(\%) = \frac{\rho}{\rho_f} \times 100 \quad ...(15.27)$$

or

$$(U\,\%) = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} \varepsilon^{\frac{-(2N+1)^2 \pi^2}{H^2} c_v . t} \right] \times 100 \quad ...(15.28)$$

Let us introduce a dimensionless parameter, called the *time factor* T_v defined by following equation:

$$T_v = \frac{c_v t}{d^2} \quad ...(15.29)$$

where d = *drainage path* (The drainage path represents the maximum distance which the water particles have to travel for reaching the free drainage layer. For the present case of double drainage $d = H/2$).

Eq. 15.28 may be written as:

$$U(\%) = \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{N=\infty} \frac{1}{(2N+1)^2} \varepsilon^{\frac{-(2N+1)^2 \pi^2}{4} T_v} \right] \times 100 \quad ...(15.30)$$

or

$$U(\%) = f(T_v) \quad ...(15.31\ a)$$

Thus, the degree of consolidation is a function of time factor. The time factor contains the physical constants of soil layer influencing its time-rate of consolidation Eq. 15.29 can be written as:

$$T_v = \frac{k}{m_v \gamma_w} \frac{t}{d^2} = \frac{k(1+e_0)}{a_v \gamma_w} \frac{t}{d^2} \quad ...(15.31)$$

The time factor, and hence the degree of consolidation depends upon: (*i*) thickness of clay layer, (*ii*) number of drainage faces, (*iii*) k, (*iv*) c_v and (*v*) magnitude of consolidating pressure and the manner of its distribution across the thickness of the layer.

Eq. 15.30 gives the relation between U and T_v when the clay layer has double drainage and the distribution of consolidating pressure is *uniform* throughout the depth of the layer [Fig. 15.6 (*a*)]. The same equation also applies to all other *linear* distributions of consolidating pressure provided there is double drainage of the consolidating layer. The equation is also valid for a layer with single drainage, if it has uniform distribution of consolidating pressure (Fig. 15.6). Table 15.1 gives the values of time factor for these conditions. If the layer has single drainage, with other linear distribution of the consolidating pressure [Fig. 15.6(*c*) to (*f*)], the values of time factor are given in Table 15.2.

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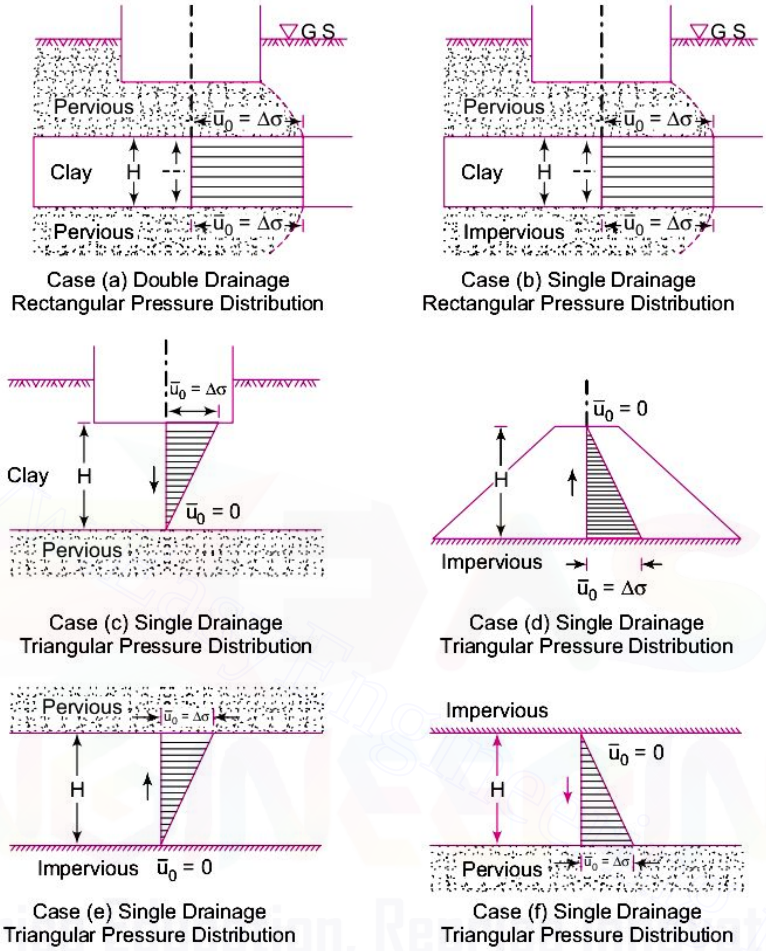

Fig. 15.6 Various cases of drainage face and consolidation pressure distribution

Approximate expression for T_v. Eq. 15.30 may also be written in the following form of an infinite series:

$$U = 1 - \frac{8}{\pi^2}\left[\varepsilon^{\frac{-\pi^2}{4}T_v} + \frac{1}{9}\varepsilon^{\frac{-9\pi^2}{4}T_v} + \frac{1}{25}\varepsilon^{\frac{-25\pi^2}{4}T_v} + ...\right] \quad ...(15.32)$$

Eq. 15.30 or Eq. 15.32 may very closely be represented by the following empirical expressions:

When $U < 60\%$: $$T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 \quad ...(15.33)$$

When $U > 60\%$: $$T_v = -0.9332 \log_{10}\left(1 - \frac{U\%}{100}\right) - 0.0851 \quad ...(15.34a)$$

or $$T_v = 1.7813 - 0.9332 \log_{10}(100 - U\%) \quad ...(15.34)$$

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Table 15.1 Values of time factor

Boundary Condition

(*i*) Double drainage with all linear distribution of consolidation pressure [Fig. 15.6 (*a*)]

or

(*ii*) Single drainage with uniform distribution of consolidation pressure [Fig. 15.6 (*b*)]

U(%)	T_v	*U(%)*	T_v
5	0.002	55	0.238
10	0.008	60	0.287
15	0.018	65	0.342
20	0.031	70	0.403
25	0.049	75	0.477
30	0.071	80	0.567
35	0.096	85	0.684
40	0.126	90	0.848
45	0.159	95	1.129
50	0.197	100	∞

Table 15.2 Values of time factor for single drainage

Boundary conditions	*Cases (c) and (d) (Fig. 15.6)*	*Cases (e) and (f) (Fig. 15.6)*
U(%)	T_y	T_y
10	0.047	0.003
20	0.100	0.009
30	0.158	0.024
40	0.221	0.048
50	0.294	0.092
60	0.383	0.160
70	0.500	0.271
80	0.865	0.440
90	0.940	0.720
100	∞	∞

Coefficient of consolidation Eq. 15.29 may also written as

$$t = \frac{d^2}{c_v} T_v \qquad ...(15.35)$$

Since T_v is constant for a given degree of consolidation and given boundary conditions of the problem under consideration, the time required to attain a certain degree of consolidation is directly proportional to the square of its drainage path and inversely proportional to the coefficient of consolidation. In the theoretical analysis, c_v is assumed constant, although it is a variable quantity. For a soil at a given voids ratio, c_v increases with increasing magnitude of the consolidating pressure, and hence it should be determined in the laboratory consolidation test for the range of pressure that would be encountered in practice.

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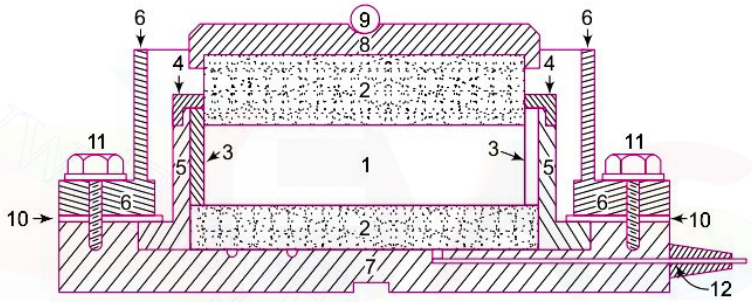
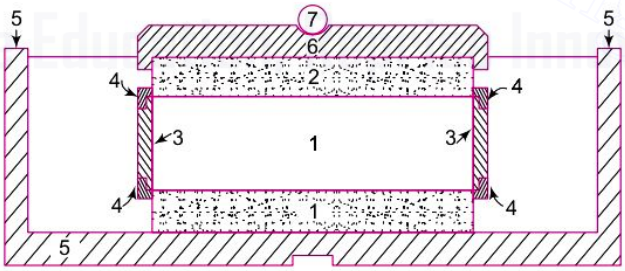
15.7 LABORATORY CONSOLIDATION TEST

The laboratory consolidation test is conducted with an apparatus known as *consolidometer* consisting essentially of a loading frame and *consolidation cell* in which the specimen is kept. Porous stones are put on the top and bottom ends of the specimen. Figs. 15.7 and 15.8 show the *fixed ring cell* and *floating ring cell,* respectively. In the fixed ring cell, only the top porous stone is permitted to move downwards as the specimen compresses. In the floating ring cell, both top and bottom porous stones are free to compress the specimen towards the middle. Direct measurement of permeability of the specimen at any stage of loading can be made only in the fixed ring type. However, the floating ring cell has the advantage of having smaller effects of friction between the specimen ring and the soil specimen.



1. SOIL SPECIMEN
2. POROUS STONES
3. SPECIMEN RING
4. GUIDE RING
5. OUTER RING
6. WATER JACKET
7. BASE
8. PRESSURE PAD
9. PRESSURE BALL
10. RUBBER GASKET
11. BOLTS
12. DRAIN TUBE..

Fig. 15.7 Fixed ring consolidation ring



1. SOIL SPECIMEN
2. POROUS STONES
3. SPECIMEN RING
4. GUIDE RINGS
5. WATER JACKET
6. PRESSURE PAD
7. PRESSURE BALL

Fig 15.8 Fixed ring consolidation ring

The loading machine is usually capable of applying steady vertical pressure upto 800 or 1000 kN/m^2 (kPa) to the soil specimen. During the test, the specimen is allowed to consolidate under a number of increments of vertical pressure, such as 10, 20, 50, 100, 200, 400, 800, 1000 kN/m^2 (kPa), and each pressure increment is maintained constant until the compression virtually ceases, generally

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24 hours. The vertical compression of the specimen is measured by means of a dial gauge. Dial gauge readings are taken after the application of each pressure increment at the following total elapsed times: 0.25, 1.00, 2.25, 4.00, 6.25, 9.00, 12.25, 16.00, 20.25, 25, 36, 49, 60 minutes, and 2, 4, 8, and 24 hours. The dial gauge readings showing the final compression under each pressure increment are also recorded. After the completion of consolidation under the desired maximum vertical pressure, the specimen is unloaded and allowed to swell. The final dial reading corresponding to the completion of swelling is recorded and the specimen is taken out, and dried to determine its water content and the weight of soil solids. The consolidation test data are then used to determine the following:

1. Voids ratio and coefficient of volume change,
2. Coefficient of consolidation, and
3. Coefficient of permeability.

These have been explained in the subsequent articles.

15.8 CALCULATION OF VOIDS RATIO AND COEFFICIENT OF VOLUME CHANGE

The equilibrium voids ratio or final voids ratio at the end of each pressure increment can be calculated by two methods:

1. 'Height of solids' method.
2. 'Change in voids ratio' method.

The *'change in voids ratio method'* is used only for fully saturated specimens, while the *height of solids method* is applicable to both saturated as well as unsaturated samples.

1. **Height of solids method.** The height H_s of the solids of the specimen is calculated from the following expression:

$$H_s = \frac{M_d}{GA\rho_w} = \frac{W_d}{GA} \qquad ...(15.36)$$

where H_s = height of solids (cm) ; M_d = mass of dried specimen (g) ; W_d = weight of dried specimen; A = cross-sectional area of specimen (cm^2) and G = specific gravity of soil. The voids ratio is calculated from the following relation:

$$e = \frac{H - H_s}{H_s} \qquad ...(15.37)$$

where H = specimen height (cm) at equilibrium under various applied pressures

$= H_0 + \Sigma\Delta H = H_1 + \Delta H$

H_0 = initial height of specimen

ΔH = change in the specimen thickness under any pressure increment

H_1 = height of specimen at the beginning of the load increment.

Knowing the voids ratio at the beginning and at the end of the test, the corresponding water contents and degrees of saturation can be calculated. Table 15.3 shows the observations and calculations by this method.

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Table 15.3 Calculation of voids ratio by height of solids method

$H_0 = 24$ mm; $A = 50$ cm^2 ; $V = 120$ cm^3; $M_d = 180.4$ g; $G = 2.68$; $w_f = 16\%$; $\rho_w = 1$ g/cm^3

$$H_s = \frac{M_d}{G.A\,\rho_w} = \frac{180.4 \times 10}{2.68 \times 50 \times 1} = 13.45 \text{ mm}$$

Applied Pressure σ' *(kN/m^2 or kPa)*	*Final dial Reading* $(10^{-2}$ *mm)*	*Dial Change* ΔH *(mm)*	*Specimen Height* $H = H_1 \pm \Delta H$ *(mm)*	*Height of Voids* $H - H_s$ *(mm)*	*Voids ratio* $e = \frac{H - H_s}{H_s}$	*Remarks*
0.0	58		24.00	10.55	0.786	
10	66	– 0.08	23.92	10.47	0.778	
20	76	– 0.10	23.82	10.37	0.771	
50	138	– 0.62	23.20	9.75	0.725	
100	233	– 0.95	22.25	9.50	0.706	
200	341	– 1.08	21.17	7.72	0.575	
400	447	– 1.06	20.11	6.66	0.495	
600	548	– 1.01	19.10	5.65	0.420	
800	550	– 0.02	19.08	5.63	0.418	
1000	578	– 0.28	18.80	5.35	0.398	Water added at the end of period
0.0	438	+ 1.40	20.20	6.75	0.502	

2. Change in voids ratio method. Assuming the specimen to by fully saturated, the voids ratio e_f at the end of the test is determined from the relation:

$$e_f = w_f G \qquad ...(15.38)$$

where e_f = final voids ratio, at the end of the test

w_f = final water content, at the end of the test.

The change of voids ratio Δe under each pressure increment is calculated from the relationship:

$$\frac{\Delta e}{1+e} = \frac{\Delta H}{H}$$

or
$$\Delta e = \frac{1+e_f}{H_f}\Delta H \qquad ...(15.39)$$

where H_f = final height of specimen, at the end of the test.

Knowing Δe and working backwards from the known value of e_f, the equilibrium voids ratio corresponding to each pressure can be evaluated, as illustrated in Table 15.4.

3. Coefficient of volume change. The coefficient of volume change m_v can be calculated either from Eq. 15.11 or from Eq. 15.13:

$$m_v = -\frac{\Delta e}{1+e_0}\frac{1}{\Delta\sigma'} \qquad ...(15.11)$$

$$m_v = -\frac{\Delta H}{H_0}\frac{1}{\Delta\sigma'} \qquad ...(15.13)$$

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Table 15.4 Calculation of voids ratio by change in voids ratio method

H_0 = 24 mm; A = 50 cm^2; V = 120 cm^3; H_f = 17.75 mm; w_f = 32%; G = 2.70; $e_f = 0.32 \times 2.70 = 0.864$;

$$\Delta e = \frac{H + 0.864}{17.75} \Delta H = 0.105\, \Delta H$$

Applied Pressure σ' *(kN/m^2 or kPa)*	*Final dial Reading*	*Change in thinkness* ΔH *(mm)*	*Specimen height* $H = H_1 \pm \Delta H$ *(mm)*	*Change n voids ratio* $\Delta e = 0.102\, \Delta H$	*Voids ratio* e
0.0	82		24.00		1.521
10	104	– 0.22	23.78	– 0.023	1.498
20	152	– 0.48	23.30	– 0.050	1.448
50	260	– 1.08	22.22	– 0.114	1.334
100	424	– 1.64	20.58	– 0.172	1.162
200	578	– 1.54	19.04	– 0.162	1.000
400	730	– 1.52	17.52	– 0.160	0.840
800	851	– 1.21	16.31	– 0.127	0.713
0.0	707	+ 1.44	17.75	+ 0.151	0.864 ↑

The method of calculation with the help of Eq. 15.11 is known as the *voids ratio method,* while that by Eq. 15.13 is known as *change in the thickness method.* It may be noted that m_v is the reciprocal of compressibility modulus. It is expressed in the units of m^2/kN or more commonly in m^2/MN.

15.9 DETERMINATION OF COEFFICIENT OF CONSOLIDATION

The coefficient of consolidation c_v can be determined by comparing the characteristics of the theoretical relationship between T_v and U to the relationship between elapsed time t and degree of consolidation of the specimen obtained in the laboratory. Out of the many methods available, the following two methods will be described here: (1) square root of time fitting method, and (2) logarithm of time fitting method.

1. Square root of time fitting method. Figure 15.9 shows a theoretical characteristic curve between $\sqrt{T_v}$ and U. The curve is straight up to $U = 60$ % and the abscissa at $U = 90\%$ is equal to 1.15 times the abscissa at $U = 60\%$. The use of this characteristic of the theoretical curve to determine the 90% U point on a laboratory time consolidation curve was suggested by Taylor (1948).

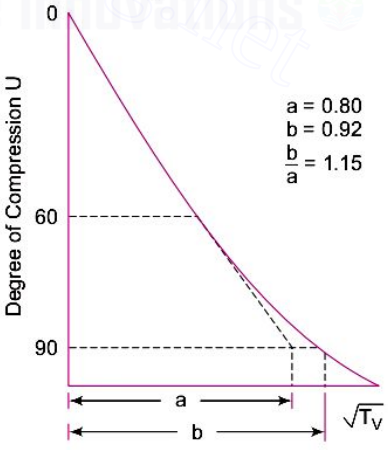

Fig. 15.9 Theoretical curve between $\sqrt{T_v}$ and U

The method consists of drawing the curve between root of time $\sqrt{t}$ as abscissa and the dial reading R, representing the compression of the specimen, as ordinate for any pressure increment in the consolidation test. Figure 15.10 shows such a plot drawn for test data shown in Table 15.5.

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Table 15.5. Test data: Determination of coefficient of consolidation

Pressure Range: 100 kN/m^2 (kPa) to 200 kN/m^2 (kPa)

Initial height of specimen (under pressure of 100 kN/m^2) = 17.38 mm

Final height of specimen (under pressure of 200 kN/m^2) = 16.11 mm

Elapsed time t (min.)	*Dial Reading R(10^{-2} mm)*	$\sqrt{t}$	*Elapsed time t (min.)*	*Dial Reading R (10^{-2} mm)*	$\sqrt{t}$
0	340	0	25	426	5
0.25	360	0.5	36	434	6
1.00	370	1.0	49	440	7
2.25	378	1.5	60	445	7.75
4.00	386	2.0	120	454	10.8
6.25	394	2.5	180	456	13.4
9.00	402	3.0	300	459	17.3
12.25	410	3.5	480	462	21.9
16.00	416	4.0	1440	467	33
20.25	422	4.5			

The initial dial reading R_0 corresponds to the time $t = 0$ and $U = 0$. The straight portion (line A) is produced back to meet the ordinate at reading R_c, which is called the corrected zero reading and the consolidation between R_0 and R_c is called *initial consolidation.* From R_c, another line B is so drawn that its abscissa at every point is 1.15 times that of line A. The intersection of line B with the consolidation curve gives a point P corresponding to 90% U whose dial reading and time may be designated as R_{90} and t_{90} respectively. From the curve, we get $\sqrt{t_{90}} = 5.5$ or $t_{90} = (5.50)^2 = 30.1$ minutes. The coefficient of consolidation is calculated from Eq. 15.35:

$$c_v = \frac{(T_v)_{90}\, d^2}{t_{90}}$$

where

$t_{90} = 30.1$ minutes

$= 30.1 \times 60$ seconds

$(T_v)_{90}$ = times factor corresponding to 90% $U = 0.848$ (Table 15.1)

d = average drainage path for the pressure increment

$$= \frac{H}{2} = \frac{1}{2}\left[\frac{H_1 + H_f}{2}\right]$$

$$= \frac{1.738 + 1.611}{4} = 0.837 \text{ cm}$$

$$c_v = \frac{0.848\,(0.837)^2}{30.1 \times 60}$$

$$= 3.94 \times 10^{-4} \text{ cm}^2/\text{sec.}$$

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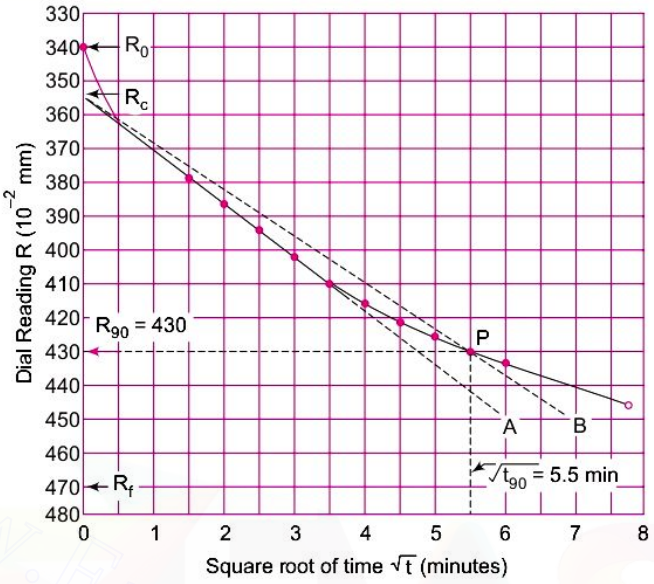

Fig. 15.10 Time consolidation curve (Table 15.5)

2. Logarithm of time fitting method. This method suggested by *A*. Casagrande (1930), is based on the characteristic of the $U - \log_{10} T_v$ plot in which intersection of a tangent at the point of inflection and the asymptote of the lower portion is at the ordinate of 100% *U*. This characteristic is used to determine the point of 100% *U* on the semi-log plot of laboratory time consolidation curve. Figure 15.11 shows the theoretical curve while Fig. 15.12 shows the plotting of the test data of Table 15.5.

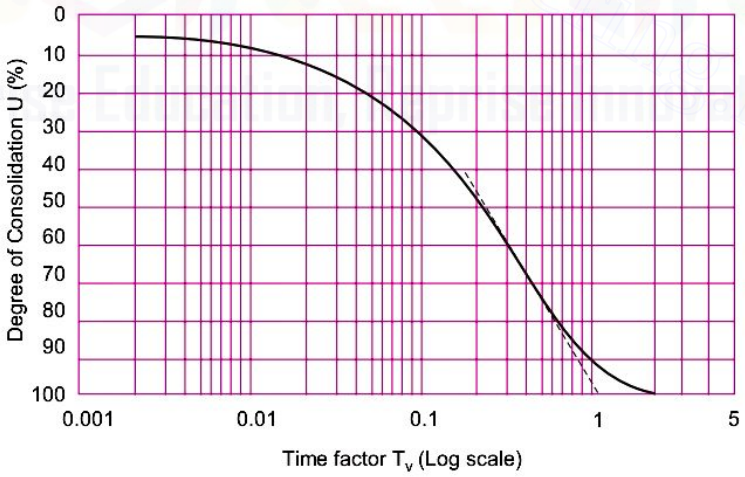

Fig. 15.11 Theoretical curve log T_v and U (Ref. Table 15.1)

The corrected zero reading R_c in the laboratory curve is obtained on the assumption that the curve portion at the beginning of the graph (on semi-log plot) is parabola. A time $t_1 = 1$ minute is selected and its corresponding point *A* is marked on the curve. Another point *B* is so selected on the curve that

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its corresponding time is $\frac{t_1}{4} = \frac{1}{4}$ minute. A horizontal line is then drawn at a vertical height z above B, where z is the vertical height between A and B. The ordinate corresponding to this horizontal line is the correct reading R_c corresponding to $U = 0$.

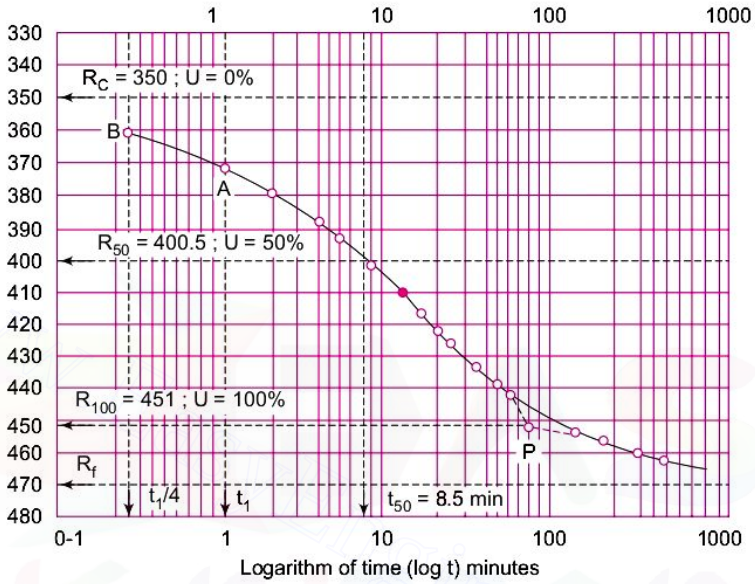

Fig. 15.12 Time consolidation curve (Table 15.5)

The dial reading R_{100} corresponding to $U = 100\%$ is given by extending the straight portion of the curve to meet the point P which is the point of 100% consolidation. The consolidation from R_c to R_{100} is the *primary consolidation* while the consolidation from R_{100} to R_f is the *secondary consolidation.* After locating R_c and R_{100}, the dial reading R_{50}, and hence t_{50}, corrseponding to 50% U can be found out from the plot. The calculations are shown below:

For $t_1 = 1.0$ min. $R = 370$; For $t_{1/4} = 0.25$ min., $R = 360$

$\therefore$ $z = 370 - 360 = 10$

For $U = 0\%$, $R_c = 360 - 10 = 350$; For $U = 100\%$, $R_{100} = 451$

For $U = 50\%$, $R_{50} = 350 + \frac{451 - 350}{2} = 400.5$;

$\therefore$ $t_{50} = 8.5 \text{ min} = 6.5 \times 60$ seconds

$$d = \frac{1.738 + 1.611}{4} = 0.837 \text{ cm} ; (T_v)_{50} = 0.197;$$

The co-efficient of consolidation is obtained from the relation:

$$c_v = \frac{(T_v)_{50}\, d^2}{t_{50}} = \frac{0.197\, d^2}{t_{50}} = \frac{0.197 \times (0.837)^2}{8.5 \times 60} = 2.71 \times 10^{-4} \text{ cm}^2/\text{sec}$$

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Determination of coefficient of permeability. A falling head permeability test can be performed on the consolidation-specimen attaching a stand pipe to fixed ring consolidometer, when the consolidation of the specimen is complete under a particular pressure increment. Alternatively, the coefficient of permeability can be determined from Eq. 15.18 or Eq. 15.19:

$$k = c_v m_v \gamma_w = \frac{c_v a_v \gamma_w}{1 + e_0}.$$

Thus, knowing c_v and m_v (or a_v), k can be computed.

SOLVED EXAMPLES

Example 15.1. *An undisturbed sample of clay, 24 mm thick, consolidated 50% in 20 minutes, when tested in the laboratory with drainage allowed at top and bottom. The clay layer, from which the sample was obtained, is 4 m thick in the field. How much time will it take to consolidate 50%, with double drainage? If the clay stratum has only single drainage, calculate the time to consolidate 50%. Assume uniform distribution of consolidation pressure.*

Solution. For the same degree of consolidation, T_v is the same.

Hence $t \propto \frac{d^2}{c_v}$. Also, since both the soils are the same, $t \propto d^2$.

(a) For the same case of double drainage:

$$\left(\frac{t_2}{t_1}\right) = \left(\frac{d_2}{d_1}\right)^2$$

where d_2 = drainage path in the field = 4/2 m = 200 cm

d_1 = drainage path in laboratory specimen = 2.4 / 2 = 1.2 cm

t_1 = time for 50% consolidation in the laboratory = 20 min.

$$\therefore \quad t_2 = t_1\left(\frac{d_2}{d_1}\right)^2 = 20\left(\frac{200}{1.2}\right)^2 \text{ minutes} = \textbf{386 days.}$$

(b) For the case of single drainage:

$$d_2 = 4 \text{ m} = 400 \text{ cm}$$

$$\therefore \quad t_2 = 20\left(\frac{400}{1.2}\right)^2 \text{ minutes} = \textbf{1544 days}$$

Example 15.2. *An undisturbed sample of a clay stratum, 2 m thick, was tested in the laboratory and the average value of coefficient of consolidation was found to be 2×10^{-4} cm^2/sec. If a structure is built on the clay stratum, how long will it take to attain half the ultimate settlement under the load of the structure? Assume double drainage.*

Solution. For $U < 60\%$, $T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$;

Hence, T_v for $U = 50\%$ is given by

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$$(T_v)_{50} = \frac{\pi}{4}\left(\frac{50}{100}\right)^2 = \frac{\pi}{16} = 0.1956 \approx 0.197$$

(*Alternatively,* from Table 15.1, $T_v = 0.197$ for $U = 50\%$)

$$t_{50} = \frac{(T_v)_{50}\, d^2}{c_v} = \frac{0.197\,(100)^2}{2\times10^{-4}} = 9.85\times10^6 \text{ sec} = 114 \text{ days.} \quad \left[d = \frac{1}{2}\times 2 = 1\text{ m} = 100\text{ cm}\right]$$

Example 15.3. *Two clay specimens A and B, of thickness 2 cm and 3 cm, have equilibrium voids ratios 0.68 and 0.72 respectively under a pressure of 200 kN/m^2. If the equilibrium voids ratios of the two soils reduced to 0.50 and 0.62 respectively, when the pressure was increased to 400 kN/m^2, find the ratio of the co-efficients of permeability of the two specimens. The time required by the specimen A to reach 40 per cent degree of consolidation is $\frac{1}{4}$ of that required by specimen B for reaching 40% degree of consolidation.*

Solution. Let subscripts A and B be used for the two soils. We have

$$m_v = -\frac{\Delta e}{1+e_0}\frac{1}{\Delta\sigma'}$$

For soil A, $\quad e_0 = 0.68\ ;\ e = 0.50\ ;\ \Delta e = e_0 - e = 0.50 - 0.68 = -0.18$

$$\Delta\sigma' = 400 - 200 = 200 \text{ kN/m}^2$$

For soil B, $\quad e_0 = 0.72\ ;\ e = 0.62\ ;\ \Delta e = 0.62 - 0.72 = -0.10$

$$\Delta\sigma' = 400 - 200 = 200 \text{ kN/m}^2$$

Hence,
$$(m_v)_A = \frac{0.18}{1+0.68}\times\frac{1}{200} = 5.36\times10^{-4} \text{ m}^2/\text{kN} = 0.536 \text{ m}^2/\text{MN}$$

$$(m_v)_B = \frac{0.10}{1+0.72}\times\frac{1}{200} = 2.91\times10^{-4} \text{ m}^2/\text{kN} = 0.291 \text{ m}^2/\text{MN}$$

$$\therefore \quad \frac{(m_v)_A}{(m_v)_B} = \frac{5.36\times10^{-4}}{2.91\times10^{-4}} = 1.845 \qquad ...(1)$$

Also, for the same degree of consolidation, we have

$$\frac{t_A}{t_B} = \frac{(d_A)^2}{(d_B)^2}\times\frac{(c_v)_B}{(c_v)_A}$$

$$\therefore \quad \frac{(c_v)_A}{(c_v)_B} = \frac{(d_A)^2}{(d_B)^2}\times\frac{t_B}{t_A} \approx \frac{(2/2)^2}{(3/2)^2}\times\frac{4}{1} = \frac{16}{9} \qquad ...(2)$$

Now,
$$k = c_v\, m_v\, \gamma_w$$

$$\therefore \quad \frac{k_A}{k_B} = \frac{(c_v)_A}{(c_v)_B}\times\frac{(m_v)_A}{(m_v)_B} = \frac{16}{9}\times 1.845 = \mathbf{3.28.}$$

Example 15.4. *A clay layer, whose total settlement under a given loading is expected to be 12 cm settles 3 cm at the end of 1 month after the application of load increment. How many months will be*

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required to reach a settlement of 6 cm? How much settlement will occur in 10 months? Assume the layer to have double drainage.

Solution. When $t_1 = 30$ days,

$$U_1 = \frac{\rho}{\rho_f} \times 100 = \frac{3 \times 100}{13} = 25\%$$

$$t_2 = ?$$

$$U_2 = \frac{6 \times 100}{12} = 50\%$$

The corresponding values of time factors can either be known from Table 15.1 or calculated from the approximate expression for $U < 60\%$:

$$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$$

When $U_1 = 25\%, (T_v)_1 = \frac{\pi}{4}(0.25)^2 = 0.0492;$

When $U_2 = 50\%, (T_v)_2 = \frac{\pi}{4}(0.5)^2 = 0.1965$

$$\therefore \quad \frac{t_2}{t_1} = \frac{(T_v)_2}{(T_v)_1} = \frac{0.1965}{0.0492}$$

$$\therefore \quad t_2 = \frac{0.1965}{0.0492} \times 30 = 120 \text{ days} = 4 \text{ months}$$

Also, when $t = 10$ months, $T_v = \frac{t}{t_1}(T_v)_1 = \frac{10}{1} \times 0.0492 = 0.492.$

When $U = 60\%$, $T_v = 0.287$, and hence the approximate expression between T_v and U is valid for a maximum value of $T_v = 0.287$. In the present case, $T_v = 0.492$, and hence U cannot be found out from the approximate expression of Eq. 15.33. However, from approx. Eq. 15.34,

We have $T_v = 0.492 = 1.7813 - 0.9332 \log_{10}(100 - U\%)$ or $\log_{10}(100 - U\%) = 1.3816$, from which $U \approx 76\%$

Also, from the table 15.1 when $T_v = 0.492$, we get $U \approx 76\%$

$$\therefore \quad \rho = U \,.\, \rho_f = 0.76 \times 12 = \mathbf{9.12\ cm.}$$

Example 15.5. *Calculate the co-efficient of volume change for pressure range 100 to 200 kN/m^2, from the data given in Table 15.4, by (i) change in voids ratio method, (ii) change in the thickness method.*

Solution.

(a) Change in voids ratio method:

$$\sigma_0' = 100 \text{ kN/m}^2;$$

$$e_0 = 1.121; \sigma' = 200 \text{ kN/m}^2; e = 0.964$$

$$\therefore \quad \Delta\sigma' = 100 \text{ kN/m}^2; \Delta e = -0.157$$

$$m_v = -\frac{\Delta e}{1 + e_0}\frac{1}{\Delta\sigma'} = \frac{0.157}{1 + 1.121} \times \frac{1}{100} = 7.4 \times 10^{-4} \text{ m}^2/\text{kN} = \mathbf{0.74\ m^2/MN}$$

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(b) Change in thickness method:

$\sigma_0' = 100$ kN/m^2;
$H_0 = 2.080$ cm;
$\sigma' = 200$ kN/m^2 ; $H = 1.926$ cm
$\therefore$ $\Delta\sigma' = 100$ kN/m^2 ; $\Delta H = -0.154$

$$m_v = -\frac{\Delta H}{H_0}\frac{1}{\Delta\sigma'} = \frac{0.154}{2.08}\times\frac{1}{100} = 7.4\times10^{-4}\ \text{m}^2/\text{kN} = \mathbf{0.74\ m^2/MN}$$

Example 15.6. *An undisturbed sample of clay, 24 mm in thickness, is tested in a consolidometer. The total change in the thickness of the specimen in 4.74 mm when the applied pressure is 200 kN/m^2 and 6.13 mm when the applied pressure is 400 kN/m^2. On reducing the pressure to zero, the specimen swells to a thickness of 18.50 mm. The final water content of the specimen is 32.6% and specific gravity is 2.72. The clay stratum, 1 m thick, is sandwiched between pervious strata, and is subjected to an overburden pressure of 200 kN/m^2. Estimate the probable settlement of a proposed structure, if due to its construction, the effective pressure at the centre of the clay layer is expected to be increased to 400 kN/m^2.*

Solution. The settlement is given by $\rho_f = m_v\,\Delta\sigma' H$ ...(1)

where m_v = coefficient of volume change for the pressure range 200 to 400 kN/m^2

$\Delta\sigma' = 400 - 200 = 200$ kN/m^2 ; $H = 1$ m $= 100$ cm

When $\sigma_0' = 200$ kN/m^2 ; $H_0 = 24 - 4.74 = 19.26$ mm;
$\sigma' = 400$ kN/m^2 ; $H = 24 - 6.13 = 17.87$ mm
$\therefore$ $\Delta\sigma' = 200$ kN/m^2 ; $\Delta H = -0.139$ cm

$$m_v = -\frac{\Delta H}{H_0}\frac{1}{\Delta\sigma'} = \frac{0.139}{19.26}\times\frac{1}{200} = 3.61\times10^{-4}\ \text{m}^2/\text{kN}$$

$$\rho_f = 3.61\times10^{-4}\times200\times1 = 7.22\times10^{-2}\ \text{m} = 7.22\ \text{cm}$$

Alternatively, $\rho_f = \dfrac{e_0 - e}{1+e_0}H$ ...(2)

Now, $e_f = w_f G = 0.326\times2.72 = 0.886$; $\Delta e = \dfrac{1+e_f}{H_f}\Delta H = \dfrac{1+0.886}{18.50}\Delta H = 0.102\,\Delta H$ ...(2)

The voids ratio are calculated in the table below:

σ'(kN/m^2)	ΔH (mm)	$\Delta e = 0.102\,\Delta H$	e
0			1.448
200	− 4.74	− 0.484	0.964
400	− 1.39	− 0.142	0.822
0	+ 0.63	+ 0.064	0.886↑

Substituting the value in (2), $\rho_f = \dfrac{0.964 - 0.822}{1+0.964}\times100 = \mathbf{7.22\ cm}$

Example 15.7. *A 2 m wide strip footing is to be placed in a sand layer 2 m thick, at a depth of 1 m below the ground surface. The sand is underlain by a layer of saturated clay which is 1 m thick. The clay overlies a bed of dense sand. The ground water table is at the level of the top of the clay layer. The bulk unit weight of sand above the layer is 20 kN/m^3 and submerged unit weight of clay is 8 kN/m^3. The footing is designed to carry a load of 220 kN/m^2. Compute the probable ultimate settlement of the footing below its centre. Also determine*

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the elapsed time in which 10 per cent and 90 per cent of the ultimate settlement will occur. It is known from a graph between pressure and voids ratio for clay that the voids ratio corresponding to vertical pressure of 44 kN/m^2 and 181 kN/m^2 are respectively 1.16 and 1.02. The coefficient of consolidation for the soil is 4×10^{-4} cm^2/sec.

Solution. σ_0' (at middle of clay) $= (2 \times 20) + (0.5 \times 8) = 44$ kN/m^2 ...(1)

Pressure at the footing level before excavation $= 1 \times 20 = 20$ kN/m^2

Pressure at the footing level after construction $= 220$ kN/m^2.

$\therefore$ Net increase in the pressure intensity at the footing level $= q = 220 - 20 = 200$ kN/m^2.

From the elastic analysis (Boussinesq theory), the pressure distribution between the centre of the strip load of width B varies with depth under (see Table 13.6)

Depth	σ_z
0.1 B	0.997 *q*
0.5 B	0.817 *q*
B	0.530 *q*
2 B	0.306 *q*

In our case, the depth of top level of the clay layer, below the footing level = 1 m = $0.5 \times 2 = 0.5\ B$.

Hence, $\sigma_z = 0.817\ q$.

Also, the depth of the bottom level of the clay layer below the footing level = 2 m = B.

Hence, $\sigma_z = 0.550\ q$.

Hence the average pressure increment at centre of the clay layer

$$= \frac{1}{2}(0.817 + 0.550)\ q$$

$$= 0.6835\ q$$

$$= 0.6835 \times 200 = 136.7 \text{ kN/m}^2 \quad ...(2)$$

(**Note**. If the thickness of the layer is more, it should be divided into a number of thin strips, and the vertical pressure at the centre of each strip should be found, and each strip should be analysed separately.)

In absence of Table 13.6, σ_z can be computed *analytically,* using Eq. 13.20

$$\sigma_z = \frac{q}{\pi}(\theta + \sin\theta).$$

For the top point P_1 of clay layer, $\theta = \theta_1$ (Fig. 15.13) and is given by

$$\theta_1 = 2 \tan^{-1}(1/1) = 90° = 1.5708 \text{ rad.}$$

$\therefore$ $$\sigma_z = \frac{q}{\pi}(1.5708 + \sin 90°) = 0.8183\ q$$

Also, for the bottom point P_2 of clay layer, $\theta = \theta_2$ given by

$$\theta_2 = 2 \tan^{-1}(1/2) = 53.13° = 0.9273 \text{ rad.}$$

$\therefore$ $$\sigma_z = \frac{q}{\pi}(0.9273 + \sin 53.13°) = 0.5498\ q$$

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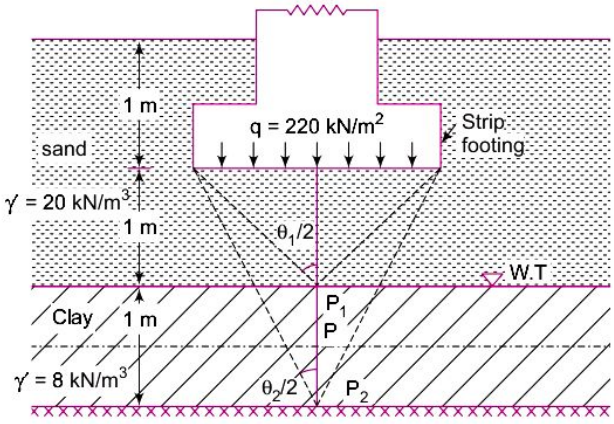

Fig. 15.13

$\therefore$ Average $\sigma_z = \frac{1}{2}(0.8183 + 0.5498)\, q = 0.684\, q = 136.8$ kN/m^2

This is practically the same as found above.

$\therefore$ $\sigma' = \sigma_0' + \Delta\sigma' = 44 + 136.7 = 180.7$ kN/m^2

The corresponding voids ratio are: e_0 (at $\sigma_0' = 44$) = 1.16; e_0 (at $\sigma' = 180.7$) $\approx$ 1.02

$\therefore$ $\rho_f = \frac{e_0 - e}{1 + e_0} H = \frac{1.16 - 1.02}{1 + 1.16} \times 1 = 0.065$ m = 6.5 cm.

Again, from Table 15.1, When $U = 10$ %, $(T_v)_{10} = 0.008$ and $U = 90$%, $(T)_{90} = 0.848$

$$t_{10} = \frac{(T_v)_{10}\, d^2}{c_v} = \frac{0.008 \times (50)^2}{4 \times 10^{-4}} \text{ sec} = \textbf{13.9 hours ;}$$

$$t_{90} = \frac{(T_v)_{90}\, d^2}{c_v} = \frac{0.848 \times (50)^2}{4 \times 10^{-4}} \text{ sec} = \textbf{61.34 days.}$$

Example 15.8. *A clay layer has a thickness of 5 m and is subjected to a pressure of 60 kN/m^2. If the layer has a double drainage and undergoes 50% consolidation in one year, determine the coefficient of consolidation taking $T_v = 0.197$. Also if the coefficient of permeability is 0.025 m/year, determine the settlement in one year and rate of flow of water per unit area in one year.*

Solution.

(*a*) We have: $c_v = T_v d^2/t = \frac{0.197(5/2)^2}{1} = 1.231$ m^2/year

Also, from Eq. 15.18, $m_v = \frac{k}{c_v \gamma_w} = \frac{0.025}{1.231 \times 9.81} = 2.07 \times 10^{-3}$ m^2/kN

Final settlement $\rho_f = m_v \,.\, \Delta\sigma' .\, H = 2.07 \times 10^{-3} \times 60 \times 5 = 0.621$ m

$\therefore$ Settlement after 1 year = $0.5\, \rho_f = 0.5 \times 0.621 =$ **0.3105 m**

(*b*) For $U < 60$%, U is proportional to $\sqrt{t}$; hence settlement (ρ) is also proportional to $\sqrt{t}$

i.e. $\rho \propto \sqrt{t}$ or $\rho^2 = t$.

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Let $t = C\rho^2$, where C is a constant

When $t = 1$ year, $\rho = 0.3105$ m

$\therefore$ $C = t/\rho^2 = 1/(0.3105)^2 = 10.37$

Hence, $t = 10.37\,\rho^2$

or $$\frac{d\rho}{dt} = \frac{1}{2 \times 10.37 \times \rho} = \frac{0.0482}{\rho}$$

since $\rho = 0.3105$ in 1 year, $\dfrac{d\rho}{dt} = \dfrac{0.0482}{0.3105} = \mathbf{0.3106\ m/year}$

(*c*) Discharge per unit area per face = 0.3106/2 = **0.1503 m^3/yr/m^2**

Example 15.9. *The loading period for a new building extended from July 1980 to July 1982. In July 1985, the average measured settlement was found to be 6.78 cm. If it is known that ultimate settlement will be about 25 cm, estimate the settlement in July 1991. Assume double drainage to occur.*

Solution. Since the loading period was between July 1980 to July 1982, the *datum* will be the middle of the period, *i.e.* July 1981. All the computations will be done with reference to this datum. Hence the settlement of 8.78 cm, observed in July 1985, will be after 4 years (*i.e.*, July 1985 – July 1981).

The settlement = 25 cm, after a period of 1991 – 1981 = 10 years.

Thus, when time $t_1 = 4$ years, settlement $\rho_1 = 8.78$ cm and when time $t_2 = 10$ years, settlement = ρ_2

$\therefore$ Degree of settlement U_1 after t_1 (= 4 years) = $\dfrac{6.78}{25} = 0.2712$

and Degree of settlement U_2 after t_2 (= 10 years) = $\dfrac{\rho_2}{25} = 0.04\,\rho_2$

Assuming the degree of settlement to be less than 60% in both the cases, we have, from Eq. 15.33

$$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$$

or $U = 1.1284\ \sqrt{T_v}$, when U is expressed as a fraction.

Hence $$\frac{\rho_1}{\rho_2} = \frac{U_1}{U_2} = \sqrt{\frac{T_{v1}}{T_{v2}}} = \sqrt{\frac{t_1}{t_2}}$$, since $T_v = \dfrac{c_v\,t}{d^2}$ where $\dfrac{c_v}{d^2}$ is a constant.

$\therefore$ $$\frac{6.78}{\rho_2} = \sqrt{\frac{4}{10}} = 0.6325$$ from which $\rho_2 = 10.72$ cm

Check. After $t_2 = 10$ years, $U_2 = 0.04\,\rho_2 = 0.04 \times 10.72 = 0.429$

Since the value of U_2 is less than 60%, the above use of approximate equation (*i.e.* Eq. 15.33) is valid.

Example 15.10. *Solve example 15.9 if the observed settlement in July 1985 was 10.55 cm.*

Solution. U_1 after t_1, (= 4 years) = $\dfrac{10.55}{25} = 0.422$ ...(*i*)

U_2 = degree of settlement after t_2 (= 10 years) = $\dfrac{\rho_2}{25} = 0.04\,\rho_2$

For the first period, since $U_1 = 0.422$ (*i.e.* < 0.6), ...(*ii*)

$$T_{v1} = \frac{\pi}{4}\left(\frac{U_1}{100}\right)^2 = \frac{\pi}{4}(0.422)^2 = 0.1399$$ ...(*iii*)

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Now, $\frac{T_{v1}}{T_{v2}} = \frac{t_1}{t_2}$, since $t_v = \frac{c_v t}{d^2}$ where $\frac{c_v}{d^2}$ is a constant.

$\therefore$ $T_{v2} = T_{v1} \times \frac{t_2}{t_1} = 0.1399 \times \frac{10}{4} = 0.3596$ ...(*iv*)

For the second period t_2 (= 10 years), U_2 is likely to exceed 60%. Hence Eq. 15.33 will not be valid.

From Eq. 15.34, when $U > 60\%$, $T_v = 1.7813 - 0.9332 \log_{10} (100 - U\%)$

$\therefore$ $T_{v2} = 0.3596 = 1.7813 - 0.9332 \log_{10} (100 - U_2)$

$\therefore$ $\log_{10}(100 - U_2) = 1.5235$ or $100 - U_2 = 33.38$

From which $U_2 = 100 - 33.38 = 66.62$ %

Hence, $\rho_2 = \frac{U_2}{0.04} = \frac{0.6662}{0.04}$

$= \mathbf{16.655\ cm}$

Example 15.11. *A column, carrying a load of 2400 kN transfer this load to sub-soil through a footing of size 4 m × 4 m founded at a depth of 2 m below ground level (Fig. 15.14). The soil below the base of footing is fine sand upto a depth of 4 m and below this, there is a 5 m thick layer of soft compressible clay. The water table is 4 m below the ground level. The specific gravities of solids of sand and clay are 2.66 and 2.72 and the natural water contents are 20 and 40 per cent respectively. The liquid limit of clay stratum is 60%. Determine the probable settlement of the footing. The sand above the water table may be assumed to remain saturated.*

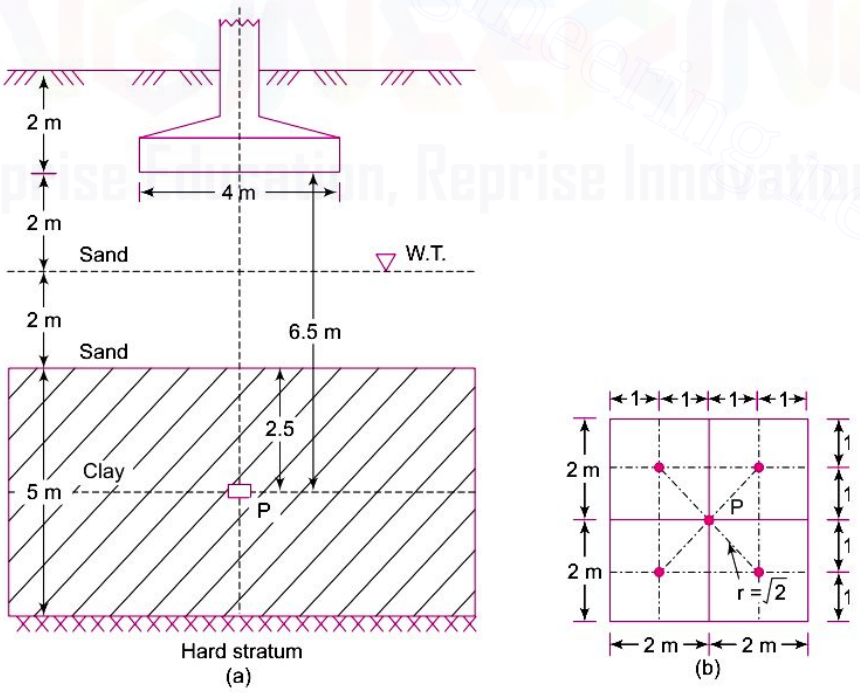

Fig. 15.14

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Solution.

(a) Computation of excess pressure $\Delta\sigma'$ at the middle of clay layer: Depth D of point P (situated at the middle of clay layer) below footing base = 6.5 m.

Width B of loaded area = 4 m; $\therefore$ $D/B = 6.5/4 = 1.625 < 3.0$

Divide the footing into 4 equal parts so that $D/B = 6.5/2 = 3.25 > 3.0$ (Fig. 15.14 *b*).

Concentrated load at the centre of each part = 2400/4 = 600 kN

Radial distance $r = 1\sqrt{2} = 1.414$ m ; $z = 4 + 5/2 = 6.5$ m

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$= \frac{3}{2\pi}\left[\frac{1}{1+(1.414/6.5)^2}\right]^{5/2} = 0.4253$$

$\therefore$ Excess pressure $\Delta\sigma' = 4\dfrac{Q}{z^2}K_B = \dfrac{4\times 600}{(6.5)^2}\times 0.4253 = \mathbf{24.16\ kN/m^2}$

(b) Computations for unit weights and voids ratio: The computations for unit weights and voids ratio, for both sand as well as clay, are done with reference to the soil element shown in Fig. 15.15.

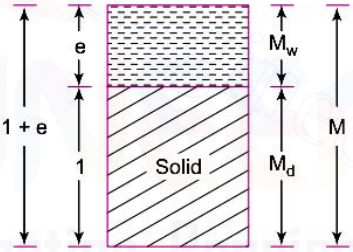

Fig. 15.15

Sand	*Clay*
$M_d = \rho_s V_s = V_s G \gamma_w = G\gamma_w$	$M_d = \rho_s V_s = V_s G\gamma_w = G\gamma_w$
$= 2.66 \times 1 = 2.66$ g	$= 2.72 \times 1 = 2.72$ g
$M_w = wM_d = 0.2 \times 2.66 = 0.532$ g	$M_w = wM_d = 0.4 \times 2.72 = 1.088$ g
$V_w = 0.532$ cm^3	$V_w = 1.088$ cm^3
$e = \dfrac{V_w}{V_s} = \dfrac{0.532}{1} = 0.532$	$e = \dfrac{V_w}{V_s} = \dfrac{1.088}{1} = 1.088$
$M = M_d + M_w = 2.66 + 0.532 = 3.192$ g	$M = M_d + M_w = 2.72 + 1.088 = 3.808$ g
$\rho_{sat} = \dfrac{M}{1+e} = \dfrac{3.192}{1+0.532} = 2.084$ g/cm^3	$\rho_{sat} = \dfrac{M}{1+e} = \dfrac{3.808}{1+1.088} = 1.824$ g/cm^3
$\gamma_{sat} = 9.81\ \rho_{sat} = 9.81 \times 2.084 = 20.44$ kN/m^3	$\gamma_{sat} = 9.81\ \rho_{sat} = 17.89$ kN/m^3
$\gamma' = 20.44 - 9.81 = 10.63$ kN/m^3	$\gamma' = 17.89 - 9.81 = 8.08$ kN/m^3

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(c) Computation of overburden pressure

$$\sigma'_o = 2 \times 2.084 + 2 \times 10.63 + 2.5 \times 8.08 = 45.628 \text{ kN/m}^2$$

$$\therefore \quad \sigma' = \sigma'_o + \Delta\sigma = 45.628 + 16.16 = 61.788 \text{ kN/m}^2$$

(d) Compression index: $C_c = 0.009\ (w_L - 10\%) = 0.009\ (60 - 10) = 0.45$

(e) Computation of settlement:

$$\rho_f = H \frac{C_c}{1+e_0} \log_{10} \frac{\sigma'}{\sigma'_0} = \frac{500 \times 0.45}{1+1.088} \log_{10} \frac{61.788}{45.628} = 14.19 \text{ cm}$$

Example 15.12. *A clay stratum has 3 m thickness and has an initial overburden pressure of 40 kN/m^2. The clay is over-consolidated, with a pre-consolidation pressure of 60 kN/m^2. Determine the final settlements due to an increase in pressure of 50 kN/m^2 at the middle of the clay layer. Take the following values:*

(i) Recompression or swelling index = 0.05
(ii) Compression index = 0.28
(iii) Initial voids ratio = 1.3

Solution.

From Eq. 15.15 *b*, we have:

$$\rho_f = \frac{C_s}{1+e_0} H \log_{10} \frac{\sigma'_p}{\sigma'_0} + \frac{C_c}{1+e_0} H \log_{10} \frac{\sigma'}{\sigma'_p}$$

$\therefore$ Here $\sigma'_o = 40$ kN/m^2; $\sigma'_p = 60$ kN/m^2; $\sigma' = 40 + 50 = 90$ kN/m^2

$$\therefore \quad \rho_f = \frac{0.05}{1+1.3} \times 3 \log_{10} \frac{60}{40} + \frac{0.28}{1+1.3} \times 3 \log_{10} \frac{90}{60}$$

or $$\rho_f = 0.01148 + 0.06431 = 0.07579 \text{ m} \approx \mathbf{75.8\ mm}$$

It is seen that the first part of the settlement is quite small. Hence it is some times neglected.

Example 15.13. *A building column has a footing area of 2 m × 3 m, and transmits a pressure increment of 150 kN/m^2 at its base embedded 1.6 m below ground level, as shown in Fig. 15.16. Assuming a pressure distribution of 2 vertical to 1 horizontal, determine the consolidation settlement at the middle of clay layer. Consider the pressure variation across the thickness of clay layer also. Given the following.*

*(i) For sand : $\gamma = 16.5$ kN/m^3 **and** $\gamma_{sat} = 18.5$ kN/m^3*
(ii) For clay: $\gamma_{sat} = 16$ kN/m^3 ; $e_0 = 0.95$; $C_c = 0.26$

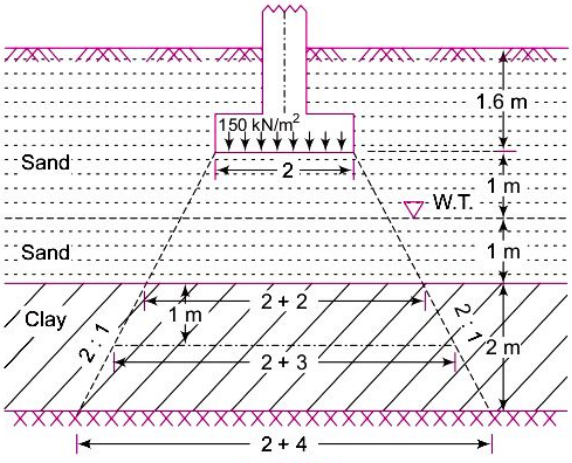

Fig. 15.16

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Solution.

(*i*) ***Initial pressure at the centre of clay layer***

$$\sigma'_o = 2.6 \times 16.5 + 1\,(18.5 - 9.81) + 1\,(16 - 9.81) = 57.78 \text{ kN/m}^2$$

(ii) ***Pressure increase at the top middle and bottom of clay layer***

$$(\Delta\sigma)_t = \frac{150\,(2 \times 3)}{(2+2)\,(3+2)}$$

$$= 45 \text{ kN/m}^2$$

$$(\Delta\sigma)_m = \frac{150\,(2 \times 3)}{(2+3)\,(3+3)}$$

$$= 30 \text{ kN/m}^2$$

$$(\Delta\sigma)_b = \frac{150\,(2 \times 3)}{(2+4)\,(3+4)}$$

$$= 21.43 \text{ kN/m}^2$$

Average pressure is found using Simpson's rule as under:

$$\Delta\sigma = \frac{1}{6}\left[(\Delta\sigma)_t + 4\,(\Delta\sigma_m) + (\Delta\sigma)_b\right] = \frac{1}{6}\,(45 + 4 \times 30 + 21.43)$$

$$= 31.07 \text{ kN/m}^2$$

This is slightly greater than $\Delta\sigma_m$

(iii) ***Final settlement***

$$\rho_f = \frac{C_c}{1+e_0} H \log_{10} \frac{\sigma'_0 + \Delta\sigma}{\sigma'_0} = \frac{0.26}{1+0.95} \times 2 \log_{10} \frac{57.78 + 31.07}{57.78} = 0.0498 \text{ m} = \mathbf{49.8 \text{ mm}}$$

Example 15.14. *Fig. 15.17 shows a soil profile consisting of two layers of clay, and two layers of sand, all of which are completely submerged. Compute the total settlement under a uniform load of 120 kN/m^2, well distributed over a large area. Given the following properties:*

(a) Sand layers

$$\gamma_{sat} = 20.86 \text{ kN/m}^2$$

(b) Clay layers:

$$w = 38\%$$

$$C_c = 0.26\ ;\ G = 2.72$$

Solution.

The total settlement will be the sum of the settlement of top clay layer and the settlement of bottom clay layer.

For clay, $\gamma' = \dfrac{(G-1)\gamma_w}{1+e}$

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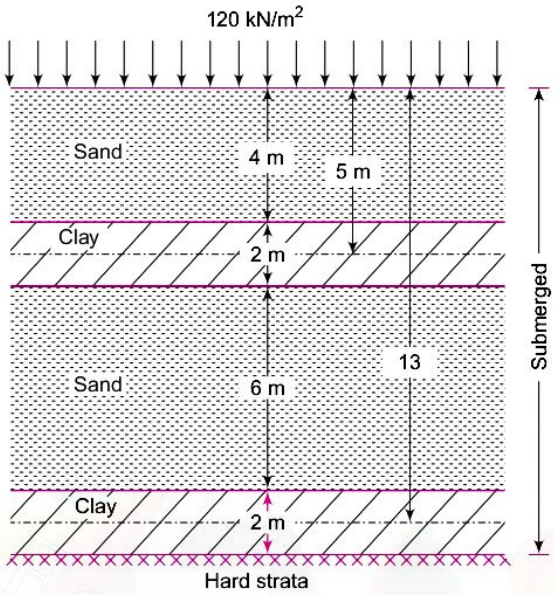

Fig. 15.17

where $e = w_{sat}\, G = 0.38 \times 2.72 = 1.0336$

$$\therefore \quad (\gamma')_{clay} = \frac{(2.72-1)\,9.81}{1+1.0336} \approx 8.3 \text{ kN/m}^3$$

Also, $$(\gamma')_{sand} = \gamma_{sat} - \gamma_w$$
$$= 20.86 - 9.81 = 11.05 \text{ kN/m}^3$$

(a) Settlement of top clay layer

$$\sigma_o' = 11.05 \times 4 + 8.3 \times 1 = 52.5 \text{ kN/m}^2$$

$$\therefore \quad \rho_{1f} = \frac{C_c H}{1+e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma}{\sigma_0'} = \frac{0.26 \times 2}{1+1.0336} \log_{10} \frac{52.5+120}{52.5} = 0.1321 \text{ m}$$

(b) Settlement of bottom clay layer

$$\sigma_o' = (4+6)\ 11.05 + (2+2/2)\ 8.3 = 135.4 \text{ kN/m}^2$$

$$\therefore \quad \rho_{2f} = \frac{0.26 \times 2}{1+1.0336} \frac{\log_{10}(135.4+120)}{135.4} = 0.0705 \text{ m}$$

(c) Total settlement

$$\rho_f = \rho_{1f} + \rho_{2f} = 0.1321 + 0.0705 = 0.2026 \text{ m} = \mathbf{20.26\ cm}$$

15.10 SECONDARY CONSOLIDATION

When the excess pore pressure due to consolidation has been dissipated, the change in voids ratio continues, but generally at a reduced rate. This phenomenon is known as *secondary compression* or *secondary consolidation* during which some of the highly viscous water between the point of contact is

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forced out from between the grains. During the secondary consolidation, there is plastic readjustment of soil particles and of the absorbed water due to the continued stress, and the progressive fracture of some of the particles. In many soil deposits, the secondary compression is so small that it can be neglected. Since secondary consolidation is not governed by the dissipation of excess hydrostatic pressure, Terzaghi's theory of consolidation cannot be applied to find the rate of secondary consolidation.

Secondary consolidation is believed to start coming into play in the range of primary consolidation itself, although its magnitude may be very small. This is also the reason why the experimental time consolidation curve is in agreement with Terzaghi's theory of consolidation only upto $U = 60$ per cent. A pressure voids ratio diagram based on final voids ratio in a consolidation test includes also a part of secondary consolidation which occurs under the various pressure increments. For highly organic soils, highly micaceous soils, and some loosely deposited clays, secondary consolidation may constitute a substantial percentage of total settlement.

It is often assumed that secondary compression proceeds linearly with the logarithm of time. The secondary compression usually appears as a straight line, or a series of straight lines, with different slopes in the plot of voids ratio as a function of logarithm of time, and may be represented by the following equation:

$$\Delta e = -\alpha \log_{10} \frac{t_2}{t_1} \qquad ...(15.40)$$

where α = a coefficient representing rate of secondary compression, t_1 and t_2 = total elapsed time since load was applied to soil.

Time t_1 is the period required for the hydrodynamic consolidation (i. e. primary consolidation) to be nearly complete, and may be taken as the time corresponding to 90% consolidation.

15.11 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 15.15. *A 2.5 cm thick sample of clay was taken from field for predicting the times of settlement for a proposed building which exerts a uniform pressure of 10.5 tonnes/m^2 over the clay stratum. The sample was loaded to 10.5 tonnes/m^2 and proper drainage was allowed from top and bottom. It was seen that 50 per cent of the total settlement occurred in 3 minutes. Find the time required for 50 per cent of the total settlement of the building if it is to stand on 6 m thick layer of clay which extends from ground surface and is undertain by sand.* **(Civil services Exam. 1984)**

Solution. Use suffix 1 for lab and 2 for field. For the same degree of consolidation and similar drainage conditions $T_v = \dfrac{C_v t_1}{d_1^2} = \dfrac{C_v t_2}{d_2^2}$ from which $\dfrac{t_1}{d_1^2} = \dfrac{t_2}{d_2^2}$

$$\therefore \qquad t_2 = \frac{t_1}{d_1^2} \times d_2^2 = \frac{3}{\left(\frac{2.5}{2}\right)^2} = \left(\frac{600}{2}\right)^2 = 172800 \text{ min} = \textbf{120 days}$$

Example 15.16. *A compressible layer is expected to have total settlement of 15 cm under a given loading. It settles by 3 cm at the end of two months after the application of load increment? How many months will be required to reach a settlement of 7.5 cm? What is the settlement in 18 months? The layer has double drainage.* **(Civil Services Exam. 1986)**

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Solution. Given $\rho_f = 15$ cm

At $t_1 = 60$ days, $U_1 = \dfrac{\rho_1}{\rho_f} \times 100 = \dfrac{3}{15} \times 100 = 20\% = 0.2$

At $t_2 = ?\ U_2 = \dfrac{\rho_2}{\rho_f} \times 100 = \dfrac{7.5}{15} \times 100 = 50\% = 0.5$

At $t_3 = 18 \times 30 = 540$ days, $\rho_3 = ?$

For $U < 60\%$, $T_v = \dfrac{\pi}{4}\left(\dfrac{U}{100}\right)^2$

$\therefore$ $T_{v1} = \dfrac{\pi}{4}\left(\dfrac{20}{100}\right)^2 = 0.03142;\ T_{v2} = \dfrac{\pi}{4}\left(\dfrac{50}{100}\right)^2 = 0.1963$

Now, $\dfrac{t_2}{t_1} = \dfrac{T_{v2}}{T_{v1}}$ or $t_2 = 60 \times \dfrac{0.1963}{0.03142} = \mathbf{374.95\ days}$

Also, $T_{v3} = \dfrac{t_3}{t_1} \times T_{v1} = \dfrac{540}{60} \times 0.03142 = 0.2827$

$\therefore$ $U_3 = \left(\dfrac{4}{\pi} T_{v3}\right)^{1/2} = \left(\dfrac{4}{\pi} \times 0.2827\right)^{1/2} = 0.6$

Hence the approximate expression, given by (1) is just valid.

$\therefore$ $\rho_3 = U_3\, \rho_f = 0.6 \times 15 = \mathbf{9\ cm}$

Example 15.17. *Two clay specimens A and B, of thickness 2 cm and 3 cm, has equilibrium voids ratios 0.65 and 0.70 respectively under a pressures of 200 kN/m^2. If the equilibrium voids ratio of the two soils reduced to 0.48 and 0.60 respectively when the pressure was increased to 400 kN/m^2, find the ratio of coefficients of permeability of the two specimens. The time required by the specimen A to reach 40 degree of consolidation is one fourth of that required by specimen B for reaching 40% degree of consolidation.* **(Civil services Exam. 1989)**

Solution. This problem is practically the same as example 15.3. Refer Example 15.3

Now, $\dfrac{k_A}{k_B} = \dfrac{(C_v)_A}{(C_v)_B} \times \dfrac{(m_v)_A}{(m_v)_B}$ ...(1)

For soil A, $e_0 = 0.65;\ e = 48;\ \Delta e = e - e_0 = 0.48 - 0.65 = -0.17$

For soil B, $e_0 = 0.70;\ e = 0.60;\ \Delta e = e - e_0 = 0.6 - 0.7 = -0.10$

For both soils, $\Delta\sigma' = 400 - 200 = 200$ kN/m^2

$\therefore$ $(m_v)_A = -\dfrac{\Delta e}{1 + e_0} \times \dfrac{1}{\Delta\sigma'} = \dfrac{0.17}{1 + 0.65} \times \dfrac{1}{200} = 5.1515 \times 10^{-4}$

$(m_v)_B = -\dfrac{\Delta e}{1 + e_0} \times \dfrac{1}{\Delta\sigma'} = \dfrac{0.10}{1 + 0.70} \times \dfrac{1}{200} = 2.9412 \times 10^{-4}$

Also, $\dfrac{(C_v)_A}{(C_v)_B} = \dfrac{(d_A)^2}{(d_B)^2} \times \dfrac{t_B}{t_A} = \left(\dfrac{2/2}{3/2}\right)^2 \times \dfrac{4}{1} = 1.7778$

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$$\therefore \quad \frac{k_A}{k_B} = 1.7778 \times \frac{5.1515}{2.9412} = \mathbf{3.114}$$

Example 15.18. *A stratum of clay with an average liquid limit of 45% is 6 m thick. Its surface is located at a depth of 8 m below the ground surface. The natural water content of clay is 40% and the specific gravity is 2.7. Between ground surface and clay the subsoil consists of fine sand.*

The water is located at a depth of 4 m below the ground surface. The average submerged unit weight of sand is 10.5 kN/m^3 and the unit weight of sand above the water table is 17 kN/m^3. The weight of building that will be constructed on the sand above clay increases the over burden pressure on the clay by 40 kN/m^2. Estimate the settlement of building. **(Engg. Services Exam. 1990)**

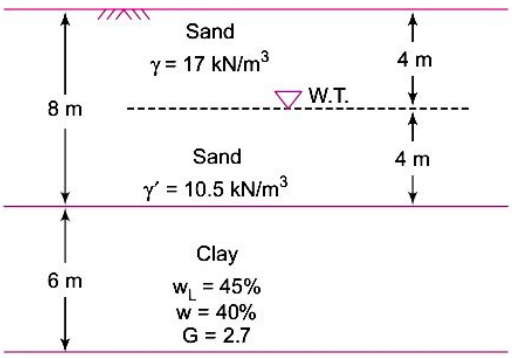

Fig. 15.18

Solution. Pressure on the top of clay due to over burden = σ

$$= 17 \times 4 + 10.5 \times 4 = 110 \text{ kN/m}^2$$

Increase in pressure due to construction of building = 40 kN/m^2 = Δσ

$$C_c = 0.009\,(w_L - 10) = 0.009\,(45 - 10) = 0.315$$

$$e_0 = w_{sat}\, G = 0.4 \times 2.7 = 1.08$$

Settlement $$S = \frac{C_c\, H}{1 + e_0} \log_{10} \frac{\sigma' + \Delta\sigma}{\sigma'} = \frac{0.315 \times 6}{1 + 1.08} \log_{10} \left(\frac{110 + 40}{110} \right) = 0.1224 \text{ m}$$

$$= 12.24 \text{ cm}$$

Example 15.19. *A stratum of clay with an average liquid limit of 45% is 6 m thick. Its surface is located at a depth of 8 m below the ground surface. The natural water content of the clay is 40% and the specific gravity is 2.7. Between ground surface and clay, the subsoil consists of fine sand.*

The water is located at a depth of 4 m below the ground surface. The average submerged unit weight of sand is 10.5 kN/m^3 and unit weight of sand above the water table is 17.0 kN/m^3. The weight of the building that will be constructed on the sand above clay increases the overburden pressure on the clay by 40 kN/m^2. Estimate the settlements of the building.

(Civil Services Exam. 1991)

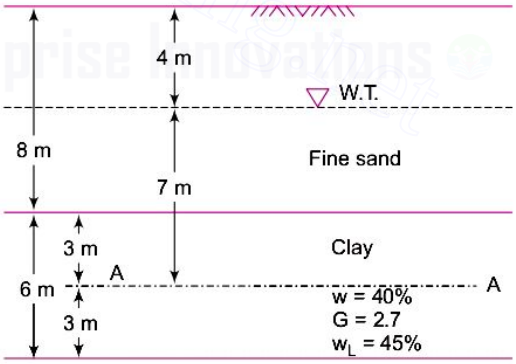

Fig. 15.19

Solution. For clay, $$e = w_{sat}\, G = 0.4 \times 2.7 = 1.08$$

$$\therefore \quad \gamma'_{clay} = \frac{G - 1}{1 + e} \gamma_w = \frac{2.7 - 1}{1 + 1.08} \times 9.81 = 8.02 \text{ kN/m}^3$$

Now, $$\sigma'_0 = 17.0 \times 4 + 10.5 \times 4 + 8.02 \times 3 = 134.06 \text{ kN/m}^2 \text{ at level } A\text{-}A.$$

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$\Delta\sigma' = 40$ kN/m^2; H = depth of clay layer = 6 m

e_0 = voids ratio of clay layer = 1.08

$C_c = 0.009\ (w_L - 10\%) = 0.009\ (45 - 10) = 0.315$

$$\therefore \quad \Delta H = \rho = \frac{C_c H}{1+e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'} = \frac{0.315 \times 6}{1+1.08} \log \frac{134.06+40}{134.06} = \mathbf{0.103\ m}$$

Example 15.20. *Boring at a foundation site shows 6 m of fine sand having a unit weight of 17.8 kN/m^3 over 6 m thick clay layer which has unit weight of 16.25 kN/m^3. The clay layer lies over dense sand. The boring was continued upto 18 m below ground level. Water table was found at a depth of 5.0 m below ground level. Consolidation test gave the following relation between void ratio and pressure.*

$$e = 1.45 - 0.5 \log_{10} p$$

The foundation of a building is to placed at 3 m below ground level due to which the pressure increment at top and bottom layers of the clay layer will be 80 kN/m^2 and 14.50 kN/m^2 respectively. Estimate the amount of settlement expected due to the construction of the building. The pressure release due to excavation for foundation may be taken as 24.0 kN/m^2 and 12.0 kN/m^2 respectively at the top and bottom layers respectively.

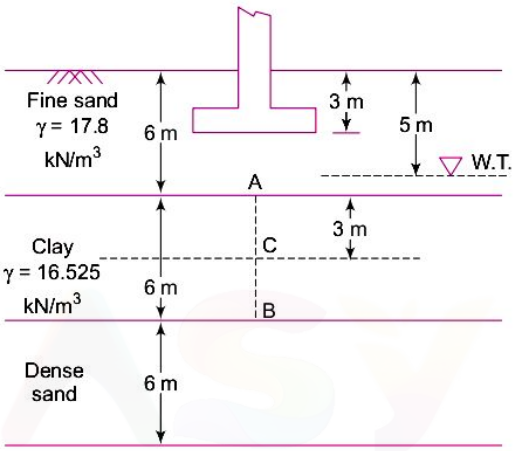

Fig. 15.20

(Civil Services Exam. 1992)

Solution. σ_0' at point C, the centre of clay layer is

$$= 5 \times 17.8 + 1\ (17.8 - 9.81) + 3\ (16.25 - 9.81) = 116.31\ \text{kN/m}^2$$

Release of pressure, due to excavation, at $C = \frac{1}{2}(24 + 12) = 18$ kN/m^2

$$\therefore \quad \sigma_0' p_0 = 116.31 - 18 = 98.31\ \text{kN/m}^2$$

$$\Delta\sigma' = \frac{1}{2}\ (80 + 14.5) = 47.25$$

Now, $e = 1.45 - 0.5 \log_{10} p$

$\therefore$ $e_1 = 1.45 - 0.5 \log_{10} p_1$

and $e_2 = 1.45 - 0.5 \log_{10} p_2$

$\therefore$ $e_1 - e_2 = 0.5\ (\log_{10} p_2 - \log_{10} p_1)$

or $$\frac{e_1 - e_2}{\log_{10}\ p_2 / p_1} = 0.5,$$

From which we have $C_c = 0.5$

Also, $e = 1.45 - 0.5 \log_{10} p$, from which $e_0 = 1.45 - 0.5 \log_{10} 98.31 = 0.454$

Hence settlement $$S = \frac{C_c\ H}{1+e_0} \log_{10} \frac{p_0 + \Delta p}{p_0}$$

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$$= \frac{0.5 \times 6}{1 + 0.454} \log_{10} \frac{98.31 + 47.25}{98.31} = \mathbf{0.352\ m}$$

Example 15.21. *A saturated soil stratum 6 metres thick lies above an impervious stratum and below a pervious stratum. It has a compression index of 0.28 and a coefficient of permeability of 3.5×10^{-4} cm/sec. Its void ratio at a stress of 150 kN/m^2 is 1.95. Determine:*

(i) the change in void ratio due to an increase in stress to 210 kN/m^2

(ii) settlement of the soil stratum due to the above increase in stress and

(iii) time required for 50 per cent consolidation.

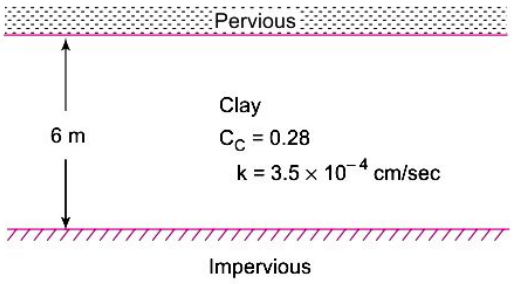

Fig. 15.21

Assume time factor T for 50 per cent consolidation as 0.20. **(Civil Services Exam. 1993)**

Solution. Given: $C_c = 0.28$; $e_0 = 1.95$; $k = 3.5 \times 10^{-4}$ cm/sec $= 3.5 \times 10^{-6}$ m/sec.

(i) Change in voids ratio

$$\Delta e = C_c \log_{10} \frac{\sigma_2}{\sigma_1} = 0.28 \log_{10} \frac{210}{150} \approx \mathbf{0.0409\ (decrease)}$$

(ii) Settlement of soil stratum: $\frac{\Delta H}{H_0} = \frac{\Delta e}{1 + e_0}$ or $\Delta H = 6 \times \frac{0.0409}{1 + 1.95} = 0.0832$ m $= \mathbf{8.32\ cm}$

(iii) Time required for 50% consolidation: Given $(T_v)_{50} = 0.2$

Now, $$t_{50} = \frac{T_v \cdot d^2}{c_v},$$

where $d = 6$ m for single drainage

$$c_v = \frac{k}{m_v \gamma_w} = \frac{k(1 + e_0)}{a_v \gamma_w} = \frac{k(1 + e_0)\Delta\sigma}{\Delta e \cdot \gamma_w}$$

$$= \frac{3.5 \times 10^{-6}(1 + 1.95)(210 - 150)}{0.0409 \times 9.81}$$

$$= 1.544 \times 10^{-3}\ \text{m}^2/\text{sec}.$$

Hence $$t_{50} = \frac{0.2\,(6)^2}{1.544 \times 10^{-3}} = 4663\ \text{sec}.$$

$$= \mathbf{1\ hour\ 17.72\ min.}$$

Example 15.22. *A soil stratum is 10 m thick with pervious stratum on top and bottom. Determine the time required for 50% consolidation. Given that coefficient of permeability = 10^{-7} cm/s.*

Coefficient of compression = 0.0003 cm^2/gm

Void ratio = 2; Time factor = 0.197 **(Civil Services Exam. 1994)**

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Solution. Given $k = 10^{-7}$ cm/s; $a_v = 0.0003$ cm^2/gm ; $e_0 = 2$; $T_v = 0.197$

$$t_{50} = \frac{(T_v)_{50}\, d^2}{c_v}, \text{ where } d = 10/2 = 5 \text{ m} = 500 \text{ cm} ; \gamma_w = 1 \text{ gm/cm}^3$$

$$c_v = \frac{k}{m_v\, \gamma_w} = \frac{k(1+e)}{a_v\, \gamma_w} = \frac{10^{-7}(1+2)}{0.0003 \times 1} = 1 \times 10^{-3} \text{ cm}^2/\text{s}$$

$\therefore$ $$t_{50} = \frac{0.197\,(500)^2}{10^{-3}} = 49250 \times 10^3 \text{ sec} \approx \mathbf{570 \text{ days}}$$

Example 15.23. *A 6.0 m thick layer of clay is located between two layers of free-draining sand. Also, there is a thin drainage layer within the clay at a depth of 1.50 m from its top surface (see Fig. 15.22). The average value of c_v is found as 4.92×10^{-2} mm^2/sec. If a structure is constructed above the clay layer, how many days would be required for it to attain half the ultimate settlement? Assume that the expression $T = \frac{\pi}{4} U^2$ is applicable for the entire range of consolidation.* **(Engg. Services Exam. 1995)**

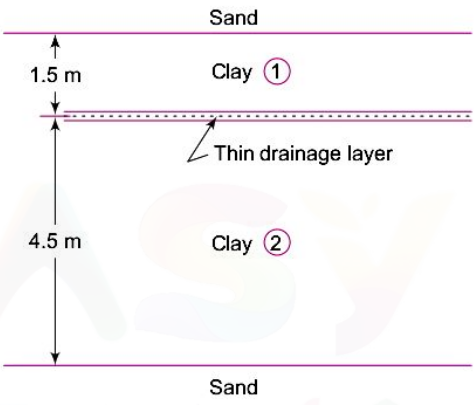

Fig. 15.22

Solution. Let t be the time required to attain half the ultimate settlement. Let us use suffix 1 for upper layer of clay and 2 for the lower layer of clay. Because of existence of intermediate layer of sand, both the layers of clay will have *double drainage.*

Now, $$T_{v1} = \frac{\pi}{4} U_1^2 \quad ...(i) \quad \text{and} \quad T_{v2} = \frac{\pi}{4} U_2^2 \quad ...(ii)$$

Also, $$T_{v1} = \frac{c_v}{d_1^2}\, t = \frac{c_v}{(75)^2}\, t = \frac{4.92 \times 10^{-4}}{(7.5)^2}\, t = 8.747 \times 10^{-8}\, t \quad ...(iii)$$

and $$T_{v2} = \frac{c_v}{d_2^2}\, t = \frac{4.92 \times 10^{-4}}{(225)^2}\, t = 9.7185 \times 10^{-9}\, t \quad ...(iv)$$

From (i) and (ii), $\frac{\pi}{4} U_1^2 = 8.747 \times 10^{-6}\, t$ ...(a)

Similarly, from (ii) and (iv), $\frac{\pi}{4} U_2^2 = 9.7185 \times 10^{-9}\, t$ ...(b)

$\therefore$ $$\frac{U_1^2}{U_2^2} = \frac{8.747 \times 10^{-8}}{9.7185 \times 10^{-9}} = 9, \text{ or } \frac{U_1}{U_2} = 3 \quad ...(1)$$

Now, $$\Delta H_1 = U_1 \cdot H_1 = U_1 \times 1.5 = 1.5\, U_1 \text{ and } \Delta H_2 = U_2 \times 4.5 = 4.5\, U_2$$

$\therefore$ $$\Delta H = \Delta H_1 + \Delta H_2 = 1.5\, U_1 + 2.25\, H_2 \quad ...(c)$$

However, total settlement corresponding to 50% consolidation is

$$\Delta H = U \times H = 0.5 \times 6 = 3 \text{ m}$$

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Substituting in (c), we get $1.5\ U_1 + 4.5\ U_2 = 3$

or $\quad 1.5\ (3\ U_2) + 4.5\ U_2 = 3$ from which $U_2 = \dfrac{1}{3}$

Substituting the value of U_2 in (b), we get $\dfrac{\pi}{4}\left(\dfrac{1}{3}\right)^2 = 9.7185 \times 10^{-9}\ t$

From which $\quad t = 8.979 \times 10^6$ sec = **103.93 days**

Example 15.24. *A soil profile is shown in Fig. 15.23. Calculate the settlement due to primary consolidation for the 6 m clay layer due to a surcharge of 100 kN/m^2. Ground water table is flush with the top of the clay layer.* **(Civil Services Exam. 1996)**

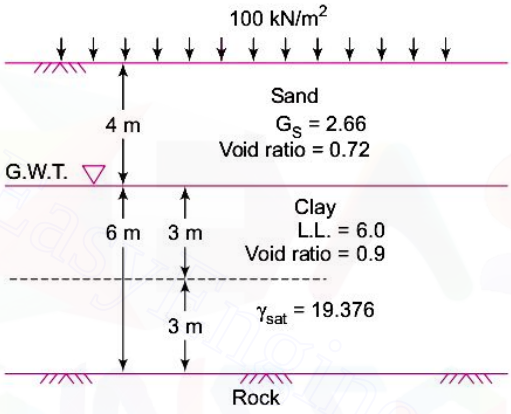

Fig. 15.23

Solution. $\quad C_c = 0.009\ (60 - 10) = 0.45$

For sand, $\quad \gamma = \gamma_d = \dfrac{G\,\gamma_w}{1+e} = \dfrac{2.66 \times 9.81}{1 + 0.72} = 15.17\ \text{kN/m}^3$

For clay, $\quad \gamma' = \gamma_{sat} - \gamma_w = 19.376 - 9.81$

$= 9.566\ \text{kN/m}^3$

$\therefore \quad \sigma_0 = 15.17 \times 4 + 9.566 \times 3$

$= 89.378\ \text{kN/m}^2;\ \Delta\sigma = 100\ \text{kN/m}^2$

$\therefore \quad \Delta H = \dfrac{C_c \,.\, H}{1 + e_0} \log_{10} \dfrac{\sigma_0 + \Delta\sigma}{\sigma_0} = \dfrac{0.45 \times 6}{1 + 0.9} \log_{10} \dfrac{89.378 + 100}{89.378} = \mathbf{0.463\ m}$

Example 15.25. *A homogeneous clay layer 12 m thick is expected to have an ultimate settlement of 332 mm. After a time span of 3 years, the average settlement was measured to be 152 mm. How much longer will it take for the average settlement to attain 237 mm?* **(Civil Services Exam. 1997)**

Solution. Given $\quad \rho_f = 332\ \text{mm} = 0.332\ \text{m}$

Also, when $\quad t_1 = 3$ years, $\rho_1 = 0.152$ mm

When $\quad t_2 = t_2(?),\ \rho_2 = 0.237$ mm

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Hence after t_1, $$U_1 = \frac{\rho_1}{\rho_f} = \frac{0.152}{0.332} = 0.4578$$

and after t_2 $$U_2 = \frac{\rho_2}{\rho_f} = \frac{0.237}{0.332} = 0.7139$$

For $U \le 0.6$, $$T_{v1} = \frac{\pi}{4}(U_1)^2 = \frac{\pi}{4}(0.4578)^2 = 0.1646$$

When $U > 60\%$, $$T_v = -0.9332 \log_{10}\left(1 - \frac{U}{100}\right) - 0.0851$$

$\therefore$ $$T_{v2} = -0.9332 \log_{10}(1 - 0.7139) - 0.0851 = 0.4221$$

Alternatively, From Table 15.1, we get $T_v \approx 0.4236$ for $U = 0.7139$.

Now, $$\frac{c_v}{d^2} = \frac{T_v}{t} = \frac{T_{v1}}{t_1} = \frac{0.1646}{3} = 0.0549$$

Hence, $$t_2 = \frac{d^2}{c_v} \cdot T_{v2} = \frac{T_{v2}}{c_v/d^2} = \frac{0.4221}{0.0549} \approx \textbf{7.69 years}$$

Example 15.26. *A saturated soil has a compression index of 0.28. The void ratio at a stress of 12 kN/m^2 is 2.05 and its permeability is 35×10^{-7} mm/s.*

Compute: (i) change is void ratio if the stress is increased to 21.6 kN/m^2

(ii) the settlement in (i) above if the soil stratum is 6 m thick. **(Civil Services Exam. 1998)**

Solution. Given $$C_c = 0.283;\ \sigma_0 = 12 \text{ kN/m}^2;\ \sigma = 21.6 \text{ kN/m}^2$$

(*i*) $$\Delta e = C_c \log_{10}\frac{\sigma}{\sigma_0} = 0.28 \log_{10} = \frac{21.6}{12} = \textbf{0.0715}$$

(*ii*) $$\Delta H = \frac{C_c H}{1 + e_0} \log_{10}\frac{\sigma}{\sigma_0} = \frac{0.28 \times 6}{1 + 2.05} \log_{10}\frac{21.6}{12} = \textbf{0.141 m}$$

Example 15.27. *A normally consolidated clay settled by 2 cm when the effective stress was increased from 100 kPa to 200 kPa. Calculate the settlement when the effective stress is increased to 400 kPa and 800 kPa.* **(Civil Services Exam. 2000)**

Solution. In general, $$\Delta H = \frac{C_c H}{1 + e_0} \log_{10}\frac{\sigma'}{\sigma'_0} = K \log_{10}\frac{\sigma'}{\sigma'_0}$$

$\therefore$ $$K = \frac{\Delta H}{\log_{10}\dfrac{\sigma'}{\sigma'_0}} = \frac{2}{\log_{10}\dfrac{200}{100}} = 6.6439$$

Hence, $$\Delta H_1 = K \log_{10}\frac{\sigma'_1}{\sigma'_0} = 6.6439 \log_{10}\frac{400}{100} = \textbf{4 cm}$$

and $$\Delta H_2 = K \log_{10}\frac{\sigma'_2}{\sigma'_0} = 6.6439 \log_{10}\frac{800}{100} = \textbf{6 cm}$$

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Example 15.28. *At a given site, with the soil profile shown below (Fig. 15.24), calculate the settlement that will occur, if the water table drops to El. – 6 m.*

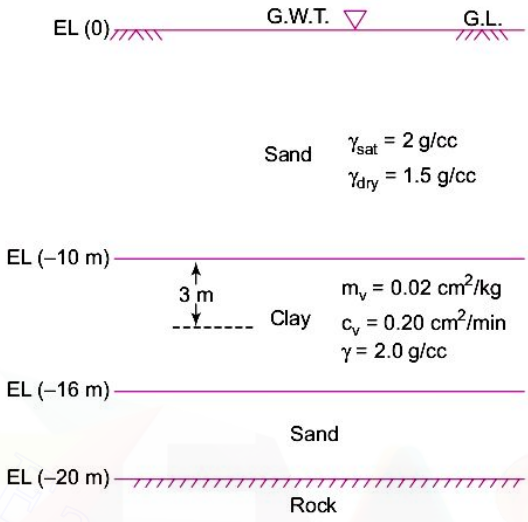

Fig. 15.24

Solution. For sand, $\gamma_{sat} = 2 \text{ g/cm}^3 = 2 \text{ } t/\text{m}^3$

$= 2 \times 9.81 \text{ kN/m}^3 = 19.62 \text{ kN/m}^3$

$\gamma_d = 1.5 \text{ g/cm}^3 = 1.5 \text{ } t/\text{m}^3 = 14.715 \text{ kN/m}^3$

$\gamma' = \gamma_{sat} - \gamma_w = 19.62 - 9.81 = 9.81 \text{ kN/m}^3$

For clay, $\gamma_{sat} = 2 \text{ g/cm}^3 = 2 \text{ } t/\text{m}^3$

$= 2 \times 9.81 \text{ kN/m}^3 = 19.62 \text{ kN/m}^3$

$\gamma' = 19.62 - 9.81 = 9.81 \text{ kN/m}^3$

Initially, with W.T. at 0.00 level, $\sigma'_0 = 9.81 \times 10 + 9.81 \times 3 = 127.53 \text{ kN/m}^2$

Finally, with W.T. at – 6.0 m level,

$$\sigma' = 14.715 \times 6 + 9.81 \times 4 + 9.81 \times 3 = 156.96 \text{ kN/m}^2$$

Now, $$m_v = 0.02 \text{ cm}^2/\text{kg} = 0.02 \times 10^{-4} \frac{-4 \text{ m}^2}{9.81 \text{ N}}$$

$$= \frac{0.02 \times 10^{-4}}{9.81 \times 10^{-3}} \frac{\text{m}^2}{\text{kN}} = 2.0387 \times 10^{-4} \text{ m}^2/\text{kN}$$

Now, $$\Delta H = m_v \Delta\sigma' H_0 = 2.0387 \times 10^{-4} (156.96 - 127.53) \times 6 = 0.036 \text{ m} = \mathbf{3.6 \text{ cm}}$$

Example 15.29. *At a given site (site A) having a clay layer 8 m thick with single drainage, a uniform fill that was constructed, resulted in a settlement of 5 cm in one year, corresponding to 50% average degree of consolidation. It was found that at a neighbouring site (site B), the same type of clay exists but having a thickness of 16 m. How long it will take for 50% consolidation to occur and what will be the magnitude of settlement, if a fill of the same magnitude as at site A was to be constructed at site B?*

(Civil Services Exam. 2000)

Solution. Given: $H_1 = 8 \text{ m}; \Delta H_1 = 5 \text{ cm} = 0.05 \text{ m} ; t_1 = 1 \text{ year} ; U_1 = 50\%$

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$$H_2 = 16 \text{ m} ; \Delta H_2 = ? ; t_2 = ? ; U_2 = 50\%$$

For the same degree of consolidation, $\frac{t_1}{t_2} = \left(\frac{H_1}{H_2}\right)^2$

$$\therefore \quad t_2 = t_1\left(\frac{H_2}{H_1}\right)^2 = 1\left(\frac{16}{8}\right)^2 = \mathbf{4 \text{ years}}$$

Also, $\Delta H = m_v \cdot \Delta\sigma' \cdot H_0$

$$\therefore \quad \frac{\Delta H_1}{\Delta H_2} = \frac{H_1}{H_2} \quad \text{or} \quad \Delta H_2 = \Delta H_1 \cdot \frac{H_2}{H_1} = 5 \times \frac{16}{8} = \mathbf{10 \text{ cm}}$$

Example 15.30. *A building was constructed on a clay stratum. Preliminary analysis indicated settlement of 50 mm in 6 years and an ultimate settlement of 250 mm. The average increase of pressure in clay stratum was 24 kN/m^2.*

The following variations occurred from the assumptions used in the preliminary analysis.

(i) The loading period was 3 years, which was not considered in the preliminary analysis.
(ii) Borings indicated 20% more thickness for the clay stratum than originally assumed.
(iii) During construction, the water table got lowered permanently by 1 m.

Estimate: 1. The ultimate settlement 2. The settlement at the end of the loading period 3. The settlement at 2 years after completion of building. **(Civil Services Exam. 2001)**

Solution.

(a) ***Effect of lowering water table:*** When water table is lowered, the effective stress gets increased by the amount $(\gamma - \gamma')\,\Delta h$, where γ is the bulk density after lowering water table and Δh is the depth of lowering W.T. Let us assume that $\gamma \approx \gamma_{sat}$ in absence of any other data available.

Hence, $\Delta\sigma' = (\gamma_{sat} - \gamma')\,\Delta h = \gamma_w\,\Delta h = 9.81 \times 1 = 9.81 \text{ kN/m}^2$

Now as per preliminary assumptions, $\rho f_1 = 250$ mm; $\Delta\sigma' = 24$ kN/m^2 and $(H_0)_1 = H_0$(say).

For final/actual conditions, $\rho f_2 = ? ; \Delta\sigma' = 24 + 9.81 = 33.81$ kN/m^2 and $(H_0)_2 = 1.2\,H_0$

Now, in general, $\rho_f = m_v \cdot \Delta\sigma'\,H_0$ or $\rho f_1 = 250 = m_v(24)\,H_0$

or $$m_v H_0 = \frac{250}{24} = 10.42 \frac{\text{mm}}{\text{kN/m}^2} \qquad ...(i)$$

Also, $\rho f_2 = m_v(33.81)\,(1.2\,H_0) = m_v H_0\,(33.81 \times 1.2) = 10.42\,(33.81 \times 1.2)$

$= \mathbf{422.63 \text{ mm}}$

Hence ultimate settlement under actual site conditions will be 422.63 mm instead of 250 mm.

(b) ***Settlement at the end of construction period:*** Settlement is assumed to start from mid-time of construction period. Hence settlement at the end of construction period will be that occurring in 3/2= 1.5 years.

Now, at $t_1 = 6$ years, $U_1 = 50/250 = 0.2$

Hence at $t_2 = 1.5$ years, $U_2 = U_1\sqrt{t_2/t_1} = 0.2\sqrt{1.5/6} = 0.1$

$\therefore \quad \rho_2 = U_2 \cdot \rho_f = 0.1 \times 422.63 = \mathbf{42.26 \text{ mm}}$

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(c) *Settlement 2 years after construction period*

Here $t_3 = 2 + 1.5 = 3.5$

$\therefore \quad U_3 = U_1\sqrt{t_3/t_1} = 0.2\sqrt{3.5/6} = 0.1527$

$\therefore \quad \rho_3 = U_3 \cdot \rho_f = 0.1527 \times 422.63 = \mathbf{64.56\ mm}$

Example 15.31. *A layer of clay 2 m thick is subjected to a loading of 0.5 kg/cm^2. One year after loading, the average consolidation is 50%. The layer has double drainage, (i) What is the coefficient of consolidation ? (ii) If the coefficient of permeability is 3 mm/year, what is the settlement after one years, and how much time will the layer take to reach 90% consolidation?* **(Civil Services Exam. 2002)**

Solution. $H = 2$ m; $d = 2/2 = 1$ m (for double drainage)

$U = 50\%$; $t = 1$ year; $\Delta\sigma = 0.5$ kg/cm$^2 = 0.5 \times 10^4$ kg/m^2

(a) Determination of c_v: $U < 0.6$, $T_{v1} = \dfrac{\pi}{4}U_1^2 = \dfrac{\pi}{4}(0.5)^2 = 0.1963$

Now, $c_v = T_{v1} \cdot \dfrac{d^2}{t} = 0.1963 \times \dfrac{(1)^2}{1} = 0.1963$ m^2/year

(b) Determination of settlement after 1 year

From Eq. 15.18, $m_v = \dfrac{k}{c_v \cdot \gamma_w} = \dfrac{3 \times 10^{-3}\ m/yr}{0.1963\ \text{m}^2/\text{year} \times 1000\ \text{kg/m}^3} = 15.283 \times 10^{-6}$ m^2/kg

$\therefore \quad \rho_f = m_v\, \Delta\sigma \cdot H_0 = 15.283 \times 10^{-6}$ m^2/kg $\times 0.5 \times 10^4$ kg/m$^2 \times 2$ m

$= 0.1528$ m

$\therefore$ Settlement after one year $= \rho_f \cdot U_1 = 0.1528 \times 0.5 = 0.0764$ m $= \mathbf{76.4\ mm}$

(c) Time taken for 90% settlement

$U_2 = 90\% > 60\%$.

$\therefore$ Hence from Eq. 15.34, $T_{v2} = -0.9332 \log_{10}(1 - 0.9) - 0.0851 = 0.8481$

(Alternatively, from Table 15.1, $T_v = 0.848$ for $U = 90\%$)

Now, $\dfrac{t_2}{t_1} = \dfrac{T_{v2}}{T_{v1}}$; Hence $t_2 = t_1 \dfrac{T_{v2}}{T_{v1}} = 1 \times \dfrac{0.8481}{0.1962} = \mathbf{4.326\ years}$

Example 15.32. *A 20 m thick isotropic clay stratum overlies an impervious rock. The coefficient of consolidation of soil is 5×10^{-4} cm^2/sec.*

(i) Find the time required for 50% and 90% consolidation. Time factor for U = 50% is 0.2 and for U = 90% is 0.85, where U is the degree of consolidation.

(ii) In order to accelerate the settlement ratio, vertical sand drains of 1 m diameter were made at 5 m centre to centre in the soil stratum throughout the area.

Calculate how much the settlement is accelerated due to provision of sand drains. For the provided ratio of five between the spacing of sand drains (centre to centre) and diameter of drains, values of time factor for U = 50% is 0.078 and for U = 90% is 0.28. **(Engg. Services Exam. 1986)**

Solution. Given $c_v = 5 \times 10^{-4}$ cm^2/sec

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(i) Time required to 50% and 90% consolidation

$d = H = 20$ m $= 2000$ cm for single drainage.

For 50% U, $$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2 = \frac{\pi}{4}\left(\frac{50}{100}\right)^2 = 0.1963$$

$$\therefore \quad t_{50} = \frac{T_v d^2}{c_v} = \frac{0.1963}{5\times10^{-4}}(2000)^2 \text{ seconds} = 1.57\times10^9 \text{ sec} = \mathbf{49.8 \text{ years}}$$

For 90% U, $$T_v = 1.7183 - 0.9332 \log_{10}(100 - U\%)$$
$$= 1.7813 - 0.9332 \log_{10}(100 - 90) = 0.8481$$

$$\therefore \quad t_{90} = \frac{0.8481}{5\times10^{-4}}(2000)^2 \text{ sec} = 6.785\times10^9 \text{ sec} = \mathbf{215.1 \text{ years}}$$

(ii) Acceleration of consolidation due to provision of sand drains

$$T_r = \frac{c_{vr}}{(2R)^2} t_r \quad \text{Take} \quad c_{vr} = c_{vz} = c_v, \text{ assuming isotropic case.}$$

For square arrangement $R = 0.564\, l = 0.564 \times 5 = 2.82$ m $= 282$ cm

For a given ratio of $\frac{l}{d_w} = 5$, $(T_r)_{50} = 0.078$ and $(T_r)_{90} = 0.28$

Hence, $$(t_r)_{50} = \frac{T_r (2R)^2}{c_{vr}} = \frac{0.078(2\times282)^2}{5\times10^{-4}} \text{ sec.} = 0.496\times10^8 \text{ sec}$$

$$\therefore \quad \left(\frac{t_r}{t}\right)_{50} = \frac{0.496\times10^8}{1.57\times10^9} = \frac{1}{31.7} \quad \text{or} \quad (t_r)_{50} = \mathbf{\frac{1}{31.7}} \times t_{50}$$

Similarly, $$(t_r)_{90} = \frac{T_r (2R)^2}{c_{vr}} = \frac{0.28(2\times282)^2}{5\times10^{-4}} = 0.178\times10^9 \text{ sec.}$$

$$\therefore \quad \left(\frac{t_r}{t}\right)_{90} = \frac{0.178\times10^9}{6.785\times10^9} = \frac{1}{38.1} \quad \text{or} \quad (t_r)_{90} = \mathbf{\frac{1}{38.1}} \times t_{90}$$

Example 15.33. *The time to reach 40% consolidation on a two-way drained laboratory 1 cm thick saturated clayey soil sample is 35 seconds. Determine the time required for 60% consolidation of the same soil 10 m thick on the top of a rocky surface subjected to the same loading condition as the laboratory sample.* **(Engg. Services Exam. 1987)**

Solution. Let us use suffix 1 for lab and 2 for field condition.

$$\therefore \quad T_{v_1} = \frac{\pi}{4}\left(\frac{40}{100}\right)^2 = 0.1257 \quad \text{and} \quad T_{v_2} = \frac{\pi}{4}\left(\frac{60}{100}\right)^2 = 0.2827$$

For laboratory test having two way drainage, $d_1 = H/2 = 1/2 = 0.5$ cm $= 5\times10^{-3}$ m

$$\therefore \quad c_v = \frac{T_v d^2}{t} = \frac{T_{v_1} d_1^2}{t_1} = \frac{0.1257(5\times10^{-3})^2}{35} = 8.979\times10^{-8} \text{ m}^2/\text{sec.}$$

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For field condition of one way drainage, $d_2 = 10$ m

$$\therefore \quad t_2 = \frac{T_{v_2} d_2^2}{c_v} = \frac{0.2827(10)^2}{8.979 \times 10^{-8}} = 3.148 \times 10^8 \text{ sec.} = \textbf{9.98 years}$$

Example 15.34. *A 2 m wide strip footing is to be placed in a sand layer 2 m thick at a depth of 1 m below the ground surface. The sand is underlain by a layer of saturated clay which is 1 m thick. The clay overlies a bed of dense sand. The ground water table is at the level of the top of the clay layer. The bulk unit weight of sand above the clay layer is 2 t/m^3 and the submerged unit weight of clay is 0.8 t/m^3. The footing is designed to carry a load of 22 t/m^3. Compute the probable ultimate settlement of the footing below its centre. Also, determine the elapsed time in which 10 per cent and 90 per cent of the ultimate settlement will occur.*

It is known from a graph between pressure and void ratio for clay that the void ratio corresponding to the vertical pressure of 4.4 t/m^2 and 18.1 t/m^2 are respectively 1.16 and 1.02. The coefficient of consolidation for the soil is 4×10^{-4} cm^2/s.

The vertical pressure (σ_z) below the centre line of uniformly loaded strip footing (intensity q) of width B may be taken as

Depth	σ_z
0.1 B	*0.997 q*
0.5 B	*0.817 q*
1.0 B	*0.550 q*
2.0 B	*0.306 q*

The time factor (T_v) corresponding to the degree of consolidation (U %) is as follows.

(U %)	*10*	*90*
T_v	*0.008*	*0.848*

(Engg. Services Exam. 1991)

Solution. ***This problem was set from author's present book, 10th edition which was in MKS units. For solution please see Example 15.7, which is the SI version of the above problem.***

Answers: $\rho_f = 6.5$ cm; $t_{10} = 13.9$ hours; $t_{90} = 61.34$ days.

Example 15.35. *A layer of clay 5 m thick lies under a newly constructed building. The effective pressure due to overlying strata on the clay layer is 3.0 kg/cm^2, and the new construction increases the overburden by 1.2 kg/cm^2. If the compression index of clay is 0.45, compute the settlement assuming the natural water content of the clay layer to be 43% and the specific gravity of its soil grains as 2.7.*

(Engg. Services Exam. 1993)

Solution. Given: $\sigma_0' = 3$ kg/cm^2; $\Delta\sigma' = 1.2$ kg/cm^2; $C_c = 0.45$; $w = 0.43$; $G = 2.7$

$e_0 = w \cdot G = 0.43 \times 2.7 = 1.161$, assuming clay to be saturated

$$\Delta H = \frac{C_c H}{1+e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'} = \frac{0.45 \times 5}{1+1.161} \log_{10} \frac{3+1.2}{3} = 0.1521 \text{ m} = \textbf{15.21 cm}$$

Example 15.36. *A 3 m thick clay layer beneath building is over-lain by a permeable stratum and is underlain by an impervious rock. The coefficient of consolidation of the clay was found to be 0.025 cm^2/min. The final expected settlement for the layer is 8 cm.*

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(*i*) *How much time will it take for 80% of total settlement to take place?*
(*ii*) *Determine the time required for a settlement of 2.5 cm.*
(*iii*) *What will be settlement in 6 months?*

U	T_v	U	T_v
0	0	0.6	0.287
0.1	0.008	0.7	0.403
0.2	0.031	0.8	0.567
0.3	0.071	0.9	0.848
0.4	0.126	1.0	∞
0.5	0.196		

(Engg. Services Exam. 1994)

Solution. Given $\rho_f = 8$ cm ; $c_v = 0.025$ cm^2/min.

(*i*) ***Time required for 80% consolidation***

From given table, $T_v = 0.567$ for $U = 0.8$. For one way drainage $d = 3/1 = 3$ m $= 300$ cm

$$t_{80} = (T_v)_{80} \cdot \frac{d^2}{c_v} = 0.567 \frac{(300)^2}{0.025} = 2041200 \text{ min.} = 1417.5 \text{ days}$$

$$\approx \mathbf{3.88 \text{ years}}$$

(*ii*) ***Time required for $\Delta H = 2.5$ cm***

$$U = \frac{\Delta H}{\rho_f} = \frac{2.5}{8} = 0.3125$$

Hence from the given table, $T_v \approx 0.0779$ for $U = 0.3125$

$$\therefore \quad t = 0.0779 \times \frac{(300)^2}{0.025} = \mathbf{194.69 \text{ days}}$$

(*iii*) ***Settlement in 6 months***

$t = 6$ months $= 365/2$ days $= 182.5$ days.

For $U < 60\%$, we have $\frac{\Delta H_1}{\Delta H_2} = \sqrt{\frac{T_{v_1}}{T_{v_2}}} = \sqrt{\frac{t_1}{t_2}}$

Here, when $t_1 = 194.69$ days, $\Delta H_1 = 2.5$ cm; $t_2 = 182.5$ days, $\Delta H_2 = ?$

$$\therefore \quad \Delta H_2 = \Delta H_1 \sqrt{t_2/t_1} = 2.5\sqrt{182.5/194.69} = \mathbf{2.42 \text{ cm}}$$

Check: $U_1 = \frac{\Delta H_1}{\rho_f} = \frac{2.5}{8} = 0.3125 < 0.6; U_2 = \frac{\Delta H_2}{\rho_f} = \frac{2.42}{8} = 0.3025 < 0.6$

Hence $U < 0.6$ in both the cases.

Example 15.37. *The subsoil profile at a proposed site of construction is shown below. A footing 2 m square carries a total load of 1000 kN and is laid with its base at 1 m depth below the ground surface.*

Assuming that the post construction settlement in sand is negligible, determine the consolidation settlement of the clay layer on account of the construction. There is geological evidence to indicate that

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the clay is normally consolidated. Use approximate 2:1 load spread to estimate the stress increase in the clay layer. **(Engg. Services Exam. 1997)**

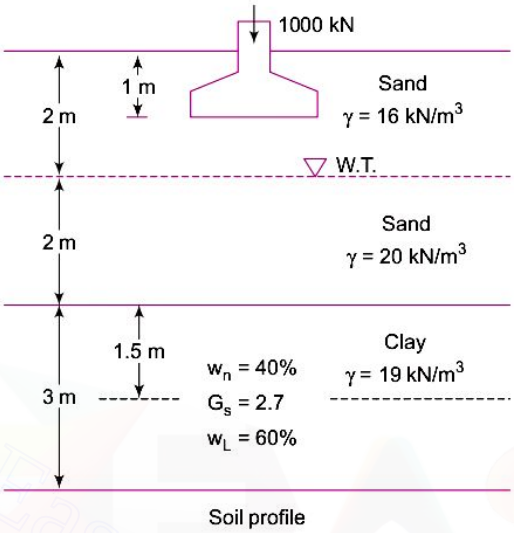

Fig. 15.25

Solution. Increase in load at the base of footing = $1000 - (16 \times 1) \times 2 \times 2 = 936$ kN

At the centre of clay layer, area of spread = $\left(2 + \frac{4.5}{2} \times 2\right)^2 = 42.25 \text{ m}^2$

$\therefore \quad \Delta\sigma = 936/42.25 = 22.154 \text{ kN/m}^2$

For clay, $\gamma' = \frac{G-1}{1+e}\gamma_w = \frac{G-1}{1+wG}\gamma_w = \frac{2.7-1}{1+0.4\times 2.7} \times 9.81 = 8.018 \text{ kN/m}^3$

$\sigma_0' = 2 \times 16 + 2\,(20 - 9.81) + 1.5 \times 8.018 = 64.41 \text{ kN/m}^2$

$C_c = 0.009\,(w_L - 10) = 0.009\,(60 - 10) = 0.45$

$e_0 = wG = 0.4 \times 2.7 = 1.08$; $H_0 = 3$ m

$$\Delta H = \frac{C_c H_0}{1+e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma}{\sigma_0'} = \frac{0.45 \times 3}{1+1.08} \log_{10} \frac{64.41 + 22.154}{64.41}$$

$= 0.0833 \text{ m} = \mathbf{8.33 \text{ cm}}$

Example 15.38. *Undrained soil sample 30 m thick got 50% consolidation in 20 minutes with drainage allowed at top and bottom in the laboratory. If the clay layer from which the sample was obtained is 3 m thick in field condition, estimate the time it will take to consolidate 50% with (i) double surface drainage (ii) single surface drainage, if in both cases, consolidation pressure is uniform.*

(Engg. Services Exam. 2001)

Solution.

(*i*) ***Double drainage:*** For $U = 50\%$ ($< 60\%$), we have $\frac{t_1}{t_2} = \left(\frac{H_1}{H_2}\right)^2$

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$\therefore \quad t_2 = t_1\left(\frac{H_2}{H_1}\right)^2$, where suffix 1 stands for lab condition while suffix 2 stands for field condition

$\therefore \qquad t_2 = 20\left(\frac{3000}{30}\right)^2 = 200000 \text{ min.} = \mathbf{138.89 \text{ days}}$

(*ii*) ***Single drainage:*** For $U < 0.6$, $T_v = \frac{\pi}{4}U^2$

$\therefore \qquad (T_v)_{50} = \frac{\pi}{4}(0.5)^2 = 0.1963$. Again, in general, $T_v = \frac{c_v}{d^2} \cdot t$

Let us use suffix 2 for double drainage in the field and suffix 3 for single drainage in the field. T_v remains the same for both the cases.

$\therefore \qquad \frac{c_v}{d_2^2} t_2 = \frac{c_v}{d_3^2} t_3, \quad \text{or} \quad \frac{t_2}{d_2^2} = \frac{t_3}{d_3^2}$

or $\qquad t_3 = t_2\left(\frac{d_3}{d_2}\right)^2 = t_2\left(\frac{2d_2}{d_2}\right)^2 = 4\, t_2 = 4 \times 138.89 = \mathbf{555.56 \text{ days}}$

Example 15.39. *In a consolidation test, the void ratio of the specimen which was 1.068 under the effective pressure of 214 kN/m^2, changed to 0.994 when the pressure was increased to 429 kN/m^2. Calculate the coefficient of compressibility, compression index, and the coefficient of volume compressibility. Find the settlement of foundation resting on above type of clay, if thickness of layer is 8 m and the increase in pressure is 10 kN/m^2.* **(Engg. Services Exam. 2002)**

Solution. Given: $e_0 = 1.068$; $e = 0.994$; $\sigma_0' = 214$ kN/m^2 ; $\sigma' = 429$ kN/m^2

(*i*) $\qquad a_v = \frac{\Delta e}{\Delta \sigma'} = \frac{e_0 - e}{\sigma' - \sigma_0'} = \frac{1.068 - 0.994}{429 - 214} = 3.442 \times 10^{-4}$ m^2/kN

(*ii*) $\qquad C_c = \frac{\Delta e}{\log_{10}\frac{\sigma'}{\sigma_0'}} = \frac{1.068 - 0.994}{\log_{10}\frac{429}{214}} = \mathbf{0.245}$

(*iii*) $\qquad m_v = \frac{a_v}{1 + e_0} = \frac{3.442 \times 10^{-4}}{1 + 1.068} = \mathbf{1.664 \times 10^{-4}}$ **m^2/kN**

(*iv*) $\qquad \Delta H = \frac{C_c H_0}{1 + e_0} \log_{10}\frac{\sigma_0' + \Delta\sigma}{\sigma_0'} = \frac{0.245 \times 8}{1 + 1.068} \log_{10}\frac{214 + 10}{214} = 0.0188$ m

$= \mathbf{18.8 \text{ mm}}$

Example 15.40. *The time to reach 60% consolidation is 30 seconds for a sample of 1 cm thick, tested in the laboratory under condition of double drainage. How many years will the corresponding layer in nature require to reach the same degree of saturation if it is 10 m thick and drained on one side only?* **(Gate Exam. 1988)**

Solution. $\qquad c_v = T_v \frac{d^2}{t} = T_{v_1} \frac{d_1^2}{t_1} = T_{v_2} \frac{d_2^2}{t_2}$

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$$\therefore \quad t_2 = t_1\left(\frac{d_2}{d_1}\right)^2, \text{ for the same degree of consolidation}$$

Now $d_1 = 1/2 = 0.5$ cm $= 5 \times 10^{-3}$ m ; $d_2 = 10$ m (one way drainage only)

$$\therefore \quad t_2 = 30\left(\frac{10}{5\times10^{-3}}\right)^2 \text{ seconds} = \mathbf{120 \times 10^6\ s} = \frac{120\times10^6}{365\times24\times3600} \text{ years}$$

$$= \mathbf{3.805 \text{ years}}$$

Example 15.41. *A saturated clay layer of 5 m thickness takes 1.5 years for 50% primary consolidation when drained on both sides. Its coefficient of volume change m_v is 1.5×10^{-3} m^2/kN. Determine the coefficient of compressibility (in m^2/yr.) and the coefficient of permeability (in m/yr.) Assume $\gamma_w = 10$ kN/m^3.* **(Gate Exam. 1991)**

Solution. Given: $t = 1.5$ years ; $U = 0.5$; $m_v = 1.5 \times 10^{-3}$ m^2/kN

$d = H/2 = 5/2 = 2.5$ m, for both side drainage.

For $U \le 0.6$, $\quad T_v = \frac{\pi}{4}U^2 = \frac{\pi}{4}(0.5)^2 = 0.1963$

$$\therefore \quad c_v = d^2\frac{T_v}{t} = (2.5)^2 \times \frac{0.1963}{1.5} = \mathbf{0.818\ m^2/yr.}$$

$$k = c_v \,.\, m_v \,.\, \gamma_w = 0.818 \times 1.5 \times 10^{-3} \times 10 = \mathbf{0.0123\ m/yr}$$

Example 15.42. *A footing 1.5 m square is located 1.5 m below the surface of a uniform soil deposit of density 20 kN/m^3. The void ratio of the soil is 0.8 and its compression index is 0.07. If the total thickness of the deposit, which is underlain by rock strata is 3.5 m, compute the primary consolidation settlement of the footing when it carries a load of 225 kN. Use traperzoidal stress distribution (2:1 horizontal to vertical) and consider four layers.* **(Gate Exam. 1992)**

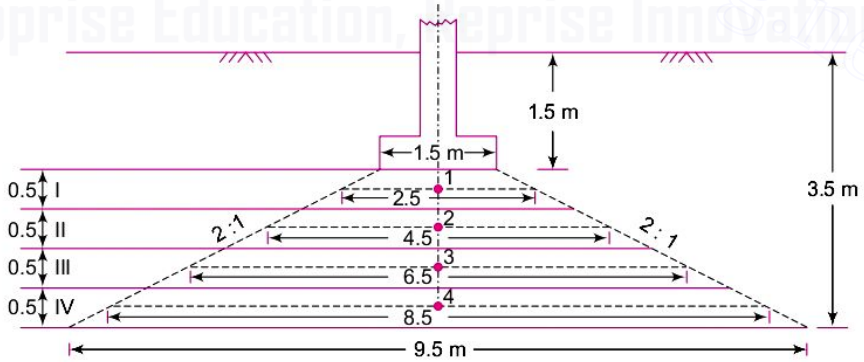

Fig. 15.26

Solution. Let us compute the settlement at the centre of each layer, as illustrated in the table below. For any layer, settlement is given by

$$\Delta H = \frac{C_c H_0}{1+e_0}\log_{10}\frac{\sigma_0' + \Delta\sigma}{\sigma_0'} = \frac{0.07\times0.5\times10^3}{1+0.8}\log_{10}\frac{\sigma_0' + \Delta\sigma}{\sigma_0'} \text{ mm}$$

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$$= 19.44 \log_{10} \frac{\sigma_0' + \Delta\sigma}{\sigma_0'} \text{ mm}$$

Foundation load, $P = 225$ kN;

Layer No.	*Mid point of layer*	*Base area A at the mid point of the layer (m^2)*	*$\Delta\sigma$ due to load of P P/A kN/m^2*	*σ_0' due to original soil deposit = γH (kN/m^2)*	*ΔH (mm)*
I	1	$2.5 \times 2.5 = 6.25$	36	$20 \times 1.75 = 35$	5.97
II	2	$4.5 \times 4.5 = 20.25$	11.11	$20 \times 2.25 = 45$	1.86
III	3	$6.5 \times 6.5 = 42.25$	5.33	$20 \times 2.75 = 55$	0.78
IV	4	$8.5 \times 8.5 = 72.25$	3.11	$20 \times 3.25 = 65$	0.39
				Sum	**9.00 mm**

$\therefore$ Total settlement = **9.0 mm**

Example 15.43. *A building is supported on a raft 60 m × 30 m, the net foundation pressure (assumed to be uniformly distributed) being 150 kPa. The soil profile is as shown in the following figure (Fig. 15.27). The value of m_v for the clay is 0.45 m^2/MN. Determine the ultimate consolidation settlement. You may need the following data for computing the vertical stress at depth z under a corner of a raft of dimensions mz and nz.*

For m = 1, n = 1: the influence factor I_z = 0.175. For m = 1, n = 2 and m = 2, n = 1, the influence factor I_z = 0.2. For m = 2, n = 2, the influence factor, I_z = 0.232. **(Gate Exam. 1994)**

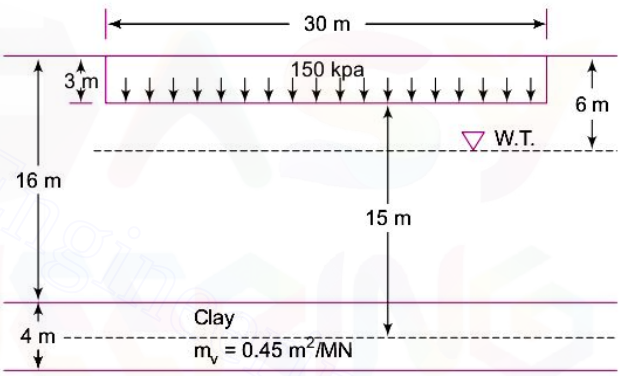

Fig. 15.27

Solution. Consolidation settlement is given by

$$\Delta H = m_v \cdot \Delta\sigma \cdot H_0$$

where $H_0 = 4$ m

and $m_v = 0.45 \text{ m}^2/\text{MN} = 0.45 \times 10^{-3} \text{ m}^2/\text{kN}$

Divide the loaded area (30 m × 60 m) into four rectangles, so that point P lies at the corner of each rectangle.

Then $\sigma_z = 4\, q \cdot I_z$

where $q = 150$ kPa $= 150$ kN/m^2

$z = 15$ m (upto centre of clay layer)

$mz = 30$ and $nz = 15$

$$m = \frac{30}{15} = 2 \quad \text{and} \quad n = \frac{15}{15} = 1$$

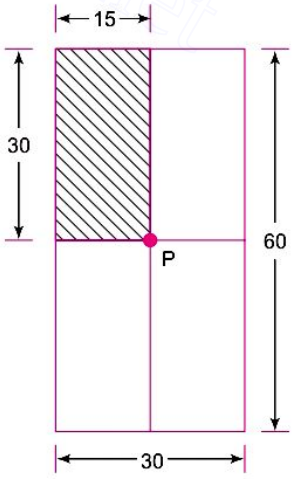

Fig. 15.28

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Hence for $m = 2$ and $n = 1$, we have $I_z = 0.2$ (given)

$\therefore \quad \Delta\sigma = \sigma_z = 4 \times 150 \times 0.2 = 120 \text{ kN/m}^2$

$\therefore \quad \Delta H = m_v \cdot \Delta\sigma \cdot H_0 = 0.45 \times 10^3 \times 120 \times 4 = \mathbf{0.215 \text{ m} = 21.6 \text{ cm}}$

Example 15.44. *In the laboratory test on a clay sample of thickness 25 mm drained at top only, 50% consolidation occurred in 11 minutes. Find the time required for the corresponding clay layer in the field, 2 m thick and drained at top and bottom to undergo 70% consolidation. Assume $T_{50} = 0.197$; $T_{70} = 0.405$.* **(Gate Exam. 1995)**

Solution.

(*a*) ***Laboratory consolidation:*** $H = 25$ mm; $d = H = 25$ mm $= 0.025$ m for one way drainage

$U = 50\% = 0.5$; $t = 11$ minutes;

$$c_v = (T_v)_{50} \frac{d^2}{t} = 0.197 \frac{(0.025)^2}{11} = 1.1193 \times 10^{-5} \text{ m}^2/\text{min}$$

(*b*) ***Field consolidation:*** H = 2 m ; $d = H/2 = 2/2 = 1$ m for two way drainage.

$$t = (T_v)_{70} \cdot \frac{d^2}{c_v} = 0.405 \frac{(1)^2}{1.1193 \times 10^{-5}} = 36183 \text{ min.} = \mathbf{25.13 \text{ days}}$$

Example 15.45. *A square footing is to be established in a clayey soil at a depth of 2 m, where water table has risen upto the ground level, as shown in Fig. 15.28. Determine the width of the footing if it is permitted to settle by 120 mm for the given data. Assume that the net load given is a constant, and that the same is dispersed into clay as shown. Take $\gamma_w = 10$ kN/m^3.* **(Gate Exam. 1995)**

Solution. Let the size of the footing be B m $\times$ B m.

Width of spread at the centre of the depth below footing $= B + 2$

$$\therefore \quad \Delta\sigma = \frac{500}{(B+2)^2} \text{ kN/m}^2$$

Also, at $P = 4\gamma' = 4(19.3 - 190)$

$= 37.2 \text{ kN/m}^2$

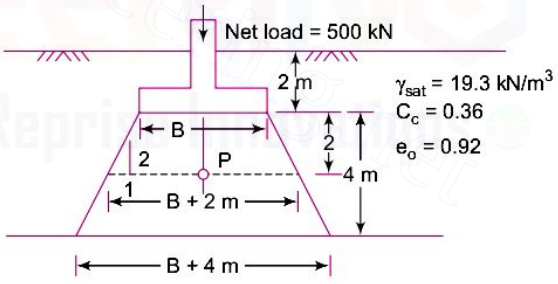

Fig. 15.29

Now, $$\Delta H = \frac{C_c H_0}{1 + e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma}{\sigma_0'}$$

where $\Delta H = 120$ mm $= 0.12$ m;

$C_c = 0.36$; $e_0 = 0.92$; $H_0 = 4$ m

$$\therefore \quad 0.12 = \frac{0.36 \times 4}{1 + 0.92} \log_{10} \frac{\sigma_0' + \Delta\sigma}{\sigma_0'}$$

From which, we get $\dfrac{\sigma_0' + \Delta\sigma}{\sigma_0'} = 1.4454$ or $\Delta\sigma = 1.4454\,(37.2) - 37.2 = 16.57$

$$\therefore \quad \frac{500}{(B+2)^2} = 16.57 \text{ from which } B = 3.493 \text{ m} \approx \mathbf{3.5 \text{ m}}$$

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Example 15.46. *A clay layer, 8 metre thick, is subjected to a pressure of 70 kN/m^2. If the layer has a double drainage and under goes 50% consolidation (T_v = 0.196) in one year, determine the coefficient of consolidation. If the coefficient of permeability is 0.40 m/year, determine the settlement in one year. Use γ_w = 9.81 kN/m^3.* **(Gate Exam. 1996)**

Solution. $H = 8$ m ; $d = H/2 = 8/2 = 4$ m; $U = 50\%$, $(T_v)_{50} = 0.196$; $t = 1$ year.

$$\therefore \quad c_v = T_v \frac{d^2}{t} = \frac{0.196(4)^2}{1} = \mathbf{3.136\ m^2/years}$$

$$m_v = \frac{k}{c_v \gamma_w} = \frac{0.040}{3.136 \times 9.81} = 1.3 \times 10^{-3}\ \text{m}^2/\text{kN}$$

$$\therefore \quad \rho_f = m_v \cdot \Delta\sigma \cdot H_0 = 1.3 \times 10^{-3} \times 70 \times 8 = 0.728\ \text{m}$$

This is the total settlement.

$\therefore$ Settlement in 1 year = 50% of total settlement = $0.5 \times 0.728 = \mathbf{0.364\ m}$

Example 15.47. *A settlement analysis carried out for a proposed structure indicates that 9 cm of settlement will occur in 5 years and the final settlement will be 45 cm based on double drainage condition. A detailed site investigation indicates that only single, drainage exists. Estimate the settlement at the end of 5 years for the changed condition. Use* $T = \frac{\pi}{4} U^2$. **(Gate Exam. 1997)**

Solution.

(a) ***Initial analysis using double drainage:*** $t = 5$ years ; $\Delta H = 9$ cm; $\rho_f = 45$ cm

$$\therefore \quad U = \frac{9}{45} = 0.2\ ; T_v = \frac{\pi}{4} U^2 = \frac{\pi}{4} (0.2)^2 = 0.0314$$

For double drainage, $d = H/2$

$$\therefore \quad c_v = T_v \frac{d^2}{t} = \frac{0.0314}{5} \left(\frac{H}{2}\right)^2 = 1.571 \times 10^{-3}\ H^2\ \text{m}^2/\text{years}$$

(b) ***Actual field condition:*** Single drainage: $d = H$; $t = 5$ years.

c_v does not change. Hence $c_v = 1.571 \times 10^{-3}\ H^2$

$$\therefore \quad T_v = \frac{c_v}{d^2} t = \frac{1.571 \times 10^{-3}\ H^2}{(H)^2} \times 5 = 7.854 \times 10^{-3}$$

Hence, $$U = \sqrt{\frac{4}{\pi} T_v} = \sqrt{\frac{4}{\pi} \times 7.854 \times 10^{-3}} = 0.1$$

But $$U = \frac{\Delta H}{\rho_f}; \quad \text{Hence } \Delta H = U \cdot \rho_f = 0.1 \times 45 = \mathbf{4.5\ cm}$$

Example 15.48. *A soft normally consolidated clay layer is 20 m thick with a moisture content of 45%. The clay has a saturated unit weight of 20 kN/m^3, a particle specific gravity of 2.7 and a liquid limit of 60%. A foundation load will subject the centre of layer to a vertical stress increase of 10 kPa. Ground water level is at the surface of the clay estimate.*

(a) The initial and final effective stresses at the centre of the layer
(b) The approximate value of the compression index (C_c)

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(c) *The consolidation settlement of the foundation if the initial effective stress at the centre of the soil is 100 kPa.*

Assume unit weight of water = 10 kN/m^3. **(Gate Exam. 1998)**

Solution.

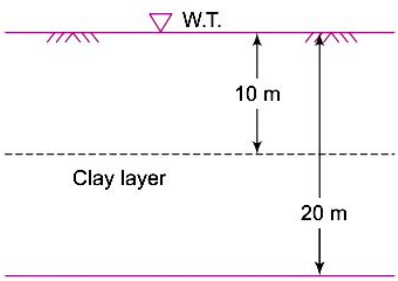

Fig. 15.30

(a) $\sigma_0' = \gamma' \times 10 = (20 - 10)\ 10 = 100\ \text{kN/m}^2$

$\Delta\sigma' = 10\ \text{kPa} = 10\ \text{kN/m}^2$

$\therefore \quad \sigma' = \sigma_0' + \Delta\sigma' = 100 + 10 = 110\ \text{kN/m}^2$

(b) $C_c = 0.009\ (w_L - 10) = 0.009\ (60 - 10) = \mathbf{0.45}$

(c) $$\Delta H = \frac{C_c H_0}{1 + e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma}{\sigma_0'}$$

where $e_0 = w_{sat}\, G = 0.45 \times 2.7 = 12.15$

$$= \frac{0.45 \times 20}{1 + 1.215} \log_{10} \frac{100 + 10}{100} = 0.1682\ \text{m} = \mathbf{16.82\ cm}$$

Example 15.49. *A building is constructed on the ground surface beneath which there is a 2 m thick saturated clay layer sandwiched between two highly pervious layers. The building starts settling with time. If the average coefficient of consolidation of clay is 2.5×10^{-4} cm^2/s, in how many days will the building reach half of its final settlement? $T_{50} = 0.197$* **(Gate Exam. 1999)**

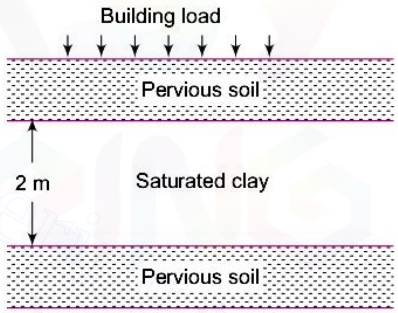

Fig. 15.31

Solution. Clay has two way drainage

$\therefore \quad d = H/2 = 2/2 = 1\ \text{m} = 100\ \text{cm}$

$U = 50\%,\ (T_v)_{50} = 0.197$

$c_v = 2.5 \times 10^{-4}\ \text{cm}^2/\text{s}$

$$t_{50} = (T_v)_{50} \cdot \frac{d^2}{c_v} = 0.197\ \frac{(100)^2}{2.5 \times 10^{-4}}\ \text{seconds}$$

$= 7880000\ \text{s} = \mathbf{91.2\ days}$

Example 15.50. *In an odometer test, a specimen of saturated clay 19 mm thick reaches 50% consolidation in 20 minutes. How long it would take a layer of this clay 5 m thick to reach the same degree of consolidation under the same stress and drainage conditions ? How long would it take the clay layer to reach 30% consolidation?* **(Gate Exam. 2000)**

Solution. Let us use suffix 1 for lab conditions and 2 for filed conditions

Give $H_1 = 19$ mm; $U = 50\%$; $t_1 = 20$ min. ; $H_2 = 5$ m.

$$\frac{t_1}{t_2} = \left(\frac{H_1}{H_2}\right)^2$$

From which $$t_2 = t_1 \left(\frac{H_2}{H_1}\right)^2 = 20 \left(\frac{5000}{19}\right)^2\ \text{min.} = \mathbf{961.83\ days}$$

For $U = U_3 = 30\%$, we have

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$$\frac{U_2}{U_3} = \sqrt{\frac{T_{v2}}{T_{v3}}} = \sqrt{\frac{t_2}{t_3}}$$

or $$\frac{50}{30} = \sqrt{\frac{961.83}{t_3}}$$

from which $t_3 = \mathbf{346.26\ days}$

15.12 LABORATORY EXPERIMENTS

EXPERIMENT 16 : DETERMINATION OF CONSOLIDATION PROPERTIES

Object and Scope

The object of the experiment is to determine the consolidation properties of disturbed or undisturbed soil by conducting one dimensional consolidation test using either fixed ring or floating ring type consolidometer. The test covers the calculations of (*i*) rate of consolidation of soil under a normal load, (*ii*) degree of consolidation at any time, (*iii*) pressure voids ratio relationship, (*iv*) coefficient of consolidation at various successively increasing pressure, (*v*) permeability of soil at various stages of loading, (*vi*) compression index, and (*vii*) coefficient of compressibility.

Material and Equipment

(*i*) Fixed ring type consolidometer, consisting of the following: (1) Specimen ring, with highly polished interior surface, and top edge bevelled (2) porous stone, 2 nos., of silicon carbide, aluminium oxide or porous metal ; the diameter of top stone should be about 0.2 to 0.5 mm less than the internal diameter of the ring and the diameter of bottom stone should be equal to the external diameter of the ring (3) guide ring (4) outer ring (5) water jacket with base (6) pressure pad (7) steel ball (8) rubber gasket and (9) bolts, or (*ii*) Floating ring type consolidometer consisting of (1) specimen ring (2) porous stones two nos. (3) guide rings two nos. (4) presssure pad (5) steel ball and (6) water trough (*iii*) Suitable loading device for applying vertical loading to the soil specimen, loading being done either by a jack with weight, of known magnitude can be hung (*iv*) dial gauge, accurate to 0.002 mm (*v*) balance sensitive to 0.01 g (*vi*) thermostatically controlled oven (*vii*) containers for determining water content (*viii*) mixing basin (*ix*) soil trimming tools, like fine wire-saw, knife spatula etc. (*x*) glass plate (*xi*) filter paper (Whatman no. 1 or equivalent) (*xii*) stop watch (*xiii*) water reservoir.

Test Procedure

(*a*) Preparation of the Soil Specimen

1. **Preparation of specimen from undisturbed soil samples.** The undisturbed sample from the field may be circular (at least 10 cm in diameter) or a block sample. Clean the specimen ring and weigh it empty. Cut off about 3 to 5 cm of soil specimen from one end of the field sample. Gradually insert the specimen ring into the sample, by pressing with hands and carefully removing the material around the ring. The soil specimen so obtained should project about one centimetre from either side of the ring. Trim the sample smooth and flush with the top and bottom of the ring

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by using glass plates. Clean the ring from outside and weigh. Keep three specimens from the soil trimmings, for water content determination.

2. **Preparation of specimen from representative soil samples.** If the consolidation properties are to be determined from a disturbed soil sample, soil is compacted at the desired water content and density in a separate large mould, and then the specimen is cut as explained in step (1) above.

(*b*) Preparation of Mould Assembly and Sample Saturation

1. Saturate the porous stone either by boiling in distilled water for about 15 minutes or by keeping them submerged in distilled water for 4 to 8 hours. Wipe away excess water. Moisten all surfaces of the consolidometer which are to be enclosed.
2. Assemble the consolidometer, with the soil specimen (in the ring) and porous stone at top and bottom of the specimen, providing a filter paper between the soil specimen and the porous stone. Position the pressure pad centrally on the top porous stone.
3. Mount the mould assembly on the loading frame, and centre it such that the load applied is axial.
4. Position the dial gauge to measure vertical compression of the specimen. The dial gauge holder should be so set that the dial gauge is near the beginning of its release run, allowing sufficient margin for the swelling of the soil, if any.
5. Connect the mould assembly to the water reservoir, and the sample is allowed to saturate. The level of water in the reservoir should be at about the same level as the soil specimen.
6. Apply an initial seating load to the assembly. The magnitude of this load should be chosen by trial such that there is no swelling. It should not less than 5 kN/m^2 for ordinary soils (or 2.5 kN/m^2 for every soft soils). The load should be allowed to stand until there is no change in dial gauge reading for two consecutive hours, or for maximum of 24 hours.

(*c*) Consolidation Test

1. Note the final dial reading under the initial seating load. Apply first load of intensity 10 kN/m^2, and start the stop watch simultaneously with loading. Record the dial gauge readings at various time intervals indicated in Table 15.6. The dial gauge reading are taken until 90% consolidation is reached. Primary consolidation is generally reached within 24 hours.
2. At the end of the period, specified above, take the dial reading and time reading. Double the load intensity and take dial readings at various time intervals. Repeat this procedure for successive load increments. The usual loading intensities are as follows : 10, 20, 50, 100, 200, 400 and 800 kN/m^2 (kPa).
3. After the last loading is completed, reduce the load to 1/2 of the value of the last load and allow it to stand for 24 hours. Reduce the load further in steps of l/4th the previous intensity till an intensity of 10 kN/m^2 is reached. Take the final reading of the dial gauge.
4. Reduce the load to the initial setting load, keep it for 24 hours and note the final dial readings.
5. Quickly dismantle the specimen assembly and remove the excess surface water on the soil specimen by blotting. Weigh the ring with consolidation specimen. Dry the soil specimen in oven and determine its dry weight.

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Table 15.6 Data and observation sheet for consolidation test : pressure, compression and time

Empty mass of ring : Area of ring : Dia. of ring:

Volume of ring : Ht. of ring : Specific gravity of soil sample :

Pressure intensity (kN/m^2)		*10*	*20*	*50*	*100*	*200*	*400*	*800*
Elapsed time (min.)	$\sqrt{t}$	*Dial Gauge Readings*						
0	0							
0.25	0.5							
1	1							
2,25	1.5							
4	2							
6.25	2.5							
9	3							
12.25	3.5							
16	4							
20.25	4.5							
25	5							
36	6							
49	7							
64	8							
81	9							
100	10							
121	11							
144	12							
169	13							
196	14							
225	15							
256	16							
289	17							
324	18							
361	19							
400	20							
500	22.4							
600	24.5							
1440	38							

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Tabulation of Observations

The test observations and other data are recorded as illustrated in Tables 15.6, 15.7, and 15.8.

Table 15.8 Observation sheet for consolidation test : water contents

	Before test	*After test*
1. Mass of ring + wet soil (g)		
2. Mass of ring + dry soil (g)		
3. Mass of ring (g)		
4. Mass of dry soil (M_d) (g)		
5. Mass of water (g)		
6. Water content (w)		
7. Degree of saturation $= \frac{wG}{e}$		
Ht. of Solids $\left(H_s = \frac{M_d}{GA\rho_w}\right)$		

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Applied pressure σ' (kN/m^2)	Final dial Reading	Dial change ΔH	Specimen height $H = H_1 \pm \Delta H$	Drainage path $d = \frac{1}{4}(H_1 + H)$	Ht. of voids $H - H_s$	Voids ratio $e = \frac{H - H_s}{H_s}$	Fitting time		$c_v(cm^2/min)$		a_v	C_c	k	Remarks
							t_{50}	t_{90}	$\frac{0.197\ d^2}{t_{50}}$	$\frac{0.848\ d^2}{t_{90}}$				
10														
20														
50														
100														
200														
400														
800														
0														

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Calculations:

1. ***Height of solids (H_s)*** is calculated from Eq. 15.36 : $H_s = \dfrac{M_d}{GA\rho_w}$
2. ***Voids ratio:*** The voids ratios at the end of various pressure are calculated from Eq. 15.37: $e = \dfrac{H - H_s}{H_s}$. Table 15.3 illustrates the method of calculating voids ratios at various applied pressure. Results are tabulated as shown in Table 15.7.
3. ***Coefficient of consolidation:*** The coefficient of consolidation at each pressure increment is calculated by the following equation:

   $c_v = \dfrac{0.197\, d^2}{t_{50}}$ (Log fitting methods) and $c_v = \dfrac{0.848\, d^2}{t_{90}}$ (Square root fitting method)

   In the log fitting method, a plot is made between dial reading and logarithm of time (Fig. 15.12) and the corresponding to 50% consolidation is determined. In the square root fitting method, a plot is made between dial reading and square to roof time (Fig. 15.10) and the time corresponding to 90% consolidation is determined. The values of c_v are recorded in Table 15.7.
4. ***Compression index:*** To determine the compression index, a plot of voids ratio e versus log σ' is made (Fig. 15.3). The initial compression curve would be found to be a straight line, and the slope of this line would give the compression index C_c.
5. ***Coefficient of compressibility:*** The coefficient of compressibility is calculated as follows:

   $a_v = \dfrac{0.435\, C_c}{\sigma'}$ where a_v = coefficient of compressibility ; σ' = average pressure for the increment
6. ***Coefficient of permeability:*** The coefficient of permeability k is calculated as follows:

$$k = \frac{c_v a_v \gamma_w}{1 + e}$$

Note:

1. ***Specimen size:*** IS : 2720 (part XV)-1986 recommends specimen diameter of 60 mm, though specimens of diameters 50, 70 and 100 mm may also be used in special cases. The specimen should be at least 20 mm thick in all cases. However, the thickness should not be less than 10 times the maximum diameter of the grain in the soil specimen. The diameter of thickness ratio should be a minimum of 3.
2. ***Sample preparation:*** While preparing the specimen, attempt should be made to have the soil strata oriented in the same direction in the consolidation apparatus as they were or will be oriented in practice. The orientation should also be such that the laboratory test normally compresses the soil in the same direction relative to soil strata as the applied load in the field. In trimming the soil, great care should be taken to handle the specimen delicately with the least pressure applied to the soil.

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PROBLEMS

1. The foundation of a new structure is to be laid on a bed of sand overlying a layer of saturated clay 1.5 metres thick. Below the clay layer is another bed of coarse sand. The effective overburden pressure at the middle of the clay layer before construction of the foundation is 100 kN/m^2. Due to foundation loading an additional increase of 100 kN/m^2 in vertical pressure is expected at the middle of the clay layer.

   An undisturbed sample of clay, 2 cm thick was tested in a floating ring consolidation apparatus with opportunity of drainage both ways. Under a pressure of 100 kN/m^2, the total change in thickness was 0.06 cm and under 200 kN/m^2, it was 0.1 cm. The time taken for 50% of the total settlement to occur under the 200 kN/m^2 pressure was 1 hour. Compute (1) Coefficient of volume decrease for the range of 100 to 200 kN/m^2, (2) ultimate settlement of clay bed, (3) time required for 50% of the ultimate settlement.

   [**Ans.** (1) 0.206 m^2/MN; (2) 3.09 cm; (3) 234 days]

2. During a consolidation test, a sample of fully saturated clay, 2.0 cm thick is consolidated under a pressure increment of 200 kN/m^2. When equilibrium is reached, the sample thickness gets reduced to 1.78 cm. The pressure is then removed and the sample is allowed to expand and take water. The final thickness is observed as 1.95 cm and final water content is 25%. If G =2.72, find voids ratio before and after consolidation. Also find m_v over the range of pressure applied. [**Ans.** (*i*) 0.7233 and 0.5335; (*ii*) 0.55 m^2/MN]

3. A two metre thick saturated clay layer is sandwitched between two highly pervious layers of coarse sand. When a building is constructed on the ground surface, it starts settling due to the consolidation of the clay layer. If the average coefficient of consolidation of clay is 4.5×10^{-4} cm^2/sec, in how many days will the building reach half of its final settlement? [**Ans.** 91 days]

4. Sub-surface exploration at the site of a proposed building reveals the existence of a 2.4 metre thick layer of soft clay below a stratum of coarse sand which is 4 metre thick and extends from the ground surface upto the top of the clay layer. The ground water table is at 2.5 metre below the ground surface. Laboratory tests indicate the natural water content of clay as 40%, average liquid limit 45% and specific gravity 2.75. Also, the clay is proved to be normally consolidated. The unit weights of sand above and below the water table are respectively 17.8 kN/m^3 and 21.0 kN/m^3. Estimate the probable final settlement of the building, if its construction will increase the average vertical pressure on the clay layer by 71 kN/m^2. [**Ans.** 10.8 cm]

5. During a consolidation test, a specimen of clay, 50 cm^2 in cross-sectional area, has height of 1.868 cm after full consolidation under an effective pressure 400 kN/m^2. Keeping the specimen fully submerged, the specimen is allowed to expand. When fully expanded under zero load, the specimen is 1.988 cm thick and its water content is 15%. On oven drying, the specimen weighs 190 g and the specific gravity of solids is determined as 2.7. Determine the degree of saturation of specimen, when it was at equilibrium under the effective pressure of 400 kN/m^2. Assume the air voids of the specimen constant under-consolidated and expanded conditions. [**Ans.** 98%]

6. The thickness of a saturated specimen of clay under a consolidation pressure of 100 kN/m^2 is 22.12 mm and its water content is 14 per cent. On increase of the consolidation pressure to 200 kN/m^2, the specimen thickness decrease by 1.28 mm. Determine the compression index for the soil. Take G for the soil as 2.70. [**Ans.** 0.266]

7. Two clay layers A and B are respectively 4 m and 5 m thick. The time taken for layer A to reach 50% consolidation is 6 months. Calculate the time taken by the layer B to reach the same degree of consolidation. The coefficient of consolidation of layer B is half the coefficient of consolidation for layer A. Both layers have double drainage. [**Ans.** 18.75 months]

8. A clay layer 3.6 m thick is sandwitched between layers of sand. Calculate the time the clay layer will take to reach 50% consolidation. The coefficient of consolidation, found by laboratory tests is 4×10^{-4} cm/sec. [**Ans.** 184 days]

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9. A 6 m layer of sand is underlain by 3 m thick layer of clay. There is another layer of dense sand below the clay layer. The water table is upto the ground surface. The submerged unit weight of sand is 10 kN/m^2. The clay layer has a compression index of 0.35, water content = 40% and specific gravity 2.7. Calculate the ultimate settlement of clay layer. If in a period of 2 months a settlement of 4.4 cm is observed, how much time will be required to reach half the ultimate settlement? [**Ans.** ρ_f = 22 cm; 12.5 months]

10. A 20 mm thick undisturbed sample of a saturated clay is tested in the laboratory with drainage being allowed both through the top and bottom faces. The sample reaches 50 per cent degree of consolidation in 60 minutes. If the clay layer from which the sample was obtained is 4 m thick and is free to drain through both top and bottom surface, calculate the time required by the clay layer to undergo the same degree of consolidation. What would have been the time of consolidation if the clay layer were free to drain only through its top surface ? Assume uniform distribution of consolidating pressure. [**Ans.** 1665 days; 6650 days]

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CHAPTER 16

Three-Dimensional Consolidation

16.1 INTRODUCTION

In the Terzaghi's theory of one-dimensional consolidation, it was assumed that the soil is laterally confined so that the excess hydrostatic pressure varies vertically only and the strains are also vertical. In most of the actual problems surface loadings cause excess pore pressure which may vary both horizontally and vertically. The resulting consolidation will involve horizontal as well as vertical flow and horizontal as well as vertical strains. Such a consolidation is called *three-dimensional consolidation*. A more common example of three-dimensional consolidation is that due to installation of sand drains (Fig. 16.3), where there is a combination of radial (two dimensional) consolidation in a horizontal plane and vertical consolidation. The theory of the primary design of sand drains is based upon the extension of the Terzaghi's basic work on the consolidation of clays, and was largely developed by Barron (1940–42). Prior to his work, Rendulic (1935) solved the differential equation for consolidation by radial flow to a well. Carillo also worked independently on the problem of sand drains and published his results in 1942.

16.2 THREE-DIMENSIONAL CONSOLIDATION EQUATION

Assumptions:

1. The soil is completely saturated.
2. Solid particles and the water are incompressible.
3. Darcy's law is valid and can be generalised for anisotropic medium.
4. The soil mass is homogeneous.
5. Pressure increment $\Delta\sigma'$ is applied instantaneously so that t is independent of time.

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As the change in the total head h can be due only to a change in the excess hydrostatic pressure $\bar{u}$, we have

$$\partial h = \frac{1}{\gamma_w}\partial \bar{u} \qquad ...(vi)$$

Now, $v_x = k_x i_x = k_x \frac{\partial h}{\partial x} = \frac{k_x}{\gamma_w}\frac{\partial \bar{u}}{\partial x}$; $v_y = k_y i_y = k_y \frac{\partial h}{\partial y} = \frac{k_y}{\gamma_w}\frac{\partial \bar{u}}{\partial y}$; $v_z = k_z i_z = k_z \frac{\partial h}{\partial z} = \frac{k_z}{\gamma_w}\frac{\partial \bar{u}}{\partial z}$ ...(16.2)

Substituting in Eq. 16.1, we get

$$\frac{\partial e}{\partial t} = \frac{1+e}{\gamma_w}\left(k_x \frac{\partial^2 \bar{u}}{\partial x^2} + k_y \frac{\partial^2 \bar{u}}{\partial y^2} + k_z \frac{\partial^2 \bar{u}}{\partial z^2}\right) \qquad ...(16.3)$$

When a pressure increment is applied instantaneously, it is taken entirely by pore water and the excess hydrostatic pressure becomes equal to the applied pressure increment. As the time passes, water is squeezed out and the excess hydrostatic pressure is dissipated. In turn the effective pressure is increased and the rate of increase of effective pressure becomes equal to rate of decrease of excess hydrostatic pressure. Hence

$$\frac{\partial \sigma'}{\partial t} = -\frac{\partial \bar{u}}{\partial t} \qquad ...(vii)$$

Also, since any increase in σ' is equal to decrease $\bar{u}$, we have

$$\frac{\partial e}{\partial \sigma'} = -\frac{\partial e}{\partial \bar{u}} \qquad ...(viii)$$

And, by definition, $a_v = -\frac{\partial e}{\partial \sigma'}$; $\therefore \quad \frac{\partial e}{\partial \bar{u}} = a_v$ ...(16.4)

Again, $$\frac{\partial e}{\partial t} = \frac{\partial e}{\partial \bar{u}}\frac{\partial \bar{u}}{\partial t} = a_v \frac{\partial \bar{u}}{\partial t} \qquad ...(ix)$$

Substituting in Eq. 16.3, we get $\frac{\partial \bar{u}}{\partial t} = \frac{1+e}{a_v \gamma_w}\left(k_x \frac{\partial^2 \bar{u}}{\partial x^2} + k_y \frac{\partial^2 \bar{u}}{\partial y^2} + k_z \frac{\partial^2 \bar{u}}{\partial z^2}\right)$ ...(16.5)

Writing in integral, $c_v = \frac{k(1+e)}{a_v \gamma_w}$ = coefficient of consolidation, we have

$$\frac{\partial \bar{u}}{\partial t} = c_{vx}\frac{\partial^2 u}{\partial x^2} + c_{vy}\frac{\partial^2 u}{\partial y^2} + c_{vz}\frac{\partial^2 u}{\partial z^2} \qquad ...(16.6)$$

where c_{vx} = coefficient of consolidation in x-direction = $\frac{k_x(1+e)}{a_v \gamma_w}$

c_{vy} = coefficient of consolidation in y-direction = $\frac{k_y(1+e)}{a_v \gamma_w}$

c_{vz} = coefficient of consolidation in z-direction = $\frac{k_z(1+e)}{a_v \gamma_w}$

Eq. 16.6 is the basic three dimensional consolidation equation.

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16.3 CONSOLIDATION EQUATION IN POLAR COORDINATES FOR AXIS SYMMETRIC CASE

In the case of sand drains, where the process of three-dimensional consolidation is symmetrical about vertical axis, it is more convenient to express Eq. 16.6 into polar co-ordinates. For transformation from cartesian to polar co-ordinates, we have

$$x = r\cos\theta\ ;\ y = r\sin\theta\ ;\ z = z. \quad \text{Also, } r^2 = x^2 + y^2 \text{ and } \theta = \tan^{-1} y/x$$

Hence $\dfrac{\partial r}{\partial x} = \dfrac{x}{r} = \cos\theta;\ \dfrac{\partial r}{\partial y} = \dfrac{y}{r} = \sin\theta\ ;\ \dfrac{\partial \theta}{\partial x} = -\dfrac{y}{r^2} = -\dfrac{\sin\theta}{r}\ ;\ \dfrac{\partial \theta}{\partial y} = \dfrac{x}{r^2} = \dfrac{\cos\theta}{r}$ ...(1)

Using these and considering excess-hydrostatic pressure $\bar{u}$ as function of r and θ, we get

$$\frac{\partial \bar{u}}{\partial x} = \frac{\partial \bar{u}}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial \bar{u}}{\partial \theta}\frac{\partial \theta}{\partial x} = \frac{\partial \bar{u}}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial \bar{u}}{\partial \theta}\sin\theta \qquad ...(2)$$

To get the second derivative with respect to x, it is only neccessary to repeat the above operation ; hence

$$\frac{\partial^2 \bar{u}}{\partial x^2} = \left(\frac{\partial}{\partial r}\cos\theta - \frac{1}{r}\sin\theta\frac{\partial}{\partial \theta}\right)\left(\frac{\partial \bar{u}}{\partial r}\cos\theta - \frac{1}{r}\frac{\partial \bar{u}}{\partial \theta}\theta\sin\theta\right)$$

$$= \frac{\partial^2 \bar{u}}{\partial r^2}\cos^2\theta - 2\frac{\partial^2 \bar{u}}{\partial \theta\,\partial r}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial \bar{u}}{\partial r}\frac{\sin^2\theta}{r^2} + 2\frac{\partial \bar{u}}{\partial \theta}\frac{\sin\theta\ \cos\theta}{r^2} + \frac{\partial^2 \bar{u}}{\partial \theta^2}\frac{\sin^2\theta}{r^2} \qquad ...(3)$$

In the same manner, we get

$$\frac{\partial^2 \bar{u}}{\partial y^2} = \frac{\partial^2 \bar{u}}{\partial r^2}\sin^2\theta + 2\frac{\partial^2 \bar{u}}{\partial \theta\,\partial r}\frac{\sin\theta\cos\theta}{r} + \frac{\partial \bar{u}}{\partial r}\frac{\cos^2\theta}{r} - 2\frac{\partial \bar{u}}{\partial \theta}\frac{\sin\theta\cos\theta}{r^2} + \frac{\partial^2 \bar{u}}{\partial \theta^2}\frac{\cos^2\theta}{r^2} \qquad ...(4)$$

Adding (3) and (4), we get $\dfrac{\partial^2 \bar{u}}{\partial x^2} + \dfrac{\partial^2 \bar{u}}{\partial y^2} = \dfrac{\partial^2 \bar{u}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \bar{u}}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 \bar{u}}{\partial \theta^2}$ ... (5)

Now for the case of radial symmetry (axis-symmetric case), $\bar{u}$ is independent of θ and

hence, we get $\dfrac{\partial^2 \bar{u}}{\partial x^2} + \dfrac{\partial^2 \bar{u}}{\partial y^2} = \dfrac{\partial^2 \bar{u}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \bar{u}}{\partial r}$ ...(16.7)

Again, for the case of radial symmetry, $c_{vx} = c_{vy} = c_{vr}$ (say)

Hence, we get from Eqs. 16.6 and 16.7,

$$\frac{\partial \bar{u}}{\partial t} = c_{vr}\left(\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r}\frac{\partial \bar{u}}{\partial r}\right) + c_{vz}\frac{\partial^2 \bar{u}}{\partial z^2} \qquad ...(16.8)$$

Eq. 16.8 is the governing consolidation equation for the case of a sand *drain which* is a special case of three-dimensional consolidation having radial symmetry.

16.4 SOLUTION OF THREE-DIMENSIONAL CONSOLIDATION EQUATION

The general solution for consolidation by three-dimensional flow for a given set of boundary conditions may become mathematically involved. However, the method of separation of variables can be applied to this problem by considering Eq. 16.8 to consist of two parts:

One-dimensional flow : $c_{vz}\dfrac{\partial^2 \bar{u}}{\partial z^2} = \dfrac{\partial \bar{u}}{\partial t}$ ...(16.9)

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Radial flow : $$c_{vr}\left(\frac{\partial^2 \bar{u}}{\partial r^2}+\frac{1}{r}\frac{\partial \bar{u}}{\partial r}\right)=\frac{\partial \bar{u}}{\partial t} \qquad ...(16.10)$$

Carillo (1942) has shown that the solution of Eq. 16.8 is given by combination of the solutions for Eqs. 16.9 and 16.10 as follows:

$$(1-U)=(1-U_z)(1-U_r) \qquad ...(16.11)$$

where U = degree of consolidation for the one-dimensional flow.

U_z = degree of consolidation for one-dimensional flow (in the vertical direction).

U_r = degree of consolidation for radial flow.

Solution of Eq. 16.9 was obtained by Terzaghi, giving the relation between the time t and degree of consolidation U_z by the characteristic equation:

$$U_z = f(T_v) \qquad ...(16.12)$$

where T_v = Time factor for the vertical flow $= \frac{c_{vz}}{H^2}t$ . ...(16.13)

For the radial flow, solution of Eq. 16.10 was obtained by Rendulic (1935), expressing the relation between t and the degree of consolidation U_r by the characteristic equation:

$$U_r = F(T_r) \qquad ...(16.14)$$

where T_r = Time factor for radial flow $= \frac{c_{vr}}{(2R)^2}t$ ...(16.15)

$2R$ = Effective diameter of soil cylinder from which water will flow into the sand drain (see Fig. 16.3). The value of R is found by replacing the vertical prismatic blocks which surround the drain wells by cylindrical blocks with equal horizontal cross-sectional area. Thus, for square arrangement [Fig. 16.3 (*c*)] having well spacing l, we get $R = 0.564\,l$; for triangular arrangement [Fig. 16.3(*b*)], we get $R = 0.525\,l$.

Table 16.1 gives the value of T_v corresponding to different degrees of consolidation in the vertical direction.

Barron gave the solution of the radial flow towards a sand drain. He considered two types of vertical strains which might occur in the clay layer : (*i*) 'free vertical strain' resulting from a uniform distribution of surface load, and (*ii*) 'equal vertical strains' resulting from imposing the same vertical deformation at all points on the surface.

(*a*) **Free strain condition.** The solutions for the *free strain conditions* were obtained by Glover (1930) and Rendulic (1935) assuming the following boundary and initial conditions:

(*i*) Initial excess hydrostatic pressure $\bar{u}_0$ is constant.

(*ii*) Excess hydrostatic pressure at the drain well is zero.

(*iii*) At the external radius R, $\frac{\partial u}{\partial r}=0$ because of symmetry.

The solution is given by the expression

$$U_r = 1-\sum_{\alpha_1,\alpha_2\ldots}^{\alpha=\infty}\frac{4U_1^2(\alpha)}{\alpha^2(n^2-1)[n^2U_0^2(\alpha n)-U_1^2(\alpha)]}\varepsilon^{-4\alpha^2 n^2 T_r} \qquad ...(16.16)$$

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Table 16.1 Solution of radial flow equation (Equal vertical strain case)

U(%)	Time factor T_r								
	$n = \frac{r}{r_w} = 5$	10	15	20	25	30	50	80	
5	0.006	0.010	0.013	0.014	0.016	0.017	0.020	0.023	0
10	0.012	0.021	0.026	0.030	0.032	0.035	0.042	0.048	0.
15	0.019	0.032	0.040	0.046	0.050	0.054	0.064	0.074	0.
20	0.026	0.044	0.055	0.063	0.069	0.074	0.088	0.101	0.
25	0.034	0.057	0.071	0.081	0.089	0.096	0.144	0.131	0.
30	0.042	0.070	0.088	0.101	0.110	0.118	0.141	0.162	0.
35	0.050	0.085	0.106	0.121	0.133	0.143	0.170	0.196	0.
40	0.060	0.101	0.125	0.144	0.158	0.170	0.202	0.232	0.
45	0.070	0.118	0.147	0.169	0.185	0.198	0.236	0.291	0.
50	0.081	0.137	0.170	0.195	0.214	0.230	0.274	0.315	0.
55	0.094	0.157	0.197	0.225	0.247	0.265	0.316	0.363	0.
60	0.107	0.180	0.226	0.258	0.283	0.304	0.362	0.416	0.
65	0.123	0.207	0.259	0.296	0.325	0.348	0.415	0.477	0.
70	0.137	0.231	0.289	0.330	0.362	0.389	0.463	0.532	0.
75	0.162	0.273	0.342	0.391	0.429	0.460	0.548	0.629	0.
80	0.188	0.317	0.397	0.453	0.498	0.534	0.636	0.730	0.
85	0.222	0.373	0.467	0.534	0.587	0.629	0.750	0.861	0.
90	0.270	0.455	0.567	0.649	0.712	0.764	0.911	1.046	1.
95	0.351	0.590	0.738	0.844	0.926	0.994	1.185	1.360	1.
99	0.539	0.907	1.135	1.298	1.423	1.528	1.821	2.091	2.

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where $U_1(\alpha) = J_1(\alpha)\, Y_0(\alpha) - Y_1(\alpha)\, J_0(\alpha)$; $U_0(n\alpha) = J_0(\alpha n) Y_0(\alpha) - Y_0(\alpha n)\, J_0(\alpha)$

J_0, J_1 = Bessel functions of first kind, of zero order and first order respectively

Y_0, Y_1 = Bessel functions of second kind, of zero order and first order, respectively

α_1, α_2 = Roots of the Bessel functions which satisfy $J_1(\alpha n) Y_0(\alpha) - Y_1(\alpha n)\, J_0(\alpha) = 0$

n = Ratio $\frac{R}{r_w}$ and r_w = Radius of sand drain.

(*b*) **Equal strain condition.** Under the condition of free strain, it was implied that the settlements
he surface did not change the distribution of load to the soil. However, in an actual installation, the fact t
he consolidation proceeds faster near the sand drain thereby causing greater surface settlement in t
egion, could very well cause a redistribution of the surface loading. This would be specially true if t
oading material had any tendency to arch across such depressions. As an extreme case, the arching a
ion could redistribute the loads to the surface in such a fashion that the surface settlement is the same
ll points. This is the condition of equal vertical strains of which Barron (1948) developed the followi
nalytical solution:

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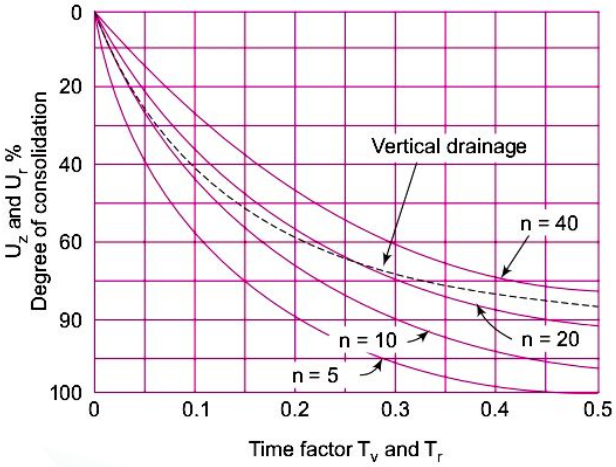

Fig. 16.2

$$U_r = 1 - \varepsilon^{\frac{-8T_r}{F(n)}} \qquad ...(16.17)$$

and

$$n = \frac{R}{r_w}; F(n) = \frac{n^2}{n^2 - 1} \log \varepsilon^{n - \frac{3n^2 - 1}{4n^2}} \qquad ...(16.18)$$

Values of T_r for various values of U_r and $n = \frac{R}{r_w}$ are given in Table 16.1. Fig. 16.2 gives curves between U_r and T_r for various values of n. Richart (1957) has pointed out the fact that for $n > 10$, there is very little difference between the results obtained by the two extreme conditions of 'varying vertical strain' and 'equal vertical strain'. Since the results are identical, but the time needed to evaluate Eq. 16.16 is of the order of ten to fifteen times that needed to evaluate Eq. 16.17, the equal strain solution is preferable.

16.5 CONSTRUCTION OF VERTICAL SAND DRAINS

Vertical sand drains accelerate the rate of consolidation, and are very helpful in the construction of high-way or airport embankments or dams over compressible soils. Due to the provision of sand drains in the compressible soil, most of the settlement will occur during, and not after construction. It permits more rapid construction because the rate of gain of shear strength of the soil is very rapid. The purpose of a *sand drain* or a *drain well* is to provide an easier path for the excess water to flow as it is squeezed out of a soil layer during construction. Vertical sand drains represent convenient device for the stabilisation of soft and compressible soil which is either too weak to support a proposed structure or is so incompressible that large and long-drawn-out settlements would occur following completion of construction. One such example of sand drain installation in India is at the 'Eastern Express Highway Project' Bombay.

Major benefits derived from the use of sand drains are:

(*i*) expediting the rate of settlement,
(*ii*) consequently, acceleration in the rate of gain in shear strength, and
(*iii*) reduction in the lateral transmission of excess hydrostatic pressure.

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Figure 16.3 shows sand-drain installation. For the construction or installation of sand drains, a sand blanket of about 1/2 m thickness is placed over the ground surface (top of the clay layer). This blanket serves as a drainage layer and as a working platform besides functioning like a graded filter preventing the soft clay below from permeating into the embankment material above. In order to obviate the possibility of shearing off the sand drains due to failure of embankment by spreading or sinking, it is considered desirable to build the embankment before installation of sand drains. However, the inadequate shear strength of the subsoil may not allow the embankment to be built in single stage construction. In that circumstances, the embankment has to be built to partial height only, the sand drains are subsequently installed and sufficient lapse of time given to permit sufficient increase in strength in the subsoil to enable the remaining height of the embankment before commencing placement of the pavement.

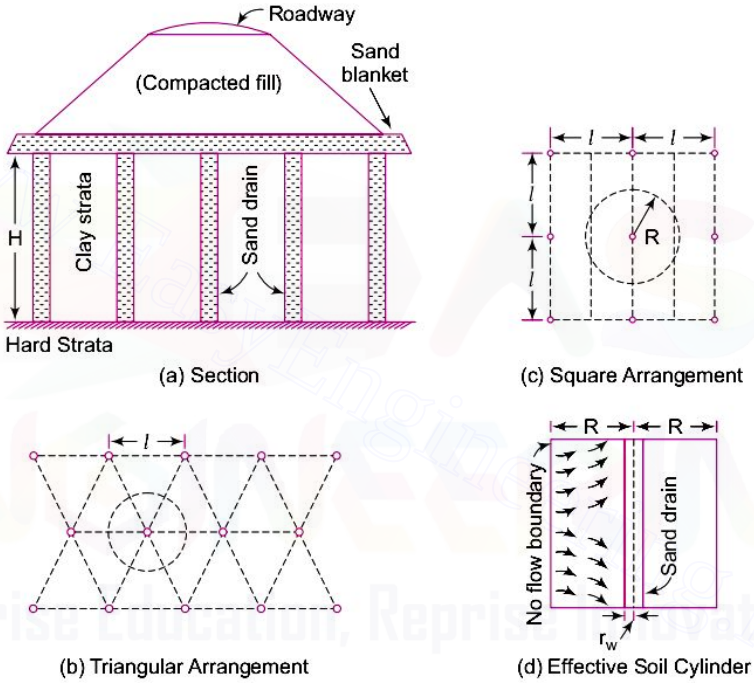

Fig. 16.3 Layout of sand drains

After a partial height of embankment is built, hollow steel mandrel about 25 cm internal diameter and 6 m long, with a detachable concrete shoe, is driven vertically through the working table and soft clay, by a pile driver, to the firm bottom. After the hole is back filled with clean well-draining sand (permeability greater than 5 m per day), the mandrel is removed. After the sand drains are installed, the rest of the embankment is constructed.

SOLVED EXAMPLES

Example 16.1 *A clay layer, 5 m thick, is consolidated with the help of drain well of diameter 30 cm and spaced at 2.7 metres centre to centre. Determine the influence of the wells on the average degree of consolidation at the time when the degree of consolidation in the clay without wells would be equal to 20 per cent. Make computations under two cases of permeability : (i)* $k_r = k_z$ *or* $c_{vr} = c_{vz}$ *and (ii)* $k_r = 5k_z$ *or* $c_{vr} = 5c_{vz}$

The well may be assumed to be arranged in a square pattern.

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Solution. $l = 2.7$

$R = 0.564\, l = 0.564 \times 2.7$ m $= 1.52$ m ; r_w = radius of drain well = 15 cm = 0.15 m

$$\therefore \quad n = \frac{R}{r_w} = \frac{1.52}{0.15} = 10.15 \approx 10 \text{ (for calculation purposes)}$$

From Table 15.1 for $U_z = 20\%$, $T_v = 0.031$

$$\therefore \quad T_v = 0.031 = \frac{c_{vz}}{H^2} t \; ; \quad \text{or} \quad t = \frac{0.031\, H^2}{c_{vz}} \qquad ...(i)$$

The time factor for radial flow at time t is $T_r = \dfrac{c_{vr}}{4R^2} t$

Substituting the value of t from (1), we get

$$T_r = \frac{c_{vr}}{4R^2} \times \frac{0.031\,(H)^2}{c_{vz}} = \frac{c_{vr}}{c_{vz}} \times \frac{0.031\, H^2}{4R^2} = \frac{c_{vr}}{c_{vz}} \times \frac{0.031\,(5)^2}{4\,(1.52)^2}$$

$$= 0.081 \frac{c_{vr}}{c_{vz}}$$

(*i*) Isotropic case: $c_{vr} = c_{vz}$; $T_r = 0.081$

From Table 16.1, for $\dfrac{R}{r_w}$ (or n) $= 10$ and, $T_r = 0.081$, we get

$$U_r \approx 34\% = 0.34$$

Now, from Eq. 16.11,

$$(1 - U) = (1 - U_z)\,(1 - U_r) = (1 - 0.2)\,(1 - 0.34) = 0.8 \times 0.66 = 0.528$$

$$\therefore \quad U = 1 - 0.528 = 0.472 = \mathbf{47.2\%.}$$

Thus, for the isotropic case, the presence of wells increases the degree of consolidation at time t from 20 to 47.2 per cent.

(*ii*) Anisotropic case: $c_{vr} = 5\, c_{vz}$

$$T_r = 0.081 \frac{c_{vr}}{c_{vz}} = 0.081 \times 5 = 0.405$$

Hence, from Table 16.1, $U_r = 87\% = 0.87$

$$\therefore \quad (1 - U) = (1 - U_z)\,(1 - U_r) = (1 - 0.2)\,(1 - 0.87) = 0.8 \times 0.13 = 0.104$$

$$\therefore \quad U = 1 - 0.104 = 0.896 = \mathbf{89.6\%.}$$

Thus, for the anisotropic case, the presence of wells increases the degree of consolidation at time t from 20 to 89.6%

Example 16.2 *An embankment of 10 m height is to be constructed on a 8 m thick layer of clay overlying rock. The embankment will increase the mean effective vertical stress in the clay after consolidation from a value of 80 kN/m^2 to 200 kN/m^2. The embankment will carry a road, and will be laid in 5 months. The surface will be laid 13 months after the commencement of construction. Only 3 cm of settlement can be accepted after the surfacing of the road. Design a suitable sand-drain installation to achieve the above requirements. Take $c_{vz} = c_{vr} = 8 \times 10^{-4}$ cm^2/sec and $m_v = 0.3$ m^2/MN.*

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Solution. The final settlement is given by $\rho_f = m_v \Delta\sigma' H$ where m_v and $\Delta\sigma'$ are assumed to be constant over the depth.

Here, $m_v = 0.3 \text{ m}^2/\text{MN} = 0.3 \times 10^{-3} \text{ m}^2/\text{kN}$; $\Delta\sigma' = 200 - 80 = 120 \text{ kN/m}^2$; $H = 8$ m

$\therefore$ $\rho_f = 0.3 \times 10^{-3} \times 120 \times 8 = 0.288 \text{ m} = 28.8 \text{ cm}$

Settlement that can be tolerated after 10 months = 3 cm.

$\therefore$ The settlement to be achieved at the end of 13 months = 28.8 – 3 = 25.8 cm

Degree of settlement in this time $= U = \dfrac{25.8}{28.8} = 89.5\%$ ...(1)

Effective time within which this settlement is to occur

$$= 13 - \frac{5}{2} = 10.5 \text{ months} = \frac{365}{12} \times 10.5 = 320 \text{ days}$$

Now, $T_v = \dfrac{c_v t}{d^2}$, where $t = 320 \times 3600 \times 24$ seconds ; $d = H = 8$ m

$c_v = 8 \times 10^{-4} \text{ cm}^2/\text{sec} = 8 \times 10^{-8} \text{ m}^2/\text{sec}$

$\therefore$ $T_v = \dfrac{8 \times 10^{-8} (320 \times 3600 \times 24)}{8 \times 8} = 0.0346$

$\therefore$ From Table 15.1, $U_z = 21\%$ ...(2)

Let us try 30 cm diameter wells spaced at 2 m centre to centre, and let these be arranged in a square pattern.

$\therefore$ $R = 0.564 \times 2 = 1.128 \text{ m} = 112.8 \text{ cm}$

$r_w = 15$ cm ; $n = \dfrac{R}{r_w} = \dfrac{112.8}{15} = 7.5$

$$T_r = \frac{c_{vr}}{4R^2} t = \frac{8 \times 10^{-8}}{4(1.128)^2} \times (320 \times 3600 \times 24) = 0.435$$

From Table 16.1, for $T_r = 0.435$ and $n = 7.5$, we get

$U_r = 93\%$ ...(3)

Now, $(1 - U) = (1 - U_z)(1 - U_r) = (1 - 0.21)(1 - 0.93) = 0.055$

$\therefore$ $U = 0.945 = 94.5\%$.

This is greater than the desired minimum value of $U = 89.5\%$. Hence the design is satisfactory. The spacing may, however, be increased to 2.2 m or so to reduce U to the desired value.

Example 16.3. *Modify the design of example 16.2 if it is desired to accelerate further the settlement by construction of additional height of embankment which would increase the effective vertical pressure to 250 kN/m^2.*

Solution. Let us assume that the pressure increase due to the surcharge is not going to change the value of coefficient of consolidation and the coefficient of volume change. The ultimate settlement will be:

$$\rho_f = m_v \Delta\sigma' H = 0.3 \times 10^{-3} (250 - 80) \times 8 = 0.418 \text{ m} = 41.8 \text{ cm}$$

The surcharge will, however, be removed at a settlement of 25.8 cm, which is expected to occur at 320 days. Therefore, the per cent consolidation produced by the installation

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$$= \frac{25.8}{41.8} \times 100 = 61.7\% \text{ in 320 days}$$

Now, $$T_v = \frac{c_v t}{d^2} = \frac{8 \times 10^{-8}}{8 \times 8} \times 320 \times 3600 \times 24 = 0.0346$$

$\therefore$ $U_z = 21\%$ (as earlier)

Let us try 30 cm diameter wells spaced at 3.3 m centre to centre and arranged in square pattern.

$$R = 0.564 \times 3.3 = 1.865 \text{ m} = 186.5 \text{ cm} \; ; \; r_w = 15 \text{ cm} \; ; \; n = \frac{186.5}{15} \approx 12.5$$

$$T_r = \frac{8 \times 10^{-8}}{4(1.86)^2}(320 \times 3600 \times 24) = 0.16$$

From Table 16.1 for $T_r = 0.16$ and $n = 12.5$, we get $U_r = 51\%$

$\therefore$ $(1 - U) = (1 - U_z)(1 - U_r) = (1 - 0.21)(1 - 0.51)$

$\therefore$ $U = 0.613 = 61.3\%$

This is nearly equal to the desired minimum value of $U = 61.7\%$.

Hence the layout will be satisfactory.

$$\text{Saving in the drains} = \left\{1 - \left(\frac{2.0}{3.3}\right)^2\right\} \times 100 = \mathbf{55.5\%.}$$

16.6 EFFECT OF PERIPHERAL SMEAR

'*Smear*' is the term used to define the wiping action caused by the casing or hollow mandrel used to form the well as it is driven down into the soil and then pulled out after it has been filled with sand. The action tends to smear the soil at the well periphery. For a soil originally having a greater permeability in the horizontal than in the vertical direction, the smeared zone forms a barrier to the horizontal flow of water, thereby slowing down considerably the process of consolidation.

In order to include the effects of smear and remoulding, Barron has considered that the smeared zone contains a homogeneous material having soil properties different from those in the remaining material in the soil cylinder. Barron also assumed that the smeared zone will consolidate very fast, thus its consolidation can be ignored and zone can be treated as an incompressible material. Barron's solution for excess pore water pressure in soil cylinder undergoing equal vertical strains and containing a smeared zone around the drain well is

$$\bar{u}_r = \bar{\bar{u}}_r \frac{\left[\log_\varepsilon \frac{r_w}{r_s} - \frac{r_w^2 - r_s^2}{2 r_s^2} + \frac{k_h}{k_s}\left(\frac{n^2 - s^2}{n^2}\right)\log_\varepsilon s\right]}{v} \qquad ...(16.19)$$

in which $$v = \left[\frac{n^2}{n^2 - s^2}\log_\varepsilon\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{s^2}{4n^2} + \frac{k_h}{k_s}\left(\frac{n^2 - s^2}{n^2}\right)\log_\varepsilon s\right] \qquad ...(16.20)$$

and $\bar{\bar{u}}_r$ can be determined from $\bar{\bar{u}}_r = u_0 \varepsilon^{\xi}$ ...(16.21)

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in which $$\xi = -\frac{8T_r}{v} \qquad ...(16.22)$$

Hence, $$U_r = 1 - \varepsilon^{-\frac{8T_r}{v}} \qquad ...(16.23)$$

r_s = radius of smeared zone ; $s = \dfrac{r_s}{r_w}$

k_h = horizontal permeability of undisturbed soil

k_s = horizontal permeability of smeared zone

$\bar{u}_r$ = excess pore water pressure at a point in the clay layer, as a result of radial flow of water

$\bar{\bar{u}}_r$ = average value of excess pore water pressure in the layer, due to radial flow.

Eqs. 16.17 and 16.23 can now be compared. For ideal wells (*i.e.*, having no smear), the position of the consolidation-time curves for any particular value of n depends on the value of $f(n)$ which is a function of n only (Eqs. 16.17 and 16.18). For wells with smear (Eq. 16.23), the position of the resulting consolidation time curves depends upon v which is a function of h, s and k_h/h_s. *Thus, it is possible to interpret various combinations of n, s and k_h/k_s in the treatment of a well with smear as if they define an ideal well having a large value of n.* For given radius of influence, a larger value of n determines the radius of an *equivalent ideal* well which is smaller than the radius of the actual well surrounded by zone of smear (Fig. 16.4). The effect on consolidation of the soil cylinder caused by the introduction of a smeared zone at the well periphery is identical to the effect caused by reducing the size of the ideal well (Richart : 1959).

In order to illustrate the effect of the smear let us take the example of sand drain installation shown in Fig. 16.4, with the following dimensions:

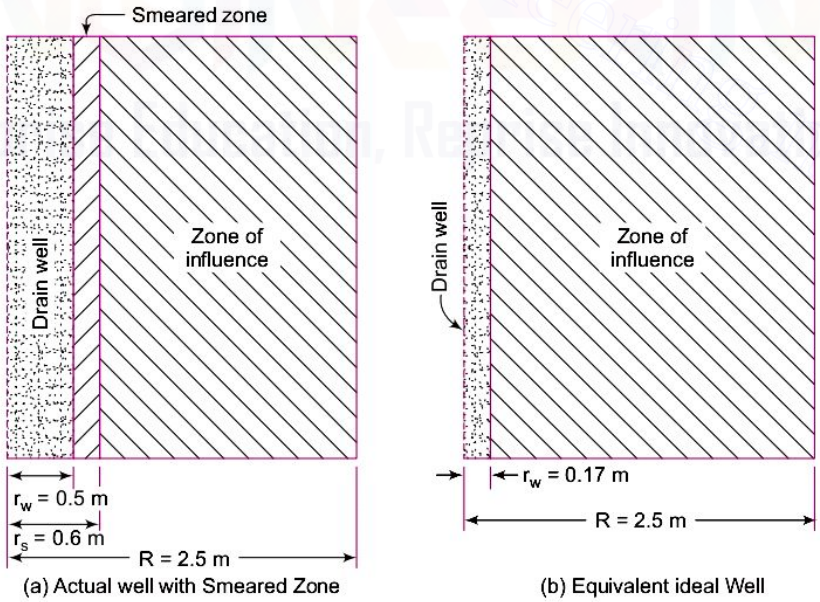

Fig. 16.4 Effect of smear

Radius of drain well = 50 cm ; Radius of smeared zone = 60 cm

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Radius of influence = 2.5 m ; and $\frac{k_h}{k_s} = 7$

Hence, $n = \frac{2.5}{0.5} = 5; s = \frac{6}{5} = 1.2$

$\therefore$ v (from Eq. 16.20) = 1.97

This corresponds to $f(n)$ =1.97 for a well having no smear (s = 1)

$$f(n) = 1.97 = \frac{n^2}{n^2 - 1} \log_e n - \frac{3n^2 - 1}{4n^2} \text{ (from Eq. 16.18)}$$

from which n = 15

$\therefore$ Effective radius of drain well = $\frac{2.5}{15}$ = **0.17 m.**

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CHAPTER 17

Compaction

17.1 INTRODUCTION

Compaction is a process by which the soil particles are artificially rearranged and packed together into a closer state of contact by mechanical means in order to decrease the porosity (or voids ratio) of the soil and thus increase its dry density. The compaction process may be accomplished by rolling, tamping, or vibration. Compaction is somewhat different from consolidation. While consolidation is a gradual process of volume reduction under sustained loading, compaction refers to a more or less rapid reduction *mainly* in the air voids under a loading of short duration. An example of compaction is the reduction in voids produced in a layer of the sub-grade by a rubber-tyred or steel-tyred roller during construction.

For compaction of any particular soil in the field, the engineer can vary water content, amount of compaction, and type of compaction. In 1933, Proctor showed that there existed a definite relationship between the soil water content and degree of dry density to which a soil might be compacted, and that for a specific amount of compaction energy applied on the soil there was a water content termed as *'optimum water content'* at which a particular soil attained maximum density (Fig. 17.1). The compaction characteristics are first determined in the laboratory by various compaction tests. These tests are based on any one of the following methods or types of compaction: dynamic or impact, kneading, static and vibration. Some of the usual compaction tests used in the laboratory to determine water-density relationships of soils are: Standard and Modified Proctor tests, Harvard Miniature compaction test, Abbot compaction test, and Jodhpur-mini compactor test. The main aim of these tests is to arrive at a standard which may serve as a guide and a basis of comparison for field compaction.

17.2 STANDARD PROCTOR TEST

The *Standard Proctor Test* was developed by R.R. Proctor (1933) for the construction of earth fill dams in the state of California. The test equipment consists of (*i*) cylindrical metal mould, having an internal

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diameter of 4 inches (10.15 cm), an internal effective height of 4.6 in. (11.7 cm), and a capacity of 1/30 cu. ft. (0.945 litre), (*ii*) detachable base plate, (*iii*) collar 2 in. (5 cm) in effective height, and *(iv)* rammer 5.5 lb (2.5 kg) in mass falling through a height of 12 in. (30.5 cm). The test consists in compacting soil at various water contents in the mould, in three equal layers, each layer being given 25 blows of the 5.5 lb rammer dropped from a height of 12 inches. The dry density obtained in each test is determined by knowing the mass of the compacted soil and its water content. The compactive energy used for this test is 6065 kg cm per 1000 mL of soil (or 595 kJ/m^3). IS : 2720 (Part VII) 1980/87 recommends a mould of 1000 mL capacity with an internal diameter 100 mm and internal effective height of 127.5 mm. The rammer has a mass of 2.6 kg with a drop of 310 mm. Figure 17.1 shows the dimensions of the IS version of Proctor test mould and the rammer.

About 3 kg of air-dried and pulverised soil, passing a 4.75 mm sieve, is mixed thoroughly with a small quantity of water. The mixture is covered with wet cloth, and left for a maturing time of about 5 to 30 minutes (or more for clay soils) to permit proper absorption of water. The quantity of water to be added for the first test depends upon the probable optimum water content for the soil. The initial water content may be taken 4% for coarse grained soils and 10% for fine grained soils. The empty mould attached to the base plate is weighed without collar. The collar is then attached to the mould. The mixed and mature soil is placed in the mould and compacted by giving 25 blows of the rammer uniformly distributed over the surface, such that the compacted height of soil is about 1/3 the height of the mould. Before putting the second instalment of soil, the top of the first compacted layer is scratched with the help of any sharp edge. The second and third layers are similarly compacted, each layer being given 25 blows. The last

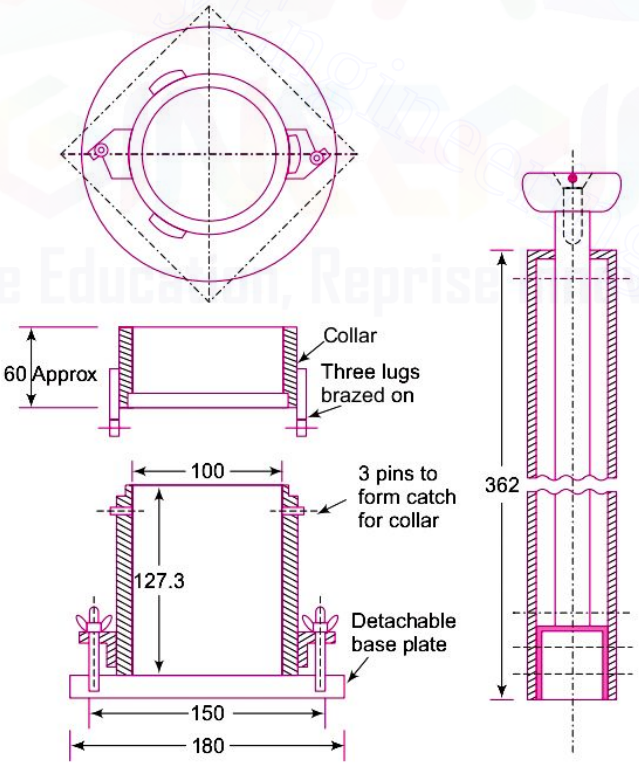

Fig. 17.1 Equivalent for standard proctor test

[IS : 2720 a(Part VII): 1965: light compaction]

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compacted layer should project not more than 6 mm into the collar. The collar is removed and the excess soil is trimmed off to make it level with the top of mould. The weight of the mould, base plate and the compacted soil is taken. A representative sample is taken from the centre of the compacted specimen and kept for water content determination. The bulk density ρ and the corresponding dry density ρ_d for the compacted soil are calculated from the following relation:

$$\rho = \frac{M}{V}\left(\text{g/cm}^3\right) \; ; \; \rho_d = \frac{\rho}{1+w}\left(\text{g/cm}^3\right) \qquad ...(17.1)$$

where M = mass of wet compacted specimen (g); w = water content (ratio);

V = volume of the mould = $\frac{1}{30}$ cu ft. (945 mL)

The compacted soil is taken out of the mould, broken with hand and remixed with raised water content (by 2 or 4 per cent). After allowing for the maturing time, the soil is compacted in the mould in three equal layers, as described above, and the corresponding dry density ρ_d and water content w are thus determined. The test is repeated on soil samples with increasing water contents, and the corresponding dry density ρ_d obtained is thus determined. A *compaction curve* is plotted between the water contents as abscissae and the corresponding dry densities as ordinates (Fig. 17.2). The dry density goes on increasing as the water content is increased, till maximum density is reached. The water content corresponding to the maximum density is called the *optimum water content* w_0.

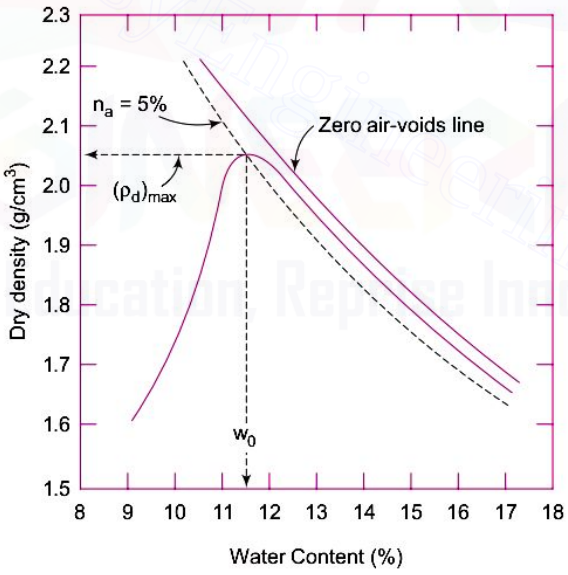

Fig. 17.2 Water content density relationship

Zero air voids line. A line which shows the water content dry density relation for the compacted soil containing a constant percentage air voids is known as an *air-voids line,* and can be established from the following relation (Eq. 2.35):

$$\rho_d = \frac{(1-n_a)\,G\rho_w}{1+wG} \qquad ...(17.2)$$

where n = per cent air voids ; w = water content of compacted soil

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ρ_d = dry density corresponding to w; g = specific gravity

ρ_w = density of water = 1 g/cm^3

The theoretical maximum compaction for any given water content corresponds to zero air voids condition ($n_a = 0$). The line showing the dry density as a function of water content for soil containing no air voids is called the *zero air voids line* or the *saturation line,* and is established by the Equation:

$$\rho_d = \frac{G\rho_w}{1 + wG} \qquad ...(17.3)$$

Alternatively, a line showing the relation between water content and dry density for a constant degree of saturation S is established from Equation:

$$\rho_d = \frac{G\rho_w}{1 + \frac{wG}{S}} \qquad ...(17.4)$$

For S =100%. Eq. 17.4 reduces to Eq. 17.3. The air voids line or degree of saturation lines when drawn across a compaction curve give a direct indication of the percentage air voids or of the degree of saturation existing at different points on the curve.

17.3 MODIFIED PROCTOR TEST

Higher compaction is needed for heavier transport and military aircraft. The modified Proctor test was developed to give a higher standard of compaction. This test was standardised by the American Association of State Highway Officials and is known as the modified AASHO test. In this test, the soil is compacted in the Standard Proctor mould (capacity 1/30 cu. ft. or 945 mL), but in five layers, each layer being given 25 blows of a 10 lb (4.5 kg) rammer dropped through a height of 18 inches. The compactive energy given to the soil in this test is 27260 kg-cm per 1000 cm^3 (or 2674 kJ/m^3) of soil, which is about $4\frac{1}{2}$ times * that of the Standard Proctor test. IS : 2720 (Part VII) – 1980/87 recommends the use of a 4.9 kg rammer with a drop of 45 cm for heavy compaction. In the modified proctor test, the water content dry density curve lies above the standard Proctor test curve, and has its peak relatively placed towards the left. Thus, for a same soil, the effect of heavier compaction is to increase in the maximum dry density and to decrease the optimum water content.

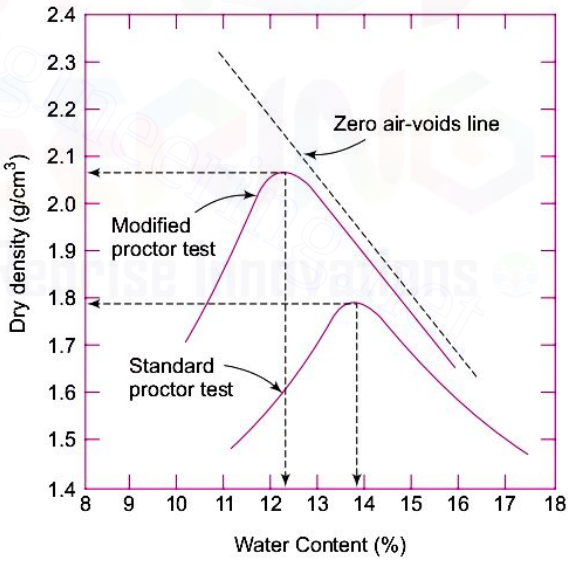

Fig. 17.3 Modified proctor test curve

* $\left[\frac{5 \text{ layers}}{3 \text{ layers}} \times \frac{10 \text{ lb}}{5.5 \text{ lb}} \times \frac{18''}{12''} \times \frac{25 \text{ blows}}{25 \text{ blows}} = 4.545 \approx 4\frac{1}{2}\right]$

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17.4 HARVARD MINIATURE COMPACTION TEST

In this test, the soil is compacted by *kneading action* of a cylindrical tamping foot 0.5 in (12.7 mm) in diameter. The soil is compacted in a small cylindrical mould having a capacity of 1/454 cu. ft. (62.4 mL) with an internal diameter of $\frac{21}{16}$ in. and a height of 2.816 in. The tamping foot operates through a preset compression spring so that the tamping force does not exceed appreciably a predetermined value. The number of layers, number of tamps per layer and the tamping force are variables depending upon the type of soil and the amount of compaction required.

17.5 THE DIETERT TEST

The apparatus consists of a 2 in. diameter mould supported on a metal base by two pegs. The soil is compacted by means of a piston on which a cylindrical weight of 18 lb falls through a height of 2 in. The weight is operated by means of a cam.

An air dried sample, weighing 150 g and passing through a 2.36 mm IS sieve, is mixed with water and then compacted in the mould by the application of 10 blows of the weight. The mould is then inverted and further 10 blows are applied. The weight and the length of the compacted soil cylinder are then measured to know its volume and bulk density. A specimen is put in the oven for its water content determination. The same procedure is adopted with the soil having varying water contents and the curve between water content and dry density is plotted.

17.6 ABBOT COMPACTION TEST

The apparatus consists of a metal cylinder 5.2 cm in internal diameter and 40 cm in effective height, clamped to the base. Oven dried soil, weighing 200 g is mixed with water and compacted in the cylinder with the blows of 2.5 kg rammer with a 5 cm diameter circular face falling through a height of 35 cm above the base. The number of blows to be used for the compaction are decided by calibration tests either with respect to Proctor's compaction or field compaction methods. The upper portion of the stem of the rammer is graduated in millimetres. The height of the compacted specimen may be determined from the reading on the graduated stem of the rammer. The volume of the compacted specimen is calculated from the known values of its height and cross-section. Knowing the dry mass *(i.e.,* 200 g) of the sample, the dry density is known. A number of such specimen are compacted at varying water contents, and a graph is plotted between dry density and water content.

17.7 JODHPUR MINI-COMPACTOR TEST

The Jodhpur Mini-compactor (Singh 1965), consists of (*i*) compaction mould having an internal diameter of 79.8 mm (cross-sectional area 50 cm^2) and internal height of 60 mm and a capacity of 0.3 litre, (*ii*) a dynamic ramming tool, called the 2.5 kg DRT consisting of a 2.5 kg drop weight which freely slides down a stem through a height of 25 cm and falls over a foot, 50 mm in diameter and 75 mm in height. The soil is compacted in 2 equal layers, each layer being given 15 blows of the ramming tool. The gross compactive energy used for the test is 6250 kg cm per 1000 cm^3 of soil (613 kJ/m^3). Laboratory calibration test (Singh and Punmia, 1965) have shown that in the close limits of experimental error, the optimum

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water content and the maximum dry density obtained by the Jodhpur Mini-compactor are the same as in the Standard Proctor compaction Test (Fig. 17.4). Figure 17.5 shows the Jodhpur Mini-compactor.

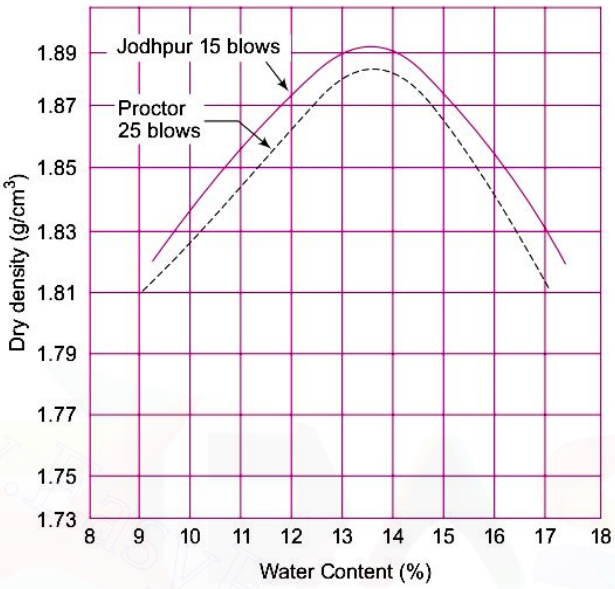

Fig. 17.4 A typical comparison of the standard proctor test and Jodhpur mini-compactor test (Singh and punmia 1965)

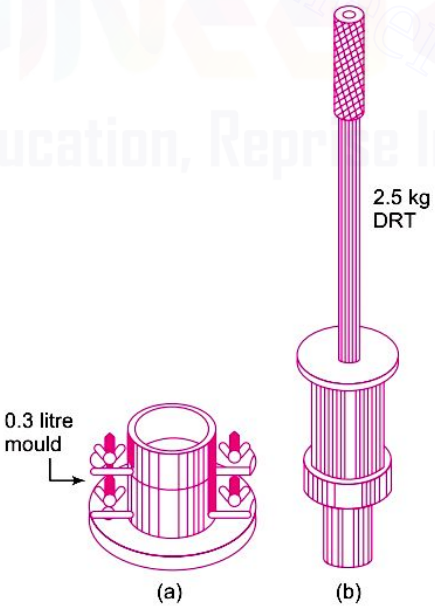

Fig. 17.5 Jodhpur mini-compactor

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17.8 FIELD COMPACTION METHODS

Various types of soils can be compacted in the field by three methods: rolling, ramming (by impact) and vibration. Corresponding to these, the various compacting equipments can be grouped under three categories: rollers, rammers and vibrators. The *rolling equipments* are of five types: (*i*) smooth wheel rollers, (*ii*) pneumatic tyred rollers, (*iii*) sheep foot rollers, (*iv*) lorries and pneumatic tyred contruction plant, and (*v*) track laying vehicles. The *ramming equipment* consists of three types: (*i*) dropping weight type (including piling equipment), (*ii*) internal combustion type and (*iii*) pneumatic type. The *vibrating equipment,* mounted on screeds, plates or rollers are of two types: (*i*) dropping weight type, and (*ii*) pulsating hydraulic type.

The *smooth wheel rollers* are of three types : (*i*) the conventional three-wheel type with two large smooth-faced steel wheels in the rear and one smaller smooth-faced drum in the front weighing from 20 to 150 kN, (*ii*) tandem rollers weighing from 10 to 140 kN, and (*iii*) the three axle tandem rollers weighing from 120 to 180 kN. Smooth wheel rollers are usually self-propelled and are equipped with a clutch-type reversing gear so that they can be operated back and forth without turning.

The *pneumatic type rollers* range in size from the smaller wobble wheel rollers to the very heavy rollers. A common form of pneumatic roller consists of a box or platform mounted between two axles, the rear of which has one more wheel than the front, the wheel mounted on the front axle being arranged to track in between those mounted on the rear axle. The tyre pressures in the small rollers are of the order of 250 kN/m^2 and in the heavier rollers, the pressure ranges from 400 to 1050 kN/m^2. Smaller rollers are having tyre loads of about 7.5 kN while the bigger rollers have tyre load ranging from 100 kN to 500 kN per tyre. The pneumatic tyred rollers are loaded with kentledge such that when the tyres are inflated to their desired pressure, the sum of the contact widths of the tyres approximately equals 80 per cent of the width of the roller. The *wobble-wheel-roller* has wheels mounted at slight angle with respect to the axle so as to provide a kneading action. The rollers are normally towed by either a track laying or a pneumatic tyred tractor.

The *sheep foot rollers* consist of hollow cylindrical steel drum on which projecting feet are mounted. The weight of the drum can be varied by filling it partly or fully with water or sand and they are mounted either singly or in pairs on a steel frame which is towed by either track-laying or pneumatic-tyred tractors. The *loaded weight* per drum ranges from about 15 to 130 kN, and the foot pressure range from 800 to 3500 kN/m^2. *Rammers* for compacting the soil comprise of pneumatic and internal combustion type weighing from 300 to 1500 N. Internal combustion type jumping rammers, known as frog rammers, weigh up to one tonne. The *vibrators* consist of a vibrating unit of either the out-of balance weight type or a pulsating hydraulic type mounted on a screed, plate or roller.

Suitability of various compaction equipments. The performance of a compaction equipment depends upon the soil type, its particle size distribution, and its water content. In general, smooth wheel rollers are most suited to crushed rock, hard core, mechanically stable gravel sand sands. They can also be used satisfactorily on moderately cohesive soils. In cohesionless sands and gravels, vibrating type equipment, crawler tractors and rubber-tyred rollers are effective in producing densities upto about 100 per cent of modified AAHSO. Sheep-foot-rollers are recommended for compacting cohesive soils, but are not considered effective on coarse grained cohesionless soils. The kneading action of the sheep-foot roller results in a better bond between compacted layers compared to other types of rollers. The action of pneumatic-tyred rollers is a combination of pressure and kneading, and they are suitable both on cohesionless sand and gravels, and on cohesive soils. Rammers are used for compacting soils in confined places. Vibratory rollers are useful for cohesionless soils.

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17.9 PLACEMENT WATER CONTENT

The water content used in the field compaction is called the *placement water content* which may be equal to, lower than or higher than the optimum water content determined in the laboratory. Cohesive subgrades under pavements should preferably be compacted wet of optimum so that they may not exhibit large expansions and swelling pressure on submergence. Highway embankments on cohesive soils should be compaced somewhat dry of optimum in order to achieve high strength and resistance to deformation and low volume compressibility. High earth dams should be compacted at a placement water content 1 to 2.5 per cent less than the optimum to reduce the probability of the development of high pore pressure. However, the impervious cores of the earth dams should desirably be compacted on the wet-side of the optimum in order to achieve low permeability, and greater safety against cracking due to differential settlements or other causes. For a given soil, the laboratory optimum water content and the field optimum water content may differ, depending upon the type of the compaction equipment used. U.S. Bureau of Reclamation has found that the laboratory optimum water content corresponding to the standard Proctor test is approximately equal to the compaction achieved in the field by 12 passes of the 20 ton dual-drum sheepfoot roller used by it on 15 cm compacted lifts.

17.10 FIELD COMPACTION CONTROL

The field compaction control consists of the determination of (*i*) water content at which the soil has been compacted, and (*ii*) the dry density and hence the *degree of compaction* achieved. For proper compaction control as the work progresses, rapid methods of testing must be used.

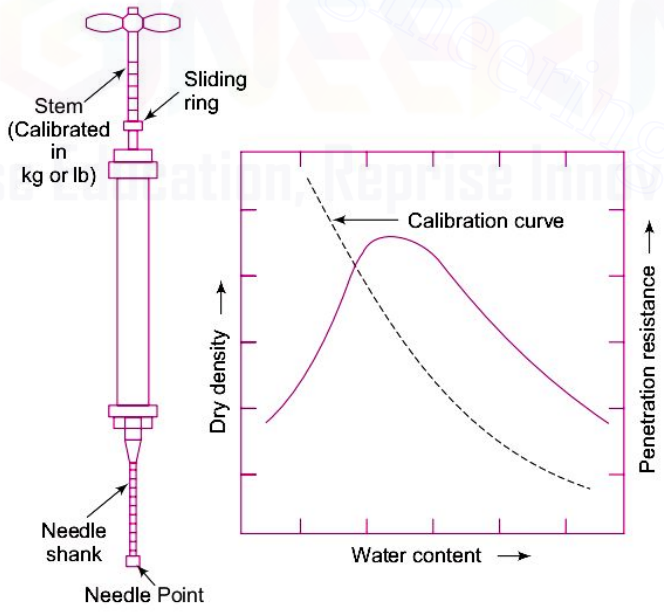

Fig. 17.6 Proctor needle and calibration curve

The rapid determination of water content can be done by two methods: (*i*) the calcium carbide method discussed in § 3.2 and *(ii)* the Proctor needle method. The second method is commonly used in the field.

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The Proctor needle consists of a needle point, attached to graduated needle shank which in turn is attached to a spring loaded plunger. The needle point of varying cross-sectional area are available so that a wide range of penetration resistance can be measured. The penetration force is read on a loaded gauge fixed over the handle [Fig. 17.6 (*a*)]. To use the needle in the field, a calibration curve [Fig. 17.6 (*b*)] is plotted in the laboratory between the penetration resistance as the ordinate and the water content as the abscissa. The laboratory penetration resistance is measured by inserting the Proctor needle in the compacted soil in the Proctor mould. The penetration resistances corresponding to various water contents are thus noted at the end of each Proctor compaction, and a calibration curve is plotted. This curve may be used to determine the placement water content. The penetration resistance of the compacted soil in the field is determined with the Proctor's needle, and its water content is read off from the calibration curve.

The bulk density of the compacted soil in the field can be determined by any of the methods described in Chapter 3. The degree of compaction obtained in the field is measured by the *relative compaction* or the *per cent compaction* which is defined as the ratio, expressed as a precentage, of the field dry density to the maximum density obtained in a laboratory compaction test. In free-draining cohesionless soils, the degree of compaction is sometimes expressed in terms of density index, particularly in earth dam construction.

17.11 FACTORS AFFECTING COMPACTION

The various factors which affect the compacted density are as follows : (*i*) water content, (*ii*) amount and type of compaction, (*iii*) method of compaction, (*iv*) type of soil, and (*v*) addition of admixtures.

1. Water content: It has been seen by laboratory experiments that as the *water content* is increased, the compacted density goes on increasing, till a maximum dry density is achieved after which further addition of water decreases the density. When only a relatively small amount of water is present in soil, it is firmly held by the electrical forces at the surface of soil particles with a high concentration of electrolyte which prevents the diffuse double layer surrounding the particles from developing fully. The double layer depression leads to a low inter-particle repulsion and the particles do not move over one another easily when compactive energy is applied and hence high percentage air voids and low density is achieved. The increase in water content results in an expansion of double layer and a reduction in the net attractive forces between particles or in an increased inter-particle repulsion which permits the particles to slide more easily past one another into a more oriented and denser state of packing together, and hence higher density. After the optimum water content is reached, the air voids approach approximately a constant value as further increase in water content does not cause any appreciable decrease in them, even though a more orderly arrangement of particles may exist at higher water contents. The total voids due to water and air combination go on increasing with increase of water content beyond the optimum and hence the dry density of the soils falls.

2. Amount of compaction: The *amount of compaction* greatly affects the maximum dry density and optimum water content of a given soil. The effect of increasing the compactive energy results in an increase in the maximum dry density and decrease in the optimum water content as shown in Fig. 17.7. However, the increase in maximum dry density does not have a linear relationship with increase of compactive effort.

3. Method of compaction: The density obtained during compaction, for a given soil, greatly depends upon the type of compaction or the manner in which the compactive effort is applied. The various variables in this aspect are (*i*) weight of the compacting equipment, (*ii*) the manner of operation such as

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dynamic or impact, static, kneading or rolling, and (*iii*) time and area of contact between the compacting element and the soil.

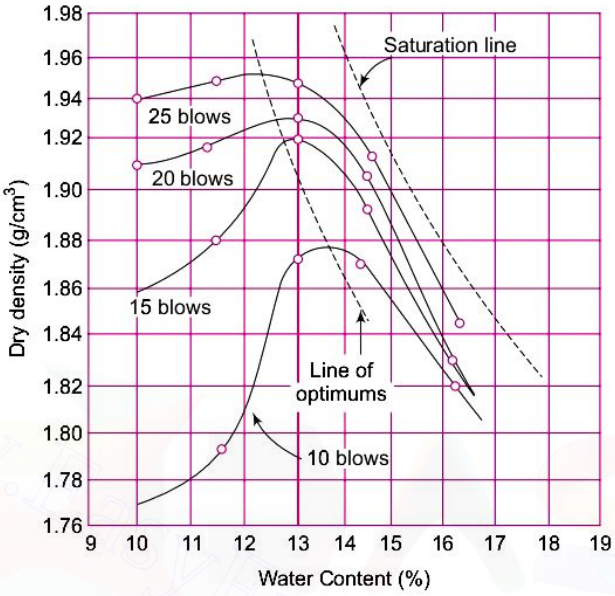

Fig. 17.7 Effect of compactive effort on compaction, obtained by the Jodhpur mini-compactor (Singh and punmia, 1965)

4. Type of soil : The maximum dry density achieved corresponding to a given compactive energy largely depends upon the *type of soil*. Well graded coarse-grained soils attain a much higher density and lower optimum water contents then fine grained soils which require more water for lubrication because of the greater specific surface.

Figure 17.8 show dry density water content curves for a range of soil types. In general, coarse grained soils can be compacted to higher dry densities than fine grained soils.

***(i) Compaction curve for cohesionless sands*:** In the case of cohesionless soils which are devoid of fines, the water content has very little influence on the compacted density. For such soils, the dry density decreases with an increase in the water content, in the initial stage of the curve, particularly under a low compactive effect (Fig. 17.9). This is due to the *'bulking of sands'* wherein the capillary tension resists the tendency of soil particles to take a dense state. In other words, the capillary tension developed in the sandy soil is not fully counteracted by the compactive effort and this capillary tension holds the particles in a loose state resisting compaction. It is interesting to note that the same soil, in air-dried or oven dried condition, achieves greater density under the same compactive effort. The maximum bulking (and hence the least density) occurs at a water content between 4 to 5%. On further addition of water, the meniscus is destroyed and the soil particles are able to shift to take a closer packing, resulting in increase in dry density. The density reaches the maximum value when the cohesionless soil is fully saturated. On further addition of water, the dry density again decreases. Also, the maximum density, attained under full saturation condition, is not very much higher than that corresponding to air dried or oven dried condition. Secondly, the attainment of maximum density at full saturation is not due to *lubrication* action of water but rather it is due to the reduction of effective pressure between soil particles by hydrostatic pressure. Such soils do not display a distinct optimum water content.

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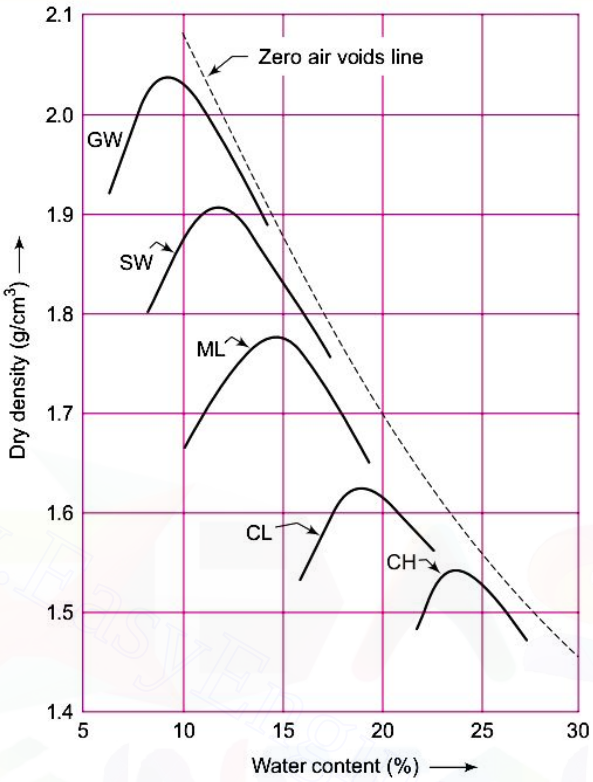

Fig. 17.8 Compaction curves for a range of soil types

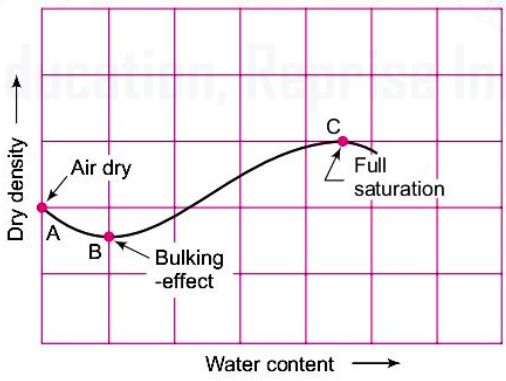

Fig. 17.9 Typical compaction curve for cohesionless sand

(ii) Compaction curve for fat clay: Figure 17.10 shows a typical compaction curve for a fat clay. Here also, we see that there is initial decrease of dry density at lower water contents. This is the characteristic feature of black cotton soils, highly swelling clays and some fat clays. The optimum water content for such soils range between 20 to 25%.

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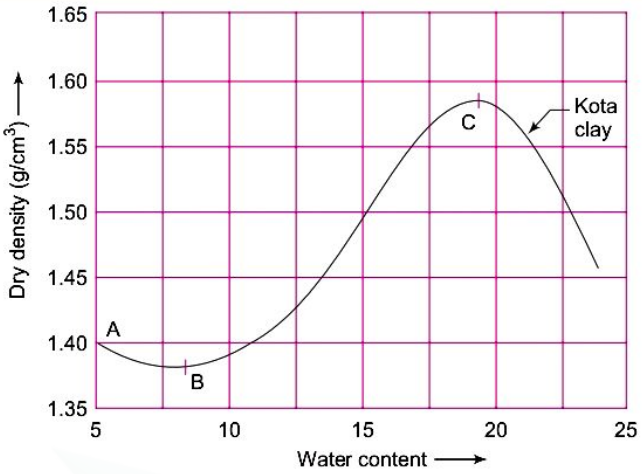

Fig. 17.10 Typical compaction curve for fat clay

5. Addition of admixtures: The compaction properties/characteristics of a soil can be modified by a number of admixtures other than soil material. These admixtures have special application in stabilised soil construction. Chapter 31 describes various types of admixtures, along with their effects on the compaction characteristics of soils.

17.12 EFFECT OF COMPACTION ON SOIL PROPERTIES

The main aim of compacting a soil is to improve some desirable properties of the soil, such as reduction of compressibility, water absorption and permeability, increase in soil strength, bearing capacity, etc. and change in swelling and shrinkage characteristics. However, the effect of compaction on soil properties very much depends on the structure attained by the soil during compaction.

1. Change in structure of soil: The structure of a soil during compaction depends upon (*i*) type of soil, (*ii*) moulding water content, and (*iii*) type and amount of compaction. For the purposes of discussion, let us divide the soils into three types (*a*) coarse grained soils with little or no fines, (*b*) composite soils, and (*c*) purely cohesive soils (*i.e.*, clays). The soils of first type (*i.e.*, coarse grained soils), maintain a *single grained structure* at any possible voids ratio or water content. However, the structure of composite soils, after being compacted depend upon the relative proportion of coarse particles and fines and their structure can either be *coarse grained skeleton structure* or *cohesive matrix structure.*

In contrast to these, the structure of compacted clay is complicated. At the same compactive effort, the soil structure becomes increasingly oriented (*i.e.*, dispersed) with increasing water content. When compacted dry of optimum, the structure of clay is always *flocculated.* Figure 17.11 shows two compaction curves for a clay – one at lower compactive effort and other at higher compactive effort. At water content higher than optimum, the soil at points *B* and *D* of the two curve is more oriented than at point *A*. However, the structure is more oriented at *C* than at *A*.

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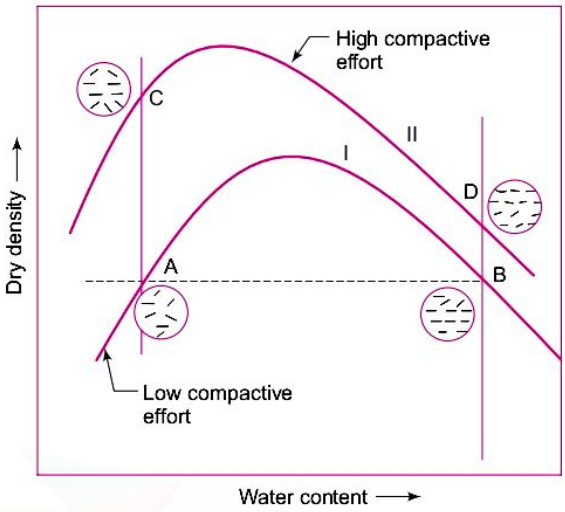

Fig. 17.11 Effects of compaction on structure of clay (After lambe, 1958)

2. Permeability: The following points are note worthy:

(*i*) As the dry density increases due to compaction, the voids go on reducing and hence the permeability goes on decreasing.

(*ii*) For the same density, fine grained samples compacted dry of optimum are more permeable than those compacted wet of optimum. This is so because these soils have *flocculated* structure when compacted dry of optimum, and have dispersed structure (*i.e.*, parallel orientation) when compacted wet of optimum.

(*iii*) For a given voids ratio, greater the size of individual pores, greater is the permeability.

(*iv*) As the compactive effort is increased, the permeability of soil decreases because of the increased dry density and better orientation of particles.

3. Shrinkage: For the same density, soil sample compacted dry of optimum shrink appreciably less than the sample compacted wet of optimum. This is so because the soil particles having dispersed structure have nearly parallel orientation and can pack more efficiently.

4. Swelling: A clayey soil sample compacted dry of optimum water content has high water deficiency and more radom orientation and hence exert greater swelling pressure and swell to higher water content than the sample of the same density obtained from wet side compaction.

5. Pore pressure: Saturated sample of clay, compacted dry of optimum, tend to develop substantially lower pore pressure at low strains in undrained shear test than the sample of the same soil of the same density and water content, compacted wet of optimum. However, at higher strains, both the samples exhibit the same pore pressure.

6. Compressibility: Saturated sample of clay, compacted wet side of optimum is more compressible than another sample of the same soil, having the same voids ratio, but compacted dry of the optimum, when the applied pressure is in low pressure range. This is so because the sample compacted dry of optimum has *flocculated* structure and requires extra pressure to cause parallel orientation of particles. However, in the high pressure range, a sample compacted dry of optimum is more compressible than the one compacted wet of the optimum.

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7. Stress-strain characteristics: For a given soil, a sample compacted dry side of optimum has a steeper stress-strain curve and hence has a higher modulus of elasticity, than the one which is compacted wet of optimum (Fig. 17.12), at the same density. Soil compacted wet of optimum have *brittle failure* while soil compacted wet of optimum, and having dispersed structure, continue to increase in strength even at higher strains.

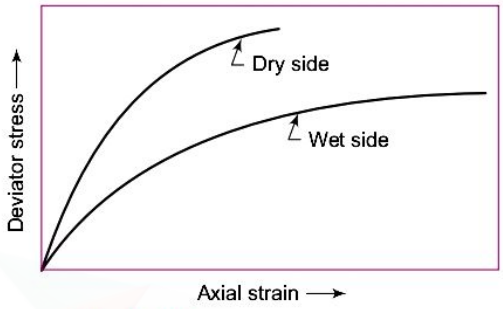

Fig. 17.12 Stress-strain curves

8. Shear strength: The shear strength of compacted clays depend upon (*i*) dry density, (*ii*) moulding water content, (*iii*) soil structure, (*iv*) method of compaction, (*v*) strain used to defined strength, (*vi*) drainage condition, and (*vii*) type of soil.

In general, at low strains, strength of cohesive soils compacted dry of optimum is higher than those compacted wet of optimum. Figure 17.13 shows the failure envelope of two samples of the same soil, one compacted dry of optimum and the other compacted wet of the optimum, but both compacted at the same density. However, of higher strains, the flocculated structure of the same compacted on the dry side is broken, giving rise to ultimate strength for both the samples. The manner of compaction also influences the strength of soil sample compacted wet of optimum. It is interesting to note that the clay cores in earth dams are usually compacted wet of optimum to tolerate large settlements without cracking.

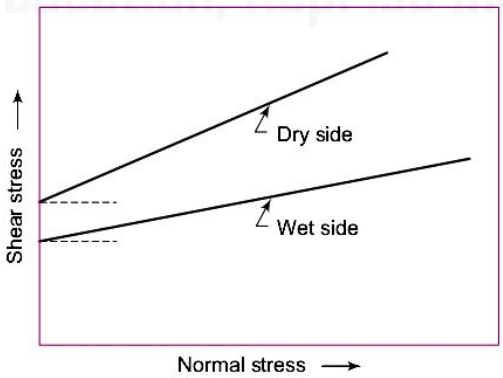

Fig. 17.13 Failure envelopes

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Example 17.1. *A laboratory compaction test on soil having specific gravity equal to 2.68 gave a maximum dry density of 1.82 g/cm³ and a water content of 17 per cent. Determine the degree of saturation, air content and percentage air voids at the maximum dry density. What would be theoretical maximum dry density corresponding to zero air voids at the optimum water content?*

Solution.

$$\rho_d = \frac{G\rho_w}{1+\frac{wG}{S}}$$

$$\therefore \quad 1+\frac{0.17\times 2.68}{S} = \frac{G\rho_w}{\rho_d} = \frac{2.68}{1.82} = 1.473$$

$$\therefore \quad S = \frac{0.17\times 2.68}{0.473} = 0.96 = 96\% \; ; \; a_c = 1 - S = 1 - 0.96 = 0.04 = 4\%$$

$$\rho_d = \frac{(1-n_a)\,G\rho_w}{1+wG}$$

$$\therefore \quad (1-n_a) = \frac{1.82\,(1+0.17\times 2.68)}{2.68} = 0.99 \text{ or } n_a = 1 - 0.99 = 0.01 = 1\%$$

When, $n_a = 0$ ($S = 1$), theoretical dry density at $w = 17\%$ is given by

$$\rho_d = \frac{G\rho_w}{1+wG} = \frac{2.68\times 1}{1+0.17\times 2.68} = \mathbf{1.84\ g/cm^3.}$$

The corresponding dry unit weight is

$$\gamma_d = 9.81\ \rho_d = 9.81\times 1.84 = \mathbf{18.05\ kN/m^3.}$$

17.13 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 17.2. *The following are the results of a compaction test:*

Mass of mould + wet soil (g)	*2925*	*3095*	*3150*	*3125*	*3070*
Water content (%)	*10.0*	*12.0*	*14.3*	*16.1*	*18.2*

Volume of mould = 1000 mL; Mass of mould = 1000 g

Specific gravity of solids = 2.70

(*i*) *Find the compaction curve showing the optimum moisture content and maximum dry density* (*ii*) *Plot the zero air void line* (*iii*) *Determine the degree of saturation at the maximum dry density.*

(Gate Exam. 1993)

Solution. The zero-air voids line, corresponding to $S = 1$, is plotted by the equation

$$(\rho_d)_0 = \frac{G\rho_w}{1+wG} = \frac{2.7\times 1}{1+2.7w} = \frac{2.7}{1+2.7w}\ \text{g/cm}^3$$

Also,

$$\rho_d = \frac{\rho}{1+w} = \frac{M}{V}\times\frac{1}{1+w} = \frac{M}{1000\,(1+w)}\ \text{g/cm}^3$$

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The computations are arranged in a tabular form below:

S.No.	Water content w	Mass of soil (M) g	$\rho_d = \frac{M}{1000(1+w)}$ g/cm³	$(\rho_d)_0 = \frac{2.7}{1+2.7w}$ g/cm³
1	0.10	1925	1.75	2.13
2	0.12	2095	1.87	2.04
3	0.143	2150	1.88	1.95
4	0.161	2125	1.83	1.88
5	0.182	2070	1.75	1.81

Figure 17.14 shows the compaction curve along with the zero-air voids line. From the curve we get $\rho_{d,\max} = 1.89$ g/cm³ and $w = 0.135$. The degree of saturation is given by

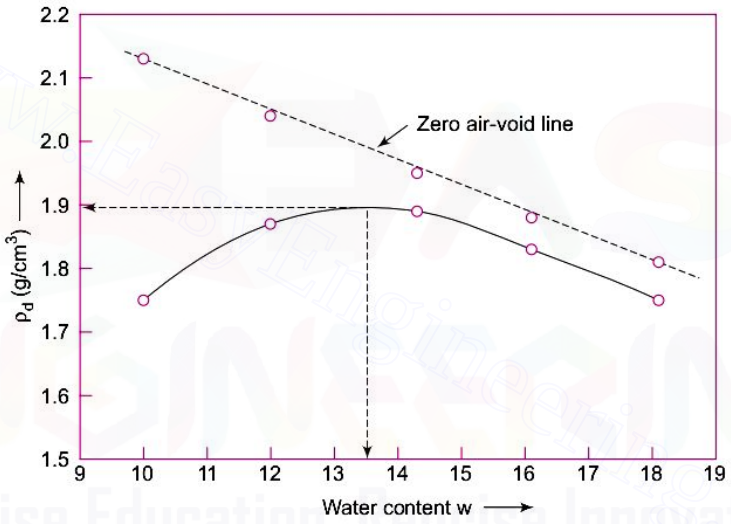

Fig. 17.14

$$\rho_d = 1.89 = \frac{G\rho_w}{1+\frac{wG}{S}} = \frac{2.7\times 1}{1+\frac{0.135\times 2.7}{S}} \text{ or } \frac{0.135\times 2.7}{S} = \frac{2.7}{1.89} - 1 = 0.428$$

From which $$S = \frac{0.135\times 2.7}{0.428} = 0.852 = \mathbf{85.2\%.}$$

Example 17.3. *Work out theoretical maximum dry density for a soil sample having sp. gr. of 2.7 and OMC = 16%. Also explain the difference in OMC value in case of Proctor test and modified proctor test for cohesive soils and granular soils.* **(Engg. Services Exam. 2001)**

Solution. $\gamma_{d,\max}$ occurs when S is maximum, *i.e.*, when $S = 1$

$$\gamma_{d,\max} = \frac{G\gamma_w}{1+\frac{wG}{S}} = \frac{G\gamma_w}{1+w.G}$$

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Hence, $$\rho_{d,\max} = \left(\frac{G\rho_w}{1+wG} = \frac{2.7\times1}{1+0.16\times2.7}\right) = \mathbf{1.885\ g/cm^3}.$$

Example 17.4. *A cohesive soil yields a maximum dry density of 1.8 g/cc at an OMC of 16% during a standard proctor test. If the values of G is 2.65, what is the degree of saturation? What is the maximum dry density it can further compacted to?* **(Gate Exam. 1992)**

Solution. Given: $\rho_d = 1.8\ g/cm^3$; $w = 0.16$; $G = 2.65$

$$e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.65\times1}{1.8} - 1 = 0.4722$$

$$\therefore \quad S = \frac{wG}{e} = \frac{0.16\times2.65}{0.4722} = 0.8979 = \mathbf{89.79\%}$$

Now $$\rho_d = \frac{G\rho_w}{1+\frac{wG}{S}}$$; when $S = 1$, we get

$$\rho_{d,\max} = \frac{G\rho_w}{1+wG} = \frac{2.65\times1}{1+0.16\times2.65} = \mathbf{1.861\ g/cm^3}.$$

17.14 LABORATORY EXPERIMENTS

EXPERIMENT 17: DETERMINATION OF COMPACTION PROPERTIES

Object and Scope. The object of the experiment is to determine the relationship between water content and dry density of soil using Standard Proctor Test (light compaction) or Modified Proctor Test (heavy compaction), and then to determine the optimum water content and the corresponding maximum dry density for a soil. The test also covers the determination of relationship between penetration resistance and water content for the compacted soil.

(A) Light Compaction (Standard Proctor Test)

Materials and Equipment. (*i*) Cylindrical metal mould of capacity 1000 cc, with an internal diameter of 100 ± 0.1 mm and an internal effective height of 127.3 ± 0.1 mm, or mould of capacity 2250 cc, with an internal diameter 150 ± 0.1 mm and an internal effective height of 127.3 ± 0.1 mm , each mould fitted with a detachable base and a removable extension (collar) approximately 60 mm high, (*ii*) Metal rammer, 50 mm diameter circular face, weighing 2.6 kg and having drop of 310 mm, (*iii*) Steel straight edge (*iv*) 20 mm and 4.75 mm IS sieves, (*v*) Balances, 10 kg capacity sensitive to 1 g, and 200 capacity sensitive to 0.01 g, (*vi*) Thermostatically controlled oven (105°C – 110°C), (*vii*) Water content containers, (*viii*) Mixing equipment, such as mixing pan, spoon, trowel spatula etc., (*ix*) Measuring cylinder of glass, 100 mL capacity, (*x*) Sample extruder (optional).

Test Procedure

1. Take a representative sample weighing approximately 20 kg of thoroughly mixed air dried material passing 4.75 mm (or 20 mm) IS sieve. Add enough water to bring its water content to about 7 per cent (sandy soils) or 10 per cent (clayey soils) less than the estimated optimum moisture content. Keep this soil in an air tight container for about 20 hours, for maturing.

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2. Clean the mould and fix it to the base. Take the empty mass of the mould and the base, nearest to 1 g.
3. Attach the collar to the mould. The inside of the mould may be greased thoroughly.
4. Mix the matured soil thoroughly. Take out about $2\frac{1}{2}$ kg of the soil and compact it is the mould in three equal layers, each layer being given 25 blows from the rammer weighing 2.6 kg dropping from a height of 310 mm, if 1000 mL mould is used. If however, the 2250 mL mould is used, about 5 kg of soil should be taken and should be compacted in three equal layers, each layer being given 56 blows from the rammer weighing 2.6 kg dropping from a height of 310 mm. The blows should be uniformly distributed over the surface of each layer. Each layer of compacted soil should be scored with a spatula before putting the soil for the succeeding layer. The amount of soil used should be just sufficient to fill the mould leaving about 5 mm to be struck off when collar is removed. Find the penetration resistance of compacted soil, using the Proctor's needle.
5. Remove the collar, and cut the excess soil with the help of a straight edge. Clean the mould from outside, and weigh it to the nearest gram. Eject out the soil from the mould, cut it in the middle and keep a representative soil specimen for water content determination.
6. Repeat steps 4 and 5 for about five or six times, using a fresh part of the soil specimen and after adding a higher water content than the precceeding specimen.

Tabulation of observations. The observations are tabulated as illustrated in Table 17.1.

Table 17.1 Data and observation sheet for proctor's test (Light compaction)

Determination No.	*1*	*2*	*3*	*4*	*5*	*6*	7
(*a*) Density							
Mass of mould + compacted soil (g)	6607	6644	6723	6795	6837	6842	6829
Mass of mould (g)	4944	4944	4944	4944	4944	4944	4944
Mass of compacted soil (g)	1663	1720	1779	1851	1893	1898	1885
Bulk density (g/cm^3)	1.76	1.82	1.88	1.96	2.00	2.01	1.99
Dry density (g/cm^3)	1.55	1.57	1.58	1.64	1.63	1.62	1.58
(*b*) Water content							
Container No.	23	94	8	9	159	71	10
Mass of container + wet soil (g)	45.3	59.9	38.8	52.2	46.8	45.6	44.9
Mass of container + dry soil (g)	42.5	54.7	36.2	47.4	42.4	41.3	40.2
Mass of water (g)	2.8	5.2	2.6	4.8	4.4	4.3	4.7
Mass of container (g)	22.5	22.9	22.5	22.6	23.1	22.9	22.5
Mass of dry soil	20.0	31.8	13.7	24.8	19.3	18.4	17.7
Water content (%)	13.9	16.3	18.8	19.4	22.8	23.4	26.6

Calculations: 1. The dry density of the compacted soil is calculated as follows:

$$\rho_d = \frac{\rho}{1+w}.$$

A curve showing the relationship between dry density and water content is plotted (Fig. 17.2). The water content corresponding to the maximum dry density is found from the curve. For the data

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tabulated above, the optimum water content is found to be 20% corresponding to maximum dry density of 1.64 g/cm^3. The corresponding dry unit weight = $1.64 \times 9.81 = 16.09$ kN/m^3.

2. The voids ratio for each determination is found from the equation:

$$e = \frac{G\rho_w}{\rho_d} - 1$$

3. On the same plot, a curve is drawn between penetration resistance and water content (Fig. 17.6 *b*).

(B) Heavy Compaction (Modified Proctor's Test)

The equipment required for the heavy compaction test is the same as that required for the light compaction except that the rammer has a falling mass of 4.9 kg and has a drop of 45 cm. The soil is compacted in *five* equal layers, instead of three. Each layer is given 25 blows of the rammer if the 1000 mL mould is used, and 55 blows if the 2250 mL mould is used.

PROBLEMS

1. The following are results of standard compaction test performed on a sample of soil:

Water content %	5	10	14	20	25
Bulk density (g/cm^3)	1.77	1.98	2.1	2.18	2.16

Plot the water content-dry density curve and obtain the optimum water content and maximum dry density. Calculate the water content necessary to completely saturate the sample at its maximum dry density, assuming no change in the volume. Take $G = 2.7$. [**Ans.** 1.84 g/cm^3, 45% ; 17.4%]

2. To determine the in-situ density of a compacted embankment a small hole is dug into the embankment and the excavated soil is weighed. Uniformly graded dry soil is then poured into the hole from a weighed sand pouring cylinder fitted with a pouring cone at the bottom. The cylinder after filling the hole is weighed. The mass of sand filling the pouring cone is determined afterwards. The density of sand used in cylinder is determined by filling it in 1000 m calibrating can whose empty mass is 944 g. If the water content of the embankment soil is 8% and the other test data are as given below, determine the in-situ dry density :

(*i*) mass of excavated soil = 925 g
(*ii*) mass of cylinder + sand before the test = 5332 g
(*iii*) mass of cylinder + sand after filling the hole = 4152 g
(*iv*) mass of sand filling the pouring cone = 432 g
(*v*) mass of calibrating can plus sand = 2483 g. [**Ans.** 1.76 g/cm^3]

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Part V
Strength and Stability

18. Shear Strength
19. Failure Envelopes
20. Earth Pressure
21. Bulkheads and Cofferdams
22. Shafts, Tunnels and Conduits
23. Stability of Slopes

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CHAPTER 18

Shear Strength

18.1 INTRODUCTION

When soil is loaded, shearing stresses are induced in it. When the shearing stresses reach a limiting value, shear deformation takes place, leading to the failure of the soil mass. The failure may be in the form of sinking of a footing, or movement of a wedge of soil behind a retaining wall forcing it to move out, or the slide in an earth embankment. The *shear strength* of soil is the resistance to deformation by continuous shear displacement of soil particles or on masses upon the action of a shear stress. The failure conditions for a soil may be expressed in terms of limiting shear stress, called shear strength, or as a function of the principal stresses. The two forms of the failure conditions are often but not always interchangeable. All stability analysis in soil mechanics involve a basic knowledge of the shearing properties and shearing resistance of the soil. The shear strength is the most difficult to comprehend and one of the most important of the soil characteristics. The method used to determine the shear characteristics in the laboratory must be understood in detail in order to permit an intelligent application of laboratory results to field conditions.

The shearing resistance of soil is constituted basically of the following components:

1. the *structural resistance* to displacement of the soil because of the interlocking of the particles,
2. the *frictional resistance* to translocation between the individual soil particles at their contact points, and
3. *cohesion* or adhesion between the surface of the soil particles.

The shear strength in cohesionless soil results from intergranular friction alone, while in all other soils it results both from internal friction as well as cohesion. However, plastic undrained clay does not possess internal friction.

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18.2 THEORETICAL CONSIDERATIONS : MOHR'S STRESS CIRCLE

Through a point in a loaded soil mass, innumerable planes pass and stress components on each plane depends upon the direction of the plane. It can be shown that there exists three typical planes, mutually orthogonal to each other, on which the stress is wholly normal and no shear stress acts. These planes are called the *principal planes* and the normal stresses acting on these planes are called the *principal stresses*. In the order of decreasing magnitude of the normal stress, these planes are called *major, intermediate* and *minor* principal planes and the corresponding normal stresses on them are called *major principal stress* σ_1, *intermediate principal stress* σ_2 and *minor principal stress* σ_3. Many problems in soil engineering can be approximated by considering two dimensional stress conditions.

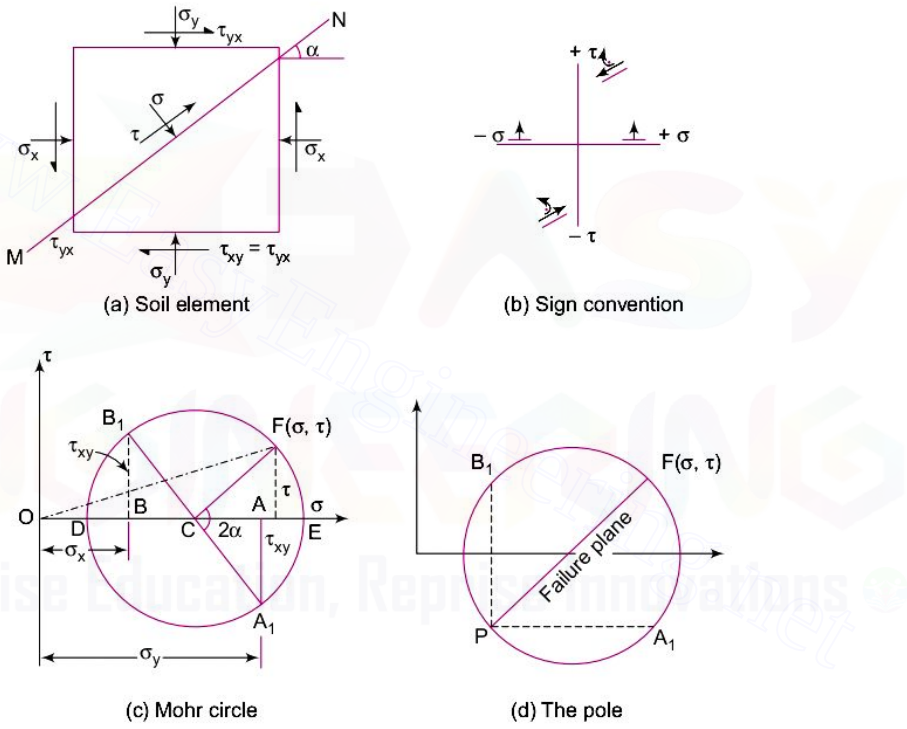

Fig. 18.1 Mohr's stress circle

Figure 18.1 (*a*) shows a soil element subjected to two-dimensional stress system. From the consideration of the equilibrium of the element, one gets the following expressions for the normal stress σ and shearing stress τ on any plane *MN* inclined at an angle α with the *x* direction:

$$\sigma = \frac{\sigma_y + \sigma_x}{2} + \left(\frac{\sigma_y - \sigma_x}{2}\right)\cos 2\alpha + \tau_{xy} \sin 2\alpha \qquad ...(18.1)$$

and

$$\tau = \frac{\sigma_y + \sigma_x}{2} \sin 2\alpha - \tau_{xy} \cos 2\alpha \qquad ...(18.2)$$

where σ_y and σ_x = normal stresses on planes perpendicular to *y* and *x*-axes, respectively ($\sigma_y > \sigma_x$)

τ_{xy} ($= \tau_{yx}$) = shear stresses on these two planes

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Squaring equations 18.1 and 18.2 and adding, we get the following results:

$$\left(\sigma - \frac{\sigma_y + \sigma_x}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2 \qquad ...(18.3)$$

Eq. 18.3 is the equation of a circle whose centre has co-ordinates $\left(\frac{\sigma_y + \sigma_x}{2}, 0\right)$, and whose radius is equal to $\sqrt{\left\{\frac{1}{2}(\sigma_y - \sigma_x)\right\}^2 + \tau_{xy}^2}$

The co-ordinates of points on the circle represent the normal and shearing stresses on inclined planes at a given point. This circle is known as *Mohr's circle of stress* (Mohr, 1870).

To draw the Mohr circle, the normal stresses σ_x and σ_y are marked on the abscissa, at points B and A and a circle is drawn with point C, mid-way between A and B, as the centre, with radius equal to $CB_1 = CA_1$ where BB_1 and AA_1 are the perpendiculars drawn at B and A of magnitude equal to τ_{xy}. The sign conventions are shown in Fig. 18.1(b). Figure 18.1 (c) shows the Mohr circle so drawn. The co-ordinates of any point $F(\sigma, \tau)$ represent the stress conditions on plane which makes an angle α with the x direction.

If from a point B_1 [Fig. 18.1 (d)] on a circle representing the state of stress on vertical plane, a line is drawn parallel to this plane (*i.e.*, vertical), it intersects the circle at a point P. Also, if from the point A_1 on the circle representing the stresses on the horizontal plane, a line is drawn parallel to this latter plane (*i.e.*, horizontal) it will also intersect the circle in the same point P. In general, if through a point F representing the stresses on a given plane, a line is drawn parallel to that plane, it will also intersect the circle in the point P. The point P is therefore, a unique point called the *origin of planes* or the *pole*.

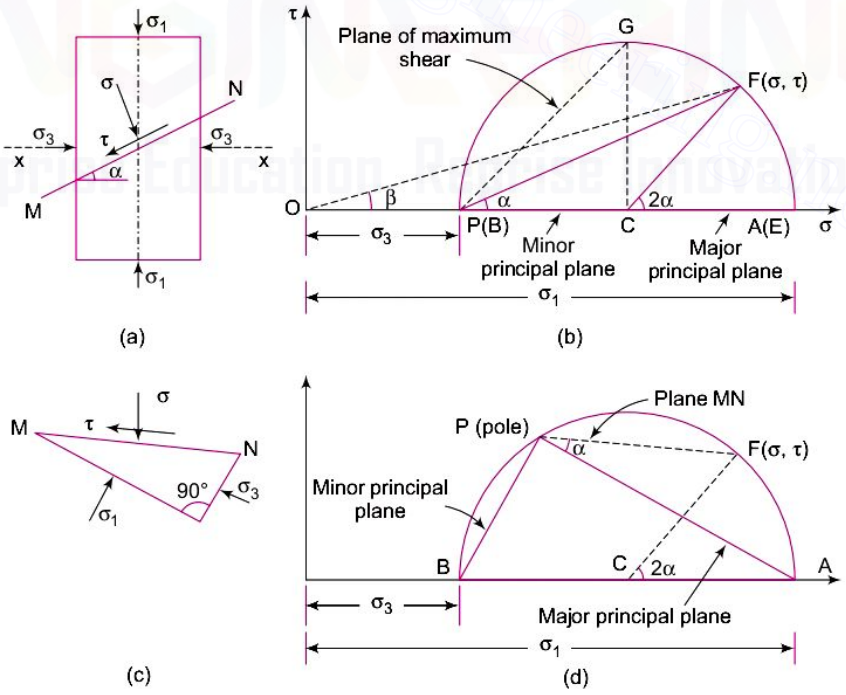

Fig. 18.2 Mohr circle of stresses

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Let us now take the case of soil element whose sides are the principal planes, *i.e.*, consider the state of stress where only normal stresses are acting on the faces of the element. Figure 18.2 (*a*) shows the element, and Fig. 18.2 (*b*) shows the Mohr circle.

In Fig. 18.2 (*a*) the major principal plane is horizontal. Hence the pole *P* is located by drawing a horizontal line through point *A* [Fig. 18.2 (*b*)] representing the major principal stress σ_1. This intersects the circle at *B*. If a line *PF* is drawn through *P* at an angle α with the horizontal, it will intersect the circle at *F* which represents the stress conditions on a plane inclined at an angle α with the direction of the major principal plane.

Figure 18.2 (*c*) shows an element in which the principal planes are not horizontal and vertical, but are inclined to *y* and *x*-directions. Figure 18.2 (*d*) shows the corresponding stress circle. Point *A* represents the major principal stress (σ_1, 0) and *B* represents the minor principal stress (σ_3, 0). Hence to get the position of the pole, a line is drawn through *A*, parallel to the major principal plane, to intersect the circle in *P*. Evidently, *PB* gives the direction of minor principal plane. To find the stress components on any plane *MN* inclined at an angle α with the major principal plane, a line is drawn through *P*, at an angle α with *PA*, to intersect the circle at *F*. The co-ordinates (σ, τ) of point *F* give the stress components on the plane *MN*. Analytical expression for σ, τ are:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha \qquad ...(18.4)$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha \qquad ...(18.5)$$

The resultant stress on any plane is $\sqrt{\sigma^2 + \tau^2}$ and its angle of obliquity β is equal to $\tan^{-1}\left(\frac{\tau}{\sigma}\right)$

The maximum shear stress (point *G*) τ_{max} is equal to $\frac{\sigma_1 - \sigma_3}{2}$ and it occurs on planes with $\alpha = 45°$. In Fig. 18.2 (*b*), *PG* shows the direction of plane having maximum shear stress. The normal stress on this plane will be equal to $\frac{\sigma_1 + \sigma_3}{2}$.

18.3 MOHR-COULOMB FAILURE THEORY

Of the many theories of failure that have been proposed, only that formulated by Mohr (1900) has been useful in case of soils. The following are essential points of Mohr's strength theory:

1. Material fails essentially by shear. The critical shear stress causing failure depends upon the properties of the material as well as on normal stress on the failure plane.
2. The ultimate strength of the material is determined by the stresses on the potential failure plane (or plane of shear).
3. When the material is subjected to three-dimensional principal stress (*i.e.*, σ_1, σ_2, σ_3) the intermediate principal stress does not have, any influence on the strength of material. In other words, the failure criterion is independent of the intermediate principal stress.

Note: For detailed discussions on various theories of failure, see Chapter 19, where the effect of the intermediate principal stress has also been discussed.

The theory was first expressed by Coulomb (1776) and later generalised by Mohr. The theory can be expressed algebraically by the equation.

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$$\tau_f = s = F(\sigma) \qquad ...(18.6)$$

where $\tau_f = s$ = shear stress on failure plane, at failure = shear resistance of material

$F(\sigma)$ = function of normal stress

If the normal and shear stress corresponding to failure are plotted, then a curve is obtained. The plot or the curve is called the *strength envelope.* Coulomb defined the function $F(\sigma)$ as a linear function of σ and gave the following strength equation:

$$s = c + \sigma \tan \phi \qquad ...(18.7)$$

where the empirical constants c and ϕ represent respectively, the intercepts on the shear axis, and the slope of the straight line of Equation 18.7 [Fig 18.3 (*a*)]. These parameters are usually termed as *cohesion* and *angle of internal friction* or *shearing resistance* respectively.

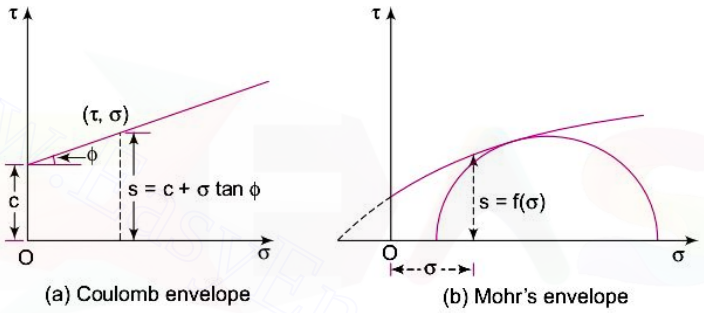

Fig. 18.3 Failure envelopes

Figure 18.3 (*b*) shows the Mohr's envelope, which is the graphical representation of Eq. 18.6. Coulomb considered that the relationship between shear strength and normal stress could be adequately represented by the straight line. The generalised Mohr theory also recognises that the shear strength depends on the normal stress, but indicates that the relation is not linear. The strength theory upon which the Coulomb and Mohr strength lines are based indicates that definite relationship exists among the principal stresses, the angle of internal friction and the inclination of the failure plane. The curved failure envelope of Mohr is often referred to as a straight line for most of the calculations regarding the stability of soil mass. For an ideal pure friction material, such a straight line passes through the origin [Fig. 18.4 (*a*)]. However, dense sands exhibit a slightly curved strength line, indicated by dashed line. Figure 18.4 (*b*) represents purely cohesive (plastic) material, for which the straight line is parallel to the σ-axis. The strength of such a material is independent of the normal stress acting on the plane of failure. The way in which a straight line is fitted to a Mohr envelope will depend on the range of α which is of interest.

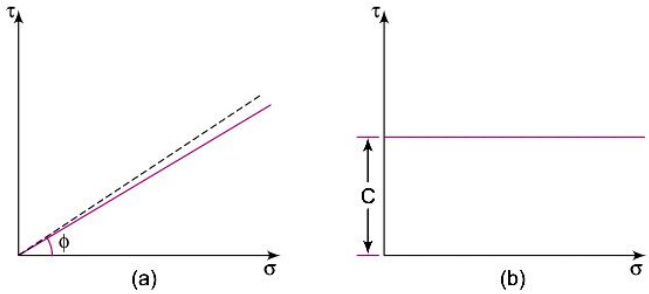

Fig. 18.4 Straight failure envelope

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It can, therefore; be concluded that the Mohr envelope can be considered to be straight if the angle of internal friction ϕ is assumed to be constant. Depending upon the properties of a material the failure envelope may be straight or curved, and it may pass through the origin of stress or it may intersect the shear stress axis.

18.4 THE EFFECTIVE STRESS PRINCIPLE

In Eq. 18.7, it is assumed that the *total* normal stress governs the shear strength of soil. This assumption is not always correct. Extensive tests on remoulded clays have sustained beyond doubt Terzaghi's early concept that the effective normal stresses control the shearing resistance of soils. Therefore, a failure criterion of greater general applicability is obtained by expressing the shear strength as a function of the effective normal stress σ', given by the equation:

$$\tau_f = c' + \sigma' \tan \phi' \qquad ...(18.8)$$

or

$$\tau_f = c' + (\sigma - u) \tan \phi' \qquad ...(18.9)$$

where c' = effective cohesion intercept; and ϕ' = effective angle of shearing resistance

In terms of total stresses, the equation takes the form:

$$\tau_f = c_u + \sigma \tan \phi_u \qquad ...(18.10)$$

where c_u = apparent cohesion ; ϕ_u = apparent angle of shearing resistance.

The normal stress σ' and shear stress τ on any plane inclined at an angle α to the major principal plane can be expressed in terms of effective major principal stress σ_1' and effective minor principal stress σ_3' from Eqs. 18.4. and 18.5 as under:

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \qquad ...(18.11)$$

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha \qquad ...(18.12)$$

Substituting the values of σ' in Eq. 18.8, we get

$$\tau_f = c' + \tan \phi' \left[\frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \right] \qquad ...(18.13)$$

The most dangerous plane *i.e.*, the plane on which failure will take place is the one on which the difference $(\tau_f - \tau)$, between the shear strength and shear stress is minimum.

$$(\tau_f - \tau) = c' + \frac{\sigma_1' + \sigma_3'}{2} \tan \phi' + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\alpha \tan \phi' - \frac{\sigma_1' - \sigma_3'}{2} \sin 2\alpha$$

Differentiating this with respect to α, we get

$$\frac{d}{d\alpha}(\tau_f - \tau) = -(\sigma_1' - \sigma_3') \sin 2\alpha \tan \phi' - (\sigma_1' - \sigma_3') \cos 2\alpha$$

For a minimum $(\tau_f - \tau)$, $\frac{d}{d\alpha}(\tau_f - \tau) = 0$

This gives $\cos 2\alpha = -\sin 2\alpha \tan \phi'$ or $\cot 2\alpha = -\tan \phi' = \cot (90° + \phi')$

$\therefore$

$$\alpha = \alpha_f = 45° + \frac{\phi'}{2} \qquad ...(18.14)$$

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The above expression for the location of the failure plane can be directly derived from the Mohr circle (Fig. 18.5). *J F* represents the *failure envelope* given by the straight line $\tau_f = c' + \sigma' \tan \phi'$. The pole *P* will be the point with stress co-ordinates as (σ'_3, 0). The Mohr circle is tangential to the Mohr envelope at the point *F*. *PF* represents the direction of the failure plane, inclined at an angle α_f with the direction of the major principal plane. From the geometry of Fig. 18.5, we get from triangle *JFK*

$$2\alpha_f = 90° + \phi' \text{ or } \alpha_f = 45° + \frac{\phi'}{2}$$

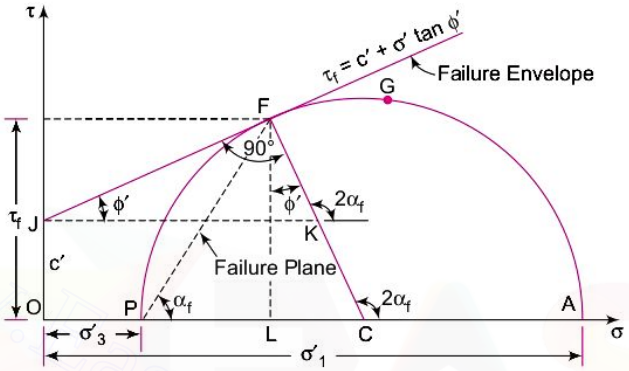

Fig. 18.5

It should be noted that for any combination of the applied principle effective stress σ'_1 and σ'_3, failure will occur only if the stress circle touches the failure envelope. Also, the co-ordinates of the failure point *F* represent the stress components σ' and τ_f at failure. As it is evident from Fig. 18.5, the τ_f at failure is less than the maximum shear stress, corresponding to the point *G*, acting on the plane *PG*. Thus, the failure plane does not carry maximum shear stress, and the plane which has the maximum shear stress is *not* the failure plane.

18.5 MEASUREMENT OF SHEAR STRENGTH

The measurement of shear strength of soil involves certain test obsevations at failure with the help of which the *failure envelope* or *strength envelope* can be plotted corresponding to a given set of conditions (specially the drainage conditions). Shearing resistance can be determined in the laboratory by the following four methods:

1. Direct shear test
2. Triaxial shear test
3. Unconfined compression test
4. Vane shear test

Again depending upon the *drainage conditions,* three types of shear tests have been developed:

(*a*) Undrained test or quick test
(*b*) Consolidated undrained test and
(*c*) Drained test

In the *undrained* or *quick test*, no drainage of water is permitted. Hence there is no dissipation of pore pressure during the entire test. In the *drained test,* drainage is permitted throughout the test during the application of both normal and shear stresses, so that full consolidation occurs and no excess pore pressure is set up at any stage of the test. In the consolidated-undrained test, drainage is permitted under the initally applied normal stress only and full primary consolidation or softening is allowed to take place. No drainage is allowed afterwards. The parameters *c* and ϕ are not fundamental properties of the soil; they may simply be considered merely coefficients derived from the geometry of the graph obtained by the plotting shear stress at failure against normal stress. They vary with drainage conditions of the test.

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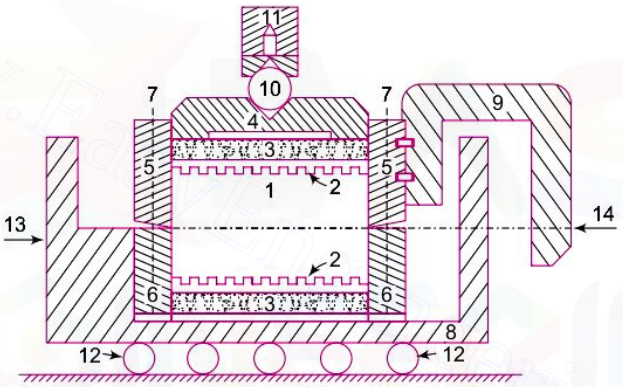
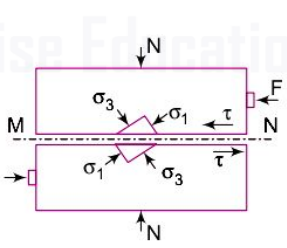
18.6 DIRECT SHEAR TEST

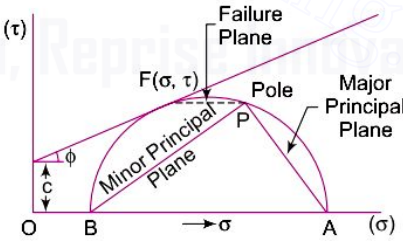
This is a simple and commonly used test and is performed in a shear-box apparatus (Fig. 18.6). The apparatus consists of a two piece shear box of square or circular cross-section. The lower half of the box is rigidly held in position in a container which rests over slides or rollers and which can be pushed forward at a constant rate by geared jack, driven either by electric motor or by hand. The upper half of the box butts against a proving ring. The soil sample is compacted in the shear box, and is held between metal grids and porous stones (or plates). As shown in Fig. 18.6 (*a*), the upper half of the specimen is held in the upper box and the lower half in the lower box, and the joint between the two parts of the box is at the level of the centre of the specimen. Normal load is applied on the specimen from a loading yoke bearing upon steel ball of pressure pad. When a shearing force is applied to the lower box through the geared jack, the movement of the lower part of the box is transmitted through the specimen to the upper part of the box and hence on the proving ring. The deformation of proving ring indicates the shear force.



(a) Parts of direct shear box



(b) Principle of direct shear box



(c) Mohr's envelope and principal stresses during the test

1. SOIL SPECIMEN
2. METAL GRIDS
3. POROUS STONES
4. LOADING PAD
5. UPPER PART
6. LOWER PART
7. SCREWS TO FIX TWO HALVES OF SHEAR BOX
8. CONTAINER FOR SHEAR BOX
9. U-ARM
10. STEEL BALL
11. LOADING YOKE
12. ROLLERS
13. SHEAR FORCE APPLIED BY JACK
14. SHEAR RESISTANCE MEASURED BY PROVING RING

Fig. 18.6 Shear box test

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The volume change during the consolidation and during the shearing process is measured by mounting a dial gauge at the top of the box. The soil specimen can be compacted in the shear box by clamping both the parts together with the help of two screws. These screws are, however, removed before the shearing force is applied. Metal grids, placed above the top and below the bottom of the specimen may be perforated if drained test is required, or plain if undrained test is required. The metal grids have linear slots or serrations to have proper grip with the soil specimen, and are so oriented that the serrations are perpendicular to the direction of the shearing force.

The specimen of the shear box is sheared under a normal load N. The shearing strain is made to increase at a constant rate, and hence the test is called the *strain controlled* shear box test. The other type of test is the *stress controlled* shear box test, in which there is an arrangement to increase the shear stress at a desired rate and measure the shearing strain. Figure 18.6 (a) shows the strain controlled shear box. The shear force, F, at failure, corresponding to the normal load N is measured with the help of the proving ring. A number of identical specimens are tested under increasing normal loads and the required maximum shear force is recorded. A graph is plotted between the shear force F as the ordinate and the normal load N as the abscissa. Such a plot gives the failure envelope for the soil under the given test conditions. Figure 18.6. (c) shows such a failure envelope plotted as a function of the shear stress s and the normal stress σ. The scales of both s and σ are kept equal so that the angle of shearing resistance can be measured directly from the plot. Any point F (σ, τ) on the failure envelope represents the state of stress in the material during failure, under a given normal stress. In the direct shear test, the failure plane MN is predetermined, and is horizontal. Figure 18.6 (b) shows the stress conditions during failure. In order to find the direction of principal planes at failure, we first locate the position of the pole on the Mohr circle [Figure 18.6 (c)] on the principle that the line joining any point on the circle to the pole P gives the direction of the plane on which the stresses are those given by the co-ordinates of that point. Hence, through point F a horizontal line (representing the direction of the failure plane) is drawn to intersect the circle at the point P which is the pole. Since points A and B represent respectively, the major and minor principal stresses, PA and PB give the directions of major and minor principal planes.

Tests can be performed under all the three conditions of drainage. To conduct undrained test, plain grids are used. For the drained test, perforated grids are used. The same is first consolidated under the normal load, and then sheared sufficiently slowly so that complete dissipation of pore pressure takes place. The drained test is therefore also known as the *slow test,* and the shearing of cohesive soil may sometimes require 2 to 5 days. Cohesionless soils are sheared in relatively less time. For the consolidated undrained test, perforated grids are used. The sample is permitted to consolidate under the normal load. After the completion of consolidation, the specimen is sheared quickly in about 5 to 10 minutes.

Comments on the shear box test. The direct shear test is a simple test. The relatively thin thickness of sample permits quick drainage and quick dissipation of pore pressure developed during the test. However, the test has the following disadvantages:

1. The stress conditions across the soil sample are very complex. The distribution of normal stresses and shearing stresses over the potential surface of sliding is not uniform. The stress is more at the edges and less in the centre. Due to this there is *progressive failure* of the specimen *i.e.,* the entire strength of the soil is not mobilised simultaneously.
2. As the test progresses, the area under shear gradually decreases. The corrected area (A_f) at failure should be used in determining the values of σ and, τ.
3. As compared to the triaxial test, there is little control on the drainage of soil.
4. The plane of shear failure is predetermined, which may not be the weakest one.
5. There is effect of lateral restraint by the side walls of the shear box.

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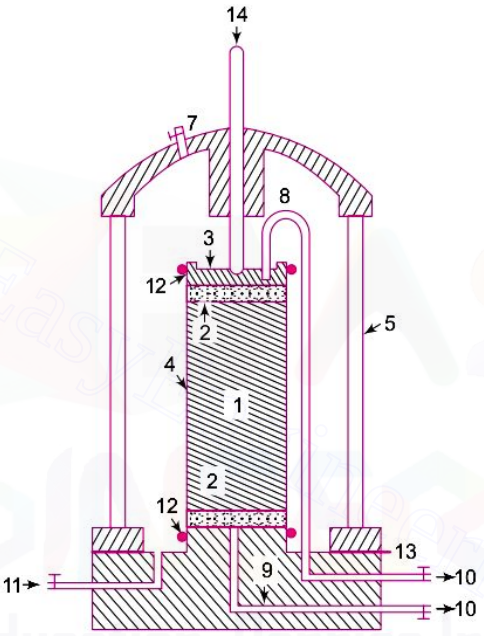
18.7 TRIAXIAL COMPRESSION TEST

The strength test more commonly used in a research laboratory today is the triaxial compression test, first introduced in the U.S.A. by A. Casagrande and Karl Terzaghi in 1936–37. The solid specimen, cylindrical in shape, is subjected to direct stresses acting in three mutually perpendicular directions. In the common solid cylindrical specimen test, the major principal stress σ_1 is applied in the vertical direction, and the other two principal stresses σ_2 and σ_3 ($\sigma_2 = \sigma_3$) are applied in the horizontal direction by the fluid pressure round the specimen.



1. SOIL SPECIMEN
2. POROUS DISC
3. TOP CAP
4. RUBBER MEMBRANE
5. PERSPEX CYLINDER
6. LOADING RAM
7. AIR RELEASE VALVE
8. TOP DRAINAGE TUBE
9. BOTTOM DRAINAGE TUBE
10. CONNECTIONS FOR DRAINAGE OR PORE PRESSURE MEASUREMENT
11. CELL FLUID INLET
12. RUBBER RINGS
13. SEALING RING
14. AXIAL LOAD THROUGH PROVING RING

Fig. 18.7 The triaxial cell

The test equipment specially consists of a high pressure cylindrical cell, made of perspex or other transparent material, fitted between the base and the top cap. Three outlet connections are generally provided through the base: cell fluid inlet, pore water out let from the bottom of the specimen and the drainage outlet from the top of the specimen. A separate compressor is used to apply fluid pressure in the cell. Pore pressure developed in the specimen during the test can be measured with the help of a separate pore pressure measuring equipment, such as Bishop's apparatus shown in Fig. 18.8. The

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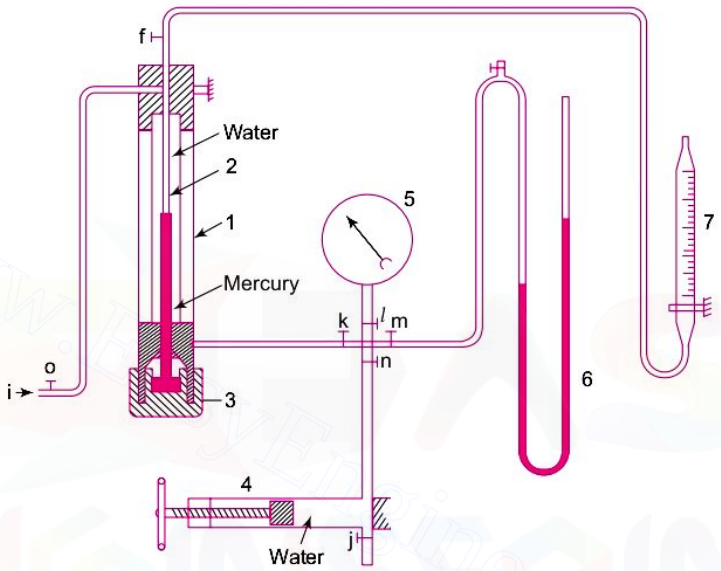
cylindrical specimen is enclosed in a rubber membrane. A stainless steel piston running through the centre of the top cap applies the vertical compressive load (called the *deviator stress)* on the specimen under test. The load is applied through a proving ring, with the help of a mechanically operated load frame. Depending upon the drainage conditions of the test, solid nonporous discs or end caps, or porous discs are placed on the top and bottom of the specimen and the rubber membrane is sealed on to these end caps by rubber rings.



1. NULL INDICATOR
2. GLASS CAPILLARY TUBE
3. MERCURY TROUGH
4. CONTROL CYLINDER
5. PRESSURE GAUGE
6. MERCURY MANOMETER
7. BURETTE
8. CONNECTION FROM PORE FLUID OUTLET OF TRIAXIAL CELL

Fig. 18.8 Layout of Bishop's pore pressure apparatus (Bishop 1950, 1961)

The length of the specimen is kept about 2 to 2.5 times its diameter. The cell pressure σ_3 ($= \sigma_2$) acts all round the specimen; it acts also on the top of the specimen as well as the vertical piston meant for applying the deviator stress. The vertical stress applied by the loading frame, through the proving ring is equal to $(\sigma_1 - \sigma_3)$, so that the total stress on the top of the specimen = $(\sigma_1 - \sigma_3) + \sigma_3 = \sigma_1$ = major principal stress. This principal stress difference $(\sigma_1 - \sigma_3)$ is called the *deviator stress* recorded on the proving ring dial. Another dial measures the vertical deformation of the sample during testing. It is desirable to maintain the cell pressure reservoir and mercury control apparatus, devised by Skempton and Bishop (1950), as shown in Fig. 18.9 (*a*). For long duration test (lasting about a week or more), self-compensating mercury control can be used [Fig. 18.9 (*c*)].

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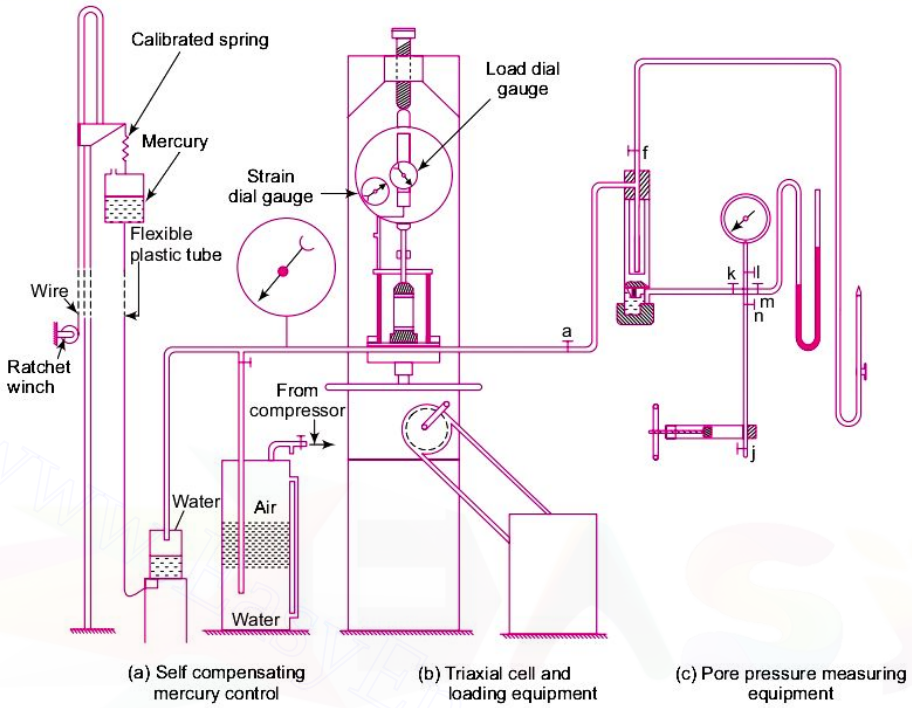

Fig. 18.9 Diagrammatical layout of triaxial cell, water-air reservoir, mercury

A particular confining pressure σ_3 is applied during one observation, giving the value of the other stress σ_1 at failure. A Mohr circle corresponding to this set of (σ_1, σ_3) can thus be plotted. Various sets of observations are taken for different confining pressures σ_3 and the corresponding values of σ_1 are obtained. Thus, a number of Mohr circles, corresponding to failure conditions, are obtained. A curve, tangential to these stress circles, gives the failure envelope for the soil under the given drainage conditions of the test.

Shear tests can be performed in the triaxial apparatus under all the three drainage conditions. For undrained test, solid (nonporous) end caps are placed on the top and bottom of the specimen. In the consolidated-undrained test, porous discs are used. The specimen is allowed to consolidate under the desired confining pressure by keeping the pore water outlet open. When the consolidation is complete, the pore water outlet is closed, and the specimen is sheared under undrained conditions. The pore water pressure can be measured during the undrained part of the test. In the drained test, porous discs are used, and the pore water outlet is kept open throughout the test. The compression test is carried out sufficiently slowly to allow for the full drainage during the test.

Measurement of pore pressure during the test. Figure 18.8 shows the layout of Bishop's pore pressure apparatus. It mainly consists of (*i*) the null indicator, (*ii*) the control cylinder, (*iii*) pressure gauge, (*iv*) mercury manometer, and (*v*) burette. The null indicator consisting of a single straight section of glass capillary tube dipping into an enclosed trough of mercury, is connected to the trixial cell through valve *a* by a copper tube, and to the control cylinder, etc. through valve *k*. An increase in pore pressure in the sample during the test will tend to depress the mercury in the limb of the null indicator. This can be immediately balanced by adjusting the piston in the control cylinder to increase the pressure in the limb by an equal amount which is registered in the pressure gauge. Valves *m*, *f* and *j* are kept closed during the

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pore pressure measurements. In addition to the pressure gauge, a mercury manometer is also provided. This is used (*i*) for negative pore pressure, (*ii*) for accurate measurement of low positive pore pressure, and (*iii*) for checking the zero error of the pressure gauge. When this manometer is connected through valves *k* and *m*, valves *l* and *n* are kept closed. The graduated tube or burette connected to the valve *f* is used for determining the gauge and manometer readings corresponding to zero pore pressure. In the case of fully saturated samples, this graduated tube can also be used to measure volume change during the consolidation stage of test in which drainage is permitted through the base of specimen (Bishop and Henkel 1957).

Stress conditions in soil specimen during triaxial testing. Figure18.10 (*a*) shows the effective stresses acting on the soil specimen during triaxial testing. The minor principal stress and the intermediate principal stress are equal. The effective minor principal stress is equal to the cell pressure minus the pore pressure. The major principal stress is equal to the deviator stress plus the cell pressure. The effective major principal stress σ_1' is equal to the major principal stress minus the pore pressure. The stress components on the failure plane *MN* are σ' and τ_f, and the failure plane is inclined at an angle α' to the major principal plane. Figure 18.10 (*b*) shows the failure envelope *JF* and a Mohr circle corresponding to any failure point *F*. Since $\angle JFC = 90°$ and the failure envelope cuts the abscissa at an angle ϕ', the angle α' of the failure plane is given by:

$$\alpha' = \frac{1}{2}\angle FCA = \frac{1}{2}(90° + \phi') = 45° + \phi'/2 \qquad ...(18.14\ a)$$

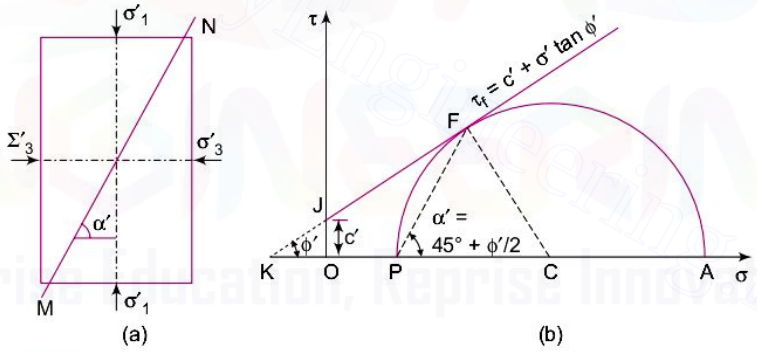

Fig. 18.10 Stress conditions and failure envelope in triaxial compression tests

The principal stress relationship at failure can be found with the help of Fig. 18.10(*b*).

$$FC = \text{radius of Mohr circle} = \frac{1}{2}(\sigma_1' - \sigma_3');\ OC = \frac{1}{2}(\sigma_1' + \sigma_3');\ OK = c' \cot \phi'$$

Hence,
$$\sin \phi' = \frac{FC}{KC} = \frac{FC}{KO + OC} = \frac{\frac{1}{2}(\sigma_1' - \sigma_3')}{c' \cot \phi' + \frac{1}{2}(\sigma_1' + \sigma_3')} = \frac{(\sigma_1' - \sigma_3')}{2c' \cot \phi' + (\sigma_1' + \sigma_3')}$$

$$\therefore \quad \sigma_1' - \sigma_3' = 2\ c' \cos \phi' + (\sigma_1' + \sigma_3') \sin \phi' \qquad ...(18.15\ a)$$

or
$$\sigma_1' (1 - \sin \phi') = \sigma_3' (1 + \sin \phi') + 2c' \cos \phi'$$

$$\therefore \quad \sigma_1' = \sigma_3' \frac{1 + \sin \phi'}{1 - \sin \phi'} + 2c' \frac{\cos \phi'}{1 - \sin \phi'} \qquad ...(18.15)$$

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or $$\sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi'}{2}\right) + 2c' \tan\left(45° + \frac{\phi'}{2}\right) \quad ...(18.16)$$

or $$\sigma_1' = \sigma_3' \tan^2 \alpha' + 2c' \tan \alpha' \quad ...(18.17)$$

or $$\sigma_1' = \sigma_3' N_\phi' + 2c' \sqrt{N_\phi'} \quad ...(18.17a)$$

where $$N_\phi' = \tan^2 \alpha' = \tan^2 (45° + \phi'/2)$$

Equations 18.16. or 18.17 gives principal stress relationship. When the soil is in the state of stress defined by the equations 18.16 or 18.17, it is said to be in *plastic equilibrium*. In terms of total stresses, 18.17 is written as

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c_u \tan \alpha \quad ...(18.18)$$

or $$\sigma_1 = \sigma_3 N_\phi + 2 c_u \sqrt{N_\phi} \quad ...(18.18a)$$

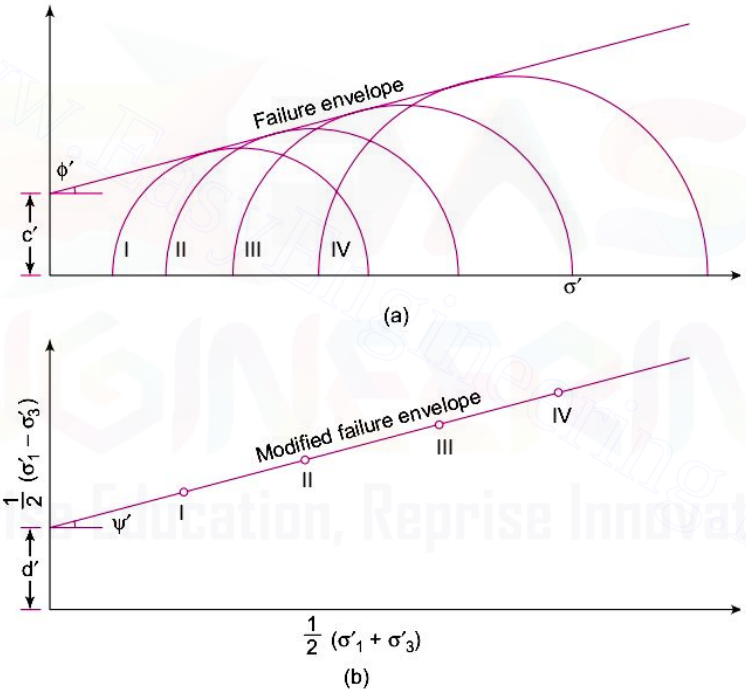

Fig. 18.11 Failure envelopes

where $$\alpha = 45° + \frac{\phi_u}{2} \text{ and } N_\phi = \tan^2 \alpha = \tan^2\left(45° + \frac{\phi_u}{2}\right)$$

In equation 18.16, σ_1' and σ_3' are known, and the two unknowns are ϕ' and c'. Hence two sets of observations are required to determine these two unknown parameters. In practice, a number of sets of (σ_1', σ_3') at failure are observed, and Mohr circles are plotted for each set. A curve drawn tangential to these circles gives the failure envelope [Fig. 18.11(*a*)].

Another method of plotting the test results is in the form of the *modified failure envelope* which is a function of $\frac{1}{2}(\sigma_1' + \sigma_3')$ and $\frac{1}{2}(\sigma_1' - \sigma_3')$. Rewriting equation 18.15 (*a*) in the form

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$$\frac{1}{2}(\sigma_1' - \sigma_3') = d' + \frac{1}{2}(\sigma_1' + \sigma_3') \tan \psi' \qquad ...(18.19)$$

and comparing it with equation 18.15 *a*, we observe that

$$\sin \phi' = \tan \psi' \qquad ...(18.20\ a)$$

and

$$c' = \frac{d'}{\cos \phi'} \qquad ...(18.20\ b)$$

Equation 18.19 representing the principal stress relationship, is the equation of a straight line having its y-coordinate represented by $\frac{1}{2}(\sigma_1' - \sigma_3')$ and x-coordinate represented by $\frac{1}{2}(\sigma_1' + \sigma_3')$. Figure 18.11(*b*) shows the modified failure envelope, represented by equation 18.19, in which the slope ψ' and the intercept d' are related to ϕ' and c' through equation 18.20. The line so obtained is often called the k_f line (Lambe, 1969). The advantage of this method of plotting the failure envelope is that the averaging of scattered test results is facilitated to a great extent, giving the mean value of the parameters.

The calculation of the deviator stress must be done on the basis of the changed area of cross-section at failure, or during any stage of the test. The area A_2 at failure, or during any stage of the test can be found by the relation.

$$A_2 = \frac{V_1 \pm \Delta V}{L_1 - \Delta L} \qquad ...(18.21)$$

where, V_1 = initial volume of the specimen ; L_1 = initial length of the specimen

ΔV = change in the volume of the specimen

ΔL = change in the length of the specimen

The deviator stress σ_d is given by $\sigma_d = \dfrac{\text{Additional axial load}}{A_2}$; σ_3 = fluid pressure

$$\therefore \qquad \sigma_1 = \sigma_3 + \sigma_d$$

Knowing σ_1, σ_3 and pore pressure, σ_1' and σ_3' can be determined.

Advantages of triaxial test:

1. The shear tests under all the three drainage conditions can be performed with complete control.
2. Precise measurements of the pore pressure and volume change during the test are possible.
3. The stress distribution on the failure plane is uniform.
4. The state of stress within the specimen during any stage of the test, as well as at failure is completely determinate.

18.8 UNCONFINED COMPRESSION TEST

The unconfined compression test is a special case of triaxial compression test in which $\sigma_2 = \sigma_3 = 0$. The cell pressure in the triaxial cell is also called the confining pressure. Due to the absence of such a confining pressure, the uniaxial test is called the unconfined compression test. The cylindrical specimen of soil is subjected to major principal stress σ_1 till the specimen fails due to shearing along a critical plane of failure.

In its simplest form, the apparatus consists of a small load frame fitted with a proving ring to measure the vertical stress applied to the soil specimen. Figure 18.12 (*a*) shows an unconfined compression tester

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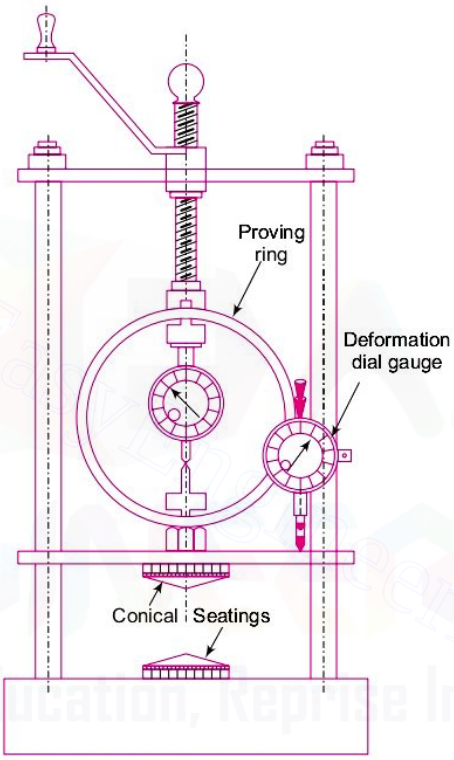
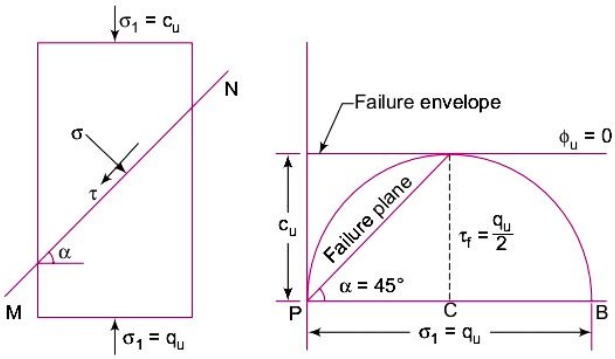
(Goyal and Singh, 1958). The deformation of the sample is measured with the help of a separate dial gauge. The ends of the cylindrical specimen are hollowed in the form of cone. The cone seatings reduce the tendency of the specimen to become barrel shaped by reducing end-restraints. During the test, load versus deformation readings are taken and a graph is plotted. When a brittle failure occurs, the proving ring dial indicates a definite maximum load which drops rapidly with the further increase of strain. In the plastic failure, no definite maximum load is indicated. In such a case, the load corresponding to 20% strain is arbitrarily taken as the failure load.



(a) The unconfined compression tester



(b) (c)

Fig. 18.12 Unconfined compression test

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Figure 18.12 (*b*), (*c*) shows the stress conditions, at failure, in the unconfined compression test which is essentially an undrained test (if it is assumed that no moisture is lost from the specimen during the test). Since, $\sigma_3 = 0$, the Mohr circle passes through the origin which is also the *pole*.

From Eq. 18.18, we get $$\sigma_1 = 2\, c_u \tan \alpha = 2c_u \tan\left(45° + \frac{\phi_u}{2}\right) \quad ...(18.22)$$

In the above equation, there are two unknowns c_u and ϕ_u which cannot be determined by the unconfined test since a number of tests on the identical specimens give the same value of σ_1. Therefore, the unconfined compression test is generally applicable to saturated clays for which the apparent angle of shearing resistance ϕ_u is zero. Hence

$$\sigma_1 = 2\, c_u \quad ...(18.23)$$

When the Mohr circle is drawn, its radius is equal to $\sigma_1/2 = c_u$. The failure envelope is horizontal. *PF* is the failure plane, and the stresses on the failure plane are

$$\sigma = \frac{\sigma_1}{2} = \frac{q_u}{2} \quad ...(18.24)$$

and $$\tau_f = \frac{\sigma_1}{2} = \frac{q_u}{2} = c_u \quad ...(18.25)$$

where, q_u = unconfined compressive strength at failure. The compressive stress is calculated on the basis of changed cross-sectional area A_2 at failure, which is given by

$$A_2 = \frac{V}{L_1 - \Delta L} = \frac{A_1}{1 - \frac{\Delta L}{L_1}} \quad ...(18.26)$$

where, V = initial volume of the specimen; L_1 = initial length of the specimen

ΔL = change in length at failure.

SOLVED EXAMPLES

Example 18.1. *Table 18.1 gives observations for normal load and maximum shear force for the specimens of sandy clay tested in the shear box, 36 cm^2 in area under undrained conditions. Plot the failure envelope for the soil and determine the values of apparent angles of shearing resistance and the apparent cohesion.*

Table 18.1

Normal load (N)	*Maximum shear force (N)*
100	110
200	152
300	193
400	235

Solution. Figure 18.13 shows the plot between the shear force F and the normal load N. From the plot, we get $\phi_u = 22°$, and total cohesive force = 70 N.

$\therefore$ Unit apparent cohesion $c_u = \frac{70}{36} = 1.95$ N/cm^2 = 19.5 kN/cm^2 = **19.5 kPa.**

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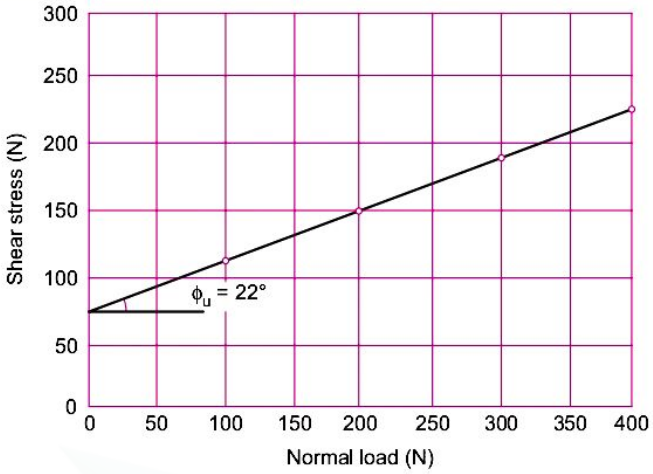

Fig. 18.13

Example 18.2. *Samples of compacted, clean dry sand were tested in a shear box, 6 cm × 6 cm and the following results were obtained:*

Normal load (N)	*: 100*	*200*	*300*	*400*
Peak shear load (N)	*: 90*	*181*	*270*	*362*
Ultimate shear load (N)	*: 55*	*152*	*277*	*300*

Determine the angle of shearing resistance of the sand in (a) the dense, and (b) the loose state.

Solution. The value of the shearing resistance of sand, obtained from the peak stress represents the value of ϕ in its initial compacted state, while that obtained from the ultimate shear corresponds to the sand when loosened by the shearing action.

Figure 18.14 shows the two plots. The values of angles of shearing resistance are found to be

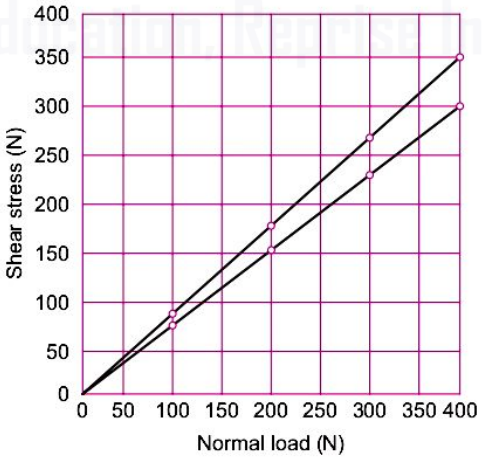

Fig. 18.14

(*a*) dense state : $\phi = 42°$
(*b*) loose state : $\phi = 37°$

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Example 18.3. *A specimen of clean, dry, cohesionless sand is tested in shear box and the soil failed at a shear stress of 40 kN/m^2 when the normal load on the specimen was 50 kN/m^2. Determine (a) the angle of shearing resistance, (b) the principal stress during the failure, (c) the directions of the principal planes with respect to the direction of the plane of shearing.*

Solution. The failure envelope will pass through (*i*) origin, and (*ii*) a point *F* whose co-ordinates are: shear stress = 40 kN/m^2 (40 kPa) and normal stress = 50 kN/m^2 (50 kPa). The angle of shearing resistance (Fig. 18.15) is found to be 38.7°

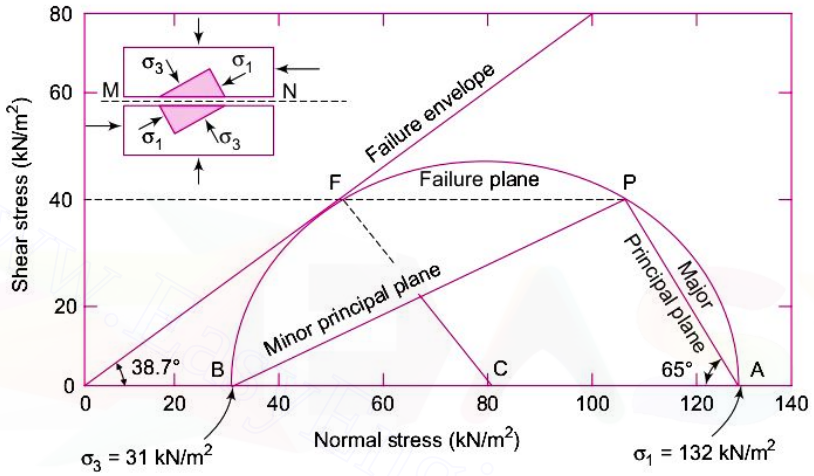

Fig. 18.15

The Mohr's stress circle is so drawn that it is tangential to the envelope at point *F*. To do this, a line *FC* is drawn perpendicular to the envelope. With *C* as the centre and *CF* as the radius, a circle is drawn, which intersects the normal load axis at points *A* and *B*. Point *A* corresponds to the major principal stress σ_1 which comes out to be 132 kN/m^2 and point *B* corresponds to the minor principal stress σ_3 which comes out to be 31 kN/m^2 (31 kPa).

To locate the position of the pole, a line *FP* is drawn parallel to the plane of failure in the shear box (*i.e., horizontal*), *P* is the pole. *PA* is the direction of the major principal plane which makes an angle of 65° in the clockwise direction with the plane of shear, while *PB* is the minor principal plane, making an angle of 25° in the anti-clockwise direction with the plane of shearing.

Example 18.4. *A cylindrical specimen of saturated clay, 4 cm in diameter and 9 cm in overall length is tested in an unconfined compression tester. The specimen has coned ends and its length between the apices of cones is 8 cm. Find the unconfined compressive strength of clay, if the specimen fails under an axial load of 46.5 N. The change in the length of specimen at failure is 1 cm.*

Solution. Original length of specimen = 9 cm overall, and 8 cm to apices of cones.

Length of cylinder of the same volume and diameter (average length) $L_1 = 8.66$ cm.

Initial cross-sectional area $A_1 = \dfrac{\pi \times 4^2}{4} = 12.57$ cm^2

Change in length at failure, $\Delta L = 1$ cm

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$\therefore$ Area of failure, $A_2 = \dfrac{A_1}{1-\dfrac{\Delta L}{L_1}} = \dfrac{12.57}{1-\dfrac{1}{8.66}} = 14.2 \text{ cm}^2$

$\therefore$ Unconfined compressive strength, $q_u = \dfrac{\text{failure load}}{A_2} = \dfrac{465}{14.2} = 32.8 \text{ kN/cm}^2$

$= 328 \text{ kN/m}^2 = 328 \text{ kPa}$

$\therefore$ Shear strength, $c_u = \dfrac{q_u}{2} = \dfrac{328}{2} = 164 \text{ kN/m}^2$ **=164 kPa.**

Example 18.5. *A cylinder of soil fails under an axial vertical stress of 160 kN/m^2, when it is laterally unconfined. The failure plane makes an angle of 50° with the horizontal. Calculate the value of cohesion and the angle of internal friction of the soil.*

Solution. $\alpha = 50° = 45° + \dfrac{\phi_u}{2}$; $\therefore \quad \phi_u = 2\,(50° - 45°) = 10°$

$$\tan \alpha = \tan\left(45° + \frac{\phi_u}{2}\right) = \tan 50° = 1.192$$

As the sample is unconfined, $\sigma_3 = 0$

Now, $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$

$\therefore$ $160 = 2c \tan 50° = 2c \times 1.192$

$\therefore$ $c = \dfrac{160}{2 \times 1.192} = 67.1 \text{ kN/m}^2 = \mathbf{67.1 \text{ kPa}}$.

Example 18.6. *Two identical specimens, 4 cm in diameter and 8 cm high, of partly saturated compacted soil are tested in a triaxial cell under undrained conditions. The first specimen failed at an additional axial load (i.e., deviator load) of 720 N under a cell pressure of 100 kN/m^2. The second specimen failed at an additional axial load of 915 N under a cell pressure of 200 kN/m^2. The increase in volume of the first specimen at failure is 1.2 ml and it shortens by 0.6 cm, at failure. The increase in volume of the second specimen at failure is 1.6 ml, and it shortens by 0.8 cm at failure. Determine the value if apparent cohesion and the angle of shearing resistance (a) analytically, (b) graphically by Mohr's circle.*

Solution.

(*a*) ***For the first specimen:***

Initial area, $A_1 = \dfrac{\pi \times 4^2}{4} = 12.57 \text{ cm}^2$

Initial volume, $V_1 = 12.57 \times 8 = 100.56 \text{ cm}^3$

$\Delta L = 0.6 \text{ cm}, \Delta V = 1.2 \text{ cm}^3$

$\therefore$ Area of failure, $A_2 = \dfrac{V_1 + \Delta V}{L_1 - \Delta L} = \dfrac{100.56 + 1.2}{8 - 0.6} = 13.75 \text{ cm}^2$

$\therefore$ Deviator stress at failure, $\sigma_d = \dfrac{720}{13.75} = 52.4 \text{ N/cm}^2 = 524 \text{ kN/m}^2$

$\sigma_3 = 100 \text{ kN/m}^2$; $\sigma_1 = \sigma_3 + \sigma_d = 100 + 524 = 624 \text{ kN/m}^2$

Substituting the value of σ_1 and σ_3 in equation 18.18 *a*, we get

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$$624 = 100N_\phi + 2c_u\sqrt{N_\phi} \qquad ...(1)$$

(b) ***For the second specimen:***

$$A_1 = 12.57 \text{ cm}^2 \; ; \; V_1 = 100.56 \text{ cm}^2 \; ; \; \Delta L = 0.8 \; ; \; \Delta V = +1.6 \text{ cm}^3$$

$$\therefore \quad A_2 = \frac{100.56 + 1.6}{8 - 0.8} = 14.2 \text{ cm}^2 \text{ and } \sigma_d = \frac{915}{14.2} = 64.4 \text{ N/cm}^2 = 644 \text{ kN/m}^2$$

$$\sigma_3 = 200 \text{ kN/m}^2 \; ; \quad \therefore \quad \sigma_1 = \sigma_3 + \sigma_d = 200 + 64.4 = 844 \text{ kN/m}^2$$

Substituting the values of σ_1 and σ_3 in equation 18.18 *a*, we get

$$844 = 200N_\phi + 2c_u\sqrt{N_\phi} \qquad ...(2)$$

Solving (1) and (2), we get

$$c_u = 13.6 \text{ kN/m}^2 \text{ (136 kPa)}$$

and $$N_\phi = 2.2$$

Now, $$N = \tan^2\left(45° + \frac{\phi_u}{2}\right) = 2.2$$

$$\therefore \quad \phi_u = 22°.$$

Graphical solution. The Mohr circles at failure for the two specimens are drawn as shown in Fig. 18.16. The failure envelope is drawn tangential to the two circles. Thus, from Fig. 18.16, we get c_u = 132 kN/m^2 (132 kPa) and $\phi_u = 22.5°$.

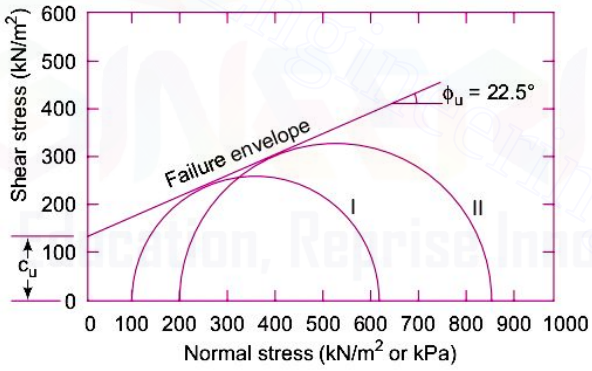

Fig. 18.16

Example 18.7. *A saturated specimen of cohesionless sand was tested in triaxial compression and the sample failed at a deviator stress of 482 kN/m^2 when the cell presure was 100 kN/m^2, under the drained conditions. Find the effective angle of shearing resistance of sand. What would be the deviator stress and the major principal stress at failure for another identical specimen of sand, if it is tested under cell pressure of 200 kN/m^2?*

Solution. In the drained tests, the effective stresses are equal to the total stress.

$$\therefore \quad \sigma_3' = \sigma_3 = 100 \text{ kN/m}^2 \text{ (kPa)}$$

$$\sigma_1' = \sigma_3' + \sigma_d = 100 + 482 = 582 \text{ kN/m}^2 \text{ (kPa)}$$

Figure 18.17 shows the Mohr circle (circle 1). The failure envelope will pass through the origin, since $c' = 0$ for sand, and will be tangential to the circle. The angle of inclination of the failure envelope gives $\phi' = 45°$.

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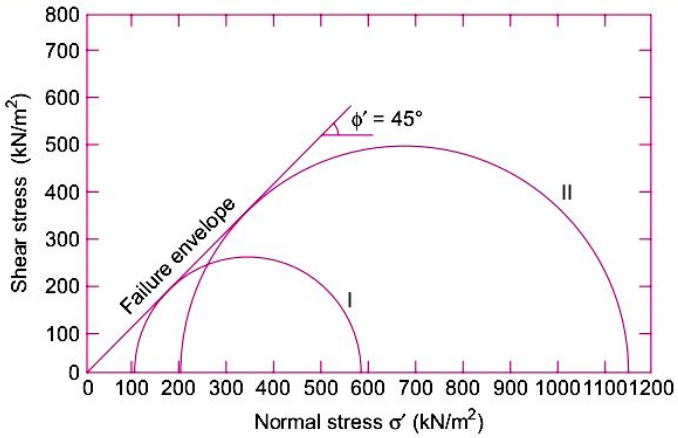

Fig. 18.17

Alternatively, from equation 18.16,

$$\therefore \quad \sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi'}{2}\right)$$

$$\therefore \quad 582 = 100 \times \tan^2\left(45° + \frac{\phi'}{2}\right)$$

$$\therefore 45° + \frac{\phi'}{2} = \tan^{-1}\left(\sqrt{5.82}\right) = 67.5°$$

$$\therefore \quad \phi' = 45°$$

For the second specimen with $\sigma_3' = 200$ kN/m^2, the centre of the Mohr circle is so selected that the circle passes through $\sigma_3' = 200$ kN/m^2 and is tangential to the failure envelope. Circle II corresponds to this, from which $\sigma_d' = 1160$ kN/m^2

$$\therefore \quad \sigma_2 = \sigma_1' - \sigma_3' = 960 \text{ kN/m}^2 \text{ (960 kPa)}$$

Alternatively, σ_1' can be calculated from the relation:

$$\sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi'}{2}\right) = 200 \tan^2\left(45° + \frac{45°}{2}\right) = 1160 \text{ kN/m}^2 \text{ (1160 kPa)}$$

$$\sigma_d = 1160 - 200 = 960 \text{ kN/m}^2 \text{ (960 kPa).}$$

Example 18.8. *Three clay specimens, having a small air-void content were tested in a shear box under undrained conditions and the following observations were made:*

Normal stress (kN/m^2)	*100*	*200*	*300*
Shear stress (kN/m^2) (at failure)	*90*	*102*	*108*

Find the apparent cohesion and angle of shearing resistance of the clay. What value of apparent cohesion would be obtained from an unconfined compression test on the same soil?

Solution. Figure 18.18 shows the failure envelope from which

$$c_u = 80 \text{ kN/m}^2 \text{ (kPa)} \; ; \; \phi_u \approx 6°$$

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To locate the Mohr circle corresponding to unconfined compression test, a line OF at an angle $\alpha = 45° + \frac{\phi_u}{2} = 45° + \frac{6}{3} = 48°$, is drawn to represent the direction of the failure plane. At F, a line FC is drawn perpendicular to the envelope. With C as centre, and CO as the radius, Mohr circle is drawn, which will be tangential to the Mohr envelope; *alternatively,* the centre C can be located by trial so that the circle, passing through O is tangential to the failure envelope. The value of cohesive strength given by the unconfined compressive strength test is represented by the radius of a Mohr circle passing through the origin ($\sigma_3 = 0$) and tangent to the Coulomb line. The diameter of the circle $= q_u =$ 176 kN/m^2. This shows that if a soil shows an apparent angle of shearing resistance in an undrained test, the unconfined compression test will give a high value of apparent cohesion, which is approximately equal to 176/2 = 88 kN/m^2 (kPa).

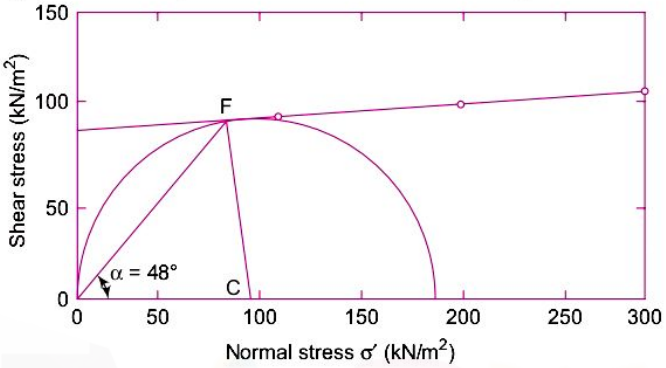

Fig. 18.18

Example 18.9. *Following are the results of undrained triaxial compression test on two identical soil specimens, at failure:*

Lateral pressure σ_3 (kN/m^2)	*100*	*300*
Total vertical pressure σ_1 (kN/m^2)	*440*	*760*
Pore water pressure u (kN/m^2)	*–20*	*60*

Determine the cohesion and angle of shearing resistance (a) referred to total stress, (b) referred to effective stress.

Solution. In Fig 18.19, the circles A and B with dark lines correspond to the total stress, and from failure envelope, drawn tangential to the two circles, we get

$$\phi_u = 14° \text{ and } c_u = 110 \text{ kN/m}^2 \text{ (kPa)}$$

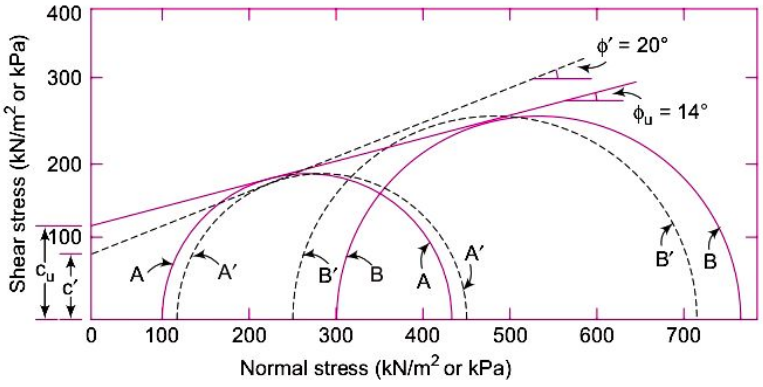

Fig. 18.19

For the effective stress analysis, we have

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σ_1' : 440 + 20 = 460

and 760 – 60 = 700 kN/m^2

σ_3' : 100 + 20 = 120

and 300 – 60 = 240 kN/m^2

The Mohr circles A' and B' corresponding to the effective stresses are shown by dotted lines. From the dotted failure envelope, we get

$$\phi' = 20°, c' = 76 \text{ kN/m}^2 \text{ (kPa)}.$$

Example 18.10. *Undrained triaxial tests are carried out on four identical specimens of silty clay, and the following results are obtained:*

Cell pressure (kN/m^2)	*50*	*100*	*150*	*200*
Deviator stress at failure (kN/m^2)	*350*	*440*	*530*	*610*
Pore pressure (kN/m^2)	*5*	*10*	*12*	*18*

Determine the value of the effective angles of shearing resistance and the cohesion intercept by plotting (a) conventional failure envelope from Mohr circles, (b) modified failure envelope.

Solution. Table 18.2 shows the necessary calculations of plotting the failure envelope:

Table 18.2

Specimen No.	σ_3	σ_3'	σ_d	σ_1'	$\frac{1}{2}(\sigma_1' + \sigma_3)$	$\frac{1}{2}(\sigma_1' - \sigma_3')$
1	50	45	350	395	220	175
2	100	90	440	530	310	220
3	150	138	530	668	403	265
4	200	182	610	792	487	305

Figure 18.20 shows the conventional failure envelope from Mohr's circles, from which we get $\phi' = 29.5°$ and $c' = 8$ kN/m^2 (kPa).

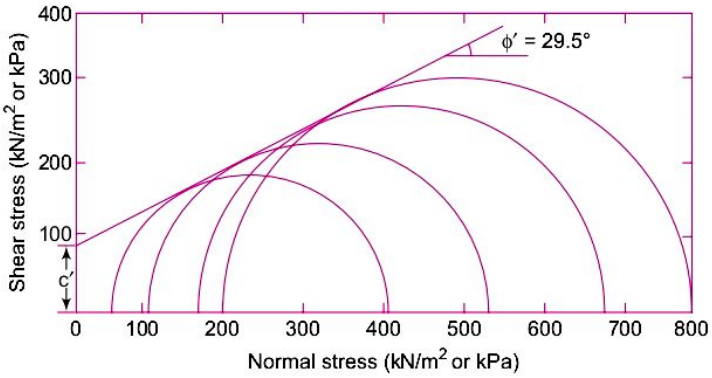

Fig. 18.20

Figure 18.21 shows the modified failure envelope, from which, we get $\psi' = 26.5°$ and $d' = 70°$

$\therefore$ $\sin \phi' = \tan \psi' = \tan 26.5°$ or $\phi' = 30°$

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$$c' = \frac{d'}{\cos \phi'} = \frac{70}{0.86} = 81 \text{ kN/m}^2 \text{ (kPa)}.$$

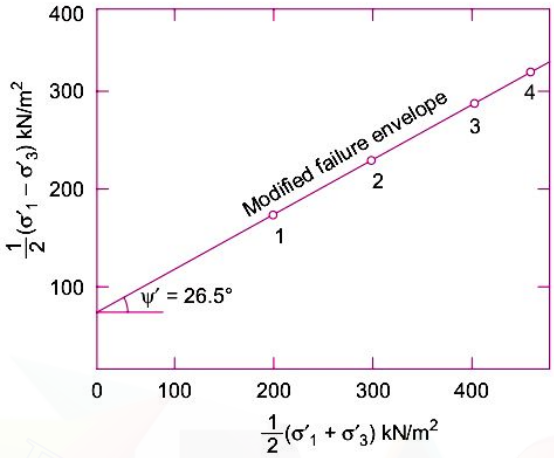

Fig. 18.21

18.9 VANE SHEAR TEST

Vane shear test is a quick test, used either in the laboratory or in the field, to determine the undrained shear strength of cohesive soil. The vane shear tester consists of four thin steel plates, called vanes, welded orthogonally to a steel rod. A torque measuring arrangement, such as a calibrated torsion spring, is attached to the rod which is rotated by a worm gear and worm wheel arrangement. After pushing the vanes gently into the soil, the torque rod is rotated at a uniform speed (usually at 1° per minute). The rotation of the vane shears the soil along a cylindrical surface. The rotation of the spring in degrees is indicated by a pointer moving on a graduated dial attached to the worm wheel shaft. The torque T is then calculated by multiplying the dial reading with the spring constant. A typical laboratory vane is 20 mm high and 12 mm in diameter with blade thickness from 0.5 to 1 mm, the blades being made of high tensile steel. The field shear vane is from 10 to 20 cm in height and from 5 to 10 cm in diameter, with blade thickness of about 2.5 mm.

Let τ_f = unit strength of the soil

H = height of the vane

d = diameter of the vane

Let us assume that the top end of the vane is embedded in the soil, so that both top and bottom ends partake in the shearing of the soil. Assuming that the shear resistance of the soil is developed uniformly on the cylindrical surface, the maximum total shear resistance, at failure, developed along the cylindrical surface = $\pi\, dH\, \tau_f$ ...(*i*)

To find the maximum shear resistance developed at top and bottom ends, consider a radius r of the sheared surface. The shear strength of a ring of thickness dr will be $2\pi r\, dr\, \tau_f$. Hence the total resistance of both top and bottom faces will be

$$= 2\int_0^{\pi} (2\pi r dr)\tau_f \qquad ...(ii)$$

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The total shear strength developed will be equal to the sum of (*i*) and (*ii*). The maximum moment of the total shear resistance about the axis of torque rod equals the torque T at failure.

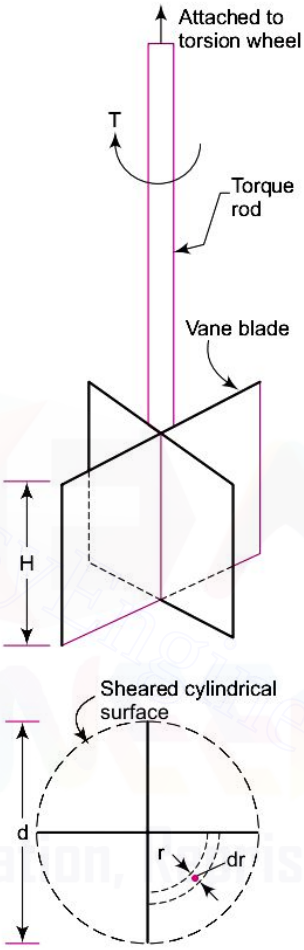

Fig. 18.22 Vane shear tests

Hence, $$T = \left(\pi d\, H\, \tau_f\right)\frac{d}{2} + 2\int_0^{d/2}\left(2\pi r\, dr\, \tau_f\right) r = \pi\tau_f\left[\frac{d^2H}{2} + \frac{d^3}{6}\right] = \pi d^2\,\tau_f\left[\frac{H}{2} + \frac{d}{6}\right] \quad ...(18.27)$$

If only the bottom end partakes in the shearing the above equation takes the from:

$$T = \pi d^2\,\tau_f\left[\frac{H}{2} + \frac{d}{12}\right] \quad ...(18.28)$$

Knowing T, H and d, the shear strength τ_f can be determined.

Example 18.11. *A vane, 10 cm long and 8 cm in diameter, was pressed into soft clay at the bottom of a bore hole. Torque was applied and gradually increased to 45 N-m when failure took place. Subsequently, the vane rotated rapidly so as to completely remould the soil. The remoulded soil was sheared at a torque of 18 N-m. Calculate the cohesion of the clay in the natural and remoulded states and also the value of the sensitivity.*

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Solution.

(*a*) *Natural state:* $T = 4500$ N-cm ; $H = 10$ cm ; $d = 8$ cm

From equation 18.27, $T = \pi d^2 \tau_f \left[\frac{H}{2} + \frac{d}{6}\right]$ $\therefore$ $4500 = \pi(8)^2 \tau_f \left[\frac{10}{2} + \frac{8}{6}\right]$

$\therefore$ $\tau_f = \frac{4500}{64\pi(6.33)} = 3.54$ N/cm^2 $\therefore$ $c = \tau_f = \mathbf{35.4\ kN/m^2}$ ($\because$ $\phi = 0$)

(*b*) *Remoulded state:* $T = 1800$ N-cm

$\therefore$ $1800 = \pi(8)^2 \tau_f \left[\frac{10}{2} + \frac{8}{6}\right]$ $\therefore$ $\tau_f = \frac{1800}{64\pi(6.33)} = 1.41$ N/cm^2 $= 14.1$ kN/m^2

$c = \tau_f = \mathbf{14.1\ kN/m^2\ (kPa)}$

$\therefore$ Sensitivity $= \frac{35.4}{14.1} = \mathbf{2.5}$.

18.10 SKEMPTON'S PORE PRESSURE PARAMETERS*

The change in the pore pressure due to change in the applied stress, during an undrained shear, may be explained in terms of empirical coefficients called the *pore pressure parameters*. A pore pressure parameter may be defined as a dimensionless number that indicates the fraction of total stress increment that show up an excess pore pressure for the condition of no drainage.

Let us consider a soil mass or skeleton subject to increase in the three principal stresses $\Delta\sigma_1$, $\Delta\sigma_2$ and $\Delta\sigma_3$, resulting in a volume decrease ΔV and a consequent increase in pore pressure of Δu. The increase in the effective stresses will be

$$\Delta\sigma_1' = \Delta\sigma_1 - u\ ;\ \Delta\sigma_2' = \Delta\sigma_2 - u\ ;\ \Delta\sigma_3' = \Delta\sigma_3 - u \qquad ...(1)$$

If ε_1, ε_2 and ε_3 denote the strains in the three directions, we have

$$\begin{aligned} E \cdot \varepsilon_1 &= \Delta\sigma_1' - \mu(\Delta\sigma_2' + \Delta\sigma_3') \\ E \cdot \varepsilon_2 &= \Delta\sigma_2' - \mu(\Delta\sigma_3' + \Delta\sigma_1') \\ E \cdot \varepsilon_3 &= \Delta\sigma_3' - \mu(\Delta\sigma_1' + \Delta\sigma_2') \end{aligned} \qquad ...(2)$$

$\therefore$ $E(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = E \cdot \varepsilon_v = E\frac{\Delta V}{V} = (1 - 2\mu)(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3')$

$\therefore$ $\frac{\Delta V}{V} = \frac{1 - 2\mu}{E}(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3') = \frac{3(1 - 2\mu)}{E}\frac{1}{3}(\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3')$

Putting $\frac{3(1 - 2\mu)}{E} = C_c$ = compressibility of the soil skeleton, and substituting the values of effective stresses in terms of the total stresses, we have

$$\frac{\Delta V}{V} = C_c \frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u) = C_c\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right] \qquad ...(I)$$

If n is the porosity, the volume of voids or the volume of pore fluid equals nV. If the pore fluid (air + water) is assumed to show a linear relationship between volume change and stress, and its coefficient of volume compressibility is represented by C_v, the change in volume of the pore fluid ΔV_w due to the increase in the pore pressure Δu under the codition of no drainage is given by $\Delta V_w = nVC_v\,\Delta u$ ...(II)

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The decrease in the volume of the soil skeleton is almost entirely due to the decrease in the volume of the voids. Hence (I) and (II) are equal:

$$\therefore \quad nVC_v \Delta u = C_v \left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3) - \Delta u\right]$$

$$\therefore \quad \Delta u = \frac{1}{1 + \dfrac{nC_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)\right] \qquad (4)$$

In a conventional triaxial test, $\Delta\sigma_2 = \Delta\sigma_3$

$$\therefore \quad \Delta u = \frac{1}{1 + \dfrac{nC_v}{C_c}}\left[\frac{1}{3}(\Delta\sigma_1 + 2\Delta\sigma_3)\right] = \frac{1}{1 + \dfrac{nC_v}{C_c}}\left[\Delta\sigma_3 + \frac{1}{3}(\Delta\sigma_1 - \Delta\sigma_3)\right] \qquad ...(18.29)$$

The above equation is obtained on the assumption that the soil mass is elastic and isotropic. Since the behaviour of soil is by no means is accordance with the elastic theory, the above equation may be written in the form

$$\Delta u = B\,[\Delta\sigma_3 + A\,(\Delta\sigma_1 - \Delta\sigma_3)] \qquad ...(18.30)$$

where B and A are called Skempton's pore pressure parameters. These parameters are to be determined experimentally.

In an undrained triaxial test, stress changes are usually made in two, stages : (*i*) an increase in the cell pressure $\Delta\sigma_3$ resulting in an all round change in the stress, and (*ii*) an increase in the axial load resulting in a change in the deviator stress $\Delta\sigma_d$ $(= \Delta\sigma_1 - \Delta\sigma_3)$ are represented in Fig. 18.23.

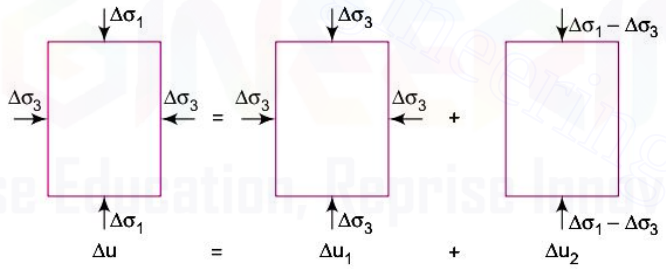

Fig. 18.23

Let Δu_1 be the change in the pore pressure during the first stage of the test when the cell pressure is applied, and Δu_3 be the change in the pore pressure when deviator stress is applied.

Then $$\Delta u = \Delta u_1 + \Delta u_2 \qquad ...(18.31)$$

Comparing equations 18.30 and 18.31, we get $\Delta u_1 = B\Delta\sigma_3$

or $$B = \frac{\Delta u_1}{\Delta\sigma_3} \qquad ...(18.32)$$

and $$\Delta u_2 = AB\,(\Delta\sigma_1 - \Delta\sigma_3) = \bar{A}\,(\Delta\sigma_1 - \Delta\sigma_3) \quad \therefore \quad \bar{A} = \frac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3} \qquad ...(18.32a)$$

Factors affecting *A* and *B*: From Eq. 18.29, $B = \dfrac{1}{1 + \dfrac{nC_v}{C_c}}$ ...(18.33)

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In a fully saturated soil, the compressibility of water alone (C_v) is very much smaller than C_c. Then ratio C_v/C_c is approximately equal to zero. Hence $B = 1$. If, however, the soil is perfectly dry C_v/C_c approaches infinity, since compressibility of air is far greater than that of solids. Hence $B = 0$. For partially saturated soils, $0 < B < 1$. At Proctor's optimum water content and density, the values of B ranges typically from about 0.1 to 0.5. Figure 18.24 shows the variation of B with the degree of saturation.

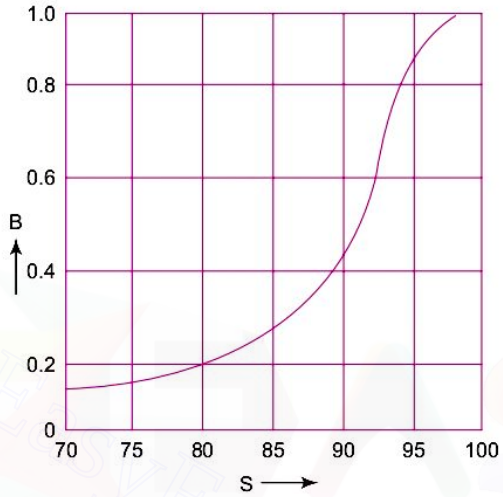

Fig. 18.24 Variation of B with degree of saturation

For any given soil, the coefficient A varies with the stresses and strains (Fig. 18.25a). Hence its value may be quoted at failure (maximum deviator stress), at maximum effective principal stress ratio or at any other required points. Its value also depends upon whether the total stresses are increasing or decreasing.

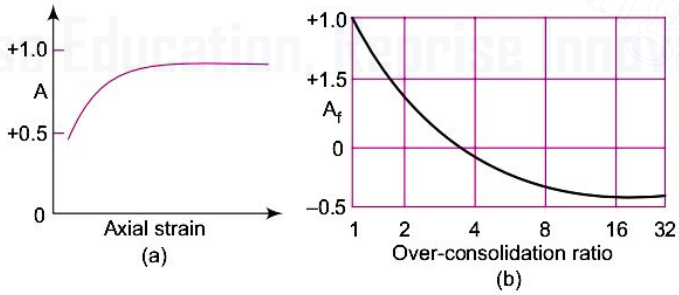

Fig. 18.25 Variation of A

The stress history of the soil also has a significant effect on parameter A. Preconsolidation of test specimen tends to reduce considerably the parameter A. Table 18.3 gives the value of A at failure (A_f). Figure 18.25 (b) shows the variation of A_f with overconsolidation ratio. Other factors affecting the parameters A are type of shear, sample disturbance and environment during shear (such as temperature and nature of the fluid).

A pore pressure parameter greater than one is associated with a loose structure, either in sand or clay, which collapses upon load application. On the other hand, if we load a sample of sand or clay which tends to expand upon loading, we can develop negative pore pressure.

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Table 18.3 Approximate values of pore pressure parameter A at failure

Soil type	A_f
Very loose, fine saturated sand, satuarted clay:	2 to 3
Extra-sensitive to quick	1.2 to 2.5
Normally consolidated	0.7 to 1.3
Lightly preconsolidated	0.3 to 0.7
Heavily preconsolidated	– 0.5 to 0
Compacted sandy clays	0.25 to 0.75
Compacted clay gravels	– 0.25 to 0.35

Determination of parameters A and B. The parameter B is experimentally determined in the laboratory by measuring the change in the pore pressure Δu_1 due to change in the cell pressure $\Delta\sigma_3$, in the first part of the test. Then $B = \Delta u_1/\Delta\sigma_3$. The parameter $\bar{A}$ is measured during the second stage of the test when deviator stress $(\Delta\sigma_1 - \Delta\sigma_3)$ is applied at a constant cell pressure, when Δu_2 is measured. Then $\bar{A} = A \cdot B = \dfrac{\Delta u_2}{\Delta\sigma_1 - \Delta\sigma_3}$. Knowing B and $\bar{A}$, A can be computed. Since parameter B varies with the stress change, the value of B must correspond to the pressure range in the deviator part of the test. A direct way to determine A is from the equation:

$$A = \frac{\Delta u - \Delta\sigma_3}{\Delta\sigma_1 - \Delta\sigma_3} \qquad ...(18.34)$$

For the usual undrained triaxial text, $\Delta\sigma_3 = 0$, when deviator stress is applied.

$$\therefore \quad A = \frac{\Delta u}{\Delta\sigma_1} \qquad ...(18.35)$$

Thus, during the undrained triaxial text, σ_3 is held constant and σ_1 is increased. The change in the pore pressure Δu is measured and then A is computed. (See also § 18.14 for the application of stress path method for the determination of A).

Example 18.12*. *An embankment, 5 m high, is made up of soil whose effective stress parameters are $c' = 50$ kN/m^2 and $\phi' = 16°$, and $\gamma = 16.2$ kN/m^3. The pore pressure parameters as found from triaxial test are $A = 0.40$ and $B = 0.92$. Find the shear strength of the soil at the base of the embankment just after the fill has been raised from 5 m to 9 m. Assume that the dissipation of pore pressure during this stage of construction is negligible, and that the lateral pressure at any point is one-half of the vertical pressure.*

Solution. $\gamma = 16.2$ kN/m^3

$\Delta\sigma_1$ = increase in the vertical stress due to 4 m construction

$= \gamma\Delta H = 16.2 \times 4 = 64.8$ kN/m^2

$$\therefore \quad \Delta\sigma_3 = \frac{1}{2}\Delta\sigma_1 = 32.4 \text{ kN/m}^2$$

The relation between the increase in the pore pressure and increase of principal stress is given by $\Delta u = B\,[\Delta\sigma_3 + A\,(\Delta\sigma_1 - \Delta\sigma_3)] = 0.92\,[32.4 + 0.4\,(64.8 - 32.4)] = 41.73$ kN/m^2

Now, original pressure $= \sigma_1 = 5 \times 16.2 = 81$ kN/m^2

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$\therefore$ Effective stress $\sigma' = \sigma_1 + \Delta\sigma_1 - \Delta u = 81 + 64.8 - 41.73 = 104.07 \text{ kN/m}^2 = 104.07 \text{ kPa}$

$\therefore$ Shear strength $= c' + \sigma' \tan \phi' = 50 + 104.07 \tan 16° = 50 + 29 = 79 \text{ kN/m}^2 = 79 \text{ kPa}.$

18.11 SHEAR STRENGTH OF COHESIVE SOILS

(*a*) Undrained test on saturated cohesive soils. The undrained test is carried out on undisturbed sample of clay, silt and peat to determine the strength of the natural ground. It is also carried out on remoulded samples of clay to measure its sensitivity. Figure 18.26 shows typical results of undrained shear test on a saturated soil.

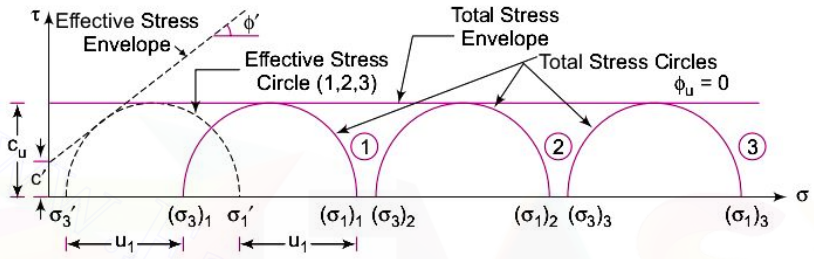

Fig. 18.26 Mohr stress circles for undrained test on saturated cohesive soil

Mohr circles 1, 2 and 3 in terms of total stresses correspond to σ_1 and σ_3 stresses at failure for three identical specimens of a saturated clay. For the first circle, the diameter is equal to $(\sigma_1)_1 - (\sigma_3)_1$. If u is the pore pressure measured at failure, the effective stress circle will have a diameter $(\sigma_1')_1 - (\sigma_3')_1 = [(\sigma_1)_1 - u_1] - [(\sigma_3)_1 - u_1] = (\sigma_1)_1 - (\sigma_3)_1$. Thus, the diameter of the effective stress circle will be the *same* as that of the total stress circle. However, it is located to the left of the total stress circle, as shown by dotted lines. If a second identical sample is tested at a cell pressure of $(\sigma_3)_2$ as represented by circle 2, the increase in the cell pressure $\Delta\sigma_3$ simply results in corresponding change in the pressure under the conditions of no drainage, because the pore pressure parameter B equals unity. Hence $\Delta u = \Delta\sigma_3$. Hence the corresponding minor principal effective stress does not change, because the whole increase of cell pressure is borne completely by the pore water. Similarly, any subsequent increase $\Delta\sigma_1$ in the major principal stress (to cause failure under the increased cell pressure) also results in the same change in pore pressure . In other words, the major principal effective stress also does not change. *It may, therefore, be concluded that in an undrained test on saturated clays (B = 1), both the major principal effective stress σ_1' and the minor principal effective stress σ_3' are independent of magnitude of the cell pressure applied.* We will therefore, get *only one* Mohr circle in terms of effective stress (shown dotted), for all the identical specimens tested under increased pressure. It will also be observed that since the deviator stress $= \sigma_1 - \sigma_3 = \sigma_1' - \sigma_3'$, we have

$$\sigma_d = (\sigma_1)_1 - (\sigma_3)_1 = (\sigma_1)_2 - (\sigma_3)_2 = (\sigma_1)_3 - (\sigma_3)_3 = \sigma_1' - \sigma_3' \qquad ...(18.36)$$

Hence the diameters of the Mohr circles in terms of total stress will be the same, though the corresponding Mohr circles will be laterally shifted as shown in Fig. 18.26. The failure envelope for total stresses will thus be horizontal giving $\phi_u = 0$ and its intercept with the τ-axis will give c_u which will be equal to half the deviator stress. Since only one Mohr circle, in terms of effective stresses, is obtained for all tests, *effective stress failure envelope cannot be obtained from this test.* Consolidated undrained or drained tests are used for the latter purpose. Pore pressure measurements are not, usually made during undrained tests on saturated sample.

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Figure 18.27 shows the modified failure envelope for undrained test on cohesive soils. Since $\frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}\sigma_d$ = constant, the failure envelope will be horizontal, giving. $\psi = 0$

$$\therefore \qquad \sin \phi_u = \tan \psi = 0 \quad \text{or} \quad \phi_u = 0 \quad \text{Also,} \quad c_u = \frac{d}{\cos \phi_u} = d.$$

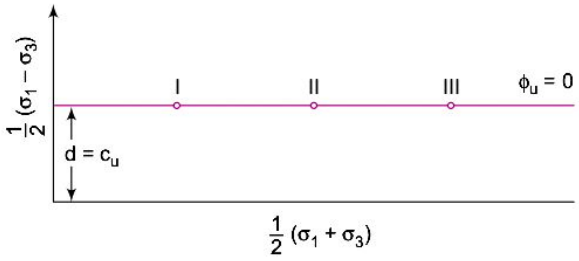

Fig. 18.27 Modified failure envelope for undrained test on saturated cohesive soil

***(b)* Undrained test on partly saturated cohesive soil.** In the case of earth embankments, which are compacted at optimum water content, the soil remains partly saturated and it is necessary to conduct undrained test to determine the shear parameters of the soil. This test is also sometimes applied to undisturbed samples of soil taken from existing rolled fills or trail sections. Figure 18.28 shows the Mohr stress circle, in terms of total stresses as well as effective stresses for undrained tests on partly saturated soil.

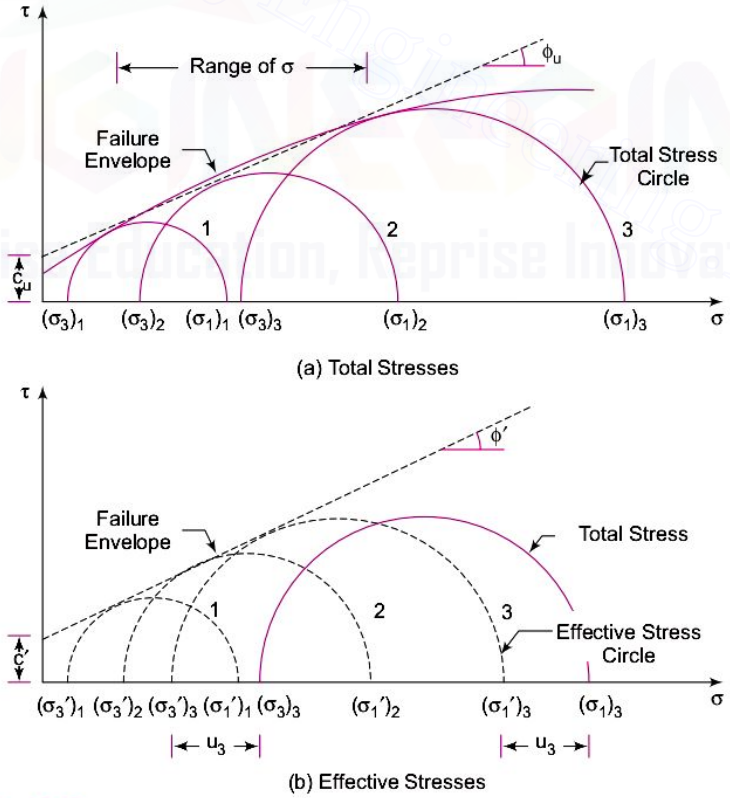

Fig. 18.28 Mohr stress circles for undrained tests on partly saturated soil

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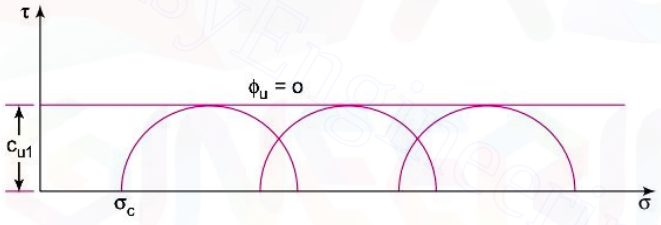
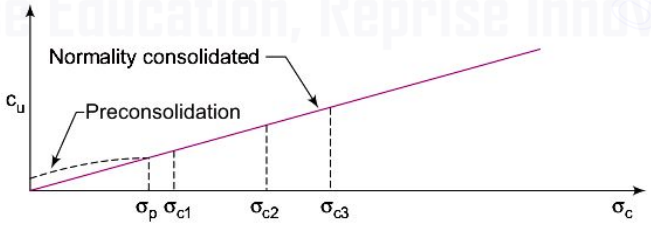
As the cell pressure is increased, the deviator stress at failure also increases, though this increase in deviator stress becomes progressively smaller as the air in soil voids is compressed and dissolved. The increase in the deviator stress later ceases when the large stresses cause full saturation. Due to this reason, the failure envelope in terms of total stresses [Fig. 18.28 (*a*)] is *non-linear*. The value of c_u and d_u can, therefore, the quoted only for a certain pressure range. However, the failure envelope corresponding to effective stress is very closely a straight line over a wide range of pressure.

(*c*) Consolidated-undrained test on saturated cohesive soils. The consolidated undrained test is carried out on undisturbed sample of clay, silt and peat, on remoulded samples of clay and silt, and on redeposited samples of cohesionless soils such as sand and gravel. The consolidated-undrained tests are performed by two methods: (*i*) the moulded specimens are first consolidated under the *same* cell pressure, and then sheared under undrained conditions with *different* cell pressure by increasing the axial stress, (*ii*) the remoulded specimens are sheared under a cell pressure equal to the consolidation pressure.

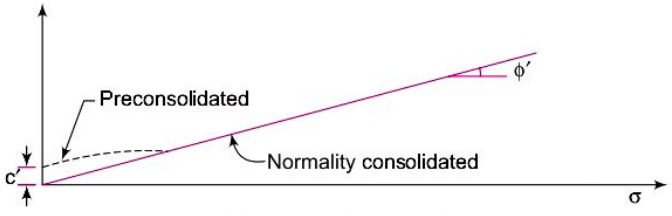
(*i*) *First method*: In the first method, where remoulded specimens of saturated clay with the same initial voids ratio are first consolidated under the same cell pressure (σ_c) and then sheared under undrained conditions with different cell pressure by increasing the axial stress, the deviator stress is found to be independent of the cell pressure (except in the case of dilatant soils). Figure 18.29 (*a*) shows a series of Mohr circle at failure having the same diameter, for identical specimens consolidated under a consolidation pressure σ_{c_1}. The diameter of each circle is equal to deviator stress ($\sigma_1 - \sigma_3$), which is found



(a) Failure envelope for pre-consolidation pressure σ_{c1}



(b) Variation of C_u with σ_c



(c) Effective stress envelope

Fig. 18.29 Consolidated undrained tests on saturated clay

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to be independent of the cell pressure. The failure envelope is horizontal, giving $\phi_u = 0$ and the apparent cohesion intercept c_{u_1}. If the test is repeated with increased effective consolidation pressure σ_{c_2}, the corresponding Mohr circles will have an increased diameter equal to increased deviator stress, and the new failure envelope will give cohesion intercept equal to c_{u_2}. The test result, in terms of total stresses, may be expressed by plotting the value of c_u against the consolidation pressure σ_c. Figures 18.29 (*b*) shows the variation of c_u with preconsolidation pressure σ_c. It is seen that the apparent cohesion increases linearly with the increasing effective consolidation pressure; for normally consolidated soil the straight line passes through the origin. If the clay is preconsolidated (over-consolidated), the curve (shown dotted) is nonlinear up to the preconsolidation pressure (σ_p); however at higher consolidation pressure ($\sigma_c > \sigma_p$), the soil behaves as a normally consolidated soil and a linear relationship between c_u and σ_c is obtained. For normally consolidated soils, the ratio of c_u/σ_c is found to be constant, its value depending on soil type.

However, if the pore pressure is measured during the undrained stage of the test, the results can be expressed in terms of effective parameters c' and ϕ'. Figure 18.29 (*c*) shows the failure envelope. For normally consolidated samples, the effective stress envelope is a straight line with c' equal to zero. Over consolidation results in an envelope lying a little above the straight line; the section of this envelope relevant to any particular practical problem can generally be represented with sufficient accuracy by a slightly modified value of ϕ' and a cohesion intercept c'.

(*ii*) *Second method*: In the second (and more usual) method of consolidated undrained test, each specimen is consolidated with different cell pressure ($\sigma_c = \sigma_3$) and then sheared under deviator stress to failure. The cell pressure σ_3 during the failure is kept the same as the consolidation pressure σ_c. (In the case of shear box test, the normal pressure during shear is kept equal to the normal consolidation pressure). The triaxial test results in terms of total stresses (firm lines) and effective stresses (dotted lines) for normally consolidated specimens are shown in Fig. 18.30 (*a*). The diameter of the Mohr circles in terms of effective stresses remain the same as in terms of total stresses (since the deviator stress $\sigma_d = \sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$) but the effective stress circles are shifted to the left, because of positive pore pressure. The failure envelopes pass through the origin in both the cases, giving $c_{cu} = c' = 0$. The equation for consolidated undrained shear strength for normally consolidated condition, in terms of total stress is given by

$$\tau_f = \sigma \tan \phi_{cu} \qquad ...(18.37)$$

The effective angle of shearing resistance ϕ' is found to be greater than the apparent angle ϕ_{cu} for normally consolidated soil, and as a rough approximation, ϕ_{cu} may be taken to be about half of ϕ'.

Figure 18.30 (*a*) shows the failure envelopes for preconsolidated saturated clay, both in terms of effective stress as well as total stress. The most marked effect of over-consolidation is however, on the value of the pore pressure co-efficient *A*, which, with increasing over consolidation ratio, drops from a value typically about 1 at failure to values in the negative range. These low *A*-values are, in turn, largely responsible for high undrained strength values resulting from over-consolidation. When *A* is negative, the pore pressure developed with increasing axial load applied during the test, will also be negative. Due to the development of negative pore pressure, the effective stress Mohr circle will be shifted towards the right relative to the total stress circle, as shown in Fig. 18.30 (*b*). The failure envelope in terms of total stresses is curved in the initial portion (up to preconsolidation pressure). The equation for the consolidated undrained strength, of the preconsolidated clay in terms of total stress may be approximately expressed as follows:

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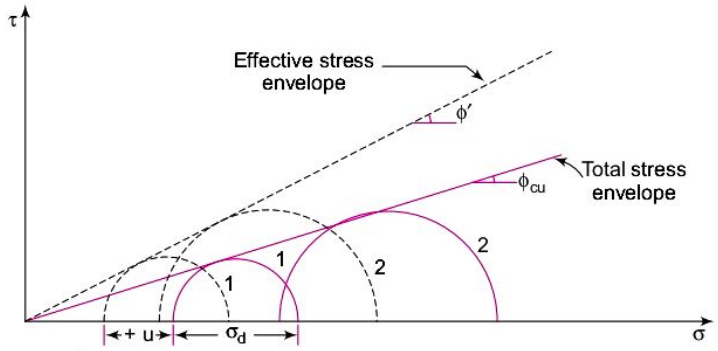
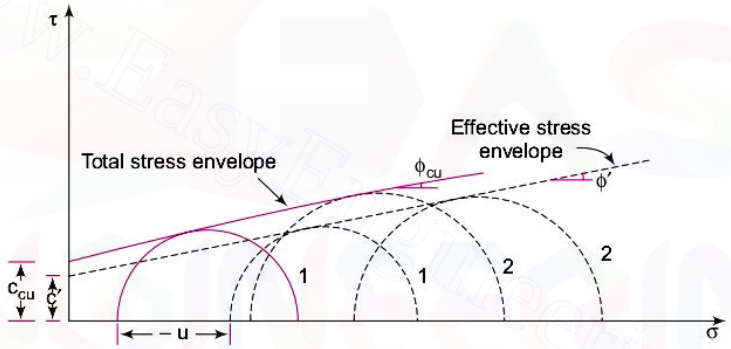
$$\tau_f = c_{cu} + \sigma \tan \phi_{cu} \quad ...(18.38)$$

where σ is the normal pressure under which the soil is consolidated prior to shearing.



(a) Failure envelopes for normally consolidated clay



(b) Failure envelopes for pre-consolidated clay

Fig. 18.30 Failure envelopes for consolidated undrained tests on saturated clays

(*d*) Consolidated undrained test on partly saturated cohesive soils. Consolidated undrained tests are required to determine the shear parameters of undisturbed samples or of compacted samples of earth fill where the soil is partly saturated. The test may also be conducted to examine the effect on c' and ϕ' of flooding foundation strata and earth fill materials, by applying back pressure to the pore space to ensure full saturation. Since full saturation in the laboratory is not possible, the shear strength measured during the undrained stage of the test is not independent of changes in cell pressure at this stage, and cannot be expressed simply by a value of c_{cu} as in the case of a saturated soil. In other words, incomplete saturation will mean that unique value of c_{cu} and ϕ_{cu} will only be obtained if no change in cell pressure is made after consolidation stage, before sample is sheared. Due to this limitation, the total stress analysis is found to be quite impracticable (Bisphop and Henkel, 1962). The value of c' and ϕ' are determined by measuring the pore pressure and getting the value of effective stresses at failure.

(*e*) Drained tests. Drained tests are carried out on both undisturbed and remoulded samples of cohesive soils to obtain directly the shear strength parameters relevant to the condition of long term stability, when the pore pressure have decreased (or increased) to their equilibrium values. In the drained test, the specimen is first consolidated under a cell pressure ($\sigma_c = \sigma_3$) and is then sheared sufficiently slowly so

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that the pore pressure developed during shearing is dissipated. The minor principal stress σ_3' at failure is thus equal to σ_c, the consolidation pressure ; the major principal stress σ_c' is the axial stress. Since pore pressure is zero, the total stresses and effective stresses are equal. The value of c' and ϕ' obtained from drained tests are often denoted by c_d and ϕ_d respectively. Figure 18.31 shows the failure envelopes for both normally consolidated and over consolidated saturated clay samples.

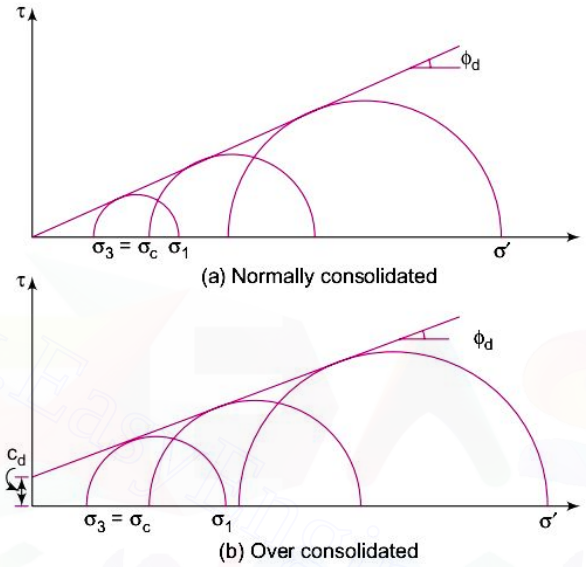

Fig. 18.31 Failure envelopes for drained test

18.12 HVORSLEV SHEAR STRENGTH PARAMETERS

Based on the experimental work on remoulded cohesive soil, Hvorslev (1937) found that irrespective of the stress history of a soil, its shear strength can be represented by:

$$\tau_f = F(e, \sigma') \qquad ...(18.39)$$

or more explicitly,

$$\tau_f = c_e + \sigma' \tan \phi_e \qquad ...(18.40)$$

where, $F(e, \sigma')$ = function of voids ratio e at failure and the normal stress σ' at failure

c_e = true cohesion

ϕ_e = true angle of internal friction

σ' = effective normal stress on the failure plane at failure.

The true cohesion c_e is function of water content of the soil at failure. The quantity tan ϕ_e may be defined as the rate of increase of strength with effective normal stress at constant water content (w_f) at failure. In order to separate the two components of shear strength, it is necessary to compare the results of tests in which specimens have the same water content at failure but significantly different effective stress. This should be done by conducting tests on normally and over consolidated specimens. The values

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of shear strength and effective stress for tests having the same water content at failure (w_f) is then found by interpolation.

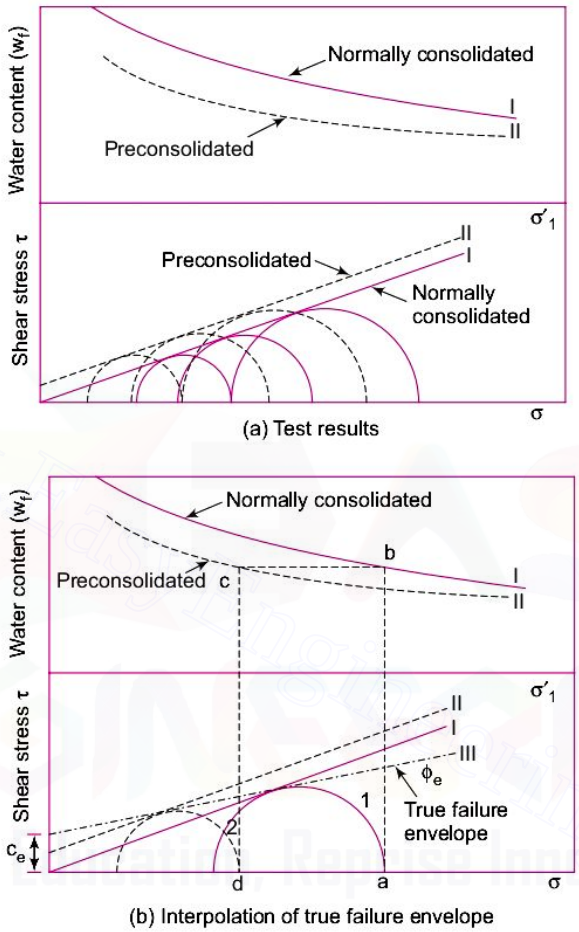

Fig. 18.32 Hvoslev's true failure envelope

Figure 18.32 (*a*) shows the test results on normally consolidated and preconsolidated specimens; the upper part shows the variations of water content at failure (w_f) versus maximum principal stress (σ'_1) while the lower part shows the failure envelopes for both the soil specimens. Figure 18.32 (*b*) shows the method of interpolation of the true failure envelope; the upper part shows the two $w_f - \sigma'_1$ curves while the lower part shows the two failure envelopes obtained in Fig. 18.32 (*a*). Curves I correspond to normally consolidated specimens while curves II correspond to preconsolidated specimens. A circle (1) is drawn so as to touch the failure envelopes (I) for the normally consolidated clay, and the point *a* corresponding to its major principal stress is projected up to point *b* on the $w_f - \sigma'_1$ curve (I) for the normally consolidated specimens. Point *b* is projected across the *constant* water content, to point *c* on curve (II). Point *c* is then projected down to point *d* from which a Mohr circle (2) is drawn touching the failure envelope (II) for the preconsolidated specimens. A common tangent (III) is drawn to touch both the Mohr circles (1, 2). This common tangent is the Hvorslev's true failure envelope, the slope of which gives the true angle of friction ϕ_e, and its intercept on the vertical axis gives the true cohesion c_e.

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Example 18.13*. *Given the relationship between τ' and σ' on failure plane $\tau = c' + \sigma' \tan \phi'$, express the relationship between $\frac{\sigma_1' - \sigma_3'}{2}$ and $\frac{\sigma_1' + \sigma_3'}{2}$.*

Solution. (Refer to Fig. 18.5)

Radius of Mohr Circle, $FC = \frac{\sigma_1' - \sigma_3'}{2}$; $FL = \frac{\sigma_1' - \sigma_3'}{2} \cos \phi' = \tau$ ...(1)

and $LC = \frac{\sigma_1' - \sigma_3'}{2} \sin \phi'$

Also, $OC = \frac{\sigma_1' + \sigma_3'}{2}$; $\because \quad OL = OC - LC$

$\therefore \quad \sigma' = \frac{\sigma_1' + \sigma_3'}{2} - \frac{\sigma_1' - \sigma_3'}{2} \sin \phi'$ ...(2)

Substituting the value of τ and σ' in the Mohr-Coulomb equation, we get

$$\frac{\sigma_1' - \sigma_3'}{2} \cos \phi' = c' + \left[\frac{\sigma_1' + \sigma_3'}{2} - \frac{\sigma_1' - \sigma_3'}{2} \sin \phi' \right] \tan \phi'$$

or $$\frac{\sigma_1' - \sigma_3'}{2} \cos^2 \phi' = c' \cos \phi' + \frac{\sigma_1' + \sigma_3'}{2} \sin \phi' - \frac{\sigma_1' - \sigma_3'}{2} \sin^2 \phi'$$

$\therefore$ $$\frac{\sigma_1' - \sigma_3'}{2} = c' \cos \phi' + \frac{\sigma_1' - \sigma_3'}{2} \sin \phi'$$ ...(18.41)

which is the required relationship.

Denoting $\frac{\sigma_1' - \sigma_3'}{2} = q$ and $\frac{\sigma_1' + \sigma_3'}{2} = p'$, the above equation is also sometimes written as

$$q = c' \cos \phi' = p' \sin \phi'$$ ...(18.42)

This is the equation of the K_f line (Fig. 18.36).

Example 18.14*. *Find an expression for the unconfined compressive strength q_u in terms of c', ϕ' and A_f (pore pressure parameter at failure). Take parameter $B = 1$ and initial capillary tension $= u_c$.*

Solution. We have $\Delta u = B[\sigma_3 + A_f (\sigma_1 - \sigma_3)]$ ...(1)

Putting $B = 1$ and $\sigma_3 = 0$ (unconfined test), $\Delta u = A_f \sigma_1$

If u_f = pore pressure at failure, we have ; $\Delta u = u_f - (-u_c) = u_f + u_c$ ...(2)

Substituting in (1), we have $\Delta u = u_f + u_c = A_f \sigma_1$; or $u_f = A_f \sigma_1 - u_c$ ...(3)

From equation 18.41, we have $\frac{\sigma_1' - \sigma_3'}{2} = c' \cos \phi' + \frac{\sigma_1' + \sigma_3'}{2} \sin \phi'$

Putting $\sigma_3' = -u_f$ and $\sigma_1' = \sigma_1 - u_f$, we get

$$\frac{\sigma_1}{2} = c' \cos \phi' + \left(\frac{\sigma_1}{2} - u_f \right) \sin \phi'$$ ...(4)

Substituting the value of u_f from (3), we have

$$\frac{\sigma_1}{2} = c' \cos \phi' + \left(\frac{\sigma_1}{2} - A_f \sigma_1 + u_c \right) \sin \phi'$$

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or $\frac{\sigma_1}{2}\left[(1-\sin\phi')+2\,A_f\,\sin\phi'\right] = c'\cos\phi' + u_c\sin\phi'$

or $$\sigma_1 = q_u = 2\,\frac{c'\cos\phi' + u_c\sin\phi'}{(1-\sin\phi')+2\,A_f\,\sin\phi'} \qquad ...(18.43)$$

which is the required expression.

Example 18.15*. *In the case of normally consolidated clays, prove that*

$$\frac{c_u}{p} = \frac{\sin\phi'}{1+\sin\phi'\,(2\,A_f - 1)}$$

where p = *consolidation pressure and* $c_u = \frac{1}{2}q_u$.

Solution. Putting $c_u = \frac{1}{2}q_u$, $c' = 0$ and $p = u_c$ in Eq. 18.43, we get

$$\frac{q_u}{2} = c_u = \frac{p\sin\phi'}{(1-\sin\phi')+2\,A_f\,\sin\phi'}$$

or $$\frac{c_u}{2} = \frac{\sin\phi'}{1+\sin\phi'\,(2\,A_f - 1)} \qquad ...(18.44)$$

Example 18.16*. *The equation of Mohr envelope is as follows :* $\tau^2 = \sigma + 9$ *(kg/cm*^{2}*) Determine the unconfined compressive strength, torsional shear strength and the tensile strength.*

Solution.

(*a*) *Unconfined compression and tensile strengths* (Fig. 18.33).

Let σ_1 be the major principal stress. The equation of the *failure circle* in each case may be written as

$$\left(\frac{\sigma_1}{2}-\sigma\right)^2 + \tau^2 = \left(\frac{\sigma_1}{2}\right)^2 \text{ or } \left(\frac{\sigma_1}{2}\right)^2 + \sigma^2 - \sigma\sigma_1 + \tau^2 = \left(\frac{\sigma_1}{2}\right)^2$$

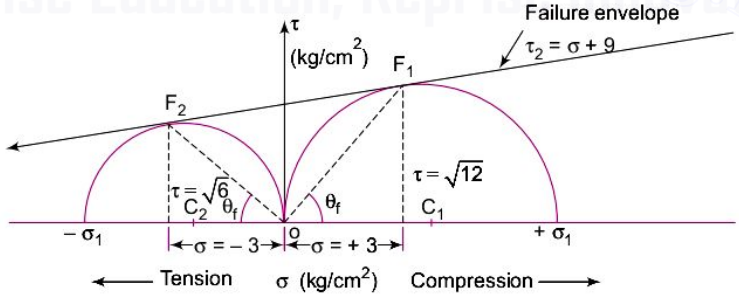

Fig. 18.33

or $$\sigma^2 - \sigma\sigma_1 + \tau^2 = 0 \qquad ...(1)$$

Also, $$\tau^2 = \sigma + 9 \quad \text{(given)} \qquad ...(2)$$

From (1), $2\tau\frac{\partial\tau}{\partial\sigma} = -2\sigma + \sigma_1$; From (2), $2\tau\frac{\partial\tau}{\partial\sigma} = 1$

Equating the two, we get $\sigma_1 - 2\sigma = 1$ or $\sigma_1 = 1 + 2\sigma$ ...(3)

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Substituting the value of σ_1 and τ^2 in (1), we get

$$\sigma^2 - \sigma(1 + 2\sigma) + (\sigma + 9) = 0$$

From which $\sigma^2 = 9$ or $\sigma = \pm 3$

$$\tau = \sqrt{\pm 3 + 9} = \sqrt{12} \text{ or } \sqrt{6}$$

Hence, for compression test, co-ordinates of F_1 at failure are :

$$\sigma = +3 \text{ kg/cm}^2 ; \tau = \sqrt{12} \text{ kg/cm}^2$$

and $$\theta_f = \tan^{-1}\frac{\tau}{\sigma} = \tan^{-1}\frac{\sqrt{12}}{3} = \mathbf{49° 10'}$$

The unconfined compressive strength $= \sigma_1 = 1 + 2\sigma = 1 + 6 = 7 \text{ kg/cm}^2$

For tension test, co-ordinates of F_2 at failure are :

$$\sigma = -3 \text{ kg/cm}^2 ; \tau = \sqrt{6} \text{ and } \theta_f = \tan^{-1}\frac{\sqrt{6}}{3} = \mathbf{39° 12'}$$

The unconfined tensile strength $= \sigma_1 = 1 + 2\sigma = 1 - 6 = -5 \text{ kg/cm}^2$.

(*b*) *Torsion test* (Fig. 18.34): For failure corresponding to F_3

$$\tau^2 = S^2 - \sigma^2 \text{ where S = torsion} \quad ...(1)$$

Also, $$\tau^2 = \sigma + 9 \quad \text{(given)} \quad ...(2)$$

From (1), $$2\tau\frac{\partial\tau}{\partial\sigma} = -2\sigma$$; From (2), $$2\tau\frac{\partial\tau}{\partial\sigma} = 1$$

Equating the two, we get $\sigma = -\frac{1}{2} = -0.5 \text{ kg/cm}^2$

$$\tau = \sqrt{-0.5 + 9} = 2.92 \text{ kg/cm}^2$$

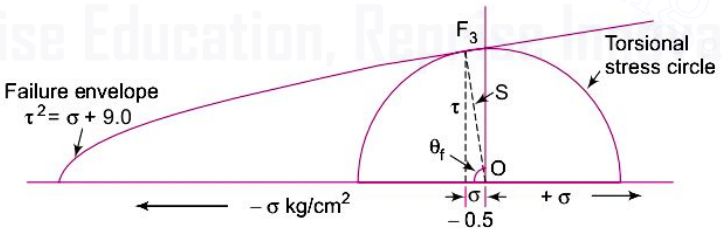

Fig. 18.34

$$\therefore \quad S = \sqrt{\sigma^2 + \tau^2} = \sqrt{(-0.5)^2 + 8.5} = \mathbf{2.96 \text{ kg/cm}^2}$$

$$\theta_f = \tan^{-1}\frac{\tau}{\sigma} = \tan^{-1}\frac{2.92}{0.5} = \mathbf{80° 18'}$$

18.13 STRESS PATH METHOD

(*a*) Rendulic's unique stress path. *Unique stress path* is a path or line in the stress-space, which shows the history of the stress. The effective stress path, first used by Rendulic (1936), can be used to prove

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the unique relationship between the water content and the effective stress. His work was continued and extended by Henkel (1958).

Figure 18.35 (*a*) shows a three dimensional stress space in which the plane *ABCO*, containing the space diagonal ($\sigma'_1 = \sigma'_2 = \sigma'_3$) and σ_1-axis, the stress plane that can be investigated in standard triaxial test. Figure 18.35 (*b*) shows the projections (normal view) on the plane *ABCO*.

In Fig. 18.35 (*b*), *OB* is the space diagonal, *PQ* is the failure envelope for compression and *RS* is the failure envelope for tension (see chapter 19). For any point, such as point *a* on the space diagonal, the stress conditions are such that $\sigma'_1 = \sigma'_2 = \sigma'_3$ *i.e.*, it represents hydrostatic compression ; hence the space diagonal *OB* is also known as hydrostatic pressure line. The points representing the stresses in the specimens after consolidation and before the commencement of the shear test will all lie on this hydrostatic pressure line. As shearing forces are applied to the specimen, the points representing the effective stresses will move away from the hydrostatic pressure line and the path traced out in the stress space by these points is called a *stress path*. The stress paths for compression test appear above the hydrostatic pressure line,

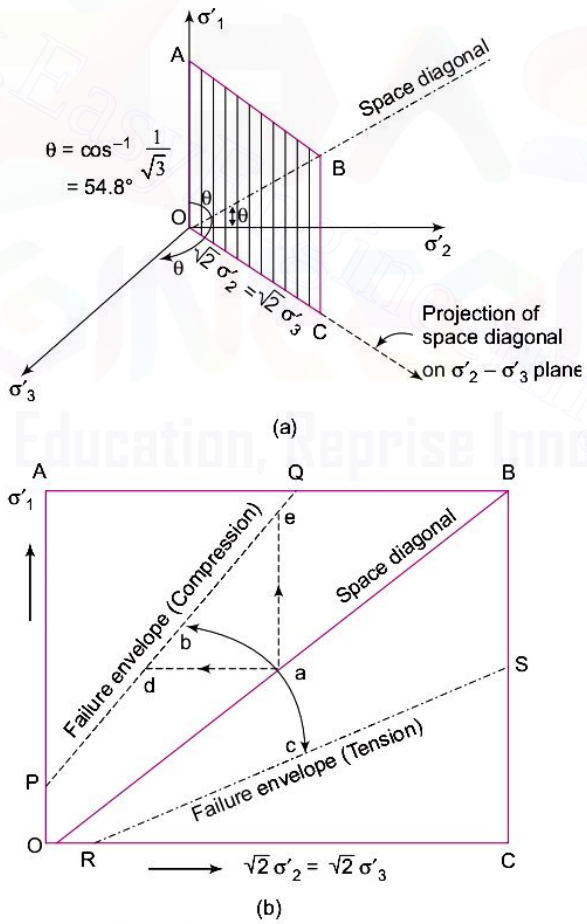

Fig. 18.35 Rendulic's stress path

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while the stress paths followed in the expansion tests lie below the space diagonal. Point *a* represents a hydrostatic compression (*i.e.*, cell pressure). *ab* represents the stress path for failure of the specimen in an undrained test with increasing vertical stress, *ae* represents the stress path, when the sample is failed in a drained test with increasing vertical stress, *ad* represents a stress path when the specimen is failed in a drained test with lateral stress decreasing. Similarly, *ac* represents the stress path in an extension test.

(b) Lambe's stress path (1967). Lambe's stress path is a line drawn through points on a plot of stresses. The particular stress point used in the stress path is that of maximum shear stress (Fig. 18.36). *Thus, stress path is the locus of points of maximum shear stress experienced by an element in going from one stress to another.*

There are three basic stress paths in use :

(*i*) *Effective stress path (ESP):*

$$\frac{\sigma_1 - \sigma_3}{2} \text{ versus } \frac{\sigma_1' + \sigma_3'}{2} \text{ or } q \text{ versus } p'$$

(*ii*) *Total stress path (TSP):*

$$\frac{\sigma_1 - \sigma_3}{2} \text{ versus } \frac{\sigma_1 + \sigma_3}{2} \text{ or } q \text{ versus } p$$

(*iii*) *Path of total stress minus static pore pressure:* $(T - u_s)\,SP$

$$\frac{\sigma_1 - \sigma_3}{2} \text{ versus } \left(\frac{\sigma_1 + \sigma_3}{2} - u_s\right) \text{ ; or } q \text{ versus } p - u_s$$

where u_s = static pore pressure.

The stress path method has been used only in those situations when the vertical and horizontal stresses are principal stresses. For these situations, it is more convenient to use vertical and horizontal stresses rather than major and minor principal stresses. Thus, the three basic stress paths become:

(*i*) $\frac{\sigma_v - \sigma_h}{2}$ versus $\frac{\sigma_v' + \sigma_h'}{2}$; (*ii*) $\frac{\sigma_v - \sigma_h}{2}$ versus $\frac{\sigma_v + \sigma_h}{2}$

(*iii*) $\frac{\sigma_v - \sigma_h}{2}$ versus $\left(\frac{\sigma_v + \sigma_h}{2} - u_s\right)$

The basic *strain plot* is $\frac{\sigma_v - \sigma_h}{2}$ versus ε_v.

Figure 18.36 (*a*) shows the stress points *A*, *B*, *C*, *D*, *E*, *F* etc., *i.e.*, the points of maximum shear, and the line *ABCDEF* joining these points is the stress path. Figure 18.36 (*b*), shows the same stress path plotted on

$$p\left(=\frac{\sigma_1 + \sigma_3}{2} = \frac{\sigma_v + \sigma_h}{2}\right) \text{versus } q\left(=\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_v - \sigma_h}{2}\right)$$

Figure 18.36 (*c*) and (*d*) correspond to three dimensional compression in the laboratory. The line joining the origin to the point *P* of maximum shear stress corresponding to the Mohr circle at failure, represents the K_f - line. It is a line joining all such points of maximum shear of Mohr circles at failures to the origin. Its inclination ψ' to the horizontal bears a fixed relation to the inclination ϕ' of the failure envelope (called the ϕ' line)

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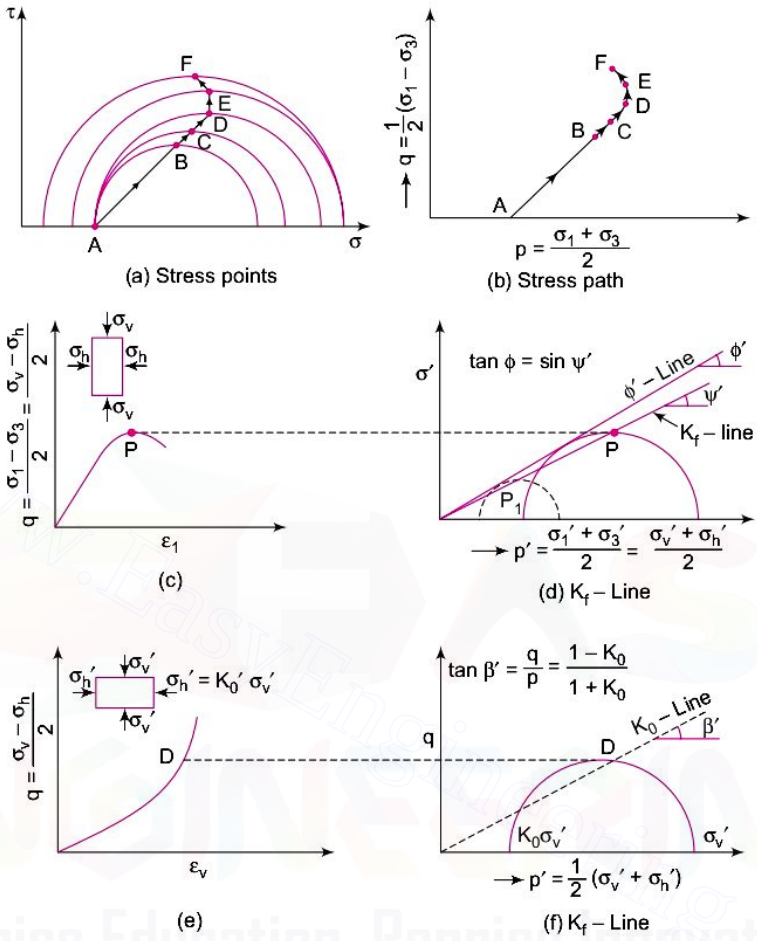

Fig. 18.36 Lambe's stress path : K_f and K_0 lines

$$\sin \psi' = \tan \phi' \qquad ...(18.45)$$

Figure 18.36 (*e*) and (*f*) correspond to the case of confined compression (oedometer) with no horizontal movement. The corresponding stress path K_0 indicates the way in which the stresses within a normally consolidated soil increase during the consolidation process. K_0 is the co-efficient of lateral earth pressure at rest defined by

$$K_0 = \frac{\sigma_h'}{\sigma_v'}; \quad \therefore \quad \sigma_h' = K_0 \ \sigma_v' \qquad ...(18.46)$$

The slope of the stress path K_0 [Fig. 18.36 (*f*)] is denoted by β', *i.e.*, for K_0 loading,

$$\tan \beta' = \frac{q}{p'} = \frac{\sigma_v' - \sigma_h'}{\sigma_v' + \sigma_h'} = \frac{1 - K_0}{1 + K_0} \qquad ...(18.47)$$

From Eq. 18.46, we get $\quad p' = \dfrac{1 + K_0}{2} \qquad ...(18.48)$

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and $$q = \frac{1 - K_0}{2} \qquad ...(18.49)$$

Figure 18.37 shows all three stress paths in a oedometer test, both during loading as well as unloading. In Fig. 18.37 (*a*), point *A* corresponds to the effective stress conditions at the beginning of any stress increment (*i.e.*, it has certain total vertical stress and a static pore pressure) while point *D* corresponds to the total stress conditions. When the vertical stress is quickly increased, the change in the pore pressure is equal to the change in total vertical stress, so that the effective vertical stress remains unchanged . The horizontal effective stress also remains unchanged (since K_0 is constant), but the horizontal total stress increase by the amount of the pore pressure increases Δu. Thus the effective stress remains at a point *A* during this undrained loading while the total stress path is *DE*. During the process of consolidation (*i.e.*, drained compression), the vertical total stress remains constant. As drainage occurs, pore pressure decreases, and the vertical and horizontal effective stresses increase. Thus, the effective stress path during drainage is *AB*. Since the shear is increasing, the horizontal stress must decrease, and hence the total stress path rises to left along *EF*.

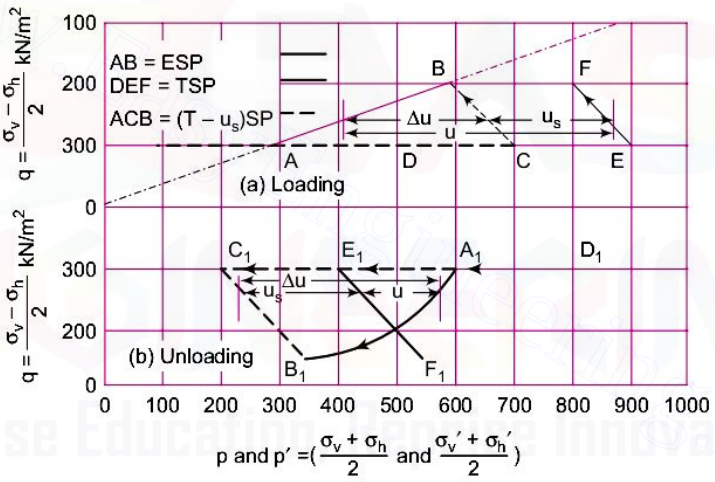

Fig. 18.37 Stress path for oedometer test

When the consolidation is complete, point *F* will reach the K_0-line. Thus, in Fig. 18.37 (*a*) during loading test, we have:

$AB = ESP$ (effective stress path)

$DEF = TSP$ (Total stress path)

$ACB = (T - u_s)\ SP$ (path of total stress minus pore pressure.)

Similarly, Fig. 18.37 (*b*) shows the three stress paths during an increment of undrained unloading followed by dissipation of excess pore pressure. Point D_1 corresponds, to the total stress, while A_1 corresponds to the effective stress, at the beginning of unloading. The total stress path is $D_1\ E_1$ followed by $E_1\ F_1$. The effective stress path remains at A_1 during the undrained unloading, and then follows A_1 B_1. $A_1\ C_1\ B_1$ represents the path of total stress minus static pore pressure, *i.e.*, $(T - u_s)\ SP$. Along the stress path $A_1\ B_1$, the value of K_0 increases. In an oedometer test of clay, line $A_1\ B_1$ can continue until it

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reaches $q = 0$ (*i.e.*, $K_0 = 1$) and, in fact, can go to negative values of q (*i.e.*, $K_0 > 1$). By running a series of oedometer tests in which both σ'_v and σ'_h were measured, Brooker and Ireland (1965) developed the relationship between K_0, over consolidation ratio and I_P. As an average for all normally consolidated clay, $K_0 \approx 0.95 \sin \phi'$. For normally consolidated sand, $K_0 \approx 1.0 - \sin \phi'$.

Figure 18.38 shows effective stress path for four types of tests that can be conducted readily in the triaxial apparatus. For a drained test, $ESP = (T - u_s)\ SP$. A stress path along the horizontal axis is obtained when $\sigma_v = \sigma_h$. Point A corresponds to the stress conditions during an all round cell pressure. AC represents the stress path for failure due to unloading, when $\Delta\sigma_h$ = negative and $\Delta\sigma_v = 0$. Similarly, AE represents the stress path due to unloading when $\sigma_h = 0$ and $\Delta\sigma_v$ = negative. AB represents the stress path for failure, due to loading, when $\Delta\sigma_h = 0$ and $\Delta\sigma_v$ = positive. Similarly, AD is the stress path for failure due to loading, when $\Delta\sigma_v = 0$ and $\Delta\sigma_h$ = positive. All these four stress paths are useful since they approximately correspond to the following four situations in soil mechanics:

(*i*) Path AC : case of active earth pressure.
(*ii*) Path AD : case of passive earth pressure.
(*iii*) Path AB : case of foundation loading.
(*iv*) Path BE : case of foundation excavation.

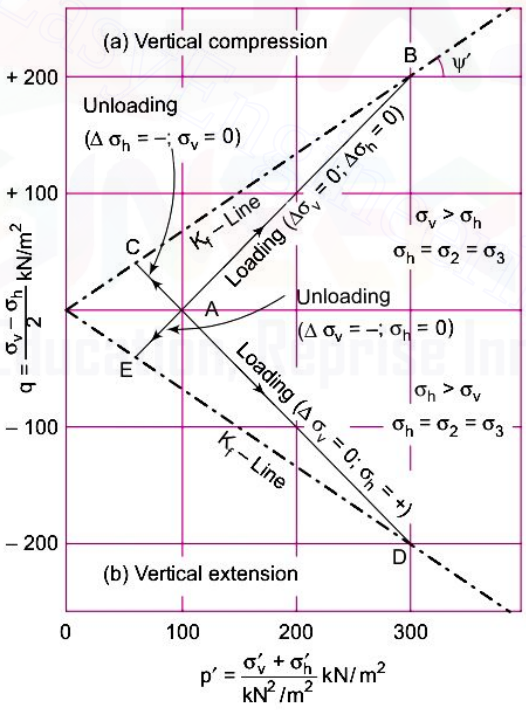

Fig. 18.38 Stress paths for triaxial test

Figure 18.39 (*a*) shows the actual total stress paths for the Rankine cases of active and passive pressure. Point A corresponds to the at-rest condition for normally consolidated soil. AC is the stress path for the active case, during which failure occurs due to decrease of σ_h (*i.e.*, σ_h decreases from its at-rest value to the active earth pressure value). AB is the stress path for passive case in which the failure occurs due to

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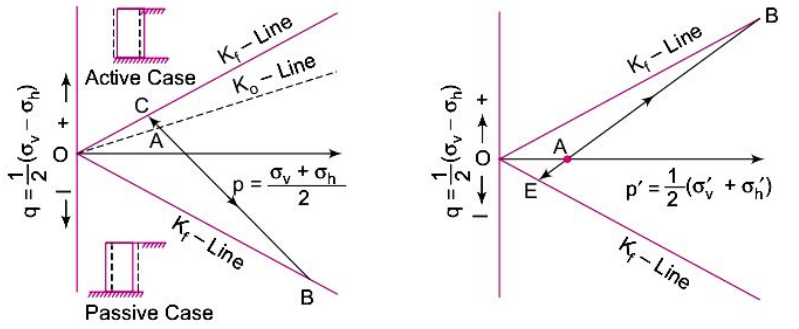
increase of σ_h (and hence increase in negative q and increase in p).



(a) Stress paths for rankine active and passive conditions

(b) Stress paths for foundation loading and excavation

Fig. 18.39

In Fig. 18.39 (*b*), *AB* is the effective stress path for foundation loading, and *AB* is the stress path for foundation excavation. In Fig. 18.39 (*b*), the stress path for foundation loading and the stress path for excavation are not exactly at 45° to the horizontal as was true stress paths *AB* and *AE* in Fig. 18.38. In the foundation loading, the most significant stress change is an increase in the vertical stress, but there is also an increase in the horizontal stress. In the excavation, the most important stress change is a decrease in vertical stress, but there is also a decrease in the horizontal stress. Thus, to perform a triaxial test that precisely duplicates the field situation, the experiment must vary both the horizontal and vertical stress during the test (Lambe: 1967).

The following important points about the stress-path are worth noting (Lambe : 1967):

1. Lines from any location on a stress path, drawn 45° with the vertical, intersect the horizontal axis at the vertical stress and horizontal stress acting on the soil element.
2. A negative value of q indicates that the horizontal stress is greater than the vertical stress.
3. A value of K_0 greater than one indicates that the horizontal effective stress is greater than the vertical effective stress.
4. *TSP* to the right of *ESP* indicates a positive pore-water pressure and *TSP* to the left of *ESP* indicates negative pore water pressure.
5. $(T - u_s)$ *SP* to right of *ESP* indicates positive excess pore pressure and $(T - u_s)$ *SP* to the left of *ESP* indicates negative excess pore pressure.

18.14 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 18.17. *In a direct shear test on sand, the normal stress was 2.0 kg/cm^2 and shear stress at failure was 0.8 kN/cm^2. Determine the orientations of the principal planes at failure.*

(Civil Services Exam. 1981)

Solution. Figure 18.40 shows the graphical solution. The failure envelope will pass through (*i*) the origin *O* and (*ii*) a point *F* whose coordinates are:

shear stress = 0.8 kg/cm^2 and normal stress = 2.0 kg/cm^2. Mohr circle is so drawn that it is tangential to the failure envelope at point *F*. Then *PA* is the direction of major principal plane which makes an angle

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of 57° in the clockwise direction with the plane of shear. Similarly, *PB* is the direction of minor principal plane, which makes angle of 33° in the anticlockwise direction, with the plane of shear.

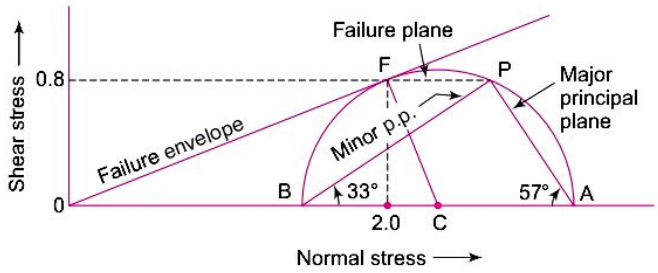

Fig. 18.40

Example 18.18. *A cylindrical sample of soil, having cohesion of 0.8 kg/cm^2 and angle of internal friction of 20°, is subjected to a cell pressure of 1.0 kg/cm^2. Calculate the maximum deviator stress at which the sample will fail and the angle made by the failure plane with the axis of the sample.*

(Civil Services Exam. 1984)

Solution. $\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$

where $\sigma_3 = 1.0$ kg/cm^2; $\alpha_f = 45° + \varphi/2 = 45° + 20/2 = 55°$; $c = 0.8$ kg/cm^2

$\therefore$ $\sigma_1 = 1 \tan^2 55° + 2 \times 0.8 \tan 55° = 4.32$ kg/cm^2

$\therefore$ $\sigma_d = \sigma_1 - \sigma_3 = 4.32 - 1 = \mathbf{3.32\ kg/cm^2}$

Angle made by the failure plane with the axis of the sample

$= 90° - \alpha_f = 90° - 55° = \mathbf{35°}$.

Example 18.19. *An embankment 5 m high is made up of soil whose effective stress parameter are $c' = 50$ kN/m^2 and $\phi' = 16$ deg. and $\gamma = 16.2$ kg/m^3. The pore pressure parameters as found from triaxial tests are $A = 0.4$ and $B = 0.92$. Find the shear strength of the soil at the base of the embankment just after the soil has been raised from 5 m to 8 m. Assume that the dissipation of pore pressure during the stage of construction is negligible and that the lateral pressure at any point is one half of the vertical pressure.*

(Civil Services Exam. 1989)

Solution. This problem is based exactly on example 18.12 of this book. All the data is the same, except that the embankment has been raised from 5 to 8 m, instead of 5 to 9 m given in example 18.12.

$$\Delta\sigma_1 = \gamma\, \Delta H = 16.2\ (8 - 5) = 48.6 \text{ kN/m}^2; \quad \Delta\sigma_3 = \frac{1}{2} \Delta\, \sigma_1 = 24.3 \text{ kN/m}^2$$

Now, $\Delta u = B\,[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] = 0.92\ [24.3 + 0.4\ (48.6 - 24.3)] \approx 31.3$ kN/m^2

Now, original pressure, $\sigma_1 = 16.2 \times 5 = 81$ kN/m^2

$\therefore$ Effective stress, $\sigma' = \sigma_1 + \Delta\sigma_1 - \Delta u = 81 + 48.6 - 31.3 = 98.3$ kN/m^2

$\therefore$ Shear strength $= c' + \sigma' \tan \phi' = 50 + 98.3 \tan 16° = \mathbf{78.19\ kN/m^2}$.

Example 18.20. *A standard specimen of cohesionless sand was tested in triaxial compression and the sample failed at a deviator stress of 482 kN/m^2, when the cell pressure was 100 kN/m^2, under drained condition. Find the effective angle of shearing resistance of sand. What would be the deviator stress and the major principal stress at failure for another identical specimen of sand if it is tested under a cell pressure of 200 kN/m^2 ?*

(Civil Services Exam. 1990)

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Solution Given. $\sigma_d = 482$ kN/m^2 ; $\sigma_3 = 100$ kN/m^2 ; $c = 0$

$\therefore$ $\sigma_1 = \sigma_3 + \sigma_d = 100 + 482 = 582$ kN/m^2

Now, $\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f = \sigma_3 \tan^2 \alpha_f$

$\therefore$ $\tan^2 \alpha_f = \dfrac{\sigma_1}{\sigma_3} = \dfrac{582}{100} = 5.82$, from which $\alpha_f = 67.485 = 45° + \varphi/2$

$\therefore$ $\varphi = 2\,(67.485° - 45°) = \mathbf{44.97°}$

For another specimen, $\sigma_3 = 200$ kN/m^2 $\therefore$ $\sigma_1 = \sigma_3 \tan^2 \alpha_f = 200 \times 5.82 = \mathbf{1164\ kN/m^2}$

and $\sigma_d = \sigma_1 - \sigma_3 = 1164 - 200 = \mathbf{964\ kN/m^2}$.

Example 18.21. *A lateral pressure in a triaxial compression test in a cohesive soil gave the following results: angle of shearing resistance $\varphi = 17.5$ deg.; cohesion = 3.0 kg/cm^2; total axial stress at failure= 18 kg/cm^2. Determine the lateral pressure.* **(Civil Services Exam. 1991)**

Solution. Given $\varphi = 17.5°$; $c = 3.0$ kg/cm^2; $\sigma_1 = 18$ kg/cm^2; $\sigma_3 = ?$

$\alpha_f = 45° + \varphi/2 = 45° + 17.5°/2 = 53.75°$; $\tan \alpha_f = 1.3638$

Now, $\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$ or $18 = \sigma_3\,(1.3638)^2 + 2 \times 3\,(1.3638)$

From which $\sigma_3 = \mathbf{5.28\ kg/cm^2}$.

Example 18.22. *A cohesive soil specimen was tested under triaxial compression with a cell pressure of p_s. Failure of the specimen occurred under a total pressure of 35 N/cm^2. With the same soil, direct shear test was also done. Shearing forces at failure were 450 N and 325 N under normal loads of 1000 N and 500 N respectively. The sectional area of the shear box was 36 cm^2. Find the cell pressure p_s at failure.* **(Civil Services Exam. 1992)**

Solution. Figure 18.41 shows the plotting of the shear box test data. If C is the cohesion, and m is the slope of the line, we have

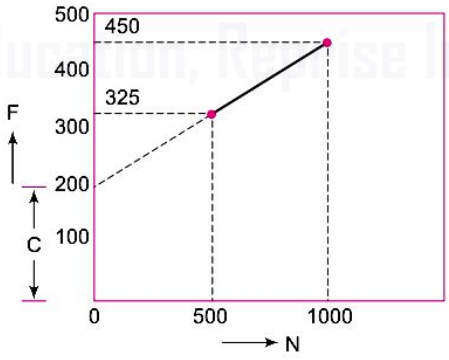

Fig. 18.41

$$m = \frac{450 - 325}{1000 - 500} = \frac{450 - C}{1000 - 0}$$

or $$450 - C = 1000 \times \frac{125}{500}$$

From which $C = 200$ N.

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$\therefore$ Unit cohesion $c = 200/36 = 5.6$ N/cm^2

Now, for the triaxial test, we have

$\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$ where $\alpha_f = 45° + \varphi/2 = 45°$, since the soil is cohesive

$\therefore$ $\sigma_1 = \sigma_3 \tan^2 45° + 2c \tan 45° = \sigma_3 + 2c$

$\therefore$ $\sigma_3 = \sigma_1, - 2c = 35 - 2 \times 5.56 = \mathbf{23.89\ N/cm^2}$

Example 18.23. *A shear box test carried out on a sandy clay gave the following results:*

Vertical load (kg)	*Division of proving ring dial gauge (one division = 1 μm)*
36.8	*17*
73.5	*26*
110.2	*35*
146.9	*44*

Shear box is 60 mm × 60 mm and the proving ring constant is 20 N/μm. Determine the apparent cohesion and angle of internal friction for this soil. **(Civil Services Exam. 1995)**

Solution. The dial gauge readings are converted into shear force at failure, as tabulated below

S.No.	*Vertical load N (N)*	*Shear force F (N)*
1.	$36.8 \times 9.81 = 361$	$17 \times 20 = 340$
2.	$73.5 \times 9.81 = 721$	$26 \times 20 = 520$
3.	$110.2 \times 9.81 = 1081$	$35 \times 20 = 700$
4.	$146.9 \times 9.81 = 1441$	$44 \times 20 = 880$

Figure 18.42 shows a plot between F and N.

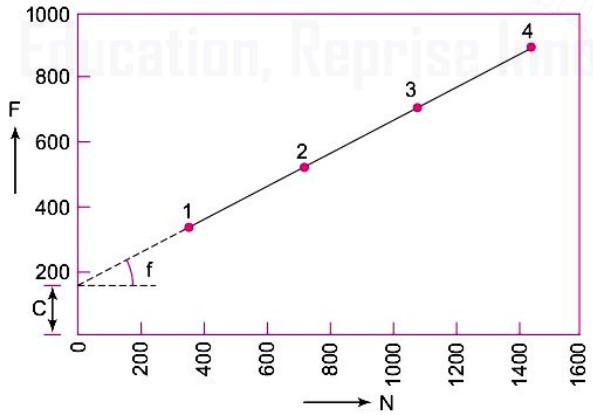

Fig. 18.42

$$\tan \varphi = \frac{F_3 - F_2}{N_3 - N_2} = \frac{700 - 520}{1081 - 721} = \frac{180}{360} = 0.5$$

From which, we get $\varphi = \mathbf{26.6°}$

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Now, $F_2 = C + N_2 \tan \varphi$

$\therefore$ $C = F_2 - N_2 \tan \varphi = 520 - 721 \times 0.5 = 159.5$ N

$\therefore$ $c = \dfrac{C}{\text{area}} = \dfrac{159.5}{60 \times 60} = 0.0443$ N/mm^2 = **44.3 kN/m^2**

Example 18.24. *A consolidated undrained triaxial test was conducted on a normally consolidated clay yielding the following data:*

$$\sigma_3 = 250 \text{ kN/m}^2 \text{ ; } (\Delta\sigma_d)_f = 275 \text{ kN/m}^2$$

Determine (i) the angle of friction, (ii) angle which the failure plane makes with the major principal plane, and (iii) normal stress and shear stress on the failure plane. **(Civil Services Exam. 1996)**

Solution. Given $\sigma_3 = 250$ kN/m^2 ; $\sigma_d = 275$ kN/m^2

$\therefore$ $\sigma_1 = \sigma_3 + \sigma_d = 250 + 275 = 525$ kN/m^2

Now, for undrained test on normally consolidated clay, we have $c = 0$.

Hence, $\sigma_1 = \sigma_3 \tan^2 \alpha_f$ or $\tan \alpha_f = \sqrt{\sigma_1/\sigma_3} = \sqrt{525/250} = 1.4491$

From which $\alpha_f = \mathbf{55.4°}$ and $\varphi = 2\,\alpha_f - 90° = 2 \times 55.4 - 90° = \mathbf{20.8°}$

Now, normal stress, $\sigma = \dfrac{\sigma_1 + \sigma_3}{2} + \dfrac{\sigma_1 - \sigma_3}{2} \cos 2\alpha_f$ (From Eq. 18.4)

$$= \frac{525 + 250}{2} + \frac{525 - 250}{2} \cos 2 \times 55.4° = \mathbf{338.67 \text{ kN/m}^2}$$

and shear stress, $\tau = \dfrac{\sigma_1 - \sigma_3}{2} \sin 2\alpha_f = \dfrac{525 - 250}{2} \sin 2 \times 55.4° = \mathbf{128.54 \text{ kN/m}^2}$.

Example 18.25. *A particular soil failed under a major principal stress of 300 kN/m^2 with a corresponding minor principal stress of 100 kN/m^2. If, for the same soil, the minor principal stress has been 200 kN/m^2, determine the major principal stress for (a) $\varphi = 30°$ (b) $\varphi = 0°$.*

(Civil Services Exam. 1997)

Solution. $\sigma_1 = \sigma_3 \tan^2 \alpha_f + 2c \tan \alpha_f$ ...(1)

(*a*) **When $\varphi = 30°$** $\alpha_f = 45° + \varphi/2 = 45° + 30/2 = 60°$; $\tan \alpha_f = 1.732$; $\tan^2 \alpha_f = 3$

$300 = 100 \times 3 + 2c\,(1.732)$, from (1)

From which $c = 0$. Hence Eq. (1) reduces to $\sigma_1 = \sigma_3 \tan^2 \alpha_f$

When $\sigma_3 = 200$ kN/m^2, $\sigma_1 = 200\,(3) = \mathbf{600 \text{ kN/m}^2}$

(*b*) When $\varphi = 0°$ $\alpha_f = 45° + 0/2 = 45°$; $\tan \alpha_f = 1$; $\tan^2 \alpha_f = 1$

$\therefore$ $300 = 100 \times 1 + 2c\,(1)$

From which $c = 100$ kN/m^2. Hence Eq. (1) reduces to

$\sigma_1 = \sigma_3 \tan^2 \alpha_f + 200 \tan \alpha_f = \sigma_3 + 200$

When $\sigma_3 = 200$ kN/m^2, $\sigma_1 = 200 + 200 = \mathbf{400 \text{ kN/m}^2}$.

Example 18.26. *In an unconfined compression test, a sample of clay 100 mm long and 50 mm in diameter fails under a load of 150 N at 10% strain. Calculate the shearing resistance taking into account the effect of change in cross-section of the sample.* **(Civil Services Exam. 1998)**

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Solution. Given $\Delta L/L = 0.1$

From Eq. 18.26, $$A_f = \frac{A}{1 - \frac{\Delta L}{L}} = \frac{\frac{\pi}{4}(50)^2}{1 - 0.1} = 2181.7 \text{ mm}^2$$

$\therefore$ $$q_u = \frac{P_f}{A_f} = \frac{150}{2181.7} = 0.06875 \text{ N/mm}^2 = 68.75 \text{ kN/m}^2$$

$\therefore$ Shear resistance $= \frac{q_u}{2} = \frac{68.75}{2} \approx \mathbf{34.38 \text{ kN/m}^2}$.

Example 18.27. *Calculate the potential shear strength on a horizontal plane at a depth of 3 m below the surface in a formation of cohesionless soil when the water table is at a depth of 3.3 m. The degree of saturation may be taken as 0.5 on the average; void ratio = 0.5; grain specific gravity =2.7; angle of internal friction = 30°. What will be the modified values of shear strength if water table reaches the ground surface?* **(Civil Services Exam. 1999)**

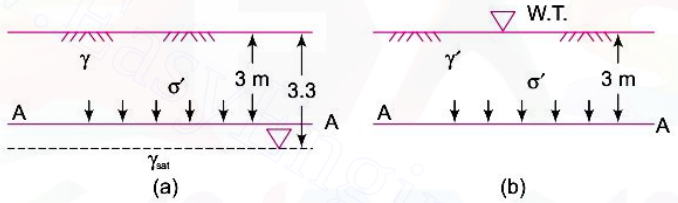

Fig. 18.43

Solution.

(*a*) ***Water table at 3.3 m depth*** **(Fig. 18.43 *a*)**

$$w = \frac{es}{G} = \frac{0.5 \times 0.5}{2.7} = 0.0926$$

$\therefore$ $$\gamma = \frac{G\gamma_w}{1+e}(1+w) = \frac{2.7 \times 9.81}{1+0.5}(1+0.0926) = 19.293 \text{ kN/m}^3$$

$\therefore$ σ' at AA, at 3 m depth $= \gamma H = 19.293 \times 3 = 57.879 \text{ kN/m}^2$

$\therefore$ $$\tau_f = c + \sigma' \tan \varphi = 0 + 57.879 \tan 30° = \mathbf{33.42 \text{ kN/m}^2}$$

(*b*) ***Water table at G.L. (Fig. 18.43 b)***

$$w_{sat} = \frac{es}{G} = \frac{0.5 \times 1}{2.7} = 0.1852$$

$$\gamma_{sat} = \frac{G\gamma_w}{1+e}(1+w_{sat}) = \frac{2.7 \times 9.81}{1+0.5}(1+0.1852) = 20.928 \text{ kN/m}^3$$

$$\gamma' = 20.928 - 9.81 = 11.118 \text{ kN/m}^2$$

$\therefore$ σ' at $AA = \gamma' H = 11.118 \times 3 = 33.354 \text{ kN/m}^2$

Hence, $$\tau_f = c + \sigma' \tan \varphi = 0 + 33.354 \tan 30° = \mathbf{19.26 \text{ kN/m}^2}.$$

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Example 18.28. *In a formation of cohesionless soil, the water table is at a depth of 3 m. The degree of saturation may be taken as 0.5 on the average void ratio 0.5; grain specific gravity 2.70; angle of internal friction = 30°. Calculate the potential shear strength on a horizontal plane at a depth of 2.5 m below the surface. Also, calculate the modified value of the shear strength if the water table reaches the ground surface.* **(Civil Services Exam. 2001)**

Solution. Given $e = 0.5$; $S = 0.5$;

$G = 2.7$; $\varphi = 30°$

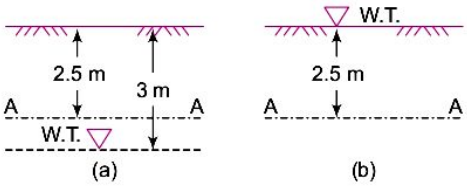

Fig. 18.44

(a) ***Water table 3 m depth*** **(Fig. 18.44 *a*)**

$$w = \frac{es}{G} = \frac{0.5 \times 0.5}{2.7} = 0.0926$$

$$\gamma = \frac{G\gamma_w}{1+e}(1+w) = \frac{2.7 \times 9.81}{1+0.5}(1+0.0926)$$

$$= 19.293 \text{ kN/m}^3$$

$\therefore$ σ' at 2.5 m depth $= 2.5 \times 19.293 = 48.233 \text{ kN/m}^2$

$$\tau_f = c + \sigma' \tan \varphi = 0 + 48.233 \tan 30° = \mathbf{27.85 \text{ kN/m}^2}$$

(b) ***Water table at Ground surface*** **(Fig. 18.44 *b*)**

$$w_{sat} = \frac{e}{G} = \frac{0.5}{2.7} = 0.185$$

$$\gamma_{sat} = \frac{2.7 \times 9.81}{1+0.5}(1+0.185) = 20.928 \text{ kN/m}^2 \text{ ; } \gamma' = 20.928 - 9.81 = 11.118$$

$\therefore$ σ' at 2.5 m depth $= 11.118 \times 2.5 = 27.795 \text{ kN/m}^2$

$\therefore$ $\tau_f = 27.795 \tan 30° = \mathbf{16.05 \text{ kN/m}^2}$

Example 18.29. *A direct shear test is run on a medium dense sandy silt. At failure, normal stress is 65 kPa and shear stress is 41 kPa. Draw Mohr's circle and determine (i) the angle of shearing resistance, (ii) principal stresses at failure, (iii) orientation of failure plane, principal planes and the plane of maximum shear stress at failure.* **(Civil Services Exam. 2002)**

Solution. Refer Example 18.3 and Fig. 18.15

The failure envelope will pass through origin O and a point F having the following coordinates: (*i*) normal stress = 65 kPa and (*ii*) shear stress = 41 kPa. Hence plot point F and draw line OF, which represents failure envelope. At F, draw FC at 90° to OF, getting the centre C of Mohr circle. Draw the Mohr circle, cutting the abscissa at points A and B. Thus $OA = \sigma_1 = 140$ kPa and $OB = \sigma_3 = 43$ kPa.

From F draw a horizontal line FP representing the failure plane. The point P so obtained is the *pole*, the origin of planes. Thus AP is the major principal plane which is inclined at an angle $\alpha_f = 61°$ with the

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failure plane. Similarly, *BP* represents the minor principal plane, inclined at 90° to the major principal plane. Also, we get $\varphi = \angle FOC = 32°$ and $2\,\alpha_f = \angle FCA = 122°$.

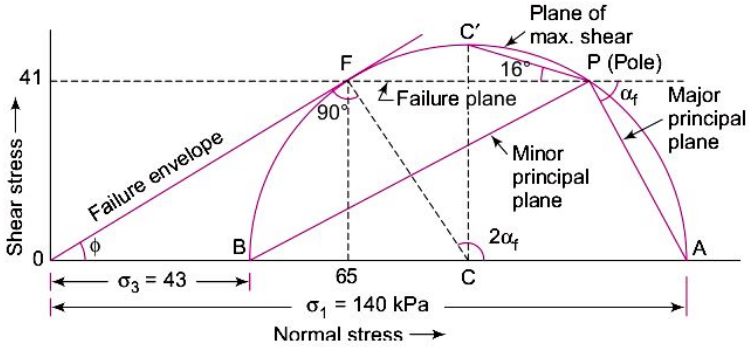

Fig. 18.45

Draw vertical line *CC′*. Then *C′* represents the point corresponding to the maximum shear stress. Evidently *C′P* is the direction of plane of maximum shear stress, which is inclined at an angle = 16° with the failure plane *FP*.

Example 18.30. *Two samples of a soil were tested in a triaxial machine. The all-round pressure maintained for the first was 2.0 kg/cm^2 and failure occurred at additional axial stress of 7.7 kg/cm^2, while for the second, these values were 5.0 kg/cm^2 and 13.7 kg/cm^2 respectively. Find c and φ of the soil.*

(Engg. Services Exam. 1979)

Solution. In general, we have $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$ ...(1)

For the first test, $\sigma_3 = 2$ kg/cm^2 and $\sigma_1 = \sigma_3 + \sigma_d = 2 + 7.7 = 9.7$ kg/cm^2

For the second test, $\sigma_3 = 5$ kg/cm^2 and $\sigma_1 = \sigma_3 + \sigma_d = 5 + 13.7 = 18.7$ kg/cm^2

$\therefore \quad 9.7 = 2 \tan^2 \alpha + 2c \tan \alpha$

or $2a^2 + 2ca - 9.7 = 0$ where $a = \tan \alpha$ ...(2)

and $5a^2 + 2\,ca - 18.7 = 0$ ...(3)

From (3) and (2), $3a^2 = 9$, from which $a = 1.732$

Also, from (2), $2\,(3) + 2\,c\,(1.732) = 9.7$ from which $c = \mathbf{1.068\ kg/cm^2}$

Now, $\tan \alpha = a = 1.732$ from which $\alpha = 60° = 45° + \varphi/2$

$\therefore \quad \varphi = \mathbf{30°}$

Example 18.31. *A cohesive soil has an angle of shearing of 15° and a cohesion of 35 kN/m^2. If the specimen of this soil is subjected to a triaxial compression test, find the value of the lateral pressure in the cell for failure to occur at total stress of 300 kN/m^2.* **(Engg. Services Exam. 1983)**

Solution. $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$

Here, $\varphi = 15°$, $\therefore \quad \alpha = 45° + 15°/2 = 52.5°$; $\tan \alpha = 1.303$; $\tan^2 \alpha = 1.698$

$\therefore \quad 300 = \sigma_3\,(1.698) + 2\,(35)\,(1.303)$ from which $\sigma_3 = \mathbf{122.96\ kN/m^2}$

Example 18.32. *A cylindrical specimen of dry sand was tested in a triaxial test. Failure occurred under a cell pressure of 1.2 kg/cm^2 and at a deviator stress of 4.0 kg/cm^2.*

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(i) What is the angle of shearing resistance of the soil?
(ii) What were the normal and shear stresses on the failure plane?
(iii) What angle did the failure plane make with the minor principal plane?
(iv) What was the maximum shear stress on any plane in the specimen at the instant of failure and how was the plane, in question, be oriented with the major principal plane?

(Engg. Services Exam. 1987)

Solution. Given: $\sigma_3 = 1.2$ kg/cm^2 ; $\sigma_d = 4.0$ kg/cm^2

$$\sigma_1 = \sigma_3 + \sigma_d = 1.2 + 4 = 5.2 \text{ kg/cm}^2$$

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$$

Since $c = 0$, $\sigma_1 = \sigma_3 \tan^2 \alpha$

$$\therefore \quad \tan \alpha = \sqrt{\sigma_1/\sigma_3} = \sqrt{5.2/1.2} = 2.082$$

$$\therefore \quad \alpha = 64.34° = 45° + \varphi/2 \quad \text{or} \quad \varphi = 2\,(64.34° - 45°) \approx 38.7°$$

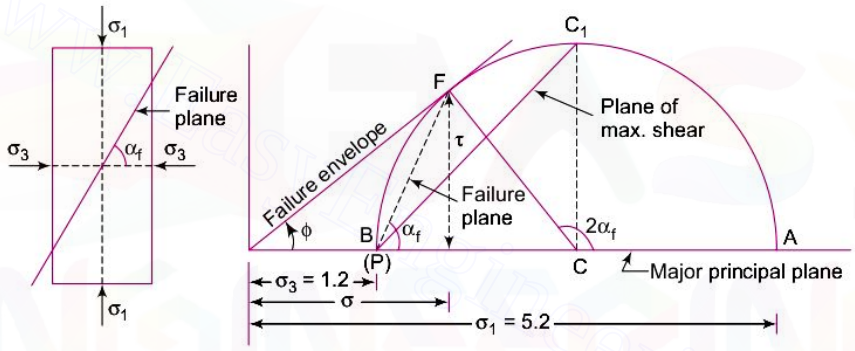

Fig. 18.46

(*i*) Hence angle of shearing resistance = **38.7°**

(*ii*) Shear stress, $$\tau = FF_1 = \frac{\sigma_1 - \sigma_3}{2} \sin 2\alpha = \frac{4}{2} \sin (2 \times 64.34°) = \mathbf{1.56 \text{ kg/cm}^2}$$

Normal stress, $$\sigma = OF_1 = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha$$

$$= \frac{5.2 + 1.2}{2} + \frac{5.2 - 1.2}{2} \cos (2 \times 64.34°) = \mathbf{1.95 \text{ kg/cm}^2}$$

(*iii*) Angle made by the failure plane with the major principal plane, $\alpha_f = 64.34°$

$\therefore$ Angle made by the failure plane with the minor principal plane = $90 - \alpha$

$$= 90 - 64.34° = \mathbf{25.66°}$$

(*iv*) Maximum shear stress, $$\tau = \frac{\sigma_1 - \sigma_3}{2} = \frac{5.2 - 1.2}{2} = \mathbf{2 \text{ kg/cm}^2}$$

Angle made by the plane of maximum shear stress with the major principal plane

$$= \angle C_1 BC = \tan^{-1} \frac{CC_1}{BC} = \tan^{-1} \frac{2}{2} = \mathbf{45°}$$

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Example 18.33. *A shear vane of 7.5 cm diameter and 11.0 cm length was used to measure the shear strength of soft clay. If torque of 600 kg-cm was required to shear the soil, calculate the shear strength. The vane was then rotated rapidly to cause remoulding of the soil. The torque required in the remoulded state was 200 kg-cm. Determine the sensitivity of the soil and offer your comments on the same.*

(Engg. Services Exam. 1992)

Solution. Refer to Example 18.11.

$$\tau_f = \frac{T}{\pi d^2\left[\frac{H}{2}+\frac{d}{6}\right]} = \frac{600}{\pi(7.5)^2\left[\frac{11}{2}+\frac{7.5}{6}\right]} = \mathbf{0.503\ kg/cm^2}$$

$\therefore$ $c = \tau_f = \mathbf{0.503\ kg/cm^2}$.

***For remoulded state*:** The shear strength will be in direct proportion to the torque

$$\therefore \quad \text{Sensitivity} = \frac{c \text{ in undisturbed condition}}{c \text{ in disturbed condition}} = \frac{T \text{ in undisturbed condition}}{T \text{ in disturbed condition}} = \frac{600}{200} = 3$$

***Comments*:** Since the sensitivity is between 2 to 4, the clay is *normal clay* with honeycomb structure.

Example 18.34. *laboratory results on a soil have shown that its unconfined compressive strength is 1.2 kg/cm^2. In a triaxial compression test, a specimen of the soil when subjected to a chamber pressure of 0.4 kg/cm^2 failed at an additional stress of 1.6 kg /cm^2.*

Estimate the shearing strength of the same soil along a horizontal plane at a depth of 4 m in a deposit. The ground water table is at a depth of 2.5 m from the ground level. Take dry unit weight of soil as 1.7 g/cc and specific gravity as 2.7. **(Engg. Services Exam. 1995)**

Solution.

(*a*) ***From unconfined compression test***

$\sigma_1 = 2c \tan\alpha$ (since σ_3 is zero) $\quad\therefore\quad 1.2 = 2c \tan\alpha$ ...(1)

(*b*) ***Triaxial test***

$\sigma_3 = 0.4$ kg/cm^2 ; $\sigma_d = 1.6$ kg/cm^2 ;

$\therefore$ $\sigma_1 = 0.4 + 1.6 = 2$ kg/cm^2

Now, $\sigma_1 = \sigma_3 \tan^2\alpha + 2c \tan\alpha$

or $2 = 0.4 \tan^2\alpha + 2c \tan\alpha$ ...(2)

Substituting the value of $2c \tan\alpha$ from (1), we have

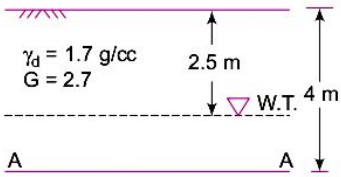

Fig. 18.47

$2 = 0.4 \tan^2\alpha + 1.2$

From which $\tan\alpha = 1.4142$

or $\alpha = 54.74° = 45° + \varphi/2$

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$\therefore \quad \varphi = 2\,(54.74° - 45°) = 19.47°$

Substituting the value of tan α in (1) we get $c = 0.424$ kg/cm^2 = 4.24 t/m^2

(c) $$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 1}{1.7} - 1 = 0.588$$

$$\therefore \quad \gamma' = \frac{G-1}{1+e}\gamma_w = \frac{2.7-1}{1+0.588} \times 1 = 1.07 \text{ g/cm}^3 = 1.07 \text{ t/m}^3$$

Now, $\sigma' = 2.5 \times 1.7 + 1.07 \times 1.5 = 5.855$ t/m^2

$\therefore$ Shear strength $\tau = c + \sigma' \tan \varphi = 4.24 + 5.855 \tan 19.47 =$ **6.31 t/m^2.**

Example 18.35. *A CU (consolidated undrained) test was conducted on a soil sample with cell pressure $\sigma_c = 100$ kN/m^2. The deviator stress at failure $(\sigma_1 - \sigma_c)_f$ was observed to be 60 kN/m^2. The soil is known to have a cohesion $c' = 0$ and angle of shearing resistance $\varphi' = 30°$ (referred to effective stress) and an undrained cohesion $c_u = 0$ and angle of shearing resistance $\varphi_u = 13.3°$ (referred to total stress). What was the pore pressure at failure ?* **(Engg. Services Exam. 1997)**

Solution.

$$\sigma_c = \sigma_3 = 100 \text{ kN/m}^2$$

$$\sigma_d = \sigma_1 - \sigma_c = 60 \text{ kN/m}^2 ; \quad \therefore \quad \sigma_1 = 60 + \sigma_c = 60 + 100 = 160 \text{ kN/m}^2$$

(a) ***Total stress analysis:*** Given $c_u = 0$; $\varphi_4 = 13.3°$

$$\therefore \quad \sin \varphi c_u = \sin \varphi_u = \frac{FC}{OC} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

$$\sin \varphi c_u = \frac{160 - 100}{160 + 100} = \frac{60}{260}$$

or $\varphi c_u = 13.34°$ (equal to the value given)

(b) ***Effective stress analysis:***

Given $c' = 0$; $\varphi' = 30°$

$$\therefore \quad \sin \varphi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

where $\sigma_1' = \sigma_1 - u$ and $\sigma_3' = \sigma_3 - u$

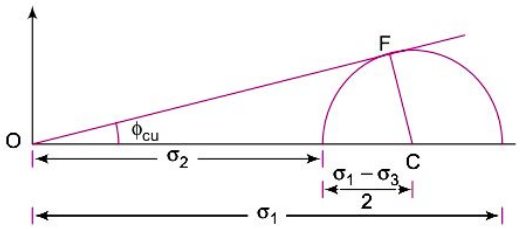

Fig. 18.48

$$\therefore \quad \sin \varphi' = \frac{(\sigma_1 - u) - (\sigma_3 - u)}{(\sigma_1 - u) + (\sigma_3 - u)} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2u}$$

$$\therefore \quad \sin 30° = \frac{160 - 100}{160 + 100 - 2u} = \frac{60}{260 - 2u} \text{ from which } u = \textbf{70 kN/m}^2.$$

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Example 18.36. *When an unconfined compression test is conducted on a cylinder of soil, it fails under axial stress of 1.2 kg/cm^2. The failure plane makes an angle of 50° with the horizontal. Determine the cohesion and angle of internal friction.* **(Gate Exam. 1988)**

Solution. $\alpha_f = 50° = 45° + \varphi/2$; $\therefore \quad \varphi = (50 - 45)\,2 = 10°$

Now, $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$; Here $\sigma_3 = 0$

$\therefore$ $\sigma_1 = 2c \tan \alpha$ or $c = \dfrac{\sigma_1}{2 \tan \alpha_f} = \dfrac{1.2}{2 \tan 50°} = \mathbf{0.503\ kg/cm^2.}$

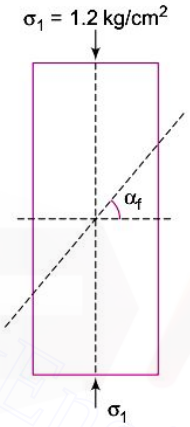

Fig. 18.49

Example 18.37. *In a consolidated drained triaxial test, a specimen of clay fails at a cell pressure of 60 kN/m^2. The effective shear strength parameters are $c' = 15$ kN/m^2 and $\varphi' = 20°$. Determine the compressive strength.* **(Gate Exam. 1991)**

Solution. Given $\sigma_3 = 60$ kN/m^2; $c' = 15$ kN/m^2 ; $\varphi' = 20$

$\alpha_f = 45° + \varphi'/2 = 55°$; $\tan \alpha_f = 1.428$; $\tan^2 \alpha_f = 2.04$

$\therefore$ $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c' \tan \alpha = 60 \times 2.04 + 2 \times 15 \times 1.428 = 165.24$ kN/m^2

$\therefore$ Compressive strength = vertical compressive stress $\sigma_d = \sigma_1 - \sigma_3$

$= 165.24 - 60 = \mathbf{105.24\ kN/m^2.}$

Example 18.38. *A given saturated clay is known to have effective strength parameters of $c' = 10$ kPa and $\varphi' = 28°$. A sample of this clay was brought to failure quickly so that no dissipation of the pore water pressure, could occur. At failure, it was known that $\sigma_1' = 60$ kPa, $\sigma_3' = 10$ kPa and $u_f = 20$ kPa.*

(a) Estimate the value of σ_1 and σ_3 at failure

(b) What was the effective normal stress on the failure plane ?

(c) What was the value of undrained shear strength c_u (s_u) ? **(Gate Exam. 1996)**

Solution.

(*a*) $\sigma_1 = \sigma_1' + u = 60 + 20 = \mathbf{80\ kPa}$; $\sigma_3 = \sigma_3' + u = 10 + 20 = \mathbf{30\ kPa}$

(*b*) $\varphi' = 28°$; Hence $\alpha_f = 45° + \varphi'/2 = 45 + 28/2 = 59°$

Now, $\sigma' = \dfrac{\sigma_1' + \sigma_3'}{2} + \dfrac{\sigma_1' + \sigma_3'}{2} \cos 2\alpha_f = \dfrac{60 + 10}{2} + \dfrac{60 - 10}{2} \cos 2 \times 59° = \mathbf{23.26\ kPa}$

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(c) ***Undrained shear strength parameters*:** While *drained* shear strength is worked out corresponding to effective stress parameters (c', φ'), the *undrained* shear strength parameters are worked out corresponding to total stress parameters c_u, φ_u.

$\therefore$ Undrained shear strength, $s_u = c_u + \sigma \tan \varphi_u$

For undrained shear strength, $\varphi_u = 0$ so that $s_u = c_u$

$\therefore$ $\tan \alpha = \tan 45° + \varphi_u/2 = \tan 45° = 1$

Now, $\sigma_1 = \sigma_3 \tan^2 \alpha + 2c_u \tan \alpha$

or $80 = 30(1)^2 + 2c_u(1)$, from which $c_u = \mathbf{25\ kN/m^2}$.

Example 18.39. *Two identical soil specimen were tested in a triaxial apparatus. First specimen failed at a deviator stress of 770 kN/m^2 when the cell pressure was 200 kN/m^2. Second specimen failed at a deviater stress of 1370 kN/m^2 under a cell pressure of 400 kN/m^2. Determine the value of c and φ analytically. If the same soil is tested in a direct shear apparatus with a normal stress of 600 kN/m^2, estimate the shear stress at failure.* **(Gate Exam. 1997)**

Solution. First test : $\sigma_3 = 200$ kN/m^2; $\sigma_d = 770$ kN/m^2; $\sigma_1 = 200 + 770 = 970$ kN/m^2

$\therefore$ $970 = 200 \tan^2 \alpha + 2c \tan \alpha$ ...(1)

Second test: $\sigma_3 = 400$ kN/m^2; $\sigma_d = 1370$ kN/m^2; $\therefore$ $\sigma_1 = 400 + 1370 = 1770$ kN/m^2

$\therefore$ $1770 = 400 \tan^2 \alpha + 2c \tan \alpha$ ...(2)

From (1) and (2), $200 \tan^2 \alpha = 800$ from which $\tan \alpha = 2$,

$\therefore$ $\alpha = 63.43° = 45° + \varphi/2$ or $\varphi = \mathbf{36.87°}$

Again, from (1), $970 = 200\ (2)^2 + 2c\ (2)$, from which $\boldsymbol{c = 42.5\ kN/m^2}$

For shear box test, $\tau = c + \sigma \tan \varphi$

$\therefore$ $\tau = 42.5 + 600 \tan 36.87° = \mathbf{492.50\ kN/m^2}$.

Example 18.40. *What is the shear strength in terms of effective stress on a plane within a saturated soil mass at a point where the total normal stress is 295 kPa and the pore water pressure 120 kPa? The effective shear strength parameter are $c' = 12$ kPa and $\varphi' = 30°$.* **(Gate Exam. 1998)**

Solution. Given: $\sigma = 295$ kPa; $u = 120$ kPa

$\therefore$ Effective stress, $\sigma' = \sigma - u = 295 - 120 = 175$ kPa

$\therefore$ Shear strength, $s = c' + \sigma' \tan \varphi' = 12 + 175 \tan 30° = 113.04$ kPa (113.04 kN/m^2)

Example 18.41. *A c-u triaxial compression test was performed on a saturated sand at a cell pressure of 100 kPa. The ultimate deviator stress was 350 kPa and the pore pressure at the peak stress was – 40 kPa (suction) Estimate the total and effective stress shear strength parameters.* **(Gate Exam. 2001)**

Solution. $\sigma_3 = 100$ kPa ; $\sigma_d = 350$ kPa ; $u = -40$ kPa

$\sigma_1 = \sigma_3 + \sigma_d = 100 + 350 = 450$ kPa

$\sigma_1' = \sigma_1 - u = 450 - (-40°) = 490$ kPa

$\sigma_3' = \sigma_3 - u = 100 - (-40°) = 140$ kPa

(a) ***Total stress parameters*:** For saturated sand $c = 0$

$$\sin \varphi_u = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{450 - 100}{450 + 100} = \frac{350}{550} = 0.6364 \quad \therefore \quad \varphi_u = \mathbf{39.52°}$$

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(*b*) ***Effective stress parameters*:** $c' = 0$

$$\therefore \quad \sin\varphi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{490 - 140}{490 + 140} = 0.5556\,; \quad \therefore \quad \varphi' = \mathbf{33.75°}$$

18.15 LABORATORY EXPERIMENTS

EXPERIMENT 18 : DETERMINATION OF SHEAR PARAMETERS BY DIRECT SHEAR TEST

Object and Scope. The object of the test is to determine the shear parameters of a soil with the help of shear box test.

Materials and Equipment (*i*) Shear box equipment consisting of : (*a*) Shear box, 60 mm square and 50 mm deep, so that samples of size 60 × 60 × 24 mm may be tested. The box should be divided into two parts horizontally, with suitable spacing screws, (*b*) container for shear box, (*c*) grid plates, two pairs, one pair plain and other pair perforated : depth of serrations to be 1.5 mm, (*d*) porous stones, one pair, 6 mm thick, (*e*) base plate, with cross-grooves on its top face to fit into the shear box, (*f*) loading pad, with a steel ball on its top, (*ii*) Loading frame, to distribute the load from the yoke over the specimen, normal to the shear plane, (*iii*) Set of weights for normal load, (*iv*) Proving ring with dial gauge accurate to 0.002 mm to measure the shear force, (*v*) Micrometer dial gauge, two nos., accurate to 0.01 mm to measure horizontal and vertical displacements during shear, (*vi*) Spatula, straight edge, sample trimmer etc, (*vii*) Stop watch.

Test Procedure

(*i*) Preparation of specimen

1. The undisturbed specimen is prepared by pushing a cutting ring of size 10 cm in diameter and 2 cm high, in the undisturbed soil sample obtained from field. The square specimen of size 6 cm × 6 cm is then cut from the circular specimen so obtained.
2. In order to obtain remoulded specimen of cohesive soil, the soil may be compacted to the required density and water content, in a separate bigger mould. The sample is then extracted and trimmed to the required size. Alternatively, the soil may be compacted at the required density and water content directly into the shear box after fixing two halves of the shear box together by means, of the fixing screws.
3. Non-cohesive soils may be tamped in the shear box itself with the base plate and grid plate or porous stone as required in place at the bottom of the box.
   In all the three cases mentioned above, water content and dry density of the soil compacted in the shear box should be determined.

(*ii*) Undrained test

1. The shear box with the specimen *plain grid plate* over the base plate at the bottom of the specimen, and plain grid plate over the top of the specimen, should be fitted into position. The serrations of the grid plates should be placed at right angles to the direction of shear. As the porous stones are not used in the undrained tests, plain plates of equal thickness should be placed, one at the bottom and the other at the top of the two grids, so as to maintain the shear plane in the sample in the middle of its thickness. Place the loading pad on the top of the plain grid plate. Both the parts of the box should be tightened together by the fixing screws.
2. Put water inside the water jacket so that the sample does not get dried during the test.

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3. Mount the shear box assembly on the load frame (or shearing machine). Set the lower part of the shear box to bear against the load jack and the upper part of the box to bear against the proving ring. Set the dial of the proving ring to zero.
4. Put the loading yoke on the top of the loading pad, and adjust the dial gauge to zero to measure the vertical displacement in the soil sample. Put proper normal weight on the hanger of the loading yoke, so that this weight plus the weight of the hanger equals the required normal load. Note the reading of the vertical displacement dial gauge.
5. Remove the locking screws so that the parts are freed to move against each other. By turning the spacing screws, raise the upper part slightly above the lower parts by about 1 mm.
6. Conduct the test by applying horizontal shear load to failure or to 20 per cent longitudinal displacement, whichever occur first. The rate of strain may vary from 1 to 2.5 mm per minute. Start the stop watch immediately at the start of the application of the shear load. Take the readings of proving ring dial gauge, longitudinal displacement gauge and vertical displacement gauge at regular time intervals.
7. At the end of the test, remove specimen from the box and determine its final water content. Repeat the above steps on three or four identical specimens, undervarying normal loads.

(*iii*) Consolidated undrained test

1. Assemble the box in the similar manner as described above for the undrained test, except that instead of plain grid plates, perforated grid plates should be used, and saturated porous stones, should be used one at the top of the top perforated grid plate and the other at the bottom of the bottom perforated grid plate.
2. Step (2), (3) and (4) same as in undrained test.
3. The shear test should be conducted only after *complete* consolidation has occurred under a particular normal stress. After the application of the normal load, the vertical compression of the soil with time should be recorded, as is done in the consolidation test.
4. Apply the horizontal shear load. The rate of shear should be such that water does not drain from the specimen at the time of application of shear load. Take the reading of the three dial gauges at constant interval of time.
5. Remove the specimen from the box, at the end of the test, and determine its final water content. Repeat the test on three or four identical specimens, under varying normal loads.

(*iv*) Consolidated drained test

1. Assemble the shear box with sample, perforated grid plates and porous stones, as in consolidated undrained test. Allow the sample to consolidate under the normal load. After the application of the normal load, vertical compression of the soil with time should be recorded, as is done in the consolidation test.
2. When consolidation has completely occurred, the shear test should be done at a slow rate so that complete drainage can occur and at least 95 per cent pressure dissipation occurs during the test . IS : 2720 (Part XIII)-1986 recommends a rate of strain of 0.2 mm/min. for sandy soils, and 0.01 mm/min or less for clay. Actual rate of strain suitable for the soil under test may be ascertained by first determining the value of c_v from the consolidation data (obtained in step 1) using the following expression:

$$c_v = \frac{0.197h^2}{t_{50}}$$

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where $2h$ = initial thickness of the specimen

t_{50} = time corresponding to 50% consolidation determined from the plot of vertical dial reading (R) versus log of time.

The requisite time to failure, when theoretically 95 per cent dissipation is ensured may be obtained from the following equation:

$$t_f = \frac{h^2}{nc_v(1-U_c)} = \frac{20h^2}{3c_v}$$

where t_f = time to failure ; n = constant for drainage from both ends = 3

U_c = degree of pore pressure dissipation.

Knowing t_f, and the approximate strain expected at failure, the rate of strain for the test may be calculated. In case of cohesive soils, the failure may be assumed as taking place at 20 per cent deformation 3. At the end of the test, remove the specimen from the box and determine its final water content. Repeat the test on three or four identical specimens, under varying normal loads.

Tabulation of observations. Test observations are tabulated as illustrated in Table 18.4.

Table 18.4 Data and observation sheet for direct shear test

1. **Soil Specimen Measurements:**
   - Dimensions: Area of specimen:
   - Thickness: Volume of specimen:
   - Initial mass of specimen: Initial water content:
   - Bulk density: Dry density:
   - Final wet mass of specimen: Final water content at shear zone,
2. **Consolidation data:**
   - Hanger load:
   - Normal stress: Applied load:

Date and time	*Vertical dial reading*	*Vertical dial difference*	*Thickness of specimen*

3. **Shearing data:**

Date and time	*Displace-ment dial reading*	*Displa-cement δ*	*Area correction*	*Corrected area*	*Stress dial reading*	*Shear force*	*Shear stress*	*Vertical dial reading*	*Vertical dial difference*	*Thickness of specimen*

Note : Use separate table for each test. Plot the stress-shear displacement curve and find (*a*) Maximum shear stress and (*b*) corresponding shear displacement.

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4. Summary of results

Test No.	*Normal stress*	*Shear stress at failure*	*Shear displacement at failure*	*Initial water content*	*Final water content*	*Remarks*
1.						
2.						
3.						
4.						

Calculations and Results

The loads corresponding to proving ring dial readings can be calculated from the calibration curve of the proving ring. The load divided by the corrected cross-sectional area of the specimen gives the shear stress in the sample. The corrected cross-sectional area is calculated from the following equation:

$$\text{Corrected area} = A_0 \left(1 - \frac{\delta}{3}\right)$$

where A_0 = initial area of the specimen; δ = displacement, at any time.

Plot the stress-longitudinal displacement readings and obtain (*a*) maximum shear stress and (*b*) corresponding longitudinal displacement.

Plot the applied normal stress as abscissa and maximum shearing stress as ordinate. The slope of the straight line so obtained would give the angle of shearing resistance and the vertical intercept of the line will give the cohesion intercept. In case the plot is not a straight line, shear parameters are obtained by drawing a tangent to the normal stress-maximum shear stress curve at the points of maximum normal stress expected in the field.

Notes: 1. The choice of spacing between the two halves of the shear box is dependent on the size of the largest soil grain and the denseness of the soil which is being tested. A larger spacing is required for loose soils.

2. The normal stresses to be selected for the test should correspond to the field conditions and design requirements.

3. In general, failure in direct shear may be considered to take place either at maximum shear or at the maximum obliquity of the Mohr failure envelope. If the failure is assumed to take place at maximum shear and not at maximum obliquity, the angle of shearing resistance thus obtained will be smaller, giving an error, if any, on the safe side. It should, however, be noted that differences in the values of the angle of shearing resistance obtained by using the two criteria are more important for sands than for clays.

EXPERIMENT 19: DETERMINATION OF UNCONFINED COMPRESSIVE STRENGTH OF SOIL

Object and Scope. The object of the experiment is to determine the unconfined compressive strength of clayey soil using controlled strain. The purpose of the test is to obtain a quantitative value of compressive and shearing strength of soils in an undrained state. The test may be performed on both undisturbed and remoulded soil specimen.

Materials and Equipment. (*i*) Compression device of any suitable type, (*ii*) Sample ejector, (*iii*) Strain measuring dial gauge with 0.01 mm graduations, (*iv*) Stop watch, (*v*) Oven, (*vi*) Balance, (*vii*) Miscellaneous equipment, such as specimen trimming and curving tools, remoulding apparaturs, moisture cans, etc., (*viii*) Split mould 3.5 cm diameter and 7 cm long (or 3.75 cm diameter and 7.5 cm long).

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Test procedure

(*i*) Preparation of test specimen:

1. Undisturbed cylindrical specimen may be cut from the bigger undisturbed sample obtained from the field. A wire saw may be used to trim the ends parallel to each other. A lathe or trimmer may be used to trim the specimen to circular cross-section. Alternatively, field sample may be obtained directly in a thin sampling tube having the same internal diameter as the specimen to be tested. The split mould is oiled lightly from inside and the sample is then pushed out of the tube into the split mould. The split mould is opened carefully and sample taken is out.
2. Remoulded sample may be prepared by compacting the soil at the desired water content and dry density in a bigger mould, and then cut by the sampling tube. Alternatively, remoulded specimen may be prepared directly in the split mould.

   In both the cases, the density and water content of the specimen is determined.

(*ii*) Compression test:

1. Measure the initial length and diameter of the specimen.
2. Put the specimen on the bottom plate of the loading device. Adjust the upper plate to make contact with the specimen. Set the load dial gauge and the strain (compression) dial gauge to zero.
3. Compress the specimen until cracks have definitely developed or the stress strain curve is well past its peak or until a vertical deformation of 20 per cent is reached. Take the load dial readings approximately at every 1 mm deformation of the specimen.
4. Sketch the failure pattern; measure the angle between the cracks and the horizontal, if possible, and if the specimen is homogeneous.

Tabulation of observations. The test results are tabulated as shown in Table 18.5.

Table 18.5 Data and observation sheet for determination of unconfined compressive strength

1. Initial diameter or specimen (D_0);
2. Initial length (L_0);
3. Initial area (A_0);
4. Initial density;
5. Initial water content.

S.No.	*Elapsed time in min.*	*Load*	*Deformation in mm*	*Strain* ϵ *(%)*	*Area A* (cm^2) $A = \frac{A_0}{1-\epsilon}$	*Stress*

Calculations: The axial strain ϵ is determined by the following equation:

$$\epsilon = \frac{\Delta L}{L_0}$$

where ΔL = change in specimen, as read from the strain dial

L_0 = initial length of the specimen. The average cross-sectional area, A, at a particular strain is determined from the following relation:

$$A = \frac{A_0}{1-\epsilon}$$ where A_0 = initial average area of specimen.

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The compressive stress is determined by:

$$\sigma = \frac{P}{A} \text{ ; where } P = \text{compressive force.}$$

A plot is made between σ and ϵ. The maximum stress from this curve gives the values of the unconfined compressive strength q_u. Where no maximum occurs, the unconfined compressive strength is taken as the stress at 20 per cent axial strain.

EXPERIMENT 20: DETERMINATION OF SHEAR PARAMETERS BY TRIAXIAL TEST

Object and Scope. The object of the experiment is to determine shear parameters of undisturbed (or remoulded) soil specimen in the triaxial compression apparatus by unconsolidated undrained test without the measurement of pore pressure.

Material and Equipment (*i*) Triaxial cell without transparent chamber capable of resisting internal fluid pressure of 1000 kN/m^2 complete with all accessories, (*ii*) Apparatus for applying and maintaining the desired fluid pressure in the cell, to an accuracy of 10 or 5 kN/m^2, (*iii*) Compression machine, capable of applying axial compression to the specimen at convenient speeds to cover the range 0.05 to 7.5 mm per minute, (*iv*) Dial gauge to measure axial compression, accurate to 0.01 mm, (*v*) Seamless rubber membranes, (*vi*) Membrane stretcher, (*vii*) rubber rings, (*viii*) Split mould, trimming knife, wire saw-meter straight-edge, sample extruder, thin walled tubes, soil lathe, meter box etc., (*ix*) Water content determination tins, (*x*) Balance, (*xi*) Stop watch.

Test Procedure

(*i*) Sample Preparation

1. ***Undisturbed specimen.*** If the undisturbed sample has been collected in a thin walled tube having the same internal diameter as that of the specimen required for testing, the sample may be extruded out with the help of sample extruder, and pushed into the split mould. The sample should be extruded from the tube *pusing from the cutting edge side.* The ends of the specimen are trimmed flat and normal to its axis. The split mould should be lightly oiled from inside. The specimen is then taken out, carefully, from the split mould, and its length, diameter, weight should be measured to an accuracy enabling the bulk density to be calculated to an accuracy of ±1.0 per cent. A portion of the soil trimmings is placed for water content determination. The specimen is then placed on one of the end caps and the other end cap is put on the top of the specimen. The rubber membrane is then placed around the specimen using the membrane stretcher. The membrane is sealed to the end caps by means of rubber rings. The specimen is then ready to be placed on the pedestal in the triaxial cell.
2. ***Undisturbed specimen.*** If the undisturbed soil sample brought from the field is of large diameter than the specimen diameter, the sample may be cut to size either by means of thin walled tube or hand trimming or by a soil lathe. If a block sample has been obtained from the field, a rectangular prism slightly larger than the required final dimensions of the specimen is cut from the block sample. The ends of prism are made plane and parallel using meter box. The prism is then placed in the soil lathe, and excess soil is cut off in thin layers. The trimming operation, rotating the sample between end cutting operation, is continued until a cylindrical specimen results. The rest of the procedure for enclosing the specimen in the membrane etc., is the same as described in step-1.

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3. ***Remoulded specimen.*** Remoulded specimens may be prepared by compacting the soil, at required water content and dry density, in a big size mould by static or dynamic method, and then preparing the cylindrical specimen of required dimensions by the method described in step-2 above.

(*ii*) Undrained triaxial compression test

1. Cover the pedestal in the triaxial cell with a solid end cap or keep drainage valve closed. Place the specimen assembly centrally on the pedestal. Assemble the cell, with the loading ram initially clear of the top of the specimen, and place it in the loading machine.
2. Admit the operating fluid in the cell, and raise its pressure to the desired value. Adjust the loading machine to bring the loading ram a short distance away from the set on the top cap of the specimen. Read the initial reading to the load measuring gauge. Adjust the loading machine further so that the loading ram comes just in contact with the seat on the top of the specimen. Note the initial reading of the dial measuring axial compression.
3. Apply the compressive force at constant rate of axial compression, such that failure is produced in a period of approximately 5 to 15 minutes. Take the simultaneous reading of load and deformation dials, define the stress-strain curve. Continue the test until the maximum value of stress has been passed or until an axial strain of 20 per cent has been passed.
4. Unload the specimen and drain off the cell fluid. Dismantle the cell and take out the specimen. Remove the rubber membrane and note down the mode of failure. Weigh the specimen. Keep samples for water content determination.
5. Repeat the test on three or more identical specimens under different cell pressure.

Tabulation of observations. The test results are tabulated as shown in Table 18.6.

Table 18.6 Data and observation sheet for undrained triaxial test

(*i*) Soil specimen measurement:

Height: Area:
Diameter: Volume:
Initial mass: Water content:
Final mass: Final water content:

(*ii*) Undrained triaxial test:

Cell pressure (σ_3)
Load gauge constant:

Comp. dial reading	*Load gauge reading*	*Comp. of sample*	*Strain*	*Corrected area*	*Load*	*Vertical stress* σ_1	*Deviator stress* $\sigma_1 - \sigma_3$	*Stress Ratio* σ_1/σ_3

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(iii) Summary of results:

Test No.	*Cell pressure (σ_3)*	*Deviator stress at failure ($\sigma_1 - \sigma_3$)*	*σ_1 at failure*	*$\frac{\sigma_1}{\sigma_3}$ at failure*
1.				
2.				
3.				
4.				

Calculations and Results

1. The difference between the initial pressure and subsequent reading of the load measuring device is the axial load applied to the specimen in addition to that due to cell pressure.
2. The area A of the specimen at any stage of the test is determined by

$$A = \frac{A_0}{1-\varepsilon} = \frac{A_0 L_0}{L}$$

where A_0 = initial area of specimen; $\varepsilon = \frac{L_0 - L}{L_0}$

L_0 = initial length of specimen

L = length of specimen at the stage of the test at which area A is to be determined.

3. The deviator stress ($\sigma_1 - \sigma_3$) or the principal stress difference at any stage of the test is determined by dividing the additional axial load by the area A.
4. For each test, a plot may be made between ($\sigma_1 - \sigma_3$) and ε.
5. The shear parameters are obtained from a plot of Mohr circles for which purpose peak value of principal stress difference ($\sigma_1 - \sigma_3$) or principal stress-ratio (σ_1/σ_3) or the ultimate value as desired may be used.

PROBLEMS

1. Two identical specimens of dry sand are tested in the triaxial cell apparatus with confining pressure of 150 kN/m^2 and 250 kN/m^2 respectively. If the angle of internal friction of sand is 35°, what are the values of the additional axial stress at failure of the specimens? [**Ans.** 550 kN/m^2 ; 920 kN/m^2]

2. Two identical specimen of a soil were tested in triaxial apparatus. First specimen failed at a deviator stress 770 kN/m^2 when the cell pressure was 200 kN/m^2 , while the second specimen failed at a deviator stress of 1370 kN/m^2 under a cell pressure of 400 kN/m^2. Determine the value of c and ϕ for the soil.
If the same soil is tested in a direct shear apparatus, estimate the shear at which the sample will fail under a normal stress of 600 kN/m^2. [**Ans.** $\phi = 30°$; $c = 50$ kN/m^2 ; $s = 396$ kN/m^2]

3. Following are the results of a shear box test on a soil:

Normal stress (kN/m^2 or kPa)	*Shear stress at failure (kN/m^2 kPa)*
25	60
75	80
150	105
250	145

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Determine the values of c and ϕ. If a specimen of the same soil is tested in a triaxial apparatus, estimate the deviator stress at which the sample will fail when the cell pressure is 100 kN/m^2 (100 kPa).

[**Ans.** 50 kN/m^2 (kPa); $\phi = 20°$; c_d = 247 kN/m^2 (kPa)]

4. A soil specimen having c = 86 kN/m^2 and $\phi = 30°$ is tested in a triaxial apparatus. Estimate (*a*) the deviator stress at which the sample will fail when the cell pressure is 60 kN/m^2 ; (*b*) the cell pressure if the soil sample fails at a major principal stress of 900 kN/m^2. [**Ans.** (*a*) 720 kN/m^2 ; (*b*) 100 kg/m^2]
5. If the major and minor principal stresses through a mass of soil at the instant of failure are 600 kN/m^2 and 200 kN/m^2 respectively, calculate the values of normal and shear stress on a plane making an angle of 30°with the direction of minor principal stress. [**Ans.** σ = 500 kN/m^2 ; τ = 173 kN/m^2]
6. A cylinder of soil fails under an axial stress of 80 kN/m^2. The failure plane makes an angle of 48° with the horizontal. Calculate the value of cohesion and the angle of internal friction of the soil.

[**Ans.** $\phi = 6°$; c = 360 kN/m^2]

7. A sample of cohesionless sand in a direct shear test fails under a shear stress of 160 kN/m^2 when the normal stress is 140 kN/m^2. Find the angle of shearing resistance and the principal stress failure.

[**Ans.** $\phi = 34°$; σ_1 = 538 kN/m^2 ; σ_3 = 155 kN/m^2]

8. Consolidated undrained triaxial tests are performed on two identical specimens of saturated, remoulded clay with pore pressure measurements. The observations are recorded in the table below:

Test No.	*Consolidation pressure (kN/m^2 or k Pa)*	*Cell pressure at failure (kN/m^2 or kPa)*	*Deviator stress at failure (kN/m^2 or kPa)*	*Pore pressure at failure (kN/m^2 or kPa)*
1.	250	250	179	101
2.	350	350	242	145

Determine the values of the shear strength parameters, cohesion and friction, for the clay both in terms of total and effective stresses. If in the consolidated undrained test, an identical specimen is first consolidated under a cell pressure of 400 kN/m^2 (kPa), what would be the deviator stress at failure?

[**Ans.** $\phi_{cu} = 15°$; $\phi' = 22°$; $c = 0$; σ_d = 280 kN/m^2 (280 kPa)]

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CHAPTER 19

Failure Envelopes*

19.1 INTRODUCTION

For the case of plain strain, the condition of limiting equilibrium can be expressed by the following equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \qquad ...(19.1(a))$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} = \gamma \qquad ...(19.1(b))$$

where, σ_x, σ_z are the normal stress, and τ_{xz} is the shearing stress on the plane. From equation 19.1, it is clear that conditions of equilibrium alone are not sufficient to provide the stress field within the soil body since there are three unknown stresses. In order to relate these three stresses in a unique way, another independent expression should exist, so that the resulting system could fulfil the needs for assessing the adequacy of a structure subjected to loading. Since such an expression has to provide a measure of the circumstances governing motion of a part of the soil body, it is said to be a *yield* or *failure criterion.* The yield criterion may be expressed as

$$f(I_1, I_2, I_3) = 0 \qquad ...(19.2)$$

where, I_1, I_2 and I_3 are the invariants of stress. For the assumptions of homogenity and isotropy, it can also be expressed as

$$F(\sigma_1, \sigma_2, \sigma_3) = 0$$

where, σ_1, σ_2 and σ_3 are the principal stresses given by the roots of Segner's equation

$$\sigma^3 - I_1 \sigma^2 + I_2 \sigma - I_3 = 0 \qquad ...(19.3)$$

If all the stress combinations sufficient to cause failure are represented by the above fundamental relationship, they could trace out a surface called the *yield* or *failure surface* or *failure envelope.* In as much as any combination of stress falling on the surface represents a limiting condition or state of failure, the determination of its shape is of paramount importance.

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The shape of the failure surface, in a three-dimensional system of co-ordinates (σ_1, σ_2, σ_3) that Eq. 19.3 represents, can be seen from Fig. 19.1. *OA* represents the space diagonal or the hydrostatic axis along which $\sigma_1 = \sigma_2 = \sigma_3$ inclined at $\alpha = \cos^{-1}\frac{1}{\sqrt{3}} = 54° 44'$ with the three axes. Experiments conducted by Crossland (1965) have demonstrated that a pure state of hydrostatic pressure ($\sigma_1 = \sigma_2 = \sigma_3$) is unable to produce failure even under enormous pressure. Hence the failure surface may not intersect the hydrostatic axis for the +ve (compressive) stresses.

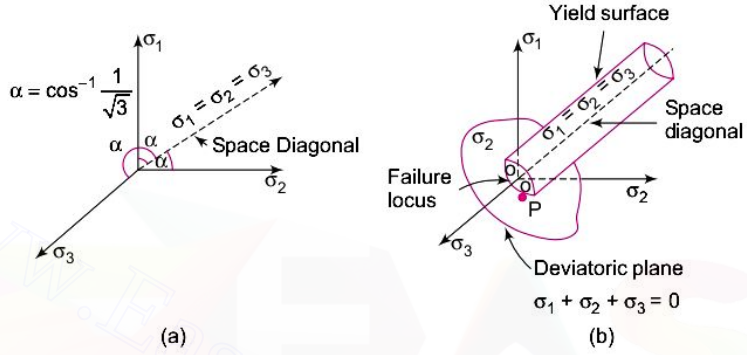

Fig. 19.1

Let us now introduce an auxiliary plane perpendicular to the hydrostatic axis. Evidently, the auxiliary plane will be an octahedral plane the equation of which is

$$\sigma_1 + \sigma_2 + \sigma_3 = \text{constant} \qquad ...(19.4)$$

If the auxiliary plane passes through the origin, as shown in Fig. 19.1 (*b*) its equation is

$$\sigma_1 + \sigma_2 + \sigma_3 = 0 \qquad ...(19.4a)$$

Since all the points in the plane represent deviatorial state of stress, the plane is also called the *deviatoric plane*. The intersection of the yield surface or envelope with octahedral plane is called the *failure locus*. For convenience, in Fig. 19.1 (*b*), the yield surface is shown as a cylinder, and failure locus as a circle. The point *P* on the failure locus represents the stress state at incipient failure, while a point *Q*, completely within the failure locus corresponds to a stable condition.

19.2 THE MOHR-COULOMB FAILURE ENVELOPE

Mohr's theory of failure can be symbolically expressed by:

$$\sigma_{max} - \sigma_{min} = f(\sigma_{max} + \sigma_{min}) \qquad ...(19.5a)$$

or

$$\tau_{max} = F(\sigma_n) \qquad ...(19.5b)$$

The major drawback of Mohr criterion is that the curved shape of the envelope, and hence functional form of Eq. 19.5 is not amenable to mathematical computations. The use of coulomb equation $\tau = c + \sigma_n \tan\phi$ (a straight line relationship) to represent the Mohr envelope in the Mohr diagram is termed the Mohr-Coulomb theory, and can be expressed in its most general form as follows:

$$[(\sigma_1 - \sigma_2)^2 - \{2c\cos\phi + (\sigma_1 + \sigma_2)\sin\phi\}^2] \times [(\sigma_2 - \sigma_3)^2 - \{2c\cos\phi + (\sigma_2 + \sigma_3)\sin\phi\}^2] \times$$
$$[(\sigma_3 - \sigma_1)^2 - \{2c\cos\phi + (\sigma_3 + \sigma_1)\sin\phi\}^2] = 0 \qquad ...(19.6)$$

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The principal stresses (σ_1, σ_2, σ_3) can in turn become the major, intermediate and minor principal stresses. Equation 19.6 can also be expressed in terms of effective stress parameters.

The Mohr-Coulomb criterion implicity assumes that the strength is independent of the intermediate principal stress σ_2. The effect of the intermediate principal stress was studied by D.C. Drucker and W. Prager (1952), who presented the concept of a three-dimensional yield envelope. Under the assumption that the soil yields plastically at failure and no strain hardening occures, the Mohr-Coulomb criterion can be extended to the three-dimensional stress space. If, in addition, the failure criterion is independent of intermediate principal stress, the resulting envelope is a hexagonal pyramid, as illustrated in Fig. 19.5.

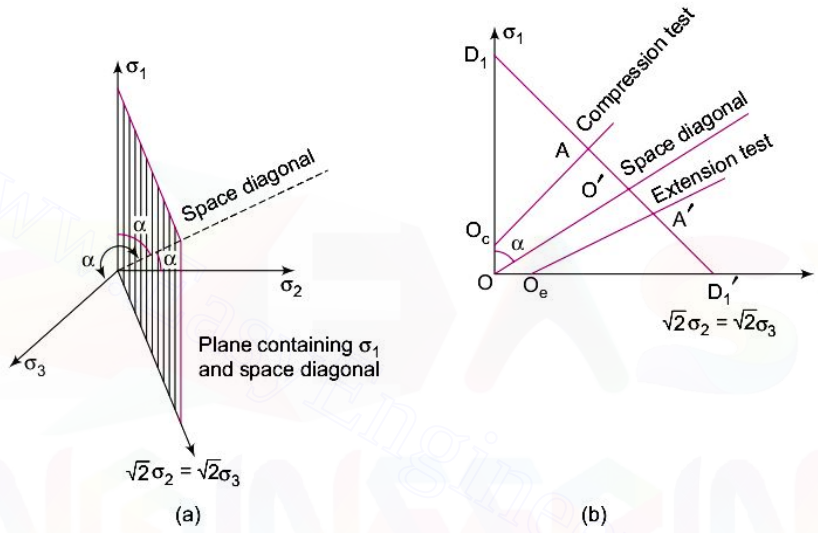

Fig. 19.2 Projection on plane containing σ_1 and space diagonal

In order to trace the failure surface governed by Mohr-Coulomb criterion, consider the principal stress co-ordinate axes σ_1, σ_2, σ_3 shown in Fig. 19.2(*a*), and the section of the surface formed by a plane containing both the axis and the space diagonal [Fig. 19.2(*b*)]. Let us now examine the view of a right section of the surface *i.e.*, section taken at right angles to the space diagonal. It is assumed that the failure lines $O_c A$ and $O_c A'$ are straight and represent the Mohr-Coulomb failure condition and that the parameters c and ϕ are independent of the intermediate principal stress. The right section is shown in Fig. 19.5, where D_1 , D_2 and D_3 are the points of penetration of the three axes; the point A represents failure in compression tests ($\sigma_1 > \sigma_2 = \sigma_3$) and A' represents failure in extension tests ($\sigma_2 = \sigma_3 > \sigma_1$) in a conventional triaxial apparatus. Similarly, point B and B' (Fig. 19.5) can be obtained by considering the right section of the plane containing σ_2 axis and the space diagonal (Fig. 19.3), and points C and C' by taking the right section of the plane containing σ_3 axis and the space diagonal (Fig. 19.4). It can be shown mathematically that the straight lines connecting these points represent failure conditions for intermediate values of the second principal stress. Thus, the shape of the right section (*i.e.*, failure locus) is a hexagon with sides of equal length but opposits sides not being parallel. The surface in space is a hexagonal pyramid with its apex at a origin of the system of co-ordinates and extending in the compressive direction of axes. The right sections are similar in shape but increase in dimension with the distance along the space diagonal at which they are taken. For special case $\phi = 0$, the yield stresses in tension and compression are equivalent and the hexagon becomes regular.

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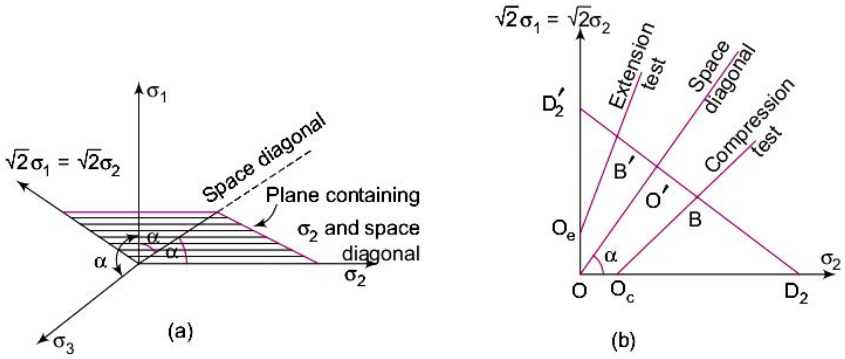

Fig. 19.3 Projection on plane containing σ_2 and space diagonal

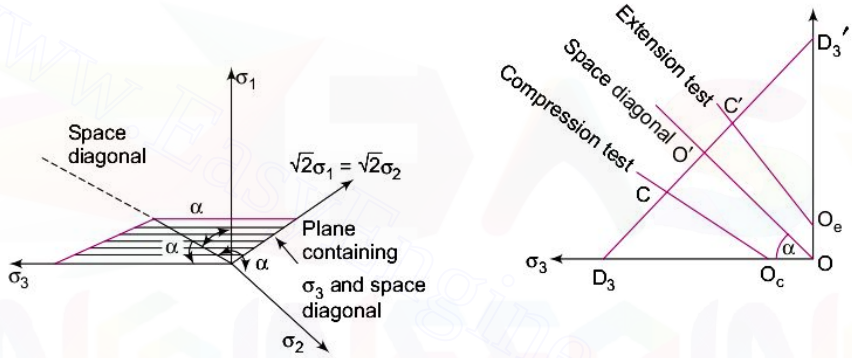

Fig. 19.4 Projection on plane containing σ_3 and space diagonal

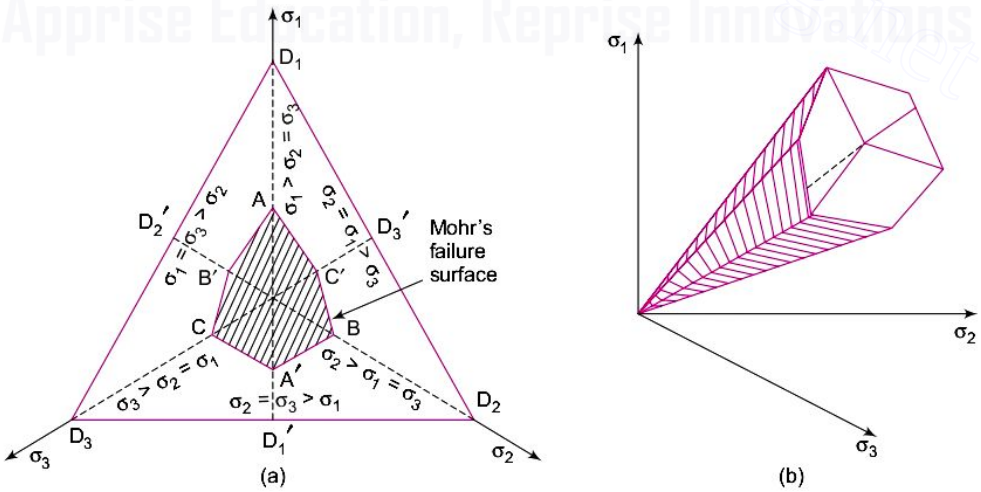

Fig. 19.5 Failure envelope : Mohr-coulomb criterion

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19.3 THE TRESCA FAILURE ENVELOPE

Hvorslev (1960) has shown that the radii $O'A$ (Fig. 19.2 (b)] for compression test and $O'A'$ for extension test are related as follows:

$$\frac{O'A}{O'A'} = \frac{1-\frac{1}{3}\sin\phi}{1+\frac{1}{3}\sin\phi} \qquad ...(19.7)$$

It is clear that the ratio is independent of c and approaches unity with decreasing value of ϕ as shown in Table 19.1.

Table 19.1 Ratio of $O'A$ and $O'A'$ for Mohr-coulomb failure surface

ϕ	0°	5°	10°	15°	20°	25°	30°	35°	40°
$\frac{O'A}{O'A'}$	1.000	0.943	0.889	0.842	0.796	0.753	0.715	0.678	0.647
$\frac{O'A'}{O'A}$	1.000	1.061	1.123	1.188	1.258	1.328	1.400	1.473	1.546

For $\phi = 0$ the Mohr-Coulomb criterion reduces to $\sigma_1 - \sigma_3 = c$ ...(19.8 a)

or $\sigma_{max} - \sigma_{min} = \text{constant} = 2k \text{ (say)}$ ...(19.8 b)

or $\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = k$ ...(19.8 c)

This criterion was first given by Tresca in 1968. He assumes that failure occurs for given material when the maximum shear stress equals to a critical constant value. The corresponding failure surface is a prism, for which the right section is a regular hexagon with $O'A = O'A'$. The extended Tresca condition, credited to Sandels, is expressed by

$$[(\sigma_1-\sigma_2)^2 - \{c + k_1 \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\}^2][(\sigma_2-\sigma_3)^2 - \{c + k_1 \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\}^2]$$

$$\times[(\sigma_3-\sigma_1)^2 - \{c + k_1 \cdot \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)\}^2] = 0 \qquad (19.8\ d)$$

where k_1 is a constant related to $f(\sin\phi)$.

The corresponding failure surface is a pyramid with a regular hexagonal right section, as shown in Fig. 19.6. Points A, B and C correspond the compression test ($\sigma_i > \sigma_j = \sigma_k$) and A_1, B_1 and C_1 correspond to the extension test ($\sigma_j = \sigma_k > \sigma_i$).

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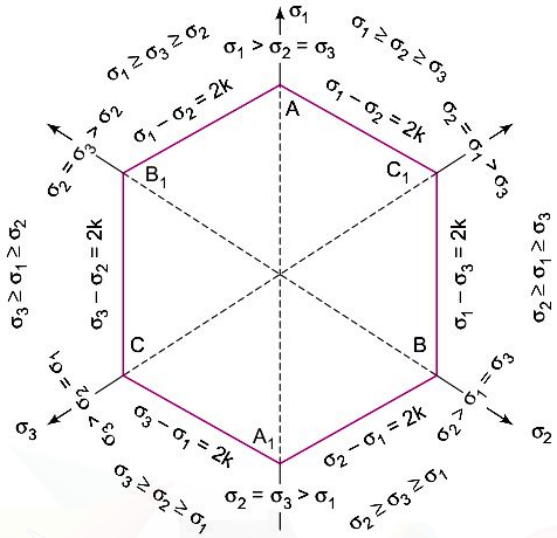

Fig. 19.6 The tresca failure envelope

19.4 VON MISES FAILURE ENVELOPE

The functional form for the yield surface given by Von Mises (1913) is expressed as $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \text{constant}$ ...(19.9)

The above form of the Von Mises criterion was introduced by Nadai (1950). Schleicher suggested the following extended Von Mises condition of failure to include the dependency on the mean normal stress:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \left[c + k_2 \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)^2\right] \quad ...(19.10)$$

where k_2 is constant related to $f(\sin \phi)$.

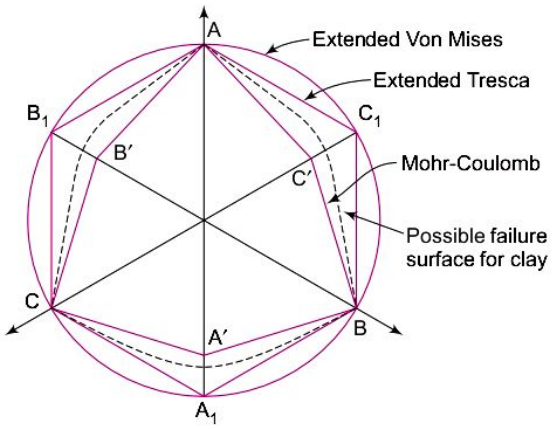

Fig. 19.7 Various failure envelopes

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The failure surface represented by equation 19.9 is a cylinder with its axis lying along the space diagonal [Fig. 19.1 (*b*)]. The failure surface represented by equation 19.10 is a cone with its apex at the origin, and its axis lying along the space diagonal. The right section (or failure locus) is circle which circumscribes the Tresca hexagon, as shown in Fig. 19.7.

19.5 COHESIONLESS SOIL : KIRKPATRICK FAILURE SURFACE

Kirkpatrick (1957) developed equipment for triaxial tests on thick walled cylinders of sand, to obtain the variation of σ_2 between the values of σ_1, and σ_3 and to investigate the conditions of failure for drained tests on sands. The thick cylindrical sample of sand can be constrained by both a bore pressure and an outside pressure, the bore pressure being increased until failure is induced. The dimensions of the sample were so arranged. that the difference of bore and outside pressure acting on the sealed end of the sample would bring the axial stress to a value approximately midway between the values of tangential and radial stresses. An element of material is thus subjected to radial, tangential and axial compressive stresses of different amounts. The radial stress is the major principal stress and the tangential stress is the minor principal stress. The axial stress is the intermediate principal stress caused by load difference on the sealed ends of the sample, or alternatively, the axial stress can be varied by applying external loads to the sample.

In his tests, failure was brought by first setting the outside fluid pressure (p_0) and then slowly increasing the inside pressure (p_i). If the ratio (σ_1/σ_3) of the major and minor principal stresses is λ, we have [Fig. 19.8(*a*)]:

$$\sigma_{r(i)} = p_i = \sigma_1 = p_0 \left(\frac{r_i}{r_0}\right)^{\lambda-1} ; \sigma_{t\,(i)} = \sigma_3 = \lambda\sigma_1 = \lambda p_0 \left(\frac{r_i}{r_0}\right)^{\lambda-1} ; \sigma_{\text{axial}} = \sigma_2 = \frac{p_0 r_0^2 - p_i r_i^2}{r_0^2 - r_i^2} \quad ...(19.11)$$

where $\lambda = \dfrac{\sigma_{t(i)}}{\sigma_{r(i)}} = \dfrac{\sigma_3}{\sigma_1} = \tan^2 (45° - \phi/2)$; σ_r = radial stress and σ_i = tangential stress

It is evident from above that $\left(\dfrac{p_i}{p_0}\right)_{\text{failure}} = \left(\dfrac{r_i}{r_0}\right)^{\lambda-1}$ ...(19.12)

Kirkpatrick measured p_i and p_0 at failure, obtained λ, from Eq. 19.12, and calculated the stresses ($\sigma_1 = p_0 : \sigma_3 = \lambda\sigma_1$) at outside, ($\sigma_1 = p_i : \sigma_2 = \lambda\sigma_1$) at inside, and the corresponding axial stresses for each value of λ.

The reduced right section on failure locus produced by these results is shown in Fig. 19.8(*c*). To investigate the shape of the yield surface in space, Kirkpatrick obtained the corresponding β angle [Fig. 19.8 (*b*)] by the relation

$$\cos\beta = \frac{OO'}{OP} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)/\sqrt{3}}{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)^{1/2}}$$

$$= \frac{\sigma_1 + \sigma_2 + \sigma_3}{\sqrt{3}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)^{1/2}} \quad ...(19.12a)$$

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The resulting angles (β) when plotted against distance $\left(\frac{\sigma_1+\sigma_2+\sigma_3}{\sqrt{3}}\right)$ along the space diagonal give a constant relation, *i.e.,* a straight line is obtained, except for very minor deviations. It can, therefore, be said that the shape of the surface in space is a straight sided pyramid as predicted by Mohr-Coulomb theory and that the other two theories *(i.e.,* extended Von Mises and Tresca) are not applicable.

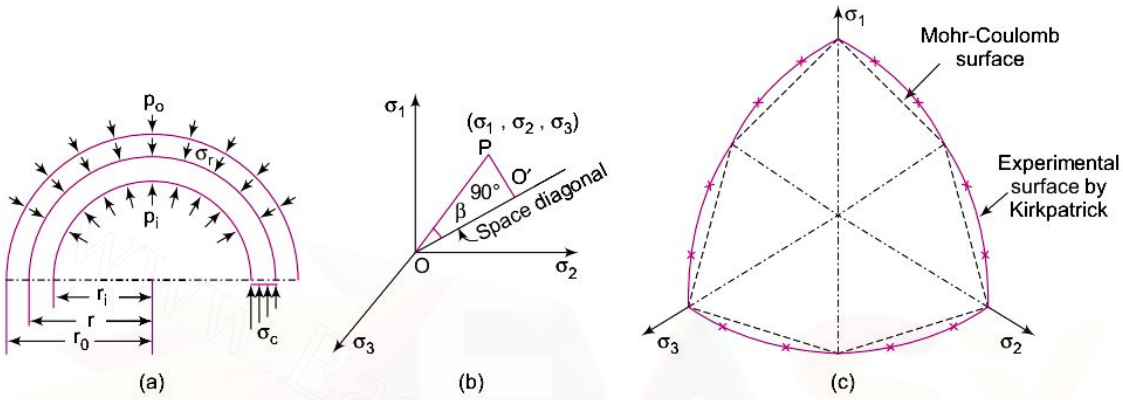

Fig. 19.8 Kirkpatrick failure surface

The experimental results by Kirkpatrick lie close to the theoretical Mohr-Coulomb surface based on the results of triaxial compression [Fig. 19.8 (*c*)]. They show an increase in strength over that predicted by the theory but the difference is small amounting to approximately 2° in the angle of internal friction. He concluded that the Mohr-Coulomb theory provides a reliable means of predicting failure for sands under conditions of full drainage. The close agreement between the angles of internal friction found in the compression and extension tests was also obtained by Bishop and Eldin. In contrast thereto, Habib (1935) and Haythornthwaite (1960) investigated the problem by means of torsion tests. Habib tested cylindrical specimens of sand and clay, and Haythornthwaite used hollow cylindrical specimen of a silt-clay mixture. Both found considerable departure from the Mohr-Coulomb criterion. They found that the value of ϕ in extension tests is considerably smaller than that obtained in compression tests. Haythornthwaite concludes that Mohr-Coulomb failure condition in some cases may furnish unsafe values and suggests that it be replaced with a criterion based on triangular failure locus (Fig. 19.9).

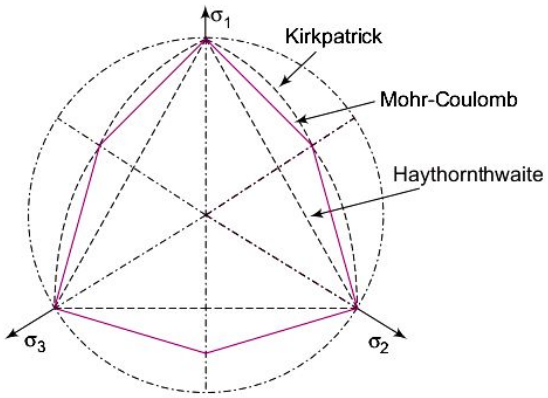

Fig. 19.9 Various failure envelopes

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19.6 COHESIVE SOILS : FAILURE SURFACE STUDY BY WU, LOH AND MALVERN

Wu, Loh and Malvern (1963) carried out hollow cylinder triaxial test permitting independent variation of all the three principal stresses, in addition to conventional triaxial compression and extenstion tests on cylindrical specimens of remoulded clay and sand. Pore water pressure measurement was made, which made possible the evaluation of the strength in terms of effective stress.

In its most general form, the Mohr-Coulomb criterion ($\tau = c + \sigma_n \tan \phi$) can be written in terms of major and minor principal stress, σ_1 and σ_3 as follows :

$$\sigma_1 = \sigma_3 N^2 + 2cN \qquad ...(19.13\ a)$$

where $$N = \tan\left(45° + \frac{\phi}{2}\right) \text{ and } \frac{1}{N} = \tan\left(45° - \frac{\phi}{2}\right)$$

When the failure is brought by increasing the outside pressure progressively, the tangential stress (σ_t or σ_θ) is the major principal stress, radial stress (σ_r) is the intermediate principal stress and the axial stress (σ_a) is the minor principal stress. The Mohr-Coulomb yield critertion (Eq. 19.13 *a*) can be rewritten as

$$\sigma_\theta = \sigma_r N^2 + 2cN \qquad ...(19.13\ b)$$

For an element in cylindrical co-ordinates, assuming radial symmetry and no variation in the axial direction, the equilibrium, condition is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad ...(19.13\ c)$$

Combining Eq. 19.13(*b*) and 19.3(*c*), integrating, and applying the boundary conditions (*i.e.*, at $r = r_1$, $\sigma_r = p_i$), we get the following expressions for radial and tangential stress at failure:

$$\sigma_3 = \sigma_r = \left(p_i + \frac{2cN}{N^2 - 1}\right)\left(\frac{r}{r_i}\right)^{N^2 - 1} - \frac{2cN}{N^2 - 1} \qquad ...(19.14\ a)$$

$$\sigma_1 = \sigma_\theta = \left(p_i + \frac{2cN}{N^2 - 1}\right)(N)^2\left(\frac{r}{r_i}\right)^{N^2 - 1} - \frac{2cN}{N^2 - 1} \qquad ...(19.14\ b)$$

$$= \frac{1}{\lambda}\left(p_i + \frac{2cN}{N^2 - 1}\right)\left(\frac{r}{r_i}\right)^{N^2 - 1} - \frac{2cN}{N^2 - 1} \qquad ...(19.14\ c)$$

where $$\lambda = \frac{\sigma_3}{\sigma_1} = \frac{\sigma_r}{\sigma_\theta} = \tan^2\left(45° - \frac{\phi}{2}\right) = \frac{1}{N^2}$$

At $r = r_0$, $\sigma_{r_0} = \sigma_{ri} = p_0$ and we have, from Eq. 19.14(*a*), the following relationship between the inside and outside pressure at failure:

$$p_0 = \sigma_{r\theta} = \left(p_i + \frac{2cN}{N^2 - 1}\right)\left(\frac{r_\theta}{r_i}\right)^{N^2 - 1} - \frac{2cN}{N^2 - 1} \qquad ...(19.15\ a)$$

It terms of effective stresses, the above equation becomes:

$$p_0' = \left(p_i' + \frac{2c'N'}{N'^2 - 1}\right)\left(\frac{r_\theta}{r_i}\right)^{N'^2 - 1} - \frac{2c'N'}{N'^2 - 1} \qquad ...(19.15\ b)$$

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where $N' = \tan(45° + \phi'/2)$.

The analysis by Wu, Loh and Malvern led to the conclusion that, when the intermediate principal stress is different from the major or minor principal stress, the measured ϕ' differs somewhat from that obtained in triaxial compression and extension tests. The nature of these departures are examined by projecting the measured stresses at failure onto an octahedral plane, using the apex of the failure envelope as a centre of projection. These projected stress points may then be compared with the trace of the failure surface on the octahedral plane.

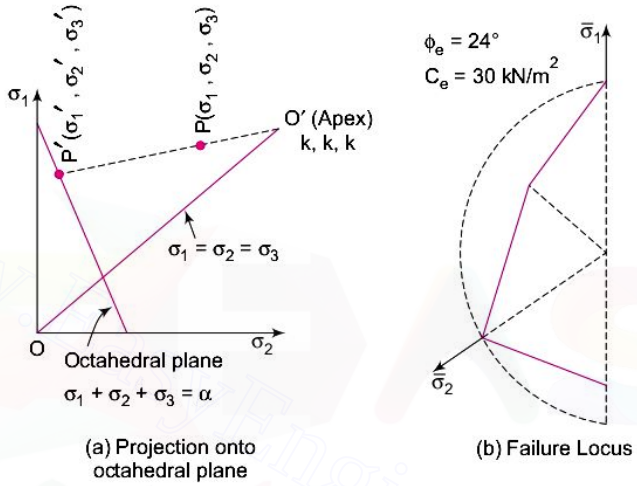

Fig. 19.10 Failure envelope ; Wu, Loh and Malvern

Let $(\sigma_1, \sigma_2, \sigma_3)$ be the principal stresses at specimen at failure represented by point P (Fig. 19.10 *a*). Let P' be the projection of the point P along the straight line joining the stress point (P) and the apex (O') having the co-ordinates $\sigma_1 = \sigma_2 = \sigma_3 = k$. If $(\sigma_1', \sigma_2', \sigma_3')$ are the co-ordinates of the projected point P', the equation of the straight line $P'PO'$ is

$$\frac{\sigma_1 - \sigma_1'}{k - \sigma_1} = \frac{\sigma_2 - \sigma_2'}{k - \sigma_2} = \frac{\sigma_3 - \sigma_3'}{k - \sigma_3} \qquad ...(i)$$

The projection plane (or the octahedral plane) is defined by

$$\sigma_1' + \sigma_2' + \sigma_3' = \alpha \qquad ...(ii)$$

where α is the distance of the plane from the origin O. Eqs (i) and (ii) may be combined to yield

$$\sigma_1' = \frac{k(2\sigma_1 - \sigma_2 - \sigma_3) + \alpha(k - \sigma_1)}{3k - (\sigma_1 + \sigma_2 + \sigma_3)} \qquad ...(19.16\ a)$$

$$\sigma_2' = \frac{k(2\sigma_2 - \sigma_3 - \sigma_1) + \alpha(k - \sigma_2)}{3k - (\sigma_1 + \sigma_2 + \sigma_3)} \qquad ...(19.16\ b)$$

$$\sigma_3' = \frac{k(2\sigma_3 - \sigma_1 - \sigma_2) + \alpha(k - \sigma_3)}{3k - (\sigma_1 + \sigma_2 + \sigma_3)} \qquad ...(19.16\ c)$$

To determine the failure envelope, the value of N is calculated from equation 19.15, in which p_i and p_0 are pressures at failure, and are known. The radial and tangential stresses are calculated with equation 19.14.

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The average axial stress ($\sigma_a = \sigma_2$) is known from the applied load. The principal stress point (P) can the projected onto the octahedral plane by calculating (σ_1', σ_2', σ_3') from equation 19.16. Wu, Loh and Malvern observed that when the plot is made in terms of effective stresses for sand, the higher value of ϕ' obtained for large p_0/p_i ratio, and this is represented by the stress points outside the failure envelope. The pattern agreed with behaviour of sand reported by Kirkpatrick. For clay, they also obtained the failure surface based on Hvorslev parameters (Fig. 19.10 *b*). Their experimental data on hollow specimen of clay substantiate the extension of the Mohr-Coulomb failure theory to the three-dimensional stress space of the strength as defined by Hvorslev parameters. The intermediate principal stress is found to exercise no influence over the strength of clay. For sands, however, the intermediate principal stress can increase the angle of friction by as much as 5°.

19.7 CONCLUSIONS

The failure envelope obtained from the results of triaxial compression and extension tests together with thick cylinder tests indicates that the extended Tresca and Von Mises theories are not applicable. The results of the drain tests on sand lie close to the theoretical Mohr-Coulomb surface based on the results of the triaxial compression tests. They show an increase in strength over that predicted by the theory but the difference is small amounting to approximately 2° in the angle of internal friction. The experimental data on hollow cylindrical specimens of clay substantiate the extension of the Mohr-Coulomb failure theory to the three-dimensional stress space if the strength is defined by Hvorslev parameters. The intermediate principal stress is found to exercise no influence over the strength of clay. However, definite conclusions cannot be formulated on the basis of the test data summarised above since clays vary widely in structure and stress history, and further investigation is needed.

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CHAPTER 20

Earth Pressure

20.1 INTRODUCTION

In the design of retaining walls, sheet piles, or other earth-retaining structures, it is necessary to compute the lateral pressure exerted by the retained mass of soil. The question of finding out the lateral earth pressure against retaining walls is one of the oldest in the civil engineering field. The *plastic state of stress* when the failure is imminent was investigated by Rankine in 1860. A lot of theoretical and experimental work has been done in this field, and many theories and hypotheses have been proposed.

A *retaining wall* or retaining structure is used for maintaining the ground surfaces at different elevations on either side of it. The material retained or supported by the structure is called *backfill* which may have its top surface horizontal or inclined. The position of the backfill lying above a horizontal plane at the elevation of the top of a wall is called the *surcharge,* and its inclination to the horizontal is called *surcharge angle* β.

20.2 PLASTIC EQUILIBRIUM IN SOILS: ACTIVE AND PASSIVE STATES

A body of soil is said to be in plastic equilibrium if every point of it is on the verge of failure. Rankine investigated the stress conditions corresponding to those states of plastic equilibrium which can be developed simultaneously throughout a semi-infinite mass of soil acted on by no force other than gravity. The stress condition during plastic equilibrium can be represented by the following Mohr-coulomb equation:

$$\frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2} \sin \phi = c \cos \phi \qquad ...(20.1)$$

or
$$\sigma_1 = 2c \tan (45° + \phi/2) + \sigma_3 \tan^2 (45° + \phi/2) \qquad ...(20.2)$$

or
$$\sigma_1 = 2c \sqrt{N_\phi} + \sigma_3 N_\phi \qquad ...(20.3)$$

where
$$N_\phi = \tan^2 (45° + \phi/2) = \text{flow value}$$

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σ_1 and σ_3 = major and minor principal stresses at any point in the soil mass, at failure.

In terms of stress components in x-z plane, equation 20.1 may be written as:

$$\sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{xz}^2} - \frac{\sigma_z + \sigma_x}{2} \sin \phi = c \cos \phi \qquad ...(20.4)$$

The theory on which the computation of the stress in a state of plastic equilibrium is based is called the *theory of plasticity*. The theory of plasticity pertaining to soils is based on Mohr's theory of rupture. When the material is just on the point of flowing plastically, it is still in static equilibrium, satisfying the following equilibrium equations in x-z plane:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \qquad ...(20.5)$$

and

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} + \gamma = 0 \qquad ...(20.6)$$

Combining equations 20.4, 20.5 and 20.6 lead to an equation, called Kotter's equation, the solution of which for a given boundary condition gives the orientation of slip lines together with the stress at each point of the failure zone (Sokolovsky, 1965).

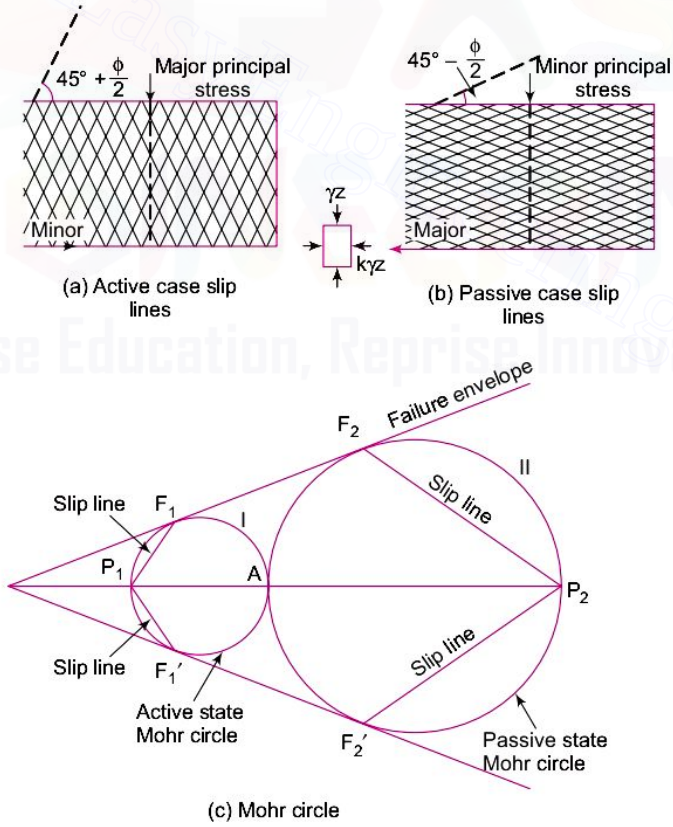

Fig. 20.1 Active and passive states of plastic equilibrium

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The equations of equilibrium in two dimensions are automatically satisfied by the Mohr-circle. Also, the state of plastic equilibrium (Eqs. 20.1 to 20.4) can be represented by two straight lines at an angle ϕ to the σ-axis (Fig. 20.1 *c*). Thus the Mohr diagram is a convenient tool for studying the stress conditions at equilibrium both before and at yield in the soil.

Figure 20.1 show the active and passive state of plastic equilibrium in a non-cohesive soil with the horizontal ground surface. In an active state, the major principal stress σ_1 is vertical and the minor principal stress σ_3 is horizontal. Circle I represents such a state, in which the pole P_1 corresponds to the minor principal stress (*i.e.*, horizontal stress or the lateral pressure) while point, A corresponds to the major principal stress. The circle touches the failure envelopes at F_1 and F_1', and hence $P_1 F_1$ and $P_1 F_1'$ show the directions of failure planes or *slip lines*. These slip lines have also been shown in Fig. 20.1(*a*). Similarly, for the passive state, the minor principal stress is in the vertical direction while the major principal stress is horizontal. Circle II corresponds to the passive state, in which the pole P_2 corresponds to the major principal stress. The circle touches the failure envelope at F_2 and F_2' and hence $P_2 F_2$ and $P_2 F_2'$, are the directions of slip lines corresponding to the passive case. From equation. 20.2 putting $c = 0$, we get

$$\frac{\sigma_1}{\sigma_3} = \tan^2 (45° + \phi/2) = N_\phi \qquad ...(20.7)$$

The ratio of the horizontal stress σ_h to the vertical stress σ_v is called the *co-efficient of earth pressure K*. When the soil is in the active state of plastic equilibrium, $\sigma_h = \sigma_3$ and $\sigma_v = \sigma_1$.

$$\frac{\sigma_1}{\sigma_3} = \frac{\sigma_v}{\sigma_h} = \tan^2 (45° + \phi/2)$$

$$\therefore \quad \frac{\sigma_h}{\sigma_v} = K_a = \frac{1}{\tan^2 (45° + \phi/2)} = \cot^2 (45° + \phi/2) = \frac{1 - \sin \phi}{1 + \sin \phi} \qquad ...(20.8)$$

where K_a = co-efficient of active earth pressure

Similarly, in passive state, $\sigma_h = \sigma_1$; $\sigma_v = \sigma_3$

$$\therefore \quad \frac{\sigma_1}{\sigma_3} = \frac{\sigma_h}{\sigma_v} = \tan^2 (45° + \phi/2)$$

$$\therefore \quad K_p = \frac{\sigma_h}{\sigma_v} = \tan^2 (45° + \phi/2) = \frac{1 + \sin \phi}{1 - \sin \phi} \qquad ...(20.9)$$

When the soil is at elastic equilibrium (*i.e.*, at rest) the ratio of horizontal to vertical stress is called the co-efficient of earth pressure of rest

$$\frac{\sigma_h}{\sigma_v} = K_0$$

Figure 20.2 shows the active and passive states of plastic equilibrium behind a retaining wall. During the active state, the wall moves away from backfill and a certain portion of the backfill located immediately behind the wall breaks away from the rest of the soil mass. This wedge-shaped portion of the backfill tending to move with the wall is called the *failure wedge*. The resisting force due to the shear strength of soil is developed in an upward direction along the failure plane (or slip lines). Figure 20.2(*b*) shows the passive case in which the wall moves *towards* the fill, due to some thrust etc.

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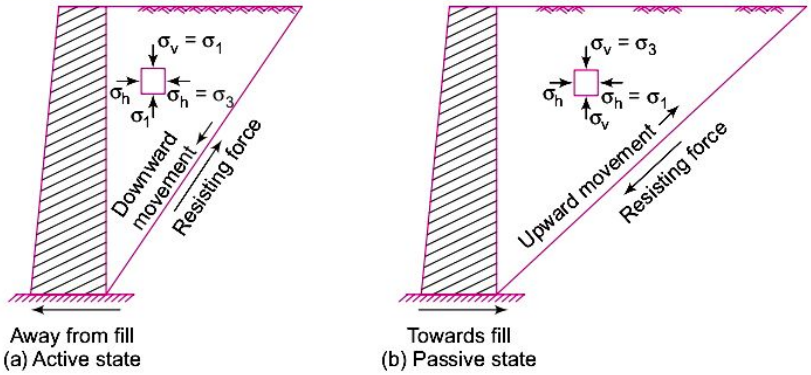

Fig. 20.2 Active and passive earth pressure

The magnitude of the lateral earth pressure depends upon the movement of the retaining wall relative to the backfill and upon the nature of the soil. Figure 20.3(*a*) shows the variation of lateral earth pressure with movement of the retaining wall. Figure 20.3(*b*) shows the variation of co-efficients of earth pressure with the horizontal strain. Point *A* represents earth pressure at rest. When the wall moves away from the backfill, mobilisation of the internal resistance of soil, which builds up in directions away from the walls takes place and hence the earth pressure on the wall decreases. The decrease of earth pressure continues up to *B* until the full resistance has been mobilised. The earth pressure does not decrease beyond this point with further movement of the wall. This minimum pressure is called the *active earth pressure*. If on the other hand; the wall moves towards the fill, the earth pressure increases, because the shearing resistance builds up in directions towards the wall. The pressure reaches a maximum point (point *C*) when the shearing resistance of soil has been fully mobilised. Any further movement of the wall does not increase the pressure. This maximum pressure is called the *passive earth pressure*. For dense sand, the following important conclusions have been drawn (Lambe, 1969): (1) Very little horizontal strain, less than –0.5%, is required to reach the active state (2) Little horizontal compression, about 0.5% is

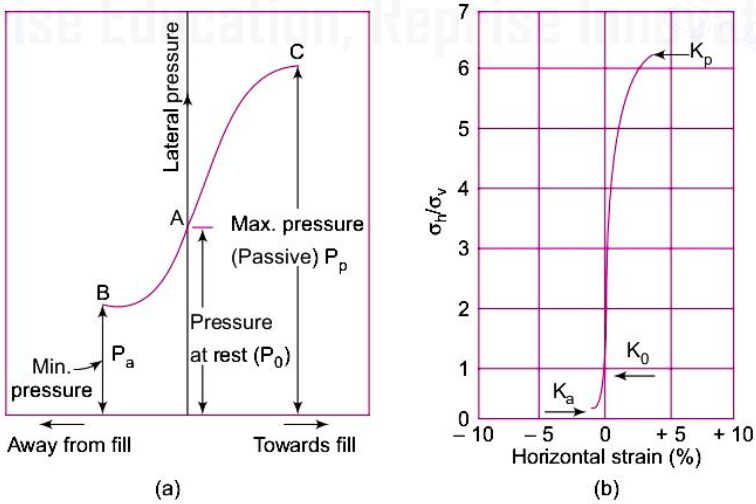

Fig. 20.3 Variation of earth pressure and co-efficients of earth pressure with the movement of the wall

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required to reach one-half of the maximum passive resistance (3) Much more horizontal compression, about 2% is required to reach the full maximum passive resistance. For loose sands, the first two conclusions remain valid, but the horizontal compression required to reach full passive resistance may be as large as 15% .

Effective stress analysis. If the analysis is based on effective stresses, Eqs. 20.8 and 20.9 are modified as under:

$$K_a = \frac{\sigma_h'}{\sigma_v'} = \frac{1}{\tan^2(45° + \phi'/2)} = \frac{1 - \sin\phi'}{1 + \sin\phi'} \qquad ...(20.8\ a)$$

$$K_p = \frac{\sigma_h'}{\sigma_v'} = \frac{1 + \sin\phi'}{1 - \sin\phi'} \qquad ...(20.9\ a)$$

20.3 EARTH PRESSURE AT REST*

The earth pressure at rest, exerted on the back of a rigid, unyielding retaining structure, can be calculated using theory of elasticity, assuming the soil to the semi-infinite, homogeneous, elastic and isotropic. Consider an element of soil at a depth z, being acted upon by vertical stress σ_v and horizontal stress σ_h. There will be no shear stress. The lateral strain $\in_h$ in the horizontal direction is given by:

$$\in_h = \frac{1}{E}\left[\sigma_h - \mu(\sigma_h + \sigma_v)\right]$$

The earth pressure at rest corresponding to the condition of zero lateral strain ($\in_h = 0$).

Hence $$\sigma_h = \mu(\sigma_h + \sigma_v) \quad \text{or} \quad \frac{\sigma_h}{\sigma_v} = K_0 = \frac{\mu}{1 - \mu} \qquad ...(20.10)$$

where K_0 is *coefficient of the earth pressure at rest.*

Designating the lateral earth pressure (σ_h) at rest by p_0 and substituting $\sigma_v = \gamma z$, we have

$$p_0 = K_0 \gamma z \qquad ...(20.11)$$

The pressure distribution diagram is thus triangular with zero intensity at $z = 0$ and an intensity of $K_0 \gamma H$ at the base of the wall, where $z = H$. The total pressure P_0 per unit length for the vertical height H is given by

$$P_0 = \int_0^H K_0 \gamma z \, . \, dz = \frac{1}{2} K_0 \gamma H^2 \qquad ...(20.12)$$

The behaviour of soil is not in accordance with the elastic theory and do not have a well-defined value of the Poission's ratio. Table 20.1 gives some value of K_0 available from experience.

Table 20.1 Values of K_0 for different soils

S.No	*Soil Type*	K_0
1.	Loose sand	0.4
2.	Dense sand	0.6
3.	Sand compacted in layers	0.8
4.	Soft clay	0.6
5.	Hard clay	0.5

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K_0 may also be computed from the following approximate expression given by Jaky (1944).

$$K_0 = 1 - \sin \varphi \qquad ...(20.10\ a)$$

It is to be noted very carefully that the earth pressure at rest (P_0) is always greater than active earth pressure (P_a) but lesser than passive earth pressure (P_p), as is evident from Fig. 20.3(a).

Example 20.1. *A rigid retaining wall, 6 m high is restrained from yielding. The backfill consists of cohesionless soil having $\varphi = 26°$ and $\gamma = 19$ kN/m^3. Compute the total earth pressure per metre length of the wall?*

Solution. Since the wall is restrained from yielding, the wall will be subjected to earth pressure (P_0), *at rest,* given by Eq. 20.12

$$P_0 = \frac{1}{2} K_0 \gamma H^2$$

Here, K_0 can be estimated from Eq. 20.10(a) by Jacky.

$$K_0 = 1 - \sin 26° = 0.5616$$

(**Note:** This value corresponds to sand in nearly dense state)

$$P_0 = \frac{1}{2} \times 0.5616 \times 19\ (6)^2 = \mathbf{192.1\ kN/m}\ \text{length of wall}$$

20.4 ACTIVE EARTH PRESSURE: RANKINE'S THEORY

As originally proposed, Rankine's theory of lateral earth pressure is applied to uniform cohesionless soils only. Later, it was extended to include cohesive soils, by Resal (1910) and by Bell (1915). The theory has also been extended to stratified, partially immersed and submerged soils. Following are the assumptions of the Rankine's theory:

1. The soil mass is semi-infinite, homogeneous, dry and cohesionless.
2. The ground surface is a plane which may be horizontal or inclined.
3. The back of the wall is vertical and smooth. In other words, there are no shearing stresses between the wall and the soil and the stress relationship for any element adjacent to the wall is the same as for any other element far away from the wall.
4. The wall yields about the base and thus satisfies the deformation condition for plastic equilibrium.

However, the retaining walls are constructed of masonry or concrete, and hence the back of the wall is never smooth. Due to this, frictional forces develop. As a consequence of Rankine's assumption of no-existence of frictional forces at the wall face, the resultant pressure must be parallel to the surface of the backfill. The existence of the friction makes the resultant pressure inclined to the normal to the wall at an angle that approaches the friction angle between the soil and the wall.

The following cases of cohesionless backfill will now be considered:

1. Dry or moist backfill with no surcharge.
2. Submerged backfill.
3. Backfill with uniform surcharge.
4. Backfill with sloping surface.
5. Inclined back and surcharge.

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1. Dry or Moist Backfill with no Surcharge. Consider an element at a depth z below the ground surface. When the wall is at the point of moving outwards (*i.e.*, away from the fill), the active state of plastic equilibrium is established. The horizontal pressure σ_h is then the minimum principal stress σ_3 and the vertical pressure σ_v is the major principal stress σ_1. From the stress relationship (Eq. 20.2) we have

$$\sigma_1 = \sigma_3 \tan^2 (45° + \phi/2) \qquad \text{(since } c = 0\text{)}$$

$$\frac{\sigma_3}{\sigma_1} = \frac{\sigma_h}{\sigma_v} = \frac{1}{\tan^2 (45° + \phi/2)} = \cot^2 (45° + \phi/2)$$

Now σ_h = lateral earth pressure = p_a

σ_v = vertical pressure on the element = $\gamma \,.\, z$

$\therefore$ $$p_a = \gamma \,.\, z \cot^2 (45° + \phi/2) = K_a \gamma z \qquad ...(20.13)$$

where K_a = co-efficient of active earth pressure = $\cot^2 (45° + \phi/2) = \dfrac{1 - \sin \phi}{1 + \sin \phi}$ ...(20.13 *a*)

When $\varphi = 30°$; $$K_a = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3}$$

Figure 20.4 shows the distribution of active earth pressure p over the retaining wall. At $z = H$, the earth pressure is:

$$p_a = K_a \gamma H \qquad ...(20.14)$$

The total active earth pressure P_a or the resultant pressure per unit length of the wall is found by integrating Eq. 20.13, or from the triangular pressure distribution diagram.

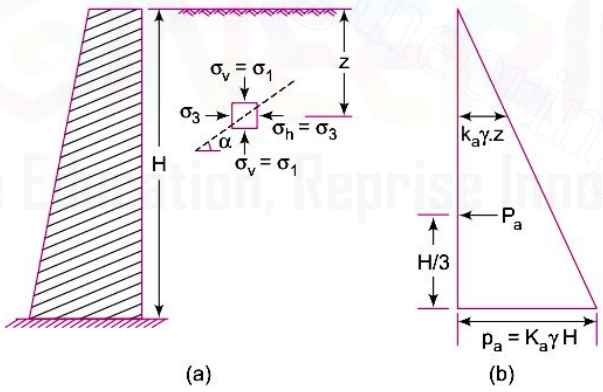

Fig. 20.4 Active earth pressure of dry or moist cohesionless soil

$\therefore$ $$P_a = \frac{1}{2} K_a \gamma H^2 \qquad ...(20.15)$$

acting at $H/3$ above the base of the wall.

If the soil is dry, γ is the dry weight of the soil, and if wet, γ is the moist weight, to be substituted in Eqs. 20.14 and 20.15.

2. Submerged Backfill. In this case, the sand fill behind the retaining wall is saturated with water. The lateral pressure is made up of two components:

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(*i*) lateral pressure due to submerged weight γ' of the soil, and
(*ii*) lateral pressure due to water. Thus, at any depth z below the surface,

$$p_a = K_a \gamma' z + \gamma_w z$$

The pressure at the base of the retaining wall ($z = H$) is given by

$$p_a = K_a \gamma' H + \gamma_w H \qquad ...(20.16)$$

If the free water stands to both sides of the wall [Fig. 20.5(*b*)] the water pressure need not be considered, and the net lateral pressure is given by

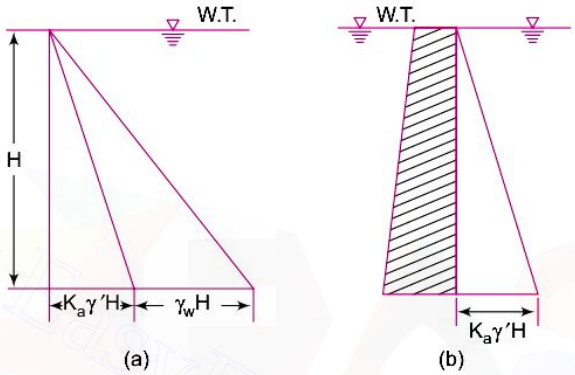

Fig. 20.5 Submerged backfill

$$p_a = K_a \gamma' H \qquad ...(20.17)$$

If the backfill is partly submerged, *i.e.*, the backfill is moist to a depth H_1 below the ground level, and then it is submerged, the lateral pressure intensity at the base of the wall is given by

$$p_a = K_a \gamma H_1 + K_a \gamma' H_2 + \gamma_w H_2 \qquad ...(20.18)$$

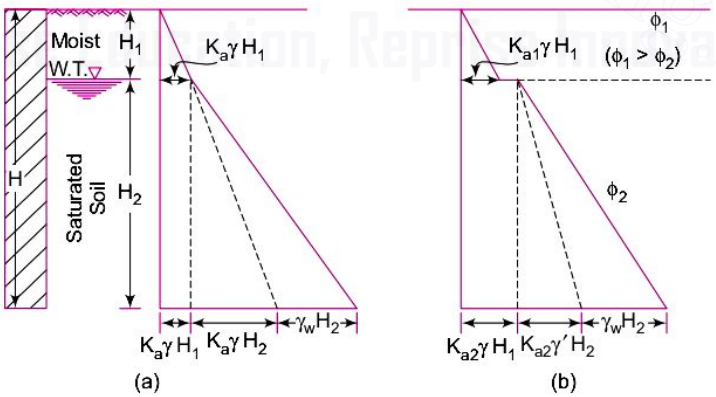

Fig. 20.6 Partly submerged backfill

The above expression is on the assumption that the value of ϕ is the same for the moist as well as submerged soil. If it is different, say ϕ_1 and ϕ_2 respectively, the earth pressure co-efficient, K_{a1} and K_{a2} for both the portions will be different. As ϕ decreases, K_a increases. The lateral pressure intensity at the base of wall is given by:

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$$p_a = K_{a2}\,\gamma H_1 + K_{a2}\,\gamma'\,H_2 + \gamma_w H_2 \qquad ...(20.19)$$

3. Backfill with Uniform Surcharge. If the backfill is horizontal and carries a surcharge of uniform intensity q per unit area, the vertical pressure increment, at any depth z, will increase by q. The increase in the lateral pressure due to this will be $K_a\,q$. Hence the lateral pressure at any depth z is given by

$$p_a = K_a \gamma z + K_a\,q \qquad ...(20.20)$$

At the base of the wall, the **pressure intensity** is

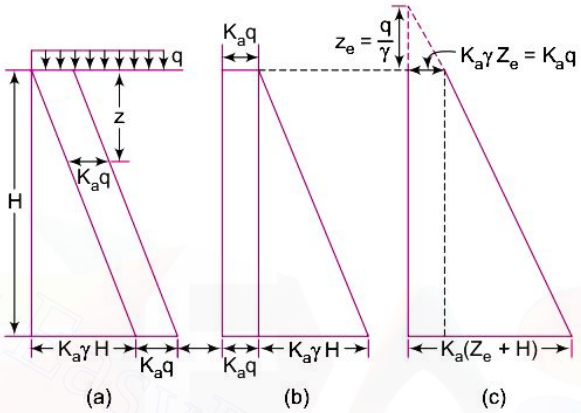

Fig. 20.7 Backfill with uniform surcharge

$$p_a = K_a \gamma H + K_a\,q \qquad ...(20.21)$$

Figure 20.7(a) and (b) show the two alternative methods of plotting the lateral pressure diagram for this case. The lateral pressure increment due to the surcharge is the same at every point of the back of the wall, and does not vary with depth z. The height of fill z_e, equivalent to the uniform surcharge intensity is given by the relation,

$$K_a \gamma z_e = K_a\,q \quad \text{or} \quad z_e = \frac{q}{\gamma} \qquad ...(20.22)$$

This means that the effect of the surcharge of intensity q is the same as that of a fill of height z_e above the ground surface [Fig. 20.7(c)].

4. Backfill with Sloping Surface. Let the sloping surface behind the wall be inclined at an angle β with the horizontal; β is called the surcharge angle. In finding out the active earth pressure for this case by Rankine's theory, an additional assumption that the vertical and lateral stresses are *conjugate* is made. It can be shown that if the stress on a given plane at a given point is parallel to another plane, the stress on the latter plane at the same point must be parallel to the first plane. Such planes are called the *conjugate planes* and the stresses acting on them are called conjugate stresses.

Consider a soil element at point A at depth z within a backfill with a sloping surface, as shown in Figure 20.8(a). The top plane of the element is parallel to the ground plane, and the other plane conjugate to this is vertical. Let σ and p be the conjugate stresses, σ being vertical and p being parallel to the sloping backfill. Being conjugate, both the vertical pressure σ and lateral pressure p have the same angle of obliquity β, which is equal to the surcharge angle. Note that σ and p are the resultant stresses on the two conjugate planes, and they are not the principal stresses. Let σ_1 and σ_3 be the major and minor principal stresses on the soil element at A. From stability requirement, we have, for cohesionless soil,

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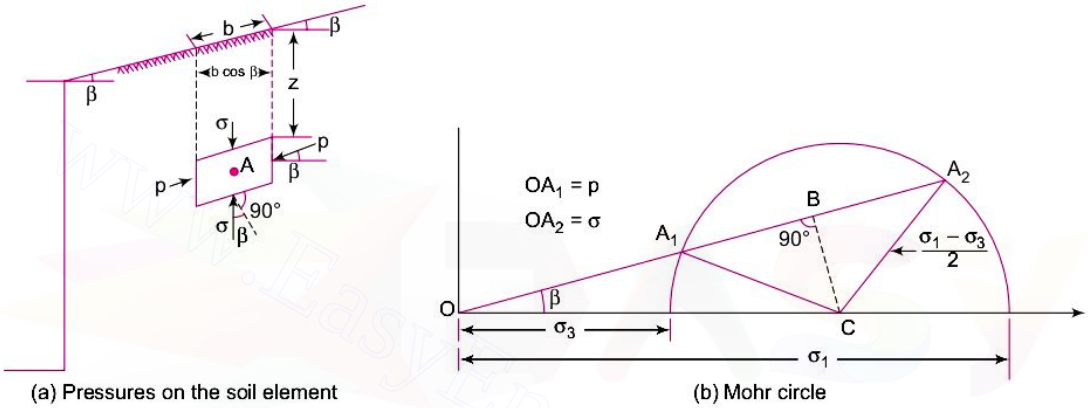
$$\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \sin \phi \qquad ...(1)$$

Figure 20.8(b) shows that Mohr circle corresponding to the principal stress intensities σ_1 and σ_3 at A. The Mohr's circle is the locus of a point that represents that resultant stress intensity at all planes passing through the point under consideration. Hence we can find the resultant stress intensities σ and p (*i.e.*, conjugate stresses) on the two conjugate planes at point A. The obliquity of σ and p is β. Hence through origin O, draw a line at obliquity β to cut the circle at A_1 and A_2. Then OA_1 represents the resultant stress p and OA_2 represents the resultant stress σ. Draw OB perpendicular to $A_1 A_2$.



(a) Pressures on the soil element (b) Mohr circle

Fig. 20.8 Conjugate stress relationship

$$OB = OC \cos \beta$$

$$= \frac{\sigma_1 + \sigma_3}{2} \cos \beta \qquad ...(2)$$

$$BC = OC \sin \beta = \frac{\sigma_1 + \sigma_3}{2} \sin \beta \qquad ...(3)$$

$$A_1 B = B A_2 = \sqrt{A_1 C^2 - BC^2} = \sqrt{\left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 - \left(\frac{\sigma_1 + \sigma_3}{2}\right)^2 \sin^2 \beta}$$

But $(\sigma_1 - \sigma_3) = (\sigma_1 + \sigma_3) \sin \phi$, from (1) ;

Hence
$$A_1 B = BA_2 = \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta} \qquad ...(4)$$

Now stress
$$\sigma = OB + BA_2 = \frac{\sigma_1 + \sigma_3}{2} \cos \beta + \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta} \qquad ...(5)$$

and stress
$$p = OB - A_1 B = \frac{\sigma_1 + \sigma_3}{2} \cos \beta - \frac{\sigma_1 + \sigma_3}{2} \sqrt{\sin^2 \phi - \sin^2 \beta} \qquad ...(6)$$

Dividing (5) and (6), we get

$$\frac{p}{\sigma} = K = \frac{\cos\beta - \sqrt{\sin^2 \phi - \sin^2 \beta}}{\cos\beta + \sqrt{\sin^2 \phi - \sin^2 \beta}} = K = \frac{\cos\beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos\beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} \qquad ...(20.23)$$

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The ratio K is called the *conjugate ratio* or Rankine's lateral pressure ratio.

For the present case, $$\sigma = \frac{\gamma z \,.\, b \cos\beta}{b} = \gamma z \cos\beta \qquad ...(20.23\ a)$$

p = lateral earth pressure = p_a

Substituting these in Eq. 20.23, we get

$$p_a = \gamma z \,.\, \cos\beta \, \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \qquad ...(20.24)$$

or $$p_a = K_a \gamma z \qquad ...(20.24\ a)$$

where $$K_a = \cos\beta \, \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \qquad ...(20.25)$$

where $\beta = 0$ (*i.e.*, horizontal ground surface), Eq. 20.25 reduces to:

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

which is the same as Eq. 20.13(a).

The pressure distribution given by Eq. 20.24 will be triangular.

The total active pressure P_a for the wall of height H is given by

$$P_a = \frac{1}{2} K_a \gamma H^2$$

The resultant acts at $\frac{H}{3}$ above the base in direction parallel to the surface, as shown in Fig. 20.9.

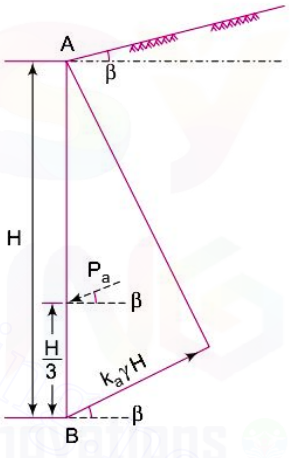

Fig. 20.9 Lateral pressure distribution for sloping surcharge

If the backfill is submerged, the lateral pressure due to the submerged weight of the soil will act at β with horizontal, while the lateral pressure due to water will act *normal* to the wall.

5. Inclined Back and Surcharge. Figure 20.10 shows a retaining wall with an inclined back supporting a backfill with horizontal ground surface. The total active pressure P_1 is first calculated on a vertical plane BC passing through the heel B. The total pressure P is the resultant of the horizontal pressure P_1 and the weight W of the wedge ABC:

$$P = \sqrt{P_1^2 + W^2} \qquad ...(20.26)$$

where $$P_1 = \frac{1}{2} K_a \gamma H^2$$

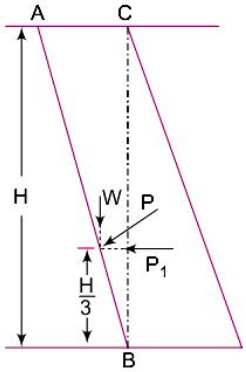

Fig. 20.10 Inclined back

Figure 20.11 shows the case of a retaining wall with inclined back and surcharge. The active earth pressure is first calculated on a vertical plane passing through the heel and intersecting the surface of the backfill or its

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extension in point *C*. The height *H* of vertical plane is represented by *BC*. The resultant of *P* is the vector sum of P_1 and *W*, where *W* is the weight of the soil contained in the triangle *ABC*

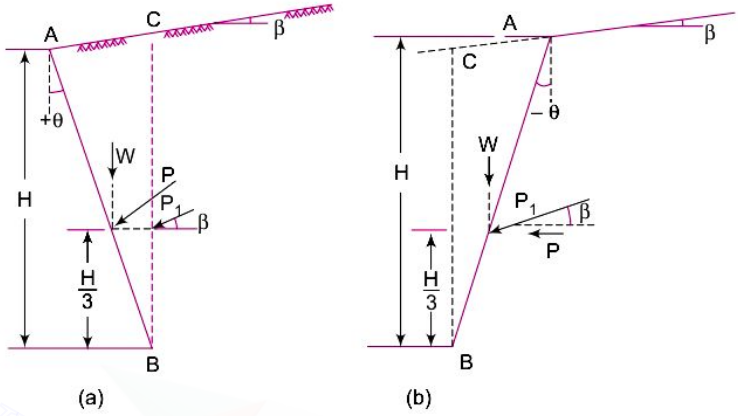

Fig. 20.11 Pressure on inclined back with sloping ground surface

20.5 ACTIVE EARTH PRESSURE OF COHESIVE SOILS

The principal stress relationship on a failure plane is given by equation 20.2:

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha \qquad (a)$$

In the case of cohesionless soils, *c* was equated to zero in the Rankines's theory. To extend the theory to the cohesive soil, *c* has to be taken into account.

Consider a retaining wall with a smooth, vertical back retaining a horizontal backfill. At any depth *z* we have

$$\sigma_1 = \gamma z \text{ and } \sigma_3 = \text{lateral pressure } p_a$$

Substituting in (1), we have

$$\gamma z = p_a \tan^2 \alpha + 2c \tan \alpha$$

or
$$p_a = \gamma z \cot^2 \alpha - 2c \cot \alpha \qquad ...(20.27)$$

where
$$\alpha = 45° + \phi/2$$

Equation 20.27 is known as Bell's equation for the lateral pressure of cohesive soils.

At $z = 0$, we get from Eq. 20.27,

$$p_a = -2c \cot \alpha$$

When
$$p_a = 0, z = z_0 = \frac{2c}{\gamma} \tan \alpha = \frac{2c}{\gamma} \frac{1}{\sqrt{K_a}} \qquad ...(20.28)$$

This shows that negative pressure (*i.e.*, tension) is developed at top level of the retaining wall. The tension decreases to zero at a depth $z_0 = \frac{2c}{\gamma} \tan \alpha$.

For depth $z > z_0$, the pressure p_a, given by equation 20.27 is positive.

At $z = H, p_a = \gamma H \cot^2 \alpha - 2c \cot \alpha \qquad ...(20.29)$

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Figure 20.12 shows the variation of lateral pressure along the height of the retaining wall. It will be seen that if c were equal to zero, the pressure at the base would have been equal to $\gamma H \cot^2 \alpha$: *thus the effect of cohesion in soil is to reduce the pressure intensity everywhere by 2c cot α.*

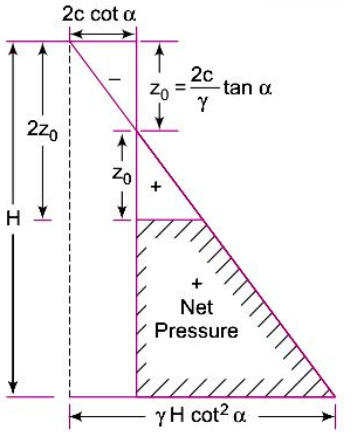

Fig. 20.12 Active earth pressure of cohesive backfill

The total net pressure is given by

$$P_a = \int_0^H p_a \, . \, dz = \int_0^H \left(\gamma z \cot^2 \alpha - 2c \cot \alpha\right) dz$$

or
$$P_a = \frac{1}{2} \gamma H^2 \cot^2 \alpha - 2cH \cot \alpha \qquad ...(20.30)$$

Because of negative pressure, a tension crack is usually developed in the soil near the top of the wall, up to a depth z_0. Also, the total *net* pressure upon a depth 2 z_0 is zero. This means that a cohesive soil should be able to stand with a vertical face up to a depth 2 z_0 without any lateral support. The critical height H_c of an unsupported vertical cut in cohesive soil is thus given by

$$H_c = 2z_0 = \frac{4c}{\gamma} \tan \alpha \qquad ...(20.31)$$

As cracks do occur and soil does not necessarily remain adhered to the top portion of the wall, up to height z_0, it is usual to neglect the negative pressure and consider whole of the positive pressure below z_0. The total lateral thrust on the wall is then obtained by integrating equation 20.27 between the limits z_0 to H:

$$P_a = \int_{z0}^H \left(\gamma z \cot^2 \alpha - 2c \cot \alpha\right) dz = \frac{1}{2} \gamma \left(H^2 - z_0^2\right) \cot^2 \alpha - 2c \left(H - z_0\right) \cot \alpha \qquad ...(20.32)$$

Substituting $z_0 = \dfrac{2c}{\gamma} \tan \alpha$, the above equation reduces to :

$$P_a = \frac{1}{2} \gamma H^2 \cot^2 \alpha - 2cH \cot \alpha + \frac{2c^2}{\gamma} \qquad ...(20.33)$$

Backfill with surcharge. If the backfill carries a surcharge of uniform intensity q per unit area, the lateral pressure is increased by $K_a q$ or $q \cot^2 \alpha$ everywhere. Hence equation 20.27 for the lateral pressure is modified to :

$$p_a = \gamma z \cot^2 \alpha - 2c \cot \alpha + q \cot^2 \alpha \qquad ...(20.34)$$

At $z_0 = 0, p_a = q \cot^2 \alpha - 2c \cot \alpha$,

The depth z_0 at $p_a = 0$ is given by

$$z_0 = \frac{2c}{\gamma} \tan \alpha - \frac{q}{\gamma} \qquad ...(20.35)$$

Submerged backfill. If the water table exists at a depth H_1 below the top of the wall, the lateral pressure at depth z ($z > H_1$) is given by

$$p_a = [\gamma H_1 + \gamma' (z - H_1)] \cot^2 \alpha - 2c \cot \alpha + \gamma_w (z - H_1) \qquad ...(20.36)$$

Backfill of intact saturated clay. The active lateral pressure of intact saturated clays for temporary works or immediately after construction of retaining wall is calculated by assuming $\phi = \phi_u = 0$.

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Then $$K_a = \frac{1-\sin\phi}{1+\sin\phi} = 1$$

$\therefore$ $$p_a = \gamma_{sat} \cdot z - 2c_u \qquad ...(20.37)$$

$$p_a = \frac{1}{2}\gamma_{sat}\left(H^2 - z_0^2\right) - 2c_u\left(H - z_0\right) \qquad ...(20.38)$$

and $$z_0 = \frac{2c_u}{\gamma_{sat}} \qquad ...(20.39)$$

20.6 PASSIVE EARTH PRESSURE: RANKINE'S THEORY

***(a)* Cohesionless backfill:** In the case of passive state of plastic equilibrium the lateral pressure is the major principal stress while the vertical pressure is the minor principal stress. Thus.

$$\sigma_h = p_p = \sigma_1 \; ; \; \sigma_v = \sigma_3 = \gamma z$$

Substituting this in the principal stress relationship: $\sigma_1 = \sigma_3 \tan^2 \alpha$, we get

$$p_p = \gamma z \tan^2 \alpha = K_p \gamma z \qquad ...(20.40)$$

where p_p = passive earth pressure intensity

K_p = Rankine's co-efficient of passive earth pressure

or $$K_p = \tan^2 \alpha = N_\phi = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1}{K_a} \qquad ...(20.41)$$

Also ratio $$\frac{K_p}{K_a} = \frac{\tan^2\left(45° + \varphi/2\right)}{\cot^2\left(45° + \varphi/2\right)} = \tan^4\left(45° + \varphi/2\right)$$

For example, if $\varphi = 30°$, $\frac{K_p}{K_a} = \tan^4 (45° + 30°/2) = \tan^4 60° = 9$

or $K_p = 9\, K_a$

The distribution of passive earth pressure, given by Eq. 20.40 is triangular, with maximum value of $K_p\, \gamma H$ at the base of the retaining wall of height H. The total pressure P_p for a depth H is given by

$$P_p = \int_0^H K_p \gamma z \, . \, dz = \frac{1}{2} K_p \gamma H^2 \qquad ...(20.42)$$

If a uniform surcharge intensity q per unit area acts over the surface of the backfill, the increase in the passive pressure will be equal to $K_p \, . \, q$. The passive pressure intensity at a depth z is then given by

$$p_p = K_p (\gamma z + q) \qquad ...(20.43)$$

If the backfill is having its top surface inclined at an angle β, the passive pressure is given by

$$p_p = \gamma z \cos\beta \, \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \qquad ...(20.44)$$

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or $$p_p = K_p \gamma z \quad \text{where} \quad K_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}} \qquad ...(20.45)$$

(b) Cohesive backfill: For the case of cohesive soil, the principal stress relationship at failure is given by

$$\sigma_1 = \sigma_3 \tan^2\alpha + 2c \tan\alpha$$

For the case of passive pressure,

$$\sigma_1 = \sigma_h = p_p$$

$$\sigma_3 = \sigma_v = \gamma z$$

Substituting these values of σ_1 and σ_3, we get

$$p_p = \gamma z \tan^2\alpha + 2c \tan\alpha \qquad ...(20.46)$$

or $$p_p = \gamma z N_\phi + 2c\sqrt{N_\phi}$$

At $z = 0$, $p_p = 2c \tan\alpha$

At $z = H$, $p_p = \gamma H \tan^2\alpha + 2c \tan\alpha$

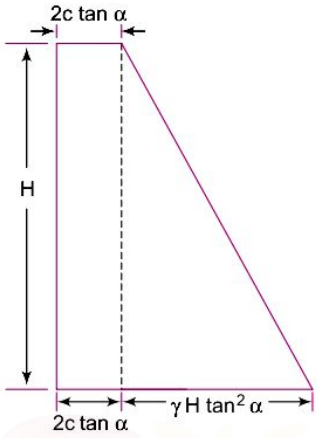

Fig. 20.13 Variation of passive pressure cohesive backfill

Figure 20.13 shows the pressure distribution diagram. The total pressure is given by

$$P_p = \int_0^H p_p \, dz = \frac{1}{2}\gamma H^2 \tan^2\alpha + 2cH \tan\alpha \qquad ...(20.47)$$

$$= \frac{1}{2}\gamma H^2 N_\phi + 2cH\sqrt{N_\phi}. \qquad ...(20.47\ a)$$

Example 20.2. *Compute the intensities of active and passive earth pressure at depth of 8 metres in dry cohesionless sand with an angle of internal friction of 30° and unit weight of 18 kN/m³. What will be the intensities of active and passive earth pressure if the water level rises to the ground level ? Take saturated unit weight of sand as 22 kN/m³.*

Solution.

(a) Dry soil:

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3} \; ; \; K_p = \frac{1+\sin\phi}{1-\sin\phi} = \frac{1}{K_a} = 3$$

$\therefore$ $$p_a = K_a \gamma H = \frac{1}{3} \times 18 \times 8 = 48 \text{ kN/m}^2$$

$$p_p = K_p \gamma H = 3 \times 18 \times 8 = 432 \text{ kN/m}^2$$

(b) Submerged backfill:

$$\gamma' = \gamma_{sat} - \gamma_w = 22 - 9.81 = 12.19 \text{ kN/m}^3$$

$$p_a = K_a \gamma' H + \gamma_w H = \frac{1}{3} \times 12.19 \times 8 + 9.81 \times 8 \approx \mathbf{111} \text{ kN/m}^2$$

$$p_p = K_p \gamma' H + \gamma_w H = (3 \times 12.19 \times 8) + (9.81 \times 8) = \mathbf{371} \text{ kN/m}^2$$

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Example 20.3. *A retaining wall 4 m high, has a smooth vertical back. The backfill has a horizontal surface in level with the top of the wall. There is uniformly distributed surcharge load of 36 kN/m^2 intensity over the backfill. The unit weight of the backfill is 18 kN/m^3; its angle of shearing resistance is 30° and cohesion is zero. Determine the magnitude and point of application of active pressure per metre length of the wall.*

Solution. $K_a = \dfrac{1-\sin\phi}{1+\sin\phi} = \dfrac{1}{3}$

The lateral pressure intensity due to the surcharge is given by

$$p_1 = K_a \,.\, q = \frac{1}{3} \times 36 = 12 \text{ kN/m}^2$$

The pressure intensity due to the backfill, at depth $H = 4$ m is given by,

$$p_2 = K_a \gamma H = \frac{1}{3} \times 18 \times 4 = 24 \text{ kN/m}^2$$

The total pressure intensity at the base of the wall is given by

$$p_a = p_1 + p_2 = 12 + 24 = 36 \text{ kN/m}^2$$

Figure 20.14 shows the pressure distribution diagrams for p_1 and p_2. The resultant total pressure P_1 due to intensity p_1 is given by:

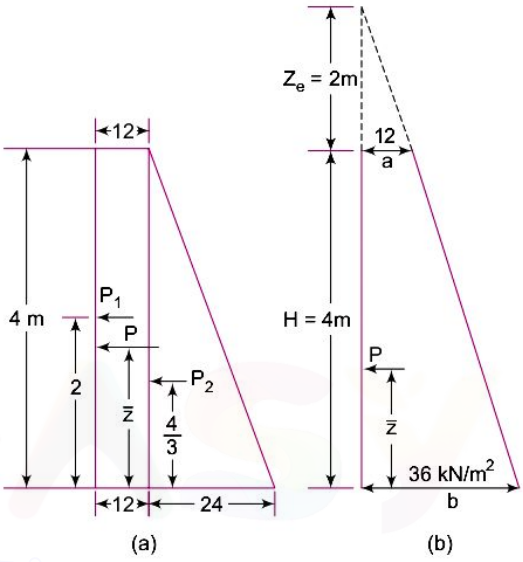

Fig. 20.14

$$P_1 = p_1 \times H = 12 \times 4 = 48 \text{ kN/m},$$

acting at $\dfrac{4}{2} = 2$ m from the base.

The resultant total pressure P_2 due to intensity p_2 is given by

$$P_2 = \frac{1}{2} p_2 H = \frac{1}{2} \times 24 \times 4 = 48 \text{ kN/m, acting at } \frac{1}{3} \times 4 = 1.33 \text{ m from the base.}$$

$\therefore$ $P = P_1 + P_2 = 48 + 48 = 96$ kN per meter length of the wall.

The resultant P acts at a distance $\bar{z}$ above the base, given by taking the moments about the base :

$$\bar{z} = \frac{(48 \times 2) + \left(48 \times \dfrac{4}{3}\right)}{96} = \mathbf{1.67 \text{ m.}}$$

Alternative solution. Equivalent height of surcharge is given by Eq. 20.22 :

$$z_e = \frac{q}{\gamma} = \frac{36}{18} = 2 \, m$$

Thus the problem reduces to the calculation of active earth pressure due to height

$$Z = H + z_e = 4 + 2 = 6 \text{ m.}$$

The pressure intensity at the base of the wall is given by

$$p_a = K_a \gamma Z = \frac{1}{3} \times 18 \times 6 = 36 \text{ kN/m}^2$$

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Pressure intensity at the top of the wall is evidently 12 kN/m^2. The pressure distribution is thus trapezoidal, having pressure intensities 12 and 36 kN/m^2.

$$P = \frac{1}{2}(12 + 36) \times 4 = 96 \text{ kN/m}^2$$

The distance of point of application from the base given by

$$\bar{z} = \frac{H}{3} \cdot \frac{2a + b}{a + b} = \frac{4}{3} \cdot \frac{2 \times 12 + 36}{12 + 36} = \mathbf{1.67\ m.}$$

Example 20.4. *In example 20.3, if the water table rises behind the wall to an elevation 1.5 m below the top, determine the total active pressure and its point of application. Take submerged weight of sand as 12 kN/m^3. Assume that there is no change in the angle of shearing resistance due to submergence.*

Solution. Let p_1 = lateral pressure intensity due to surcharge.

p_2 = lateral pressure intensity due to dry soil.

p_4 = lateral pressure intensity due to submerged soil.

p_5 = lateral pressure intensity due to water.

$$\therefore \quad p_1 = K_a \cdot q = \frac{1}{3} \times 36 = 12 \text{ kN/m}^2$$

$$p_2 = K_a \gamma H_1 = \frac{1}{3} \times 18 \times 1.5 = 9 \text{ kN/m}^2$$

$$p_4 = K_a \gamma' H_2 = \frac{1}{3} \times 12(2.5) = 10 \text{ kN/m}^2$$

$$p_5 = \gamma_w \cdot H_2 = 9.81 \times 2.5 = 24.53 \text{ kN/m}^2.$$

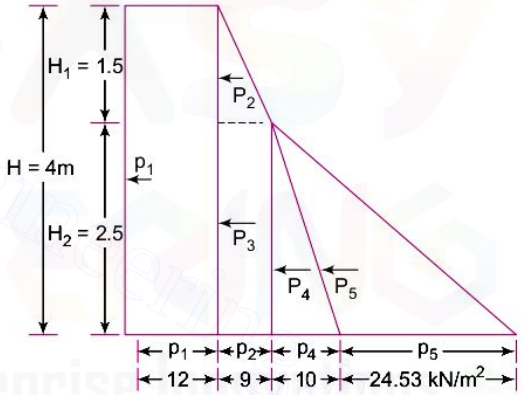

Fig. 20.15

Figure 20.15 shows the pressure distribution diagram with the resultant pressure P_1, P_2, P_3, P_4 and P_5.

$$P_1 = p_1 \cdot H = 12 \times 4 = 48 \text{ kN/m}$$

acting @ $\frac{4}{2}$ = 2 m from base.

$$P_2 = \frac{1}{2} p_2 H_1 = \frac{1}{2} \times 9 \times 1.5$$

$$= 6.75 \text{ kN/m},$$

acting @ $2.5 + \frac{1.5}{3}$ = 3 from base.

$P_3 = p_2 H_2 = 9 \times 2.5 = 22.5$ kN/m, acting @ 1.25 m from base.

$$P_4 = \frac{1}{2} p_4 H_2 = \frac{1}{2} \times 10 \times 2.5 = 12.5 \text{ kN/m, acting @ 0.833 m from base.}$$

$$P_5 = \frac{1}{2} p_5 H_2 = \frac{1}{2} \times 24.53 \times 2.5 = 30.66 \text{ kN/m, acting @ 0.833 m from base.}$$

$\therefore$ Total pressure $= P = P_1 + P_2 + P_3 + P_4 + P_5$

$= 48 + 6.75 + 22.5 + 12.5 + 30.66 = \mathbf{120.41\ kN/m}$

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The distance $\bar{z}$ of the point of application of P above the base is obtained by taking moments about the base

$$\bar{z} = \frac{1}{120.41}[(48\times 2)+(6.75\times 3)+(22.5\times 1.25)+(12.5\times 0.833)+(30.66\times 0.833)]$$

$$= \mathbf{1.50\ m.}$$

Example 20.5. *A vertical excavation was made in a clay deposit having weight of 20 kN/m^3. It caved in after the depth of digging reached 4 metres. Taking the angle of internal friction to be zero, calculate the value of cohesion. If the same clay is used as a backfill against a retaining wall, up to a height of 8 metres, calculate (i) total active earth pressure, (ii) total passive earth pressure. Assume that the wall yields far enough to allow Rankine deformation conditions to establish.*

Solution. The critical height H_c of an unsupported vertical cut in cohesive soil is given by equation 20.31:

$$H_c = \frac{4c}{\gamma}\tan\alpha$$

As $\phi = 0$, $\quad \tan\alpha = \tan(45° + \phi/2) = 1$

$$\therefore \quad c = \frac{H_c\gamma}{4} = \frac{4\times 20}{4} = 20\ \text{kN/m}^2 \qquad ...(1)$$

(*i*) Total active earth pressure is given by equation 20.30

$$P_a = \frac{1}{2}\gamma H\cot^2\alpha - 2cH\cot\alpha = \frac{1}{2}\times 20\times(8)^2\times 1 - (2\times 20\times 8)\times 1$$

$$= 640 - 320 = \mathbf{320\ kN/m}$$

(*ii*) Total passive earth pressure is given by equation 20.47

$$P_p = \frac{1}{2}\gamma H\tan^2\alpha + 2cH\tan\alpha = \frac{1}{2}\times 20\times(8)^2\times 1 + (2\times 20\times 8)\times 1$$

$$= 640 + 320 = \mathbf{960\ kN/m.}$$

Example 20.6. *A 6 m high vertical wall supports a saturated, cohesive backfill ($\phi_u = 0$) with horizontal surface. The top 3 m of the backfill weighs 18 kN/m^3 and has an apparent cohesion of 18 kN/m^2. The bulk unit weight and apparent cohesion of the bottom 3 m of the backfill are respectively 20 kN/m^3 and 24 kN/m^2. What is the likely depth of tension cracks behind the wall? If tension cracks develop, what will be the total active pressure? Draw the pressure distribution diagram and determine the point of application of the resultant pressure.*

Solution. The active earth pressure for a saturated clay ($\phi_u = 0$) is given by equation 20.37.

$$p_a = \gamma_{sat}\cdot z - 2c_u \qquad ...(1)$$

and the height of tension cracks is given by equation 20.39

$$z_0 = \frac{2c_u}{\gamma_{sat}} \qquad ...(2)$$

For the top soil, $\quad \gamma_{sat} = 18\ \text{kN/m}^3$ and $c_u = 18\ \text{kN/m}^2$.

For the bottom soil, $\quad \gamma_{sat} = 20\ \text{kN/m}^3$ and $c_u = 24\ \text{kN/m}^2$

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From Eq. (1), we get

when $z = 0$, $p_a = -2c_u = -2 \times 18 = -36$ kN/m^2

$$z_0 = \frac{2c_u}{\gamma_{sat}} = \frac{2 \times 18}{18} = 2 \text{ m}$$

$z = 3$ m, $p_a = (18 \times 3) - (2 \times 18) = 18$ kN/m^2 (upper soil)

$z > 3$ m, $p_a = (18 \times 3) + 20(z - 3) - (2 \times 24) = 54 + 20(z - 3) - 48$ ...(3)

$z = 3$ m, $p_a = 54 - 48 = 6$ kN/m^2 (lower soil)

$z = 6$ m, $p_a = 54 + 20(6 - 3) - 48 = 66$ kN/m^2

The pressure distribution is shown in Fig. 20.15 (*a*).

Assuming that the tension cracks have been developed the *net* active pressure will be equal to the area of the positive pressure diagram.

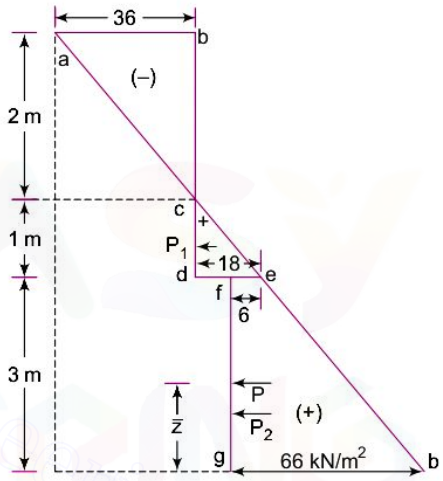

Fig. 20.15(a)

$$P_1 = \frac{1}{2} \times 1 \times 18 = 9 \text{ kN/m}$$

Acting at $3 + \frac{1}{3} = 3.33$ m from the base

$$P_2 = \frac{6 + 66}{2} \times 3 = 108 \text{ kN/m}$$

Acting at $\frac{3}{3}\left(\frac{2 \times 6 + 66}{6 + 66}\right) = 1.083$ m from the base.

Total pressure $P = P_1 + P_2 = 9 + 108$

$= \mathbf{117\ kN/m}$ length.

Distance of the point of application of P from base is given by

$$\bar{z} = \frac{(9 \times 3.33) + (108 \times 1.083)}{117} = \mathbf{1.26\ m.}$$

Example 20.7. *A retaining wall, 4 m high supports a backfill ($c = 20$ kN/m^2; $\phi = 30°$; $\gamma = 20$ kN/m^3) with horizontal top, flush with the top of the wall. The backfill carries a surcharge of 20 kN/m^2. If the wall is pushed towards the backfill, compute the total passive pressure on the wall, and its point of application.*

Solution. $c = 2$ kN/m^2

$\phi = 30°$; $K_p = N_\phi = \tan^2 (45° + \phi/2) = \tan^2 60° = 3$

The passive pressure intensity due to surcharge is given by

$$p_1 = K_p \,.\, q = N_\phi\, q = 3 \times q = 3 \times 20 = 60 \text{ kN/m}^2$$

The passive pressure intensity due to backfill is given by Eq. 20.46 *a*:

$$p = 2c\sqrt{N_\phi} + \gamma H N_\phi = p_2 + p_3$$

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where $p_2 = 2c\sqrt{N_\phi} = 2 \times 20\ \sqrt{3} = 69.28\ \text{kN/m}^2$

and $p_3 = \gamma H N_\phi = 20 \times 4 \times 3 = 240\ \text{kN/m}^2$

The pressure distribution diagram consists of two rectangles and one triangle corresponding to p_1, p_2 and p_3, as shown in Fig. 20.16.

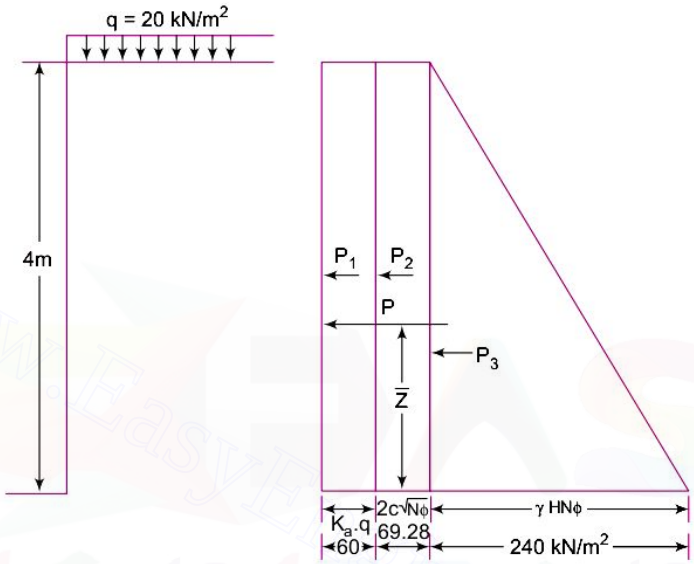

Fig. 20.16

$\therefore$ $P_1 = H\,(q\,.\,N_\phi) = 4 \times 60 = 240$ kN/m acting at 2 m above base

$P_2 = H\,(2c\sqrt{N_\phi}) = 4 \times 69.28 = 277.1$ kN/m, acting at 2 m above base

$P_3 = \frac{1}{2}\,H\,(\gamma H N_\phi) = \frac{1}{2} \times 4 \times 240 = 480$ kN/m, acting at $\frac{4}{3}$ m above base

$\therefore$ $P = P_1 + P_2 + P_3 = 240 + 277.1 + 480 = \mathbf{997.1\ kN/m}$

Its point of application above the base is given by

$$\bar{z} = \frac{(240 \times 2) + (277.1 \times 2) + \left(480 \times \frac{4}{3}\right)}{997.1} = \mathbf{1.68\ m.}$$

Example 20.8. *A wall of 6 m height retains sand having a unit weight of 20 kN/m³ and angle of internal friction of 30°. If the surface of the backfill slope upwards at 15° to the horizontal, find the active thrust per unit length of the wall, using Rankines' theory. Solve the problem both analytically as well as graphically. Also, draw the general failure pattern diagram (i.e., slip lines) for such a case.*

Solution. *(a) Analytical solution*

$$P_a = \frac{1}{2}\,K_a\,\gamma\,H^2 \text{ where } K_a = \cos\beta\,\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$$

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Here $\beta = 15°$, $\cos \beta = 0.9659$ and $\cos^2 \beta = 0.9330$

$\varphi = 30°$, $\cos^2 \varphi = 0.75$

$$\therefore \quad K_a = 0.9659 \times \frac{0.9659 - \sqrt{0.9330 - 0.75}}{0.9659 + \sqrt{0.9330 - 0.75}} = 0.3729$$

$$\therefore \quad P_a = \frac{1}{2} \times 0.3729 \times 20\ (6)^2 = \mathbf{134.2\ kN/m}$$

(b) Graphical solution (Fig. 20.17)

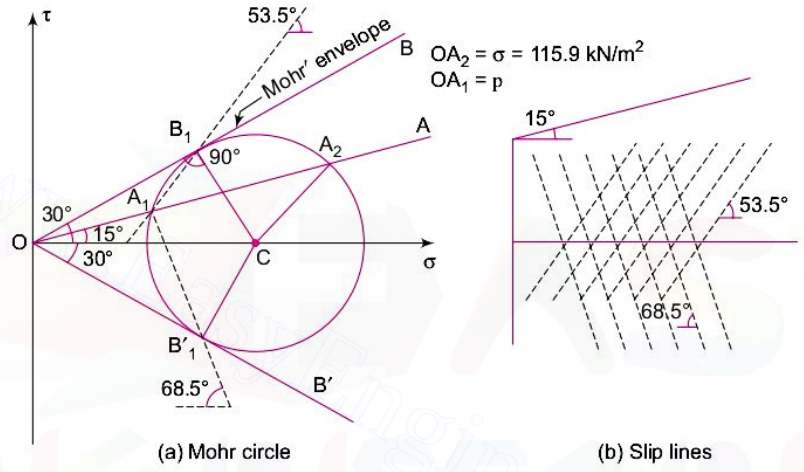

Fig. 20.17 Graphical solution

From Eq. 20.23(*a*)

$$\sigma = \gamma H \cos \beta = 20 \times 6 \cos 15° = 115.9\ \text{kN/m}^2$$

Step 1: Draw Mohr envelope *OB* at an angle 30 ° and ground line *OA* at an angle of 15° with horizontal axis (Fig. 20.17). Also, draw line OB_1', at – 30°.

Step 2: On ground line *OA*, mark $OA_2 = \sigma = 115.9$ to some scale.

Step 3: Draw Mohr circle, such that (*i*) its centre *C* is on the horizontal axis, (*ii*) it passes through point A_2 and (*iii*) it is tangential to the line *OB*. In other words the centre *C* is selected on horizontal axis in such a way that $CA_2 = CB_1$.

Step 4: The point A_1 at which the circle cuts the ground line, represents the earth pressure. Thus, $p_a = OA_1 \approx 45\ \text{kN/m}^2$ (by measurement).

The failure planes are parallel to $A_1 B_1$ and AB_1'. The directions of these are measured as 53.5° and 68.5° respectively to the horizontal (Fig. 20.17 *b*).

Example 20.9. *A counterfort retaining wall of 8 m height retains non-cohesive backfill. The void ratio and angle of internal friction of the backfill respectively are 0.8 and 28° in loose state and they are 0.45 and 36° in the dense state. Calculate and compare active and passive earth pressures in both cases, taking G = 2.68. Give your comments on the result.*

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Solution. The computations are done in the tabular form below:

S.No.	*Case 1: Loose state*	*Case 2 : Dense state*
1.	$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.68 \times 9.81}{1+0.8} = 14.606 \text{ kN/m}^2$	$\gamma_d = \frac{2.68 \times 9.81}{1+0.45} = 18.132 \text{ kN/m}^2$
2.	$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 28°}{1+\sin 28°} = 0.361$	$K_a = \frac{1-\sin 36°}{1+\sin 36°} = 0.2596$
3.	$K_p = 1/0.361 = 2.77$	$K_p = 1/0.2596 = 3.852$
4.	$p_a = K_a \gamma_d H = 0.361 \times 14.606 \times 8 = 42.18 \text{ kN/m}^2$	$p_a = 0.2596 \times 18.132 \times 8 = 37.66 \text{ kN/m}^2$
5.	$p_p = K_p \gamma_d H = 2.77 \times 14.606 \times 8 = 323.67 \text{ kN/m}^2$	$p_p = 3.852 \times 18.132 \times 8 = 558.76 \text{ kN/m}^2$
6.	Ratio $\frac{p_{a1}}{p_{a2}} = \frac{42.18}{37.66} = 1.12$	
7.	Ratio $\frac{p_{p1}}{p_{p2}} = \frac{323.67}{558.76} = 0.58$	

Comments: *Densification of soil decreases active earth pressure and increases passive earth pressure. Since active earth pressure is a disturbing force, it is advantageous to reduce it by compacting the backfill.*

Example 20.10. *In order to estimate the greatest height to which a vertical cut can be made, without support in a clay soil deposit, a triaxial test was run on the soil and the following observations were made:*

σ_3 *(kN/m²)*	*4.5*	*9*	*13.8*	*18.2*
σ_1 *(kN/m²)*	*39.5*	*53*	*66.8*	*79.2*

Calculate the height of the cut, if the unit weight of clay is 19 kN/m³.

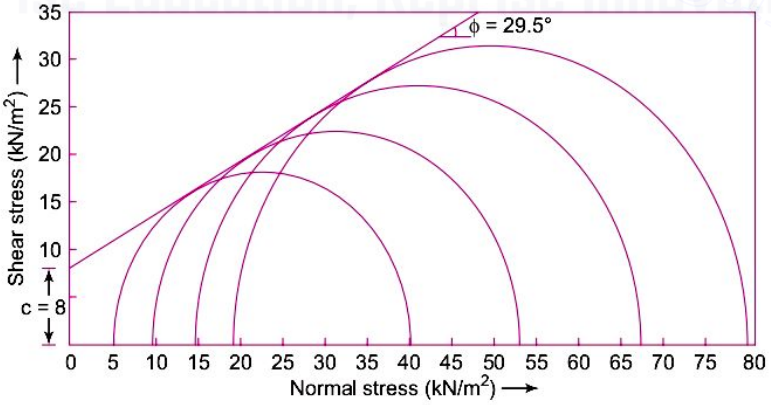

Fig. 20.18

Solution. Figure 20.18 shows the conventional failure envelope from the four Mohr's circles. From the plot, we get $\varphi = 29.5°$ and $c = 8 \text{ kN/m}^2$

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Hence, $$\alpha = 45° + \frac{\varphi}{2} = 45° + \frac{29.5°}{2} = 59.75°$$

Now, $$z_0 = \frac{2c}{\gamma} \tan \alpha \text{ (Eq. 20.28)} = \frac{2 \times 8}{19} \tan 59.75° \approx 1.44 \text{ m}$$

$\therefore$ Critical height H_c of unsupported vertical cut $= 2\, z_0 = 2 \times 1.44 \approx$ **2.9 m**

20.7 RANKINE'S ACTIVE THRUST BY TRIAL WEDGES*

The Rankine's active earth pressure (*i.e.,* assuming the wall to be smooth) can also be found by method of trial wedges. Figure 20.19 shows a trial wedge *ABC*, considered as a free body. The three forces acting on it are (*i*) the resultant force *P* that must exist between the free body and the wall, (*ii*) the weight *W* of the wedge, and (*iii*) the resultant reaction *F* between the wedge and the rest of the soil along the surface *BC*.

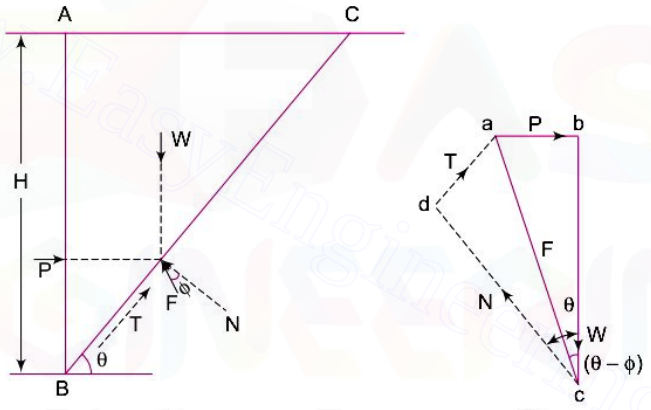

Fig. 20.19 Rankine's thrust by trial wedge method

A trial wedge is taken, with θ as the inclination of the surface *BC* with horizontal. Figure 20.19(*b*) shows the force diagram, in which $F(= ca)$ is composed of the normal component *N* and the tangential component *T*. Applying conditions of equilibrium on the wedge, *P* can be calculated in terms of weight *W* and angle θ. A number of trials may be made to get the corresponding value of thrust *P*. The actual force P_a against the wall will be *largest* value of *P* found as above as the result of considering all possible free bodies. Even though the active thrust is the minimum possible thrust for which the backfill can be in equilibrium, we must seek the free body that gives the largest value of this thrust consistent with the assumption that the full shear strength of the soil is mobilised. The equations of equilibrium are:

$\Sigma V = 0$; $\quad \therefore \quad F = \dfrac{W}{\cos(\theta - \phi)}$ ...(1)

$\Sigma H = 0$ $\quad \therefore \quad P = W \tan(\theta - \phi)$ ...(2)

But $$W = \text{weight of soil} = \frac{1}{2} \gamma H^2 \cot \theta$$

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$\therefore$ From (2), we get $P = \frac{1}{2}\gamma H^2 \cot\theta \tan(\theta - \phi)$ ...(20.48)

For maximum value of P, $\frac{\partial P}{\partial \theta} = 0 \quad \therefore \quad \frac{\partial P}{\partial \theta} = \frac{1}{2}\gamma H^2 \left[-\frac{\tan(\theta-\phi)}{\sin^2\theta} + \frac{\cot\theta}{\cos^2(\theta-\phi)} \right] = 0$

or $\frac{-\sin(\theta-\phi)\cos(\theta-\phi) + \sin\theta\cos\theta}{[\sin\theta\cos(\theta-\phi)]^2} = 0$

or $-\sin\theta\cos\theta(\cos^2\phi - \sin^2\phi - 1) - \sin\phi\cos\phi(\sin^2\theta - \cos^2\theta) = 0$

or $\sin 2\theta \sin^2\phi + \sin\theta\cos\phi\cos 2\theta = 0$ or $\sin\phi\cos(2\theta - \phi) = 0$

$\therefore \quad \cos(2\theta - \phi) = 0 = \cos 90°$

$\therefore \quad \theta = \theta_{cr} = 45° + \phi/2 = \alpha$ ...(20.49)

Substituting in Eq. 20.48, we get

$P = P_a = \frac{1}{2} . \gamma H^2 \cot(45° + \phi/2) \tan(45° - \phi/2) = \frac{1}{2}\gamma H^2 \tan^2(45° - \phi/2) = \frac{1}{2}\gamma H^2 K_a$ which is the same as Eq. 20.15.

20.8 COULOMB'S WEDGE THEORY

Instead of considering the equilibrium of an element within the mass of the material, Coulomb (1776) considered the equilibrium of whole of the material supported by a retaining wall when the wall is on the point of moving slightly away from the filling. The *wedge theory* of earth pressure is based on the concept of a sliding wedge which is torn off from the rest of the backfill on movement of the wall. In the case of active earth pressure, the sliding wedge moves downwards and outwards on a *slip surface* relative to the intact backfill and in the case of passive earth pressure, the sliding wedge moves upwards and inwards. The pressure on the wall is, in fact, a force of reaction which it has to exert to keep the sliding wedge in equilibrium. Factors such as wall friction, irregular soil surfaces and different soil strata can easily be taken into account in this method. Following are the basic *assumptions* of the wedge theory:

1. The backfill is dry, cohesionless, homogenous, isotropic and elastically undeformable but breakable.
2. The slip surface is plane which passes through the heel of the wall.
3. The sliding wedge itself acts as a rigid body and the value of earth pressure is obtained by considering the limiting equilibrium of the sliding wedge as a whole.
4. The position and direction of the resultant earth pressure are known. The resultant pressure acts on the back of the wall at one-third the height of the wall from the base and is inclined at an angle δ (called the angle of *wall friction)* to the normal to the back. (The assumption means that the pressure distribution is hydrostatic, *i.e.*, triangular). The back of the wall is rough and a relative movement of the wall and the soil on the back takes place which developes frictional forces that influence the direction of the resultant pressure.

The forces acting on a wedge of soil are: its weight W, the reaction R along the plane of sliding and the active thrust P_a against the retaining wall. R will act at an angle ϕ to the normal of the plane of sliding. The pressure P is inclined at an angle of wall friction δ to the normal which is considered positive as marked in Fig. 20.20. Both R and P_a will be inclined in a direction so as to oppose the movement of the

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wedge. For the condition of the yield of the wall from the backfill the most dangerous or the *critical* slip surface is that for which the wall reaction is maximum, *i.e.,* the wall must resist the maximum lateral pressure before it moves away from the fill.

Condition for maximum pressure from a sliding wedge. *BD* shows a plane inclined at an angle ϕ to the horizontal at which the soil is expected to stay in the absence of any lateral support. The line *BD,* therefore, is called the natural slope line, *repose line* or the ϕ - line. *AD,* inclined at β to the horizontal, is called the *ground line* or *surcharge line.* Plane *BC*, inclined at angle λ (to be determined) is the *S* line or rupture plane or *slip plane;* the angle λ is called the *critical slip angle.* The reaction *R* inclined at an angle ϕ to the normal to the slip line; *R* is also inclined at an angle $(\lambda - \phi)$ to the vertical. The wall reaction P_a is inclined at an angle δ to the normal to the wall. The inclination of P_a to vertical is represented by angle $\psi = 90° - \theta - \delta$ (= constant for given value of θ and δ). The force triangle is shown in Fig 20.20(*b*). The value of P_a depends upon the slip angle λ. P_a is zero when $\lambda = \phi$. As λ increases beyond ϕ, *P* also increases and after reaching a maximum value it again reduces to zero when λ equals $90° + \theta$. Thus, the critical slip plane lies between ϕ-line and the back of the wall.

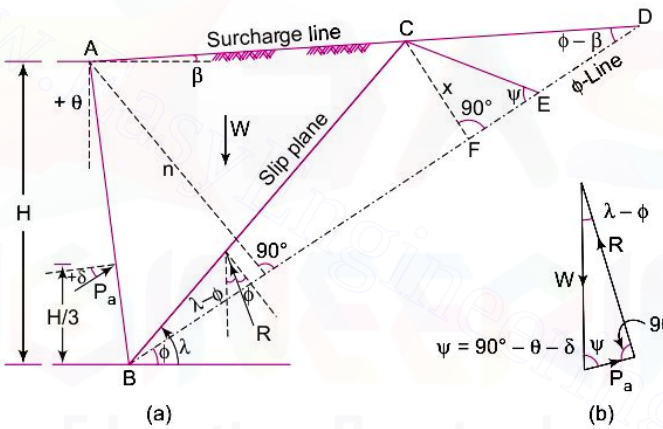

Fig. 20.20 Coulomb's wedge theory

In order to derive the condition for maximum active pressure P_a from the sliding wedge, draw line *CE* at an angle ψ to the ϕ-line, Let *x* and *n* be the perpendicular distances of points *C* and *A* from the ϕ-line, and *m* be the length of line *BD*. It will be seen that triangle *BCE* and the force triangle [Fig. 20.20 (*b*)] are similar.

Hence
$$\frac{P_a}{W} = \frac{CE}{BE} \quad \therefore \quad P_a = W \cdot \frac{CE}{BE} \qquad ...(1) \quad ...(20.50)$$

Now,
$$CE = x \operatorname{cosec} \psi = A_1 x, \text{ (where } A_1 = \operatorname{cosec} \psi = \text{constant)}$$
$$BE = BD - (DF - FE) = m - x\,[\cot(\phi - \beta) - \cot\psi] = m - A_2 x$$

where
$$A_2 = \cot(\phi - \beta) - \cot\psi = \text{constant}$$
$$W = \gamma(\Delta ABD) = \gamma(\Delta ABD - \Delta BCD) = \frac{1}{2}\gamma m(n - x)$$

Substituting the values of *CE*, *BE* and *W* in (1), we get
$$P_a = \frac{1}{2}\gamma m(n - x)\frac{A_1 x}{m - A_2 x} = \frac{1}{2}\gamma m A_1 \frac{nx - x^2}{m - A_2 x} \qquad ...(2)$$

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In the above expression, x is the only variable which depends upon the position of slip plane BC. For maxima, $\frac{dP_a}{dx} = 0$.

$$\therefore \quad \frac{dP_a}{dx} = \frac{1}{2}\gamma m A_1 \frac{(n-2x)(m-A_2 x)-(-A_2)(nx-x^2)}{(m-A_2 x)^2} = 0$$

$$\therefore \quad (n-2x)(m-A_2 x) = -A_2 (nx-x^2)$$

or $$mn - mx = mx - A_2 x^2 = x(m - A_2 x) \text{ or } \frac{mn}{2} - \frac{mx}{2} = \frac{x \cdot BE}{2}$$

or $$\Delta ABC = \Delta BCE \qquad ...(20.51)$$

Thus the criterion for maximum active pressure is that the slip-plane is so chosen that ΔABC and ΔBCE are equal in area. Rebhann (1971) is credited to have presented this criterion.

20.9 REBHANN'S GRAPHICAL METHOD FOR ACTIVE PRESSURE

Rebhann (1871) presented a graphical method for the location of the slip plane and the total active earth pressure according to the Coulomb's Wedge theory.

Procedure (Fig. 20.21):

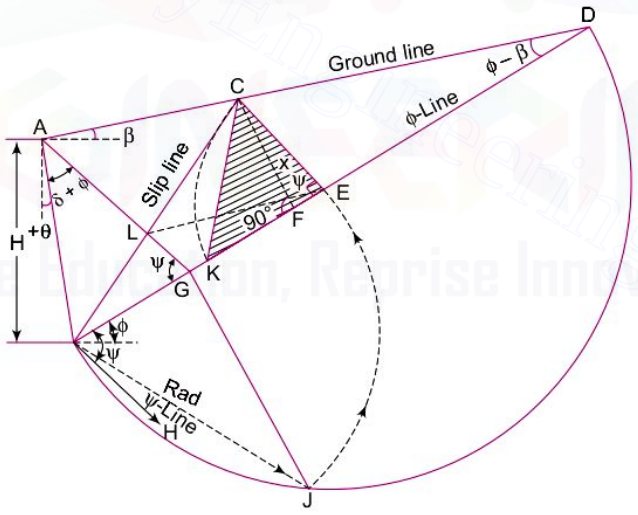

Fig. 20.21 Rebhann's graphical method for active pressure

1. Draw the ground line and ϕ-line at angles β and ϕ respectively with the horizontal to meet in point D.
2. Draw a semi-circle on BD as diameter.
3. Through B, draw a line BH at an angle ψ with BD. Line BH is called the *earth pressure line* or the ψ-line.
4. Through A, draw line AG parallel to the ψ-line.
5. Draw GJ perpendicular to BD, to meet the semi-circle in J.

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6. With B as centre, and BJ as radius, draw an arc to cut BD in E.
7. Through E, draw EC parallel to the ψ-line. BC then represents the slip plane.
8. With E as the centre and EC as radius, draw an arc to cut BD in K. Join CK.
9. Calculate the total active earth pressure from the relation:

$$P_a = \gamma\,(\Delta\, KCE) = \frac{1}{2}\,\gamma\,(KE)\,x \qquad ...(20.52)$$

Proof of Eq. 20.52 is as follows: From the properties of a circle,

$$BG \times GD = GJ^2 \qquad \therefore \quad BG \times GD + BG^2 = GJ^2 + BG^2$$

$$\therefore \quad BG(BG + GD) = BJ^2 = BE^2 \qquad \therefore \quad \frac{BG}{BE} = \frac{BE}{BD} \qquad ...(1) \quad ...(20.53)$$

Again, since AG is parallel to CE, $\qquad \therefore \quad \dfrac{BG}{BE} = \dfrac{BL}{BC} \qquad ...(2)$

Hence, from (1) and (2), $\dfrac{BE}{BD} = \dfrac{BL}{BC}$

Hence LE is parallel to AD and figure $ALEC$ becomes a parallelogram. Hence perpendiculars on diagonal LC, drawn from A and E are of equal length. In other words, triangles ABC and BCE have equal altitudes. As they have common base BC and have equal altitudes, they become equal in area. Rebhann's construction, therefore, satisfies the condition of Eq. 20.51 of maximum pressure, and hence BC represents the critical slip plane. Now, the total active pressure is given by Eq. 20.50.

$$P_a = W.\,\frac{CE}{BE} = \gamma\,(\Delta\, ABC)\,.\,\frac{CE}{BE}$$

or $$P_a = \gamma\left(\frac{1}{2}\,BE\,.\,x\right).\frac{CE}{BE} = \frac{1}{2}\,\gamma\,.\,CE\,.\,x = \gamma\,(\Delta\, KCE) \qquad ...(20.52)$$

Also, since $x = CE \sin\psi$ Eq. (20.52) may also be written as

$$P_a = \frac{1}{2}\,\gamma\,(CE)^2 \sin\psi \qquad ...(20.54)$$

Expression for co-efficient of active earth pressure.* Refer Fig. 20.21. Let us determine the value of CE (appearing in Eq. 20.54) analytically in terms of height H and angles θ, β, δ and ϕ.

From similar triangle AGD and CED,

$$\frac{CE}{CD} = \frac{AG}{AD} \quad \text{or} \quad (CE)^2 = \left(\frac{AG}{AD} \cdot CD\right)^2 \qquad ...(1)$$

From triangle GAB, $\dfrac{GA}{AB} = \dfrac{\sin(90° - \phi + \theta)}{\sin\psi} = \dfrac{\cos(\phi - \theta)}{\sin\psi}$

or $$GA = AB\,\frac{\cos(\phi - \theta)}{\sin\psi} \qquad ...(2)$$

Also, $$\frac{AD}{CD} = \frac{AC + CD}{CD} = \frac{AC}{CD} + 1 = \frac{LE}{CD} + 1 = \frac{BE}{BD} + 1$$

(from Δs BLE and BCD) ...(3)

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Also, $BE^2 = BG \cdot BD$ (Eq. 20.53) $\therefore \quad \dfrac{BE}{BD} = \sqrt{\dfrac{BG}{BD}}$

Substituting in (3), we get $\dfrac{AD}{CD} = \sqrt{\dfrac{BG}{BD}} + 1$ ...(4)

Let us now find the values of BG and BD

From $\Delta\, GAB$, $\dfrac{BG}{AB} = \dfrac{\sin(\phi+\delta)}{\sin\psi} \quad \therefore \quad BG = AB \,.\, \dfrac{\sin(\phi+\delta)}{\sin\psi}$ ...(5)

From $\Delta\, ABD$, $\dfrac{BD}{AB} = \dfrac{\sin(90°+\beta-\theta)}{\sin(\phi-\beta)} \quad \therefore \quad BD = AB \,.\, \dfrac{\cos(\beta-\theta)}{\sin(\phi-\beta)}$ ...(6)

Substituting the values of BG and BD in (4), we get $\dfrac{AD}{CD}$. Substituting the values of $\dfrac{CD}{AD}$ and AG (Eq. 2) in Eq. 1, we get

$$(CE)^2 = AB^2 \frac{\cos^2(\phi-\theta)}{\sin^2\psi} \times \frac{1}{\left[\sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin\psi\cos(\beta-\theta)}} + 1\right]^2}$$

Substituting this value of $(CE)^2$ in equation 20.54, and noting that $AB = H \sec\theta$ and $\psi = 90° - \theta - \delta$ we get

$$P_a = \frac{1}{2}\gamma H^2 \left[\frac{\sec\theta \,.\, \cos(\phi-\theta)}{\sqrt{\cos(\theta+\delta)} + \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\cos(\beta-\theta)}}}\right]^2 \quad ...(20.54(a))$$

or $$P_a = \frac{1}{2} K_a \gamma H^2 \quad ...(20.55)$$

where, $$K_a = \left[\frac{\sec\theta \,.\, \cos(\phi-\theta)}{\sqrt{\cos(\theta+\delta)} + \sqrt{\dfrac{\sin(\phi+\delta)\sin(\phi-\beta)}{\cos(\beta-\theta)}}}\right]^2 \quad ...(20.55(a))$$

When θ is zero and $\delta = \beta$, Eq. 20.55 reduces to Eq. 20.25 developed by Rankine's theory.

The value of δ is difficult to be correctly known, and as an approximation, following values may be assumed:

(*i*) For smooth walls, $\delta = \dfrac{1}{3}\phi$

(*ii*) For ordinary retaining walls, $\delta = \dfrac{2}{3}\phi$

(*iii*) For rough walls with well drained backfill, $\delta = \dfrac{3}{4}\phi$

(*iv*) For backfill subjected to vibrations, $\delta = 0$

Rebhann's method is said to be based on Poncelet's solution, and hence the method is named after Poncelet.

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Special case. β and ϕ nearly equal. A special case in Rebhann's construction arises when β and ϕ are nearly equal so that ground line and ϕ line meet at too great a distance to be accommodated in the drawing. The following method of construction is adopted (Fig. 20.22):

1. Draw the ground line and ϕ-line. Through B draw the ψ line BH and an angle ψ to the ϕ-line.
2. Choose any point D_1 on the ϕ-line and draw semi-circle on $B\,D_1$ as diameter.
3. Draw $D_1\,A_1$ parallel to the ground line.
4. Draw $A_1\,G_1$ parallel to the ψ-line.
5. Draw $G_1\,J_1$ perpendicular to BD_1 to meet the semi-circle in J_1.
6. With B as centre and $B\,J_1$ as radius, draw arc BE_1. Join A_1E_1.
7. Draw AE parallel to $A_1\,E_1$. Thus, the required point E is obtained.

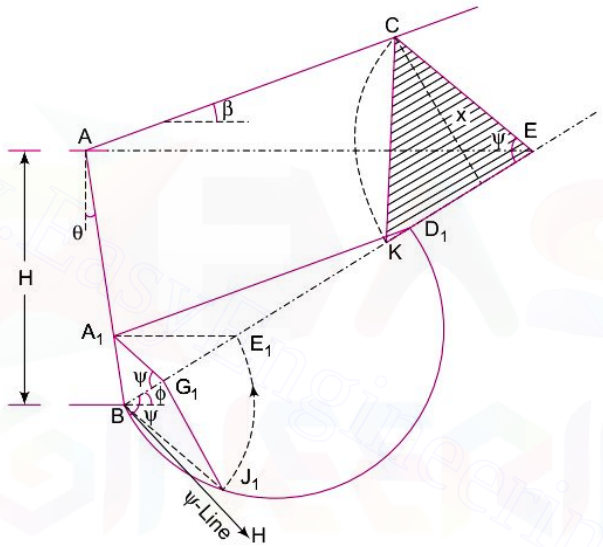

Fig. 20.22 Rebhann's construction when β and ϕ are nearly equal

8. Draw line EC parallel to the ψ-line. EC is then the required slip plane.
9. With E as centre and EC as radius, draw arc EK. Join C and K. Then

$$P_a = \gamma\,(\Delta\,KEC) = \frac{1}{2}\,\gamma\,.\,(CE)^2\,.\,\sin\psi.$$

Special case: β = ϕ. When β is exactly equal to ϕ, the ground line and ϕ-line will meet at infinity. The points C, D and the Δ CKE exist at infinity. However, Δ CKE can be constructed anywhere between the ground line and the ϕ-line, as shown in Fig. 20.23. The area of triangle remains constant.

Steps:

1. Draw the ground line and ϕ-line.
2. Choose any point E on the ϕ-line, and draw EC parallel to ψ-line.
3. Complete Δ CKE.

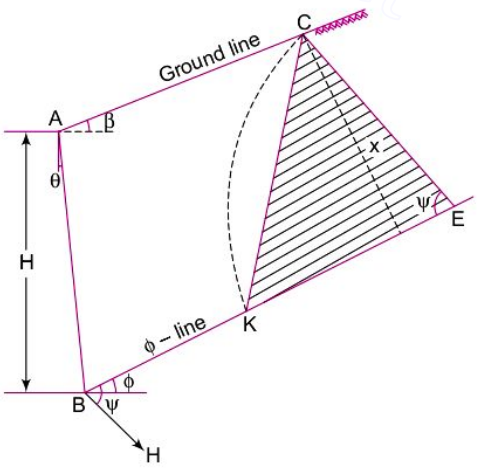

Fig. 20.23 Rebhann's construction when β = ϕ

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Backfill with uniform surcharge load

Figure 20.24 shows a backfill, with uniform surcharge load. Let BC be the failure plane.

Let $AC = L$; H = height of wall

Draw BA_1 perpendicular to the surcharge line Let $BA_1 = H_1$

We have $H = AB \cos \theta$

$$H_1 = AB \cos (\beta - \theta)$$

$$\therefore \quad H_1 = H \frac{\cos (\beta - \theta)}{\cos \theta} \qquad ...(20.56)$$

Weight of sliding wedge,

$$W = \frac{1}{2} \gamma HL \frac{\cos (\beta - \theta)}{\cos \theta} \qquad ...(20.57)$$

Increase in weight due to surcharge load = $L \times q$

$\therefore$ Total weight of wedge

$$W_1 = \frac{1}{2} \gamma HL \frac{\cos (\beta - \theta)}{\cos \theta} + L \, . \, q \qquad ...(20.58)$$

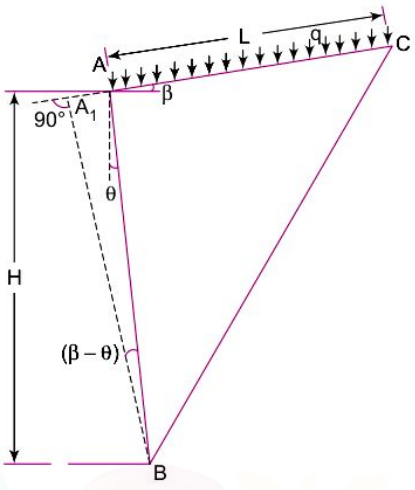

Fig. 20.24 Backfill with uniform surcharge load

Let γ_1 be the equivalent density of a sliding wedge of the same volume as ABC, and the weight of which is given by Eq. 20.58.

Then $$\frac{1}{2} \gamma_1 HL \frac{\cos (\beta - \theta)}{\cos \theta} = \frac{1}{2} \gamma HL \frac{\cos (\beta - \theta)}{\cos \theta} + L \, . \, q$$

or $$\gamma_1 = \gamma + \frac{2q}{H} \frac{\cos \theta}{\cos (\beta - \theta)} \qquad ...(20.59)$$

Thus, the effect of surcharge can be taken into account by taking a value of density γ_1, given by Eq. 20.59, in Eq. 20.52, and the usual method of construction can be used for obtaining the earth pressure.

Broken ground surface

Figure 20.25 shows the case of a broken ground surface AMD, having a break at M. The graphical construction is carried out in the following steps:

1. Join BM.
2. Through A, draw AA_1 parallel to BM, to meet DM prolonged in A_1.
3. Join $A_1 B$.

The break in the surface is thus eliminated by replacing area ABM by area $BA_1 M$ and the Rebhann construction can be performed in the usual manner by considering $A_1 B$ as the back of the wall.

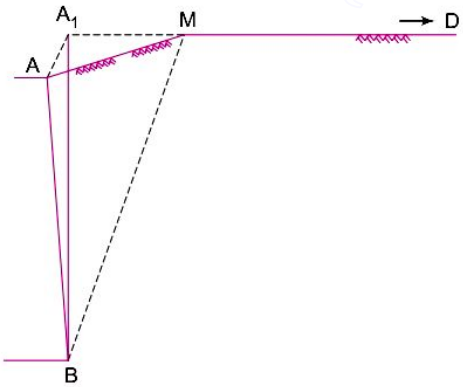

Fig. 20.25 Broken ground surface

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Example 20.11. *A retaining wall, 3 m high supports a dry cohesionless backfill with a plane ground surface sloping upwards at a surcharge angle of 10° from the top of the wall. The back of the wall is inclined to the vertical at a positive batter angle of 8°. The backfill weighs 19 kN/m³ and has an angle of shearing resistance of 30°. Assuming an angle of wall friction of 10°, determine by Rebhann's method the total active pressure, and the pressure distribution.*

Solution. Figure 20.21 shows the Rebhann's construction with the given data.

By measurement $CE = 2$ m and $CF = x = 1.9$ m

$$\therefore \quad P_a = \frac{1}{2}\gamma\,(CE \cdot x) = \frac{1}{2} \times 19\,(2 \times 1.9) = \mathbf{36.1\,kN/m}$$

It acts at a height $= \frac{H}{3} = \frac{3}{3} = 1$ m above the base and has an obliquity $= \delta = 10°$.

The intensity of pressure at the top A is zero and that at the base B is p_a. The average intensity of pressure over the length AB is thus $\frac{1}{2}(0 + p_a) = \frac{p_a}{2}$, and the total pressure on the wall is thus equal to average pressure × length $AB = \frac{1}{2}\,p_a\ (AB)$.

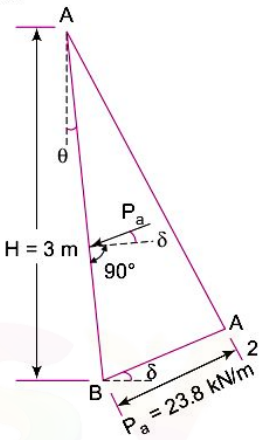

Fig. 20.26

Hence $\frac{1}{2}\,AB\,(p_a) = p_a$

$$\therefore \quad p_a = \frac{2\,P_a}{AB} = \frac{2\,P_a\,.\cos\theta}{H} = \frac{2 \times 36 \times \cos 8°}{3} = \mathbf{23.8}\ \text{kN/m}^2$$

Figure 20.26 shows the pressure distribution diagram.

Example 20.12. *Solve example 20.11 taking $\beta = 20°$.*

Solution. Figure 20.22 shows the Rebhann's construction with the given data. By measurement,

$CE = 2.13$ m and $CF = x = 2.06$ m

$$\therefore \quad P_a = \frac{1}{2} \times \gamma\ (CE\,.\,x) = \frac{1}{2} \times 19\ (2.13 \times 2.06) = \mathbf{41.684\ kN/m\ length.}$$

Example 20.13. *Solve example 20.11 taking $\beta = \phi = 20°$.*

Solution. Figure 20.23 shows the Rebhann's construction with the given data. By measurement, we get

$CE = 3$ m $\quad CF = x = 2.9$ m.

$$\therefore \quad P_a = \frac{1}{2} \times 19 \times 3 \times 2.9 = \mathbf{82.65\ kN/m.}$$

Example 20.14. *A masonry retaining wall 6 m high is back filled with granular soil having angle of internal friction of 32°. The back face (i.e., soil face) of the wall is inclined to the vertical at a positive batter angle of 12° and the backfill is sloping upward from the top of the wall at a slope of 10 : 1. Assuming the angle of wall friction as 16°, calculate the total active earth pressure on the wall per metre length. The backfill has water content of 16%, degree of saturation of 70% and specific gravity of 2.68.*

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Solution.

$$\tan \beta = \frac{1}{10}, \text{ which gives } \beta = 5.71°$$

$$e = \frac{wG}{S} = \frac{0.16 \times 2.68}{0.7} = 0.6125$$

$$\therefore \quad \gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.68 \times 9.81}{1+0.613}$$

$$= 16.3 \text{ kN/m}^3$$

and $$\gamma = \gamma_d (1 + w) = 16.3 (1 + 0.16)$$

$$= 18.91 \text{ kN/m}^3$$

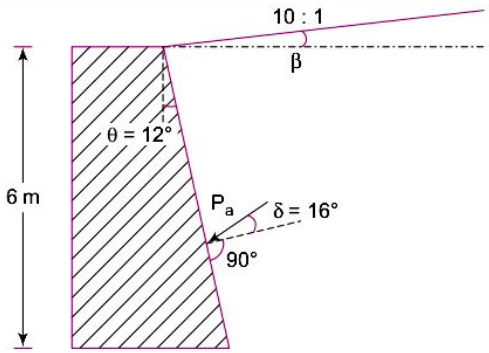

Fig. 20.27

$$K_a = \left[\frac{\sec \theta \cos (\varphi - \theta)}{\sqrt{\cos (\theta + \delta)} + \sqrt{\dfrac{\sin (\varphi + \delta) \sin (\varphi - \beta)}{\cos (\beta - \theta)}}} \right]$$

$\theta = 12°$, $\sec \theta = 1.0223$; $\varphi - \theta = 32 - 12° = 20°$; $\cos (\varphi - \theta) = 0.9397$

$\theta + \delta = 12 + 16 = 28°$; $\cos (\theta + \delta) = 0.8829$; $\varphi + \delta = 32° + 16° = 48°$; $\sin (\varphi + \delta) = 0.7431$

$\varphi - \beta = 32° - 5.71° = 26.29°$; $\sin (\varphi - \beta) = 0.4429$; $\beta - \theta = 5.71° - 12°$; $\cos (\beta - \theta) = 0.994$

$$\therefore \quad K_a = \left[\frac{1.0223 \times 0.9397}{\sqrt{0.8829} + \sqrt{\dfrac{0.7431 \times 0.4429}{0.994}}} \right]^2 = 0.402$$

$$\therefore \quad P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times 0.402 \times 18.91 (6)^2 = \mathbf{136.85 \text{ kN/m.}}$$

Example 20.15. *A wall of 5 m height retains a backfill of dry granular soil that has a level surface and weighs 20 kN/m^3. When there is no surcharge above the fill, the overturning moment caused by the total active pressure at a point at the base of the wall is 125 kN-m/m length of the wall. The specifications permit certain amount of uniformly distributed surcharge but state that surcharge must not increase the overturning moment by more than 50%. What surcharge can be allowed if the angle of wall friction is 20°?*

Solution. Since θ is not given, the backface of the wall is vertical, *i.e.*, $\theta = 0$; $\delta = 20°$; $\beta = 0$ in the *first case*; φ is not known to start with.

Now, $$M_a = \frac{1}{6} K_a \gamma H^3, \text{ where } M_a = 125 \text{ kN-m/m.}$$

$$\therefore \quad 125 = \frac{1}{6} K_a \times 20 (5)^3, \text{ from which } K_a = 0.3$$

Substituting this value in equation 20.55 (*a*), we have

$$K_a = 0.3 = \left[\frac{\sec 0 \cos (\varphi - 0)}{\sqrt{\cos (0 + 20°)} + \sqrt{\dfrac{\sin (\varphi + 20°) \sin (\varphi - 0)}{\cos (0 - 0)}}} \right]^2$$

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or $$\sqrt{(0.3)} = 0.5477 = \frac{\cos\varphi}{\sqrt{\cos 20^\circ} + \sqrt{\frac{\sin(\varphi + 20^\circ)\sin\varphi}{1}}}$$

or $$\cos\varphi = 0.5477\left[0.9694 + \sqrt{\sin(\varphi + 20^\circ)\sin\varphi}\right]$$

The above equation, in which φ is unknown, has to be solved by trial and error. In the first trial, substituting $\varphi = 30^\circ$ in the right hand side, we get

$$\varphi = \cos^{-1} 0.5477\left\{0.9694 + \sqrt{\sin(30^\circ + 20^\circ)\sin 30^\circ}\right\} = 29.55^\circ$$

In the second trial, substituting $\varphi = 29.55^\circ$ in the right hand side, we get

$$\varphi = \cos^{-1}\left[0.5477\left\{0.9694 + \sqrt{\sin 49.55 \sin 29.55}\right\}\right] = 29.948^\circ$$

Hence adopt $\varphi = 29.55^\circ$.

For the second case: $M_a = 1.5 \times 125$

Since $M_a \propto K_a$, $K_a = 1.5 \times 0.3 = 0.45$

Hence substituting in equation 20.55 (a) and noting that $\beta \neq 0$, we get

$$K_a = \sqrt{0.45} = 0.6708 = \left[\frac{\sec 0 \cos(29.95^\circ - 0)}{\sqrt{\cos(0 + 120^\circ)} + \sqrt{\frac{\sin(29.95 + 20)\sin(29.95 - \beta)}{\cos(\beta - 0)}}}\right]$$

or $$0.6708 = \frac{0.8665}{0.9694 + \sqrt{\frac{0.7655\sin(29.95 - \beta)}{\cos\beta}}}$$

or $$\sqrt{\frac{0.7655\sin(29.95^\circ - \beta)}{\cos\beta}} = \frac{0.8665}{0.6708} - 0.9694 = 0.3223$$

or $$\frac{\sin(29.95^\circ - \beta)}{\cos\beta} = \frac{(0.3223)^2}{0.7655} = 0.1357 \quad \text{or} \quad \sin(29.95^\circ - \beta) = 0.1357\cos\beta \qquad ...(1)$$

The above equation is solved by trial and error, noting that β can have a maximum value of 29.95° only. To start with, substituting $\beta = 25^\circ$ in the RHS, we get $\sin(29.95 - \beta) = 0.1357 \cos 25^\circ = 0.123$,

From which $\beta = 29.95^\circ - \sin^{-1} 0.123 = 22.89^\circ$.

In the second trial, putting $\beta = 22.89^\circ$ in the RHS of eq. (1) above, we get

$$\sin(29.95^\circ - \beta) = 0.1357 \cos 22.89^\circ = 0.1250$$

From which $\beta = 29.95 - \sin^{-1} 0.1250 = 22.77^\circ$

Hence adopt $\beta \approx \mathbf{22.8^\circ}$ = surcharge angle.

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20.10 CULMANN'S GRAPHICAL METHOD FOR ACTIVE PRESSURE

Culmann (1886) also gave a graphical solution to evaluate the active earth pressure by the Coulomb's theory. His method is more general than Rebhann's method, and can be conveniently used for ground surface of any shape, for various types of surcharge loads, and for a layered backfill of different densities.

Procedure:

1. Draw the ground line, ϕ-line and the ψ-line as usual.
2. Take a slip plane BC_1. Calculate the weight of the wedge ABC_1 and plot it as BE_1, to some scale, on the ϕ-line.
3. Through E_1, draw $E_1 F_1$ parallel to the ψ-line, to cut the slip plane BC_2 in F_1.

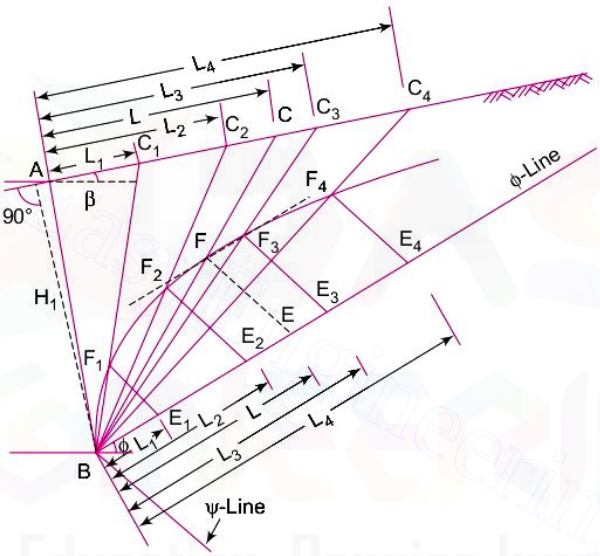

Fig. 20.28 Culmann's graphical method for active pressure

4. Similarly take another slip plane BC_2, calculate the weight of wedge ABC_2 and plot it as BE_2 on the ϕ-line. Draw $E_2 F_2$ parallel to the ψ-line to cut the slip plane BC_2 in F_2.
5. Take a number of such slip planes BC_3, BC_4 etc., plot the weight of the corresponding wedges on the ψ-line and obtain points F_3, F_4, etc.
6. Draw a smooth curve through points B, F_1, F_2, F_3, F_4 etc. This curve is known as the *Culmann's line.*
7. Draw a tangent to the Culmann's line, parallel to the ϕ-line. The maximum value of the earth pressure is represented by the intercept EF, on the adopted scale, EF being drawn through the point of tangency parallel to the ψ-line. BFC represents the critical slip plane.
8. To locate the point of application of the resultant pressure, draw a line (not shown) parallel to the critical slip plane BC, through the centre of gravity of the sliding wedge ABC and obtain its intersection on the back AB.

When the ground line is a plane, the weights of the wedges ABC_1, ABC_2, ABC_3, ABC etc. are proportional to the length AC_1 $(= L_1)$, AC_2 $(= L_2)$, AC_3 $(= L_3)$, AC $(= L)$ etc., since the height of each soil wedge is constant being equal to H_1. Hence the weights of these wedges are plotted as their base lengths L_1, L_2, L_3, L etc. on the ϕ-line.

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Let L = base length of the critical wedge = BE

The total active pressure is calculated from the relation

$$\frac{P_a}{W} = \frac{EF}{BE} \qquad ...(20.60)$$

$$\therefore \quad P_a = W \cdot \frac{EF}{BE} = \frac{1}{2} \gamma H_1 L \frac{EF}{BE}.$$

or
$$P_a = \frac{1}{2} \gamma H_1 (EF) \qquad ...(20.61)$$

If the backfill also carries a surcharge of intensity q, γ_1 given by Eq. 20.59 is used in Eq. 20.61 instead of γ.

Effect of line load. Culmann's graphical method can also be used to take into account the effect of line load, such as a railway track or a long wall of a building etc., running parallel to the retaining wall. Figure 20.29 shows a line load of intensity q per unit length, acting at a point C_1 distant a from the top of the wall. $BFF_1 F_n$ shows the Culmann line and BC is the failure plane in absence of the line load. Let W_1 be the weight of the wedge ABC_1 which is plotted as BE_1 on the ϕ-line, and point F_1 is obtained, if there were no line load. However, when the line load is there, the weight of the wedge ABC_1 increases by q. Thus BE' represents $(W_1 + q)$ and a point F' is obtained by drawing $E'F'$ parallel to the ψ-line. There is an abrupt change in the Culmann line, the change being proportional to q. For all other failure wedges to the right, the weight q is added to the weight of the wedges, and then plotted on the ϕ-line. The *modified Culmann's line* is thus represented by $BFF_1 F' F_n'$. When the slip plane is BC, the pressure on the wall is represented by EF and when the slip plane is BC_1 the pressure is represented by $E'F'$. If $F'E' < FE$, slip occurs along BC and there is no influence of the line load on the total pressure P_a. If $F'E' > FE$, slip occurs along BC_1 and the pressure on the wall is increased by:

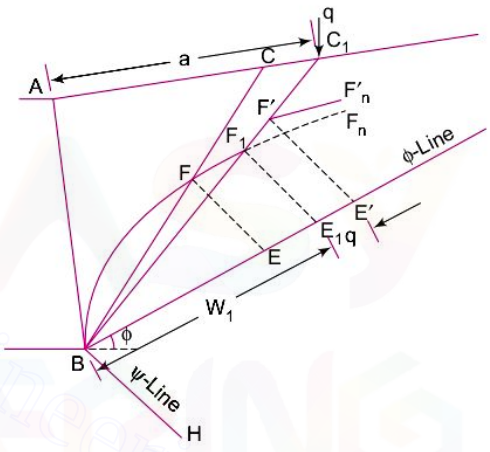

Fig. 20.29 Effect of line load

$$\Delta P_a = E'F' - EF \qquad ...(20.62)$$

Figure 20.30 shows the influence diagram of ΔP_a for various positions (*i.e.*, various values of a) of the line load. The Culmann line BFF_2 is plotted by ignoring the line load. The modified Culmann line $BF'F_2$ is then plotted by taking into account the line load, when the load q is added to the weight of each soil wedge considered. By drawing tangents to the two Culmann's lines parallel to ϕ-line, intercepts FE and $F'E'$ are obtained. The intercept $E'F'$ gives the greatest value of pressure due to backfill acted upon by q, whereas FE gives the maximum pressure in

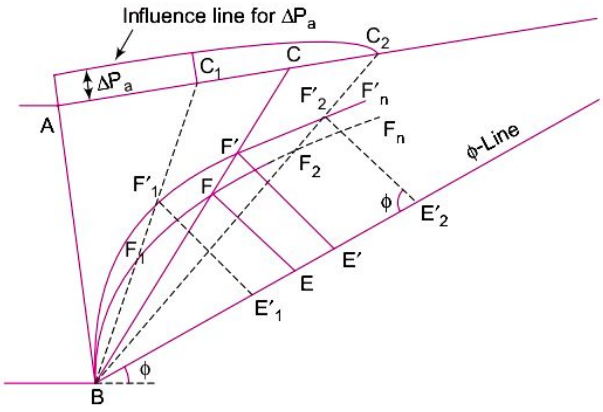

Fig. 20.30

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the absence of the line load. If the tangent at F is prolonged to meet the modified Culmann line in F_2', the intercept $E_2'F_2'$ equals to FE. This means that if the line load is placed beyond C_2, there is no effect of the line load on the pressure (*i.e.*, $\Delta P_a = E_2' F_2' - EF = 0$). For other positions of q between A and C_2, ΔP_a can be calculated from Eq. 20.62, and plotted as shown in Fig. 20.30. It will be seen that ΔP_a is maximum when the load is just at the face of the wall, it remains constant with the position of q up to point C_1 and then decreases gradually to zero at C_2. For load positions beyond C_2, the pressure on the wall is not due to q. This method is very much used in locating the position of the railway line or the footing of building on the backfill at such a safe distance that the earth pressure on the (existing) wall does not increase.

Example 20.16. *Solve example 20.11 by Culmann's graphical method.*

Solution. Figure 20.28 shows the Culmann's graphical construction with the given data. By measurement, we get

$$FE = 1.25 \text{ m and } H_1 \approx 3 \text{ m}$$

$$\therefore \quad P_a = \frac{1}{2}\gamma (FE) H_1 = \frac{1}{2} \times 19 \times 1.25 \times 3 = \mathbf{35.6\ kN/m}$$

Example 20.17. *A retaining wall 4 m high, with vertical back supports a backfill with horizontal ground surface. The soil has unit weight of 18 kN/m^3 and angle of internal friction of 35°. The angle of wall friction may be taken as 20°. A footing, running parallel to the retaining wall and carrying a load intensity of 18 kN/m, is to be constructed. Find the safe distance of the footing from the face of the wall so that there is no increase in lateral pressure on the wall due to the load of footing.*

Solution.

A wedge ABC_1 with $AC_1 = 1$ m is selected. Its weight $W_1 = \frac{1}{2} \times 4 \times 1 \times 18 = 36$ kN is plotted as BE_1 on the ϕ-line, and the point F_1 on the Culmann's line is obtained. Similarly, other points F_2, F_3, F_4 etc. are obtained and Culmann line drawn, ignoring the line load q = 18 kN/m. The line load q = 18 kN is now placed at C_1 and the weight of ABC_1 (= W_1) plus the load q(= 18 kN), is plotted as BE_1' (*i.e.*, = BE_1' = 36 + 18 = 54 kN), and the point F_1' on the Culmann's line is obtained. Similarly, other points F_2', F_3', F_4' are obtained and the Culmann line $BF_1' F_2' F_3' F_4'$ is obtained taking into consideration the line load q. It should be noted that

$$E_1 E_1' = E_2 E_2' = E_3 E_3' = E_4 E_4' = 18 \text{ kN}$$

A line FF' is drawn parallel to the ϕ-line in such a way that it is tangential to the Culmann line without the line load. The line FF' cuts the Culmann line with the load at F'. Join BF' and prolong it to cut the ground line in C. AC is then the safe distance beyond which q does not increase the lateral pressure on the retaining wall. By measurement, $AC = L = 3.5$ m.

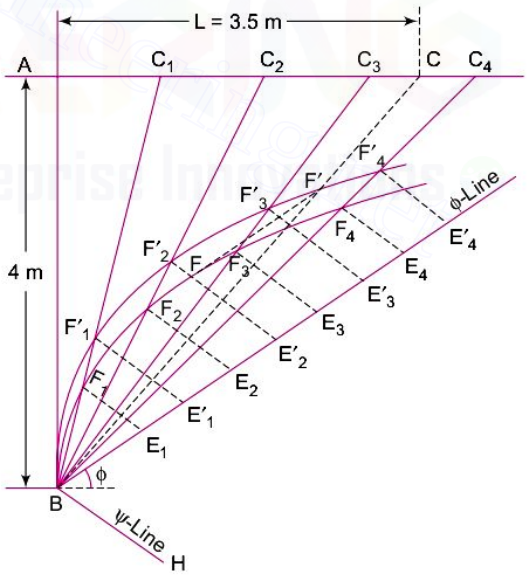

Fig. 20.31

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20.11 EARTH PRESSURE COMPUTATION FOR PRACTICAL CASES

We have seen in the previous articles that the pressures behind retaining walls can be computed *analytically* as well as *graphically,* using Rankine's, Culmann's, or Coulomb's solutions. In practice, however, earth pressures for retaining walls of less than 6 m height are obtained from graphs or tables. Almost all such graphs and tables, available in the literature, are based on Rankine's theory. One such type of graphs, developed by Peck, Hanson and Thornburn (1974) are shown in Figs. 20.32 and 20.33.

Peck, Hanson and Thornburn considered the following *five types* of backfills. The numerals on the curves indicate there soil types, as described below:

1. Coarse grained soil without admixture of fine soil particles, very free-draining (clean sand, gravel or broken stone).

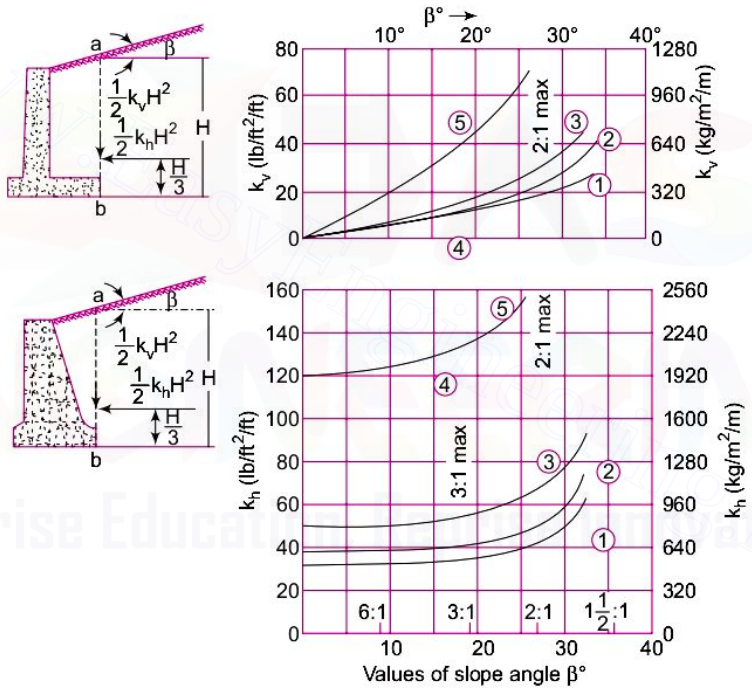

Fig. 20.32 Charts for earth pressure computation

2. Coarse grained soil of low permeability, owing to admixture of particles of silt size.
3. Fine silty sand; granular materials with conspicuous clay content; or residual soil with stones.
4. Soft or very soft clay; organic silt; soft silty clay.
5. Medium or stiff clay that may be placed in such a way that a negligible amount of water will enter the spaces between the chunks during floods or heavy rains.

Note: For material of type (5) computation should be based on value of H, 1.2 m (4 ft.) less than the actual value.

Figure 20.32 is useful for retaining wall having backfill with sloping surface at an angle β with horizontal. Values of K_V and K_H in the units of $lb/ft^2/ft$ or $kg/m^2/m$ can be read from the corresponding charts, for various values of angle β, and for corresponding type of backfill.

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Figure 20.33 is useful for retaining wall have sloping backfill surface for some distance and then the backfill becomes horizontal (*i.e.*, backfill with broken ground surface). Due to this, one more variable parameter, H_1/H ratio is involved. Separate curves have been provided for each of the five types of backfills. In these charts, solid curves are for the values of K_H while dotted curves are for the values of K_V.

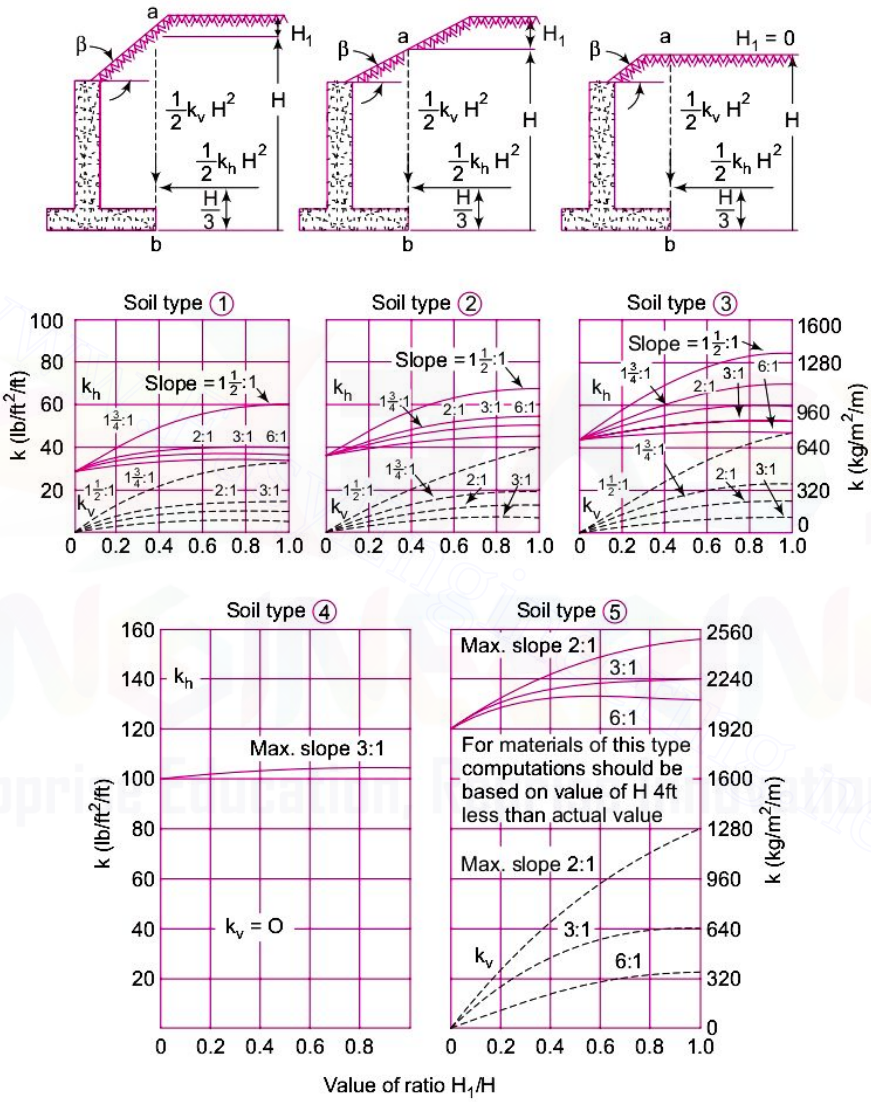

Fig. 20.33 Charts for earth pressure computation

Example 20.18. *A retaining wall 4 m high, shown in Fig. 20.34, has cohesionless backfill, at an inclination of 10°. Using the charts of Fig. 20.32, determine the active lateral thrust.*

Solution. We have, $CD = AE = 4$ m

$BC = AC \tan 10° = 2 \tan 10° \approx 0.35$ m

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$\therefore$ $H = BD = 4 + 0.35 \text{ m} = 4.35 \text{ m}$

From Fig. 20.32, for $\beta = 10°$, we have

$K_V \approx 80 \text{ kg/m}^2\text{/m}$ and $K_H \approx 510 \text{ kg/m}^2\text{/m}$

$\therefore$ $P_V = \frac{1}{2} K_V H^2 = \frac{1}{2} \times 80 (4.35)^2 = 757 \text{ kg/m}$

and $P_H = \frac{1}{2} K_H . H^2 = \frac{1}{2} \times 510 (4.35)^2 = 4825 \text{ kg/m}$

$\therefore$ $P_a = \sqrt{(757)^2 + (4825)^2} = \mathbf{4884 \text{ kg/m}}$

Inclination of P_a with horizontal $= \tan^{-1} \frac{757}{4825} \approx 8.92°$

P_a acts at H/3 = 4.35/3 = **1.45 m** above the base of the wall, as shown in Fig. 20.34.

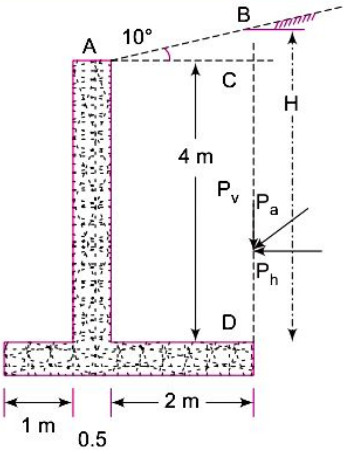

Fig. 20.34

20.12 DESIGN OF GRAVITY RETAINING WALL

A gravity retaining wall is the one which resists the lateral earth pressure by its weight in contrast to the cantilever and counterfort retaining walls in which the pressure is resisted by bending action. A gravity retaining wall is therefore, thicker in section. They are constructed of mass concrete, brick or stone masonry. The criteria of design of gravity retaining walls are as follows:

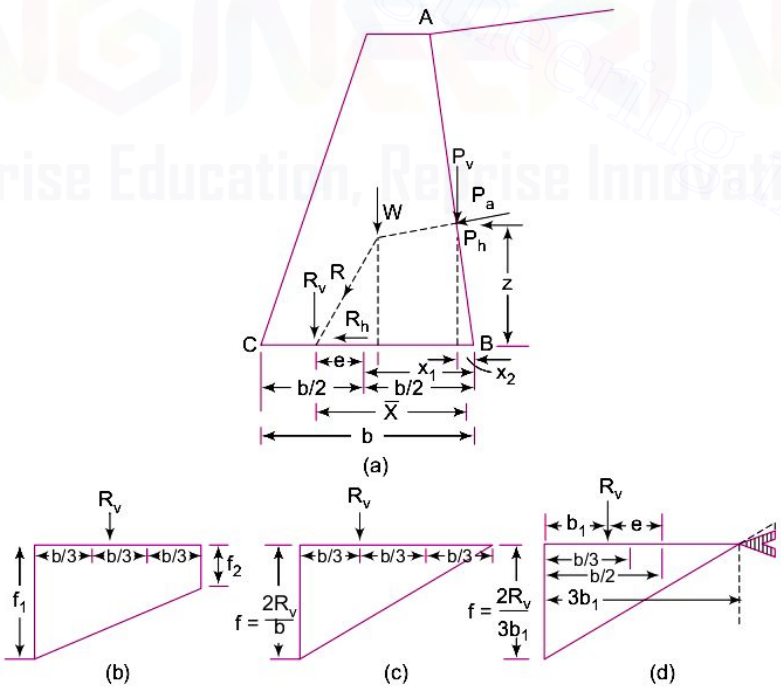

Fig. 20.35

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1. The base width of the wall must be such that the maximum pressure exerted on the foundation soil does not exceed the safe bearing capacity of the soil.
2. No tension should be developed anywhere in the wall.
3. The wall must be safe against sliding.
4. The wall must be safe against overturning.

Let P_a be the resultant active pressure acting on the wall face. P_h and P_v are its components in horizontal and vertical directions. If W is the weight of the wall acting through its centroid, the resultant R will pass through the point of intersection of W and P_a. The resultant R striking the base at some point, can be resolved into vertical and horizontal components R_v and R_h.

Resolving the forces into vertical and horizontal directions, we get

$$R_v = W + P_v \qquad ...(20.63)$$

and

$$R_h = P_h \qquad ...(20.64)$$

To find the distance of the point of application of R, equate the moment of the forces to the moment of their resultant about point B:

$$R_v\, \bar{x} = W x_1 + P_v x_2 + P_h \bar{z}$$

$$\therefore \quad \bar{x} = \frac{W x_1 + P_v x_2 + P_h \bar{z}}{W + P_v} = \frac{\Sigma M}{\Sigma V} \qquad ...(20.65)$$

where ΣM = sum of moments of all forces about $B = W x_1 + P_v x_2 + P_h \,.\, \bar{z}$

ΣV = sum of the vertical force $= R_v = W + P_v$

Since the resultant vertical force $R_v\ (= \Sigma V)$ acts eccentrically on the base, the bearing pressure on the soil beneath the base are the combination of direct and bending stress. Assuming a linear distribution of pressure, the pressure intensities at the ends on the base are given by

$$f_1 = \frac{R_v}{b}\left(1 + \frac{6e}{b}\right) \qquad ...(20.66\ a)$$

and

$$f_2 = \frac{R_v}{b}\left(1 - \frac{6e}{b}\right) \qquad ...(20.66\ b)$$

where b = base width of wall

e = eccentricity or the distance from the mid-point of the base to the point of applicaton of $R = \bar{x} - b/2$

Figure 20.35(b), (c), (d) shows three cases of pressure distribution below the base. When R strikes the base within the *middle third* (*i.e.*, $e < b/6$), f_1 and f_2 will be compressive as shown in Fig. 20.35(b). Figure 20.35 (c) shows the special case when $e = b/6$, so that

$$f_1 = \frac{2R_v}{b} \text{ and } f_2 = 0$$

When $e > b/6$ (Fig. 20.35 d) tension is developed at B. Since soil is generally considered incapable to resist tension, the pressure will be redistributed along the intact base, of width $3b_1$ where b_1 is the distance of the point of application of R from end C. The intensity f_1 will then be given by

$$f_1 = \frac{2R_v}{3b_1} \qquad ...(20.67\ a)$$

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Substituting $b_1 = \frac{b}{2} - e, \quad f_2 = \frac{2R_v}{3(b-2e)}$ ...(20.67 *b*)

The following are the criteria of design of gravity retaining wall:

1. The maximum pressure f_1 should not exceed the bearing capacity of soil.
2. For no tension to develop, $e < b/6$, or $\bar{x} \le \frac{2b}{3}$ ...(20.68)
3. For no sliding to occur, $R_h < R_v \cdot \mu$ ...(20.69)
   where $\mu = \tan \delta'$ = coefficient of friction between the base of the wall and the soil. The factor of safety F against sliding is given by
   $$F = \frac{R_v \cdot \mu}{R_h} \quad ...(20.70)$$
   The minimum value of F should be 1.5.
4. For the wall to be stable against overturning, R must pass within the base width. However, if the requirement of no tension is fulfilled, complete safety against overturning is automatically assured.

Example 20.19. *Design a gravity retaining wall, 5 m high with vertical back to retain a dry cohesionless backfill of unit weight 18 kN/m^3 and angle of shearing resistance 30°. Find also the factor of safety against sliding assuming the angle of friction between the base of the wall and the foundation soil as 30°. The wall is to be 1 m wide at top, and to be constructed of brick masonry having unit weight 20 kN/m^3. Use Rankine's theory.*

Solution.

$$\phi = 30° \quad \therefore \quad K_a = \frac{1}{3}$$

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times \frac{1}{3} \times 18 (5)^2 = 75 \text{ kN/m}$$

It acts horizontally at a height of $\frac{5}{3}$ m from the base.

Let b_1 = width of wall at its top

b = width of wall at its bottom

Since P_a acts horizontally, we have

$$R_v = W \text{ and } R_h = P_a$$

For no tension to develop, R_v must act at a distance $2b/3$ from heel B.

From statics, the moment of the forces W and P_a about B must be equal to the moment of the resultant about B. For calculating W, it is convenient to divide the trapezoidal section of the wall into rectangular and triangular portions as shown in Fig. 20.36. Let W_1 be the weight of rectangular portion, and W_2 of the triangular portion.

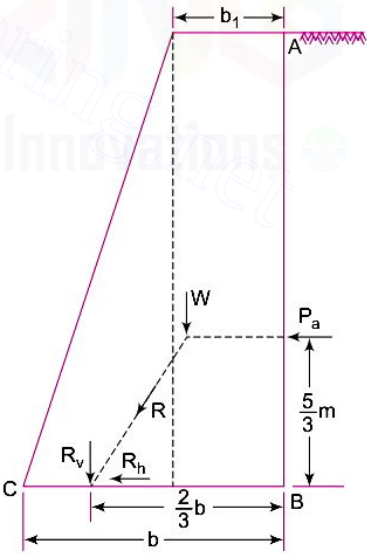

Fig. 20.36

$$\therefore \quad R_v \times \frac{2b}{3} = W_1 \times \frac{b_1}{2} + W_2 \left(b_1 + \frac{b - b_1}{3} \right) + P_a \times \frac{H}{3} \quad ...(20.71)$$

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The above equation is valid only for the case when P_a acts horizontally.

Now, $$R_v = W = \frac{b + b_1}{2} H \cdot \rho = \frac{b+1}{2} \times 5 \times 20 = 50\,(b+1)$$

$$W_1 = (b_1 \times H)\,\rho = 1 \times 5 \times 20 = 100 \text{ kN}$$

$$W_2 = \frac{1}{2}(b - b_1)\,H\rho = \frac{1}{2}(b-1)\ 5 \times 20 = 50\,(b-1)$$

$$P_a = 75 \text{ kN}$$

Substituting these values in Eq. 20.71, we get

$$50\,(b+1) \times \frac{2b}{3} = 100 \times \frac{1}{2} + 50\,(b-1)\left(1 + \frac{b-1}{3}\right) + \frac{75 \times 5}{3}$$

or $$b^2 + b - 8.5 = 0$$

$\therefore$ $b = 2.45$ m. Hence adopt $b = 2.5$

$\therefore$ $$W = \frac{b + b_1}{2} \times H \times \rho = \frac{2.5+1}{2} \times 5 \times 20 = 175 \text{ kN/m}$$

$\therefore$ Factor of safety against sliding is given by

$$F = \frac{\mu W}{P_a} = \frac{175 \times \tan \delta'}{75} = \frac{175}{75} \tan 30° = 1.35.$$

This is lesser than the recommended value of 1.5. Hence increase the base width b to 3 m. Then F comes out to be 1.54.

20.13 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 20.20. *A vertical wall, 5 m high, supports a saturated cohesive back fill ($\varphi = 0$) with horizontal surface. The top 3 m of the backfill weighs 1.76 g/cm^3, and has an apparent cohesion of 0.15 kg/cm^2. The bulk density and apparent cohesion of the bottom 2 m of the back fill are respectively 1.92 g/cm^3 and 0.2 kg/cm^2. Determine the likely depth of tension cracks behind the wall. If tension cracks develop, what will be the total active pressure? Sketch the pressure distribution diagram and locate the point of application of the resultant pressure.* **(Civil Services Exam. 1986)**

Solution. Refer Example 20.6.

For $\varphi = 0$, $$K_a = \frac{1 - \sin 0°}{1 + \sin 0°} = 1$$

(a) Active pressure distribution due to top soil:

$$\gamma_1 = 1.76 \text{ g/cm}^3 = 1.76 \text{ t/m}^3 \ ; \ \gamma_2 = 1.92 \text{ g/cm}^3 = 1.92 \text{ t/m}^3$$

$$c_1 = 0.15 \text{ kg/cm}^2 = 1.5 \text{ t/m}^2 \ ; \ c_2 = 0.2 \text{ kg/cm}^2 = 2 \text{ t/m}^2$$

$$p_a = K_a \gamma_1 z_1 - 2 c_{u1} \sqrt{K_a} = \gamma_1 z_1 - 2 c_{u1} \text{ or } p_a = 1.76 z_1 - 2 \times 1.5 = 1.76 z_1 - 3 \quad ...(1)$$

At A, $z_1 = 0$ $\therefore$ $(p_a)_A = -3$ t/m^2

At B, $z_1 = 3$ m $\therefore$ $(p_a)_B = p_1 = 1.76 \times 3 - 3 = 2.28$ t/m^2

p_a is zero at $z_1 = z_0 = 3/1.76 = 1.705$ m

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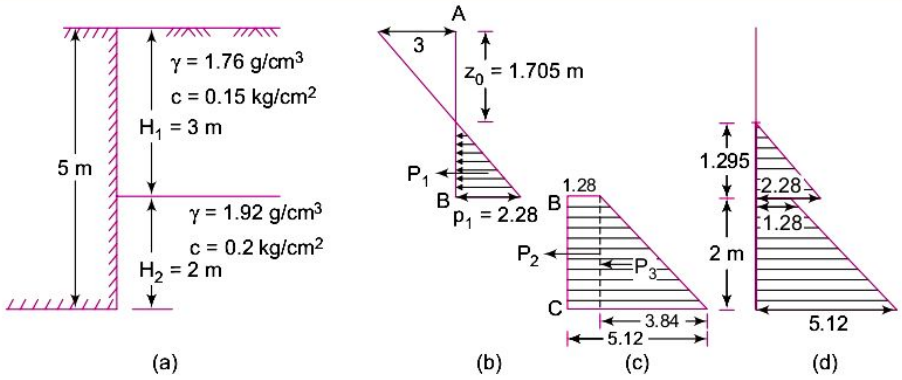

Fig. 20.37

If tension crack develops, the active pressure will be for the remaining depth = 3 – 1.705 = 1.295 m, as shown in Fig. 20.37(*a*).

Total pressure $P_1 = \frac{1}{2} p_1 (H_1 - z_0) = \frac{1}{2} \times 2.28 \times 1.295 = 1.476$ t/m acting at $2 + \frac{1}{3} \times 1.295 = 2.432$ above the base of the wall.

(*b*) *Active pressure distribution due to bottom soil*: The weight of top soil ($= \gamma_1 H_1 = 1.76 \times 3 = 5.28$ t/m^2) will act as surcharge on the bottom soil.

$\therefore \quad p_a = K_a \gamma_2 z_2 - 2 c_{u2} \sqrt{K_a} + K_a q = \gamma_2 z_2 - 2 c_{u2} + 5.28$

or $\quad p_a = 1.92 z_2 - 2 \times 2 + 5.28 = 1.92 z_2 + 1.28$ ...(2)

At B, $\quad z_2 = 0 \quad \therefore \quad (p_a)_B = 1.28$ t/m^2

At C, $\quad z_2 = 2$ m $\quad \therefore \quad (p_a)_C = 1.92 \times 2 + 1.28 = 3.84 + 1.28 = 5.12$ t/m^2

The pressure distribution due to bottom soil shown in Fig. 20.37(*c*). Fig. 20.37(*d*) shows the final pressure distribution due to both soils.

Now $\quad P_2 = 1.28 \times 2 = 2.56$ t/m, acting at 2/2 = 1 m above base

and $\quad P_3 = \frac{1}{2} \times 3.84 \times 2 = 3.84$ t/m, acting at 2/3 = 0.667 m above base.

Total $\quad P_a = P_1 + P_2 + P_3 = 1.476 + 2.56 + 3.84 = 7.876$ t/m

Acting at $\quad z = \dfrac{1.476 \times 2.432 + 2.56 \times 1 + 3.84 \times 0.667}{7.876} =$ **1.106 m** above base.

Example 20.21. *A wall of 6 m height retains backfill of dry granular soil that weighs 18.5 kN/m^3, has a level surface. When there is no surcharge above fill, the overturning moment caused by the total active pressure at a point at the base of the wall is 150 kN/metre length of wall. The specifications permit certain amount of uniformly distributed surcharge but state that surcharge must not increase overturning moment by more than 75%. What surcharge can be allowed of the angle of wall friction is 25°?*

(Civil Services Exam. 1990)

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Solution.

In general, $$K_a = \left[\frac{\sec\theta\cos(\varphi-\theta)}{\sqrt{\cos(\theta+\delta)}+\sqrt{\frac{\sin(\varphi+\delta)\sin(\varphi-\beta)}{\cos(\beta-\theta)}}}\right]^2$$ Eq. ...20.55 (*a*)

Here, take $\theta = 0°$ (wall face vertical) ; $\delta = 25°$

Hence $$K_a = \left[\frac{\cos\varphi}{\sqrt{\cos 25°}+\sqrt{\frac{\sin(\varphi+25°)\sin(\varphi-\beta)}{\cos\beta}}}\right]^2 = \left[\frac{\cos\varphi}{0.952+\sqrt{\frac{\sin(\varphi+25°)\sin(\varphi-\beta)}{\cos\beta}}}\right]^2$$

(a) First case: without surcharge: $\beta = 0$

$$M_a = K_a \gamma H^3/6 = 150 \text{ kN/m (given)} \quad \therefore \quad K_a = \frac{6\times150}{18.5\,(6)^3} = 0.2252$$

$$\therefore \quad 0.2252 = \left[\frac{\cos\varphi}{0.952+\sqrt{\frac{\sin(\varphi+25°)\sin\varphi}{\cos 0}}}\right]^2$$

or $$\frac{\cos\varphi}{0.952+\sqrt{\sin(\varphi+25°)\sin\varphi}} = 0.4746$$

or $$\cos\varphi = 0.4518 + 0.4746\sqrt{\sin(\varphi+25°)\sin\varphi}$$

Solve the above equation by trial and error as under:

Trial No.	φ	*LHS*	*RHS*
1	30°	0.866	0.755
2	35°	0.819	0.786
3	40°	0.766	0.814
4	38°	0.788	0.803
5	37.5°	0.793	0.800

Hence $\varphi \approx 37.5°$

(b) Second case : Surcharge angle β

$$M_A = 1.75 \times 150 \; ; K_a = 1.75 \times 0.225 = 0.39375$$

$$\therefore \quad 0.39375 = \left[\frac{\cos 37.5}{0.952+\sqrt{\frac{\sin(37.5°+25)\sin(\varphi-\beta)}{\cos\beta}}}\right]^2$$

or $$\frac{0.79335}{0.952+\sqrt{\frac{0.8944\sin(37.5-\beta)}{\cos\beta}}} = 0.6275$$

$\sin(37.5 - \beta) = 0.109 \cos\beta$. Solving this by trial and error, we get $\beta \approx 32°$

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Example 20.22. *A retraining wall 6 m high retains sand with $\varphi = 30°$ and unit weight 24 kN /m^3 up to a depth of 3 m from top. From 3 m to 6 m, the material is a cohesive soil with $c = 20$ kN/m^2 and $\varphi = 20°$. Unit weight of cohesive soil is 18 kN/m^3. A uniform surcharge of 100 kN/m^2 acts on the top of soil. Determine the total lateral pressure acting on the wall and its point of application.*

(Civil Services Exam. 1991)

Solution.

(a) Lateral pressure due to top soil

$$K_{a1} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}.$$ Hence at any depth z_1 below top,

$$P_{a1} = K_{a1}\, q_1 + K_{a1}\, \gamma_1\, z_1 = \frac{1}{3} \times 100 + \frac{1}{3} \times 24\, z_1 = 33.33 + 8\, z_1 \qquad ...(1)$$

At $z_1 = 0$, $p_{a1} = 33.33$ kN/m^2

At $z_1 = 3$ m, $p_{a1} = 33.33 + 8 \times 3 = 33.33 + 24 = 57.33$ kN/m^2

The active pressure distribution diagram is shown in Fig. 20.38(*b*)

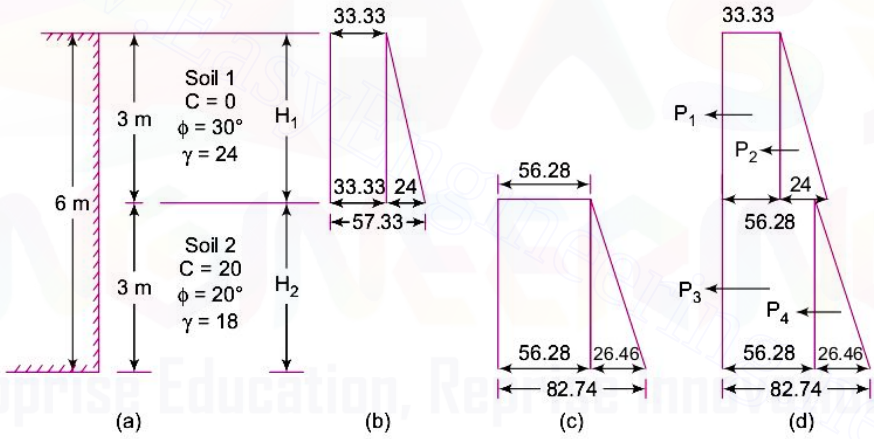

Fig. 20.38

(b) Lateral pressure due to bottom soil: For the bottom soil, the weight of the top soil and the initial surcharge ($q_1 = 100$ kN/m^2) become the surcharge.

$$\therefore \quad q_2 = q_1 + \gamma_1 H_1 = 100 + 24 \times 3 = 172 \text{ kN/m}^2$$

$$\cot \alpha_2 = \cot (45 + \varphi/2) = \cot (45 + 20/2) = 0.7$$

The active earth pressure at any depth z_2 below the junction of the two soils is given by Eq. 20.34:

$$p_{a2} = \gamma_2 z_2 \cot^2 \alpha_2 - 2\, c \cot \alpha_2 + q_2 \cot^2 \alpha_2$$

or $$p_{a2} = 18\, z_2\, (0.7)^2 - 2 \times 20\, (0.7) + 172\, (0.7)^2$$

or $$p_{a2} = 8.82\, z_2 - 28 + 84.28 = 8.82\, z_2 + 56.28 \qquad ...(2)$$

At $z_2 = 0$, $p_{a2} = 56.28$ kN/m^2

At $z_2 = 3$ m, $p_{a2} = 8.82 \times 3 + 56.28 = 26.46 + 56.28 = 82.74$ kN/m^2

The active pressure distribution diagram for bottom soil is given in Fig. 20.38(*c*)

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(*c*) ***Total lateral pressure:*** The lateral pressure diagram for both the soils is shown in Fig. 20.38(*d*) with component diagrams completely marked.

Force $P_1 = 33.33 \times 3 = 100$ kN/m acting at $z_1 = 3 + 3/2 = 4.5$ m above base

$P_2 = \frac{1}{2} \times 24 \times 3 = 36$ kN/m acting at $z_2 = 3 + 3/3 = 4$ m above base

$P_3 = 56.28 \times 3 = 168.84$ kN/m acting at $z_3 = 3/2 = 1.6$ above base

$P_4 = \frac{1}{2} \times 26.46 \times 3 = 39.69$ kN/m acting at $z_4 = 3/3 = 1$ m above base

$\therefore$ Total $P = 100 + 36 + 168.84 + 39.69 = \mathbf{344.53\ kN/m}$

Acting at $\bar{z} = \dfrac{100 \times 4.5 + 36 \times 4 + 168.84 \times 1.5 + 39.69 \times 1}{344.53} = \mathbf{2.57\ m}$ above base.

Example 20.23. *A 5 m high retaining wall has a granular soil backfill with a level top. The retaining face makes an angle of 85° with the base. Soil parameters γ, φ and δ are 16 kN/m³, 35° and 10° respectively. Demonstrate the Coulomb's method and obtain the active thrust.* **(Civil Services Exam. 1994)**

Solution. Given: $\gamma = 16$ kN/m^3 ; $\varphi = 35°$;

$\delta = 10°$; $\theta = 90° - 85° = 5°$, $\beta = 0°$

From Eq. 20.55(*a*)

$$K_a = \left[\frac{\sec\theta \cos(\varphi - \theta)}{\sqrt{\cos(\theta + \delta)} + \sqrt{\dfrac{\sin(\varphi + \delta)\sin(\varphi - \beta)}{\cos(\beta - \theta)}}} \right]^2$$

$$= \left[\frac{\sec 5° \cos(35° - 5°)}{\sqrt{\cos(5° + 10°)} + \sqrt{\dfrac{\sin(35 + 10)\sin(35 - 0)}{\cos(0 - 5)}}} \right]^2 = 0.2876$$

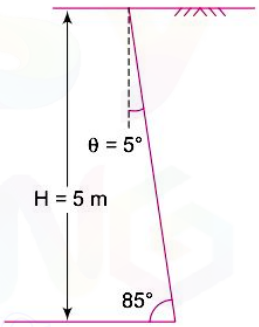

Fig. 20.39

$\therefore$ $P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times 0.2876 \times 16\,(5)^2 = \mathbf{57.572\ kN/m}$ run.

Example 20.24. *A retaining wall with soft saturated clay backfill is 7 m high. For the undrained condition (φ = 0) of the backfill, determine:*

(i) the maximum depth of the tensile crack, (ii) the active force before the tensile crack occurs, and (iii) the active force after the occurrence of the tensile crack.

$\gamma = 16$ *kN/m*^{3} ; $c_u = 17$ *kN/m*^{2} **(Civil Services Exam. 1994)**

Solution. Given: $\varphi = 0$; $c_u = 17$ kN/m^2 ; $\gamma = 16$ kN/m^3

$$\cot^2\alpha = K_a = \frac{1 - \sin 0}{1 + \sin 0} = 1$$

$$z_0 = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2 \times 17}{16 \times \sqrt{1}} = 2.125 \text{ m.}$$

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(a) Before the crack develops (Fig. 20.40 a)

$$P_a = \frac{1}{2}\gamma H^2 \cot^2 \alpha - 2cH \cot \alpha$$

$$= \frac{1}{2}\gamma H^2 K_a - 2cH\sqrt{K_a}$$

$$= \frac{1}{2} \times 16\,(7)^2 \times 1 - 2 \times 17 \times 7 \times \sqrt{1}$$

$$= 154 \text{ kN/m}.$$

(b) After the crack develops (Fig. 20.40 c)

When the crack develops, the soil in depth z_0 becomes *inactive*

$$\therefore \quad P_a = \frac{1}{2} K_a \gamma (H - z_0)^2$$

$$= \frac{1}{2} \times 1 \times 16\,(7 - 2.125)^2 = \mathbf{190.125\ kN/m}$$

Alternatively, from equation 20.33

$$P_a = \frac{1}{2}\gamma H^2 \cot^2 \alpha - 2cH \cot \alpha + \frac{2c^2}{\gamma}$$

$$= \frac{1}{2} \times 16\,(7)^2 \times 1 - 2 \times 17 \times 7 \times \sqrt{1} + \frac{2\,(17)^2}{16} = \mathbf{190.125\ kN/m}$$

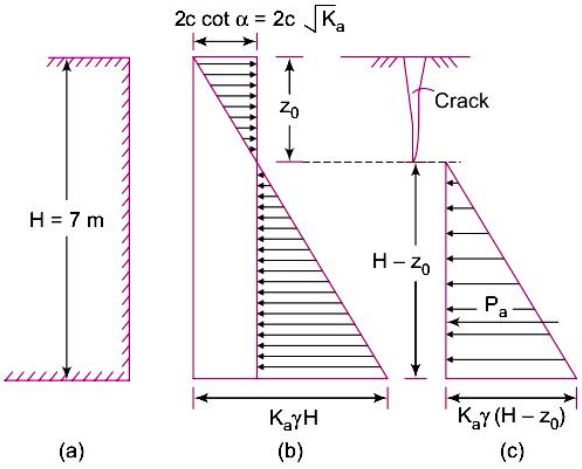

Fig. 20.40

Example 20.25. *For an earth retaining wall shown in Fig. 20.41(a), sketch the earth pressure diagram under active state and find the total thrust (per unit length of wall) and its location.*

(Civil Services Exam. 1995)

Solution. $K_a = \dfrac{1 - \sin 30°}{1 + \sin 30°} = \dfrac{1}{3}$

The components of active earth pressure are shown in Fig. 20.41(*b*)

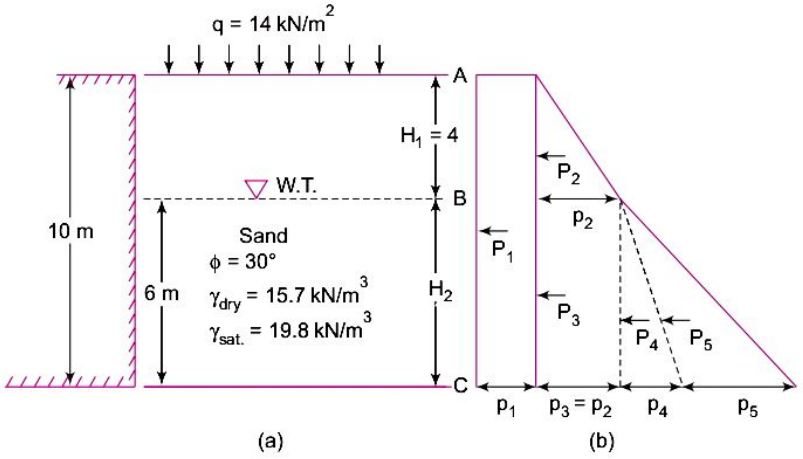

Fig. 20.41

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$$p_1 = K_a\, q = \frac{1}{3} \times 14 = 4.67 \text{ kN/m}^2$$

$$p_2 = K_a\, \gamma_d\, H_1 = \frac{1}{3} \times 15.7 \times 4 = 20.93 \text{ kN/m}^2$$

$$p_3 = p_2 = 20.93 \text{ kN/m}^2$$

$$p_4 = K_a\, \gamma'\, H_2 = \frac{1}{2}\,(19.8 - 9.81) \times 6 = 19.98 \text{ kN/m}^2$$

$$p_5 = \gamma_w\, H_2 = 9.81 \times 6 = 58.86 \text{ kN/m}^2$$

$\therefore$ $P_1 = 4.67 \times 10 = 46.7$ kN/m acting at $z_1 = 10/2 = 5$ m above base

$P_2 = \frac{1}{2} \times 20.93 \times 4 = 41.86$ kN/m acting at $z_2 = 6 + 4/3 = 7.33$ m above base

$P_3 = 20.93 \times 6 = 125.58$ kN/m acting at $z_3 = 6/2 = 3$ m above base

$P_4 = \frac{1}{2} \times 19.98 \times 6 = 59.94$ kN/m acting at $z_4 = 6/3 = 2$ m above base

$P_5 = \frac{1}{2} \times 58.86 \times 6 = 176.58$ kN/m acting at $z_5 = 6/3 = 2$ m above base

$\therefore$ Total $P = 46.7 + 41.86 + 125.58 + 59.94 + 176.58 =$ **450.66 kN/m**

$$z = \frac{46.7 \times 5 + 41.86 \times 7.33 + 125.58 \times 3 + 59.94 \times 2 + 176.58 \times 2}{450.66} = \textbf{3.085 m} \text{ above base.}$$

Example 20.26. *A retaining wall 10 m high retains a cohesionless soil having an angle of internal friction of 30°. The surface of the soil is level with the top of the wall. The top 3 m of the fill has a unit weight of 20° kN/m^3 and that of the rest is 30 kN/m^3. Find the magnitude per metre run and point of application of the resultant active thrust. Assume φ the same for both the strata.*

(Civil Services Exam. 1998)

Solution. (Refer Fig. 20.42)

$$K_a = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3};$$

$\gamma_1 = 20$ kN/m^3 ; $H_1 = 3$ m;

$$p_1 = K_a\, \gamma_1\, H_1 = \frac{1}{3} \times 20 \times 3$$

$= 20$ kN/m^2 ;

$p_2 = p_1 = 20$ kN/m^2

$$p_3 = K_a\, \gamma_2\, H_2 = \frac{1}{3} \times 30 \times 7$$

$= 70$ kN/m^2

$$P_1 = \frac{1}{2}\, p_1\, H_1 = \frac{1}{2} \times 20\,(3) = 30 \text{ kN/m; at } z_1 = 7 + \frac{1}{3} \times 3 = 8 \text{ m above base}$$

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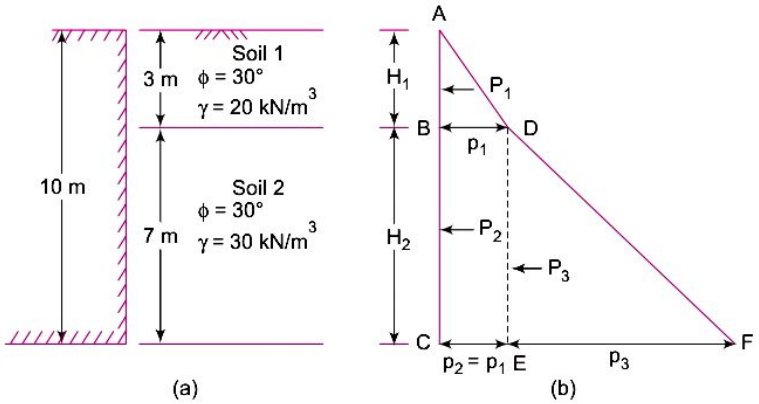

Fig. 20.42

$$P_2 = p_2 H_2 = 20 \times 7 = 140 \text{ kN/m; at } z_2 = 7/2 = 3.5 \text{ above base}$$

$$P_3 = \frac{1}{2} p_3 H_2 = \frac{1}{2} \times 70 \times 7 = 245 \text{ kN/m; at } z_3 = 7/3 = 2.33 \text{ above base}$$

Total $P = 30 + 140 + 245 = \mathbf{415\ kN/m\ run}$

Acting at $z = \dfrac{30 \times 8 + 140 \times 3.5 + 245 \times 2.33}{4.5} = \mathbf{3.137\ m}$ above base.

Example 20.27. *A retaining wall 10 m high retains a cohesionless soil with an angle of internal friction 35°. The surface is level with the top of the wall. The unit weight of the top 3 m of the fill is 1.6 t/m^3 and that of the rest is 2.0 t/m^3. Find the magnitude and application of the resultant active thrust.*

(Civil Services Exam. 1999)

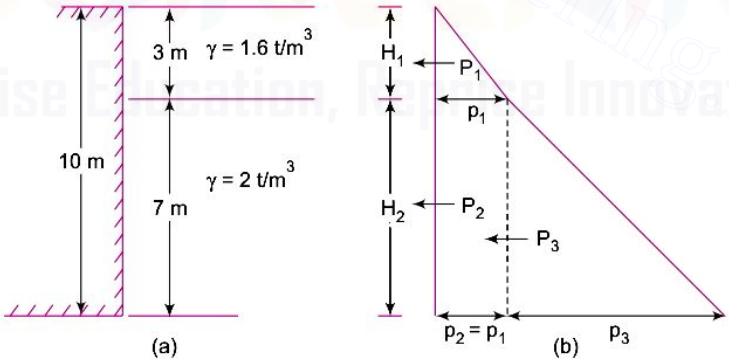

Fig. 20.43

Solution.

$$K_a = \frac{1 - \sin 35°}{1 + \sin 35°} = 0.271$$

$$P_1 = \frac{0.271 \times 1.6(3)^2}{2} = 1.951 \text{ kN/m ; acting at } z_1 = 7 + \frac{3}{3} = 8 \text{ m above base}$$

$$P_2 = 0.271 \times 1.6 \times (3)7 = 9.107 \text{ kN/m ; acting at } z_2 = 7/2 = 3.5 \text{ m above base}$$

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$$P_3 = \frac{1}{2} \times 0.271 \times 2\,(7)^2 = 13.279 \text{ kN/m: acting at } z_3 = 7/3 = 2.333 \text{ m above base}$$

$\therefore$ Total $P = 1.951 + 9.107 + 13.279 = \mathbf{24.337\ kN/m}$ run

Acting at $z = \dfrac{1.951 \times 8 + 9.107 \times 3.5 + 13.279 \times 2.333}{24.337} = \mathbf{3.224\ m}$ above base

Example 20.28. *For the frictionless retaining wall shown in Fig. 20.44 (a), plot the distribution of active earth pressure after the occurrence of the tensile rack.* **(Civil Services Exam. 2000)**

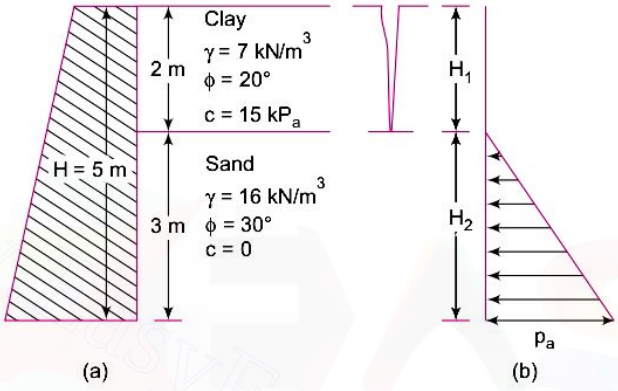

Fig. 20.44

Solution.

For Clay: $K_a = \dfrac{1 - \sin 20°}{1 + \sin 20°} = 0.49$; $z_0 = \dfrac{2c}{\gamma} \tan \alpha = \dfrac{2c}{\gamma \sqrt{K_a}} = \dfrac{2 \times 15}{17\sqrt{0.49}} = 2.52$ m

Since this is more than height ($H_1 = 2$ m) of clay, there will be no earth pressure due to clay. The earth pressure will be entirely due to sand.

For Sand: $K_a = \dfrac{1 - \sin 30°}{1 + \sin 30°} = \dfrac{1}{3}$; $c = 0$

$\therefore$ $p_a = K_a \gamma_2 H_2 = \dfrac{1}{3} \times 16 \times 3 = \mathbf{16\ kN/m^2}$

The earth pressure diagram will be triangular as shown in Fig. 20.44(*b*)

Example 20.29. *A masonry retaining wall has overall height of 2.4 m, width at top = 0.4 m. If the earth face is vertical, determine minimum width of the wall for the following data:*

Unit weight of earth retained = 1600 kg/m^3; Angle of shearing friction $\varphi = 30°$

Coefficient of friction between the masonry and ground = 0.7

Unit weight of masonry = 2200 kg/m^3; Maximum allowable base pressure = 12000 kg/m^2

The top surface of the backfill is level. Assume no. cohesion between particles of backfill. **(Civil Services Exam. 2000)**

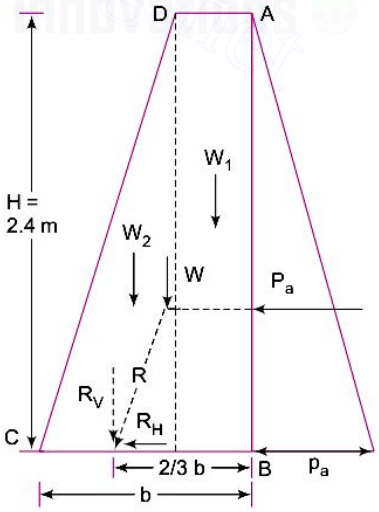

Fig. 20.45

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Solution.

$$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

$$p_a = K_a \gamma H = \frac{1}{3} \times 1600 \times 2.4 = 1280 \text{ kg/m}^2$$

$$P_a = \frac{1}{2} \times 1280 \times 2.4 = 1536 \text{ kg/m acting at 0.8 m above base}$$

Let R_v = total vertical weight

$$= W_1 + W_2 = W = \frac{b+0.4}{2} \times 2.4 \times 2200 = 2640\,(b+0.4)$$

$$W_1 = 0.4 \times 2.4 \times 2200 = 2112 \text{ kN}$$

and $$W_2 = \frac{1}{2}\,(b-0.4)\,2.4 \times 2200 = 2640\,(b-0.4)$$

For no tension to develop, the resultant R_v should pass through outer third point M_1. Hence Taking moments about B, we have

$$R_v \cdot \frac{2b}{3} = W_1 \times \frac{b_1}{2} + W_2 \left(b_1 + \frac{b-b_1}{3}\right) + P_a \times \frac{H}{3}$$

Hence, $$2640\,(b+0.4) \times \frac{2}{3} b = 2112 \times \frac{0.4}{2} + 2640\,(b-0.4)\left(0.4 + \frac{b-0.4}{3}\right) + \frac{1536+2.4}{3}$$

Which simplifies to $b^2 + 0.4\,b - 1.556 = 0$ which gives $b = 1.063$ m

Check

$$\bar{x} = \frac{2}{5} \times 1.063 = 0.709\,;\; e = \bar{x} - \frac{b}{2} = 0.709 - \frac{1.063}{2} = 0.177 \text{ m}$$

$$R_v = 2640\,(1.063 + 0.4) = 3862.3 \text{ kg}$$

$\therefore$ $$f = \frac{R_v}{b}\left(1 \pm \frac{6e}{b}\right) = \frac{3862.3}{1.063}\left(1 \pm \frac{6 \times 0.177}{1.063}\right) = 3633.4\,(1 \pm 1)$$

$\therefore$ $f_1 = 3633.4 \times 2 = 7266.8 \text{ kg/m}^2 < 12000$. Hence OK

$f_2 = 0$ (Hence OK)

Also, Factor of safety against sliding is

$$F = \frac{\mu R_v}{P_a} = \frac{3862.3}{1536} \times 0.7 = 1.76 > 1.5 \text{ Hence OK}$$

Example 20.30. *A masonry retaining wall of trapezoidal section has its top width equal to 0.75 m and height 5 m. Its face which is in contact with the retained earth is vertical. The earth retained is level at top. The soil weighs 16 kN/m^3 and its angle of internal friction is 30°. The masonry weighs 24 kN/m^3. Determine the minimum width of the base to avoid tensile stresses and determine the maximum compressive stresses for this base with. If the coefficient of friction between base and the soil is 0.60, check the stability of the retaining wall against sliding.* **(Civil Services Exam. 2001)**

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Solution. Taking moments about B,

$$R_v \times \frac{2b}{3} = W_1 \times \frac{b_1}{2} + W_2\left(b_1 + \frac{b-b_1}{3}\right) + P_a \times \frac{H}{3}$$

$$R_v = W = \frac{b+b_1}{2} H\rho = \frac{b+0.75}{2} \times 5 \times 24 = 60\,(b+0.75)$$

$$W_1 = (b_1 \times H)\,\rho = 0.75 \times 5 \times 24 = 90 \text{ kN}$$

$$W_2 = \frac{1}{2}(b-b_1)\,H\rho = \frac{1}{2}\,(b-0.75)\,5 \times 24 = 60\,(b-0.75)$$

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times \frac{1}{3} \times 16(5)^2 = 66.67 \text{ kN}$$

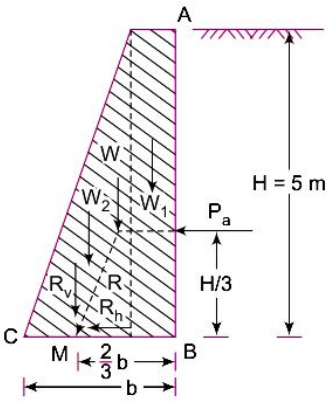

Fig. 20.46

$$\therefore \quad 60\,(b+0.75) \times \frac{2}{3} b = 90 \times \frac{0.75}{2} + 60\,(b-0.75)\left(0.75 + \frac{b-0.75}{2}\right) + 66.67 \times \frac{5}{3}$$

or $\quad b^2 + 0.75\,b - 6.1185 = 0$ from which $b \approx 2.13$ m

$$\therefore \quad W = \frac{2.13+0.75}{2} \times 5 \times 24 = 172.8 \text{ kN}$$

$$\therefore \quad \text{Factor of safely against sliding} = \frac{\mu W}{P_a} = \frac{0.6 \times 172.8}{66.67} = 1.56 > 1.5 \text{ OK.}$$

$$\bar{x} = \frac{2}{3} b = \frac{2}{3} \times 2.13 = 1.42 \text{ m}; \; e = \bar{x} - \frac{b}{2} = 1.42 - \frac{2.13}{2} = 0.355$$

$$p_n = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right) = \frac{172.8}{2.13}\left(1 \pm \frac{6 \times 0.355}{2.13}\right) = \frac{172.8}{2.13}(1 \pm 1)$$

$$\therefore \quad p_{nt} = \frac{172.8}{2.13}(1-1) = 0 \text{ and } p_{nc} = \frac{172.8}{2.13}(1+1) = \mathbf{162.25\ kN/m^2}$$

Example 20.31. *A soil mass is retained by a smooth backed vertical wall of 6.0 m height. The soil has a bulk unit weight of 20 kN/m³ and φ – 16°. The top of the soil is level with the top of the wall and is horizontal. If the soil surface carries a uniformly distributed load of 4.5 kN/m², determine the total active thrust on the wall per lineal metre of the wall and its point of application.* **(Civil Services Exam. 2001)**

Solution. $K_a = \dfrac{1-\sin 16°}{1+\sin 16°} = 0.568$

$$p_1 = K_a\, q = 0.568 \times 4.5 = 2.555 \text{ kN/m}^2$$

$$p_1 = 2.555 \times 6 = 15.332 \text{ kN/m}$$

acting at $\quad z_1 = 6/2 = 3$ m above base.

$$p_2 = K_a \gamma H = 0.568 \times 20 \times 6 = 68.16 \text{ kN/m}^2$$

$$p_2 = \frac{1}{2} \times 68.16 \times 6 = 204.48 \text{ kN/m}$$

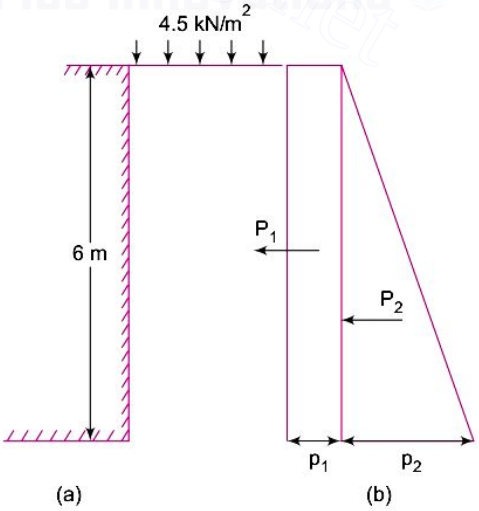

Fig. 20.47

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acting at 6/3 = 2 m above the base.

Total $P = 15.332 + 204.48 = \mathbf{219.812\ kN/m}$

$$z = \frac{15.332 \times 3 + 204.48 \times 2}{219.812} = 2.07 \text{ m above base}$$

Example 20.32. *Calculate the total active earth pressure on the retaining wall 6 m high shown in Fig. 20.48(a). Also calculate the line of action of lateral force from the base of the wall.*

(Civil Services Exam. 2002)

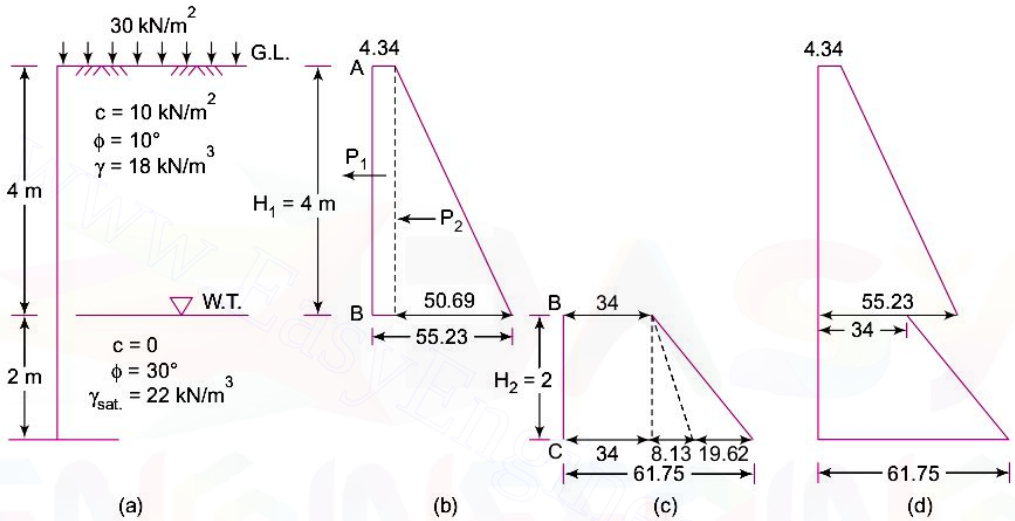

Fig. 20.48

Solution.

(a) Active earth pressure due to top c – φ soil

$$K_{a1} = \frac{1 - \sin 10°}{1 + \sin 10°} = 0.704 = \cot^2 \alpha \ ; \cot \alpha = 0.839$$

$$p_{a1} = \gamma_1 z_1 \cot^2 \alpha - 2c \cot \alpha + q \cot^2 \alpha = 18 z_1 (0.704) - 2 \times 10 \times 0.839 + 30 \times 0.704$$

or $p_{a1} = 12.672 z_1 - 16.78 + 21.12 = 12.672 z_1 + 4.34$ ...(1)

At $z_1 = 0, (p_a)_A = 4.34\ \text{kN/m}^2$; At $z_1 = 4$ m, $(p_a)_B = 12.672 \times 4 + 4.34 = 55.23$

The earth pressure diagram is shown in Fig. 20.48 (*b*)

(b) Active earth pressure due to bottom soil

$$c = 0; K_{a2} = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3}; \gamma' = 22 - 9.81 = 12.19\ \text{kN/m}^3$$

The surcharge q_2 for bottom soil $= q + \gamma_1 H_1 = 30 + 18 \times 4 = 102\ \text{kN/m}^3$

Hence $p_{a2} = K_{a2} q_2 + K_{a2} \gamma' z_2 + \gamma_w z_2 = \frac{1}{3} \times 102 + \frac{1}{3} \times 12.19 z_2 + 9.81 z_2 = 34 + 4.063 z_2 + 9.81 z_2$ ...(2)

At $B, z_2 = 0$, $\therefore$ $(p_{a2})_B = 34 + 0 + 0 = 34\ \text{kN/m}^2$

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At C, $z_2 = 2$m; $\therefore \quad (p_{a2})C = 34 + 4.063 \times 2 + 9.81 \times 2 = 34 + 8.13 + 19.62 = 61.75$

The earth pressure diagram for bottom soil is shown in Fig. 20.48(c). The composite earth pressure diagram for the whole height of wall is shown in Fig. 20.48(d).

Now $P_1 = 4.34 \times 4 = 17.36$ kN/m at $z_1 = 2 + 4/2 = 4$ m above base

$P_2 = \frac{1}{2} \times 50.69 \times 4 = 101.38$ kN/m at $z_2 = 2 + 4/3 = 3.333$ m above base

$P_3 = 34 \times 2 = 68$ kN/ m at $z_3 = 2/2 = 1$ m above base

$P_4 = \frac{1}{2} \times 8.13 \times 2 = 8.13$ kN/m at $z_4 = 2/3 = 0.667$ m above base

$P_5 = \frac{1}{2} \times 19.62 \times 2 = 19.62$ kN/m at $z_5 = 2/3 = 0.667$ m above base

Total $P = 17.36 + 101.38 + 68 + 8.13 + 19.62 =$ **214.49 kN/m** run.

Acting at $z = \dfrac{17.36 \times 4 + 101.38 \times 3.333 + 68 \times 1 + 8.13 \times 0.667 + 19.62 \times 0.667}{214.49} =$ **2.302 m** above base

Example 20.33. *A 20 m high vertical retaining wall supports a cohesionless fill of unit weight 1.8 t/m. The upper surface of the fill rises from the edge of the wall at an angle of 10° to the horizontal. Assuming φ to be 30° and angle of wall friction $\delta = 20°$, determine the active earth pressure.*

(Engg. Services Exam. 1997)

Solution. Given $\beta = 10°$; $\varphi = 30°$; $\theta = 0$; $\delta = 20°$

$$K_a = \left[\frac{\sec\theta \cos(\varphi - \theta)}{\sqrt{\cos(\theta + \delta)} + \sqrt{\dfrac{\sin(\theta + \delta)\sin(\varphi - \beta)}{\cos(\beta - \theta)}}}\right]^2$$

$$= \left[\frac{\sec 0 \cos(30° - 0)}{\sqrt{\cos(0 + 20°)} + \sqrt{\dfrac{\sin(30° + 20°)\sin(30° - 10°)}{\cos(10° - 0°)}}}\right]^2$$

$$= \left[\frac{1 \times 0.866}{0.9694 + 0.5158}\right]^2 \approx 0.34$$

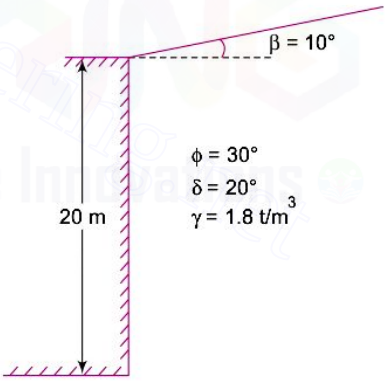

Fig. 20.49

$\therefore \quad P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times 0.34 \times 1.8 (20)^2 =$ **122.4 t/m** run

Example 20.34. *Determine the stresses at the top and bottom of a vertical cut, 4.5 m deep in soil with $\varphi' = 16°$ and $c' = 1.91$, t/m^2 (19.1 kN/m^2) and $\gamma = 1.85$ t/m^3 (18.5 kN/m^3). What could be the depth of the potential crack? What is the maximum depth of excavation that can be left unsupported.*

(Engg. Services Exam. 1990)

Solution. For a $c - \varphi$ soil, the stress (*i.e.*, active earth pressure) at any depth z is given by

$$p_a = \gamma z \cot^2 \alpha' - 2 c' \cot \alpha' = \gamma z K_a - 2 c' \sqrt{K_a}$$

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Here, $K_a = \dfrac{1-\sin 16°}{1+\sin 16°} = 0.5679$

$\therefore$ $p_a = 18.5\, z\,(0.5679) - 2 \times 19.1\ \sqrt{0.5679} = 10.505\, z - 28.787$ ...(1)

At top, $z = 0$. Hence $p_a = -$ **28.787 kN/m^2** (*i.e.*, tension)

At bottom, $z = 4.5$ m, $p_a = 10.505 \times 4.5 - 28.784 =$ **18.489 kN/m^2**

Depth of potential crack is at z_0 where $p_a = 0$

$\therefore$ $z_0 = \dfrac{2\,c'}{\gamma\sqrt{K_a}} = \dfrac{2 \times 19.1}{18.5\sqrt{0.5679}} =$ **2.74 m**

Max. depth of unsupported excavation $= 2\,z_0 = 2 \times 2.74 =$ **5.48 m**

Example 20.35. *A smooth vertical wall 4 m high is pushed against a mass of soil having a horizontal surface and a shearing resistance given by Coulomb's equation in which $c = 20$ kN/m^2 and $\varphi = 30°$. The unit weight of the soil is 20 kN/m^3. Its surface carries a uniform load of 20 kN/m^2. What is the total passive Rankine's pressure? What is the distance from the base of the wall to the centre of the pressure?*

(Engg. Services Exam. 1994)

Solution ***This problem is exactly the same as Example 20.6. It was therefore set from Authors present book. For solution, see Example 20.6***

Example 20.36. *A vertical bank was formed during the excavation of a soil having $\varphi = 15°$ and unit weight of 1800 kg/m^3. When the depth of excavation reached 5.5 m the bank failed. What was the approximate value of cohesion of clay ?* **(Engg. Services Exam. 1994)**

Solution. $z_0 = \dfrac{2c}{\gamma\sqrt{K_a}}$ where $K_a = \dfrac{1-\sin 15°}{1+\sin 15°} = 0.589$

$$z_0 = \frac{2c}{1800\sqrt{0.589}} = 1.448 \times 10^{-3}\, c$$

$\therefore$ Safe depth of cutoff $= 2\,z_0 = 2.896 \times 10^{-3}\, c$

$\therefore$ $2.896 \times 10^{-3}\, c = 5.5$ From which $c =$ **1899 kg/m^2**

Example 20.37. *A counterfort wall of 10 m height retains non-cohesive backfill. The void ratio and angle of internal friction of the backfill respectively are 0.7 and 30 ° in the loose state and they are 0.40 and 40° in the dense state. Calculate and compare active and passive earth pressure in both the states. Take specific gravity of soil grains as 2.7.* **(Engg. Services Exam. 1995)**

Solution.

(a) Loose state: $e = 0.70$; $\varphi = 30°$; $G = 2.7$;

$$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}\ ;\ K_p = 3;\quad \gamma_d = \frac{G\,\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.7} = 15.58\ \text{kN/m}^3$$

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times \frac{1}{3} \times 15.58\,(10)^2 =$$ **259.67 kN/m**

$$P_p = \frac{1}{2} K_p \gamma H^2 = \frac{1}{2} \times 3 \times 15.58\,(10)^2 =$$ **2337 kg/m**

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(b) Dense state: $e = 0.40$ and $\varphi = 40°$; $G = 2.7$ as before.

$$\therefore \quad K_a = \frac{1-\sin 40°}{1+\sin 40°} = 0.2174 \; ; \; K_p = \frac{1}{0.2174} = 4.5989.$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.4} = 18.919 \text{ kN/m}^3$$

$$P_a = \frac{1}{2} K_a \gamma H^2 = \frac{1}{2} \times 0.2174 \times 18.919 (10)^2 = \mathbf{205.65 \text{ kN/m}^3}$$

$$P_a = \frac{1}{2} K_p \gamma H^2 = \frac{1}{2} \times 4.5989 \times 18.919 (10)^2 = \mathbf{4350.4 \text{ kN/m}}$$

Thus, compaction of backfill results in *decrease in active pressure* and *increase in passive pressure*.

Example 20.38. *A 4 m high vertical wall supports a saturated cohesive soil ($\varphi = 0$) with horizontal surface. The top 2.5 m of the backfill has bulk density of 17.6 kN/m^3 and apparent cohesion of 15 kN/m^2. The bulk density and apparent cohesion of the bottom 1.5 m is 19.2 kN/m^3 and 20 kN/m^2 respectively. If tension cracks develop, what would be the total active pressure on the wall ? Also draw the pressure distribution diagram.* **(Engg. Services Exam. 1996)**

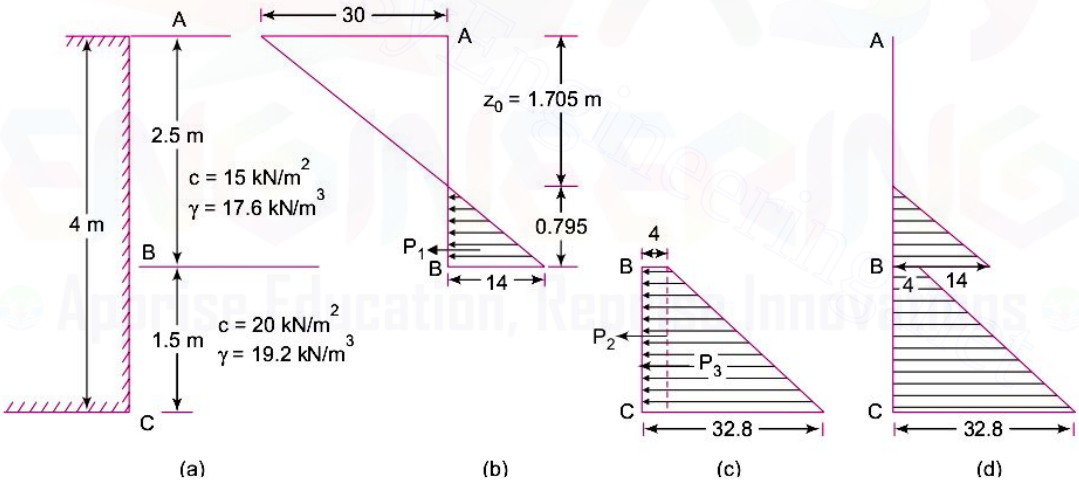

Fig. 20.50

Solution. $\varphi_u = 0$ for both soils. $K_{a1} = K_{a2} = 1 = \cot^2 \alpha$

(a) P_a – diagram for top soil

$$p_{a1} = \gamma_1 z_1 K_a - 2 c_1 \sqrt{K_{a1}} = 17.6 z_1 (1) - 2 \times 15 \times 1 \quad \text{or} \quad p_{a1} = 17.6 z_1 - 30$$

At A, $z_1 = 0$ $\therefore$ $(p_{a1}) = -30$ kN/m^2; At B, $z_1 = 2.5$ m. $(p_{a1})_B = 17.6 \times 2.5 - 30 = 14$ kN/m^2

p_{a1} is zero at $z_0 = \dfrac{30}{17.6} = 1.705$ m

The p_a – diagram due to top soil is shown in Fig. 20.50(b)

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(b) ***p_a – diagram for bottom soil:*** For bottom soil, the weight of top soil will act as surcharge at B.

$$\therefore \quad q = \gamma_1 H_1 = 17.6 \times 2.5 = 44 \text{ kN/m}^2$$

At any depth z_2 below B, we have

$$p_{a_2} = \gamma_2 z_2 K_{a_2} - 2 c_2 \sqrt{K_{a_2}} + q K_{a_2}$$

or $$p_{a_2} = 19.2 z_2 \times 1 - 2 \times 20 \times \sqrt{1} + 44 \times 1 = 19.2 z_2 - 40 + 44$$

At B, $z_2 = 0$ $\quad \therefore \quad (p_{a_2})_B = -40 + 44 = 4 \text{ kN/m}^2$

At C, $z_2 = 1.5\text{m}$ $\quad \therefore \quad (p_{a_2})_C = 19.2 \times 1.5 - 40 + 44 = 28.8 + 4 = 32.8$

The p_a diagram due to bottom soil is shown in Fig. 20.50(c)

(c) ***Total active pressure:*** The combined active pressure diagram is shown in Fig. 20.50(d). Since the tension cracks develop, active earth pressure from $z_1 = 0$ to $z_1 = z_0 = 1.705$ m will be zero.

$$P_1 = \frac{1}{2} \times 14(2.5 - 1.705) = 4.935 \text{ acting at } z_1 = 1.5 + \frac{1}{3} \times 0.795 = 1.765 \text{ m above base.}$$

$$P_2 = 4 \times 1.5 = 6 \text{ kN/m acting at } z_2 = 1.5/2 = 0.75 \text{ m above base.}$$

$$P_3 = \frac{1}{2} \times 28.8 \times 1.5 = 21.6 \text{ kN/m acting at } 1.5/3 = 0.5 \text{ m above base.}$$

$\therefore$ Total $P = 4.935 + 6 + 21.6 = \mathbf{32.535 \text{ kN/m}}$

Example 20.39. *A retaining wall 6 m high supports earth with its face vertical. The earth is cohesionless with particle specific gravity 2.69, angle of internal friction 35° and porosity 40.5%. The earth surface is horizontal and level with the top of the wall. Determine the earth thrust and its line of action on the wall, if the earth is water logged to level 2.5 m below the top surface. Neglect wall friction. Drawn the pressure diagrams.* **(Engg. Services Exam. 1999)**

Solution. Given: $G = 2.69$

$\varphi = 35°$; $\quad c = 0$; $\quad n = 40.5\%$

$$e = \frac{n}{1-n} = \frac{0.405}{1 - 0.405} = 0.681;$$

$$\gamma_d = \frac{G \gamma_w}{1+e} = \frac{2.69 \times 9.81}{1 + 0.681} = 15.701 \text{ kN/m}^2$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{2.69 + 0.681}{1 + 0.681} \times 9.81$$

$$= 19.674 \text{ kN/m}^2;$$

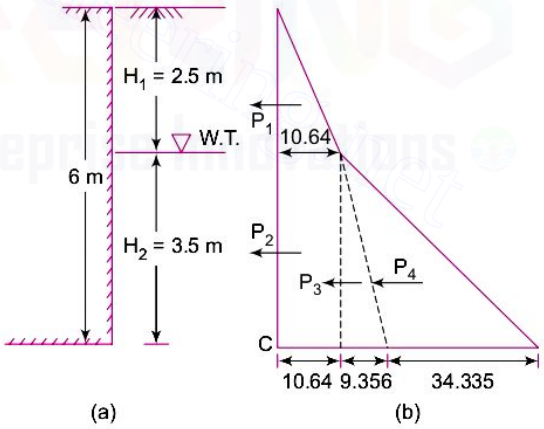

Fig. 20.51

$$\gamma' = 19.674 - 9.81 = 9.864 \text{ kN/m}^3 \text{ ; } K_a = \frac{1 - \sin 35°}{1 + \sin 35°} = 0.271$$

$$p_{aB} = K_a \gamma_d H_1 = 0.271 \times 15.701 \times 2.5 = 10.64 \text{ kN/m}^2$$

$$p_{aC} = 10.64 + K_a \gamma' H_2 + \gamma_w H_2 = 10.64 + 0.271 \times 9.864 \times 3.5 + 9.81 \times 3.5$$

$$= 10.64 + 9.356 + 34.335 = 54.331 \text{ kN/m}^2$$

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$$P_1 = \frac{1}{2} \times 10.64 \times 2.5 = 13.3 \text{ kN/m acting at } 3.5 + 2.5/3 = 4.33 \text{ m above base}$$

$$P_2 = 10.64 \times 3.5 = 37.24 \text{ kN/m acting at } 3.5/2 = 1.75 \text{ m above base}$$

$$P_3 = \frac{1}{2} \times 9.356 \times 3.5 = 16.373 \text{ kN/m acting at } 3.5/3 = 1.167 \text{ m above base}$$

$$P_4 = \frac{1}{2} \times 34.335 \times 3.5 = 60.086 \text{ kN/m acting at } 3.5/3 = 1.167 \text{ m above base.}$$

Total $P_a = 13.3 + 37.24 + 16.373 + 60.086 \approx \mathbf{127\ kN/m}$

Acting at $z = \dfrac{13.3 \times 4.333 + 37.24 \times 1.75 + 16.373 \times 1.167 + 60.086 \times 1.167}{127} = \mathbf{1.67\ m}$ above base

Example 20.40. *For an earth retaining structure shown in Fig. 20.52(a), construct earth pressure diagram for active state and final the total thrust per unit length of the wall.*

(Engg. Services Exam. 2000)

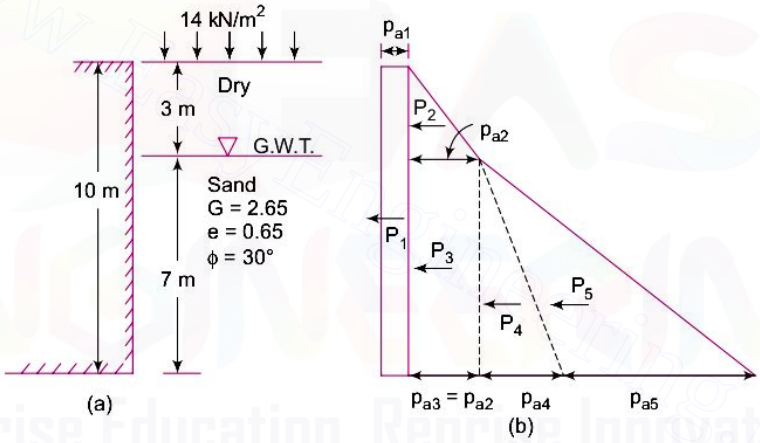

Fig. 20.52

Solution.

$$K_a = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3}$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1 + 0.65}$$

$$= 15.755 \text{ kN/m}^3$$

$$\gamma' = \frac{(G-1)\gamma_w}{1+e} = \frac{(2.65-1)9.81}{1+0.65}$$

$$= 9.81 \text{ kN/m}^3$$

$$p_{a1} = K_a\, q = \frac{1}{3} \times 14 = 4.67 \text{ kN/m}^2 ; \; p_{a2} = K_a\, \gamma_d\, H_1 = \frac{1}{3} \times 15.755 \times 3 = 15.755 \text{ kN/m}^2$$

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$$p_{a3} = p_{a2} = 15.755 \text{ kN/m}^2 ; p_{a4} = K_a \gamma' H_2 = \frac{1}{3} \times 9.81 \times 7 = 22.89 \text{ kN/m}^2$$

$$p_{a5} = \gamma_w H_2 = 9.81 \times 7 = 68.67 \text{ kN/m}^2; \therefore P_1 = p_{a1} H = 4.67 \times 10 = 46.7 \text{ kN/m}$$

$$P_2 = \frac{1}{2} p_{a2} H_1 = \frac{1}{2} \times 15.755 \times 3 = 23.632 \text{ kN/m} ; P_3 = P_{a3} H_2 = 15.755 \times 7$$

$$= 110.285 \text{ kN/m};$$

$$P_4 = \frac{1}{2} p_{a4} H_2 = \frac{1}{2} \times 22.89 \times 7 = 80.115 \text{ kN/m} ; P_5 = \frac{1}{2} p_{a5} H_2 = \frac{1}{2} \times 68.67 \times 7$$

$$= 240.345 \text{ kN/m}$$

$\therefore$ Total $P_a = 46.7 + 23.632 + 110.285 + 80.115 + 240.345 = \mathbf{501.08 \text{ kN/m}}$

Example 20.41. *A retaining wall 2 m in height has a smooth vertical surface. The backfill has a horizontal levelled surface with the top of retaining wall. The density of the backfill is 1.8 t/m^3, shearing resistance angle of 30° and cohesion zero. A uniformly distributed surcharge load of 3 t/m^2 intensity is acting on the backfill.*

(i) Calculate the magnitude and point of application of active earth pressure per metre length of the retaining wall.

(ii) If during rainy season, water table rises behind the wall to a height of 1 m above the base of the retaining wall, work out the effect on the value of active earth pressure if there is no change in the angle of shearing resistance. Submerged unit weight of backfill is 1.25 t/m^3.

(Engg. Services Exam. 2001)

Solution. $K_a = \dfrac{1 - \sin 30°}{1 + \sin 30°} = \dfrac{1}{3}$; $\gamma = 1.8$ t/m^3 ; $\gamma' = 1.25$ t/m^3

(i) Water table below the base of wall (Fig. 20.53 a)

$$p_{a1} = K_a q = \frac{1}{3} \times 3 = 1 \text{ t/m}^2; p_{a2} = K_a \gamma H = \frac{1}{3} \times 1.8 \times 2 = 1.2 \text{ t/m}^2$$

$\therefore$ $P_1 = 1 \times 2 = 2$ t/m acting at $z_1 = 2/2 = 1$ m above base

$$P_2 = \frac{1}{2} \times 1.2 \times 2 = 1.2 \text{ t/m acting at } z_2 = 2/3 = 0.667 \text{ m above base}$$

Total $P_a = 2 + 1.2 = \mathbf{3.2 \text{ t/m}}$

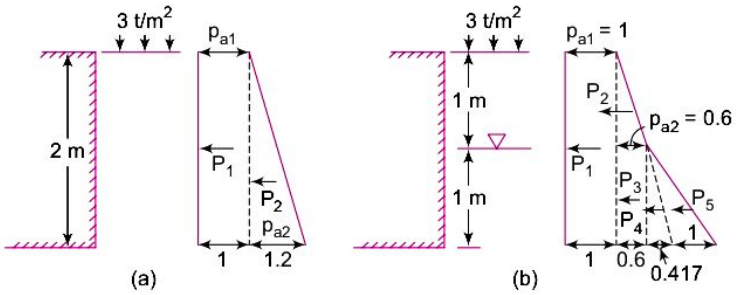

Fig. 20.53

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Acting at $z = \dfrac{2\times1 + 1.2\times0.667}{3.2} = \mathbf{0.875\ m}$ above base.

(ii) Water table at 1 m above base of wall [(Fig. 20.53 b)]

$$p_{a1} = K_a\, q = \frac{1}{3}\times 3 = 1\ \text{t/m}^3; \quad \therefore \quad p_{a2} = K_a\,\gamma\, H_1 = \frac{1}{3}\times 1.8\times 1 = 0.6\ \text{t/m}^2$$

$$p_{a3} = p_{a2} = 0.6\ \text{t/m}^2; p_{a4} = K_a\,\gamma'\,H_2 = \frac{1}{3}\times 1.25\times 1 = 0.417\ \text{t/m}^2; p_{a5} = \gamma_w H_2 = 1\,(1) = 1\ \text{t/m}^2$$

$\therefore$ $P_1 = 1\times 2 = 2$ t/m acting at $2/2 = 1$ m above base

$P_2 = \dfrac{1}{2}\times 0.6\times 1 = 0.3$ t/m acting at $1 + 1/3 = 1.333$ m above base

$P_3 = 0.6\times 1 = 0.6$ t/m acting at $1/2 = 0.5$ m above base

$P_4 = \dfrac{1}{2}\times 0.417\times 1 = 0.209$ t/m acting at $1/3 = 0.333$ m above base

$P_5 = \dfrac{1}{2}\times 1\times 1 = 0.5$ t/m acting at $1/3 = 0.333$ m above base

Total $P_a = 2 + 0.3 + 0.6 + 0.209 + 0.5 = 3.609$ t/m

Acting at $z = \dfrac{2\times1 + 0.3\times1.333 + 0.6\times0.5 + 0.209\times0.333 + 0.5\times0.333}{3.609} = \mathbf{0.814\ m}$ above base

Example 20.42. *A smooth retaining wall 6 m high retains dry granular backfill weighing 16 kN/m^3 to its level surface. The active thrust on the wall is 96 kN/m of wall. What will be the total active thrust if the water table comes up to backfill surface? Take specific gravity of backfill = 2.65.*

(Engg. Services Exam. 2002)

Solution.

***(a) Water below the base of wall* (Fig. 20.54 *a*)**

$$P_{a1} = \frac{1}{2}K_a\,\gamma_d\,H^2 = \frac{1}{2}K_a\,(16)\,(6)^2$$

$= 96$ kN/m (given)

$\therefore$ $K_a = \dfrac{96\times 2}{16(6)^2} = 0.333 = \dfrac{1}{3}$

***(b) Water at top surface* (Fig. 20.54 *b*)**

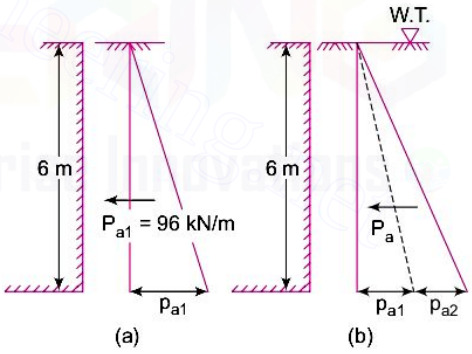

Fig. 20.54

$$\gamma_d = 16 = \frac{G\,\gamma_w}{1+e} = \frac{2.65\times 9.81}{1+e}$$

From which $e = 0.625$

$\therefore$ $\gamma_{sat} = \dfrac{(G+e)\gamma_w}{1+e} = \dfrac{2.65 + 0.625}{1 + 0.625}\times 981 \approx 19.77\ \text{kN/m}^2$

$\therefore$ $\gamma' = 19.77 - 9.81 = 9.96\ \text{kN/m}^3$

$\therefore$ $p_{a1} = K_a\,\gamma'\,H = \dfrac{1}{3}\times 9.96\times 6 = 19.92; \quad p_{a2} = \gamma_w H = 9.81\times 6 = 58.86$

$\therefore$ Total $p_a = \dfrac{1}{2}\,(19.92 + 58.86)\,6 = \mathbf{236.34\ kN/m}$

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Example 20.43. *An unsupported excavation is to be made in a clay layer. If $\gamma_t = 18\ kN/m^3$, $c = 30\ kN/m^2$ and $\varphi = 10°$,*

(a) Calculate the depth of tension cracks
(b) Calculate the maximum possible unsupported depth, and
(c) Draw the active pressure distribution diagram. **(Gate Exam. 1991)**

Solution.

$$K_a = \frac{1-\sin 10°}{1+\sin 10°} = 0.704 = \cot^2 \alpha$$

$$p_a = \gamma z K_a - 2c\sqrt{K_a}$$

or $$p_a = 18 \times 0.704\, z - 2 \times 30\ \sqrt{0.704}$$

or $$p_a = 12.672\, z - 50.343 \quad ...(1)$$

At top $z = 0$,

$\therefore$ $(p_a)_A = -50.343$ kN/m^2 (*i.e.*, tension).

p_a is zero at $z = z_0 = \dfrac{50.343}{12.672} = \mathbf{3.973\ m}$

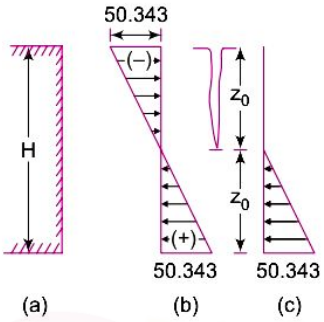

Fig. 20.55

Hence depth of tension crack = **3.973 m**

Max. possible unsupported depth = $2 z_0 = 2 \times 3.973 = \mathbf{7.946\ m}$

$\therefore$ Pressure at bottom

$$= (p_a)_B = 12.672 \times 7.946 - 50.343 = 50.343\ \text{kN/m}^2$$

The active pressure diagram, before the occurrence of crack is shown in Fig. 20.55(*b*). However, after the formation of tension crack, the pressure diagram will be as shown in Fig. 20.55(*c*).

Example 20.44. *The consolidated drained shear tests on silt yielded the following data:*

$\sigma_3 = \sigma_c$ *(N/mm^2)*	σ_1 *(N/mm^2)*
0.2	*0.46*
0.4	*0.88*

If this material is used as a back fill for a smooth vertical retaining wall of 10 m height, what is the active earth pressure at the base of the wall? Density of the soil is 1.6 g/cc. **(Gate Exam. 1992)**

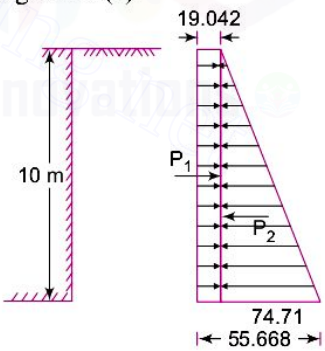

Fig. 20.56

Solution.

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha$$

$\therefore$ $$0.46 = 0.2 \tan^2 \alpha + 2c \tan \alpha \quad ...(1)$$

and $$0.88 = 0.4 \tan^2 \alpha + 2c \tan \alpha \quad ...(2)$$

Substracting (1) from (2), we get

$$0.2 \tan^2 \alpha = 0.88 - 0.46 = 0.42$$

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$\therefore \quad \tan^2 \alpha = 2.1 \quad \therefore \quad K_a = \cot^2 \alpha = 1/2.1 = 0.476$

Also, from (1), we get $0.46 = 0.2\,(2.1) + 2\,c\,\sqrt{2.1}$

From which $c = 0.0138\ \text{N/mm}^2 = 13.8\ \text{kN/m}^2$

$\gamma = 1.6\ \text{g/cc} = 1.6\ \text{t/m}^3 = 1.6 \times 9.81 = 15.696\ \text{kN/m}^3$

The earth pressure at any depth z is given by

$$p_a = \gamma z K_a - 2c\sqrt{K_a} = 15.696 \times 0.476\, z - 2 \times 13.85\ \sqrt{0.476}$$

or $$p_a = 7.471\, z - 19.042$$

At top, $z = 0 \quad \therefore \quad (p_a)_A = -19.042\ \text{kN/m}^2$

At bottom, $z = 10$ m $\quad \therefore \quad (p_a)_B = 7.471 \times 10 - 19.042 = 74.71 - 19.042 = 55.668$

$\therefore$ Total $$P_a = P_2 - P_1 = \frac{1}{2} \times 74.71 \times 10 - 19.042 \times 10 = 373.55 - 19.042$$

$$= \mathbf{183.13\ kN/m}$$

Alternatively, $$P_a = \int_0^{10} p_a = \int_0^{10} 7.471\, z - 19.042 = \frac{7.471}{2}(10)^2 - 19.042 \times 10$$

$$= \mathbf{183.13\ kN/m}$$

Example 20.45. *Plot (not to scale) the active earth pressure distribution on the retaining wall shown (Fig. 20.57a) by Rankine's theory for the data given. Take $\gamma_w = 10\ kN/m^3$.* **(Gate Exam. 1995)**

Solution. $K_a = \dfrac{1 - \sin 26°}{1 + \sin 26°} = 0.39$; $\gamma' = 20.4 - 10 = 10.4\ \text{kN/m}^2$

At any depth z, $p_a = K_a \gamma z - 2c\sqrt{K_a}$, where γ is the appropriate unit weight

From A to B, $p_a = 0.39 \times 16.2\, z - 2 \times 15\ \sqrt{0.39} = 6.318\, z - 18.735$

At A, $z = 0$; $(p_a)_A = -18.735 = p_1$ (say)

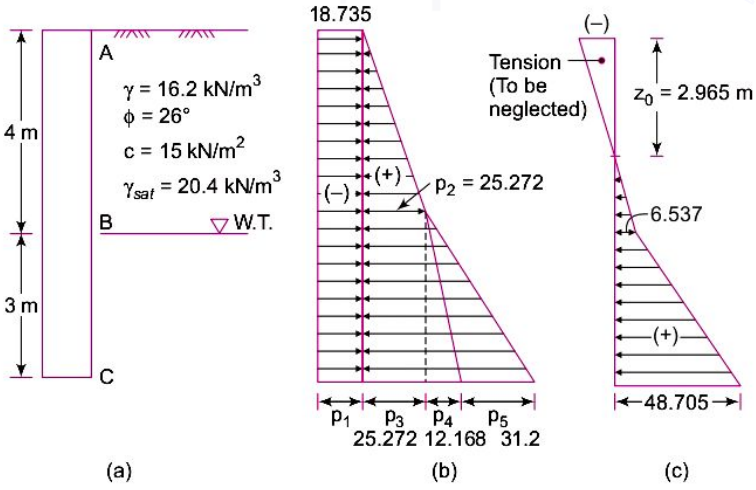

Fig. 20.57

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At B, $z = 4$ m ; $(p_a)_B = 6.318 \times 4 - 18.735 = p_2 - p_1 = 25.272 - 18.735 = 6.537$ kN/m

The earth pressure is zero at $z = z_0 = 18.735/6.318 = 2.965$ m

For B to C, measuring z from B, we have

$$p_a = (p_a)_B + K_a \gamma' z + \gamma_w z = 6.537 + 0.39 \times 10.4\, z + 10\, z$$
$$= 6.537 + 4.056\, z + 10\, z$$

At C, $z = 3$ m ; $(p_a)_C = 6.537 + 12.168 + 30 = 48.705$ kN/m

The *component* pressure distribution in shown in Fig. 20.57(b) while the *composite* pressure distribution diagram is shown in Fig. 20.57(c).

Example 20.46. *Compute the intensity of passive earth pressure at a depth of 8 m in a cohesionless sand with an angle of internal friction of 30° when water rises to the ground level. Saturated unit weight of sand is 21 kN/m^3 ; $\gamma_w = 9.81$ kN/m^3.* **(Gate Exam. 1999)**

Solution. Given $\gamma_{sat} = 21$ kN/m^3 ; $\varphi = 30°$.

$$K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \frac{1+\sin 30°}{1-\sin 30°} = 3 \; ; \gamma' = 21 - 9.81 = 11.19 \text{ kN/m}^2$$

$\therefore$ $P_p = K_p \gamma' H + \gamma_w H = 3 \times 11.19 \times 8 + 9.81 \times 8 = \mathbf{347.04\ kN/m^2}$

Example 20.47. *A vertical excavation was made in a clay deposit having unit weight of 22 kN/m^3. It caved-in after the digging reached 4 m depth. Assuming $\varphi = 0$, calculate the magnitude of cohesion.* **(Gate Exam. 1999)**

Solution. $K_a = \dfrac{1-\sin\varphi}{1+\sin\varphi} = 1$

$$P_a = K_a \gamma z - 2c\sqrt{K_a} = 22z - 2c \; ; \quad \therefore \quad p_a = 0 \text{ at } z = z_0 = \frac{2c}{22} = \frac{c}{11}$$

$\therefore$ Max. unsupported depth $= 2z_0 = \dfrac{2c}{11} = 4$ m (given)

$\therefore$ $c = \dfrac{4 \times 11}{2} = \mathbf{22\ kN/m^2}$

Example 20.48. *For the retaining wall shown in Fig. 20.58 (a), assume that wall can yield sufficiently to develop active state. Use Rankines' active pressure theory and determine (a) active force per metre of the wall and (b) the location of the resultant line of action.* **(Gate Exam. 2000)**

Solution. $K_{a1} = \dfrac{1-\sin 30°}{1+\sin 30°} = \dfrac{1}{3}$; $K_{a2} = \dfrac{1-\sin 36°}{1+\sin 36°} = 0.2596$

(a) Active pressure due to top soil

At B, $p_1 = K_{a1} \gamma_1 H = \dfrac{1}{3} \times 16 \times 3 = 16$ kN/m^2

(b) Active pressure due to bottom soil

At B, $p_2 = K_{a2} \cdot \gamma_1 H_1 = 0.2596 \times 16 \times 3 = 12.46$

At C, $p_a = K_{a2}\gamma_1 H_1 + K_{a2}\gamma' H_2 + \gamma_w H_2 = 12.46 + 0.2596\,(19 - 9.81) \times 3 + 9.81 \times 3$

$= 12.46 + 7.157 + 29.43 = 49.047$

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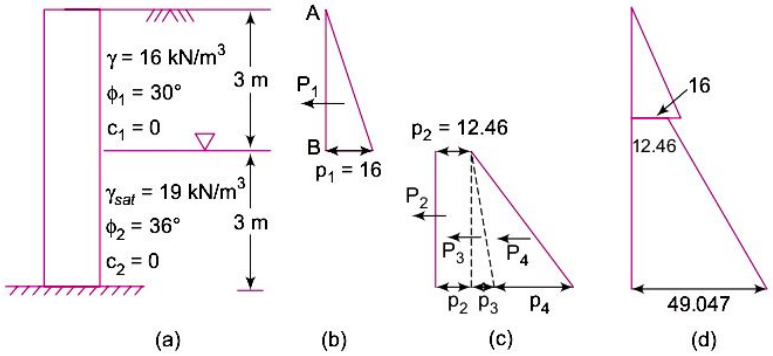

Fig. 20.58

(c) *Total active pressure*

$$P_1 = \frac{1}{2} \times 16 \times 3 = 24 \text{ kN/m acting at } 3 + 3/3 = 4 \text{ m above base}$$

$$P_2 = 12.46 \times 3 = 37.38 \text{ kN/m acting at } 3/2 = 1.5 \text{ m above base}$$

$$P_3 = \frac{1}{2} \times 7.157 \times 3 = 10.736 \text{ kN/m acting at } 3/3 = 1 \text{ m above base}$$

$$P_4 = \frac{1}{2} \times 29.43 \times 3 = 44.145 \text{ kN/m acting at } 3/3 = 1 \text{ m above base}$$

$\therefore$ Total $P_a = 24 + 37.38 + 10.736 + 44.145 = \mathbf{116.261\ kN/m}$

Acting at $z = \dfrac{24 \times 4 + 37.38 \times 1.5 + 10.736 \times 1 + 44.145 \times 1}{116.261}$

$= \mathbf{1.78\ m}$ above base.

Example 20.49. *A concrete gravity type retaining wall shown (Fig. 20.59a) retains granular soil having a friction angle of 35° and dry and saturated unit weight of 16 kN/m³ and 20 kN/m³. The unit weights of concrete and water are 24 kN/m³ and 10 kN/m³ respectively. The friction factor at the base of the wall against lateral sliding is 0.47. Calculate the following quantities for the retaining wall.*

(i) Factor of safety against lateral sliding
(ii) Factor of safety against over turning and
(iii) Bearing pressure on foundation soil using Meyerhofs' method **(Gate Exam. 2001)**

Solution. $K_a = \dfrac{1 - \sin 35°}{1 + \sin 35°} = 0.271$

(a) *Computation of total active force*

$p_1 = 0.271 \times 16 \times 2 = 8.672 \text{ kN/m}^2$; $p_2 = p_1 = 8.672 \text{ kN/m}^2$

$p_3 = 0.271\ (20 - 10) \times 3 = 8.13 \text{ kN/m}^2$; $p_4 = 10 \times 3 = 30 \text{ kN/m}^2$

$P_1 = \frac{1}{2} \times 8.672 \times 2 = 8.672$ kN acting at $z_1 = 3 + 2/3 = 3.667$ m above base

$\therefore$ $P_2 = 8.672 \times 3 = 26.016$ kN/m acting at $z_2 = 3/2 = 1.5$ m above base

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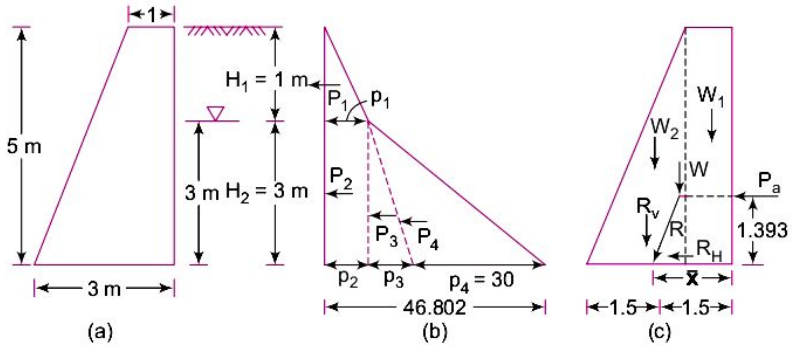

Fig 20.59

$$P_3 = \frac{1}{2} \times 8.13 \times 3 = 12.195 \text{ kN acting at } z_3 = 3/3 = 1 \text{ m above base}$$

$$P_4 = \frac{1}{2} \times 30 \times 3 = 45 \text{ kN acting at } z_4 = 3/3 = 1 \text{ m above base}$$

$\therefore$ Total $P_1 = 8.672 + 26.016 + 12.195 + 45 = 91.883$ kN

Acting at $z = \dfrac{8.672 \times 3.667 + 26.016 \times 1.5 + 12.195 \times 1 + 45 \times 1}{91.883} = 1.393$ m above base.

(b) Stability analysis (Fig. 20.59c)

$$W_1 = 1 \times 5 \times 1 \times 24 = 120 \text{ kN/m} \; ; \; W_2 = \frac{1}{2} \times 2 \times 5 \times 24 = 120 \text{ kN}$$

$$\bar{x} = \frac{W_1 x_1 + W_2 x_2 + P_a . z}{W_1 + W_2} = \frac{120 \times 0.5 + 120 \times 1.667 + 91.883 \times 1.393}{120 + 120} = 1.617 \text{ m}$$

$$e = \bar{x} - B/2 = 1.617 - 3/2 = 0.117 < B/6$$

(*i*) Factor of safety against sliding $= \dfrac{\mu \Sigma W}{\Sigma H} = \dfrac{0.47 \times 240}{91.883} = \mathbf{1.23}$

(*ii*) Factor safety against overturning $= \dfrac{M_V}{M_H} = \dfrac{120 \times 0.5 + 120 \times 1.667}{91.883 \times 1.393} = \mathbf{2.03}$

(*iii*) Bearing pressure by *Meyerhof's* method:

Effective width $B' = B - 2e = 3 - 2 \times 0.117 = 2.766$ m

$\therefore$ Bearing pressure $= \dfrac{W_1 + W_2}{B' \times 1} = \dfrac{120 + 120}{2.766 \times 1}$

$= \mathbf{86.77 \text{ kN/m}^2}$

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Example 20.50. *A retaining wall with a stratified backfill and a surcharge load is shown in Fig. 20.60(a). Draw the earth pressure diagram detailing the values at the critical points. Also estimate the resultant thrust on the wall and its position.* **(Gate Exam. 2002)**

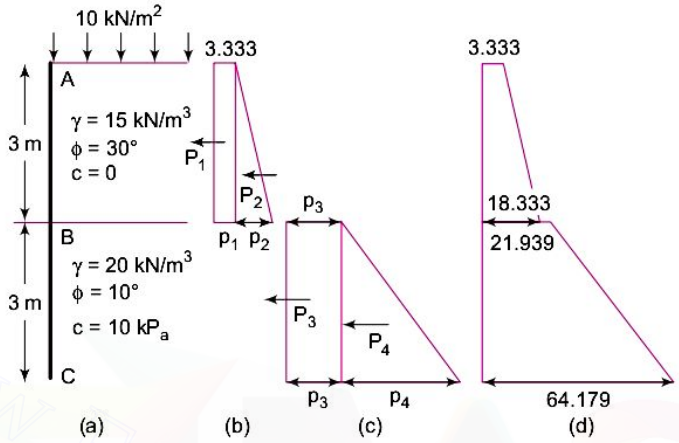

Fig. 20.60

Solution.
$$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$
$$K_{a2} = \frac{1-\sin 10°}{1+\sin 10°} = 0.704$$

(a) p_a – diagram for top soil (Fig. 20.60 a)

$$p_a = K_{a1}\, q + K_{a1}\, \gamma_1\, z_1 = p_1 + p_2$$

where $p_1 = \frac{1}{3} \times 10 = 3.333 \text{ kN/m}^2$; $p_2 = \frac{1}{3} \times 15 \times 3 = 15 \text{ kN/m}^2$

$\therefore$ $(p_a)_B = 3.333 + 1.5 = 18.333 \text{ kN/m}^2$

(b) p_a – diagram for bottom soil (Fig. 20.60 c)

$$p_a = K_{a2}\,(q + \gamma_1 H_1) + K_{a2}\,\gamma_2 z_2 - 2c\sqrt{K_{a2}}$$

or $P_a = 0.704\,(10 + 15 \times 3) + 0.704 \times 20 z_2 - 2 \times 10\sqrt{0.704} = 21.939 + 14.08 z_2$

At B, $z_2 = 0$; $\therefore$ $p_a = p_3 = 21.939 \text{ kN/m}^2$

At C, $z_2 = 3$ m $\therefore$ $p_a = 21.939 + 14.08 \times 3 = 21.939 + 42.24 = p_3 + p_4 = 64.179 \text{ kN/m}^2$

The composite p_a – diagram is shown in Fig. 20.60(d)

Now $P_1 = 3.333 \times 3 = 10$ kN/m acting at $z_1 = 3 + 3/2 = 4.5$ m above base

$P_2 = \frac{1}{2} \times 15 \times 3 = 22.5$ kN/m acting at $z_2 = 3 + 3/3 = 4$ m above base

$P_3 = 21.939 \times 3 = 65.817$ kN/m acting at $z_3 = 3/2 = 1.5$ m above base

$P_4 = \frac{1}{2} \times 42.24 \times 3 = 63.36$ kN/m acting at $z_4 = 3/3 = 1$ m above base

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$\therefore$ Total $P_a = 10 + 22.5 + 65.817 + 63.36 = \mathbf{161.677\ kN/m}$

Acting at $z = \dfrac{10 \times 4.5 + 22.5 \times 4 + 65.817 \times 1.5 + 63.36 \times 1}{161.677} = \mathbf{1.837\ m}$ above base.

Example 20.51. *A cohesionless soil with a void ratio of e = 0.6 and specific gravity of soil solids G_s = 2.65 exists at a site where the water table is located at a depth of 2 m below the ground surface. Assuming a value of coefficient of earth pressure at rest K_0 = 0.5, calculate the following quantities at a depth of 5 metres below the ground surface: total stresses σ_v and σ_H, effective Stresses σ_v' and σ_H' and pore water pressure u.* **(Gate Exam. 1996)**

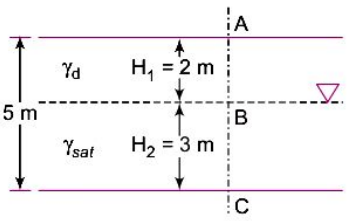

Fig. 20.61

Solution. Given : $e = 0.6;\ G = 2.65;\ K_0 = 0.5$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.6} = 16.248\ \text{kN/m}^3;$$

$$\gamma_{sat} = \frac{(G+e)\gamma_w}{1+e} = \frac{(2.65+0.6)9.81}{1+0.6} = 19.927\ \text{kN/m}^3$$

$\therefore$ $\gamma' = 19.927 - 9.81 = 10.117\ \text{kN/m}^3$

(a) Vertical pressure

Now $\sigma_v' = \sigma_z' = \gamma_d \cdot H_1 + \gamma' H_2$

$= 16.248 \times 2 + 10.117 \times 3 = \mathbf{62.847\ kN/m^2}$

$\sigma_v = \sigma_z = \gamma_d H_1 + \gamma_{sat} H_2 = 16.248 \times 2 + 19.927 \times 3 = \mathbf{92.277\ kN/m^2}$

(b) Pore pressure

$u = \sigma_u = \gamma_w H_2 = 9.81 \times 3 = 29.43\ \text{kN/m}^2$

(c) Horizontal pressure

σ_H' = Earth pressure at rest $= p_0 = K_0\, \gamma_d \cdot H_1 + K_0\, \gamma' H_2$

$= 0.5 \times 16.248 \times 2 + 0.5 \times 10.117 \times 3 = \mathbf{31.424\ kN/m^2}$

$u_H = \gamma_w H_2 = 9.81 \times 3 = 29.43$

$\sigma_H = \sigma_H' + u_H = 31.424 + 29.43 = \mathbf{60.854\ kN/m^2}.$

PROBLEMS

1. A retaining wall 4.2 m high with a smooth vertical back retains a dry sandy backfill of unit weight 18 kN/m^3 and angle of shearing resistance of 30°. The backfill carries a uniformly distributed load of 10 kN/m^2. Find by Rankine's theory the total active pressure per metre length of the wall and its point of application above the base. If the water table rises behind the back of the wall to an elevation of 2.1 m below the top of the wall, what is the change in the total active pressure per metre of the wall? Assume no change in angle of shearing resistance and submerged unit weight of sand as 9.5 kN/m^3.

[**Ans.** (*a*) 67 kN; 1.55 m above the base, (*b*) Increase in pressure = 15.72 kN]

2. A retaining wall with a vertical back 6 m high supports a cohesionless backfill of unit weight 19.6 kN/m^3. The upper surface of the backfill rises at an angle of 10° with the horizontal, from the crest of the wall. The angle of internal friction for the soil is 35° and the angle of wall friction is 20°. Find the total active pressure per metre length of the wall and mark the direction and point of application of the resultant pressure. Use Rebhann's graphical method. [**Ans.** 94 kN/m]

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3. A retaining wall 5 metres high with a sloping back at 10° to the vertical supports a cohesionless backfill rising from the crest at an angle of 5° with the horizontal. The backfill weighs 19 kN/m^3 and carries a uniformly distributed load of 5 kN/m^2. The angle of shearing resistance of the backfill is 30° and the angle of wall friction is 30°. Find by Rebhann's graphical method the total active pressure per metre.
[**Ans.** 106.5 kN/m^2]

4. Solve problem 3 if there is no surcharge load of 5 kN/m^2. [**Ans.** 89.2 kN/m^2]

5. A retaining wall 6 m high, with vertical back, supports a cohesive backfill having unit weight = 19 kN/m^3, apparent cohesion = 26 kN/m^2 and angle of internal friction zero. Calculate (*a*) internal pressure intensity at the top of the wall, (*b*) depth of tension cracks, and (*c*) lateral pressure intensity at the base.
[**Ans.** (*a*) – 50 kN/m^2/m; (*b*) 2.64 m; (*c*) 64 kN/m^2/m length]

6. A retaining wall has the following data:
   (*i*) Height of wall = 6 m
   (*ii*) Inclination of face of wall with vertical = 0°
   (*iii*) Inclination of backfill (surcharge line) with horizontal = 0°
   (*iv*) Angle of internal friction = 30°
   (*v*) Angle of wall friction = 20°
   (*vi*) Unit weight of cohesionless backfill = 17 kN/m^3

   Using Rebhann's method, compute the total active pressure per metre length of the wall. What will be the corresponding value of Rankine's active earth pressure ? [**Ans.** (*i*) 92.5 kN/m ; (*ii*) 102 kN/m]

7. A cohesive soil has unit weight of 19.2 kN/m^3, unit cohesion as 12 kN/m^2 and angle of internal friction as 10°. Calculate the critical height of vertical excavation that can be made without any lateral support.
[**Ans.** 2.98 m]

8. A 5 m high vertical wall supports a cohesive backfill with upper horizontal surface. The properties of the backfill are as follows : (*i*) upper 3 m depth. γ_{sat} = 17.6 kN/m^3 ; c = 7.5 kN/m^2 and ϕ = 20°; (*ii*) lower 2 m depth. γ_{sat} = 19.2 kN/m^3; c = 10 kN/m^2 and ϕ = 20°. The free water level stands behind the wall at a depth of 4 m from top. Neglecting negative pressure up to a depth of tension cracks, determine the total active pressure and its point of application. [**Ans.** 58.75 kN; 1.238 m from base]

9. A retaining wall is 5 m high and has its back inclined at a positive batter angle of 14° . The surface of the backfill rises from the top of the wall at a slope of 4 horizontal to 1 vertical up to a horizontal distance of 4 m away from the top; thereafter, the surface of the backfill becomes horizontal. The bulk density of soil is 17.8 kN/m^3, its angle of shearing resistance 36° and the cohesion is negligible. The angle of wall friction is 20°. Find the total active pressure per metre length of the wall. What is the critical slip angle λ?
[**Ans.** 92.5 kN/m; λ = 63°]

10. A soil has the following properties : c = 9 kN/m^2; ϕ = 20° and λ = 18 kN/m^3. Calculate the critical depth of a vertical excavation that can be made in the soil without any lateral support. The ground carries a surcharge load of 3 kN/m^2. [**Ans.** 2.25 m]

11. Calculate the passive earth pressure per metre length against a 5 m high retaining wall if it is pushed towards a clayey backfill (c = 25 kN/m^2 and ϕ = 30°) with horizontal top, flush with the top of the wall. The backfill carries a surcharge of intensity 40 kN/m^2 and weighs 18 kN/m^3. Find also the point of application of this pressure. [**Ans.** 1700 kN/m; 2.17 m above base]

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CHAPTER 21

Bulkheads and Cofferdams

21.1 INTRODUCTION : SHEET PILE WALLS

A *sheet pile wall* consists of a series of sheet piles driven side by side into the ground, thus forming a continuous vertical wall for the purpose of retaining an earth bank. They are commonly used for (*i*) water front structures, (*ii*) temporary construction and (*iii*) light weight construction where subsoil is poor for supporting a retaining wall. A sheet pile wall may be of three types (*i*) *cantilever* sheet piling (*ii*) *anchored* sheet piling, and (*iii*) *braced* sheeting. Certain configurations of sheet piling are termed as *bulkheads* or *cofferdams*. A *bulkhead* is a sheet pile retaining wall of water front, backed up by ground. A *cofferdam* is a reasonably water-tight enclosure made of sheet pile walls, usually temporary, built around a working area for the purpose of excluding water during construction. Sheet pile walls are employed as bulkheads in piers, docks and harbours, and in sea walls, break waters, and other shore protection works. Figure 21.1 shows various sheet piling structures.

The sheet piles can be of timber, reinforced concrete or steel. Timber sheet piling is used for short spans, light lateral loads, and commonly for temporary structures in the form of braced sheeting. Wooden sheet piles may be considered permanent if they are permanently under water, or if they are impregnated with preservatives. Concrete sheet piles are precast members, usually with a tongue and groove joint, designed to withstand the permanent stresses during service and the handling stresses during construction. They are heavy and bulky, and require heavier equipment to drive and handle. Steel sheet piling is the most common type used because of the following advantages over other materials: (*i*) resistant to high driving stresses, (*ii*) light weight, (*iii*) can be used several times, (*iv*) long service, (*v*) easier to increase the pile length (*vi*) lesser deformation of joints when wedged full with soil and stones during driving.

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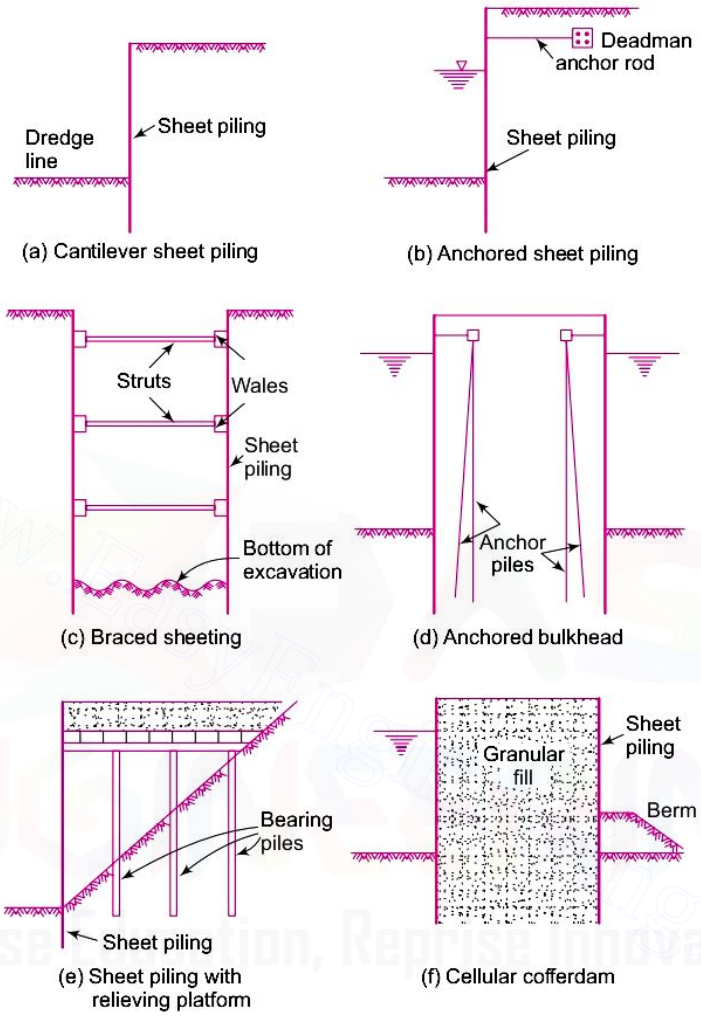

Fig. 21.1 Common types of sheet piling structures

21.2 CLASSIFICATION OF BULKHEADS

A bulkhead is a sheet pile retaining wall of water front, backed up by ground. They are widely used for dock and harbour structures. A bulkhead provides a vertical wall so that ships may tie-up along side. A bulkhead may also serve as a pier structure which may jut out into the water. In contrast to the retaining walls, which are rigid bulky structures, bulkheads are flexible, generally composed of a single row of partially embedded sheet piles with major portion of the lateral support furnished externally. The upper and lower edges of bulkhead are practically fixed on account of the anchorage at the top and the passive resistance of the soil adjoining the lowest part of the bulkhead. The use of the anchor member tends to reduce the lateral deflection, the bending moment, and the depth of penetration of the pile. Bulkheads may be classified in the number of ways:

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(*a*) according to mode of establishing the retaining system,
(*b*) according to structural type and loading scheme,
(*c*) according to sheet pile material, and
(*d*) according to end support.

(*a*) *Classification according to mode of establishing the retaining system.* According to the mode of establishing the retaining system, a bulkhead may be classified as (*i*) *fill bulkhead* or (*ii*) *dredged bulkhead*.

A *fill bulkhead* [Fig. 21.2 (*a*)] is the one which is constructed in open water and then backfilled. If the bulkhead is constructed in natural ground (by driving) and then the earth is removed from its face by dredging, it is called *dredged bulkhead* [Fig. 21.2 (*b*)]

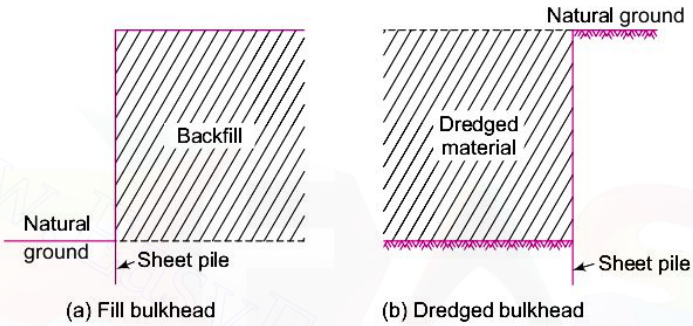

Fig. 21.2 Fill and dredged bulkheads

(*b*) *Classification according to structural type and loading scheme.* This is an important classification, according to which a bulkhead may be cantilever type or may be anchored. A *cantilever bulkhead* is the one which derives its stability entirely from the lateral resistance of the soil into which the sheet piling is driven [Fig. 21.1 (*a*)]. An *anchored bulkhead* is one which is held above its driven depth by one or more tie rods or anchors at or near its top [Fig. 21.1 (*b*)]. The stability of the cantilever bulkhead or sheet pile wall depends upon its embedment into soil below the dredged line. The stability of the anchored bulkhead depends more upon the anchors than on the inducement of large resultant earth pressure.

(*c*) *Classification according to sheet pile material.* Depending upon the material of sheet pile, a bulkhead may be classified as *flexible or rigid*. A *flexible bulkhead,* generally constructed of steel piles, deflects and bends under the influence of a lateral force. Its stability depends upon the development of sufficient lateral resistance of soil adjacent to the embedded length of the bulkhead, and upon the anchor rods. A *rigid bulkhead,* constructed of reinforced concrete sheet pile derives its stability against lateral forces basically from its inherent structural strength.

(*d*) *Classification according to condition of end support.* According to this, bulkheads may be classified into two categories: (*i*) bulkhead with free earth support, and (*ii*) bulkhead with fixed earth support. This classification applies only to anchored bulkheads. In the case of bulkhead with *free earth support,* the sheet piling is driven to a shallow depth only so that the deflection of the bulkhead is somewhat similar to that of a vertical elastic beam whose lower end is simply supported. In the case of bulkhead with *fixed earth support,* the sheet piles are driven to a considerable depth so that lower end of the bulkhead is practically fixed in position.

Forces acting on a bulkhead. A number of forces are known to be acting over a bulkhead, some of which are constantly in play while others come in effect only occasionally and intermittently. A sheet pile wall or bulkhead may be subjected to some or all of the following types of lateral pressures:

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1. active and passive earth pressures,
2. lateral pressure due to surcharge load,
3. unbalanced water pressure and seepage pressure,
4. mooring pull, ship impact etc.
5. earth-quake forces.

The classical earth pressure theories of Rankine or Coulomb, which are based on the condition of full mobilisation of the shear strength of soil due to yielding of the wall laterally by sliding or by rotation about its bottom, do not apply in the case of bulkheads where the deformation conditions are different. The earth pressure against sheet pile wall can be determined by theories which take into account the conditions of yield of the wall (Hansen, J.Brinch, 1953). In practice, several empirical and semi-empirical methods have been developed all of which use the classical earth pressure theories.

21.3 CANTILEVER SHEET PILE WALL

A cantilever sheet pile wall or bulkhead derives its stability entirely from the lateral resistance of the soil into which it is driven. The bulkhead is adequately embedded into the soil below the dredge line so that a driven line of sheeting acts as wide cantilever beam in resisting the lateral earth pressures developed above the dredge line. A cantilever bulkhead is used for moderate height only.

Cantilever sheet piling in granular soils. The action of earth pressure against the cantilever sheet piling is illustrated in Fig. 21.3(*a*), assuming the sheet piling to be perfectly rigid. Under the influence of the active pressure of the backfill, the wall tends to rotate, developing active pressure behind the wall (portion *ad*) and passive pressure in front of the wall (portion *db*). *At the pivot point (b), the soil behind the wall goes from active to passive pressure, while the soil in front of the wall goes from passive to active for the remaining distance bc.* Fig. 21.3 (*b*) shows the probable qualitative pressure distribution, while Fig. 21.3 (*c*) shows that simplified pressure diagram for computational purposes.

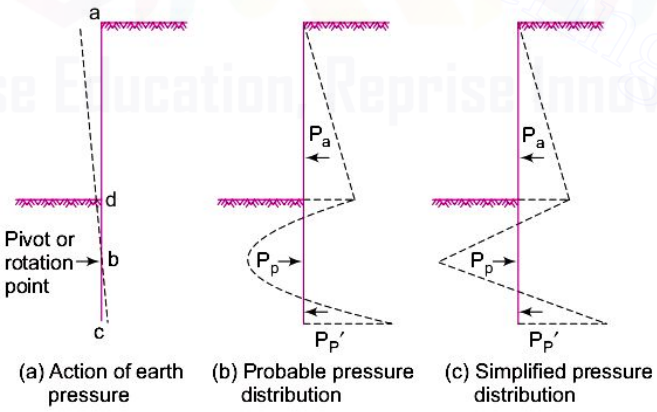

Fig. 21.3 Cantilever sheet piling in granular soils

Figure 21.4 shows the pressure distribution diagram for a cantilever sheet piling in granular soil. The height of wall above the dredge line is H and its embedment is D. The water table stands up to a height h_1 above the dredge line. Let the soil properties for the height h_1 above water table be γ, ϕ, K_p and K_a (where K_a and K_p are the earth pressure coefficients, usually Rankine's) and the soil properties for the height h_2, corresponding to the *submerged conditions* are γ', ϕ', K_p' and K_a'.

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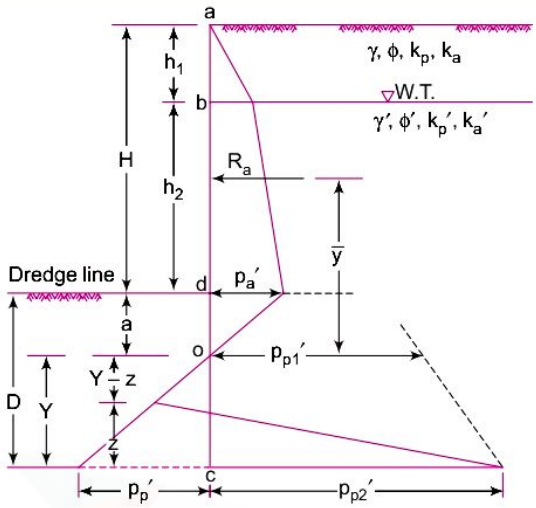

Fig. 21.4 Cantilever sheet piling: pressure distribution diagram for a granular soil

Let p_a' = active earth pressure intensity at dredge line

$$= K_a \gamma h_1 + K_a' \gamma' h_2 \quad ...(21.1)$$

p_p' = effective passive earth pressure at the base of pile, acting towards the fill

$$= \gamma' (K_p' - K_a') Y = \gamma' K' Y$$

where $K' = K_p' - K_a'$;

$K = K_p - K_a$

and Y = distance oc ...(21.2)

Let p_{p1}' = effective earth pressure at point o

$$= \gamma h_1 K_p + (h_2 + a) \gamma' K_p' - \gamma' a K_a' \quad ...(21.3\ a)$$

$$= \gamma h_1 K_p + \gamma' h_2 K_p' + \gamma' a K' \quad ...(21.3)$$

p_{p2}' = effective passive pressure at base of piling acting against backfill side of wall

$$= p_{p1}' + \gamma' K' Y + p_{p1}' + p_p' \quad ...(21.4)$$

If the value of ϕ is the same for both the portions,

$$K_p = K_p', \quad K_a = K_a' \quad \text{and} \quad K = K'$$

Let R_a = resultant of all forces above point o, acting at a distance $\bar{y}$ above o.

The point o is located at a distance a below the dredge line in such a way that the pressure intensity at o is zero .This requirement gives the relation:

$$a\gamma' (K_p' - K_a') = p_a' \quad \text{or} \quad a = \frac{p_a'}{\gamma' (K_p' - K_a')} = \frac{p_a'}{\gamma' K'} \quad ...(21.5)$$

Thus, the distance a is known. The distance z can be found in terms of Y, by using the equation $\Sigma F_H = 0$ (by statics):

$$R_a + (p_p' + p_{p2}') \frac{z}{2} - p_p' \cdot \frac{Y}{2} = 0 \quad \text{which gives} \quad z = \frac{p_p' Y - 2R_a}{p_p' + p_{p2}'} \quad ...(21.6)$$

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In the above equation, Y is unknown. An additional equation can be obtained in terms of z and Y by assuming the moments at the bottom of the pile to zero ($\Sigma M = 0$):

$$R_a\left(Y+\bar{y}\right)+\frac{z}{3}\left(p_p{}'+p_{p2}{}'\right)\frac{z}{2}-p_a{}'\left(\frac{Y}{2}\right)\left(\frac{Y}{3}\right)=0$$

which simplifies to : $6R_a\left(Y+\bar{y}\right)+z^2\left(p_p{}'+p_{p2}{}'\right)-p_p{}'\,Y^2=0$ ...(21.7)

Substituting value of z (Eq. 21.6) in Eq. 21.7, and replacing $p_{p2}{}'$ by $p_{p1}{}'+p_p{}'$ (Eq. 21.4) and simplifying, we get

$$Y^4+Y^3\left(\frac{p_{p1}{}'}{\gamma' K'}\right)-Y^2\left(\frac{8R_a}{\gamma' K'}\right)-Y\left[\frac{6R_a}{\left(\gamma' K'\right)^2}\left(2\bar{y}\,\gamma' K'+p_{p1}{}'\right)\right]-\frac{6R_a\,.\,\bar{y}\,p_{p1}{}'+4R_a^2}{\left(\gamma' K'\right)^2}=0 \quad ...(21.8)$$

This is a fourth order equation in Y, which can be solved by trial and error. If there is no water, $\gamma' K'$ simplifies to γK. Knowing Y, D can be computed from the relation $D = a + Y$. In order to provide for factor of safety, two alternative methods are adopted : (i) the value of D so obtained is increased arbitrarily by 20 – 40%, or (ii) a factor of safety of 1.5 is used with the terms K_p and $K_p{}'$ and then Eq. 21.8 is solved. The first method is sometimes preferred.

Approximate analysis. In the approximate analysis, the active pressure distribution on the back, and the passive pressure distribution in the front is assumed to extend up to the bottom end of the wall (Fig. 21.5), and that the passive pressure P_{p2} of the small length be (Fig. 21.4) near the bottom, also acts at its bottom c.

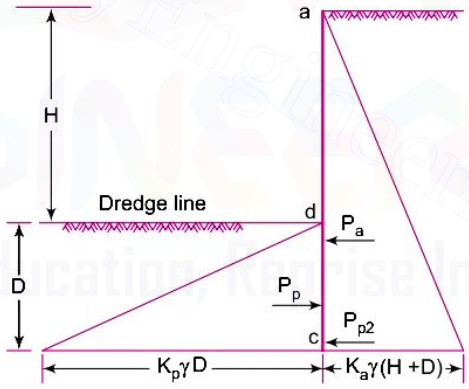

Fig. 21.5

For equilibrium, summing the moment of all forces about c and equating it to zero, we have

$$\frac{1}{2}K_p\,\gamma\,D^2\times\frac{D}{3}=\frac{1}{2}K_a\,\gamma\left(H+D\right)^2\times\left(\frac{H+D}{3}\right) \quad ...(21.9)$$

This is a cubic equation in terms of D, which is required to maintain the equilibrium.

21.4 CANTILEVER SHEET PILE WALL IN COHESIVE SOIL

Figure 21.6 (a) shows the pressure distribution for cantilever sheet pile wall in purely cohesive soil ($\phi = 0$). The active and passive pressure intensities for a purely cohesive soil are given by

$$p_a=\sigma K_a-2c\sqrt{K_a}$$

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$$= \sigma - 2c \qquad ...(21.9)$$

(since $K_a = K_p = 1$ when $\phi = 0$; and $\sigma = \gamma z$)

and
$$p_p = \sigma K_p + 2c\sqrt{K_p}$$
$$= \sigma + 2c \qquad ...(21.10)$$

Thus, active pressure intensity at point a is equal to $-2c$ and that at the dredge line is given by

$$p_a = \gamma' H - 2c = q - 2c \qquad ...(21.11)$$

where q = effective pressure at dredge line = $\sigma = \gamma' H$

The passive pressure diagram is shown hatched in Fig. 21.6 (a). At the point d, on the left of the sheet piling at the dredge line, the over burden pressure $\sigma = 0$. Hence net pressure at d is

$$(p_p - p_a)_d = (0 + 2c) - (q - 2c) = 4c - q \qquad ...(21.12)$$

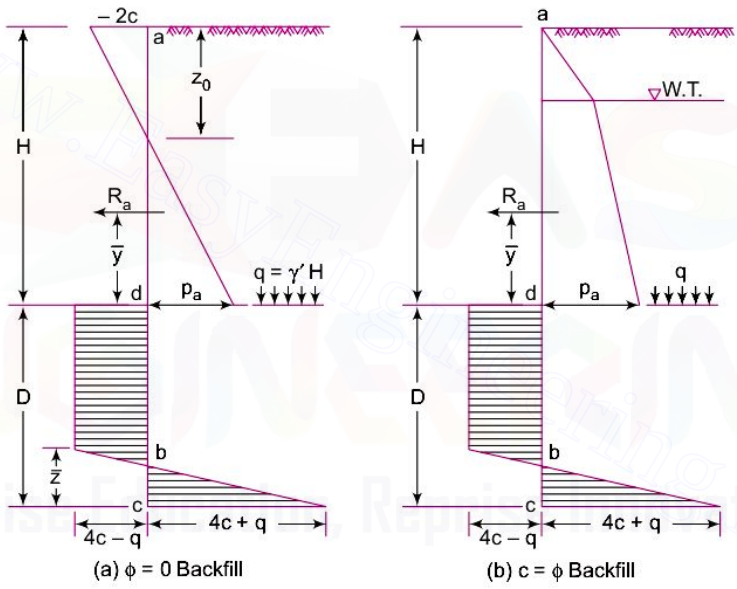

Fig. 21.6 Cantilever sheet pile wall in cohesive soil

Also, at point c the net pressure is given by

$$(p_p - p_a)_d = (q + \gamma D + 2c) - (\gamma D - 2c) = 4c + q \qquad ...(21.13)$$
$$\leftarrow \qquad \rightarrow$$

In order to find the height $\bar{z}$ above the base c, equate the sum of horizontal forces to zero ($\Sigma H = 0$).

$$\therefore \qquad R_a + \frac{\bar{z}}{2}(4c + q + 4c - q) - D(4c - q) = 0$$

(where, R_a = resultant of active pressure above the dredge line active at $\bar{y}$ above the dredge line.)

Solving for $\bar{z}$
$$\bar{z} = \frac{D(4c - q) - R_a}{4c} \qquad ...(21.14)$$

In order to get another equation of $\bar{z}$ and D, sum the moment of all forces about the base and equate it to zero.

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$$R_a\left(\bar{y}+D\right)-\frac{D^2}{2}\left(4c-q\right)+\frac{\bar{z}}{3}.\frac{\bar{z}}{2}\left(4c-q+4c+q\right)=0$$

Substituting $\bar{z}$ and simplifying, we get

$$D^2\left(4c-q\right)-2\,DR_a-\frac{Ra\left(12\,c\,\bar{y}+R_a\right)}{2\,c+q}=0 \qquad ...(21.15)$$

Note. If the backfill is of granular soil (and not clay), but the soil of embedment is clay, ($\phi = 0$), the above analysis is valid, provided the active earth pressure diagram is adopted as shown in Fig. 21.6 (*a*). In that case, q is to be taken as the effective pressure at the dredge line. Because of the uncertainty of clay, and since many of the sheet pile walls are for the purpose of containing a fill on which a structure may be constructed, the piling may be driven in clay and then backfilled with a freely draining granular material.

Example 21.1. *Compute the embedment length D of the sheet pile wall shown in Fig. 21.7 (a).*

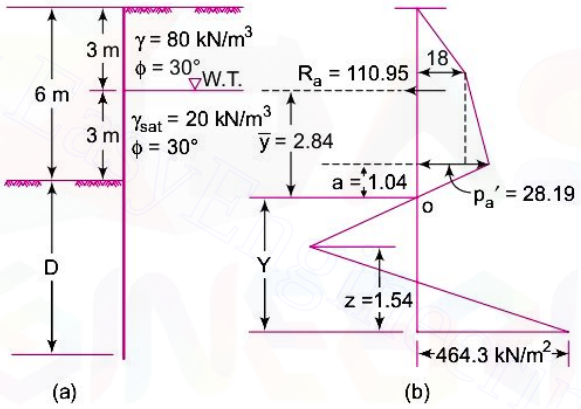

Fig. 21.7

Solution.

$$K_a = K_a' = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3} = 0.333$$

$$K_p = K_p' = 3;$$

$$K = K' = 3-\frac{1}{3} = \frac{8}{3} = 2.67$$

$$\gamma' = 20 - 9.81 = 10.19 \text{ kN/m}^2$$

Refer Fig. 21.4,

$$p_a' = \frac{1}{3}\times 18\times 3+\frac{1}{3}\times 10.19\times 3 = 28.19 \text{ kN/m}^2$$

$$a = \frac{p_a'}{\gamma' K} = \frac{28.19}{10.19\times 2.67} = 1.04 \text{ m (Eq. 21.5)}$$

$$R_a = \left(\frac{1}{2}\times 18\times 3\right)+(18\times 3)+\frac{1}{2}(10.19\times 3)+\left(\frac{1}{2}\times 28.19\times 1.04\right)$$

$$= 27 + 54 + 15.29 + 14.66 = 110.95 \text{ kN/m length.}$$

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Its distance $\bar{y}$ is found by equating moment of R_a to the sum of moments of individual pressures, about O.

$$110.95\ \bar{y} = (27 \times 5.04) + (54 \times 2.54) + (15.29 \times 2.04) + (14.66 \times 0.69)$$

$$\therefore \quad \bar{y} = 2.84 \text{ m}$$

From Eq. 21.3, p_{p1}' is given by

$$p_{p1}' = \gamma h_1 K_p + \gamma' h_2 K_p + \gamma' aK$$
$$= (18 \times 3 \times 3) + (10.19 \times 3 \times 3) + (10.19 \times 1.04 \times 2.67) = 282 \text{ kN/m}^2$$
$$\gamma' K = 10.19 \times 2.67 = 27.21$$

The coefficients of Eq. 21.8 are as under (putting $K' = K$)

$$\frac{p_{p1}'}{\gamma' K} = \frac{282}{27.21} = 10.36 \qquad ...(1)$$

$$\frac{8 R_a}{\gamma' K} = \frac{8 \times 110.95}{10.19 \times 2.67} = 32.6 \qquad ...(2)$$

$$\frac{6 R_a}{(\gamma' K)^2}\left(2 \bar{y} \gamma' K + p_{p1}'\right) = \frac{6 \times 110.95}{(10.19 \times 2.67)^2}\left[(2 \times 2.84 \times 10.19 \times 2.67) + 282\right] = 392.6 \qquad ...(3)$$

and

$$\frac{6 R_a \bar{y}\, p_{p1}' + 4 R_a^2}{(\gamma' K)^2} = \frac{(6 \times 110.95 \times 2.84 \times 282) + 4(110.95)^2}{(10.19 \times 2.67)^2} = 786.8$$

Substituting in Eq. 21.8, we get

$$Y^4 + Y^3 (10.36) - Y^2 (32.6) - Y (392.6) = 786.8$$

The trial and error solution is shown in a tabular form below:

Y	Y^4	$10.36\ Y^3$	$-32.6\ Y^2$	$-392.6\ Y$	$= 786.8$
7	2401	3552	– 1591	– 2737	+ 1625
6.5	1650	2763	– 1375	– 2552	+ 486
6.7	1940	2980	– 1463	– 2630	+ 827

Hence adopt $Y \approx 6.7$ m

***Check*:**

$$z = \frac{p_p' Y - 2 R_a}{p_p' + p_{p2}'} \quad \text{(Eq. 21.6)}$$

where

$$p_p' = \gamma' K Y = 10.19 \times 2.67 \times 6.7 = 182.3 \text{ kN/m}^2$$
$$p_{p2}' = p_{p1}' + p_p' = 282 + 182.3 = 464.3 \text{ kN/m}^2$$

$$\therefore \quad z = \frac{(182.3 \times 6.7) - (2 \times 110.95)}{182.3 + 464.3} \approx 1.55$$

$$\Sigma F = 0 = R_a + (p_p' + p_{p2}')\frac{z}{2} - p_p' \frac{Y}{2}$$

$$\therefore \quad 110.95 + (182.3 + 464.3)\frac{1.55}{2} - 182.3 \times \frac{6.7}{2} = 1.36 \approx 0$$

Hence O.K. $\therefore$ $D = Y + a = 6.7 + 1.04 = 7.74$ m

Increasing it by 30 % $D = 7.74 \times 1.30 \approx 10$ m.

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Example 21.2. *Compute the embedment length D for the sheet pile wall shown in Fig. 21.8.*

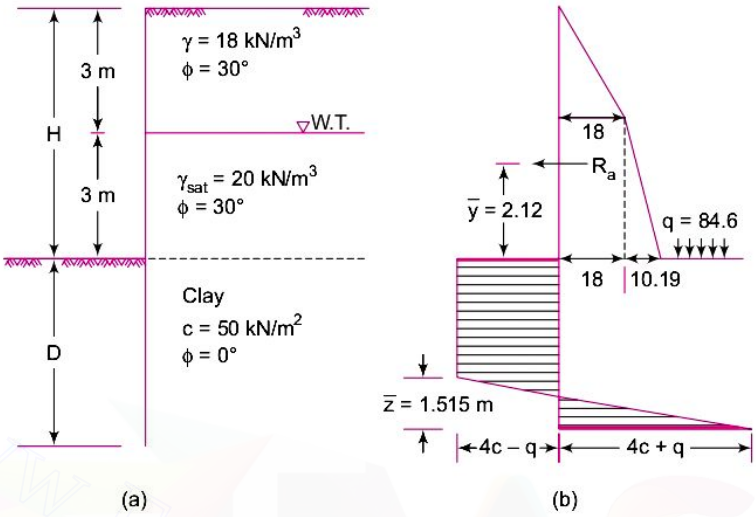

Fig. 21.8

Solution. (Refer Figs. 21.6 and 21.8)

$$K_a = K_a' = \frac{1}{3}\,;\; K_p = K_p' = 3\,;\; \gamma' = 20 - 9.81 = 10.19 \text{ kN/m}^3$$

The effective pressure q at the dredge line is given by

$$q = (18 \times 3) + (10.19 \times 3) = 84.6 \text{ kN/m}^2$$

From the lateral earth pressure diagram [Fig. 21.8 (*b*)]

$$R_a = (\frac{1}{2} \times 18 \times 3) + (18 \times 3) + (\frac{1}{2} \times 10.19 \times 3) = 27 + 54 + 15.3 = 96.3$$

and
$$\bar{y} = \frac{(27 \times 4) + (54 \times 1.5) + (15.3 \times 1)}{96.3} = 2.12 \text{ m}$$

The depth of embedment is given by Eq. 21.15

$$D^2 (4c - q) - 2D R_a - \frac{R_a (12c\,\bar{y} + R_a)}{2c + q} = 0$$

Substituting the values
$$D^2 (4 \times 50 - 84.6) - 2D (96.3) - \frac{96.3 (12 \times 50 \times 2.12 + 96.3)}{2 \times 50 + 84.6} = 0$$

or
$$115.4\, D^2 - 192.6\, D - 713.8 = 0$$

or
$$D^2 - 1.67\, D - 6.19 = 0, \quad \text{from which } D = 3.46 \text{ m.}$$

Check: From Eq. 21.14,

$$\bar{z} = \frac{D(4c - q) - R_a}{4c} = \frac{3.46 (4 \times 50 - 84.6) - 96.3}{4 \times 50} = 1.515 \text{ m.}$$

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Equating $\Sigma H = 0$, we get

$$R_a + \frac{\bar{z}}{2}(8c) - D(4c - q) = 0$$

or $96.3 + (1.515 \times 4 \times 50) - 3.46(4 \times 50 - 84.6) = 0$

or $399.3 - 399.3 = 0$

Hence, O.K. Providing 30% extra embedment.

$D = 3.46 \times 1.3 = \mathbf{4.498\ m}$

Hence provide $D = \mathbf{4.5\ m.}$

21.5 DESIGN OF ANCHORED BULKHEAD: FREE EARTH SUPPORT METHOD

An anchored bulkhead is the one which is held above its driven depth by one or more tie rods or anchors at or near its top. The design of anchored bulkhead consists of the determination of (*i*) the length of embedment, (*ii*) the magnitude of the tensile force in the anchor or tie rod, and (*iii*) design of the section of the sheet. The various methods of analysis of anchored bulkheads can be broadly grouped under two categories: (1) method of *free earth support* and (2) method of *fixed earth support*.

DESIGN OF ANCHORED BULKHEAD BY FREE-EARTH SUPPORT METHOD:

(A) Granular soil with variation of properties along the depth: This method is based on the following assumptions:

1. The sheet pile is perfectly rigid as compared to the surrounding soil.
2. The sheet pile is free to rotate at the anchor rod level with failure occurring by a rotation about the anchor rod.
3. The active and passive soil pressure acting on the sheet pile may be computed by the Rankine theory, and their distribution is hydrostatic.

Figure 21.9 shows a simplified pressure distribution diagram.

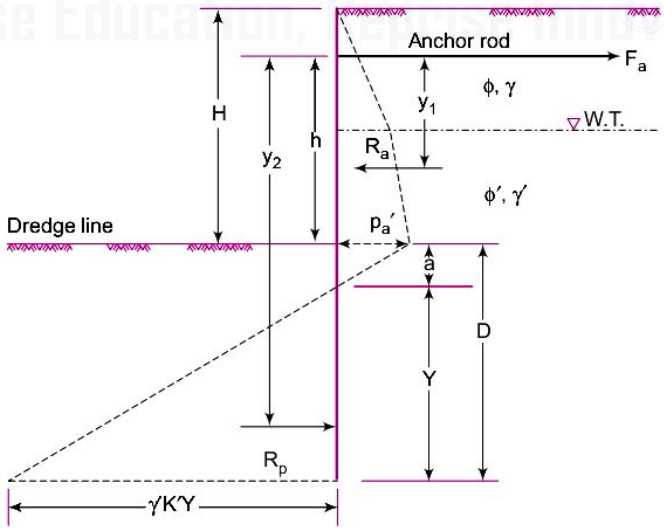

Fig. 21.9 Free earth method of analysis of anchored bulkhead : Granular soil

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Let F_a = tensile force in the anchor rod

K_a' and K_p' = co-efficients of active and passive pressures, based on ϕ'

$K' = K_p' - K_a'$

a = depth of point of zero pressure below the dredge line

If p_a' = active pressure intensity at the dredge line, we have

$$a\gamma' (K_p' - K_a') = p_a'$$

$$a = \frac{p_a'}{\gamma' (K_p' - K_a')} = \frac{p_a'}{\gamma' K'} \quad ...(21.16)$$

Let R_a = resultant active earth pressure, acting at y_1 below the anchor rod level.

R_p = resultant passive earth pressure, acting at y_2 below the anchor rod level.

Applying $\Sigma M = 0$ at the tie rod level, we get

$$y_1 R_a = y_2 R_p \quad ...(21.17)$$

But $$R_p = \frac{1}{2} Y \left[\gamma' (K_p' - K_a') Y\right] = \frac{1}{2} Y^2 \gamma' K' \quad ...(21.18)$$

and $$y_2 = h + a = \frac{2}{3} Y \quad ...(21.19)$$

Substituting in Eq. 21.17.

$y_1 R_a = \gamma' K' \frac{Y^2}{2} \left(h + a + \frac{2}{3} Y\right)$. Rearranging the terms, we get

$$Y^3 \left(\frac{\gamma' K'}{3}\right) + Y^2 \left(\frac{\gamma' K'}{2}\right)(h + a) - R_a y_1 = 0 \quad ...(21.20)$$

Solving this, Y is obtained. Hence $D = Y + a$.

This value of D may either be increased by 20 to 40% or K_p' may be divided by an appropriate factor of safety before substituting in Eq. 21.20.

The force in the tie rod is obtained by equating $\Sigma H = 0$

$$\therefore \quad F_a = R_a - R_p \quad ...(21.21)$$

(B) Free earth support method: Cohesive soil below dredge line: If the soil is cohesive ($\phi = 0$) below the dredge line, but the backfill is granular, the passive pressure distribution will be rectangular, as shown in Fig. 21.10, with the previous notation. To find the intensities of passive pressure, we use the equation

$$p_p = \sigma \tan^2 \alpha + 2c \tan \alpha$$

$$= \sigma K_p + 2c \sqrt{K_p}$$

But for $\phi = 0$, $K_a = K_p = 1$; also,

$$\sigma = \gamma' H = q$$

$$\therefore \quad p_p = \sigma + 2c \quad ...(i)$$

Also $$p_a = \sigma K_a - 2c \sqrt{K_a}$$

$$= \sigma - 2c \quad ...(ii)$$

At the dredge line, to the left of d, $\sigma = 0$

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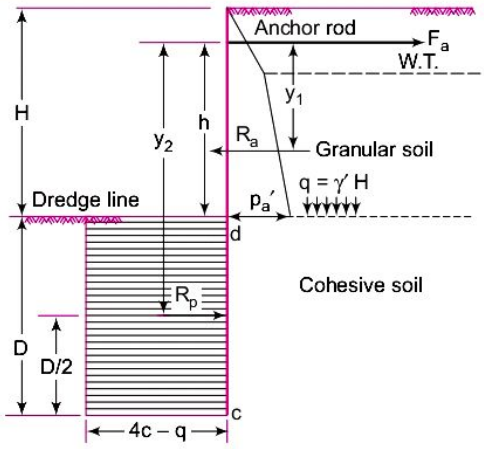

Fig. 21.10 Free earth support method cohesive soil below dredge line

$$(p_p)_d = 2c \qquad ...(iii)$$

At the dredge line, to the right of d, $\sigma = q = \gamma' H$

$$\therefore \quad (p_a)_d = q - 2c = \gamma' H - 2c \qquad ...(iv)$$

$\therefore$ Net passive pressure at d is given by

$$(p_p - p_a)_d = 2c - (q - 2c) = 4c - q \qquad ...(21.22)$$

Let R_p = resultant pressure = $(4c - q)\, D$ ...(21.23)

Acting at $y_2 = h + \dfrac{D}{2}$ below the anchor rod

Putting $\Sigma M = 0$ at the anchor rod,

$$R_a y_1 - D\,(4c - q)\left(h + \frac{D}{2}\right) = 0$$

or $$D^2 + 2Dh - \frac{2\, y_1\, R_a}{4c - q} = 0 \qquad ...(21.24)$$

Solving this, D is obtained. This may be increased by 20–40%. The wall will be unstable if the passive pressure $4c - q = 0$.

Hence if $\dfrac{c}{q} = \dfrac{c}{\gamma' H} = \dfrac{1}{4}$, the wall will be unstable.

The term $\dfrac{c}{q}$ is called *stability number* S_n.

If the wall adhesion c_a is taken into account, the stability number is given by

$$S_n = \frac{c}{q}\sqrt{1 + \frac{c_a}{c}}$$

or $$S_n = \frac{1.25\, c}{q} \text{ (at passive failure)} \qquad ...(21.25)$$

For factor of safety $F = 1$ and $\dfrac{c}{q} = \dfrac{1}{4}$ $S_n \approx 0.3\, F$ ...(21.26)

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(C) Free earth support method: Uniform granular soil: If the back fill has uniform properties (*i.e.*, if γ and φ are the same throughout the depth), analysis becomes relatively simpler. Fig. 21.11 shows the active and passive pressure distribution diagrams, with maximum values of pressure intensities at point D, as under:

$$p_a = K_a \gamma (H + D) \quad ...(i)$$

and

$$p_p = K_p \gamma D \quad ...(ii)$$

$$\therefore \quad P_a = \frac{1}{2} K_a \gamma (H + D)^2$$

and

$$P_p = \frac{1}{2} K_p \gamma D^2$$

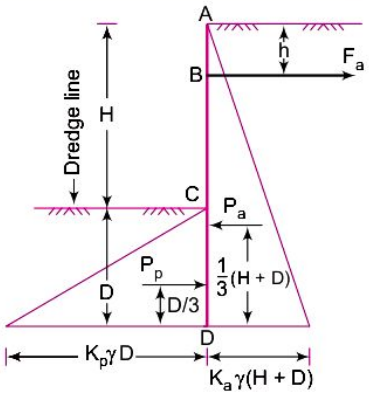

Fig. 21.11

Taking moments about point B (*i.e.*, level of the anchor rod), we get

$$\frac{1}{2} K_p \gamma D^2 \left(H - h + \frac{2}{3} D \right) = \frac{1}{2} K_a \gamma (H + D)^2 \left[\frac{2}{3} (H + D) - h \right]$$

or

$$K_p D^2 \left(H - h + \frac{2}{3} D \right) = K_a (H + D)^2 \left[\frac{2}{3} (H + D) - h \right] \quad ...(21.27)$$

Equation 21.27 is a cubic equation in terms of D, the value of which can be found by trial and error procedure. See example 21.6 for illustration.

21.6 DESIGN OF ANCHORED BULKHEAD: FIXED EARTH METHOD

When the anchored bulkhead penetrates to a considerable depth so that the lower end of the wall is perfectly fixed and the wall acts as a vertical propped cantilever, the wall is said to be of fixed earth support type. Figure 21.12 (*a*) shows the deflected shape (called the elastic line). The point g is the point of fixidity, below which the sheet pile remains vertically straight. Point j is the point of contraflexure located at a distance x below the dredge line. For reasons of simplicity, the lower part of the pressure diagram, *egcf is* replaced by a force R_b acting at g. The fixed-earth-support problem can be solved by two methods: (1) elastic line method and (2) equivalent beam method; In the *elastic line method,* the distance Y is first estimated, from which R_b is found by statics. Then the bending moment diagram is computed by any other method, and the tangential deviation of g is computed with respect to point t at the anchor rod level. The distance Y is then revised till the computed tangential deviation is zero. The *equivalent beam method* is described below.

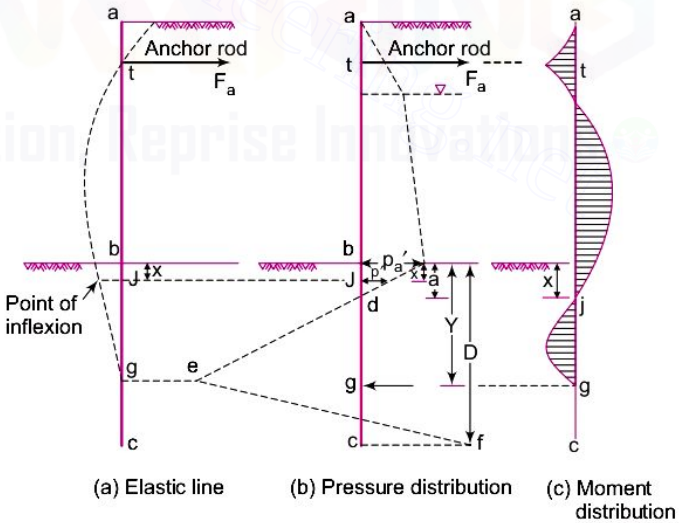

Fig. 21.12 Fixed earth method

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Equivalent beam method. The equivalent beam method was developed by *H.* Blum (1931). In this method, the pressure diagram [Fig. 21.14 (*a*)] is the same as in the elastic line method, but the point of contraflexure, located at x below the dredge line, is considered as a function of ϕ, and may be taken from Fig. 12.13 plotted for $K_p = \dfrac{2}{K_a}$ (Verdegen and Rolsin). Blum's equivalent beam method consists in assuming a hinge at the point of contraflexure (point *j*), where the bending moment is zero. The part of the bulkhead above the hinge can then be treated as a separate freely supported beam with over-hanging end. The beam reaction R_b', acting at the hinge (*i.e.*, at point of contraflexure) can be evaluated by taking the sum of the moments of the equivalent beam about the tie rod, and equating it to zero. Further, it is assumed, as in the elastic method, that the distance $gc = 0.2\ Y$ or $D = 1.2\ Y$.

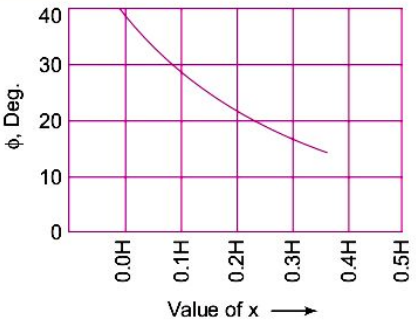

Fig. 21.13 Chart for *X*

The design computations are done in the following *steps*:

(1) Draw the pressure distribution diagram, as shown in Fig. 21.14 (*a*) and compute the value of p_a'. Calculate and K_a', K_p' and K.

(2) Find *a,* the distance of point of zero pressure, below the dredge line by the relation:

$$a = \frac{p_a'}{\gamma' \left(K_p' - K_a'\right)} = \frac{p_a'}{\gamma' K'} \qquad ...(21.28)$$

(3) Find x from Fig. 21.13, and compute the pressure intensity p' at j (point of inflection)

$$p' = \frac{(a - x)\, p_a'}{a} \qquad ...(21.29)$$

(4) Find R_b' by equating ΣM, about tie rod, to zero [consider forces of the upper part only, Fig. 21.14 (*c*)].

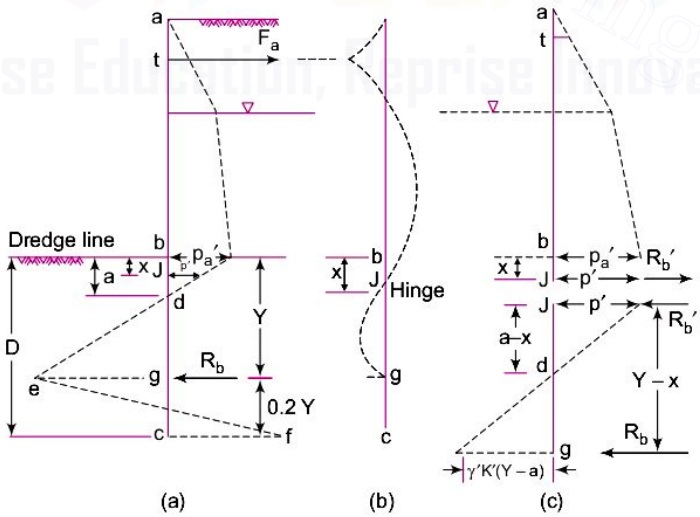

Fig. 21.14 Equivalent beam method

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(5) Find the distance $(Y - x)$ by equating ΣM about point g to zero [consider forces of the lower part only, of Fig. 21.14 (c)]. Solve for Y.

(6) Find $D = 1.2\ Y$.

Example 21.3. *Compute the embedment length and the pull in the anchor rod for the sheet pile structure shown in Fig. 21.15. The soil of the backfill and that below the dredge line is the same, having the following properties:*

$$\phi = \phi' = 30°\ ;\ c = 0;\ \gamma_{sat} = 20\ kN/m^3;\ \gamma = 18\ kN/m^3$$

Use free earth support method.

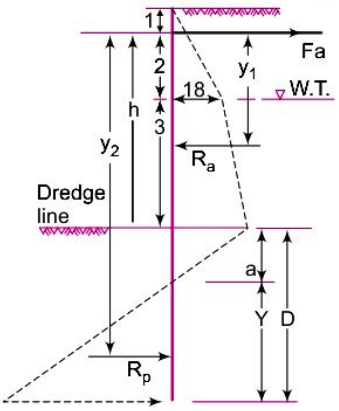

Fig. 21.15

Solution.

$$K_a = K_a' = \frac{1}{3};\ K_p = K_p' = 3 \text{ and}$$

$$K' = 3 - \frac{1}{3} = 2.67$$

The earth pressure diagram is shown in Fig. 21.15, with all values marked.

$$p_a' = \left(\frac{1}{3} \times 3 \times 18\right) + \frac{1}{3} \times 3\,(20 - 9.81)$$

$$= 18 + 10.19 = 28.19 \text{ kN/m}^2$$

$$a = \frac{p_a'}{\gamma' K} \text{ (Eq. 21.16)} = \frac{28.19}{10.19 \times 2.67} = 1.04 \text{ m}$$

$$R_a = \left(\frac{1}{2} \times 3 \times 18\right) + (18 \times 3) + \left(\frac{1}{2} \times 3 \times 10.19\right) + \left(\frac{1}{2} \times 28.19 \times 1.04\right)$$

$$= 27 + 54 + 15.3 + 14.7 = 111 \text{ kN}$$

$$y_1 = \frac{(27 \times 1) + (54 \times 3.5) + (15.3 \times 4) + (14.7 \times 5.35)}{111} = 3.2 \text{ m}$$

The depth Y is calculated from Eq. 21.20:

$$Y^3\left(\frac{\gamma' K'}{3}\right) + Y^2\left(\frac{\gamma' K'}{2}\right)(h + a) - R_a\, y_1 = 0$$

or

$$Y^3\left(\frac{10.19 \times 2.67}{3}\right) + Y^2\left(\frac{10.19 \times 2.67}{2}\right)(5 + 1.09) - (111 \times 3.2) = 0$$

or $\quad 9.07\ Y^3 + 82.3\ Y^2 - 355.2 = 0$

$$Y^3 + 9.06\ Y^2 = 39.2$$

By trial and error solution, $Y = 1.9$ m

$\therefore \quad D = Y + a = 1.9 + 1.04 = 2.94$ m

Providing 30% extra embedment,

$$D = 2.94 \times 1.3 = 3.82 \text{ m}$$

Provide $D = 4$ m, $\quad R_p = \frac{1}{2} Y^2\, \gamma'\, K'$ (Eq. 21.18) $= \frac{1}{2}\ (1.9)^2 \times 10.19 \times 2.67 = 49.1$ kN.

$\therefore \quad F_a = R_a - R_p = 111 - 49.1 = \mathbf{61.9\ kN.}$

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Example 21.4. *Solve example 21.3 if the soil below the dredge line is purely cohesive, having the following properties: $c = 50$ kN/m^2; $\gamma_{sat} = 19$ kN/m^3; $\phi = 0$.*

Solution. As before: $K_a = K_a' = \frac{1}{3}$; $K_p = K_p' = 3$

$K' = 2.67$, $\gamma' = 20 - 9.81 = 10.19$

q (at the dredge line) $= (18 \times 3) + (10.19 \times 3) = 84.57$ kN/m^2

$R_a = 111$ kN (as before), $y_1 = 3.2$ m (as before)

The depth D is determined from Eq. 21.24:

$$D^2 + 2Dh - \frac{2y_1 R_a}{4c - q} = 0 \quad \text{or} \quad D^2 + (2 \times 5) D - \frac{2 \times 3.2 \times 111}{(4 \times 50) - 84.57} = 0$$

$$D^2 + 10D - 6.15 = 0 \quad \text{or} \quad D = 0.58 \text{ m}$$

Increasing D by 30%, $D = 1.3 \times 0.58 = 0.76$

Hence provide $D = 0.8$ m

$R_p = (4c - q) D = (4 \times 50 - 84.57)\, 0.58 = 66.95$

$\therefore$ $F_a = R_a - R_p = 111 - 66.95 = \mathbf{44.05\ kN}$

Example 21.5. *Solve example 21.3 by the equivalent beam method.*

Solution. Refer Fig. 21.13. From example 21.3, we have

$p_a' = 28.19$ kN/m^2

$$a = \frac{p_a'}{\gamma' K} = \frac{28.19}{10.19 \times 2.67} = 1.04 \text{ m}$$

For $\phi = 30°$, $x = 0.08$, H $= 0.08 \times 6 = 0.48$ m

$$p' = \frac{(a - x)\, p_a'}{a} = \frac{(1.04 - 0.48)\, 28.19}{1.04} = 15.2 \text{ kN/m}^2$$

Figure 21.16 shows the pressure intensities marked for the upper portion *aj* and lower portion *jg*, as free body diagrams.

Considering beam *aj* as a free body and putting

$\Sigma M = 0$ at the anchor rod, we get

$$\left(\frac{1}{2} \times 18 \times 3 \times 1\right) + (18 \times 3 \times 3.50)$$

$$+ \left(\frac{1}{2} \times 10.19 \times 3 \times 4\right) + (15.2 \times 0.48 \times 5.24)$$

$$+ \left(\frac{1}{2} \times 12.99 \times 0.48 \times 5.16\right) - (R_b' \times 5.48) = 0$$

$\therefore$ $R_b' = 60.5$ kN

In order to get an equation for Y, take moments of the forces on the lower portion *jg*, about *g*.

Let $Y - a = A$

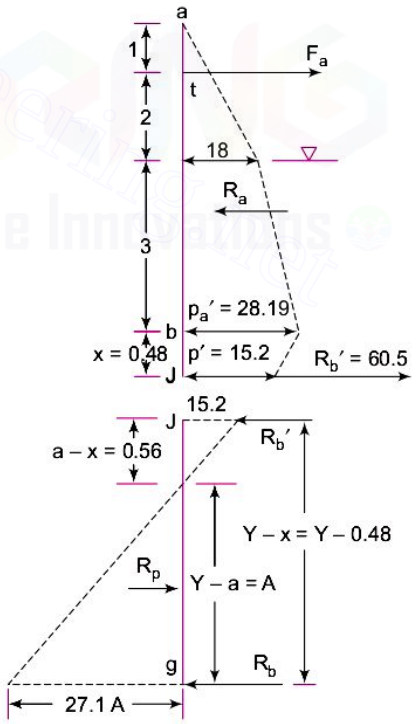

Fig. 21.16

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The slope of the pressure diagram $= \frac{p_{a'}}{a} = \frac{28.19}{1.04} = 27.1$

$$R_b' (A + 0.56) + (\frac{1}{2} \times 15.2 \times 0.56)(A + 0.37) - R_p \left(\frac{A}{3}\right) = 0 \quad ...(1)$$

where $R_b' = 60.5$ kN; $R_P = \frac{1}{2}(27.1\, A)\, A = 13.55\, A^2$

Substituting in (1), we get $60.5\,(A + 0.56) + 4.256\,(A + 0.37) - \frac{13.55\, A^3}{3} = 0$

Simplifying, $A^3 - 14.34\, A = 7.85$ By trial and error, we get $A = 4.03$ m

$\therefore \quad Y = A + a = 4.03 + 1.04 = 5.07$ m

$\therefore \quad D = 1.2\, Y = 6.08$ m

$$R_a = (\frac{1}{2} \times 18 \times 3) + (18 \times 3) + (\frac{1}{2} \times 10.19 \times 3) + (15.2 \times 0.48) + (\frac{1}{2} \times 12.99 \times 0.48) = 106.7 \text{ kN}$$

$\therefore \quad F_a = R_a - R_b' = 106.7 - 60.5 =$ **46.2 kN**

Example 21.6. *An anchored bulkhead 5.1 m high retains sand on both sides. The anchor rods are 1 m below the top and depth of embedment is 1.5 m. Determine the factor of safety against failure. Angle of internal friction of sand is 30° and the sand is wet throughout having a unit weight of 16 kN/m³. Comment if the depth of embedment is adequate. If not, obtain the depth of embedment using a factor of safety of 2.*

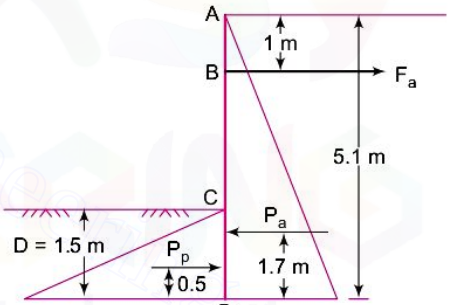

Fig. 21.17

Solution. $K_a = \frac{1 - \sin 30°}{1 + \sin 30°} = \frac{1}{3}$; $K_p = 3$

$$P_a = \frac{1}{2} \times \frac{1}{3} \times 16 (5.1)^2 = 69.36 \text{ kN/m}$$

$$P_p = \frac{1}{2} \times 3 \times 16 (1.5)^2 = 54 \text{ kN/m}$$

Taking moments about B, we get

Disturbing moment, $M_D = P_a \left(\frac{2}{3} \times 5.1 - 1\right) = 69.36 \left(\frac{2}{3} \times 5.1 - 1\right) = 166.464$ kN – m/m

Restoring moment, $M_R = P_p \left(5.1 - \frac{D}{3} - 1\right) = 54 \left(5.1 - \frac{1.5}{3} - 1\right) = 194.4$ kN – m/m

$\therefore$ Factor of safety, $F = \frac{M_R}{M_D} = \frac{194.4}{166.464} = 1.17$

Thus, factor of safety is 1.17 which is *inadequate*. The desired factor of safety is 2. Let us find the value of D for $F = 2$.

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Then $P_a = \frac{1}{2} \times \frac{1}{3} \times 16(5.1)^2 = 69.36$ and $M_D = 166.464$ kN-m/m, as before.

However, $P_p = \frac{1}{2} \times 3 \times 16\, D^2 = 24\, D^2$ kN/m

and $M_R = P_p\left(5.1 - 1 - \frac{D}{3}\right) = 24\, D^2\left(4.1 - \frac{D}{3}\right)$

$\therefore$ $F = 2 = \frac{M_R}{M_D} = \frac{24\, D^2\left(4.1 - \frac{D}{3}\right)}{166.464}$

or $D^2\left(4.1 - \frac{D}{3}\right) = 13.872$ or $D^2(12.3 - D) = 41.616$

Solving the above cubic equation by trial and error, we get $D \approx 2.02$ m.

Providing additional 20% embedment for scour etc.; $D = 1.2 \times 2.02 \approx$ **2.4 m**

21.7 DESIGN OF ANCHORS FOR BULKHEAD

The tie rods used in anchored bulkheads are anchored at the far end in beams, plates or concrete anchor block of depth b and length l the values of which depend upon the depth D of embedment and spacing (s) of the tie rods. If DE is Rankine's failure plane and EF is a line drawn normal to it then the anchor block should be placed well beyond the live EF, as shown in Fig. 21.18. Also if the depth b is chosen more than $h_a/2$, the anchors can be assumed to develop the active and passive thrusts over the full depth ha.

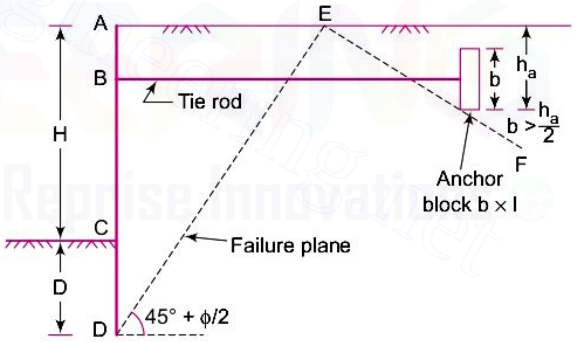

Fig. 21.18

Let F_a be the force in tie rod. If s is the spacing of the tie rods, total force in each tie rod $= F_a \,.\, s$ ...(1)

The above force is resisted by anchor block of length l. If tie anchor block is continuous, then $l = s$ for each tie rod. The net passive and active forces developing at the anchor on a full depth h_a and l is

$$= \frac{1}{2}\left(K_p - K_a\right)\gamma\, h_a^2\,.\,l \qquad \ldots(2)$$

Using a factor of safety F, we have

$$\frac{\text{Resisting force}}{\text{Disturbing force}} = \text{Factor of safety, } F$$

or $$\frac{\frac{1}{2}\left(K_p - K_a\right)\gamma\, h_a^2\, l}{F_a\,.\,s} = F \quad \text{or} \quad F_a\,.\,s = \frac{\gamma\, h_a^2\, l}{2F}\left(K_p - K_a\right) \qquad \ldots(21.30)$$

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where s = spacing of tie rods and l = length of anchor. For continuous anchor, $l = s$. Design force in the rod = $\left(P_a - \frac{P_p}{F}\right)$, with a factor of safety F.

Example 21.7. *For an anchored sheet pile shown in Fig. 21.19., determine the minimum penetration depth of pile (D) and force in tie (T) under free earth support condition. The properties of the soil behind and in front of the wall are: $\gamma = 18$ kN/m^3; $\varphi = 30°$; $c = 0$. (b) Also, determine the safe depth of embedment (D) and tension in each tie rod. Also design a continuous anchor to support the tie rods, if the tie rods are spaced 2 m centres.*

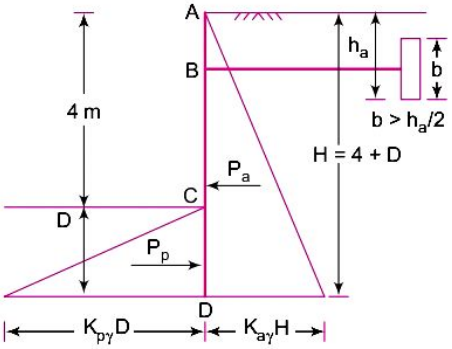

Fig. 21.19

Solution. $K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$; $K_p = 3$; $\gamma = 18$ kN/m^3

$$P_a = \frac{1}{2} \times \frac{1}{3} \times 18 (4+D)^2 = 3 (4+D)^2 \text{ kN/m}$$

$$P_p = \frac{1}{2} \times 3 \times 18\, D^2 = 27\, D^2 \text{ kN/m}.$$ Taking moments about B,

$$M_D = M_a = 3 (4+D)^2 \left[\frac{2}{3}(4+D) - 1\right] = (4+D)^2 (2D+5)$$

$$M_R = M_p = 27\, D^2 \left(3 + \frac{2}{3} D\right) = 9\, D^2 (9 + 2D)$$

Equating the two, we get $(4+D)^2 (2D+5) = 9\, D^2 (2D+9)$

Solve the above equation by trial and error as under

Trial No.	*D*	*LHS*	*RHS*
1	1.4	227.4	208
2	1.45	234.6	225.2
3	1.5	242	243

Hence $D \approx 1.5$ m

Force in tie rod $= T = P_a - P_p = 3 (4 + 1.5)^2 - 27 (1.5)^2 = 30$ kN/m

Since the tie rods are spaced 2 m centres, actual force in each tie rod = $2 \times 30 = 60$ kN

(a) Safe design: In the above analysis we have $M_D = M_R$ *i.e.*, $F = 1$

However, for safe design $F = 2$, corresponding to which $M_R = 2M_D$

$\therefore$ $9\, D^2 (9 + 2D) = 2 (4+D)^2 (2D+5)$

or $4.5\, D^2 (9 + 2D) = (4+D)^2 (2D+5)$

Solving this by trial and error, we get $D \approx$ **2.66 m**

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Corresponding to this depth of embedment, we get $P_a = 3\,(4 + 2.66)^2 = 133.1$ kN/m

$$P_p = \frac{27}{2}\,(2.66)^2 = 95.5 \text{ kN/m with a factor of safety of 2.}$$

$\therefore$ Force in tie rod $= 133.1 - 95.5 = 37.6$ kN/m

Since the tie rods are spaced @ 2 m c/c.

Force in each tie rod $= 2 \times 37.6 =$ **75.2 kN**. For continuous anchor, $s = L$.

Hence from Eq. 21.30, $T = \dfrac{\gamma h_a^2}{2F}\left(K_p - K_a\right)$, where $T = 37.6$ kN/m

$$\therefore \quad 37.6 = \frac{18\left(h_a\right)^2}{2 \times 2}\left(3 - \frac{1}{3}\right) \text{ From which we get } h_a = 1.77 \text{ m}$$

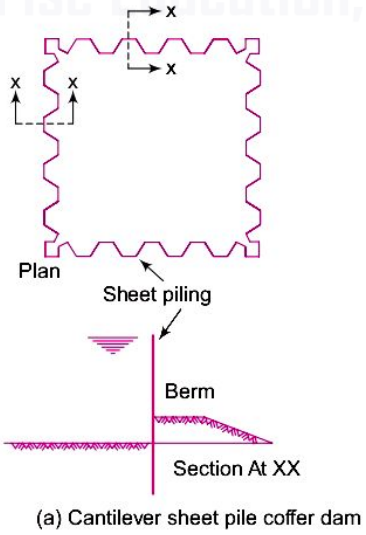
$$\therefore \quad b_{\min} = \frac{h_a}{2} = \frac{1.77}{2} = 0.885 \text{ m}$$

Hence use continuous anchor of height $b =$ **0.9 m**. Also, from scour point of view, keep $D = 1.2 \times 1.5 =$ **1.8 m.**

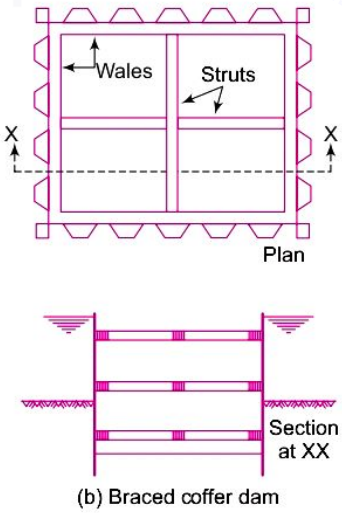
21.8 COFFER DAMS

A coffer dam is a temporary enclosure in a river, lake, etc., built round a working area for the purpose of excluding water during construction. During the construction period, a certain amount of pumping is constantly needed because some water will leak through the coffer dam and the foundation. A coffer dam may be made of earth material, timber or steel sheet piling, or a combination thereof. The following are some of the common types of coffer dams:

1. Cantilever sheet pile coffer dam
2. Braced coffer dam
3. Embankment protected coffer dam
4. Double wall coffer dam
5. Cellular coffer dam



(a) Cantilever sheet pile coffer dam



(b) Braced coffer dam

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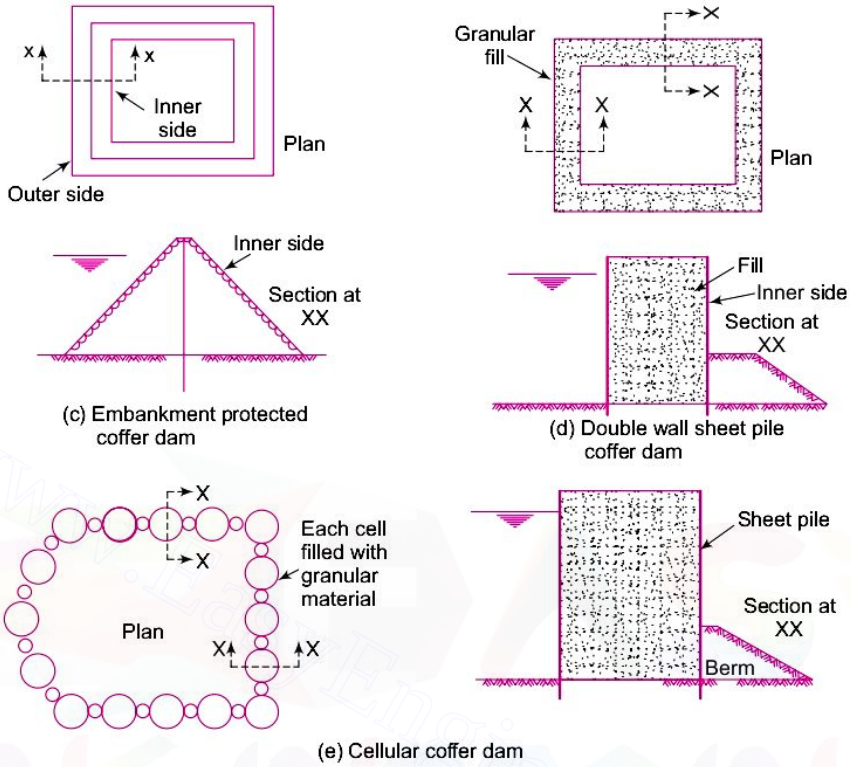

Fig. 21.20 Common types of coffer dams

These have been shown in Fig. 21.20.

Cantilever sheet pile coffer dams are suitable for small heights, since these are susceptible to large leakage and flood damage. Braced coffer dams are economical for small to moderate height. For earth embankment type coffer dams, there is no height limitations, but since they occupy large base area, they are adopted only when the area to be excavated is very large. Double wall coffer dams are suitable for moderate and large heights. A cellular coffer dam consists of a series of adjoining cells of circular or other curved shape, made of sheet piling. Each cell is a huge vertical cylinder, 9–12 m more in lateral dimension, and is filled with rock, gravel and sand.

21.9 SOIL PRESSURE ON BRACED COFFER DAMS OR STRUTTED EXCAVATION

Figure 21.21 (*a*) shows a braced coffer dam or a strutted excavation. Figure 21.21 (*b*) shows the deflection line of sheeting as well as the theoretical active failure zone, and the corresponding resulting pressure distribution (in sand) is shown in Fig. 21.21 (*c*). Figure 21.21 (*d*), (*e*) and (*f*) show the empirical diagrams for dense sand, loose and clay, proposed by Terzaghi and Peck, based on measurements on actual job sites.

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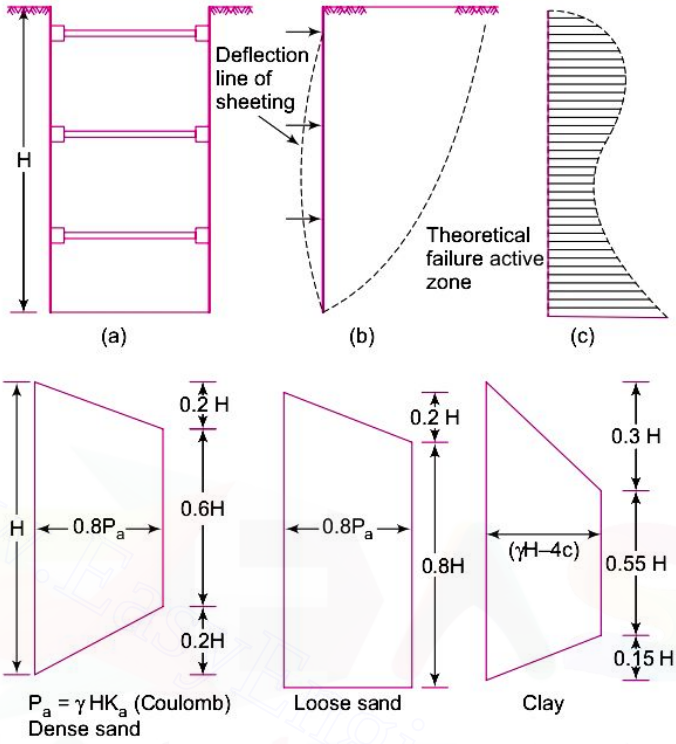

Fig. 21.21 Soil pressure on strutted excavations

21.10 QUESTIONS BASED ON COMPETITIVE EXAMINATIONS

Example 21.8. *An anchored sheet pile is to support a mass of cohesionless soil up to a height of 6 m above ground level with horizontal anchor ties spaced at 1 m intervals and located at 1.0 m below ground surface. If the unit weight of the soil is 21 kN/m^3 and its angle of internal friction is 30°, determine the minimum depth of embedment of the sheet pile for stability.*

(Civil Services Exam. 1992)

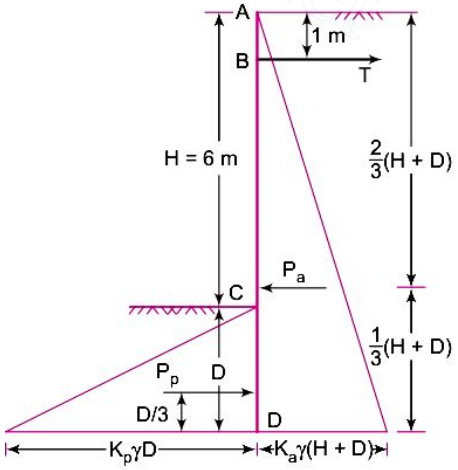

Fig. 21.22

Solution. Let D be the depth of embedment.

$$K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3}$$

$$K_p = 3$$

$$\therefore \quad P_a = \frac{1}{2} K_a \gamma (H+D)^2$$

$$= \frac{1}{2} \times \frac{1}{3} \times 21 (6+D)^2$$

$$= 3.5\,(6+D)^2 \qquad ...(1)$$

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$$P_p = \frac{1}{2} K_p \gamma D^2$$

$$= \frac{1}{2} \times 3 \times 21 D^2$$

$$= 31.5 D^2 \qquad ...(2)$$

Taking moments about the tie rod level B, we get

$$31.5 D^2 \left(5 + \frac{2}{3} D\right) = 3.5 (6 + D)^2 \left[\frac{2}{3}(6 + D) - 1\right]$$

The above, cubic equation can be solved by trial and error as under:

Trial No.	*D*	*LHS*	*RHS*
1	2	798	970.7
2	3	1984.5	1417.5
3	2.5	1312.5	1180
4	2.3	1089	1093

Hence **D ≈ 2.3 m.**

Example 21.9. *An anchored sheet pile wall is to support a mass of cohesionless soil, up to a height of 5 metres above the ground level with the horizontal surface. The anchor ties are 1 m below the top at a horizontal spacing of 1 m. Density of the soil is 24 kN/m^3 and φ = 28 deg. Find the minimum depth of anchor for the pile neglecting the friction on the surface of the pile.* **(Civil Services Exam. 1993)**

Solution. Let D be the depth of embedment:

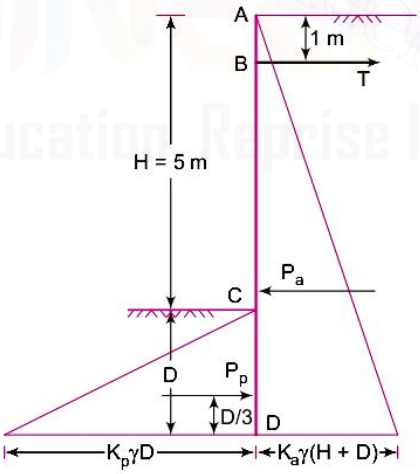

Fig. 21.23

$$K_a = \frac{1 - \sin 28°}{1 + \sin 28°} = 0.361$$

$$K_p = 1/K_a = 1/0.361 = 2.77$$

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$$P_a = \frac{1}{2} \times 0.361 \times 24\,(5+D)^2 = 4.332\,(5+D)^2$$

$$P_p = \frac{1}{2} \times 2.77 \times 24\,(D)^2 = 33.24\,D^2$$

Taking moments about B, we get

$$33.24\,D^2\left(4+\frac{2}{3}D\right) = 4.332\,(5+D)^2\left[\frac{2}{3}(5+D)-1\right]$$

or

$$7.673\,D^2\left(4+\frac{2}{3}D\right) = (5+D)^2\left(\frac{2}{3}D+2.33\right)$$

The above equation is solved by trial and error as under

Trial No.	*D*	*LHS*	*RHS*
1	1.5	86.3	140.7
2	2.0	163.7	179.5
3	2.1	182.7	188.0
4	2.2	203.0	196.8

Hence $D \approx$ **2.15 m.**

Example 21.10. *An anchored bulkhead 4.8 m high retains sand on both sides. The anchor rods are 1 m below the top and depth of embedment is 1.44 m. Determine the factor of safety against failure. Angle of internal friction of sand is 35° and the sand is submerged throughout with a unit weight of 10 kN/m^3. Comment if the depth of embedment is adequate. If not, obtain the depth of embedment which render the design adequate.* **(Civil Services Exam. 1996)**

Solution.

Given: $D = 1.44\,\text{m}$; $h = 4.8\,\text{m}$; $\varphi = 35°$; $\gamma' = 10\,\text{kN/m}^3$

$$H = h - D = 4.8 - 1.44 = 3.36\,\text{m}$$

$$K_a = \frac{1-\sin 35°}{1+\sin 35°} = 0.271;$$

$$K_p = 1/K_a = 1/0.271 = 3.69$$

$$P_a = K_a\,\gamma'\,\frac{h^2}{2} = \frac{0.271 \times 10(4.8)^2}{2}$$

$$= 31.22\,\text{kN/m}$$

$$P_p = K_p\,\gamma'\,\frac{D^2}{2} = \frac{3.69 \times 10(1.44)^2}{2} = 38.26\,\text{kN/m}$$

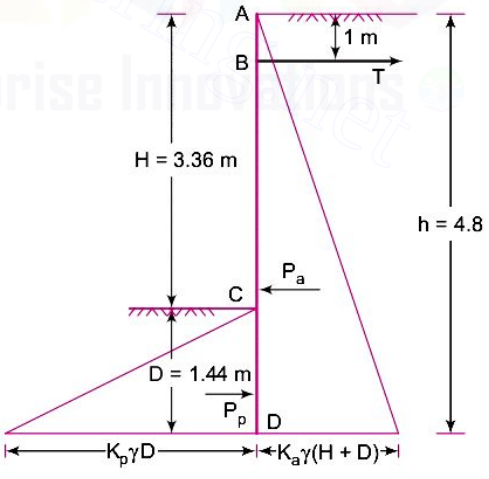

Fig. 21.24

Taking moments about B, we have

Disturbing moment

$$M_D = P_a\left(\frac{2}{3}h-1\right) = 31.22\left(\frac{2}{3}\times 4.8-1\right) = 68.68\ \text{kN-m/m}$$

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Restoring moment

$$M_R = P_p\left(h - \frac{1}{3}D - 1\right) = 38.26\left(4.8 - \frac{1}{3}\times 1.44 - 1\right) = 127.02 \text{ kN – m/m}$$

$\therefore$ Factor of safety $$F = \frac{M_R}{M_D} = \frac{127.02}{68.68} = \mathbf{1.85}$$

Thus, factor of safety is 1.85, which is *inadequate*. The desired factor of safety $F = 2$. Let D be the depth of embedment corresponding to this F.S.

Then $$P_a = K_a \gamma' \frac{h^2}{2} = 31.22 \text{ and } M_D = M_a = 68.68 \text{ as before.}$$

But $$P_p = K_p \gamma' \frac{D^2}{2} = \frac{3.69\times 10}{2} D^2 = 18.45\, D^2 \text{ kN/m}$$

and $$M_R = M_p = P_p\left(4.8 - \frac{1}{3}D - 1\right) = 18.45\, D^2\left(3.8 - \frac{1}{3}D\right)$$

$\therefore$ $$F = 2 = \frac{M_R}{M_D} = \frac{18.45\, D^2\left(3.8 - \frac{1}{3}D\right)}{68.68}$$

or $$D^2(3.8 - \frac{1}{3}D) = 7.445 \quad \text{or} \quad D^2(11.4 - D) = 22.335$$

Solving this by trial and error, we get $D \approx 1.50$ m

Providing additional 20% embedment for scour etc., $D = 1.2 \times 1.5 = \mathbf{1.8\ m}$

Example 21.11(*a*). *For an anchored sheet pile shown in Fig. 21.25 (a), determine the minimum penetration depth of the pipe (d) and forces in the tie (T) under free earth support condition. The properties of soil both behind and in front of the wall are:*

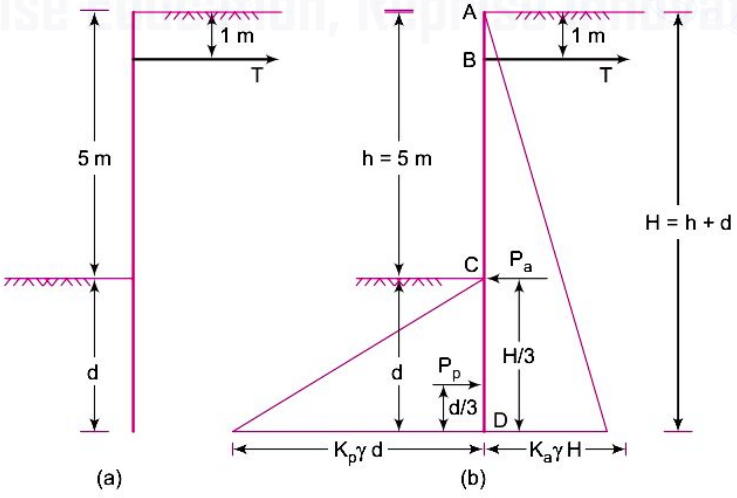

Fig. 21.25

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$$\gamma = 21\ kN/m^3;\ \varphi = 30^\circ;\ c = 0$$

(b) Also determine the safe depth of embedment (d) and tension in each tie rod. Also, design a continuous anchor to support the tie rods, if the tie rods are spaced 2 m centres. **(Civil Services Exam. 1997)**

Solution (a).

$$K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1}{3};\ K_p = 1/K_a = 3.$$

$$P_a = \frac{1}{2} K_a\, \gamma\, H^2 = \frac{1}{2} \times \frac{1}{3} \times 21 (5+d)^2 = 3.5\,(5+d)^2 \text{ kN/m}$$

$$P_p = \frac{1}{2} K_p\, \gamma\, d^2 = \frac{1}{2} \times 3 \times 21 d^2 = 31.5\, d^2 \text{ kN/m}$$

Taking moments about B,

$$M_D = M_a = 3.5(5+d)^2 \left[\frac{2}{3}(5+d) - 1\right] = 1.167\,(5+d)^2\,[2d+7]$$

$$M_R = M_p = 31.5\, d^2 \left(4 + \frac{2}{3} d\right) = 21\, d^2 (6+d)$$

Equating the two, we get $1.167\,(5+d)^2\,(7+2d) = 21d^2\,(6+d)$

Solve the above equation by trial and error as under:

Trial No.	*d*	*LHS*	*RHS*
1	1.6	518	408.5
2	1.7	544.8	467.3
3	1.8	572	530.7
4	1.9	600	598.9

Hence d = **1.9 m.**

$$P_a = 3.5\,(5+1.9)^2 = 166.4 \text{ kN/m and } P_p = 31.5\,(1.9)^2 = 113.72 \text{ kN/m}$$

Force in tie rod = $T = P_a - P_p = 166.64 - 113.72 =$ **52.92 kN/m.**

(*b*) ***Safe Design:*** In the above analysis, we have $M_D = M_R$ *i.e.*, $F = 1$. For safe design, $F = 2$, corresponding to which $M_R = 2\,M_D$

$$\therefore \qquad 21\, d^2\,(6+d) = 2 \times 1.167\,(5+d)^2\,(2d+7)$$

or $$(5+d)^2\,(2\,d+7) = 8.997\, d^2\,(6+d)$$

Solving this by trial and error, we get d = **3.4 m**. Corresponding to this depth of embedment, we get

$$P_a = 3.5\,(5+3.4)^2 = 246.96 \text{ kN/m}$$

$$P_p = \frac{1}{2} \times 31.5\,(3.4)^2 = 182.07 \text{ kN/m, with a factor of safety of 2.}$$

$$\therefore \qquad T = P_a - P_p = 246.96 - 182.07 =$$ **64.89 kN/m**

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Since the tie rods, are spaced @ 2m c/c, force in each tie rod = $2 \times 64.89 =$ **129.78 kN.**

For continuous anchor, $s = L$, $T = \dfrac{\gamma h_a^2}{2F}\left(K_p - K_a\right)$ or $64.89 = \dfrac{21 h_a^2\left(3 - \dfrac{1}{3}\right)}{2 \times 2}$

From which we get $h_a = 2.15$ m

Min. height of anchor is $b_{\min} = \dfrac{h_a}{2} = \dfrac{2.15}{2} \approx 1.08$ m

However, use continuous anchor of height **1.1 m**

Also from scour point of view, keep $d = 1.2 \times 3.4 \approx 4$ m.

Example 21.12. *An anchored sheet pile wall is to retain soil to a height of 5.5 m. The soil including that into which the pile is driven, is cohesion-less soil with $\varphi = 30°$ and $\gamma = 20.8$ kN/m^3 The surface of the retained soil is horizontal and level with the top of the wall. Determine the minimum penetration depth of the pile to achieve the free earth support.* **(Gate Exam. 1997)**

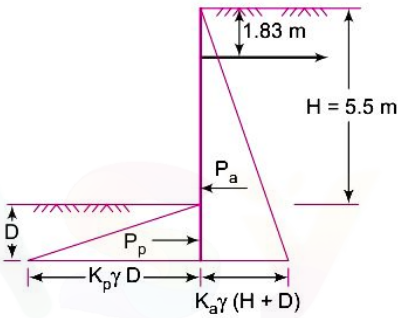

Fig. 21.26

Solution. $K_a = \dfrac{1 - \sin 30°}{1 + \sin 30°} = \dfrac{1}{3}$; $K_p = 3$

$$\therefore \quad P_a = \frac{1}{2} K_a \gamma \left(H + D\right)^2 = \frac{1}{2} \times \frac{1}{3} \times 20.8 \left(5.5 + D\right)^2$$

$$= 3.467\ (5.5 + D)^2$$

$$P_p = \frac{1}{2} K_p \gamma D^2 = \frac{1}{2} \times 3 \times 20.8 \left(D\right)^2 = 31.2\ D^2$$

Taking moments at B, we have

$$31.2\ D^2 \left(5.5 - 1.83 + \frac{2D}{3}\right) = 3.467 \left(5.5 + D\right)^2 \left[\frac{2}{3}\left(5.5 + D\right) - 1.83\right]$$

or $$9\ D^2\ (11.01 + 2\ D) = (5.5 + D)^2 (5.51 + 2\ D)$$

The above cubic equation can be solved by trial and error as under:

Trial No.	*D*	*LHS*	*RHS*
1	1.5 m	283.7	417
2	1.8 m	426.0	485.5
3	2 m	540.4	535

Hence $D \approx 2$ **m.**

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PROBLEMS

1. An anchored bulkhead is to be designed to retain a granular backfill of 9 m height above the dredge line. The anchor rod is to be provided at a depth of 1 m below the top level of the fill. Assuming that the water table is 2 m below the top of the fill and that the soil of the fill as well as that below the dredge line have the same soil properties ($c = 0$; $\phi = 33°$; $\gamma = 17$ kN/m^3 and $\gamma_{sat} = 20$ kN/m^3) compute the depth of embedment and the tensile force in the anchor rod of the bulkhead. Use the free earth method, and increase computed depth embedment by 40%. [**Ans.** 4.6 m; 87 kN]
2. Solve problem 1, if the soil below the dredge line is clay having $c = 70$ kN/m^2, $\phi = 0$ and $\gamma_{sat} = 19$ kN/m^3. [**Ans.** 0.7 m; 64 kN]
3. Solve problem 2, by applying safety factor of 2 to the cohesion of the soil below the dredge line, instead of increasing the embedment depth by 40%. [**Ans.** 2.2 m; 72.8 kN]
4. Solve problem 1, by the simplified equivalent beam method. [**Ans.** 7.2 m; 66.8 kN]

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CHAPTER 22

Shafts, Tunnels and Conduits

22.1 STRESS DISTRIBUTION IN THE VICINITY OF SHAFTS

A soil engineer is commonly encountered with the problem of stress distribution in the vicinity of vertical or inclined shafts in an elastic semi-infinite solid with a horizontal or inclined surface. Westergaard (1940) solved the problem by theory of elasticity, using a suitable stress function. The method given here is that suggested by Biot, (1935) using the method of superposition (Terzaghi, 1943).

Figure 22.1 shows the section of vertical shaft of radius r_0. Let z be the vertical co-ordinate of any point measured downward from the horizontal surface, and r be its horizontal, radial distance from the Z-axis which is directed along the centre line of the shaft. Let σ_z, σ_r and σ_θ be the vertical stress, horizontal radial stress and horizontal circumferential stress acting at any point P (r, z) ; all the three stresses are normal stress. Let τ_{rz} be the shearing stress in the direction r and z.

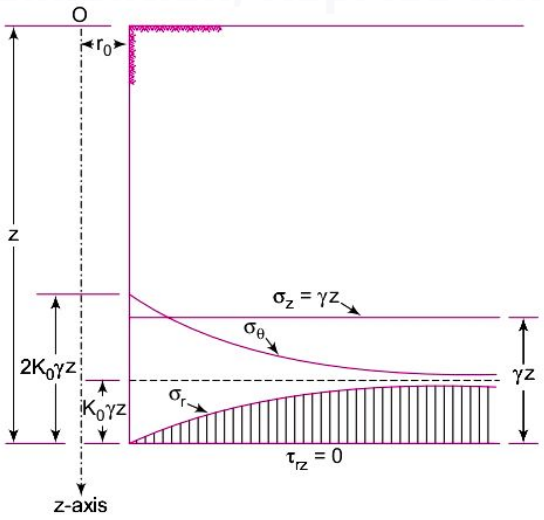

Fig. 22.1 Stress distribution around shaft elastic equilibrium

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The stress components at any point $P(r, z)$ before the excavation of the shaft are given by

$$(\sigma_z)_1 = \gamma \cdot z \qquad ...(22.1\ (a))$$

$$(\sigma_r)_1 = K_0\, \gamma\, z \qquad ...(22.1\ (b))$$

$$(\sigma_\theta)_1 = K_0\, \gamma\, z \qquad ...(22.1\ (c))$$

and $$(\tau_{rz})_1 = 0 \qquad ...(22.1\ (d))$$

where K_0 = coefficient of earth pressure at rest.

Since the shearing stresses on cylindrical surface are zero, the material located within the boundaries of the future shaft can be replaced by liquid of unit weight, $\rho = K_0\, \gamma$, without changing the state of stress in the surrounding material. This is so because the horizontal fluid pressure p due to the equivalent liquid is given by $p = \rho z = K_0 \cdot \gamma \cdot z$, which is the same as $(\sigma_r)_1$ or $(\sigma_\theta)_1$ given by Eq. 22.1(b). Therefore, stress components (Eq. 22.1) which act at any point in the material before the excavation of the shaft can be resolved into two parts: (i) stresses due to weight of material, and (ii) stresses due to pressure exerted by the equivalent liquid, and the sum of the stresses due to both these two will give stress components of Equation 22.1. Now the stresses due to equivalent fluid pressure can be found by using the Lame's formulae for the state of stress in thickwalled cylinders subjected to internal pressure:

$$(\sigma_z)_2 = 0 \qquad ...(22.2\ (a))$$

$$(\sigma_r)_2 = p\,\frac{r_0^2}{r^2} = K_0\, \gamma \cdot z\,\frac{r_0^2}{r^2} \qquad ...(22.2\ (b))$$

$$(\sigma_\theta)_2 = -\,p\,\frac{r_0^2}{r^2} = -\,K_0 \cdot \gamma \cdot z\,\frac{r_0^2}{r^2} \qquad ...(22.2\ (c))$$

and $$(\tau_{rz})_2 = 0 \qquad ...(22.2\ (d))$$

where suffix 2 denotes the stress components at the points $P(r, z)$ due to the equivalent fluid filled in the shaft.

After the shaft has been excavated, the shearing stresses along the walls of the shaft are equal to zero, and the radial normal stresses are also equal to zero. Therefore, the effect of excavating the shaft on the stresses in the surrounding material is identical with the effect of pumping the heavy liquid out of a cylindrical hole whose dimensions are identical with those of the shaft (Biot, 1935). Hence the stress components at any point, after the excavation of the shaft, will be equal to the difference between the initial stresses (Eq. 22.1) and those given in Eq. 22.2.

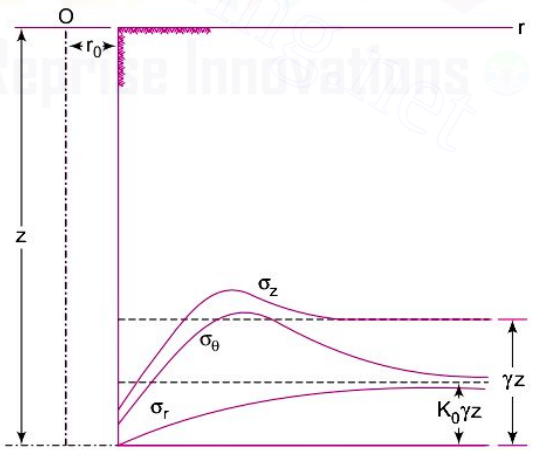

Fig. 22.2 Stress distribution around the shaft plastic equilibrium

$$\sigma_z = (\sigma_z)_1 - (\sigma_z)_2 = \gamma_z \qquad ...(22.3\ (a))$$

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$$\sigma_r = (\sigma_r)_1 - (\sigma_r)_2 = K_0 \,.\, \gamma \,.\, z \left(1 - \frac{r_0^2}{r^2}\right) \qquad ...(22.3\,(b))$$

$$\sigma_\theta = (\sigma_\theta)_1 - (\sigma_\theta)_2 = K_0 \,.\, \gamma \,.\, z \left(1 + \frac{r_0^2}{r^2}\right) \qquad ...(22.3\,(c))$$

and $$\tau_{rz} = 0 \qquad ...(22.3\,(d))$$

Figure 22.1 shows the variations of the stress components (Eq. 22.3) on a horizontal section around the shaft. At $r = r_0$, we have $\sigma_r = 0$ and $\sigma_\theta = 2\,K_0 \,.\, \gamma \,.\, z$. At $r \to \infty$, $\sigma_r \to K_0 \,.\, \gamma \,.\, z$ and $\sigma_\theta \to K_0\,\gamma z$. Thus, at infinite radial distance, the stress components of Eq. 22.3 reduce to those given by Eq. 21.1.

The above stress components have been derived on the assumption that the material is elastic. However, the high value of σ_θ $(= 2K_0 \,.\, \gamma \,.\, z)$ in the vicinity of the walls of a shaft may exceed the compressive strength of the soil, resulting in a plastic flow of the soil which continues until a state of plastic equilibrium or of rupture is reached. Figure 22.2 shows the stress distribution if the surrounding shaft is in a state of plastic equilibrium.

22.2 STRESS DISTRIBUTION AROUND TUNNELS

The principle of superposition discussed in the previous article can also be used in determining the stress distribution around a tunnel. Let r_0 be the radius of the tunnel and, D be the depth of its centre line, measured vertically below the horizontal ground surface. Let σ_r and σ_θ be the radial stress and circumferential stress at any point P at a radial distance r from the centre line of the tunnel. Before the excavation of the tunnel, the stress components at the point P, *assuming* $K_0 =$ l, are given by equation 22.1 :

$$(\sigma_r)_1 = K_0 \,.\, \gamma \,.\, z = \gamma z \qquad ...(22.4\,(a))$$

$$(\sigma_\theta)_1 = K_0 \,.\, \gamma \,.\, z = \gamma z \qquad ...(22.4\,(b))$$

Substituting $z = D$ (*i.e.*, point P situated on a horizontal plane passing through the centre line of the tunnel), we get :

$$(\sigma_r)_1 = \gamma D \qquad ...(22.5\,(a))$$

$$(\sigma_\theta)_1 = \gamma D \qquad ...(22.5\,(b))$$

Let us imagine that the future tunnel is filled by a liquid, of unit weight γ, with a pressure such that the liquid would rise in a piezometric tube to the level of the ground surface. The fluid pressure p on the walls of the tunnel will be equal to γz. If $z = D$ is large in comparison to the diameter $2\,r_0$ of the tunnel, the fluid pressure is approximately equal to γD. The stress components due to the fluid pressure can be computed by Lame's equation (Eq. 22.2) using $z = D$ and $K_0 = 1$

$$(\sigma_r)_2 = \gamma D \frac{r_0^2}{r^2} \qquad ...(22.6\,(a))$$

and $$(\sigma_\theta)_2 = -\gamma D \frac{r_0^2}{r^2} \qquad ...(22.6\,(b))$$

Hence the stresses after excavation of the tunnel are given by:

$$\sigma_r = (\sigma_r)_1 - (\sigma_r)_2 = \gamma D \left(1 - \frac{r_0^2}{r^2}\right) \qquad ...(22.7\,(a))$$

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and $$\sigma_\theta = (\sigma_\theta)_1 - (\sigma_\theta)_2 = \gamma D \left(1 + \frac{r_0^2}{r^2}\right) \quad \text{...(22.7 (}b\text{))}$$

Figure 22.3 (*a*) shows the stress distribution around the tunnel, situated at a great depth below ground surface, in an elastic material. At $r = r_0$, σ_r is zero and $\sigma_\theta = 2D\gamma$ *i.e.*, equal to twice the overburden pressure. At $z \to \infty$ both σ_r and σ_θ tend to approach a value equal to γD. Figure 22.3 (*b*) shows the stress distribution in the state of plastic equilibrium. The above analysis has been made on two important assumptions : (*i*) $K_0 = 1$ and (*ii*) the depth D is very great so that the equivalent liquid pressure γD is uniform in the tunnel. However, if the tunnel is located close to the surface, the distribution of the hydrostatic presssure exerted by the liquid on the walls of the tunnel is not even approximately uniform. The problem of computing the stress in the vicinity of a cylindrical tunnel corresponding to the state of elastic equilibrium has been solved more rigorously by Mindlin(1939), using the following values of K_0:

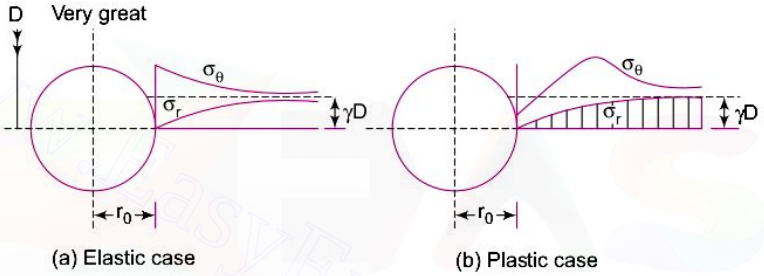

Fig. 22.3 Stress distribution around tunnel situated at a great depth below ground surface

(*i*) $K_0 = 1$; (*ii*) $K_0 = \dfrac{\mu}{1-\mu}$ and (*iii*) $K_0 = 0$

However, the final equations are so involved that they cannot be applied to practical problems until they have been condensed into tables or graphs.

22.3 ARCHING IN SOILS

Arching is defined as the phenomenon of the transfer of pressure from a yielding part of a soil mass to adjoining less yielding or restrained part of the mass. When one part of the support of a soil mass yield while the adjoining parts remain stationary, the yielding part has the tendency to move out of its original position between the non-yielding parts. This action is resisted by the shearing strength of the zone of contact between the yielding and non-yielding parts, and hence the pressure on the yielding part is reduced while the pressure on the non-yielding part is increased. In other words, the soil is said to *arch over* the yielding part or the support and redistribution of pressure takes place.

Figure 22.4 shows the yielding of a long narrow section of the base of a layer of soil. The actual failure surface or slip surfaces are curved one. However, the analysis that follows is based on the assumption of a vertical slip surface (Terzaghi, 1945).

Consider the equilibrium of a yielding slice of soil of width B, and of unit length normal to the plane of the figure. The vertical forces on the top and bottom of the strip are $B.\ \sigma_z \downarrow$ and $B\ .\ [\sigma_z + d(\sigma_z)] \uparrow$, in addition to the weight $dW\ (= \gamma B dz)$ of the strip acting vertically downward. If τ_f is the intensity of shearing resistance along the vertical strip plane, the total resistance, acting vertically upwards, is equal to $2dz\ .\ \tau_f$.

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Hence equating the total downward force to the total upward force at the critical equilibrium, we get

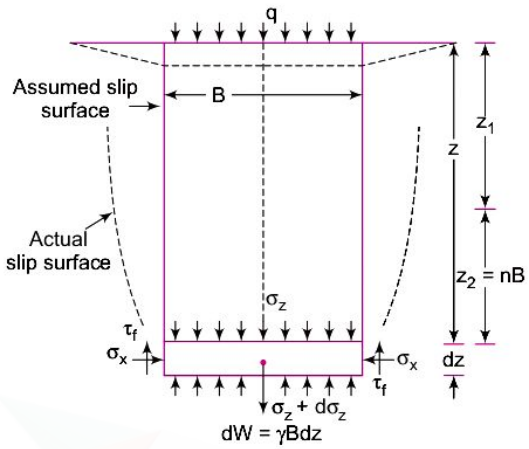

Fig. 22.4 Yielding of horizontal strip

$$B\sigma_z + dW = B(\sigma_z + d\sigma_z) + 2dz \,.\, \tau_f \qquad ...(22.8)$$

Now $dW = \gamma B dz$; $\tau_f = c + \sigma_x \tan\phi$; σ_x = lateral pressure = $K\sigma_z$

when K = empirical constant.

Substituting these values in Eq. 22.8, we get

$$B \,.\, \sigma_z + \gamma B dz = B\sigma_z + B \,.\, d\sigma_z + 2dz(c + K \,.\, \sigma_z \tan\phi)$$

$$\therefore \quad \frac{d\sigma_z}{dz} = \gamma - \frac{2c}{B} - \frac{2K\tan\phi}{B}\sigma_z \qquad ...(22.9)$$

Also, $\sigma_Z = q$ when $z = 0$ ...(22.9 (*a*))

The solution of the above equations gives the following result:

$$\sigma_z = \frac{B\left(\gamma - \frac{2C}{B}\right)}{2K\tan\phi}\left[1 - e^{\frac{-2K\,.\,z}{B}\tan\phi}\right] + q_e^{\frac{-2K\,.\,z}{B}\tan\phi} \qquad ...(22.10)$$

When $c = 0$ and $q = 0$, $\sigma_z = \dfrac{B\gamma}{2K\tan\phi}\left[1 - e^{\frac{-2K\,.\,z}{B}\tan\phi}\right]$ ...(22.11)

At, $z = \infty$, the above equation reduces to : $\sigma_z = \dfrac{B\gamma}{2K\tan\phi}$ ...(22.12)

Terzaghi's experiments show that the value of K varies from about unity immediately above the centre of strip to a maximum of about 1.5 at a height of approximately B above the strip. His experiments on dry sand further indicate that the shearing resistance of sand of height equal to 2.5 B above the yielding strip is mobilised, and not the full height z. If z_1 = depth of sand in which there are no shearing stresses, and z_2 (= nB = 2.5 B) is the depth, above the yielding strip in which shear resistance develops along the slip surfaces, the vertical pressure σ_z on the yielding strip can be calculated from Eq. 22.10 by substituting $c = 0$, $q = \gamma z_1$ and $z = z_2 = nB$:

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$$\sigma_z = \frac{B\gamma}{2K \tan\phi}\left[1 - e^{-2Kn\tan\phi}\right] + \gamma z_1 e^{-2Kn\tan\phi} \quad ...(22.13)$$

where $n = 2.5$ for sands.

22.4 CLASSES OF UNDERGROUND CONDUITS

Underground conduits are commonly used as sewers, drains, water mains, gas lines, culverts etc. They can be divided into the following classes.:

1. *Ditch conduits*
2. *Projecting conduits:*
   (*a*) Positive projecting conduits : (*i*) Complete ditch condition (*ii*) Incomplete ditch condition (*iii*) Complete projection (*iv*) Incomplete projection
   (*b*) Negative projecting conduits : (*i*) Complete ditch condition (*ii*) Incomplete ditch condition.
3. *Special conduits:* Imperfect ditch conduits.

Figure 22.5 shows the three main classes of conduits commonly used.

A ditch conduit [Fig. 22.5 (*a*)] is the one which is installed in a narrow ditch, excavated in undisturbed soil below ground level, which is then covered with earth backfill. *A positive projecting conduit* [Fig. 22.5 (*b*)] is the one which is installed in a shallow bedding, with its top projecting above the natural ground and which is then covered with an embankment. In the case of a *negative projecting conduit* [Fig. 22.5 (*c*)], the conduit is installed in a narrow, shallow ditch with its top below the ground level, and which is then covered with an embankment. Type (*a*) is more commonly used as sewers, water mains etc., while types (*b*) and (*c*) are used in highway or railway culverts.

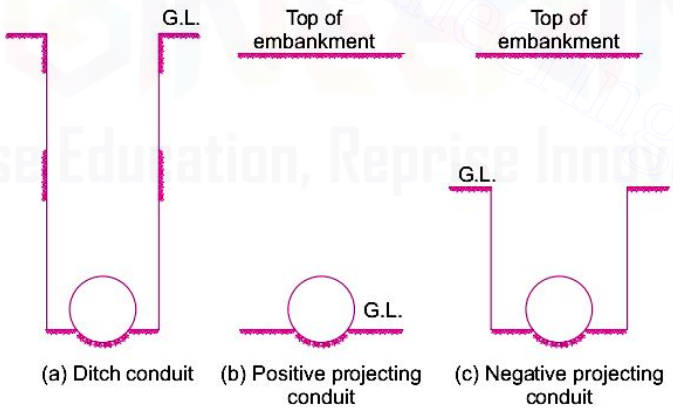

Fig. 22.5 Classes of common conduits

22.5 LOAD ON A DITCH CONDUIT

Figure 22.6 shows a free body diagram of a ditch conduit:

Let B_c = diameter of conduit, metres; B_d = width of the ditch, metres

H = depth of top of conduit below ground surface, metres

W_c = load on conduit, in kg per linear metre

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γ = unit weight of filling material

$\mu = \tan \phi$ = coefficient of internal friction of fill material

$\mu' = \tan \phi_1$ = coefficient of friction between fill material and sides of ditch ($\mu' \leq \mu$)

$$K = \frac{\text{Lateral unit pressure}}{\text{Vertical unit pressure}}$$

Consider an elemental strip of height dh, at h below the ground surface. Let V be the vertical load on it, at its top, and $V + dV$ be the load on its bottom acting in an upward direction. The weight of the strip is equal to $\gamma B_d\, dh$. The horizontal pressure on the sides will be $K\,(V/B_d)$. dh and the shear resistance developed vertically upward against the surfaces of the ditch will be $K\mu' \dfrac{V}{B_d}.dh$.

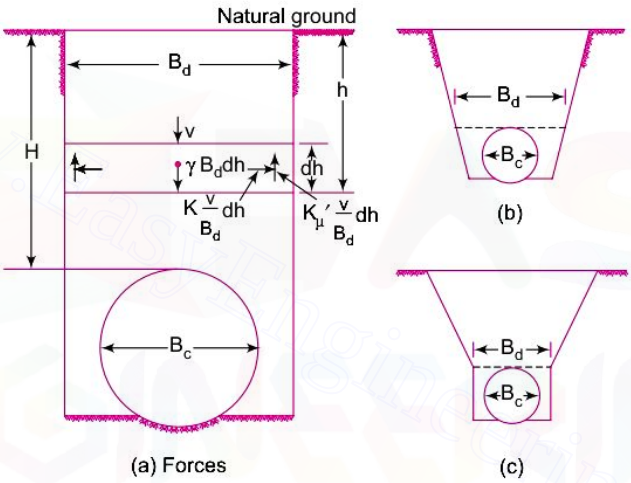

Fig. 22.6 Ditch conduit

At the time of critical equilibrium, when the elemental strip has a tendency to move downwards, we obtain,

$$(V + dV) + 2\,.\,K\,\mu' \frac{V}{B_d}.dh = V + \gamma B_d\, dh \qquad ...(22.14)$$

The solution of the above linear differential equation is

$$V = \gamma B_d^2 \frac{1 - e^{-2K\mu' \frac{h}{B_d}}}{2K\mu'} \quad \text{(where } e = \text{base of natural logarithm)} \qquad ...(22.15)$$

The above equation gives the variation of V with h . At $h = H$, $V = W_c$ given by

$$W_c = \gamma B_d^2 \frac{1 - e^{-2K\mu' \frac{H}{B_d}}}{2K\mu'} \qquad ...(22.16)$$

or

$$W_c = C_d \gamma B_d^2 \qquad ...(22.17)$$

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where $$C_d = \left(1 - \frac{e^{-2K\mu' \frac{H}{B_d}}}{2K\mu'}\right) \quad \text{...(22.18)}$$

= load coefficient for ditch conduits.

Equation 22.7 gives the load on a *rigid ditch conduit* with relatively compressible side fills.

Figure 22.7 gives curves for coefficient C_d for various values of $\frac{H}{B_d}$, plotted for several kinds of filling materials having different values of $K\mu'$ (Spanglar,1963). For curve (*a*), $K\mu' = 0.1924$ (minimum for granular materials without cohesion) ; for curve (*b*), $K\mu' = 0.165$ (maximum for sand and gravel); for curve (*c*) $K\mu' = 0.150$ (maximum for saturated top soil); for curve (*d*), $K\mu' = 0.130$ (ordinary maxiumum for clay) and for curve (*e*) $K\mu' = 0.110$ (maximum for saturated clay)

If the ditch has sloping sides [Fig. 22.6 (*b*), (*c*)], the values of B_d for use in Eq. 22.17, will be equal to the width of horizontal plane tangential to the top of the conduit.

Fig. 22.7 Curves for C_d for ditch conduit

22.6 LOAD ON POSITIVE PROJECTING CONDUITS

Positive projecting conduits are of four types:

1. Complete ditch condition
2. Complete projection condition
3. Incomplete ditch condition
4. Incomplete projection condition

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In order to understand the basic difference between all the four types, consider Fig. 22.8.

When a positive projecting conduit is installed, the plane along which relative movements occur, and on which shearing forces are generated are considered to be imaginary vertical planes extending upwards from the sides of the conduits as shown in Fig. 22.8. The magnitude and direction of relative movements between the interior and exterior prisms are dependent upon the *settlement ratio* r_{sd} defined by the following equation:

$$r_{sd} = \frac{(s_m + s_g) - (s_f + d_c)}{s_m} \qquad ...(22.19)$$

where s_m = compressive strain of side prisms of soils of height $p.B$

s_g = settlement of natural ground surface adjacent to conduit

s_f = settlement of conduit into its foundation

d_c = shortening for vertical height of conduit

p = projection ratio = ratio of the distance from the natural ground surface to the top of the conduit to its width B_c

The *critical plane* is a horizontal plane, tangential to the top of conduit. If the critical plane settles more than the top of the conduit [Fig. 22.8 (*a*)] *i.e.*, if $(s_m + s_g)$ is more than $(s_f + d_c)$, the settlement ratio is *positive*. Hence the exterior prisms move downward relative to the interior prism (or the interior prisms move downward relative to the exterior prisms), the shearing forces are directed downward and hence the load on the conduit is increased. This is known as the *projection condition*. If, however, the critical plane settles less than the top of the conduit [Fig. 22.8 (*b*)], *i.e.*, if $(s_m + s_g) < (s_f + d_c)$, the settlement ratio is negative. The interior prism moves downwards relative to the exterior prisms, shearing forces are directed upwards and hence the load is less than the weight of soil directly above the structure. This is called the *ditch condition* because the shearing forces act in the same direction as in the case of the ditch conduit.

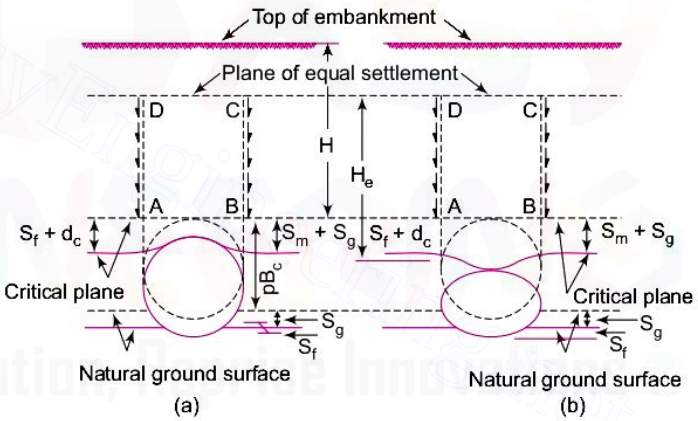

Fig. 22.8 Positive projecting conduit

Both the projection conditions and the ditch conditions of a positive projecting conduit may be *complete* or *incomplete*, depending upon the position of *plane of equal settlement*. Above the plane of equal settlement, the interior and exterior prisms settle equally, and hence no shearing forces are generated above this plane. If the embankment is sufficiently high, the plane of equal settlement may be within the embankment. This condition is then called *incomplete projection condition* or *incomplete ditch condition,* as the case, may be. If, however, the embankment is not of sufficient height, the plane of equal settlement does not fall within the embankment (*i.e.*, $H_e > H$); such a condition is called *complete projection condition* or *complete ditch condition*. Marston gave the following expression for the load on complete ditch condition or complete projection condition of a positive projecting conduit:

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$$W_c = C_c \gamma B_c^2 \qquad ...(22.20)$$

where
$$C_c = \frac{e^{\pm 2K\mu \frac{H}{B_c} - 1}}{\pm 2K\mu} \qquad ...(22.21)$$

Use (+) sign for complete projection and minus (–) sign for complete ditch condition (*i.e.*, use +ve sign for +ve settlement ratio, and –ve sign for negative settlement ratio). The load for the incomplete ditch condition or incomplete projection condition is given by.

$$C_c = \frac{e^{\pm K\mu \frac{H}{B_c} - 1}}{\pm 2K\mu} + \left(\frac{H}{B_c} - \frac{H_e}{B_c}\right) \cdot e^{\pm 2K\mu (H_e/B_c)} \qquad ...(22.22)$$

Use + sign for the incomplete projection conduit and minus (–) sign for incomplete ditch condition (*i.e.*, use positive (+) sign for positive settlement ratio, and –ve sign for negative settlement ratio). The height of plane of equal settlement, H_e, above the top of the conduit, is evaluated from the following expression:

$$\left[\frac{1}{2K\mu} \pm \left(\frac{H}{B_c} - \frac{H_e}{B_c}\right) \pm \frac{r_{sd}\, p}{3}\right] \frac{e^{\pm K\mu (H_e/B_c) - 1}}{\pm 2K\mu} \pm \frac{1}{2}\left(\frac{H_e}{B_c}\right)^2$$
$$\pm \frac{r_{sd}\, p}{3}\left(\frac{H}{B_c} - \frac{H_e}{B_c}\right) e^{\pm K\mu (H_e/B_c)} - \frac{1}{2K\mu} \cdot \frac{H_e}{B_c} \pm \frac{H}{B_c} \cdot \frac{H_e}{B_c}$$
$$= \pm r_{sd} \cdot p \frac{H}{B_c} \qquad ...(22.23)$$

Figure 22.9. gives curves for the determination of coefficient C_c.

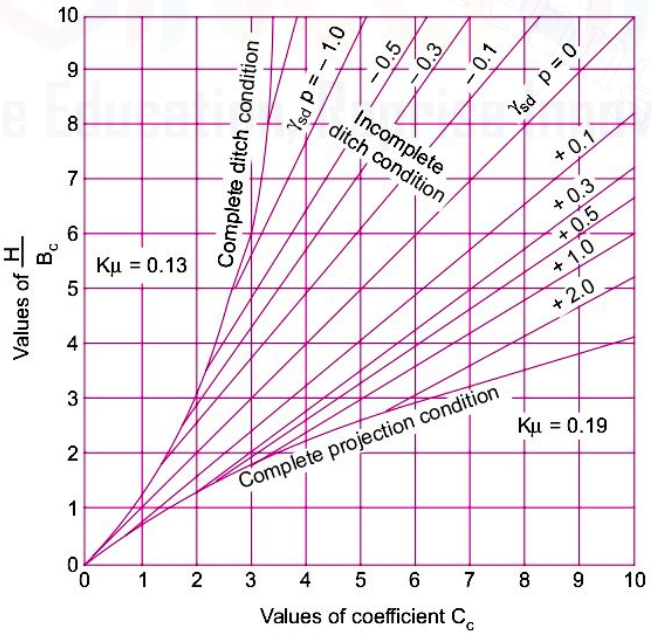

Fig. 22.9 Curves for determination of C_c for positive projecting conduits (Spanglar 1963)

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Table 22.1 gives the design values for the settlement ratio for use in design.

Table 22.1 Values of r_{sd} recommended for use in design (After spanglar)

Conduit type	*Construction*	*Projection condition*	*Foundation soil condition*	*Recommended value*
Rigid	Wide trench	Positive	Any	+ 1.0
	Embankment	Positive	Rock or hard unyielding soil	+ 1.0
			Ordinary	+ 0.5 to 0.8
			Yielding soil	0 to 0.5
	Any	Positive	Pipeline supported on piles in soft soil	Uncertain (Use complete projection value of C_c)
	Wide trench	Negative	Any	– 0.3
	Embankment	Negative	Any	– 0.3
	Induced trench	Negative	Any	0.0 to – 0.3
Flexible	Wide trench	Positive	Any with poorly compacted side fill	0.0 to – 0.4
	Embankment	Positive	Any with well compacted side fill	0
	Wide trench	Negative	Any with well compacted side fill	0
	Embankment	Negative	,,	0
	Induced trench	Negative	,,	0

22.7 LOAD ON NEGATIVE PROJECTING CONDUIT

In the case of negative projecting conduits, the exterior prisms of embankment material are not so high as the prism within the width of ditch above the conduit. Due to this, the interior prism always settles more, and the shearing stresses on it act in the upward direction, thereby reducing the load on the conduit to a value lesser than the weight of the interior prism.

The critical plane passes through the top of the ditch. The settlement ratio r_{sd} for the negative projecting conduit is given is by :

$$r_{sd} = \frac{s_g - (s_d + s_f + d_c)}{s_d} \qquad ...(22.24)$$

where s_d = deformation of fill in ditch within height $p' B_d$

p' = Projection ratio or negative projecting conduit

= ratio of height of the ditch above the top of the conduit to the width of the ditch.

The settlement ratio for a negative projecting conduit is always *negative* since the settlement of the critical plane is more than the settlement of the natural ground.

Hence $r_{sd} P'$ is also negative. The load on the conduit is given by

$$W_c = C_n \gamma B_d^2 \qquad ...(22.25)$$

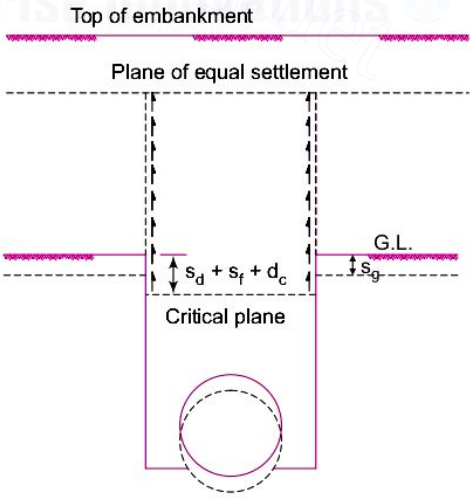

Fig. 22.10 Negative projecting conduit

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C_n = load coefficient for negative projecting conduit,

which is a function of $\frac{H}{B_d}$ ratio, p' and r_{sd}.

Coefficient C_n can be found from curves. One such set of curves, for $p' = 1$, is given in Fig. 22.11.

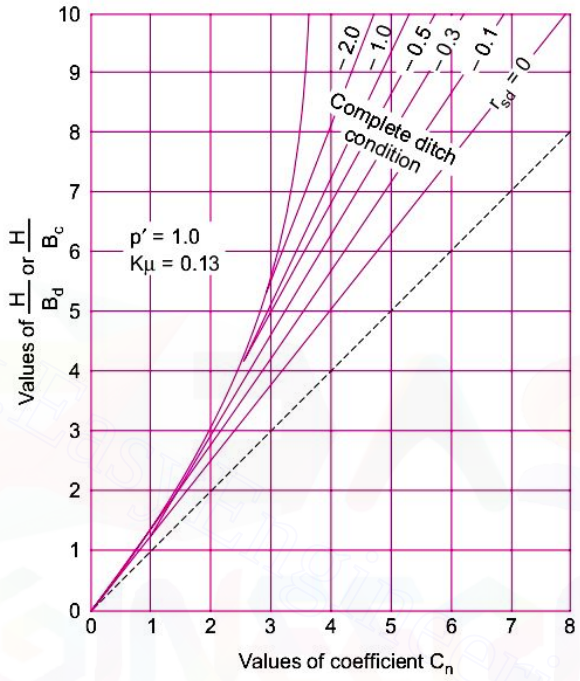

Fig. 22.11 Diagram for the determination of C_n ($p' = 1$)

22.8 IMPERFECT DITCH CONDUIT

Imperfect ditch conduit, somewhat similar to a negative projecting conduit, is a special technique of construction by which the load on the conduit is very much reduced so that the conduit can support a greater height of embankment. Figure 22.12 shows two stages of construction of such a conduit.

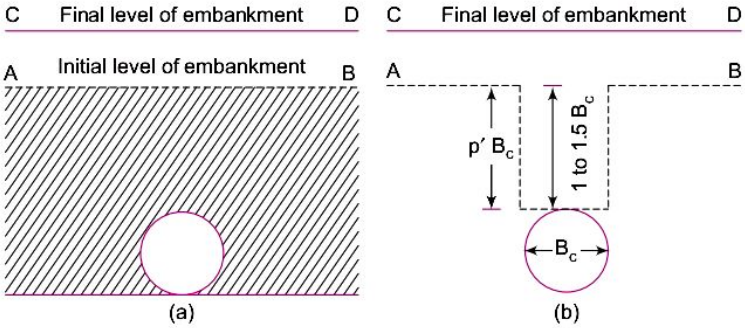

Fig. 22.12 Construction of imperfect ditch conduit

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In the first stage of construction, the conduit is intalled as a positive projection conduit, and a fill is constructed over it up to a height $p' B_c$ (1 to 1.5 times B_c). The fill is well compacted. In the second stage, a trench, of width B_c is dug above the conduit, down to the top level of the conduit. The trench is refilled with material placed in the loosest possible manner up to the top, and the rest of the embankment is constructed as usual. Marston recommended to use hay, straw, cornstalks etc., to be filled in trench, in order to increase the compressibility of this prism of soil and thus decrease the load on the conduit. This method of construction was used in Atlanta, Georgia, where the initial embankment of 35 ft, above a sewer constructed in 1937 was proposed to be raised to about 96 ft, in 1955. The sewer was protected from the increased load, by excavating a trench to a depth of 15 ft below natural ground, directly over the pipe. The trench was refilled with loose soil in which three 1 ft layers of tree leaves and pine straw were incorporated at approximately 5 ft intervals. Then as the embankment was constructed, shallow trenches were excavated over the sewer in each 15 ft lift of the compact fill. The depth of trench was about one half of the lift in which it was excavated. Then the trench was refilled with loose, uncompacted soil.

22.9 LOAD ON CONDUIT DUE TO LIVE LOADS

Marston proposed the following formula for computing additional load on the conduits due to loads applied to the surface of the fill.

$$W_t = \frac{1}{L_c} I_e \, C_t \, . P \qquad ...(22.26)$$

where W_t = Average load on conduit, due to wheel load, (kN/m)

L_e = Effective length of conduit

(It is taken equal to the actual length of a precast segment section of pipe if less than 1 m. If the precast length is more than 1 m, the maximum value of I_e is recommended as 1 m)

I_e = Impact factor

C_t = Load coefficient

P = Concentrated wheel load, on the embank (kg).

The load coefficient C_t is dependent on the length and width of the conduit cross-section and the depth of cover over it may be evaluated in accordance with the Boussinesq formula of stress distribution.

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CHAPTER 23

Stability of Slopes

23.1 INTRODUCTION

Earth embankments are commonly required for railways, roadways, earth dams, levees and river training works. The stability of these embankments or *slopes,* as they are commonly called, should be very thoroughly analysed since their failure may lead to loss of human life as well as colossal economic loss.

The failure of a mass of soil located beneath a slope is called a *slide*. It involves a downward and outward movement of the entire mass of soil that participates in the failure. The failure of slopes takes place mainly due to (*i*) the action of gravitational forces, and (*ii*) seepage forces within the soil. They may also fail due to excavation or undercutting of its foot, or due to gradual disintegration of the structure of the soil. Slides may occur in almost every conceivable manner, slowly or suddenly, and with or without any apparent provocation.

Every mass of soil which is bounded by a sloping surface is subjected to shearing stresses on nearly all its internal surfaces because of the gravitational forces which tend to pull the upper portions of the mass downward towards a more nearly level surface. An analysis of stability of slope consists of two parts: (*i*) the determination of the most severely stressed internal surface and the magnitude of the shearing stress to which it is subjected, (*ii*) the determination of the shearing strength along this surface. The shearing stress to which any slope can be subjected depends upon the unit weights of the material and the geometry of the slope, while shearing strength which can be mobilised to resist the shearing stress depends on the character of the soil, its density and drainage conditions.

Slopes may be of two types : infinite slope and finite slope. If a slope represents the boundary surface of a semi-infinite soil mass, and the soil properties for all identical depths below the surface are constant, it is called an *infinite slope*. If the slope is of limited extent, it is called a *finite slope*. Slopes extending to infinity do not exist in nature. The examples of finite slopes are the inclined faces of earth dam, embankments, and cuts etc.

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23.2 STABILITY ANALYSIS OF INFINITE SLOPES

Figure 23.1 shows an infinite slope *AB*, inclined at angle *i* to the horizontal. For an infinite slope, the soil properties and the soil stresses on any plane parallel to the slope surface are identical, and therefore, the failure of the slope usually involves a sliding of soil mass along a plane parallel to the slope at some depth. Let *CD* represents such a failure plane at depth *z* below the surface.

Consider a prism of soil, of inclined length *b* along the slope, and depth *z* up to the critical surface. The horizontal length of prism is *b* cos *i*, and its volume per unit length of prism is *z.b.* cos *i.*

$\therefore$ Weight of prism = $W = \gamma \,.\, z \,.\, b \cos i$.

$\therefore$ Vertical stress σ_z on the surface *CD* is given by

$$\sigma_z = \frac{W}{b} = \gamma \,.\, z \,.\, \cos i \qquad ...(23.1)$$

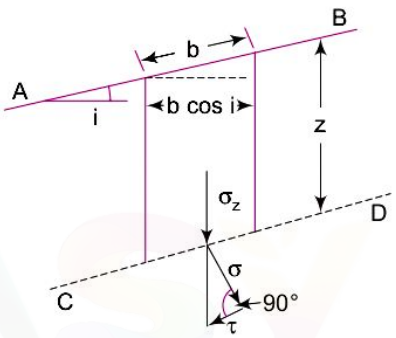

Fig. 23.1 Infinite slope

If σ and τ are the stress components normal and tangential to the surface *CD*, we have

$$\sigma = \sigma_z \cos i = \gamma z \cos^2 i \qquad ...(23.2a)$$

and

$$\tau = \sigma_z \sin i = \gamma z \cos i \sin i \qquad ...(23.2b)$$

The tangential component τ is called the shear stress which induces failure along *CD* and which is resisted by shear strength τ_f of the soil. The factor of safety of the slope, against sliding due to shear is given by

$$F = \frac{\tau_f}{\tau} \qquad ...(23.3)$$

The shear strength τ_f, in general, consists of both cohesion and internal friction. We shall consider two cases of soil : (*i*) cohesionless soil, and (*ii*) cohesive, soil.

(*i*) **Case (*i*) : Cohesionless soil:** In Fig. 23.2, *OA* is the failure envelope for a cohesionless soil, defined by the equation.

$$\tau_f = \sigma \tan \phi \qquad ...(23.4)$$

OB represents the locus of the stress components (σ, τ) acting on the critical surface *CD* (Fig. 23.1) for various values of *z*. For a given slope *i*, both σ and τ vary with *z*, but their ratio

$$\frac{\sigma}{\tau} = \frac{\cos i}{\sin i} = \cot i = \text{constant}$$

The line *OB*, drawn at inclination *i* with the σ-axis, therefore, represents the equation

$$\sigma = \tau \,.\, \cot i$$

or

$$\tau = \sigma \,.\, \tan i \qquad ...(23.5)$$

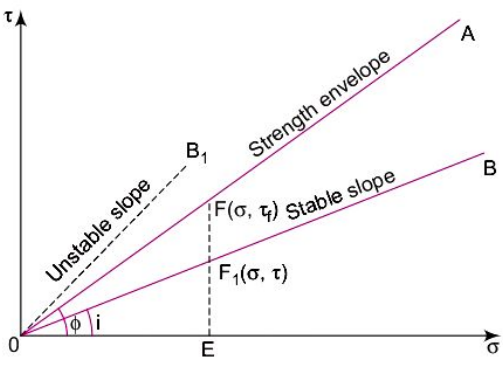

Fig. 23.2 Failure condition for an infinite slope of cohesionless soil

For a given value of normal stress σ failure will not occur so long as τ is smaller than τ_f, *i.e.*, so long as *i* is less

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than ϕ. In the limiting case of stability, the angle of slope is referred to as the angle of repose. The factor of safety against sliding is given by

$$F = \frac{\tau_f}{\tau} = \frac{\tan \phi}{\tan i} \qquad ...(23.6)$$

Submerged slope : If the slope is submerged, the bulk unit weight γ should be replaced by submerged unit weight γ' and σ and τ should be calculated using γ'. Also, φ should be determined corresponding to submerged condition.

$$\sigma = \gamma' z \cos^2 i \; ; \; \tau = \gamma' z \cos i \sin i$$

Hence,
$$F = \frac{\tau_f}{\tau} = \frac{(\gamma' z \cos^2 i) \tan \varphi}{\gamma' z \sin i \cos i} = \frac{\tan \varphi}{\tan i}$$

This shows that factor safety of a submerged slope is the same as that in dry state.

Steady seepage along the slope

If the infinite slope is subjected to steady seepage parallel to the slope, the weight W of the prism should be taken corresponding to saturated weight of the soil. Thus,

$$W = \gamma_{sat} \cdot z \, b \cos i,$$

$$\sigma_z = W/b = \gamma_{sat} z \cos i;$$

$$\therefore \quad \sigma = \sigma_z \cos i = \gamma_{sat} z \cos^2 i \qquad ...(i)$$

and
$$\tau = \sigma_z \sin i = \gamma_{sat} z \cos i \sin i \qquad ...(ii)$$

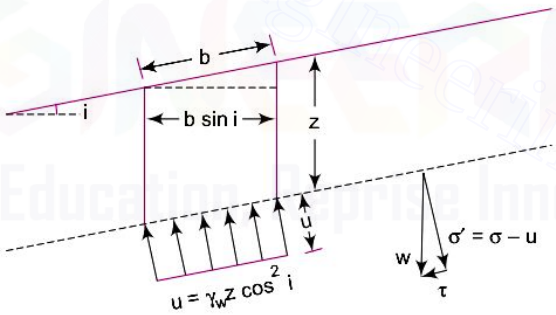

Fig. 23.3 Steady seepage along infinite slope

In addition to these, there is an upward force u due to seeping water, given by

$$u = \gamma_w z \cos^2 i \qquad ...(iii)$$

Hence the effective normal stress, $\sigma' = \sigma - u = \gamma_{sat} z \cos^2 i - \gamma_w z \cos^2 i$

$$= \gamma' z \cos^2 i \qquad ...(iv)$$

Hence,
$$F = \frac{\tau_f}{\tau} = \frac{\sigma' \tan \varphi}{\tau} = \frac{\gamma' z \cos^2 i \tan \varphi}{\gamma_{sat} z \cos i \sin i} = \frac{\gamma' \tan \varphi}{\gamma_{sat} \tan i} \qquad ...(23.6a)$$

Since γ'/γ_{sat} is *nearly* half, the factor of safety is reduced to nearly one-half when there is seepage parallel to the slope.

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Case (*ii*) : Cohesive soil: In Fig. 23.4, *DA* is strength envelope defined by the equation

$$\tau_f = c + \sigma \tan \phi \qquad ...(23.7a)$$

If the slope angle is equal to or less than ϕ, represented by line *OB*, no critical state of stress is reached and the slope will be stable. If a line *OF*, at a slope $i > \phi$ is drawn, it will cut the strength envelope at some point *F*, and a state of incipient failure is reached because the shear stress corresponding to the depth represented by point *F* equals to shear strength τ_f. For any depth *z* less than that represented by point *F*, the shear stress τ is less than the shear strength τ_f and the slope remains stable. For example, the depth *z* corresponding to point C_1 is stable for the slope of angle $i > \phi$. Hence, if *i* is greater than ϕ, the slope can be stable only up to a limited depth known as the *critical depth* H_c.

The factor of safety against failure, for any depth *z* corresponding to point C_1 of the slope angle $i > \phi$, is given by

$$F = \frac{\tau_f}{\tau} = \frac{c + \sigma \tan \phi}{\tau}$$

Putting $\sigma = \gamma z \cos^2 i$ and $\tau = \gamma z \cos i \sin i$, we get

$$F = \frac{c + \gamma z \cos^2 i \tan \phi}{\gamma z \cos i \sin i}$$

$$= \frac{c}{\gamma z \sin i \cos i} + \frac{\tan \phi}{\tan i} \qquad ...(23.7)$$

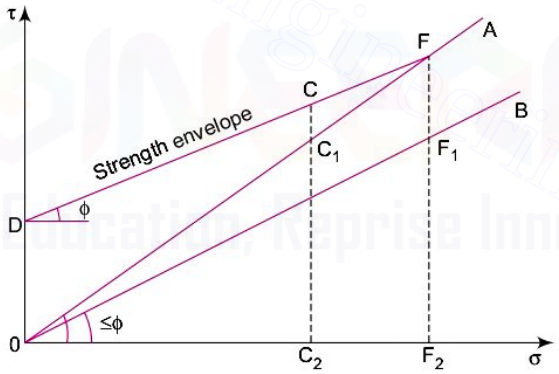

Fig. 23.4 Failure conditions of an infinite slope of cohesive soil

Note that for non-cohesive soil, $c = 0$

and hence Eq. 23.7 reduces to Eq. 23.6.

For the critical depth $z = H_c$ corresponding to point *F*, τ_f equals τ (*i.e.*, $F = 1$). Hence we get, from Eq. 23.7, $\gamma H_c \cos i \sin i = c + \gamma H_c \cos^2 i \tan \phi$

or
$$H_c = \frac{c}{\gamma} \frac{1}{(\tan i - \tan \phi) \cos^2 i} \qquad ...(23.8)$$

Equation 23.8 indicates that for given values of *i* and ϕ, H_c is proportional to cohesion.

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Rewriting Eq. 23.8, $\frac{c}{\gamma H_c} = (\tan i - \tan \phi) \cos^2 i$ ...(23.9)

The dimensionless quantity $\frac{c}{\gamma H_c}$ is called the *stability number* S_n:

$$S_n = \frac{c}{\gamma H_c} \quad ...(23.10)$$

Let F_c represents the factor of safety with respect to cohesion, and let c_m be the mobilised cohesion, at depth H, given by $c_m = \frac{c}{F_c}$ ...(23.11)

The stability number is then written as

$$S_n = \frac{c}{\gamma H_c} = \frac{c_m}{\gamma H} = \frac{c}{F_c \gamma H} = (\tan i - \tan \phi) \cos^2 i \quad ...(23.12)$$

From Eqs. 23.9 and 23.12, we get

$$F_c = \frac{H_c}{H} \quad ...(23.13)$$

Thus, the factor of safety F_c with respect to cohesion, also represents the factor of safety with respect to height. It is based on the assumption that the frictional resistance of the soil is fully developed. However, the true *factor of safety* is different from F_c, and is equally applied to both cohesion and frictional resistance of soil.

Submerged slope : If the slope is submerged, γ should be replaced by γ' ; also c and φ should be determined corresponding to submerged condition.

Thus, $$F = \frac{c + \gamma' z \cos^2 i \tan \varphi}{\gamma' z \cos i \sin i} \quad ...(23.7a)$$

$$H_c = \frac{c}{\gamma'} \frac{1}{(\tan i - \tan \varphi) \cos^2 i} = \frac{c}{\gamma'} \cdot \frac{\sec^2 i}{\tan i - \tan \varphi} \quad ...(23.8a)$$

Steady seepage along the slope : As in the case of non-cohesive soil, τ should be taken with respect to saturated weight (γ_{sat}) while σ should be computed with respect to the submerged weight. Hence, equation 23.7 is modified as under

$$F = \frac{c + \gamma' z \,.\cos^2 i \tan \varphi}{\gamma_{sat} z \cos i \sin i} = \frac{c}{\gamma_{sat} \cos i \sin i} + \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan \varphi}{\tan i}$$

For the critical height $z = H_c$ corresponding to $F = 1$, we have from above

$$\gamma_{sat} H_c \cos i \sin i = c + \gamma' H_c \cos^2 i \tan \varphi$$

which gives $$H_c = \frac{c}{(\gamma_{sat} \tan i - \gamma' \tan \varphi) \cos^2 i} = \frac{c}{\gamma_{sat} \left\{ \tan i - \frac{\gamma'}{\gamma_{sat}} \tan \varphi \right\} \cos^2 i} \quad ...(23.8b)$$

Comparing it with Eq. 23.8, it is seen that the effect of shearing resistance φ is reduced as compared to Eq. 23.8.

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Example 23.1. *A long natural slope of cohesionless soil is inclined at 12° to the horizontal. Taking φ = 30°, determine the factor of safety of the slope. If the slope is completely submerged, what will be change in the factor of safety?*

Solution. From equation 23.6, $F = \frac{\tau_c}{\tau} = \frac{\tan \varphi}{\tan i}$

Here, $\varphi = 30°, i = 12° \quad \therefore \quad F = \frac{\tan 30°}{\tan 12°} = \mathbf{2.72}$

***Effect of submergence*:** When the slope is submerged, γ is replaced by γ′

$$\therefore \quad F = \frac{\tau_c}{\tau} = \frac{(\gamma' z \cos^2 i) \tan \varphi}{\gamma' z \sin i \cos i} = \frac{\tan \varphi}{\tan i} = \frac{\tan 30°}{\tan 12°} = \mathbf{2.72}$$

Thus, the F.S. will remain the same, except that φ is to be determined under submerged condition.

Example 23.2. *A long natural slope of sandy soil (φ = 25°) is inclined at 10° to the horizontal. The water table is at the surface and the seepage is parallel to the slope. If the saturated unit weight of the soil is 19.5 kN/m³, determine the factor of safety of the slope.*

Solution. From Eq. 23.6 (*a*), $F = \frac{\gamma' \tan \varphi}{\gamma_{sat} \tan i} = \frac{(19.5 - 9.81) \tan 25°}{19.5 \tan 10°} = \mathbf{1.31}$

Example 23.3. *A long natural slope in a c – φ soil is inclined at 12° to the horizontal. The water table is at the surface and the seepage is parallel to the slope. If a plane slip has developed at a depth of 4 m, determine the factor of safety.*

Take $c = 8\ kN/m^2$, $\varphi = 22°$ *and* $\gamma_{sat} = 19\ kN/m^3$.

Solution.

From Eq. 23.7 (*b*) $$F = \frac{c + \gamma' z \cos^2 i \tan \varphi}{\gamma_{sat} z \cos i \sin i} = \frac{8 + (19 - 9.81) \times 4 \cos^2 12° \tan 22°}{19 \times 4 \cos 12° \sin 12°} = \mathbf{1.44}$$

23.3 STABILITY ANALYSIS OF FINITE SLOPES

Failure of finite slopes occurs along a surface which is a curve. In stability computations, the curve representing the real surface of sliding is usually replaced by an arc of a circle or logarithmic spiral. Two basic types of failure of a finite slope may occur: (*i*) slope failure, (*ii*) base failure.

If the failure occurs along a surface of sliding that intersects the slope at or above its toe, the slide is known as *slope failure*. Slope failure is called as a *face failure* [Fig. 23.5 (*a*)] if the arc passes above the toe, or *toe failure* [Fig. 23.5 (*b*)] if the arc passes through the toe. On the other hand, if the soil beneath the toe of the slope is weak the failure occurs along a surface that passes at some distance below the toe of the slope. Such a type of failure is called *base failure* [Fig. 23.5(*c*)]. The ratio of the total depth ($H + D$) to depth H is called the *depth factor* D_f. For toe failure, $D_f = 1$; for base failure, $D_f > 1$.

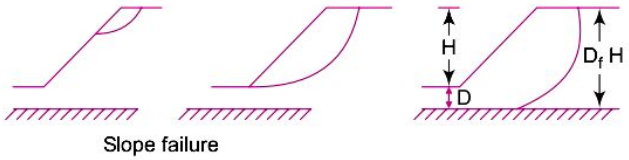

Fig. 23.5 Types of failure of finite slope

Type of slip surfaces or failure surfaces: The rupture of a finite slope may take place along one of the following failure surfaces

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1. Planar failure surface 2. Circular failure surface 3. Non-circular failure surface

Planar failure surface may commonly occur in a soil deposit or embankment with a specific plane of weakness. Excavation in stratified deposit quite often leads to a planar failure surface along a plane parallel to the strata. Similarly, in composite earth dams with sloping cores, planes of weakness within the bank may consist of two or three planar surfaces. However, in most cases, actual failure surfaces are *curved*. Collin observed that the rupture mass slide down a sliding surface, in a definite pattern resembling that of a *cycloid*. Generally, the failure surfaces have arcs somewhat flatter at the ends and sharper at the centre. Circular rupture surface was first proposed by Petterson (1916). Further field investigations by Swedish Geotechnical Commission justified *circular arcs* as close approximation of actual slip surface in homogeneous and isotropic soil conditions. *For simple idealised problems, the assumption of a circular failure surface is sufficiently accurate.*

Non-circular (or composite) failure surface occurs in many practical cases. According to Bennett (1951), non-circular (or composite) slip surface may arise in homogeneous dams having one or more of the following:

(*i*) Foundation of infinite depth,
(*ii*) Rigid boundary planes of maximum or zero shear,
and (*iii*) Presence of relatively stronger or weaker layer.

Similarly, according to Morgenstern and Price (1965), conditions for which non-circular (or composite) slip surface may occur in non-homogeneous earth dams may occur are :

(*i*) Presence of a soft layer in foundation,
(*ii*) Use of different types of soil or rock in the dam section with varying strength and pore pressure condition
and (*iii*) Use of drainage blankets to facilitate dissipation of pore pressures.

Methods of Analysis: The stability of a finite slope can be investigated by a number of methods. We shall deal with the following common methods:

1. Culmann's method of planer failure surface 2. The Swedish circle method (slip circle method) 3. The friction circle method 4. Bishop's method.

23.4 PLANAR FAILURE SURFACE: CULMANN'S METHOD

Culmann (1866) considered a simple failure mechanism of a slope of homogeneous soil with plane failure surface passing through the toe of the slope.

Let AB be any probable slip plane. The wedge ADB is in equilibrium under the action of three forces:

(*i*) Weight of the wedge, $W = \frac{1}{2} AB \,.\, h\,\gamma = \frac{1}{2} L \,.\, h \,.\, \gamma$ ...(*i*)

(*ii*) The cohesive force C along the surface AB, resisting motion $= c_m \,.\, L$

(*iii*) The reaction R, inclined at angle φ_m to the normal

Now, $AD = \dfrac{h}{\sin(i-\theta)} = \dfrac{H}{\sin i}$ Hence, $h = \dfrac{H \sin(i-\theta)}{\sin i}$

Substituting in (*i*), we get $W = \dfrac{1}{2} L\gamma \,.\, H \dfrac{\sin(i-\theta)}{\sin i}$ ...(*ii*)

If c and φ are the appropriate shear strength parameters, the *shear strength* (τ_f) along the slip plane is $\tau_f = c \,.\, L + W \cos\theta \,.\, \tan\varphi$ ...(*iii*)

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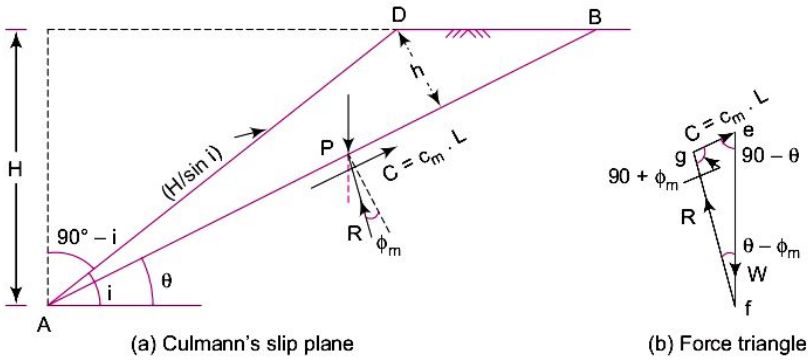

Fig. 23.6 Culmann's method using planar failure surface

The weight component parallel to the plane AC causing sliding is $\tau = W \sin \theta$.

$\therefore$ Factor of safety, $$F = \frac{\tau_f}{\tau} = \frac{cL + W \cos \theta \tan \varphi}{W \sin \theta}$$

Substituting the value of W in the above expression, we get

$$F = \frac{c + \frac{1}{2} \gamma H [\sin (i - \theta)/\sin i] \cos \theta \tan \varphi}{\frac{1}{2} \gamma H [\sin (i - \theta)/\sin i] \sin \theta} \quad ...(23.14)$$

Let c_m be the mobilised cohesion $= \frac{c}{F}$ and φ_m = angle of mobilised friction $= \tan^{-1}\left(\frac{\tan \varphi}{F}\right)$.

Considering AB to be the failure plane, we have, from force polygon shown in Fig. 23.7 (b),

$$\frac{c_m \, . \, L}{\sin (\theta - \varphi_m)} = \frac{W}{\sin (90° + \varphi_m)} = \frac{W}{\cos \varphi_m}$$

Substituting for the weight W, we get $\dfrac{c_m \, . \, L}{\sin (\theta - \varphi_m)} = \dfrac{\frac{1}{2} L \gamma H \sin (i - \theta)}{\sin i \cos \varphi_m}$

or $$\frac{c_m}{\gamma H} = \frac{1}{2}\left[\frac{\sin (\theta - \varphi_m) \sin (i - \theta)}{\sin i \cos \varphi_m}\right] = \frac{1}{2} \operatorname{cosec} i \sec \varphi_m \sin (i - \theta) \sin (\theta - \varphi_m) \quad ...(23.15)$$

where $c_m/\gamma H = S_n$ is known as the *stability number*.

For failure to occur, the stability number (S_n) has to be maximum. In other words, the most dangerous plane is that for which the angle θ ($= \theta_c$) is the such that S_n becomes maximum.

$\therefore$ $$\frac{d(S_n)}{d\theta} = \frac{d}{d\theta} [\sin (i - \theta) \sin (\theta - \varphi_m)] = 0$$

or $\cos (\theta - \varphi_m) \sin (i - \theta) - \sin (\theta - \varphi_m) \cos (i - \theta) = 0$

or $\tan (\theta - \varphi_m) = \tan (i - \theta)$ or $\theta - \varphi_m = i - \theta$.

Making $\theta = \theta_c$ = critical slope angle, we get $\theta_c = \frac{1}{2}(i + \varphi_m)$ ...(23.15a)

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This gives the angle of inclination of *critical slip plane.*

Substituting this value of θ_c in Eq. 23.15 we get

$$\left(\frac{c_m}{\gamma H}\right)_{\max} = \frac{1}{2}\operatorname{cosec} i \sec \varphi_m \left[\sin\left\{i - \frac{i+\varphi_m}{2}\right\}\right] \times \left[\sin\left\{\frac{i+\varphi_m}{2} - \varphi_m\right\}\right]$$

$$= \frac{1}{2}\operatorname{cosec} i \sec \varphi_m \sin\left(\frac{i-\varphi_m}{2}\right)\sin\left(\frac{i-\varphi_m}{2}\right)$$

$$= \frac{1}{2}\operatorname{cosec} i \sec \varphi_m \left[\frac{1-\cos(i-\varphi_m)}{2}\right] = \frac{1-\cos(i-\varphi_m)}{4\sin i \cos \varphi_m}$$

From which $$H_{safe} = \frac{4 c_m \sin i \cos \varphi_m}{\gamma\left[1-\cos(i-\varphi_m)\right]} \qquad ...(23.15b)$$

In the above equation, c_m and φ_m are known corresponding to a factor of safety F.

Culmann's method is suitable for very steep slopes.

23.5 THE SWEDISH SLIP CIRCLE METHOD

The method, developed by Swedish engineers assumes that the surface of sliding is an arc of a circle. We shall consider two cases: (*i*) analysis of purely cohesive soil ($\phi_u = 0$ analysis) and (*ii*) analysis of a soil possessing both cohesion and friction ($c - \phi$ analysis).

(*i*) **$\phi_u = 0$ analysis:** Figure 23.7 shows a slope AB, the stability of which is to be determined. The method consists in assuming a number of trial slip circles, and finding the factor of safety of each. The circle corresponding to the minimum factor of safety is the *critical slip circle*. Let AD be a trial slip circle, with r as the radius and O as the centre of rotation. Let W be the weight of the soil of the wedge $ABDA$ of unit thickness, acting through its centroid. The driving moment M_D will be equal to $W\bar{x}$, where $\bar{x}$ is the distance of line of action of W from the vertical line passing through the centre of rotation. If c_u is the unit cohesion and $\hat{L}$ = length of the slip arc $AD = \dfrac{2\pi r\delta}{360°}$, the shear resistance developed along the slip surface will be equal to $c_u \hat{L}$, which acts at a radial distance r from the centre of rotation O. Hence, the resisting moment M_R will be equal to $r \cdot c_u \hat{L}$. The factor of safety F is then given by

$$F = \frac{M_R}{M_D} = \frac{c_u \hat{L} r}{W \bar{x}} \qquad ...(23.16a)$$

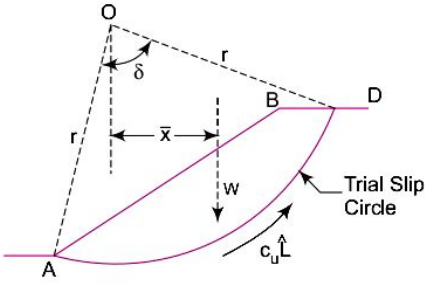

Fig. 23.7 $\phi_u = 0$ Analysis

Alternatively, Let c_m = mobilised shear resistance of soil ($\phi = 0$), necessary for equilibrium.

Then $W\bar{x} = c_m \hat{L}.r$ or $c_m = \dfrac{W\bar{x}}{\hat{L}}.r$

Hence, $$F = \frac{c_u}{c_m} = \frac{c_u \hat{L}.r}{W\bar{x}} \qquad ...(23.16b)$$

The distance $\bar{x}$ of the centroid of the wedge, from centre of rotation O, can be determined by dividing the wedge into

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a number of vertical slices and dividing the algebraic sum of moment of weight of each slice by the weight of the wedge.

Effect of tension crack : If a tension crack of depth $z_0 \left(= \frac{2c}{\gamma}\right)$ develops, water will enter in the crack, exerting a hydrostatic pressure force P_w acting on the portion DE at a height $z_0/3$ from E. Hence, the arc portion DE will be ineffective in resisting the slide. The modified arc length ($\hat{L}$), for use in Eq. 23.16, will be equal to AE only.

(*ii*) **$c - \phi$ analysis:** In order to test the stability of the slope of a $c - \phi$ soil, trial slip circle is drawn, and the material above the asssumed slip surface is divided into a convenient number of vertical strips or slices. The forces between the slices are neglected, and each slice is assumed to act independently as a column of soil of unit thickness and of width b. The weight W of each slice is assumed to act at its centre. If this weight of each slice is resolved into normal (N) and tangential (T) components, the normal components will pass through the centre of rotation (O), and hence do not cause any driving moment on the slice. However, the tangential component T causes a driving moment $M_D = T \times r$, where r is the radius of the slip circle. The tangential components of the few slices at the base may cause resisting moment; in that case T is considered negative.

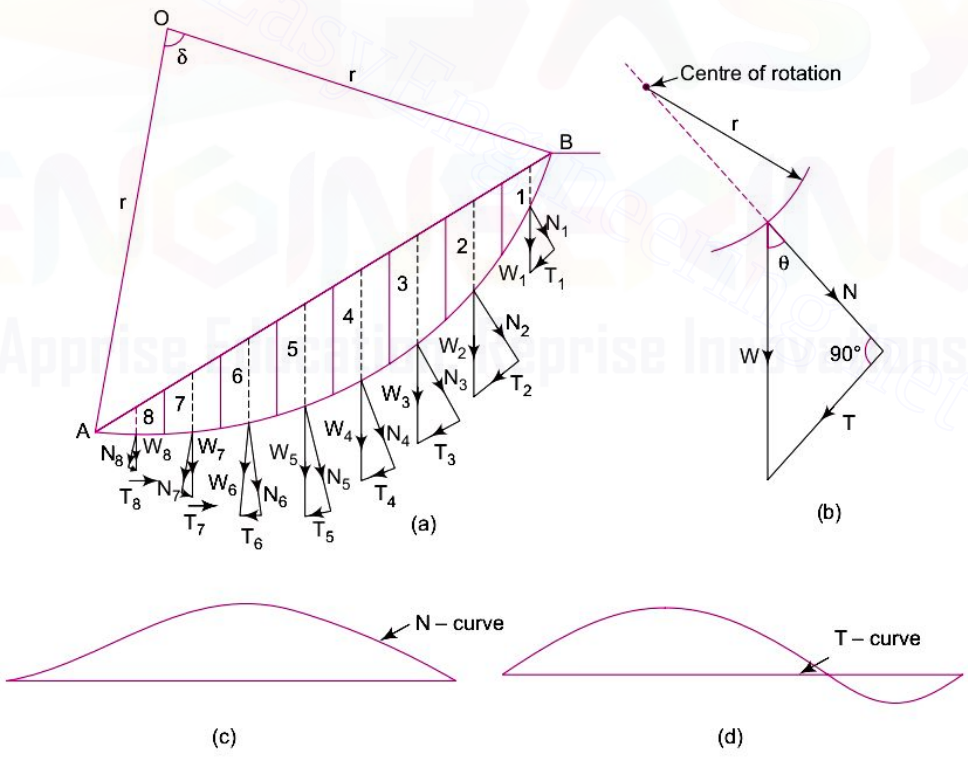

Fig. 23.8 Slip circle method: c–ϕ analysis

If c is the unit cohesion and ΔL is the curved length of each slice then the resisting force, from Coulomb's equation is equal to ($c\,\Delta L + N \tan \phi$)

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For the entire slip surface AB, we have

Driving moment $M_D = r\Sigma T$; Resisting moment $M_R = r[c\Sigma\Delta L + \tan\phi\,\Sigma N]$

where ΣT = algebraic sum of all tangential components

ΣN = sum of all normal components

$$\Sigma\Delta L = \hat{L} = \frac{2\pi r\delta}{360^\circ} = \text{length } AB \text{ of slip circle}$$

Hence, factor of safety against sliding is $F = \dfrac{M_R}{M_D} = \dfrac{c\,\hat{L} + \tan\phi\,\Sigma N}{\Sigma T}$ ...(23.17)

A number of trial slip circles are chosen and factor of safety of each is computed. The circle giving the minimum factor of safety is the *critical* slip circle.

In order to find ΣN and ΣT, N-curves and T-curves are drawn [Figs. 23.8 (*c*), (*d*)] by making the ordinates of these diagrams equal to N and T values for different strips and joining them by smooth curves. The area of these diagrams can be measured with the help of planimeter and ΣN and ΣT can then be computed.

A simplified *rectangular plot method* has been suggested by Singh (1962). Referring to Fig. 23.9, let z_1, z_2 ... be the *boundary ordinates* of the slices. Let the width of each slice be b and the width of the last slice be mb. The total weight ΣW of the sliding sector, in terms of area, may be written as

$$\Sigma W = b\left\{z_1 + z_2 + \ldots\left(\frac{1+m}{2}\right)z_n\right\} \quad \ldots(23.18)$$

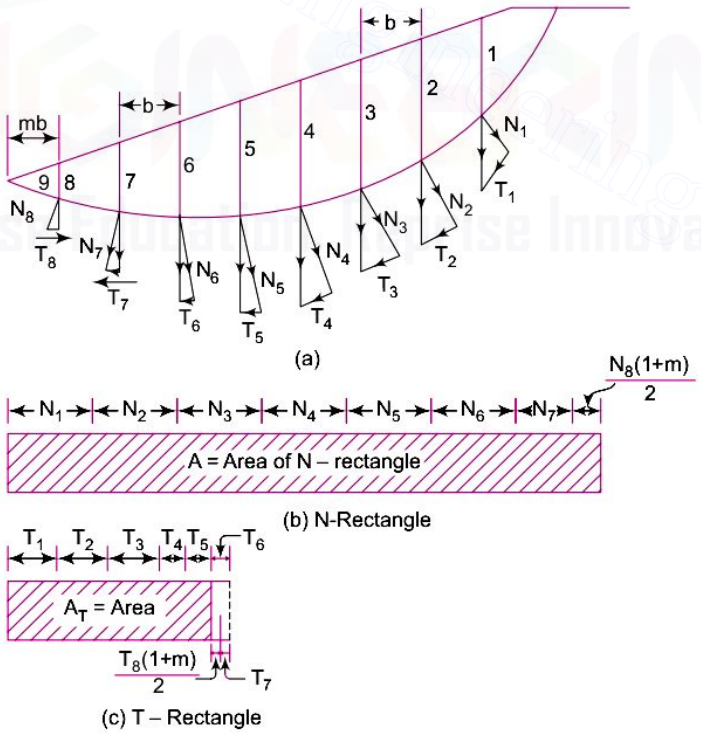

Fig. 23.9 Rectangular plot method

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This shows that the weight can be taken corresponding to the end ordinate of each slice. This weight can be resolved into corresponding N and T components. Then the N-rectangle is plotted by measuring various N-values (*i.e.*, N_1, N_2 ... etc) as abscissa and the constant width b as the ordinate. Similarly, T rectangle can be constructed.

It should be clearly noted that in the rectangular plot method if there are n-slices, the total number of ordinates will be $(n - 1)$, the ordinates at the end of the last strip (*e.g.*, strip 9) being zero. If A_N = area of N-rectangle in sq. cm, γ = unit weight of the soil of (kN/m^3) and if the scale of drawing is 1 cm = x metres, then the area of N-rectangle in square metres = $A_N \cdot x^2$ and its volume = $A_N \times x^2 \times 1$ m^3 per metre length of the dam.

Hence, $\Sigma N = A_N \cdot x^2 \gamma$ kN, similarly $\Sigma T = A_T \cdot x^2 \gamma$ kN

The third method of computing ΣN and ΣT is with the help of table shown below:

Table 23.1

Slice No.	*W*	$N = W \cos \alpha$	$T = W \sin \alpha$
1	W_1	N_1	T_1
2	W_2	N_2	T_2
3	W_3	N_3	T_3
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.
n	W_n	N_n	$-T_n$
	Sum	$\Sigma N =$	$\Sigma T =$

The method of slices is a general method which is equally applicable to homogeneous soils, stratified soils, fully or partly submerged soils, non-uniform slopes and to cases when seepage and pore pressure exist within the soil.

Method of locating centre of critical slip circle: In order to reduce the number of trials to find the centre of critical slip circle, Fellinious has given a method of locating the locus on which the probable centre may lie. For a homogeneous c–ϕ soil, the centre of slip circle lie on a line PQ, in which the point Q has its co-ordinates H downwards from toe and 4.5 H horizontally away, as shown in Fig. 23.10. The other point P is located with the help of directional angle α and β given in Table 23.2.

When the line PQ is obtained, trial centres are obtained on it and factor of safety corresponding to each centre is calculated from Eq. 23.17. These various factors of safety so obtained are plotted as ordinates on

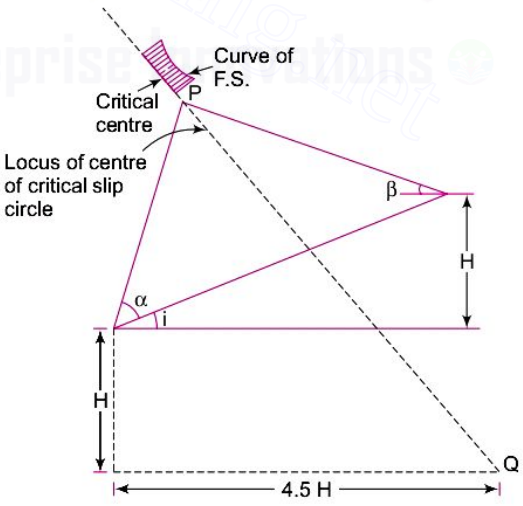

Fig. 23.10 Fellinious method of locating centre of critical slip circle

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the corresponding centres (Fig. 23.10) and a smooth curve is obtained. The centre corresponding to the lowest factor of safety is the critical circle centre. For a $\phi = 0$ soil, P represents the centre of the critical circle.

Table 23.2

Slope	*Slope angle (i)°*	*Directional angles*	
		(α)°	(β)°
0.58 : 1	60	29	40
1 : 1	45	28	37
1.5 : 1	33.8	26	35
2 : 1	26.6	25	35
3 : 1	18.4	25	35
5 : 1	11.3	25	35

23.6 STABILITY OF SLOPES OF EARTH DAM

The stability of slopes of an earth dam is tested under the following conditions:

1. Stability of downstream slope during steady seepage.
2. Stability of upstream slope during sudden drawdown.
3. Stability of upstream and downstream slopes during and immediately after construction.

1. **Stability of downstream slope during steady seepage :** Critical condition of downstream slope occurs when the reservoir is full and percolation is at its maximum rate. The direction of seepage forces tends to decrease the stability. In other words, the pore water pressure (U) acting on the soil mass below the saturation line reduces the effective stress responsible for mobilising shearing resistance. The factor of safety in this case is given by

$$F = \frac{c'\hat{L} + \tan\phi' \Sigma (N - U)}{\Sigma T} \quad ...(23.19)$$

where ΣU = total pore pressure on the slip surface and c' and ϕ' are the shear parameters based on the effective stress analysis.

The pore water pressure at any point is represented by the piezometric head h_w at that point. Thus, the variations of pore water pressure along a likely slip surface is obtained by measuring at each of its intersections with an equipotential line, the vertical height from that intersection to the level at which the equipotential line cuts the phreatic line. The pore pressure represented by vertical height so obtained are plotted to scale in a direction normal to the sliding surface at the respective points of intersection. The distribution of pore water pressure on the vertical slip surface during steady seepage is shown in Fig. 23.11. The area of U-diagram can be measured with the help of a planimeter, or else the rectangular plot method can be utilised. If A_U is the area of U-diagram (cm^2), then $\Sigma U = A_U \times x^2 \times \gamma_w$ kN, where x is the scale of drawing (*i.e.*, 1 cm = x metres), and γ_w is the unit weight of the water in kN/m^3.

In the absence of a flow net, the factor of safety of the downstream slope can be approximately calculated from the equation

$$F = \frac{c'\hat{L} + \tan\phi' \Sigma N'}{\Sigma T} \quad ...(23.20)$$

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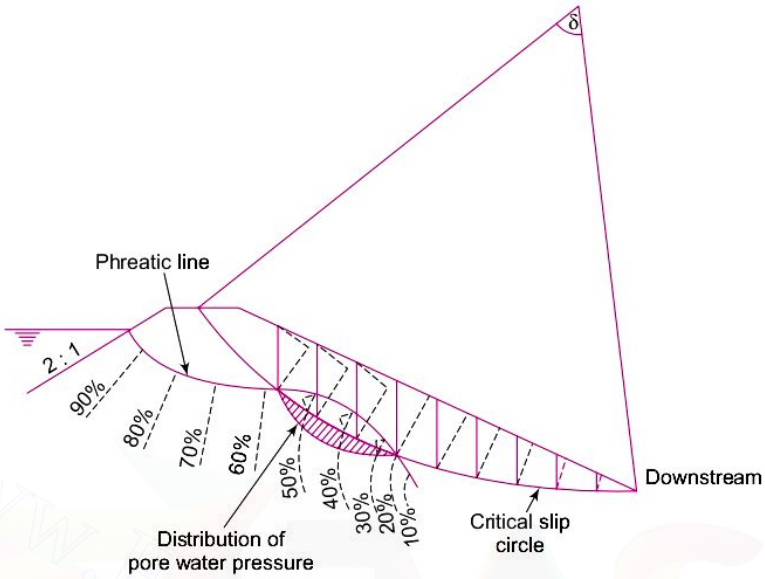

Fig. 23.11 Stability of downstream slope during steady seepage

where the normal components N' are to be calculated on the basis of buoyant unit weight γ' of the dam while T components are to be calculated on the basis of saturated unit weight.

2. **Stability of upstream slope during sudden drawdown :** For the upstream slope, steady seepage does not represent the critical state, because the seepage pressure then acts inwards from this slope and tends to increase the stability on the upstream side. For the upstream slope, the critical condition is when the reservoir is suddenly emptied without allowing any appreciable change in the water level within the saturated mass of soil. This state is known as sudden drawdown. With complete drawdown, hydrostatic force acting along the upstream slope at the time of full reservoir is emptied. The weight of water within the soil now tends to help a sliding failure without the hydrostatic pressure on the slope to counteract it. Or, by another interpretation, while the disturbing force is tangential component of the saturated weight, the shear resistance is considerably reduced on account of the development of pore water pressure on a likely slip surface. The effect of drawdown on slope stability varies appreciably with opportunity of drainage at the base. If the base is quite impervious, flow lines tend to be horizontal and in outward direction towards the slope, which is quite unfavourable to stability. On the other hand if the base material is pervious, flow pattern tends to be downwards and this is more favourable with respect to stability.

The magnitude and distribution of pore water pressure on a likely slip surface is estimated from *pressure net* which is developed from the flow net. The pressure net gives the line of equal piezometric heads h_w expressed in terms of percentage of total hydraulic head when the reservoir is full. In order to plot the distribution of pore water pressure on the assumed slip surface, the pressure net is superimposed on the diagram showing the location of slip surface. At the intersection point of the slip surface with the contours of pressure net, radial ordinates of the values of piezometric heads h_w of the corresponding contours are erected. The curve joining these ordinates gives the distribution of pore water pressure. The factor of safety is calculated from the equation

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$$F = \frac{c'\hat{L} + \tan\phi' \Sigma(N - U)}{\Sigma T}$$

In the absence of the flow net, the factor of safety can be calculated from the expression

$$F = \frac{c'\hat{L} + \tan\phi' \Sigma N'}{\Sigma T}$$

in which N' components are computed with respect to the submerged density γ' of impervious upstream slope while T components are computed with respect to saturated density.

3. **Stability of upstream and downstream slopes during or immediately after construction:** When a dam is built of relatively impervious compressible soil, excess pore pressure develops in the air and water entrapped in the pore space. This is because the soil mass undergoes a change in volume due to compaction of the dam during construction and also due to its down weight. In due course of time this initial excess pore pressure gets gradually dissipated. As the pore pressure greatly affects the shear strength of soil, it is essential to know its magnitude for stability analysis. The estimation of the construction pore pressure is made with the help of Hilf's equation:

$$u = \frac{p_a \Delta}{V_a + h_c \gamma_w - \Delta} \qquad ...(23.21)$$

where u = induced pore pressure.

p_a = air pressure in the voids of soil mass after initial compaction (absolute; considered to be atmospheric; should be corrected to site elevation).

Δ = embankment compression in per cent of original total embankment volume.

V_a = volume of free air voids, in per cent of original total embankment volume.

V_w = volume of pore water, in per cent of original total embankment.

h_c = Henry's constant of solubility of air in water by volume (= 0.02 at 68°F).

For use in this method, a graph is plotted between effective stress σ' and per cent consolidation Δ, as obtained from consolidation test. Fig. 23.12 (*a*) gives a sample consolidation-load curve. The value of pore pressure u for various values of Δ is calculated from Eq. 23.21. Knowing effective stress σ' and pore pressure u, the total stress σ is obtained by the relation, $\sigma = \sigma' + u$. The results are tabulated as shown in Table 23.3. A plot is then made between u and total stress σ as shown in Fig. 23.12 (*b*).

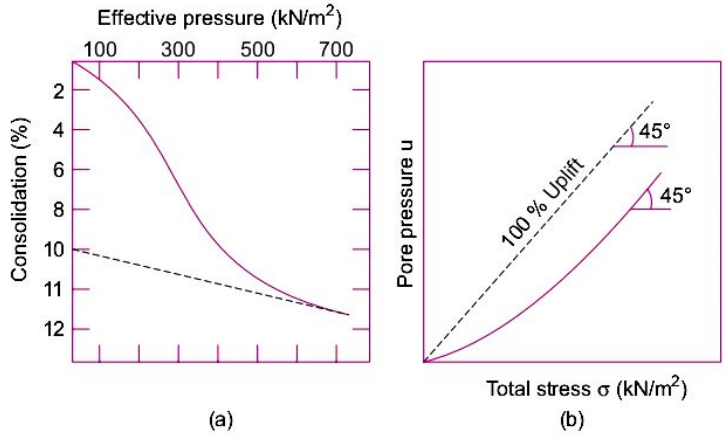

Fig. 23.12 Consolidation test data for estimating construction pore-pressure

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Table 23.3 Consolidation test data

Consolidation Δ*(%)*	*Effective stress* σ' *(kg/cm²)*	*Pore pressure* u *(kg/cm²)*	*Total stress* σ *(kg/cm²)*
1	0.7	0.065	0.765
2	1.3	0.139	1.439
4	2.3	0.322	2.622
6	3.0	0.576	3.576
8	3.6	0.948	4.548
10	4.1	1.550	5.650
12	4.9	2.680	7.580
14	6.0	6.160	12.160
16	7.9	38.2	46.10

In order to find the construction pore pressure at the bottom of various slices (Fig. 23.8) the mid-heights $z_1, z_2, \ldots, z_n$ of the strips are measured and the total stresses σ (*i.e.*, $\sigma_1, \sigma_2, \ldots, \sigma_n$) are calculated from the relation $\sigma = \gamma z$. Then from Fig. 23.12 (*b*) the values of u corresponding to various values of σ are found and results tabulated as shown in Table 23.4.

Table 23.4 Determination of pore pressure

Slice No.	*Height* z	*Total stress* $\sigma = \gamma z$ *(kg/cm²)*	*Pore pressure*	
			u *(kg/cm²)*	$U = u \times b \times l$*(kg)*
			Sum :	$\Sigma U =$

Knowing ΣU, the factor of safety of the *u/s* or *d/s* slope is calculated by the equation

$$F = \frac{c' \hat{L} + \tan \phi' \Sigma (N - U)}{\Sigma T}$$

Example 23.4. *In order to find the factor of safety of d/s slope of an earth dam, during steady seepage, the section of the dam was drawn to a scale of 1 cm = 4 cm, and the following results obtained on a critical slip circle:*

Area of N-rectangle = 14.4. sq. cm.

Area of T-rectangle = 6.4 sq. cm.

Area of U-rectangle = 6.9 sq. cm

Length of arc = 12.6 cm.

Laboratory tests have furnished values of 26° for effective angle of shear resistance and 19.5 kN/m² for cohesion. Determine the factor of safety of the slope. Unit wt. of soil = 19 kN/m³.

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Solution. Scale 1 cm = 4 cm ∴ $x = 4$

Consider 1 cm length of the dam.

∴ $\Sigma N = A_N \,.\, x^2\, \gamma$, where $\gamma = 19$ kN/m^3 $= 14.4(4)^2 \times 19 = 4378$ kN

$\Sigma T = A_T \,.\, x^2\, \gamma = 6.4\,(4)^2 \times 19 = 1946$ kN

$\Sigma U = A_U \,.\, x^2\, \gamma_w = 6.9\,(4)^2 \times 9.81 = 1083$ kN

$\hat{L} = 12.6 \times 4 = 50.4$ m ; $c' = 19.5$ kN/m^2

∴ $$F = \frac{c'\hat{L} + \tan\phi' \Sigma (N - U)}{\Sigma T} = \frac{(19.5 \times 50.4) + \tan 26°(4378 - 1083)}{1946} = \mathbf{1.33.}$$

23.7 FRICTION CIRCLE METHOD

The friction circle method also assumes the failure surface as the arc of a circle. Figure 23.13 (*a*) shows a failure arc of radius *r* with *O* as the centre of rotation. If a small concentric circle is drawn with *O* as the centre and *r* sin ϕ as the radius, any line *EF* tangential to this smaller circle will cut the failure arc *AD* at an obliquity ϕ. Conversely, any vector representing reaction Δ*R* at an obliquity ϕ to an element of the failure arc *AD* will be tangential to the small circle. This small circle of radius *r* sin ϕ is, therefore, called *friction circle or* ϕ*-circle.*

The forces acting on the sliding wedge *ABDA* are : (*i*) the weight *W* of the wedge, (*ii*) the total frictional resistance or resultant *R*, and (*iii*) total cohesive resistance $c\hat{L}$ developed along the slip circle. If we divide the slip arc into elementary arcs of length Δ*L*, the elementary reaction Δ*R* of each arc, acting at an obliquity ϕ to the normal to the arc, will be tangential to the friction circle. Thus, in Fig. 23.13 (*a*), the elementary reaction Δ*R*, at points *E* and E_1 are tangential to the friction circle at point *F* and F_1 respectively. Also, the point *G* of intersection of *EF* and $E_1 F_1$ is slightly away from the circle. The resultant of these two elementary reactions *EF* and $E_1 F_1$, will thus pass through point *G*, and hence will not be tangential to the friction circle. If *R* is the resultant reaction, [Fig. 23.13 (*b*)], at an obliquity ϕ to the normal direction, it will miss the friction circle by a small margin; in fact it will be tangential to another circle (shown dotted), concentric to the friction circle, and of radius *K r* sin ϕ, where *K* is a factor depending upon the central angle δ of the slip arc. The value of *K* can be determined from Fig. 23.14. *However, in the friction circle method, it is assumed that the resultant reaction is tangential to the friction circle*. The error involved in this assumption is of small magnitude.

Let c_m = mobilised unit cohesion, assumed constant along the arc.

∴ Mobilised cohesion on elementary arc of length $\Delta L = c_m \,.\, \Delta L$

∴ Total cohesive resistance = $c_m \hat{L} = c_m \Sigma \Delta L$

If the total cohesive resistance $c_m \hat{L}$ is assumed to consist of elementary resistance $c_m\, \Delta L$, the arc *AD*, divided into a number of elementary arcs of length Δ*L*, represents a force polygon, and the chord *AD*, representing the closing side of the polygon, represents the magnitude as well as direction of the resultant of all the elementary cohesive forces.

Let $\bar{L}$ = length of chord *AD*

Total cohesive force represented by $AD = c_m\, \bar{L}$

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The position of this resultant cohesive force $c_m \bar{L}$, acting parallel to the chord AD, is such that its moment about the centre of rotation is equal to the sum of moment of elementary cohesive forces. Thus, if a is the perpendicular distance of direction of $c_m \bar{L}$ from O [Fig. 23.13 (c)], we have

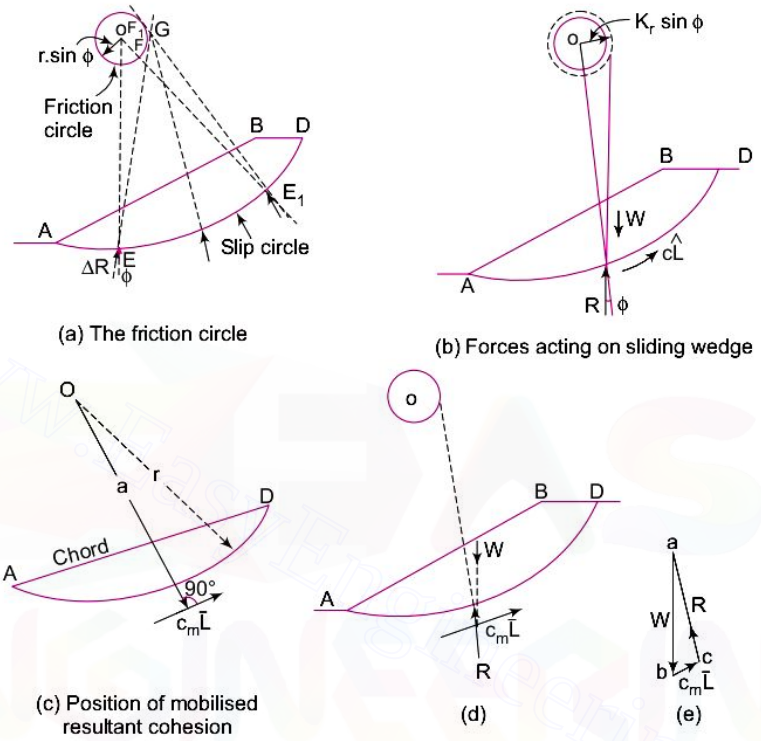

Fig. 23.13 Friction circle method

$$(c_m \bar{L})\, a = (c_m \Sigma \Delta L) r = c_m \hat{L} \cdot r \quad \therefore \quad a = r \frac{\hat{L}}{L} \qquad ...(23.22)$$

Thus, the *direction* and *location* of the resultant cohesive force $c_m \bar{L}$ is known, as marked in Fig. 23.13 (d) along with the force W. The reaction R is drawn through the point of intersection of W and $c_m \bar{L}$ in such a way that its line of action is tangential to the friction circle. In order to know that magnitude $c_m \bar{L}$ and R, force triangle abc [Fig. 23.13(e)] is constructed, in which the magnitude of W is known. Thus, from the force triangle, $c_m \bar{L}$ is known. The factor of safety F_c with respect to cohesive strength, based on the assumption that frictional strength has been fully mobilised, is given by

$$F_c = \frac{c}{c_m} \qquad ...(23.23)$$

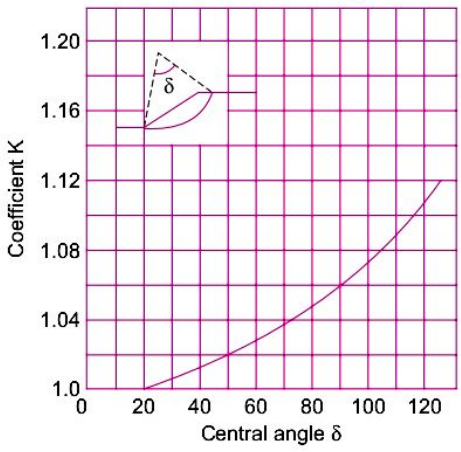

Fig. 23.14 Value of coefficient K

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A number of slip circles are taken and factor of safety for each is found. The circle giving minimum factor of safety is the critical slip circle.

23.8 TAYLOR'S STABILITY NUMBER AND STABILITY CURVES

The total cohesive force $c\hat{L}$, which resists the slipping along the slip arc at critical equilibrium, is proportional to the cohesion c and the height H of the slope. The force causing instability is the weight of the wedge which is equal to unit weight γ and the area of the wedge which is proportional to the square of the height H. Hence, the weight of the wedge is proportional to $\gamma \times H^2$. If F_c is the factor of safety with respect to cohesion,

We have $$\frac{c \times H}{F_c \times \gamma H^2} = \frac{c}{F_c \gamma H} = S_n \quad \text{...(23.24)}$$

The dimensionless quantity $\dfrac{c}{F_c \gamma H}$ is called Taylor's *Stability number* S_n

Let c_m = mobilised unit cohesion necessary for equilibrium of a slope of height H

or $$c_m = \frac{c}{F_c} \quad \text{...(23.25)}$$

Equation 23.24 may be written as $S_n = \dfrac{c}{F_c \cdot \gamma H} = \dfrac{c_m}{\gamma H}$ ...(23.26)

Also, if H_c is the critical height, the factor of safety with respect to height is also equal to the factor of safety with respect to cohesion,

i.e., $$F_c = \frac{H_c}{H} \quad \text{...(23.27}a\text{)}$$

Hence, $$S_n = \frac{c}{F_c \gamma H} = \frac{c_m}{\gamma H_c} = \frac{c_m}{\gamma H} \quad \text{...(23.27}b\text{)}$$

Using the friction circle method along with an analytical procedure, Taylor determined the values of S_n, for finite slopes, and presented tables (Table 23.5) and charts (Fig. 23.15) giving values of S_n for values of ϕ and slope angle i. Table 23.6 gives the stability number for slopes in cohesive soils ($\phi = 0$) with different depth factors.

Table 23.5 Taylor's stability number

$i \downarrow$ \ $\phi \rightarrow$	0°	5°	10°	15°	20°	25°
90°	0.261	0.239	0.218	0.199	0.182	0.166
75°	0.219	0.195	0.173	0.152	0.134	0.117
60°	0.191	0.162	0.138	0.116	0.097	0.079
45°	(0.170)	0.136	0.108	0.083	0.062	0.044
30°	(0.156)	(0.110)	0.075	0.046	0.0625	0.009
15°	(0.145)	(0.068)	(0.023)	–	–	–

[**Note.** Figures in brackets are the most dangerous circle through the toe when a more dangerous circle passing below the toe exists.]

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Table 23.6 Values of S_n for slopes in cohesive soils ($\phi = 0$) with different depth factors

Slope angle	*Stability number* S_n				
	Depth factor D_f				
	1	*1.5*	*2*	*3*	∞
90	0.261				
75	0.219				
60	0.191				
53	0.181	0.181	0.181	0.181	0.181
45	0.164	0.174	0.177	0.180	0.181
30	0.133	0.164	0.172	0.178	0.181
22.5	0.113	0.153	0.166	0.175	0.181
15	0.083	0.128	0.150	0.167	0.181
7.5	0.054	0.080	0.107	0.140	0.181

It should be noted that Taylor's stability number is based on the factor of safety F_c with respect to cohesion assuming that the frictional resistance has been fully mobilised (*i.e.*, the factor of safety F_ϕ with respect to friction is unity). The stability number does not include a friction term. When a soil possesses both cohesion and friction, the factor of safety F is to be provided with respect to cohesion as well as friction. The average shear stress s which would be developed for a given factor of safety F would be

$$s = \frac{c}{F} + \sigma \frac{\tan \phi}{F} \qquad ...(23.28)$$

In using Taylor's stability curves, and effective value $\phi_m = \tan^{-1}\left(\frac{\tan \phi}{F}\right)$ should therefore be taken instead of ϕ. As an approximation, however, ϕ_m may be taken equal to $\frac{\phi}{F}$. For example, if $\phi = 18°$ and $F = 1.5$, $\phi_m = \frac{18°}{1.5} = 12°$. The value of S_n is then taken from the table (or chart) corresponding to ϕ_m.

For purely frictional soil ($c = 0$), the stability number is zero, and Taylor's stability curves do not apply. The stability of slope then entirely depends upon the slope angle i, irrespective of the height of the slope.

In Eq. 23.28, F is the factor of safety with respect to shear strength, *i.e.*, $F = F_c = F_\varphi$. In order to determine the value F, a trial and error procedure is adopted.

To start with a value of F_φ is *assumed* and the value of $\tan \varphi_m = \tan^{-1}\left(\frac{\tan \varphi}{F_\phi}\right)$ is found. Then from Table 23.5 (or Fig. 23.15), S_n is found for this value of φ_m and slope i.

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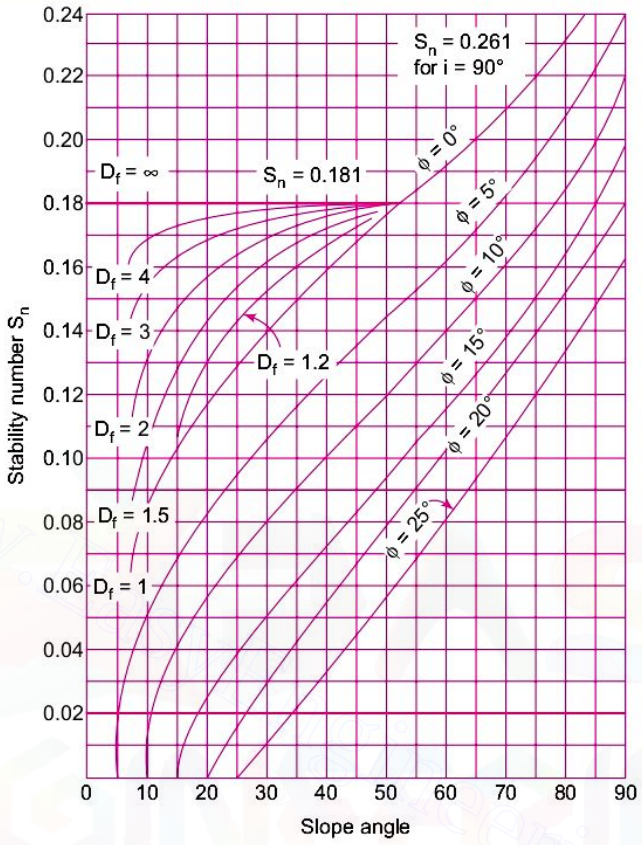

Fig. 23.15 Chart of taylor's stability number

Then from Eq. 23.27 F_c is determined by the relation, $F_c = \dfrac{c}{S_n \gamma H}$.

If this value of F_c is not equal to F_φ, another value of F_φ is assumed and the above steps are repeated to get the corresponding value of F_c. Three or Four such trials will give three or four pairs of values of (F_φ, F_c), from which a plot is made between F_φ and F_c as shown in Fig. 23.16. The point P of intersection of this curve with a 45° line gives $F_\varphi = F_c = F$, the required factor of safety with respect to shear strength defined by Eq. 23.26. See example 23.9 for illustration.

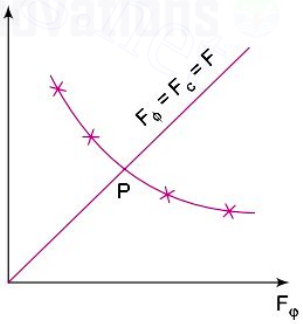

Fig. 23.16

Long term stability: For long term stability, the drained cohesion and angle of shearing resistance has to be used. Assuming no change in water content, and neglecting seepage effect, an approximate idea of the long term stability may be obtained from Taylor's stability curves.

Stability of submerged slope: When the slope is fully submerged, the submerged density γ', and *weighted frictional angle* φ_w given by the following equation, should in used for computing stability number S_n.

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$$\tan \varphi_w = \frac{\gamma'}{\gamma_{sat}} \left(\frac{\tan \varphi}{F_\varphi} \right) \qquad ...(23.29)$$

or
$$\varphi_w = \frac{\gamma'}{\gamma_{sat}} \varphi_m = \frac{\gamma'}{\gamma_{sat}} \left(\frac{\varphi'}{F_\varphi} \right) \qquad ...(23.29a)$$

If $F_\varphi = 1$, $\varphi_m = \varphi$, and hence, $\varphi_w = \frac{\gamma'}{\gamma_{sat}} \varphi$ ...(23.29*b*)

For long term stability, the drained cohesion and angle of shearing resistance have to be used. Assuming no change in water content and neglecting seepage effects, an approximate idea of the long term stability may be obtained from Taylor's stability curves. When the slope is fully submerged, submerged density γ' should be used in Eq. 23.27 and if the slope is saturated by capillary water, γ_{sat} should be used. In the case of sudden drawdown, the *weighted frictional angle* ϕ_w (given by Eq. 23.30) should be used in finding the stability number, and γ_{sat} should be used in Eq. 23.28.

$$\phi_w = \frac{\gamma'}{\gamma_{sat}} \phi \qquad ...(23.29c)$$

Example 23.5. *Calculate the factor of safety with respect to cohesion, of a clay slope laid at 1 in 2 to a height of 10 m, if the angle of internal friction $\phi = 10°$; $c = 25$ kN/m^2 and $\gamma = 19$ kN/m^3. What will be the critical height of the slope in this soil?*

Solution. $i = \tan^{-1}\left(\frac{1}{2}\right) = 26.5°$ For $i = 26.5°$ and $\phi = 10°$, $S_n = 0.064$

But $S_n = \frac{c}{F_c \gamma H}$ $\therefore$ $F_c = \frac{c}{S_n \gamma H} = \frac{25}{0.064 \times 19 \times 10} = \mathbf{2.06}$

The critical height H_c is given by

$$H_c = F_c \cdot H = 2.06 \times 10 = \mathbf{20.6}\ \text{m}$$

Alternatively : $H_c = \frac{c}{\gamma S_n} = \frac{25}{19 \times 0.064} = \mathbf{20.6\ m}.$

Example 23.6. *A slope is to be constructed at an inclination of 30° with the horizontal. Determine the safe height of the slope at factor of safety of 1.5. The soil has the following properties: $c = 15$ kN/m^2, $(\phi) = 22.5°$ and $\gamma = 19$ kN/m^3.*

Solution. The mobilised frictional angle ϕ_m is given by

$$\phi_m = \frac{\phi}{F} = \frac{22.5}{1.5} = 15° \qquad ...(23.30)$$

For $i = 30°$ and $\phi_m = 15°$, $S_n = 0.046$

Now, $S_n = \frac{c}{F \cdot \gamma H}$ $\therefore$ $H = \frac{c}{S_n F \cdot \gamma} = \frac{15}{0.046 \times 1.5 \times 19} \approx \mathbf{11.5\ m}.$

Example 23.7. *A new canal is excavated to a depth of 5 m below ground level, through a soil having the following characteristics: $c = 14$ kN/m^2; $\phi = 15°$; $e = 0.8$ and $G = 2.70$. The slope of banks is 1 in 1. Calculate the factor of safety with respect to cohesion when the canal runs full. If it is suddenly and completely emptied, what will be the factor of safety ?*

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Solution.
$$\gamma_{sat} = \frac{G+e}{1+e}\gamma_w = \frac{2.7+0.8}{1+0.8} \times 9.81 = 19.08 \text{ kN/m}^3$$
$$\gamma' = 19.08 - 9.81 = 9.27 \text{ kN/m}^3$$
$$i = 45° \; ; \phi = 15°$$

(*i*) ***Submerged case:*** For $i = 45°; \phi = 15° ; S_n = 0.083$

$$\therefore \quad F_c = \frac{c}{\gamma' H S_n} = \frac{14}{9.27 \times 5 \times 0.083} = \mathbf{3.64}$$

(*ii*) ***Drawdown case:*** Taking $F_\phi = 1$ and $\phi_m = \phi$

$$\phi_w = \frac{\gamma'}{\gamma_{sat}}\phi_m = \frac{\gamma'}{\gamma_{sat}}\phi = \frac{9.27}{19.08} \times 15 \approx 7.3°$$

For $i = 45°$ and $\phi = 7.3, S_n = 0.122$

$$\therefore \quad F_c = \frac{c}{\gamma_{sat} H S_n} = \frac{14}{19.08 \times 5 \times 0.122} = \mathbf{1.2.}$$

Example 23.8. *A temporary cutting 8 m deep is to be made in a clay having a unit weight of 18 kN/m^3 and an average cohesion of 20 kN/m^2. A hard stratum of rock exists at a depth of 12 m below the ground surface. Use Taylor's stability curves to estimate if a 30° slope is safe. If a factor of safety of 1.25 is considered necessary, find the safe slope angle.*

Solution. Depth factor $D_f = \frac{12}{8} = 1.5$

From Taylor's stability curves for $D_f = 1.5, i = 30°, S_n = 0.163$

$$\therefore \quad F_c = \frac{c}{S_n \gamma H} = \frac{20}{0.163 \times 18 \times 8} = \mathbf{0.85}$$

The proposed slope is therefore, unsafe. For $F_c = 1.25$, the required stability number is

$$S_n = \frac{c}{F_c \gamma H} = \frac{20}{1.25 \times 18 \times 8} = 0.11$$

Form Taylor's curves, for $S_n = 0.11$ and $D_f = 1.5$, we get $i = \mathbf{12°}$.

Example 23.9. *A slope is 12 m high, and has an inclination of 30°. If the soil of the slope has c = 25 kN/m^2, φ = 12° and γ = 18.6 kN/m^3, determine the factor of safety with respect to shear strength.*

Solution. The factor of safety F ($= F_c = F_\varphi$) is found by trial.

***First trial*:** Let $F_\varphi = 1.2$

$$\tan \varphi_m = \frac{\tan \varphi}{F_\varphi} = \frac{\tan 12°}{1.2} = 0.1771$$

$$\therefore \quad \varphi_m = 10.04°$$

From Table 23.5, for $i = 30°$ and $\varphi = \varphi_m = 10.04$

we get $S_n = 0.0745 = \frac{c}{F_c \gamma H}$

$$\therefore \quad F_c = \frac{25}{0.0745 \times 18.6 \times 12} = 1.503$$

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Second trial: Similarly, for $F_\varphi = 1.4$, we get $F_c = 1.32$

Third trial: For $F_\varphi = 1.3$, we get $F_c = 1.4$

Fourth trial: For $F_\varphi = 1.35$, we get $F_c = 1.358$

From these four sets of values of F_φ and F_c, a curve is drawn. The intersection of this curve with the 45° line gives $F_c = F_\varphi = 1.36$

Hence, $F = F_c = F_\varphi \approx \mathbf{1.36.}$

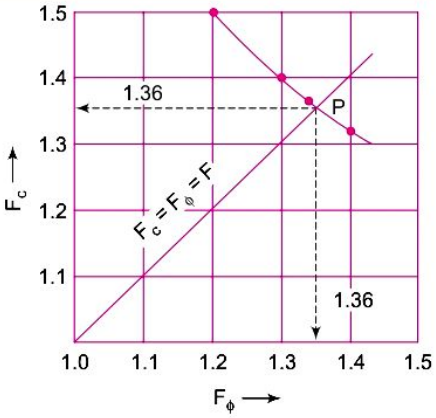

Fig. 23.17

Example 23.10. *Figure 23.18 shows an earth slope of clayey soil having $c = 55$ kN/m^2 and $\varphi = 0$. Corresponding to a trial slip circle AB, we have the following data:*

(i) Radius of slip circle = 19 m
(ii) Weight of wedge ABD = 2050 kN
(iii) Distance of W from AO = 9 m
(iv) Angle δ subtended by the arc AB at the centre = 64.5°

Determine the factor of safety against sliding along the slip surface.

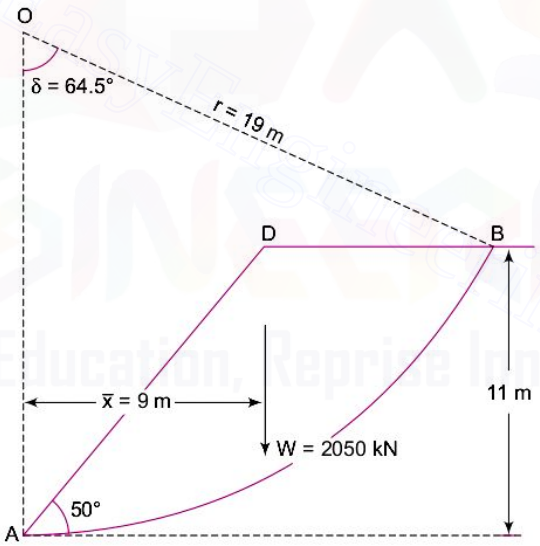

Fig. 23.18

Solution. Length of arc AB,

$$\hat{L} = \frac{2\pi r\delta}{360°} = \frac{2\pi(19)\,64.5°}{360°}$$

$$= 21.39 \text{ m}$$

Disturbing moment $M_D = W\bar{x} = 2050 \times 9 = 18450$ kN m

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Resisting moment $M_R = C\hat{L}\ .\ R = 55 \times 21.39 \times 19 = 22353$ kN m

$\therefore$ $F = \dfrac{M_R}{M_D} = \dfrac{22353}{18450} = \mathbf{1.212}$

Alternatively : $F = \dfrac{c_u}{c_m}$

where c_m = mobilised shear resistance of soil ($\varphi_u = 0$) necessary for equilibrium

Now, $c_m\,\hat{L}\,R = W\bar{x}$ or $c_m = \dfrac{W\bar{x}}{\hat{L}R} = \dfrac{2050 \times 9}{21.39 \times 19} = 45.4$ kN/m^2

$\therefore$ $F = \dfrac{c}{c_m} = \dfrac{55}{45.4} = \mathbf{1.212}$

Example 23.11. *A Soil mass EBD, having c = 8 kN/m^2, φ = 20° and γ = 19 kN/m^3 is resting on an inclined impermeable clay layer, as shown in Fig. 23.19. Determine the factor of safety against wedge failure along the interface EB.*

Solution. $a = EB_1 = 3.5/\tan 30° = 6.062$ m

$b = EB_1 - ED_1$

$= 6.062 - 3.5/\tan 50° = 3.125$ m

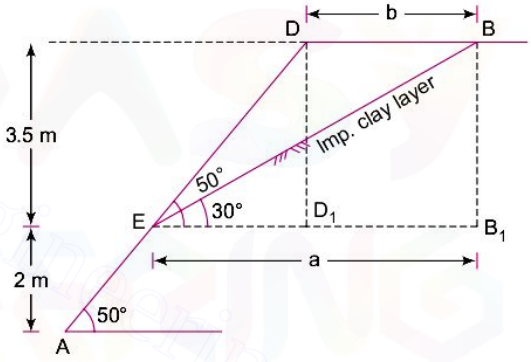

Fig. 23.19

Weight of triangular wedge EBD

$= W = \dfrac{1}{2} \times 3.5 \times 3.125 \times 19$

$= 103.91$ kN

$\therefore$ Sliding component of W parallel to sliding plane EB

$= T = W \sin 30° = 103.91 \sin 30°$

$= 51.96$ kN ...(1)

Now, length L of plane EB

$= \dfrac{3.5}{\sin 30°} = 7$ m

Resistance to sliding $S = cL + N \tan \varphi = 8 \times 7 + (103.91 \cos 30°) \tan 20°$

$= 56 + 32.75 = 88.75$ kN ...(2)

$\therefore$ Factor of safety, $F = \dfrac{S}{T} = \dfrac{88.75}{51.96} = \mathbf{1.71}$

Example 23.12. *A 6 m high embankment is required for the construction of a railway track. The clay to be used for the construction of the embankment has the following values: c = 18 kN/m^2, φ = 0 and γ = 19.5 kN/m^3. Hard rocky stratum is found to exist at 3 m below the ground level. Determine the critical maximum side slope angle for the embankment.*

Solution. $S_n = \dfrac{c}{F\gamma H}$

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Since maximum critical slope angle is required, $F = 1$

$$\therefore \quad S_n = \frac{c}{\gamma H} = \frac{18}{19.5 \times 6} = 0.154$$

Also, since hard stratum is available at 3 m below G.L., we have

$$D_f = \frac{6+3}{6} = 1.5$$

Hence, for $D_f = 1.5$ and $S_n = 0.154$, the slope angle i can be found either from Table 23.6 or from Fig. 23.15. Thus, we get $i \approx 22.5°$.

Example 23.13. *A 25 m high earth dam is to be built from a soil having an effective angle of internal friction of 30° and a cohesive strength of 20 kN/m^2. The compacted moist unit weight of the soil is 19 kN/m^3 and the saturated unit weight is 20.2 kN/m^3. Calculate the steepest angle at which the upstream slope of the dam may be inclined to the horizontal, for a factor of safety of 1.3.*

Solution. The worst case of the design of the upstream slope of an earth dam is the sudden drawdown case. For such a case, the weighted angle (φ_w) for angle of friction should be used. Taking $F_\varphi = F_1$, we get from Eq. 23.29 (a).

$$\varphi_w = \frac{\gamma'}{\gamma_{sat}} \phi_m, \quad \text{where} \quad \varphi_m = \tan^{-1}\left(\frac{\tan\varphi}{F}\right) \tan^{-1}\left(\frac{\tan 30°}{1.3}\right) = 23.95°$$

Hence, $$\varphi_w = \frac{20.2 - 9.81}{20.2} \times 23.95 = 12.32°$$

Also, $$c_m = \frac{c}{F} = \frac{20}{1.3} = 15.38 \text{ kN/m}^2$$

$$\therefore \quad S_n = \frac{c_m}{\gamma_{sat} \cdot H} = \frac{15.38}{20.3 \times 25} \approx 0.03$$

Hence, for $S_n = 0.03$ and $\varphi = \varphi_w = 12.32°$, the value of i can be found either from Fig. 23.15 or from Table 23.5. Thus, we get $\boldsymbol{i \approx 18°}$.

Example 23.14. *A 45° slope has been excavated to a depth of 6 m in a saturated clay having the following properties : $c_u = 50$ kN/m^2, $\varphi_u = 0$ and $\gamma = 19$ kN/m^3. For a trial slip surface shown in Fig. 23.20 determine the factor of safety.*

Also determine the minimum value of factor of safety for the given slope.

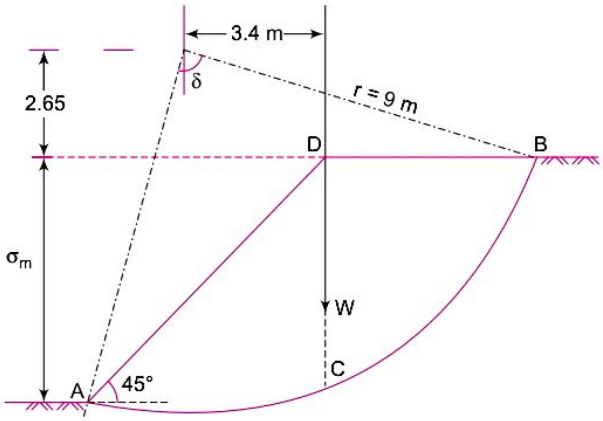

Fig. 23.20

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Solution. In order to find the area of the wedge *ABD*, let us plot all the given geometrical data on a graph paper, to some suitable scale, say 1 cm = 1 m. From the plot, we get central angle $\delta = 90°$ and area of wedge = 39 m^2.

$\therefore$ $W = \text{area} \times \text{length} \times \gamma = 39 \times 1 \times 19 = 741$ kN/m run.

Length of arc $AB = r \,.\, \theta \text{ (rad.)} = 9 \times \dfrac{\pi}{180} \times 90° = 14.14$ m

$\therefore$ Total cohesive force developed along the arc $= C = c\hat{L} = 50 \times 14.14 = 706.8$ kN/m

Taking moments about the centre *O*, we get

Disturbing moment, $MD = 3.4\ W = 3.4 \times 741 = 2519.4$ kN m/m run

Restoring moment, $M_R = r \,.\, C = 9 \times 706.8 = 6361.2$ kN m/m run

$\therefore$ Factor of safety, $F_c = \dfrac{M_R}{M_D} = \dfrac{6361.2}{2519.2} = \mathbf{2.53}$

***(a) Minimum factor of safety*:** Evidently, the above circle may not be critical and hence the above F.S. may not be minimum. The minimum factor of safety is computed with respect to the Taylor's stability number S_n corresponding to $i = 45°$, $\varphi = 0$ and $D_f = \infty$, assuming the hard stratum to be available at very large depth. Hence, from Table 23.6, we get $S_n = 0.181$.

But $S_n = \dfrac{c_m}{\gamma H} = \dfrac{c}{F_c \cdot \gamma H}$ $\therefore$ $F_c = \dfrac{c}{S_n \cdot \gamma H} = \dfrac{50}{0.181 \times 19 \times 6} = \mathbf{2.423}$

Hence, minimum factor of safety = 2.423

Example 23.15. *Figure 23.21 shows a slip surface with radius of 16.5 m in a slope with a height of 10.5 m and an angle of inclination of 45°. If $c = 36$ kN/m^2, $\varphi = 16°$ and $\gamma = 19$ kN/m^3, determine the factor of safety with respect to cohesion, using friction circle method. The weight of soil wedge ABD is 900 kN and it acts at a horizontal distance of 7.7 m from A.*

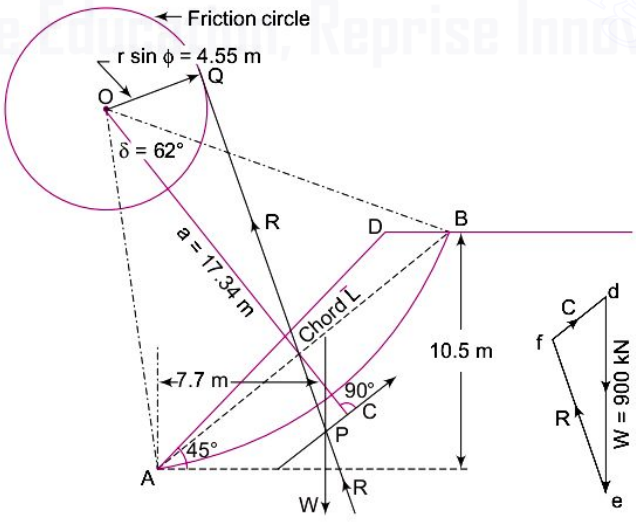

Fig. 23.21

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Solution. Radius of φ-circle = $r \sin \varphi = 16.5 \sin 16° = 4.55$ m

Hence, on the drawing showing the slope and slip surface, draw the φ circle with radius = 4.55 m.

Now, $$\hat{L} = 2\pi r \times \frac{62°}{360°} = 2\pi \times 16.5 \times \frac{62°}{360°} = 17.855 \text{ m}$$

$$\bar{L} = 2r \sin \delta/2 = 2 \times 16.5 \sin 62°/2 \approx 16.996$$

$\therefore$ $$a = r \frac{\hat{L}}{\bar{L}} = 16.5 \times \frac{17.855}{16.996} = 17.34 \text{ m}$$

Hence, draw a line parallel to the chord AB, at a distance of $a = 17.34$ m. This line, representing $C = c_m \bar{L}$, will intersect in a point P with the vertical line representing weight W. Through P, draw a line PQ tangential to the friction circle. Thus, the direction of reaction R will be along the line PQ.

Now, in order to plot the force triangle (Fig. 23.21(b)), draw vertical line de representing $W = 900$ kN to some scale. Now, through e draw a line parallel to R, and similarly through d, draw a line parallel to the force C; both these lines will intersect in point f. Thus, fd represents cohesive force C to the same scale.

By measurement, we get $C = fd = 300$ kN. Now, $C = c_m \cdot \bar{L}$

$\therefore$ $$c_m = \frac{C}{\bar{L}} = \frac{300}{16.996} = 17.65 \text{ kN/m}^2$$

$\therefore$ Factor of safety $$F_c = \frac{c}{c_m} = \frac{36}{17.65} = \mathbf{2.04}$$

Example 23.16. *For a given failure surface shown in Fig. 23.22, determine the factor of safety in terms of total stress. The unit weight for both soils 1 and 2 is 19 kN/m^3. For soil 1, the relevant shear strength parameters are $c_u = 20$ kN/m^2 and $\varphi_u = 0$ and for soil 2, $c_u = 35$ kN/m^2 and $\varphi_u = 0$. What is the factor of safety if allowance is made for the development of tension crack which fills with water?*

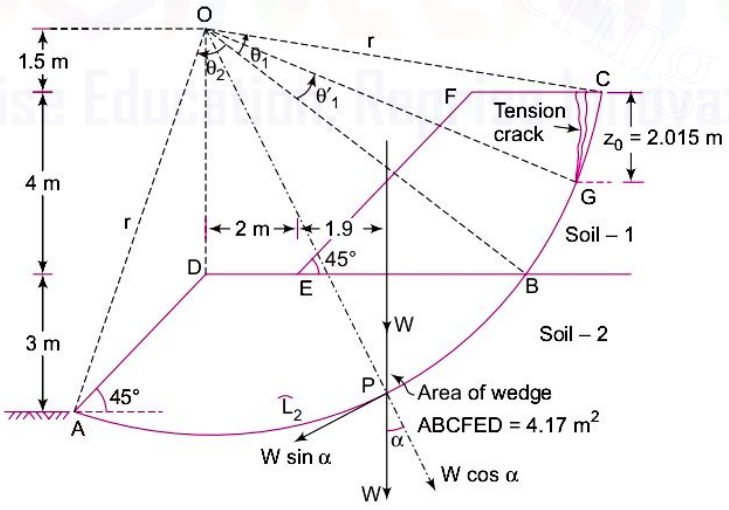

Fig. 23.22

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Solution. The problem is solved graphically by plotting the given geometrical data. From the plot, we get the following :

$$r = 9 \text{ m} ; \theta_1 = 28° ; \theta_2 = 72°; \alpha = 25.5°$$

Length of arc $$BC = \hat{L}_1 = 2\pi r \frac{\theta_1}{360°} = 2\pi\,(9) \times \frac{28°}{360°} = 4.4 \text{ m}$$

Length of arc $$AB = \hat{L}_2 = 2\pi r \frac{\theta_2}{360°} = 2\pi\,(9) \times \frac{72°}{360°} = 11.31 \text{ m}\,.$$

Weight of wedge, W = area × length × unit weight = $41.7 \times 1 \times 19 = 792.3$ kN/m run.

$$\therefore \quad M_R = r\left[c_1 \hat{L}_1 \times 1 + c_2 \hat{L}_2 \times 1\right]$$

$$= 9\,[20 \times 4.4 \times 1 + 35 \times 11.31 \times 1] = 4355 \text{ kN-m/m}$$

Disturbing moment, $M_D = (W \sin \alpha)\,.\,r = (792.3 \sin 25.5°)\,9 = 3070$ kN-m/m

$$\therefore \quad \text{Factor of safety} = \frac{M_R}{M_D} = \frac{4355}{3070} = \mathbf{1.42}$$

(*a*) ***Factor of safety under tension cracks:*** If tension cracks develop and water fills in these, hydrostatic pressure P_w will act on the portion GC at a height $z_0/3$ from point G. Hence, shear resistance will not be available for the arc portion GC. The effective arc length for soil 1 will be BG only, for which the angle θ_1', will be lesser than θ_1.

Now, $$z_0 = \frac{2c}{\gamma} \text{ (when } \varphi = 0) = \frac{2 \times 20}{19} = 2.105 \text{ m.}$$

Hence, locate point G at 2.105 m below the level FC.

Then angle $\theta_1' \approx 14°$ (By measurement from Fig. 23.22)

$$\therefore \quad \hat{L}_1' = 2\pi r \frac{\theta_1'}{360} = 2\pi\,(9) \frac{14°}{360°} = 2.2 \text{ m}$$

$$\therefore \quad M_R = 9\,(20 \times 2.2 \times 1 + 35 \times 11.31 \times 1) = 3958 \text{ kN-m/m run}$$

The disturbing moment will however remain the same.

$$\therefore \quad \text{Factor of safety} = \frac{3958}{3070} = \mathbf{1.29.}$$

23.9 BISHOP'S METHOD OF STABILITY ANALYSIS

Bishop (1955) took into consideration the forces acting on the sides of the slices, which were neglected in the Swedish method. The slip surface is assumed to be an arc of a circle and the factor of safety against sliding is defined as the ratio of the actual shear strength of soil to that required to maintain limiting equilibrium (*i.e.*, mobilised shear strength):

$$F = \frac{\tau_f}{\tau} \qquad \ldots(23.31a)$$

where τ_f = shear strength ; τ = mobilised shear

His method also takes into account the pore pressure acting on the slice.

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Let E_n and E_{n+1} = resultant horizontal forces on the section n and $n + 1$ respectively

X_n and X_{n+1} = resultant vertical shear forces

W = weight of slice

P = total normal force acting at the base of the slice

S = shear force acting along the base of the slice

z = height of the slice

l = length of the arc ab of the slice

b = horizontal width of the slice

θ = angle of the base ab of the slice with horizontal

x = horizontal distance of the slice from the centre of the rotation.

The normal stress σ on the slice = $\frac{P}{l}$.

If u is the pore pressure, the effective stress is

$$\sigma' = \sigma - u = \frac{P}{l} - u \qquad ...(23.31)$$

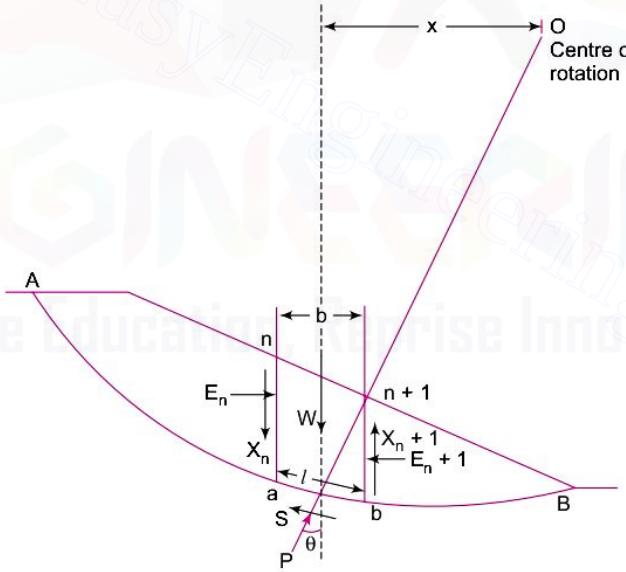

Fig. 23.23 Bishop's analysis

Now, $$F = \frac{\tau_f}{\tau} \quad \therefore \quad \tau = \frac{1}{F} \cdot \tau_f = \frac{1}{F}\left[c' + (\sigma - u)\tan\phi'\right] \qquad ...(23.32)$$

Hence, shear force $$S = \tau . l = \frac{1}{F}\left[c' + \left(\frac{P}{l} - u\right)\tan\phi'\right] l$$

or $$S = \frac{1}{F}\left[c' l + (P - ul)\tan\phi'\right] \qquad ...(23.33)$$

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Resolving forces on the slice vertically, we get

$$P\cos\theta + S\sin\theta = W + X_n - X_{n+1} \qquad ...(23.34)$$

Substituting $P = P' + ul$ and the value of S from Eq. 23.33 and simplifying we get

$$P' = \frac{W + X_n - X_{n+1} - ul\cos\theta - \dfrac{c'}{F} l\sin\theta}{\cos\theta + \sin\theta\dfrac{\tan\phi'}{F}} \qquad ...(23.35)$$

Consider the equilibrium of the whole sliding wedge $AabB$. Since no external force acts on the surface of the slope, we have

$$\Sigma W x = \Sigma S r = \Sigma \tau\, lr$$

$$\therefore \qquad \Sigma S = \frac{\Sigma W x}{r} \qquad ...(23.36)$$

Hence, from Eqs. 23.33 and 23.36, we have

$$F = \frac{r}{\Sigma W x}\Sigma\left[c'l + (P - ul)\tan\phi'\right] \qquad ...(23.37)$$

$$= \frac{r}{\Sigma W x}\Sigma\left[c'l + P'\tan\phi'\right] \qquad ...(23.38)$$

Substituting the value of P' from Eq. 23.35, we get

$$F = \frac{r}{\Sigma W x}\sum\left[c'l + \tan\phi'\frac{\left\{W + X_n - X_{n+1} - ul\cos\theta - \dfrac{c'}{F} l\tan\theta\right\}}{\cos\theta + \dfrac{\sin\theta\tan\phi'}{F}}\right] \qquad ...(23.39)$$

Substituting $x = r\sin\theta$, $b = l\cos\theta$

$$\frac{ub}{W} = r_u \qquad \left(\text{where } r_u = \frac{u}{\gamma z}.\right)$$

and $X_n - X_{n+1} \approx 0$, we get

$$F = \frac{1}{W\sin\theta}\sum\left[\left\{c'b + W(1 - r_u)\tan\phi'\right\}\frac{\sec\theta}{1 + \dfrac{\tan\theta\tan\phi'}{F}}\right] \qquad ...(23.40)$$

For partly submerged slope, a similar treatment leads to the following expression :

$$F = \frac{1}{\Sigma(W_1 + W_2)\sin\theta}\sum\left[\left\{c'b + (W_1 + W_2 - \bar{u}b)\tan\phi'\right\} \times \frac{\sec\theta}{1 + \dfrac{\tan\theta\tan\phi'}{F}}\right] \qquad ...(23.41)$$

where W_1 = full weight of the slice above the free water surface

W_2 = submerged weight of soil in the soil below the free water surface

$\bar{u}$ = pore water pressure expressed as an excess over the hydrostatic pressure corresponding to the water level outside the slope.

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Equations 23.40 and 23.41 contain factor of safety in both sides. The solution is that of trial and error. To avoid necessary trial computations, Bishop and Morgenstern (1960) gave the following expression for the factor of safety :

$$F = m - nr_u \qquad ...(23.42)$$

when m and n are *stability coefficients,* to be obtained from charts prepared by them, for various values of $\frac{c}{\gamma H}$, D_f (depth factor) and r_u.

During and immediately after the end of construction, the pore pressure ratio r_u may be written as

$$r_u = \frac{u}{\gamma z} = \frac{u_i}{\gamma z} + \frac{\Delta u}{\gamma z} = \frac{u_i}{\gamma z} + \frac{\bar{B} \cdot \Delta \sigma_1}{\gamma z}$$

Taking $u_i \approx 0$ at optimum water content, and substituting $\Delta \sigma_1 = \gamma z$, we get

$$r_u \approx \bar{B} \qquad ...(23.43)$$

where $\bar{B}$ = overall pore pressure coefficient = $\frac{\Delta u}{\Delta \sigma_1}$

From the Skempton's pore pressure equation,

$$\Delta u = B\,[\Delta \sigma_3 + A\,(\Delta \sigma_1 - \Delta \sigma_3)]$$

$$\bar{B} = \frac{\Delta u}{\Delta \sigma_1} = B\left[\frac{\Delta \sigma_3}{\Delta \sigma_1} + A\left(1 - \frac{\Delta \sigma_3}{\Delta \sigma_1}\right)\right] \qquad ...(23.44)$$

Putting $K = \frac{\Delta \sigma_3'}{\Delta \sigma_1'} = \frac{\Delta \sigma_3 - \Delta u}{\Delta \sigma_1 - \Delta u}$, we get $\bar{B} = \frac{B\left[K + A(1-K)\right]}{1 - B(1-A)(1-K)} = r_u$ ...(23.45)

23.10 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 23.17. *Stability analysis by the method of slices for 1:1 slope on the critical slip gave the following results :*

Sum of tangential forces = 150 kN; sum of normal forces = 320 kN

Sum of neutral forces = 50 kN; length of failure surface = 18 m

Effective angle of shearing resistance = 15°, Effective cohesion = 20 kN/m^2

Calculate the factor safety with respect to the shear strength. **(Civil Services Exam. 2002)**

Solution. $$\text{F.S.} = \frac{c\hat{L} + (\Sigma N - \Sigma U)\tan\varphi}{\Sigma T} = \frac{20 + 18 + (320 - 50)\tan 15°}{150} = \mathbf{2.882}$$

Example 23.18. *A slope is to be constructed in a soil for which $\bar{c} = 0$ and $\bar{\varphi} = 36°$. It is to be assumed that the water level may occasionally reach the surface of a slope, with seepage taking place parallel to the slope. Determine the maximum slope angle for a factor of safely 1.5, assuming a potential failure surface parallel to the slope. What would be the factor of safety of the slope, constructed at this angle, if the water table should be well below the surface? The saturated unit weight of the soil is 19 kN/m^3.*

(Gate Exam. 1994)

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Solution.

(a) Seepage parallel to slope

$$\sigma_z = \frac{W}{b} = \gamma_{sat} \cdot z \cos i$$

$$\sigma = \sigma_z \cos i = \gamma_{sat}\, z \cos^2 i$$

$$\tau = \sigma_z \sin i = \gamma_{sat}\, z \cos i \sin i$$

u = pore pressure $= \gamma_w z \cos^2 i$

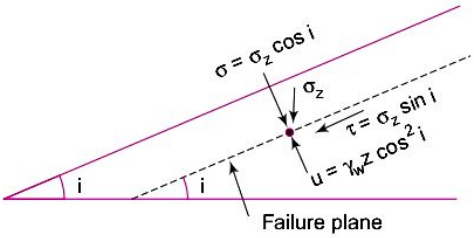

Fig. 23.24

$$\therefore \quad F = \frac{\tau_f}{\tau} = \frac{(\sigma - u)\tan\varphi}{\tau} = \frac{\gamma' z \cos^2 i \tan\varphi}{\gamma_{sat}\, z \cos i \sin i} = \frac{\gamma' \tan\varphi}{\gamma_{sat} \tan i} \qquad ...(23.46a)$$

Here, $\gamma_{sat} = 19\ \text{kN/m}^3$; $\gamma' = 19 - 9.81 = 9.19\ \text{kN/m}^3$

$$\therefore \quad F = 1.5\ \text{(given)} = \frac{9.19}{19} \times \frac{\tan 36°}{\tan i}$$

$$\therefore \quad \tan i = \frac{9.19}{19} \times \frac{\tan 36°}{1.5} = 0.2343, \text{ from which } i = \mathbf{13.19°}$$

(b) No seepage : Water table well below the surface : For no seepage, $u = 0$

$$F = \frac{\tau_f}{\tau} = \frac{\sigma \tan\varphi}{\tau} = \frac{\gamma_{sat} \cdot z \cos^2 i \tan\varphi}{\gamma_{sat}\, z \cos i \sin i} = \frac{\tan\varphi}{\tan i} = \frac{\tan 36°}{\tan 13.19°} = \mathbf{3.10}$$

Example 23.19. *For the failure surface, determine the factor of safety in terms of total stress for the slope given in Fig. 23.25. The unit weight for both soils 1 and 2 is 19 kN/m³. For soil 1, the relevant shear strength parameter are $c_u = 20\ \text{kN/m}^2$ and $\varphi_u = 0$ and for soil 2, $c_u = 35\text{kN/m}^2$ and $\varphi_u = 0$.*

(Gate Exam. 1996)

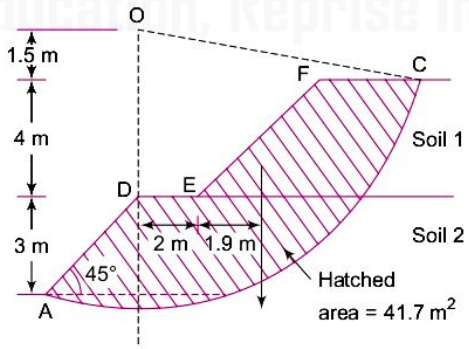

Fig. 23.25

Solution. This problem is exactly the same as first part of example 23.16. For solution, see first part of example 23.16.

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PROBLEMS

1. A 6 m deep cut is to be made in cohesive soil with a slope of 1:1. The soil has c_u = 30 kN/m^2 ; ϕ_u = 10° and γ = 18 kN/m^3. Find the factor of safety with respect to cohesion. What will be the critical height of the slope in this soil ? [**Ans.** 2.57 ; 15.4 m]
2. A cutting 5 m deep, is made in a clay at a slope of 45°. The bulk-density of clay is 18.2 kN/m^3 and the angle of shearing resistance is 10°. What is the value of cohesion necessary to give a factor of safety of 1.5 with respect to cohesion ? [**Ans.** 13.65 kN/m^2]
3. A canal, 3 m deep runs through a soil having the following properties: c_u = 10 kN/m^2 ; ϕ_u = 10° ; e = 0.8 and G = 2.72. The angle of slope of the banks is 45°. Determine the factor of safety with respect to cohesion when the canal is full up to the top of banks. What will be the factor of safety in case of sudden drawdown? [**Ans.** 3.23 ; 1.24]
4. A trench with vertical sides is to be excavated in a purely cohesive soil (ϕ = 0) whose bulk density is 19 kN/m^3, and apparent cohesion is 25 kN/m^2. Determine the intial depth up to which the trench can be excavated without any lateral support. Solve the problem according to Taylor's Stability number and also according to Bell's equations. [**Ans.** 5.03; 5.26 m]

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Part VI
Foundation Engineering

24. Bearing Capacity
25. Shallow Foundations
26. Pile Foundations
27. Well Foundations
28. Machine Foundations

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CHAPTER 24

Bearing Capacity

24.1 DEFINITIONS

1. **Footing.** A footing is a portion of the foundation of a structure that transmits loads directly to the soil.
2. **Foundation.** A foundation is that part of the structure which is in direct contact with and transmits loads to the ground.
3. **Foundation soil.** It is the upper part of the earth mass carrying the load of the structure.
4. **Bearing capacity.** The supporting power of a soil or rock is referred to as its bearing capacity. The term bearing capacity is defined after attaching certain qualifying prefixes, as defined below.
5. **Gross pressure intensity (q).** The gross pressure intensity q is the total pressure at the base of the footing due to the weight of the superstructure, self-weight of the footing and the weight of the earth fill, if any.
6. **Net pressure intensity (q_n).** It is defined as the excess pressure, or the difference in intensities of the gross pressure after the construction of the structure and the original overburden pressure. Thus, if D is the depth of footing

$$q_n = q - \bar{\sigma} = q - \gamma D \qquad ...(24.1)$$

where γ is the average unit weight of soil above the foundation base.

7. **Ultimate bearing capacity (q_f).** The ultimate bearing capacity is defined as the minimum gross pressure intensity at the base of the foundation at which the soil fails in shear. (When the term bearing capacity is used in this book without any prefix, it may be understood to be ultimate bearing capacity).
8. **Net ultimate bearing capacity (q_{nf}).** It is the minimum net pressure intensity causing shear failure of soil. The *ultimate bearing capactiy* q_f and the *net ultimate capacity* are evidently connected by the following relation:

$$q_f = q_{nf} + \bar{\sigma} \qquad ...(24.2)$$

or

$$q_{nf} = q_f - \bar{\sigma} \qquad ...(24.2a)$$

where $\bar{\sigma}$ is the *effective surcharge* at the base level of the foundation.

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9. **Effective surcharge at the base level of foundation ($\bar{\sigma}$).** It is the intensity of vertical pressure at the base level of foundation, computed assuming total unit weight for the portion of the soil above the water table and submerged unit weight for the portion below the water table.
10. **Net safe bearing capacity (q_{ns}).** The net safe bearing capacity is the net ultimate bearing capacity divided by a factor of safety F

$$q_{ns} = \frac{q_{nf}}{F} \qquad ...(24.3)$$

11. **Safe bearing capacity (q_s).** The maximum pressure which the soil can carry safely without risk of shear failure is called the safe bearing capacity. It is equal to the net safe bearing capacity plus original overburden pressure:

$$q_s = q_{ns} + \gamma D = \frac{q_{nf}}{F} + \gamma D \qquad ...(24.4)$$

Sometimes, the safe bearing capacity is also referred to as the ultimate bearing capacity q_f divided by a factor of safety F.

12. **Safe bearing pressure or Net soil pressure for specified settlement.** It is the intensity of loading that will cause a permissible settlement or specified settlement for the structure.
13. **Allowable bearing capacity or pressure (q_a).** It is the net loading intensity at which neither the soil fails in shear nor there is excessive settlement detrimental to the structure.

24.2 MINIMUM DEPTH OF FOUNDATION: RANKINE'S ANALYSIS

Rankine considered the equilibrium of two soil elements, one immediately below the foundation (element I), and the other just beyond the edge of the footing (element II), but adjacent to element I. When the load on the footing increases, and approaches a value q_f, a state of plastic equilibrium is reached under the footing. For the shear failure of element I, element II must also fail by lateral thrust from element I. During the state of shear failure (plastic equilibrium), the following principal stress relationship exists.

$$\sigma_1 = \sigma_3 \tan^2 \alpha + 2c \tan \alpha \qquad ...(24.5)$$

For cohesionless soil $\sigma_1 = \sigma_3 \tan^2 \alpha$ ...(24.5*a*)

For element II : $\sigma_3 = \sigma_v = \gamma D$

$\therefore$ $\sigma_1 = \sigma_h = \gamma D \tan^2 \alpha$ ...(1)

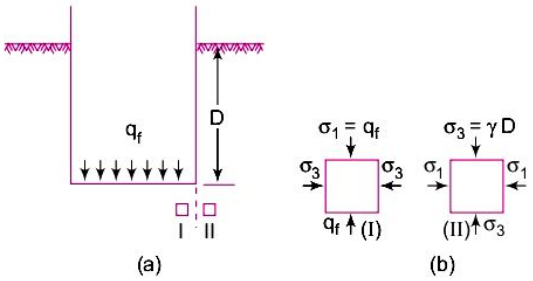

Fig. 24.1 Rankine's analysis

For element I :

$\therefore$ $\sigma_3 = \sigma_h = \sigma_1$ of element II

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$$\sigma_3 = \gamma D \tan^2 \alpha$$

$$\sigma_1 = \sigma_3 \tan^2 \alpha = \gamma D \tan^4 \alpha$$

But $\sigma_1 = q_f$

$$\therefore \quad q_f = \gamma D \tan^4 \alpha = \gamma D \left[\frac{1+\sin\phi}{1-\sin\phi}\right]^2 \qquad ...(24.6)$$

In the effective stress analysis the above expression reduces to

$$q_f = \gamma D \left[\frac{1+\sin\phi'}{1-\sin\phi'}\right]^2 \qquad ...(24.7)$$

Equation 24.6 gives the bearing capacity of cohesionless soils as zero at the ground surface. This is not consistent with the general experience. Equation 24.6 may be used in the following form to get the minimum depth of foundation.

$$D_{min} = \frac{q}{\gamma}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]^2 \qquad ...(24.8)$$

where q = intensity of loading.

24.3 TYPES OF BEARING CAPACITY FAILURES

Experimental investigations have indicated that when a footing fails due to insufficient bearing capacity, distinct failure patterns are developed, depending upon type of failure mechanism. Failure is accompanied by appearance of *failure surfaces* and by building of sheared mass of soil. Vesic (1963) observed three types of bearing capacity failures (Fig. 24.2).

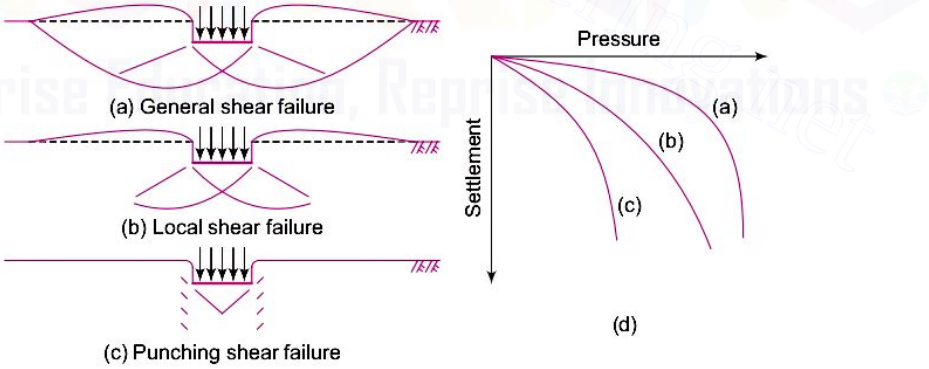

Fig. 24.2 Modes of bearing capacity failures

1. General shear failure, 2. Local shear failure and 3. Punching shear failure.

1. ***General shear failure*:** In the case of *general shear failure,* continuous failure surfaces develop between the edges of the footing and the ground surface, as shown in Fig. 24.2 (*a*). When the pressure approaches the value of q_f, the state of *plastic equilibrium* is reached initially in the soil around the edges of the footing, and it then gradually spreads downwards and outwards. Ultimately, the state of plastic equilibrium is fully developed throughout the soil above the failure surfaces. The

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failure is accompanied by appearance of failure surfaces and by considerable bulging of sheared mass of soil. However, the final slip movement would occur only on one side, accompanied by *tilting* of the footing. Such a failure occurs in soils of *low compressibility*, *i.e.*, dense or stiff soil, and the pressure-settlement curve is of the general form as shown is curve *a* of Fig. 24.2 (*d*). Following are the typical *characteristics* of general shear failure.

(*i*) It has well defined failure surfaces, reaching up to ground surface
(*ii*) There is considerable bulging of sheared mass of soil adjacent to the footing
(*iii*) Failure is accompanied by tilting of the footing
(*iv*) Failure is sudden, with pronounced peak resistance
(*v*) The ultimate bearing capacity is well defined

2. ***Local shear failure:*** In *local shear failure,* there is significant *compression* of the soil under the footing and only partial development of state of plastic equilibrium. Due to this reason, the failure surfaces do not reach the ground surface and only *slight* heaving occurs. The pressure-settlement curve is represented by curve *b* of Fig. 24.2 (*d*), where the peak of the base resistance may never be reached. In such a failure, tilting of foundation is not expected. Local shear failure is associated with soils of high compressibility and in sands having relative density lying between 35 and 70%. The failure is not sudden, and it is characterised by occurrence of relatively large settlements which would not be acceptable in practice. Also, ultimate bearing capacity in such a failure is not well defined. Following are typical *characteristics* of local shear failure.
   1. Failure pattern is clearly defined only immediately below the footing
   2. The failure surfaces do not reach ground surface.
   3. There is only slight bulging of soil around the footing.
   4. Failure is not sudden and there is no tilting of footing.
   5. Failure is defined by large settlements.
   6. Ultimate bearing capacity is not well defined.
3. ***Punching shear failure:*** *Punching shear failure* occurs where there is relatively high compression of soil under the footing, accompanied by shearing in the vertical direction around the edges of the footing. Punching shear may occur in relatively loose sand with relative density less than 35%. Punching shear failure may also occur in a soil of low compressibility if the foundation is located at considerable depth. The failure surface, which is vertical or slightly inclined and follows the perimeter of the base, never reaches the ground surface. There is no heaving of the ground surface away from the edges and no tilting of the footing. Relatively large settlements occur in this mode. The ultimate bearing capacity is not well defined. Following the *characteristics* of punching shear failure:

(*i*) No failure pattern is observed,
(*ii*) The failure surface, which is vertical or slightly inclined, follows the perimeter of the base.
(*iii*) There is no bulging of soil around the footing.
(*iv*) There is no tilting of footing.
(*v*) Failure is characterised in terms of very large settlements.
(*vi*) The ultimate bearing capacity is not well defined.

Conditions for typical mode of failure:

Table 24.1 gives the conditions under which a typical mode of failure may occur.

Modes of failure of footings in sand

Figure 24.3 shows the modes of failure of footings in sand. As the relative depth/width ratio increases, the limiting relative densities at which failure type change increases.

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Table 24.1 Conditions of typical mode of failure

	Condition	*Mode of failure*
1.	Footings on ground surface or at shallow depth, in very dense sand	General shear failure
2.	Footing on saturated, normally consolidated clay, under undrained loading	General shear failure
3.	Very deep footing in dense sand	Punching shear failure
4.	Footing on surface or shallow depth in loose sand	Punching shear failure
5.	Footing in very dense sand, loaded by transient dynamic loads	Punching shear failure
6.	Footing on very dense sand underlain by loose sand or soft clay	Punching shear failure
7.	Footing on saturated, normally consolidated clay under drained loading	Punching/local shear failure
8.	Footing on ground surface or at shallow depth, in soils of high compressibility	Local shear failure
9.	(*a*) Footing at ground surface or at shallow depth, in sands of relative density between 0.3 to 0.7 (*b*) Footing at great depth, in sands of relative density between 0.7 to 0.9	Local shear failure Local shear failure.

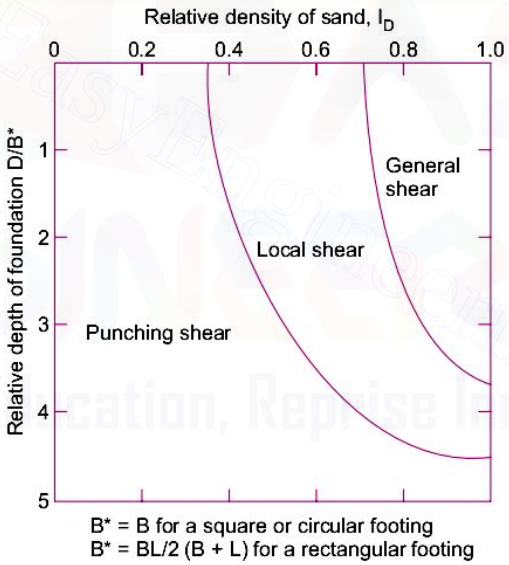

Fig. 24.3 Expected modes of failure of footings in sand (De beer, 1976)

24.4 TERZAGHI'S ANALYSIS

An analysis of the condition of complete bearing capacity failure, usually termed *general shear failure,* can be made by assuming that the soil behaves like an ideally plastic material. The concept was first developed by Prandtl, and later extended by Terzaghi, Meyerhof and others. Terzaghi derived a general bearing capacity equation from a modification of equations proposed by Prandtl. Figure 24.4 shows a footing of width B, and subjected to loading intensity q_f to cause failure. The footing is shallow, *i.e.*, the depth D of the footing is equal to or less than width B of the footing. Also, the footing is continuous, *i.e.*, it is a strip footing ($L > 5B$).

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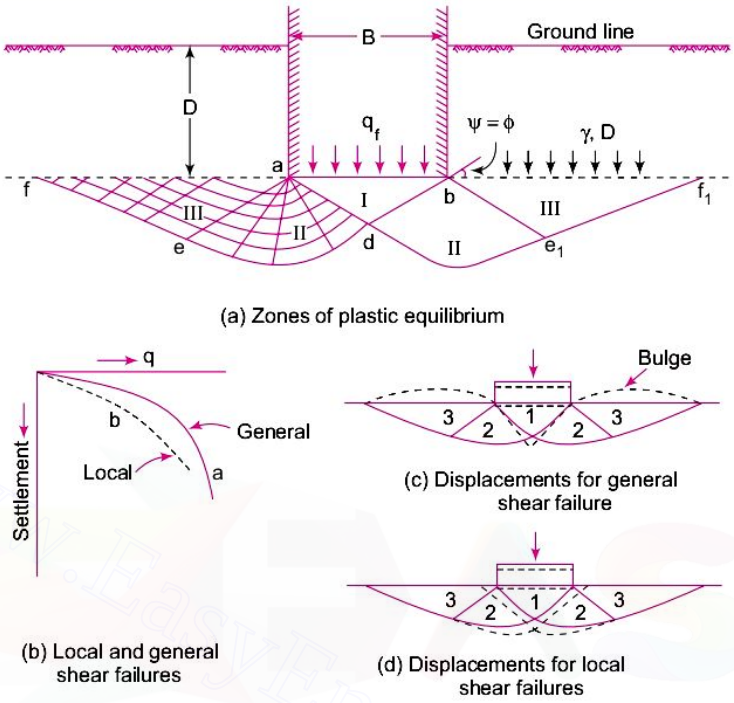

Fig. 24.4 Terzaghi's analysis

The loaded soil fails along the composite surface $fede_1f_1$. This region can be divided into five zones: zone I, two pairs of zone II and two pairs of zone III. When the base of the footing *ab* sinks into the ground, zone I (soil wedge *abd*) immediately beneath the footing is prevented from undergoing any lateral yield by the friction and adhesion between the soil and the base of the footing. Thus, zone I remains in a state of elastic equilibrium, and it acts as if it were part of the footing. Its boundaries *db* and *da* are assumed as plane surfaces, rising at an angle $\psi = \phi$ with the horizontal. Zone II is called the zone of *radial shear,* as the lines that constitute one set in the shear pattern radiate from the outer edge of the base of the footing. These radial lines are straight while the lines of the other set are the logarithmic spirals with their centres located at the outer edges of the base of the footing. Zone III is called the zone of *linear shear,* and is identical with that for passive Rankine state. The boundaries of zone III rise at $45° - \phi/2$ with the horizontal. The failure zones are assumed not to extend above the horizontal plane through the base *ab* of the footing. This implies that the shear resistance of the soil above the horizontal plane through the base of the footing is neglected, and the soil above this plane is replaced with a surcharge $q = \sigma = \gamma D$.

The application of the load intensity q_f on the footing tends to push the wedge of the soil *abd* into the ground with lateral displacement of zones II and III, but this lateral displacement is resisted by forces on the plane *db* and *da*. These forces are: (*i*) the resultant of the passive pressure P_p and (*ii*) the cohesion *c* acting along the surface *da* and *db*. The passive pressure resultant makes an angle ϕ with the normal to the surfaces *da* and *db*. If it is assumed that surfaces *db* and *da* intersect the horizontal line at an angle ϕ, the passive pressure acts vertically. At the instant of failure, the downward and upward forces on the wedge *adb* of unit length must balance. The downward forces are (*i*) $q_f B$ and (*ii*) the weight $\frac{1}{4}\gamma B^2 \tan \phi$ of the wedge. The upward forces are (*i*) the resultant passive pressure P_p on each of the surfaces *db* and *da,* and

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(*ii*) the vertical component of cohesion acting along the lengths *ad* and *bd*. The length $db = da = \dfrac{B/2}{\cos\phi}$

and hence vertical component of cohesion on *each* of the surfaces *db* and $da = c\,\dfrac{B/2}{\cos\phi}\sin\phi = \dfrac{B}{2}c\tan\phi$.

Hence, $q_f B + \dfrac{1}{4}\gamma B^2 \cdot \tan\phi = 2P_p + 2\cdot\dfrac{B}{2}c\tan\phi$

or $$q_f B = 2P_p + B\cdot c\tan\phi - \frac{1}{4}\gamma B^2\tan\phi \qquad ...(24.9)$$

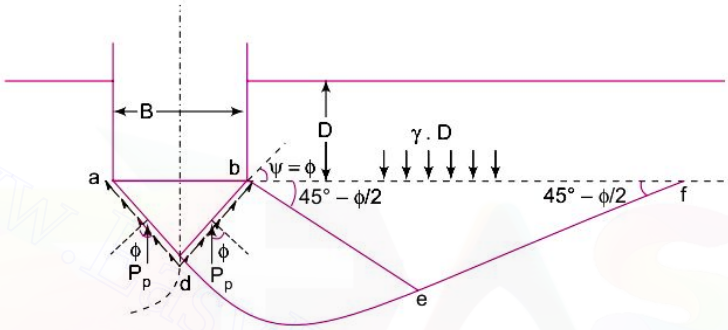

Fig. 24.5 Forces acting to restrain failure

The resultant of passive earth pressure P_p can be divided into three components: (*i*) $P_{p\gamma}$ produced by weight of the shear zone *dbfe*, (*ii*) P_{pc} produced by soil cohesion, and (*iii*) P_{pq} produced by surcharge. These components of passive pressure are computed separately and then added to obtain the value of P_p. Substituting these components in Eq. 24.9, we get

$$q_f B = 2(P_{p\gamma} + P_{pc} + P_{pq}) + B\cdot c\tan\phi - \frac{1}{4}\gamma B^2\tan\phi \qquad ...(24.10)$$

or $$q_f B = \left(2P_{p\gamma} - \frac{1}{4}\gamma B^2\tan\phi\right) + \left(2P_{pc} + B\cdot c\tan\phi\right) + 2P_{pq} \qquad ...(24.11)$$

Let $2P_{p\gamma} - \dfrac{1}{4}\gamma B^2\tan\phi = B\times\dfrac{1}{2}\gamma B N_\gamma$

$2P_{pc} + B\cdot c\tan\phi = B\times c\,N_c$ and $2P_{pq} = B\times\bar{\sigma}\,N_q$

where $\bar{\sigma}$ = effective over burden pressure above the base of footing.

Substituting in Eq. 24.11, $q_f = cN_c + \bar{\sigma}\cdot N_q + 0.5\,\gamma B N_\gamma$ ...(24.12)

If the water table is below the base of footing, $\bar{\sigma} = \gamma D$ and hence

$$q_f = cN_c + \gamma D N_q + 0.5\,\gamma B N_\gamma \qquad ...(24.12a)$$

Hence, $$q_{nf} = cN_c + \bar{\sigma}\,(N_q - 1) + 0.5\,\gamma B N_\gamma \qquad ...(24.13)$$

and $$q_s = \frac{1}{F}\left[c\cdot N_c + \bar{\sigma}\,(N_q - 1) + 0.5\,\gamma B N_\gamma\right] + \bar{\sigma} \qquad ...(24.14)$$

If the water table is below the base of footing, the above equations reduce to the forms :

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$$q_{nf} = cN_c + \gamma D\,(N_q - 1) + 0.5\,\gamma BN_\gamma \qquad ...(24.13a)$$

and

$$q_s = \frac{1}{F}\left[cN_c + \gamma D(N_q - 1) + 0.5\,\gamma BN_\gamma\right] + \gamma D \qquad ...(24.14a)$$

For purely cohesive soil, the bearing capacity is given by

$$q_f = cN_c + \bar{\sigma}\,N_q = 5.7\,c + \bar{\sigma} \qquad ...(24.15)$$

where $\bar{\sigma} = \gamma D$ if the water table is below the base of the footing.

Equation 24.12 is known as Terzaghi's general bearing capacity equation for a continuous footing. The parameters N_c, N_q and N_γ are the dimensionless numbers, known as *bearing capacity factors* depending only on the angle of shearing resistance of the soil. Terzaghi gave the following expressions for the bearing capacity factors :

$$N_q = \frac{a^2}{2\cos^2(45° + \varphi/2)}, \text{ where } a = e^{(0.75\,\pi - \varphi/2)\tan\varphi} \qquad ...(24.12b)$$

$$N_c = (N_q - 1)\cot\varphi \qquad ...(24.12c)$$

and

$$N_\gamma = \frac{\tan\varphi}{2}\left(\frac{K_{p\gamma}}{\cos^2\varphi} - 1\right) \qquad ...(24.12d)$$

$K_{p\gamma}$ = passive earth pressure coefficient, dependent on φ

Terzaghi never explained very well how he obtained $K_{p\gamma}$ used to compute N_γ. He did, however, give a curve of φ versus N_γ and three specific values of N_γ at $\varphi = 0$, 34° and 48°. The values of the bearing capacity factors N_q, N_c and N_γ can be obtained from Table 24.2. The values of N_γ for φ of 34° and 48° are the original Terzaghi values which were used by Bowles to back compute $K_{p\gamma}$. The back computed values of $K_{p\gamma}$ are also given in Table 24.2.

General and local shear failure: The type of failure analysed above is called the *general shear failure,* and the bearing capacity factors N_c, N_q and N_γ correspond to the general shear failure. In such a failure, the soil properties are assumed to be such that a slight downward movement of footing develops fully plastic zones and the soil bulges out [Fig. 24.4 (*c*)]. In the case of fairly soft or loose and compressible soil, large deformations may occur below the footing before the failure zones are fully developed. Such a failure is called a *local shear failure* [Fig. 24.4 (*d*)] which is associated with considerable vertical soil movement before soil bulging takes place. In the absence of analytical solution, Terzaghi has proposed the following local shear-failure soil parametres.

$$c_m = \frac{2}{3}c \qquad ...(24.16a)$$

$$\tan\phi_m = \frac{2}{3}\tan\phi \qquad ...(24.16b)$$

The bearing capacity factors are determined with respect to these reduced parameters c_m and ϕ_m. The bearing capacity factors corresponding to the local shear failure are indicated with dashes N_q' and N_γ' and the bearing capacity equation (Eq. 24.12) reduces

to

$$q_f = \frac{2}{3}c \cdot N_c' + \bar{\sigma}\,N_q' + 0.5\,\gamma BN_\gamma' \qquad ...(24.17)$$

It is difficult to define the limiting conditions for which general or local shear failure should be assumed at a given site. However, the following points may be used as a guide:

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(*i*) ***Stress strain test*** (*c*–ϕ *soil*): General shear failure at low strain, say < 5%, while for local shear failure, stress strain curve continues to rise at strains of 10 to 20%.

(*ii*) ***Angle of shear resistance:*** For $\phi > 36°$, general shear failure
$\phi < 28°$, local shear failure.

(*iii*) ***Penetration test:*** $N \geq 30$: General shear failure
$N \leq 5$: Local shear failure.

(*iv*) ***Plate load test:*** Shape of the load settlement curve decides whether it is general shear failure or local shear failure.

(*v*) ***Density Index:*** $I_D > 70$: General shear failure
$I_D < 20$: Local shear failure.

For purely cohesive soil, local shear failure may be assumed to occur when the soil is soft to medium, with an unconfined compressive strength $q_u \leq 100$ kN/m^2 (or $c_u \leq 50$ kN/m^2).

Table 24.2 Terzaghi's bearing capacity factors

ϕ	*General shear failure*			*Local shear failure*		
	N_c	N_q	N_γ	N_c'	N_q'	N_γ'
0	5.7*	1.0	0.0	5.7	1.0	0.0
5	7.3	1.6	0.5	6.7	1.4	0.2
10	9.6	2.7	1.2	8.0	1.9	0.5
15	12.9	4.4	2.5	9.7	2.7	0.9
20	17.7	7.4	5.0	11.8	3.9	1.7
25	25.1	12.7	9.7	14.8	5.6	3.2
30	37.2	22.5	19.7	19.0	8.3	5.7
34	52.6	36.5	35.0	23.7	11.7	9.0
35	57.8	41.4	42.4	25.2	12.6	10.1
40	95.7	81.3	100.4	34.9	20.5	18.8
45	172.3	173.3	297.5	51.2	35.1	37.7
48	258.3	287.9	780.1	66.8	50.5	60.4
50	347.5	415.1	1153.2	81.3	65.6	87.1

* = $(1.5\,\pi + 1)$

Assumptions in Terzaghi's Analysis

1. The soil is homogeneous and isotropic and its shear strength is represented by Coulomb's equation.
2. The strip footing has a rough base, and the problem is essentially two dimensional.
3. The elastic zone has straight boundaries inclined at $\psi = \phi$ to the horizontal, and the plastic zones fully develop.,
4. P_p consists of three components which can be calculated separately and added, although the critical surface for these components are not identical.
5. Failure zones do not extend above the horizontal plane through the base of the footing, *i.e.*, the shear resistance of soil above the base is neglected and the effect of soil around the footing is considered equivalent to a surcharge $\sigma = \gamma D$.

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Limitations (1) As the soil compresses, ϕ changes; slight downward movement of footing may not develop fully the plastic zones. (2) Error due to assumption 4 is small and on the safe side (3) Error due to assumption 5 increases with depth of foundation, and hence the theory is suitable for shallow foundation only.

Specialization of Terzaghi's Equations

In order to take into account the shape of the footing (*i.e.*, strip, round, square etc.), Terzaghi used only *shape factors* with cohesion (s_c) and base (s_γ) terms. Taking into account these factors, Terzaghi's original equation (Eq. 24.12) can be expressed as under.

$$q_f = c\,N_c s_c + \bar{\sigma}\,N_q + 0.5\,\gamma\,B\,N_\gamma s_\gamma \qquad ...(24.12e)$$

The values of s_c and s_γ are given below.

Shape	*Strip*	*Round*	*Square*	*Rectangle*
s_c	1.0	1.3	1.3	$1 + 0.3\ B/L$
s_γ	1.0	0.6	0.8	0.8 or $1 - 0.3\ B/L$

Based on these values, Terzaghi gave the following semi-empirical equations for square, circular and rectangular footings:

(*a*) Frictional cohesive soil (*c*-ϕ soil)

For circular footing : $q_f = 1.3\ cN_c + \bar{\sigma} \cdot N_q + 0.3\ \gamma BN_\gamma$ ...(24.18)

where B = diameter of the footing

For square footing : $q_f = 1.3\ cN_c + \bar{\sigma} \cdot N_q + 0.4\ \gamma BN_\gamma$ ...(24.19)

where B = width (or length of footing)

For rectangular footing : $q_f = cN_c\left(1 + 0.3\,\frac{B}{L}\right) + \bar{\sigma} \cdot N_q + 0.4\,\gamma BN_\gamma$ ...(24.20*a*)

Alternatively, $q_f = cN_c\left(1 + 0.3\,\frac{B}{L}\right) + \bar{\sigma} \cdot N_q + 0.5\,\gamma BN_\gamma\left(1 - 0.2\,\frac{B}{L}\right)$ ...(24.20*b*)

(*b*) Cohesive soil ($\phi = 0$; $c > 0$)

For circular footing: $q_f = 1.3\ cN_c + \bar{\sigma} = 7.4\ c + \bar{\sigma}$

This is considerably greater than the value of $q_f = 5.1\ c + \bar{\sigma}$ for strip footing (Eq. 24.15). *For rectangular and square footing;*

$$q_f = cN_c\left(1 + 0.3\,\frac{B}{L}\right) + \bar{\sigma}$$

(*c*) Non-cohesive soil ($\phi > 0$; $c = 0$)

For strip footing : $q_f = \bar{\sigma}N_q + 0.5\,\gamma BN_\gamma$

For rectangular and square footing :

$$q_f = \bar{\sigma}N_q + 0.4\,\gamma BN_\gamma$$

For circular footing : $q_f = \bar{\sigma}N_q + 0.3\,\gamma BN_\gamma$

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General comments on Terzaghi's Theory: Terzaghi assumed the value of angle $\psi = \phi$ (Fig. 24.4), which is not true. In actual practice, ψ has been found to vary from $45° - \phi/2$ for perfectly smooth base to $45° + \phi/2$ for perfectly rough base. Since footings are normally rough, ψ has been found close to $45° + \phi/2$ than to ϕ. Also Terzaghi's bearing capacity equations do not have provision for including depth effects, inclination factors, etc.

Mixed State of Local and General shear failure: As stated earlier, local shear failure generally occurs in loose sand while general shear failure occurs in dense sand. When $\phi < 28°$, local shear failure may be assumed, and values of bearing capacity factors may be taken from Table 24.2. For $\phi > 36°$, the general shear failure may be assumed and the values of the bearing capacity factors may be taken from Table 24.2. For intermediate value of ϕ, there is a *mixed state of local and general shear failure*, for which no separate equations are available. As ϕ increases above 28°, there is gradual transition from local to general shear failure. Peck, Hansen and Thornburn give curves for N_q and N_γ for the transition state from $\phi = 28°$ to $\phi = 38°$, in which they assumed general shear failure when $\phi > 28°$. These values can be obtained from curves for N_γ and N_q shown in Fig. 24.6. It will be seen that when $\phi > 38°$, the values are the same as given in Table 24.1.

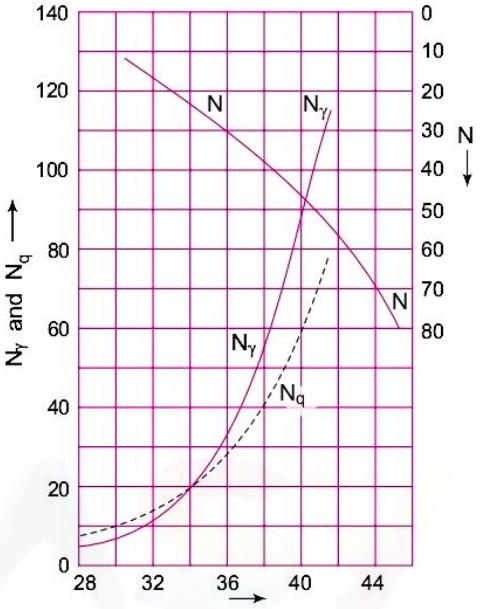

Fig. 24.6 Terzaghi's bearing capacity factors for transitional state

24.5 SKEMPTON'S VALUES FOR N_c

Based partly on theory and partly on laboratory tests, Skempton observed that the factor N_c increases with the ratio D/B. He found that for purely cohesive soil ($\phi = 0$), N_c has a maximum value of 9 for square or circular footing and 7.5 for strip footing. His recommendations may be summarised as follows:

1. ***When D = 0.*** $N_c = 5.14$ (for strip footing) ...(24.21*a*)

   $N_c = 6.20$ for square or circular footing ...(24.21*b*)

2. ***At depths*** $\frac{D}{B} < 2.5$, $N_c = \left(1 + 0.2\,\frac{D}{B}\right)[N_c]_{surface}$ ...(24.22)

   Value of $[N_c]_{sufarce}$ may be roughly taken as 5 for surface strip footing and as 6 for square or circular footing.

3. ***At depths*** $\frac{D}{B} > 2.5$, $N_c = 1.5\,[N_c]_{surface}$ ...(24.23)

4. ***For rectangular footings:***

$$[N_c]_{rect.} = \left(1 + 0.2\,\frac{B}{L}\right)[N_c]_{strip} \quad \text{...(24.24)}$$

or

$$[N_c]_{rect.} = 5\left(1 + 0.2\,\frac{B}{L}\right)\left(1 + 0.2\,\frac{D}{B}\right) \quad \text{...(24.24}a\text{)}$$

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Since $$[N_c]_{strip} = \left(1 + 0.2\,\frac{D}{B}\right)[N_c]_{surface}$$

$$= 5\left(1 + 0.2\,\frac{D}{B}\right) \qquad ...(24.24b)$$

Alternatively, for $B > D$,

$$[N_c]_{rect.} = \left(0.84 + 0.16\,\frac{B}{L}\right)[N_c]_{surface} \qquad ...(24.25)$$

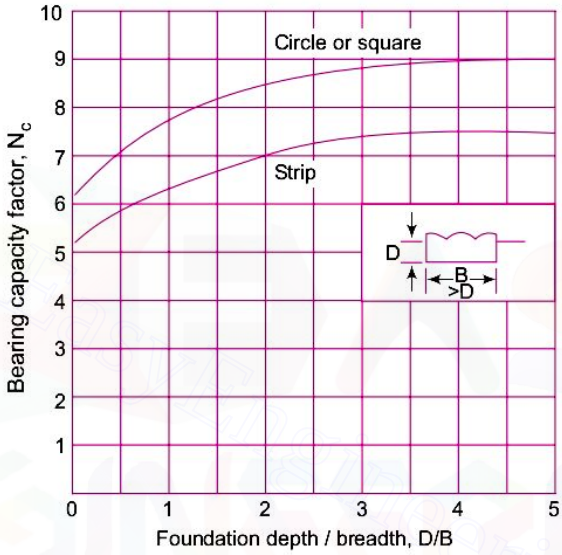

Fig. 24.7 Bearing capacity factor N_c for cohesive soil ($\phi = 0$) (After skempton)

The bearing capacity factor N_c for cohesive soil ($\phi = 0$) are given in Fig. 24.7 (Skempton, 1951)

The Skempton values of N_c for vertical loading on strip or circular (and rectangular) foundations are more widely used. The Brinch Hansen shape factors can be applied to these N_c values for the case of rectangular footing or else, simple relationship given in Eq. 24.25 can be used.

24.6 GENERAL BEARING CAPACITY EQUATION: BRINCH HANSEN'S ANALYSIS

In recent years, many new equations have been proposed by various workers, giving the ultimate bearing capacity of foundations. These equations are based on the assumption of various values of angle ψ and on the shape of failure surfaces. Notable amongst the workers are Hansen (1970), Hu (1964), Chen and Davidson (1973) and Balla (1962). Out of these the one proposed by Hansen gives better results.

According to Hansen, the ultimate bearing capacity is given by

$$q_f = c \,.\, N_c \,.\, s_c \,.\, d_c \,.\, i_c \,.\, g_c \,.\, b_c + \sigma_0 \,.\, N_q \,.\, s_q \,.\, d_q \,.\, i_q \,.\, g_q \,.\, b_q + \frac{1}{2}\,\gamma \,.\, B \,.\, N_\gamma \,.\, s_\gamma \,.\, d_\gamma \,.\, i_\gamma \,.\, g_\gamma \,.\, b_\gamma \qquad ...(24.26)$$

where $\sigma_0 = \bar{\sigma}$ = effective overburden pressure at foundation level

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s = shape factor, to account for the effect of the shape of the foundation in developing a failure surface.

d = depth factor to account for the embedment depth and the additional shearing resistance in the top soil.

i = inclination factor to account for both horizontal and vertical components of foundation loads

g = ground factor ; b = base factor.

γ = density of soil below the foundation level.

For $\phi = 0$ or undrained condition, the above equation is modified to:

$$q_f = 5.14\, c\, \left[1 + s_c' + d_c' - i_c' - b_c' - g_c'\right] + \sigma_0 \qquad ...(24.27)$$

where the various factors with dashes (primed) are for $\phi = 0$ (undrained) condition.

In equation 24.26, the bearing capacity factors are computed by the following equations:

$$N_q = \tan^2\left(45° + \frac{\phi}{2}\right)\left(\epsilon^{\pi \tan \phi}\right) = K_p\, \epsilon^{\pi \tan \phi} \qquad ...(24.28a)$$

$$N_c = (N_q - 1) \cot \phi \qquad ...(24.28b)$$

and $$N_\gamma = 1.5\,(N_q - 1) \tan \phi \qquad ...(24.28c)*$$

The bearing capacity factors are given in Table 24.3 and Fig. 24.8. Hansen considered the bearing capacity as a plane strain problem. If ϕ is found by triaxial tests, its corresponding value for the plane strain case can be computed from the expression

$$\phi_{plain\ strain} = 1.1\, \phi_{triaxial} \qquad ...(24.29)$$

Bowel suggests that this increase in ϕ be computed only for ϕ greater than 25°.

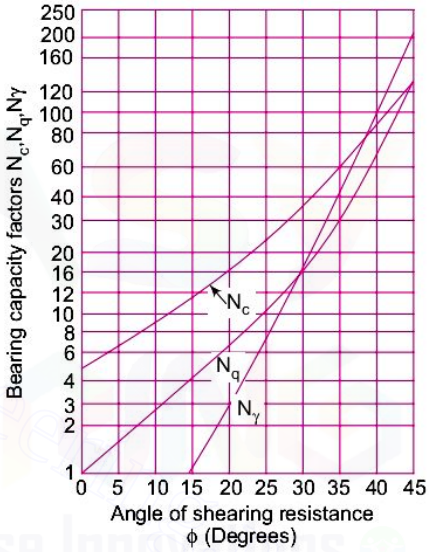

Fig. 24.8 Brinch-Hansen's factors N_c, N_q, N_γ

The values of *shape factors, ground factors* and *base factors* are given in Table 24.4, while the values of depth factors and inclination factors are given in Table 24.5. The following symbols are used in these tables:

A' = effective footing contact area = $B' \times L'$

L' = effective footing length = $L - 2e_y$

B' = effective footing width $B - 2\, e_x$

e_y, e_x = eccentricity of load, with respect to the centre of footing area (Fig. 24.10)

D = depth of footing in ground

H, V = load components parallel and perpendicular to the footing respectively

η = inclination of the base (supporting the footing) with horizontal, reckoned positive for up-slopes and negative for down slopes

β = ground slope with horizontal, reckoned positive for down slopes and negative for up slopes

$\tan \delta$ = coefficient of friction between footing and base soil.

* Some workers use a factor of 1.8 in place of 1.5, in Eq. 24.28 (*c*), for bearing capacity factor N_f

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According to Schultze and Horn (1967), take

$\delta = \phi$ for concrete poured on ground

L = length of footing ; B = width of footing.

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Tables 24.4 and 24.5 have the following limitations:

$H \leq V \tan \delta + c \times A'$; $i_q, i_\gamma > 0.0$

$\beta \leq \phi$; $\eta + \beta \leq 90°$.

Table 24.3 Hansen's bearing capacity factors

ϕ	N_c	N_q	N_γ	N_q/N_c	$2 \tan \phi \times (1 - \sin \phi)^2$
0	5.14	1.0	0.0	0.19	0.000
5	6.49	1.6	0.1	0.24	0.146
10	8.34	2.5	0.4	0.30	0.241
15	10.98	3.9	1.2	0.36	0.294
20	14.83	6.4	2.9	0.43	0.315
25	20.72	10.7	6.8	0.51	0.311
26	22.25	11.9	7.9	0.53	0.308
28	25.80	14.7	10.9	0.57	0.299
30	30.14	18.4	15.1	0.61	0.289
32	35.49	23.2	20.8	0.65	0.276
34	42.16	29.4	28.8	0.70	0.262
36	50.59	37.8	40.1	0.75	0.247
38	61.35	48.9	56.2	0.80	0.231
40	75.31	64.2	79.5	0.85	0.214
45	133.87	134.9	200.8	1.01	0.172
50	266.38	319.1	563.6	1.20	0.130

Table 24.4 Shape factors, ground factors and base factors

Shape factors (s)	*Ground factors (g)*	*Base factors (b)*
$s'_c = 0.2 \frac{B}{L}$	$g'_c = \beta°/147°$	$b'_c = \eta°/147°$
$s_c = 1 + \frac{N_q}{N_\gamma} \frac{B}{L}$	$g_c = 1 - (\beta°/147)$	$b_c = 1 - (\eta°/147)$
$s_q = 1 + \frac{B}{L} \tan \phi$	$g_q = g_\gamma = (1 - 0.5 \tan \beta°)^5$	$b_q = \epsilon^{-2\eta \tan \phi}$ and $b_\gamma = \epsilon^{-2.7 \eta \tan \phi}$
$s_\gamma = 1 - 0.4 \frac{B}{L}$	For horizontal ground, $g'_c = 0.0$ and $g_q = g_\gamma = 1.0$	For horizontal base, $b'_c = 0.0$ and $b_q = b_\gamma = 1.0$

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Note. (*i*) Primed factors are for undrained conditions and $\phi = 0$ (*ii*) η is in radians for b_q and b_γ.

Table 24.5 Depth factors and inclination factors

Depth factors	*Inclination factors*
$d'_c = 0.4 \dfrac{D}{B}$ *for* $D \le B$ $d'_c = 0.4 \tan^{-1} \dfrac{D}{B}$ for $D > B$	$i'_c = 0.5 - 0.5 \sqrt{1 - \dfrac{H}{cA'}}$ $i_c = i_q - \dfrac{i - i_q}{N_q - 1}$
$d_c = 1 + 0.4 \dfrac{D}{B}$ for $D \le B$ $d_c = 1 + 0.4 \tan^{-1} \dfrac{D}{B}$ for $D > B$	$i_q = \left(1 - \dfrac{0.5\,H}{V + cA' \cot \phi}\right)^5$
$d_q = 1 + 2 \tan \phi\,(1 - \sin \phi)^2 \dfrac{D}{B}$ for $D \le Bt$ $d_q = 1 + 2 \tan \phi\,(1 - \sin \phi)^2 \tan^{-1} \dfrac{D}{B}$ for $D > B$	$i_y = \left(1 - \dfrac{0.7\,H}{V + cA' \cot \phi}\right)^5$ for horizontal ground $i_y = \left[\dfrac{1 - \left(0.7 - \dfrac{\eta^\circ}{450^\circ}\right) H}{V + cA' \cot \phi}\right]^5$ for sloping ground
$d_\gamma = 1.0$ for all ϕ	

24.7 MEYERHOF'S ANALYSIS

Meyerhof extended the analysis of plastic equilibrium of a surface footing to shallow and deep foundations. Figure 24.9 (*a*) and (*b*) show the failure mechanisms for shallow and deep foundations according to both Terzaghi and Meyerhof analysis. In the Meyerhof's analysis, *abd* is the elastic zone, *bde* is the radial shear zone and *befg* is the zone of mixed shear in which shear varies between radial and plane shear, depending largely upon the depth and roughness of the foundation. The plastic equiliburium in these zones can be established from the boundary conditions starting from the foundation shaft. To simplify the analysis, Meyerhof introduced a parameter β, the angle to define the line *bf,* joining point *b* to *f* where the assumed boundary failure slip line intersects the soil surface. The resultant effects of the wedge of soil *bfg* is represented by the normal and tangential stresses, p_0 and s_0 on *bf.* The plane *bf* is termed as the *equivalent free surface,* and p_0 and s_0 are termed as the *equivalent free surface stresses.* The angle β increases with depth, and become 90° for deep foundations.

Meyerhof (1963) gave the following equations for ultimate bearing capacity, taking into account the shape, depth and inclination factors:

Vertical load: $$q_f = cN_c \,.\, s_c\, d_c + \bar{\sigma} N_q\, s_q\, d_q + 0.5\gamma\, BN_\gamma\, s_\gamma \,.\, d_\gamma \quad ...(24.30)$$

Inclined load : $$q_f = cN_c \,.\, d_c \,.\, i_c + \bar{\sigma} N_q\, d_q\, i_q + 0.5\gamma\, BN_\gamma\, s_\gamma \,.\, d_\gamma \quad ...(24.31)$$

where $$N_q = e^{\pi \tan \phi} \tan^2 (45^\circ + \phi/2) \quad ...(24.32a)$$

$$N_c = (N_q - 1) \cot \phi \quad ...(24.32b)$$

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$$N_\gamma = (N_q - 1) \tan (1.4\,\phi) \qquad ...(24.32c)$$

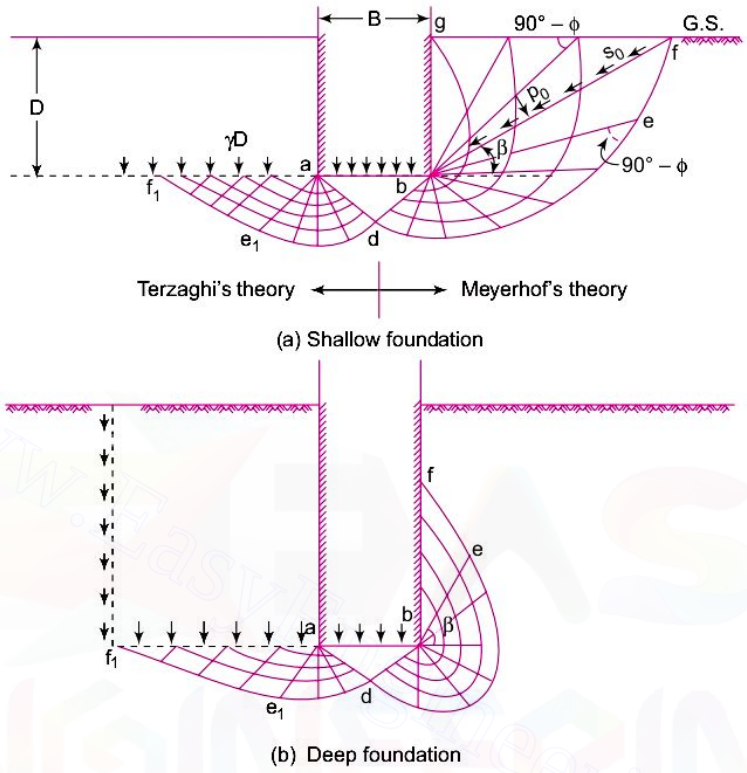

Fig. 24.9 Meyerhof's analysis

The bearing capacity factors N_c, N_q and N_γ are given in Table 24.7. It will be seen that the factors N_c and N_q are the same as given by Hansen (Table 24.3) while the factor N_γ is different.

s = shape factors ; d = depth factors ; i = inclination factors.

The values of these factors are given in Table 24.6.

Table 24.6 Shape, Depth and Inclination Factors for the Meyerhof Equation

Shape factors	*Depth factors*	*Inclination factors*
$s_c = 1 + 0.2\,K_p \dfrac{B}{L}$	$d_c = 1 + 0.2\sqrt{K_p}\,\dfrac{D}{B}$	$i_c = i_q = 1 - \dfrac{\alpha}{90°}$
(*i*) for $\phi = 0°$ $s_q = s_\gamma = 1.0$	(*i*) for $\phi = 0°$ $d_q = d_\gamma = 1.0$	$i_\gamma = \left(1 - \dfrac{\alpha}{\phi}\right)^2$
(*ii*) for $\phi \geq 10°$ $s_q = s_\gamma = 1 + 0.1\,K_p \dfrac{B}{L}$	(*ii*) for $\phi \geq 10°$ $d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\,\dfrac{D}{B}$	α = angle of resultant measured from vertical axis $K_p = \tan^2\left(45° + \dfrac{\phi}{2}\right)$

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It may be noted that the bearing capacity equations have been derived, treating the problem as a plane strain problem. Meyerhof therefore proposed the following equation to obtain ϕ:

$$\phi_{plain\ strain} = \left(1.1 - 0.1\frac{B}{L}\right)\phi_{triaxial} \qquad ...(24.33)$$

Table 24.7 Meyerhof's bearing capacity factors

ϕ°	N_e	N_q	N_γ
0	5.14	1.0	0
5	6.5	1.6	0.1
10	8.3	2.5	0.4
15	11.0	3.9	1.1
20	14.8	6.4	2.9
25	20.7	10.7	6.8
30	30.1	18.4	15.7
35	46.1	33.3	37.1
40	75.3	64.2	93.7
45	133.9	134.9	262.7
50	266.9	319.0	873.7

24.8 VESIC'S BEARING CAPACITY EQUATION

Vesic's bearing capacity equation is similar to Hansen's equation (Eq. 24.26).

The essential differences in Vesic's and Hansens' procedures are (*i*) use of slightly different value of N_γ and (*ii*) a variation on some of Hansen's inclination, base and ground factors. Vesic's bearing capacity factors are given by the following equations:

$$N_q = \tan^2(45° + \varphi/2)\, e^{\pi \tan \varphi}$$

$$N_c = (N_q - 1)\cot \varphi$$

and

$$N_\gamma = 2(N_q + 1)\tan \varphi \qquad ...(24.34)$$

Thus, the values of N_q and N_c are the same as in Hansen's analysis while the values of N_γ are different.

24.9 COMPARISON OF BEARING CAPACITY FACTORS

The values of bearing capacity factors N_c, N_q and N_γ, originally given by Terzaghi have now been superseded by the analysis given by other workers such as Reissner, Hansen, Meyerhof, Vesic etc. Following are commonly followed expressions:

(*i*) ***Expressions for N_c and N_q***

$$N_c = (N_q - 1)\cot \varphi \ ; \ N_q = \tan^2(45° + \varphi/2)\, e^{\pi \tan \varphi}$$

When $\varphi = 0$, $N_c = 5.14$ (instead of 5.7 given by Terzaghi)

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The above two equations have been adopted by (*i*) Terzaghi and Peck, (*ii*) Meyerhof, (*iii*) Hansen, (*iv*) Vesic and (*v*) Bureau of Indian Standards.

(*ii*) ***Expressions for N_γ***

However, the expression for N_γ as given by different workers are given below :

(*a*) Meyerhof : $N_\gamma = (N_q - 1) \tan (1.4\ \varphi)$

(*b*) Hansen : $N_\gamma = 1.5\ (N_q - 1) \tan \varphi$

(*c*) Vesic (Also by BIS) : $N_\gamma = 2(N_q + 1) \tan \varphi$

(*d*) Spangler and Handy (1982) : $N_\gamma = 1.1\ (N_q - 1) \tan 1.3\ \varphi$

The N_γ factor has the widest suggested range of values of any of N factors. A literature search reveals that

For $\varphi = 40°$, $38 \le N_\gamma \le 192$

Fortunately, this term N_γ does not contribute significantly to q_f so that almost any reasonable value can be used Bureau of Indian Standards (BIS) recommends to use values of N_γ given by Vesic.

Table 24.8 Bearing capacity factors for Meyerhof, hansen and vesic bearing capacity equations

$\varphi°$	N_c	N_q	N_γ			N_q/N_c	$2 \tan \varphi\ (1 - \sin \varphi)^2$
			$N_{\gamma\,(H)}$	$N_{\gamma\,(M)}$	$N_{\gamma\,(V)}$		
0	5.14	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10,9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

Table 24.8 gives the values of bearing capacity factors for Meyerhof, Hansen and Vesic bearing capacity equations. It is to be noted that values of N_c and N_q are same for all the three methods, while subscripts for N_γ identify the author, *i.e.*, $N_{\gamma\,(H)}$ = values by Hansen, $N_{\gamma\,(M)}$ = values by Meyerhof, while $N_{\gamma\,(V)}$ = values by Vesic.

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24.10 EFFECT OF WATER TABLE ON BEARING CAPACITY

When the water table is above the base of the footing, the submerged weight γ' should be used for the soil below the water table for computing the *effective pressure* or the surcharge. When the water table is located somewhat below the base of the footing, the elastic wedge is partly of moist soil and party of submerged soil, and a suitable reduction factor should be used with the wedge term $\frac{1}{2}\gamma BN_\gamma$, since it uses effective unit weight.

As a thumb rule, this term may be reduced to half (since $\gamma' \approx \frac{1}{2}\gamma_{sat}$) if the water table is just at the base of the footing, and no reduction to be made if the water table is at a depth equal to the width of the footing below base of the footing. For intermediate positions, linear interpolation of the reduction be made.

(*i*) **First Method.** For any position of the water table, Eq. 24.12, may be modified as under:

$$q_f = c\,.\,N_c + \gamma_1\,DN_q\,R_{w_1} + \frac{1}{2}\gamma_2\,BN_\gamma\,.\,R_{w_2} \qquad ...(24.35)$$

where R_{w_1} and R_{w_2} are the reduction factors for water table, computed as follows :

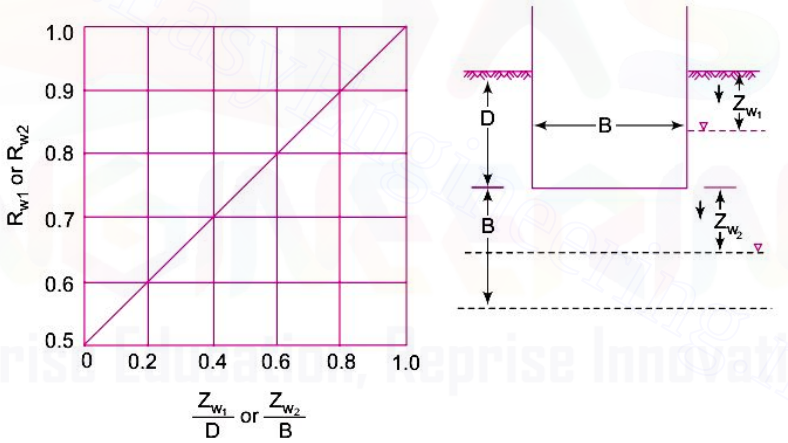

Fig. 24.10 Water reduction factors

$$R_{w_1} = 0.5\left(1 + \frac{Z_{w_1}}{D}\right) \qquad ...(24.35a)$$

At $Z_{w_1} = 0, R_{w_1} = \frac{1}{2}$; At $Z_{w_1} = D, R_{w_1} = 1$

and

$$R_{w_2} = 0.5\left(1 + \frac{Z_{w_2}}{B}\right) \qquad ...(24.35b)$$

At $Z_{w_2} = 0, R_{w_2} = 0.5$; At $Z_{w_2} = B, R_{w_2} = 1$

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In Eq. 24.34,

γ_1 = average unit weight of the surcharge soil situated above the water table.

γ_2 = average unit weight of the soil in the wedge zone, situated within a depth B below the base of the footing.

When the water table is just at the base of the footing, $\gamma_2 = \gamma_{sat}$.

When the water table is at the ground surface, both γ_1 and γ_2 are the saturated weights. For intermediate locations of water table, γ_1 and γ_2 can be computed, using rule of three as illustrated in example 24.4.

***(ii)* Second Method: IS Code Method:** Alternatively, q_f can be computed using only one reduction factor, by rewriting Eq. 24.34 as under:

$$q_f = cN_c + \bar{\sigma} N_q + \frac{1}{2}\gamma B N_\gamma R_w$$

where $\bar{\sigma}$ = effective unit weight of the soil situated above the base

$$R_w = R_{w_2} = 0.5\left(1 + \frac{Z_{w_2}}{B}\right) \qquad ...(24.36)$$

Equation 24.36 has been recommended by Indian Standard. When the water table is situated at a depth D_1 below the ground level ($D_1 < D$) or D_2 above the base of the footing (such that $D_1 + D_2 = D$), we have

$$\bar{\sigma} = (\gamma D_1 + \gamma_{sat} D_2) - \gamma w \,.\, D_2 = \gamma D_1 + \gamma' D_2 \qquad ...(24.36a)$$

Knowing $\bar{\sigma}$ and R_w ($= R_{w_2}$), q_f can be computed.

This alternative method is perferred to the method of Eq. 24.34 in which two reduction factors R_{w_1} and R_{w_2} are used. See example 24.4 for illustration and comparison.

(iii) **Third Method.** In the third method (Bowles, 1988), no water reduction factor is used, but effective unit weight (γ_e) is used for the soil in the wedge zone.

Thus, we have $$q_f = cN_c + \bar{\sigma} N_q + \frac{1}{2}\gamma_e B N_\gamma \qquad ...(24.37a)$$

where $\bar{\sigma}$ = effective overburden pressure

and γ_e = effective unit weight of soil in the wedge zone.

The wedge zone has a depth $H = 0.5\ B \tan(45° + \phi/2)$. Hence, when the water table is within this wedge zone, γ_e can be computed from the expression

$$\gamma_e = \left(2H - z_{w_2}\right)\frac{z_{w_2}}{H^2}\gamma + \frac{\gamma'}{H^2}(H - z_{w_2})^2 \qquad ...(24.37b)$$

See example 24.6 for illustration

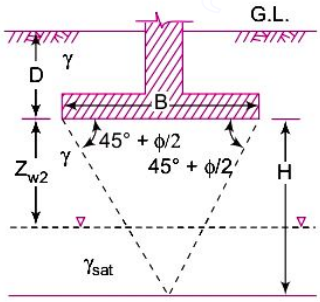

Fig. 24.11

Alternatively, taking $H = B$, the effective unit weight γ_e in the wedge term can be computed as under :

$$\gamma_e = \frac{\gamma z_{w_2} + \gamma'\left(B - z_{w_2}\right)}{B} = \gamma' + (\gamma - \gamma')\frac{z_{w_2}}{B} \qquad ...(24.37c)$$

when $Z_{w_2} = 0$, $\gamma_e = \gamma'$ (as expected)

when $Z_{w_2} = B$, $\gamma_e = \gamma'$ (as expected). See example 24.6 for illustration.

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24.11 EFFECT OF ECCENTRICITY OF LOADING

If the total load V on the footing acts eccentrically (*i.e.*, line of action of V not passing through the centre of gravity of the footing area), the width B and length L (= unity in the case of strip footing) should be reduced as under:

$$B' = B - 2e_x \qquad ...(24.38a)$$

$$L' = L - 2\,e_y \qquad ...(24.38b)$$

$$A' = B' \times L' \qquad ...(24.38c)$$

The area A' is now such that its centroid coincides with the line of action of V.

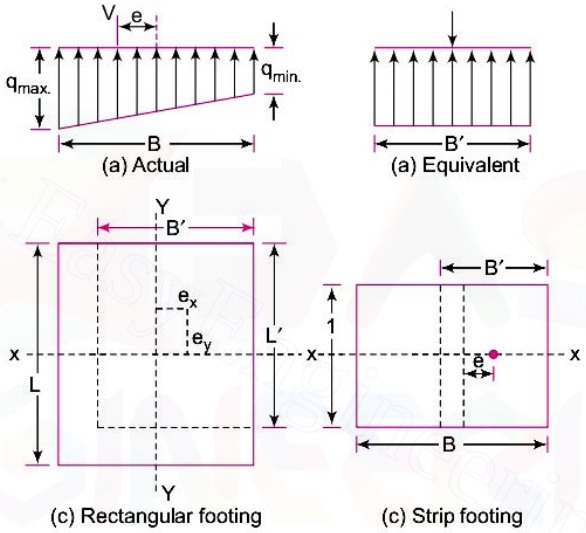

Fig. 24.12 Eccentric loading

24.12 IS CODE METHOD FOR COMPUTING BEARING CAPACITY

1. General: IS Code (IS : 6403–1981), recommends a bearing capacity equation which is similar in natureto those given by Meyerhof and Brinch Hansen. The Code recognises, depending upon the deformations associated with the load and the extent of development of failure surface, *three types* of failures of soil support beneath the foundations, They are: (*a*) general shear failure; (*b*) local shear failure; and (*c*) punching shear failure. The first two types of failures have already been described § 24.3. The *punching shear failure,* occurs on soils of high compressibility. In such a failure, there is vertical shear around the footing, perimeter and compression of soil immediately under the footing, with soil on the sides of the footing remaining practically un-involved.

2. Bearing capacity equation for strip footing for c-ϕ soils: The ultimate net bearing capacity of *strip footing* is given by the following equations:

(*i*) For the case of general shear failure:

$$q_{nf} = cN_c + \bar{\sigma}\ (N_q - 1) + \frac{1}{2} B\gamma\, N_\gamma \qquad ...(24.39)$$

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(*ii*) For the case of local shear failure :

$$q_{nf} = \frac{2}{3} cN'_c + \bar{\sigma}\left(N'_q - 1\right) + \frac{1}{2} B\gamma N'_\gamma \qquad ...(24.40)$$

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where $\bar{\sigma}$ = *effective* surcharge at the base level of foundation.

N_c, N_q, N_γ = Bearing capacity factors.

These factors are computed from the following equations.

$$N_c = (N_q - 1) \cot \varphi$$

$$N_q = \tan^2 (45° + \varphi/2)\, e^{\pi \tan \varphi} \quad \text{and} \quad N_\gamma = 2 (N_q + 1) \tan \varphi \qquad ...(\text{Vesic})$$

These factors are given in Table 24.9, for various values of φ at 1° interval. Original table by IS : 6403 – 1981 gives these values at 5° interval of φ.

Table 24.9 Bearing capacity factors (IS : 6403–1981)

φ°	N_c	N_q	N_γ	φ°	N_c	N_q	N_γ
0	5.14	1.00	0.00				
1	5.38	1.09	0.07	26	22.25	11.85	12.54
2	5.63	1.20	0.15	27	23.94	13.20	14.47
3	5.90	1.31	0.24	28	25.80	14.72	16.72
4	6.19	1.43	0.34	29	27.86	10.44	19.34
5	6.49	1.57	0.45	30	30.14	18.40	22.40
6	6.81	1.72	0.57	31	32.67	20.63	25.99
7	7.16	1.88	0.71	32	35.49	23.18	30.22
8	7.53	2.06	0.86	33	38.64	26.09	35.19
9	7.92	2.25	1.03	34	42.16	29.44	41.06
10	8.35	2.47	1.22	35	46.12	33.30	48.03
11	8.80	2.71	1.44	36	50.59	37.75	56.31
12	9.28	2.97	1.69	37	55.63	42.92	66.19
13	9.81	3.26	1.97	38	61.35	48.93	78.03
14	10.37	3.59	2.29	39	67.37	55.96	92.25
15	10.98	3.94	2.65	40	75.31	64.20	109.41
16	11.63	4.34	3.06	41	83.86	73.90	130.22
17	12.34	4.77	3.53	42	93.71	85.38	155.55
18	13.10	5.26	4.07	43	105.11	99.02	186.54
19	13.93	5.80	4.68	44	118.37	115.31	224.64
20	14.83	6.40	5.39	45	133.88	134.88	271.76
21	15.82	7.07	6.20	46	152.10	158.51	330.35
22	16.88	7.82	7.13	47	173.64	187.21	403.67
23	18.05	8.66	8.20	48	199.26	222.31	496.01
24	19.32	9.60	9.44	49	229.93	265.51	613.16
25	20.72	10.66	10.88	50	266.89	319.07	762.89

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For obtaining N_c', N_q', N_γ' corresponding to local shear failure, calculate $\phi' = \tan^{-1}(0.67\,\phi)$ and read N_c, N_q and N_γ from Table 24.9 corresponding to the value of ϕ' instead of ϕ.

3. Shape factor, depth factor and inclination factor: The above bearing capacity equations, applicable for strip footing, shall be modified to take into account, the shape of the footing, inclination of loading, depth of embedment and effect of water table. The modified bearing capacity formulae are given below:

(i) For general shear failure

$$q_{nf} = cN_c \cdot s_c \cdot d_c \cdot i_c + \bar{\sigma}\left(N_q - 1\right) s_q \cdot d_q \cdot i_q + \frac{1}{2} B\gamma \cdot N_\gamma \cdot s_\gamma d_\gamma \cdot i_\gamma W' \quad ...(24.41)$$

(ii) For local shear failure

$$q_{nf} = \frac{2}{3} cN_c' \cdot s_c \cdot d_c \cdot i_c + \bar{\sigma}\left(N_q' - 1\right) s_q \cdot d_q \cdot i_q + \frac{1}{2} B \cdot \gamma \cdot N_\gamma' \cdot s_\gamma d_\gamma \cdot i_\gamma W' \quad ...(24.42)$$

The shape factors s_c, s_q and s_γ are given in Table 24.10.

The *depth factors* are given as under:

$$d_c = 1 + 0.2 \frac{D}{B} \cdot \sqrt{N_\phi} \quad ...(24.43a)$$

$$d_q = d_\gamma = 1 \text{ for } \phi < 10° \quad ...(24.43b)$$

and $$d_q = d_\gamma = 1 + 0.1 \frac{D}{B} \cdot \sqrt{N_\phi} \text{ for } \phi > 10° \text{ where } N_\phi = \tan^2(45° + \phi/2).$$

Table 24.10 Shape factors (IS : 6403–1981)

S.No.	Shape of base	Shape factors		
		s_c	s_q	s_γ
1.	Continuous strip	1.0	1.0	1.0
2.	Rectangle	$\left(1 + 0.2 \frac{B}{L}\right)$	$\left(1 + 0.2 \frac{B}{L}\right)$	$\left(1 - 0.4 \frac{B}{L}\right)$
3.	Square	1.3	1.2	0.8
4.	Circle	1.3	1.2	0.6

Note. In case of circular footing, use B as the diameter.

The depth factors are to be applied only when the back filling is done with proper compaction. The *inclination factors are* given as under :

$$i_c = i_q = \left(1 - \frac{\alpha}{90}\right)^2 \quad ...(24.44a)$$

and $$i_\gamma = \left(1 - \frac{\alpha}{\phi}\right)^2 \quad ...(24.44b)$$

where α = inclination of the load to the vertical, in degrees.

4. Effect of water table: The effect of water table is taken into account in the form of a correction factor W' applied to the 'wedge term' in Eqs. 24.41 and 24.42. The value of W' may be chosen as indicated below:

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(*a*) If the water table is likely to permanently remain at or below a depth of $(D + B)$ beneath the ground level surrounding the footing, then $W' = 1$.

(*b*) If the water table is located at a depth D or likely to rise to the base of footing or above, then the value of W' shall be taken as 0.5.

(*c*) If the water table is likely to permanently get located at depth $D < D_w < (D + B)$, then W' be obtained by linear interpolation. Thus, for the intermediate case, we get the following expression for W'

$$W' = 0.5\left[1 + \frac{D_w - D}{B}\right] = 0.5\left[1 + \frac{Z_w}{B}\right] \quad ...(24.45)$$

When $Z_w = 0$, $W' = 0.5$, when $Z_w = B$, $W' = 1$

The validity of the above equation is within the range $D < D_w < (D + B)$. If we compare the above expression with Eq. 24.36, it is clear that W' is same as factor R_{w_2}.

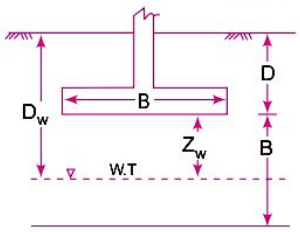

Fig. 24.13 Effect of water table

It may be noted that if the water table rises *above* the base of footing, W' will remain at its minimum value of 0.5, but $\bar{\sigma}$ (effective overburden pressure) will be computed of the basis of total unit weight of the portion of soil above the water table and submerged unit weight for the portion below the water table.

5. **Cohesionless soils ($c = 0$) :** For cohesionless soils, having $c = 0$, Indian Standard Code recommends that the bearing capacity be calculated (*a*) based on relative density or (*b*) based on standard penetration resistance value, and (*c*) based on static cone penetration test.

(*a*) ***Based on relative density*:** In this method, bearing capactiy may be calculated by Eqs. 24.39, 24.40, 24.41, 24.42, together with relevant shear strength parameter. In these formulae, c is taken equal to zero. The *relative density* given in Table 24.11 is used as a guide to determine the method of analysis.

Table 24.11 Method of analysis based on relative density

	Relative density (Density Index)	*Voids Ratio*	*Condition*	*Method of analysis*
1.	Greater than 70%	Less than 0.5	Dense	General shear
2.	Less than 20%	Greater than 0.75	Loose	Local shear (as well as punching shear)
3.	20% to 70%	0.55 to 0.75	Medium	Interpolate between (1) and (2)

(*b*) ***Based on standard penetration resistance value:*** The standard penetration resistance (§ 24.14) is determined at a number of selected points at intervals of 75 cm in the vertical direction or change of strata if it takes place earlier and the average value beneath each point is determined between the level of base of the footing and the depth equal to 1.5 to 2 times the width of foundation. In computing the value, any individual value more than 50% of the average calculated shall be neglected and average recalculated (the values for all loose seams shall however be included). Knowing the value of N, the value of ϕ is read from Fig. 24.14. The ultimate net bearing capactiy is then calculated from the formula :

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$$q_{nf} = \bar{\sigma}\left(N_q - 1\right) s_q \cdot d_q \cdot i_q + \frac{1}{2}\gamma B N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma \cdot W' \qquad ...(24.46)$$

where the shape factors, depth factors and inclination factors are determined as described earlier, and the bearing capactiy factors N_q and N_γ are taken from Table 24.7.

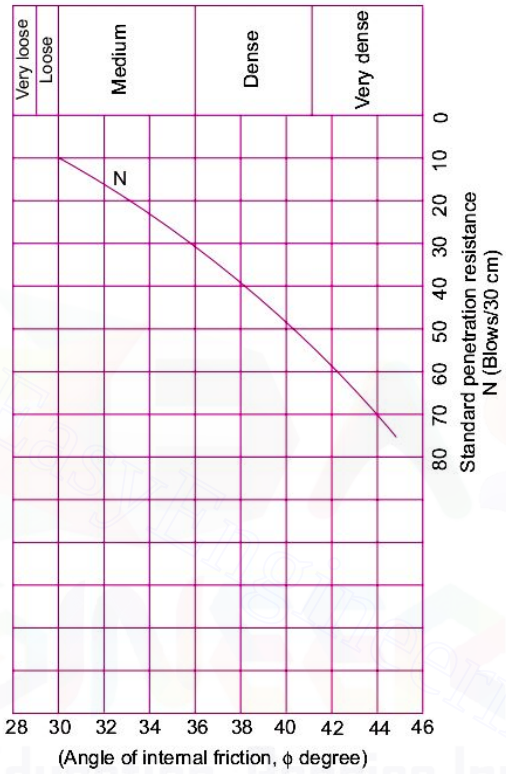

Fig. 24.14 Relationship between ϕ and *N*

(*c*) ***Based on static cone penetration test*:** The static cone point resistance q_c (§ 24.15) is determined as per IS: 4968 (Part III)-1976 at a number of selected points at invervals of 10 to 15 cm. The observed values are corrected for the dead weight of sounding rods. Then the average value at each one of the location is determined between the level of the base of the footing and the depth equal to 1.5 to 2 times the width of the footing. The averag of static cone point resistance value is determined for each one of the location and the minimum of the average value is used in the design. The net ultimate bearing capacity of shallow strip footing on cohesionless soil deposit is then determined from Fig. 24.15.

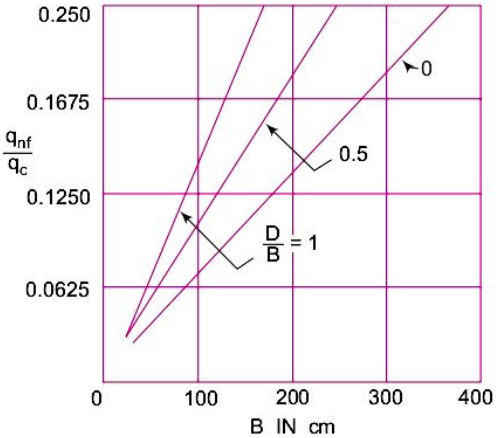

Fig. 24.15 Relation between q_{nf} and q_c

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6. **Cohesive soils ($\phi = 0$):** The net ultimate bearing capacity *immediately* after construction on fairly saturated homogeneous cohesive soils can be calculated from the expression:

$$q_{nf} = cN_c \cdot s_c \cdot d_c \cdot i_c \qquad ...(24.47)$$

where $N_c = 5.14$ (Table 24.7, for $\phi = 0$)

The value of c is obtained from unconfined compressive strength test. Alternatively, cohesion c may be determined from the static cone point resistance q_c using the empirical relationship given in Table 24.12.

Table 24.12 Relationship between q_c and c

	Soil Type	*Point Resistance Values q_c (kgf/cm^2)*	*Range of undrained cohesion (kgf/cm^2)*
1.	Normally consolidated clays	$q_c < 20$	$\frac{q_c}{18}$ to $\frac{q_c}{15}$
2.	Over consolidated clays	$q_c > 20$	$\frac{q_c}{26}$ to $\frac{q_c}{22}$

Note. 1 kgf/cm^2 = 100 kN/m^2.

Example 24.1. *A square footing 2.5 m by 2.5 m is built in a homogeneous bed of sand of unit weight 20 kN/m^3 and having an angle of shearing resistance of 36°. The depth of the base of footing is 1.5 m below the ground surface. Calculate the safe load that can be carried by a footing with a factor of safety of 3 against complete shear failure. Use Terzaghi's analysis.*

Solution. Given : $B = 2.5$ m ; $D = 1.5$ m ; $\gamma = 20$ kN/m^3. Since the soil is dense, and $\phi = 36°$, the footing is likely to fail by general shear failure.

From Table 24.2, the values of bearing capacity factors are :

$N_c = 65.4$, $N_q = 49.4$ and $N_\gamma = 54.0$. Also, $\bar{\sigma} = \gamma D$

Since $c = 0$, $q_f = \gamma D N_q + 0.4\,\gamma\, B\, N_\gamma$

or $q_{nf} = \gamma D(N_q - 1) + 0.4\gamma\, BN_\gamma = 20 \times 1.5(49.4 - 1) + 0.4 \times 20 \times 2.5 \times 54.0$

$= 1452 + 1080 = 2532$ kN/m^2

$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma D$ (Eq. 24.14) $= \frac{2532}{3} + (20 \times 1.5) = 844 + 30 = 874$ kN/m^2

Maximum safe load $= B^2 \times q_s = (2.5)^2 \times 874 =$ **5462.5 kN**

Example 24.2. *What will be the maximum safe load for example 24.1 if the soil is loose sand of unit weight 16 kN/m^3 and the angle of shearing resistance is 25°?*

Solution. Since the soil is loose and $\phi < 28°$, the footing is likely to fail by local shear failure. From Table 24.2, for $\phi = 25°$

$N'_q = 5.6$ and $N'_\gamma = 3.2$

$q_{nf} = \gamma D\,(N'_q - 1) + 0.4\,\gamma B\, N'_\gamma = 16 \times 1.5\,(5.6 - 1) + 0.4 \times 16 \times 2.5 \times 3.2$

$= 110.4 + 51.2 = 161.6$ kN/m^2

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$$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{16.18}{3} + (16 \times 1.5) = 77.87 \text{ kN/m}^2$$

$\therefore$ Safe load $= B^2 \times q_s = (2.5)^2\ (77.87) =$ **486.7 kN**.

Example 24.3. *A strip footing, 1 m wide at its base is located at a depth of 0.8 m below the ground surface. The properties of the foundation soil are : $\gamma = 18$ kN/m^3, $c = 30$ kN/m^2 and $\phi = 20°$. Determine the safe bearing capacity, using a factor of safety of 3. Use Terzaghi's analysis. Assume that the soil fails by local shear.*

Solution. The bearing capacity is given by Eq. 24.17, taking $\bar{\sigma} = \gamma D$

$$q_f = \frac{2}{3} cN'_c + \gamma D N'_q + 0.5\, \gamma\, B\, N'_\gamma$$

For $\phi = 20°$, the bearing capacity factors taken from Table 24.2 are :

$$N'_c = 11.8\ ;\ N'_q = 3.9\ ;\ N'_\gamma = 1.7$$

$\therefore$ $$q_f = (\frac{2}{3} \times 30 \times 11.8) + (18 \times 0.8 \times 3.9) + (0.5 \times 18 \times 1 \times 1.7)$$

$$= 236 + 56.2 + 15.3 = 307.5 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma D = 307.5 - 18 \times 0.8 = 293.1 \text{ kN/m}^2$$

$$q_f = \frac{q_{nf}}{F} + \gamma D = \frac{293.1}{3} + (18 \times 0.8) = 112.1 \text{ kN/m}^2.$$

Example 24.4. *Solve example 24.3 if the water table is located at the base of the footing. Assume saturated unit weight of soil as equal to 19.5 kN/m^3.*

Solution. Since the water table is at the base of footing, $R_{w_1} = 1$.

Z_{w_2} = depth of water table below the base of footing = zero.

$\therefore$ $R_{w_2} = 0.5 \left(1 + \frac{Z_{w_2}}{B}\right) = 0.5$. In the wedge term, take $\gamma = \gamma_{sat} = 19.5$ kN/m^3

$\therefore$ $$q_f = \frac{2}{3} cN'_c + \gamma D N'_q + 0.5 \gamma_{sat} \cdot B N'_\gamma R_{w_2}$$

$$= (\frac{2}{3} \times 30 \times 11.8) + (18 \times 0.8 \times 3.9) + (0.5 \times 19.5 \times 1 \times 1.7 \times 0.85)$$

$$= 236 + 56.16 + 14.09 = 306.25 \text{ kN/m}^2$$

The net ultimate bearing capacity is $q_{nf} = q_f - \gamma D = 306.25 - 18 \times 0.8 = 291.85$ kN/m^2

$\therefore$ Net safe bearing capacity is $q_{ns} = \frac{q_{nf}}{F} = \frac{291.85}{3} = 97.28$ kN/m^2

$\therefore$ Safe bearing capacity (also known as gross safe bearing capactiy)

$$q_s = q_{ns} + \gamma D = 97.28 + 18 \times 0.8 = \mathbf{111.68\ kN/m^2.}$$

Example 24.5 *A strip footing 2 m wide carries a load intensity of 400 kN/m^2 at a depth of 1.2 m in sand. The saturated unit weight of sand is 19.5 kN/m^3 and unit weight above water table is 16.8 kN/m^3. The shear strength parameters are $c = 0$ and $\phi = 35°$. Determine the factor of safety with respect to shear failure for the following cases of location of water table:*

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(*a*) *Water table is 4 m below G.L.* (*b*) *Water table is 1.2 m below G.L.*
(*c*) *Water table is 2.5 m below G.L.* (*d*) *Water table is 0.5 m below G.L.*
(*e*) *Water table is at G.L. itself. Use Terzaghi's equations.*

Solution. For a strip footing, the bearing capacity is given by

$$q_f = cN_c + \bar{\sigma} N_q + \frac{1}{2} B\gamma N_\gamma$$

Taking into account the water reduction factor, we have, from Eq. 24.34.

$$q_f = cN_c + \gamma D\, N_q\, R_{w_1} + \frac{1}{2}\, B\gamma\, N_\gamma\, .\, R_{w_2}$$

For the present case $c = 0$. $\therefore \quad q_f = \gamma D\, N_q\, .\, R_{w_1} + \frac{1}{2}\, B\gamma\, N_\gamma\, .\, R_{w_2}$

For $\phi = 35°$ assuming general shear failure, $N_q = 41.4$ and $N_\gamma = 42.4$

$$\therefore \quad q_f = 41.4 \times 1.2 \times \gamma\, R_{w_1} + \frac{1}{2} \times 2 \times 42.2\, \gamma\, R_{w_2}$$

$$q_f = 49.68\, \gamma\, R_{w_1} + 42.3\, \gamma\, R_{w_2} \qquad ...(1)$$

Case (a) Water table is 4 m below G.L.

$$Z_{w_2} = 4 - 1.2 = 2.8 \text{ m} \; ; \; R_{w_1} = 1$$

Since $Z_{w_2} > B$, $\quad R_{w_2} = 1.$

Hence there will be no effect of water table. Also $\gamma = 16.8$

$$\therefore \quad q_f = 49.68 \times 16.8 \times 1 + 42.4 \times 16.8 \times 1$$

$$= 1546.9 \text{ kN/m}^2$$

Now actual footing load $\quad = q_a = 400 \text{ kN/m}^2$

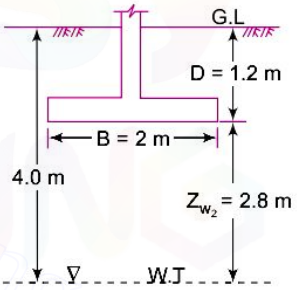

Fig. 24.16

$$\therefore \quad \text{F.S} = \frac{q_f}{q_c} = \frac{1546.9}{400} \approx 3.87$$

Case (b) Water table is just at the base of footing.

$$\therefore \quad R_{w_1} = 0.5\left(1 + \frac{Z_{w_1}}{D}\right) = 0.5\,(1+1) = 1$$

$$R_{w_1} = 0.5\left(1 + \frac{Z_{w_2}}{B}\right) = 0.5\,(1+0) = 0.5$$

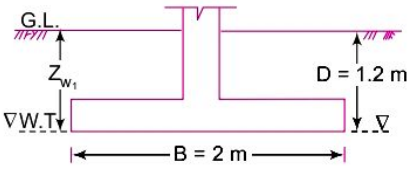

Fig. 24.17

$\therefore$ For the surcharge term, use $\gamma = 16.8$ kN/m^2, because the surcharge soil is situated above water table. For the 'wedge term', use $\gamma = \gamma_{sat} = 19.5$ kN/m^3, since the wedge soil is situated below water table.

$$\therefore \quad q_f = 49.68\, \gamma\, R_{w_1} + 42.4\, \gamma_{sat}\, .\, R_{w_2}$$

or $\quad q_f = 49.68 \times 16.8 \times 1 + 42.4 \times 19.5 \times 0.5 = 1248 \text{ kN/m}^2$

$$\therefore \quad \text{F.S.} = \frac{q_f}{q_a} = \frac{1248}{400} = \mathbf{3.12.}$$

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Case (c) Water table at 2.5 m below the G.L. (Fig. 24.18*a*)

$$Z_{w_2} = 2.5 - 1.2 = 1.3 \text{ m} < \text{B}. \quad \therefore \quad R_{w_1} = 1$$

$$R_{w_2} = 0.5\left(1 + \frac{Z_{w_2}}{B}\right) = 0.5\left(1 + \frac{1.3}{2}\right) = 0.825.$$

For the surcharge term, $\gamma = 16.8 \text{ kN/m}^3$

For the *wedge term*, γ will be taken as *average unit weight* of soil situated below the footing level, since the soil up to depth B below the footing is partly above water table and partly below the water table.

$$\gamma_{av} = \frac{(16.8 \times 1.3) + (19.5 \times 0.7)}{(1.3 + 0.7)} = 17.75 \text{ kN/m}^3$$

Hence from Eq. 1, we have $q_f = 49.68\, \gamma R_{w_1} + 42.4\, \gamma_{av} R_{w_2}$

$$= (49.68 \times 16.8 \times 1) + (42.4 \times 17.75 \times 0.825) = 1455.5 \text{ kN/m}^2$$

$$\therefore \quad \text{F.S.} = \frac{q_f}{q_a} = \frac{1455.5}{400} = \mathbf{3.64}$$

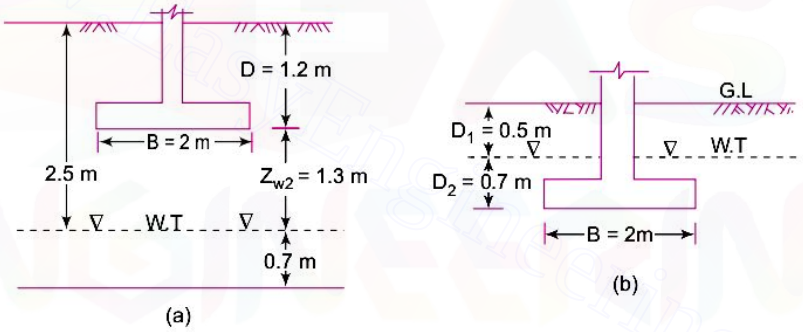

Fig. 24.18

Case (d) Water table is 0.5 m below G.L. (Fig. 24.18*b*)

$$Z_{w_1} = 0.5$$

$$\therefore \quad R_{w_1} = 0.5\left(1 + \frac{Z_{w_1}}{D}\right) = 0.5\left(1 + \frac{0.5}{1.2}\right) = 0.708$$

$R_{w_2} = 0.5$. Since water table is above footing.

For the wedge term, $\gamma = \gamma_{sat}\ 19.5 \text{ kN/m}^3$

For the surcharge term, take γ = average unit weight of soil situated above the base of footing, since this soil is located partly above the water table and partly below the water table.

$$\therefore \quad \gamma_{av} = \frac{(16.8 \times 0.5) + (19.5 \times 0.7)}{(0.5 + 0.7)} = 18.38$$

Hence from Eq. 1. $q_f = 49.68\, \gamma_{av} R_{w_1} + 42.4\, \gamma_{sat} R_{w_2}$

$$= (49.68 \times 18.38 \times 0.708) + (42.4 \times 19.5 \times 0.5) = 1060 \text{ kN/m}^2$$

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$$\therefore \qquad \text{F.S.} = \frac{q_f}{q_a} = \frac{1060}{400} = \mathbf{2.65.}$$

Note. Alternatively, q_f can be calculated by taking $\bar{\sigma}$ in place of γD term, *without* considering reduction factor R_{w_1} as recommended by IS code (see § 24.12)

$$q_f = \bar{\sigma} N_q + \frac{1}{2} B \gamma N_\gamma \, . R_{w_2}$$

or
$$q_f = \bar{\sigma} \ (41.4) + \left(\frac{1}{2} \times 2 \times 42.4\right) \gamma_{sat} R_{w_2}$$

where $\bar{\sigma}$ = effective overburden pressure = $(\gamma_1 D_1 + \gamma_2 D_2) - \gamma_w D_2$ (Eq. 24.36 *a*)

$$= (16.8 \times 0.5) + (19.5 \times 0.7) - (9.81 \times 0.7) = 15.183$$

$$\therefore \qquad q_f = 15.183 \times 41.4 + \frac{1}{2} \times 2 \times 42.4 \times 19.5 \times 0.5 = 1042 \text{ kN/m}^2$$

against a value of 1060 kN/m^2.

Case (e) Water table at the ground level

When water table is at ground level,

$$R_{w_1} = 0.5 \ (1 + 0) = 0.5; \ R_{w2} = 0.5$$

$\gamma = \gamma_{sat} = 19.5$ for both the surcharge soil as well as the wedge soil.

Hence from Eq.(l) $q_f = 49.68 \ \gamma_{sat} R_{w_1} + 42.4 \ \gamma_{sat} R_{w_2}$

$$= 49.68 \times 19.5 \times 0.5 + 42.4 \times 19.5 \times 0.5 = 897.78 \text{ kN/m}^2$$

$$\therefore \qquad \text{F.S.} = \frac{q_f}{q_a} = \frac{897.78}{400} = 2.24.$$

Example 24.6. *Solve part (c) of example 24.5 considering effective weight of soil in the wedge zone.*

Solution. $Z_{w_2} = 1.3$ m ;

$$\gamma' = \gamma_{sat} - \gamma_w = 19.5 - 9.81 = 9.69 \text{ kN/m}^3$$

Height H of the wedge zone

$$= 0.5 \, B \tan\left(45° + \frac{\phi}{2}\right) = 0.5 \times 2 \tan\left(45° + \frac{35}{2}\right) = 1.92 \text{ m}$$

Hence the water table is within wedge zone

$$\gamma_e = \left(2H - Z_{w_2}\right) \frac{Z_{w_2}}{H^2} \gamma + \frac{\gamma'}{H^2} \left(H - Z_{w_2}\right)^2$$

$$= (2 \times 1.92 - 1.3) \frac{1.3}{(1.92)^2} \times 16.8 + \frac{9.69}{(1.92)^2} [1.92 - 1.3]^2 = 16.06 \text{ kN/m}^3$$

$$\bar{\sigma} = \gamma D = 16.8 \times 1.2 = 20.16$$

Hence substituting in Eq. 24.37 (*a*)

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$$q_f = cN_c + \bar{\sigma} N_q + \frac{1}{2}\gamma_e B N_\gamma = 0 + 20.16 \times 41.4 + \frac{1}{2} \times 16.06 \times 2 \times 42.4$$

$$= 1515.6 \text{ kN/m}^2 \text{ (against a value of 1455.5 found in Example 24.4)}$$

$$\therefore \quad \text{F.S.} = \frac{q_f}{q_a} = \frac{1515.6}{400} = \mathbf{3.79.}$$

Alternatively, from Eq. 24.37 (*c*),

$$\gamma_e = \gamma' + (\gamma - \gamma') \frac{Z_{w_2}}{B} = 9.69 + (16.8 - 9.69) \frac{1.3}{2} = 14.31 \text{ kN/m}^3$$

$$\bar{\sigma} = \gamma D = 16.8 \times 1.2 = 20.16 \text{ kN/m}^2$$

Hence substituting in Eq. 24.37 (*a*)

$$q_f = cN_c + \bar{\sigma} N_q + \frac{1}{2}\gamma_e B N_\gamma = 0 + (20.16 \times 41.4) + (\frac{1}{2} \times 14.31 \times 2 \times 42.4)$$

$$= 1441.4 \text{ kN/m}^2$$

$$\therefore \quad \text{F.S.} = \frac{q_f}{q_a} = \frac{1441.4}{400} = 3.6.$$

This value is very near to the one found in Example 24.5 (*c*).

Example 24.7. *Design a strip footing to carry a load of 750 kN/m at a depth of 1.6 m in a c-ϕ soil having a unit weight of 18 kN/m³ and shear strength parameters as c = 20 kN/m² and ϕ = 25°. Determine the width of footing, using a factor of safety of 3 against shear failure. Use Terzaghi's equations.*

Solution. Assume general shear failure. From Eq. 24.12.

$$q_f = cN_c + \bar{\sigma}\ N_q + 0.5\ \gamma B N_\gamma$$

For $\phi = 25°$ we have $N_c = 25.1$, $N_q = 12.7$ and $N_\gamma = 9.7$ from Table 24.2. Also, $\bar{\sigma} = \gamma D = 18 \times 16$. Substituting the values, we get

$$q_f = (20 \times 25.1) + (18 \times 1.6) \times 12.7 + 0.5 \times 18 \times B \times 9.7$$

or $$q_f = 502 + 365.8 + 87.3\ \text{B} = 867.8 + 87.3\ B \qquad ...(1)$$

$\therefore$ Intensity of pressure, at F.S. of 3 at footing level

$$= \frac{q_f}{3} = \frac{867.8 + 87.3\ B}{3} \text{ kN/m}^2 \qquad ...(2)$$

Applied load intensity $$= \frac{750}{B \times 1} = \frac{750}{B} \text{ kN/m}^2 \qquad ...(3)$$

Equating the two, we get $\frac{750}{B} = \frac{867.8 + 87.3B}{3}$. Simplifying, we get

$$87.3\ B^2 + 867.8\ \text{B} - 2250 = 0$$

or $$B^2 + 9.94\ \text{B} \times 25.77 = 0$$

From which, we get $B = \mathbf{2.134\ m}$

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Example 24.8. *A rectangular footing 2 m × 3 m rests on a c–ϕ soil, with its base at 1.5 m below the ground surface. Calculate the safe bearing capacity, using a factor of safety of 3 on (i) net ultimate bearing capacity, and (ii) ultimate bearing capacity. The soil has following parameters : $\gamma = 18$ kN/m^3; $c = 10$ kN/m^2; $\phi = 30°$. Use Terzaghi's analysis.*

Solution. There are two expressions [Eqs. 24.20 (*a*) and (*b*)] given for q_f for rectangular footing. We will use here Eq. 24.20 (*b*) which uses *shape factors* with both N_c and N_γ terms. Taking, $\bar{\sigma} = \gamma D$, we get

$$\therefore \quad q_f = cN_c\left(1 + 0.3\frac{B}{L}\right) + \gamma D N_q + 0.5\,\gamma B N_\gamma\left(1 - 0.2\frac{B}{L}\right)$$

For $\phi = 30°$ we get

$$N_c = 37.2\ ;\ N_q = 22.5 \text{ and } N_\gamma = 19.7$$

$$\therefore \quad q_f = 10 \times 37.2\,(1 + 0.3 \times \frac{2}{3}) + 18 \times 1.5 \times 22.5 + 0.5 \times 18 \times 2 \times 19.7 \times (1 - 0.2 \times \frac{2}{3})$$

or $$q_f = 446.4 + 607.5 + 307.3 = 1361.2 \text{ kN/m}^2$$

(*i*) $$q_{nf} = q_f - \gamma D = 1361.2 - 18 \times 1.5 = 1334.2 \text{ kN/m}^2$$

$$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{1334.2}{3} + 18 \times 1.5 = \mathbf{471.7 \text{ kN/m}^2}$$

(*ii*) Applying F.S. on q_f, we get

$$q_s = \frac{q_f}{3} = \frac{1361.2}{3} = \mathbf{453.7 \text{ kN/m}^2}.$$

Example 24.9. *A square footing 2 m × 2 m, is founded on a depth of 1.2 m below the surface of a deep stratum of soft saturated clay having unit weight of 19.5 kN/m^3.*

The soil has undrained soil parameters as $\phi_u = 0$ and $c_u = 25$ kN/m^2 and consolidated undrained parameters (triaxial test) as $\phi' = 22.5°$ and $c' = 0$.

Determine the ultimate bearing capacity of the foundation, (i) immediately after construction, and (ii) few years after construction.

Solution. Saturated cohesive soils remain in undrained state during and immediately after construction. Hence the relevant soil parameters are c_u and ϕ_u where ϕ_u is generally zero. If long term stability is required, undrained parameters c' and ϕ' may be used, where c' is generally taken as zero.

(*i*) *Immediately after construction:* $\phi = \phi_u = 0$ and $c = c_u = 25$ kN/m^2

For square footing, we have from Eq. 24.19

$$q_f = 1.3\,cN_c + \bar{\sigma} N_q + 0.4\,\gamma_{sat}\,BN_\gamma \qquad ...(i)$$

where $\bar{\sigma} = \gamma_{sat} \,.\, D = 19.5 \times 1.2\ ;\ \phi = 0$

$\therefore \quad N_c = 5.7\ ;\ N_q = 1.0\ ;\ N_\gamma = 0.0$

$\therefore \quad q_f = 1.3 \times 25 \times 5.7 + 19.5 \times 1.2 \times 1 + 0 = \mathbf{208.65 \text{ kN/m}^2}.$

(*ii*) *Few years after construction:* $\phi = \phi' = 22.5°\ ;\ c = 0$

$\therefore \quad N_c = 21.4\ ;\ N_q = 10.1\ ;\ N_\gamma = 7.3$

Hence from (*i*)

$$q_f = 0 + (19.5 \times 1.2) \times 10.1 + 0.4 \times 19.5 \times 2 \times 7.3 = \mathbf{350.22 \text{ kN/m}^2}.$$

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DEPARTMENT OF CIVIL ENGINEERING

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ACADEMIC YEAR: 2013-2014

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	78	SHILPI RATHOD		
	01	SHAKAMBARI ALLE		
2	53	KAPIL MOKASHI	SCOURING DEPTH	Prof Avinash More
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	54	MOHSIN FAHIM		
	49	LAMTURE WAJID		
3	9	SHRUSHTI CHORMALE	MUMBAI METROLINE (UNDERGROUND TUNNELING)	Prof Avinash More
	82	HEMITA SAWANT		
	119	SHASHWAT SINGH		
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Example 24.10. *A square footing located at a depth of 1.3 m below the ground has to carry a safe load of 800 kN. Find the size of the footing if the desired factor of safety is 3. The soil has the following properties :*

Voids ratio = 0.55 ; Degree of saturation = 50% ; Specific gravity = 2.67

$c = 8$ kN/m^2 ; $\phi = 30°$. Use Terzaghi's analysis.

Solution. The bulk unit weight of soil is given by

$$\gamma = \frac{(G + eS)\gamma_w}{1+e} = \frac{(2.67 + 0.55 \times 0.5)\,9.81}{1+0.55} = 18.64 \text{ kN/m}^3.$$

For $\phi = 30°$, $N_c = 37.2$, $N_q = 22.5$ and $N_\gamma = 19.7$

Terzaghi's bearing capacity equation for square footing is

$$q_f = 1.3\, cN_c + \bar{\sigma} N_q + 0.4\, \gamma B N_\gamma \text{ where } \bar{\sigma} = \gamma D = 18.64 \times 1.3.$$

$$\therefore \quad q_f = 1.3 \times 8 \times 37.2 + 18.64 \times 1.3 \times 22.5 + 0.4 \times 18.64\, B \times 19.7$$

or $$q_f = 932.1 + 146.88\, B \text{ kN/m}^2$$

$$\therefore \quad q_{nf} = (932.1 + 146.88\, B) - 18.64 \times 1.3 = 907.87 + 146.88\, B$$

$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma D = \frac{907.87 + 146.88\, B}{3} + (18.64 \times 1.3)$$

$$= 326.86 + 48.96\, B \text{ kN/m}^2 \qquad ...(1)$$

Actual load intensity $$q_a = \frac{800}{B^2} \qquad ...(2)$$

Equating the two, we get $\frac{800}{B^2} = 326.86 + 48.96\, B$

or $$B^3 + 6.676\, B^2 = 16.34$$

Solving this by trial and error, we get $B \approx$ **1.42 m.**

Example 24.11. *Determine the depth at which a circular footing of 2 m diameter be founded to provide a factor of safety of 3, if it has to carry a safe load of 1600 kN. The foundation soil has $c = 10$ kN/m^2, $\phi = 30°$ and unit weight = 18 kN/m^3. Use Terzaghi's analysis.*

Solution. For $\phi = 30°$, we have $N_c = 37.2$; $N_q = 22.5$; $N_\gamma = 19.7$.

The Terzaghi's bearing capacity equation for circular footing is

$$q_{nf} = 1.3\, cN_c + \gamma D\,(N_q - 1) + 0.3\, \gamma B N_\gamma$$

$$= 1.3 \times 10 \times 37.2 + 18\, D\,(22.5 - 1) + 0.3 \times 18 \times 2 \times 19.7 = 696.36 + 387\, D$$

$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma D = \frac{696.36 + 387\, D}{3} + 18\, D$$

or $$q_s = 232.12 + 147\, D \qquad ...(1)$$

Actual load intensity, $$q_a = \frac{1600}{\frac{\pi}{4}(2)^2} = 509.3 \qquad ...(2)$$

Equating the two, we get $232.12 + 147\, D = 509.3$

$\therefore$ $D = 1.89$ m Keep $\boldsymbol{D = 1.9}$ **m**

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Example 24.12. *A rectangular footing has a size of 1.8 m × 3 m has to transmit the load of a column at a depth of 1.5 m. Calculate the safe load which the footing can carry at a factor of safety of 3 against shear failure. Use IS code method. The soil has following properties : n = 40%; G = 2.67; w = 15% c = 8 kN/m² ; ϕ = 32.5°.*

Solution.

$$e = \frac{n}{1-n} = \frac{0.4}{1-0.4} = 0.667$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.67 \times 9.81}{1+0.667} = 15.71 \text{ kN/m}^3.$$

$$\gamma = \gamma_d (1 + w) = 15.71 (1 + 0.15) = 18.07 \text{ kN/m}^2$$

For $\phi = 32.5°$ we get, from Table 24.9

$$N_c = 38.13 ; N_q = 25.85 ; N_\gamma = 35.21$$

For general shear failure, we have

$$q_{nf} = cN_c \,.\, s_c \,.\, d_c \,.\, i_c + \bar{\sigma} (N_q - 1)\, s_q \,.\, d_q \,.\, i_q + \frac{1}{2} B\gamma N_\gamma \,.\, S_\gamma \,.\, d_\gamma \,.\, i_\gamma W' \quad ...(24.41)$$

Since the load is vertical $i_c = i_q = i_\gamma = 1$; Take $W' = 1$

For rectangular footing, shape factors will be as under (Table 24.10) :

$$s_c = 1 + 0.2 \frac{B}{L} = 1 + 0.2 \times \frac{1.8}{3} = 1.12$$

$$s_q = 1 + 0.2 \frac{B}{L} = 1.12$$

$$s_\gamma = 1 - 0.4 \frac{B}{L} = 1 - 0.4 \times \frac{1.8}{3} = 0.76$$

The depth factors will be as under :

$$d_c = 1 + 0.2 \frac{D}{B} \sqrt{N_\phi} ,$$

where $$\sqrt{N_\phi} = \tan (45° + \phi/2) = \tan \left(45° + \frac{32.5}{2} \right) = 1.823$$

$$d_c = 1 + 0.2 \times \frac{1.5}{1.8} \times 1.823 = 1.304$$

$$d_q = d_\gamma = 1 + 0.1 \frac{D}{B} \sqrt{N_\phi} \text{ , for } \phi > 10° = 1 + 0.1 \times \frac{1.5}{1.8} \times 1.823 = 1.152$$

Also, $\bar{\sigma}$ = effective overburden pressure = $\gamma D = 18.07 \times 1.5 = 27.105 \text{ kN/m}^2$.

Substituting the values, we get

$$q_{nf} = \{8 \times 38.13 \times 1.12 \times 1.304 \times 1\} + \{27.105 (25.85 - 1) \times 1.12 \times 1.152 \times 1\}$$
$$+ \{\tfrac{1}{2} \times 1.8 \times 18.07 \times 35.21 \times 0.76 \times 1.52 \times 1\}$$

or $$q_{nf} = 445.50 + 869.05 + 501.34 = 1815.89 \text{ kN/m}^2$$

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$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma D = \frac{1815.89}{3} + (18.07 \times 1.5) = 632.4 \text{ kN/m}^2$$

$$\therefore \quad \text{Safe load} = q_s (B \times L) = 632.4 \times 1.8 \times 3 \approx \mathbf{3415\ kN.}$$

Example 24.13. *What will be the per cent reduction in the load capacity of the footing of example 24.12 if the water table rises up to (a) base of footing (b) 1 m below the base of the footing. Assume that saturated unit weight of soil is 19.5 kN/m^3.*

Solution.

(*a*) ***When the water table rises up to the base of footing***

In this case, $W' = 0.5$. However, γ with the last terms of bearing capacity equation will become γ_{sat}. The first two terms of the bearing capacity equation will remain unaffected.

$$\text{Now, } \frac{1}{2} B \gamma N_\gamma . s_\gamma . d_\gamma . i_\gamma . W' = \frac{1}{2} \times 1.8 \times 19.5 \times 35.21 \times 0.76 \times 1.152 \times 1 \times 0.5$$

$$= 270.51 \text{ kN/m}^2 \text{ (as against earlier value of 501.34)}$$

$$\therefore \quad q_{nf} = 445.50 + 869.05 + 270.51 = 1585.06$$

$$q_s = \frac{1585.06}{3} + (18.07 \times 1.5) = 555.46 \text{ kN/m}^2$$

$$\therefore \quad \text{Safe load} = 555.46 \times 1.8 \times 3 = \mathbf{2999.6\ kN}$$

$$\therefore \quad \% \text{ reduction} = \frac{3415 - 2999.5}{3415} \times 100 = \mathbf{12.2\%}$$

(*b*) ***When the water table rises up to 1 m below the base of footing.***

Here $Z_w = 1$. Hence from Eq. 24.45

$$W' = 0.5\left[1 + \frac{Z_w}{B}\right] = 0.5\left[1 + \frac{1}{1.8}\right] \approx 0.78$$

The unit weight γ to be substituted in the last term (wedge term) may be taken as average unit weight of the soil in the wedge zone.

$$\gamma_{av} \approx \frac{(1 \times 18.07) + (0.8 \times 19.5)}{1 + 0.8} = 18.7 \text{ kN/m}^3$$

$$\therefore \quad \frac{1}{2} B \gamma N_\gamma . s_\gamma . d_\gamma . i_\gamma W' = \frac{1}{2} \times 1.8 \times 18.7 \times 35.21 \times 0.76 \times 1.152 \times 1 \times 0.78$$

$$= 404.68 \text{ kN/m}^2$$

$$\therefore \quad q_{nf} = 445.50 + 869.05 + 404.68 = 1719.23 \text{ kN/m}^2$$

$$\therefore \quad q_s = \frac{1719.23}{3} + (18.07 \times 1.5) = 600.18 \text{ kN/m}^2$$

$$\therefore \quad \text{Safe laod} = 600.18 \times 1.8 \times 3 \approx \mathbf{3241\ kN}$$

$$\% \text{ reduction} = \frac{3415 - 3241}{3415} \times 100 = \mathbf{5.1\%.}$$

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Example 24.14. *A square footing 1.5 m × 1.5 m is located at a depth of 1 m. The soil has the following properties: $\gamma = 17.5$ kN/m^3, $c = 20$ kN/m^2 and $\phi = 20°$. Using Hansen's method, compute the ultimate bearing capacity of the soil. The footing base and ground are horizontal.*

Solution. Since base and ground are horizontal, factors g and b are each unity.

Hence, $$q_f = cN_c\, s_c\, d_c\, i_c + q_0\, N_q\, s_q\, d_q\, i_q + 0.5\,\gamma\, BN_\gamma\, s_\gamma\, d_\gamma\, i_\gamma \qquad ...(1)$$

(*i*) The value of bearing capacity factors (Table 24.3) for $\phi = 20°$ are

$$N_c = 14.83,\ N_q = 6.4 \text{ and } N_\gamma = 2.9.$$

(*ii*) Shape factors (Table 24.4)

$$s_c = 1 + \frac{N_q}{N_c} \cdot \frac{B}{L} = 1 + \frac{6.4}{14.83} \times \frac{1.5}{1.5} = 1.43$$

$$s_q = 1 + \frac{B}{L} \tan\phi = 1 + \frac{1.5}{1.5} \tan 20° = 1.36$$

$$s_\gamma = 1 - 0.4\,\frac{B}{L} = 1 - 0.4\,\frac{1.5}{1.5} = 0.6\,.$$

(*iii*) Depth factors (Table 24.5) : $D < B$

$$d_c = 1 + 0.4\,\frac{D}{B} = 1 + 0.4 \times \frac{1}{1.5} = 1.27$$

$$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2\,\frac{D}{B} = 1 + 2\tan 20°\,(1 - \sin 20°)^2 \times \frac{1}{1.5} = 1.21$$

$$d_\gamma = 1.0$$

(*iv*) Inclination factors: Since the load is vertical

$$i_c = i_q = i_\gamma = 1$$

Substituting these values, we get

$$q_f = (20 \times 14.83 \times 1.43 \times 1.27 \times 1) + (17.5 \times 1 \times 6.4 \times 1.36 \times 1.21) + (0.5 \times 17.5 \times 1.5 \times 2.9 \times 0.6 \times 1 \times 1)$$

$$= 538.7 + 184.3 + 22.8 = \mathbf{745.8\ kN/m^2}.$$

Example 24.15. *If in example 24.14, the horizontal component of weight = 20 kN acting at height of 4 m parallel to the size of footing, and vertical component is 500 kN, determine q_f and factor of safety.*

Solution. (Fig. 24.19) $V = 500$ kN ; $H = 20$ kN

Eccentricity, $$e = \frac{H}{V} \times z = \frac{20}{500} \times 4 = 0.16 \text{ m}$$

$\therefore$ $$B' = B - 2e = 1.5 - 2 \times 0.16 = 1.18 \text{ m}$$

$$L' = B = 1.5 \text{ m}$$

$\therefore$ $$A' = L' \times B' = 1.5 \times 1.18 = 1.77 \text{ m}^2$$

The inclination factors are given by Table 24.5.

$$i_q = \left(1 - \frac{0.5H}{V + cA' \cot\phi}\right)^5$$

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$$= \left(1 - \frac{0.5 \times 20}{500 + 20 \times 1.77 \cot 20°}\right)^5 = 0.92$$

$$i_\gamma = \left(1 - \frac{0.7H}{V + qA' \cot \phi}\right)^5$$

$$= \left(1 - \frac{0.7H}{500 + 20 \times 1.77 \cot 20°}\right)^5 = 0.89$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1} = 0.92 - \frac{1 - 0.92}{6.4 - 1} = 0.905$$

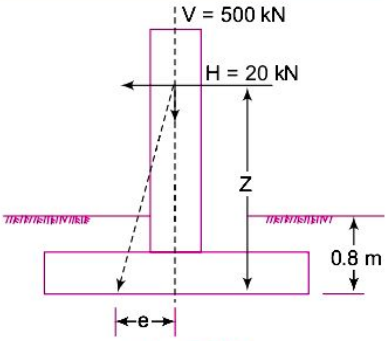

Fig. 24.19

All other factors remain the same, as in the previous example. Hence

$$q_f = (20 \times 14.83 \times 1.43 \times 1.27 \times 0.905) + (17.5 \times 1 \times 6.4 \times 1.26 \times 1.21 \times 0.92) + (0.5 \times 17.5 \times 1.5 \times 2.9 \times 0.6 \times 1 \times 0.89)$$

$$= 487.5 + 169.6 + 20.3 = 677.4 \text{ kN/m}^2$$

$\therefore$ Ultimate load $= 677.4 \times 1.77 \approx 1200$ kN

$\therefore$ $\text{F.S.} = \dfrac{1200}{500} = 2.4$

24.13 PLATE LOAD TEST (IS 1888 : 1982)

Plate load test is a field test to determine the ultimate bearing capacity of soil, and the probable settlement under a given loading. The test essentially consists in loading a rigid plate at the foundation level, and determining the settlements corresponding to each load increment. The ultimate bearing capacity is then taken as the load at which the plate starts sinking at a rapid rate. The method assumes that down to the depth of influence of stresses, the soil strata is reasonably uniform.

1. **Bearing plate :** The bearing plate is either circular or square, made of mild steel of not less than 25 mm in thickness and varying in size from 300 to 750 mm with chequered or grooved bottom (Fig. 24.20). The plate is provided with handles for convenient setting and centre marked. As an alternative, cast *in situ* or precast concrete blocks may be used with depth not less than two thirds the width.

   Except in case of road problems or circular footings, square plates may be adopted. For clayey and silty soils and for loose to medium dense sandy soils with $N < 15$, a 450 mm square plate or concrete block shall be used. In case of dense sandy or gravely soils ($15 < N < 30$), three plates of sizes 300 mm to 750 mm shall be used depending upon the practical considerations of reaction loading and maximum grain size. The size of the plate shall be at least four times the maximum size of the soil particles present at the test location.

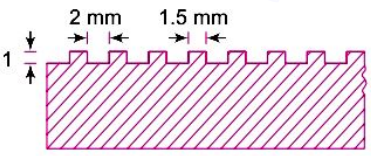

Fig. 24.20 Details of chequers or grooves in test plate

2. **Test pit :** The test pit, usually at the foundation level, having in general normally of width equal to five times the test plate (B_p) or block, shall have a carefully levelled and cleaned bottom at the foundation level, protected against disturbances or changes in natural formation. The test pits should preferably have steps to conveniently go in the pit for setting and for taking observations.

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3. **Loading arrangement :** The loading to the test plate may be applied with the help of a hydraulic jack. The reaction of the hydraulic jack may be borne by either of the following two methods:

(*a*) Gravity loading platform method (Fig. 24.22)

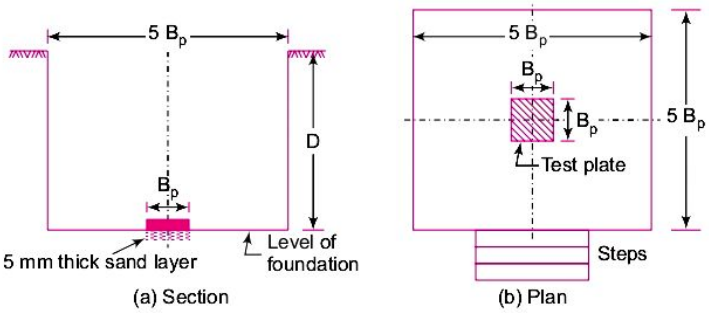

Fig. 24.21

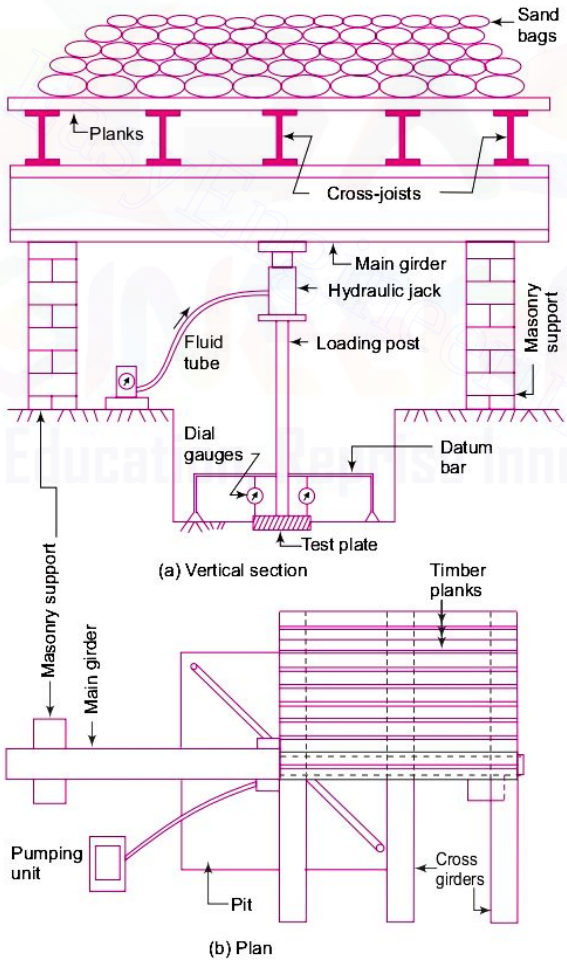

Fig. 24.22 Plate load test : Reaction by gravity loading

(*b*) Reaction truss method. (Fig. 24.23)

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(*a*) ***Gravity loading* method :*** In the case of gravity loading method, a platform is constructed over a vertical column resting on the test plate, and the loading is done with the help of sand bags, stones or concrete blocks. The general arrangement of the test set-up for this method is shown in Fig. 24.22.

When load is applied to the plate, it sinks or settles. The settlement of the plate is measured with the help of sensitive dial gauges. For square plate, two dial gauges are used. The dial gauges are mounted on independently supported *datum bar.* As the plate settles, the ram of the dial gauge moves down and settlement is recorded. The load is indicated on the load-gauge of the hydraulic jack.

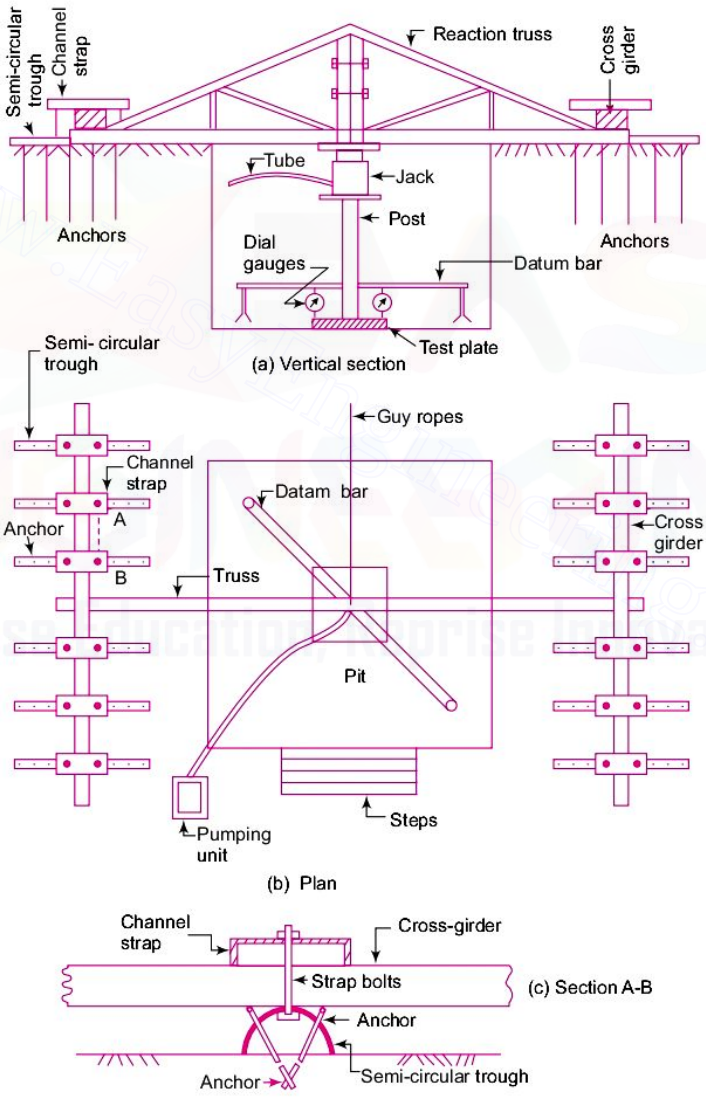

Fig. 24.23 Plate load test : Reaction by truss

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(*b*) ***Reaction truss method :*** Figure 24.23 shows the arrangement when the reaction of the jack is borne by a *reaction truss*. The truss is held to the ground through soil *anchors*. These anchors are firmly driven in the soil with the help of hammers. The reaction truss is usually made of mild steel sections. Guy ropes are used for the lateral stability of the truss.

Indian Standard Code (IS: 1888–1982) recommends that the loading of the plate should *invariably* be borne either by *gravity loading platform* (Fig. 24.22) or by the *reaction truss* (Fig. 24.23). The use of the *reaction truss* is more popular now-a-days since this is simple, quick and less clumsy. No support of loading platform should be located within a distance of 3.5 times the size of the test plate from its centre.

4. **Setting of the plate :** The test plate shall be placed over a *fine sand layer* of maximum thickness 5 mm, so that the centre of the plate coincides with the centre of the reaction girder/beam, with the help of a plumb and bob and *horizontally levelled* by a spirit level to avoid eccentric loading. The hydraulic jack should be centrally placed over the plate with the loading column in between the jack and the reaction beam so as to transfer the load to the plate. A ball and socket arrangement shall be inserted to keep the direction of the load vertical throughout the test. *A minimum seating pressure of 70 g/cm^2 (0.7 t/m^2) shall be applied and removed before starting the load test.*
5. **Load increments :** Apply the load to soil in cumulative equal increments up to 1 kg/cm^2 (10 *t*/m^2) or one fifth of the estimated ultimate bearing capacity, whichever is less. The load is applied without any impact, fluctuation or eccentricity and, in case of hydraulic jack, load is measured over the pressure gauge, attached to the pumping unit kept over the pit, away from the testing plate, through extending pressure pipes.
6. **Settlement and observations :** Settlement should be observed for *each increment of load* after an interval of 1, 2.25, 4, 6.25, 9, 16 and 25 minutes and thereafter at hourly intervals to the nearest 0.02 mm. In case of clayes soils, the time-settlement curve shall be plotted at each load stage and load shall be increased to next stage either when the curve indicates that the settlement has exceeded 70 to 80% of probable ultimate settlement at that stage or at the end of 24 hour period. For soils other than clayes soils, each load increment shall be kept for not less than one hour or up to a time when the rate of settlement gets appreciably reduced to a value of 0.02 mm/min. The next increment of load shall then be applied and the observations repeated. The test shall be continued till a settlement of 25 mm under the normal circumstances or 50 mm in special cases such as dense gravel, gravel and sand mixture is obtained or till failure occurs, whichever is earlier. Alternatively, where settlement does not reach 25 mm, the test should be continued to at least two times the estimated design pressure. If needed, rebound observations may be taken while releasing the load.
7. **Load settlement curve and ultimate bearing capacity :** A load settlement curve is plotted out to arithmetic scale. From this load settlement curve, *zero correction* which is given by the intersection of the early straight line or nearly straight line of the curve with zero load line shall be determined and subtracted from the settlement readings to allow for the perfect seating of the bearing plate and other causes. Four typical curves are shown in Fig. 24.24. Curve *A* is typical for loose to medium cohesionless soil: it is straight line in the earlier stages but flattens out at later stage, and there is no clear point of failure. Curve *B* is for cohesive soil: it may not be quite straight in the early part and leans towards settlement axis as the settlement increases. For partially cohesive soils, curve *C* possessing the characteristics of both curves *A* and *B* is obtained, while curve *D* is purely for dense cohesionless soils. For curves *B* and *D,* no difficulty will be experienced is arising at the ultimate bearing capacity as the failure is well defined. However, in cases of curves *A* and *C*,

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where yield point is not well defined, settlements are plotted as abscissa against corresponding load intensities as ordinate, both to logarithmic scale (Fig. 24.25). Such a plotting will give two straight lines, the intersection of which will be considered as yield value of soil.

IS: 1888 : 1982 has not specified any factor of safety. In order to determine the safe bearing capacity, it would be normally sufficient to use a factor of safety of 2 or 2.5 on the ultimate bearing capacity.

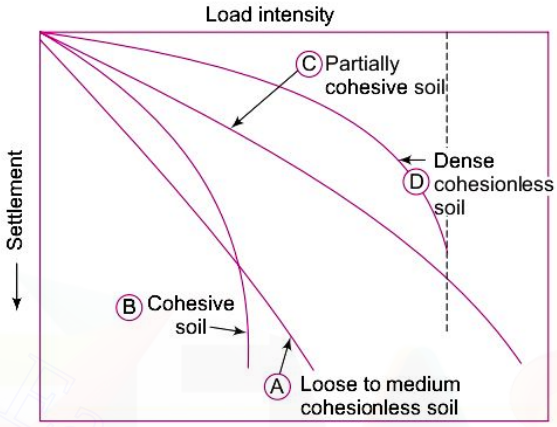

Fig. 24.24 Load settlement curves

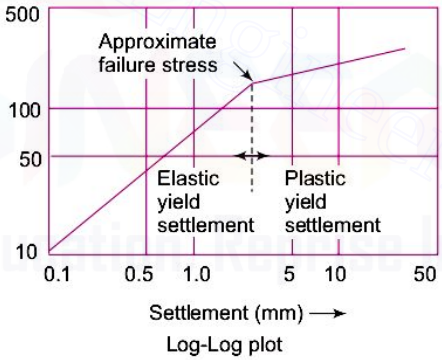

Fig. 24.25 Load settlement curve on log-log plot

Limitations of Plate Load Test : The plate loading test has the following *limitations:*

1. The test results reflect only the character of the soil located within a depth less than twice the width of the bearing plate (corresponding to an isobar of one-tenth the loading intensity at the test plate). Since the foundations are generally larger, the settlement and resistance against shear failure will depend on the properties of a much thicker stratum.
2. It is essentially a short duration test, and hence the test does not give the ultimate settlement, particularly in the case of cohesive soil.
3. Another limitation is the effect of the size of foundation. For clayey soils the ultimate pressure for a large foundation is the same as that for the test plate. But in dense sandy soils, the bearing capacity increases, with the size the foundation, and the test on smaller size bearing plates tend to given conservative values.

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Effect of the size of plate on bearing capactiy: As stated in limitation 3 above, the bearing capacity of sands and gravels increases with the size of the footing. The relationship can be expressed as under:

$$q_F = M + N \frac{B_F}{B_P} \qquad ...(24.48)$$

In the above relation, M term includes the N_c and N_q termes while N includes N_γ portion of Eqs. 24.12 to 24.14.

Equation. 24.48 can be solved graphically by using more than one size plates. By extrapolating the plate load test data, one may use the following equation for all practical purposes:

$$q_F = q_P \cdot \frac{B_F}{B_P} \qquad ...(24.49)$$

where q_F = bearing capacity of the actual footing ; B_F = width of actual footing.

q_P = bearing capacity, obtained from plate load test

B_P = width of plate.

However, for clays, the bearing capacity is almost independent of the footing size or the plate size.

$$q_F \approx q_P \qquad ...(24.50)$$

For a c–ϕ soil, Housel (1929) suggested the following expression :

$$Q = A \cdot q + P \cdot s \qquad ...(24.51)$$

where Q = total load on bearing area ; A = contact area of footing or plate

P = perimeter of footing ; q = bearing pressure beneath area A

s = perimeter shear.

The above equation can be used for the design of a footing to carry a given load, by conducting plate load tests on two different size plates and determining loads Q_1 and Q_2 corresponding to a given fixed settlement. The test data can then be used in Eq. 24.51 to determine q and s. When once q and s are known, Eq. 24.51 can be reused to get foundation size to carry a given load Q.

Effect of size of plate on settlements: The settlement of a footing varies with its size. Terzaghi and Peck have suggested the following relationship between the settlement of plate (ρ_P) and settlement of actual footing (ρ_F) for granular soils:

$$\rho_P = \rho_F \left[\frac{B_P (B + 0.3)}{B(B_P + 0.3)} \right]^2 \qquad ...(24.52)$$

If $\Delta\rho$ is the *permissible settlement* of the foundation, the maximum settlement, of the largest footing should be restricted to $\frac{4}{3}\Delta\rho$. The corresponding settlement of the test plate (ρ_P) on sand, soil is given by

$$\rho_P = \frac{4}{3}\Delta\rho \left[\frac{B_P (B + 0.3)}{B(B_P + 0.3)} \right]^2 \qquad ...(24.53)$$

The net loading intensity q_P corresponding to ρ_P, determined from the load settlement curve from the plate is then the bearing pressure for a specified settlement for the plate and $\frac{4}{3}\Delta\rho$ for the large footing. For clayey soils, the following relation can be used :

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$$\rho_P = \rho_F \cdot \frac{B_P}{B} \qquad ...(24.54)$$

Bond (1961) has proposed the following equation for settlements :

$$\frac{\rho_F / B}{\rho_P / B_P} = \left(\frac{B}{B_P}\right)^n \qquad ...(24.55a)$$

or

$$\frac{\rho_F}{\rho_P} = \left(\frac{B}{B_P}\right)^{n+1} \qquad ...(24.55)$$

where n = coefficient depending on the types of soil

The value of index n can be determind by carrying out two or more plate load tests on different size plates. In absence of test data, the following values of n can be adopted:

Clay : 0.03 to 0.05 Sandy clay : 0.08 to 0.10

Dense sand : 0.4 to 0.5 Medium to dense sand : 0.25 to 0.35

Loose sand : 0.20 to 0.25

24.14 PENETRATION TESTS

These tests involve the measurement of the resistance to penetration of a sampling spoon, a cone or other shaped tools under dynamic or static loadings. The resistance is empirically correlated with some of the engineering properties of soil such as density index, consistency, bearing capacity, etc. The values of these tests lie in the amount of experience behind them. These tests are useful for general exploration of erratic soil profiles, for finding depth to bed rock or hard stratum, and to have an approximate indication of the strength and other properties of soils, particularly the cohesionless soils, from which it is difficult to obtain undisturbed samples. The two commonly used tests are the *standard penetration test* and *the cone penetration test*.

1. Standard Penetration Test (IS 2131 : 1981) : The test (IS: 2131–1981) is performed in a clean hole, 55 to 150 mm in diameter. A casing or drilling mud is used to support the sides of the hole. A thick-wall split-tube sampler, 50.8 mm OD and 35 mm ID, is driven into the undisturbed soil at the bottom of the hole under the blows of a 63.5 kg drive weight with 75 cm free fall. The minimum open length of the sample should be 60 cm.

Test procedure : The split tube sampler, commonly known as *split spoon sampler* resting on the bottom of the bore hole is allowed to sink under its own weight. It is then *seated* 15 cm with the blows of the hammer falling through a height of 75 cm. Thereafter, the split spoon sampler is further driven by 30 cm or 50 blows (except that driving shall cease before the split spoon sampler is full). The number of blows required to affect each 15 cm penetration is recorded. The first 15 cm of drive may be considered to *be seating drive. The total blows required for the second and third 15 cm of penetration is termed as the* penetration resistance *N*. If the split spoon sampler is driven less than 45 cm (total), then the penetration resistance shall be for the last 30 cm of penetration. The entire sampler may sometimes sink under its own weight when very soft subsoil stratum is encountered. Under such conditions, it may not be necessary to give any blow to be sampler and SPT value should be indicated as zero.

Corrections to be observed N values : The observed value of N is corrected for (*i*) overburden and (*ii*) dilatancy/submergence.

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(*i*) ***Correction for overburden :*** The N-value for cohesionless soil shall be corrected for overburden, according to Fig. 24.26, to get the corrected value N_0. The curve of Fig. 24.26 is after Peck (1974).

Thus, we have $N_0 = C_n \, . \, N$ ...(24.56)

where C_n = normalising correction factor

(*ii*) ***Correction due to dilatancy/submergence :*** The values N_0 obtained after applying overburden correction is corrected further for dilatancy if the stratum consists of fine sand and silt below water table, for values of N greater than 15, using the following expression:

$$N_e = 15 + (N_0 - 15) \qquad ...(24.57)$$

Overburden pressure correction by other workers : For a constant density index, the N value increase with increasing effective overburden pressure for which correction have been proposed by Gibbs and Holtz, Peck, Thornburn, Whitman and others.

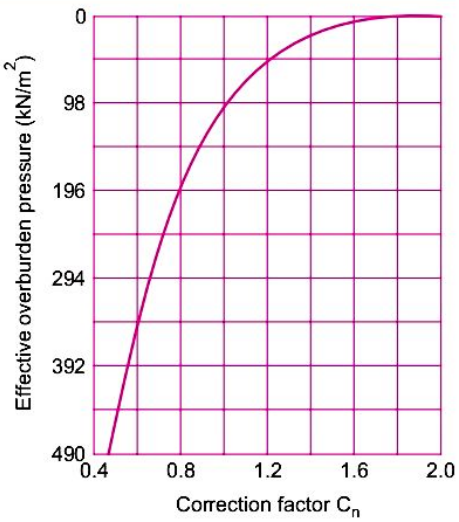

Fig. 24.26 Normalising correction

Gibbs and Holtz (1957) have experimentally studied the effect of overburden pressure on the values of N. Their modification for air dry or moist sand can be represented by the relation :

$$N_0 = N \frac{500}{1.42\,\sigma' + 100} \qquad ...(24.58)$$

where N_0 = corrected value for overburden effect ; N = actual values (observed)

σ' = effective overburden pressure (not to exceed 282 kN/m^2).

Peck (1974) proposed that N values be reported at a reference overburden pressure of 100 kN/m^2, and the normalised value of N (corrected for overburden pressure) be expressed as follows:

$$N_0 = c_n \, . \, N \qquad ...(24.56)$$

where c_n = normalizing factor = $0.77 \log_{10} \frac{2000}{\sigma'}$ ...(24.59)

where σ' = effective overburden pressure (kN/m^2) at the test level.

The above correction is valid for $\sigma' \geq 25$ kN/m^2. It may be noted that at reference overburden pressure of 100 kN/m^2, $c_n = 1$. If σ' decreases below reference pressure, c_n increases; at $\sigma' = 50$ kN/m^2 $c_n = 1.234$. However if σ' increases above the reference overburden pressure of 100 kN/m^2, c_n becomes less than unity ; at $\sigma' = 400$ kN/m^2, $c_n = 0.54$. Eq. 24.59 is represented by curves of Fig. 24.26 adopted by BIS.

Lio and Whitman (1986) have proposed the following expression for c_n :

$$c_n = \sqrt{\frac{2000}{\sigma'}} \qquad ...(24.60)$$

Here again at $\sigma' = 100$ kN/m^2, $c_n = 1$. At $\sigma' = 50$ kN/m^2, $c_n = 1.414$ while at $\sigma' = 400$ kN/m^2, $c_n = 0.5$.

Correlation of *N* with soil properties : Table 24.13 gives the relation between corrected N value and angle of shearing resistance ϕ, as suggested by Peck (1974).

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Table 24.13 Relation between N and ϕ (PECK, 1974)

Corrected N	5	10	15	20	25	30	35	40	45	50
ϕ	28.5	30	32	33	35	36	37.5	39	40	43

Figure 24.14 gives the graphical relationship between coreected N value and angle ϕ as adopted by Indian Standard (IS : 6403–1981). This value of ϕ can then be used for finding capacity factors N_c, N_q and N_γ.

Tables 24.14 and 24.15 give some empirical correlations of the soil properties with corrected penetration resistance. The approximate values of ϕ are after Meyerhof (1956).

Table 24.14 Penetration resistance and empirical correlations for cohesionless soils

Penetration resistance N (blows)	*Approx. ϕ (degrees)*	*Density Index (%)*	*Description*	*Approx. moist unit weight (kN/m^3)*
–	25–30	0		
4	27–32	15	Very loose	1.12–1.6
10	30–35	35	Loose	1.44–1.84
30	35–40	65	Medium	1.76–2.08
50	38–43	85	Dense	1.76–2.24
–		100	Very dense	2.08–2.40

Also, $\phi = 25 + 0.15\, I_D$, with fines greater than 5%.

$\phi = 30 + 0.15\, I_D$, with fines less than 5%.

Larger values should be used for granular soil with 5% or less fine sand and silt.

Table 24.15 Penetration resistance and empirical correlation for cohesive soils

Penetration resistance (blows)	*Unconfined compressive strength (t/m^2)*	*Saturated density (t/m^3)*	*Consistency*
0	0	–	Very soft
2	2.5	1.6–1.92	Soft
4	5		Medium
8	10	1.76–2.08	Stiff
16	20	1.92–2.24	Very stiff
32	40		Hard

2. **Static Cone Test: IS 4968 (Part 3): 1976/1987 :** *Dutch static cone penetration test,* commonly abbreviated as CPT is used for getting a continuous record of the resistance of soil by penetrating steadily under static pressure a cone with a base of 10 cm^2 (3.6 cm in dia.) and an angle of 60° at the vertex. The cone is carried at the lower end of a steel driving rod which passes through a steel tube (mantle) with external diameter equal to the base of the cone. Either the cone or the tube, or both together can be forced into the soil by means of jacks. To know the cone resistance, the cone alone is first forced down for a distance of 8 cm and the maximum value of resistance is recorded. The steel tube is then pushed down up to the cone, and both together are further penetrated through a depth of 20 cm to give the total of the cone resistance and the frictional resistance along the tube.

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Normalisation of q_c values : Similar to SPT values (N), the CPT values (q_c) also require *normalisation* to the standard overburden pressure of 100 kN/m^2, using the relation

$$q_{cn} = C_n \cdot q_c \qquad ...(24.56a)$$

where q_{cn} = normalised value of q_c, corrected for overburden effect. Table 24.16 gives the values of C_n for various overburden pressures, as suggested by Robertson and Campanella (1985).

Table 24.16 Values of C_n for CPT (Robertson and companella, 1985)

Effective overburden pressure, kN/m^2	C_n	*Effective overburden pressure, kN/m^2*	C_n
20	2.80	220	0.59
40	1.76	240	0.56
50	1.58	250	0.54
60	1.40	260	0.53
80	1.18	280	0.52
100	**1.00**	**300**	**0.51**
120	0.90	320	0.48
140	0.81	340	0.47
150	0.77	350	0.46
160	0.73	360	0.46
180	0.68	380	0.45
200	0.63	400	0.45

Correlation between CPT and SPT : The cone test is considered very useful in determining the bearing capacity of pits in cohesionless soils, particularly in fine sands of varying density. The cone resistance q_c (kg/cm^2) is approximately equal to 5 to 10 times the penetration resistance N.

Table 24.17 gives the correlations between CPT and SPT, in the form of q_c/N ratio, for various types of soils

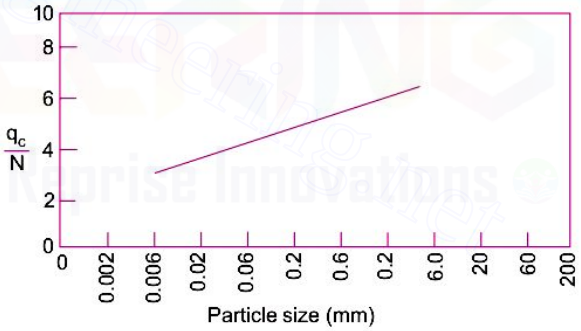

Fig. 24.27 Relation between q_c and N

Table 24.17 Correlation between CPT and SPT: (Schmermann, 1970) : q_c in kg/cm^2

S.No.	*Soil Type*	q_c/N
1.	Silts, sandy silts, slightly cohesive silt sand mixtures	2
2.	Clean, fine to medium sands and slightly silty sands	3.5
3.	Coarse sands and sands with little gravel	5
4.	Sandy gravel and gravel	6

However, Fig. 24.27 shows the relationship between CPT (q_c) and SPT (N), where q_c is in kg/cm^2 (Thornburn, 1971).

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24.15 SAFE BEARING PRESSURE BASED ON TOLERABLE SETTLEMENT, *N* VALUES AND q_c VALUES

The analytical methods for the determination of bearing capacity of footings are based essentially on shear failure. However, permissible settlements play an important role in determining the safe bearing capacity.

The safe bearing pressure or net soil pressure q_ρ for specified settlements is the intensity of loading which causes a permissible or specified settlement (ρ) of the foundation (IS 6403 : 1981)

(*a*) **Use of *N*-values :** Figure 24.28 shows empirical charts giving relationship between allowable bearing pressure and *N* values, for a permissible settlement of 25 mm (Terzaghi and Peck, 1967). The allowable bearing pressures are based on the assumption that water table is at a depth of at least *B* below foundation level.

The above relationships limit the maximum settlement of individual footing to 25 mm and a differential settlement of 20 mm, assuming that a differential settlement of 20 mm can be tolerated by most of the ordinary structures. The empirical relationships are represented by the equations (Teng, 1969) :

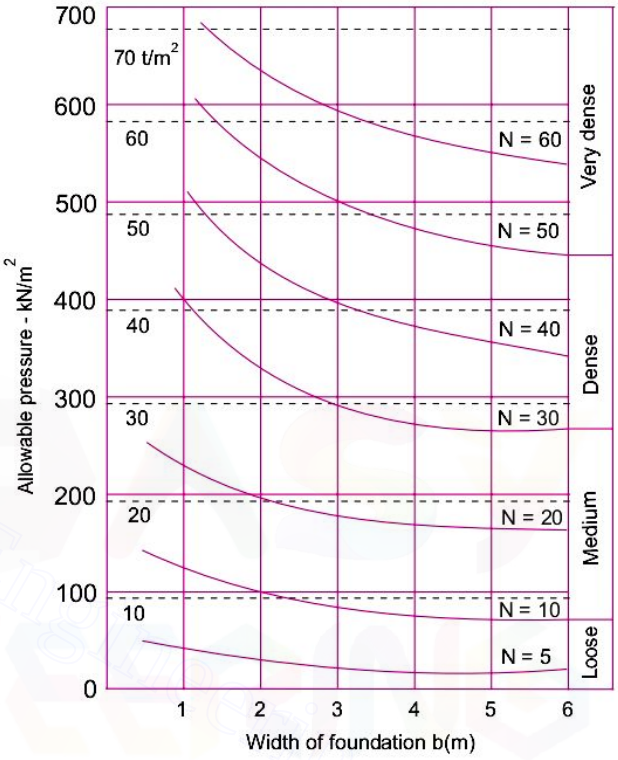

Fig. 24. 28 Charts for determining allowable bearing pressure for foundation of sand

$$q_\rho = 34.3\,(N-3)\left(\frac{B+0.3}{2B}\right)^2 R_{w_2}\,.\,R_d \text{ kN/m}^2 \qquad ...(24.61)$$

Allowable *gross safe pressure will be*

$$q_g = q_\rho + \gamma D$$

where,

q_ρ = allowable *net* increase in soil pressure over existing soil pressure for settlement of 25 mm.

N = standard penetration number with applicable corrections.

B = width of footing (or least lateral dimensions), in metres

ρ = permissible settlement = 25 mm

$$R_{w_2} = \text{water reducton factor} = 0.5\left(1+\frac{Z_{w_2}}{B}\right)$$

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$$R_d = \text{depth factor} = \left(1 + \frac{0.2\,D}{B}\right) \le 1.20$$

Meyerhof (1956) has proposed slightly different equation, but they yield, for usual size of footings, almost the same results.

$$q_\rho = 12.5\,N\,R_{w2}\,R_d, \text{ for } B < 1.2 \text{ m} \qquad ...(24.62a)$$

$$q_\rho = 8.3\,N\left(\frac{B+0.3}{B}\right)^2 R_{w2}\,R_d \text{ for } B > 1.2 \text{ m} \qquad ...(24.62b)$$

where $$R_d = \left(1 + \frac{0.33\,D}{b}\right) \le 1.33 \text{ and } q_\rho \text{ is in kN/m}^2$$

The above equations (24.61, 24.62) are considered conservative.

Bowles (1988) adjusted the Meyerhof equations for an approximate 50% increase in allowable bearing capacity to obtain the following.

$$q_\rho = 20\,N\,R_d \text{ for } B \le 1.2 \text{ m} \qquad ...(24.62c)$$

and $$q_\rho = 12.5\,N\left(\frac{B+0.3}{B}\right)^2 R_d \text{ for } B > 1.2 \text{ m} \qquad ...(24.62d)$$

where $$R_d = 1 + 0.33\,\frac{D}{B} \le 1.33$$

(as suggested by Meyerhof, 1965)

Figure 24.29 shows a plot of equations 24.62 (*c*), (*d*).

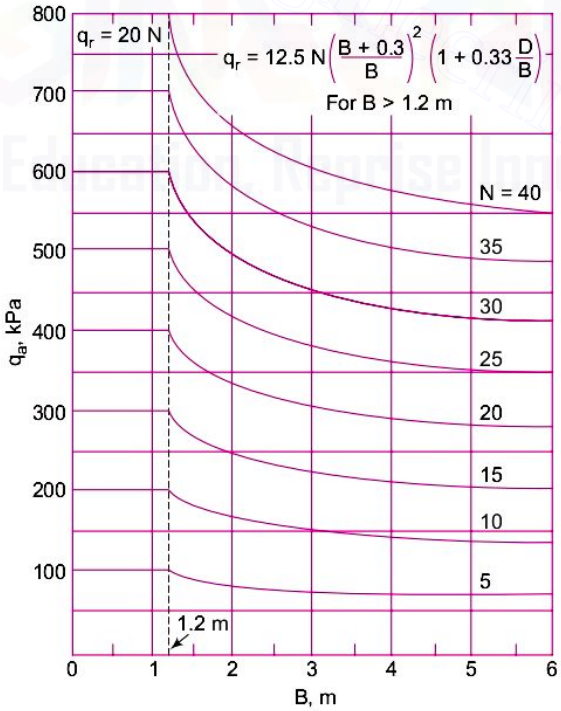

Fig. 24.29 Allowable bearing capacity for 25 mm settlement

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From the above plot, we observe that *footing width* is significant parameter. For mats, Meyerhof equation, adjusted by Bowles for a 50% increase, can be expressed by the following equation

$$q = 12.5\, N R_d \,(\text{kN/m}^2) \qquad ...(24.62e)$$

If tolerable settlement is other than 25 mm, say ρ′, the corresponding net pressure increase is given by

$$q_\rho' = q_\rho \frac{\rho'}{\rho} = q_\rho \times \frac{\rho'}{25} \qquad ...(24.63)$$

Thus, Teng's equation, applicable for 40 mm settlement takes the following form :

$$q_\rho = 55\,(N-3)\left(\frac{B+0.3}{2B}\right)^2 R_{w_2} \cdot R_d \ \text{kN/m}^2 \qquad ...(24.61a)$$

IS code equation : IS 6403 – 1971 accepted Teng's equation for 40 mm settlement, without depth factor R_d, expressed as follows :

$$q_p = 55\,(N-3)\left(\frac{B+0.3}{2B}\right)^2 R_w \qquad (24.61b)$$

However, IS 6403–1981 dropped this equation.

(*b*) **Use of cone penetration resistance (q_c) values**

Meyerhof (1956) suggested the following simplified formulae for determining allowable bearing pressure for a permissible settlement of 25 mm for foundations on *dry* sands:

(*i*) ***Square or Strip foundation equal to or less than 1.2 m wide***

$$q_\rho = 3.6\, q_c \ \text{kN/m}^2 \qquad ...(24.64a)$$

(*ii*) ***Square or Strip footing greater than 1.2 m wide***

$$q_\rho = 2.1\, q_c \left(1+\frac{1}{B}\right)^2 \ \text{kN/m}^2 \qquad ...(24.64b)$$

An approximate formula to cover all foundations, irrespective of width is

$$q_\rho = 2.7\, q_c \ \text{kN/m}^2 \qquad ...(24.64c)$$

In all the above formulae, q_e is in kg/cm^2. The above formulae are based on the assumption that *N*-value is one-quarter of static cone resistance (in kg/cm^2). However, Figure 24.23 shows the relationship between static cone (q_c) resistance and *N* (Thornburn, 1971), where q_c is in kg/cm^2.

The value of q_ρ in Eq. 24.62 should be *halved* if the sand within the stressed zone is submerged.

IS code method : Figure 24.15 gives the ratio of q_{n_f}/q_c for various values of *D/B* ratio and width *B* as recommended by IS code (IS : 6403–1981). From these curves, the net ultimate bearing capacity can be determined corresponding to a known value of q_c.

24.16 PERMISSIBLE TOTAL AND DIFFERENTIAL SETTLEMENTS

The vertical downward movement of the base of a structure is called settlement and its effect upon the structure depends on its magnitude, its uniformity, the length of the time over which it takes place and the nature of the structure itself.

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It is suggested that allowable pressure should be selected such that maximum settlement of any individual foundation is 25 mm. It has also been suggested that differential settlement of uniformly loaded continuous foundation and of equally loaded spread foundation of approximately the same size, is unlikely to exceed half the maximum settlement, and that normal structures such as office buildings and flats can satisfactorily withstand differential settlements of about 18 mm between adjacent columns spaced 6 to 8 m apart.

According to National Building Code of India (SP: 7–1970), the differential settlement shall be kept within limits to which the super-structure can accommodate itself without harmful distortion, by suitably designing the foundation. Total settlements shall be so restricted or special arrangements made so that connections to the building, such as drains, are not damaged. For simple spread footing on sands, the allowable bearing pressure should be such that differential settlement does not exceed 1/300. This condition is generally satisfied if the total settlement is limited to 50 mm. For simple spread footing on clayey soils, the allowable bearing pressure should be such that the differential seetlement does not exceed 1/300. This condition is generally satisfied if the total settlement is limited to 75 mm. The recommendations of American Code are based upon the simple logic that if the maximum total settlement is kept within a reasonable limit, the differential settlement will be only a fraction (generally about three-quarters of this limit), depending upon the type of structure and pattern of loading. The allowable maximum settlement values are given below:

Type of structure	Allowable maximum settlement (*mm*)
Commercial and Industrial buildings	25
Industrial building	38
Warehouses	50
Special machinery foundations	Often less than 0.5 mm.

According to Polshin and Tokar (1957), brick masonry will crack (due to differential settlement) when the unit elongation amounts to 0.005. Based on this criterion, the *permissible differential settlement* of brick walls is shown in Fig. 24.30, and is as follows :

For $\frac{L}{H} \leq 2$

Rate of differential settlement

$= 0.0003$ cm/cm

For $\frac{L}{H} = 8$

Rate of differential settlement

$= 0.0010$ cm/cm

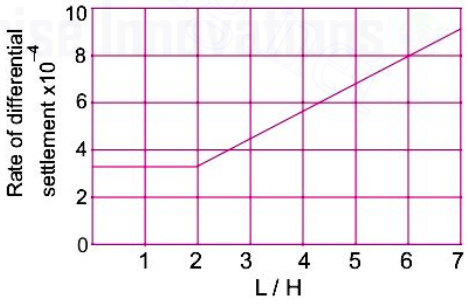

Fig. 24.30 Permissible differential settlements of brick walls

where L is the wall length and H is the height of wall measured above the base of footing. The rate of differential settlement is defined as the slope or the relative settlement between two points divided by the horizontal distance.

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24.17 BEARING CAPACITY FROM BUILDING CODES

Table 24.18 gives the bearing capacity for various soils according to National Building Code.

Note 1. Compactness or Looseness of cohesionless materials may be determined by driving a wooden picket of dimensions 5 × 5 × 70 cm with a sharp point. The picket shall be pushed vertically into the soil by the full weight of a person weighing at least 70 kg. If the penetration of the picket exceeds 20 cm, the loose state shall be assumed to exist.

Note 2. Dry means that the ground water level is at a depth not less than the width of foundation below the base of the foundation.

Table 24.18 Values of safe bearing capacity according to national building code of india (1983)

Rocks and Cohesionless soils			*Cohesive soils*		
Description		*Safe Bearings capacity (kN/m^2)*	*Description*		*Safe Bearings capacity (kN/m^2)*
(*a*) ***Rocks***			(*c*) ***Cohesive soils***		
1.	Rocks (hard) without lamination and defects, for example, granite, trap and diorite	3240	1.	Soft shale, hard or stiff clay in deep bed, dry	440
2.	Laminated rocks, for example sandstone and limestone in sound condition	1620	2.	Medium clay readily indented with a thumb nail	245
3.	Residual deposits of shattered and broken bedrock and hard shale, cemented material	880	3.	Moist clay and sand clay mixture which can be indented with strong thumb pressure	150
4.	Soft rock	440	4.	Soft clay indented with moderate thumb pressure	100
(*b*) ***Cohesionless soils***					
5.	Gravel, sand and gravel, compact and offering resistance to penetration when excavated by tools (See Note 1)	440	5.	Very soft clay which can be penetrated several centimetre, with the thumb	50
6.	Coarse sand, compact and dry	440	6.	Black cotton soils or other shrinkable or expansive clay in dry condition (50%) saturation.	130–160
7.	Medium sand, compact and dry	245			
8.	Fine sand, silt (dry lumps easily pulverised by fingers)	150			
9.	Loose gravel or sand gravel mixture : Loose coarse to medium sand, dry	245			
10.	Fine sand, loose and dry	100			

Note 3. The bearing capacity of peat, fills or made-up ground shall be determined after investigation.

Note 4. Cohesive soils are susceptible to long term consolidation settlement.

Note 5. Increase or Decrease the safe bearing capacity as follows :

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(*a*) The safe bearing capacity may be increased by an amount equal to the weight of the material removed from above the bearing level, *i.e.*, the base of the foundation.

(*b*) For cohesionless soils, the safe bearing capacity shall be reduced by 50% if the water table is above or near the bearing surface of the soil. If the water table is below the bearing surface of the soil at a distance at least equal to the width of the foundation, no such reduction shall apply. For intermediate depth of the water table, proportional reduction of the safe bearing capacity may be made.

Example 24.16. *A 30 cm square bearing plate settles by 8 mm in the plate load test on cohesionless soil, when the intensity of loading is 180 kN/m^2. Estimate the settlement of a shallow foundation of 1.6 m square under the same intensity of loading.*

Solution.

From Eq. 24.52, $$\rho_F = \rho_p \left[\frac{B(B_P + 0.3)}{B_p (B + 0.3)}\right]^2 = 8\left[\frac{1.6(0.3 + 0.3)}{0.3(1.6 + 0.3)}\right]^2 = \mathbf{22.7\ mm.}$$

Example 24.17. *A plate load test was conducted with a 30 cm square plate at a depth of 1.2 m below the ground level, in a cohesive soil having $\phi = 0$. The failure was observed at a load of 36 kN. The water table was observed to be at a depth of 4.7 m below ground surface.*

Compute the ultimate bearing capacity for a strip footing, 1 m wide, with its base located at the same level as the test plate, and in the same soil. Take the bulk unit weight of the soil as 16.8 kN/m^3. Also, calculate the safe bearing capacity of factor at a safety of 3.

Solution. Depth of water table below the test level = 4.7 – 1.2 = 3.5 m. Since this depth is much more than the width of footing, there will be no influence of water table.

For the plate load test on square plate:

$$q_f = 1.3\, cN_c + \bar{\sigma} N_q + 0.4\, B\, .\, \gamma N_\gamma \qquad ...(1)$$

where $\bar{\sigma} = \gamma D$. For $\phi = 0$, we get from Table 24.2,

$$N_c = 57\ ;\ N_q = 1 \text{ and } N_\gamma = 0$$

Also, $$q_f = \frac{36}{0.3 \times 0.3} = 400\ \text{kN/m}^2.$$

∴ Substituting the values in (1),

$$400 = 1.3\, c \times 5.7 + 16.8 \times 1.2 \times 1 + 0$$

From which $$c = 51.26\ \text{kN/m}^2$$

For the strip footing

$$q_f = cN_c + \bar{\sigma} N_q + 0.5\, B_\gamma N_\gamma = (51.26 \times 5.7) + (16.8 \times 1.2 \times 1) + 0 = \mathbf{312}\ \text{kN/m}^2.$$

$$q_{nf} = q_f - \gamma D = 312.3 - (16.8 \times 1.2) = 292.14\ \text{kN/m}^2$$

∴ $$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{292.14}{3} + (16.8 \times 1.2) = \mathbf{117.5\ kN/m^2.}$$

Example 24.18. *Plate load tests were conducted in a c-ϕ soil, on plates of two different sizes and the following results were obtained:*

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Load	*Size of plate*	*Settlement*
40 kN	*0.3 m × 0.3 m*	*25 mm*
100 kN	*0.6 m × 0.6 m*	*25 mm*

Find the size of square footing to carry a load of 800 kN at the same specified settlement of 25 mm.

Solution. For c-ϕ soil, we have Housel equation

$$Q = A \cdot q + P \cdot s \qquad ...(24.51)$$

where Q = total load on bearing area ; A = area of plate (or footing)

P = perimeter of the footing; q = bearing pressure beneath the footing

and s = perimeter shear.

In the above equation q and s are the two unknowns.

From the first test, we have $40 = (0.3 \times 0.3)\, q + (4 \times 0.3)s$

or $$40 = 0.09\, q + 1.2\, s \qquad ...(1)$$

From the second test, we have $100 = (0.6 \times 0.6)\, q + (4 \times 0.6)s$

or $$100 = 0.36\, q + 2.4\, s \qquad ...(2)$$

Solving (1) and (2), we get $q = 111.11$ kN/m^2

and $s = 25$ kN/m^2

Let B be the size of the square footing.

$\therefore$ $$800 = (B \times B) \times 111.11 + 4B \times 25$$

or $B^2 + 0.9B - 7.2 = 0$. From which $\boldsymbol{B \approx 2.4}$ **m**

Example 24.19 *A strip footing 1.2 m wide is located at a depth of 1.5 m in a non-cohesive soil deposit for which the corrected N-value of SPT is 20. Water table is located at a depth of 2 m below the ground surface. Find the allowable bearing pressure for the soil.*

Solution. Depth of water table below base of foundation

$$= Z_{w_2} = 2 - 1.5 = 0.5$$

$\therefore$ $$R_{w_2} = 0.5\left(1 + \frac{Z_{w2}}{B}\right) = 0.5\left(1 + \frac{0.5}{1.2}\right) \approx 0.71$$

From Terzaghi's empirical relationship,

$$q_\rho = 35(N - 3)\left(\frac{B + 0.3}{2B}\right)^2 . R_{w_2} . R_d$$

where $$R_d = \text{depth factor} = \left(1 + \frac{0.2D}{B}\right) \le 1.2 = \left[1 + \frac{0.2 \times 1.5}{1.2} = 1.25\right] \le 1.2$$

Hence, take $R_d = 1.2$. Substituting the values,

$\therefore$ $$q_\rho = 35(20 - 3)\left(\frac{1.2 + 0.3}{2 \times 1.2}\right)^2 \times 0.71 \times 1.2 = \mathbf{198\ kN/m^2}.$$

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Alternatively, from Meyerhof's equation (Eq. 24.62*a*)

$$q_\rho = \frac{5}{4} N R_{w2} \,.\, R_d \text{ t/m}^2$$

where $$R_d = \left(1 + \frac{0.33\,D}{B}\right) \le 1.33 = \left[\left(1 + \frac{0.33 \times 1.5}{1.2}\right) = 1.4125\right] \le 1.33$$

Hence, take $R_d = 1.33$.

$\therefore$ $$q_\rho = \frac{5}{4} \times 20 \times 0.71 \times 1.33 = 23.6 \text{ t/m}^2 \approx \mathbf{231.6\ kN/m^2.}$$

Example 24.20. *Determine the net bearing pressure for a 2 m × 2 m footing at a depth of 1.5 m in a medium dense sand so that the total settlement does not exceed 25 mm. The average SPT blows below the footing are 20 per 30 cm. The average moist unit weight of soil is 17.0 kN/m^3. The water table is at 5 m below the ground level. What will be the bearing pressure if the water table rises up to the base of the footing.*

Solution.

(*a*) Let us apply the correction for overburden pressure, using Peck's formula:

$$N_0 = c_n \,.\, N \qquad \text{...(24.58)}$$

where $$c_n = 0.77 \log_{10} \frac{2000}{\sigma'}$$

where σ' (at test level) $= \gamma D = 17 \times 1.5 = 25.5$ kN/m^2

$\therefore$ $$c_n = 0.77 \log_{10} \frac{2000}{22.5} \approx 1.5$$

$\therefore$ $N_0 = 1.5 \times 20 = 30$ | Using Terzaghi's formula (Eq. 24.61*b*)

$$q_{\rho = 25} = 35\,(n - 3)\left(\frac{B + 0.3}{2B}\right)^2 R_{w_2} \,.\, R_d$$

where $R_{w_2} = 1$ (since $Z_w > B$)

$$R_d = \left(1 + \frac{0.2\,D}{B}\right) \le 1.20 = \left(1 + \frac{0.2 \times 1.5}{2}\right) \le 1.20 = 1.15$$

$\therefore$ $$q_{\rho = 25} = 35\,(30 - 3)\left(\frac{2 + 0.3}{2 \times 2}\right)^2 \times 1 \times 1.15 = 312.4 \text{ kN/m}^2.$$

(*b*) When the water table rises up to the base of footing,

$$R_{w_2} = 0.5\left(1 + \frac{Z_w}{B}\right) = 0.5\,(1 + 0) = 0.5$$

Hence, the bearing capacity is reduced to half.

$\therefore$ $q_{\rho = 25} = 0.5 \times 312.4 = \mathbf{156.2\ kN/m^2.}$

Example 24.21. *Static cone penetration test was conducted in dry sand formation, and the average cone resistance was found to be 50 kg/cm^2. Estimate the safe bearing pressure for a strip footing of width 1.5 m and depth 1.5 m, for a permissible settlement of 25 mm. Use a factor of safety for 3 of shear failure. The soil has a unit weight of 17 kN/m^3.*

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Solution. Using Meyerhof's equation, for $B > 1.2$, we have

$$q_\rho = 2.1\ q_c \left(1 + \frac{1}{B}\right)^2 \text{ kN/m}^2 \qquad ...(24.64\ b)$$

where q_c = cone penetration value in kg/cm^2 = 50

$$\therefore \quad q_{\rho=25} = 2.1 \times 50 \left(1 + \frac{1}{1.5}\right)^2 = \mathbf{292\ kN/m^2}.$$

For bearing capacity based on shear failure, let us use IS charts, Fig. 24.15.

We have $\dfrac{D}{B} = \dfrac{1}{1} = 1. \quad \therefore \quad \dfrac{q_{nf}}{q_c} \approx 0.209.$

$$\therefore \quad q_{nf} = 0.209 \times 50 = 10.45 \text{ kN/cm}^2 \approx 1025 \text{ kN/m}^2$$

$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma D. \text{ Using a F.S. of 3,}$$

$$q_s = \frac{1025}{3} + (17 \times 1.5) = 367 \text{ kN/m}^2$$

Hence, the safe bearing pressure will be governed on the basis of permissible settlement, and its values is **292 kN/m^2.**

Example 24.22. *Use the following data to plot the allowable bearing pressure against width of footing for a standard penetration resistance N = 20 as per design chart of Terzaghi and Peck for allowable settlement of 25 mm :*

B(m)	*1*	*2*	*3*	*4*	*5*	*6*
Allowable soil pressure q_a *(kN/m^2)*	250	210	180	170	165	160

Using this chart, determine the width of a square footing to carry an imposed load of 800 kN on the soil at depth of 2 m. The ground water level is 1 m below the ground level and the unit weight of the soil is 20 kN/m^3.

Solution. The plot is shown in Fig. 24.31.

The solution will be done by trial and error.

At $D = 2$ m

$\bar{\sigma} = \gamma D = 20 \times 2$

$= 40$ kN/m^2.

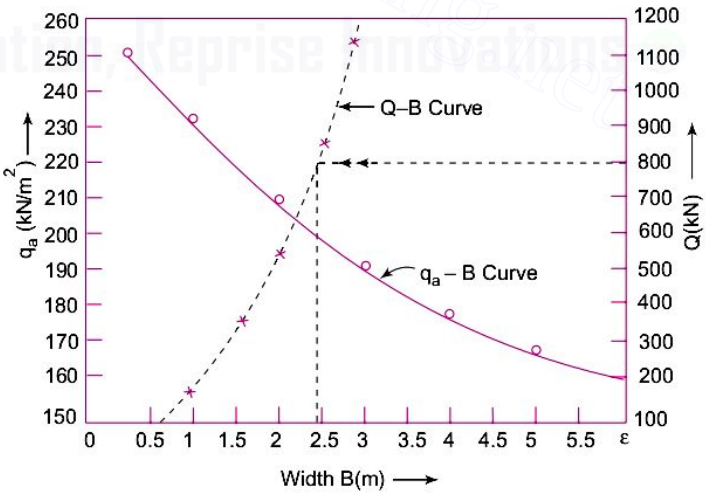

Fig. 24.31

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(a) *Let* $B = 2m$

$q_a = 210$ kN/m^2. Use a water reduction factor of 0.5 because the water table is above the base.

$$\therefore \quad q_a' = \frac{1}{2} \times 210$$
$$= 105 \text{ kN/m}^2$$

Gross safe pressure

$$q_{\rho g} = q_a' + \gamma D = 105 + 40 = 145 \text{ kN/m}^2$$

$$\therefore \quad Q = q_{\rho g} \times B^2 = 145 \times 2^2 = 580 \text{ kN} < 800.$$

(b) *Let* $B = 2.5$ *m*

$$q_a = 194 \text{ kN/m}^2 \; ; \; q_a' = \frac{1}{2} \times 194 = 97 \text{ kN/m}^2$$

$$q_{\rho g} = 97 + 40 = 137 \text{ kN/m}^2$$

$$Q = 137\,(2.5)^2 = 856 \text{ kN/m}^2 > 800$$

(c) *Let* $B = 2.4$ m

$$q_a = 196 \text{ kN/m}^2, \; q_a' = \frac{1}{2} \times 196 = 98 \text{ kN/m}^2$$

$$q_{\rho g} = 98 + 40 = 138 \text{ kN/m}^2$$

$$\therefore \quad Q = 138\,(2.4)^2 = 794.9 \text{ kN}.$$

Hence, adopt $B = 2.5$ m

The variation of Q with B is shown by dotted curve in Fig. 24.31.

Example 24.23. *Solve example 24.20, using Terzaghi's analytical expression.*

Solution. From Eq. 24.61

$$q_\rho = 34.3(N - 3)\left(\frac{B + 0.3}{2B}\right)^2 . R_{w_2} . R_d$$

Here, $R_{w_2} = 0.5\left(1 + \dfrac{Z_{w_2}}{B}\right) = 0.5$ since water table is above the base of the footing.

$$R_d = \left(1 + \frac{0.2\,D}{B}\right) \le 1.20 = \left(1 + \frac{0.2 \times 2}{B}\right) \le 1.20$$

Assuming that B will be greater 2 m, R_d will always be less than 1.2.

$\bar{\sigma}$ = existing overburden pressure = $\gamma D = 20 \times 2 = 40$ kN/m^2

$$\therefore \quad q_{\rho g} = \text{gross increase in bearing pressure} = q_\rho + \gamma D = q_\rho + 40$$

$$Q = q_{\rho g}\,(B)^2 = 800 \text{ kN}$$

Hence, substituting the value, we get

$$800 = \left[\left\{34.3(20 - 3)\left(\frac{B + 0.3}{2B}\right)^2 \left(1 + \frac{0.4}{B}\right) \times 0.5\right\} + 40\right] B^2$$

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$$800 = \left[72.89\left(\frac{B+0.3}{B}\right)^2\left(\frac{B+0.4}{B}\right)+40\right]B^2$$

Solving this by trial and error, we get $B \approx 2.3$ m.

The reduction in B is due to mobilization of depth factor.

Check. For $B = 2.3$ m

$$R_d = \left(1+\frac{0.2\times 2}{2.3}\right) = 1.174$$

$$\therefore \quad q_\rho = 34.3(20-3)\left(\frac{2.3+0.3}{2\times 2.3}\right)^2 \times 0.5 \times 1.174 = 111.6 \text{ kN/m}^2$$

$$q_{\rho g} = 111.6 + 40 = 151.6 \text{ kN/m}^2$$

$$Q = 151.6\ (2.3)^2 \approx 802 \text{ kN}$$

24.18 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 24.24. *What will be the gross and net safe bearing pressure of sand having $\varphi = 36°$ and effective unit weight 1.8 tonnes/m^3 under the following cases:*

(i) 1 m wide strip footing (ii) 1 m × 1 m square footing

Consider the footings are placed at a depth of 1 m from ground surface and water table is at a great depth. Assume a factor of safety of 3.0. Use Terzaghi's theory. Given: for $\varphi = 36°$ from Terzaghi's chart $N_q = 47$ and $N_\gamma = 43$. **(Civil Services Exam. 1984)**

Solution. For sand, $c = 0$; hence $N_c = 0$

(i) 1 m wide strip footing

$$q_f = \gamma D N_q + 0.5\, B\gamma N_\gamma = 1.8\times 1\times 47 + 0.5\times 1\times 1.8\times 43 = 123.3 \text{ t/m}^2$$

$$q_{nf} = q_f - \gamma_D = 123.3 - 1.8\times 1 = 121.5 \text{ t/m}^2$$

Net safe bearing capacity $q_{ns} = \dfrac{q_{nf}}{F} = \dfrac{121.5}{3} = \mathbf{40.5\ t/m^2}$

Gross safe bearing capacity $q_s = \dfrac{q_{nf}}{F} + \gamma D = \dfrac{121.5}{3} + 1.8\times 1 = \mathbf{42.3\ t/m^2}$

(ii) For square footing

$$q_f = \gamma D\, N_q + 0.4\,\gamma B\, N_\gamma = 1.8\times 1\times 47 + 0.4\times 1.8\times 1\times 43 = \mathbf{115.56\ t/m^2}$$

$$q_{nf} = 115.56 - 1.8\times 1 = 113.76 \text{ t/m}^2$$

Net safe bearing capacity $= q_{ns} = q_{nf}/F = \dfrac{113.76}{3} = \mathbf{37.92\ t/m^2}$

Gross safe bearing capacity $q_s = \dfrac{q_{nf}}{F} + \gamma D = \dfrac{113.76}{3} + 1.8\times 1 = \mathbf{39.72\ t/m^2}$

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Example 24.25. *A square mass concrete footing supporting a load of 3250 kN extends from ground level to 3.5 m deep into a clay stratum. What will be the size of the footing allowing for a factor of safety of 4.0 ? Unit weight of concrete 25 kN/m^3. Shear strength of soil 0.12 N/mm^2. Adhesion of clay with footing is 25 kN/m^2. The adhesion may be supposed to act over a depth of 2 m from the bottom of the foundation. For $\varphi = 0$ deg.; the value of $N_c = 5.7$; $N_q = 1$ and $N_\gamma = 0$.* **(Civil Services Exam. 1991)**

Solution.

Given : $\varphi = 0$; $N_c = 5.7$; $N_q = 1$; $N_\gamma = 0$

$c = 0.12$ N/mm^2 $= 0.12 \times 10^3$ kN/m^2

$= 120$ kN/m^2

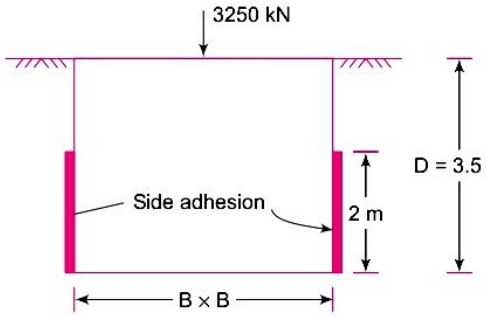

Fig. 24.32

For square footing

$$q_{f_1} = 1.3\, cN_c + \gamma DN_q + 0.4\, \gamma BN_\gamma$$

$$= (1.3 \times 120 \times 5.7) + (21 \times 3.5 \times 1) + (0.4 \times 21\, B \times 0)$$

$$= 889 + 73.5 = 962.5 \text{ kN/m}^2$$

$$\therefore \quad q_{nf_1} = 962.5 - \gamma D = 962.5 - 21 \times 3.5 = 889 \text{ kN/m}^2$$

$$\therefore \quad q_{ns_1} = \frac{q_{nf_1}}{F} + \gamma D = \frac{889}{4} + 21 \times 3.5 = 295.75 \text{ kN/m}^2$$

$\therefore$ Safe load $= Q_{s_1} = q_{ns_1} \times$ base area $= 295.75\, B^2$ kN ...(1)

Additional load capacity by side adhesion $= Q_{f_2} = 4\, B \times 2 \times 25 = 200\, B$ kN ...(2)

$\therefore$ Additional safe load capacity $= Q_{s_2} = Q_{f_2}/F = 200\, B/4 = 50\, B$

$\therefore$ Total safe load capacity, $Q_s = Q_{s_1} + Q_{s_2} = 295.75\, B^2 + 50\, B$ ...(3)

Actual load $= 3250 + 3.5\, B^2 \times 25 = 3250 + 87.5\, B^2$ ...(4)

Equating (3) and (4), $295.75\, B^2 + 50\, B^2 = 3250 + 87.5\, B^2$

or $208.5\, B^2 + 50\, B - 3250 = 0$

From which $B = 3.83$ m.

Hence, provide **3.85 m × 3.85 m** size square footing.

Example 24.26. *A strip footing is needed to carry a load of 1000 kN/m at a depth of 1 m. Shear strength parameters for the soil are $c = 0$ and $\varphi = 56°$. Determine the minimum width of footing for a factor of safety of 3 against shear failure. The water table may rise to the base of the footing. Take $G_s = 2.65$, $e = 0.6$ and $\gamma = 16$ kN/m^3.* **(Civil Services Exam. 1994)**

Solution.

$$\gamma_{sat} = \frac{G + e}{1 + e}\gamma_w = \frac{2.65 + 0.6}{1 + 0.6} \times 9.81 = 19.93 \text{ kN/m}^3$$

$$\therefore \quad \gamma' = 19.93 - 9.81 = 10.12 \text{ kN/m}^2$$

Now, $$q_{nf} = cN_c + \bar{\sigma}(N_q - 1) + \frac{1}{2} B\gamma N_\gamma$$

Since the water table is at the base of the footing, $\bar{\sigma} = \gamma D$, where γ is the wet weight, while γ will be taken as γ' with the wedge term.

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$$\therefore \quad q_{nf} = cN_c + \gamma D\,(N_q - 1) + \frac{1}{2} B\gamma' N_\gamma \qquad ...(1)$$

Let us use IS Code method, which is based on Vesic's equation

$$\therefore \quad N_q = \tan^2\left(45° + \frac{\varphi}{2}\right) e^{\pi \tan \varphi} = \tan^2\left(45° + \frac{36°}{2}\right) e^{\pi \tan 36°} = 37.75$$

$$N_c = (N_q - 1) \cot \varphi = (37.75 - 1) \cot 36° = 50.58$$

$$N_\gamma = 2\,(N_q + 1) \tan \varphi = 2\,(37.75 + 1) \tan 36° = 56.31$$

$$\therefore \quad q_{nf} = 0 + 16 \times 1(37.75 - 1) + \frac{1}{2} B \times 10.12 \times 56.31 = 588 + 284.9\,B$$

$$q_s = \frac{q_{nf}}{F} + \gamma \,.\, D = \frac{588 + 284.9\,B}{3} + 16 \times 1 = 212 + 95\,B \qquad ...(2)$$

Actual load intensity, $$q = \frac{1000}{B+1} = \frac{1000}{B}\, m^2 \qquad ...(3)$$

$$\therefore \quad \frac{1000}{B} = 212 + 95\,B \quad \text{or} \quad B^2 + 2.23\,B - 10.53 = 0$$

From which $B \approx 2.32$ m

Example 24.27. *Explain briefly how the bearing capacity is determined by field load test. The result of two field tests on a given location are as follows:*

(i) Diameter = 750 mm ; S =15 mm; Q = 150 kN
(ii) Diameter = 300 mm ; S =15 mm ; Q = 50 kN
Determine the load on a circular footing of 1.2 m diameter that will cause a settlement of 15 mm.
(Civil Services Exam. 1995)

Solution. Housel (1992) equation : $Q = A \,.\, q + P \,.\, s$

$$\therefore \quad Q_1 = 150 = \frac{\pi}{4} (0.75)^2\, q + \pi\,(0.75) \,.\, s$$

or $$0.4418\,q + 2.3562\,s = 150 \qquad ...(1)$$

and $$Q_2 = 50 = \frac{\pi}{4} (0.3)^2\, q + \pi\,(0.3)\, s$$

or $$0.0707\,q + 0.9425\,s = 50 \qquad ...(2)$$

Solving (1) and (2), we get $q = 94.22$ kN/m^2 and $s = 46$ kN/m

$$\therefore \quad Q = 94.22\,A + 46\,P$$

when $D = 1.2$ m, we get

$$Q = 94.22 \times \frac{\pi}{4} (1.2)^2 + 46 \times \pi\,(1.2) \approx \mathbf{280\ kN}$$

Example 24.28. *Foundation in loose sand is 5 × 3 × 1.5 m deep. The soil weighs 16 kN/m^3 and has a φ = 35°. Compute the safe bearing capacity.*

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$$N_q = e^{\pi \tan \varphi}\left[\tan^2\left(45° + \frac{\varphi}{2}\right)\right]\ ;\ N_c = (N_q - 1)\cot\varphi$$

$$N_\gamma = 1.8\,(N_q - 1)\tan\varphi$$ **(Civil Services Exam. 1996)**

Solution. $$N_q = e^{\pi \tan 35°}\left[\tan^2\left(45° + \frac{35°}{2}\right)\right] = 33.3$$

$$N_c = (33.3 - 1)\cot 35° = 46.13$$

$$N_\gamma = 1.8\,(33.3 - 1)\tan 35° = 40.71$$

For rectangular footing, $$q_f = c\left(1 + 0.3\frac{B}{L}\right)N_c + \gamma D N_q + \frac{1}{2}\left(1 - 0.2\frac{B}{L}\right)\gamma B N_\gamma$$

or $$q_f = 0 + 16 \times 1.5 \times 33.3 + \frac{1}{2}\left(1 - 0.2 \times \frac{3}{5}\right) 16 \times 3 \times 40.71 = 1649 \text{ kN/m}^2$$

Also, $$q_{nf} = q_f - \gamma D = 1659 - 16 \times 1.5 = 1659 - 24 = 1635 \text{ kN/m}^2$$

Assuming a factor of safety of 3 on q_f, we have

$$q_s = \frac{q_f}{F} = \frac{1659}{3} = 553 \text{ kN/m}^2$$

Alternatively, using a factor of safety of 3 on q_{nf}, we have

$$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{1635}{3} + 16 \times 1.5 = \mathbf{569 \text{ kN/m}^2}$$

Example 24.29. *A square footing has dimensions of 2 m × 2 m and a depth of 2 m. Determine its ultimate bearing capacity in pure clay with an unconfined strength of 0.15 N/mm², φ = 0° and γ = 1.7 g/cm³. Assume Terzaghi's factors for φ = 0, as N_c = 5.7; N_q = 1 and N_γ = 0* **(Civil Services Exam. 1998)**

Solution. $$c = q_u/2 = 0.15/2 \text{ N/mm}^2 = 0.075 \times 10^3 \text{ kN/m}^2 = 75 \text{ kN/m}^2$$

$$\gamma = 1.7 \text{ g/cm}^3 = 1.7 \text{ t/m}^3 = 16.677 \text{ kN/m}^3$$

For square footing $$q_f = 1.3\,CN_c + \bar{\sigma} N_q + 0.3\,\gamma B N_\gamma$$

$$= 1.3 \times 75 \times 5.7 + 16.677 \times 2 \times 1 + 0.3 \times 1.677 \times 2\,(0) = \mathbf{589.2 \text{ kN/m}^2}$$

Example 24.30. *A plate load test was conducted in a sandy soil with a plate of size 0.3 m × 0.3 m. The ultimate load per unit area was found to be 2.0 kg/cm². Find the allowable load for a footing of 2 m × 2 m, using a factor of safety as per IS.* **(Civil Services Exam. 2000)**

Solution. From Eq. 24.49, $$q_F = q_P \cdot \frac{B_F}{B_P}$$

$$\therefore \quad q_F = 2 \times \frac{2 \text{ m}}{0.3 \text{ m}} = 13.333 \text{ kg/cm}^2 = 133.33 \text{ t/m}^2$$

$$\therefore \quad \text{Using a factor of safety of 3, } q_a = \frac{q_F}{F} = \frac{133.33}{3} \approx \mathbf{44.4 \text{ t/m}^2}$$

Example 24.31. *Determine the ultimate net bearing capacity of the circular footing shown in Fig. 24.33. Also, compute the change in ultimate net bearing capacity, if the entire region is flooded, due to which the ground water level reaches ground level.* **(Civil Services Exam. 2000)**

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Solution. For circular foundation,

$$q_f = 1.3\, cN_c + \bar{\sigma} N_q R_{w_1} + 0.3\, \gamma B N_q R_{w_2} \qquad ...(1)$$

(a) Water table at 3.5 m below base of footing

Since $z_{w_2} > B$, water table will have no effect. Hence, both R_{w_1} and R_{w_2} will be equal to 1.

$$\gamma = 1.8 \text{ g/cm}^3 = 1.8 \text{ t/m}^3$$
$$= 1.8 \times 9.81 \text{ kN/m}^3 = 17.658 \text{ kN/m}^3$$

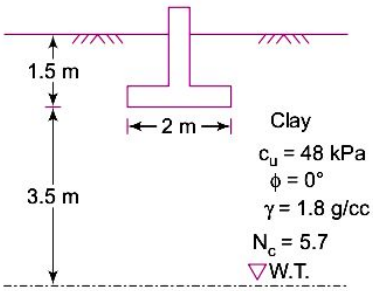

Fig. 24.33

Here $c = c_u = 48 \text{ kPa} = 48 \text{ kN/m}^2$

$D = 1.5\text{m}$

$\bar{\sigma} = \gamma D = 17.658 \times 1.5 = 26.487 \text{ kN/m}^2$

For $\varphi = 0, N_q = \tan^2\left(45° + \frac{0}{2}\right) e^{\pi \tan 0°} = 1$

$N_\gamma = (N_q - 1)\cot 0 = 0$; $N_c = 5.7$ (given)

$\therefore \quad q_f = 1.3 \times 48 \times 5.7 + 26.487 \times 1 \times 1 + 0$

$= 355.68 + 26.487 = 382.167 \text{ kN/m}^2$

$\therefore \quad q_{nf} = 382.167 - \gamma D = 382.167 - 26.487 = \mathbf{355.68 \text{ kN/m}^2}$

(b) Water table at G.L.

$$R_{w_1} = 0.5\left(1 + \frac{z_{w_1}}{D}\right)^2 = 0.5;\ R_{w_2} = 0.5\left(1 + \frac{z_{w_2}}{B}\right)^2 = 0.5$$

$\therefore \quad q_f = 1.3 \times 48 \times 5.7 + 26.487 \times 1 \times 0.5 + 0 = 355.68 + 13.24 = 368.92 \text{ kN/m}^2$

$\therefore \quad q_{nf} = 368.92 - 0.5 \times 26.487 = \mathbf{355.68 \text{ kN/m}^2}$

Hence, q_{nf} remains the same.

Example 24.32. *What is the ultimate bearing capacity of a circular footing of 1.5 m diameter resting on the surface of a saturated clay of unconfined compressive strength of 100 kN/m^2? What is the safe value, if the factor of safety is 3 ?* **(Civil Services Exam. 2001)**

Solution. $q_u = 100 \text{ kN/m}^2$; $\therefore \quad c = q_u/2 = 50 \text{ kN/m}^2$

For saturated clay $N_c = 5.7$; $N_q = 1$; $N_\gamma = 0$; $\bar{\sigma} = \gamma D = 0$ since $D = 0$

Hence, for circular foundation on saturated clay,

$$q_f = 1.3\, cN_c + \bar{\sigma} = 1.3 \times 5.7\, c + 0 = 7.41\, c \text{ (Terzaghi)}$$
$$= 7.41 \times 50 = 370.5 \text{ kN/m}^2$$

$$\therefore \quad q_s = \frac{q_f}{F} = \frac{370.5}{3} = \mathbf{123.5 \text{ kN/m}^2}$$

Note : As per IS code, $N_c = 5.14$ for $\varphi = 0$

$\therefore \quad q_f = 1.3\, cN_c = 1.3 \times 50 \times 5.14 = \mathbf{334.1 \text{ kN/m}^2}$

and $$q_s = \frac{q_f}{F} = \frac{334.1}{3} = \mathbf{111.4 \text{ kN/m}^2}$$

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Example 24.33. *A footing is to be constructed at a depth of 1.2 m in a uniform deposit of stiff clay having unconfined compressive strength of 145.8 kN/m^2 and density of 18.82 kN/m^3. The footing will support a wall that imposes a loading of 152 kN/m of wall length. Calculate the width of the footing using a factor of safety of 3.0.* **(Civil Services Exam. 2002)**

Solution. Given $q_u = 145.8$ kN/m^2; Hence $c = 145.8/2 = 72.9$ kN/m^2

For strip footing $q_{nf} = cN_c + \bar{\sigma}(N_q - 1) + \frac{1}{2} B \gamma N_\gamma$

For clay, $\varphi = 0$. Hence $N_c = 5.14$, $N_q = 1$ and $N_\gamma = 0$ (IS code)

$\therefore$ $q_{nf} = 5.14\, c = 5.14 \times 72.9 = 374.706$ kN/m^2

$\therefore$ $q_s = \frac{q_{nf}}{3} = \frac{374.706}{3} \approx 124.9$ kN/m^2

$\therefore$ $B = \frac{P}{q_s} = \frac{152}{124.9} = 1.217 \approx \mathbf{1.22\ m}$

Example 24.34. *A purely cohesive soil was tested by unconfined compression. The mean unconfined compression strength was obtained as 50 kN/m^2. Estimate the ultimate bearing capacity, utilising Terzaghi's concept (Bearing capacity factor = 5.7).* **(Engg. Services Exam. 1983)**

Solution. $c = q_u/2 = 50/2 = 25$ kN/m^2.

For purely cohesive soil $q_f = cN_c + \gamma D = 25 \times 5.7 + \gamma D = 142.5 + \gamma D$

and $q_{nf} = cN_c = 25 \times 5.7 = \mathbf{142.5\ kN/m^2}$

Example 24.35. *A square footing is to be constructed at a depth of 4 m below the ground surface on a sandy clay soil for which the cohesion is 500 g/cm^2 and the bulk density is 1.68 g/cm^3. The total load applied to the soil is 425 tonnes uniformly distributed. Find out the size of the footing using Terzaghi's formula. Use a factor of safety of 3 and $N_c = 10$, $N_q = 2$ and $N_\gamma = 2$.* **(Engg. Services Exam. 1987)**

Solution. Given $D = 4$ m ; $\gamma = 1.68$ g/cm^3 $= 1.68$ t/m^3; $c = 500$ g/cm^2 $= 5$ t/m^2

$q_f = 1.3\, cN_c + \gamma D N_q + 0.4\, \gamma B N_\gamma$

$= 1.3 \times 5 \times 10 + 1.68 \times 4 \times 2 + 0.4 \times 1.68\, B \times 2 = 78.44 + 1.344\, B$

$q_{nf} = 78.44 + 1.344\, B - \gamma D = 78.44 + 1.344\, B - 1.8 \times 4 = 71.24 + 1.344\, B$

$\therefore$ $q_s = \frac{q_{nf}}{F} + \gamma D = \frac{71.24 + 1.344\, B}{B} + 1.8 \times 4 = 0.448\, B + 30.95$ ...(1)

Actual load intensity $q_a = \frac{420}{B \times B}$ t/m^2 ...(2)

Equating (1) and (2), we get $0.448\, B + 30.95 = \frac{420}{B^2}$ ...(3)

This is a cubic equation, which can be solved by trial and error as under. Neglecting first term of LHS, we get $B = \sqrt{420/30.95} = 3.68$ m. Actual B will be lesser than this.

Trail No.	*B*	*LHS*	*RHS*
1.	3.5	32.52	34.29
2.	3.6	32.56	32.41

Hence, $\boldsymbol{B \approx 3.6}$ **m**

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Example 24.36. *The load settlement curve data from a plate load test on a sandy soil are as under :*

Load, t/m²	10	20	30	40	50	60	70	80
Settlement, mm	4.5	10	15.5	22	29	38.5	50	64

The size of the plate used was 0.3 m × 0.3 m. Determine the size of the square column footing to carry a net load of 250 t with a maximum settlement of 25 mm. **(Engg. Services Exam. 1989)**

Solution. Given $\rho_F = 25$ mm. We will use following two equations, relating bearing capacity, as well as settlements of actual footing and test plate :

$$q_F = q_P \frac{B_F}{B_P} \qquad ...(1)$$

and

$$\rho_P = \rho_F \left[\frac{B_P (B_F + 0.3)}{B_F (B_P + 0.3)}\right]^2 \qquad ...(2)$$

Since B_F is not known, we will adopt a trial and error procedure so that both the above equations are satisfied.

***Trial 1*:** Let $B = 1.5$ m. $\therefore \quad q_F = 250/(1.5)^2 = 111.11$ t/m²

From (1), $$q_p = q_F \cdot \frac{B_P}{B_F} = 111.11 \times \frac{0.3}{1.5} = 22.22 \text{ t/m}^2$$

Corresponding to $q_P = 22.22$ t/m², we get $\rho_P = 10 + \frac{15.5 - 10}{10} \times 2.22 = 11.22$ mm, by linear interpolation from the values of the above table.

Hence, from (2), $$\rho_F = \rho_P \left[\frac{B_F (B_P + 0.3)}{B_P (B_F + 0.3)}\right]^2 = 11.22 \left[\frac{1.5 (0.3 + 0.3)}{0.3 (1.5 + 0.3)}\right]^2 = 31.17 \text{ mm}$$

Since this is more than 25 mm, the width $B = 1.5$ m will be inadequate.

***Trial 2*:** Hence, increase B in the ratio of $\sqrt{31.17/25}$

$\therefore$ Next value of $$B = 1.5 \times \sqrt{\frac{31.17}{25}} = 1.67 \text{ m.}$$

$\therefore$ $$q_F = \frac{250}{(1.67)^2} = 89.64 \text{ t/m}^2 \text{ and } q_P = 89.64 \times \frac{0.3}{1.67} = 16.1 \text{ t/m}^2$$

$\therefore$ $$\rho_P = 4.5 + \frac{10 - 4.5}{10} \times 6.1 = 7.86 \text{ mm}$$

$\therefore$ $$\rho_F = 7.86 \left[\frac{1.67 (0.3 + 0.3)}{0.3 (1.67 + 0.3)}\right]^2 = 22.6 \text{ mm} < 25 \text{ mm}$$

A final trial will yield a value of $B =$ **1.62 m**

Note: The linear interpolation may be avoided by plotting the data of the above table and taking the exact value ρ_P from the curve, corresponding to the computed value of q_P.

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Example 24.37. *A footing 2 m square, rests on a soft clay soil with its base at a depth of 1.5 m from ground surface. The clay stratum is 3.5 m thick and is underlain by a firm sand stratum. The clay soil has L.L. = 30%, G =2.7, water content at saturation = 40%, cohesion = 0.5 kg/cm^2 ($\varphi = 0$). It is known that the clay stratum is normally consolidated. Compute the settlement that would result if the load intensity equal to safe bearing capacity of soil were allowed to act on the footing. Natural water table is quite close to the ground surface. For given conditions, bearing capacity factor (N_c) is obtained as 6.9. Take factor of safety as 3. Assume load spread of 2 (vertical) to 1 (horizontal).* **(Engg. Services Exam. 1995)**

Solution. $c = 0.5 \text{ kg/cm}^2 = 5 \text{ t/m}^2$

(a) Determination of submerged unit weight γ'

$$\gamma' = \frac{G-1}{1+e}\gamma_w \quad \text{where} \quad e = w_{sat} G = 0.4 \times 2.7 = 1.08 \text{ and } \gamma_w = 1 \text{ t/m}^3$$

$$\therefore \quad \gamma' = \frac{2.7-1}{1+1.08} \times 1 = 0.817 \text{ t/m}^3$$

(b) Determination of footing load

$$q_{nf} = cN_c = 5 \times 6.9 = 34.5 \text{ t/m}^2$$

$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma' D = \frac{34.5}{3} + 0.817 \times 1.5$$

$$\approx 12.726 \text{ kN/m}^2$$

$$\therefore \quad Q_s = q_s \times \text{Area} = 12.726 \times 2 \times 2 = 50.9 \text{ t}$$

(c) Determination of settlement : Consider level *AA* at mid depth of clay.

$$\sigma_0' = \gamma' (2.5) = 0.817 \times 2.5 = 2.043 \text{ t/m}^2$$

Area of spread at level *AA* = $3 \times 3 = 9 \text{ m}^2$.

$$\therefore \quad \Delta\sigma' = \frac{Q_s}{A} = \frac{50.9}{9} = 5.656 \text{ t/m}^2$$

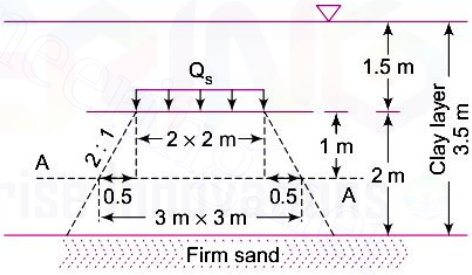

Fig. 24.34

$$C_c = 0.009 (w_L - 10) = 0.009 (30 - 10)$$

$$= 0.18 \; ; \; e_0 = e = 1.08$$

$$\therefore \quad \Delta H = \frac{C_c \, . \, H}{1+e_0} \log_{10} \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'} = \frac{0.18 \times 2}{1+1.08} \log_{10} \frac{2.043 + 5.656}{2.043} \approx 0.1 \text{ m} = \mathbf{10 \text{ cm}}$$

Example 24.38. *The following data was obtained from plate load test carried out on a 60 cm square test plate at a depth of 2 m below ground surface on a sandy soil which extends up to a large depth.*

Load intensity (t/m^2)	*Settlement (mm)*	*Load intensity (t/m^2)*	*Settlement (mm)*
0	0	25	16.3
5	2.0	30	23.5
10	4.0	35	34.0
15	7.5	40	45.0
20	11.0		

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Determine the settlement of foundation 3.0 m × 3.0 m carrying a load of 110 t and located at a depth of 2 m below ground surface. Water table is located at a large depth from the ground surface.

(Engg. Services Exam. 1996)

Solution. Figure 24.35 shows the plot between load intensity and settlement.

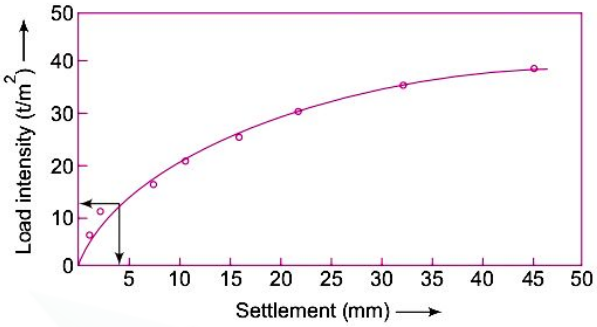

Fig. 24.35

Actual load intensity

$$= \frac{110}{3 \times 3} = 12.22 \text{ t/m}^2$$

Hence, from the graph,

$$\rho_P = 4.5 \text{ mm}$$

Now,
$$\rho_F = \rho_P \left[\frac{B\,(B_P + 0.3)}{B_P\,(B + 0.3)}\right]^2 = 4.5 \left[\frac{3\,(0.6 + 0.3)}{0.3\,(3 + 0.3)}\right]^2 \approx \mathbf{33.5 \text{ mm}}$$

Example 24.39. *A 3.0 m square footing is located in a dense sand at a depth of 2.0 m. Determine the ultimate bearing capacity for the following water table positions:*

(i) at ground surface, (ii) at footing level, and (iii) at 1 m below the footing.

The moist unit weight of sand above the water table is 18 kN/m³ and the saturated weight is 20 kN/m³. φ = 35° ; c = 0; N_q = 33 and N_γ = 34.0. **(Engg. Services Exam. 1998)**

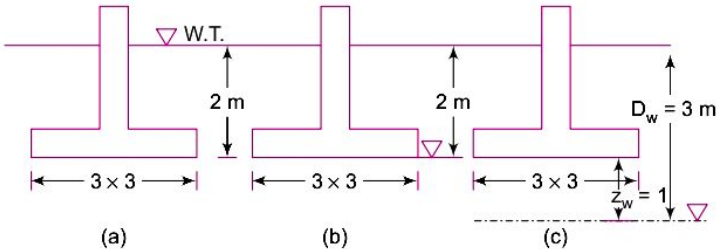

Fig. 24.36

Solution. Given $\gamma_{sat} = 20 \text{ kN/m}^3$; $\gamma' = 20 - 9.81 = 10.19 \text{ kN/m}^3$

For a square footing $q_f = 1.3\, c\, N_c + \bar{\sigma}\, N_q + 0.4\, \gamma B\, N_\gamma$

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Taking into account water reduction factor, we have, as per IS code

$$q_f = 1.3\, cN_c + \bar{\sigma} N_q + 0.4\, \gamma B\, N_\gamma W'$$

(a) *Water table at G.L.* : (Fig. 24.36 *a*)

Here $\gamma = \gamma_{sat} = 20\ \text{kN/m}^3$; $\sigma' = \gamma' D = 10.19 \times 2$ and $W' = 0.5$

$$q_f = 0 + 10.19 \times 2 \times 33 + 0.4 \times 20 \times 3 \times 34 \times 0.5 = 1080.54\ \text{kN/m}^2$$

(b) *Water table at footing level* (Fig. 24.36*b*)

Here also, $\gamma = \gamma_{sat} = 20\ \text{kN/m}^3$; $W' = 0.5$ and $\bar{\sigma} = \gamma_{wet} \cdot D = 18 \times 2$

$\therefore$ $$q_f = 0 + 18 \times 2 \times 33 + 0.4 \times 20 \times 3 \times 34 \times 0.5 = 1596\ \text{kN/m}^2$$

(c) *Wafer table 1 m below footing* (Fig. 24.36*c*)

$$D_w = 2 + 1 = 3\ \text{m}\ ;\ W' = 0.5\ [1 + z_w/B] = 0.5\ [1 + 1/3] = 0.667$$

$$\bar{\sigma} = \gamma_{sat}\, D = 18 \times 2$$

$\therefore$ $$q_f = 0 + 18 \times 2 \times 33 + 0.4 \times 20 \times 3 \times 34 \times 0.667 = \mathbf{1732.3\ kN/m^2}$$

Example 24.40. *A square footing located at a depth 1.5 m from the ground surface carries a column load of 150 kN. The soil is submerged having an effective unit weight of 11 kN/m^3 and an angle of shearing resistance of 30°. Find the size of the footing using Terzaghi's theory, if $F_s = 3$; for $\varphi = 30°$, $N_q = 10$ and $N_\gamma = 6.0$.* **(Engg. Services Exam. 2000)**

Solution. Refer example 24.10. Since c and N_c are not given, assume that $c = 0$.

$$q_f = 1.3\, c\, N_c + \bar{\sigma} N_q + 0.4\, \gamma B\, N_\gamma$$

or $$q_f = 0 + 11 \times 1.5 \times 10 + 0.4 \times 11 \times B\ (6.0) = 165 + 26.4\, B$$

$$q_{nf} = (165 + 26.4\, B) - 11 \times 1.5 = 148.5 + 26.4\, B$$

$\therefore$ $$q_s = \frac{q_{nf}}{F} + \gamma' D = \frac{148.5 + 26.4B}{3} + 11 \times 1.5 = 66 + 8.8\, B \qquad ...(1)$$

Actual load intensity $$q_a = \frac{150}{B^2} \qquad ...(2)$$

Equating the two, we get $66 + 8.8\, B = \dfrac{150}{B^2}$

or $$8.8\, B^3 + 66\, B^2 = 150 \text{ or } B^3 + 7.5\, B^2 = 17.045$$

Solving this by trial and error, we get $\boldsymbol{B \approx 1.4\ m}$

Example 24.41. *A footing 2 m × 2 m in plan is founded at 1 m depth below G.L. in sand having angle of shearing resistance of 36°. Compute the allowable load at the base of the footing, if (i) factor of safety against shear is 3, and (ii) the maximum settlement of footing is not exceed 50 mm. The water table is 1 m below G.L. The allowable soil pressure for 50 mm settlement with water table at very great depth is 50 t/m^2. Saturated and dry unit weight of sand are 2 t/m^3 and 1.6 t/m^3.*

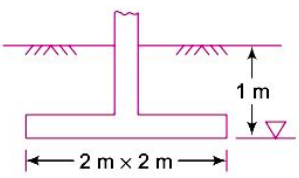

Fig. 24.37

For $\varphi = 36°$; $N_c = 50$; $N_q = 42$ and $N_\gamma = 46$. If the load on the footing is eccentric with $e_x = e_y = 0.25$ m, what will be the allowable load ? Take soil above water table as dry. **(Engg. Services Exam. 2002)**

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Solution. For square footing,

$$q_f = 1.3\, cN_c + \gamma D\, N_q + 0.4\, \gamma B\, N_\gamma$$

(*a*) ***Shear failure criterion:*** Since the water table is at the base of footing, γ with N_q term will be γ_d (= 1.6 t/m^3) while that with N_γ term will be γ' (= 2 – 1 = 1 t/m^3). Also, $c = 0$

$$\therefore \quad q_{nf} = \gamma_d D (N_q - 1) + 0.4\gamma' BN_\gamma = 1.6 \times 1 (42 - 1) + 0.4 \times 1 \times 2 \times 46 = 102.4 \text{ t/m}^2$$

$$q_a = \frac{q_{nf}}{F} + \gamma_d D = \frac{102.4}{34} + 1.6 \times 1 = 35.73 \text{ t/m}^2$$

(*b*) ***Allowable settlement criteria:*** Allowable soil pressure for 50 mm settlement with water table at very great depth is 50 t/m^2. Let us find the value of q_s on shear failure basis when the water table is at very great depth. In that case γ with both N_q and N_γ terms will be equal to γ_d (= 1.6 t/m^3).

$$\therefore \quad q_{nf} = 1.6 \times 1\ (42 - 1) + 0.4 \times 1.6 \times 2 \times 46 = 124.48 \text{ t/m}^2$$

$$\therefore \quad q_a = \frac{124.48}{3} + 1.6 \times 1 = 43.09 \text{ t/m}^2 < 50 \text{ t/m}^2$$

Since the settlement at 50 t/m^2 is 50 mm, the settlement at q_a = 43.09 t/m^2 will be much less than 50 mm. Hence, the total allowable pressure q_a computed in step (*a*) is OK from permissible settlement criterion also.

(*c*) ***Allowable load for given eccentricities***

$$B' = B - 2\, e_x = 2 - 2 \times 0.25 = 1.5 \text{ m} \; ; \; L' = L - 2\, e_y = 2 - 2 \times 0.5 = 1.5 \text{ m}$$

$$\therefore \quad Q_a = q_a \times B' \times L' = 35.73 \times (1.5)^2 = \mathbf{80.39\ t}$$

Example 24.42. *A large scale bearing capacity test on a footing of size 1.05 m × 1.05 m at a depth of 1.5 m yielded an ultimate value of 141.0 kN. Unconfined compression test on the soft saturated clay yielded a strength of 0.03 N/mm^2. If the unit weight of the soil is 1.6 g/cc, how much does the test value differ from that obtained using Terzaghi's bearing capacity equation?* **(Gate Exam. 1992)**

Solution.

$$\gamma = 1.6 \text{ g/cc} = 1.6 \text{ t/m}^3 = 1.6 \times 9.81 \text{ kN/m}^3 = 15.596 \text{ kN/m}^3$$

$$q_u = 0.03 \text{ N/mm}^3 = 30 \text{ kN/m}^2 \; ; \; c = q_u/2 = 15 \text{ kN/m}^2$$

Taking $\varphi = 0$, we have $N_c = 5.7$, $N_q = 1.0$ and $N_\gamma = 0$

For square footing $q_f = 1.3\, c_u N_c + \bar{\sigma}\, N_q = 1.3\, c_u N_c + \gamma D\, N_q$

For soft saturated clay, we will assume *local shear failure* for which

$$c = c_m = \frac{2}{3} c_u = \frac{2}{3} \times 15 = 10 \text{ kN/m}^2$$

$$\therefore \quad q_f = 1.3 \times 10 \times 5.7 + 15.596 \times 1.5 \times 1 = 97.49 \text{ kN/m}^2$$

$$\therefore \quad Q_f = q_f \times A = 97.49 \times 1.05 \times 1.05 \approx \mathbf{107.5\ kN}$$

$$Q_{test} = 141 \text{ kN}$$

$$\therefore \quad \% \text{ difference} = \frac{141 - 107.5}{107.5} \times 100 = \mathbf{31.2\%}$$

Example 24.43. *A 2 m wide strip footing is placed at 1 m below the ground surface of a clay soil having the following properties :*

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$$c_u = 80\ kN/m^2\ ;\ c' = 0\ ;\ \varphi_u = 0;\ \varphi' = 30°$$

Unit weight (above the water table) = 16 kN/m³

Saturated unit weight = 20 kN/m³

(i) If the water table is at foundation level, calculate the net safe bearing capacity of the footing using a factor of safety of two, for short term and long term conditions.

(ii) Also indicate the change that may occur in the short term value, when water table rises to ground level.

φ	N_c	N_q	N_γ
0°	6.0 (considering depth effect)	1.0	0.0
30°	37.2	22.5	19.7

(Gate Exam. 1993)

Solution. We will use IS code method

(a) Water table at foundation level

$$q_{nf} = cN_c + \gamma D\,(N_q - 1) + \frac{1}{2}\gamma' B N_\gamma$$

(i) For short term condition : $c_u = 80\ kN/m^2$ and $\varphi_u = 0$

$\therefore$ $N_c = 6.0$, $N_q = 1$ and $N_\gamma = 0$

$\therefore$ $q_{nf} = 80 \times 6 = 480\ kN/m^2$

$\therefore$ $q_{ns} = \dfrac{q_{nf}}{F} = \dfrac{480}{2} = \mathbf{240\ kN/m^2}$

(ii) For long term conditions : $c' = 0$ and $\varphi' = 30°$

$\therefore$ $N_c = 37.2$; $N_q = 22.5$ and $N_\gamma = 19.7$

$\therefore$ $q_{nf} = 0 + 16 \times 1\,(22.5 - 1) + \dfrac{1}{2}\,(20 - 9.81)\,2 \times 19.7 = 544.7\ kN/m^2$

$\therefore$ $q_{ns} = \dfrac{q_{nf}}{F} = \dfrac{544.7}{2} \approx \mathbf{272.4\ kN/m^2}$

(b) Water table at ground level: Short term value: $c_u = 80$ and $\varphi_u = 0$

$$q_{nf} = cN_c + \gamma'\ D\,(N_q - 1) + \frac{1}{2}\gamma' B N_\gamma$$

For short term case ($\varphi_u = 0$), $(N_q - 1)$ is zero and N_γ is also zero. Hence, q_{nf} will be equal to cN_c. Since this term does not have any effect due to rise of water table, q_{nf} and hence q_{ns} will not change. Hence $q_{ns} = \mathbf{240\ kN/m^2}$, as before.

Example 24.44. *A strip footing is to be designed to carry a gross load of 900 kN/m at a depth of 1 m is a gravelly mud. The appropriate shear strength parameters are* $\bar{c} = 0$ *and* $\varphi = 38°$. *Determine the width of the footing if a factor of safety of 3 against shear failure is to be assured. Water table is found to be at the foundation level. Above the water table, the unit weight is 18 kN/m³ and below the water table the saturated unit weight is 20 kN/m³. For* $\varphi = 38°$, *the bearing capacity factors are* $N_q = 49$ *and* $N_\gamma = 67$. *Unit weight of water is 9.8 kN/m³.* **(Gate Exam. 1994)**

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Solution. $q_f = cN_c + \gamma_1 DN_q + 0.5\,\gamma_2'\,B\,N_\gamma$

$= 0 + 18 \times 1 \times 49 + 0.5\,(20 - 9.8)\,B \times 67 = (882 + 341.7\,B)$ kN/m^2

Using $F = 3$ on q_f, we get

$$q_s = \frac{q_f}{3} = \frac{1}{3}\,(882 + 341.7\,B) = 294 + 113.9\,B \qquad ...(a)$$

Actual load intensity, $q_a = \dfrac{900}{B \times 1} = \dfrac{900}{B}$

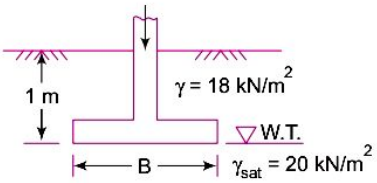

Fig. 24.38

Equating the two, we get $294 + 113.9\,B = \dfrac{900}{B}$ ...(b)

or $B^2 + 2.581\,B - 7.902 = 0$, from which $B = \mathbf{1.8\ m}$

Alternatively: Using $F = 3$ on q_{nf}, we get

$$q_{nf} = 882 + 341.7\,B - 18 \times 1 = 864 + 341.7\,B$$

$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma_{1\,sat}\,D = \frac{864 + 341.7\,B}{3} + 18 \times 1$$

$$= 306 + 113.9\,B \qquad ...(c)$$

Equating (*b*) and (*c*), $\dfrac{900}{B} = 306 + 113.9\,B$

or $B^2 + 2.687\,B - 7.902 = 0$, from which $5 \approx \mathbf{1.78\ m}$

Example 24.45. *Using Terzaghi's theory, determine the ultimate bearing capacity of a strip footing 1.5 m wide resting on a saturated clay ($c_u = 30$ kN/m^2, $\varphi_u = 0$ and $\gamma_{sat} = 20$ kN/m^3), at a depth of 2 below ground level. The water table is also at a depth of 2 m from the ground level. If the water table rises by 1 m, calculate the percentage reduction in the ultimate bearing capacity.* **(Gate Exam. 1997)**

Solution. For $\varphi = \varphi_u = 0$, $N_c = 5.7$, $N_q = 1.0$ and $N_\gamma = 0$. Also, $c_u = 30$ kN/m^2

$q_f = cN_c + \bar{\sigma} N_q + 0.5\,B\,\gamma\,N_\gamma = 5.7\,c \times \bar{\sigma}$

(a) Water table at foundation level

$$\bar{\sigma} = \gamma_{sat} \cdot D = 20 \times 2 = 40 \text{ kN/m}^2$$

$$\therefore \quad q_f = 5.7 \times 30 + 40 = \mathbf{211\ kN/m^2}$$

(b) Water table at 1 m above foundation level

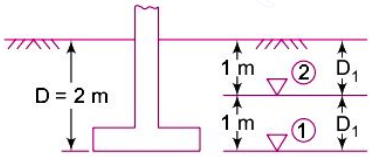

Fig. 24.39

$$\bar{\sigma} = \gamma_{sat}\,D_1 + \gamma'\,D_2 = (20 \times 1) + (20 - 9.81)\,1$$

$$= 30.19 \text{ kN/m}^2$$

$$\therefore \quad q_f = 5.7 \times 30 + 30.19 = \mathbf{201.19\ kN/m^2}$$

$$\therefore \ \%\text{ Reduction} = \frac{211 - 201.19}{211} \times 100 = \mathbf{4.75\%}$$

Example 24.46. *For a general $c - \varphi$ soil, cohesion c is 50 kPa, the total unit weight γ_t is 20 kN/m^3 and the bearing capacity factors are $N_c = 8$, $N_q = 3$ and $N_\gamma = 2$. Using Terzaghi's formula only, estimate the safe total load on a footing 10 m long by 2 m wide strip footing, using a factor of safety of 3.*

(Gate Exam. 1998)

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Solution. $q_{nf} = cN_c + \bar{\sigma}(N_q - 1) + 0.5\, B\, \gamma\, N_\gamma$

or $q_{nf} = 50 \times 8 + (20 \times 1)(3 - 1) + 0.5 \times 2 \times 20 \times 2 = \mathbf{480\ kN/m^2}$

$\therefore \quad q_s = \dfrac{q_{nf}}{F} + \gamma_t\, D = \dfrac{480}{3} + 20 \times 1 = 180\ \text{kN/m}^2$

$\therefore \quad Q_s = q_s \times \text{area} = 180\,(2 \times 10) = \mathbf{3600\ kN}$

Example 24.47. *A footing 2.25 m square is located at a depth of 1.5 m in a sand of unit weight 18 kN/m³. The shear strength parameters are c′ = 0 and φ = 36°. Calculate the safe load carried by the footing against complete shear failure. For of safety against shear failure is 3. Use Terzaghi's analysis, $N_c = 65.4$, $N_q = 49.4$, $N_\gamma = 54.0$.* **(Gate Exam. 1999)**

Solution. For square footing, $q_f = 1.3\, cN_c + \bar{\sigma}\, N_q + 0.4\, \gamma\, B\, N_\gamma$

For $c' = 0$, $q_{nf} = \bar{\sigma}(N_q - 1) + 0.5\, \gamma\, B\, N_\gamma$

or $q_{nf} = 18 \times 1.5\,(49.4 - 1) + 0.4 \times 18 \times 2.25 \times 54 = 2181.6\ \text{kN/m}^2$

$\therefore \quad q_s = \dfrac{q_{nf}}{F} + \gamma D = \dfrac{2181.6}{3} + 18 \times 1.5 = 754.2\ \text{kN/m}^2$

$\therefore \quad Q_s = q_s \times \text{area} = 754.2\,(2.25)^2 = \mathbf{3818\ kN}$

Example 24.48. *A footing 3 m square carries of gross pressure of 350 kN/m² at a depth of 1.2 m in sand. A saturated unit weight of sand is 20 kN/m³, and the unit weight above the water table is 17 kN/m³. The shear strength parameters are c′ = 0 and φ′ = 30° [for φ′ = 30°, $N_q = 22$, $N_\gamma = 20$]. Determine the factor of safety with respect to shear failure for the following cases:*

(a) Water table is 5 m below the ground level

(b) Water table is 1.2 m below the ground level **(Gate Exam. 2000)**

Solution. We will follow IS code method. For square footing in soil having $c = 0$,

$$q_f = \bar{\sigma} N_q + 0.4\, \gamma\, B\, N_\gamma\, W'$$

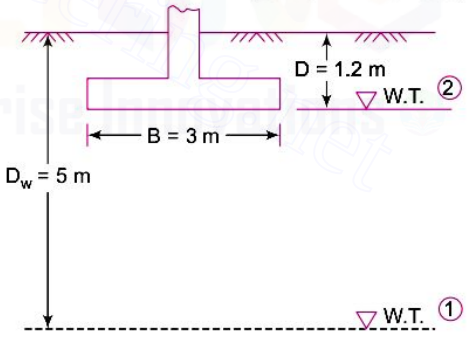

Fig. 24.40

(a) Water table at 5 m below footing

$\bar{\sigma} = 17 \times 1.2 = 20.4\ \text{kN/m}^2$

$D_w = 5$ m; $D + B = 3 + 1.2 = 4.2$ m

Since, $D_w > (D + B)$, $W' = 1$.

Also, $\gamma = 17\ \text{kN/m}^3$

$\therefore \quad q_f = 20.4 \times 22 + 0.4 \times 17 \times 3 \times 20 \times 1$

$= 856.8\ \text{kN/m}^2$

$q_{nf} = 856.8 - 17 \times 1.2 = 836.4\ \text{kN/m}^2$

$\therefore \quad q_s = \dfrac{q_{nf}}{F} + \gamma\, D = \dfrac{836.4}{F} + 17 \times 1.2 = \dfrac{836.4}{F} + 20.4$ ...(1)

Actual load intensity, $q_a = 350\ \text{kN/m}^2$ ...(2)

Equating the two we get $\dfrac{836.4}{F} + 20.4 = 350$ or $F = \dfrac{836.4}{350 - 20.4} = 2.54$

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(b) Water table at footing level

$$W' = 0.5 \text{ and } \gamma = \gamma_{sat} = 20 \text{ kN/m}^3$$

$$\sigma' = 17 \times 1.2 = 20.4 \text{ kN/m}^2 \text{ as before.}$$

$$\therefore \quad q_f = 20.4 \times 22 + 0.4 \times 10.19 \times 3 \times 20 = 693.4 \text{ kN/m}^2$$

$$q_{nf} = 693.4 - 17 \times 1.2 = 673 \text{ kN/m}^2$$

$$\therefore \quad q_s = \frac{q_{nf}}{F} + \gamma D = \frac{673}{F} + 20.4$$

$$\therefore \quad \frac{673}{F} + 20.4 = 350$$

From which $F = 2.04$

PROBLEMS

1. A strip footing is laid on the ground surface. The subsoil consists of a deep stratum of clay, having $c_u = 60$ kN/m^2. Calculate q_{nf} taking $N_c = 5.14$. [**Ans.** 308.4 kN/m^2]
2. A strip footing is laid to a depth of 1 m. The soil consists of clay having c_u (undrained) = 55 kN/m^2 and $\gamma = 17.6$ kN/m^3. Using Terzaghi's analysis, calculate q_{nf} and q_f. [**Ans.** $q_{nf} = 313.5$ kN/m^2 ; $q_f = 331.1$ kN/m^2]
3. A strip footing 1.4 m wide is laid at depth of 4.5 m in purely cohesive soil having $c = 144$ kN/m^2 and $\gamma = 17.7$ kN/m^3. Calculate q_f by (*a*) Terzaghi, and (*b*) Skempton analysis. [**Ans.** (*a*) 900 kN/m^2 (*b*) 1160 kN/m^2]
4. A strip footing 1 m wide is laid at a depth of 2 m in c-ϕ soil having the following characteristics. Calculate q_f by Terzaghi analysis. Take the following values of bearing capacity factors: Terzaghi ($\phi = 28°$) ; $N_c = 32$, $N_q = 18$; $N_\gamma = 16$. Take $c = 20$ kN/m^2, $\gamma = 18$ kN/m^3 and $\phi = 28°$. [**Ans.** 1432 kN/m^2]
5. A strip foundation 1.5 m wide has depth of 10 m in a c-ϕ soil having $\phi = 15°$; $c = 85$ kN/m^2 and $\gamma = 18$ kN/m^3. Using Meyerhof's analysis for deep foundations, calculate q_f. Take $N_c = 35$; $N_q = 10$; $N_\gamma = 6$. [**Ans.** 4856 kN/m^2]
6. A strip footing 1.6 m wide rests on the surface of a dry cohesionless soil having $\phi = 17°$ and $\gamma = 18$ kN/m^3. If the flood causes the water table to rise temporarily to the surface, what percentage of q_f is reduced. Take $N_\gamma = 4$ (Terzaghi) [**Ans.** 53.2%]

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CHAPTER 25

Shallow Foundations

25.1 TYPES OF FOUNDATIONS

Foundations may be broadly classified under two heads: *shallow foundations* and *deep foundations*. According to Terzaghi, a foundation is shallow if its depth is equal to or less than its width. In the case of deep foundations, the depth is equal to or greater than the width. Apart from deep strip, rectangular or square foundations, other common forms of deep foundations are: *pier foundation, pile foundation* and *well foundation*. The shallow foundations are of the following types: spread footing (or simply, footing), strap footing, combined footing, and mat or raft footing. Figure 25.1 shows the common types of shallow foundations.

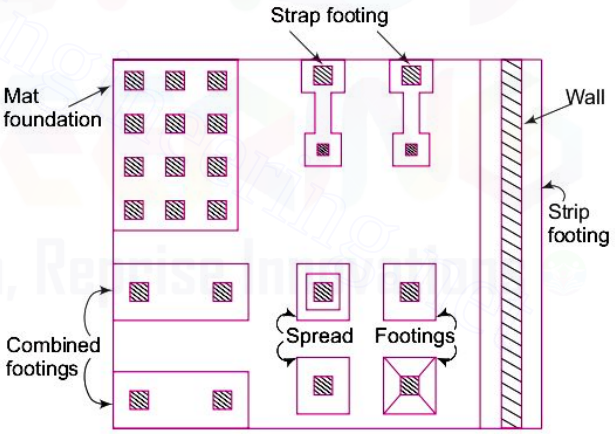

Fig. 25.1 Various types of shallow foundations

25.2 SPREAD FOOTING

A *spread footing* or simply *footing* is a type of shallow foundation used to transmit the load of an isolated column, or that of a wall to the subsoil. This is most common type of foundation. The base of the column or wall is enlarged or spread to provide individual support for the load. Figure 25.2 shows some typical spread footings.

Pressure Distribution: Both from observations as well as the analytical studies from theory of elasticity, it is known that the pressure distribution beneath footing, symmetrically loaded is not uniform.

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The pressure intensities depend upon the rigidity of footing, the soil type, and the condition of soil. Figure 25.3(a) and (b) shows the probable pressure distribution beneath a rigid footing on a loose cohesionless soil and on cohesive soil. Figure 25.3(c) shows the usually assumed uniform pressure distribution.

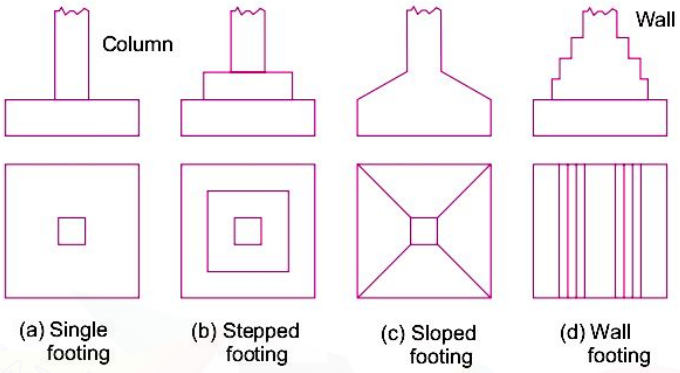

Fig. 25.2 Typical spread footings

When a rigid footing rests on cohesionless soil, the soil grains at the outer edge have no lateral restraint whereas in centre the soil is relatively confined, resulting in a pressure distribution shown in Fig. 25.3 (a). For the footings on cohesive soil, the edge stresses may be very large. However, the pressure distribution may be considered to be linear, as shown in Fig. 25.3 (c), for the purpose of the design of reinforced concrete footing. Once the pressure distribution is known, the bending moment and shear force in the footing can be calculated, and the thickness of the structural member of the footing along with the reinforcement etc., can be calculated using the usual principles of reinforced concrete.

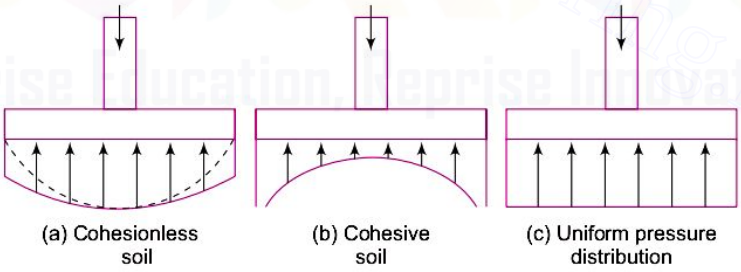

Fig. 25.3 Pressure distribution beneath a rigid footing

25.3 SAFE BEARING PRESSURE

In conventional design, the allowable bearing capacity should be taken as the smaller of the following two values: (i) the safe bearing capacity based on ultimate capacity, and (ii) the allowable bearing pressure on tolerable settlement.

The safe bearing capacity q_s may be computed from the following equations based as Terzaghi's analysis:

For strip footing: $$q_s = \frac{1}{F}\left[cN_c + \gamma D\left(N_q - 1\right)R_{w1} + 0.5\gamma\, BN_\gamma\, R_{w2}\right] + \gamma D \qquad \ldots(25.1)$$

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considered to consist of three components (Skempton and Bjerrum, 1957)

$$S = S_i + S_c + S_s \qquad ...(25.8)$$

where S = total settlement ;

S_i = immediate elastic settlement

S_c = consolidation settlement

S_s = settlement due to secondary consolidation of clay.

The immediate settlements is the elastic settlement and can computed from the following expression based on the theory of elasticity,

$$S_i = q\, B \left(\frac{1 - \mu^2}{E_s} \right) I_w \qquad ...(25.9)$$

where q = Intensity of contact pressure;

B = Least lateral dimension of footing

E_s = Modulus of elasticity of soil

I_w = Influence factor

= 0.88 for rigid circular footing

= 0.82 for rigid square footing

= 1.06 for rigid rectangular footing with $L/B = 1.5$

= 1.70 for rigid rectangular footing, with $L/B = 5$

Table 25.1 gives the values of *influence factor* I_w as suggessed by BIS Code IS: 8009 Part I, 1976. According to this code, the total settlement of a rigid footing is taken as 0.8 times the settlement at the centre of a flexible foundation. Accordingly, column 5 of Table 25.1 gives the values of I_w by multiplying the values of column (2) by 0.8.

Table 25.1 Values of influence factor I_w for use in EQ 25.9 (IS : 8009 Part I-1976)

Shape of loaded area	*Flexible Footing*			*Rigid Footing* Col. (5) = 0.8 × Col. (2)
	Centre	*Corner*	*Average*	
(1)	*(2)*	*(3)*	*(4)*	*(5)*
1. Circular 2. Rectangular (L/B)	1.00	0.64	0.85	0.80
1.0	1.12	0.56	0.95	0.90
1.5	1.36	0.68	1.20	1.09
2.0	1.52	0.77	1.31	1.22
5.0	2.10	1.05	1.83	1.68
10.0	2.52	1.26	2.25	2.02
100.0	3.38	1.69	2.96	2.70

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It is difficult to determine μ and E_s for soils. However, the entire term $\frac{1-\mu^2}{E_s} I_w$ for Eq. 25.6 may be determined from plate load test by using two or three different size plates of the same shape. A plot between S_i and q. B gives a straight line, the slope of which is equal to $\frac{1-\mu^2}{E_s} I_w$.

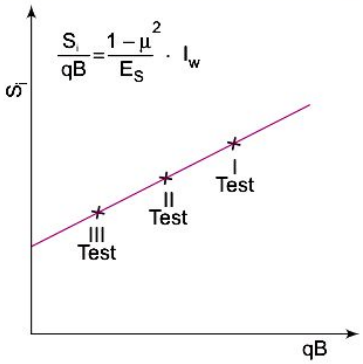

Fig. 25.4 Graphical solution to find $\left(\frac{1-\mu^2}{E_s}\right) I_w$

Janbu, Bjerrum and Kjaernsli (1966) have proposed the following equation for computing the imediate settlement

$$S_i = \mu_0 \mu_1 qB \frac{1-\mu^2}{E_s} \quad \ldots(25.10)$$

where the modulus of elasticity E_s is computed from triaxial test data

$$E_s = \frac{\sigma_1 - \sigma_3}{\Delta L / L} \quad \ldots(25.11)$$

and μ_0 and μ_1, are taken from Fig. 25.5.

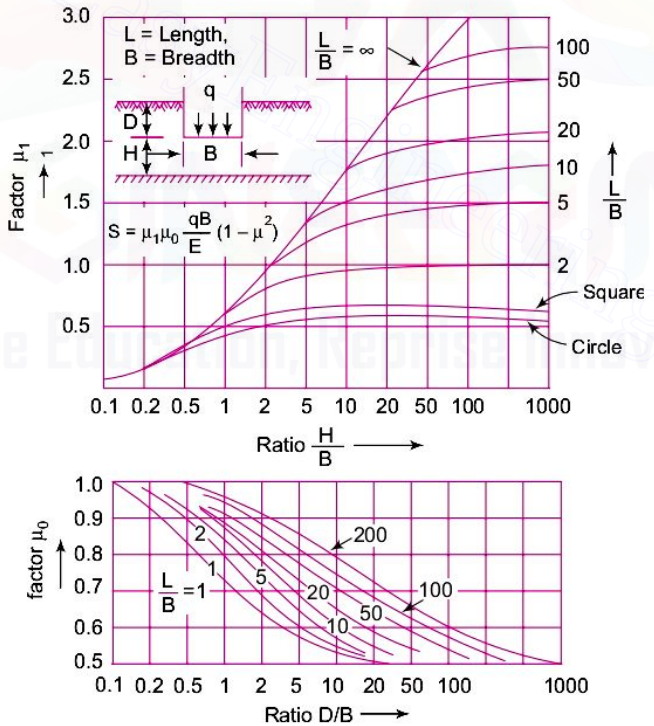

Fig. 25.5 Values of μ_0 and μ_1. (After janbu, bjerrum and kjaernsli, 1966)

The *consolidation settlement* S_c is computed from the following equation:

$$S_c = C \frac{C}{1+e_0} . H \log_{10} \frac{\sigma_0 + \Delta\sigma}{\sigma_0} \quad \ldots(25.12)$$

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where σ_0 = effective initial overburden pressure due to soil overburden, measured at the centre of the layer

$\Delta\sigma$ = vertical stress increment due to footing load, at the centre of layer

C_c = compression index = $0.009\ (w_L - 10)$

e_0 = initial voids ratio; H = thickness of compressible layer; C = a coefficient or correction factor depending upon the geometry of the footing and the history of loading on the clay *(i.e.,* on the pore pressure coefficient A), given in Fig. 25.6. In the absence of any other data, C may be taken as unity for the clay which is being compressed for the first time.

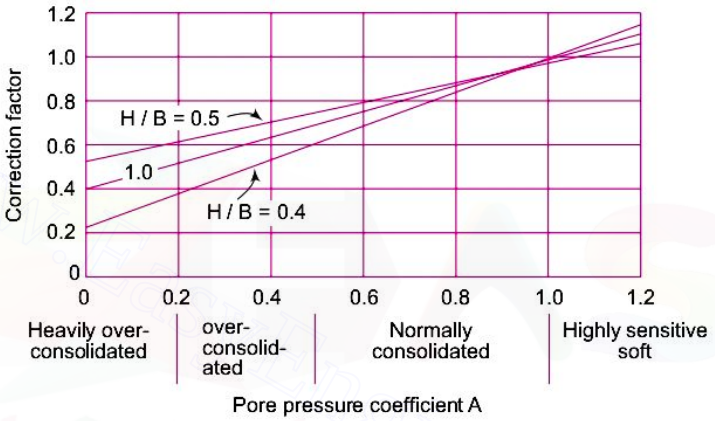

Fig. 25.6 Value of factor C. (After skempton and bjerrum, 1957)

25.5 COMBINED FOOTING AND STRAP FOOTING

A spread footing which supports two or more columns is termed as a *combined footing*. If the independent spread footing of two columns are connected by a beam, it is called a strap footing. The combined footing may be rectangular in shape if both the columns carry equal loads, or may be trapezoidal if they carry unequal loads. Generally, combined footings are constructed of reinforced concrete. It is assumed that the footing is *rigid* and rests on a homogeneous soil, so as to give rise to a *linear stress distribution* on the bottom of the footing. If the resultant of the soil pressure coincides with the resultant of the loads (and centre of gravity of the footing), the soil pressure is assumed to be uniformly distributed.

(a) Rectangular Combined Footing: The design of rigid rectangular combined footing consists in determining the location of centre of gravity of the column loads and using length and width dimensions such that centroid of the footing and the centre of gravity of column loads coincide. The resulting pressure distribution will be rectangular, as shown in Fig. 25.7 (*a*), with the pressure intensity $q = \dfrac{P_1 + P_2}{B}$ (per unit area).

The column loads may be considered to be concentrated loads, and the resulting shear force and bending moment diagrams can be plotted. The maximum bending moment should be adopted as the design value for the reinforced concrete footing, which should also be checked for maximum shear and bond etc.

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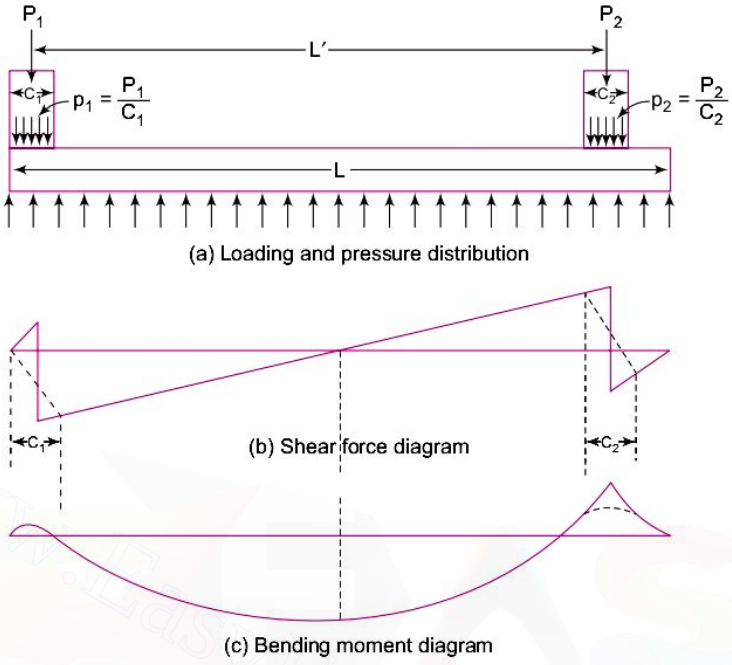

Fig 25.7 Rectangular combined footing

***(b) Trapezoidal Combined Footing*:** When the two column loads are unequal, with the outer column carrying heavier load, and when there is space limitation beyond the outer column, a trapezoidal combined footing is provided. The resulting pressure distribution is linear (and not rectangular) as shown in Fig. 25.8 (*b*). In order to determine widths *a* and *b*, the following relations are used:

Area $$A = \left(\frac{a+b}{2}\right)L = \frac{P_1 + P_2}{q_s}$$

or $$a + b = \frac{2}{L}\left(\frac{P_1 + P_2}{q_s}\right) \qquad ...(25.13)$$

where q_s = allowable soil pressure.

Also, taking moments about the centre of the column (1), we have

$$(P_1 + P_2)\,x' = P_2\,L_1 \text{ or } x' = \frac{P_2 L_1}{P_1 + P_2}$$

$$\therefore \quad \bar{x} = \frac{C_1}{2} + \frac{P_2 L_1}{P_1 + P_2} \qquad ...(25.14)$$

But for a trapezoidal section, $\bar{x}$ is given by

$$\bar{x} = \frac{L}{3}\cdot\left(\frac{2a+b}{a+b}\right) \qquad ...(25.15)$$

Equating Eqs. 25.11 and 25.12,

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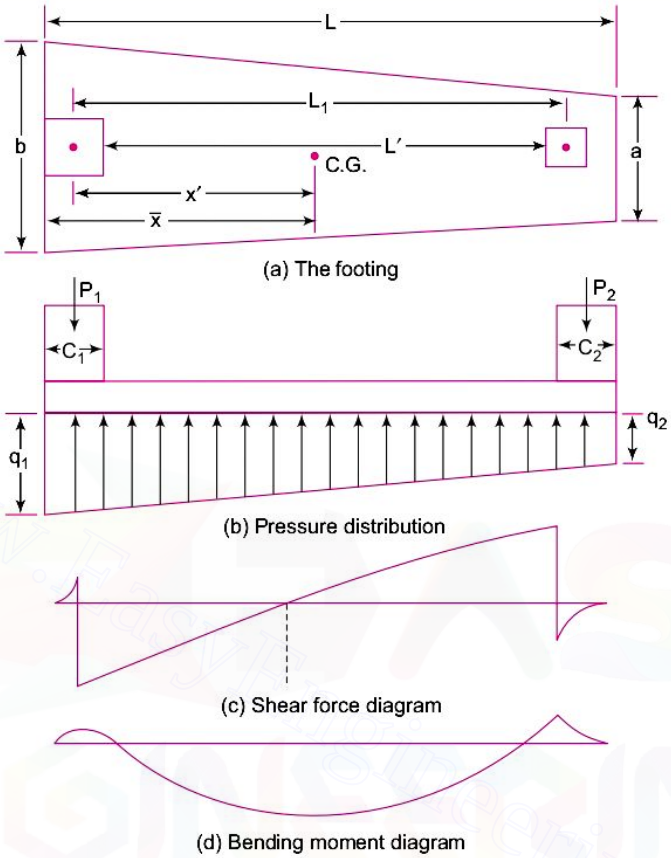

Fig. 25.8 Trapezoidal combined footing

$$\frac{L}{3}\left(\frac{2a+b}{a+b}\right) = \frac{C_1}{2} + \frac{P_2 L_1}{P_1 + P_2}$$

or
$$\frac{2a+b}{a+b} = \frac{3}{L}\left[\frac{C_1}{2} + \frac{P_2 L_1}{P_1 + P_2}\right] \quad ...(25.16)$$

The RHS of equation 25.16 is completely known. Similarly, the RHS of equation 25.13, is completely known. Hence simultaneous solution of equations 25.13 and 25.16 give the values of a and b.

Knowing a and b, the pressure intensities q_1 and q_2 are calculated as under:

$$q_1 = b \cdot q_s \quad ...(25.17)$$

and
$$q_2 = a \cdot q_s$$

The bending moment and shear force diagrams can now be drawn.

For a rectangular combined footing $\bar{x} = \frac{L}{2}$.

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If $a = 0$ we have a triangle with $\bar{x} = \frac{L}{3}$.

Hence a trapezoidal combined footing solution exists between the following limits.

$$\frac{L}{3} < \bar{x} < \frac{L}{2}.$$

(c) Strap or Cantilever Footing: A *strap footing* may be used where the distance between the columns is so great that the combined trapezoidal footing becomes quite narrow, with high bending moments, or where $\bar{x} < \frac{L}{3}$. A strap *footing* consists of spread footings of two columns, connected by a strap beam.

The strap beam does not remain in contact with soil, and thus does not transfer any pressure to the soil. The strap, assumed to be infinitely stiff, serves to transfer the column loads onto the soil, with equal and uniform soil pressure under both footings.

Figure 25.9 shows the assumed loading and reaction for a strap footing the reactions R_1 and R_2 act centrally over the length L_1 and L_2 giving a uniform pressure distribution.

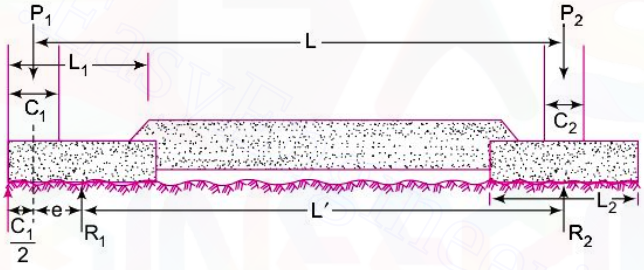

Fig. 25.9 Strap footing

Taking moment about P_2, $R_1 = \frac{P_1 L}{L'}$ ...(25.18)

$$R_1 + R_2 = P_1 + P_2 \quad ...(25.19)$$

Length $$L_1 = 2\left(e + \frac{C_1}{2}\right) \quad ...(25.20)$$

Width of footing $$B = \frac{R_1}{L_1 q_s} \quad ...(25.21)$$

The design procedure is essentially that of trial and error. It begins with a trial value of e, so that L_1 is known from Eq. 25.20 and B from Eq. 25.21.

The length $L_2 = \frac{P_2}{B \times q_s}$, Knowing L, L_1 and L_2, L' is known. Hence R_2 and R_1 are calculated from Equations 25.18 and 25.19. But R_1 is also equal to $q_s \times L_1 \times B$ and $R_2 = q_s \times L_2 \times B$. Hence the calculations are repeated with another value of e till values R_1 and R_2 calculated by both the procedures coincide. After knowing the final values of the various components, the shear and moment in the strap are determined, and the strap is designed to withstand the shear and moments. The footings are assumed to be subjected to uniform soil pressure and designed as simple spread footings.

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25.6 MAT OR RAFT FOOTING

A raft or mat is a combined footing that covers the entire area beneath a structure and supports all the walls and columns. When the allowable soil pressure is low, or the building loads are heavy, the use of spread footings would cover more than one-half of the area and it may prove more economical to use mat or raft foundation. They are also used where the soil mass contains compressible lenses or the soil is sufficiently erratic so that the differential settlement would be difficult to control. The mat or raft tends to bridge over the erratic deposits and eliminates the differential settlement. Raft foundation is also used to reduce settlement above highly compressible soils, by making the weight of structure and raft approximately equal to the weight of the soil excavated.

Ordinarily, rafts are designed as reinforced concrete flat slabs. If the C.G. of loads coincide with the centroid of the raft, the upward load is regarded as a uniform pressure equal to the downward load divided by the area of the raft. The weight of raft is not considered in the structural design because it is asssumed to be carried directly by the subsoil. Since this method does not take into acount moments and shear caused by differential settlements, it is customary to reinforce the raft more heavily than required according to analysis.

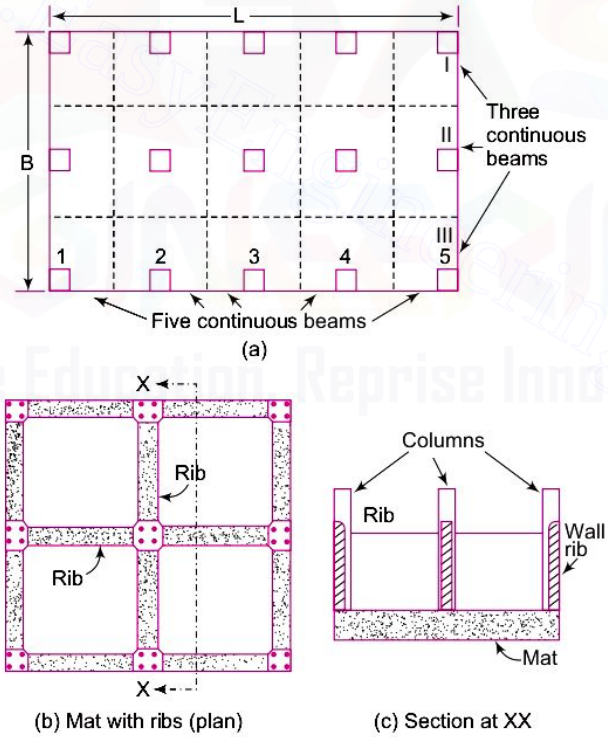

Fig. 25.10 Raft foundation

The raft is subdivided into a series of continuous beams (strips) centred on the appropriate column rows. With the raft now a series of beams, the shear and moment diagrams may be established, and the mat can be designed. A conservative design generally results from this, since the effect of adjacent strip is neglected. The perpendicular direction is similary analysed. The raft may also be designed as inverted

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flat slab, using heavy beams from column to column [Fig. 25.10(*b*), (*c*)]. The portion between the beam is then designed as a conventional one way or two way slab.

A raft on clay may fail in shear. The net ultimate bearing capacity may be determined from the following expression.

(*i*) ***Skempton's equation:*** $q_{nf} = 5\left[1 + 0.2\,\frac{D}{B}\right]\left[1 + 0.2\,\frac{B}{L}\right]c$ ...(25.22 *a*)

subject in the condition that $D/B \leq 2.5$ or the factor $5\left[1 + 0.2\,\frac{D}{B}\right]$ does not exceed 7.5.

(*ii*) ***Terzaghi's equation:*** $q_{nf} = 5.7\left[1 + \frac{0.3\,B}{L}\right]c$ ...(25.22 *b*)

A factor of safety of 3 should be used to arrive at the safe bearing capacity. The danger of shear failure on sands is very remote and the settlement criterion governs the design. A raft may undergo large settlements without causing harmful differential settlements. For this reason, almost double settlement of that permitted for footings is acceptable for raft. If a maximum settlement of 50 mm (2 in.) is permitted for a raft, the differential settlement is not likely to exceed 20 mm (0.75 in). Based on this consideration, the net pressure can be calculated from following equation for rafts having width greater than 6 m.

$$q_p = 2(N - 3)\,R_{w2}\ (t/m^2) \approx 20(N - 3)\,R_{w2}\ \text{kN/m}^2 \qquad ...(25.23)$$

The penetration resistance N values should be taken at 75 cm intervals for depths equal to width of the raft, below the base of the raft. The minimum average value of N for the holes should be used in Eq. 25.23. If N is less than 5, sand should be compacted by artificial means to raise N above 10, or else piles or piers should be used. N should be corrected for fine sand below water table.

25.7 IS CODE OF PRACTICE FOR DESIGN OF RAFT FOUNDATIONS

IS: 2950–1965 gives the code of practice for design and construction of raft foundations. The maximum differential settlement should not exceed 40 mm in foundation on clayey soils and 25 mm in foundations on sandy soils. The maximum settlement should generally be limited to the following values:

Raft foundation on clay: 65 to 100 mm.

Raft foundation on sand: 40 to 65 mm.

The behaviour of a raft foundation being complicated, a number of simplifying assumptions have to be made in the design. There are two approaches for design—*conventional method* and the *elastic method or the soil line method.*

1. Conventional method: The conventional method is based on the following two basic assumptions:

(*i*) The foundation is infinitely rigid and, therefore, the actual deflection of the raft does not influence the pressure distribution below the raft.

(*ii*) The soil pressure is assumed to be planer such that the centroid of the soil pressure coincides with the line of action of the resultant force of all the loads acting on the foundation.

In the conventional method, the allowable bearing pressure is found using the following formulae:

$$q_1 = 21.4\,N^2\,B\,Rw_1 + 64(100 + N^2)\,DRw_2 \qquad ...(25.24\ a)$$

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$$q_2 = 1950\ (N - 3)\ R_{w_2} \qquad ...(25.24\ b)$$

where q_1 and q_2 = allowable soil pressure under raft foundation in kg/m^2 (using a factor of safety of three).

R_{w1} and R_{w2} = reduction factors on account of subsoil water as explained in chapter 24. The smaller of the two values of q_1 and q_2 should be used for design. The effect of overburden pressure on the N value should also be considered as explained in chapter 24. In the case of saturated silts, the equivalent penetration resistance N_e, for the values of N greater than 15 should be taken for design as explained in Chapter 24.

The pressure distribution (q) under the raft should be determined by the following formula:

$$q = \frac{Q}{A} \pm Q \frac{e_y'}{I_x'} y \pm Q \frac{e_x'}{I_y'} x \qquad ...(25.25)$$

where, Q = total vertical load on the raft

x , y = co-ordinates of any given point on the raft with respect of the x and y axes passing through the centroid of the area of the raft.

I_x', I_y', e_x', e_y' = moments of inertia and eccentricities about the principal axes through the centroid of the section

A = total area of the raft

I_x', I_y', e_x' and e_y' can be calculated from the following equations

$$I_x' = I_x - \frac{I_{xy}^2}{I_y} \; ; \; I_y' = I_y - \frac{I_{xy}^2}{I_x}$$

$$e_x' = e_x - \frac{I_{xy}}{I_x} e_y \; ; \; e_y' = e_y - \frac{I_{xy}}{I_y} e_x$$

where I_x , I_y = moment of inertia of the area of the raft respectively about the x and y axes through the centroid

$I_{xy} = \int xy\, dA$ for the whole area about x and y axes through the centroid e_x, e_y = eccentricities in the x and y directions of the load from the centroid.

If one or more of the values of q are negative as calculated by the above formula, it indicates that the whole area of foundation is not subjected to pressure and only a part of the area is in contact with soil, and the above formula will still hold good provided, the appropriate values of I_x , I_y , e_x and e_y are used with respect to the area in contact with the soil instead of the whole area.

Thus, the pressure distribution is known from Eq. 25.25. The raft is analysed as a whole in each of the two perpendicular directions. The total shear force acting on any section cutting across the entire raft is equal to the arithmetic sum of all forces and reaction to the left or right of the section. Similarly the total bending moment acting on such section is equal to the sum of all moments to the left or right of this section. The structural design of the reinforced concrete raft can now be done, considering the foundation as an inverted floor. The actual reinforcement provided should be twice of that worked out theoretically in the case of design by conventional method.

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2. Elastic Method (Soil line method): A number of methods have been proposed based primarily on two approaches of simplified and truly elastic foundations.

(*i*) *Simplified elastic foundation.* The soil in this method is replaced by an infinite number of isolated springs.

(*ii*) *Truly elastic foundation.* The soil is assumed to be continuous elastic medium obeying Hooke's law.

But these methods are to be used in case the foundation is comparatively flexible and the loads tend to concentrate over small areas. The method assumes in addition to other factors that the modulus of subgrade reaction, determined from tests, is known. The modulus of subgrade reaction (K_s) as applicable to the case of load through a plate of size 30 cm × 30 cm or beam 30 cm wide on soil area given in Table 25.2 for cohesionless soil and Table 25.3 for cohesive soils.

Table 25.2 Modulus of subgrade reaction K_s for cohesionless soils

Soil Characteristics			K_s *(kg/cm²)*	
Relative Density		*Value of N*	*Dry or Moist State*	*Submerged State*
1.	Loose	< 10	1.5	0.9
2.	Medium	10 to < 30	4.7	2.9
3.	Dense	30 and over	18.0	10.8

Table 25.3 Modulus of subgrade reaction K_s for cohesive soils

Soil Characteristics			K_s *(kg/cm²)*
Consistency		*Unconfined compressive strength (kg/cm²)*	
1.	Stiff	1 to < 2	2.7
2.	Very Stiff	2 to < 4	5.4
3.	Hard	4 and over	10.8

The above values of K_s are corresponding to a square plate of size 30 cm × 30 cm. For finding the values of K corresponding to different *sizes* and *shapes,* the following relationships should be used:

(i) Effect of size:

$$K = K_s\left(\frac{B+30}{2B}\right) \text{ for cohesionless soil} \qquad \ldots(25.26)$$

$$K = x\,\frac{K'}{B'}, \text{ for cohesive soil} \qquad \ldots(25.27)$$

where K = modulus of subgrade reaction for footing of width B cm

K_s = modulus of subgrade reaction for a square plate 30 cm × 30 cm

K' = modulus of subgrade reaction for footing width x cm.

***(ii) Effect of shape*:** For cohesive soils $K_1 = K_2\,\frac{2}{3}\left(1+\frac{B}{2L}\right)$ …(25.28)

where K_1 = modulus of subgrade reaction for a rectangular footing having a length L and width B.

K_2 = modulus of subgrade reaction for square footing of side B.

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The effect of shape is negligible in the case of footing on cohesionless material.

The method utilises the principle of superposition. The column loads and bearing pressure distribution are divided into *three systems of forces* such that each system balances within itself so that the sum of forces is zero and the sum of moments of these forces about any point is zero.

First system. The first system of forces consists of (*i*) uniformly distributed bearing pressure *w* acting upwards along the entire area of the raft due to the total load, and (*ii*) the column support reactions acting vertically downwards obtained by treating the raft as an inverted floor carrying the uniformaly distributed load *w* with the reactions at the point of loading.

Second system. The second system of forces consists of the difference between the actual column loads and the column support reactions worked out in the first system above.

Third system. The earth pressure varies from point to point of a raft, and hence the resultant pressure at any point is considered to be $w + mw$, where w is constant and m varies throughout the raft. *This additional varying pressure mw forms the third system of forces* and therefore consists of a varying distributed load acting downwards in some parts of the raft and upwards in others. The total value of this third system when added to *w* gives the resultant soil reaction pressure at the point. The upward and downward pressure, of which the third system is composed of should be distributed throughout the raft in such a way as to form a balanced system.

The exact bearing pressure distribution may be cubic parabola, but sufficiently accurate results will be obtained by assuming a linear variation.

In Fig. 25.11 (*a*), the rectangle of ordinate *w* shows the uniform soil reaction pressure initially assumed for the first system while the hatched portion shows the varying pressure on account of deflection, the minimum pressure being $(1 - m)w$ and maximum pressure $(1 + m)w$. Fig. 25.11 (*b*) shows the pressure difference *mw* between uniform soil reaction pressure and varying pressure (third system).

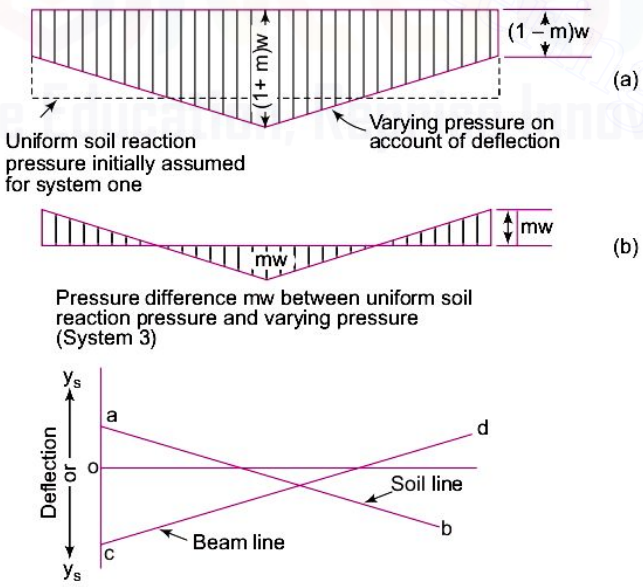

Fig. 25.11 Soil line method

Let y_s = difference between the compression of the soil at the end and the centre of the bearing area of the beam.

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y_b' = total, central deflection of the raft beam due to the action of the first two force systems.

y_b = total deflection at the centre of the raft beam, which is the sum of y_b' and the deflection caused by the additional varying soil pressure mw.

The relationship between y_s and m can be represented graphically by the *soil line* [*i.e.*, line *ab* in Fig. 22.11 (*c*)]. The relationship between y_b and m can be represented graphically by the *beam line* [line *cd* in Fig. 25.11 (*c*)]. The value of m compatible with soil and beam deflections may be taken as the value of m corresponding to the point of intersection of the soil line and the beam line. When once m is known, the bearing presssure distribution can be determined and the section of the raft can be designed. The actual reinforcement provided should be 1.5 times, the design reinforcement.

SOLVED EXAMPLES

Example 25.1. *A soft, normally consolidated clay layer is 6 m thick with a natural water content of 30 per cent. The clay has a saturated unit weight of 17.4 kN/m^3, a specific gravity of 2.67 and a liquid limit of 40 percent. The ground water level is at the surface of the clay. Determine the settlement of the foundation if the foundation load will subject the centre of the clay layer to a vertical stress increase of 8 kN/m^2.*

Solution. In Eq. 25.12, we have

σ_0 = effective initial overburden pressure, measured at the centre of the layer

$$= \gamma' Z = (17.4 - 9.81)\left(\frac{1}{2} \times 6\right) = 22.77 \text{ kN/m}^2$$

$\Delta\sigma$ = Vertical stress increment, due to foundation load = 8 kN/m^2

C_c = Compression index = $0.009\,(w_L - 10) = 0.009\,(40 - 10) = 0.27$

e_0 = initial voids ratio = $wG = 0.3 \times 2.67 = 0.801$

$\therefore$
$$S_c = C\,\frac{C_c}{1+e_0}\,H \log_{10}\frac{\sigma_0 + \Delta\sigma}{\sigma_0}$$

Taking $C = 1$, we get
$$S_c = \frac{0.2}{1+0.801} \times 6 \log_{10}\frac{22.77+8}{22.77} = 0.087 \text{ m} = \mathbf{87\ mm}$$

Example 25.2. *A square footing 1.2 m × 1.2 m rests at a depth of 1 m in a saturated clay layer 4 m deep. The clay is normally consolidated, having an unconfined compressive strength of 40 kN/m^2.*

The soil has a liquid limit of 30%, γ_{sat} = 17.8 kN/m^3, w = 28% and G = 2.68.

Determine the load which the footing can carry safely with a factor of saftey of 3 against shear. Also, determine the settlement if the footing is loaded with this safe load. Use Terzaghi's analysis for bearing capacity.

Solution. Since $\phi = 0$, $N_c = 5.7$; $N_q = 1$ and $N_\gamma = 0$. Also, $\bar{\sigma} = \gamma_{sat} D = 17.8 \times 1 = 17.8$

$$c = \frac{qu}{2} = \frac{40}{2} = 20 \text{ kN/m}^2$$

$$q_s = \frac{q_{nf}}{F} + \gamma_{sat} D = \frac{1}{F}\,[\,1.3\,cN_c + \bar{\sigma}\,(N_q - 1) + 0.04\,B\,.\,\gamma N_\gamma\,] + \gamma_{sat} D$$

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or $$q_s = \frac{1}{3}\,[\,1.3\times20\times5.7+17.8(1-1)+0]+17.8=49.4+17.8=67.2\,\text{kN/m}^2.$$

$$\therefore \quad Q_s = q_s \times B^2 = 67.2\times1.2\times1.2 = 96.77\ \text{kN}.$$

Thickness of clay layer = 4 m. Depth of centre of clay layer, below footing level = $\frac{4}{2} - 1 = 1$ m. Assuming load dispersion at 45° width of load spread = 1.2 + 2(1) = 3.2 m.

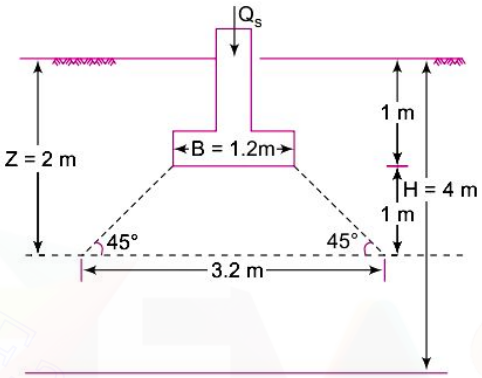

Fig. 25.12

$\therefore$ Vertical stress increment due to foundation load

$$= \Delta\sigma = \frac{96.77}{3.2\times3.2} = 9.45\text{kN/m}^2$$

Now, the consolidation settlement of footing is given by

$$S_c = C\,\frac{C_c}{1+e_0}\,H\log_{10}\frac{\sigma_0+\Delta\sigma}{\sigma_0}$$

Assume $$C = 1\ ;\ e_0 = w_{sat}\ G = 0.28\times2.68 = 0.75$$

$$C_c = 0.009\ (w_L - 10) = 0.009(30-10) = 0.18$$

Initial over burden pressure at the centre of clay layer

$$= \sigma_0 = \gamma_{sat}\,Z = 17.8\times2 = 35.6\ \text{kN/m}^2$$

$$\therefore \quad S_c = \frac{0.18}{1+0.75}\times4\log_{10}\frac{35.6+9.45}{35.6} = \mathbf{0.042\ m = 42\ mm}$$

Example 25.3. *A rectangular footing 2 m × 3 m carries a column load of 600 kN at a depth of 1 m. The footing rests on a c – ϕ soil strata 6 m thick, having Poisson 's ratio of 0.25 and Young's modulus of elasticity as 20000 kN/m^2. Calculate the immediate elastic settlement of the footing.*

Solution. $S_i = qB\left(\frac{1-\mu^2}{E_s}\right)I_w$

Here q = intensity of contact pressure = $\frac{600}{2\times3} = 100\ \text{kN/m}^2$

B = least lateral dimension of footing = 2 m

μ = Poisson's ratio = 0.25; $E_s = 20000\ \text{kN/m}^2$

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I_w = influence factor = 1.06 for rigid rectangular footing having $\frac{L}{B}$ =1.5

$$\therefore \quad S_i = 100 \times 2 \left[\frac{1-(0.25)^2}{20000}\right] \times 1.06 = 9.94 \times 10^{-3} \text{ m} = \mathbf{9.94 \text{ mm}}$$

Example 25.4. *A footing 3 m × 1.5 m in plan transmits a pressure of 160 kN/m² on a cohesive soil having $E = 8 \times 10^4$ kN/m² and $\mu = 0.48$. Determine the immediate settlement at the centre, assuming the footing to be (a) flexible, and (b) Rigid.*

Solution. $L/B = 3/1.5 = 2$. Hence from Table 15.1, we get $I_w = 1.52$ for flexible footing and 1.22 for rigid footing.

(*a*) $S_i = 160 \times 1.5 \left[\frac{1-0.48^2}{8 \times 10^4}\right] \times 1.52 = \mathbf{3.51 \text{ mm}}$

(*b*) $S_i = 160 \times 1.5 \left[\frac{1-0.48^2}{8 \times 10^4}\right] \times 1.22 = \mathbf{2.82 \text{ mm}}$

Example 25.5. *A building is to be supported on a R. C. C. raft foundation of dimensions 12 m × 18 m. The subsoil is clay which has an average unconfined compressive strength of 15 kN/m². The pressure on the soil due to the weight of the building and the loads it will carry is expected to be 130 kN/m² at the base of the raft. If the unit weight of the excavated soil is 18 kN/m³, determine the depth at which the bottom of the raft should be placed to provide a factor of safety, of 3 against shear failure.*

Solution. The soil is a $\varphi_u = 0$ soil having $c_u = q_u/2 = 15/2 = 7.5$ kN/m²

From Eq.25.22 *a*, $q_{nf} = 5\left[1 + 0.2\frac{D}{B}\right]\left[1 + 0.2\frac{B}{L}\right]c_u$, with $\frac{D}{B} \le 2.5$

$$\therefore \quad q_{nf} = 5\left[1 + 0.2\frac{D}{12}\right]\left[1 + 0.2 \times \frac{12}{18}\right] \times 7.5 = 42.5\left[1 + \frac{D}{60}\right]$$

$$\therefore \quad q_{ns} = \frac{q_{nf}}{F} = \frac{42.5}{3}\left[1 + \frac{D}{60}\right] = 14.17\left[1 + \frac{D}{60}\right] \qquad \ldots(1)$$

Since the building will have basement, the excavated soil is not going to be replaced. Hence it is possible to allow an increase of γD in the safe loading intensity, i.e. $q_s = q_{ns} + \gamma D$. Equating this value of q_s to the actual load intensity of 130 kN/m², we get

$$14.17\left[1 + \frac{D}{60}\right] + 18D = 130$$

or $\quad 0.245\,D + 18D = 130 - 14.17$

From which $\quad D = \mathbf{6.35 \text{ m}}$

25.8 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 25.6. *A raft foundation 10 m wide and 12 m long is to be constructed in a clayey soil having a shear strength of 12 kN/m². Unit weight of soil is 16 kN/m³. If the ground surface carries a surcharge of*

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20 kN/m^2 calculate the max. depth of foundation to ensure a factor of safety of 1.2 against base failure. N_c for clay is 5.7. **(Civil Services Exam. 1992)**

Solution. Bearing capacity of soil for rectangular footing in cohesive soil is given by

$$q_f = c\,Nc\left(1+0.3\frac{B}{L}\right)+\bar{\sigma} = cN_c\left(1+0.3\frac{B}{L}\right)+(\gamma D+q)$$

$$q_f = 12\times 5.7\left(1+0.3\times\frac{10}{12}\right)+16\,D+20 = 105.5+16\,D$$

Base failure will occur when q_f is equal to zero.

$\therefore$ $$D = -\frac{105.5}{16} = (-)\,6.59 \text{ (minus sign indicates that it is excavation)}$$

$\therefore$ Critical depth = 6.59 m

$\therefore$ Safe depth = 6.59/1.2 = **5.49 m**

Example 25.7. *A trapezoidal footing is to be produced to support two square columns of 30 cm and 50 cm sides respectively. Columns are 6 metres apart and the safe bearing capacity of the soil is 400 kN/m^2. The bigger column carries 5000 kN and the smaller 3000 kN. Design a suitable size of the footing so that it does not extend beyond the faces of the columns.* **(Civil Services Exam. 1993)**

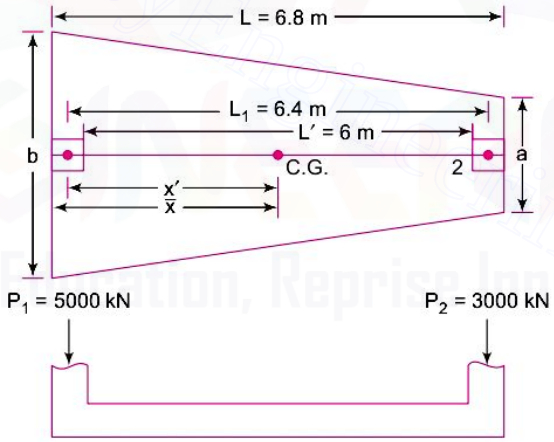

Fig. 25.13

Solution. Area $A = \dfrac{a+b}{2}L = \dfrac{P_1+P_2}{q_s}$

$\therefore$ $$a+b = \frac{2}{6.8}\times\frac{5000+3000}{400} = 5.882\text{ m} \qquad \ldots(1)$$

Also, $$\bar{x} = \frac{0.5}{2}\times x' = 0.25+\frac{3000\times 6.4}{5000+3000} = 2.65\text{ m}$$

But $$\bar{x} = \frac{L}{3}\left(\frac{2a+b}{a+b}\right)$$

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$$\therefore \quad \frac{2a+b}{a+b} = \frac{3}{6.8} \times 2.65 = 1.169$$

or $$0.831\, a = 0.169b = 0$$

or $$b = 4.917\, a \qquad ...(2)$$

Substituting this values in (1), we get $a = \dfrac{5.882}{5.917} = \mathbf{0.994\ m}$ and $\boldsymbol{b = 4.889\ m}$

Hence use trapezoidal footing of size $a = 1$ m; $b = 4.9$ m; and $L = 6.4$ m

Example 25.8. *A plan of a proposed heap is shown in Fig. 25.14 (a) The heap will stand on a thick deposit of soft clay with E-value of 15 MN/m^2. The uniform pressure on the soil may be assumed as 150 kN/m^2. Estimate the immediate settlement under the point marked X at the surface of the soil.*

(Civil Services Exam. 1999)

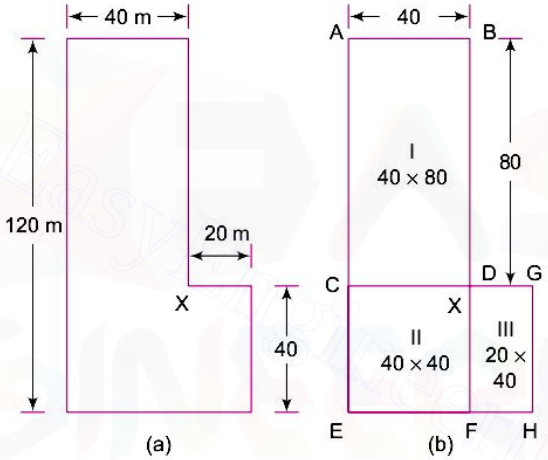

Fig. 25.14

Solution. Divide the area into three rectangles *ABDC, CDFE* and *DGHF* so that point *X* falls at a corner of each one of these.

Now, $$S_i = \frac{q \cdot B}{E_s}\left(1-\mu^2\right) I_w$$

$$= \frac{q}{E_s}\left(1-\mu^2\right)(B_1\, I_{w1} + B_2\, I_{w2} + B_3\, I_{w3})$$

where $I_{w1} = 0.77$ for $L/B = 80/40 = 2$ of first rectangle ; $B_1 = 40$ m

$I_{w2} = 0.56$ for $L/B = 40/40 = 1$ of second rectangle; $B_2 = 40$ m

and $I_{w3} = 0.77$ for $L/B = 40/20 = 2$ of third rectangle ; $B_3 = 20$ m

$q = 150$ kN/m^2; E = 15 MN/m^2 = 15000 kN/m^2 ;

$\mu = 0.5$ (assumed for soft clay).

$$\therefore \quad S_i = \frac{150}{15000}(1-0.5)^2\ [40 \times 0.77 + 40 \times 0.56 + 20 \times 0.77]$$

$$= \mathbf{0.5145\ m = 51.45\ cm}$$

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Example 25.9. *Proportion a strap footing for the following data:*

Allowable pressures: 150 kN/m^2 for DL + reduced LL

225 kN/m^2 for DL + LL.

Column loads

	Column A	Column B
DL	500 kN	600 kN
LL	450 kN	800 kN

Proportion the footing for uniform pressure under DL + reduced LL. Distance of C/C of column = 5.4 m; projection beyond column A not to exceed 0.5 m. **(Civil Services Exam. 2001)**

Solution. Refer Fig. 25.9

Column A: D.L. + Reduced L.L. = 500+ 0.5 × 450 = 725 kN = P_1

Column B: D.L. + Reduced L.L. = 600 + 0.5 × 800 = 1000 kN = P_2

Basic equations to be used are :

$$R_1 = \frac{P_1 L}{L'} \quad \ldots(1)$$

$$R_1 + R_2 = P_1 + P_2 \quad \ldots(2)$$

$$L_1 = 2\left(e + \frac{C_1}{2}\right) \quad \ldots(3)$$

and $$B = \frac{R_1}{L_1 q_s}$$

Here $\frac{C_1}{2} = 0.5$ m (given); $L = 5.4$ m; $q_s = 150$ kN/m^2

Step 1. Let $e = 0.5$ m.

Hence $L_1 = 2\ (0.5 + 0.5) = 2$ m

$$L' = L - e = 5.4 - 0.5 = 4.9 \text{ m};\ R_1 = \frac{P_1 L}{L'} = \frac{725 \times 5.4}{4.9} \approx 799 \text{kN}$$

$$B = \frac{R_1}{L_1 q_s} = \frac{799}{2 \times 150} = 2.66 \text{ m};\ L_2 = \frac{P_2}{B q_s} = \frac{1000}{2.66 \times 150} = 2.506 \text{m}$$

Thus, L_1 , L_2, L' and B are known corresponding to this value of e.

Step 2. $R_1 = \frac{P_1 L}{L'} = \frac{725 \times 5.4}{4.9} \approx 799 \text{kN}$

$$R_2 = (P_1 + P_2) - R_1 = 725 + 1000 - 799 = 926 \text{ kN}$$

$\therefore$ $$q_{s_1} = \frac{R_1}{L_1 B} = \frac{799}{2 \times 2.60} = 153.6 \text{ kN/m}^2 > 150 \text{ kN/m}^2$$

$$q_{s_2} = \frac{R_2}{L_2 B} = \frac{926}{2.50 \times 2.66} = 138.9 \text{ kN/m}^2 < 150 \text{ kN/m}^2$$

However, since $q_{s_1} > qs$, L_1 has to be increased. This can be done by increasing the value of e.

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Step 3. Let the next value of e be 0.6 m

$\therefore$ $L_1 = 2\,(0.5 + 0.6) = 2.2$ m ; $L' = 5.4 - 0.6 = 4.8$ m

$$R_1 = \frac{P_1 L}{L'} = \frac{725 \times 5.4}{4.8} = 815.6 \text{ kN};$$

$$B = \frac{R_1}{L_1 q_s} = \frac{815.6}{2.2 \times 150} = 2.47 \text{ m}$$

Keep $B = 2.5$ m

$$L_2 = \frac{P_2}{B q_s} = \frac{1000}{2.5 \times 150} = 2.67 \text{ m} \approx 2.7 \text{ m}$$

Step 4. $R_1 = \dfrac{P_1 L}{L'} = \dfrac{725 \times 5.4}{4.8} = 815.6$; $R_2 = 725 + 1000 - 815.6 = 909.4$ kN

$\therefore$ $$q_{s_1} = \frac{815.6}{2.2 \times 250} = 148.29 < 150$$

and $$q_{s_2} = \frac{909.4}{2.7 \times 2.5} = 134.7 < 150$$

Hence we have: $B = 2.5$ m; $L_1 = 2.2$ m ; $L_2 = 2.7$ m

Example 25.10. *A building has to be supported on R.C. raft foundation of dimensions 14 m × 21 m. The soil is clay, which has an average unconfined compressive strength of 15 kN/m^2. The pressure on the soil due to the weight of the building and the loads that it will carry, will be 140 kN/m^2 at the base of the raft. The building has provision for basement floors. At what depth should the bottom of the raft be placed to provide a factor of safety of 3 against shear failure? γ_{clay} = 19 kN/m^3. Use Skemptons' approach for bearing capacity calculations.* **(Engg. Services Exam. 1997)**

Solution. $q_u = 15$ kN/m^2; $c = q_u/2 = 7.5$ kN/m^2 ; $B = 14$ m; $L = 21$ m

$$q_{nf} = 5c\left(1 + 0.2\,\frac{B}{L}\right)\left(1 + 0.2\,\frac{D}{B}\right) = 5 \times 7.5\left(1 + 0.2 \times \frac{14}{21}\right)\left(1 + 0.2\,\frac{D}{14}\right)$$

or $q_{nf} = 42.5(1 + 0.01429\,D)$

$\therefore$ $$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{42.5}{3}\,(1 + 0.01429\,\text{D}) + 19\,\text{D} \qquad \ldots(1)$$

Actual load intensity $q_a = 140$ kN/m^2 … (2)

Equating the two, $\dfrac{42.5}{3}\left(1 + 0.01429\,D\right) + 19\,D = 140$

or $19.2024\,D = 125.83$ From which $D =$ **6.55 m**

Example 25.11. *A concrete strip footing rectangular in section is located at ground level and extends 1.2 m below the ground level. It carries uniformly distributed load of 15000 kg/m. The soil profile consists of homogeneous clay of 6 m thick overlying rock. The clay properties are as under:*

Saturated unit bulk weight = 1750 kg/m^3

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Shear strength (undrained) $= 8500\ kg/m^2$

Compressibility $= 1 \times 10^{-4}\ m^2/100\ kg$

Determine : (*i*) *width of footing for factor of safety* $= F$

(*ii*) *ultimate consolidation settlement for* $F = 2$

Assume bulk unit weight of concrete $= 2500\ kg/m^3$. *Neglect the spread of load beneath the footing and any side cohesion on the foundation.* **(Engg. Services Exam. 1999)**

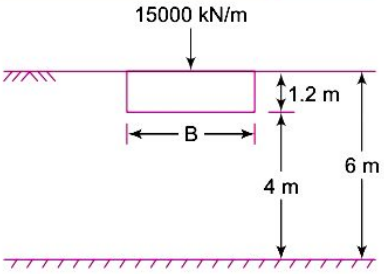

Fig. 25.15

Solution. $c = 8500\ kg/m^3$

$\gamma_{sat} = 1750\ kg/m^3$;

$\gamma' = 1750 - 1000 = 750\ kg/m^3$

(*i*) *Width of footing*: $q_{nf} = c\,N_c = 5.14\,c$ (IS Code) $= 5.14 \times 8500 = 43690$ kg

Assuming water table at the ground level and using a factor of safety F on q_{nf}, safe bearing capacity is

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma} = \frac{q_{nf}}{F} + \gamma' D = \frac{43690}{F} + 750 \times 1.2 = \left(\frac{43690}{F} + 900\right)$$

Here $$Q_s = \left(\frac{43690}{F} + 900\right) B \times 1 kg/m \qquad \ldots(1)$$

Submerged unit weight of concrete $= 2500 - 1000 = 1500\ kg/m^3$

$$q_a = 15000 \times 1 + B \times 1.2 \times 1 \times 1500 = (15000 + 1800\,B)\ kg/m \qquad \ldots(2)$$

Equating the two, we get

$$15000 + 1800\,B = \left(\frac{43690}{F} + 900\right) B$$

or $$B = \frac{15000}{\frac{43690}{F} - 900} = \frac{15000\,F}{43690 - 900\,F}$$ which is the required value of B.

Taking $F = 2$, we get $B = \dfrac{15000 \times 2}{43690 - 900 \times 2} \approx$ **0.72 m**

(*ii*) *Ultimate settlement for* $F = 2$: Consider level AA at mid-depth below footing base.

Given, coefficient of compressibility, $m_v = 1 \times 10^{-4}\ m^2/100\ kg = 1 \times 10^{-6}\ m^2/kg$

Load added by footing of width $B = 0.72$ m is

$$= 15000 + B \times D \times \gamma'_{conc} - B \times D \times \gamma'_{sat1}$$

$$= 15000 + 0.72 \times 1.2 \times 1500 - 0.72 \times 1.2 \times 750 = 15648\ kg/m$$

Neglecting load spread, $\Delta\sigma' = \dfrac{15648}{B} = \dfrac{15648}{0.72} = 21733\ kg/m^2$

$\therefore$ $$\rho_f = m_v\,\Delta\sigma'.\,H = 1 \times 10^{-6} \times 21733 \times 4.8 =$$ **0.104 m**

Example 25.12. *A square footing is required to carry a net load of 1200 kN. Determine the size of the footing if the depth of foundation is 2 m and tolerable settlement is 40 mm. The soil is sandy with* $N = 12$, *F.S.* $= 3$, *water lable is very deep. Use Teng's equation.* **(Engg. Services Exam. 2000)**

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Solution. $q_p = 55(N-3)\left[\frac{B+0.3}{2B}\right]^2 R_{w2} \cdot R_d$

Here $R_{w2} = 1$ (since water lable is very deep)

$$R_d = 1 + \frac{0.2D}{B} \le 1.2 = 1 + \frac{0.2 \times 2}{B} = 1 + \frac{0.4}{B}$$

$$\therefore \quad q_p = 55(12-3)\left[\frac{B+0.3}{2B}\right]^2 \left(1 + \frac{0.4}{B}\right) \qquad \ldots(1)$$

But $$q_a = \frac{1200}{B^2} \qquad \ldots(2)$$

$$\therefore \quad \frac{1200}{B^2} = 55(12-3)\left[\frac{B+0.3}{2B}\right]^2 \left(1 + \frac{0.4}{B}\right) \qquad \ldots(3)$$

Assuming B ≤ 2 m, $R_d = 1 + \frac{0.4}{B} = 1.2$

$$\therefore \quad \frac{1200}{B^2} = 55 \times 9 \times 1.2 \left[\frac{B+0.3}{2B}\right]^2$$

or $$B^2 \left[\frac{B+0.3}{2B}\right]^2 = 2.02$$

or $$(B+0.3)^2 = 8.08$$

From which $B = 2.54$ m

Since $B > 2$m, R_d will be less than 1.2

Hence from (3) $$\frac{1200}{B^2} = 123.75\left(\frac{B+0.3}{B}\right)^2 \left(1 + \frac{0.4}{B}\right)$$

or $$B^2 \left(\frac{B+0.3}{B}\right)^2 \left(1 + \frac{0.4}{B}\right) = 9.7$$

or $$(B + 0.3)^2 (B + 0.4) = 9.7\ B$$

Solving this by trial and error, we get $B \approx$ **2.6 m**

Example 25.13. *A building is to be supported on a reinforced concrete raft covering an area of 14 × 21 metres. The subsoil is clay with an unconfined compressive strength of 84 kN/m^2. The pressure on the soil due to weight of the building and loads it will carry, will be 120 kN/m^2, at the base of the raft. If the unit weight of excavated soil is 15 kN/m^3, at what depth should the bottom of the raft be placed to provide a factor of safety of 3 ?* **(Gate Exam. 1996)**

Solution. $q_u = 84$ kN/m^2; $c_u = q_u/2 = 42$ kN/m^2

Using Skemptons' equation for ultimate bearing capacity of raft

$$q_{nf} = 5c_u\left(1 + 0.2 \times \frac{B}{L}\right)\left(1 + 0.2\,\frac{D}{B}\right) \qquad \ldots(1)$$

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or $$q_{nf} = 5 \times 42\left(1 + 0.2 \times \frac{14}{21}\right)\left(1 + 0.2\,\frac{D}{14}\right) = 238\,(1 + 0.01429D) \quad \ldots(2)$$

$\therefore$ $$q_s = \frac{q_{nf}}{F} + \gamma D = \frac{238}{3}\,(1 + 0.01429\,D) + 15\,D = 79.33 + 16.134\,D \quad \ldots(3)$$

Actual load in density $q_a = 120\ \text{kN/m}^2$

Equating (2) and (3), we get $79.33 + 16.134\,D = 120$

From which $D = \mathbf{2.52}$ m

Check: Factor $5\left(1 + 0.2\,\frac{D}{B}\right) = 5\left(1 + 0.2 \times \frac{2.52}{14}\right) = 5.18 < 7.5$. Hence O.K.

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CHAPTER 26

Pile Foundations

26.1 TYPES OF PILES

The use of piles as a foundation can be traced since olden times. The art of driving piles was well-established in Roman times and the details of such foundations were recorded by Vitruvious in 59 AD Today, pile foundation is much more common than any other types of deep foundation. Modern pile driving started with the first steam pile drivers, invented by Nasmyth in 1845. Piles may be classified as follows:

(a) Classification based on the function. Based on the function or the use, piles may be classified as: (1) end bearing pile (2) friction pile (3) compaction pile (4) tension pile or uplift pile (5) anchor pile (6) fender pile and dolphins (7) batter pile (8) sheet pile.

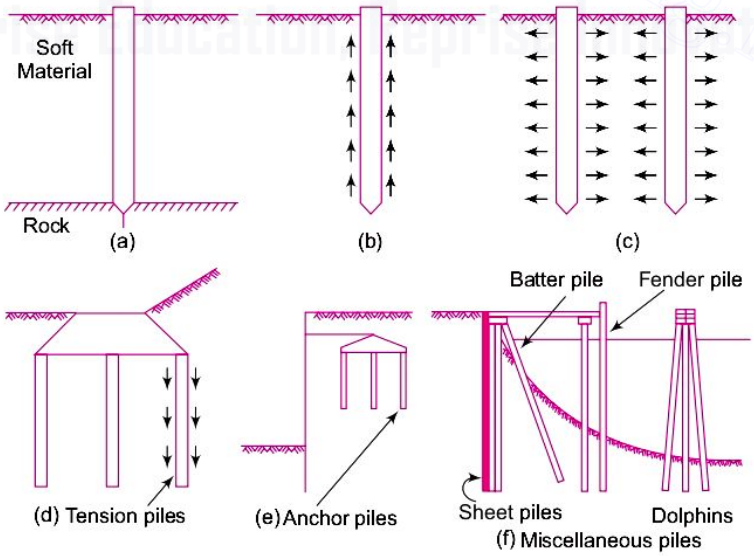

Fig. 26.1 Classification based on function

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End bearing piles: are used to transfer load through water or soft soil to a suitable bearing stratum.

Friction piles are used to transfer loads to a depth of a friction load carrying material by means of skin friction along the length of the piles.

Compaction piles are used to compact loose granular soil, thus increasing their bearing capacity. The compaction piles themselves do not carry any load. Hence they may be of weaker material-sometimes of sand only. The pile tube, driven to compact the soil, is gradually taken out and sand is filled in its place thus forming a 'sand pile'.

Tension or uplift piles anchor down the structures subjected to uplift due to hydrostatic pressure or due to overturning moment.

Anchor piles provide anchorage against horizontal pull from sheet piling or other pulling forces.

Fender piles and dolphins are used to protect water front structures against the impact from ships or other floating objects. *Sheet piles* are commonly used as bulkheads, or as impervious cutoff to reduce seepage and uplift under hydraulic structures. The *batter piles* are used to resist large horizontal or inclined forces.

(b) Classification based on materials and composition.

1. ***Concrete piles*:**
   - (*a*) Precast
   - (*b*) Cast-in-situ
     - (*i*) Driven piles: Cased or uncased
     - (*ii*) Bored piles: Pressure piles and under-reamed piles.
2. ***Timber piles*:**
3. ***Steel piles*:**
   - (*a*) H-piles (*b*) Pipe pile (*c*) Sheet pile.
4. ***Composite piles:***
   - (*a*) Concrete and timber (*b*) Concrete and steel.

The *precast concrete piles* are generally used for a maximum design load of about 80 tonnes, except for large pre-stressed piles. They must be reinforced to withstand handling stresses. They require space for casting and storage, more time to set and cure before installation and heavy equipment for handling and driving. They also incur large cost in cutting off extra length or adding more length. The *cast-in-situ* piles are generally **used** for a maximum design load of 75 tonnes, except for compacted, pedestal piles. They **are** installed by pre-excavation, thus eliminating vibration due to driving and the handling stresses. Cast-in-place piles may be classified into two classes: driven piles (cased or uncased) and bored piles (pressure piles, pedestal piles and under-reamed piles). A variety of cast-in-situ piles are in use, each bearing the name of the manufacturer. The common types are as follows:

(*i*) Raymond standard pile, (*ii*) Raymond step-taper pile,
(*iii*) Union metal pile of monotube, (*iv*) MacArthur compressed uncased pile,
(*v*) MacArthur cased pile, (*vi*) Franki standard pile,
(*vii*) Western button bottom pile etc.

Under-reamed pile is special type of bored pile having an increased diameter or bulb at some point in its length, to anchor the foundation in expansive soil subjected to alternate expansion and contraction.

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Concrete filled steel piles and steel *H-piles* rare used as long piles with high bearing capacity. They are rarely used unless they reach a stratum of exceptionally high supporting capacity, since their cost is very high. *Timber piles* have small bearing capacity, and are not permanent unless treated. They are prone to damage by hard driving, and should not be driven through hard stratum or boulders. *Composite piles* are suitable where the upper part of a pile is to project above the water table. Such a pile consists of a lower portion of untreated timber and an upper portion of concrete. In the other type of composite piles, steel piles are attached to the lower end of cast-in-place concrete pile. This type is used in case where the required length of piles is greater than that available for the cast-in-place type.

26.2 PILE DRIVING

Piles are commonly driven by means of a hammer supported by a crane or by a special device known as a *pile driver*. The hammer is guided between two parallel steel members known as leads. The leads are carried on a frame in such a way that they can be supported in a vertical position or an inclined position. Hammers are of the following types:

1. ***Drop hammer.*** If a hammer (ram or *monkey*) is raised by winch and allowed to fall by gravity on the top of a pile, it is called a drop hammer.
2. ***Single acting hammer.*** If the hammer is raised by steam, compressed air or internal combustion, but is allowed to fall by gravity alone, it is called a single acting hammer. The energy of such a hammer is equal to the weight of the ram times the height of fall.
3. ***Double acting hammer.*** The double acting hammer employs steam or air for lifting the ram and for accelerating the downward stroke. It operates with succession of rapid blows.
4. ***Diesel hammer.*** The diesel hammer is a small, light weight self-contained and self-acting type, using gasoline for fuel. The total driving energy is the sum of the impact of the ram plus the energy delivered by explosion.
5. ***Vibratory hammer.*** The driving unit vibrates at high frequency.

During pile driving, heads, helmets or caps are placed on the top of the pile to receive the blows of the hammer and to prevent damage to the head of the pile. A cushion, consisting of a pad of resilient materials, hard wood or rope, is placed between the drive cap and the top of pile to protect the pile head. Single acting hammers are advantageous when driving heavy piles in compact or hard soil, while double acting hammers are generally used to drive piles of light or moderate weight in soils of average resistance against driving.

Piles are ordinarily driven to a resistance measured by the number of blows required for the last 1 cm of penetration. Resistance of 3 to 5 blows per cm are commonly specified for concrete pile.

26.3 LOAD CARRYING CAPACITY OF PILES

The ultimate load carrying capacity, or ultimate bearing capacity, or the *ultimate bearing resistance* Q_{up} of a pile is defined as the maximum load which can be carried by a pile, and at which the pile continues to sink without further increase of load. The *allowable load* Q_a is the safe load which the pile can carry safely and is determined on the basis of: (*i*) ultimate bearing resistance divided by suitable factor of safety, (*ii*) the permissible settlement, and (*iii*) overall stability of the pile-foundation. The load carrying capacity of a pile can be determined by the following methods:

1. Dynamic formulae
2. Static formulae
3. Pile load tests
4. Penetration tests.

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26.4 DYNAMIC FORMULAE

When a pile hammer hits the pile, the total driving energy is equal to the weight of hammer times the height of drop or stroke. In addition to this, in the case of double acting hammers, some energy is also imparted by the steam pressure during the return stroke. The total downward energy is consumed by the work done in penetrating the pile and by certain losses. The various dynamic formulae are essentially based on this assumption. It is also assumed that soil resistance of dynamic penetration of pile is the same as to the penetration of pile under static or sustained loading. Following are some of the commonly used dynamic formulae:

1. Engineering News formula. The Engineering News formula was proposed by A.M. Wellington (1818) in the following general form:

$$Q_a = \frac{WH}{F(S+C)} \qquad ...(26.1)$$

where Q_a = allowable load; W = weight of hammer

H = height of fall; F = factor of safety = 6

S = final set (penetration) per blow, usually taken as average penetration, cm per blow for the last 5 blows of a drop hammer, or 20 blows of a steam hammer.

C = empirical constant

Denoting W in kg, H in cm, S in cm, and

C = 2.5 cm for drop hammer and

= 0.25 cm for single and double acting hammers, the above formula reduces to the following forms:

(*i*) *Drop hammers:* $$Q_a = \frac{WH}{6(S+2.5)} \qquad ...(26.2)$$

(*ii*) *Single acting steam hammers*: $$Q_a = \frac{WH}{6(S+0.25)} \qquad ...(26.3)$$

(*iii*) *Double acting steam hammers*: $$Q_a = \frac{(W+ap)H}{6(S+0.25)} \qquad ...(26.4)$$

where a = effective area of piston (cm^2)

p = mean effective steam pressure (kg/cm^2)

2. Hiley's formulas. Indian Standard IS: 2911 (Part I) 1964 gives the following formula based of original expression of Hiley:

$$Q_f = \frac{\eta_h \, WH \, \eta_b}{S + \frac{C}{2}} \qquad ...(26.5)$$

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where Q_f = ultimate load on pile

W = Weight of hammer, in kg

H = height of drop of hammer, in cm

S = penetration or set, in cm per blow

C = total elastic compression = $C_1 + C_2 + C_3$

C_1 , C_2 , C_3 = temporary elastic compression of dolly and packing, pile and soil respectively

η_h = efficiency of hammer, variable from 65 % for some double acting steam hammers to 100% for drop hammers released by trigger.

η_b = *efficiency of hammer blow (i.e.,* ratio of the energy after impact to striking energy of ram)

$$\eta_b = \frac{W + e^2 P}{W + P} \text{ (For the case of } W > eP) \qquad ...(26.6)$$

$$\eta_b = \frac{W + e^2 P}{W + P} - \left\{\frac{W - eP}{W + P}\right\}^2 \text{ (for the case when } W < e\,P) \qquad ...(26.7)$$

P = weight of pile, helmet, follower

e = coefficient of restitution (variable from zero for a timber pile with poor condition of head or for excess packing in the driving cap to 0.5 for double acting hammer driven steel piles without driving cap or reinforced concrete piles without helmet but with packing on top.)

Equations. 26.5, 26.6 and 26.7 are applicable for friction piles. For piles driven to refusal on rock (end bearing pile) a value of 0.5 P is substituted in above expressions. The product H is sometimes referred to as the *effective fall* of the hammer.

For double acting hammers, the *rated energy* in the same length unit as S and C is substituted for W H. The allowable load is obtained by using a factor of safety 2 or 2.5.

Comments about the use of Dynamic Formulae:

1. Dynamic formulae are best suited to coarse grained soils for which the shear strength is independent of rate of loading, because they allow no development of excess pore pressure around the pile during driving if saturated or dry.
2. The great objection to any of the pile driving formulae is the uncertainty about the relationship between the dynamic and static resistance of soil.
3. In case of submerged loose uniform fine sands, impact of driving may cause liquefaction of soil, thus showing much less resistance than that which will occur under a static load. Similarly, a very dense saturated fine sand may show an increased driving resistance which decreases.
4. For clays, the dynamic formulae are valueless because the skin friction developed in clay during driving is very much less (due to change in soil structure from flocculant to dispersed or due to thixotropic effect) than which occurs after a period of time. Also, the point resistance is much more at the time of driving because of pore pressure developed in clay, which reduces later on when the pore pressure dissipate.
5. Dynamic formulae give no indication about probable future settlement or temporary changes in soil structure.
6. The formulae do not take into account the reduced bearing capacity of pile when in a group.
7. Law of impact used for determining energy loss is not strictly valid for piles subjected to restraining influence of the surrounding soil.

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8. In Engineering News formula, the weight of the pile and hence its inertia effect is neglected.
9. Energy losses due to vibrations, heat and damage to dolly or packing are not accounted for.
10. In Hiley's formula, a number of constants are involved, which are difficult to determine.

26.5 STATIC FORMULAE

The static formulae are based on assumption that the ultimate bearing capacity Q_{up} of a pile is the sum of the total ultimate skin friction R_f and total ultimate point or end bearing resistance R_p:

$$Q_{up} = R_f + R_p \qquad ...(26.8)$$

or
$$Q_{up} = A_S \cdot r_f + A_p \cdot r_p \qquad ...(26.9)$$

where A_s = surface area of pile upon which the skin friction acts

A_p = area of cross-section of pile on which bearing resistance acts.

For tapered piles, A may be taken as the cross-sectional area at the lower one-third of the embedded length.

r_f = average skin friction

r_p = unit point or toe resistance

A factor of safety of 2.5 or 3 may be adopted for finding the allowable load.

(i) For cohesive soil. For the pile in cohesive soil, point bearing is generally neglected for individual pile action, since it is negligible as compared to frictional resistance. The unit skin friction may be taken equal to the shear strength of the soil multiplied by a reduction factor α (or m):

Thus r_f = average skin friction along the length of the pile

$$= a\bar{c}\,(or\ m\bar{c}) \qquad ...(26.10)$$

and
$$r_p = c_p N_c = 9\, c_p \qquad ...(26.11)$$

$\therefore$
$$Q_{up} = m\bar{c}\, A_s + 9\, c_p A_p \qquad ...(26.12)$$

Where m (or α) = *adhesion coefficient* (or *reduction factor)* whose value depends upon type of clay and its value can be determined by pile load test. For depth of pile less than 20 dia, its value is generally found to be 0.4. For deeper piles, its value range from 0.4 (at c = 170 kN/m^2) to 1 (for c = 90 kN/m^2). Table 26.1 gives the values of α, for different consistencies of clay.

$\bar{c}$ = Average undrained cohesion along the length of pile

c_p = Average undrained cohesion of soil at pile tip. In absence of elaborate data, both $\bar{c}$ and c_p may be taken equal to $q_u/2$

Table 26.1 Value of factor α

Consistency		*N Value*	*Value of* α	
			Bored piles	*Driven cast-in-situ piles*
1.	Soft to very soft	< 4	0.7	1.0
2.	Medium	4 – 8	0.5	0.7
3.	Stiff	8 – 15	0.4	0.4
4.	Stiff to hard	>15	0.3	0.3

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The allowable load for the pile is given by dividing Q_{up} by a suitable factor of safety F.

$$Q_a - Q_{up} = F \qquad ...(26.13a)$$

The IS Code recommends that for working out safe load, a minimum factor of safety of 2.5 should be used. If however, two different values of safety factors are adopted for shaft resistance (or skin friction) and base resistance, we have

$$Q_a = \frac{m\bar{c}\,A_s}{F_1} + \frac{9c_p A_p}{F_2} \qquad ...(26.13)$$

(ii) For non-cohesive soil

$$r_f = K \tan \phi\,(\gamma Z + q) \qquad ...(26.14)$$

$$r_p = 0.3\,\gamma\,BN_{\gamma q} \text{ (for circular piles)} \qquad ...(26.15)$$

or

$$r_p = \frac{\gamma B}{2} N_{\gamma q} \text{ (for rectangular or square piles)} \qquad ...(26.16)$$

where K = coefficient of lateral earth pressure

γ = density of soil

q = surcharge on the ground

Z = depth of centre of gravity of the pile below ground surface

B = least lateral dimension of rectangular or square pile (or diameter of circular pile)

ϕ = angle of internal friction

$N_{\gamma q}$ = Meyerhof's non-dimensional factor (resultant bearing capacity factor).

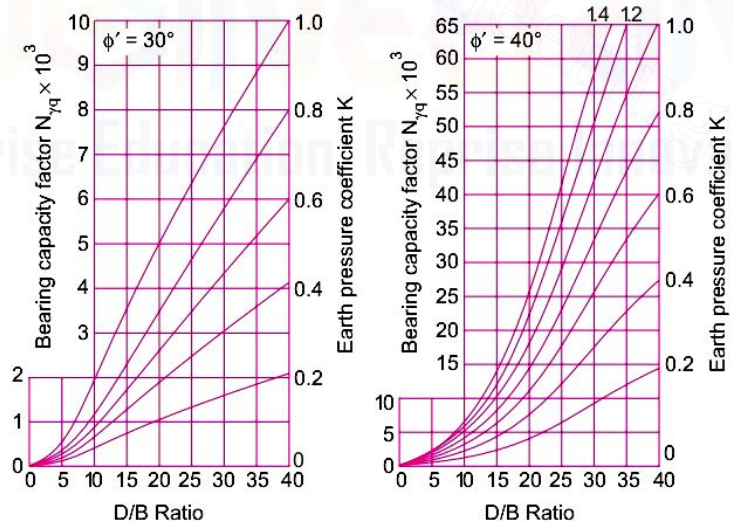

Fig. 26.2 Meyerhof's resultant bearing capacity factor $N_{\gamma q}$

Figure 26.2 gives the curves for $N_{\gamma q}$ for various values of K and D/B ratio (D = depth of pile), for $\phi = 30°$ and $\phi = 40°$. The use of these curves necessitates the determination of the coefficient of earth pressure K which is a very variable coefficient.

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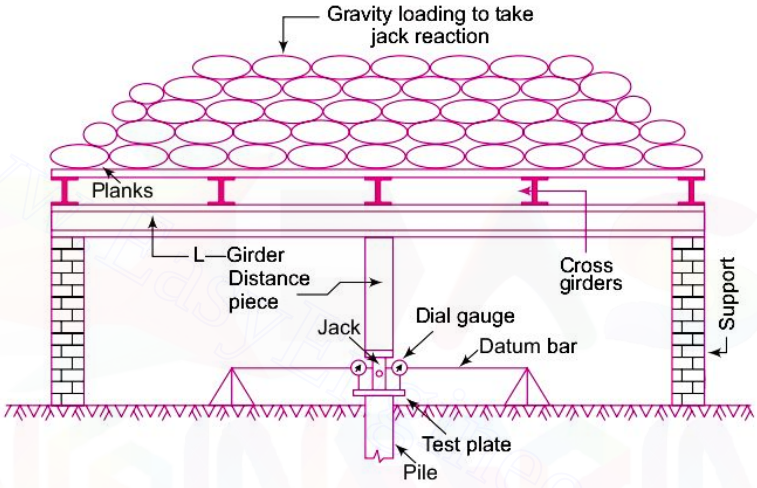
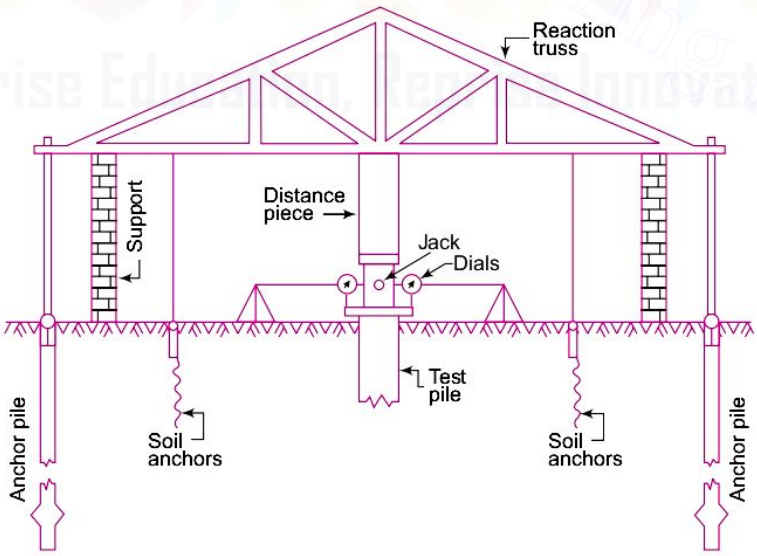
26.6 PILE LOAD TESTS

The pile load test can be performed either on a *working pile* which forms the foundation of the structure or on a *test pile.* The test load is applied with the help of calibrated jack placed over a rigid circular or square plate which in turn is placed on the head of the pile projecting above ground level. The *reaction* of the jack is borne by a *truss or platform* which may have gravity loading (in the form of sand bags etc.) or alternatively, the truss can be anchored to the ground with the help of anchor piles. In the later case, under-reamed piles or *soil anchors* may be used for anchoring the truss. Both the arrangements are shown in Fig. 26.3.



(a) Jack loading : Reaction by loaded platfrom



(b) Jack loading : Reaction by anchors

Fig. 26.3 Arrangements for pile load test

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The load is applied in equal increments of about one-fifth of the estimated allowable load. The settlements are recorded with the help of *three* dial gauges of sensitivity 0.02 mm, symmetrically arranged over the test plate, and fixed to an independent datum bar. A remote controlled pumping unit may be used for the hydraulic jack. Each load increment is kept for sufficient time till the rate of settlement becomes less than 0.02 mm per hour. The test piles are loaded until ultimate load is reached. Ordinarily, the test load is increased to a value 2.5 times the estimated allowable load or to a load which causes a settlement equal to one-tenth of the pile diameter, whichever occures earlier. The results are plotted in the form of load-settlement curve. The ultimate load is clearly indicated by load settlement curve approaching vertical. If the ulimate load cannot be obtained from the load settlement curve, the allowable load is taken as follows:

(*i*) One-half to one-third of the final load which causes settlement equal to 10% of pile diameter,
(*ii*) Two-thirds of the final load which cause a total settlement of 12 mm, or
(*iii*) Two-thirds of final load which causes a net settlement (residual settlement after the removal of load) of 6 mm.

Cyclic load test. The cyclic load test is particularly useful in separating the load carried by the pile into the skin friction and point bearing resistance. Each load increment is kept on the pile for sufficient time till the settlement decreases to a value less than 0.02 mm per hour. The load is then completely removed and the elastic rebound of the pile top is measured by means of dial gauges. The next load (increases load) is then applied and the process repeated. The cycle of loading and unloading with measurements of settlement and recovery is continued till the final load which causes a marked progressive settlement of the pile is reached. Figure 26.4 (*a*) shows the result plotted between load versus settlement.

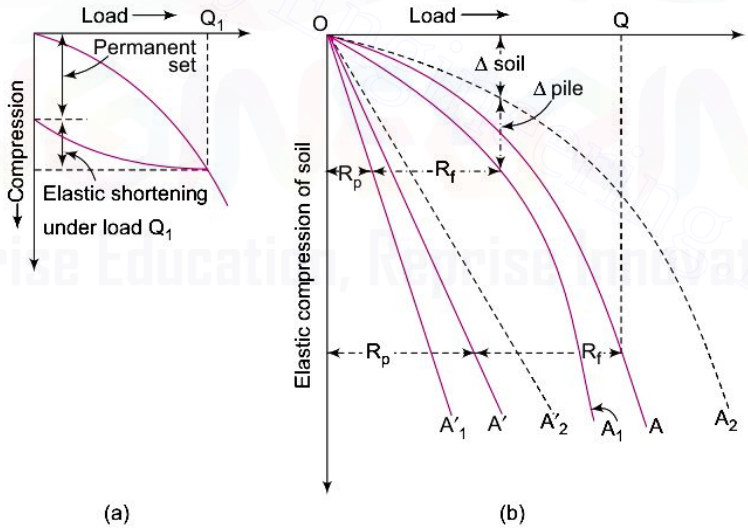

Fig. 26.4 Separation of skin friction and point resistance

The elastic compression ΔL of the pile corresponding to any load Q ($= R_f + R_p$) can be calculated from the following expression based on Hooke's law,

$$\Delta L = \frac{\left(Q - R_f / 2\right) L}{AE} \qquad ...(26.17)$$

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The separation of Q at any stage of loading into R_p and R_f is based on an experimental finding of A.F. Van Weele (1957) that the load on the pile toe (*i.e.*, R_p) increases linearly with the elastic compression of soil, and that straight line showing the relationship between point resistance and elastic compression of soil is parallel to the straight line portion of the curve drawn between the load on the pile and elastic compression of soil. The elastic compression of the soil is equal to the total elastic recovery of pile top minus the elastic compression ΔL of the pile. The procedure is described in the following steps:

Steps:

1. Since R_f is not known to start with, it is assumed that the elastic compression of the pile is zero, and hence the elastic compression of soil is equal to the total elastic recovery of pile top. A curve OA_1 [Fig. 26.4 (*b*)] is then drawn between load Q on pile top as abscissa and the elastic compression of soil as ordinate.
2. Through origin O, a line OA_1' is drawn parallel to the straight portion of the curve OA. This line is supposed to divide the load into point bearing R_p and skin friction R_f.
3. For various loads Q_1, Q_2, Q_3, etc., the skin friction R_{f1}, R_{f2}, R_{f3} etc., are determined, as marked in Fig. 26.4 (*b*).
4. Corresponding to each value of R_f, the elastic compression of pile is determined from Eq. 26.17. The elastic compression of soil is claculated from the relation.

$$\Delta_{soil} = \Delta - \Delta_{pile} \qquad ...(26.18)$$

where Δ = total elastic recovery of the pile top

(taken from Fig. 26.3 (*a*) from each load).

5. Knowing Δ_{soil} for each load Q_1, Q_2, Q_3, etc. a curve is drawn between Q and Δ_{soil}
6. Through the origin O, line OA_2' is drawn parallel to the straight line portion of curve OA_2.
7. Step 3, 4, 5 and 6 are repeated to get the final curve and straight line OA' parallel to the straight line portion of curve OA. The third trial of curves gives sufficiently accurate results. From these two, any load Q can be divided to skin friction and point resistance. The values of skin friction and point resistance corresponding to a load causing a total settlement of one-tenth of the pile diameter are divided by factors of safety of 2 and 2.5 respectively and added together to give the allowable load for the pile.

26.7 PENETRATION TESTS

The results of Dutch cone penetration test can be applied with sufficient accuracy to determine to ultimate bearing capacity of piles in cohesionless soils. The following relationships may be adopted.

$$r_p = q_c \qquad ...(26.19)$$

and $$r_f = 2f_c \qquad ...(26.19\ a)$$

where q_c = unit resistance of Dutch cone penetrometer

f_c = static skin friction on the shaft of the penetrometer.

26.8 GROUP ACTION IN PILES

1. *Efficiency of Pile Group:* When several closely spaced piles are grouped together, it is reasonable to expect that the soil pressure developed in the soil as resistance will overlap. The bearing capacity of

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a pile group may or may not be equal to the sum of the bearing capacity of individual piles constituting a group. Theory and tests have shown that the total bearing value Q_{ug} of a group of *friction piles,* particularly in clay, may be less then the product of the friction bearing value Q_{up} of an individual pile multiplied by the number of piles n in a group. However, there is no reduction due to grouping occurs in end bearing piles. For combined end bearing and friction piles, only the load carrying capacity of the frictional portion is reduced. A method of estimating the bearing capacity of a group of friction piles is to multiply the quantity nQ_{up} by a reduction factor called the *efficiency of pile group.*

$$Q_{ug} = n\, Q_{up} \cdot \eta_g \qquad ...(26.20)$$

where Q_{ug} = load carried by group of friction piles

Q_{up} = load carried by each friction pile

n = number of piles;

η_g = efficiency of pile group.

The efficiency of pile group depends upon the following factors: Characteristics of pile (*i.e.*, length, diameter, material etc.), spacing of pile, total number of piles in a row and number of rows etc. A number of formulae are available for determining the efficiency of pile group. Some of these are given below:

Converse Labarre formulae: $$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right] \qquad ...(26.21)$$

where m = number of rows; n = number of piles in a row

$\theta = \tan^{-1}\dfrac{d}{s}$ (degress); d = diameter of pile ; s = spacing of pile.

Seiler-Keeney formulae:

$$\eta_g = \left[1 - 0.479\left(\frac{s}{s^2 - 0.093}\right)\left(\frac{m+n-2}{m+n-1}\right)\right] + \frac{0.3}{m+n} \qquad ...(26.22)$$

where s = average spacing, centre to centre, in metres.

Feld's-Rule: According to this rule, the value of each pile is reduced by one-sixteenth on account of the effect of the nearest pile in each diagonal or straight row of which the pile in question is a member. This is illustrated in Fig. 26.5.

2. *Design of Pile Groups:* The bearing capacity of single pile in clay is mainly due to friction, and the point bearing resistance may be negligible. In a pile group, the piles are connected at its top by a pile cap which is rigid. Hence the failure of a pile group is likely to occur at a load which may be smaller than the ultimate load carried by each pile multiplied by the number of piles in the group. The area of the pile group, along failure surface, is approximately equal to the perimeter P of the pile group multiplied by the length L of the pile. The ultimate load will then be equal to:

$$Q_{ug} = PL\, r_f + A.r_p \qquad ...(26.23)$$

where A = Cross-sectional area of pile group, at base = $B \times B = B^2$ (Fig. 26.6)

P = Perimeter of pile group = 4 B (Fig. 26.6)

r_f = Shear strength of soil = $\bar{c} = \tau = q_u/2$ (For clayey soils)

$\therefore$ $$Q_{ug} = 4\, BL\, r_f + B^2 r_p = 4\, BL\, \bar{c} + B^2\, (9\, c_p) \qquad ...(26.24)$$

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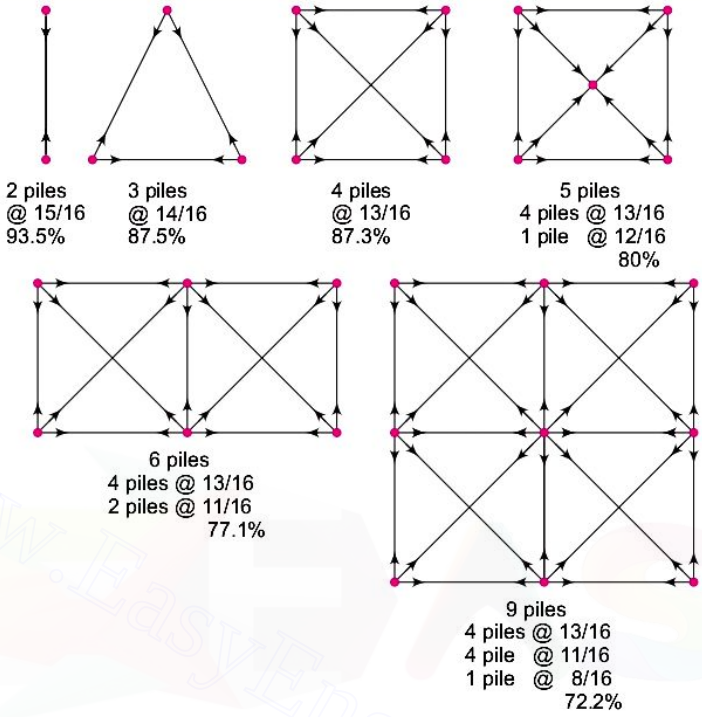

Fig. 26.5 Efficiency of pile groups by feld's rule

In absence of any other specific data, both $\bar{c}$ and c_p may be taken equal to $q_u/2$.

If, However, piles of the group are so spaced that they act individually, rather than acting in the group, the total load capacity of n piles is given by

$$Q_{un} = nQ_{up},$$

where Q_{up} = load of individual pile (Eq. 26.12)

The ultimate load (Q_u) of the pile groups will be then equal to lesser of Q_{ug} and Q_{un}, determined above, and the permissible load will be equal to Q_u/F. See Example 26.2.

The pile spacing which utilises the full capacity of each pile can be found most easily by trial. A spacing of three times the diameter of piles is commonly selected as trial spacing and checked against the criterion that the resistance Q_g obtained by Eq. 26.23 is at least equal to the capacity of single pile multiplied by the number of piles in a group. See Example 26.3.

The safe load on a group of piles driven through compressible layers to firm material (*i.e.*, point bearing piles) is equal to the number of piles in the group multiplied by the safe load per pile, and no reduction need be made on account of the close spacing of piles. However, the frictional resistance in such a case is completely neglected.

3. *Settlement of Pile Group in Clay:* As a rough approximation, the settlement of a group of friction piles can be computed on the assumption that the clay contained between the top of the piles and their lower third point is *incompressible* and that the load is applied to the soil at this lower third point of the

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pile (Fig, 26.6). The presence of pile below this level is ignored. The load is assumed to be uniformly distributed at this level, and is assumed to spread at an angle 30° with the vertical. The soil below this level is divided into a number of layers, and the σ_p and $\Delta\sigma$ are calculated at the middle of each layer. The settlement of each layer is calculated from the expression:

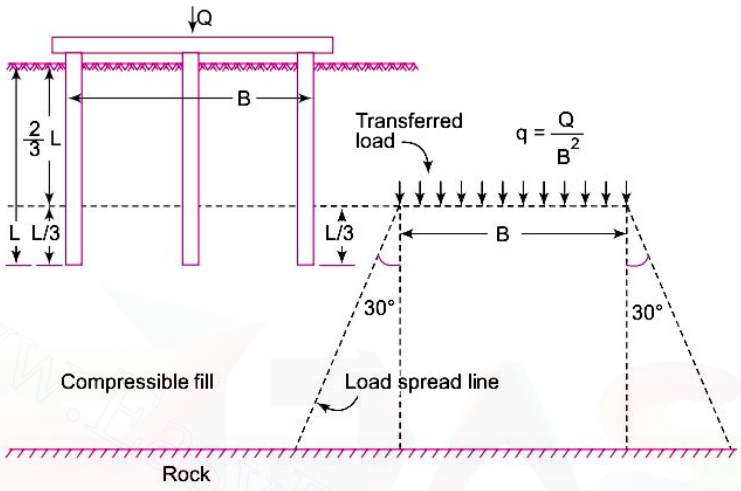

Fig. 26.6 Settlement analysis of pile group in clay

$$\rho = \frac{H \times C_c}{1 + e_0} \log_{10} \frac{\sigma_0 + \Delta\sigma}{\sigma_0} \qquad ...(26.25)$$

where H = thickness of the layer

e_0 = initial voids ratio

σ_0 = initial stress at centre of the layer = γZ

Z = depth of the centre of the layer below ground

$\Delta\sigma$ = additional stress due to piles

= Total pile load Q divided by the area of spread at the centre of layer

The total settlement = $\rho_1 + \rho_2 + \rho_3 + ... + \rho_n$. (See Example 26.6 for illustration.)

26.9 NEGATIVE SKIN FRICTION

Negative skin friction is a downward drag acting on a pile due to the downward movement of the surrounding *compressible soil* relative to the pile. This happens when the surrounding compressible soil has been recently filled or formed. As the soil consolidates, the earth fill moves downward, developing friction forces on the perimeter of the pile which tend to carry the pile farther into the ground. The negative skin friction may also be developed by the lowering of the ground water, the increase in effective stress causing consolidation of the soil, with the resultant settlement and friction force being developed on the pile.

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For individual piles, the magnitude of negative friction Q_{nf} may be taken as follows:

(*i*) for cohesive *soils*. $Q_{nf} = p \,.\, c \,.\, L_f$ ...(26.26)

(*ii*) for granular *soils*. $Q_{nf} = \frac{1}{2} L f^2 p \gamma K \,.\, f$ ...(26.27)

where

p = perimeter of the pile

L_f = depth of fill or soil which is moving vertically

c = cohesion of soil in zone of L_f

K = earth pressure coefficient ($K_a < K < K_p$)

γ = unit weight of soil

f = coefficient of friction $\approx \tan \phi$

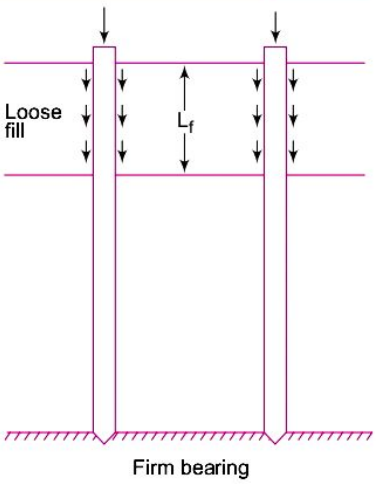

Fig. 26.7 Negative skin friction

SOLVED EXAMPLES

Example 26.1 *A wooden pile is being driven with a drop hammer weighing 20 kN and having a free fall of 1.0 m. The penetration in the last blow is 5 mm. Determine the load carrying capacity of the pile according to the Engineering News Formula.*

Solution. $Q_a = \dfrac{WH}{6(S + C)}$

where

$W = 20$ kN

$H = 1 \times 100 = 100$ cm

$S = 0.5$ cm ; $C = 2.5$ cm

$$Q_a = \frac{20 \times 100}{6(0.5 + 2.5)} = \mathbf{111.1\ kN}$$

Example 26.2 *In a 16 pile group, the pile diameter is 45 cm and centre to centre spacing of the square group is 1.5 m. If $c = 50$ kN/m^2, determine whether the failure would occur with the pile acting individually, or as a group? Neglect bearing at the tip of the pile. All piles are 10 m long. Take $m = 0.7$ for shear mobilisation around each pile*

Solution. $n = 16$; $d = 45$ cm ; $L = 10$ m

Width of group = $B = (150 \times 3) + 45 = 495$ cm $= 4.95$ m

(i) For the group:

$$Q_{ug} = c \times \text{perimeter} \times \text{length}$$

$$= c \times 4B \times L = 50 \times 4 \times 4.95 \times 10 = \mathbf{9900\ kN} \quad ...(i)$$

(ii) For the piles acting individually:

$$Q_{ug} = n\, Q_{up} = n\, \{mc\, A_p\}$$

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$$A_p = \pi\, dL = \pi \times 0.45 \times 10$$

$$\therefore \quad Q_{ug} = 16 \times 0.7 \times 50 \times \pi \times 0.45 \times 10 = \mathbf{7917\ kN} \qquad ...(ii)$$

which is less than the load carried by the group action. Hence the foundation will fail by the piles acting individually, and the load at failure would be 7917 kN.

Example 26.3 *A n-pile group has to be proportioned in a uniform pattern in soft clay with equal spacing in all directions. Assuming any value of c, determine the optimum value of spacing of piles in the group. Take n = 25 and m = 0.7. Neglect the end bearing effect and assume that each pile is circular in section.*

Solution. The optimum spacing of individual piles is based on the premise that the efficiency of the pile group is unity. That is, the total load carried by the piles, acting individually is equal to the load carried by group action.

Let the centre to centre spacing of each pile = s.

Diameter of each pile = d

$$n = 25$$

$\therefore$ Width for each pile group $4\,s + d$; Length of pile = L

$\therefore$ Load carried by group action is

$$Q_{ug1} = c\{4(4S + d)\, L\} = 4cL(4s + d) \qquad ...(i)$$

Load carried by pile acting individually is

$$Q_{ug2} = n(mc)\,(\pi dL) = 25 \times 0.7\, c\pi dL = 55dL\,.\,c \qquad ...(ii)$$

Equating (i) and (ii) for the optimum spacing

$$4cL\,(4s + d) = 55dL\,.\,c$$

or $$16s + 4d = \mathbf{55}\,d$$

$$\therefore \quad s = \frac{51}{16}d = 3.19\,d.$$

Hence the spacing should be equal to 3.19 times diameter of the pile.

Example 26.4 *A reinforced concrete pile weighing 30 kN (inclusive of helmet and dolly) is driven by a drop hammer weighing 40 kN and having an effective fall of 0.8 m. The average set per blow is 1.4 cm. The total temporary elastic compression is 1.8 cm. Assuming the co-efficient of restitution as 0.25 and a factor of safety of 2, determine the ultimate bearing capacity and the allowable load for the pile.*

Solution.

Given $P = 30$ kN ; W = 40 kN

η_h $H = 0.8$ m = 80 cm ; $S = 1.4$ cm ; $C = 1.8$ cm

$e = 0.25$; $F = 2.0$

Since $W > eP$,

$$\eta_b = \frac{W + e^2 P}{W + P} = \frac{40 + 30(0.25)^2}{40 + 30} = 0.597$$

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$$\therefore \quad Q_f = \frac{(\eta_h H) W \eta_b}{S + \frac{C}{2}} = \frac{80 \times 40 \times 0.597}{1.4 + \frac{1.8}{2}} = 830 \text{ kN}$$

$$\therefore \quad Q_a = \frac{Q_f}{F} = \frac{830}{2} = \mathbf{415\ kN}$$

Example 26.5 *Design of friction pile group to carry a load of 3000 kN including the weight of the pile cap at a site where the soil is uniform clay to a depth of 20 m, underlain by rock. Average unconfined compressive strength of the clay is 70 kN/m^2. The clay may be assumed to be of normal sensitivity and normally loaded, with liquid limit 60%. A factor of safety of 3 is required against shear failure.*

Solution.

$$c = \frac{q_u}{2} = \frac{70}{2} = 35 \text{kN/m}^2$$

Permissible $\quad c = \frac{35}{3}$ kN/m^2

Let the length of pile $\quad = 10$ m

Diameter of pile $\quad = 0.5$ m

Spacing of pile $\quad = 3d = 150$ cm (say)

Let the number of piles $\quad = n$

Considering the piles to act individually, the load at failure is given by

$$Q_{ug} = n\, c\, \pi\, dL$$

$$\therefore \quad 3000 = n \times \frac{35}{3} \times \pi \times 0.5 \times 10$$

$$\therefore \quad n = \frac{3 \times 3000}{35 \times \pi \times 0.5 \times 10} = \mathbf{16.37}$$

For a square arrangement, keep $n = 16$

The modified length L will then have to be increased by the ratio $\frac{16.37}{16}$

$$\therefore \quad L = 10 \times \frac{16.37}{16} = 10.23 \text{ m. Adopt } L = 11 \text{ m.}$$

Check for the group action: $B = 3s + d = 3 \times 150 + 50 = 500$ cm $= 5$ m

$\therefore$ Load taken by group action $\quad = 4BL\,.\,c + A_p\, cN_c$. (Taking into account the end bearing)

$$A_b = B \times B = B^2 = (5)^2 = 25\text{m}^2\ ;\ N_c = 9$$

$$\therefore \quad Q_{ug} = \left(4 \times 5 \times 11 \times \frac{35}{3}\right) + \left(25 \times \frac{35}{3} \times 9\right) = 2566.7 + 2625 = 5191.7 \text{ kN}$$

This is greater than 3000 kN. Hence safe.

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Example 26.6 *For the design data of example 26.5, compute the settlement of the group assuming the load to be transferred at* $\frac{2}{3}$ *length of the pile.*

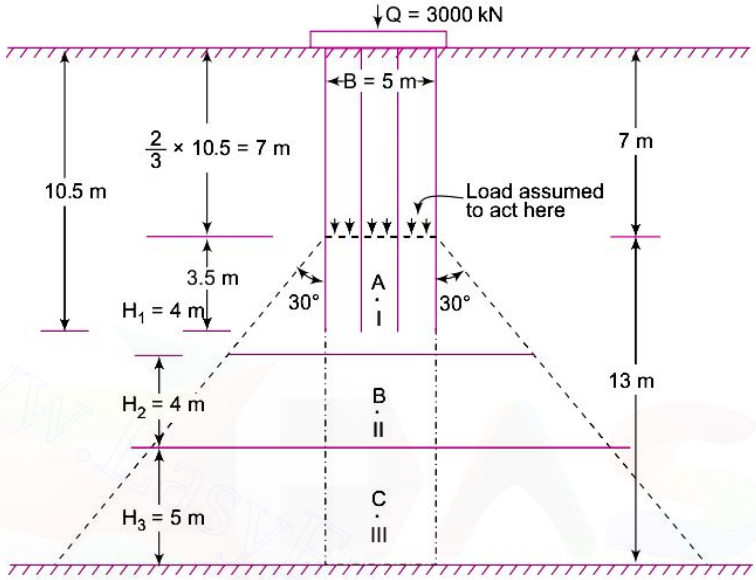

Fig. 26.8

Solution. The load is assumed to act at $\frac{2}{3}L = \frac{2}{3} \times 10.5 = 7$ m below ground surface, as shown.

Depth H below the assumed plane of acting of load = 20 – 7 = 13 m

Assume this depth to be divided into 3 layers of thickness

$$H_1 = 4 \text{ m}, H_2 = 4 \text{ m and } H_3 = 5 \text{ m}$$

Settlement of first layer

$$\rho_1 = \frac{H_1 C_c}{1 + e_0} \log_{10} \frac{\sigma_0 + \Delta\sigma}{\sigma_0}$$

Here $H_1 = 400 \text{ cm} = 4 \text{ m}$

$C_c = 0.009 (w_L - 10) = 0.009 (60 - 10) = 0.45$; $e_0 = 1$ (say)

σ_0 = initial overburden pressure at the mid-height of the layer

$= \gamma Z_1 = 16 \times (7 + 2) = 144 \text{ kN/m}^2$ (where $\gamma = 16 \text{ kN/m}^3$)

Area at the mid-point of first layer

$$= \left(B + 2\frac{H_1}{2} \tan 30\right)^2 = (B + H_1 \tan 30°)^2$$

$\Delta\sigma$ = pressure due to external load

$$= \frac{Q}{(B + H_1 \tan 30°)^2} = \frac{3000}{(5 + 4 \times 0.577)^2} = 56.4 \text{ kN/m}^2$$

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$$\therefore \quad \rho_1 = \frac{4 \times 0.45}{1+1} \log_{10} \frac{144 + 56.4}{144} = 0.129 \text{ m} = 12.9 \text{ cm} \qquad ...(i)$$

Settlement of the second layer

$$\sigma_0 = \gamma Z_2 = 16\,(7 + 4 + 2) = 208 \text{ kN/m}^2$$

Area of the mid-point of the layer $= (5 + 2 \times 6 \times 0.577)^2 = 142 \text{ m}^2$

$$\Delta\sigma = \frac{3000}{142} = 21.1 \text{ kN/m}^2$$

$$\therefore \quad \rho_2 = \frac{4 \times 0.45}{1+1} \log_{10} \frac{208 + 21.1}{208} = 0.038 \text{ m} = 3.8 \text{ cm} \qquad ...(ii)$$

Settlement of the third layer

$$\sigma_0 = \gamma Z_3 = 16(7 + 4 + 4 + 2.5) = 280 \text{ kN/m}^2$$

Area at the mid-point of layer $= (5 + 2 \times 10.5 \times 0.577)^2 = 294 \text{ m}^2$

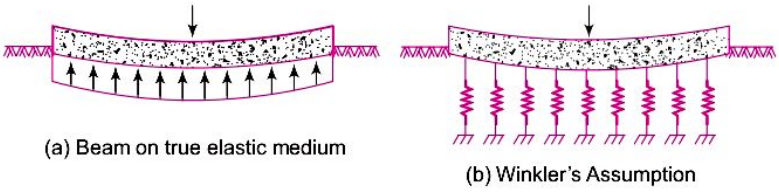
$$\therefore \quad \Delta\sigma = \frac{3000}{294} = 10.2 \text{ kN/m}^2$$

$$\therefore \quad \rho_3 = \frac{5 \times 0.45}{1+1} \log_{10} \frac{280 + 10.2}{280} = 0.017 \text{ m} = 1.7 \text{ cm}$$

$\therefore$ Total settlement $\rho = \rho_1 + \rho_2 + \rho_3 = 12.9 + 3.8 + 1.7 = \mathbf{18.4\ cm}$.

26.10 LATERALLY LOADED PILES*

The laterally loaded pile has often been analysed as though it were a beam supported by an elastic medium as shown in Fig. 26.9 (*a*). For a true elastic medium, the reaction p and the deflection y at a given point on the beam (pile) are affected by the reactions and deflections at all other points along the beam. A simplification was introduced by Winkler (1876) to approximate an elastic medium by a series of infinitely closely spaced elastic springs that operate independent of each other. Fig. 26.9 *(b)* illustrates a beam on a bed of such springs where the reaction at any point along the beam depends only on the deflection of the beam at that point.



(a) Beam on true elastic medium

(b) Winkler's Assumption

Fig. 26.9 Winkler's assumption

The ratio of the soil reaction (p) to the deflection (y) at any point is defined as the *modulus of subgrade reaction* E_s *or soil modulus,* or the *coefficient of subgrade reaction:*

$$E_s = -\frac{p}{y} \qquad ...(26.28)$$

The minus sign indicates that as p increases, y decreases.

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Dimensional analysis of elastic pile theory (Reese and Matlock). Reese and Matlock (1956, 1960) gave the following dimensional analysis of an elastic pile subjected to a horizontal shear force P_t, and moment M_t, as shown in Fig. 26.10. The modulus of subgrade reaction E_s is assumed to vary linearly with the depth x.

For very long piles, the length L of the pile loses significance because the deflection may be very nearly zero for much of the length of the pile. It is convenient to introduce some *characteristic length* dimension (T) as a substitute. T expresses a relation between the stiffness of the soil and the flexural stiffness of the pile. It is therefore, given the term *"relative stiffness factor"* and is defined by the following expression:

$$T = \left(\frac{EI}{\eta_h}\right)^{1/5} \qquad ...(26.29)$$

where η_h = unit modulus of subgrade reaction or constant of soil modulus = k_x/x

For the case of an applied shear P_t and moment M_t, the solution for deflection of the elastic curve may include relative stiffness factor and may be expressed as

$$y = f(x, T, L, E_s, EI, P_t, M_t) \qquad ...(26.30)$$

If the assumption for the elastic behaviour is introduced for the pile, and if deflection remains small relative to the pile dimension, the principle of superposition may be applied.

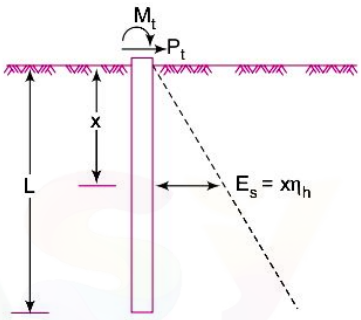

Fig. 26.10 Laterally loaded pile

Thus, $$y = y_A + y_B \qquad ...(26.31)$$

where y = total defection of the pile at any point

y_A = deflection due to shear load P_t

y_B = deflection due to moment M_t

Thus, the solution for the case of shear load (called case *A*) may be expressed as

$$\frac{y_A}{P_t} = f_A(x, T, L, E_s, EI,) \qquad ...(26.32)$$

Similarly for the case of moment (called case *B)* may be expressed as

$$\frac{y_B}{M_t} = f_B(x, T, L, E_s, EI) \qquad ...(26.33)$$

In each case there are six terms, and two dimensions (force and length) involved. Therefore, there are four independent non-dimensional groups which can be formed. The arrangements chosen are,

for case *A*: $$\frac{y_A\, EI}{P_t\, T^3}, \frac{x}{T}, \frac{L}{T} \text{ and } \frac{E_s\, T^4}{EI} \qquad ...(26.34)$$

and, for case *B*: $$\frac{y_A\, EI}{M_t\, T^2}, \frac{x}{T}, \frac{L}{T} \text{ and } \frac{E_s\, T^4}{EI} \qquad ...(26.35)$$

To satisfy conditions of similarity, each of the group must be equal for both model *(m)* and prototype *(p)*:

$$\frac{x_p}{T_p} = \frac{x_m}{T_m}; \frac{L_p}{T_p} = \frac{L_m}{T_m}; \left(\frac{E_s T^4}{EI}\right)_p = \left(\frac{E_s T^4}{EI}\right)_m \qquad ...(26.36)$$

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$$\left(\frac{y_A EI}{P_t T^3}\right)_p = \left(\frac{y_A\, EI}{P_t T^3}\right)_m \text{ and } \left(\frac{y_B\, EI}{M_t T^2}\right)_p = \left(\frac{y_B\, EI}{M_t T^2}\right)_m$$

A group of non-dimensional parameters may be defined which will have the same numerical value for any pair of structurally similar case, or for any model and its prototype. They are:

(*i*) *Depth coefficient:* $Z = \frac{x}{T}$ ...(26.36 *a*)

(*ii*) *Maximum depth coefficient* : $Z_{max} = \frac{L}{T}$ ...(26.36 *b*)

(*iii*) *Soil modulus function* : $\phi(Z) = \frac{E_s T^4}{EI}$ ...(26.36 *c*)

(*iv*) *Deflection coefficient for case A* : $Ay = \frac{y_A\, EI}{P_t T^3}$ ...(26.36 *d*)

(*v*) *Deflection coefficient for case B* : $B_y = \frac{y_B\, EI}{M_t T^2}$ ...(26.36 *e*)

Thus, from Eq. 26.31, $y = y_A + y_B = \left(\frac{P_t T^3}{EI}\right) A_y + \left(\frac{M_t T^2}{EI}\right) B_y$ ...(26.37)

where A_y and B_y are the deflection coefficients, for shear and moment respectively.

Similarly, let A_s and B_s be the slope coefficients, A_m and B_m the moment coefficients, A_v and B_v the shear coefficient *and* A_p and B_p the soil reaction coefficients respectively for the shear load P_t and moment load M_t acting on the pile at the ground level. The slope *S,* moment M, shear *V* and soil reaction *p* are given by:

$$S = S_A + S_B = \left(\frac{P_t T^2}{EI}\right) A_s + \left(\frac{M_t T}{EI}\right) B_s \quad ...(26.38)$$

where S = total slope, at any point of the pile

S_A = slope due to shear load P_t

S_B = Slope due to moment load M_t

$$M = M_A + M_B = (P_t T) A_m + (M_t) B_m \quad ...(26.39)$$

$$V = V_A + V_B = (P_t) A_v + \left(\frac{M_t}{T}\right) B_v \quad ...(26.40)$$

$$p = p_A + p_B = \left(\frac{P_t}{T}\right) A_p + \left(\frac{M_t}{T^2}\right) B_p \quad ...(26.41)$$

Now from the beam theory, the basic equation for an elastic beam (pile) is

$$EI \frac{d^4 y}{dx^4} = p \quad ...(26.42)$$

Introducing by definition $p = -E_s y$, we get

$$\frac{d^4 y}{dx^4} + \frac{E_s}{EI} y = 0 \quad ...(26.43)$$

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The applied lateral load P_t and an applied moment M_t are considered separately. According to the principle of superposition, the above equation becomes

For case A $$\frac{d^4 y_A}{dx^4} + \frac{E_s}{EI} y_A = 0 \qquad ...(26.44)$$

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For case B $$\frac{d^4 y_B}{dx^4} + \frac{E_s}{EI} y_B = 0 \qquad ...(26.45)$$

Substituting the definitions of non-dimensional parameters, a non-dimensional differential equation can be written for case A as

$$\frac{d^4 A_y}{dZ^4} + \phi(Z) A_y = 0 \qquad ...(26.46)$$

and for case B, $$\frac{d^4 B_y}{dZ^4} + \phi(Z) B_y = 0 \qquad ...(26.47)$$

The solution of these two given equations give the values of coefficients A_y and B_y. Similarly, by successive integration of these two equations, one can get the expressions, for slope, moment, shear and soil reaction and their respective coefficients (A_s, B_s), (A_m, B_m), $(A_v, B_v$ and (A_p, B_p) can be analytically computed. Table 26.2 gives the analytical values of these coefficients, computed by Reese and Matlock (1956).

Table 26.2 Non-Dimensional Coefficients for Laterally Loaded Pile (Reese and MatLock)

Z	A_y	A_s	A_m	A_v	A_p
0.0	2.435	– 1.623	0.000	1.000	0.000
0.1	2.273	– 1.618	0.100	0.989	– 0.227
0.2	2.112	– 1.603	0.198	0.956	– 0.422
0.3	1.952	– 1.578	0.291	0.906	– 0.586
0.4	1.796	– 1.545	0.379	0.840	– 0.718
0.5	1.644	– 1.503	0.459	0.764	– 0.822
0.6	1.496	– 1.454	0.532	0.677	– 0.897
0.7	1.353	– 1.397	0.595	0.585	– 0.947
0.8	1.216	– 1.335	0.649	0.489	– 0.973
0.9	1.086	– 1.268	0.693	0.392	– 0.977
1.0	0.962	– 1.197	0.727	0.295	– 0.962
1.2	0.738	– 1.047	0.767	0.109	– 0.885
1.4	0.544	– 0.893	0.772	– 0.056	– 0.761
1.6	0.381	– 0.741	0.746	– 0.193	– 0.609
1.8	0.247	– 0.596	0.696	– 0.298	– 0.445
2.0	0.142	– 0.464	0.628	– 0.371	– 0.283
3.0	– 0.075	– 0.040	0.225	– 0.349	0.226
4.0	– 0.050	0.052	0.000	– 0.106	0.201
5.0	– 0.009	0.025	– 0.033	0.013	0.046

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Z	B_y	B_s	B_m	B_v	B_p
0.0	1.623	– 1.750	1.000	0.000	0.000
0.1	1.453	– 1.650	1.000	– 0.007	– 0.145
0.2	1.293	– 1.550	0.999	– 0.028	– 0.259
0.3	1.143	– 1.450	0.994	– 0.058	– 0.343
0.4	1.003	– 1.351	0.987	– 0.095	– 0.401
0.5	0.873	– 1.253	0.976	– 0.137	– 0.436
0.6	0.752	– 1.156	0.960	– 0.181	– 0.451
0.7	0.642	– 1.061	0.939	– 0.226	– 0.449
0.8	0.540	– 0.968	0.914	– 0.270	– 0.432
0.9	0.448	– 0.878	0.885	– 0.312	– 0.403
1.0	0.364	– 0.792	0.852	– 0.350	– 0.364
1.2	0.223	– 0.629	0.775	– 0.414	– 0.268
1.4	0.112	– 0.482	0.688	– 0.456	– 0.157
1.6	0.029	– 0.354	0.594	– 0.477	– 0.047
1.8	– 0.030	– 0.245	0.498	– 0.476	– 0.054
2.0	– 0.070	– 0.155	0.404	– 0.456	– 0.140
3.0	– 0.089	0.057	0.059	– 0.213	0.268
4.0	– 0.028	0.049	– 0.042	0.017	0.112
5.0	– 0.000	0.011	– 0.026	0.029	– 0.002

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Example 26.7*. *A concrete pile, 30 cm square and 5 m long, is subjected to a horizontal load of 000 N and a moment of 4000 Nm at the ground level. Taking $\eta_h = 20$ N/cm^3, find maximum bending oment and deflection if (a) the head of the pile is considered to be free (b) the head is considered fixed ith no external moment.*

Solution. (*a*) ***Free head case***

(*i*) Maximum deflection

$$I = \frac{1}{12} bd^3 = \frac{1}{12}(30)(30^3) = 6.75 \times 10^4 \text{ cm}^4; \quad E = 3 \times 10^6 \text{ N/cm}^2 \text{ (say)}$$

$$EI = 3 \times 10^6 \times 6.75 \times 10^4 = 20.25 \times 10^{10} \text{ Ncm}^2$$

$$\eta_h = 20 \text{ N/cm}^3$$

$$\therefore \quad T = \left(\frac{EI}{\eta_h}\right)^{1/5} = \left[\frac{20.25 \times 10^{10}}{20}\right]^{1/5} \approx 100 \text{ cm}$$

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Now, $L_s = 5 \text{ cm} = 500 \text{ cm}; \quad Z_{max} = \frac{L_s}{T} = \frac{500}{100} = 5$

Hence it is a long pile.

The values of deflection coefficients for maximum deflection corresponding to $Z = 0$ (Table 26.2) re $A_y = 2.435$ and $B_y = 1.623$

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$$y_{max} = y_A + y_B$$

$$= A_y \cdot \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI} = \frac{2.435 \times 5000\,(100)^3}{20.25 \times 10^{10}} + \frac{1.623\,(4000 \times 100)\,(100)^2}{20.25 \times 10^{10}}$$

$$= 0.06 + 0.032 = \mathbf{0.092\ cm}$$

(*ii*) Maximum bending moment

$$M_{max} = A_m P_t T + B_m M_t$$

where $P_t T = 5000 \times 100 = 5 \times 10^5$

$M_t = 4000 \times 100 = 4 \times 10^5$

$\therefore \quad M_{max} = 10^5 (5 A_m + 4B_m)$ ...(*i*)

The maximum will occur at any depth Z which is not known at present. However, it will occur at such a value of Z that $(5 A_m + 4B_m)$ is maximum. The computations are done in a tabular form by taking values of A_m and B_m from Table 26.2 for various values of depth ratio $Z = \frac{x}{T}$.

$Z = x/T$	A_m	$5A_m$	B_m	$4B_m$	$5A_m + 4B_m$
0.5	0.459	3.00	0.976	3.90	6.90
0.6	0.532	2.66	0.960	3.84	6.50
0.7	0.595	2.98	0.939	3.76	6.74
0.8	0.649	3.24	0.914	3.65	6.89
0.9	0.693	3.46	0.885	3.54	7.00
1.0	0.727	3.64	0.852	3.14	7.05
1.2	0.767	3.84	0.775	3.10	6.94

Hence the maximum moment occurs when $Z = 1.0$

or at $x = ZT = 1.0 \times 100 = 100$ cm = 1 m below ground surface

$M_{max} = 10^5 (5 A_m + 4\ B_m) = 10^4 \times 7.05$ kg-cm = **7050 Nm**

(b) Fixed head case

When there is fixidity at the head, there will be no slope at the ground surface (at $Z = 0$).

$\therefore \quad S = S_A + S_B = 0$

or $A_s \cdot \frac{P_t \cdot T^2}{EI} + B_s \frac{M_t T}{EI} = 0$; or $M_s = -\frac{A_s}{B_s} P_t T$ ...(*ii*)

Equation (*ii*) gives the moment included at the ground surface due to fixidity.

From Table 26.2, for $Z = 0$, we have

$A_s = -1.623$ and $B_s = -1.750$

$\therefore \quad M_t = \frac{1.623}{1.750} P_t T = -0.93\ P_t\ T$

$= -0.93 \times 500 \times 5000$ Ncm = **–4650 Ncm** ...(*iii*)

Maximum deflection will evidently occur at the ground surface

$$y = y_A + y_B = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI}$$

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Substituting $M_t = -0.93\, P_t\, T$ from (*iii*), we get

$$y = \frac{A_y\, P_t\, T^3}{EI} - B_y\, \frac{0.93\, P_t\, T^3}{EI} = \frac{P_t\, T^3}{EI}\,(A_y - 0.93\, B_y)$$

$$= \frac{5000(100)^3}{20.25 \times 10^{10}}\,(A_y - 0.93\, B_y) \quad = 0.0247\,(A_y - 0.93\, B_y) \qquad ...(iv)$$

At $Z = 0,\ A_y = 2.435$ and $B_y = 1.623$

$y = 0.0247(2.435 - 0.93 \times 1.623) =$ **0.0228 cm**

26.11 UNDER-REAMED PILE FOUNDATIONS

Under reamed piles are bored *cast-in situ* concrete piles having one or more *bulbs* formed by enlarging the bore hole for the pile stem by an under-reaming tool. These piles find applications in widely varying situations in different types of soils where foundations are required to be taken down to a certain depth to avoid the undesirable effect of seasonal moisture changes as in expansive soils or to reach strata or to obtain adequate capacity for downward, upward and lateral loads or to take the foundations below scour level and for moments.

Figure 26.11 shows the details of the under reamed pile foundation. When the pile has only one bulb, it is known as *single under-reamed pile,* while the pile with more than one bulb is known as *multi-under-reamed* pile. Generally, the diameter of under-reamed bulbs is kept equal to 2.5 times the diameter of pile stem. However, it may vary from 2 to 3 times the stem diameter, if required, depending upon the design requirements and feasibility of construction.

Details of pile and under-reamed bulb: In deep layers of expansive soils, the minimum length of pile required is 3.5 m where the ground movements become negligible. In shallow depths of expansive soils and other poor soils depending upon the load requirements the length may be reduced and the piles may be taken up to at least 50 cm in stable zone (*i.e* the zone where there are no ground movements due to seasonal moisture changes). Pile length may be increased for higher loads.

The diameter of manually bored piles range from 20 cm to 37.5 cm. The spacing of the piles shall be considered in relation to the nature of the ground, the types of piles and the manner in which the piles transfer the loads to the ground. Generally, the centre to centre spacing for under-reamed piles should not be less than 2 D_u (where D_u is the under-reamed diameter). It may be reduced to 1.5 D_u when a reduction in load carrying capacity of 10 % should be allowed. For the spacing of 2 D_u the bearing capacity of pile group may be taken equal to the number of piles multiplied by the bearing capacity of individual pile. If the adjacent piles are of different diameters, an averge value for spacing should be taken. The maximum spacing of the under-reamed pile should not normally exceed 2.5 metres so as to avoid heavy capping beams. In buildings, the piles should generally be provided under all wall junctions to avoid point loads on beams. Positions of intermediate piles are then decided trying to keep the door openings fall in between two piles as far as possible.

In double and multi-under-reamed piles of size less than 30 cm dia., the centre-to-centre vertical spacing between the two under-reams may be kept equal to 1.5 D_u while for piles of 30 cm and more this distance may be reduced to 1.25 D_u. The upper bulb should not be placed too close to the ground. The minimum desirable depth of the centre of the bulb is 1.5m or 2 D_u whichever is greater.

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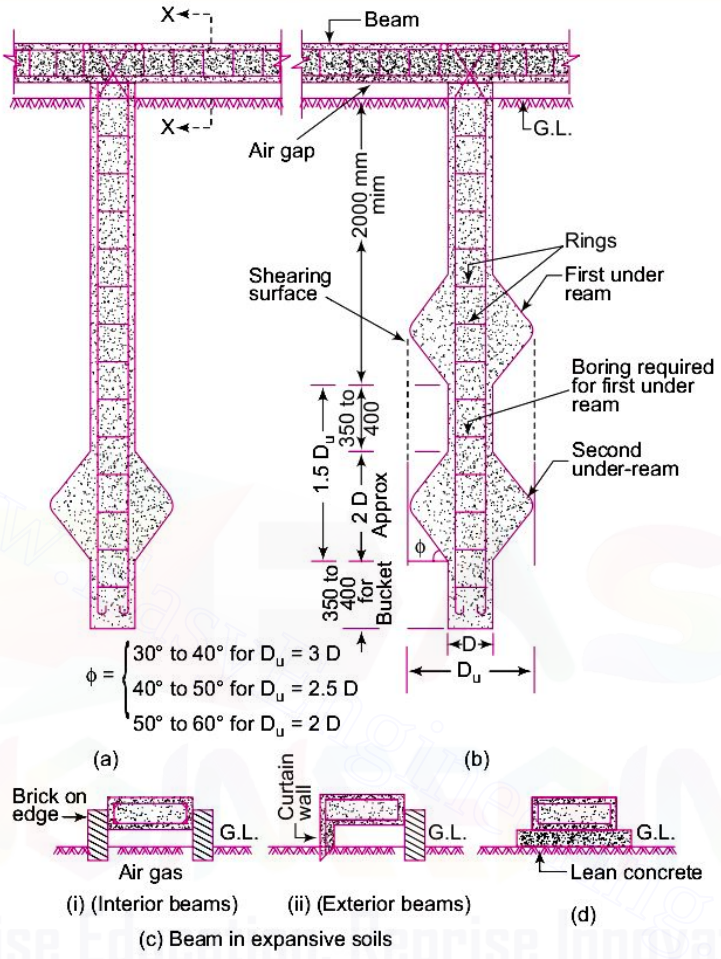

Fig. 26.11 Details of under-reamed file foundations

Under-reamed piles can be made at a batter also, for sustaining large lateral loads, thus making them suitable for tower footings, retaining walls and abutments. They have also been found useful for factory buildings, machine foundations and transmission line towers and poles. In black cotton soils and other expansive soils, the under-reamed pile anchors the structures at a depth where the volumetric changes in soils due to seasonal and other variation is negligible.

The under-reamed pile is nominally reinforced with 10 to 12 mm dia. longitudinal bars, and 6 mm ϕ rings. The details of reinforcement are shown in Table 26.3. A clear cover of 4 cm is provided.

Load carrying capacity from Soil Properties

Clayey soils. In the case of clayey soils, the ultimate load bearing capacity of an under-reamed pile may be worked out from the following expression:

$$Q_u = A_p N_c C_p + A_a N_c C_a' + C_a' A_s' + \alpha C_a A_s \quad ...(26.48)$$

where Q_u = ultimate bearing capacity of the pile

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A_p = cross-sectional area of the pile stem at toe

N_c = bearing capacity factor, usually taken as 1

C_p = cohesion of the soil around the toe

$$A_a = \frac{\pi}{4}\left(D_u^2 - D^2\right)$$

where D_u and D are the under reamed bulb and stem diameter respectively

C_a' = average cohesion of soil around the under-reamed bulbs

A_s' = surface area of the cylinder circumscribing the under-reams

α = reduction factor (usually 0.5 for clays)

C_a = average cohesion of the soil along the pile stem, and

A_s = surface area of the stem.

The above expression holds for the usual vertical spacing between under-reamed bulbs not greater than 1.5 times the diameter of the under-reamed bulb.

Sandy soils. In the case of sandy soils, the ultimate load carrying capacity of an under-reamed pile may also be worked out by the following expression:

$$Q_u = \frac{\pi}{4}\left(D_u^2 - D^2\right)\left[\frac{1}{2} D_u \,.\, n \,.\, \gamma\, N_\gamma + \gamma \,.\, N_q \sum_{r=1}^{r=n} d_r\right]$$

$$+ \frac{\pi}{4} D^2 \left(\frac{1}{2} D \,.\, \gamma \,.\, N_\gamma + \gamma \,.\, D_f \,.\, N_q\right) + \frac{1}{2} \pi D \,.\, \gamma \,.\, K \tan\delta \times \left(d_1^2 + d_f^2 - d_n^2\right) \quad \text{...(26.49)}$$

where

D_u = diameter of under-ream bulbs

D = diameter of stem

n = number of under-reamed bulbs

γ = average field density of soil

N_γ and N_q = bearing capacity factors depending on the angle of internal friction

d_r = depth of the centre of different under-ream bulbs

d_f = total depth of pile

K = earth pressure constant (usually 1.75 for sands)

δ = angle of wall friction

d_1 = depth of centre of the first under-ream bulb

d_n = depth of the centre of last under-ream bulb

Safe Load from I.S. Code Tables

The load carrying capacity of an under-reamed pile may be determined form load test. In the absence of actual tests, the safe loads allowed on piles under-reamed to 2.5 D_u may be taken from Table 26.3 (IS : 2911, Part III-1973). The safe loads given in the Table apply to both medium compact sandy soil and clayey soils of medium consistency. For dense sandy ($N \geq 30$) and stiff clayey ($N \geq 8$) soils, the loads may be increased by 25%. However, the values of the lateral thrust should not be increased unless

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stability of the top soil *(i.e.,* strata to a depth of about 3 times the stem diameter) is ascertained. On the other hand, a 25% reduction should be made in case of loose sandy ($N \leq 10$) and soft clayey ($N \leq 4$) soils.

Capping Beam or Grade Beam: The under-reamed piles are connected by a *reinforced* concrete beam, known as *capping beam* or *grade beam.* For expansive soils, the grade beam is kept above the ground, with a clear air gap of 8 to 10 cm to provide space for the expansion (swelling) of the sub-soil. In the case of non-expansive soil, however, mass concrete (1: 3: 6 or 1: 4: 8 mix) of 8 to 10 cm thickness is provided between the ground and the bottom of the beam, as shown in Fig. 26.11 (*d*). Fig. 26.11 (*c*) shows the details of the interior and exterior beams for expansive soils. For interior beams, 50 mm thick concrete slab or brick on edge is provided on both the sides, to cover the air gap. For exterior beam, the slab is provided to the inner face while the beam has sharp edge (curtain wall) penetrating the ground to the outer face. Due to this the swelling soil can easily expand without exerting any swelling pressure on the beam.

Grade beams supporting masonry walls should be designed taking into account the *panel action.* The grade beams are designed for a maximum bending moment of $(wL^2)/50$ if the beams are supported during the construction, and for a bending moment of $(wL^2)/30$ if the beams are not supported, where L is the effective span of the beam. In working out the uniformly distributed load w, a maximum load of one storey in framed structures and of two storey in structures with load bearing walls should be considered. The load of additional storey should be deemed to be transferred to the piles. For concentrated and other loads, such as suspended ground floors etc. which directly rest on these beams full bending moment should be accounted for.

It is found that the depth and reinforcement found on the basis of B.M. is very small. However, a minimum depth of 15 cm is recommended from practical considerations. Maximum positive reinforcement is provided at bottom of the beam, at the centre of the span. Negative reinforcement, equal to positive reinforcement, is provided to a distance of a quarter span both ways from pile centre. Two mild steel bars of 10 to 12 mm ϕ should be provided for full length at the top of the beam so as to support the stirrups. Stirrups of 6 mm ϕ wire are provided at 30 cm centre throughtout the length. Table 26.4 gives the details of the depth of beam and reinforcement required for various loading, as recommended by Central Building Research Institute, Roorkee. A clear cover of 3 cm should be provided for main reinforcement.

Construction of Under-Reamed Pile Foundation: The various stages involved in the construction of under reamed pile foundation are as under:

(*a*) Boring by augers
(*b*) Under-reaming by under-reamer
(*c*) Placing reinforcement cage in position
(*d*) Concreting of pile
(*e*) Concreting of pile caps, plinth beams and curtain walls

Figure 26.12 shows various stages in the formation of a under-reamed pile. The equipment required for the construction of pile are (*i*) auger boring guides, (*ii*) spiral auger with extension rod, (*iii*) under-reamer with soil bucket and (*iv*) concreting funnel. A portable tripod hoist with winch is required specially for piles larger than 4 to 5 m length and/or of diameter larger than 37.5 cm.

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Table 26.3 Safe load for vertical under-reamed piles in sandy and clayey soils including black cotton soils (IS : 2911, Part III-1973)

		Reinforcement			Safe Load									
		Longitudinal		Spacing of 6 mm dia rings	Bearing Resistance				Uplift resistance				Lateral thrust	
Dia. of pile (D)	Under-reamed Dia.(D_m)	No. of bars	Dia.		Single Under-reamed	Double Under-reamed	Increase per 30 cm length	Decrease per 30 cm length	Single Under-reamed	Double Under-reamed	Increase per 30 cm length	Decrease per 30 cm length	Single Under-reamed	Double Under-reamed
(1) cm	(2) cm	(3)	(4) mm	(5) cm	(6) t	(7) t	(8) t	(9) t	(10) t	(11) t	(12) t	(13) t	(14) t	(15) t
20	50	3	10	18	8	12	0.9	0.7	4	6	0.65	0.55	1.0	1.2
25	62.5	4	10	22	12	18	1.15	0.9	6	9	0.85	0.70	1.5	1.8
30	75	4	12	25	16	24	1.4	1.1	8	12	1.05	0.85	2.0	2.4
37.5	94	5	12	30	24	36	1.8	1.4	12	18	1.35	1.10	3.0	3.6
40	100	6	12	30	28	42	1.9	1.5	14	21	1.45	1.15	3.4	4.0
45	112.5	7	12	30	35	52.5	2.15	1.7	17.5	25.57	1.60	1.30	4.0	4.8
50	125	9	12	30	42	68	2.4	1.9	21	31.5	1.80	1.45	4.5	5.4

Note. 1. The value of bearing resistance, uplift pressure and lateral thrust given in the table are for a minimum pile length of 3.5 m except in double under-reamed piles. In double under-reamed piles, the minimum recommended lengths for 37.5 cm, 40 cm, 45 cm and 50 cm piles will normally be 3.75 m, 4.0 and 5.0 m respectively so as to suitably accommodate the bulbs at specified distance.

Note 2. Longitudinal bars should normally be provided with a clear cover of 4 cm and may be curtailed or eliminated towards the toe depending upon the stresses in pile section.

Note 3. For under-reamed piles subjected to a pull and/or lateral thrust, the requisite amount of steel should be provided.

Note 4. Values given in col. 14 and 15 for lateral thrust may not be reduced for changes in pile lengths and are fairly conservative. Higher values may be adopted after conducting lateral load test on single or group of piles.

Note 5. In 25 and 30 cm dia. normal under-reamed piles when concreting is done by a tremie, equivalent reinforcement in shape of single angle iron piece placed centrally may be used.

Note 6. When a pile designed for a certain safe load is found to be just short of the load required to be carried by it, an over-load of 10% should be allowed on it.

Note 7. For working out the safe load for a group of piles the safe load of individual piles is multiplied with the number of piles in the group. This would be applicable for piles taking lateral thrust also.

Note 8. Only 75% of the above safe loads should be taken for piles in which the bore holes are full of sub soil water during concreting. When water is confined to the bucket portion only, no such reduction need be made.

Note 9. In sandy soils, when boring and under-reaming under water, minimum size of pile recommended is 25 cm.

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Note 11. The values given should be increased by 50% for broken wire condition in the design of transmission line tower footings.

Note 12. Safe load for multi-under-reamed pile may be worked out from the table by allowing 50% of the load as per col. 6 for each additional bulb. Increase in capacity due to increase in length will be as per col. 8

Note 13. For taking very high loads, me pile shaft above the top most under-ream should be either increased in diameter and/or additional reinforcement provided as in short column.

Table 26.4 Design of grade beam

Beam width	Load on beam kg/m	Effective span of beam											
		1.5 m			1.8 m			2.1 m			2.4 m		
		Depth of beam (cm)	Bottom Reinforcement		Depth of beam (cm)	Bottom Reinforcement		Depth of beam (cm)	Bottom Reinforcement		Depth of beam (cm)	Bottom Reinforcement	
			Nos.	Dia (mm)		Nos.	Dia (mm)		Nos.	Dia (mm)		Nos.	Dia (mm)
22.5 cm (9″)	1500	15	4	10	15	4	10	15	4	10	15	4	10
	3000	15	4	10	15	4	10	15	4	10	18	4	10
	4500	15	4	10	15	4	10	18	4	10	20	4	10
	6000	15	4	10	18	4	10	20	4	10	20	4	10
	7500	18	4	10	20	4	10	20	4	12	20	4	12
34 cm (13.5″)	1500	15	4	10	15	4	10	15	4	10	15	4	10
	3000	15	4	10	15	4	10	15	4	10	18	4	10
	4500	15	4	10	15	4	10	18	4	10	18	4	10
	6000	15	4	10	18	4	10	18	4	10	20	4	10
	7500	15	4	10	18	4	10	20	4	12	20	4	12
	9000	18	4	10	18	4	12	20	4	12	20	5	12
	10500	18	4	10	18	4	12	20	4	12	20	6	12
45.7 cm (18″)	6000	15	4	12	15	4	12	15	4	12	18	4	12
	7500	15	4	12	15	4	12	18	4	12	20	4	12
	9000	15	4	12	18	4	12	20	4	12	20	4	16
	10500	15	4	12	18	4	12	20	4	16	20	4	16
	12000	18	4	12	20	4	12	20	4	16	20	4	16
	13500	18	4	12	20	4	12	20	4	16	20	4	16
	15000	18	4	12	20	4	16	20	4	16	20	5	16

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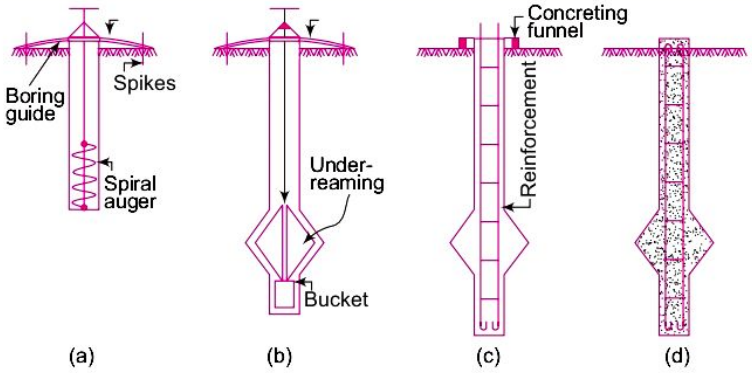

Fig. 26.12 Stages in the construction of under-reamed pile

The above equipment used for boring and under-reaming has been developed by Central Building Research Institue, Roorkee. It is covered by an Indian patent and is licenced to M/s M.S.J. (Engineers) & Co., Roorkee. Boring is done with the help of spiral auger. The use of boring guide is essential in order to keep the bore holes vertical and also in position. Each guide is provided with a circular collar and four arms. The collar is fixed to the boring guide on the lower side and it does not allow the mouth of the bore holes to widen due to frequent insertion and removal of the auger and other boring tools. After setting the guide assembly in position, the spiral auger is introduced into the circular collar of the guide by opening out the two sets of flaps of the guide assembly. The auger is pressed down and rotated manually until the spirals are half full of earth. The auger is then taken out and earth removed. The auger is again introduced and the boring process is repeated till the required depth is reached.

Under-reaming or enlarging the stem of bore hole at the required depths is achieved by means of the under-reamer consisting of a set of two collapsible blades assembly fixed around the central shaft and a detachable bucket for receiving the cut soil. The equipment is attached to extension rods and lowered down the hole which has been bored to the required depth, untill the bucket rests at the bottom of the bore hole. The guide flaps are then closed. The tool is pressed down constantly and rotated slowly. The cutting blades of the tool widen out and start cutting the sides of hole. The loose earth is collected in a bucket at the bottom. When the bucket is full, the assembly is pulled out and bucket is emptied. The depth of the bore hole is checked each time before insertion of the under-reamer so that any loose earth spilled from the bucket is removed. Otherwise, the bucket position will get shifted upwards due to loose soil lying at the bottom. This will shift the position of the bulb. The under-reamer is then lowered into the bore hole and the process repeated until the cutting blades have expanded fully and no further earth is cut by the blades. Generally, removal of eight buckets full of earth is required for completion of one under-ream.

In the case of double under-reamed pile, further boring is done, after first bulb is formed. After boring to the required depth, the under-reaming for the second bulb is carried out. The dimensions of the bulb can be checked by means of a graduated G.I. pipe assembly. After the bore and the under-ream are checked, the reinforcement cage, already fabricated, is lowered in the hole. To ensure proper cover for the; bars inside the bore hole, 4 Nos. cement mortar discs 2 cm thick and 7.5 cm dia. with a hole, in the centre are inserted into a ring of the reinforcement cage at about the middle height of the pile shaft. Each reinforcement cage is invariably raised from bottom of the bore hole while concreting to ensure proper cover at bottom. Concreting of the pile is carried out through a concreting funnel placed at the mouth of

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the bore hole. As soon as concrete is filled up to top, the funnel is removed and circular steel mould made out of mild steel sheet is placed on the top and the pile shaft for a height of about 15 cm above ground level is converted simultaneously, care being taken to see that the top level of pile shaft is not lower than the bottom of the pile cap to be cast on it. When the concrete in the pile becomes fairly hard, shuttering work for plinth beam and pile caps is started.

Load Test on Under-Reamed Piles

Piles are usually tested for determining their load carrying capacity in compression, tension and lateral loading. Two categories of tests are conducted : *(a) initial tests* and *(b) routine tests*. Initial tests should be carried out on test piles or working piles, preferably on test piles. In case the initial tests show consistently higher or lower values than the estimated safe allowable loads on piles, designs should be re-examined and neccessary modifications made. Routine tests are carried out as check load on working piles.

(a) Procedure for Initial Tests (Compression): Following are the recommendations of Indian Standard IS : 2911 (Part III) : 1973.

1. The test shall be carried out by applying series of loads to the pile unaided by any other support. Pile groups may be tested as free standing piles or piled foundations as specified.
   The load shall preferably be applied by means of hydraulic jack reacting against a loaded platform, or rolled steel joints or suitable load frame held down by soil anchors and piles or other achorage. The anchor piles may also be working piles but they shall be sufficient in number and adequately reinforced to take the full tension with proper factor of safety. The reaction available for loading should not be less than 3 times the estimated safe load carrying capacity of the piles. The jack should be of adequate capacity preferably with a remote control pump and shall have pressure gauge or other suitable device for reading the applied loads.
2. Readings of settlement and rebound shall be recorded with the help of at least 3 dial gauges of 0.02 mm sensitivity, positioned at equal distances around the pile. The dial gauges shall be fixed to datum bars resting on non-movable supports at least *5D* (subject to a maximum of 2.5 m) away from the piles where *D* is the pile stem diameter.
3. The test load shall be applied in increments of about 1/5 of the estimated safe load. At each stage of loading/unloading, the load shall be maintained till the rate of movement of the pile top is not more than about 0.02 mm per hour.
4. Loading shall generally be continued up to 2.5 times the estimated safe load or to a settlement of 7.5% of the bulb diameter whichever is earlier.
5. The *safe load* on pile shall be the least of the following:
   (*i*) two-thirds of the final load at which the total settlement attains a value of 12 mm, unless it is established that a total settlement different from 12 mm is permissible in a given case on the basis of nature and type of the structure; in the latter case the actual total settlement permissible shall be used for assessing the safe load instead of 12 mm.
   (*ii*) 50% of the final load at which the total settlement equals 7.5% on the bulb diameter.

(b) Procedure for Routine Test (Compression): Loading shall be carried out up to 1.5 times the allowable load, the reaction provided may be 2 times the allowable load.

The procedure followed for the test and determination of the allowable load shall be the same as per initial test excepting clause 5(*ii*).

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26.12 BORED COMPACTION PILES

Bored compaction piles are the modification of under-reamed pile. These piles are cast-in-situ pile which combines the advantages of both *bored* and *driven piles* The method of boring the piles and concreting the pile is the same as that for the under-reamed pile, except that the reinforcement cage is not placed in the bore hole before concreting. After the concreting is over, the reinforcement cage is *driven* through the laid concrete. Due to this feature, the compaction of surrounding soil as well as concrete are effected and the load carrying capacity is increased by 1.5 to 2 times over normal under-reamed pile. These piles are particularly suitable in loose to medium dense sandy and silty strata. Also, in case of loose strata, overlying the dense strata specially in submerged soils, these piles can be used with advantage. In such conditions, it is difficult to reach the desired depth in case of bored piles normally without loosening the strata at pile toe.

Figure 26.13 illustrates the stages of construction of such pile. The bore hole is prepared with the help of spiral auger using guides, and then under-reamed. The pile is then concreted *without* placing the reinforcement cage [Fig. 26.13 (*a*)]. The reinforcement cage, enclosing a hollow driving pipe, is then placed over the freshly laid concrete [Fig. 26.13 (*b*)]. A cast iron conical shoe, with iron cleat welded to it, is attached to the reinforcing cage. This assembly of reinforcement cage, hollow pipe and conical shoe, is driven through the freshly laid concrete to full depth [Fig. 26.13 (*c*)] by means of suitable drop weight (about 500 kg) operated with the help of mechanical winches. The movement of hammer and assembly should be controlled by suitable guiding attachment to ensure vertical penetration of the cage. As the cage is driven into the concrete, soil and concrete gets compacted. This would result in increase in the diameter of the bore hole. Extra concrete is simultaneously poured to keep it level with the ground.

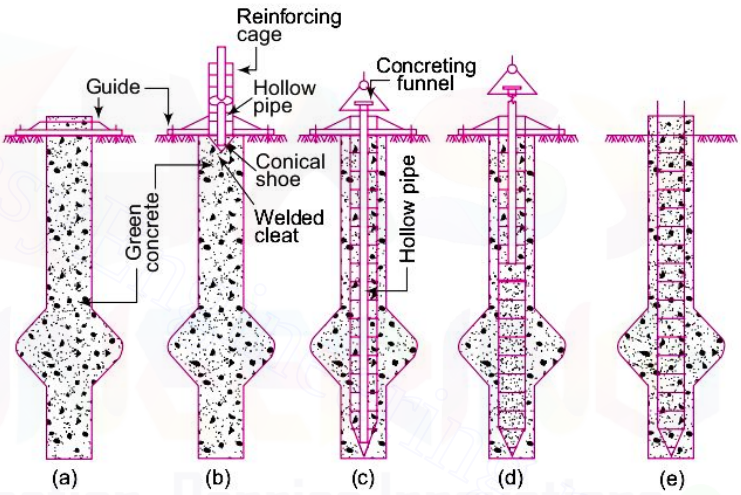

Fig. 26.13 Construction of bored compaction pile

After driving through the full depth, concrete is filled in drive pipe also. The pipe is then gradually withdrawn [Fig. 26.13 (*d*)] leaving the cage, shoe and concrete behind. Fig. 26.13 (*e*) shows the completed pile.

26.13 CASED CAST- IN-SITU CONCRETE PILES

Cased cast-in-situ piles are suitable in practically all ground conditions. The shell is driven into intimate contact with the surrounding soil and remains in place to maintain driving resistance and protect the concrete filling during the placing of other adjacent piles and during the critical setting period. Cased piles can be easily cut or extended to meet variations in shell length. One of the main advantages is that

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it is subject to internal inspection after it is driven. The following are the common types of cased cast-in-situ concrete piles:

(1) Raymond standard pile and step-taper pile
(2) McArthur cased pile
(3) Union metal monotube pile
(4) Swage pile
(5) Western button bottom pile.

1. Raymond piles: In 1897, A.A. Raymond patented the Raymond pile system and was first to develop a practical, economical way of placing cast-in-situ concrete piles. Two types of Raymond piles are in use (*a*) Raymond standard concrete pile and (*b*) Raymond step-taper concrete pile.

(*a*) *Raymond standard concrete pile*: The Raymond standard pile is used primarily as a friction pile since its uniform heavy taper of 1 in 30 usually results in shorter piles for equal driving resistance or higher driving resistance for equal lengths, than piles of lesser or no taper. The lengths of piles vary from 6 to 12 m. The diameter of piles vary from 40 to 60 cm at the top and 20 to 30 cm at the bottom. The pile consists of a thin *corrugated steel shell* closed at the bottom. The steel shell is reinforced with spirally wound hard drawn wire on 8 cm pitch. The shell is closed at the bottom with a steel boot. The shell is driven into the ground with a collapsible steel *mandrel* or *core* in it having the same taper. When the pile is driven to the desired depth, the mandrel is mechanically collapsed and withdrawn, leaving the shell inside the ground. The shell is inspected internally by using the light from a mirror of flash light or drop light. The shell is gradually filled with concrete up to the top.

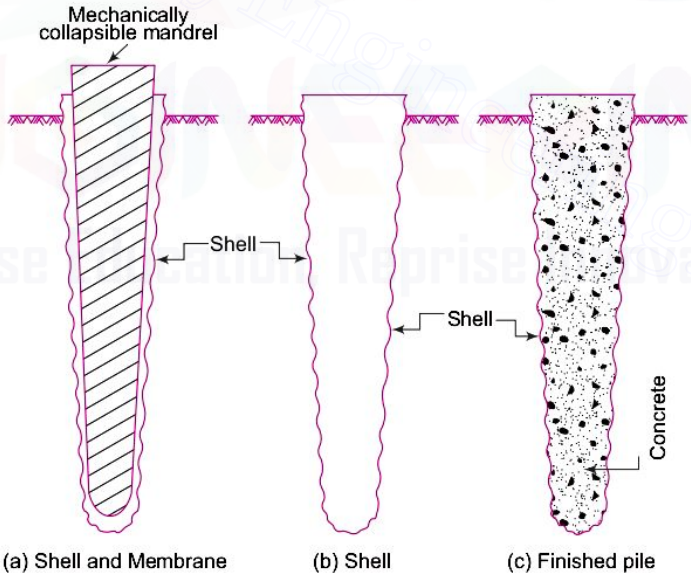

Fig. 26.14 Stages in raymond standard pile construction

(*b*) *Raymond step-taper concrete pile*: This type of mandrel-driven pile is used either as an end bearing or friction pile, and can be driven in any type of soil. The pile uses shell section driven in different lengths. The bottom most sections of shell is made of heavier gauge, and is closed by flat steel plate welded to the boot ring. The joints between sections of shell are screw-connected. The shells are driven with a rigid internal steel mandrel or core which is stepped to conform with

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the shell section used. The heavy rugged core provides a high degree of penetration and efficiently transmits hammer energy to the bearing strata. The pile diameter increases in steps at the rate of 2.5 cm for each successive shell section. These can be drawn upto a maximum depth of about 36 m, using 20 cm tip. The method of forming the pile is the same as that for the standard pile. The pile has the advantages of on-the-job length flexibility, internal inspection after being driven and a steel shell left in place to maintain driving resistance, and protect a fresh concrete filling.

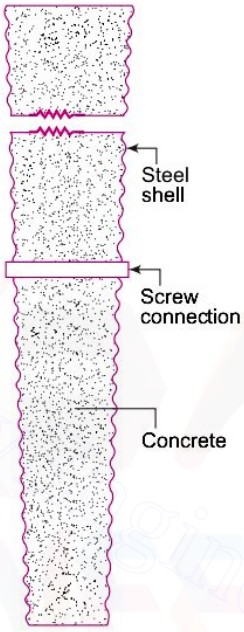

Fig. 26.15 Raymond step-taper piles

2. McArthur cased pile: McArthur cased pile is a pile of uniform diameter, using the corrugated steel shell which remains in place, as in Raymond piles. However, the driving of the pile uses an additional steel casing of heavy gauge. The heavy steel casing with a central core is driven into the ground as shown in Fig. 26.16 (*a*). After reaching the desired depth, the central core is withdrawn, and a corrugated shell is placed in the casing [Fig. 26.16 (*b*)]. Finally concrete is placed in the shell, by gradually compacting it, and withdrawing the steel casing. The completed pile, shown in Fig. 26.16 (*c*) contains concrete core and the outer corrugated shell.

3. Union metal monotube pile: Monotube piles use tapered fluted steel shell without mandrel, and are suitable for a wide variety of soil conditions, form end-bearing to friction-load-carrying soils. The shells provide rigidity, and are watertight. The pile shells are driven to the required depth, and they are inspected after driving. The stiffness of the shell against crushing from adjacent piles is very good. Shells may be driven with hammer of comparable size to those used for wood piles. The shell, after inspection is filled with concrete, and the excess length of the shell, if any, is cut.

4. Swage piles: Swage piles, are used with advantage in some soils where the driving is very hard, or where it is designed to leave watertight shell for some time before filling the concrete. The four stages of forming these piles are shown in Fig. 26.17.

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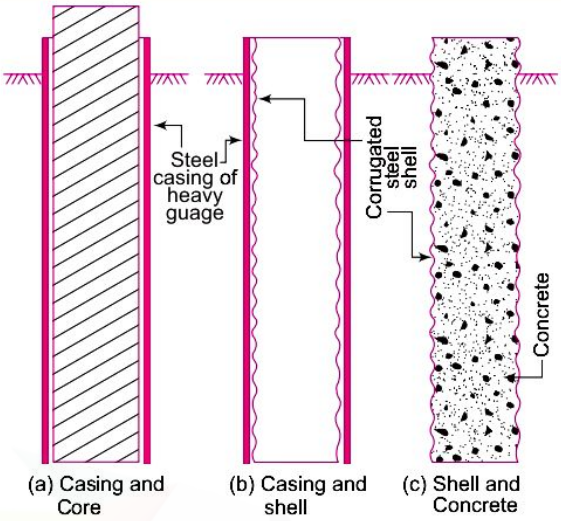

Fig. 26.16 Stages in Mcarthur cased pile

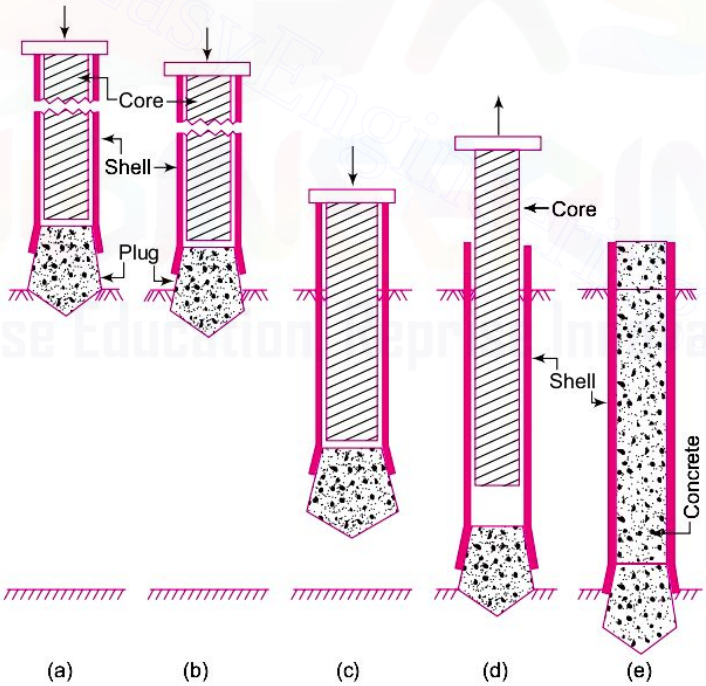

Fig. 26.17 Stages in swage pile construction

In the *first stage* [Fig.26.17 (*a*)], a thin steel pipe (known as shell) is placed on a *precast concrete* plug and a steel core, which is not long enough to reach the plug is inserted in the shell. In the *second stage* [Fig. 26.17 (*b*)], as the pipe is driven over the plug until the core reaches the plug, the pipe is *swaged out* by the taper of the plug, thus forming a watertight joint. In the *third stage* [Fig. 26.17 (*c*)], the pipe

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is driven to a specified depth. The driving force is practically all exerted by the core on the plug and the pipe is pulled down rather than driven. In the *fourth stage* [Fig. 26.17 (*d*)], after the pipe has reached the desired depth, the core is removed and the pipe left open until it is desired to fill it. In the *final stage* [Fig, 26.17 (*e*)], the pipe is filled with concrete.

5. Western button bottom pile: These piles are used in locations where increase in the end bearing area is desired. The pile uses a concrete plug, of the shape of a button. This button forms an enlarged hole in the soil during driving. Due to this, the side friction is reduced temporarily. These piles have been used up to lengths of about 23 m, and for loads up to 500 kN. The four stages in the pile driving are shown in Fig. 26.18.

In the *first stage* [Fig. 26.18 (*a*)], a steel pipe, with 12 mm thick walls and reinforced base of cast steel, is set over the concrete button. The concrete button has a diameter about 25 mm larger than the pipe. In the *second stage* [Fig. 26.18 (*b*)], the pipe and button are driven to a specified depth. In the *third stage* [Fig. 26.18 (*c*)], a corrugated steel shell is inserted in the pipe, resting on the button. A steel plate with a bolt hole in it is welded on the bottom of the shell, before lowering it, so that the hole may fit over the central bolt in button bottom. The nut may be tightened with the help of a long socket wrench. In the *fourth stage* [Fig. 26.18 (*d*)], the casing is withdrawn, leaving the button in place, and the shell is filled with concrete. Reinforcement may be used if necessary.

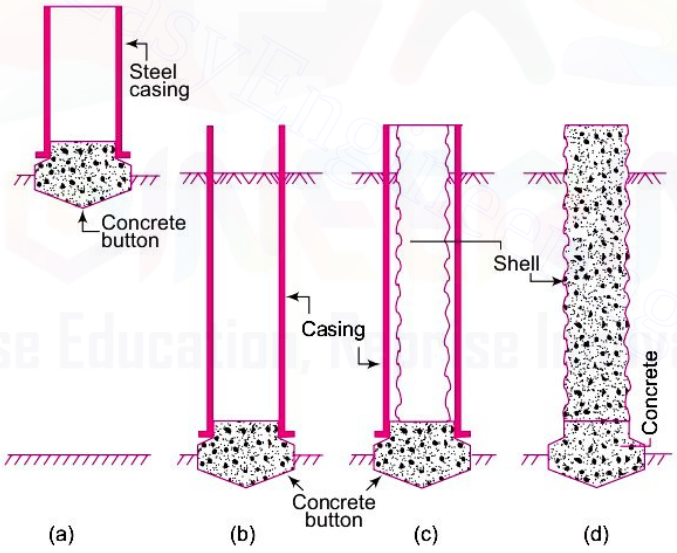

Fig. 26.18 Stages in button bottom pile construction

26.14 UNCASED CAST-IN-SITU CONCRETE PILES

Theses piles do not use casing, and hence are cheaper. However, great skill is required in their construction. These piles are used only where it is certain that neither soil nor water will fall into the hole, or squeeze into and reduce the size of the hole left after withdrawing a driven mandrel or shell before concreting and also where adjacent pile will not damage the green concrete. It is essential to have close

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installation inspection, since no inspection is possible after they are installed. These piles have the advantages that

(*i*) they need no storage space, (*ii*) they do not require cutting off excess lengths or building up short lengths, (*iii*) they do not require special handling equipment, and (*iv*) the concrete is not liable to damage from driving . The following are the common types of uncased cast-in-situ concrete piles:

(1) Simplex pile
(2) Franki pile
(3) Vibro pile and
(4) Padestal piles.

1. Simplex pile: Simplex pile can be driven through soft or hard soils. In this pile, a steel tube fitted with a cast iron shoe is driven into the ground up to the required depth, as shown in Fig. 26.19 (*a*). Reinforcement, if necessary, is put inside the tube. Concrete is then poured into the tube, and the tube is slowly withdrawn, without concrete being tamped, leaving behind the cast iron shoe.

Figure 26.19(*c*) shows the completed pile. The soil must be sufficiently firm to form a good mould for green concrete after the casing is withdrawn, or else an inner casing of slightly smaller diameter than the shell must be inserted before pouring the concrete. This pile is known as *Simplex standard pile*. If, however, tamping of concrete is done at regular interval as the tube is withdrawn, we get the *Simplex tamped pile.*

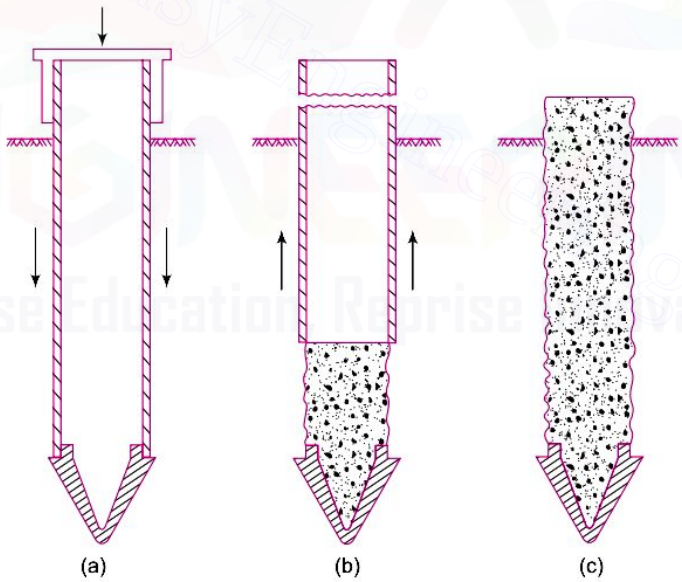

Fig. 26.19 Simplex standard pile

In the above method, the cast iron shoe remains behind, and a new shoe has to be used for each pile. But sometimes, if the soil is firm enough to stand, the cast iron shoe is provided with *alligator jaw point* [Fig. 26.20(*a*)]. This jaw point is hinged to the shell. When the concrete is poured, the jaw opens and allows concrete to flow out down into the hole.

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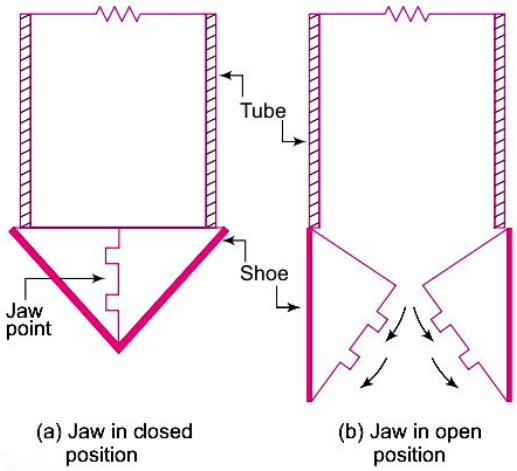

Fig. 26.20 Alligator jaw point

The jaw point is withdrawn gradually along with the steel tube as concrete is filled in. However, it would seem difficult to be certain that no soil would fall into the hole at the bottom.

2. Franki pile: This pile has an enlarged base of mushroom shape, which gives the effect of a spread footing. This pile is more useful where a bearing stratum of limited thickness can be reached at resonable depth. Also, this type of pile is best suited to granular soil. Fig. 26.21 shows various stages of forming the pile.

In the *first stage* a heavy removable pipe shell is set vertically on the ground with the help of leads [Fig. 26.21 (*a*)] and a charge of dry concrete or gravel is formed. In the *second stage,* a diesel operated drop hammer of 20 to 30 kN weight is driven on the concrete. This results in the formation of a dense plug that penetrates the ground and drags the tube with it on account of friction developed between the tube and the concrete plug [Fig. 26.21 (*b*)]. In the *third stage* [Fig. 26.21 (*c*)], when the tube has reached the desired depth, the tube is held in position by cables (leads) and the hammer is applied to the concrete, forcing it down and outward. This results in the enlargement of the base into the mushroom shape. If required, a fresh charge of semi-dry concrete is put to enlarge the bulb. In the *fourth stage,* the shaft is formed by introducing successive charges of concrete, ramming each in turn, and withdrawing gradually (about 300 mm at a time) the casing [Fig. 26.21 (*d*)]. Fig. 26.21 (*e*) shows the finally formed pile which has corrugations all along is height. Reinforcement cage can be installed, if desired, after the enlarged base has been formed (Stage 3). In that case, the hammer goes inside the cage of reinforcement. The pipe diameters in Franki piles vary from 50 cm to 60 cm, while the enlarged base may have a diameter of about 90 cm, or more. The pile has a carrying capacity of 60 t (600 kN) to 90 t (900 kN).

3. Vibro-piles: These piles are used where the ground is soft, thus offering little frictional resistance to the flow of concrete. Both *'standard'* and *'expanded'* piles are formed by the vibro-process. Vibro-piles are formed by driving a steel tube and shoe, filling with concrete, and extracting the tube, using upward extracting and downward tamping blows alternatively.

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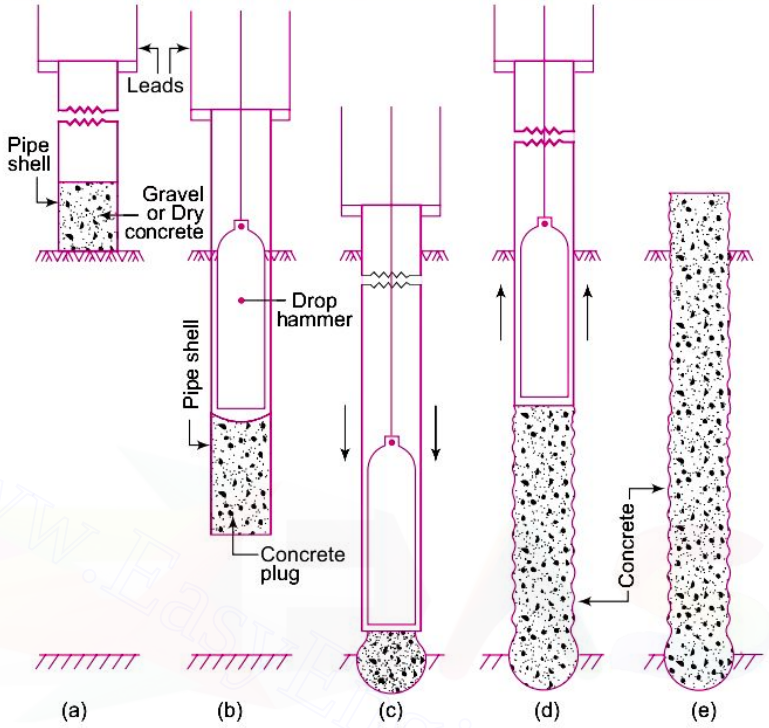

Fig. 26.21 Stages in the formation of franki pile

The three stages of formation of *standard* vibro-piles are shown in Fig. 26.22. *Standard* vibro-piles are made in size of 35, 45 and 50 cm dia., the larger for loads of 60 to 70 t (600 to 700 kN) respectively. They can be formed in the lengths of 25 m and over. A steel tube, fitted with (but not fixed to) a cast iron shoe is driven in the ground by 2 to 2.5t (20 to 25 kN) hammer, operated by steam or compressed air delivering up to 40 blows per minute with stroke of up to 1.4 m. When the shoe and the tube has reached the desired level [Fig. 26.22 (*a*)], corresponding to the desired set, extracting links are fitted to the hammer and top of the tube. The tube is now filled with concrete (usually 1:2:4 mix). The withdrawal of the tube and the ramming of the concrete are effected by hammer operating at 80 blows per minute. Each up-stroke results in 4 cm withdrawal of the tube (leaving the shoe behind) while concrete is consolidated in each downward blow. Thus, the concrete is being forced down to occupy the space left by the tube, resulting in corrugated face of pile [Fig. 26.22 (*c*)]. The corrugated face gives rise to increased frictional resistance and consequently, increase in the bearing capacity. If required, a reinforcement cage, at 150 to 200 mm pitch may be lowered after stage 1 [Fig. 26.22 (*a*)] is complete and concrete is poured. The hammer is operated through the inner space of the cage.

Vibro-expanded piles are used where the desired driving resistance is not obtained at reasonable depth due to low bearing capacity of soil. Its bearing capacity is increased by *enlarging* its diameter at the bottom. Figure 26.23 shows different stages in forming a vibro-expanded pile.

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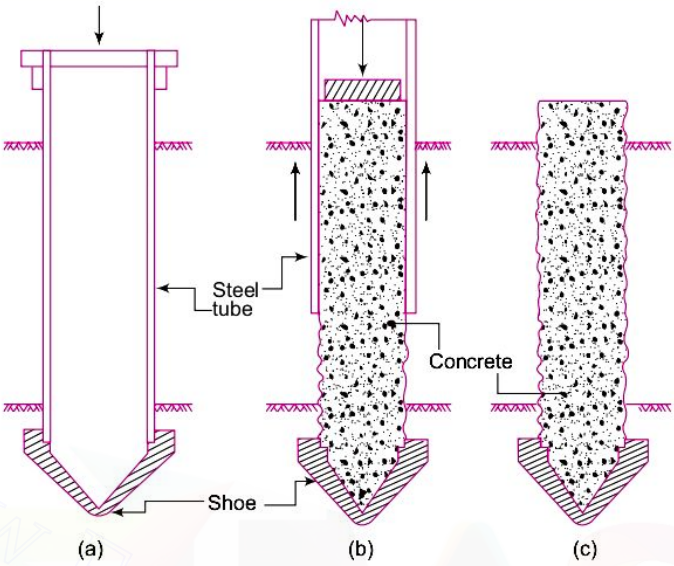

Fig. 26.22 Stages in the formation of standard vibro-piles

Procedure:

1. The tube, fitted with conical shoe is driven in the ground up to the desired depth [Fig. 26.23 (*a*)].
2. A charge of concrete is filled in the tube, up to some reasonable depth [Fig. 26.23 (*b*)].
3. The tube is completely withdrawn, in one single operation, leaving behind the conical shoe and the concrete over it [Fig. 26.23 (*c*)].
4. The tube, now fitted with a flat shoe, is again lowered in the hole until it reaches the deposited concrete [Fig. 26.23 (*d*)].

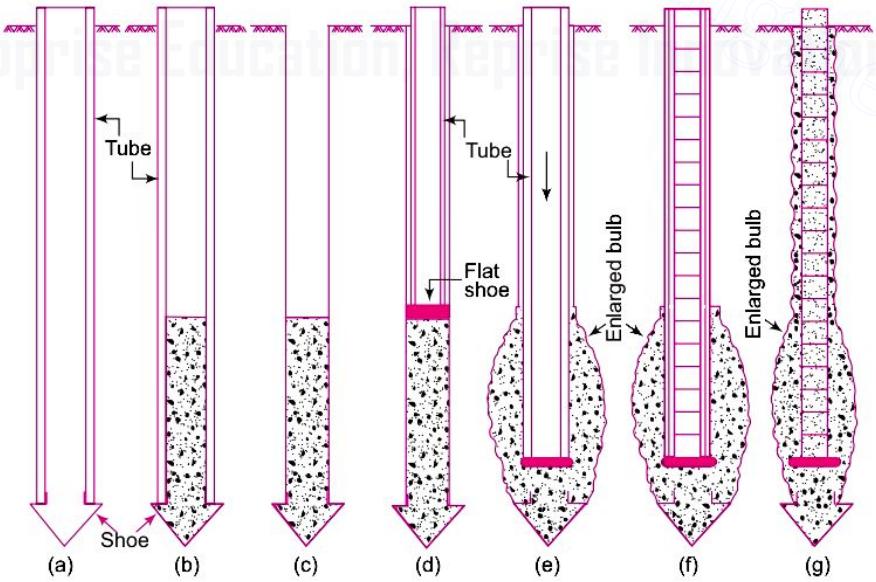

Fig. 26.23 Stages in the formation of vibro expanded piles

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5. The tube is driven down along with fiat shoe, to penetrate into the concrete, resulting in the formation of bulged end [Fig. 26.23 (*e*)].
6. The reinforcement cage is lowered in the tube, so that it rests on the flat shoe, (which is positioned at about a meter or two above the conical shoe), as shown in Fig. 26.23 (*f*).
7. The tube is now filled with concrete, and the pile is completed by succession of upward extracting and downward consolidating blows, as in the standard vibro pile. Fig. 26.23 (*g*) shows the finally formed pile.

4. McArthur padestal piles: The pile is used where thin bearing stratum is reached with reasonable depth. The padestal of the pile gives the effect of spread footing on this comparatively thin bearing. The pile uses a steel tube casing and a steel core, the lower end of the core being flush with the bottom of the casing and the end made flat. The stages of forming the pedestal piles are shown in Fig. 26.24.

Procedure:

1. The core and casing are driven together into the ground, till they reach the required level [Fig. 26.24 (*a*)].
2. The core is taken out, and a charge of concrete is placed in the tube [Fig. 26.24(*b*)].
3. The core is replaced in the casing till it rests on top of the poured concrete. While maintaining a pressure of the core and the hammer on the concrete, the casing is pulled up by 50 cm to 1 m [Fig. 26.24 (*c*)].
4. The charge of concrete is rammed out, thus resulting in the formation of padestal [Fig. 26.24 (*d*)].
5. The core is removed, the casing is filled with concrete, and core is replaced in contact with concrete [Fig. 26.24 (*e*)].
6. The casing is pulled up while maintaining the pressure of core and hammer on the concrete. The finished pile is shown in Fig. 26.24 (*f*).

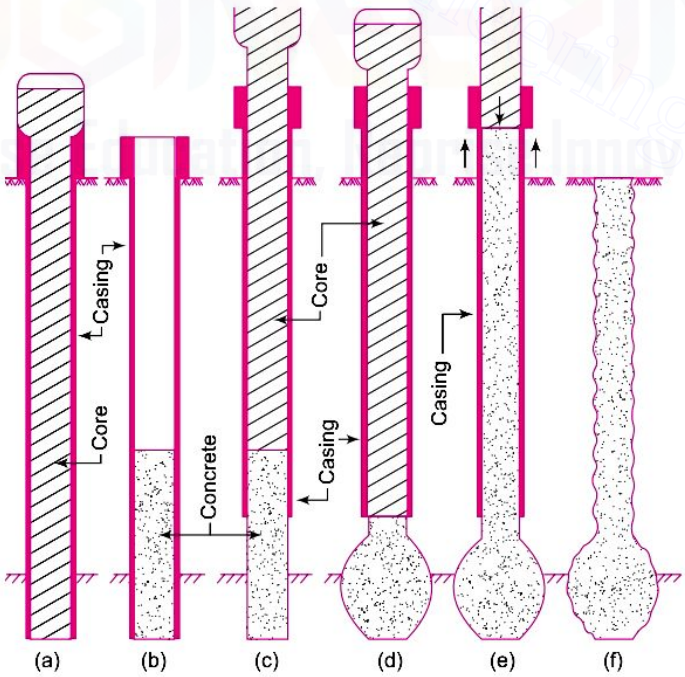

Fig. 26.24 Stages in the formation of Mcarthur padestal piles

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26.15 BORED PILES

Bored piles are those which are formed by forming a bore hole in the ground and then concreting it, either with the help of a casing tube or without a casing tube. Their procedure of construction is thus different than the cast-in-situ driven pile where a heavy pile driving equipment is required. Evidently, these piles have advantage over the driven piles, in those locations and those situations where the vibrations and noise caused by driving of piles are to be avoided or the strata of adequate bearing capacity is so deep that they are difficult to reach by *driven piles*.

Bores piles are of three types:

1. Pressure piles
2. Under-reamed piles and
3. Bored compaction piles

Under-reamed piles and bored compaction piles have been discussed in Fig. 26.11 and Fig. 26.12 respectively.

1. Pressure Piles: They are formed with the help of a casing tube, boring auger and compressed air equipment. These piles are especially suitable for those congested sites where heavy vibrations and noise are not permissible, and also where heavy pile driving machinery cannot move in. The stages in the construction of a pressure piles are shown in Fig. 2.25.

Procedure.

1. A 1.2 to 1.8 m long section of steel tube, 400 mm is dia, is sunk in the ground, whilst a boring tool, such as an auger working inside it, excavates the soil [Fig. 26.25 (*a*)].

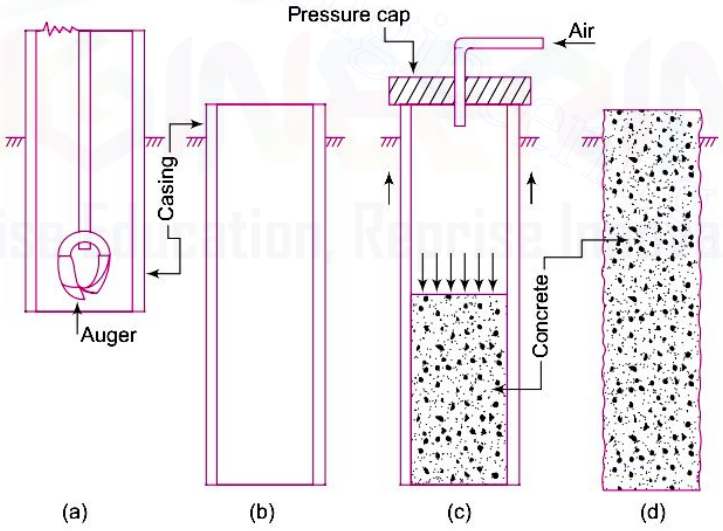

Fig. 26.25 Stages in the construction of pressure pile

2. Futher sections of 1.2 to 1.8 m long steel tubes are screwed successively and sunk in the ground, as boring proceeds, till the required depth is reached. The bored soil is continuously taken out. At the end, the boring tool is taken out and the hole is cleaned [Fig. 26.25 (*b*)].
3. A charge of concrete is placed in the tube, and the upper end of the tube is closed with the help of pressure cap. Compressed air is introduced through the air pipe of the pressure cap, thus forcing the concrete down and out against the surrounding soil. Simultaneously, the tube is slowly extracted

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with the help of a winch [Fig. 26.25 (*c*)]. The diameter of the pile exceeds that of the tube owing to the compression of soil, and rough irregular surface is formed which increases the frictional resistance of piles.

4. Fresh charges of concrete are placed in the tube, before the end of the tube comes above the previous charge of concrete, and the process of compressed air application is repeated, till the complete pile is cast and the tube is completely taken out [Fig. 26.25 (*d*)].

If it is required to increase the bearing value of the pile, an enlarged base is formed (before step 3) by introducing cement grout after the tube is sunk, and forcing it by air pressure into the adjacent soil. These piles are formed in three sizes: 340 mm dia (by using 300 mm dia. tubes), 440 mm dia. (400 mm tubes) and 500 mm dia. (460 mm tubes), up to a length of 25 m.

26.16 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 26.8. *A n pile group has to be proportioned in uniform pattern in a soft clay with equal spacing in both directions. Assuming any value for cohesion, determine the optimum spacing of the piles in the group. Take n = 25 and m = 0.7. Neglect the end bearing effect and assume that each pile is circular in section.* **(Civil Services Exam 1989)**

Solution. This question has been set from example 26.3 of this book. For solution, please see Example 26.3.

Example 26.9. *A group of 9 piles with 3 piles in a row was driven into soft clay extending from ground level to a great depth. The diameter and length of piles were 30 cm and 10 m respectively. The unconfined compressive strength of clay is 70 kN/m^2. If the piles were spaced at 90 cm centre to centre, compute the allowable load on the pile group on the basis of shear failure criteria for a factor of safety of 2.5. Neglect bearing at the tip of piles. Take m = 0.6 for shear mobilisation around each pile.* **(Civil Services Exam. 1990)**

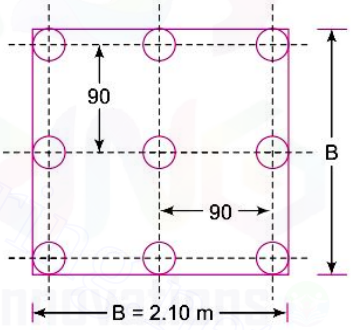

Fig. 26.26

Solution.

$$B = 90 \times 2 + 30 = 210 \text{ ; } c = c_u = 70 \text{ kN/m}^2$$

$$A_s = \pi\, d\, L = \pi\,(0.3)\ 10 = 9.425 \text{ m}^2$$

(*a*) Pile failing individually

$$Q_{un} = n\ (m\ cA_s) = 9\ (0.6 \times 70 \times 9.425) = 3562.6 \text{ kN} \quad ...(i)$$

(*b*) Piles failing in group

$$Q_{ug} = c\ (4\ BL) = 70 \times 4 \times 2.1 \times 10 = 5880 \text{ kN} \quad ...(ii)$$

Lesser of (1) and (2) = 3562.6 kN

$$\therefore \quad \text{Safe load, } Q_s = \frac{3562.6}{F} = \frac{3562.6}{2.5} = \mathbf{1425\ kN}$$

Example 26.10. *Design a friction pile group to carry a loading of 3000 kN including weight of pile cap at a site where the soil is uniform clay to a depth of 20 m under-lain by rock. Average unconfined compressive strength of clay is 70 kN/m^2. The clay may be assumed to be of normal sensitivity and normally loaded with liquid limit of 60%. Adopt a factor of safety of 3 against shear failure.*

(Civil Services Exam. 1991)

Solution. This problem has been set exactly from Example 26.5 of this book. Hence see example 26.5 for solution.

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Example 26.11. *Estimate the axial force in each of the pile beneath the footing shown in Fig. 26.27 (a).* **(Civil Services Exam. 1995)**

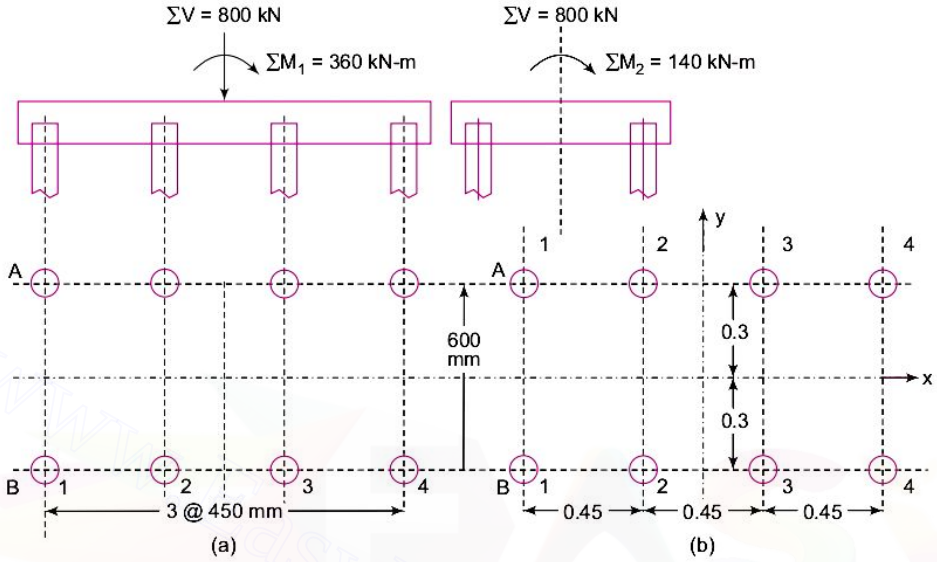

Fig. 26.27

Solution.

For biaxial BM, the force P in any pile is given by

$$P = \frac{\sum V}{n} \pm \frac{M_y \cdot x}{\sum x^2} \pm \frac{M_x \cdot y}{\sum y^2} \quad ...(i)$$

Here $\Sigma V = 800$ kN ; $n = 8$; $M_y = \Sigma M_1 = 360$ kN-m ; $M_x = \Sigma M_2 = 140$

$\Sigma x^2 = 4\,(0.675)^2 + 4(0.225)^2 = 2.025$ m^2 ; $\Sigma y^2 = 8(0.3)^2 = 0.72$ m^2

$$P_{A1} = \frac{800}{8} + \frac{360(-0.675)}{2.025} + \frac{140(+0.3)}{0.72} = \mathbf{38.33\ kN}$$

$$P_{A2} = \frac{800}{8} + \frac{360(-0.225)}{2.025} + \frac{140(+0.3)}{0.72} = \mathbf{118.33\ kN}$$

$$P_{A3} = \frac{800}{8} + \frac{360(+0.225)}{2.025} + \frac{140(+0.3)}{0.72} = \mathbf{198.33\ kN}$$

$$P_{A4} = \frac{800}{8} + \frac{360(1+0.675)}{2.025} + \frac{140(+0.3)}{0.72} = \mathbf{278.33\ kN}$$

$$P_{B1} = \frac{800}{8} + \frac{360(-0.675)}{2.025} + \frac{140(-0.3)}{0.72} = \mathbf{-78.33\ kN}$$

$$P_{B2} = \frac{800}{8} + \frac{360(-0.225)}{2.025} + \frac{140(-0.3)}{0.72} = \mathbf{1.67\ kN}$$

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$$P_{B3} = \frac{800}{8} + \frac{360(+0.225)}{2.025} + \frac{140(-0.3)}{0.72} = \mathbf{81.67\ kN}$$

$$P_{B4} = \frac{800}{8} + \frac{360(+0.675)}{2.025} + \frac{140(-0.3)}{0.72} = \mathbf{161.67\ kN}$$

Example 26.12. *A group of 9 piles arranged in a square pattern with diameter and length of each pile as 25 cm and 10 m respectively, is used as a foundation in soft clay deposit. Taking the unconfined compressive strength of clay as 120 kN /m^2 and the pile spacing as 100 cm centre to centre, find the load capacity of the group. Assume the bearing capacity factor $N_c = 9$ and adhesion factor = 0.75. A factor of safety of 2.5 may be taken.* **(Civil Services Exam. 1997)**

Solution.

(a) Piles acting individually: $c = q_{u/2} = 120/2 = 60\ \text{kN/m}^2$

$$Q_{up} = A_p \cdot r_p + A_s \cdot r_f$$

where $A_p = \frac{\pi}{4}(0.25)^2 = 0.04909\ m^2$; $A_s = \pi\ (0.25) \times 10 = 7.854\ \text{m}^2$

$r_p = cN_c = 9c = 9 \times 60 = 540\ \text{kN/m}^2$

$r_f = mc = 0.75 \times 60 = 45\ \text{kN/m}^2$

$\therefore \quad Q_{up} = 0.04909 \times 540 + 7.854 \times 45 = 380\ \text{kN}$

$\therefore$ Load capacity of 9 piles $= 9 \times 380 = 3419\ \text{kN}$ ...(*i*)

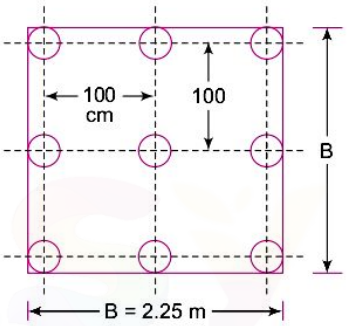

Fig. 26.28

(b) Pile acting as a group:

$$B = 2x + d = 2 \times 1 + 0.25 = 2.25\ \text{m (Fig. 26.28)}$$

$$Q_{ug} = A_p \cdot r_p + A_s \cdot r_f$$

where $A_p = B^2 = 2.25 \times 2.25 = 5.0624\ \text{m}^2$; $r_p = 9\,c = 9 \times 60 = 540\ \text{kN/m}^2$

$A_s = 4B \times 10 = 4 \times 2.25 \times 10 = 90\ \text{m}^2$; $r_f = c = 60\ \text{kN/m}^2$

$\therefore \quad Q_{ug} = 5.0625 \times 540 + 90 \times 60 = 2733.8 + 5400 = 8133.8\ \text{kN}$ ...(*ii*)

$\therefore \quad Q_{u,\,min.} = 3419\ \text{kg}$ and $Q_a = \frac{Q_{u,\min}}{F} = \frac{3419}{3.5} = \mathbf{1367.6\ kN}$

Example 26.13. *A group of 16 piles of 600 mm diameter is arranged in a square pattern with centre to centre spacing of 1.2 m. The piles are 10 m long and are embedded in soft clay with cohesion of 30 kN/m^2. Bearing resistance may be neglected for the piles. Adhesion factor is 0.6. Determine the ultimate load capacity of the pile group.* **(Civil Services Exam. 1998)**

Solution. Neglecting the bearing resistance,

$$Q_{up} = A_s \cdot r_f$$

(a) Piles acting individually

$$Q_{un} = n\,Q_{up} = n\,A_s \cdot r_f$$

where $A_s = \pi\ (0.6) \times 10 = 18.8496\ \text{m}^2$

$r_f = m\,c = 0.6 \times 30 = 18\ \text{kN/m}^2$

$\therefore \quad Q_{un} = 16 \times 18.8496 \times 18 = 5428.7\ \text{kN}$ ...(*i*)

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(b) Piles acting in group

$B = 3 \times 1.2 + 0.6 = 4.2$ m (Fig. 26.29)

$Q_{ug} = A_{sg} \cdot r_f$

$A_{sg} = 4\,Bh = 4 \times 4.2 \times 10 = 16.8\ \text{m}^2$

$r_f = c = 30\ \text{kN/m}^2$

$\therefore \quad Q_{ug} = 168 \times 30 = 5040\ \text{kN}$ ...(*ii*)

Ultimate load capacity = lesser of the two = **5040 kN**

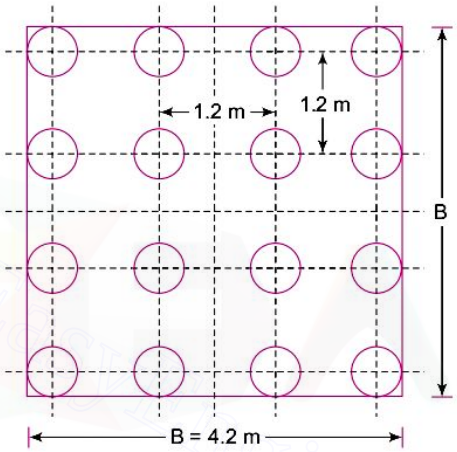

Fig. 26.29

Example 26.14. *A square group of 9 piles was driven into soft clay extending to a large depth. The diameter and length of the piles were 30 cm and 9 m, respectively. If the unconfined compression strength of clay is 9 t/m^2 and the pile spacing is 100 cm centre to centre, what is the capacity of the group? Assume factor of safety of 2.5 and adhesion factor 0.75.* **(Civil Services Exam. 1999)**

Solution. $B = 2 \times 100 + 30 = 230\ \text{cm} = 2.3$ m

$c_u = q_{u/2} = 9/2 = 4.5\ \text{t/m}^2$

(a) Piles acting individually

$Q_{un} = (A_p \cdot r_p + A_s\, r_f)\, n$

where $A_p = \dfrac{\pi}{4}(0.3)^2 = 0.07069\ \text{m}^2$

$r_p = 9\ c_u = 9 \times 4.5 = 40.5\ \text{t/m}^2$

$A_s = \pi\, dL = \pi(0.3)\ 9 = 8.4823\ \text{m}^2$

$r_f = mc_u = 0.75 \times 4.5 = 3.375\ \text{t/m}^2$

$\therefore \quad Q_{un} = 9\ (0.07069 \times 40.5 + 8.4823 \times 3.375) = 283.4\ t$ ...(*i*)

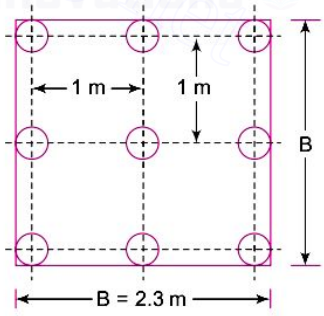

Fig. 26.30

(b) Piles acting in a group

$Q_{ug} = A_{pg} \cdot r_p + A_{sg} \cdot r_f$

where $A_{pg} = B^2 = (2.3)^2 = 5.29\ \text{m}^2$; $r_p = 9\ c_u = 9 \times 4.5 = 40.5\ \text{t/m}^2$

$A_{sg} = 4BL = 4 \times 2.3 \times 9 = 82.8\ \text{m}^2$;

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$r_f = c_u = 4.5$ t/m^2

$\therefore \quad Q_{ug} = 5.29 \times 40.5 + 82.8 \times 4.5 = 586.85$ t ...(*ii*)

Lesser of the two = 283.4 t

$\therefore$ Load capacity $= \dfrac{283.4}{2.5} =$ **113.36** *t*

Example 26.15. *A square pile group of 16 piles penetrates through a filled up soil of 3 m depth. The pile diameter is 250 mm and pile spacing is 0.75 m. The unit cohesion of the material is 18 kN/m^2 and the unit weight of soil is 15 kN/m^3. Draw plan and sectional elevation of the pile group and compute the negative skin friction on the group.* **(Civil Services Exam. 2002)**

Solution. $B = 3 \times 0.75 + 0.25 = 2.5$ m

Assume $m = 0.4$

(a) Pile acting individually

$Q_{un} = n(m\ c\ p\ L_f)$

$= 16\ (0.4 \times 18 \times \pi \times 0.25 \times 3)$

$= 271.54$ kN ...(*i*)

(b) Pile acting in a group

$Q_{ug} = c(4\ B)L_f + \gamma L_f B^2$

$= 18 \times 4 \times 2.5 \times 3 + 15 \times 3\ (2.5)^2$

$= 540 + 281.3 = 821.3$ kN ...(*ii*)

$\therefore$ Greater of the above two = **821.3 kN**

Hence negative skin friction = 821.3 kN

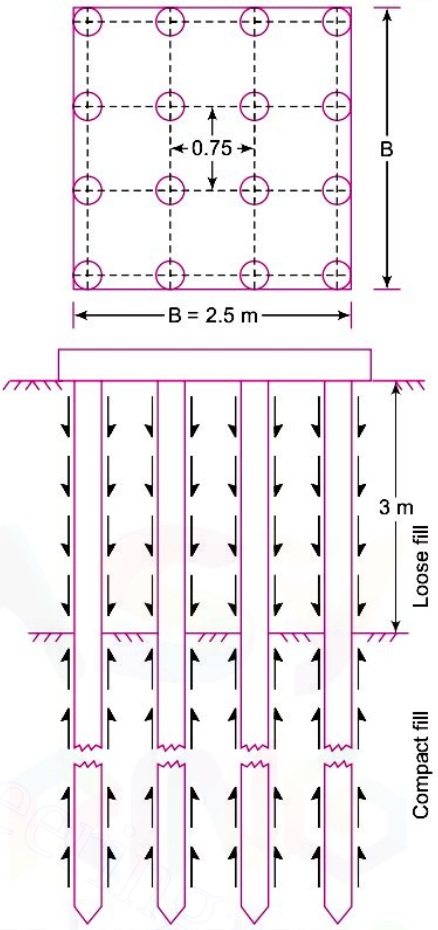

Fig. 26.31

Example 26.16. *A group of 9 piles, 10 m long is used as a foundation for a bridge pier. The piles used are 30 cm diameter with centre to centre spacing of 0.9 m. The subsoil consists of clay with unconfined compressive strength of 1.5 kg/cm^2. Determine the efficiency neglecting the bearing action. Given: adhesion factor = 0.9.* **(Engg. Services Exam. 1992)**

Solution. $B = 2 \times 0.9 + 0.3 = 2.1$ m

$c = q_{u/2} = 1.5/2 = 0.75$ kg/cm^2 $= 7.5$ *t*/m^2

(a) Piles acting individually

$Q_{un} = n\ .\ mc\ .\ A_s = 9 \times 0.9 \times 7.5\ (\pi \times 0.3 \times 10)$

$= 572.6\ t$ ...(*i*)

(b) Piles acting in a group

$Q_{ug} = c\ .\ (4\ BL) = 7.5 \times 4 \times 2.1 \times 10 = 630\ t$ ...(*ii*)

$\therefore$ Efficiency for pile group, $\eta = \dfrac{Q_{ug}}{Q_{un}} = \dfrac{630}{572.6} =$ **1.1**

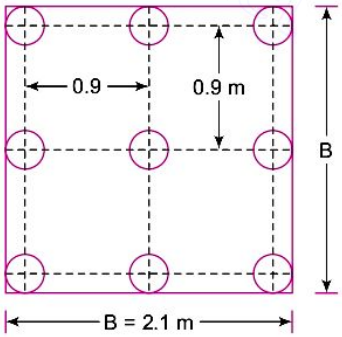

Fig. 26.32

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Example 26.17. *Design a friction pile group to carry a load of 300 tonnes including the weight of the pile cap at a site where the soil is uniform clay to a depth of 20 m underlain by rock. Average unconfined compressive strength of the clay is 0.7 kg/cm^2. The clay may be assumed to be of normal sensitivity and normally loaded with liquid limit 60%. A factor of safety of 3 is required against shear failure.*

(Engg. Services Exam. 1993)

Solution. This problem was set exactly from example 26.5 (MKS version, tenth edition). For solution, see Example 26.5 which is the SI version of the above problem.

Example 26.18. *A jetty is sited on 4.5 m^2 of very soft clayey soil overlying stiff to very stiff clay. The average shear strength from 5 to 13 m is 140 kN/m^2. Fissured shear strength at 13 m is 145 kN/m^2. For structural reasons, it is desired to use 500 mm steel tube piles. Check the penetration length of 13 m below sea bed level from the design view point, to carry a safe working load of 400 kN. For the given conditions, adhesion factor = 0.5, bearing capacity factor = N_c = 9. Give your comments on the penetration length of 13 m.* **(Engg. Services Exam. 1994)**

Solution. Average shear strength from 5 m to 13 m = 140 kN/m^2

Q_{up} = Bearing resistance + frictional resistance (Shaft resistance)

$$Q_{up} = R_p + R_f = A_p \,.\, r_p + A_s \,.\, r_f$$

where A_p = area of cross-section of pile

$$= \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$$

$$r_p = c\, N_c = 145 \times 9 = 1305 \text{ kN/m}^2$$

A_s = Surface area of pile = $\pi\, dL = \pi \times 0.5 \times 8$

$$= 12.56 \text{ m}^2$$

$$r_f = m_c = 0.5 \times 140 = 70 \text{ kN/m}^2$$

$$Q_{up} = 0.1963 \times 1305 + 12.56 \times 70 = 256.2 + 879.2$$

$$= 1135.4 \text{ kN}$$

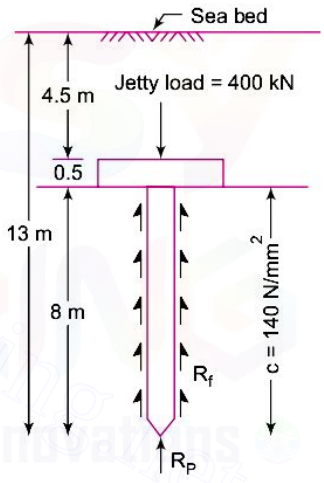

Fig. 26.33

Actual load, Q_a =400 kN

$$\therefore \text{ Available factor of safety, } F = \frac{1135.4}{400} = 2.84$$

Desirable overall F.S. = 3.

Hence the pile seems to be slightly under safe.

Example 26.19. *200 mm diameter, 8 m long piles are used as foundation for a column in a uniform deposit of medium clay (unconfined compressive strength = 100 kN/m^2 and adhesion factor = 0.9). There are nine piles arranged in a square pattern of 3 × 3. For a group efficiency = 1.0, find the spacing between the piles (neglect bearing).* **(Engg. Services Exam. 1996)**

Solution. $B = 2\,x + 0.2$;

$$c = q_{u/2} = 100/2 = 50 \text{ kN/m}^2$$

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(a) *Pile acting alone*

$$Q_{un} = n\ mc.A_s = 9 \times 0.9 \times 50\ (\pi \times 0.2 \times 8)$$
$$= 2035.8 \text{ kN} \qquad ...(i)$$

(b) *Pile acting in group*

$$Q_{ug} = c\ A_s = c\ (4\ BL) = 50 \times 4\ (2x + 0.2) \times 8$$
$$= 3200(x + 0.1) \text{ kN} \qquad ...(ii)$$

Now efficiency $\eta = \dfrac{Q_{ug}}{Q_{un}} = 1$ (given)

$\therefore$ $3200\ (x + 0.1) = 2035.8$

From which $x =$ **0.536 m**

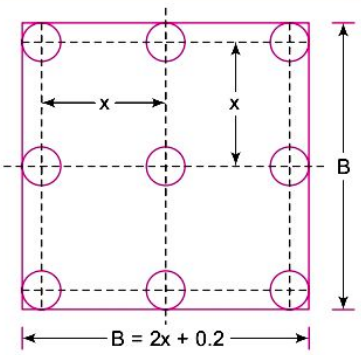

Fig. 26.34

Example 26.20. *A square group of 25 piles extends between depth of 2 m and 12 m in a deposit of 20 m thick stiff clay overlying rock. The piles are 0.5 m in diameter and are spaced at 1 m centre to centre in the group. The undrained shear strength of the clay at the pile base level is 180 kPa and the average value of the undrained shear strength over the depth of the pile is 110 kPa. The adhesion coefficient (α) is 0.45. Estimate the capacity of the pile group considering an overall factor of safety equal to 3 against shear failure. N_c corresponding to $\varphi_u = 0$ is 9.* **(Gate Exam. 1994)**

Solution.

(a) *Piles acting individually*

$$Q_{un} = n\ (A_p\ r_p + A_s\ r_f)$$

$$A_p = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$$

$$r_p = c_u\ N_c = 180 \times 9 = 1620 \text{ kN/m}^2$$

$$A_s = \pi d\ L = \pi \times 0.5\ (12 - 2) = 15.708 \text{ m}^2$$

$$r_f = \alpha\ c_u = 0.45 \times 110 = 49.5 \text{ kN/m}^2$$

$\therefore$ $Q_{un} = 25\ (0.1963 \times 1620 + 15.708 \times 49.5)$

$= 27388.8$ kN ...(*i*)

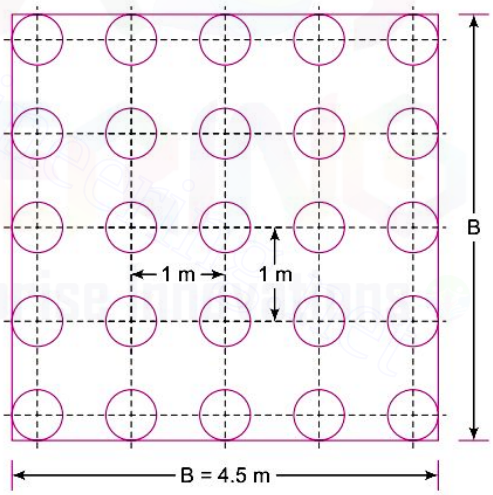

Fig. 26.35

(b) *Piles acting in a group*

$$B = 4 \times 1 + 0.5 = 4.5 \text{ m}$$

$$Q_{ug} = A_{pg}\ r_p + A_{sg}\ r_{fg}$$

$$A_{pg} = B \times B = (4.5)^2 = 20.25 \text{ m}^2$$

$$r_p = c_u\ N_c = 180 \times 9 = 1620 \text{ kN/m}^2$$

$$A_{sg} = 4\ BL = 4 \times 4.5\ (12 - 2) = 180 \text{ m}^2$$

$$r_{fg} = C_u = 110 \text{ kN/m}^2$$

$\therefore$ $Q_{ug} = 20.25 \times 1620 + 180 \times 110 = 52605$ kN ...(*ii*)

$\therefore$ Ultimate load capacity lesser of (*i*) and (*ii*) = 27388.8 kN

$\therefore$ Capacity of pile group $= \dfrac{27388.8}{3} =$ **9129.6 kN**

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Example 26.21. *Estimate the safe load carrying capacity of a single bored pile 20 m long, 500 mm diameter. The adhesion coefficient (α) is 0.4. Take a factor of safety of 2.5. The soil strata is as follows:*

Depth (m)	*Soil deposit*	*Undrained shear strength (s_u) kPa*
0 –5	*Loose fill*	*50*
5 –10	*Weathered over-consolidated clay*	*70*
10 –15	*Over-consolidated clay*	*100*
15 –20	*Highly over-consolidated clay*	*200*

Assume that $\varphi_u = 0$ is valid and $N_c = 9$ for deep foundations. **(Gate Exam. 1998)**

Solution. $Q_{up} = A_p \,.\, r_p + A_s \,.\, r_f$

where A_p = point bearing area $= \dfrac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

$A_s = \pi\, d\, L = \pi\,(0.5) \times 20 = 31.416 \text{ m}^2$

$r_p = cN_c = 9\, c_u = 9\, s_u = 9 \times 200 = 1800 \text{ kN/m}^2$

$r_f = \alpha\, c_{ua} = 0.4\, c_{ua}$

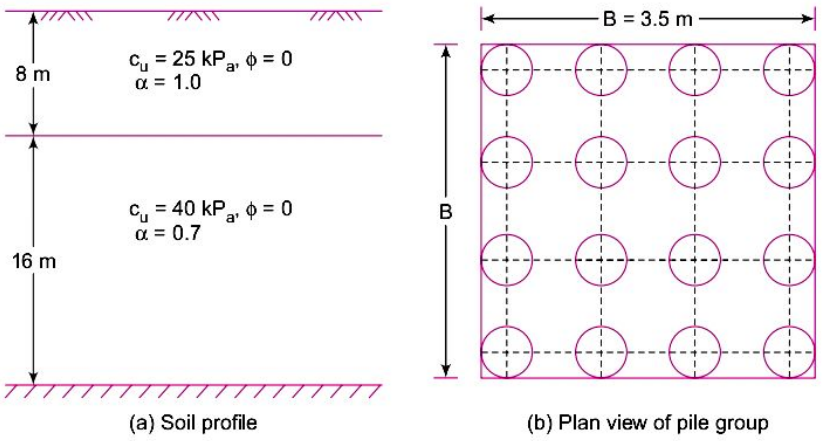
where c_{ua} = average shear strength of the depth

$$c_{ua} = \frac{50 \times 5 + 70 \times 5 + 100 \times 5 + 200 \times 5}{20} = 105 \text{ kN/m}^2$$

$\therefore$ $Q_{up} = 0.1963 \times 1800 + 31.416 \times 0.4 \times 105 = 1672.8 \text{ kN}$

$\therefore$ Safe load $Q_s = \dfrac{Q_{up}}{F} = \dfrac{1672.8}{2.5} = \mathbf{669.1\ kN}$

Example 26.22. *A group of 16 piles (4 in each row) was installed in a layered clay soil deposit as shown in Fig. 26.36. The diameter of each pile is 500 mm and their c/c distance is 1 m. The length of the pile group is 18 m. Estimate the safe load capacity of the group with a factor of safety of 2.50. The adhesion factor (α) between the pile and soil in each soil layer are shown in Fig. 26.36.* **(Gate Exam. 2001)**



(a) Soil profile (b) Plan view of pile group

Fig. 26.36

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Solution.

(a) Piles acting individually

$$Q_{un} = n\,[A_p\, r_p + A_s\, r_f] = n\,[A_p\, r_p + \pi\, d\,(L_1\, \alpha_1\, c_{u1} + L_2\, \alpha_2\, c_{u2})]$$

For piles in clay, $r_p = c_u\, N_c = 9\, c_u = 9 \times 40 = 360$ kN/m^2

$$A_p = \frac{\pi}{4}\,(0.5)^2 = 0.1963 \text{ m}^2$$

$$\therefore \quad Q_{un} = 16\,[0.1963 \times 360 + \pi \times 0.5\,(8 \times 1 \times 25 + 10 \times 0.7 \times 40)] = 13194 \text{ kN/m}^2 \quad ...(i)$$

(b) Piles acting in a group

$$B = 3 \times 1 + 0.5 = 3.5 \text{ m}$$

$$Q_{ug} = A_{pg} \cdot r_p + A_{sg}\, r_{fg}$$

where

$$A_{pg} = 3.5 \times 3.5 = 12.25 \text{ m}^2$$

$$r_p = c_u\, N_c = 9\, c_u = 9 \times 40 = 360 \text{ kN/m}^2$$

$$A_{sg} = 4\,B\,L_1 + 4\,B\,L_2$$

$r_{sg} = c_{u1}$ for first 8 m and c_{u2} for next 10 m

$$\therefore \quad Q_{ug} = 12.25 \times 360 + 4 \times 3.5\,[8 \times 25 + 10 \times 40] = 4410 + 8400 = 12810 \text{ kN} \quad ...(ii)$$

Lesser of (i) and (ii) $= 12810$ kN

$$\therefore \text{ Safe load} = \frac{12810}{2.5} = \mathbf{5124\ kN}$$

Example 26.23. *A group of 16 piles (diameter = 500 cm, length = 14 m, centre to centre spacing = 1 m) arranged in a square pattern passes through a recent fill (thickness = 3 m) overlying a soft clay deposit (thickness = 5 m) which is consolidating under the fill load and rests in a stiff clay strata (Fig. 26.37 a) All the strata are saturated. The soil properties of different strata are:*

Type of soil	*Unit weight (γ) kN/m^3*	*Strength parameters*		*Adhesion parameter (α)*
		c_u (kPa)	*φ_u*	
Fill	*16*	*50*	*0*	*0.60*
Soft clay	*17*	*20*	*0*	*0.40*
Stiff clay	*21*	*70*	*0*	*0.45*

Estimate the ultimate load capacity (Q_u) of the pile group. **(Gate Exam. 2002)**

Solution.

(a) Pile acting individually

$$Q_{un} = n\,(A_p\, r_p + A_s\, r_f)$$

where

$$A_p = \frac{\pi}{4}\,(0.5)^2 = 0.1963 \text{ m}^2\ ;\ r_p = cN_c = 9\, c_u = 9 \times 70 = 630 \text{ kN/m}^2$$

$$A_s = \pi\, d\,(L_1 + L_2 + L_3)\ ; \qquad r_f = \alpha\, C_u \text{ for various strata.}$$

$$\therefore \quad Q_{un} = 16\,[0.1963 \times 630 + \pi \times 0.5\,(L_1\, \alpha_1\, cu_1 + L_2\, \alpha_2\, cu_2 + L_3\, \alpha_3\, cu_3)]$$

$$= 16\,[123.669 + \pi \times 0.5\,(3 \times 0.6 \times 50 + 5 \times 0.4 \times 20 + 6 \times 0.45 \times 70)]$$

$$= 1978.7 + 8017.3 = 9996 \text{ kN} \quad ...(i)$$

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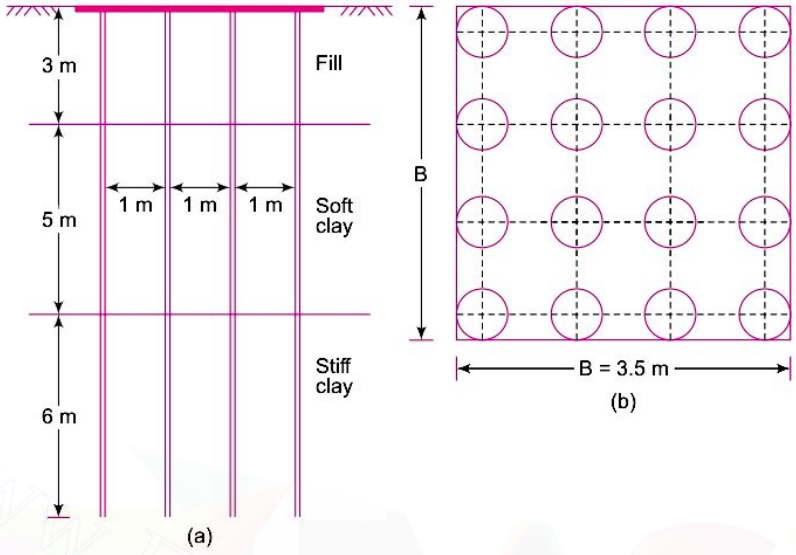

Fig. 26.37

(b) *Pile acting in group*

$$B = 3 \times 1 + 0.5 = 3.5 \text{ m}$$

$$Q_{ug} = B^2 r_p + 4B [L_1 cu_1 + L_2 cu_2 + L_3 cu_3]$$

$$= (3.5)^2 \times 9 \times 70 + 4 \times 3.5 [3 \times 50 + 5 \times 20 + 6 \times 70]$$

$$= 7717.5 + 9380 = 17097.5 \text{ kN} \quad ...(ii)$$

Lesser of the two $= 9996$ kN

$\therefore$ Ultimate load capacity = **9996 kN**

PROBLEMS

1. A single acting steam hammer weighing 2000 N and falling through a height of 80 cm drives a pile to an average penetration of 1 cm per blow under the last few blows. Determine the allowable load for the pile, using Enginering News Formula. [**Ans.** 213 kN]
2. A concrete pile 40 cm × 40 cm section and 20 m long is driven by a drop hammer having a weight of 40 kN and falling through a height of 1 m. The average penetration under last 10 blows is equal to 6 mm per blow. The efficiency of hammer = 100%, the coefficient of restitution is 0.4 and the total elastic compression 25 mm. Using Hilley's formula, determine (*i*) Q_f, (*ii*) the set to which the pile should be driven under the above conditions if Q_f desired is 1600 kN and (*iii*) $(Q_f)_{max.}$
[**Ans.** (*i*) 968 kN (*ii*) $S = -0.13$ cm (*i.e.* pile will be compressed and it will not be capable of developing Q_f=1600 kN, (*iii*) $(Q_f)_{max.} = 1432$ kN (at $S = 0$)].
3. A square pile of section 50 × 50 cm and 10 m long penetrates a deposit of clay with $c = 40$ kN/m^2. Taking $m = 0.7$, determine the load carried by the pile by skin friction. [**Ans.** 560 kN]
4. *A 25 pile square group has to be proportioned in a uniform pattern in clay with equal spacing in all directions. Taking $c = 0.7$, determine the optimum value of spacing. Neglect end bearing effect of the group. Each pile is square in cross-section, with sides of length a. [**Ans.** $s = 4.125$]

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5. *Determine the settlement in Example 26.6 if the length of piles is increased by 3 m. Other data remain unchanged. [**Ans.** 15.1 cm]
6. *A 45 cm diameter concrete pile 7 m long is subjected to a horizontal load of 20 kN at a height of 2 m above the ground. Taking $\eta_h = 20$ N/cm^3 and E for R.C.C. pile $= 3 \times 10$ N/cm^2, calculate the maximum deflection and maximum moment induced in the pile.
[**Ans** (*i*) $\gamma_{max} = 0.323$ cm (*ii*) $M_{max} = 52.7$ kNm at a depth of 1 m below ground surface]
7. *A single pile with free head is subjected to lateral load at the ground surface. The deflection under a load of 20 kN is 2 cm. Compute the deflection if the pile were fixed at the head. [**Ans.** 0.76 cm.]

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CHAPTER 27

Well Foundations

27.1 INTRODUCTION: CAISSONS

The term 'Caisson' is derived from the French word, *caisse,* meaning a chest or box. Caisson has come to mean a boxlike structure, round or rectangular, which is sunk from the surface of either land or water to some desired depth. The caissons are of three types:

(*i*) Box caissons (*ii*) Open caissons (wells) (*iii*) Peneumatic caissons.

A *box caisson* is open at top and closed at the bottom and is made of timber, reinforced concrete or steel. This caisson is built on land, then launched and floated to pier site where it is sunk in position. Such a type of caisson is used where bearing stratum is available at shallow depth, and where loads are not very heavy. Closed box caissons are also used for break waters and sea walls. An *open caisson* is a box of timber, metal, reinforced concrete or masonry which is open both at the top and at the bottom, and is used for building and bridge foundations. Open caissons are called *wells. Well foundations* form the most common type of deep foundations for bridges in India. A *pneumatic caisson* has its lower end designed as a working chamber in which compressed air is forced to prevent the entry of water and thus permit excavation in dry.

Whenever considerations for scour or bearing capacity require foundations being taken to a depth of more than 5 to 7 metres, open excavations become costly and uneconomic as heavy timbering has to be provided. Also because of the greater earth work involved due to side slopes, the progress of the work in open excavation will be very slow. Another disadvantage in adopting the ordinary type of footing is that excavated material refilled around the structure is loose and hence easily scourable as compared to natural ground. The above disadvantages are avoided in a well foundation which is a shell sunk by dredging inside of it and which finally becomes a part of the permanent structure.

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27.2 SHAPES OF WELLS AND COMPONENT PARTS

The common types of well shapes are as follows (Fig. 27.1) :

(1) Single circular
(2) Twin circular
(3) Dumb-well
(4) Double-D
(5) Twin-hexagonal
(6) Twin-octagonal
(7) Rectangular.

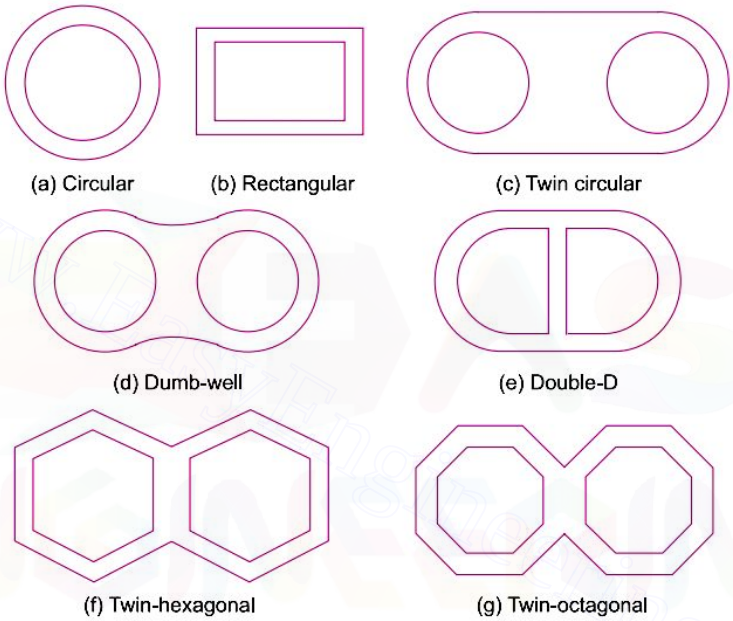

Fig. 27.1 Shapes of wells

The choice of a particular shape depends upon the dimensions of the base of the pier or abutment, the care and cost of sinking, the considerations of tilt and shift during sinking and the vertical and horizontal forces to which the well is subjected. A circular well has the minimum perimeter for a given dredge area and hence the ratio of sinking effort to skin friction is maximum. Also, since the perimeter is equidistant at all points from the centre of dredge hole, the sinking is more uniform than for other shapes. However, the disadvantage of a circular well is that in the direction parallel to the span of the bridge, the diameter of the well is much more than the minimum size required to accommodate the bridge pier and hence the circular well causes more obstruction to waterway than the bridge pier does. The disadvantage is avoided in the case of a double-D shape which conforms to the shape of the bridge pier in plan. The dredge area is also smaller for a double-D. Hence for large piers, a double-D is more economical than a single circular well. Twin-circular well aim at combining the advantage of a circular well and of a double-D, but the only snag is that the two wells sunk close to each other have a tendency to close in or move apart. However, in abutments and wing walls where the tilt and shift in position are not important, a battery of small diameter wells are adopted with advantage. The double hexagon and double octagon types, though provide efficient grabbing to all parts of the curb, suffer from the disadvantage that owing to sharp corners they can dig and are, therefore, more likely to tilt. Also, the sharp corners produce greater scour.

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Figure 27.2 shows a typical section of a well foundation with its component parts. The following components of a well have to be considered in the design of a well foundation:

(*i*) Well curb and cutting edge.
(*ii*) Staining.
(*iii*) Bottom plug.
(*iv*) Well cap.

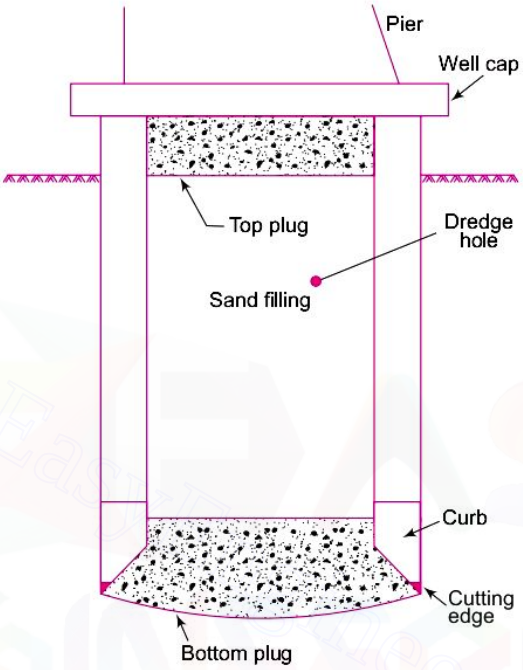

Fig. 27.2 Section of a well foundation

27.3 DEPTH OF WELL FOUNDATION AND BEARING CAPACITY

The selection of the depth of a well is based on the following two criteria:

1. There should be adequate embedded length of well, called the *grip length* below the lowest scour level. In addition to minimum Rankine depth consideration, this is required for developing sufficient passive resistance to counteract the overturning moment due to horizontal forces acting on the bridge deck, as well as those due to wind and water.
2. The well should be taken deep enough to rest on strata of adequate bearing capacity in relation to the loads transmitted.

For alluvial soils, mostly met with in North Indian rivers, the *normal scour depth* can be calculated by Lacey's Formula:

$$R_L = 1.35\left(\frac{q^2}{f}\right)^{1/3} \qquad ...(27.1)$$

where q = discharge in cumecs per linear metre of water way

f = Lacey's silt factor = $1.76\sqrt{m_d}$ ...(27.2)

m_d = mean weighted diameter in mm.

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The maximum depth of scour, at the nose of pier, is found to be twice the Lacey's value of normal scour depth:

$$R = 2R_L \qquad ...(27.3)$$

where R is measured below the high flood level (HFL).

$$\therefore \quad \text{Scour level} = \text{H.F.L.} - R = \text{H.F.L.} - 2R_L \qquad ...(27.4)$$

The grip length is taken as $\frac{1}{3}R$ below the scour level according to the code of practice of the Indian Roads Congress and as $\frac{1}{2}R$ in Railway practice. This means that the depth of foundation should be at least $1\frac{1}{3}R$ below HFL according to IRC code, and $1\frac{1}{2}R$ below HFL according to Railway practice.

It is further recommended that the minimum depth of embedment below the scour level should not be less than 2.0 m for piers and abutment with arches and 1.2 m for piers and abutments supporting other types of superstructure. According to Terzaghi and Peck, the ultimate bearing capacity can be determined from the following expression:

$$Q_f = Q_p + 2\pi R f_s D_f \qquad ...(27.5)$$

$$Q_P = \pi R^2 (1.2\, cN_c + \gamma D_f N_q + 0.6\, \gamma R N_\gamma) \qquad ...(27.6)$$

where, N_c, N_q, N_γ = Terzaghi's bearing capacity factors

R = radius of well

D_f = depth of well (depth of foundation)

f_s = average skin friction

27.4 FORCES ACTING ON A WELL FOUNDATION

In addition to the self-weight and buoyancy, a well carries the dead load of the super-structure, bearings pier and is liable to the following horizontal forces :

(*i*) bracking and tractive effort of the moving vehicles.
(*ii*) force on account of resistance of the bearings against movement due to variation of temperature,
(*iii*) force on account of water current,
(*iv*) wind forces,
(*v*) seismic forces,
(*vi*) earth pressure,
(*vii*) centrifugal forces.

The magnitude, direction and point of application of all the above forces can be found under the worst possible combinations and they can be replaced by two horizontal forces, P and Q and a single vertical force W as shown in Fig. 27.3.

P = Resultant of all horizontal forces in the direction across the pier.

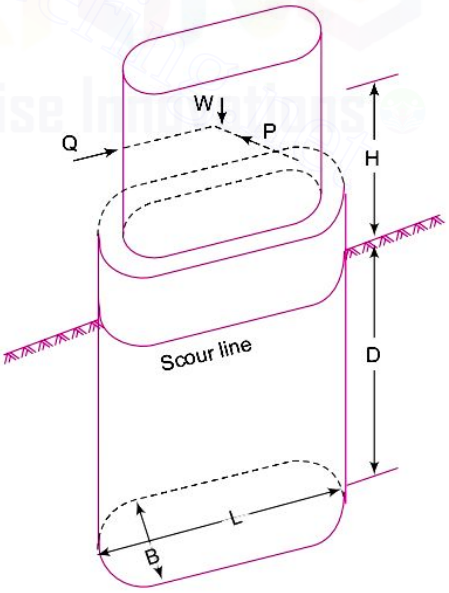

Fig. 27.3 Forces on a well

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Q = Resultant of all horizontal forces in the direction along the pier.

W = Resultant of all vertical forces.

The analysis is done on the following assumptions (Banerjee and Gangopadhyay, 1960):

1. The well is acted upon by an unidirectional horizontal force P in a direction across the pier.
2. The well is founded in sandy stratum.
3. The resultant unit pressure on soil at any depth is in simple proportion to horizontal displacement.
4. The ratio between contact pressure and corresponding displacement is independent of the pressure.
5. The co-efficient of vertical sub-grade reaction has the same value for every point of surface acted upon by contact pressure.

The analysis that follows is that suggested by Banerjee and Gangopadhyay (1960).

27.5 ANALYSIS OF WELL FOUNDATION

1. Horizontal soil reactions: When a rigid well, embedded in sand, starts moving parallel to its original position, under the action of a horizontal force P, it transforms the soil on one side to passive state of plastic equilibrium and the other side into active state. Assuming that the well movement ρ_1 is sufficient to mobilise fully the active and passive earth pressure, the resultant unit pressure at a depth z below the surface is given by

$$p_1 = \gamma \, . \, z \, (K_p - K_a) \qquad ...(27.7)$$

where γ = unit weight of soil K_p, K_a = co-efficient of passive and active earth pressure, and depend upon the angle of internal friction ϕ, and angle of wall friction δ.

Let p be the load per unit area of vertical surface of sand and ρ be the corresponding displacement. Assuming that ρ_1 is the displacement required to increase the value of resultant unit pressure form zero to p_1, we have.

$$p = p_1 \frac{\rho}{\rho_1} = \frac{\rho}{\rho_1} \gamma \, z \left(K_p - K_a\right) \qquad ...(27.8)$$

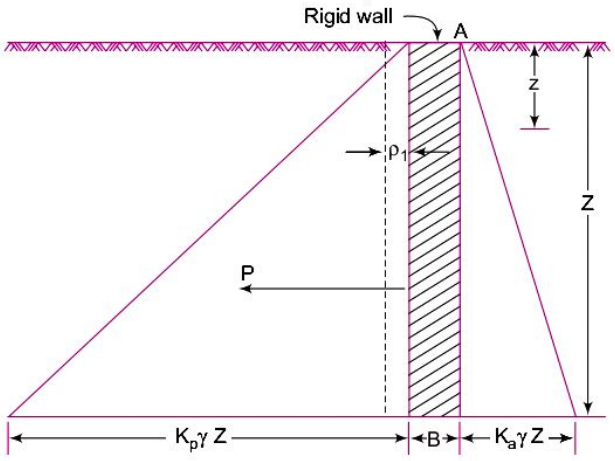

Fig. 27.4 Effect of wall movement

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or $$\frac{p}{\rho} = m.\, z \qquad \ldots(27.9)$$

where, $$m = \frac{\gamma(K_p - K_a)}{\rho_1} \qquad \ldots(27.10)$$

Factor m is called the *co-efficient of horizontal soil reaction* which depends not only on the nature of soil but also on the size and shape of the area which carries the **load.**

2. Stability of well, assuming no plastic flow. Figure 27.5 shows a well of length L and width B, acted upon by a horizontal force P per unit length of the well and a vertical load W. Let P act at a height H above the scour line, and let the depth of the well be D below the scour line. Let the well rotate at a point A situated at a depth D_1 below the scour line. The induced reactions are shown in Fig. 27.5 *(a)*.

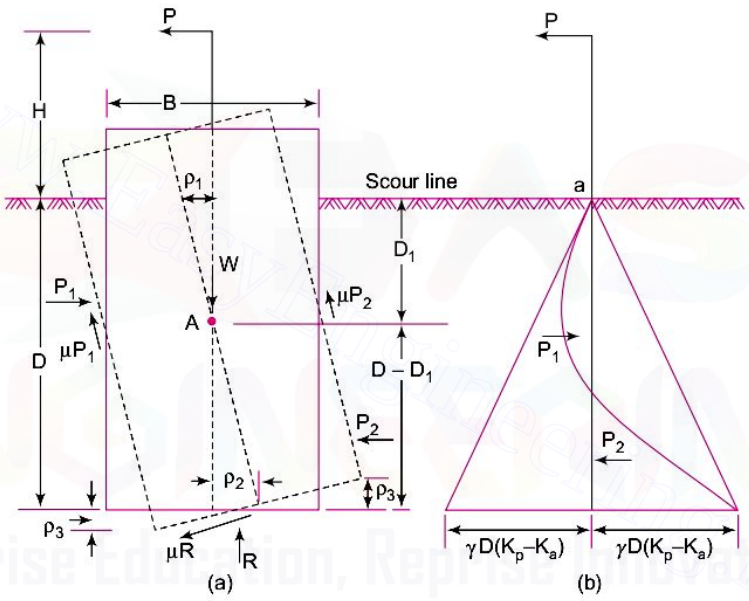

Fig. 27.5 Stability of foundation

Let ρ_1 = horizontal displacement of the centre line of the well at the scour line

ρ_2 = horizontal displacement of the centre line of the well at the base level

ρ_3 = vertical downward displacement of one edge of the well at its base

ρ'_3 = vertical upward displacement of the other edge of the well at its base (Assume $\rho_3 \approx \rho'_3$)

P_1 = resultant passive reaction of the well on the left face [Fig. 27.5 (*b*)]

P_2 = resultant vertical reaction of the well on the lower part of the right face

R = resultant vertical reaction at the base of the well

μP_1 = skin friction on the left face of the well

μP_2 = skin friction on the right face of the well

μR = frictional resistance of the soil at its base.

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From statics, the conditions of equilibrium are as follows:

$$P = P_1 - P_2 - \mu R \qquad ...(27.11)$$

$$PH = M_3 + M_2 - M_1 + \mu RD + \mu (P_1 - P_2)\frac{B}{2} \qquad ...(27.12)$$

and $$W = \mu (P_1 + P_2) + R \qquad ...(27.13)$$

where M_1 = Moment at the scour line produced by P_1

M_2 = Moment at the scour line produced by P_2

M_3 = Moment of the vertical soil reaction at the base

Let ρ_4 = Uniform vertical displacement of the well due to the resultant vertical force.

From Fig. 27.5 (*a*), we have the following relation between the various displacements:

$$\frac{\rho_1}{D_1} = \frac{\rho_1}{D - D_1} = \frac{\rho_3}{B/2} \qquad ...(27.14)$$

3. Evaluation of P_1. From Fig. 27.6 and Eq. 27.9, at any depth z,

$$p = m\rho z$$

where $$\rho = \frac{\rho_1}{D_1}(D_1 - z)$$

$$\therefore \quad p = mz\frac{\rho_1}{D_1}(D_1 - z) \qquad ...(27.15)$$

$$\therefore \quad P_1 = \int_0^{D_1} p\,dz = \int_0^{D_1} mz\frac{\rho_1}{D_1}(D_1 - z)\,dz$$

or $$P_1 = \frac{m\rho_1 D_1^2}{6} \qquad ...(27.16)$$

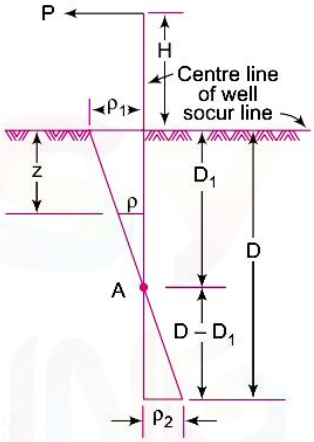

Fig. 27.6 Evaluation of P_1

4. Evaluation of P_2 (Fig. 27.7)

$$\frac{\rho}{z - D_1} = \frac{\rho_3}{D - D_1} = \frac{\rho_1}{D}$$

$$\therefore \quad \rho = \frac{z - D_1}{D}\rho_1$$

$$\therefore \quad P_2 = \int_{D_1}^{D} m\frac{\rho_1}{D_1}(z - D_1)\,z\,dz$$

or $$P_2 = \frac{m\rho_1}{6D_1}\left(2D^3 - 3D_1D^2 + D_1^3\right) \qquad ...(27.17)$$

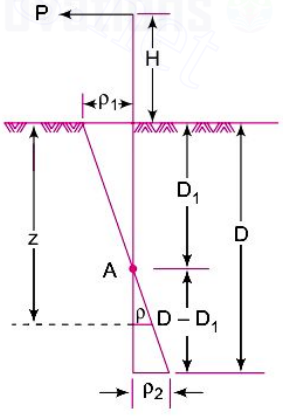

Fig. 27.7 Evaluation of P_2

5. Evaluation of moment M_1 produced by P_1

$$M_1 = \int_0^{D_1} (pdz).z$$

$$= \int_0^{D_1} m\frac{\rho_1}{D_1}(D_1 - z)\,z^2 dz = m\rho_1\frac{D_1^3}{12} \qquad ...(27.18)$$

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6. Evaluation of moment M_2 produced by P_2

$$M_2 = \int_{D_1}^{D} m \frac{\rho_1}{D_1}(z - D_1) z^2 dz = \frac{m\rho_1}{12 D_1}\left(3D^4 - 4D_1 D^3 + D_1^4\right) \quad ...(27.19)$$

7. Evaluation of vertical reaction R: Modulus of vertical subgrade reaction is

$$k = \frac{p}{\rho}$$

where ρ = vertical deflection of soil corresponding to vertical reaction $P = \rho_4$

$$\therefore \quad R = 2\int_0^{B/2} p dx = 2\int_0^{B/2} k\rho_4 \, dx$$

or
$$B = kB\rho_4 \quad ...(27.20)$$

8. Evaluation of moment M_3 produced at the base due to vertical soil reaction p: The rotation of the well is also resisted by a moment M_4 acting at the base on account of the downward deflection of the toe and upward deflection of the heel. Figure 27.8 shows the rotation of the base, with displacement ρ_3 at the ends. Let ρ be the deflection at a distance x from the centre O of the base.

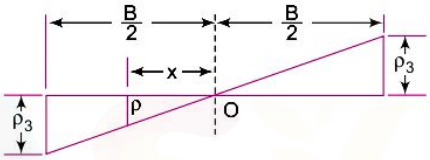

Fig. 27.8 Rotation of base

$$\therefore \quad \rho = \frac{2\rho_3}{B} x$$

$$\therefore \quad M_3 = 2\int_0^{B/2} p.\, x\, dx. = 2\int_0^{B/2} k\rho.\, x dx$$

$$= 2\int_0^{B/2} k \,.\, \frac{2\rho_3}{B} x^2 \, dx = k\rho_3 \frac{B^2}{6}$$

But
$$\rho_3 = \frac{B\rho_1}{2D_1}$$

$$M_3 = \frac{kB^3}{12D_1}\rho_1 \quad ...(27.21)$$

This moment acts in clockwise direction for the present case of Fig. 27.5 *(a)*.

9. Evaluation of $m\ \rho_1$ on the basis of maximum soil pressure: If no plastic flow is allowed in the soil, horizontal soil reaction p at any depth must not exceed $(p_z)_{max}$ for that depth, given by

$$(p_z)_{max} = \gamma_z (K_p - K_a) \quad ...(27.22)$$

$$\frac{d(p_z)}{dz} = \gamma (K_p - K_a) \quad ...(27.23)$$

It is evident that sand starts to flow as soon as the slope of the pressure parabola at the scour level becomes equal to the slope of line whose abscissa represents value of $(p_z)_{max}$ [Fig. 27.5 (*b*)]. From Eq. 27.16,

$$m\rho_1 = \frac{6P_1}{D_1^2} \quad ...(27.24)$$

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From equation 27.15, $$p = m\rho_1 \frac{z}{D_1}(D_1 - z) = \frac{6P_1}{D_1^3}(D_1 - z)z$$

$$\therefore \quad \left(\frac{dp}{dz}\right)_{z=0} = \frac{6P_1}{D_1^3} \cdot D_1 = m\rho_1 \qquad ...(27.25)$$

Equating 27.23 and 27.25, we get

$$\frac{6P_1}{D_1^2} = \gamma(K_p - K_a) = m\rho_1$$

$$\therefore \quad P_1 = \frac{\gamma(K_p - K_a)}{6} D_1^2 \qquad ...(27.26)$$

and $$m\rho_1 = \gamma(K_p - K_a) \qquad ...(27.27)$$

Substituting the value of $m\rho_1$ in Eqs. 27.17, 27.18, 27.19 and 27.21, we get

$$P_2 = \gamma(K_p - K_a)\frac{2D^3 - 3D_1 D^2 + D_1^3}{6D_1} \qquad ...(27.28)$$

$$M_1 = \gamma\left(K_p - K_a\right)\frac{D_1^3}{12} \qquad ...(27.29)$$

$$M_2 = \gamma(K_p - K_a)\frac{3D^4 - 4D_1 D^3 + D_1^4}{12 D_1} \qquad ...(27.30)$$

and $$M_3 = \frac{B^3}{12D_1}\gamma\left(K_p - K_a\right)\frac{k}{m} \qquad ...(27.31)$$

10. Evaluation of base pressure: Let p_t and p_h be the unit pressure exerted on the soil under the base, at the toe and heel respectively.

Then $$R = \frac{p_t + p_h}{2} \times B \qquad ...(27.32)$$

and $$M_3 = \frac{p_t - p_h}{12} B^2 \qquad ...(27.33)$$

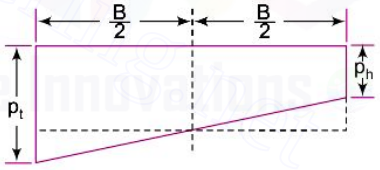

Fig. 27.9 Evaluation of base pressure

27.6 HEAVY WELLS

In the case of heavy wells, the reaction R at the base will be very high and hence sliding at the base will not take place *(i.e.,* $\rho_2 = 0$). The well will evidently rotate about its base as shown. Let F be the force required at the base to prevent sliding.

Thus, by comparison of Fig. 27.10 with Fig. 27.5,

$$\rho_2 = 0 \; ; D_1 = D$$

$$P_2 = 0 \text{ and } M_2 = 0$$

For the case of heavy well, the equations of equilibrium are as under :

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$$P = P_1 - F \qquad ...(27.34)$$

$$PH = M_3 - M_1 + FD + \mu P_1 \frac{B}{2} \qquad ...(27.35)$$

and $$W = \mu P_1 + R \qquad ...(27.36)$$

The quantities P_1, M_1 and M_3 are given by putting $D_1 = D$ in Eqs. 27.26, 27.29 and 27.40.

$$P_1 = \frac{\gamma\left(K_p - K_a\right)}{6} D^2 \qquad ...(27.37)$$

$$M_1 = \frac{\gamma\left(K_p - K_a\right)}{12} D^3 \qquad ...(27.38)$$

$$M_3 = \frac{B^3}{12D} \gamma\left(K_p - K_a\right)\left(\frac{k}{m}\right) \qquad ...(27.39)$$

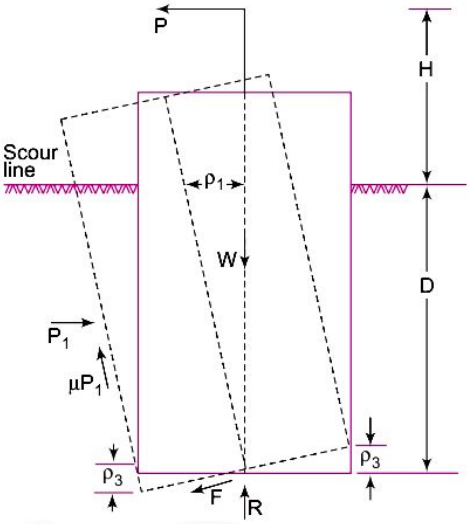

Fig. 27.10 Rotation of heavy well

Case of a non-rectangular base

Let L = maximum length of well base

B = width of well base

A = area of well base

Z = section modulus of well base

B_0 = average width of base

M_B = total moment induced in the base due to tilting

F_1 = total horizontal reaction at the base

R_1 = total vertical reaction at the base

W_T = total vertical load of the well.

The general equations of the equilibrium may be shown as

$$PL = P_1 L - F_1 \qquad ...(27.40)$$

$$PLH = M_B - M_1 L + F_1 D + \mu P_1 \frac{B_0}{2} L \qquad ...(27.41)$$

$$W_T = \mu P_1 L + R_1 \qquad ...(27.42)$$

If p_t and p_h be the maximum toe and heel pressures

$$p_t = \frac{R_1}{A} + \frac{M_B}{Z} \qquad ...(27.43)$$

$$p_h = \frac{R_1}{A} - \frac{M_B}{Z} \qquad ...(27.44)$$

Hence, from Eqs. 27.31 and 27.33

$$M_3 = \frac{B^3}{12\,D} \gamma\,(K_p - K_a)\,\frac{k}{m} = \frac{p_t - p_h}{12} B^2 \qquad ...(27.45)$$

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Example 27.1. *Find the maximum base pressure and lateral load per unit length of the well base, for a well shown in Fig. 27.11. The following data are given:*

1. *Effective downward force on well, including self-weight = 1000 t*
2. *Height of point of application of horizontal force above the scour line = 9 m*
3. *Depth of well below the scour line = 9 m*
4. *Submerged unit weight of soil = 0.78 t/m^3*
5. *Angle of internal friction = 30°*
6. *Coefficient of vertical subgrade reaction = 25×10^5 kg/m^3*
7. *Poisson's ratio = 0.5*
8. *Safe bearing capacity of soil at the base of the well = 4.5. kg/cm^2*
9. *Horizontal deformation of the well cap = 2 cm ($= \rho_1$)*
   Assume that the well rotates about its base.

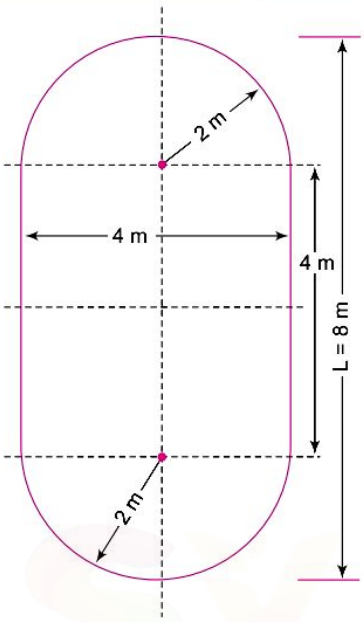

Fig. 27.11

Solution. ***(a) Properties of the Section:***

Area of cross-section $A = 4 \times 4 + \pi (2)^2 = 28.6 \text{ m}^2$

Section modulus $Z = \frac{1}{6}(4)(4)^2 + \frac{\pi}{32}(4)^3$

$= 10.67 + 6.28 \approx 17 \text{ m}^3$

B = maximum width = 4 m

L = Length = 8 m

B_0 = average width = $\frac{A}{L} = \frac{28.6}{8} = 3.6$ m.

(b) Soil properties:

$\gamma = 0.78$ t/m^3 (submerged unit weight)

$K_p = \frac{1+\sin\phi}{1-\sin\phi} = 3$; $K_a = \frac{1}{3}$

$\gamma (K_p - K_a) = 0.78 \left(3 - \frac{1}{3}\right) = 2.08$ t/m^3

$m = \frac{\gamma (K_p - K_a)}{\rho_1} = \frac{2.08}{0.02} = 104$ t/m^3

k = coefficient of vertical subgrade reaction

$= 25 \times 10^5$ kg/m^3 $= 2500$ t/m^3

$\frac{k}{m} = \frac{2500}{104} = 24$ m ; $\mu = 0.5$

Bearing capacity of soil

$= 4$ kg/cm^2 $= 40$ t/m^2.

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(c) Calculation of P_1, M_1 and M_3:

Eq. 27.37, $$P_1 = \frac{\gamma\left(K_p - K_a\right)}{6} D^2 = \frac{2.08}{6}(9)^2 = 28.1 \text{ t/m}$$

Eq. 27.38, $$M_1 = \frac{\gamma\left(K_p - K_a\right)}{12} D^3 = \frac{2.08}{12}(9)^3 = 126.5 \text{ t–m/m}$$

Eq. 27.39, $$M_3 = \frac{B^3}{12D}\gamma\left(K_p - K_a\right)\frac{k}{m} = \frac{4^3}{12 \times 9} \times (2.08)\, 24 = 30 \text{ t–m/m}$$

(d) Equilibrium Equations: If W_T is the total weight =1000 t, then we have from Eq. 27.42

$$W_T = \mu P_1 L + R_1$$

$\therefore$ $$1000 = 0.5\,(28.1)\,8 + R_1$$

$\therefore$ $$R_1 = 1000 - 112.4 = 887.6 \text{ t}$$

Also, from Eqs. 27.43 and 27.44,

$$p_t = \frac{R_1}{A} + \frac{M_B}{Z} \qquad \ldots(27.4)$$

$$p_h = \frac{R_1}{A} - \frac{M_B}{Z} \qquad \ldots(27.44)$$

$\therefore$ $$p_t + p_h = \frac{2R_1}{A} = \frac{2 \times 887.6}{28.6} = 62 \text{ t/m}^2 \qquad \ldots(i)$$

Also, from Eq. 27.33, $p_t - p_h = \dfrac{12M_3}{B^2} = \dfrac{12 \times 30}{4 \times 4} = 22.5$ t/m ...(*ii*)

From (*i*) and (*ii*) we get $p_t = 42.25$ t/m^2 (maximum base pressure)

(This is less than the safe bearing capacity of 45 t/m^2)

and $$p_h = 19.8 \text{ t/m}^2$$

From Eq. 27.43, $$M_B = \left(p_t - \frac{R_1}{A}\right)Z = (42.25 - 31)\,17 = 191.25 \text{ t-m}$$

Again, from Eq. 27.40, $$PL = P_1 L - F_1$$

$$P \times 8 = 28.1 \times 8 - F_1 \qquad \ldots(i)$$

From Eq. 27.41, $$PLH = M_B - M_1 L + F_1 D + \mu P_1 \frac{B_0}{2} L$$

$\therefore$ $$P \times 8 \times 9 = 191.25 - (126.5 \times 8) + (F_1 \times 9) + \left(0.5 \times 28.1 \times \frac{3.6}{2}\right)8$$

or $$72P = 9F_1 - 618.43$$

or $$8P = F_1 - 68.7 \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get $P = 9.76$ t

Hence answers are:

(*i*) Maximum base pressure = **42.24 t/m^2.**

(*ii*) Maximum lateral pressure/unit length = **9.76 t.**

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27.7 WELL CURB, CUTTING EDGE, STEINING AND BOTTOM PLUG

Well curb. The well curb is designed for supporting the weight of the well with partial support at the bottom of the cutting edge, *i.e.,* when only part of the cutting edge is in contact with soil and the remaining portion is only held by skin friction. A three-point support of the cutting edge resting on a log may be assumed for design purposes. The load coming on the cutting edge is uncertain as a considerable part of it is borne by skin friction. Another factor of uncertainty is in regard to the effective depth of the well curb, since the entire well acts as a deep girder to resist torsion and bending. Since the load is occasional, working stress upto 99% of yield stress may be permitted. The well curb has also to withstand stress due to sand blows, as well as due to light blasting required when boulder obstructs the sinking of the well.

Cutting edge. The cutting edge should have as sharp an angle as practicable for knifing into the soil without making it too weak to resist the various stresses induced by boulders, blows, blasting, etc. An angle to the vertical of 30°, or a slope of 1 horizontal to 2 vertical has been found satisfactory in practice. In concrete caissons, the lower portion of the cutting edge is wrapped with 12 mm steel plates which are anchored to the concrete by means of steel straps. A sharp vertical edge is generally provided along the outside face of the caisson. Such an edge facilitates the rate of sinking and prevents air leakage in the case of pneumatic caissons.

Steining thickness. The thickness of steining is designed in such a way that at all stages the well can be sunk under its own weight, as the need for weighting with kentledge takes time and retards progress considerably. For a circular well with outer diameter D and thickness t of the steining, we have

Self-weight per unit height $= \pi (D - t)\, t\, \rho$

Skin friction force per unit weight $= \pi D\, r_f$

where ρ = unit weight of concrete or masonry of the steining

r_f = unit skin friction

Equating the two, we get $\pi (D - t)t\, \rho = \pi D\, r_f$

From which
$$t = \frac{D}{2}\left[1 - \sqrt{1 - \frac{4\, r_f}{\rho D}}\right] \qquad ...(27.46)$$

It will be seen from this equation that for a given value of skin friction, the steining thickness comes out to be less with increasing value of diameter of the well. This is, however, contrary to the usual practice of providing greater thickness of steining with increasing diameter of the well as given in the following table:

D (outside diameter of well)	*t (steining thickness)*
3 m	0.75 m
5 m	1.20 m
7 m	2.00 m

This is so because large diameter wells are taken deeper and the skin friction increases with depth. Moreover, for deeper wells, water is invariably met with and the effective self-weight is reduced

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by buoyancy in the portion of the well below water level, and hence larger steining thickness is required.

Skin friction. The unit skin friction increases with depth, and at a given depth, the skin friction is equal to the coefficient of friction μ times the lateral earth pressure. However, it is not possible to evaluate the skin friction from laboratory tests as the lateral earth pressure depends upon state of stress. It is also not possible to accurately determine the value of μ. For the purpose of design, the values of skin friction given in the following table (Terazaghi and Peck, 1948) may be used:

Type of soil	*Skin friction (t/m^2)*
Silt and soft clay	0.73 – 2.93
Very stiff clay	4.9 – 19.5
Loose sand	1.22 – 3.42
Dense sand	3.42 – 6.84
Dense gravel	4.9 – 9.4

Greater skin friction requires greater sinking efforts, and hence retards the sinking of the well. Hence, methods should be used to reduce the skin friction while sinking the well. Since the frictional resistance depends on the roughness of the surface of contact, a smoothly plastered well steining surface which is in a true plane without kinks or warps, will considerably reduce skin friction. Skin friction is also reduced by flaring the well. In order to reduce skin friction on the San Francisco Oakland Bay Bridge, a coating which gave a smooth oily surface and which was tough enough not to be rubbed off during the sinking process was used on the walls of the caissons and it was estimated that this reduced the friction between the concrete and fairly stiff clay by roughly 40%. It has also been reported that bentonite solution injected on the external surface considerably reduces skin friction.

Bottom plug. The bottom plug of concrete to be designed for an upward load equal to the soil pressure (including the pore pressure) minus self-weight of bottom plug and filling. The bottom plug is made bowl-shaped so as to have inverted arch action. As generally under water concreting has to be done for bottom plug, no reinforcement can be provided. The bottom plug is generally designed as a thick plate subjected to a unit bearing pressure under the maximum vertical load which is transmitted from the vertical walls of the well. Based on theory of elasticity, the thickness of bottom plug is as follows:

$$t^2 = \frac{3W}{8\pi f_c}(3+\mu) = 1.18\, R\frac{q}{f_c} \quad \text{(For circular wells)} \qquad ...(27.47)$$

and

$$t^2 = \frac{3qb^2}{4f_c\,(1+1.61\,\alpha)} \quad \text{(For rectangular wells)} \qquad ...(27.48)$$

where t = thickness of the concrete or steel plug
W = total bearing pressure on the base of the well
f_c = flexural strength of concrete seal.
μ = Poisson's ratio = 0.15 for concrete
R = radius of well base
q = unit bearing pressure against base of well
b = width or short side of well
a = width/length or, short side/long side of well.

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27.8 WELL SINKING

1. Laying the well curb. If the river bed is dry, laying of well curb presents no difficulty. In such a case, excavation up to half a metre above subsoil water level is carried out and the well curb is laid. If, however, there is water in the river, suitable cofferdams are constructed around the site of the well and islands are made. The sizes of the island should be such as to allow free working space necessary to operate tools and plane for movement of labourer etc. When the island is made, the centre point of well is accurately marked and the cutting edge is placed in a level plane. It is desirable to insert wooden sleepers below the cutting edge at regular intervals so as to distribute the load and avoid setting of the cutting edge unevenly during concreting. These sleepers are, however, removed once the shuttering of the well curb has been stripped off. The inside shuttering of the curb is generally made of brick masonry built to proper profile and plastered. The outer shuttering is made of wood or steel. Steel lined timber shuttering is preferable. All reinforcement of the curb should be placed in position properly, and the vertical steining bars should also be placed such that they project about 2m beyond the top of the curb. All concreting in the well curb should be done in one continuous operation.

2. Masonry in well steining. The well steining should be built in intial short height of about 2m only. It is absolutely essential that the well steining is built in one straight line from bottom to top. To ensure this the steining must be built with straight edges preferably of angle iron. The lower portions of the straight edges must be kept butted with the masonry of the lower stage throughout the building of the fresh masonry. In no case should a plumb bob be used to build masonry in the well steining. Steining should not be allowed to be built more than 1 m at a time. It is desirable to keep the stages of masonry work at the location of joints in vertical steining bars. After sinking of one stage is complete, all the damaged portions of the steining at the top of the first stage should be repaired properly before masonry in the next stage is started. The well masonry is fully cured for atleast 48 hours before starting the loading or sinking operations.

3. Sinking operations. A well is ready to be 'set in' after having cast the curb and having built first short stage of masonry over it. The well is sunk by excavating material from inside under the curb. In the initial stage of sinking, the well is unstable and progress can be very rapid with only little material being excavated out. Great care should therefore be exercised during the stage to see that the well sinks to true position. To sink the well straight it should never be allowed to go out of plumb.

Excavation and scooping out of the soil inside the well can be done by sending down workers inside the wells till such a stage that the depth of water inside becomes about 1 m. After this stage, *Jhams,* worked by manual or animal power or by means of diesel, electric or steam winches, are used for excavating the material from inside and under water. When clay strata is to be pierced through, a rail chisel may be used. In case the soil is not very hard, but hard enough as not to be excavated by Jhams, the use of *phawrah Jhams* is effective. When power winches are available clayey strata can also be successfully excavated with the help of big grabs having tempered steet teeth.

As the well sinks deeper, the skin friction on the sides progressively increases. To overcome the increased skin friction and the loss in weight of the well due to buoyancy, additional loading known as *kentledge* is applied on the well.

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Pumping out the water from inside the well is effective in sinking of well under certain conditions. Pumping should be discouraged in the initial stage. Unless the well has gone deep enough or has passed through a ring of clayey strata so that chances of tilts and shifts are minimised during this process. Complete dewatering should not be allowed when the well has been sunk to about 10 m depth. Sinking thereafter should be done by grabbing, chiselling, applying kentledge and using gelignite charges. Only when these methods have failed, dewatering may be allowed upto depressed water level of 5 m and *not more.*

On certain occasions a well is struck up and normal method of kentledge and dredging fail to sink it further. In such a case frictional resistance developed on its outer periphery is reduced considerably by forcing jet of water on the outer face of the well around. This method is effective in case the well is being sunk in sand strata.

4. Tilts and Shifts. The primary aim in well sinking is to sink them straight and at the correct position. Suitable precautions should be taken to avoid tilts and shifts. Also proper records of tilts and shifts should be maintained and measure should be taken to counter-act tilts and shifts. The precasutions to avoid tilts and shifts are as follows:

1. The outer surface of the well curb and steining should be as regular and smooth as possible.
2. The radius of curb should be kept 2 to 4 cm larger than the outside radius of well steining.
3. The cutting edge of the curb should be of unifrom thickness and sharpness since the sharper edge has a grater tendency of sinking than a blunt edge.
4. The dredging should be done uniformly on all sides in a circular well and in both pockets of a twin well. The tilts and shifts of well, if any, must be carefully checked and recorded. The correct measurement of the tilts at any stage is perhaps one of the most important field observations required during well sinking.

As soon as tilt exceeds 1 in 200, the sinking should be supervised with special care and rectifying measures should be immediately taken. Any of the following measures can usefully be employed to counteract the tilts in the well during sinking operations :

(*i*) ***Regulation of grabbing:*** Unequal dredging causes tilts and hence if higher side is grabbed more by regulating the dredging, the tilt can be rectified [Fig. 27.12. (*a*)]. This method is not very effective when the well has been sunk to a great depth. In that case, a hole in the steining of the well is made on the higher side, and by hooks, the rope of the grab is pulled towards higher side to the maximum possible extent [Fig. 27.12 (*b*)]. The hole is made near the ground level. The well may be dewatered if possible and open excavation on the higher side is carried out.

(*ii*) ***Eccentric loading.*** The well normally is given kentledge in order to provide necessary sinking effort. In order to provide greater sinking effort on the higher side of well, eccentric loading is necessary by providing a suitable platform. As the sinking progresses, heavier kentledge with greater eccentricity is required in order to rectify the tilt.

In larger size wells to be sunk to great depths, eccentric loading may be as much as 400 to 600 tonnes with an eccentricity of 3 to 4 m. In such a case a welded frame bracket is used as shown in [Fig. 27.12 (*c*)].

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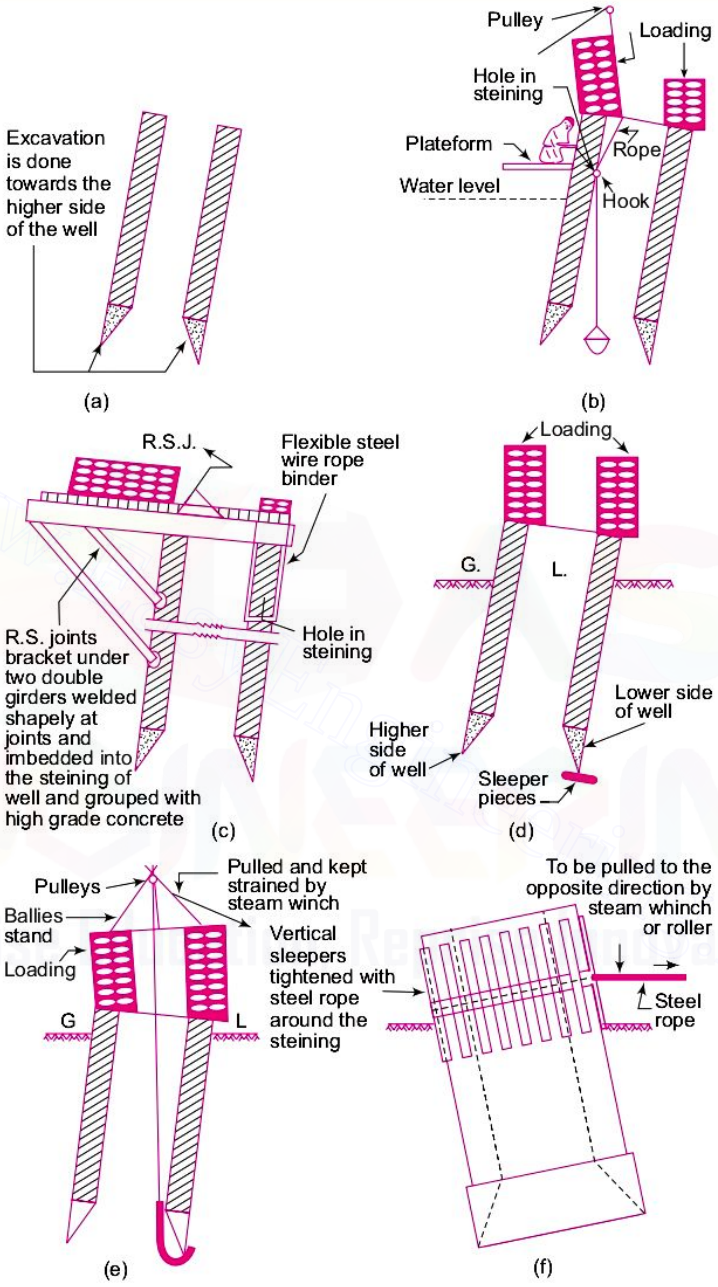

Fig. 27.12 Methods of rectifying the tilt

(iii) ***Water jetting or digging pit outside the higher side of the well.*** In this method, water jet is forced on the outer faces of the well, towards the higher side so that skin friction is reduced towards the higher side. The method if used alone is not very effective but provides a contributory effect if used with other methods.

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(*iv*) ***Excavation under the cutting edge.*** A filled well generally, refuses to straighten on account of unbroken stiff strata on the higher side of the well. In such a case, the well is dewatered, if possible and safe, an open excavation is done below the cutting edge of the higher side. If dewatering is unsafe, divers should be sent to loosen the strata.

(*v*) ***Providing temporary obstacles below the cutting edge.*** In some cases wooden sleeper pieces are put temporarily below the cutting edge of the well on the lower side to avoid further tilt of the well while various expedients are being tempted to rectify the tilt [Fig. 27.12 (*d*)]. Hooking the cutting edge on the lower side of the well with the help of the steel wire rope, pulled, and kept strained by steam which also has a similar effect [Fig. 27.12 (*c*)].

(*vi*) ***Pulling the well.*** This method is effective only in early stages of sinking. The well is pulled towards the higher side by placing one or more steel ropes round the well with vertical sleepers packed in between to distribute the pressure over larger areas of well steining. The pulling of ropes may be carried out by winches [Fig. 27.12 (*f*)].

(*vii*) ***Strutting the well.*** This method is used to avoid any further increase in the tilt of the well rather than rectifying it. The well is strutted on its tilted side with suitable logs of wood. The well steining is given covering plate to distribute pressure. The other ends of the logs rest against firm and non-yielding base by driving piles etc. Wood pieces are kept ready to be inserted and fixed in the gaps caused by the tilts of the well being rectified.

(*viii*) ***Pushing by jacks.*** The well may be pushed by force applied by hydraulic or mechanical jack on the tilted side of the wells.

27.9 PNEUMATIC CAISSONS

Pneumatic caissons are closed at the top and open (during construction) at the bottom. The essential feature of a pneumatic caisson is that compressed air is used to exclude or remove water from the working chamber at the bottom, and the excavations are thus carried out in dry conditions. The method of construction of pneumatic caisson is similar to that for open caissons (wells) except that the working chamber is kept air tight. In order that sub-soil water may not enter the working chamber, the pressure of air in the shaft is kept just higher than that of the water at that depth. However, the maximum pressure is limited from the considerations of health of persons who work inside the chamber. Normally, the tolerable, air pressure under which a man can work is limited to 3.5 kg /cm^2.

Let h be the height of water, at any stage of working. Then air pressure p required to exclude water is given by

$$p = \frac{wh}{10000}$$

where p = air pressure, in kg/cm^2

w = unit weight of water = 1000 kg/m^3

h = head of water in metres.

$$\therefore \quad p = \frac{1000\,h}{10000} = 0.1h$$

If maximum air pressure is limited to 3.5 kg/cm^2, the limiting head of water is given by

$$h_{lim} = \frac{3.5}{0.1} = 35\,\text{m}$$

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This is the maximum value. However, pneumatic caissons are adopted only if the head of water is more than 12 m. Thus, a pneumatic caisson can be used for depths of water ranging from 12 to 35 m.

Sinking of pneumatic caissons is tedious, time consuming and expensive. However, they are adopted at places where it is difficult to use bulky equipment required for sinking wells. Another advantage of pneumatic caisson is that the entire process of sinking of well is carried out under controlled conditions. It affords easy inspection work.

Figure 27.13 shows a typical section of a pneumatic caisson. The procedure for sinking the pneumatic well is as follows:

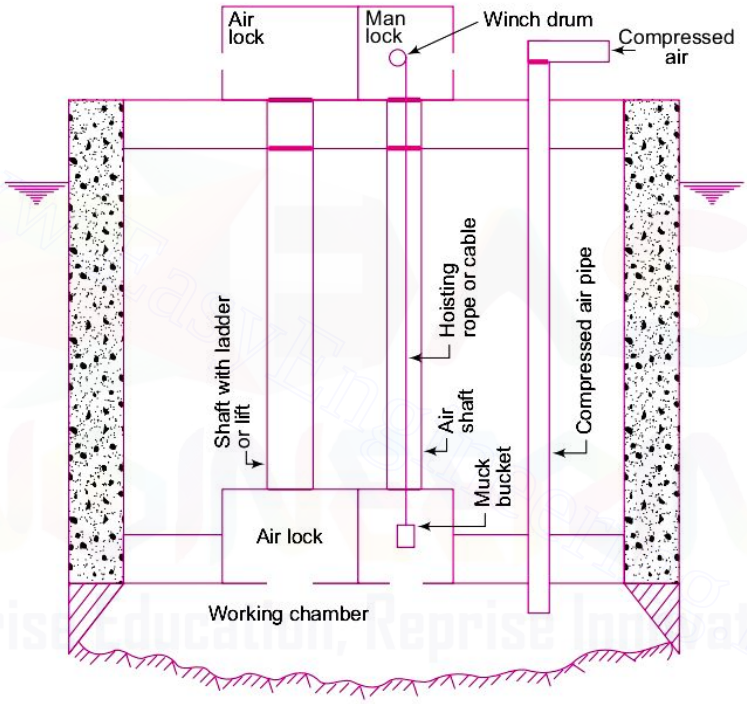

Fig. 27.13 Section of a pneumatic caisson

1. The caisson is sunk exactly in the same manner, as used for well sinking till the depth of water is shallow, and no trouble is encountered in sinking the well.
2. When the presence of water poses problems, an air lock is placed inside the well. The air lock may rest on rubber seals just above the cutting edge. The number of air locks may vary from one to three. Generally, two air locks are used-one for sending men inside and the other for moving the excavated material with the help of a much bucket and hoisting rope.
3. After properly placing the air lock in position, so that direct air entry is sealed, water is pumped out from the bottom and air pressure is gradually increased so that fresh water does not enter the working chamber.
4. Labourers are then sent down to the working chamber, through the appropriate air lock. In order to prevent leakage of air, arrangement of double gates is provided. The person enters the first gate, where pressure is atmospheric. The first door is closed and pressure is gradually increased

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to make it equal to the one in the working chamber. The height of working chamber is kept about 2 m, with proper lighting arrangement. Air is supplied through the air inlet pipe connected to an air compressor.

5. Excavation is carried out in the working chamber by the labourers sent down through air lock. The excavated material is sent up through the muck buckets lifted up by a hoisting rope operated by winch drum, through the air lock. In order to assist sinking, air pressure may be reduced for a short while. Sometimes, explosives may be employed in which case it is essential to make arrangements for the immediate removal of fall fumes.
6. When the caisson bottom has reached the desired level, concrete seal (or plug) is made by concreting upto the underside roof of the working chamber. Sufficient air pressure is maintained to force the concrete against the bottom surface till it hardens.
7. Air locks are removed, well is filled with sand or water (or even kept empty). The well cap is then formed on its top as usual.

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CHAPTER 28

Machine Foundations

28.1 SOIL DYNAMICS

Soil dynamics is defined as that constituent part of soil mechanics which deals with soil under dynamic conditions. It studies the effect of forces on soil in any way associated with causing motions in soil as well as with the mutual dynamic interaction of the foundation and soil (Jumikis, 1969). Most of the motions encountered in soil dynamics work are those of vibration, plane linear motion, motion brought about by impact, shock, elastic waves, and seismic action of geophysical forces.

The design of foundations of turbines, motors, generators, compressors, forge hammers and other machines, having a rhythmic application of unbalanced forces require special knowledge of theory of harmonic vibrations. Inertial forces of rotating elements of machines contribute, besides their static loads, additional dynamic loads. The machinery vibration influences adversely the foundation supporting soil by densifying it which may in turn, cause differential settlement of soil and foundation.

28.2 THE MASS SPRING SYSTEM

In soil dynamics problems, the analysis may be conveniently carried out by a single equivalent mass supported by a perfectly elastic system—the soil being replaced by the spring. Figure 28.1 shows mass-spring system (or *spring-mass system*), in which the weight $W = mg$ may be associated with the weight of the vibrating vibrator or foundations. The elastic spring represents the real soil support. Such a system has six degrees of freedom, and has thus six natural frequencies.

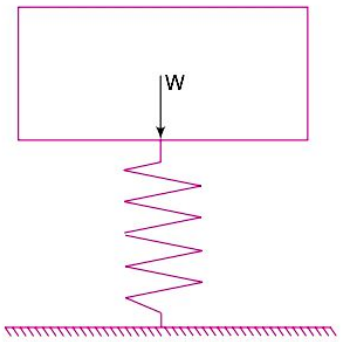

Fig. 28.1 Mass-spring system

Free-Vibrations. Let the mass spring system be set to vibration by an external force which is then removed. The system will continue to oscillate indefinitely with the same frequency and amplitude if external force or internal friction is absent. The time for one complete oscillation

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of the mass is called the *free period* and the distance up or down from the equilibrium position is called the *amplitude.*

Figure 28.2(*a*) shows a simple spring with a spring constant k kg/m. When a weight W is attached to it (without any vibrations), it extends by an amount δ_s [Fig. 28.2(*b*)]. The static deflection δ_s of the spring is given by

$$\delta_s = \frac{W}{k} \qquad ...(28.1)$$

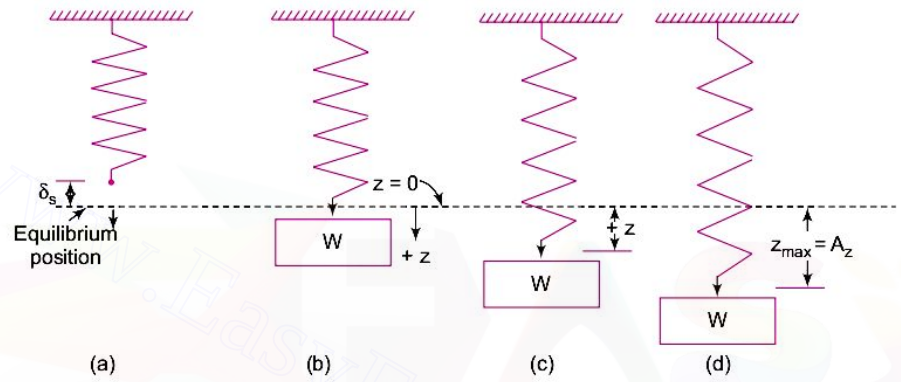

Fig. 28.2 Free vibrations

If the spring mass system is pulled down, by an external force, by a maximum distance z_{max} or A_z (called the amplitude), and then released, the whole system vibrates with a certain frequency. Let z be the displacement of the mass at any instant, with respect to the equilibrium position, the force F_s in the spring ($\uparrow$) is then given by

$$F_s = k(\delta_s + z) = k\delta_s + kz = W + kz \qquad ...(28.2)$$

The force acts in the opposite direction to the motion at any instant. The gravity force W acts downward. Hence when the motion is downward, the net downward force is equal to $W\downarrow - (W + kz)\uparrow$. This must be equal to mass × acceleration. Hence, we get

$$W - (W + kz) = \frac{W}{g}\frac{d^2z}{dt^2}$$

or
$$\frac{W}{g}\frac{d^2z}{dt^2} + kz = 0 \qquad ...(28.3\ a)$$

which is usually written as $m\ddot{z} + kz = 0$ ...(28.3)

where m = mass of the vibrating body = W/g

$\ddot{z}$ = acceleration

Equation 28.3 is called the *equation of motion,* which is similar to the following standard equation of motion:

$$\ddot{z} + \omega_n^2 z = 0 \qquad ...(28.4)$$

where ω_n = natural frequency of the system.

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Comparing Eqs. 28.3 and 28.4,

$$\omega_n^2 = \frac{k}{m}$$

or $$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{W}} \text{ rad/sec} \quad ...(28.5)$$

If f_n is the *natural frequency* of the system in cycles per second, we have

$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{kg}{W}} \text{ cycles/sec} \quad ...(28.6)$$

28.3 VIBRATING SPRING-MASS SYSTEM WITH DAMPING

If the spring mass system be provided with dash-pot, having c as the *damping factor* and $\dot{z}$ is the velocity of the vibrating sytsem at any instant, the force in the dash-pot, opposite the motion, will be equal to $c\,\dot{z}$.

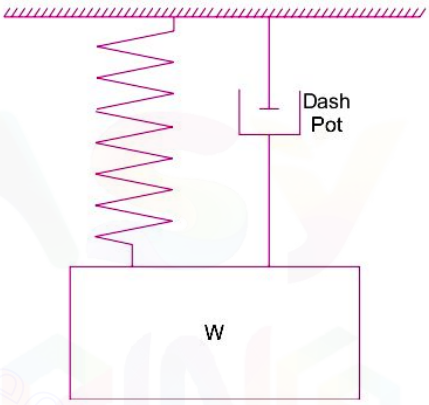

Fig. 28.3 Vibrating spring-mass system with damping

Hence the equation of the motion will be

$$W\downarrow - (W + kz)\uparrow - c\,\dot{z}\uparrow = m\,\ddot{z}$$

$$m\,\ddot{z} + c\,\dot{z} + kz = 0 \quad ...(28.7)$$

Comparing this with the standard equation for damping vibrations :

$$\ddot{z} + 2n\,\dot{z} + \omega_n^2 z = 0 \quad ...(28.8)$$

we get $$n = \frac{c}{2m} \quad ...(28.9)$$

or $$\omega_n = \sqrt{\frac{k}{m}}$$

Equation 28.8 is the standard differential equation which can be solved by putting

$$z = C' e^{\lambda \cdot t} \quad ...(i)$$

$$\dot{z} = C' \lambda e^{\lambda \cdot t} \quad ...(ii)$$

$$\ddot{z} = C' \lambda^2 \cdot e^{\lambda \cdot t} \quad ...(iii)\ (28.10\ a)$$

Putting these in equation 28.7, we get

$$mC' \lambda^2 e^{\lambda \cdot t} + c\,C' e^{\lambda \cdot t} + kC' e^{\lambda \cdot t} = 0 \quad ...(28.10)$$

or $$m\lambda^2 + c\lambda + k = 0$$

$$\lambda = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)} = -n \pm \sqrt{n^2 - \omega_n^2} \quad ...(28.11)$$

Hence, $$\lambda_1 = -n + \sqrt{n^2 - \omega_n^2}$$

and $$\lambda_2 = -n - \sqrt{n^2 - \omega_n^2}$$

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Three cases may arise from Eq. 28.11:

Case (*i*) Real, if $n^2 > \omega_n^2$

Case (*ii*) Zero, if $n^2 = \omega_n^2$

Case (*iii*) Imaginary or complex if $n^2 < \omega_n^2$

Case (*iv*) gives a value of $\lambda = -n = -\dfrac{c}{2m} = -\sqrt{\dfrac{k}{m}}$

Equation 28.11 for case (*ii*) reduces to $e^{\lambda \cdot t} = 0$, indicating that for this condition there will be no oscillation, but only a *rapid return back* to the equilibrium position of the mass [Fig.28.4 (*b*)]. The value of c for this condition is called *critical damping* c_c.

$$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk} \qquad ...(28.12)$$

For case (*i*), the radical is real $(n^2 > \omega_n^2)$, and $c \geq 2\sqrt{mk}$

Hence from (*i*), $$z = C' e^{\lambda \cdot t}\, C' e^{\left\{-\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k}{m}\right)}\right\} t} \qquad ...(28.13)$$

Equation 28.13 shows that z is not a periodic function of time. Therefore, the motion, when $n^2 - \omega_n^2 > 0$ is not a vibration, because it can only approach the equilibrium position at $t \to \infty$. However, the viscous resistance is so pronounced that the weight set in motion from its equilibrium does not vibrate but creeps gradually back to the equilibrium position at time infinity [Fig. 28.4 (*a*)].

For case (*iii*) when $n^2 < \omega_n^2$,

$$\omega_d^2 = \omega_n^2 - n^2 \text{ is positive or } c < 2\sqrt{mk}$$

Let λ_1 and λ_2 be the two roots of Eq. 28.11 :

$$\lambda_1 = -n + \omega_d i \qquad ...(28.14\ a)$$

and $$\lambda_2 = -n - \omega_d i \qquad ...(28.14\ b)$$

Now Eq. 28.10 (*a*) gives two particular solutions of Eq. 28.8. Also, the sum or difference of these two solutions multiplied by any constant is also a solution:

$$\therefore \quad z_1 = \frac{C_1}{2}(e^{\lambda_1 \cdot t} - e^{\lambda_2 \cdot t}) \qquad ...(28.15\ a)$$

and $$z_2 = \frac{C_2}{2}(e^{\lambda_1 \cdot t} + e^{\lambda_2 \cdot t}) \qquad ...(28.15\ b)$$

where C_1 and C_2 are constants

Substituting the values of λ_1 and λ_2 from Eq. 28.14, and simplifying, we get

$$z_1 = C_1 e^{-nt} \sin \omega_d t \text{ and } z_2 = C_2 e^{-nt} \cos \omega_d t$$

Summation of $z_1 + z_2 = z$ renders the general solution of Eq. 28.8 in the following form.

$$z = e^{-nt}[C_1 \sin \omega_d t + C_2 \cos \omega_d t] \qquad ...(28.16)$$

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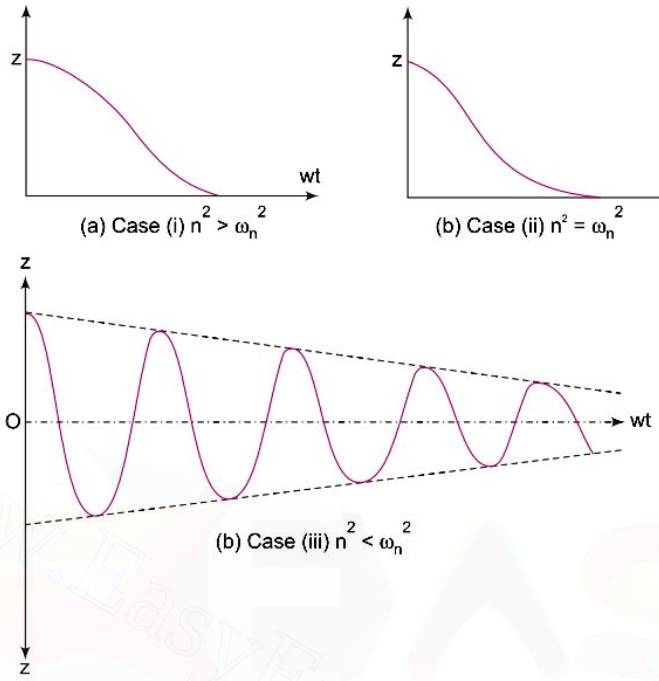

Fig. 28.4 Time displacement curves for damped vibrations

The quantity in the bracket represents the simple harmonic motion of the case of vibration without damping while e^{-nt} is the damping term. Figure 28.4 (*c*) shows the time displacement curve for this case. The peroid T of the damped vibration is given by

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{\omega_n^2 - n^2}} \qquad ...(28.17)$$

The term ω_d is called the *frequency of damped vibrations.*

$$\omega_d = \sqrt{\omega_n^2 - n^2} = \sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}$$

or

$$\omega_d = \sqrt{\frac{k}{m}}\sqrt{1 - \frac{c^2}{4\,km}} = \omega_n \sqrt{1 - \left(\frac{c}{c_c}\right)^2} \qquad ...(28.18)$$

28.4 FORCED VIBRATIONS

Forced vibrations of a system are generated and sustained by the application of an external periodic movement of the foundation of the system. Forced vibrations constitute the most important type of vibration in machine foundation design. We shall consider the case of forced vibrations with damping. Generally, for oscillating machinery (where the machinery vibrates because an unbalanced rotational force exists), the force can be expressed as a sine or cosine function, such as F_0 sin ωt. The equation of motion for such a case may be writen as

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$$m\ddot{z} + c\dot{z} + kz = F_0 \sin \omega t \qquad ...(28.19)$$

or
$$\ddot{z} + \frac{c}{m}\dot{z} + \frac{k}{m}z = \frac{F_0}{m}\sin \omega t \qquad ...(28.19\ a)$$

The solution of the above equation may be assumed in the following forms :

$$z = A \cot \omega t + B \sin \omega t \qquad ...(28.20\ a)$$

By successive differentiation, we obtain

$$\dot{z} = -A\omega \sin \omega t + B\omega \cos \omega t \qquad ...(28.20\ b)$$

$$\ddot{z} = -A\omega^2 \cos \omega t - B\omega^2 \sin \omega t \qquad ...(28.20\ c)$$

Substituting into Eq. 28.19 (a), we get

$$(-A\omega^2 \cos \omega t - B\omega^2 \sin \omega t) + \frac{c}{m}(-A\omega \sin \omega t + B\omega \cos \omega t) + \frac{k}{m}(A \cos \omega t + B \sin \omega t) = \frac{F_0}{m}\sin \omega t \qquad ...(28.21)$$

Equating the co-efficient of sin ωt to both sides,

$$-B\omega^2 - \frac{c}{m}A\omega + \frac{k}{m}B = \frac{F_0}{m} \qquad ...(28.22\ a)$$

Similarly, equating the coefficients of cos ωt to both sides,

$$-A\omega^2 + \frac{c}{m}B\omega + \frac{k}{m}A = 0 \qquad ...(28.22\ b)$$

Solving Eqs. 28.22 (a) and (b) for co-efficients A and B, we get

$$A = -\frac{F_0\, c\omega}{(k - m\omega^2)^2 + (c\omega)^2} \qquad ...(28.23\ a)$$

$$B = \frac{F_0\,(k - m\omega^2)}{(k - m\omega^2)^2 + (c\omega)^2} \qquad ...(28.23\ b)$$

Substituting these in Eq. 28.20 (a), we get the solution in the form

$$z = \frac{-F_0\, c\omega}{(c\omega)^2 + \left(k - m\omega^2\right)^2}\cos \omega t + \frac{-F_0\left(k - m\omega^2\right)\sin \omega t}{(c\omega)^2 + \left(k - m\omega^2\right)^2} \qquad ...(28.24)$$

The equation represents the components due to forced vibrations with the period of $T = \frac{2\pi}{\omega}$.

The frequency of vibrations (in cycles per second) is given by

$$f = \frac{\omega}{2\pi} \qquad ...(28.25\ a)$$

The natural frequency of vibration, as defined earlier, is given by

$$\omega_n = \sqrt{\frac{k}{m}} \text{ radians/sec} \qquad ...(28.25\ b)$$

and
$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad ...(28.25\ c)$$

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Substituting in Eq. 28.24,

$$\frac{-F_0\, c\omega}{(c\omega)^2 + \left(k - m\omega^2\right)^2} = z_2 \sin\phi \qquad ...(28.26\ a)$$

and

$$\frac{F_0\left(k - m\omega^2\right)}{(c\omega)^2 + \left(k - m\omega^2\right)^2} = z_2 \cos\phi \qquad ...(28.26\ b)$$

we get $$z = z_2\,(\sin\phi\cos\omega t + \cos\phi\sin\omega t) = z_2 \sin(\omega t + \phi) \qquad ...(28.27)$$

where the angle ϕ is termed as the phase angle between the exciting force and the motion of vibrating mass.

Noting that these terms represent a pair of vectors which must be added to obtain the displacement, the solution for the displacement due to the forced vibrations of Eq. 28.24 becomes

$$z = \sqrt{A^2 + B^2}\; F_0 \sin(\omega t + \phi) \qquad ...(28.28)$$

Substituting the values of A and B, and noting from Eq. 28.12 that

$$c_c = 2\sqrt{mk} = 2k\sqrt{\frac{m}{k}} = \frac{2k}{\omega_n} \qquad ...(28.29\ a)$$

or $$k = \frac{\omega_n C_c}{2} \qquad ...(28.29\ b)$$

we get $$z = \frac{F_0}{k}\,\frac{1}{\sqrt{\left(\frac{2c\omega}{c_c\,\omega_n}\right)^2 + \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2}}\sin(\omega t + \phi) \qquad ...(28.30\ a)$$

or $$z = \frac{F_0}{k}\,\frac{\sin(\omega t + \phi)}{\sqrt{\left(2\,\frac{c}{c_c}\cdot\frac{f}{f_n}\right)^2 + \left[1 - \left(\frac{f}{f_n}\right)^2\right]^2}} \qquad ...(28.30\ b)$$

The maximum deflection z_{max} is thus given by

$$z_{max} = A_z = \frac{F_0 / k}{\sqrt{\left(2\,\frac{c}{c_c}\cdot\frac{f}{f_n}\right)^2 + \left[1 - \left(\frac{f}{f_n}\right)^2\right]^2}} \qquad ...(28.31)$$

But $\frac{F_0}{k} = \delta_s$ = static deflection of spring

$$\therefore \quad \frac{A_z}{\delta_s} = \frac{z_{max}}{\delta_s} = \frac{1}{\sqrt{\left(2\,\frac{c}{c_c}\,\frac{f}{f_n}\right)^2 + \left[1 - \left(\frac{f}{f_n}\right)^2\right]^2}} \qquad ...(28.31\ a)$$

where $A_s = z_{max}$ = maximum dynamic deflection of the system.

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Putting $$\frac{A_z}{\delta_s} = \frac{z_{max}}{\delta_s} = \mu = \textit{magnification factor or dynamic amplification factor}$$

$$\frac{\omega}{\omega_n} = \frac{f}{f_n} = r = \text{frequency ratio}$$

and $$\frac{c}{c_c} = \xi = \textit{damping ratio,}$$ we get

$$\mu = \frac{1}{\sqrt{\left(1 - r^2\right)^2 + 4\xi^2 r^2}} \qquad ...(28.31\ b)$$

Figure 28.5 shows a plot between the magnification factor and the frequency ratio $r\left(= \frac{f}{f_n} = \frac{\omega}{\omega_n}\right)$ for various values of damping ratio $\xi\left(= \frac{c}{c_c}\right)$.

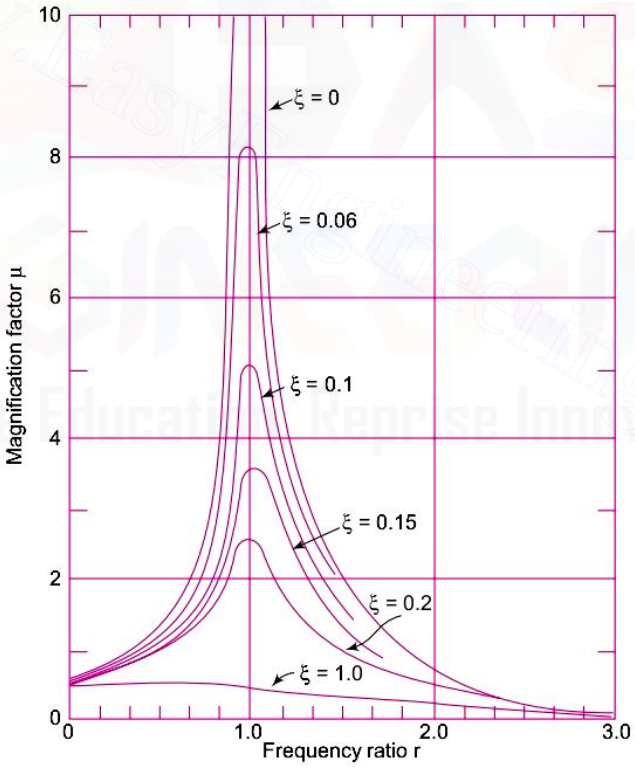

Fig. 28.5 Amplitude frequency relationship for damped forced vibrations

From Fig. 28.5, it is observed that magnification factor suddenly shoots up for the values of r between 0.6 to 1.5. At $r = 1$, *resonance* occurs for an undamped condition. Even for damped conditions, the magnification factor (and hence the amplitude) is maximum at $r < 1$. Thus, these curves show the effect of damping on shifting the frequency for maximum amplification away from the natural foundation

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frequency. The aim of the designer should be such that the frequency ratio f/f_n is either less than 0.6 or more than 1.5. However, the frequency f of the machine is always constant, and a foundation engineer has to manipulate the natural frequency f_n of the machine foundation system by suitably proportioning it.

28.5 TRANSMITTED FORCE (F_T)

In a *damped* forced vibratory system, the support is derived from the foundation by way of transmitting a force (F_T) to the foundation. This transmitted force (F_T) can be expressed as

$$F_T = c\frac{dz}{dt} + kz \qquad ...(1)$$

From Eq. 28.30 *b*,

$$z = \frac{F_0}{k}\frac{\sin(\omega t+\varphi)}{\sqrt{\left(2\frac{c}{c_c}.\frac{f}{f_n}\right)^2+\left\{1-\left(\frac{f}{f_n}\right)^2\right\}^2}} = \frac{F_0}{k}\frac{\sin(\omega t+\varphi)}{\sqrt{4\xi^2 r^2+\left(1-r^2\right)^2}}$$

or

$$z = \frac{F_0}{k}\,\mu\sin(\omega t+\varphi) \qquad ...(2)$$

where

$$\mu = \frac{1}{\sqrt{4\xi^2 r^2+\left(1-r^2\right)^2}} = \text{magnification factor}$$

Substituting this value of z in Eq. (1), we get

$$F_T = c\frac{d}{dt}\left[\left(\frac{F_0}{k}\mu\right)\sin(\omega t+\varphi)\right]+k\left(\frac{F_0}{k}\mu\right)\sin(\omega t+\varphi)$$

or

$$F_T = c\frac{F_0\mu}{k}.\cos(\omega t+\varphi)\,\omega + k\left(\frac{F_0\mu}{k}\right)\sin(\omega t+\varphi)$$

or

$$F_T = \left(\frac{F_0\mu}{k}\right)\sqrt{c^2.\omega^2+k^2}\left[\cos\left(\overline{\omega t+\varphi-\beta}\right)\right]$$

where

$$\beta = \tan^{-1}\frac{k}{c\omega}$$

Hence the magnitude of the force F_T is given by

$$|F_T| = \left(\frac{F_0\mu}{k}\right)\sqrt{k^2+c^2\omega^2} = F_0\mu\sqrt{1+\frac{c^2\omega^2}{k^2}} \qquad ...(3)$$

But

$$k = \frac{\omega_n c_c}{2} \quad \text{From Eq. 28.29 } (\mu)$$

Hence, from 3,

$$F_T = F_0\mu\sqrt{1+4\frac{\omega^2}{\omega_n^2}.\frac{c^2}{c_c^2}} \;;\; F_T = F_0\mu\sqrt{1+4r^2\xi^2} \qquad ...(28.32)$$

Also ratio

$$\frac{F_T}{F_0} = T = \mu\sqrt{1+4r^2\xi^2} = \text{force transmissibility} \qquad ...(28.32\,a)$$

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28.6 NATURAL FREQUENCY OF FOUNDATION SOIL SYSTEM

In the previous discussions, we have used the term m as the mass of the vibrating system. This includes the mass of a certain volume of soil co-vibrating with the system. The mass of co-vibrating soil is called the *apparent soil mass,* participating in the vibrations.

Thus, $$m = m_v + m_s \qquad ...(28.33)$$

where m_v = mass of the vibrator

m_s = apparent soil mass (mass of the co-vibrating soil)

$$\therefore \quad \omega_n = \sqrt{\frac{k}{m_v + m_s}} = \sqrt{\frac{\text{kg}}{W_v + W_s}} \qquad ...(28.34\ a)$$

or $$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m_v + m_s}} = \frac{1}{2\pi}\sqrt{\frac{\text{kg}}{W_v + W_s}} \qquad ...(28.34\ b)$$

where W_v = weight of the vibrator, and

W_s = weight of the apparent soil mass.

Unfortunately, the size of the co-vibrating body of soil cannot be determined exactly as yet because it depends on frequency and is influenced by the size of the base area of the vibrator (foundation) and by the elastic properties of the soil (spacing).

We shall consider here three methods of determining the natural frequency of foundation soil system: (1) Barken's method, (2) Balakrishna Rao's method, (3) Pauw's method.

28.7 BARKEN'S METHOD

Barken suggested the following equation for the natural frequency of system

$$\omega_n = \sqrt{\frac{C_u A}{m}}. \qquad ...(28.35\ a)$$

where, C_u = co-efficient of elastic uniform compression of soil

A = contact area of foundation with soil

m = mass of machine plus foundation.

The amplitude of displacement is given by

$$A_z = \frac{F}{m\,\omega_n^2\left(1 - r^2\right)} \qquad ...(28.35\ b)$$

where, $A_z = z_{max}$ = maximum displacement

F_0 = exciting force ; r = frequency ratio = $\dfrac{\omega}{\omega_n}$.

The above formulae for natural frequency takes no account of the mass of soil vibrating with the foundations.

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Barken gave the following equation for the co-efficient of elastic uniform compression of soil, obtained from the solution of theory of elasticity problem concerning the distribution of normal stresses under the base contact area of a rigid plate:

$$C_u = 1.13 \frac{E}{1-\mu^2} \frac{1}{\sqrt{A}} \qquad ...(28.36)$$

where E = Young's modulus of soil ; μ = Poisson's ratio

Thus, C_u depends not only on elastic constants E and μ but also on size of the base contact area of foundation. The co-efficient C_u changes in inverse proportion to the square root of the base area of the foundation:

$$\frac{C_{u2}}{C_{u1}} = \sqrt{\frac{A_1}{A_2}} \qquad ...(28.37)$$

Table 28.1 gives the recommended value of C_u for $A = 10\ m^2$, for various soils.

Table 28.1 Recommended design values of the co-efficient of elastic uniform compression C_u (Barken, 1962)

Soil group category	*Soil group*	*Permissible load on soil under action of static load only (kg/cm²)*	*Coefficient of elastic uniform compression C_u (kg /m²)*
I	Weak soils (clays and silty clays with sand, in a plastic state, clayey, and silty sands; also soils of categories II and III with laminae of organic silt and of peat)	up to 1.5	up to 3
II	Soil of medium strength (clays and silty clays with sand, close to the plastic limit; sand)	1.5–3.5	3–5
III	Strong soils (clays and silty clays with sands of hard consistency; gravels and gravelly sand, loess and loessial soils)	3.5–5	5–10
IV	Rocks	Greater than 5	Greater than 10

28.8 BULB OF PRESSURE CONCEPT

The calculations of natural frequency by Barken took no account of the mass of soil vibrating. But the work done by DEGEBO indicates that when a vibrating load acts on a soil, a certain mass of soil ranging from 4 to 5 times the vibratory load participates in the vibration. Balakrishna and Nagraj (1960) proposed the bulb of pressure concept of calculating the apparent mass of soil participating in the vibration. According to this, the vibrating mass of soil is assumed to be contained by the boundary of a pressure bulb. For the purpose of simplicity, the load acting on any surface is replaced by an equivalent concentrated load acting at the mass centre of the original area. If γ is the unit weight of the soil in lb/cu. ft., then according to the pressure bulb concept, the apparent mass of the soil is the mass enclosed by the pressure bulb of intensity σ_z lb/sq. ft. such that

$$\sigma_z = |\gamma| \qquad ...(28.38)$$

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For example, if the unit weight of soil is 110 lb/cu.ft., the apparent mass of the soil will be the mass of the soil contained by a pressure bulb of intensity 110 lb/sq. ft. From Boussinesq analysis, the vertical stress σ_z at a depth z and radial distance is $r = 0$ is given by

$$\sigma_z = 0.4775 \frac{Q}{z^2}$$

Hence,
$$|\gamma| = 0.4775 \frac{W_v}{z^2} \qquad ...(28.39)$$

In the above equation $|\gamma|$ and W_v are known.

Hence, z = diameter of the pressure bulb is known.

$$\therefore \quad \text{Weight } W_s \text{ of soil} = \frac{4}{3}\pi\left(\frac{z}{2}\right)^3 \gamma = \frac{4}{3}\pi\gamma\left[\frac{0.4775\, W_v}{4\gamma}\right]^{3/2} \qquad ...(28.40)$$

It should be noted that Eq. 28.40 is *not* dimensionally homogeneous, and is applicable only in F.P.S. units, where W_v and W_s are in pounds and γ is lbs/cu. ft.

28.9 PAUW'S ANALOGY OF FOUNDATION SOIL SYSTEM VERTICAL VIBRATIONS

Pauw (1953) gave an analytical approach with the help of which approximate values of both the *spring constant k* and the apparent mass m_s may be determined. He assumed a rectangular footing of size $a \times b$, acted on by a uniformly distributed load. He further assumed that the vertical component of the pressure from the footing is assumed to decrease with depth and to be uniformly distributed over a rectangular area bounded by planes sloping outward from footing at angle $\tan^{-1} \alpha/2$ with the vertical [Fig. 28.6 (*a*)].

The *effective zone*, up to a height z, is found to be volume of this prism.

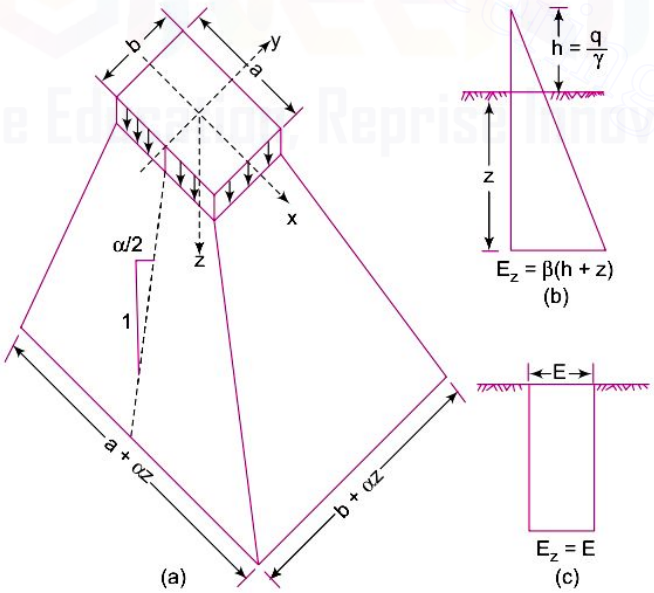

Fig. 28.6 Load dispersion

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The modulus of elasticity E_z at a depth z in the effective zone is given by

$$E_z = \beta (h + z) \qquad ...(28.41)$$

where β = rate of increase of E_z with depth

h = height of an *equilateral soil surcharge*

For a cohesive soil, however, E_z remains constant = E and may not vary with the depth. The equivalent soil surcharge is a height such that

$$h = \frac{E_0}{\beta} \qquad ...(28.42)$$

where E_0 = modulus of elasticity of soil at the base of the foundation (*i.e.*, at $z = 0$)

For cohesionless sand $E_0 = 0$ at the unconfined surface, and hence $h = 0$.

If, however, there is surcharge on the surface, such as the weight of the foundation,

$$h = \frac{q}{\gamma} \qquad ...(28.43)$$

where q = pressure intensity at foundation level ; γ = unit weight of soil.

For a partially cohesive soil, h is given by

$$h = \frac{q}{\gamma} + \frac{E_0}{\beta}$$

Table 28.2 gives the values of E_0 and β for various soils.

Table 28.2 Value of E_0 and β

Soil type	E_0 *(kg/cm²)*	β *(kg/cm²/m)*	β *(kg/cm²/m × 10⁶)*
Dense sand and gravel	0	700–1150	7–11.5
Dense sand	0	400–58	4–5.8
Loose sand	0	70–140	0.7–1.4
Loose silty sand	7	58–115	0.58–1.15
Dense silty sand	21	230–460	2.30–4.60
Clay, semi-solid	56–105	11.5–23	0.115–0.23
Clay, stiff plastic	28–56	11.5–23	0.115–0.23
Clay, weak plastic	14–28	11.5–23	0.115–0.23

The spring constant for the truncated pyramid is calculated by first of all determining the surface deformation δ_0 given by following infinite integral:

$$\delta_0 = \frac{P_0}{\beta} \int_0^\infty \frac{dz}{(a + \alpha z)(b + \alpha z)(h + z)} \qquad ...(28.44\ a)$$

or

$$\delta_0 = \frac{P_0}{\beta b^2} \int_0^\infty \frac{dm}{(l + m)(1 + m)(s + m)} \qquad ...(28.44\ b)$$

where

$$l = \frac{a}{b}(a \geq b)\,;\ s = \frac{\alpha h}{b}\,;$$

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and $$m = \frac{\alpha z}{b} \qquad ...(28.45)$$

The equivalent soil spring constant k in the vertical plane is given by

$$k = \frac{P_0}{\delta_0} \text{ (by definition)}$$

$$\frac{1}{k} = \frac{1}{\beta b^2} \int_0^\infty \frac{dm}{(l+m)(1+m)(s+m)} \qquad ...(28.46)$$

If the base of foundation is circular, a truncated cone will be considered in the place of truncated prism, and the above expression will be modified as under:

$$\frac{1}{k} = \frac{1}{\beta b^2} \int_0^\infty \frac{dm}{(1+m)^2 (s+m)} \qquad ...(28.47)$$

where $m = \frac{\alpha z}{b}$ and $s = \frac{\alpha h}{b}$; b = diameter of the foundation.

The values of k are determined by curves of Fig. 28.7 in which k is given by the equation

$$k = \beta b^2 \lambda \text{ (for rectangular base)} \qquad ...(28.48)$$

and $$k = \frac{\pi}{4} \beta b^2 \lambda \text{ (for circular base of dia. } b) \qquad ...(28.48\ a)$$

For given values of $\frac{\alpha h}{b}$ and $\frac{a}{b}$ ratios, $\frac{\lambda b}{a}$ is determined from the curves, and thus λ is known. Then k is calculated from Eq. 28.38.

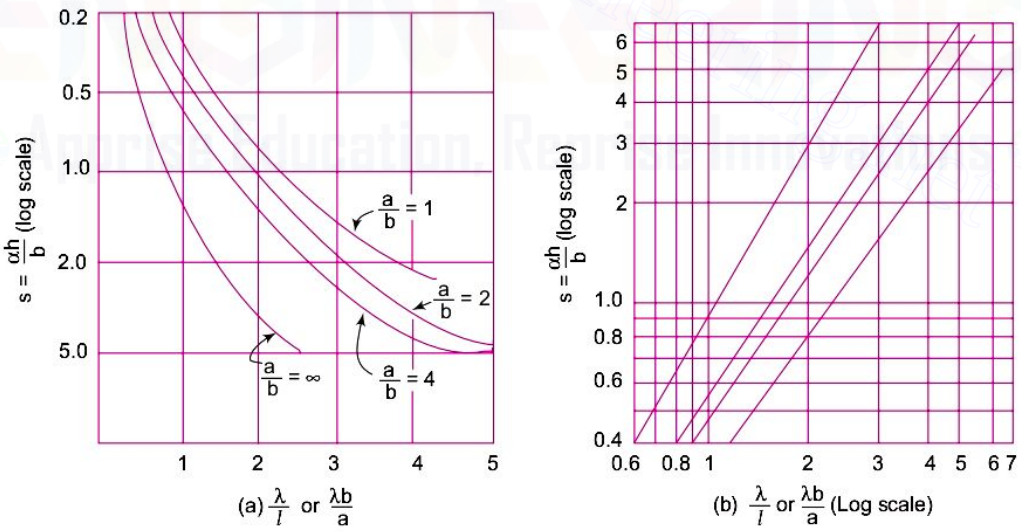

Fig. 28.7 Equivalent soil spring constant for horizontal surface

Pauw developed expression for apparent soil mass m_s by equating the kinetic energy of the effected zone to the kinetic energy of a mass assumed to be concentrated at the base of the foundation. Following is final form of the expression:

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$$m_s = \frac{\gamma b^3}{\alpha g} C_m \text{ (where the factor } C_m \text{ is read from Fig. 28.8)} \qquad ...(28.49)$$

(**Note.** The above treatment is valid only if the machine foundation has only one degree of freedom, *i.e.*, for the vibrations taking place in the vertical director. In the general case, however, the foundation may be considered as a mass having six degrees of freedom, namely, displacements in the directions of the three coordinate axes and rotations about each of these axes.)

After having determined the equivalent spring constant k and apparent soil mass m_s, the natural frequency of oscillations, and the amplitude of vibrations are determined from the following equations:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m_s + m_v}}$$

(for lineal oscillation)

and $z_{max} = A_z \qquad ...(28.34\ b)$

$$= \frac{F_0}{k\sqrt{\left\{1-\left(\frac{f}{f_n}\right)^2\right\}^2 + \left(\frac{2c}{c_c}\frac{f}{f_n}\right)^2}} \qquad ...(28.31)$$

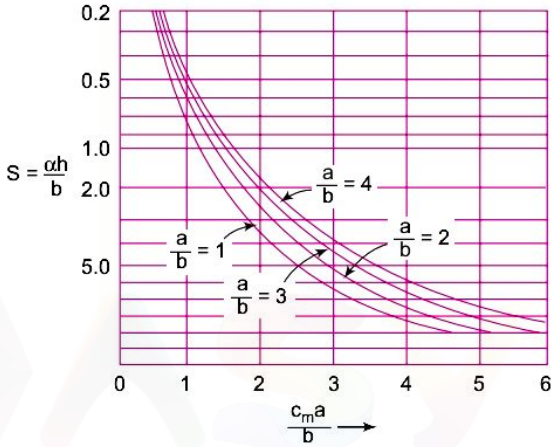

Fig. 28.8 Determination of factor C_m

Equation 28.31 for amplitude assumes the foundation rests on surface. In actual case, foundation is below the surface and hence the amplitude is less than that given by Eq. 28.31. From the few data available, the constant by which the above equation should be multiplied is roughly:

$$B = 1 - 0.6\,\frac{z}{b'} \qquad ...(28.50)$$

where z = depth of foundation below surface;

b' = average $\frac{a+b}{2}$ at the base.

The exciting force F_0 is the sum of the centrifugal force F_1 due to the rotation of eccentric weight and the dynamic force F_2 due to the acceleration of the reciprocating parts. Ranson (1943) suggests the following formulae for limiting amplitude A_{zp} in inches:

or $$A_{zp} = \frac{9.54}{f}, \text{ for } f < 1800 \text{ r.p.m.} \qquad ...(28.51)$$

$$= \frac{17600}{f^2} \text{ for } f > 1800 \text{ r.p.m} \qquad ...(28.52)$$

The excessive vibrations can be eliminated by use of heavy foundations. As a rough guide, the ratio of weight of foundation to the engine weight may be kept between 2.5 to 3.5 in most of machines. Manufacturers of machines sometime recommend the weight of foundation suitable for their machines, based largely on experience.

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SOLVED EXAMPLES

Example 28.1. *Using Barken's expressions for natural frequency and the amplitude of vibrations, calculate the change in the percentage amplitude in terms of r if the soil mass participating in the vibrations is 23% of m. Also, calculate this change for r = 0.3 and r = 2.*

Solution.

(*i*) *For the first case (without soil participation)*

$$\omega_n = \sqrt{\frac{C_u A}{m}} \qquad ...(1)$$

and $$A_z = \frac{F_0}{m\omega_n^2\left(1-r^2\right)} = \frac{a}{1-r^2} \text{ (say)} \qquad ...(2)$$

where $$r = \frac{\omega}{\omega_n}$$

(*ii*) *For the second case (with soil participation)*

$$\omega'_n = \sqrt{\frac{C_u A}{m'}} = \sqrt{\frac{C_u A}{1.23\, m}} = \omega_n \sqrt{\frac{1}{1.23}} \qquad ...(3)$$

$$A'_z = \frac{F_0}{(1.23\, m)(\omega'_n)^2 (1-r'^2)} = \frac{F_0}{(1.23\, m)\left(\frac{\omega_n^2}{1.23}\right)^2 (1-r'^2)} = \frac{a}{1-r'^2} \qquad ...(4)$$

$$\therefore \quad \% \text{ change } A_z = \frac{A_z' - A_z}{A_z} = \frac{r'^2 - r^2}{1-r'^2} \times 100 \qquad ...(5)$$

Now $$r^2 = \left(\frac{\omega}{\omega_n}\right)^2 \text{ and } r'^2 = \left(\frac{\omega}{\omega'_n}\right)^2 = \frac{\omega^2}{\frac{\omega_n^2}{1.23}} = 1.23\left(\frac{\omega}{\omega_n}\right)^2 = 1.23\, r^2$$

% change in $$A_z = \frac{1.23r^2 - r^2}{1-1.23r^2} \times 100 = \frac{23r^2}{1-1.23r^2}$$ **(Answer)**

When $$r = 0.3\text{, \% change in } A_z = \frac{23(0.3)^2}{1-1.23(0.3)^2} = 2.3\%$$

When $$r = 2\text{, \% change in } A_z = \frac{23(2)^2}{1-1.23(2)^2} = -23.5\%$$

Example 28.2. *Assuming resonance to have occurred at the frequency of 22 cycles/second in a vertical vibration of a test block, 1.0 × 1.0 × 1.0 m size, determine the coefficient of elastic uniform in compression (C_u). The weight of oscillator is 62 kg and the force produced by it at 12 cycles per second is 100 kg. Also compute the maximum amplitude in vertical direction at 12 cycles/second.*

Solution. $$\omega_n = 2\pi f_n = 2\pi \times 22 = 44\pi$$

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Mass of vibrator: $\frac{62}{g} = \frac{62}{9.81} = 6.3$

Mass of foundation block (concrete) = $\frac{1 \times 1 \times 1 \times 2.40 \times 1000}{9.81} = 244.7$

$m = 6.3 + 244.7 = 251$

A = contact area of foundations = 1 m^2

Substituting these in the expression for ω_n

$$\omega_n = \sqrt{\frac{C_u \,.\, A}{m}}, \text{ we get } (2\pi \times 22) = \sqrt{\frac{C_u \times 1}{251}}$$

or $$C_u = (44\pi)^2 (251) = 4.76 \times 10^5 \text{ kg/m}^3$$

Amplitude $$A_z = \frac{F_0}{m\omega_n^2 (1 - r^2)}$$

where F_0 = total load produced in vertical direction = 100 kg

$$m = 251 \; ; \; \omega_n = 2\pi \, (22) = 44\pi \; ; \; r^2 = \left(\frac{12}{22}\right)^2 = 0.3$$

$$A_z = \frac{100}{251(44\pi)^2 (1 - 0.3)} \text{ metres} = 3 \times 10^{-3} \text{ cm.}$$

Example 28.3. *The resonance of a test block 2 m × 1 m × 1 m occurred at 25 cycles/sec in the vertical direction. The other data are as follows :*

Weight of oscillator = 62 kg. Vertical unbalanced force = 0.5 tonnes. Unit weight of soil = 1.7 t/m^3. Calculate the apparent mass of soil by Balakrishna Rao method.

Solution. From Eq. 28.40, $W_s = \frac{4}{3} \pi\gamma \left[\frac{0.4775 \, W_v}{4\gamma}\right]^{3/2} lb$

W_v = total load in lbs (*i.e.*, weight of machine + foundation + unbalanced force) Weight of foundation block = 2 × 1 × 2.4 × 100 = 4800 kg

Weight of oscillator = 62 kg ; Vertical unbalanced force = 500 kg

Total W_v = 5362 kg =11,800 lb ; γ = 1.7 t/m^3 = 106 lb/cu ft

$$W_s = \frac{4}{3} \pi \, (106) \left[\frac{0.4775 \times 11800}{4 \times 106}\right]^{3/2} = 21500 \text{ lb} = 9750 \text{ kg}$$

$$m_s = \frac{9750}{9.81} = 994 \text{ kg-sec}^2/\text{m}$$

Example 28.4. *Design the foundation for a gas engine with a vertical cylinder and vertically oscillating parts, for the following data:*

1. *Total weight of engine = 4500 kg.*
2. *Speed of rotation = 260 r.p.m.*
3. *Unbalanced vertical force = 1 tonne*

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4. *Base dimensions of the engine = 1 m × 2.5 m*
5. *Elevation of machine base above ground = 1 m*

Weak silty sand exists to a depth of 0.5 m followed by a dense sand to a depth of 6 m. The unit weight of moist sand is 1.7 t/m³.

Solution. Let the size of the block be 1.5 × 3 m at the base of the machine, and 2 × 3.5 m at its bottom. Let the height of the concrete block be 2.0 m, so that it penetrates 1 m below the ground.

Weight of the block $= \frac{(1.5 \times 3) + (2 \times 3.5)}{2} \times 2 \times 2.4$ tonnes $= 27.6$ t

This is about 6 times the weight of machine. Hence satisfactory.

Total mass of machine and foundation, $m_v = \frac{27.6 + 4.5}{9.81} \times 1000 = 3270$ kg-sec²/m

Let us first calculate the spring constant k.

Unit soil pressure $q = \frac{27.6 + 4.5}{2 \times 3.5} = 4580$ kg/cm² ; $\gamma = 1.7$ t/m³ = 1700 kg/m³

Equivalent surcharge $= h = \frac{q}{\gamma} = \frac{4580}{1700} \approx 2.7$ m

Now $s = \frac{\alpha h}{b}$.

Assuming $\alpha = 1$ and taking $b = 2$ m

$$s = \frac{1 \times 2.7}{2} = 1.35 \; ; \; \frac{a}{b} = l = \frac{3.5}{2} = 1.75$$

From Table 28.2, assume, $\beta = 4.6 \times 10^6$ kg/m²/m.

From Fig. 28.7, when $s = 1.35$ and $\frac{a}{b} = 1.75$, we get $\frac{\lambda b}{a} = 2$

$\therefore \quad \lambda = 2 \times \frac{a}{b} = 2 \times \frac{3.5}{2} = 3.5$

$\therefore \quad k = \beta b^2 \lambda = (4.6 \times 10^6)\,(2)^2\,(3.5) = 64.4 \times 10^6$ kg/m

The apparent soil mass is given by $m_s = \frac{\gamma b^3}{g\alpha} C_m$

From Fig. 28.8, When $s = 1.35$ and $\frac{a}{b} = 1.75$, $\frac{C_m b}{a} = 1.1$

$\therefore \quad C_m = 1.1 \times \frac{3.5}{2} = 1.93$

and $m_s = \frac{1700\,(2)^3}{9.81 \times 1} \times 1.93 = \mathbf{2670\ kg\text{-}sec^2/m}$

Hence the natural frequency is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m_v + m_s}} = \frac{1}{2\pi}\sqrt{\frac{64.4 \times 10^6}{3267 + 2670}} = 16.6 \text{ c.p.s} = 996 \text{ r.p.m}$$

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The operating frequency of the machine is $\omega = 260$ r.p.m.

$$\frac{\omega}{\omega_n} = \frac{260}{996} \approx \mathbf{0.27}$$

The amplitude of vibration is determined from

$$A_z = \frac{F_0}{k\sqrt{\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\frac{c}{c_c}.\frac{\omega}{\omega_n}\right)^2}}$$

where F_0 = dynamic load = 1000 kg. Assume $\frac{c}{c_c} = 0.15$

$$A_z = \frac{1000}{64.4\times10^6\sqrt{\left\{1-(0.27)^2\right\}^2 + \{2\times0.15\times0.27\}^2}} = \frac{1}{64.4\times10^3\times0.97}\text{ m}$$

$$= 0.0016 \text{ cm}$$

Reduction factor $B = 1 - 0.6\frac{z}{b'}$

where $b' = \frac{1}{2}(2 + 2.5) = 275$ m ;

z = depth below ground surface = 1 m

$$B = 1 - 0.6\frac{1}{2.75} = 1 - 0.218 = 0.782$$

$\therefore$ $(A_z)_{max} = 0.0016 \times 0.782 = \mathbf{0.00125\ cm.}$

Permissible amplitude by Rauch equation is

$$A_{zp} = \frac{9.54}{f} \text{ (for } f < 1800 \text{ r.p.m.)}$$

$$= \frac{9.54}{260} = 0.0366'' = \mathbf{0.09\ cm.}$$ Hence the foundation is safe.

28.10 DYNAMIC ANALYSIS OF BLOCK FOUNDATIONS

The methods of Pauw and Balakrishna Rao are based on the assumption that a certain mass of soil participates in the vibration with the foundation. Barken's method is based on liner spring theory. The method neglects the effects of damping and participating soil mass. Barken's method is very much used in design offices. A summary of various formula are given below:

1. Equation of Motion: In the general case, the foundation may be considered as a mass having six degrees of freedom—displacements in the three co-ordinate axes (x, y, z) and rotation about each of these axes (Fig. 28.9). The rotations about y, z and x-axes are respectively known as *rocking, yawing* and *pitching*. Let us first consider the in oscillating forces the x-z plane. If the combined C.G. of the machine and foundation lies in the same vertical line as the centroid of the base plane, then the vertical

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and translation and twisting modes are *uncoupled,* while the sliding and rocking in each of the two vertical planes *(xz* and *yz)* passing through the common centre of gravity of the machine and foundation are separately coupled.

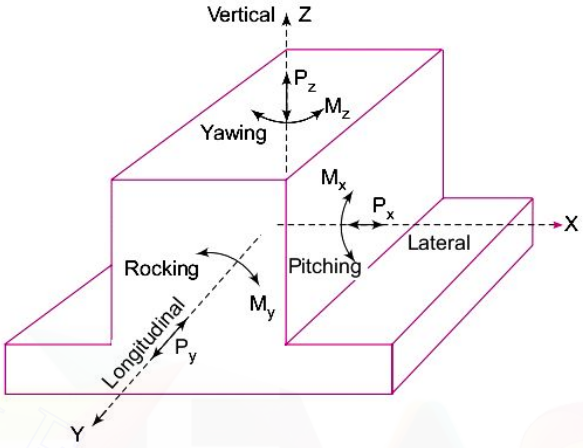

Fig 28.9 Block foundation subjected to six modes of vibrations

The following are the equations of motion for an undamped case:

Vertical: $$m\ddot{z} + k_z z = P_z(t) \quad ...(28.53)$$

Horizontal: $$m\ddot{x} + k_x (x - S\theta_y) = P_x(t) \quad ...(28.54a)$$

Rocking $$I_{my}\ddot{\theta} - k_x S + (k_{\theta y} - WS + k_x S^2)\theta_y = M_y(t) \quad ...(28.55)$$

where x, z = the displacements along x and z axes respectively

θ_y = Rotation about y-axis

S = Height of C.G. of block foundation about the *centre of elasticity* of base support. If the block is resting directly on soil, S is equal to the height of C.G. of the block measured above the soil surface. If the block is resting on elastic bedding or on springs, S is the height between the C.G. of the block and the mid-point of the elastic bedding or the springs.

k_z = Stiffness of elastic support (or spring) in vertical compression

k_x = Stiffness of elastic support against rotation about y-axis

$k_{\theta y}$ = Stiffness of elastic support against rotation about y-axis

m = Mass of the foundation block = W/g

I_{my} = Mass moment of inertia of the foundation about y-axis

$P_x(t)$ and $P_z(t)$ = Time dependent oscillating loads in horizontal and vertical directions

$M_y(t)$ = Time displacement rocking moment.

Equation 28.54 and 28.55 form a coupled set (since both contain x and θ) while Eq. 28.53 representing vertical translation can be solved independently.

2. Natural frequency and amplitude-Vertical translation: The circular natural frequencey (ω_{nz}) for uncoupled vertical translation along the z-axis is given by

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$$\omega_{nz} = \sqrt{\frac{k_z}{m}} \qquad ...(28.56)$$

For foundations resting directly on soil $\omega_{nz} = \sqrt{\frac{C_u A}{m}}$ ...(28.56*a*)

which is same as equation 28.35, in which C_u *is the coefficient of elastic uniform* compression for soil, to be taken from Table 28.1.

The vertical amplitude (A_z) under the action of an exciting force $P_z \sin \omega t$ is given by Eq. 28.35 (*b*),

$$A_z = \frac{P_z}{m(\omega_{nz}^2 - \omega^2)} = \frac{P_z}{m\omega_{nz}^2(1 - r^2)} \qquad ...(28.57)$$

where ω = circular operating frequency ;

r = frequency ratio = $\dfrac{\omega}{\omega_{nz}}$.

Sliding and rocking motion in *xz* plane. The two natural frequencies ω_{n1} and ω_{n2} representing the coupled motion (*i.e.*, sliding along *x*-axis and rocking about *y*-axis) in *xz* in plane are given by the two roots of the following quadratic equation:

$$\omega_n^4 - \left(\omega_{\theta y}^2 + \omega_x^2\right)\frac{\omega_n^2}{R_{my}} + \frac{\omega_{\theta y}^2 + \omega_x^2}{R_{my}} = 0 \qquad ...(28.58)$$

where ω_x = Natural circular frequency for *pure sliding* along *x*-axis when the foundation is assumed to possess infinite resistance to rocking.

$\omega_{\theta y}$ = Natural circular frequency for *pure rocking* when the foundation is assumed to possess infinite resistance to sliding along *x*-axis.

$$R_{my} = \frac{I_{my}}{I_{moy}}$$

I_{my} = Mass moment of inertia of the foundation block about *y*-axis.

I_{moy} = Mass moment of inertia about a parallel axis through the *centre of elasticity*.

For foundations resting on elastic support:

$$\omega_{\theta y}^2 = \frac{(k_{\theta y} - W.S)}{I_{moy}} \qquad ...(28.59)$$

and $$\omega_x^2 = \frac{k_x}{m} \qquad ...(28.60)$$

For foundations resting on soil:

$$\omega_{\theta y}^2 = \frac{C_\theta I_y - W.S}{I_{moy}} \qquad ...(28.61)$$

and $$\omega_x^2 = \frac{C_\tau A}{m} \qquad ...(28.62)$$

where C_θ = Coefficient of elastic non-uniform compression. According to Barken, its value may be taken equal to $2C_u$.

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C_τ = Coefficient of elastic uniform shear. According to Barken, its value may be taken equal to 0.5 C_u.

I_y = Second moment of contact area (A) about horizontal y-axis passing through the centroid of the base normal to the plane of rocking.

The *horizontal amplitude* (A_x) and *rotational amplitude* ($A_{\theta y}$) under the exciting force $P_x \sin \omega t$ and exciting moment $M_y \sin \omega t$ are given by :

$$A_x = [(k_{\theta y} - W.S + k_x S^2 - I_{my}\, \omega^2)\, P_x + (k_x S)\, M_y]/f(\omega^2) \quad ...(28.63)$$

and

$$A_{\theta y} = [(k_x S)\, P_x + (k_x - m\omega^2)\, M_y]/f(\omega^2) \quad ...(28.64)$$

where

$$f(\omega)^2 = m\, I_{my}\, (\omega_{n1}^2 - \omega^2)\, (\omega_{n2}^2 - \omega^2) \quad ...(28.65)$$

The net horizontal displacement at the upper edge of the foundation will be equal to $A_x + (H - S)\, A_{\theta y}$, where H is the height of foundation.

3. Stiffness of elastic supports:

(*a*) *Soil base.* The values of k_x, k_y and $k_{\theta y}$ for the elastic support of *soil* are given by the following expressions:

For vertical motion: $k_z = C_u A$ ...(28.66 *a*)

For horizontal (sliding) motion: $k_x = C_\tau A$ ...(28.66 *b*)

For rocking motion : $k_{\theta y} = C_\theta\, I_y$ ...(28.66 *c*)

Thus, knowing C_u, the values of k_z, k_x and $k_{\theta y}$ can be computed.

(*b*) *Elastic pads.* If, however, the foundation block is supported on elastic pad of contact area A and thickness t, then the values of the stiffness factors are given by the following expressions:

For vertical motion: $k_z = \dfrac{EA}{t}$ ...(28.67 *a*)

For horizontal (sliding motion): $k_x = \dfrac{CA}{t}$ ...(28.67 *b*)

For rocking motion: $k_{\theta y} = \dfrac{EI_y}{t}$ ...(28.67 *c*)

where E = modulus of elasticity of the pad material

G = shear modulus of the pad material.

(*c*) *Steel spring.* If the foundation block is supported on steel spring with d as the diameter of spring wire, D as the diameter of spring coil, n as the number of windings in each coil, h as height of each coil and G as the shear modulus of the material, the stiffness factors are given by the following expressions:

For vertical motion,

$$k_z = k_s = \frac{1}{n}\frac{Gd^4}{8D^3} \quad ...(28.68\ a)$$

If there are N springs in the coil, the resultant vertical stiffness will be $N.k_s$.

For horizontal motion (sliding motion),

$$k_x = k_s \left[\frac{1}{0.385\,\alpha\left\{1 + \dfrac{0.77}{D^2}h^2\right\}}\right] \quad ...(28.68\ b)$$

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where coefficient α depends upon $\frac{h}{D}$ and $\frac{\delta_z}{h}$ ratio, δ_z being the static compression of the spring coil under vertical load. The value of α can be found from Fig. 28.10 prepared by Major (1962).

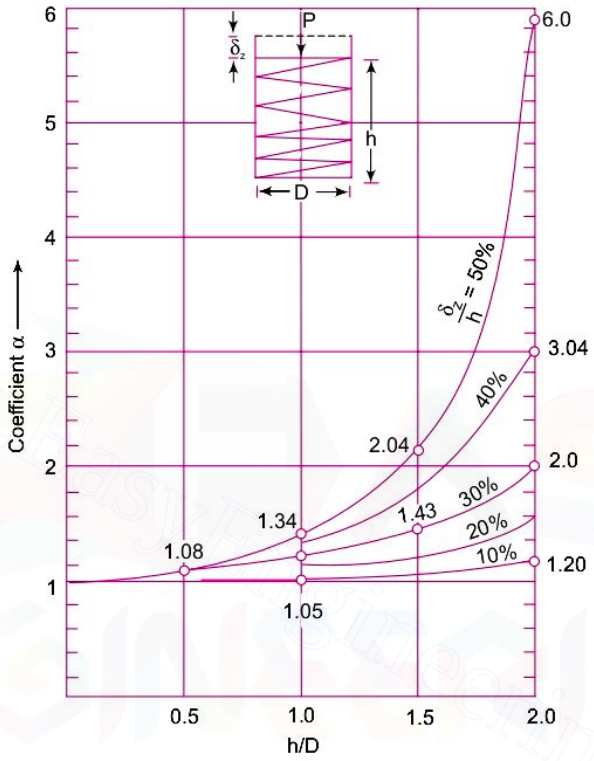

Fig. 28.10 Value of coefficient α

If there are N springs in the coil, the resultant horizontal stiffness will be equal to $N.k_x$.

For rocking motion $$k_{\theta y} = \frac{I_y'}{h}\frac{Gd^4}{8D^3} = I_y' \cdot k_s \qquad ...(28.68\ c)$$

where I_y' is the moment of intertia of die group of isolated spring supports about y-axis.

28.11 INDIAN STANDARD CODE OF PRACTICE FOR DESIGN OF FOUNDATIONS FOR RECIPROCATING TYPE MACHINES

IS code of practice (IS : 2974, Part I : 1964) gives the following criteria :

1. The size of the foundation block (in plan) should be larger than the bed plate of the machine with a minimum alround clearance of 15 cm.
2. The width of the foundation should be at least equal to the distance of the centre of gravity of the crank shaft to the bottom of foundation in all vertical machines.

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3. The depth of the foundation should be such as to rest the foundation on a good bearing strata and to ensure stability against rotation in a vertical plane.
4. The combined centre of gravity of machine and the foundation block should be as much below the top of the foundation as possible.
5. Whenever possible, the operating frequency should be lower than the natural frequency of the foundation soil system and the frequency ratio should be less then 0.5. When the operating frequency is higher than the natural frequency of the foundation soil system of the machine, the frequency ratio should be more than 2 for important machines and more than 1.5 for others.

Natural Frequency. The natural frequency (cycles per second) under the given stress intensity is given by the following equation:

$$f_n = \frac{15.8}{\sqrt{\delta}} \qquad ...(28.69)$$

where δ = elastic deflection of the foundation in milimeters, with respect to the entire area of foundation, to be determined from Fig. 28.11.

The permissible bearing pressure, for use in Fig. 28.11, under dynamic loads may be taken as $\frac{1}{3}$ to $\frac{1}{4}$ of the permissible bearing pressure under static loads.

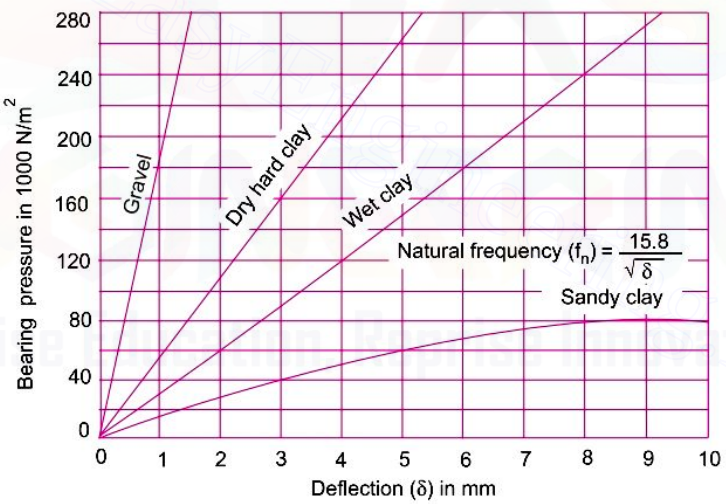

Fig. 28.11 Chart for determining δ. (IS : 2974, Part I-1964)

Amplitude of vibrations. The *permissible amplitude* should not exceed the limiting amplitude for the machine prescribed by the manufacturers. Where such data are not available, for preliminary design and for relatively unimportant structures, the limiting amplitude should be determined from Fig. 28.12.

The *probable amplitude,* in mm, of the machine at the operating frequency, is given by the following expression:

$$A_z = \frac{250 F_0}{W_v f_n^2} R \qquad ...(28.70)$$

where F_0 = dynamic force in kg

W_v = weight in kg of machine and foundation

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f_n = natural frequency in cycles per second

$$\frac{250\,F_0}{W_v\,f_n^2} = \text{frequency ratio} \qquad \text{...(28.71)}$$

R = the amplitude ratio

$$= \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \qquad \text{...(28.72)}$$

r = frequency ratio $\dfrac{\omega}{\omega_n}$ or $\dfrac{f}{f_n}$

ξ = damping ratio or damping factor $\dfrac{c}{c_c}$

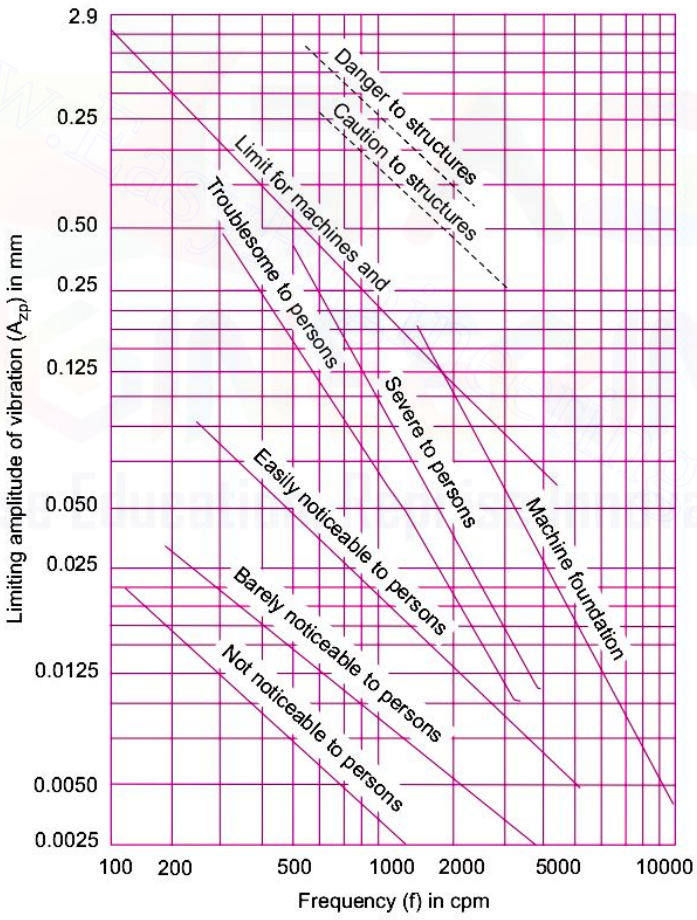

Fig. 28.12 Limiting amplitude for vertical vibrations. (IS : 2974, Part I-1964)

The value of ξ should be determined from dynamic tests. When the value of ξ is not provided, it can be taken as 0.25.

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28.12 INDIAN STANDARD CODE OF PRACTICE FOR DESIGN OF FOUNDATION FOR IMPACT TYPE MACHINES

The design requirements of the impact types machines, such as drop and forge hammers, are different than those of the reciprocating type machines discussed above. IS: 2974 (Part II): 1966 covers the design requirements for the foundations of these heavy impact type machines. Fig. 28.13 showns some typical sections of the foundations for these machines.

Definitions. (*i*) *Anvil:* Anvil is a base-block for a hammer on which material is forged into shape by repeated striking of the tup.

(*ii*) *Tup:* Tup is a weighted block which strikes the material being forged on the anvil.

(*iii*) *Foundation block:* It is a mass of reinforced concrete on which the anvil rests.

(*iv*) *Protective cushioning layer* (joint J_1) : It is an elastic cushioning of suitable material and thickness provided between the anvil and the foundation block in order to prevent bouncing of anvil and creation of large impact stresses and consequent damages to the top surface of the concrete in the foundation block.

(*v*) *Foundation support (joint J_2)* : It is a support for resting the foundation block. The block may rest directly on ground or on a resilient mounting such as timber sleepers, springs cork layer etc. The block may also be supported on pile foundations.

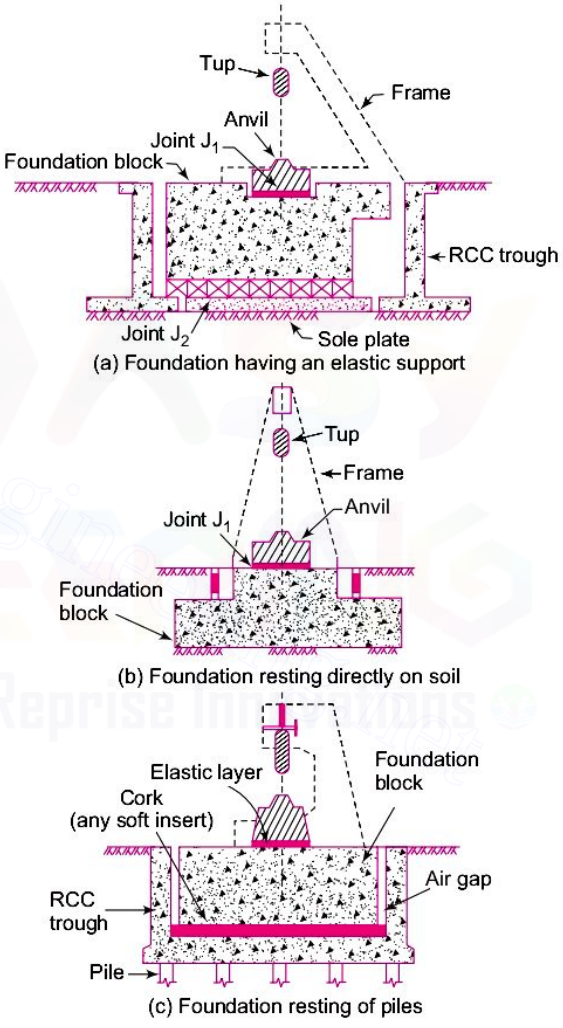

Fig. 28.13 Different types of foundation supports for impact type machines

Design Criteria:

1. The stresses produced at the time of impact in the foundation base (soil, timber, sleeper, cork, spring elements or piles) should be within 0.8 times allowable static stresses.
2. The design of the entire foundation system should be such that the centres of gravity of the anvil, and of the foundation block, as well as the joints at which the resultants of forces in the elastic joints J_1 and J_2 act, coincide with the line of fall of the hammer tup. While determining the centre of gravity of the foundation block, the weight of the frame of the tup could also be considered.
3. The maximum vertical vibrational amplitude of the foundation block should not be more than 1.2 mm. In case of foundations on sand below the ground water, the permissible amplitude should not be more than 0.8 mm.

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4. For the anvil, the permissible amplitude, which depends upon the weight of the tup should be taken from the following table:

Weight of tup	*Maximum permissible amplitude*
up to 1 t	1 mm
2 t	2 mm
more than 3 t	3 to 4 mm

5. The area of foundation block should be such that the safe loading intensity of soil is never exceeded during the operation of the hammer. The depth of the foundation block should be so designed that the block is safe both in punching shear and bending. However, the following mininum thickness of foundation block should be provided.

Weight of tup (tonnes)	*Minimum depth of foundation block*
up to 1.0	1.00 m
1.0 to 2.0	1.25 m
2.0 to 4.0	1.75 m
4.0 to 6.0	2.25 m
over to 6.0	2.50 m

6. The weight of anvil may be generally kept at 25 times the weight of tup. The weight of the foundation block W_b generally varies from 66 to 120 times the weight of the tup. Where the foundations rest on stiff clays or compact sandy deposits, the weight should be from 75 to 80 times the weight of the tup. For moderately firm to soft clays and for medium dense to loose sandy deposits, the weight of the block should be from 90 to 120 times the weight of the tup.
The approximate weight of the foundation block may also be determined from the following formula:

$$W_b = 0.08\,(1 + I_f)\,V_{tb}\,W_t - (W_a + W_f) \qquad ...(28.73)$$

where W_b = weight of the foundation block (kg)
I_f = impact factor, $0 < I_f < \lambda$, and its average value for design purposes may be taken up to 0.6
$V_{tb} = \sqrt{2gh}$ for a freely falling tup type hammer

$$= 0.65\sqrt{\frac{2g(W_t + p_s)h}{W_t}} \text{ for double acting steam hammers}$$

h = height of fall of tup (cm)
W_a = weight of anvil, kg
p_s = steam pressure, kg/cm^2
W_t = weight of the tup, (kg)
W_f = weight of frame.

7. The foundation block should be made of reinforced concrete and reinforcement should be arranged along the three axes and also diagonally to prevent shear, as shown in Fig. 28.14. More reinforcement should be provided at the top side of the foundation block than at the other side. Reinforcement at the top may be provided in the form of layers of grills made of 16 mm diameter bars suitably spaced to allow easy pouring of concrete. The reinforcement provided should be at leat 25 kg per m^3 of concrete.

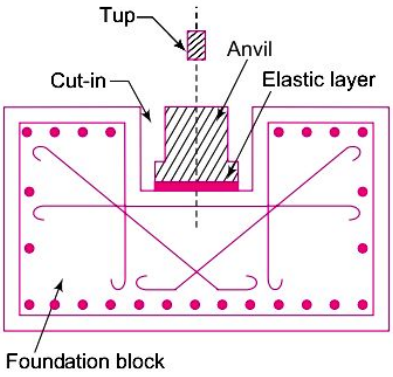

Fig. 28.14 Typical reinforcement detail

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28.13 EXAMPLES FROM COMPETITIVE EXAMINATIONS

Example 28.5. *Resonance occurred at a frequency of 22 cycles per second in a vertical vibration test of a block 1m ×1m × 1m. Determine the coefficient of elastic uniform in compression of the soil given that the weight of the oscillator in 65 kg and that the force produced by it at 12 cycles per second is 100 kg. Also compute the amplitude in vertical direction at 12 cycles per second.* **(Civil Services Exam. 1989)**

Solution. ***This problem is exactly based on example 28.2 except that the weight of the oscillator has been changed from 65 kg to 62 kg.***

$$\omega_n = 2\pi f_n = 2\pi\,(22) = 442\pi\ ;$$

Mass of vibrator $= 65/9.81 = 6.63$

Mass of test block of concrete $= \dfrac{1\times1\times2.4\times1000}{9.81} = 244.65$

$\therefore \quad m = 6.63 + 244.65 = 251.28$

Now $\omega_n = \sqrt{\dfrac{C_u \cdot A}{m}}$

$\therefore \quad C_u = \dfrac{\omega_n^2 \cdot m}{A} = \dfrac{(44\pi)^2 \times 251.28}{1m^2} = 4.8 \times 10^6 \text{ kg/m}^3$

Also, F_0 = total load produced in vertical direction = 100 kg

$r = 12/22 = 0.5455\ ;\ r^2 = 0.2975$

$\therefore$ Amplitude $A_z = \dfrac{F_0}{m\,\omega_n^2\left(1-r^2\right)} = \dfrac{100}{251.28\,(44\pi)^2\left[1-0.2925\right]}$

$= 2.96 \times 10^{-5}$ m = **0.0296 m**

Example 28.6. *Using Barken's expression for natural frequency and the amplitude of vibrations, calculate the change in the percentage amplitude in terms of frequency ratio (r) if the soil mass participating in the vibration is 23% of m.* **(Civil Services Exam. 1990)**

Solution. ***This problem has been set from Example 28.1 of this book. Hence for solution, refer Example 28.1***

Example 28.7. *Assuming resonance to have occurred at a frequency of 30 cycles/sec. in a vertical vibration of a test block 1m × 1m × 1m, determine the coefficient of elastic in compression. The weight of the oscillator is 800 N and the force produced by it after 15 cycles is 1500 N. Compute the maximum amplitude in the vertical direction at 15 cycles per sec. Weight of test block is 24 kN/m^3.*

(Civil Services Exam. 1992)

Solution. This problem is based on Example 28.2

$$\omega_n = 2\,\pi f_n = 2\,\pi\,(30) = 188.5$$

Mass of vibrator $\dfrac{800}{9.81} = 81.5$ kg

Mass of block $= \dfrac{(1\times1\times1\times24)1000}{9.81} = 2446.5$ kg

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$\therefore$ Total mass $m = 81.5 + 2446.5 = 2528$ kg

$$C_u = \frac{m\,\omega_n^2}{A} = \frac{2528\,(188.5)^2}{1} = 89.8 \times 10^6 \text{ kg/m}^3$$

Again $r = \dfrac{f}{f_n} = \dfrac{15}{30} = 0.5$; $r^2 = 0.25$; $F_0 = 1500$ N

$\therefore$ $$A_z = \frac{F_0}{m\,\omega_n^2(1-r^2)} = \frac{1500}{2528\,(188.5)^2\,(1-0.25)} = 2.23 \times 10^{-5} \text{ m} = \mathbf{0.0223\ mm}$$

Example 28.8. *A machine having a total weight of 20000 kN has an unbalance such that it is subjected to a force of magnitude 5000 kN at a frequency of 600 rpm. What should be the spring constant for the supporting springs if the maximum force transmitted into the foundation due to the machine is to be 500 kN? Assume that the damping can be neglected.* **(Civil Services Exam. 1995)**

Solution. $m = \dfrac{W}{g} = \dfrac{20000 \times 1000}{9.81} = 2.0387 \times 10^6$ kg

Transmitted force $F_T = F_0\,\mu\sqrt{1 + 4\,\xi^2 \times r^2}$ ...(1)

where r = frequency ratio $= \dfrac{f}{f_n} = \dfrac{\omega}{\omega_n}$; ξ = damping ratio = 0 (given)

$$\mu = \frac{1}{\sqrt{(1-r^2)^2 + 4\,\xi^2\,r^2}} = \frac{1}{1-r^2} \text{ for } \xi = 0$$

F_0 = exciting force = unbalanced force = 5000 kN

F_T = force transmitted = 500 kN

Hence from (1), $500 = 5000 \times \dfrac{1}{1-r^2}$ or $1 - r^2 = 10$

from which $r = 3 = \dfrac{f}{f_n} = \dfrac{\omega}{\omega_n}$

But $\omega = 600$ rpm $= 10$ r.p.s.

$\therefore$ $\omega_n = \dfrac{\omega}{3} = \dfrac{10}{3} = 3.33$ r.p.s.

Now, $\omega_n = \sqrt{k/m}$

$\therefore$ $k = m\ \omega_n^2 = 2.038 \times 10^6\ (3.33)^2 = 22.65 \times 10^6$ kg/sec^2

$\therefore$ $k = \mathbf{22.65 \times 10^6\ N/m}$ since 1 kg = 1 $\dfrac{\text{N}}{\text{m}}$ sec^2

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Part VII
Pavement Design

29. Design of Flexible Pavement
30. Design of Rigid Pavement
31. Stabilisation of Soils

Apprise Education, Reprise Innovations

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CHAPTER 29

Design of Flexible Pavement

29.1 INTRODUCTION: TYPES OF PAVEMENTS

A natural earth track is incapable of supporting modern wheel loads. A constructed pavement is required on the top of the soil in order to distribute the wheel load efficiently and to provide the necessary wearing surface. *A pavement is, therefore, defined as a relatively stable crust constructed over the natural soil for the purpose of supporting and distributing the wheel loads and providing an adequate wearing surface.* Depending upon the mode of supporting and distributing loads, pavements are classified as flexible, rigid and semi-flexible.

The *flexible pavements* consist of a relatively thin wearing surface built over a base course and sub-base course, and they rest on compacted subgrade. The flexible pavements are able to resist only very small tensile stresses. In contrast to this, *rigid pavements* are made up of Portland cement concrete and may or may not have a base course between the pavement and the subgrade. A rigid pavement can take appreciable tensile stresses and is capable of bridging small weakness and depression in the subgrade. A *semi-flexible* pavement is made of dry-lean concrete or soil cement, and possesses qualities intermediate between the flexible and rigid pavements. A semi-flexible pavement possesses appreciable flexural strength but its modulus of elasticity is considerably lower than that of concrete.

The essential difference between rigid and flexible pavements is the manner in which they distribute the load over the subgrade. The design of a flexible pavement is based on the principle that a surface load is dissipated by carrying it deep into the ground through successive layer of granular materials. Hence the strength of a flexible layer is a result of building up thick layers and thereby distributing the load over the subgrade rather than by the bending action. The thickness design of the flexible pavement is influenced primarily by the strength of the subgrade.

Because of its rigidity and high tensile strength, a rigid pavement tends to distribute the load over a relatively wide area of soil, and a major portion of the structural capacity is supplied by the slab itself. For this reason, minor variations in subgrade strength have little influence upon the structural capacity of the pavement.

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The rigid pavements are used for heavier loads and can be constructed over relatively poor subgrade such as black cotton or plastic soils, peat etc. However, since the load is taken up by the bending action of the slab, uniform subgrade support is the most essential condition for the satisfactory performance of rigid pavement. In the flexible pavements, on the contrary, a high quality, well compacted subgrade is essential.

29.2 STRUCTURAL ELEMENTS OF A FLEXIBLE PAVEMENT

A flexible pavement is usually built up in several layers as shown in Fig. 29.1, each layer having a special function. Generally, the flexible pavement thickness consists of three components: surfacing, base and subbase course. The *wearing course* or *surfacing* is the component of a pavement with which the wheels of vehicles are in actual contact. The purpose of the wearing course, made of bituminous material is to provide a smooth riding surface that is resilient and will resist pressure exerted by the tyres. It should be flexible so that it will not fail if consolidation of the subgrade or base course takes place.

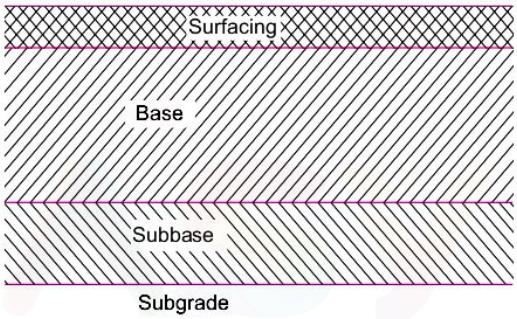

Fig. 29.1 Basic structural elements of a flexible pavement

A *base course* is defined as a layer of granular material which lies immediatly below the wearing surface of a flexible pavement. A *subbase* is a layer of material between the base and subgrade. Base course and sub-bases are used under flexible pavements primarily to increase the load supporting capacity of the pavement by distributing the load through a finite thickness of pavement. The base course lies close to the pavement surface, and hence it must possess high resistance to deformation in order to withstand the high pressures imposed upon it. However, a subbase can be of a lower quality.

The subgrade is the foundation layer, the structure which must eventually support all the loads which come on to the pavement. The performance of the pavement is affected by the characteristics of the subgrade. Desirable properties which the subgrade should possess are: strength, drainage, ease of compaction, permanency of compaction, and permanency of strength. The strength of the subgrade is increased by compaction, or in some cases by stabilisation. Stability of the subgrade is influenced by soil texture, water content, density, frost action, shrinkage and swelling, and other climatic factors.

29.3 STRESSES IN FLEXIBLE PAVEMENT

1. Stresses in homogeneous mass: We have seen in chapter 13 that the vertical stress distribution on any horizontal plane at a depth z below the ground surface, due to a concentrated load takes the form of a bell-shaped surface. The maximum stresses occur on the vertical plane passing through the point of load application. According to Boussinesq's analysis, the vertical stress σ_z at any point (r, z) is given by

$$\sigma_z = \frac{3P}{2\pi z^2} \frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}} \qquad ...(29.1)$$

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Integrating the equation over a loaded circular area of radius a, the vertical stress on a vertical line passing through the centre of the loaded area is given by

$$\sigma_z = p\left[1 - \frac{z^3}{(a^2 + z^2)^{3/2}}\right] \qquad ...(29.2)$$

where p is the unit load on the circular loaded plate, z is the depth and a is the radius of the plate. Similarly, the horizontal radial stress is given by

$$\sigma_r = \frac{p}{2}\left[1 + 2\mu - \frac{2(1+\mu)z}{(a^2 + z^2)^{1/2}} + \frac{z^3}{(a^2 + z^2)^{3/2}}\right] \qquad ...(29.3)$$

where μ = Poisson's ratio.

2. Elastic deformation under circular load: The vertical strain $\in_z$, due to the triaxial load (σ_z, σ_r, $\sigma_t = \sigma_r$) under the centre of the plate is given by Hooke's law :

$$\in_z = \frac{1}{E}(\sigma_z - 2\mu\sigma_r) \qquad ...(29.4)$$

where E = modulus of elasticity of the subgrade.

Subsituting Eqs. 29.2 and 29.3 in Eq. 29.4, and integrating between $z = z$ and $z = \infty$, the elastic strain Δ of the subgrade is given by

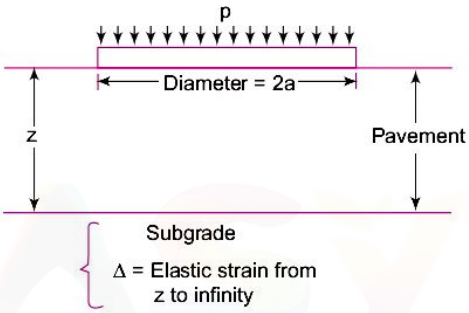

Fig. 29.2 Elastic deformation under circular load

$$\Delta = \frac{p}{E}\left[(2 - 2\mu^2)(a^2 + z^2)^{1/2} - \frac{(1+\mu)z^2}{(a^2 + z^2)^{1/2}} + (\mu + 2\mu^2 - 1)z\right] \qquad ...(29.5)$$

Taking $\mu = 0.5$, the above expression reduces to

$$\Delta = \frac{3pa^2}{2E(a^2 + z^2)^{1/2}} \qquad ...(29.6)$$

or

$$\Delta = \frac{pa}{E}F_B \qquad ...(29.7)$$

where

$$F_B = \frac{3}{2}\frac{1}{\left[1 + \left(\frac{z}{a}\right)^2\right]^{1/2}} = \text{Boussinesq settlement factor.} \qquad ...(29.8)$$

Equation 29.6 or 29.7 give the elastic deformation of the subgrade only. The elastic deformations from surface to depth z are not considered since the only significant deflections are in the subgrade. If the load is at the surface of the subgrade ($z = 0$), Eq. 29.6 reduces to

$$\Delta = 1.5\,\frac{pa}{E} \qquad ...(29.9)$$

Figure 29.3 gives the curves for deflection factor for various values of $\frac{z}{a}$ and $\frac{r}{a}$.

The $\frac{r}{a}$ ratio corresponding to Eq. 29.6 is zero and the curve of $\frac{r}{a} = 0$ in Fig. 29.3 gives the deflections according to Eq. 29.7 (Foster and Ahlvin, 1954).

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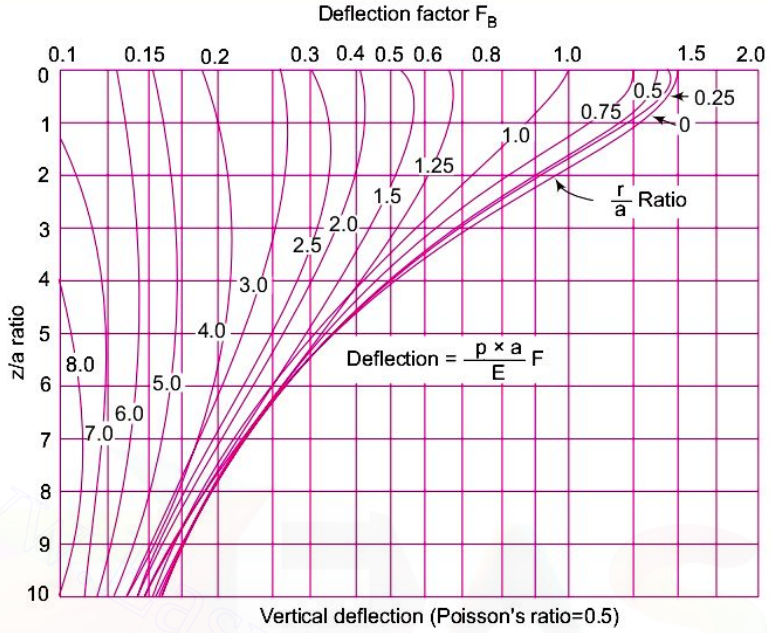

Fig. 29.3 Charts for vertical deflections (Foster and Ahlvin, 1954)

3. Burmister analysis: The flexible pavements consist of a number of layers the moduli of elasticity of which decreases with depth. In the previous analysis, the effect of the pavement components was ignored while calculating the deflections. Burmister (1943, 45, 58), took into account the effect of various layers. In the simplest case, the whole structure may be thought to be made up of two layers: the base course or pavement layer, and the subgrade layer. In the analysis of the two layer system, following assumptions are made : (*i*) the layers are homogeneous, isotropic and elastic, (*ii*) the surface layer is infinite in extent in the lateral direction, but of finite depth, (*iii*) the subgrade layer is infinite in both horizontal and vertical directions, (*iv*) both the layers are in continuous contact, and (*v*) surface layer is free from shearing and normal stress outside the loaded area. The vertical stress at any depth z, at the center of the plate is given by

$$\sigma_z = C_B p \qquad \text{...(29.10)}$$

where C_B = Burmister's influence coefficient

$$= \frac{\sigma_z}{p} \qquad \text{...(29.10 } a\text{)}$$

Figure 29.4 gives the influence curves for coefficient C_B for various values of ratio $\frac{z}{a}$ and strength ratio $\frac{E_1}{E_2}$ of the layers, E_1 is the modulus of elasticity of the upper layer and E_2 is the modulus of elasticity of the lower layer. In Fig. 29.4 the Boussinesq vertical stress *(i.e.,* for $E_1/E_2 = 1$) at base-subgrade interface, are about 70% of the applied pressure, whereas if $E_1/E_2 = 100$, the vertical stress is only 10 per cent of applied pressure. This shows that the ratio has great influence over the stress distribution in the flexible pavements. The deflections for the two-layer system are obtained by the following expressions:

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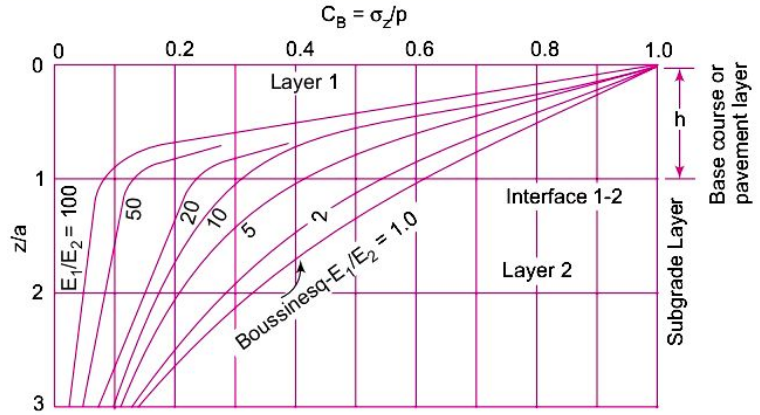

Fig. 29.4 Burmister two-layer stress influence curves (Bursmister, 1958)

$$\Delta = 1.5 \frac{pa}{E_2} F \quad \text{(for flexible plate)} \qquad ...(29.11)$$

and

$$\Delta = 1.18 \frac{pa}{E_2} F \quad \text{(for rigid plate)} \qquad ...(29.12)$$

where F = dismensionless factor called the *deflection factor* depending upon E_2/E_1 ratio and z/a ratio.

E_2 = modulus of elasticity of the lower layer (subgrade).

Figure 29.5 gives curves for the factor F.

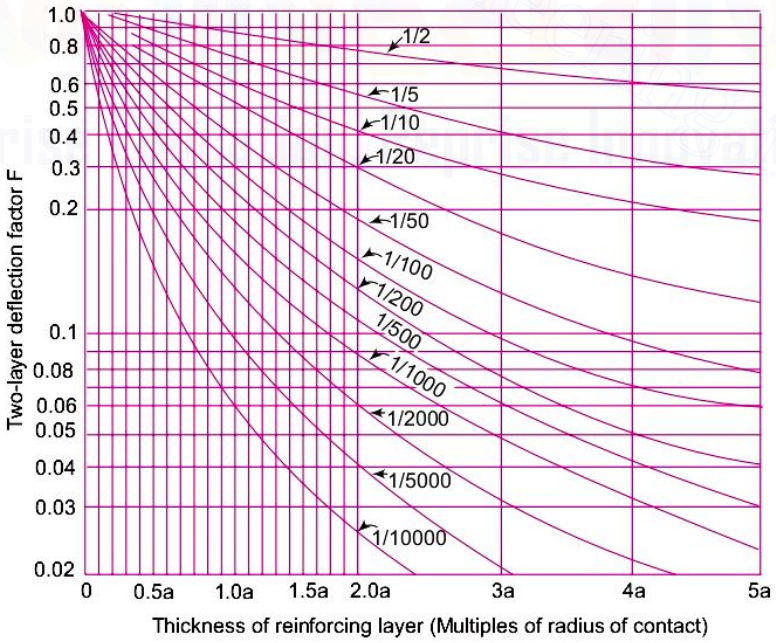

Fig. 29.5 Factor *f* for two-layer theory (Burmister, 1943)

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29.4 EQUIVALENT WHEEL AND AXLE LOADS

The pavement is usually designed from the consideration of the greatest load transmitted by a single wheel, called the *maximum wheel load*. The pressure transmitted to the surface of the pavement by the wheel is known as the *contact pressure* which depends upon the tyre pressure and the stiffness of the tyre walls. Assuming a uniform distribution of contact pressure, the *contact area* between the wheel and the pavement is the wheel load divided by the contact pressure. The contact area is approximately elliptical, but calculations are simplified by assuming it to be circular.

Dual wheels and tandem axles result in stresses at greater depths as the pressure bulbs of the tyre overlap. The depth at which stresses in the pavement resulting from the dual wheels are equal to those of a single wheel depends upon the spacing of the wheels. Figure 29.6 shows the influence of multiple wheel on stresses. At a depth approximately half the face-to-face spacing (d), the wheels cease to act independently, and at a depth equal to twice the centre to centre to spacing (S), the overlap of stresses becomes negligible. An *equivalent wheel load* can be found either from the *equal deflection criterion* or *equal-stress criterion*. Based on the equal deflection criterion, the following expression for the equivalent wheel load P_e results;

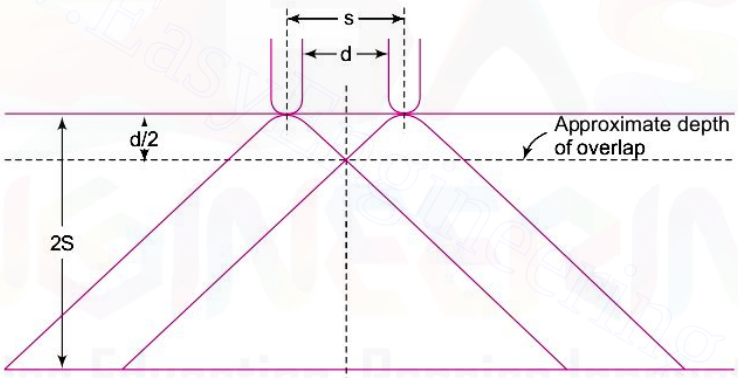

Fig. 29.6 Influence of multiple wheels on stresses

$$F_e\sqrt{P_e} = (F_1 + F_2)\sqrt{P} \qquad ...(29.13)$$

where
P_e = equivalent wheels load
P = wheel load of each of the dual tyres
F_e = settlement factor for equivalent wheel load
F_1 = settlement factor contributed by one tyre of duals
F_2 = settlement factor contributed by the other tyre.

The solution of the problem is accomplished by determining values of P_e and F_e so that the Eq. 29.13 is satisfied. The load P is known and factors F_1 and F_2 can be known from Fig. 29.5 for various values of $\frac{r}{a}$ ratios. The maximum value of $F_1 + F_2$ occurs at a small distance from the centre of the tyre. However, for practical problems, the values of $F_1 + F_2$ under the centre line, and under a tyre, need by calculated and the greatest of the two may be taken to be used in Eq. 29.13. Thus the RHS of Eq. 29.13 is known. A number of values of P_e are assumed and the values of $\sqrt{P_e}.F_e$ are computed till Eq. 29.13 is satisfied.

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29.5 DESIGN METHODS

The flexible pavement design methods can be broadly classified into three distinct groups:

(*i*) Empirical methods based on soil classification and other factors such as climate and moisture. They include the following:
  (*a*) Group index method.
  (*b*) Federal aviation agency (USA) method.

(*ii*) Empirical methods based upon some arbitrary soil strength tests which have been correlated with pavement performance. These include:
  (*a*) California Bearing Ratio (CBR) method.
  (*b*) North Dakota Cone method.
  (*c*) Plate bearing test method

(*iii*) Theoretical and semi-theoretical methods. These include:
  (*a*) Kansas triaxial method.
  (*b*) Texas triaxial method.
  (*c*) Burmister's method.

Some of these methods are discussed in the following articles.

29.6 GROUP INDEX METHOD

The group index method was suggested by the Highway Research Board (1945) for making an approximate estimation of the thickness of the pavement. The group index is an empirical quantity used for rating the strength of the subgrade and is determined from the following expression:

$$\text{Group index} = 0.2a + 0.005ac + 0.01bd \qquad ...(29.14)$$

where

a = that portion of percentage of subgrade soil passing 75 micron IS sieve greater than 35 and not exceeding 75, expressed as a positive whole number (0 to 40)

b = that portion of percentage of subgrade soil passing IS sieve greater than 15 and not exceeding 55 expressed as a positive whole number (0 to 40)

c = that portion of the numerical liquid limit greater than 40 and not exceeding 60, expressed as a positive number (0 to 20)

d = that portion of the numerical plasticity index greater than 10 and not exceeding 30, expressed as a positive whole number (0 to 20).

The higher the group index of the subgrade, the lower its strength and the greater the thickness of subbase required.

Figure 29.7 gives the tentative design curves presented by Steele (1945). The descriptions of various curves are as follows:

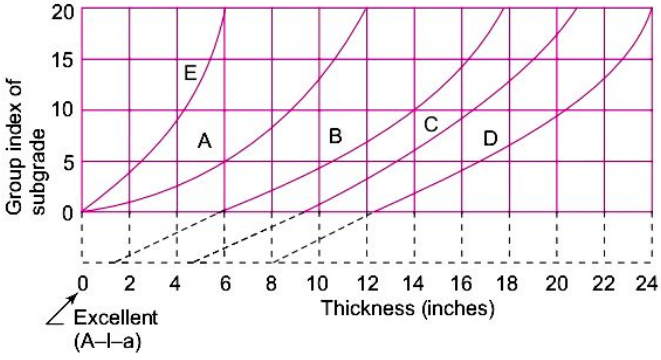

Fig. 29.7 Design curves of group index method (Steele, 1945).

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Curve A: Thickness of sub-base required.

Curve B: Total thickness of surface, base and sub-base for light traffic.

Curve C: Total thickness of surface, base and sub-base for medium traffic.

Curve D: Total thickness of surface, base and sub-base for heavy traffic.

Curve E: Thickness as additional base which may be substituted for sub-base of curve A.

Light traffic = <50 commerical vehicles/day.

Medium traffic = 50 – 300 commertical vehicles/day.

Heavy traffic = >300 commerical vehicles/day.

29.7 CALIFORNIA BEARING RATIO (CBR) METHOD

This method was originally devised by O.J. Porter, then of the California State Highway Department, but it has since been developed and modified by other authorities in U.S.A., notably the U.S. Corps of Engineers. The method combines a load penetration test performed in the laboratory or in-situ with the empirical design charts to determine the thickness of pavement and of its constituent layers. This is probably the most widely used method for the design of flexible pavement.

The thickness of the different elements comprising a pavement is determined by CBR values. The CBR test is a small scale penetration test in which a cylindrical plunger of 3 in^2 (5 cm in dia.) cross-section is penetrated into a soil mass *(i.e.,* subgrade material) at the rate of 0.05 in. per minute (1.25 mm/min.). Observations are taken between the penetration resistance (called the *test load)* versus the penetration of plunger. The penetration resistance of the plunger into a standard sample of crushed stone for the corresponding penetration is called the *standard load.* The California bearing ratio, abbreviated as CBR is defined as the ratio of the test load to the standard load, expressed as percentage, for a given penetration of the plunger.

$$\text{CBR} = \frac{\text{Test load}}{\text{Standard load}} \times 100 \quad \text{...(29.15)}$$

Table 29.1 gives the standard loads adopted for different penetrations for the standard material with a CBR value of 100%.

Table 29.1 Standard load used in California bearing ratio test

Penetration of plunger (inch)	*Standard load (lb)*	*Penetration of plunger (mm)*	*Standard load (kg)*
0.1	3000	2.5	1370
0.2	4500	5.0	2055
0.3	5700	7.5	2630
0.4	6900	10.0	3180
0.5	7800	12.5	3600

The CBR test is carried out on a compacted soil in a CBR mould 150 mm in diameter and 175 mm height, provided with a detachable collar of 50 mm in height and a detachable perforated base plate.

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A displacer disc, 50 mm deep to be kept in the mould during the specimen preparation, enables a specimen of 125 mm deep to be obtained. The moulding dry density and water content should be the same as would be maintained during field compaction. To simulate worst moisture condition of the field, the specimens are kept submerged in water for about 4 days before testing. Generally, CBR values of both soaked as well as unsoaked samples are determined. Both during soaking and penetration test, the specimen is covered with equal surcharge weights to simulate the effect of overlying pavement or the particular layer under construction. Each surcharge slotted weight, 147 mm in diameter with a central hole 53 mm in diameter and weighing 2.5 kg is considered approximately equivalent to 6.5 cm of construction. A minimum of two surcharge weights (*i.e.,* 5 kg surcharge load) is placed on the specimen. Load is applied on the penetration piston so that the penetration is approximately 1.25 mm/min. The load readings are recorded at penetrations, 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 10 and 12.5 mm. The maximum load and penetration is recorded if it occurs for a penetration of less than 12.5 mm. The load penetration curve is then plotted as shown in Fig. 29.8.

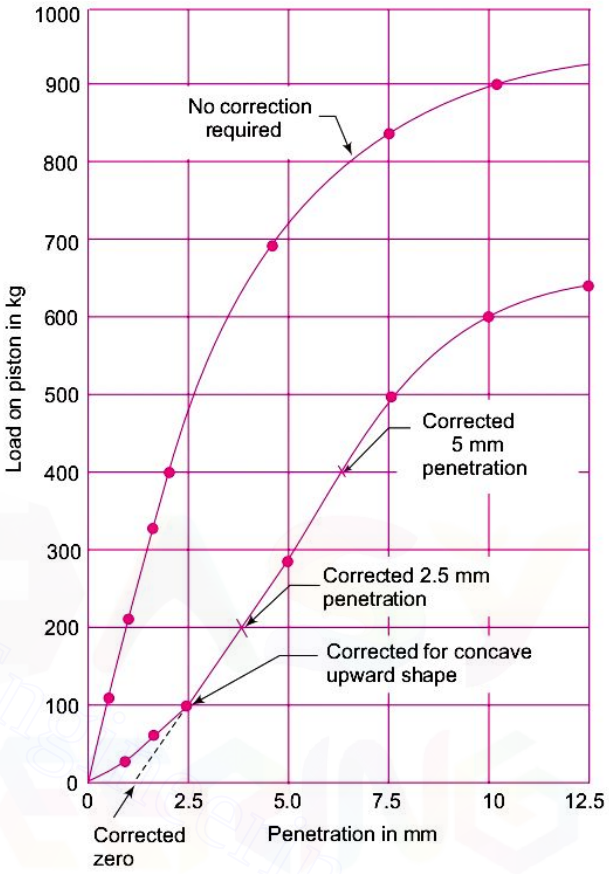

Fig. 29.8 Load penetration curves in CBR test.

The curve is mainly convex upwards although the initial portion of the curve may be concave upwards due to surface irregularities. A correction is then applied by drawing a tangent to the curve at the point of greatest slope. The corrected origin will be the point where the tangent meets the abscissa.

The CBR values are usually calculated for penetrations of 2.5 mm and 5 mm. Generally the CBR values at 2.5 mm penetration will be greater than that at 5 mm penetration and in such a case the former is to be taken as the CBR value for design purposes. If the CBR value corresponding to a penetration of 5 mm exceeds that for 2.5 mm, the test is repeated. If identical results follow, the bearing ratio correponding to 5 mm penetration is taken for design.

Figure 29.9 gives the design curves for determining the appropriate thickness of construction required *above* a material with a given CBR, for different wheel loads and traffic conditions. These design curves for roads have been proposed by the Road Research Laboratory, England, and are also followed in India.

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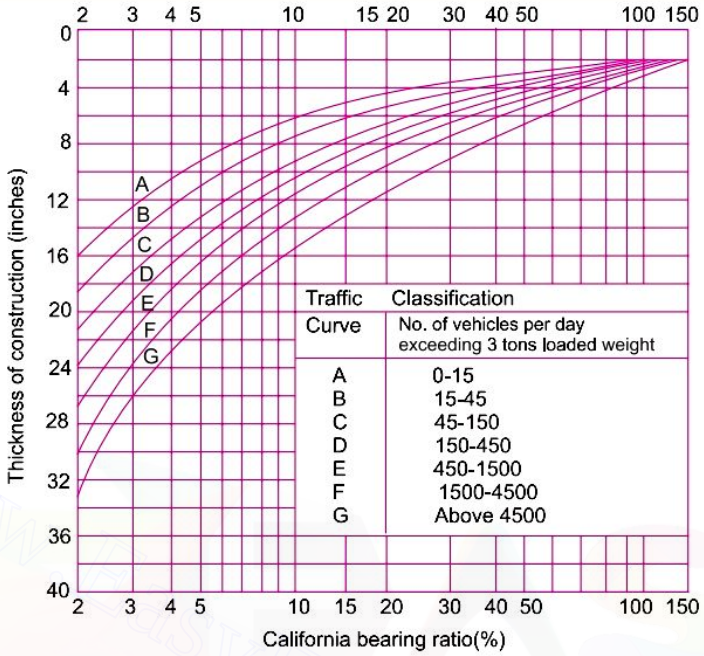

Fig. 29.9 CBR design curves: Road Research Laboratory England (Adopted by Indian Roads Congress).

29.8 NORTH DAKOTA METHOD

This method, similar to the CBR method, has been developed by the North Dakota State Highway Department. The method consists in finding out the in-situ bearing power of the subgrade by means of a cone penetrometer, of the North Dakota Cone apparatus. The thickness is then found form the design curve (Fig. 29.10).

The penetrometer (or North Dakota cone apparatus) consists essentially of a shaft with a sharp cone (half angle = 7° 45′) attached to one end. The movement of the shaft into the soil measured with the help of a vernier. The shaft can be locked to the frame when necessary. A disc is fixed to the top of the shaft, over which weights can be fixed. The load carried by the shaft, during penetration into the soil, divided by the area of the cone at the surface level is termed as the *cone bearing value* q_c.

$$q_c = \frac{Q}{\pi(\rho \tan 7°45')^2} = \frac{Q}{0.058\rho^2} \qquad \ldots(29.16)$$

where q_c = bearing value (kg/cm^2)

Q = load on cone (kg)

ρ = penetration (corrected) of cone (cm)

The loads Q used in the test are 4.5, 9, 18 and 36 kg, inclusive of the weight of the shaft and cone. Theoretically, neglecting friction, the penetration under 4.5 kg is half the penetration under 18 kg. Actually, it is never so, because of the rounding of the tip of the cone. A correction is, therefore, added to

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or subtracted from all the readings so that the penetration ρ_9 under 9 kg becomes half of the penetration ρ_{36} under 36 kg.

Correction

$$C = \rho_{36} - 2\rho_9 \qquad ...(29.17)$$

Knowing the bearing value, the thickness of pavement is determined from Fig. 29.10. A minimum thickness of 24 cm is provided for bearing values of 28 kg/cm^2 or more.

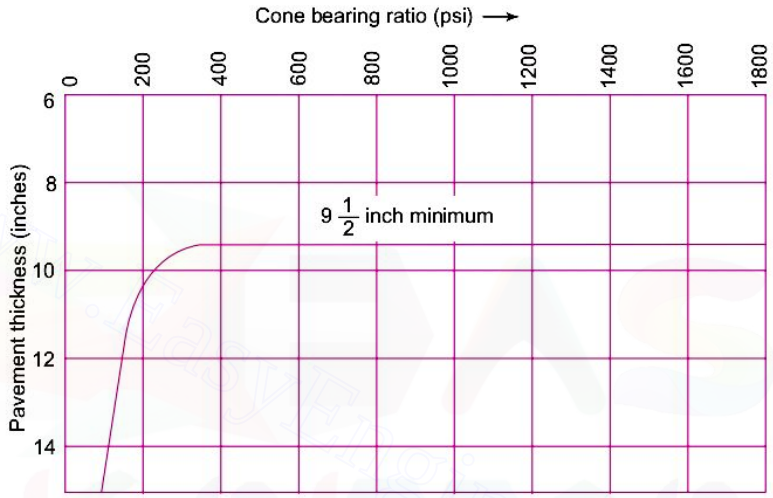

Fig. 29.10 North dakota design curve (After wise, 1955)

29.9 BURMISTER'S DESIGN METHOD

Burmister's design method is based on the concept of a two-layer system, consisting of the road surfacing, base course and the sub-base as the top layer of thickness h, and the subgrade as the bottom layer of infinite extent. The displacement of such a system, under a loaded area of radius a with load intensity p is given by Eq. 29.11.

$$\Delta = 1.5\frac{pa}{E_2}F$$

where E_2 = modulus of elasticity of the subgrade

F = deflection factor, determined from Fig. 29.5.

The method consists in selecting various values of the thickness h of the top layer and finding the value of the deflection corresponding to each value of h, from Eq. 29.11, the value of factor F being taken in each case from Fig. 29.5. The thickness h corresponding to an arbitrary deflection of $\Delta = 0.2$ inch (5 mm) has been recommended by Burmister as the required thickness of the pavement. Tentative design curves for flexibe runway pavements, using 0.2 in. as limiting deformation have been drawn assuming approximate value of modulus of elasticity for various types of subgrades.

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29.10 U.S. NAVY PLATE BEARING TEST METHOD

This method is also based on Burmister's two-layer theory. It consists of the following three steps:

Step 1. The thickness h of the base course is calculated on the basis of the two-layer theory. For this the values of modulus of elasticity E_1 and E_2 for the base course and subgrade are determined from two plate bearing tests.

Step 2. Trial sections are constructed with the pavement thickness equal to h, $\frac{2}{3}h$ and $\frac{3}{2}h$.

Step 3. Plate bearing tests are run on these trial sections and final thickness is chosen on the basis of these tests which produces a deflection of 0.2 inch (5 mm).

Step 1. In order to use the two layer theory of calculation of required pavement thickness in step 1, it is necessary first of determine the value of E_2 by the plate bearing tests on the subgrade. A 30 inch diameter plate is recommended for this test. In performing the test, it is essential to use a series of staked plates to minimize the bending of the plate upon loading. The load P corresponding to a deflection of 0.2 in is determined from the test, and the modulus of elasticity E_2 is calculated from Eq. 29.12 by taking the plate to be rigid. The deflection factor F for this test is equal to unity since the test on subgrade soil results in a one layer system.

Hence
$$\Delta = 0.2 \text{ inch}$$
$$= 1.18 \frac{pa}{E_2} \qquad ...(29.18)$$

From this, E_2 is determined. After the modulus of elasticity E_2 is known, a test section consisting of the base course material is built and plate-bearing test is made on this. The test section should be 5 m by 5 m square (or larger) and 15 to 30 cm deep. The load intensity p corresponding to $\Delta = 0.2$ inch is determined from the tests. Knowing E_2 from the previous test, and Δ (= 0.2 inch) and p from the present test, the factor F is calculated from Eq. 29.12

$$\Delta = 1.18 \frac{pa}{E_2} F$$

or
$$F = \frac{\Delta . E_2}{1.18\, pa} \qquad ...(29.19)$$

Thus, factor F is known. From Fig. 29.5, the value of E_2/E_1 is found corresponding to the value of F and h/a ratio.

After having known E_2 and E_2/E_1 (and hence E_1 also), the value of F corresponding to a given wheel load intensity p is computed from Eq. 29.11 by taking $\Delta = 0.2$ inch.

$$\Delta = 0.2 \text{ inch}$$
$$= 1.5 \frac{pa}{E_2} F \qquad ...(29.20)$$

(Considering the wheel load to be a flexible plate)

In this equation Δ, E_2, p and a (radius of the tyre contact area) are known. Knowing F, and E_2/E_1 ratio, the thickness h of the base course is determined from Fig. 29.5.

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Step 2. In the next step, trial sections are constructed of thickness h, $\frac{2}{3}h$ and $\frac{3}{2}h$ calculated in step 1. Each trial section of a given thickness is constructed for three different soil condtions: one on a typical fill section, another on a typical cut section, and a third at a position on grade. Thus, in all, nine trial sections are built. The subgrade and base courses are compacted to the densities that will be expected during construction.

Step 3. Plate-bearing tests are performed on these trial sections. The data then are used to determine the required pavement thickness which will result in the assumed deflection of 0.2 inch in making these tests, a plate is employed which has a radius corresponding to the effective tyre radius a.

In the above method, the design thickness is the total thickness of base material to sustain a given load at a given deflection, and no consideration of the type or depth of wearing surface is given. However, the structural qualities of wearing surface material are always better, and hence a certain thickness of surface material can be substituted for the base course material. This will, in effect, produce an added factor of safety.

29.11 LABORATORY EXPERIMENTS

EXPERIMENT 21 : DETERMINATION OF CALIFORNIA BEARING RATIO

Object and scope. The object of the experiment is to determine the California Bearing Ratio (CBR) of a compacted soil sample in the laboratory, both in soaked as well as unsoaked state. The method also covers the detemination of CBR of undisturbed soil sample obtained from the field.

Material and equipment. (*i*) Cylindrical mould (CBR mould) with inside diameter 150 mm and height 175 mm, provided with a detachable extension collar 50 mm height and a detachable perforated base plate 10 mm thick, (*ii*) Spacer disc, 148 mm in diameter and 47.7 mm in height, along with a handle for screwing into the disc to facilitate its removal, (*iii*) steel cutting collar which can fit flush with the mould both outside and inside, (*iv*) Metal rammers : (*a*) weight 2.6 kg with a drop of 310 mm or (*b*) weight 4.89 kg, with a drop of 450 mm, (*v*) Annular slotted weight weighing 2.5 kg each 147 mm in diameter with a centre hole 53 mm in diameter, (*vi*) Penetration piston, 50 mm diameter and minimum of 100 mm long, (*vii*) Extension measuring apparatus consisting of : *(a)* perforated plate 148 mm diameter, with a threaded stem in the centre, (*b*) adjustable contact head to be screwed over the stem, (*c*) metal tripod, (*viii*) Loading device, with a capacity of at least 5000 kg and equipped with a movable head or base that travels at a uniform rate of 1.25 mm/min; complete with load indicating device, (*ix*) Two dial gauges leading to 0.01 mm, (*x*) Sieve: 4.75 mm and 20 mm IS Sieves, (*xi*) Miscellaneous apparatus, such as a mixing bowl, straight edge, scales, soaking tank or pan, drying oven, water content determination tins, filter paper etc.

Test Procedure

(*i*) Preparation of Test Specimen

1. **Remoulded Specimen:** Remoulded specimen may be prepared at Proctors maximum dry density and optimum water content or at any other desired density and water content. The material used should pass a 20 mm IS Sieve. Allowance for large material should be made by replacing it by an equal amount of material which passes a 20 mm IS Sieve but is retained on 4.75 mm IS Sieve. The specimen may be prepared either by dynamic compaction or by static compaction.

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(*a*) ***Dynamic compaction.*** Take about 4.5 to 5.5 kg of soil and mix it thoroughly with the desired water. If the sample is to be compacted at optimum water content and the corresponding dry density determined by compaction test (light compaction or heavy compaction) take exact weight of soil required and necessary quantity of water so that the water content of the soil sample is equal to the determined optimum water content. Fix the extension collar to the top of the mould and the base plate to its bottom. Insert the spacer disc over the base (with the central hole of the disc at the lower side). Put a disc of a coarse filter paper on the top of the displacer disc. Compact the mixed soil in the mould using either the light compaction or heavy compaction. For light compaction, compact the soil in 3 equal layers, each layer being given 56 blows, uniformly distributed, by the 2.6 kg rammer. For heavy compaction, compact the soil in 5 layers, by giving 56 blows to each layer by the 4.89 kg rammer. Remove the collar and trim off excess soil. Turn the mould upside down and remove the base plate and the displacer disc. Weigh the mould with the compacted soil, so that its bulk density and dry denstity may be determined. Put filter paper on the top of the compacted soil (collar side) and clamp the perforated base plate on to it.

(*b*) ***Static compaction.*** Calculate the weight of wet soil at the required water content to give the desired density when occupying the standard specimen volume in the mould by the following expression:

$$W = \gamma_d (1 + w)\, V$$

where W = weight of wet soil ; γ_d = desired dry density

w = desired water content;

V = volume of specimen in the mould = 2250 cm^2 .

Take about 4.5 to 5.5 kg of soil and raise its water content to the desired value w. Take weight W (calculated above) of the mix soil and put it in mould filled with the base plate and filter paper at its bottom. Tamp the soil by hand during pouring. Place a filter paper and the displacer disc on the top of the soil. Keep the mould assembly in any compression machine and compact the soil by pressing the displacer disc till the level of the disc reaches the top of the mould. Keep the load for some time, and then release. Remove the displacer disc.

2. **Undisturbed specimen:** To obtain undisturbed samples, attach the cutting edge to the mould and push it gently in the ground. When the mould is sufficiently full of soil, remove it by under-digging. The top and bottom surfaces are then trimmed flat so as to give the required length of the specimen. The density of the soil should be determined by weighing the soil with the mould or by any field method (such as the sand replacement method) on the soil in the vicinity of the spot at which sample is collected.

(*ii*) Soaking of Specimen and Test for Swelling

1. Put a filter paper on the top of the soil and place the adjustable stem and perforated plate on the top of the filter paper.
2. Put annular weights to produce a surcharge equal to the weight of the base material and pavement expected in actual construction. Each 2.5 kg weight is equivalent to 7 cm of construction. A minimum of two weights should be put.
3. Immerse the mould assembly and weights etc. in a tank of water allowing free access of water to the top and bottom of the specimen.
4. Mount the tripod of the expansion measuring device on the edge of the mould and note down the initial reading of dial gauge.

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5. Keep the set-up undisturbed for 96 hours (4 days). Note down readings every day against the time of reading. Maintain constant water level in tank.
6. Take the final reading at the end of period, remove the tripod and take out the mould. Allow the specimen to drain, for 15 minutes. Remove all the free water collected in the mould taking care that the surface of the specimen is not disturbed during the process.
7. Remove the weights, perforated plate and top filter paper and weigh the mould with soaked soil specimen.

(*iii*) Penetration Test

1. Place the surcharge weights back on the top of the soaked soil specimen, and place the mould assembly on the penetration test machine (loading machine).
2. Seat the penetration piston at the centre of the specimen with the smallest possible load but in no case excess of 4 kg so that full contact is established between the surface of the specimen and the piston.
3. Set the stress and strain dial gauge to zero. Apply the load on the penetration piston so that the penetration rate is approximately 1.25 mm/min. Record the load reading at penetrations of 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 4.0, 5.0, 7.5, 10 and 12.5 mm. Record the maximum load and penetration if it occurs for a penetration of less than 12.5 mm.
4. At the end of the penetration test, detach the mould from the loading equipment. Take about 20 to 50 g of soil from the top 3 cm layer of the specimen, and keep it for water content determination.

Tabulation of observations. The test data and observations are recorded as illustrated in Table 29.2.

Table 29.2 Data and observation sheet for CBR determination

1. Compaction characteristics

(a) Dynamic Compaction

1. Optimum water content		
2. Wt. of mould + compacted specimen	(g)	
3. Wt. of empty mould	(g)	
4. Wt. of compacted specimen	(g)	
5. Volume of specimen	(cm^3)	
6. Bulk density	(g/cm^3)	
7. Dry density		

(b) Static Compaction

1. Dry density ...(g/cm^3)
2. Moulding water content ...(%)
3. Wet. wt. (W) of soil compacted ....(g)

2. Soaking and swelling test

1. Dry density before soaking
2. Bulk density before soaking
3. Bulk density after soaking
4. Surcharge weight used during soaking
5. Final expansion ratio

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Date and time	*1*	*2*	*3*	*4*
Dial reading (mm)				
Total expansion (mm)				

3. Penetration test

1. Surcharge weight used...
2. Water content after penetration test...

Penetration dial		*Load dial*		
Readings	*Penetration (mm)*	*Readings*	*Load (kg)*	*Corrected load (kg)*
	0			
	0.5			
	1.0			
	1.5			
	2.0			
	2.5			
	3.0			
	4.0			
	5.0			
	7.5			
	10.0			
	12.5			

Calculations and Test Results

1. *Expansion ratio.* The expansion ratio may be calculated as follows :

$$\text{Expansion ratio} = \frac{d_f - d_i}{h} \times 100$$

where d_f = final dial gauge reading (mm) ;

d_i = initial gauge reading (mm)

h = intial height of specimen (mm).

2. *Load penetration.* Plot the load penetration curve (Fig. 29.8). If the initial portion of the curve is concave upwards, apply the correction by drawing a tangent to the curve at the point of greatest slope.

The corrected origin will be the point where the tangent meets the abscissa. Find and record the correctecd load reading corresponding to each penetration.

Corresponding to the penetration value at which the CBR is desired, corrected load values are found from the curve and CBR is calculated as follows:

$$\text{CBR} = \frac{P_T}{P_S} \times 100$$

P_T = corrected test load corresponding to the chosen penetration from the load penetration curve.

P_S = standard load for the same penetration as for P_T taken from Table 29.1.

The CBR values are usually calculated for penetration of 2.5 mm and 5 mm. If CBR for 5 cm exceeds that for 2.5 mm, the test should be *repeated*. If identical results follow, the CBR corresponding to 5 mm penetration should be taken for design.

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CHAPTER 30

Design of Rigid Pavement

30.1 INTRODUCTION

Rigid pavements are made up of Portlant cement concrete, and may or may not have a base course between the pavement and the subgrade. Because of its rigidity and high tensile strength, a rigid pavement tends to distribute the load over a relatively wide area of soil, and a major portion of the structural capacity is supplied by the slab itself. For this reason, minor variations in subgrade strength have little influence upon the structural capacity of the pavement. The rigid pavements are used for heavier loads and can be constructed over relatively poor subgrade such as black cotton or plastic soils, peat, etc.

A base under rigid pavements may be used for the following reasons:

1. Prevention of pumping of fine grained soil (*Pumping* is defined as the ejection of water and subgrade soils through joints, cracks and along the edges of pavements caused by downward slab movement by the passage of heavy axle loads over the pavement after the accumulation of free water on or in the subgrade).
2. Protection against frost action.
3. Provides drainage.
4. Controls the shrink and swell of subgrade
5. It forms a working surface on clays and silts, and thus enables the construction to proceed expeditiously during wet weather.
6. Serves as a levelling course on irregular formations.
7. Lends some structural capacity to the pavement.

30.2 STRESSES IN RIGID PAVEMENT

The factors affecting stresses in rigid pavement can be placed in four broad categories:

1. Stresses due to restrained temperature and moisture deformations.
2. Stresses due to the externally applied loads.

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3. Stresses due to volume changes of the supporting material, including frost action.
4. Stresses due to continuity of the subgrade support as affected by permanent deformations of the subgrade or loss of support through pumping.

Relative Stiffness of Slabs: When a load is applied on a slab, it deforms in a saucer shape, arid the resistance to deformation depends upon the stiffness of the supporting medium as well as flexural stiffness of the slab. The relative stiffness of the subgrade and the slab is indicated in terms of *radius of relative stiffness,* defined by Westergaard (1927) by the following characteristic equation:

$$l = \left\{ \frac{Eh^3}{12\left(1-\mu^2\right)k_s} \right\}^{1/4} \qquad ...(30.1)$$

where l = radius of relative stiffness (cm) (a linear dimension)
E = modulus of elasticity of the pavement (kg/cm)
h = thickness of pavement (cm)
μ = Poisson's ratio of the pavement
k_s = modulus of subgrade reaction (kg/cm^3)

The *modulus of subgrade reaction* is defined as the intensity of pressure on the horizontal surface of a soil mass required to cause a unit settlement of surface:

$$k_s = \frac{p}{\rho} \qquad ...(30.2)$$

where p = subgrade reaction.
ρ = vertical deflection

30.3 STRESSES DUE TO WHEEL LOAD

Westergaard (1926) considered three cases of loading: (1) corner load, (2) load at the edge of the pavement, and (3) interior load. His original equation for interior loading was later modified by him (Westergaard, 1933). The equations for edge and corner loading were modified by Sutherland (1943). The modified equations converted into metric units, are given below:

$$\sigma_{interior} = 0.275\left(1+\mu\right)\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+\log_{10}\left\{12\left(1-\mu^2\right)\right\}-54.54\,c_2\left(\frac{1}{c_1}\right)^2\right] \qquad ...(30.3)$$

$$\sigma_{edge} = 0.529\left(1+0.54\mu\right)\frac{P}{h^2}\left[4\log_{10}\left(\frac{l}{b}\right)+\log_{10}\left(\frac{b}{2.54}\right)\right] \qquad ...(30.4)$$

$$\sigma_{corner} = \frac{3P}{h^2}\left[1-\left\{\frac{a\sqrt{2}}{l}\right\}^{1.2}\right] \qquad ...(30.5)$$

where
$(\sigma)_{interior}$ = maximum tensile stress at the bottom of slab due to loading at the interior (kg/cm^2)
$(\sigma)_{edge}$ = maximum tensile stress at the bottom of slab due to loading at the edge (kg/cm^2)
$(\sigma)_{corner}$ = maximum tensile stress at the top of the slab due to loading at corner (kg/cm^2)
P = wheel load (kg)
h = slab thickness (cm)

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μ = Poisson's ratio for concrete

l = radius of relative stiffness (cm)

E = modulus of elasticity (kg/cm^2)

k_s = modulus of subgrade reaction (kg/cm^3)

b = radius of equivalent distribution of pressure (cm), given by the following equations:

$$b = \sqrt{1.6a^2 + h^2} - 0.675\,h \text{ , when } a < 1.724\,h \quad ...(30.6)$$

$$b = a \text{ when } a \geq 1.724\,h \quad ...(30.7)$$

a = radius of area of contact between the slab (cm), the area being assumed circular in case of corner and interior loads, and semi-circular for edge loads

c_1, c_2 = correlation factors to allow for a redistribution of subgrade reactions; $c_1 \approx 5l$; $c_2 \approx 0.2$.

Pickett (1951) gave the following semi-empirical formulae for corner loading for protected and unprotected corners:

For slab corners protected by load-transfer devices such as bars etc.

$$(\sigma)_{corner} = \frac{3.36\,P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22\,a/l}\right] \quad ...(30.8)$$

For unprotected corners:

$$(\sigma)_{corner} = \frac{4.2\,P}{h^2}\left[1 - \frac{\sqrt{a/l}}{0.925 + 0.22\,a/l}\right] \quad ...(30.9)$$

The above two formulae are in FPS units in which P in lbs, σ is in lb/inch^2 , l, a and h are in inches and k_s is in lb/inch.

The formulae for the stresses in pavements are complicated for direct calculations. Use may be made of the influence diagrams and charts prepared by Docker (1948) and Pickett (1951).

30.4 STRESSES DUE TO WARPING

The surface of the slab is subjected to wide range of temperature during the daily cycle, whereas the temperature of the bottom of the slab in contact with the subgrade of the base remains relatively more constant. This temperature gradient through the slab causes differential expansion or contraction between the top and bottom of the slab.

In the day, top surface expands more than the bottom and the slab assumes a shape convex upwards. The weight of the slab and load transfer devices or friction at joint will restrain free warping and will tend to bend it back into its original shape, and hence compressive stresses at top and tensile at the bottom are created.

In the night, the sides and corners warp upwards, and might actually leave the subgrade. In this position, the weight of the raised portions of the slab tend to bend them down.

Hence tension at top and compression in the bottom is developed. Bradbury (1938) gave the following equations for edge stress and interior stress due to warping:

$$(\sigma)_{edge} = \frac{CE\,\epsilon_t\,\Delta t}{2} \quad ...(30.10)$$

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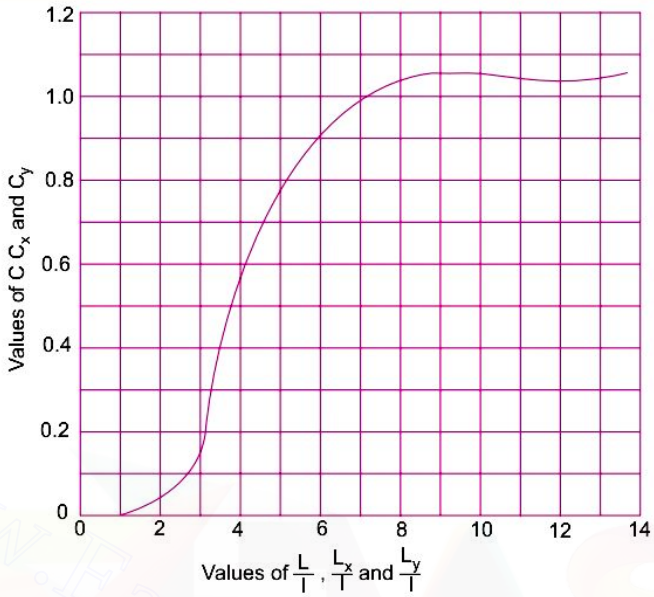

Fig. 30.1 Warping stress co-efficients (Bradbury, 1938)

$$(\sigma)_{interior} = \frac{E \in_t \Delta t}{2}\left(\frac{C_x + \mu C_y}{1-\mu^2}\right) \qquad ...(30.11)$$

where $\in_t$ = co-efficient of expansion of concrete (5×10^{-6} per °F, average)

Δt = total difference in temperature (may be taken to be equal to about 1°F per 1 cm thickness of slab)

E = modulus of elasticity of concrete

μ = Poisson's ratio

C = co-efficient given by Fig. 30.1, corresponding to L/l ratio

C_x = co-efficient in the desired direction (say, x direction)

C_y = co-efficient in the perpendicular direction (say, y direction)

L = length of the edge

L_x = free length in x direction *(i.e.,* the direction in which $\sigma_{interior}$ is sought)

L_y = free width in y direction.

30.5 STRESSES DUE TO SUBGRADE FRICTION

Stresses can also be set up in rigid pavements due to *uniform temperature change,* which cause the slab to contract or expand. If the slab is free to move and there is no friction between the slab and the subgrade, no stresses will result. However, if friction exists between the slab and subgrade, restraint results from the friction forces and the slab is stressed. During expansion, the under-side of the slab is subjected to compressive stress, while during contraction tensile stresses are induced due to the subgrade friction. Figure 30.2 (*b*) shows the distribution of frictional stresses, as suggested by Kelley (1939). Test results have shown that fully mobilised frictional resistance f_m is realised for a distance

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$\frac{L}{2} - x$, but from there to the centre of the slab, the stress distribution is parabolic in shape. The equations for the *average co-efficient of subgrade resistance f* are as follows:

For $x < \frac{1}{2}L$, $f = f_m\left(1 - \frac{2x}{3L}\right)$ ...(30.12)

For $x > \frac{1}{2}L$, $f = \frac{2f_m}{3}\sqrt{\frac{L}{2x}}$ ...(30.13)

It has been shown that the minimum amount of displacement required for friction to be fully developed is 0.06 inch (1.5 mm),

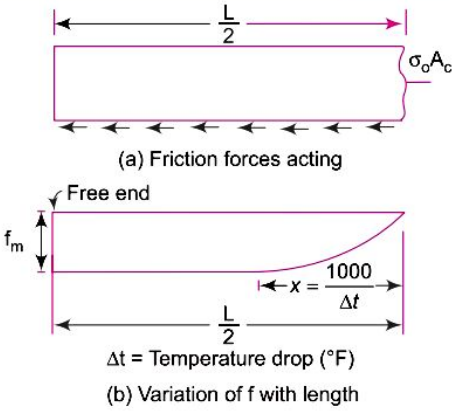

Fig. 30.2 Stresses resulting from contraction

For equilibrium conditions, the summation of the friction forces from the centres of the slab to the free end must be equal to the total tension in the concrete. The frictional resistance is assumed to be a finite value, dependent upon the weight of the slab and coefficient of sliding friction.

Let W = weight of the slab per sq. metre (kg/m^2)

L = length of the slab, in metres

h = thickness of the slab in cm

σ_0 = stress in concrete, kg/cm^2

f = average co-efficient of subgrade resistance (f = 2.3 to 1.15; average 1.5)

Then frictional resistance up to the centre of the slab

$$= W \times \frac{L}{2} \times 1 \times f \text{ (kg)} \quad \text{(per metre width of the slab)} \qquad \ldots(1)$$

Total tension in the concrete $= (1 \times 100)h\,\sigma_0$ (per metre width of the slab) ...(2)

Equating (1) and (2), we get $\frac{WLf}{2} = 100\,h\,\sigma_0$

and $$\sigma_0 = \frac{WLf}{2} \text{ kg/cm}^2 \qquad \ldots(30.14)$$

Taking unit weight of concrete as 2400 kg/m^3,

$$W = 1 \times 1 \times \frac{h}{100} \times 2400 = 24h \text{ kg} \qquad \ldots(30.15)$$

Substituting in equation 30.14, we get

$$\sigma_0 = \frac{3Lf}{25h} \qquad \ldots(30.16)$$

More exact calculations could be made by taking into account the variation in friction between the free end and the centre of the slab.

Combined stresses due to load and temperature: The critical stress condition in the slab is the one in which stresses due to temperature and stresses due to load are additive. This condition can result when the slab corners are warped down ward and a load is applied at the interior of the slab. Similarly, during a clear, cool night followed by a hot, sunny day, the edges and corners of the slab will warp upward

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and the corners may then be only-partially supported or even unsupported. This condition will give the maximum stresses due to loading.

30.6 DESIGN METHOD

Portland Cement Association (1951) has developed a design procedure for rigid highway pavements based upon formulae developed by Pickett (Eqs. 30.8 and 30.9). Figure 30.3 and 30.4 show the design charts for protected and unprotected corners, based on the formulae by Pickett for the design of highway pavements. The values of k_s are in lb/in^3.

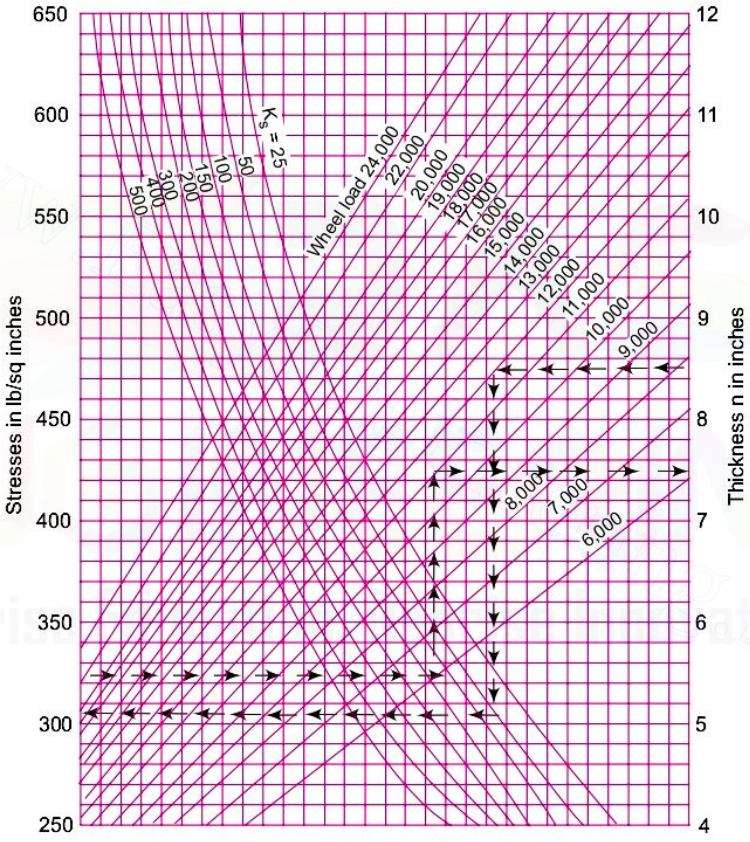

Fig. 30.3 PCA design charts for rigid highway pavements protected corners, for loads on dual tyres

The arrows shown on the diagrams indicate the procedure for finding the pavement thickness h for a given value of allowable stress in concrete, magnitude of wheel load and given value of k, and also for finding the stress in the pavement of a given thickness corresponding to a given wheel load and a given value of k. The working stress in the concrete that is used in the analysis is obtained by dividing the modulus of rupture of the concrete by a safety factor of 2.0. Loads which are applied to the pavement are increased 20 per cent to account for impact. Concrete pavements designed with the help of these charts are considered to have excess strength to offset working stresses.

Figures 30.5, 30.6 and 30.7 give the PCA (Portland Concrete Association) design charts for Rigid airport pavements, with various wheel loads and tyre pressure (indicated dotted lines).

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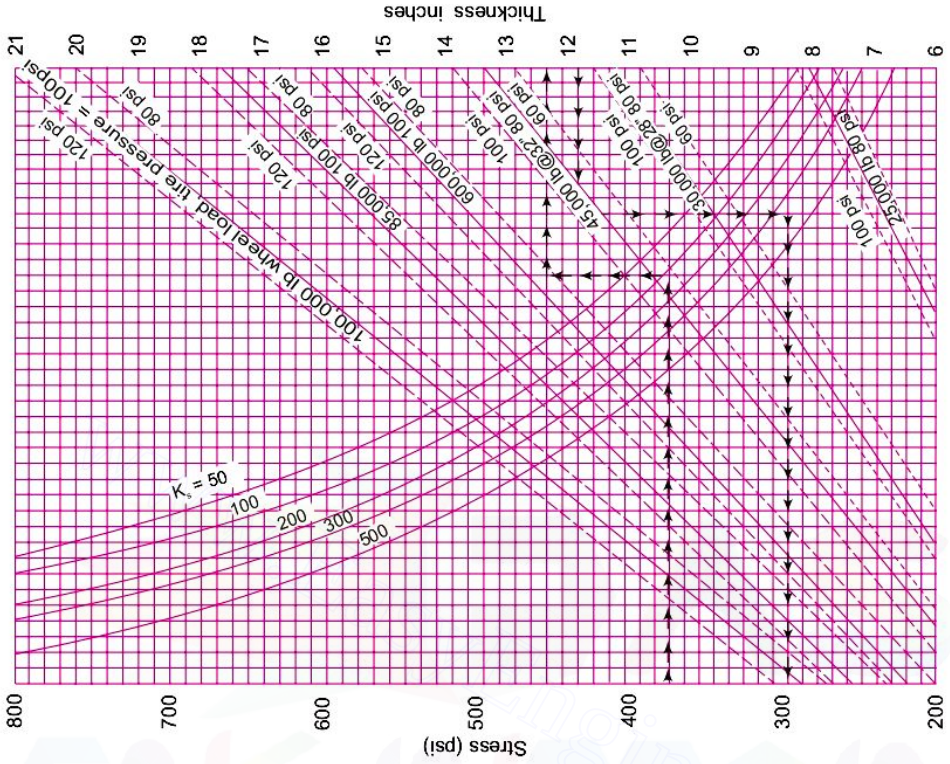

Fig. 30.5 PCA design curves for rigid airport pavements single wheel interior, load (E = 4,000,00 psi ; μ = 0.15)

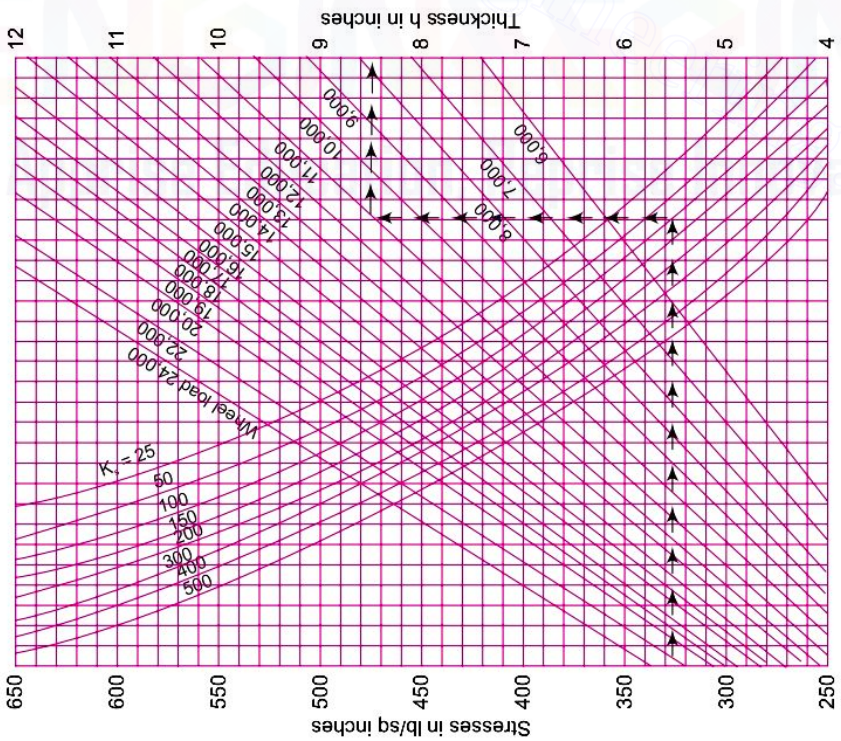

Fig. 30.4 PCA design charts for rigid highway pavements unprotected corners, for loads on dual tyres

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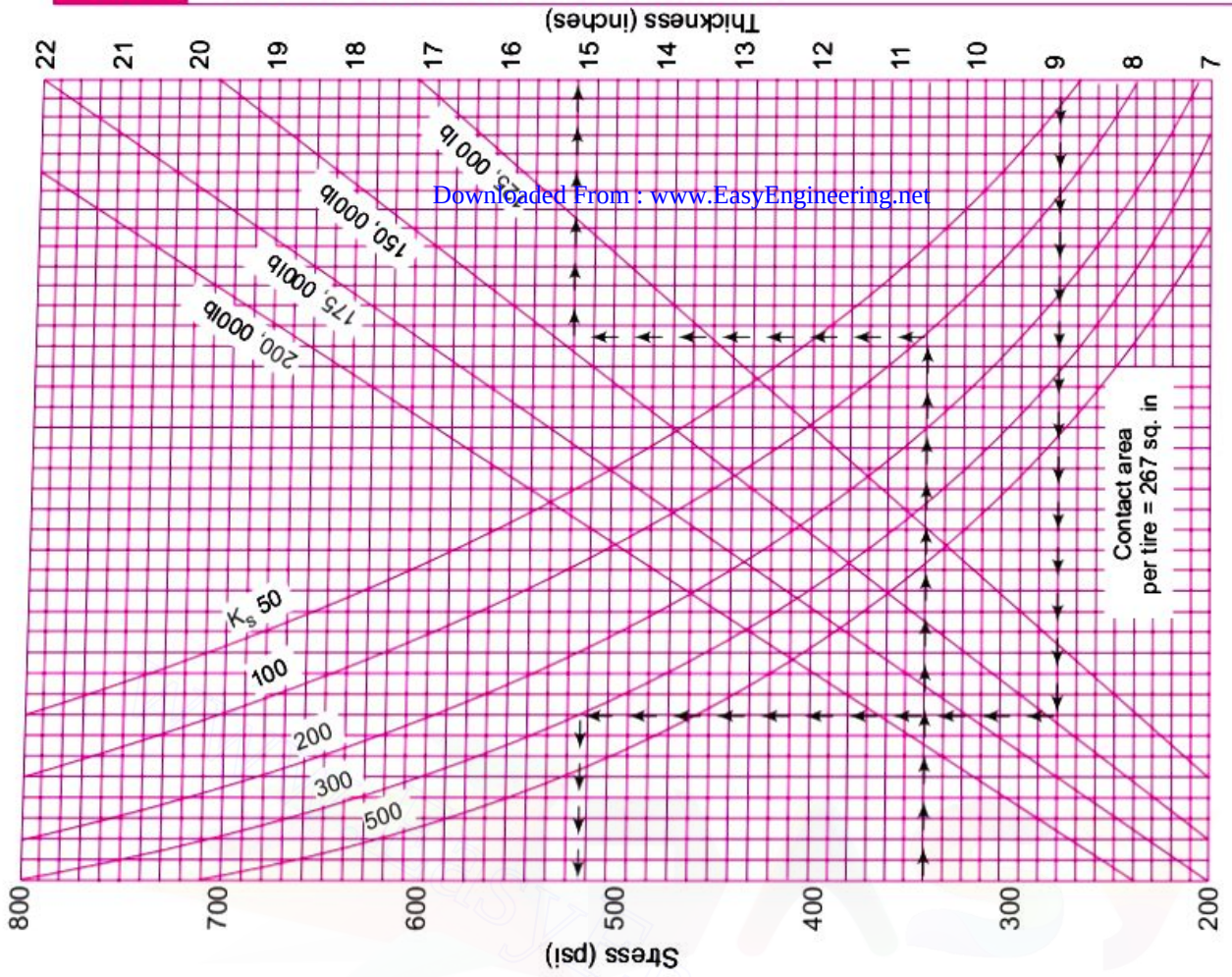

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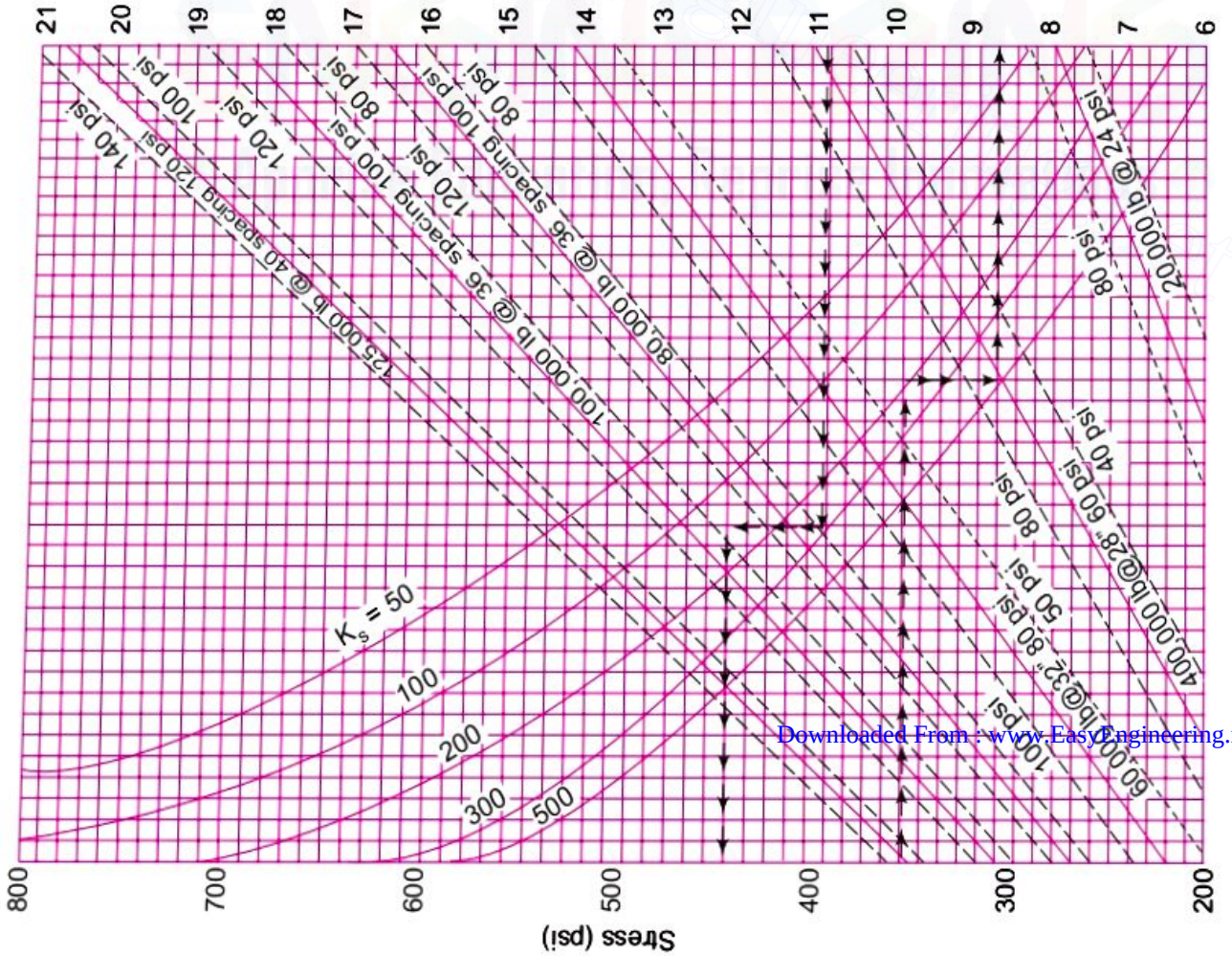

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CHAPTER 31

Stabilisation of Soils

31.1 INTRODUCTION

Stabilisation, in a broad sense, incorporates the various methods employed for modifying the properties of a soil to improve its engineering performance. Stabilisation is being used for a variety of engineering works, the most common application being in the construction of road and air-field pavements, where the main objective is to increase the strength or stability of soil and to reduce the construction cost by making best use of the locally available materials. Methods of stabilisation may be grouped under two main types: (*a*) modification or improvement of a soil property of the existing soil without any admixture, and (*b*) modification of the properties with the help of admixtures. Compaction and drainage are the examples of the first type, which improve the inherent shear strength of soil. Examples of the second type are: mechanical stabilisation, stabilisation with cement, lime, bitumen and chemicals etc. Some of the more commonly used methods will be discussed in this chapter.

31.2 MECHANICAL STABILISATION

Mechanical stabilisation involves two operations: (*i*) changing the composition of soil by addition or removal of certain constituents, and (*ii*) densification or compaction. The particle size distribution and composition are the important factors governing the engineering behaviour of a soil. Significant changes in the properties can be made by addition or removal of suitable soil fractions. For mechanical stabilisation, where the primary purpose is to have a soil resistant to deformation and displacement under loads, soil materials can be divided into two fractions: the *granular fraction* retained on a 75 micron IS sieve and the *fine soil fraction* passing a 75-micron sieve. The granular fraction impart strength and hardness. The fine fraction provides cohesion or binding property, water-retention capacity and also acts as a filler for the voids of the coarse fraction.

Mechanical stabilisation has been largely used in the construction of cheap roads. Guide of specifications have been drawn for gradation requirements of the bases and surfacing. Typical examples are given in

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Table 31.1. Instead of strictly observing the specifications, emphasis should be laid on making the maximum use of the locally available materials as many materials are found to be quite satisfactory under load conditons.

Table 31.1 Typical gradation specifications of mechanically stabilised bases and surfacings

IS Sieve	*Percentage passing*			
	Base		*Surfacing*	*Base or surfacing*
	Max. size		*Max. size*	*Max. size*
	40 mm	*20 mm*	*20 mm*	*10 mm*
40 mm	100	–	–	–
20 mm	80 – 100	100	100	–
10 mm	55 – 80	80 – 100	80 – 100	100
4.75 mm	45 – 60	50 – 75	60 – 85	80 – 100
2.0 mm	30 – 50	30 – 60	45 – 70	50 – 80
1.18 mm	–	–	35 – 60	40 – 65
600 micron	15 – 35	15 – 30	–	–
300 micron	–	–	20 – 40	20 – 40
75 micron	5 – 15	5 – 15	10 – 25	10 – 25

Note. (*i*) *For bases:* Liquid limit not exceeding 25% and plasticity index not exceeding 6.
(*ii*) *For surfacing:* Liquid limit not exceeding 35% and plasticity index between 4 and 9.

If the soil from one source does not meet the gradation and plasticity requirements of a job, it becomes necessary to mix materials from two more sources for obtaining the desired mixture. The blending of materials can be carried out by making trial combinations.

Proper compaction plays a very important role in stabilisation. Compaction has a great effect on soil properties, such as strength and stress-strain characteristics, permeability, compression, swelling and water absorption. The properties of a soil under compaction depend upon the water content, amount of compaction and the type of compaction, compared to coarse grained soils, the properties of fine grained soils are affected to a greater extent by the placement conditions. Compaction has been discussed in Chapter 17.

31.3 CEMENT STABILISATION

1. Soil cement and its influencing factors. The soil stabilised with cement (Portland) is known as *soil cement.* The cementing action is belived to be the result of chemical reaction of cement with the silicious soil during hydration. The binding action of individual particles through cement may be possible only in coarse-grained soils. In fine grained, cohesive soils, only some of the particles can be expected to have cement bonds, and the rest will be bonded, through natural cohesion.

The important factors affecting soil cement are: nature of soil, cement content, conditions of mixing, compaction and curing, and admixtures:

(*i*) ***Nature of soil.*** Almost every inorganic soil capable of pulverisation can be successfully stabilised with cement, although the cement requirement will increase with the increase in the specific

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surface of a soil. Expansive soils are particularly difficult to be stabilised. Organic matter and sodium sulphate are harmful and weaken the soil-cement. Well-graded soils, with less than 50% fraction finer than 75-micron sieve and plasticity index less than 20, are found to give best results. Figure 31.1 shows the approximate gradation limits of soil for economic stabilisation with cement (HRB, 1943).

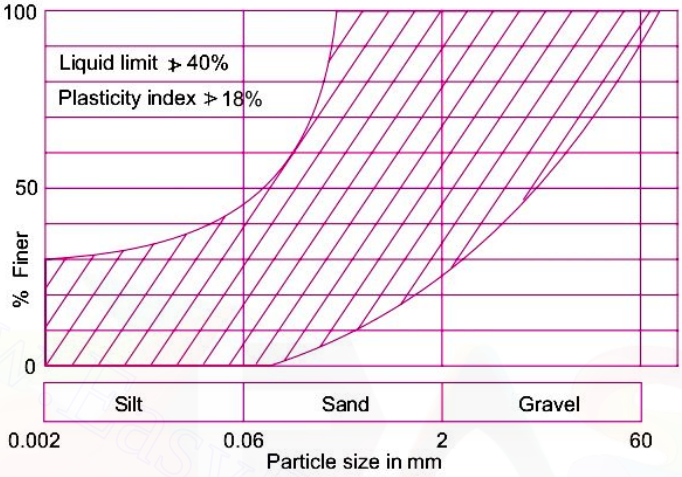

Fig. 31.1 Approximate gradation limits for soil-cement

(*ii*) ***Cement content.*** The strength of soil cement increases with an increase in the amount of cement added to a soil, and if such an increase in strength does not result, the soil may normally be considered unsuitable. The ordinary Portland cement is generally used for stabilisation. High-early-strength cement can also be used, and is usually more effective than normal cement (Lambe, 1962). The amount of cement required, expressed as a percentage by weight of dry soil, generally varies between 5 to 15%, finer soils requiring more cement. The actual amount required is determined experimentally by performing compressive strength and durability tests. The amount of cement giving a compressive strength of 25 to 30 kg/cm^2 should normally prove satisfactory for tropical climates. The approximate amounts needed for different types of soils may be as follows: gravels, 5 to 10%; sands, 7 to 12%; silts, 12 to 15% and clays, 12 to 20%. If a layer of soil cement having a surface area A (m^2), thickness H (cm) and dry density γ_d (t/m^3) is to be stabilised with p percentage of cement on the basis of dry weight of soil, the percentage of cement by weight on the basis of dry soil cement mixture will be $100p/(100 + p)$, and the amount of cement needed in tonnes for the construction will be given by the expression:

$$\text{Amount of cement} = \frac{AH\gamma_d}{100} \cdot \frac{p}{100+p}. \qquad ...(31.1)$$

The number of cement bags, weighing 50 kg each, will be given by:

$$\text{Number of cement bags} = \frac{AH\gamma_d\, p}{5(100+p)} \qquad ...(31.2)$$

(*iii*) ***Mixing, compaction and curing.*** A stronger and more durable soil-cement, will be produced, if the soil-cement water mixture is more intimately mixed. Mixing will, however, result in decreased strength if it is continued long after the cement hydration has begun.

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The amount of water to be added is decided from the considerations of good compaction and this amount is considered adequate enough for cement hydration also. It has been observed that better strength and stability develops, if the fine grained soils are compacted wet of optimum water content and the coarse grained soil on the dry side of the optimum water content (Lambe, 1962). The greater is the compacted density, the stronger and more durable will be the soil cement.

Like concrete, the strength of soil-cement increases with age. Hence it should be moist cured for at least 7 days. Curing is rapid at higher temperatures.

(*iv*) ***Admixtures.*** Certain chemicals are sometimes added to soil-cement, with the purpose either to reduce the cement consumption or to make a soil suitable for stabilisation which is not responsive to cement alone in its natural state. Lime and calcium chloride are commonly used with clays and soils containing organic matter. Sodium carbonate and sodium sulphate have also been tried. Fly ash, as an additive, has also been found very beneficial in the stabilisation of dune sand (Davidson, *et, al,* 1958). The fly ash acts as a puzzolana and also as a filler for increasing the density (Uppal and Bhalla, 1965).

2. Construction methods. The normal construction sequence for soil-cement bases is as follows: (*i*) shaping the sub-grade and scarifying the soil, (*ii*) pulverising the soil, (*iii*) adding, and mixing cement, (*iv*) adding, and mixing water, (*v*) compacting, (*vi*) finishing, (*vii*) curing and (*viii*) adding wearing surfacing. There are three methods of carrying out these operations : (*i*) mix-in place method, (*ii*) travelling plant method, and (*iii*) stationary plant method.

In the *mix-in-place* method, the sub-grade is first shaped to the required grade and is cleared of undesirable materials. It is then scarified to the required depth of treatment and the soils is pulverised, until at least 80% of the material (excluding stones) passes a 4.75 mm sieve. If another soil is to be blended, it is mixed with the loose, pulverised soil. The pulverised soil is spread and shaped to proper grade. Calculated amount of cement is then evenly distributed over the surface and intimately mixed. Water is added as required for compaction and the soil cement-water is turned into an intimate mixture. The wet mixture operation should not last more than 3 hours, after which the compaction should be completed within the next 2 hours. It is fairly easy to process coarse grained soil. Pulverisation and mixing of plastic clays can be facilitated by adding lime in proportions of 1 to 4%. The compacted soil cement is moistured for at least 7 days. A bituminous wearing surface is normally provided to protect the soil-cement base from abrasion and absorption of water in shrinkage cracks.

The mix-in-place method is considered cheaper and more adaptable to different field conditions, but the processing of soil is not so through and accurate as with other methods.

In the *travelling plant method,* the pulverised soil is heaped into a window and the cement is spread on the top. The soil and cement are lifted by an elevator to a mixer carried on a travelling platform where water is added and mixing is done. The mixture is then discharged on to the subgrade. It is spread with a grader and compacted. Uniform mixing and accurate control on added water can be ensured in the method. A uniform subgrade surface with controlled depth of treatment is possible. The plant is however, costly; In the *stationary plant method,* the excavated soil is brought to a stationary mixing plant. At the plant, cement and water are added and mixed with the soil. The mixture is then transported back to the desired location, dumped, spread and compacted. Similar to the travelling method, the method affords an accurate proportioning of materials and through mixing. The depth of treatment can be easily controlled. The method is slower and may prove expensive due to additional haulage of soil.

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31.4 LIME STABILISATION

Hydrated (or slaked) lime is very effective in treating heavy, plastic clayey soils. Lime may be used alone, or in combination with cement, bitumen or fly ash. Sandy soils can also be stabilised with these combinations. Lime has been mainly used for stabilising the road bases and sub-grades.

On addition of lime to soil, two main types of chemical reactions occur : (*i*) alteration in the nature of the absorbed layer through base exchange phenomenon, and (*ii*) cementing or puzzolanic action. Lime reduces the plasticity index of highly plastic soils making them more friable and easy to be handled and pulverised. The plasticity index of soils of low plasticity generally increases. There is generally an increase in the optimum water content and a decrease in the maximum compacted density, but the strength and durability increases.

The amount of lime required may be used on the unconfined compressive strength or the CBR test criteria. Normally 2 or 8% of lime may be required for coarse grained soils, and 5 to 10% for plastic soils. The amount of fly ash as admixture may vary from 8 to 20% of the soil weight (Lambe, 1962).

The construction procedures of lime stabilised bases are similar to those soil-cement. No strict time limitations for completion of the job are however necessary, since the soil-lime cementation reactions are respectively slow.

31.5 BITUMEN STABILISATION

Asphalts and tars are the bituminous materials which are used for stabilisation of soil, generally for pavement construction. These materials are normally too viscous to be incorporated directly with soil. The fluidity of asphalts is increased by either heating, emulsifying or by cut-back process. Tars are heated or cut-back. The bituminous materials when added to a soil impart cohesion or binding action and reduced water absorption. Thus either the binding action or the water proofing action or both the actions, may be utilised for stabilisation. Depending upon these actions and the nature of soils, bitumen stabilisation is classified under the following four types: (*i*) sand-bitumen, (*ii*) soil-bitumen, (*iii*) water-proofed mechanical stabilisation, and (*iv*) oiled earth.

1. Sand-bitumen. This term refers to bitumen-stabilised cohesionless soil, such as loose beach, dune, pit or river sand. The primary function of bitumen is to bind the soil particles. Sand should be substantially free from clay and organic matter. The gradation may vary within a wide range, but the fraction passing a 75 micron sieve should normally not exceed 12%, in case of fine dune sand the fraction may be upto 25%. Crushed stone, rock dust, gravel, etc., may be added to poorly graded sand.

The climatic conditions such as rainfall and temperature decide the type of bituminous material to be used and the method of mixing and construction to be employed. Hot mix sand asphalt is suitable in area of heavy rainfall, and emulsions are preferable in arid zones. Rapid curing cut-back are recommended for low temperatures and slow curing for high temperatures. The quantity of bituminous material required is determined by laboratory tests. The approximate proportions on dry weight basis of sand are as follows (Uppal and Bhalla, 1965); hot mix asphat, 5 to 11% ; cut-back, 4 to 10% ; emulsions, 5 to 10%. Hydrated lime, 1 to 2% is sometimes used as an admixture to assist coating of sand grains.

2. Soil bitumen. It refers to a cohesive soil in which the main function of bitumen is to preserve the natural cohesive strength by water-proofing the soil or reducing the water absorption. A large

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variety of soils can be thus stabilised. For best results the following requirements are recommended (HRB, 1946):

(*i*) Maximum size: not greater than approximately one-third the compacted thickness,
(*ii*) Passing 4.75 mm sieve: more than 50% ,
(*iii*) Passing 425 micron sieve: 35–100%,
(*iv*) Passing 75 micron sieve: 10–50%,
(*v*) Liquid limit: Less than 40%,
(*vi*) Plasticity index: Less than 18.

3. Waterproof mechanical stabilisation. Small amounts of bitumen 1 to 3%, are sometimes added to mechanically stabilised soils to make them waterproof.

4. Oiled earth. Slow and medium curing road oils are spread on the groud surface to make it water and abrasion resistant. The oils penetrate a short depth into the soil without involving any mechanical mixing.

31.6 CHEMICAL STABILISATION

There are a great many chemicals which are used for stabilisation. Only the chemicals which are commonly used for stabilising moisture in the soil and for cementation of particles will be described here.

1. Calcium chloride. Calcium chloride is used as a water retentive additive in mechanical stabilised bases and surfacing. Being hydroscopic and deliquescent, the salt absorbs moisture from the atmosphere and retains it. It makes alterations in the characteristics of pure water. The vapour pressure get lowered and the surface tension increases, and thereby the rate of evaporation decreases. The freezing point of pure water also gets lowered which results in prevention or reduction of frost heave. By depressing the electric double layer (or reducing the water deficiency), the salt reduces water pick-up, and thus the loss of strength of fine-grained soils.

Calcium chloride acts as soil flocculent. It facilitates compaction and usually causes a slight increase in the compacted density.

The salt may be spread on the surface, or incorporated into the soil by mix-in-place and plant-mix methods. The chief advantage is that the beneficial effects are lost if the salt is leached out. Frequent applications depending upon the climatic conditions are therefore necessary, which increase the cost. The relative humidity of the atmosphere should be above 30% for the salt to be effective.

2. Sodium chloride. The stabilising action of sodium chloride is somewhat similar to that of calcium chloride, but it has not been so widely used. It attracts and retains moisture and reduces the rate of evaporation. Another beneficial phenomenon is the crystallisation of the salt in the soil pores near the surface, which retards futher evaporation and also reduces the formation of shrinkage cracks. The salt is not applied on the surface, but it is mixed into the soil by mix-in place or plant-mix methods.

3. Sodium silicate. The sodium silicate solution in water, known as water glass, in combination with other chemicals, such as calcium chloride, is used as an *injection* for stabilising deep deposits of soil. The two chemicals react and precipitate in the form of an insoluble silica-gel within the soil pores making the soil impervious to water and increasing its shearing strength. These injections are found to be most successsful in fine and medium sands. The two chemicals can be injected either separately or as a single mixture.

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31.7 STABILISATION BY HEATING

Heating a fine grained soil to temperatures of the order of 400 to 600°C causes irreversible changes in clay minerals. The soil becomes non-plastic, less water sensitive and non-expansive. Also the clay clods get converted into aggregates. Soil can be baked in kilns, or *in-situ* downwards draft slow moving furnaces. The artificial aggregates so produced can be used for mechanical stabilisation.

Heating has been successfully employed in Russia for stabilising deep deposits of partly saturated loess upto about 12 m (Lambe, 1962). The method consisted in burning a mixture of liquid fuel and air injected into the ground through a net work of pipes. This way, cylinders of solidified soil, about 2.7 m in diameter, were produced which could serve as pile foundations. In Rumania, burning of liquid or gas fuel into bore holes to increase the strength and decrease the compressibility of cohesive soils (Beles and Stanculescu, 1958) was used.

31.8 ELECTRICAL STABILISATION

The stability or shear strength of fine-grained soils can be increased by draining them with the passage of direct current through them. The process is also known as *electro-osmosis,* as discussed in Chapter 11. Electrical drainage is accompanied by electro-chemical composition of the electrodes and the deposition of the metal salts in the soil pores. There may also be some changes in the structure of soil. The resulting cementing of soil due to all these reactions, is also known as *electro-chemical hardening* and for this purpose the use of aluminium anodes is recommended.

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Part VIII
Miscellaneous Topics

32. Site Investigation and Sub-Soil Exploration
33. Advanced Measuring Instruments
34. Reinforced Earth and Geotextiles

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CHAPTER 32

Site Investigation and Sub-Soil Exploration

32.1 INTRODUCTION

An investigation of site is essential for judging its suitability for the proposed engineering works and for preparing adequate and economic designs. Site investigation is equally necessary for analysing the safety or causes of failure of existing works, for selecting construction materials and for deciding upon the construction methods to be applied. In general, the purpose of a site investigation is to obtain necessary information about the soil and hydrological conditions at the site and to know the engineering properties of soil which will be affected. A timely and intelligently planned site investigation should be considered a pre-requisite for efficient, safe, economical design and construction.

The methods of site investigation are largely dependent upon the nature of the engineering project and the site. In general, any investigation should start with the collection and examination of the already existing data about the soil and geological conditions of the site. In many areas, the existing local knowledge, records of trial pits, bore holes, etc., in the vicinity and the behaviour of existing structures, particularly if they are similar to the proposed ones, are very helpful. If the existing information is not sufficient or is inconclusive, the site should be explored in detail. The exploration should be preceeded by site reconnaissance.

32.2 SITE RECONNAISSANCE

An inspection of the site and study of topographical features is often helpful in getting useful information about the soil and ground water conditions and in deciding the future programme of exploration. On going over the site, a study of the following features may be useful: local topography, excavations, cuttings, quarries, escarpments, evidence of erosion on land slides, fills, water levels in wells and streams, flood marks, and drainage pattern, etc. If there has been an earlier use of the site, information should be gathered in particular about the underground workings, if any, and about the location of fills and excavations.

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32.3 SITE EXPLORATION

The object of site exploration is to provide reliable, specific and detailed information about the soil and ground water conditions of the site which may be required for a safe and economic design and execution of the engineering work. For this purpose an exploration of the region likely to be affected by the proposed works should yield precise information about the following : (*i*) the order of occurrence and extent of soil and rock strata, (*ii*) the nature and engineering properties of the soil and rock formation, and (*iii*) the location of ground water and its variation.

Depth of exploration: Exploration, in general, should be carried out to a depth upto which the increase in pressure due to structural loading is likely to cause perceptible settlement or shear failure. Such a depth, known as the *significant depth,* depends upon the type of structure, its weight, size, shape and disposition of the loaded areas, and the soil profile and its properties. It is generally safe to assume the significant depth upto a level at which the net increase in vertical pressure becomes less than 10% of the initial overburden pressure. Alternatively, a pressure bulb bounded by an isobar of one-fifth or one-tenth of the surface loading intensity is sometimes assumed to define the minimum depth of exploration. This depth may be assumed to be equal to one-and a half to two times the width (smaller lateral dimension) of the loaded area.

The depth of exploration at the start of the work may be decided according to the following guide rules, which may need modification as exploration proceeds:

1. Isolated spread footing or raft: one and a half times the width.
2. Adjacent footings with clear spacing less than twice the width: one and a half times the length.
3. Pile foundations: 10 to 30 metres, or more, or at least one and a half times the width of structure.
4. Base of retaining wall: one and a half times the base width or one and a half times the exposed height of face of wall, whichever is greater.
5. Floating basement: depth of construction.
6. Dams: (*i*) one-half the bottom width of earth dams, twice the height from stream bed to crest for concrete dams, for dams less than 30 m high ; (*ii*) upto bed rock, or else, through all soft, unstable and permeable strata of overburden.
7. Roads, cuts and fills : (*i*) one metre where little cut or fill is required; (*ii*) in cut sections, one metre below formation level; (*iii*) in deep cuts, equal to bottom width or depth of cut, whichever is smaller; (*iv*) in fill section, two metres below the ground level or equal to the height of the fill, whichever is greater.
8. Borrow areas: convenience of excavation and thickness of available material.
9. From the considerations of weathering: (*i*) 1.5 metres in general ; (*ii*) 3.5 metres in black cotton areas.

Number and disposition of pits and boring: The number and disposition of the test pits and borings should be such as to reveal any major changes in thickness, depth or properties of the strata affected by the works, and the immediate surroundings. For compact building sites covering an area of about 0.4 hectare, one bore hole or pit in each corner and one in centre may be adequate. For smaller and less important buildings, even one bore, hole or test pit in the centre may suffice. Larger areas may be divided in a grid pattern and core penetration or other sounding tests may be performed at every 100 metres. The number of pits and borings is then decided by examining the variation in the penetration curves.

On dam sites, preliminary borings may be made at 50 m spacings along the top line of the upstream face of the dam and across one or both abutments. A few widely scattered holes are also required in the

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bottom of the reservoir on the *u/s* side of the dam. Depending upon the subsoil conditions the number and disposition of bore holes are varied, usually in the valley floor, on the abutments and in the locations of appurtenant structures. On road sites, the pits or bore holes are usually made along the proposed centre line of the road and along the ditch lines, alternating right and left ditches. The spacing may be about 100 m to start with which may be increased to 500 m on obviously uniform soils or decreased to 30 m or less, if frequent changes in the soil profile are met.

General exploration: Depending upon the details or the precision of the subsoil data to be obtained, the site exploration may be classified as *general* (or preliminary) and *detailed*. The primary aim of a general exploration is to get an approximate picture of the sub-soil conditions at a relatively low cost. The information so obtained should suffice for the design and execution of minor and routine engineering works. General exploration is also recommended on erratic soil profiles where a detailed exploration may prove very expensive. Exploratory borings, shallow test pits, subsurface penetration and soundings, and geophysical methods are used for general explorations. Exploratory borings are carried out by auger, auger and shell, and by wash boring. Disturbed, or partly disturbed, but representative samples collected from test pits and borings are subjected to simple laboratory and field tests, such as the identification and classification, *in-situ* water content, density and unconfined compressive strength. Penetration and soundings help in estimating the relative density and strength properties of coarse-grained soils and also to locate the weaker strata and lenses. The general exploration is thus able to give information about the following:

1. Depth, extent and composition of soil strata.
2. Depth of rock, when necessary.
3. Ground water level.
4. Engineering properties requiring disturbed or partly disturbed samples.
5. Approximate values of strength and compressibility of soil strata.
6. Preliminary selection of foundation type.

Detailed exploration: Detailed exploration follows as a supplement to general exploration when large engineering works, heavy loads, and complex and costly foundations are involved. A detailed exploration is meant to furnish information about soil properties such as shear strength, compressibility, density index, and permeabilty. These properties are known by procuring and testing undisturbed samples and by field tests. The field test may be conducted for determining the *in-situ* values of density, bearing capacity, shear strength, permeability, and pore pressures.

Ground water levels: The location of ground water level is important for deep excavation and foundation works, and also on sites liable to be water-logged. Open wells at the site, or in the vicinity, should be used for knowing the ground water conditions. Bore holes can also be used for recording water levels, but the bore hole observations should be made 12 to 24 hours after boring and compared with the levels in the wells. In cased holes, the casing should be raised 30 cm at the completion of drilling and water level recorded after 24 hours. The inflow or loss of water from a bore hole during drilling, or the presence of artesian water should be properly recorded. The seasonal variations in the level of water table can be studied by installing a number of observation wells over the site.

Ground water and soil may contain in certain localities constituents which cause corrosion and damage to the foundations and other buried materials. It such cases, it may be necessary to make a chemical analysis of soil and water. Corrosive action may also result from dumped, industrial waste products.

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32.4 METHODS OF SITE EXPLORATION

The various methods of the explorations may be grouped as follows:

1. Open excavations
2. Borings
3. Sub-surface soundings.
4. Geophysical methods.

The first two methods are described in this article.

Open excavations: Test pits and trenches can be used for all types of soils. Soils can be inspected in their natural condition and samples, disturbed or undisturbed can be conveniently taken.

The cost of open excavations, however, increases rapidly with depth. They are generally considered suitable for shallow depths (say upto 3 m). For greater depths (over 6 m) and below ground water table, specially in pervious soils, measures for lateral support and ground water lowering become necessary.

Boring Methods: The methods of boring or drilling are: auger, auger and shell, wash boring, percussion boring, and rotary boring.

1. **Auger boring.** Augers are used in cohesive and other soft soils above water table. Hand augers are used for depths upto about 6 m. Mechanically operated augers are used for greater depths and they can also be used in gravely soils. Figure 32.1 shows a post hole auger and a helical (spiral) auger.
   Samples recovered from the soil brought up by augers are badly disturbed and are useful for identification purposes only. Auger boring is fairly satisfactory for highway explorations at shallow depths and for exploring borrowing pits.
2. **Auger and shell boring.** Cylindrical augers and shell with cutting edge on teeth at the lower end can be used for making deep borings. Hand operated rings are used for depths upto about 25 m and the mechanised rings upto 50 m. Augers are suitable for soft to stiff clays, shells for very stiff and hard clays and shells or sand pumps for sandy soils. Small boulders, thin soft strata or rock or cemented gravel can be broken by chisel bits attached to drill rods. The hole usually requires a casing. Figure 32.2 shows a typical sand pump.
3. **Wash boring.** Wash boring is a fast and simple method for advancing holes in all types of soils. Boulders and rock cannot be penetrated by this method. The method consists in first driving a casing through which a hollow drill rod with a sharp chisel or chopping bit at the lower end is inserted. Water is forced under pressure through the drill rod which is alternatively raised and dropped, and also rotated. The resulting chopping and jetting action of the bit and water disintegrates the soil. The cuttings are forced up

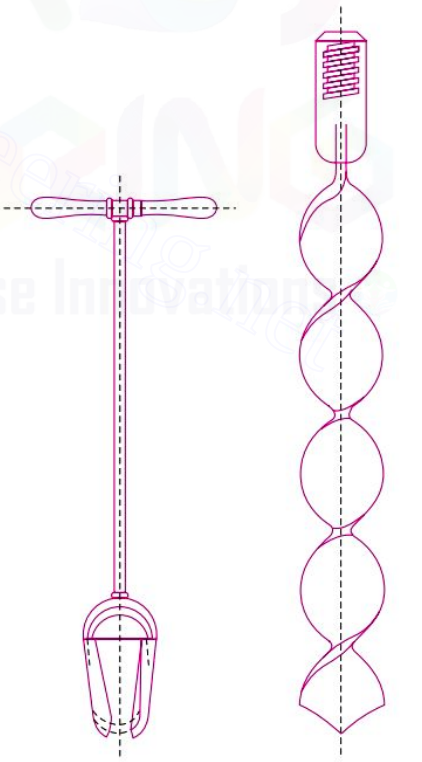

Fig. 32.1 Hand augers

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to the ground surface in the form of soil-water slurry through the annular space between the drill rod and the casing. The change of soil stratification could be guessed from the rate of progress and the colour of the wash water. The samples recovered from the wash water are almost valueless for interpreting the correct geotechnical properties of soil. Figure 32.3 shows a set-up for wash boring.

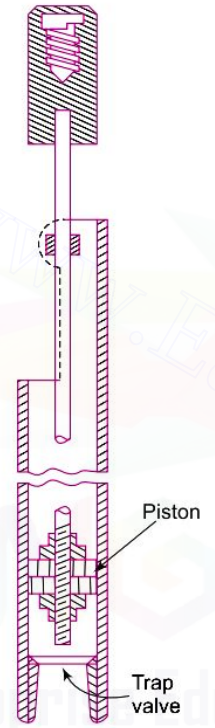

Fig. 32.2 Sand pump

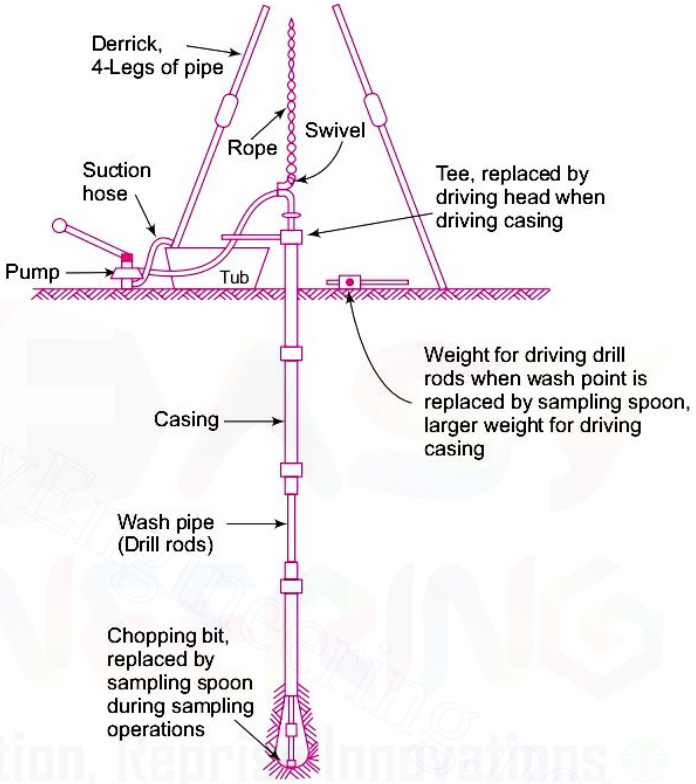

Fig. 32.3 Wash boring

4. **Percussion boring.** In this method, soil and rock formations are broken by repeated blows of heavy chisel or bit suspended by a cable or drill rod. Water is added to the hole during boring, if not already present, and the slurry or pulverised material is bailed out at intervals. The method is suitable for advancing a hole in all types of soil, boulders and rock. The formations, however, get disturbed by the impact.
5. **Rotary Boring.** Rotary boring or *rotary drilling* is a very fast method of advancing hole in both rocks and soils. A drill bit, fixed to the lower end of drill rods, is rotated by a suitable chuck and is always kept in firm contact with the bottom of the hole. A drilling mud, usually a water solution of bentonite, with or without other admixtures, is continuously forced down the hollow drillrods. The mud returning upwards brings the cuttings to the surface. The method is also known as *mud rotary drilling* and the hole usually require no casing.

Rotating core barrels, provided with commercial diamond-studded bits or a steel bit with shots, are also used for rotary drilling and simultaneously obtaining the rock cores or samples. The method is then also known as *core boring* or *core drilling*. Water is circulated down the drill rods during boring.

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Record of pits and borings: In all exploration works, it is very important to maintain an accurate and explicit record of test pits and borings. The disposition of pits and borings should be shown on a site plan.

32.5 SOIL SAMPLES AND SAMPLERS

The soil samples can be of two types: disturbed and undisturbed. A *disturbed* sample is that in which the natural structure of soils get partly or fully modified and destroyed, although with suitable precautions the natural water content may be preserved. Such a sample should, however, be *representative* of the natural soil by maintaining the original proportion of the various soil particles intact. An *undisturbed* sample is that in which the natural structure and properties remain preserved.

The sample disturbance depends upon the design of the samplers and the method of sampling. The design features governing the degree of disturbance are (*i*) cutting edge, (*ii*) inside wall friction, and non-return valve. Figure 32.4 shows a cutting edge. The following terms are defined with respect to the diameters marked in Fig. 32.4.

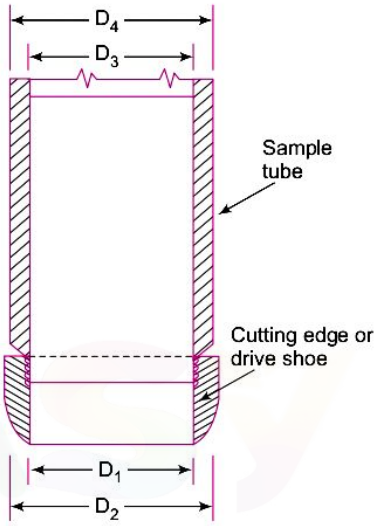

Fig. 32.4 Lower end of a sampler

$$\text{Area ratio} = \frac{D_2^2 - D_1^2}{D_1^2} \times 100\ \% \qquad \ldots(32.1)$$

$$\text{Inside clearance} = \frac{D_3 - D_1}{D_1} \times 100\ \% \qquad \ldots(32.2)$$

$$\text{Outside clearance} = \frac{D_2 - D_4}{D_4} \times 100\ \% \qquad \ldots(32.3)$$

The area ratio should be as low as possible. It should be greater than 25%; for soft sensitive soil, it should preferably not exceed 10%. The inside clearance should lie between 1 to 3% and the outside clearance should not be much greater than the inside clearance. The walls of the sampler should be smooth and should be kept properly oiled. The non-return valve should permit easy and quick escape of water and air when the sample is driven.

Types of samplers: The samplers classified as *thick wall* or *thin wall* samplers depending upon the ratio. Thick wall samplers are those having the area ratio greater than 10 to 25%. Depending upto the mode of operation, they may be classified in the following three common types: open drive stationary piston and rotary. The open drive types can be both the thick wall and thin wall samplers. The stationary piston and the rotary types are thin wall samplers.

1. **Open drive sampler.** It is a tube open at the lower end. The sampler head is provided with vents (valve) to permit water and air to escape during driving. The check valve helps to retain sample when the sampler is lifted up. The tube may be seamless or it may be split in two parts; in the latter case it is known as a *split tube* or *split spoon* sampler. The split tube may also contain an inside thin wall liner.

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2. **Stationary piston sampler.** It contains a piston or plug attached to a long piston rod extending up to the ground surface through the drill rod. During lowering of the sampler through the hole, the lower end of the sampler is kept closed with the piston. When the desired sampling elevation is reached, the piston rod is clamped, thereby keeping the piston stationary, and the sampler tube is advanced down into the soil. The sampler is then lifted up, with piston rod in the clamped position. The piston prevents the entry of water and soil into the tube, when it is being lowered, and then greatly helps to retain the sample during lifting operations. Thus the sampler is more suitable for sampling soft soils and saturated sand.
3. **Rotary samplers.** These are the core barrel types (Earth Manual, 1960) having an outer tube provided with cutting teeth and a removable thin wall liner inside. It is used for firm to hard cohesive soils and cemented soils.

Figure 32.5 shows a few typical samplers

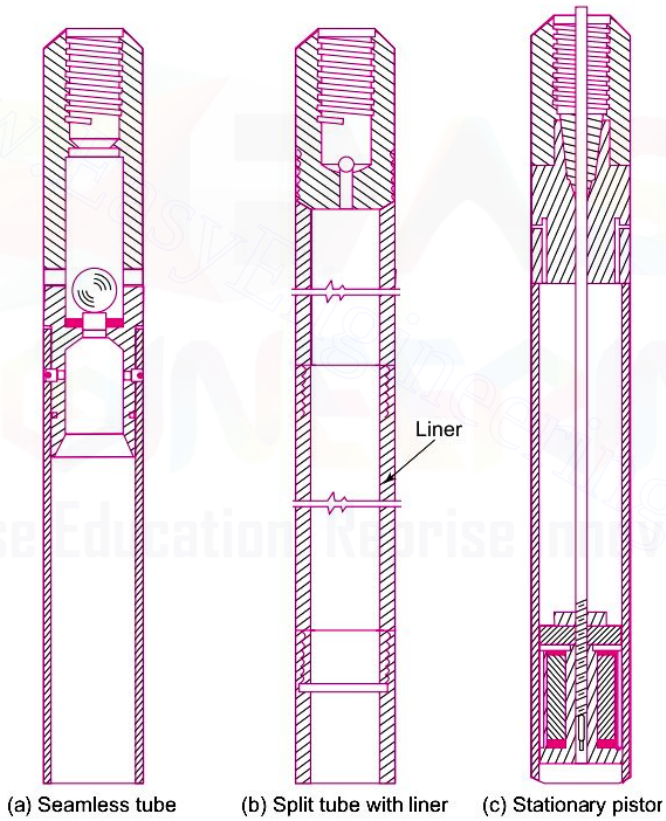

Fig. 32.5 Types of samplers

32.6 DISTURBED SAMPLING

Disturbed samples can be obtained by direct excavations, augers and thick wall samplers. For sampling saturated cohesionless soils, a trap valve or a spring sample retainer is inserted in the drive shoe (cutting edge). Sand pump and shell with trap valve (flap) are also used. The disturbed samples may be used for

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mechanical analysis, water content determination, index properties tests, compaction and stabilization tests. The split spoon samplers can be used for approximate determination, of unconfined compressive strength. The samplers should be so transported and stored that the original composition is preserved and the water content also does not change, if desired.

32.7 UNDISTURBED SAMPLING

Undisturbed samples may be required for tests on shear, consolidation, and permeability. They can also be used for other tests like the disturbed samples.

Undisturbed samples are obtained by forcing a thin wall sampler into the soil at the bottom of the bore hole or in a test pit. The penetration of the sampler into the soil should be continuous and rapid. The sampler should never be over driven so as to compress the sample. A piston sampler may be used with advantage in soft soils, specially below water table.

Undisturbed samples of cohesionless soils, specially from below the water table are difficult to be obtained. A compressed air sampler may be used. It enables the sample to be removed from the ground into an air chamber and then lifted to the ground surface without contact with water of the drill hole. A piston sampler with bore holes kept filled with drilling mud can also be used. Alternative methods may be to impart cohesion to sand by asphaltic emulsions, or to freeze the sand at the sampling depth or near the lower end of the sampler.

It is relatively easier to obtain undisturbed samples of both cohesive and cohesionless soils from open test pits and trenches. Sharp-edged thin wall tubes or cylinders can be used for this purpose. Alternatively, *hand cut samples* can be obtained. In stiff and sufficiently cohesive soil, undisturbed samples can be cut out and trimmed to regular shapes, say a cube. The sample is wrapped in wax paper or polythelene sheet and is given a wax coating on the outside. A sample may also be cut by removing the soil from the sides and left projecting like a stump. A wooden box or other suitable container with lid and bottom removed is slided over the sample. The sides of the container should project about 2 cm above the sample. The projecting portion of the container and the empty space along the sides, if any, is filled with moist sawdust or other suitable packing material. Molten wax is then poured and the lid fixed. The sample is then cut from bottom and the container with the sample is turned upside down. A portion the soil is removed and the sample is sealed with packing material and wax. The bottom cover of the container is then attached. This process of sampling is also known as *chunk sampling*.

Undisturbed samples are not usually removed from the samplers in the field. The two ends of the sample tube are hollowed out to a depth of about 2.5 cm and sealed by inserting packing material, or a wooden disc, and molten wax. The ends are then closed by screwed caps or tight-fitting lids. The top of the sample should be marked on the tube and other particulars given on a label. When the sample is taken out of the tube, its direction of movement relative to the tube should be kept the same as it was during sampling.

32.8 PENETRATION AND SOUNDING TESTS

These tests involve the measurement of the resistance to penetration of a sampling spoon, a cone or other shaped tools under dynamic or static loadings. The resistance is empirically correlated with some of the engineering properties of soil, such as density index, consistency, bearing capacity, etc. The value of these tests lies in the amount of experience behind them. These tests are useful for general exploration

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of erratic soil profiles, for finding depth to bed rock or stratum, and to have an approximate indication of the strength and other properties or soil, particularly by cohesionless soil, from which it is difficult to obtain undisturbed samples. The two commonly used tests are *standard penetration test* and *the cone penetration test.*

1. Standard penetration test: The test (IS: 2131–1981) is performed in clean hole, 55 to 150 mm in diameter. A casing or drilling mud is used to support the sides of the hole. A thick wall split-tube sampler, 50.8 mm (2 in.) OD and 35 mm (1.5 in.) ID, is driven into the undisturbed soil at the bottom of the hole under the blows of a 65 kg (140 lb) drive weight with 75 cm (30 in.) free fall. The minimum open length of the sampler should be 60 cm. The sampler is first driven through 15 cm as a seating drive. It is further driven through 30 cm (12 in.) or until 100 blows have been applied. The number of blows required to drive the sampler 30 cm beyond the seating drive, is termed the *penetration resistance N.* If required, more than 100 blows may also be used. Refusal is said to have been reached when the sampler penetrates less than 2.5 cm under 50 blows.

In very fine, silty, saturated sand an apparent increase in resistance occurs. Terzaghi and Peck have recommended the use of an equivalent penetration resistance N in place of the actually observed value of N, when N is greater than 15. N_e is given by the following relation :

$$N_e = 15 + \frac{1}{2}(N - 15) \qquad ...(32.4)$$

Gibbs and Holtz (1957) have experimentally studied the effect of overburden pressure on the value of N. Their modification for air-dry or moist sand can be represented by the following relation:

$$N_e = N \frac{50}{1.42\,\sigma' + 10} \qquad ...(32.5)$$

where N_e = corrected value for overburden effect

N = actual value of blows

σ' = effective overburden pressure (t/m^2, not to exceed 28.2 t/m^2)

Tables 32.1 and 32.2 give some of the empirical correlation of the soil properties with the penetration resistance corrected for depth and for fine staturated sand.

Table 32.1 Penetration resistance and empirical correlations for cohesionless soils

Penetration resistance N (Blows)	*Approx. ϕ (degrees)*	*Density index %*	*Description*	*Approx. moist density (t/m^3)*
–	25 – 30	0	Very loose	1.12 – 1.6
4	27 – 32	15	Loose	1.44 – 1.84
10	30 – 35	35	Medium	1.76 – 2.08
30	35 – 40	65	Dense	1.76 – 2.24
50	38 – 43	85	Very dense	2.08 – 2.4
–		100		

Note. The approximate values of ϕ are after Meyerhof (1956).

Also, $\phi = 25 + 0.15\, I_D$, with fines greater than 5%

$\phi = 20 + 0.15\, I_D$, with fines less than 5%

Larger values should be used for granular soil with 5% or less than fine sand and silt.

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Table 32.2 Penetration resistance and empirical correlations for cohesive soils

Penetration resistance N (blows)	*Unconfined compressive strength (t/m^2)*	*Saturated density (t/m^3)*	*Consistency*
0	0		Very soft
2	2.5	1.6–1.92	Soft
4	5	1.76–2.08	Medium
8	10		Stiff
16	20	1.92–2.24	Very stiff
32	40		Hard

2. Dutch cone Test: This test is used for getting a continuous record of the resistance of soil by penetrating steadily under static pressure a cone with a base of 10 cm^2 (3.6 cm in dia.) and an angle of 60° at the vertex. The cone is carried at the lower end of a steel driving rod which passes through a steel tube (mantle) with external diameter equal to the base of the cone. Either the cone or the tube, or both together can be forced into the soil by means of jacks. To know the cone resistance, the cone alone is first forced down for a distance of 8 cm and the maximum value of resistance is recorded. The steel tube is then pushed down upto the cone, and both together are further penetrated through a depth of 20 cm to give total of the cone resistance and the frictional resistance along the tube.

The cone test is considered very useful in determining the bearing capacity of pits in cohesionless soils, particularly in fine sands of varying density. The cone resistance (kg/cm^2) is approximately equal to 10 times the penetration resistance *N*.

32.9 GEOPHYSICAL METHODS

The geophysical methods of exploration were developed in connection with prospecting for useful minerals and oils. The major methods are gravitational, magnetic, seismic (refraction and reflection), and electrical resistivity. The seismic refraction and electrical resistivity methods are the most commonly used for civil engineering purposes.

1. Seismic refraction method: In this method, shock waves are created into the soil at their ground level or certain depth below it by exploding small charges in the soil or by striking a plate on the soil with a hammer. The radiating shock waves are picked up by the vibration detector (also called geophone, or seismometer), where the time of travel gets recorded. Either a number of geophones are arranged along a line, or the shock producing device is moved away from the geophone to produce shock waves at given intervals. Some of the waves, known as *direct* or *primary* waves travel directly from the shock point along the ground surface and are picked up *first* by the geophone. If the sub-soil comprises two or more distinct layers, some of the primary waves travel downwards to the lower layer and get refracted at the interface. If the underlying layer is denser, the refracted waves travel much faster. They emerge again and reach the geophone. As the distance between the shock point and the geophone increases, the refracted waves are able to reach the geophone earlier than the direct waves. Figure 32.6 shows the diagrammatical travel of the primary and refracted waves. The results are plotted as a graph (Fig. 32.7) between distance versus time of travel (ordinate). The *break* in the curve represents the point of simultaneous arrival of the primary and refracted waves, and its distance is known as the *critical distance* which is a function of the depth and the velocity ratio of the strata. The velocity V_1 of primary waves is thus $\frac{D_1}{T_1}$ and velocity V_2

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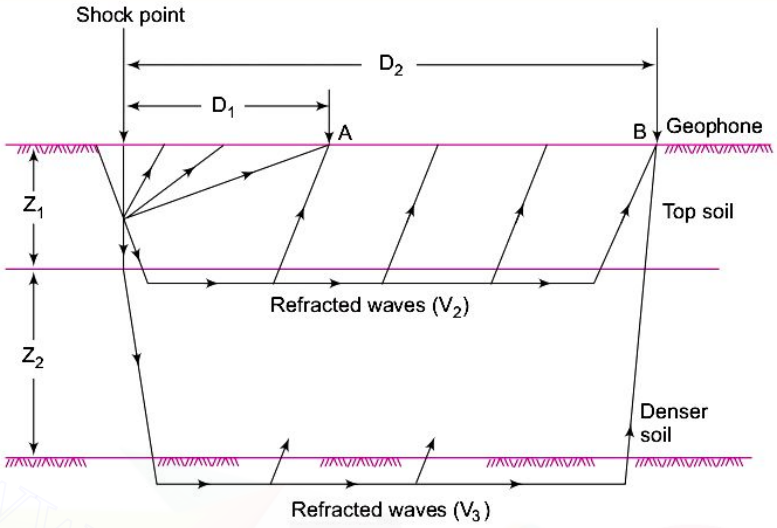

Fig. 32.6 Seismic refraction

in the denser layer is $(D_2 - D_1)/(T_2 - T_1)$ and so on. To reach me first breaking point *A*, or the *point of intersection* the refracted wave has to travel a distance of $2Z_1$ through top soil at a velocity V_1 and a distance D_1 through the second layer at velocity V_2. The time required for distance D_1, is $T_1 - t_1$, and for distance $2Z_1$ it will be t_1 (see Fig. 32.7). Hence the thickness Z_1 is given by

$$Z_1 = \frac{t_1 V_1}{2} \qquad \ldots(32.6)$$

or, more precisely (Bowles, 1968)

$$Z_1 = \frac{t_1 V_1}{2\cos\alpha} \qquad \ldots(32.6a)$$

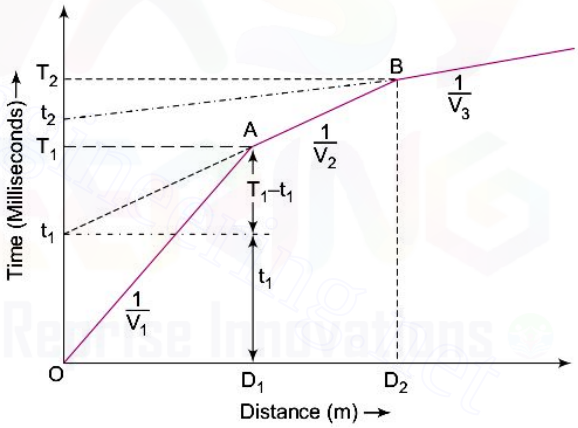

Fig. 32.7 Distance-time graph

where, the angle α is defined by the relation,

$$\sin\alpha = \frac{V_1}{V_2}$$

Similarly, the depth Z_2 of the second layer is given by

$$Z_2 = \frac{(t_2 - t_1)V_2}{2} \qquad \ldots(32.7)$$

or more precisely

$$Z_2 = \frac{(t_2 - t_1)V_2}{2\cos^2} \qquad \ldots(32.7\,a)$$

where β is defined by $\sin\beta = \frac{V_2}{V_3}$

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Seismic refraction method is fast and reliable in establishing profiles of different strata provided the deeper layers have increasingly greater density and thus higher velocities and also increasingly greater thickness. If a *blind zone i.e.,* a layer having a velocity lower than that of the upper layer, exists, it cannot be detected by seisdmic refraction (Goodman and Karol, 1968). Different kinds of materials, such as gravel, clay, hardpan, or rock have characteristic seimic velocities and hence they may be identified by the distance-time graphs. The exact type of material cannot, however, be recognised and the exploration should be supplemented by boring or soundings and sampling.

2. Electrical resistivity method: The electrical resistivity method is based on the measurement and recording of changes in the *mean resistivity* or *apparent specific resistance* of the various soils. The resistivity (ohms-cm) is usually defined as the resistance (ohms) between opposite faces of a unit cube (centimetre cube) of the material.

Each soil has its own resistivity depending upon water content, compaction and composition ; *e.g.* low for saturated silt and high for loose dry gravel or solid rock.

The test is conducted by driving four metal spikes to serve as electrodes (Fig. 32.8) into the ground along a straight line at equal distances. A direct voltage is imposed between the two outer potentiometer electrodes and the potential drop is measured between the inner electrodes. The mean resistivity is given by the following formula:

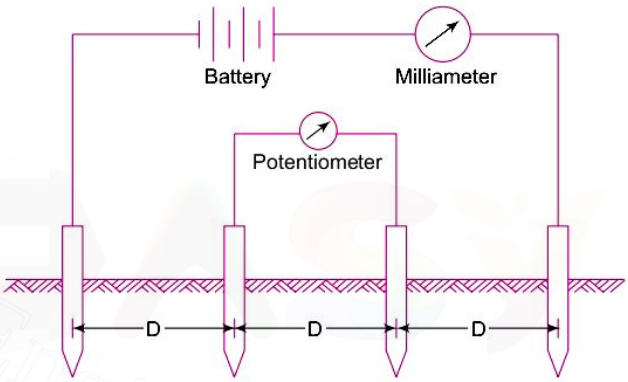

Fig. 32.8 Resistivity method

$$\rho = 2\pi D \frac{E}{I} = 2\pi DR \qquad ...(32.8)$$

where

ρ = mean resistivity (ohm-cm)

D = distance between electrodes (cm)

E = potential drop between outer electrodes (volts)

I = current flowing between outer electrodes (amperes)

R = resistance (ohms)

To correctly interpret the resistivity data for knowing the nature and distribution of soil formations, it is necessary to make preliminary trial or calibration tests on known formations.

The depth of exploration is roughly proportional to the electrode spacing. For knowing the horizontal changes in the sub-soil, the electrodes, kept at a constant spacing, are moved as a group along the line of test. This method is known as *resistivity mapping.* For studying the vertical changes, the electrode system is expanded, about a fixed central point, by increasing the spacing gradually from an initial small value to a distance roughly equal to the depth of exploration desired. The method is known as *resistivity sounding.*

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CHAPTER 33

Advanced Measuring Instruments

33.1 ELECTRONIC CONSOLIDATION APPARATUS

Electronic one dimensional consolidation apparatus basically consists of the usual loading frame with weights, consolidation cell with porous stones, pressure pad, cutting ring, water reservoir etc., as per conventional unit, but with a sensitive displacement transducer in the place of the usual displacement dial gauge. The sensitive displacement transducer, having range of 0 to 10 mm is connected to a single/multi-channel read out unit to sense the signals received from the transducer. The consolidation of the specimen is directly displayed at selectable time interval. Plate XI shows the photograph of an electronic consolidation apparatus (bench type, front loading) with read out unit, manufactured by HEICO (Hydraulic and Engineering Instruments, New Delhi). The unit has the provision to be coupled to the *data-logger* to simultaneously display and print out the readings at pre-programmed time intervals.

33.2 ELECTRONIC DIRECT SHEAR APPARATUS

Plate XII (*a*) shows an electronic direct shear apparatus manufactured by HEICO. In addition to other basic components like shear box assembly, gripper assembly, porous stones, shear box housing, loading units for application of normal load to the specimen according to the requirements as per conventional units, the electronic unit is fitted with two nos. LVDT'S—one for consolidation measurement and the other for shear strain measurement, one load cell (electronic) as per the capacity of the machine for measuring shear force and a three channel read out unit which can be connected to the two displacement transducers and the load cell for measuring consolidation, horizontal strain and shear load respectively in their direct engineering units. The system is compatible for connection to a programmable micro-processor based data-logger for continuous display and printing of the test readings at pre-set time intervals and sequences.

Plate XII (*b*) shows a HEICO multi-channel Data-Logger designed around the single chip micro-processor, incorporating latest electronic circuit. It can serve as a common module for other electronic

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instruments such as triaxial shear, direct shear and consolidation test. This can be used for all the tests running simultaneously, or for any one test with the help of channel skip facility, incorporated in the unit. In addition to continuous display of the different parameters directly in their respective engineering units, in its display screen, they can be printed by the numerical printer along with time and channel no., at programmed time intervals and sequences. The system can be programmed to shift the decimal points according to requirement. The data-logger has provision of peak hold facility which stores the peak values of any parameter, attained during the test.

33.3 ELECTRONIC TRIAXIAL SHEAR TEST EQUIPMENT

Plate XIII shows HEICO electronic triaxial test apparatus. This instrument eliminates the constant recording and computation of the data by the observer. All the above functions are carried out by continuous sensing of pore pressure, axial strain and axial load with the help of pore pressure transducer, displacement transducer and load cell respectively and their display directly in respective engineering units on the digital read out unit coupled to it through selectable switch. The unit is designed to be connected to micro-processor based data-logger [Plate XI (*b*)] which continuously scan the data received from the transducer and in addition to their display, prints them in their respective engineering units at programmed time intervals and sequences. A battery is provided to store the data programmed into the data-logger during power failure.

33.4 HYDRAULIC EARTH PRESSURE CELL

Earth pressure cells are used to measure the actual earth pressure on retaining walls, building basements, bridge abutments sheet piling surface of tunnel lining as well as for measurement of total pressure at foundations of earth dams and embankments. Such measurements help in evaluating their post construction behaviour and taking timely remedial measures for the structures showing distress.

Earth pressure cells, also called *stress cells,* are generally of two categories:

(*i*) Flexible diaphragm type, and
(*ii*) Stiff cylinder type.

The *flexible diaphragm type* earth pressure cell consists of a flexible circular or rectangular diaphragm attached to a rigid stiff case. The pressure is measured due to continuous displaced shape of the flexible diaphragm, the greatest deflection occurring at the centre. In the *stiff cylinder type cell,* the axial compression of the stiff, prismatic element, usually enclosed within a case to isolate it from the lateral stresses of the surrounding soil mass, is used to sense the total pressure.

Various systems available to measure earth pressure use the following:

(*i*) Electrial resistance strain gauges
(*ii*) Semi-conductor strain gauges
(*iii*) Vibrating wire system
(*iv*) Closed fluid system (usually called Gloetz or *hydraulic pressure cell*), and
(*v*) Pneumatic system, where air pressure is used to balance the stiffness of the cell.

Out of the above systems, the strain gauge type, the vibrating wire type and the closed fluid system type *(i.e.,* hydraulic pressure cell) are commonly used since they are most accurate. Resistance strain

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gauge type cells are easy to use and have linear rapid response, but they are susceptible to damage and are affected by the moisture of the earth fill material in long use. Vibrating wire type cells are more durable but have non-linear response; these are described in § 33.5.

The hydraulic earth pressure cell, also known as Gloetz cell, has recently come into vogue as means of measuring total pressure changes in soil, earth or rockfill, or at the interface between any of these materials. It may also be used to measure pressure changes in rock, when installed in a machined slot.

Plate XIV shows the photograph of a hydraulic pressure cell manufactured by HEICO. The pressure cell essentially consists of a sensor flat jack or fluid filled pressure pad connected to a hydraulic or pneumatic diaphragm transducer, which in turn, is connected by a flexible tubing to a read out unit. Pressure transferred from the surrounding soil to the flat jack is measured by balancing the fluid pressure in the cell by a pressure applied to the reverse side of the transducer diaphragm.

The flat jack (pressure sensor or cell) is formed from two sheets of stainless steel welded around the periphery. The narrow gap of 1.5 mm between the plates is filled with fluid of comparable deformity to that of the ground. Mercury is used in rocks and oil is used in soils. The cells can be either circular or rectangular in plan with dimensions ranging from 6 to 40 cm.

The cell is connected to a hydraulic transducer by a short length of stainless steel tubing forming a closed hydraulic circuit. Both the cell and the transducer are embedded in the structure to be monitored. Hydraulic transducer is a hydraulic valve consisting of a flexible steel, plastic or rubber diaphragm, incorporated in metal housing. The diaphragm must completely separate the cell fluid from the measuring fluid. One side of the diaphragm is connected to the cell fluid and the other to the measuring fluid delivery and return tubes. The transducer design is such that the pressure in cell fluid is slightly greater than that in the measuring fluid in order to prevent return of the measuring fluid. When the applied measuring pressure equals the cell fluid pressure, the diaphragm will displace, allowing flow along the measuring fluid return line.

Read out equipment consists of a fluid reservoir, a pump with pressure gauge to measure the applied pressure, and a detector to indicate the fluid return from the cell. Smaller applied pressures are measured by using a manometer instead of pressure gauge.

Hydraulic transducer is connected to the terminal panel or directly to the read out unit by two nylon high pressure pipes carrying quick couplings at each end. To take reading, hydraulic pressure is supplied from the read out unit to one side of the flexible diaphragm valve incorporated in the transducer. When the supply pressure is sufficient to balance the cell pressure on the reverse side of the diaphragm, the valve opens, allowing flow along the return line to a detector in the read out unit. The balancing pressure is read on the bourden type pressure gauge.

Cells are generally installed in pairs or clusters, to measure in different directions at the same location. Adjacent cell should be separated by a distance of at least one cell diameter in such a way as to prevent the presence of a cell affecting reading on adjacent cells. Typical layouts are shown in Fig. 33.1.

When installing cells in a natural soil or fill embankment, an excavation is made to accommodate a cell cluster, then individual pockets for each cell are hand dug at the correct locations with flat faces at the required inclinations. When placing cells adjacent to piers, piles, retaining walls, culverts and other structures, the cells may either be attached to the form work and placed in the structure, or fastened to the backfill, a short distance away from the structure. The contact between the cell and the backfill material should be effected by means of a layer of fine grained material.

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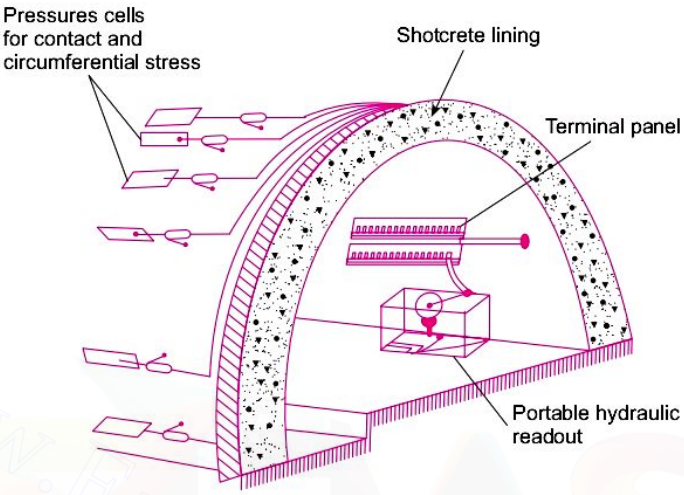

Fig. 33.1 Installation of cells in tunnel lining

33.5 VIBRATING-WIRE EARTH PRESSURE CELLS

Vibrating-wire earth pressure cell was originally developed and used for earth pressure measurements on sheet piles for strutted excavations. The primary requirement was a pressure cell that was robust enough to tolerate moderate pile driving and stable enough to permit long term measurements. Subsequently, these cells have been used to measure total pressure and/or pore pressure (with filter) acting on sheet piles, retaining walls foundations, tunnels and silos, as well as stresses in earth dams. These are also widely used in earth/rock fill dams to measure total earth pressure.

Plate XV (*a*) shows vibrating wire type earth pressure cell manufactured by AIMIL (India) in collaboration with GEONAR A/S, Norway. The principle of operation of a vibrating wire earth pressure cell or transducer is depicted in Fig. 33.2.

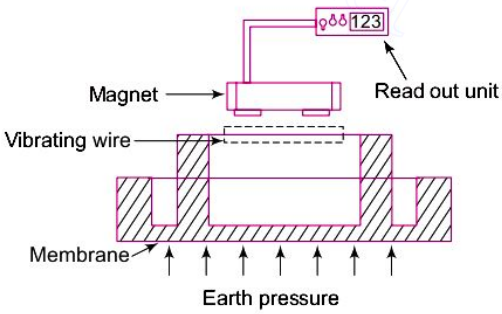

Fig. 33.2 Vibrating-wire earth pressure transducer

When an external pressure is applied to the membrane, to which the stretched *vibrating-wire* is rigidly attached, it causes a change in tension in the wire due to the deflection of the membrane posts. This causes a change in the stress in the gauge-wire and hence a change in the natural frequency of vibration. Consequently, the change in the frequency of the gauge wire vibrations is a measure of the change in external pressure acting on by the membrane surface. The frequency signal is transmitted by the *electrial cable* to a *read out instrument. The change in earth pressure acing on the membrane surface is proportional to the difference of the squared frequency of vibration for the corresponding pressures.*

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The output of a vibrating wire transducer is an alternating current which has the same frequency as the natural frequency of vibration of the gauge wire. The only measurement required is to determine to frequency of the alternating current. Once the frequency is known, the magnitude of the physical quantity which is measured, such as pressure, strain or force, can be calculated using formula and constants in calibration schemes supplied with each transducer.

All vibrating-wire type transducers contain an electrical pick-up and exciter circuit as shown in Fig. 33.3. The electrical components, which are potted in epoxy, are mounted inside the body of the transducer.

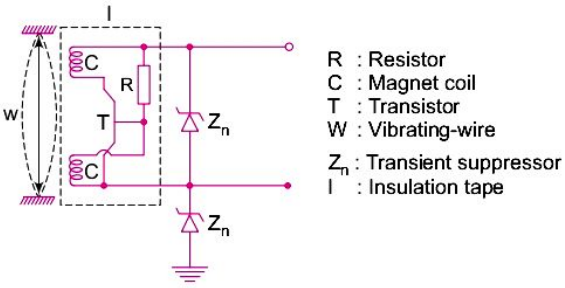

Fig. 33.3 Schematic diagram of vibrating-wire strain gauge (AIMIL)

AIMIL manufactures two types of cell: P-100 cell and P-105 cell. The active membrane diameter of P-100 cells (75 mm) limits the grain size of the soil to 1.5 mm. For larger grain size (0-2 mm), in earth/rock fill dams and off-shore applications, the P-105 cell with its active membrane diameter of 100 mm is recommended. Because of their thickness/diameter ratio (0.28 for P-100 and 0.22 for P-105), the transducers are installed in special steel frames when used in earth fills for earth pressure measurements.

The P-100 transducer is often used to measure total pressure or pore pressure acting on different kinds of walls. For low pressures, atmospheric pressure may be maintained inside the transducer through the *PE* tube encasing the lead wires. The signal cable P-540 is used along with this cell.

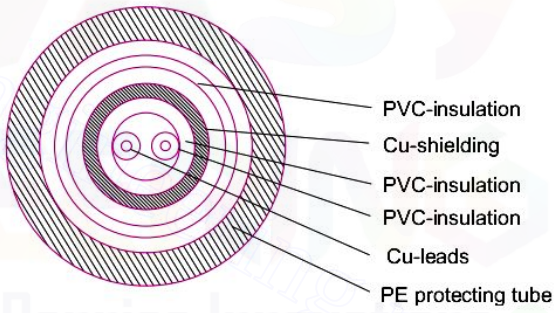

Fig. 33.4 P-540 cable

The P-540 cable is made of standard *PPOP* one-pair shielded cable with *OD* 10 mm *PE* tubing outside as an extra protection and to maintain atmospheric pressure inside the transducer.

The P-105 transducer is widely used in earth/rockfill dams to measure total earth pressure. Plate XV (*c*) shows vertical positioned P-105 cell in dam embankment. Gauge redundancy has been provided for by installing two independent vibrating-wire systems (wire and magnet system) inside the transducer. The P-105 transducer is not vented to atmosphere; hence variations in atmospheric pressure should not be ignored at low total pressures. The signal cable P-430 is used with this cell.

The P-430 cable is specially designed to withstand strong vertical forces. It is steel armoured and has a thick PE outside insulation (Fig. 33.5). The space between the cable cores and also between steel armour wire are filled with petroleum jelly to prevent longitudinal leakages caused by light surface damage to the cable. An extra protecting PE tubing for the P-430 cable (OD 25 × 3 mm) is sometimes used in adverse environments and where large settlements are expected.

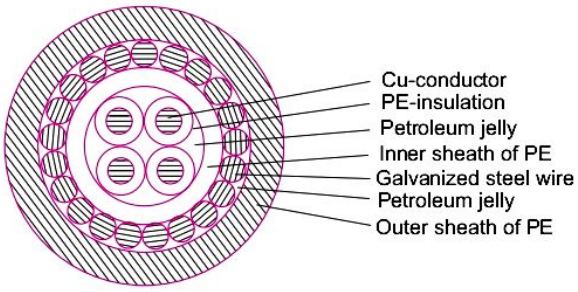

Fig. 33.5 P-430 cable

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Reading of P-100 and P-105 transducers may be performed either manually or the automatic recording equipment. Plates XV (*a*) and XVI show the P-520 F frequency indicator. It may be connected directly to the transducer cable or to a switch box. Automatic read out can be obtained by means of a micro-logger with digital print out of all data in engineering units.

33.6 VIBRATING-WIRE EXTENSOMETER

A vibrating-wire extensometer is used to monitor displacements or strains in earth structures. It is particularly useful for monitoring internal deformation and cracking of dam embankment near the abutments. The extensometers are normally linked together with steel pipe and anchor plates to form a continuous chain over the distance to be monitored. Figure 33.6 shows P-265 extensometer manufactured by AIMIL.

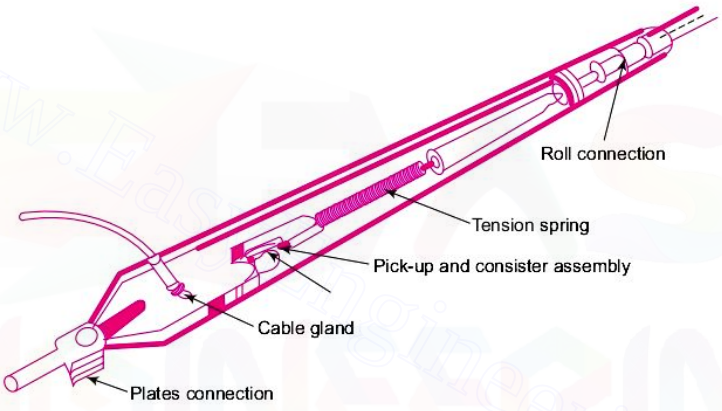

Fig. 33.6 Vibrating-wire extensometer

The complete measuring system consists of three main parts:

(*i*) Extensometer (*ii*) Cable
(*iii*) Read out instrument.

The basic principle (Fig. 33.7) of the vibrating-wire strain gauge is that the change in natural frequency of a stretched wire depends on the change of the tension in the wire. In this instrument one end of the wire is attached to the movable head of the extensometer by a steel spring. A displacement of the extensometer is thus transformed to a variation in tension of the spring and also in the vibrating-wire. Thus, the frequency of the wire is a measure of the displacement between extensometer and anchor. The square difference of frequencies is proportional to the displacement. The signal cable of P-430 type (Fig. 33.5) is used. The P-430 cable is specially designed to withstand strong external forces in hostile environments.

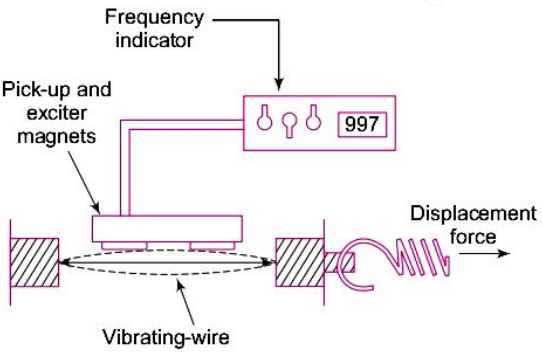

Fig. 33.7 Schematic diagram of vibrating-wire extensometer

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Reading of P-265 electronic extensometer output may be performed manually or automatically. Manually, the reading can be performed by a portable battery operated frequency indicator, P-520 F which can be connected to the extensometer cable directly, or by means of a switch box. Automatic read out can be performed by a micro-logger with the capability of digital print out of all data.

The P-265 extensometers are installed in the embankment on a level surface, either in an excavated trench, 1 m deep and 0.5 to 1 m wide, or on a compacted bedding 1 to 2 m wide. If installed in the impervious core of a dam, a 100 to 150 mm layer of screened core material should be placed between the sand and rockfill material to prevent large stones from penetrating into the fine material surrounding the instrument. The cable should be buried in a cable trench parallel to the extensometer trench. Normally, cables from an extensometer chain are connected to a multicable in a special embankment junction box. The multicable is then routed to the monitoring equipment.

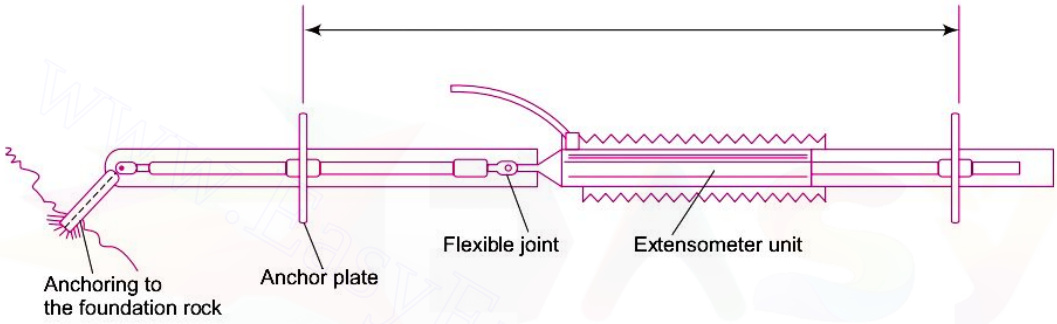

Fig. 33.8 The P-265 system

The extensometers are usually installed horizontally and laid parallel with the crest to measure strains along the longitudinal axis of the dam. The extensometers are linked together in 4 to 12 m long sections by universal joints, extension pipes and anchor plates to form continuous extensometer chains of varying lengths depending upon the geometry of the dam. If the extensometers are extended to the abutments, the end of the chain of extensometers is anchored to bedrock by means of rock-bolts (Fig. 33.8).

33.7 VIBRATING-WIRE PORE PRESSURE MEASUREMENT SYSTEM

The vibrating-wire pore pressure measurement system normally consists of (*i*) a vibrating-wire pressure transducer, (*ii*) a signal cable, and (*iii*) a frequency indicator (Plate XVI). The sytem has the advantage of high sensitivity, short time response, remote reading long possibility, distance signal transmisson and use of long cables etc.

The basic principle of the vibrating-wire transducer is that the change in natural frequency of a stretched wire depends on the change of the tension in the wire. In this instrument, one end of the gauge wire is attached to the centre of circular membrane. The other end is secured to the top of the transducer housing (Fig. 33.9).

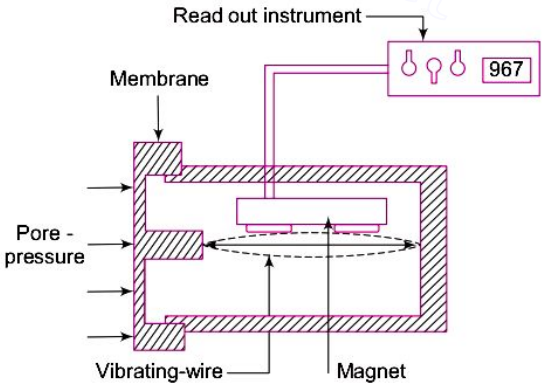

Fig. 33.9 Schematic diagram of vibrating-wire pore pressure measurement system (AIMIL-GEONAR)

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A fluid pressure applied to the membrane causes a deflection of the membrane and consequently the tension in the gauge-wire and its resonant frequency of vibration will be changed. For an increase in the fluid pressure, the frequency of vibration of the gauge-wire will be lowered and *vice versa.* Thus, the frequency of the gauge wire is a measure of the deflection of the membrane and, in turn, a measure of the pressure acting on the membrane. The difference in the square of the frequencies is proportional to the pressure change.

Two types of transducers are manufactured by AIMIL, in technical collaboration with GEONAR : (*i*) model M-600 and M-603, and (*ii*) model S-400 and S-411. The M-600 and M-603 model transducers are used as *push-in-type,* along with *EW* rods. Push-in-type are suitable for soft soils and shallow depth application and help save installation cost and time. No sealing media is required for such applications since sealing is obtained by contact established between clay and the steel rod. The S-400 and S-411 model transducers are mostly used in embankments or bore holes where bentonite cut off layers are used for sealing.

Model M-600 is the standard type, suitable for range of 0–20 bars. Model M-603 provided with in-situ zero point checking facility, has a range of 0–6 bar. The embankment/bore hole type transducer (model S-411) is a stainless steel piezometer for adverse environments. The transducer is normally delivered with high air entry resistance filter for use in unsaturated soils.

33.8 THE GDS CONSOLIDATION LOGGING SYSTEM

The GDS consolidation logging system is a computer controlled time deformation logging system for up to 80 consolidation test stations, designed and manufactured by GDS Instruments Ltd., England. The system (Fig. 33.10) is specifically designed for monitoring and recording time deformation data from multiple test stations. A test station comprises a conventional one-dimensional consolidation apparatus of the Bishop or Casagrande type with dead weight/lever arm front or rear loading.

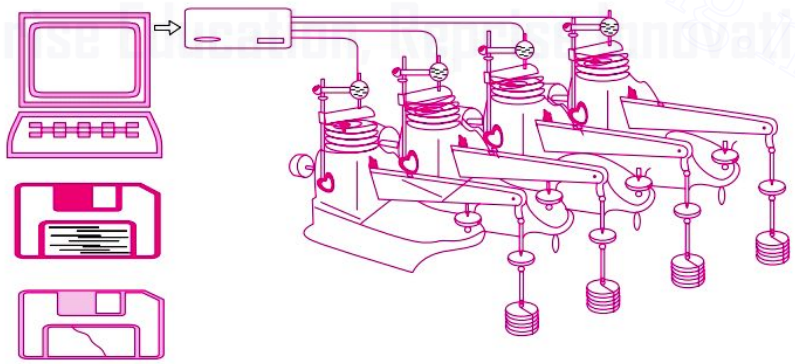

Fig. 33.10 The GDS consolidation logging system layout

In addition to logging time-deformation data to disk, the system automatically detects step loading changes and computes coefficient of consolidation for each load increment.

For the measurement of deformation, the system uses the new technology digital indicator shown in plate XVII. The digital indicator looks similar to a dial gauge. Inside the indicator, there is an

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opto-electronic encoder which gauges the displacement of a finely graduated glass plate. The indicator is provided with a digital output socket.

The advantages of this type of displacement transducer over conventional instruments of the resistive, inductive and capacitive type are as follows:

(*i*) the indicator is an inherently digital device and not subject to thermal and electronic drift,
(*ii*) the digital indicator displays its own reading,
(*iii*) the digital indicator does not require any calibration what-so-ever.

The resolution of the digital indicator is 0.001 mm (1 μ m) and the range is 12.7 mm.

The system is under computer control. Any test may start or finish at any time. The digital indicators are connected by 5 m cables to computer interface units which take up to 8 indicators each. These interface units can be stacked together giving a total capacity of 80 test stations. The interface units are linked to a desktop computer via the interface bus and cables.

From the computer keyboard, the user can define the test station set-up and the data-logging rate. Data can be saved at regular intervals with respect to time, square root of time, logarithm of time. Data can also be stored, perhaps more logically, when a given test station reading has changed by a specified amount. The default is to save the readings whenever they have changed. This has the advantage of concentrating data where they are most needed—where rapid and large changes are occurring, while avoiding filling up disks with data that are changing little.

The system automatically detects step loading changes and identifies the deformations immediately prior to the step loading changes. Before plotting or tabulating saved data, the user has the option to enter the value of axial total stress in any units at each step loading change. From these data, the system synthesises a look-up table representing the load-deformation relationship. From user-defined final voids ratio, the system computes intermediate voids ratios and plots e-log p curve.

The system carries out real time logging of time-deformation data to disk. Saved data may be tabulated or plotted or read directly into a users interpretive program. Data may be saved in a format suitable for transmission to other computers.

Saved data may be plotted in various forms. Any one of eight test parameters can be plotted against another. The test parameters are: time, square root of time, logarithm of time, axial deformation, percent axial strain, axial total stress, logarithm of axial total stress, voids ratio. In addition, the system automatically computes the coefficient of consolidation for each load increment.

33.9 HYDRAULICALLY OPERATED CONSOLIDATION CELLS

Plate XVIII shows the photograph of a hydraulically operated consolidation cell, manufactured by M/s GDS Instruments Ltd., England. The GDS hydraulically operated consolidation cell are available for test specimens of 50, 63.5, 70, 76.2 and 100 mm diameter. The cells are based on the design of Rowe & Barden's hydraulic oedometer. The GDS cells incorporate Bishop & Skinner's novel "floating" ring. Axial total stress is applied by a water-filled pressurised rubber membrane, in contact with the top of the test specimen. This, membrane, which is a perfect fit in the top chamber is not captive but is free to move with axial deformation during soil consolidation or swelling.

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Back pressure may be applied to the top of the test specimen via a tube passing through the top of the cell and fixed to the underside of the membrane. Accordingly, large variations in vertical effective stress and permeability may be avoided and field hydraulic gradients properly modelled.

Pore pressure may be measured at the base of the test specimen. GDS has standardised on Legris push-fit connectors for case of setting up. The cells do not use dead-weigths, a loading frame or compression machine and are bench mounted. The cells are each provided with a cutting shoe, dial gauge and flexible and rigid drainage discs.

The GDS Consolidation Testing System

The GDS consolidation testing system provides desk-top computer automation of test control, data logging, data reduction into SI units, on line screen graphics, and presentation of saved data by tabulation or by colour graphics plotter for direct inclusion in reports.

The hydraulic Rowe & Barden-type consolidation cell incorporates Bishop & Skinner's novel floating ring. The cell is available in a range of sizes of test specimens of 50, 63.5, 70, 76.2 and 100 mm diameter. Back pressure is applied to the cell so unrealistic and complex variations in vertical effective stress and permeability can be eliminated and field hydraulic gradients properly modelled.

Average axial stress is computed from the pressure applied to the top membrane by the axial stress controller. Accordingly, load cells are eliminated.

The test cell is linked to the dedicated controlling computer by three GDS digital controllers. Axial stress, back pressure and pore pressure are each controlled by a 200 cc/2000 kPa GDS digital controller.

Axial deformation is computed from the volume change of the back pressure controller or optionally from corrected volume change of the axial stress controller. Displacement tranducers are therefore eliminated.

The system is user-friendly with prompts, questions and information automatically appearing on the computer screen to guide the operator every step of the way.

Tests are chosen from the following test menu:

1. Controlled hydraulic gradient (CHG) by constant excess pore pressure or by excess pore pressure equal to a constant percentage of axial total stress
2. Constant rate of deformation (CRD)
3. Constant rate of loading (CRL)
4. Step loading—a staircase of axial stress versus time may be pre-programmed
5. Swelling pressure
6. Cyclic loading by sinusoidal, triangular and square wave forms; periods down to sea wave periods
7. Permeability by constant rate of flow or by controlled hydraulic gradient.

The continuous consolidation tests (CHG, CRD, CRL) cause the pore water to flow into or out of the test specimen at a more of less steady rate in contrast to the exponentially decaying flow rate of the classic step loading test where the hydraulic gradient causing the flow is itself reduced by the flow. Accordingly, these tests may be carried out in a fraction of the time taken by the conventional test.

Using the system's "loop round" facility, any number and combination of the tests may be sequentially carried out on the same test specimen. The current average height of the test specimen is passed on from the end of one test to the beginning of the next.

Tests may run unattended and continuously for any period of time. There is no minimum testing rate.

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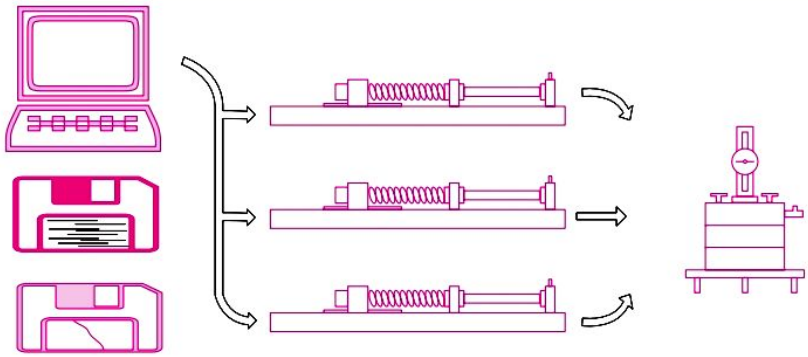

Fig. 33.11 The GDS consolidation testing system

The system facilitates the evaluation of coefficient of consolidation without curve fitting. The system provides sharp definition of preconsolidation pressure/maximum past pressure from the Janbu plot. Testing rate is automatically adjusted by the system during the controlled hydraulic gradient test.

Using the system, the conventional step loading test and the various continuous tests may be compared using the same soil and the same test apparatus.

Automatic scaling of plotted data may be over-ridden to allow details to be enlarged or families of curves to be plotted within the same set of axes.

The GDS consolidation testing system may become the GDS triaxial testing system 38 by simply changing the test cell and software package.

33.10 THE GDS 1000 CC/2000 kPa DIGITAL CONTROLLER

The GDS 1000 cc/2000 kPa digital controller is a microprocessor controlled hydraulic actuator for the precise regulation and measurement of liquid pressure and liquid volume change. (Plate XIX). The volumetric capacity is 1000 cc and the pressure range is 0–2000 kPa.

Deaerated water in a cylinder is pressurised and displaced by a piston moving in the cylinder. The piston is actuated by a ball screw turned in a captive ball nut by a stepping motor and gearbox that move rectilinearly on a ball slide. Pressure is detected by means of an integral solid state pressure transducer. Control algorithms are built into the programmable memory to cause the controller to seek to a target pressure or step to a target volume change. Volume change is detected by counting the steps of the stepping motor. In stand alone mode, the instrument is a general purpose constant pressure source, a volume change gauge, a pore pressure measuring system, a flow pump and a digital pipette. As a constant pressure source, it replaces mercury column, compressed air, pumped oil and dead weight devices.

It can be programmed through its own control panel to ramp and cycle pressure and volume change linearly with respect to time. In computer control mode, it is a full-fledged computer peripheral enabling computer-automated test control and data logging via the standard IEEE-488 computer interface. The User interface is a control panel comprising a 40 character liquid crystal display (LCD) and a 16 key membrane touch panel keypad with audio feedback.

The display is divided, into three zones : to show a continually updated display of the current pressure in kPa; to prompt for information and to display entered information; to show a continually updated display of volume change since power-on or since the volume reading was last zeroed.

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The 16 key keypad is used for entering target pressure, target volume, ramping data and other complementary functions. The keypad also gives access to on-board diagnostics for checking out each of the major hardware components of the system.

33.11 HYDRAULICALLY OPERATED TRIAXIAL CELL

Plate XX shows a GDS triaxial compression/ extension cell 100, which is a hydraulically actuated triaxial compression/extension cell for test specimens of 50, 70 and 100 mm diameter. The title cover of the book shows the photograph of GDS cell 38 for test specimens of 38 mm diameter. The cell is based on the design of Bishop & Wesley's hydraulic triaxial apparatus for controlled stress path testing developed at Imperial College of Sciene and Technology, London. It may be noted that any test, including a conventional test may be referred to as a "stress path test".

Although Bishop & Wesley set out design is a simple form of triaxial apparatus in which the stress paths encountered in engineering practice can be approximated to more readily than in conventional equipment, in a GDS computer controlled system their versatile cell is equally adept at carrying out classic "standard" tests as well as advanced tests.

Axial force is exerted on the test specimen by means of a piston fixed to the base pedestel. This piston moves vertically up and down in a "Rotolin" linear guide housed in a turret joining the cell to the base. This piston is actuated hydraulically from an integral lower chamber in the base of the cell which contains pressurised deaerated water. The piston is sealed into the cell and into the lower chamber by matched Bellofram Rolling Diaphragms which sweep equal volume of water. Cell pressure, back pressure, and pore pressure connections are made in the usual way except that GDS has standardised on Legris push-fit connectors for ease of setting-up.

In the GDS triaxial systems, and axial load cell is not used. The computer uses Bishop & Wesley's equations to compute axial stress from the cell and lower chamber pressure and from the current cross-sectional area.

In place of the load cell, the GDS extension device is fitted as standard. It is simple to set up and allows smooth transitions between compression and extension. A surgical vylastic sleeve seals a plane top cap to a fixed reaction head and prevents cell pressure from acting vertically on the test specimen. Accordingly, axial stress may be reduced below cell pressure.

The cell is supplied with interchangeable base pedestals and top caps accommodating 50, 70 and 100 mm diameter test specimens.

The cell does not use dead weights, a loading frame or compression machine and is bench-mounted.

The GDS Triaxial Testing System 38

The GDS triaxial testing system 38 provides desk-top computer automation of triaxial test control, data logging, data reduction into SI units, on-line screen graphics, and presentation of saved data by tabulation or by colour graphics plotter for direct inclusion in reports (Fig. 33.12).

The hydraulic Bishop & Wesley-type triaxial compression/extension cell tests 38 mm diameter test specimens. The test cell is linked to the dedicated controlling computer by three GDS digital controllers. Axial stress, redial stress and back pressure/pore pressure are each controlled by a 200 cc/2000 kPa GDS digital controller. Using Bishop & Wesley's equations, the system continuously computes axial stress

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by using the pressures measured by the axial and radial stress controllers. The GDS extension device is a standard fitting in place of the redundant load cell.

Axial deformation is computed from the volume change in the axial stress controller. Accordingly, displacement transducers are eliminated.

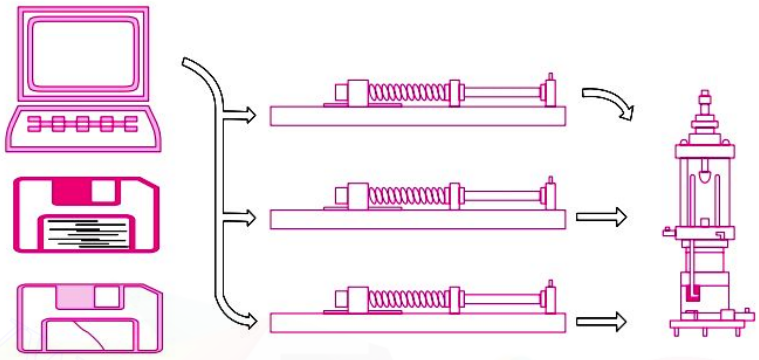

Fig. 33.12 The GDS triaxial testing system 38

The system is user-friendly with prompts, questions and information automatically appearing on the computer screen to guide the operator every step of the way.

Tests are chosen from the following test menu:

1. Isotropic consolidation and swelling.
2. Evaluation of pore pressure parameter B.
3. Saturation by simultaneous ramping of back pressure and pore pressure.
4. Unconsolidated-undrained triaxial compression/extension.
5. Consolidated-undrained triaxial compression/extension with measurement of pore pressure.
6. Drained triaxial compression/extension with measurement of volume change.
7. K-zero consolidation and swelling to an over-consolidation ratio, drained and undrained continuous linear stress paths with smooth transitions between compression and extension.
8. Cyclic loading by sinusoidal, triangular and square wave forms; periods down to sea wave periods.
9. Permeability by constant rate of flow or by controlled 'hydraulic gradient.

Using the system's "loop-round" facility, any number and combination of the tests may be sequentially carried out on the same test specimen. The current height and diameter of the test specimen are passed on from the end of one test to the beginning of the next. Tests may run unattended and continuously for any period of time.

Tests are optionally axial stress controlled or axial deformation controlled. There is no minimum testing rate. The value of the average cross-sectional area used in all computations is continuously corrected for axial deformation and for volume change. Saved data may be corrected for the effect of membrane stiffness. Automatic scaling of plotted data may be over-ridden to allow details to be enlarged or to allow families of curves to be plotted within the same set of axes. The GDS triaxial testing system 38 may become the GDS consolidation testing system by simply changing the test cell and software package.

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CHAPTER 34

Reinforced Earth and Geotextiles

34.1 REINFORCED EARTH

Reinforced earth consists of a compacted soil mass within which *reinforcing elements* or membranes, usually in the form of horizontal strips of metal (such as galvanised steel, stainless steel or aluminium alloys), rods of metals, wire grids, fibre glass strips/rods, bamboos or geotextiles, are embedded. The essential feature of the reinforced earth is that friction develops between compacted layers of the earth and the reinforcing elements. The soil tranfers the forces built up in the earth mass to the reinforcement by means of friction. Thus, tension is developed in the reinforcement. The modern form of application of reinforced earth was first applied by Henry Vidal who first constructed major retaining walls in France in 1968. It should be noted that the technique is subject to patents by Vidal and the Reinforced Earth Company, the term *'reinforced earth'* being their trade mark.

The basic principles of incorporating reinforcement members into a soil mass are much the same as those utilised in the design of reinforced concrete. The *membranes* are used to provide tensile strength in the earth mass in locations where shear stresses are generated. The provision of membrane leads to the redistribution of stresses internally within each of the membrane encapsulated layers. The reinforced soil concept/technique is essentially based in the mobilisation of inter facial shearing resistance between the soil and reinforcement which in turn *restrains* the lateral deformation of the soil.

The main application of the reinforced earth technique is the *reinforced* earth wall shown in Fig. 34.1. A *reinforced earth wall* consists of three components:

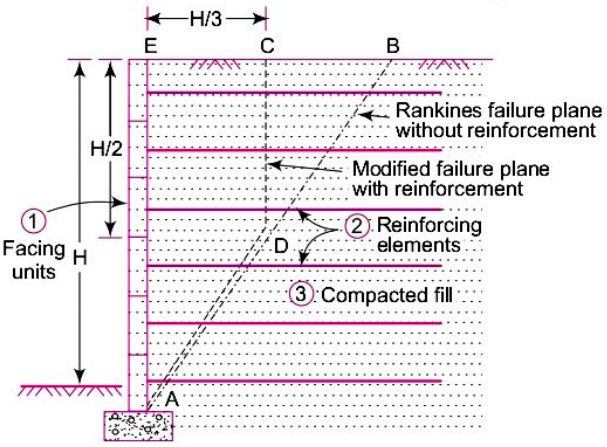

Fig. 34.1 Reinforced earth wall

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1. Wall facing elements
2. Reinforcing elements/membranes
3. Compacted backfill

1. Wall facing elements: The wall facing elements are provided at the free boundary of a reinforced earth structure, to provide some form of barrier so that the soil mass is contained. These elements, usually known as *skin,* may be either flexible or stiff; it should be strong enough to hold back the soil and should allow fastening to attach reinforcing elements. The facing units are usually prefabricated units which are small and light enough for transportation for quick and easy construction. The wall facing elements or units are generally made from steel, aluminum, plastic, fibre, glass or reinforced concrete. The facing units require a small foundation of concrete from which they can be built.

2. Reinforcing elements: The reinforcing elements consists of any or the following:

(*i*) *Galvanised steel strips,* 50 to 100 mm wide and up to 9 mm thick, several metres in length, are the most commonly used reinforcing elements. Data available on the rate of corrosion of galvanised steel in soil indicates that elements of this material are likely to have a service life of 120 years.
(*ii*) Rods of galvanised steel.
(*iii*) Strips or rods of other metal such as stainless steel, aluminium
(*iv*) Galvanised iron grids
(*v*) Fibre glass strips
(*vi*) Glass-fibre reinforced plastic (GRP)
(*vii*) Bamboos
(*viii*) Geosynthetic reinforcements such as geotextiles, geomembranes, geogrids, geostrips, geocomposits etc.

3. Compacted earth fill: The soil used for the backfill should be predominantly coarse-grained and it has been proposed that not more than 10% of the particles should pass the 63 μm sieve. The first layer of reinforcement strips is placed at the level ground surface and the backfilling is done with granular soil, compacting it in the processes of laying. The entire process of laying strips and backfilling and compacting is continued till the required height of the reinforced earth wall is attained. It is vital that the fill should be adequately drained to prevent it from becoming saturated.

Advantages of Reinforced earth structures: The following are advantages of reinforced earth structures.

1. Reinforced earth structures are quite flexible. Hence these can withstand foundation deformations/ settlements.
2. Reinforced earth structure, being flexible, can withstand earth-quake forces more efficiently than conventional rigid structures.
3. Reinforced earth structures are much more economical in comparison to the conventional structures of masonry or concrete.
4. Reinforced structural elements can be transported easily. Hence these can be constructed speedily.
5. Reinforced earth structures can also be constructed in stages.
6. The reinforcing elements used for such structures are easily available in various sizes and shapes. They can be easily stored, handled and placed during construction. **Other applications of soil reinforcement:** Jones (1985) identifies several soil reinforcement field applications, some of

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which are (*i*) Reinforced earth wall, (*ii*) bridge work, (*iii*) dams, (*iv*) embankment, (*v*) foundations, (*vi*) highways, (*vii*) root pile systems, (*viii*) water way structures, (*ix*) under ground structures. Figure 34.2 shows some of these applications.

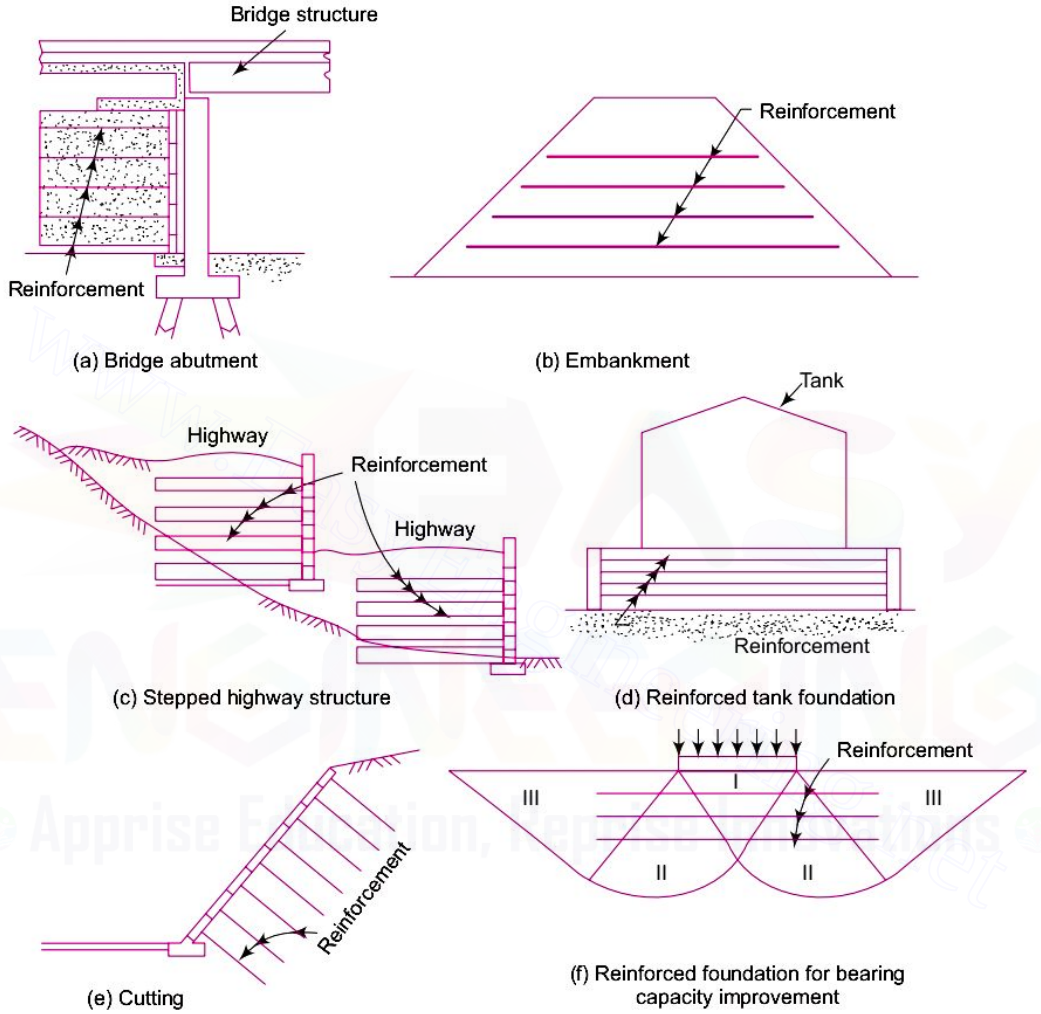

Fig. 34.2 Some applications of soil reinforcement

34.2 DESIGN OF REINFORCED EARTH WALL

The design of reinforced earth wall is made on the following assumptions:

1. The pressure acting of the wall facing elements without the reinforcements is the same as that acts on a rigid vertical retaining wall under active state.
2. Rankine's earth pressure for active state holds goods, due to which the failure plane makes an angle of $45° + \varphi/2$ with the horizontal.
3. The backfill soil is granular, and is non-cohesive.

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Figure 34.3 shows a reinforced earth retaining wall AB of height H. When the wall AB rotates about point A, away from the backfill, under Rankine active state, a failure surface AD is formed, which makes as angle $45° + \varphi/2$ with the horizontal. The active pressure acting on the wall at its base is given by

$$p_a = \gamma H K_a \qquad ...(1)$$

where K_a = Rankine's active earth pressure coefficient.

The pressure distribution along the depth is assumed to the linear, represented by the triangle ABC. The total active pressure at the base of the wall per unit length of the wall is given by

$$P_a = \frac{1}{2}\gamma H^2 K_a \qquad ...(2)$$

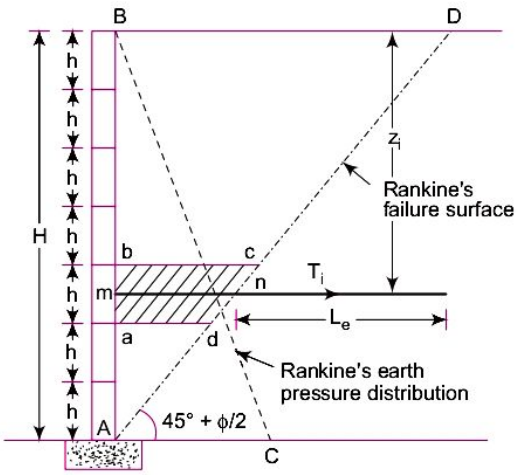

Fig. 34.3

Now let us divide the height of the wall into a number of small units of height h. Let z_i be the depth of any reinforcing strip mn. The total pressure on the strip mn is represented by the area $abcd$ of the pressure diagram, with average pressure p_i. This average pressure p_i, is given by

$$P_i = \gamma z_i K_a \qquad ...(3)$$

Let us assume that reinforcements are placed at an interval h in the vertical direction and at an interval s in the horizontal direction. The tension T_i in the reinforcement mn is thus given by

$$T_i = p_i A = \gamma z_i K_a (h \times s) \qquad ...(34.1)$$

where A = effective area of strip = $h \times s$.

Proceeding in the same way, the tension in other reinforcing strips can be found, noting the fact that the variable in equation 34.1 is the depth z_i. As the depth (*i.e.*, location) of the strip increases, the tension T_i, in the reinforcement also increases. Evidently, the sum of tension in all the reinforcing strips is equal to the total earth pressure on the wall of a length s. Thus

$$\sum_{i=1}^{n} T_i = s P_a \qquad ...(34.2)$$

Length of the reinforcement: Let us find the length of the reinforcement. The reinforcing strip (or bar or sheet) attached to the wall portion ab extends well beyond the active zone into the backfill, as shown in Fig. 34.3. The length mn of the reinforcing strip is neglected for computing the necessary grip length for resisting the tension developed in each reinforcement. In other words, the grip length L_e lying on the RHS of the failure plane AD should be such that it resists the tension so developed, with a suitable factor of safety.

Let L_e be the grip length required to resist the tension T_i developed in the reinforcement strip i located at a depth z_i $(= i\,h)$. Let F be the suitable factor of safety. If F_t is the frictional resistance of the reinforcement of length L_e located at the depth z_i we have

$$F_i = FT_i \qquad ...(4)\ ...(34.3)$$

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where F = Factor of safety against bond failure, usually taken equal to 2.

If δ is the *angle of surface friction,* we have

$$F_i = (\gamma z_i) \tan \delta \times (\text{surface area of reinforcement}) \quad \ldots(5)$$

The grip length L_e depends upon the surface frictional resistance developed between the reinforcement and soil. This resistance evidently depends upon the *shape* of the reinforcement. We will consider three shapes of the reinforcement: *(i)* strip, *(ii)* rod, and *(iii)* sheet.

(i) *Reinforcing strips:* Let b be the width of each strip, spaced at s along the length of the wall. Frictional resistance is developed on both the faces (*i.e.*, top face and bottom face) of each strip. Thus, we have $F_i = (\gamma z_i) \tan \delta \, (2\, b\, L_e)$

Substituting in Eq. 34.3, the value of F_i, we get

$$L_e = \frac{FT_i}{2\gamma z_i . b \tan \delta} \quad \ldots(34.4\ a)$$

(ii) *Reinforcing rods:* If reinforcement consists of round bars, each of diameter d, the perimeter of each bar is πd and hence surface area is $\pi d L_e$. Hence we get

$$L_e = \frac{FT_i}{\pi d \gamma z_i . \tan \delta} \quad \ldots(34.4\ b)$$

(iii) *Reinforcing sheets:* If the reinforcement consists of sheets laid along the length of the wall, the surface area will be equal to $2L_e$ per unit length of wall.

Hence we have $$L_e = \frac{FT_i}{2\gamma z_i . \tan \delta} \quad \ldots(34.4\ c)$$

Thus, the length L_e of the reinforcement is given by Eq. 34.4. This length L_e varies with the depth z_i. However, it is a common practice to use one single length L_e for the full height of the wall. Evidently, this common length L_e is calculated with respect to the maximum tension T_{max}, taking $z_i = H$ in equation 34.1.

The angle of surface friction, δ depends upon several factors such as density and type of the backfill material, *type* and of *shape* reinforcement surface etc. Usually, the value' of δ varies from $\varphi/2$ to φ, where φ is the angle of internal friction of backfill material.

Sectional area of reinforcement: Let A_{rs} be the area of reinforcing strip/bar of steel/metal

Then $$A_{rs} = \frac{F_i}{f_{rst}} = \frac{FT_i}{f_{rst}} \quad \ldots(34.5)$$

where f_{rst} = allowable tensile stress in reinforcement.

$A_{rs} = bt$ for strip of width b and thickness t.

$= \pi d^2/4$ for rod. of diameter d.

Here again, A_{rs} varies with the depth z_i. However, it is a common practice to make computation corresponding to $z_{max} = H$, and provide uniform reinforcement through out the height.

External stability of the wall: The external stability of a *reinforced earth structure* must also be considered. A reinforced earth wall, although behaving as a relatively flexible structure, should be designed as if it were a gravity wall from the point of view of external stability. *The back of the wall should be taken as the vertical plane through the inner end of the lowest reinforcing element.* The total active thrust

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on this plane should be calculated by the Rankine's theory. The factor of safety against sliding between the reinforced fill and the foundation soil should not be less than 2, the angle of shearing resistance of the weaker soil being used in the analysis. The pressure distribution on the base must be wholly compressive and must not exceed the allowable bearing capacity of the foundation soil. All potential failure surfaces encompasing the structure should be analysed, as for the slopes, and a minimum factor of safety of 1.5 should be ensured.

34.3 GEOTEXTILES: DEFINITION AND TYPES

Geotextiles are permeable or porous fabrics, made from synthetic materials, that are used with geotechnical material (such as soil or rock) as an integral part of a man made product, structure or system. As per ASTM, *geotextiles* are permeable sheets of synthetic fibres like polyester, polypropylene, polyethelene, polamide (nylon), viscose etc. while geomembranes are basically *impermeable* sheets or films made from a polymer and which may be reinforced with a textile, or may be made impermeable by spraying asphalt or resin directly on the ground or on to a geotextile. Geotextiles are available in thickness varying from 10 to 300 mils (1 mil = 1/100 inch = 0.254 mm) and in widths up to 10 m and in rolls length up to 600 m. The permeabilities of geotextiles sheets are comparable in range from coarse gravel to fine sand.

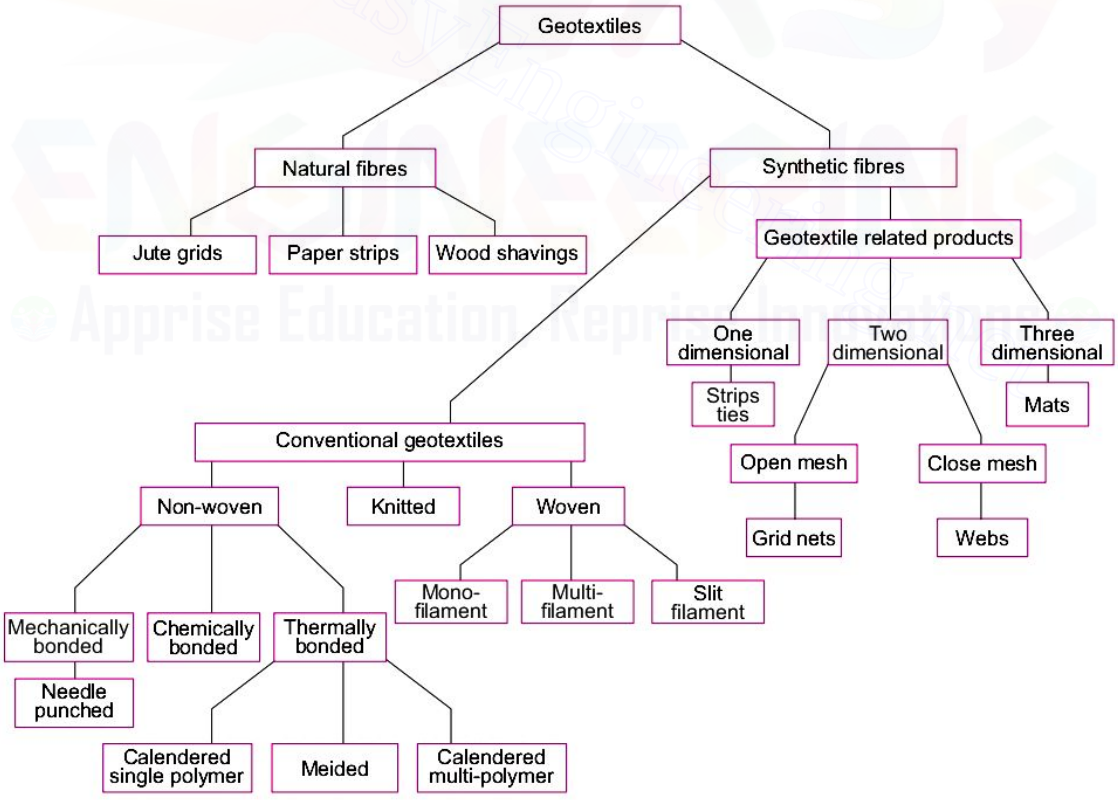

Fig. 34.4 Classification groups of geotextiles

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Geotextiles can be made of a vide variety of *natural* (such as jute paper or wood shavings etc. and *synthetic* materials (mentioned above) and can be *woven,* made up to mono-filament, multi-filament or tapes, or *non-woven i.e.*, mechanically bonded (needle punched), heat bonded or resin bonded, being made of staple fibre or continuous filament. They may also be knitted some times. Figure 34.4 gives various classification groups of geotextiles.

Common nomenclature of geosynthetics:

1. ***Geotextiles and geostrips:*** These include woven and non-woven geotextiles used for drainage, stabilization and reinforcement function (see § 34.4). *Geostrips* are in the form of cut fabric or long strips of geotextiles. Geostrips are generally produced from polypropylene and high density polyethylene. They can be connected with anchors at the ends. The anchors may be in the form of loops, rings or spirals, which may help in confining soil elements.
2. ***Geogrids:*** These include extruded, woven, flexible and stiff types of geogrids used for stabilisation and reinforcement. Geogrids are also produced by special process which align molecular chains of polymers, there by obtaining the material of high tensile strength as compared to the extruded polymeric mesh. In geogrid, the reinforcing function is achieved by positive inter-locking of the fill material into the apertures or opening while in a geotextile, the reinforcing function is achieved by the surface friction between the fabric and soil fill.
3. ***Geomembranes:*** These include HDPE, PVC, CSPE, PP liners etc., and are basically *impervious.*
4. ***Geonets:*** The include LDPE and HDPE nets and have functions similar to geogrids.
5. ***Geocells or Geoweb members:*** These include multicoloured HDPE cells of varying heights, used for stabilisation applications. They are made from prefabricated polymetric systems. These systems are made from thick HDPE strips, 50 to 200 mm wide, stitched or welded together at 200 to 50 mm intervals. The hexagonal or rhomboidal cells can be made of geosynthetic honeycomb design. The geocell or geoweb mattresses are used for footing foundations, roads and canal linings.
6. ***Geofoam:*** These include polystyrene sheets of varying dimensions used for light weight fills and other applications.
7. ***Geosynthetic clay liners:*** These include geotextile / clay / geotextile composites. Bentofix Thermal Lock Geosynthetic clay liner (GCL), a patented registered trade mark, is a sandwitched" composite of geotextiles and bentonite clay used to provide a positive barrier to liquids in containment applications.
8. ***Geocomposites:*** These are made by bonding together the geomembranes and geotextiles or any of their like forms for specie applications in drainage, erosion control, bank protection etc.

Advantages of geotextiles: As stated earlier, geotextiles can be made either from the *natural* fibres or from *synthetic* materials. The *bio-degradable nature* of natural fibres have restricted their use to some specific applications whereas synthetic materials have made their way for wide applications. Another advantage of using these fibres for geotextiles is related to their ability to be engineered chemically, physically and mechanically to suit particular geotechnical engineering applications.

The use of geotextiles is *preferred* over other construction materials because of the following reasons:

1. Ease and speed of construction.
2. Ability to withstand differential settlement.
3. Suitability for phased construction at restricted sites.
4. Ability to provide viable solution to exceptionally difficult, otherwise, intractable construction problems.

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34.4 FUNCTIONS OF GEOTEXTILES

The functions of geotextiles can be broadly divided into two heads:

(A) Hydraulic functions, and (B) mechanic functions as detailed below.

(A) ***Hydraulic functions:***

1. Fluid transmission or drainage
2. Filtration

(B) ***Mechanic functions:***

1. Separation.
2. Protection and
3. Reinforcement

Figure 34.5 diagrammatically shows the relationship between the above functions and the properties required. Table 34.1 gives the functions, requirements and the relevant properties of geotextiles.

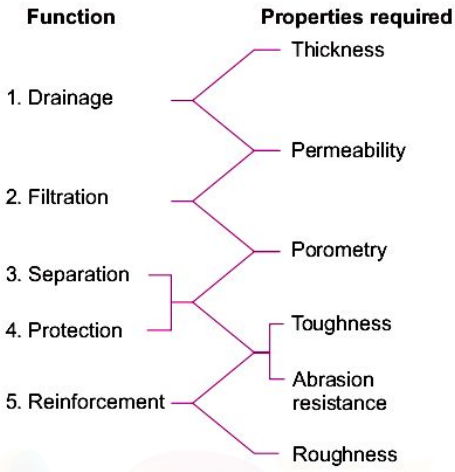

Fig. 34.5 Functions and required properties

Table 34.1 Functions and properties of geotextiles

S.No.	*Function*	*Requirements*	*Relevant Properties*
1.	*Fluid Transmission or drainage*	Geotextiles should have high porosity	1. Substantial thickness 2. High horizontal permeability 3. High transmissibility understrong compression
2.	*Filtration*	In contact with natural soil, geotextiles should prevent internal erosion and should avoid uplift	1. Structural continuity 2. Good breaking strength
3.	*Separation*	Geotextiles should preserve mechanical characteristics and properties of two soils of different particle size distribution and different behaviour which tend to inter penetrate under heavy load and marked deformation	1. Structural continuity 2. Good breaking strength; tensile strength and good static and dynamic puncture strength 3. High tearing strength
4.	*Protection.*	Geotextiles should form a fibrous system which can absorb local stresses and distribute them more evenly thereby providing protection against perforation of impervious film sheets asphaltic membranes etc., and high buried sealings	1. Adequate thickness 2. High modulus of deformation 3. Adequate plastic yield properties
5.	*Reinforcement*	Geotextiles, when placed in soil mass should take up some of the tensile stresses and help to distribute them more evenly.	1. High modulus of deformation 2. High coefficient of friction between the geotextiles and the soil.

1. Fluid transmission and drainage function: A geotextile provides fluid transmission when it collects a liquid or gas and conveys it within its own plane, towards an outlet. Water may be conveyed vertically or horizontally. Drainage is related to the role of filtration and is a function of the permeability of a geotextile and its pore opening size or porometry. The permeability involved in the *fluid transmission function* is the permeability in the plane of the geotextile. Figure 34.6 shows two typical applications of

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geotextiles as drain. In order to accomplish this function of transmission or drainage, a *bulky* geotextile or a composite system is needed. In other words, the other property, in addition to permeability, is the *thickness* of the geotextile.

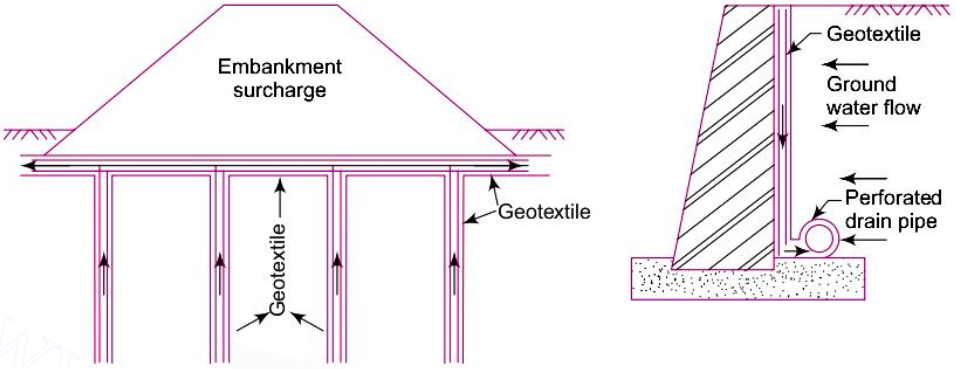

Fig. 34.6 Typical examples of fuild transmission or drainage function

2. Filtration function: A geotextile acts as a filter when it allows liquid to pass normal to its own plane while preventing most soil particles from being carried away by liquid current. Two cases can be considered: (*i*) A geotextile, placed across a flow of liquid carrying fine particles, stops most of the particles (where they accumulate on the filter) while allowing water to pass through, and (*ii*) a geotextile, placed in contact with a soil, allows water seeping from the soil to pass through, while preventing any movement of soil particles. The two important properties required in a geotextile to work as a filter are (*i*) permeability across its own plane and *(ii)* porometry. Figure 34.7 shows some typical examples of filtration function.

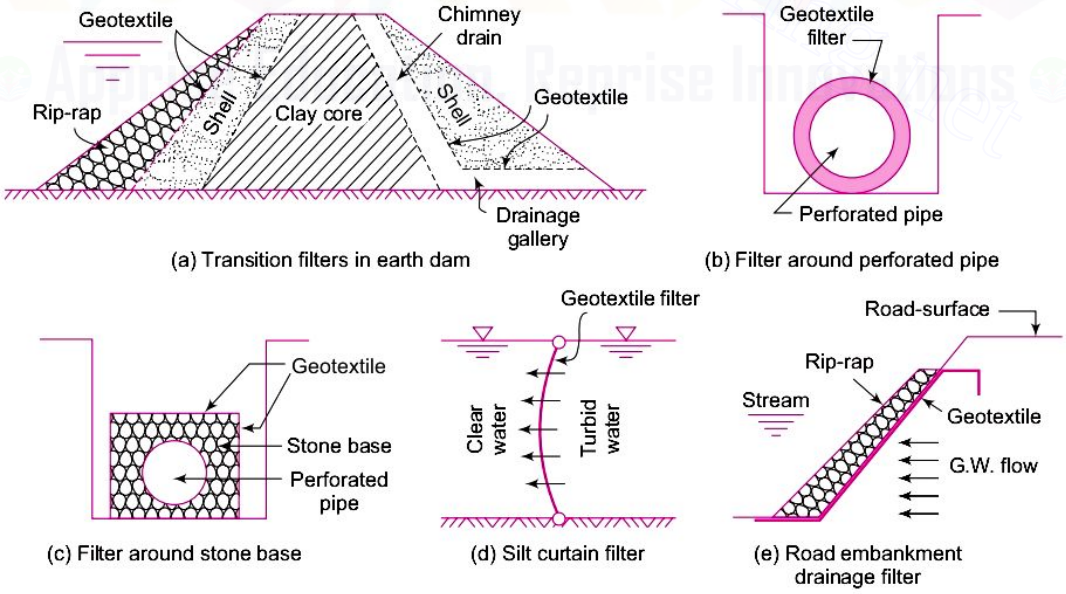

Fig. 34.7 Typical examples of filtration function of geotextiles

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3. Separation function: A geotextile acts as a separator when placed between a fine soil and a coarse material such as gravel or stone ballast. It prevents the fine soil and the coarse material from inter-mixing under the action of repeated applied loads. The key factors for a geotextile to satisfy this function are (*i*) porometry, (*ii*) toughness, and (*iii*) strength. A typical example is in the construction of a highway in which a clayey subgrade is kept to separate from a granular base course (Fig. 34.8 *a*). In absence of a geotextile, the usual procedure is to provide a layer of *transition soil* which contain grain size intermediate between the clayey subgrade and the granular base coarse. If there is a wide variation of grain sizes of the two soils, more than one intermediate grains size soil has to be provided. However, provision of a single layer geotextile between the two layers, acts as a separator. See Figs. 34.11 and 34.13 also, for the function of a geotextile as a separator. The key factors for a geotextile to satisfy this function are (*i*) porometry, (*ii*) toughness/abrasion resistance, and (*iii*) tensile strength. The advantages of using geotextiles, over a single or multiple soil layers are (*i*) simplicity in construction, (*ii*) less excavation required, (*iii*) less weight placed on the clay subgrade, and (*iv*) less time required for construction. Another example of the separator function is a geotextile envelope around a drain pipe [Eq. 34.8 (*b*)].

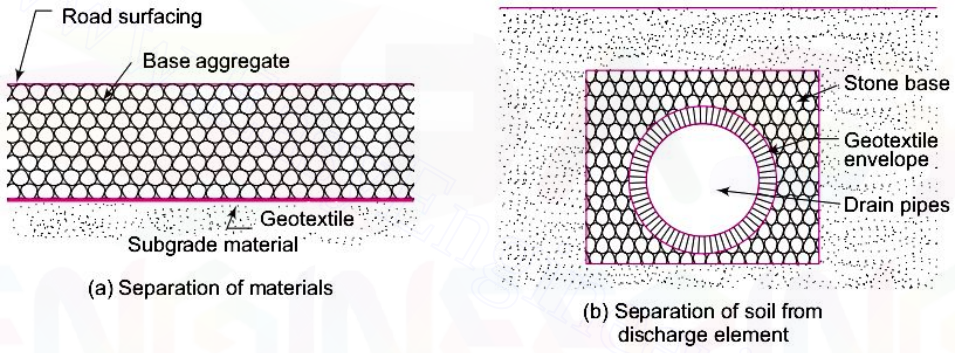

Fig. 34.8 Typical examples of separation function of geotextiles

4. Protection function: A geotextile, *protects* a material when it distributes stresses and strains transmitted to the protected material. Two cases can be considered.

(*a*) ***Surface protection:*** A geotextile, placed on the soil, prevents its surface from being damaged by such action as weather, light traffic, surface water flow etc.

(*b*) ***Interface protection:*** *A* geotextile, placed between two materials prevents one of the materials from being damaged by concentrated stresses applied by (or strains imposed by) the other material Fig. 34.9 shows typical examples of the protection function of geotextiles.

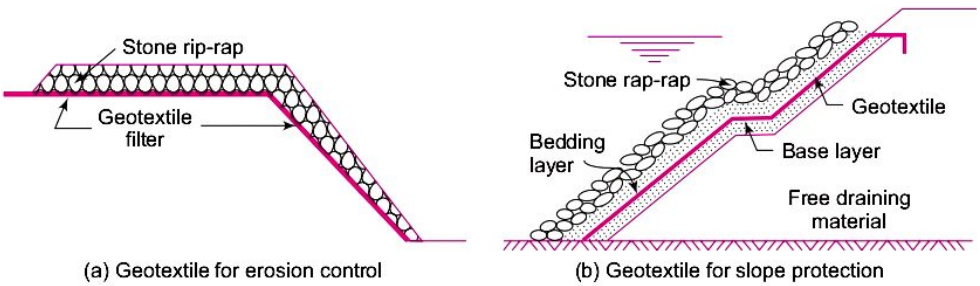

Fig. 34.9 Typical examples of protection function of geotextiles

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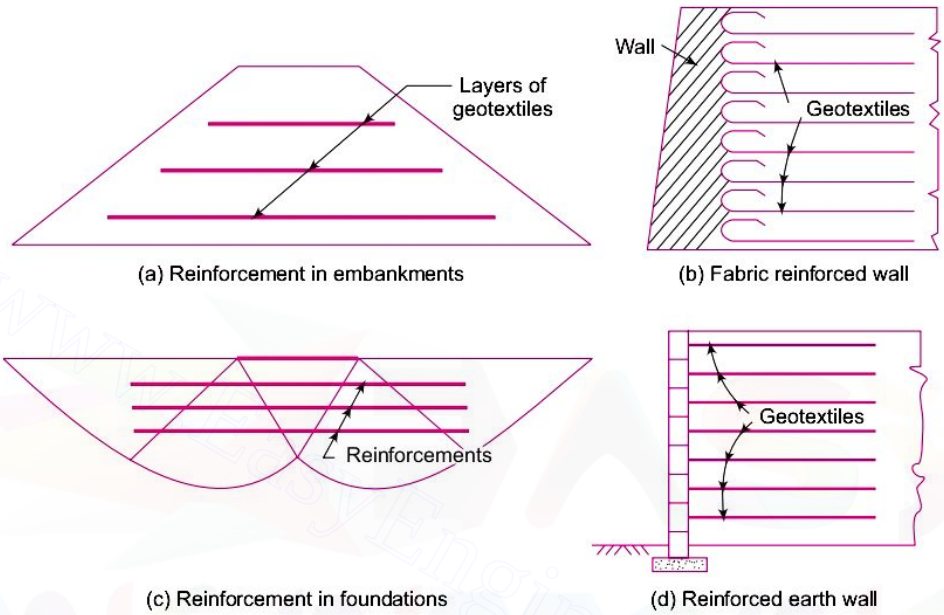
5. Reinforcement function: Reinforced earth: Since the tensile strength of soil is practically nil, geotextiles which are good in tensile strength, can contribute to the load carrying capacity of soil. In other words, geotextiles perform the function of reinforcement in soils. Figure 34.10 shows some typical examples of reinforcement function of geotextiles. See also Fig. 34.2.



(a) Reinforcement in embankments (b) Fabric reinforced wall

(c) Reinforcement in foundations (d) Reinforced earth wall

Fig. 34.10 Typical examples of reinforcement function

A Geotextile can work as reinforcement in two ways: (*a*) As tensile member, and (*b*) as tensioned membrane.

(*a*) ***As tensile member:*** A geotextile acts as a tensile member when it provides tensile modulus and tensile strength to a soil with which it is interacting through interface shear strength, *i.e.*, friction, cohesion-adhesion and/or interlocking between geotextile and soil.

(*b*) ***As tensioned membrane:*** A geotextile function as a *tensioned membrane* when it is placed between two materials having different pressures, and its tension balances the pressure difference between the two materials, thus strengthening the structure. See Fig. 34.14 also, for its use in railways.

34.5 USE OF GEOTEXTILES IN EARTH DAM CONSTRUCTION

In earthen dams, the upstream and down stream slopes are prone to erosion. Hence geotextiles can be effectively and conveniently used as filter media in erosion control of earthen dam slopes (Fig. 34.9 *a*). Case studies in this regard reported successful use of needle-punched non-woven geotextiles (Dinesh mat GPB-132) in Hiran dam (near Veraval in Gujarat) where the geotextile successfully prevented the migration of soil fines from upstream slope towards the spillway section, and thus solved the sinking/ stability problems of dam embankment.

In earthen dams, geotextiles can be used in many important positions for filtration, drainage and erosion control functions such as the follows:

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(*i*) Erosion control or slope protection function (Fig. 34.9 *a)*
(*ii*) Filtration and drainage material for seepage control system (Fig. 34.7 *a)*
(*iii*) Filtration and separation function (Fig. 34.7 *a)*
(*iv*) Separation cum filtration between natural fill and loading beam.
(*v*) Filter wraps around PVC, perforated rigid pipes (Fig. 34.7 *b)*

Needle punched non-woven geotextile (Dinesh mat GPB-127) has been successfully used in Dharol dam in Gujarat in 1982, as filter/separator between natural fill and loading berm and as wraps around PVC pipes in relief wells, thus solving the problems of sand boiling.

34.6 USE OF GEOTEXTILES IN ROAD WORKS

In many places in India, high water table of subgrade is a constant concern to road engineers. Geotextiles in subgrade-subbase *interface* of a road (Fig. 34.8 *a*) can effectively deal with this problem. Some trials have been reported by Central Road Research Institute (CRRI) using different kinds of geotextiles (Dinesh mat GPB-127 and other imported geotextiles) in road and advocated use of geotextile which passes a tentative specification made on these trials. It is seen that geotextiles offer the following :

(*i*) *Time saving*-no prior stripping of the natural soil
(*ii*) *Economy* of aggregate
(*iii*) *Reduction* of differential settlings and
(*iv*) *Speedier* consolidation of fills.

Provision of geotextiles in road works may serve three functions:

1. Separator function
2. Drainage function
3. Reinforcement function

1. *Function of geotextile as separator in road works:* Geotextile materials provides separation, which preserves the integrity and extends the life of the road surface layer. Roadway pavements are basically structures for taking high *contact pressures* from the vehicle tires and reducing this pressures through the depth of the pavement to a level that can be supported by the underlying soil/subgrade. Pressure is dissipated down through the various layers of materials within the pavement. Figure 34.11 shows the effect of wheel load on the pavement with geotextile (as separator) and without separator.

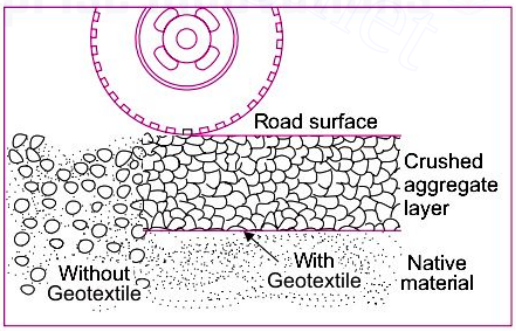

Fig. 34.11 Pavement with and without geotextile separator

If geotextile is not provided at the interface between subgrade and the aggregate base, the vehicle pressure, over time, causes subgrade soils to migrate into the aggregate base of the pavement section. Contamination of the aggregate base by the subgrade results in the reduction in the effective base thickness to less than originally designed, as illustrated in the LHS of Fig. 34.11. Reduction in base thickness results in the decrease in the load carrying capacity of the aggregate base and the consequent reduction in the pavement life. Provision of geotextiles as separation

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layer (RHS of Fig. 34.11) prevent the subgrade materials from migrating into the aggregate base, thus increasing the pavement life.

Provision of geotextile as separator helps in the construction in banked up road deck on compressible soils, construction of forest track, correction of alignment of a road on compressible ground and repair of mountain road after land slide.

Geotextiles also function as filters and separators which

(*i*) prevent the inter penetration of the natural soil (*i.e.*, subgrade) and aggregates.
(*ii*) fully preserves the properties of road making materials, and
(*iii*) allows vehicles to circulate on foundation course while work is in progress and protects the foundation course from contamination.

2. Drainage function of geotextile in road work: Also, using geotextiles to its drainage functions, geotextile road edge drains can be laid to provide drainage of roads and hence improving them.

Lastly, when a *road embankment is* to be constructed on compressible soil, the consolidation of the subgrade soil, over a period of time will cause cracks and pot holes in the road surface. In order to avoid this, usual procedure is to provide sand drains which accelerate the consolidation process prior to road surfacing. This action can be more effectively achieved by provision of geotextiles to provide vertical and horizontal drainage system, as illustrated in Fig. 34.6 (*a*).

3. Reinforcement function of geotextile in Road embankments: Figure 34.12 shows a geotextile reinforced levee or road embankment, constructed over soft soil. The geotextile layers are placed across the potential rotational failure plane to carry the tensile forces that cannot be carried by an unreinforced soil mass of the embankment.

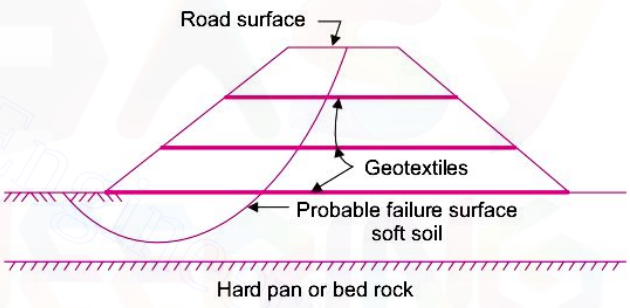

Fig. 34.12 Use of geotextile as reinforcement in road embankment constructed over soft grade soil

34.7 USE OF GEOTEXTILES IN RAILWAYS WORKS

Geotextiles can be used in railways works to provide the following:

(*a*) Separation function
(*b*) Reinforcement function and
(*c*) Water drainage function

(*a*) *Separation function:* In track foundation, geotextiles separate ballast and subgrade at interface and prevents intermixing of the two materials under dynamic loads, as shown in Fig. 34.13. Geotextile also checks mud pumping and prevents ballast fouling and thus extends deep screening cycle by several years.

(*b*) *Reinforcement function:* Geotextile provides reinforcement, through both the *tensioned membrane* and

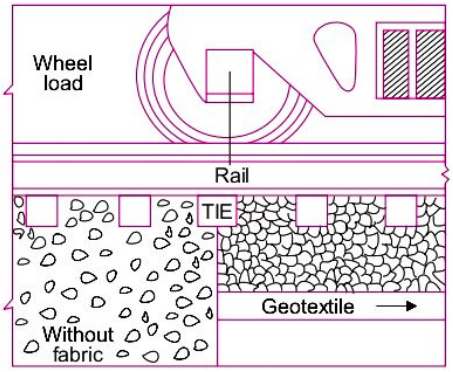

Fig. 34.13 Geotextile separates ballast and subgrade

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'tensile member' functions. Geotextile acting as reinforcement ensures better load distribution specially true for heavy weight and high modulus fabrics. In addition, it provides lateral restraint allowing the ballast to act as an elastic beam. The deformation under dynamic loads induces tension in the fabric and due to membrane action acts as reinforcement. Geotextile keeps the stress level low on the subgrade due to horizontal shear stresses mobilised by vertical loads. Fabric is placed under tension and it spreads the load in wider area and thereby decreases its intensity.

(*c*) ***Drainage function:*** Figure 34.15 illustrates the drainage action. Geotextiles provides surface and subsurface drainage by wicking action under train loads, where by water is squeezed and conveyed laterally out of track foundation. This action dries the subgrade, which in turn improve the shear parameters, thus enhancing the soil supporting strength.

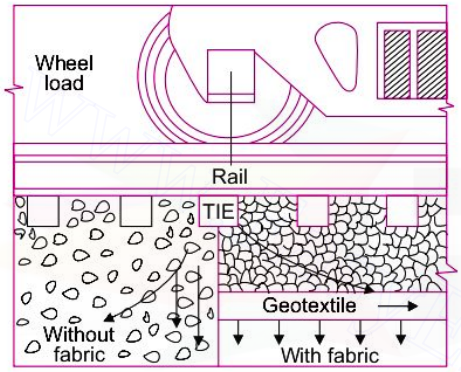

Fig. 34.14 Reinforcement action of geotextile

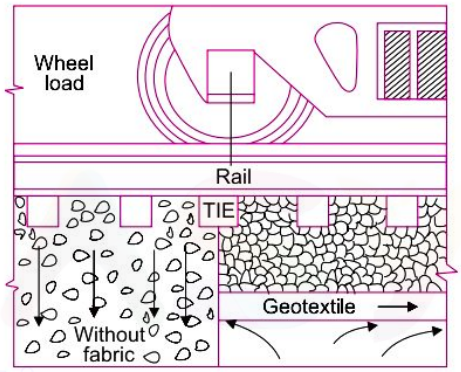

Fig. 34.15 Water drainage function

34.8 USE OF GEOTEXTILES IN EROSION CONTROL

We shall discuss here three typical uses:

(*a*) Soil slope retention.
(*b*) Slope protection under-liner.
(*c*) Grout filled mattresses.

(*a*) **Soil slope retention:** Geotextiles on a bare slope decreases water runoff velocity until seed germination can occur. Acting in place of mulch and tack coat, the geotextile has far better uniformity and tensile strength. One type has even biodegradable paper which can be designed for different life times inserted into its knit weave (Fig. 34.16).

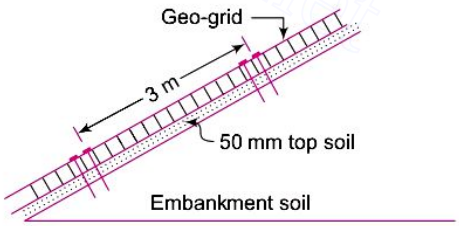

Fig. 34.16 Erosion control of slopes

(*b*) **Slope protection under - liner:** Stone rip rap, a common method of slope protection, should never be laid on a base slope. Geotextile laid on the soil under the rip rap can successfully substitute for expensive soil filters. See Fig. 34.9 (*a*) for illustration.

(*c*) **Grout filled mattresses:** A mattress of controlled thickness results from two geotextiles separated by a drop stitch and pumped full of cement grout. The shape follows the ground surface contours and when properly toed-in at the bottom and anchored at the top, provide excellent erosion protection. The technique has been used effectively under water and when in- corporating filter

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points into the mattress, it can relieve high pore water pressures which might be mobilised during the life time of a structure.

34.9 USE OF GEOTEXTILES IN BEARING CAPACITY IMPROVEMENT

Reinforced soil foundation bed is a soil foundation containing horizontally embedded geotextiles at several locations in the depth below the footing as shown in Fig. 34.17. Model studies by several investigators (*i.e.*, Binquest and Lee, 1975; Sridharan, 1988 etc.) have clearly indicated the advantage and possibilities for improvement in the bearing capacity and stiffness of the load-settlement behaviour by reinforcing the foundation soil. The reinforced soil technique / concept is essentially based on the mobilisation of the interfacial shearing resistance between the soil and reinforcement which in turn restrains the lateral deformation of the soil. Thus the effect of the reinforcement is to check the lateral flow of soil beneath the footing by introducing *lateral confinement.*

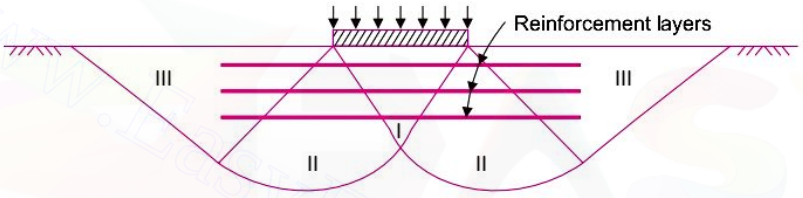

Fig. 34.17 Interaction of reinforcements with failure wedges

34.10 STORAGE, HANDLING AND PLACEMENT OF GEOTEXTILES

(*a*) Storage and Handling of geotextiles: Storage and handling geotextiles should be as per manufactures' recommendations. However, in general, geotextiles should be protected from (*i*) direct sun light, (*ii*) ultraviolet rays, (*iii*) water, (*iv*) debris. Any of these affect the strength, toughness and permeability of geotextiles. *Exposure* of the geotextiles to the elements between *lay down* and *covering* should be a maximum of 14 days. At the time of installation, geotextiles should be *rejected* and *replaced* if it is defective, ripped, flawed, deteriorated or damaged.

(*b*) Placement of geotextiles: The following points are important:

1. The ground surface that receives the geotextiles should be prepared to a relatively smooth condition, free of obstruction, depression and debris.
2. Geotextiles used along channels should be placed with the machine direction of the material parallel to the channel.
3. If successive sheets are required, they should be overlapped so that upstream sheet is placed over the downstream sheet.
4. If additional sheets are required to reach the top of the channel, the upslope sheet should overlap the downslope sheet.
5. Geotextiles used for 2 to 1 slopes or greater should be placed with the machine direction of the geotextile sheets perpendicular to the top of the slope.
6. The geotextile sheets should be over lapped in the direction of the anticipated movement of water. For example, on a fore slope the movement of water will be from the pavement. Therefore, geotextile sheets should start at the bottom of the slope and proceed upslope.

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(*c*) Overlapping and pinning:

1. Adjacent pieces of geotextiles may be joined by sewing or by overlapping, though in most cases overlapping in preferred. When geotextile sheets are *overlapped,* they must be pinned.
2. The minimum over lap should be 450 mm. When the geotextiles are placed under water, the minimum overlap should be 900 mm.
3. Overlaps should be secured by *pinning*. The securing pins should have the following specifications: (*i*) material : steel, (*ii*) Diameter : 3/16″ (5 mm), (*iii*) Length : 450 mm (*iv*) shape: pointed at one end and head at the other end.
4. Steel washer of outside diameter at least 1.5″ (40 mm) should be used along with pins.
5. Securing pins should be inserted through both the strip of overlapped textiles. The pins should be placed through the mid-point of the overlap.
6. The spacing interval of the pin depends upon the slope, as indicated below:

Slope (H:V)	*Spacing per row*
Steeper than 3:1	600 mm
3:1 to 4:1	900 mm
4:1 or flatter	1500 mm

7. Securing pins should be driven until washer bears against the geotextile. The geotextile should be secured firmly to the ground.
8. *Underdrains*: The overlap for geotextiles for under-drain is 300 mm. The upstream geotextile should always overlap the down stream geotextiles. As soon as geotextiles for the underdrains it placed, it needs to be backfilled with the aggregate for underdrains.

(*d*) General requirements during placement

1. The geotextiles should be placed such that placement of the *overlaying material* will not (*i*) excessively stretch the geotextile, (*ii*) tear the geotextile, (*iii*) pull overlap or seam apart.
2. Construction equipment should not be allowed on the exposed geotextile.
3. Placement of rip rap or stone should start at the base of the slope moving up ward and from the centre outward.
4. Rip rap should not be allowed to roll downslope.
5. The height drop for rip rap should be less than 0.3 m.

PROBLEMS

1. What do you understand by 'reinforced earth' ? Explain the mechanism of its action in resisting the active earth pressure.

2. Explain, with the help of a neat sketch, various elements of a reinforced earth wall, stating the specifications, requirements and functions of each element.

3. What do you understand by 'soil reinforcement' ? What are the types of soil reinforcements? State the advantages of reinforced earth structures.

4. Explain, with the help of neat diagrams, various application of soil reinforcement.

5. Derive expression for the following in a reinforced earth wall:
(*a*) Length of reinforcement (*b*) Sectional area of reinforcement.

6. Write a detailed note on geotextiles giving a list of various types of geotextiles.

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7. Explain, with neat sketches various functions of geotextiles.
8. Explain in a tabular form, functions, requirements and relevant properties of geotextiles.
9. Explain the use of geotextiles in road works
10. Explain the use of geotextiles in railways works.
11. Explain the use of geotextiles in earth dam construction.
12. Write notes on the following:
    (*a*) Use of geotextiles for erosion control.
    (*b*) Use of geotextiles for bearing capacity improvement.
    (*c*) Filtration and drainage function of geotextiles.
    (*d*) Placement of geotextile.

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References

1. Anon (1914). "Soil density apparatus uses rubber ballon" *Roads and streets,* 84(12), 60–3.
2. Akroyd T.N.W. (1957). *Laboratory testing in Soil Engineering.*
3. ASTM spec. Tech. Publ. 156 (July 1953). "Symposium on dynamic testing of soil."
4. Atterburg, A. (1911). "Uberdiephysikalische Bodenuntersuchung, und uber die Plasticzitat de Tonc." *Internationale* Mitteiluu gess fur Bodenkunde, Verlar fur Fachliteratur, Vol. 1, Berlin.
5. Bal Kishan H.A. and Nagraj C.N. (Oct. 1960). "A new method of predicting the natural frequency of foundations soil system." *The structural Engineer.*
6. Bannerjee A. and Gangopadhyay, S. (Dec. 1960). "Study of the stability of the well foundations for major bridges." Paper No. 226. *Journal of Indian Roads Congress,* No. 2.
7. Baron D.D. (1962). *Dynamics of Bases and Foundations,* McGraw Hill Book Co.
8. Barken R.A. (1948). "Consolidation of fine grained soils by drain wells". *Transactions of American Society of Civil Engineers,* Vol. 113.
9. Baver, L.D. (1956). "*Soil Physics*" Third Ed., John Wiley.
10. Bell, A.L. (1915). "The lateral pressure and resistance of clay and the supporting power of clay foundations." *Proc. Inst. Civil Engrs.,* Vol 199, London.
11. Biot (1935). "Distributed gravity and temperature loading in two dimensional elasticity replaced by boundary pressures and dislocations." *Journal Applied Mechanics,* Vol. 57 of Trane. Am. Soc. Mech. Engrs.
12. Bishop, A.W. and Eldin, G. (1950–50). "Undrained triaxial tests on saturated sands and their significance in the general theory of shear strength." *Geotechnique,* Vol. 2, London.
13. Bishop, A.W. (1954). "The use of pore pressure coefficient in practice." *Geotechnique,* Vol. 4, London.
14. Bishop, A.W. (1955). "The use of the slip circle in the stability analysis of slopes." *Geotechnique,* Vol. 5, No. 1, London.
15. Bishop, A.W. (1960–61). "The measurement of pore pressure in the triaxial tests." *Proc. Conf. Pore pressure and suction in soils,* published by Butterworths, London.
16. Bishop, A.W. and Bjerrum, L. (1960). "The relevance of the triaxial test to the solution of stability problems." *Proc. Res. Conf. shear strength of cohesive soil.* ASCE, Colorado.
17. Bishop, A.W. and Morgenstern, N. (Dec. 1960). "Stability coefficients for earth slopes." *Geotechnique* Vol. 10, No. 4, London.
18. Bowels J.E. (1961), *Foundation Analysis and Design,* McGraw Hill Inc., New York.
19. Bradbury, R.D. (1938). *Reinforced concrete pavements.* Wire Reinforcement Institute. Washington D.C.
20. Brooker, E.W. and Ireland H.O. (Feb 1965). "Earth pressures at rest related to Stress History." *Canadian Geotechnical Journal,* Vol. No. 1.
21. Burmister, D.M. (1943). "The theory of stresses and displacements in layered soil system." Journal of Applied Physics, Vol. 16.
22. Burmister, D.M. (1943). "The general theory of stresses and displacements in layered systems and application to the design of airport runways." *Proceeding HRB.*
23. Burmister, D.M. (1958). "Evaluation of pavement system of WASHO Road test by layered system methods." *Highway Research Board Bulletin,* 177.
24. Capper, P. L. Cassie, W.F. and Geddes J.D. (1966). *Problems in Engineering Soil,* E. and F.N. Spon Ltd. London.

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25. Casagrande, L (1932). "Naeherungsmethoden Zur Bestimmung Von Art and Menge der Sickerung durch geschuette Daemme," Thesis technische Hochschule, Veinna. Translated by U.S. Crops, of Engineers. Waterways Exp. Station. Vicksburg. Miss.
26. Casagrande, A. and Fadum, R.P. (1939–40). "Notes on Soil testing for engineering purposes." *Soil Mech., series No.* 8. publication from Graduates School of Engineering, No. 268, Hardward University, Cambridge, Mass.
27. Casagrande, A. (1940). "Seepage through dams" in *Contribution to Soil Mechanics 1925–1940,* Boston Society of Civil Engrs., Boston.
28. Chellis, R.D. (1961), *Pile Foundations* McGraw Hill Book Co. Inc., New York.
29. Cilibbron and Beresford (1902). *Expt. on the passage of water through Sand,* Government of India Central Printing Office.
30. *Code of practices for Structural Safety of Building Foundations (1961).* Indian Standard, I.S., 1904–1961, New Delhi.
31. *Concrete Pavement Design of Roads and Streets carrying all classes of Traffic (1951).* Portland Cement Association, Chicago Illinois.
32. Crossland, B. (1965). "The effect of fluid pressure on the Shear properties of Metals." *Proc. Inst. Mech. Engrs.,* Vol. 169, London.
33. Davidson, D.T. katti, R.L. and Watch, D.E. (1958). "Use of flyash with Portland Cement or Satiblisation of soil" *HRB Bulletin,* No. 198.
34. *Design of small Dams* (1960). U. S. Dept. of Interior, Bureau of Reclamation, Washington.
35. Docker, C.E. and MeFecters, J.N. (1948). "Principles of concrete runway design. "*Pro: Sec. Intern, Conf. Soil Mech. Found Engg.,* Vol. 2, Rotterdam.
36. Drucker, C.D. and Prage, W. (1952). "Soil mechanics and plastic analysis or limit design." *Quarterly of Applied Mathematics,* Vol. 10.
37. *Earth Manual* (1960). U.S. Bureau of Reclamation. Denver, Colorado.
38. Egorov, K.E. (1953). "Concerning the questions of calculations for base under foundations with footings in the form of rings. *Mekhanika Gruntov, Sb. Vr.* No. 44, Grosstroiizdat, Moscow.
39. Francher, G.H. Lewis, J.A. and Barnes. K.B. (1933). "Some physical characteristics of Oil Sand " *Mineral Industries Experiment Station,* Bulleting, 12, Pennsylvania Stae College U.S.A.
40. Fellenius W. (1940). *Erdstatische Berechnungen mit Reibung und Kohasion und unter Annahme Keissy Undrischer Gleitf lachen* (Earl Static Calculations with Friction and Cohesion and use of circular Slidiery surfaces). Wilhelm Ernst und Sohn, Berlin, 1927, II, Ed. 1940.
41. Fenske C.W. (June 1951). "Influences charts for vertical stress distribution by Westergaard's Equation *Bureau of Engs. Ries. Circular,* No. 21, University of Texas, Austin (U.S.A.).
42. Florin, V.A. (1959, 61). *Fundamentals of Soil Mechanics* Vol. 1 and 2, Gosstroiizdat Moscrow.
43. Forchheimer P. (1930). *Hydraulics,* 3rd Ed., Leipzig, Teubner.
44. Foster C.R. and Ahlvin, R.G. (1954). "Stresses and deflections induced by a uniform circular load" *Proceedings. HRB.*
45. Gibbs, H.J. and Holtz, W.G. (1957). "Research on determining the density of sands spoon penetration testing". *Proceedings Fouth International Conference Soil Mechanics and Foundation Engineering.* Vol. 1 London.
46. Gilboy, G. (May, 1953). "Influence tables for solution of Boussinesq Equation" in *Earth and Foundations,* Progress Report of Special Committee of ASCE, ***Proceedings** ASCE,* Vol. 59, No. 5.
47. Goodman L.J. and Karol K.H. (1968). *Theory of Practice of Foundation Engineering*, McMillan Co., New York.
48. Habib M.P. (1953). "Influence of the variation of Intermediate principal stress on the shearing strength of soil." *Proc. Third Int. Conf. Soil Mech. Found Engs. Vol. 1, Zurich.*

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49. Haigh F.F. (1935). "Design of weirs on sand foundations" *Punjab, Engineering congress paper No. 182.*
50. Harr. M.E. (1962). *Ground Water and Seepage,* McGraw Hill Inc.
51. Harr. M.E. (1966). *Foundations of Theoretical Soil Mechanics,* McGraw Hill Book Co.
52. Haythornthwaite, R.M. (1960). Mechanics of the triaxial tests for soils, *Prob. Am. Soc. Civil Engrs.,* SMF, Div., Vol. 80, No. SM. 5.
53. Hazen A. (1982). "Some physical properties of sands and gravels with special reference to their use in filtration" *24th Annual Report Massachusetts State Board of Health.*
54. Highway Research Board (Sept. 1964). *Current Road Problems No. 12,* Washington, D.C.
55. Hiley, A. (July-Aug. 1930). "Pile driving calculation with notes on driving force and ground resistance".
56. Hilf, J.W. (July, 1948). "Estimating construction pore pressure in Rolled Earth Dams" *Proc. 2nd Intern. Conf. Soil Mech. Found., Engg. 3,* Rotterdam.
57. HMSO (1952). *Soil Mechanics for Road Engineers.*
58. HRB (1943). "Use of soil cement mixtures for base courses" *Highway Res. Bd. Wartime Problems. No. 7,* Washington, D.C.
59. Hvorslev, M.J. (June, 1960). "Physical components of the shear strength of saturated clays" *Proc. Res. Conf. on Shear Strength of Cohesive Soils* ASCE University of Colorado, Boulder, Colorado.
60. IS : 1888 (1962). *Method of load test on soils,* Indian Standard Institution, New Delhi.
61. IS : 2131 (1963). *Method for standard penetration test for soils.*
62. IS : 2720 Part VI (1964). *Determination of shrinkage factors.*
63. IS : 2720 Part XX (1964). *Determination of linear shrinkage.*
64. IS : 2974 Part I (1964). Foundations for reciprocating type machines.
65. IS : 2974 Part II (1964). Foundations for impact type machines.
66. IS : 2720 Part IV (1965). *Method of Test for Soil : Grain Size Analysis.*
67. IS : 2720 Part VII (1965). *Determination of moisture content dry density relation using light compaction.*
68. IS : 2720 Part VIII (1965). *Determination of moisture content dry density relation using heavy compaction.*
69. IS : 2809 (1964). *Glossary of terms and symbols relating to soil mechanics.*
70. IS : 2720 Part XVI (1965). *Laboratory determination of CBR.*
71. IS : 2720 Part V (1965). *Determination of liquid and plastic limits.*
72. IS : 2720 Part XXVIII (1966). *Determination of dry density of soils, in place, by sand replacement method.*
73. Iterson, F.K. Th. Van (1961 and 1917). "Eenige Theoretiscne Beschonuingen over Kwel" *De Ingenieur.*
74. Janbu, N. and Kjaernsli, B. (1966). *Norwegian Geotech. Inst. Pub. 16, Oslo.*
75. Jumikis, A.R. (1964). *Mechanics of Soils,* D. Van Nostrand.
76. Jumikis, A.R. (1969). *Theoretical Soil Mechanics,* Van Nostrand Reinhold Company.
77. Jurgenson, L. (1984). "The application of theories of elasticity and plasticity to foundation problem" *Jour. Boston Soc. Civil Engrs.,* Vol. 21. See also : *Contribution to Soil Mechanics, 1925 to 1940,* Boston Soc. Civil Engrs. 1940.
78. Kelly E.F. (1939). "Application of the results of Research to the structural design to concrete pavements," *Public Roads* Vol. 20. '
79. Khosla, A.N., Bose, N.K. and Taylor, E.M. (1936). "Design of weirs on permeable foundation". *Publication No. 12.* Central Board of Irrigation, India.
80. Kirkpatrik, W.M. (1957). "The condition of failure for sands". *Proceedings, Fourth International Conference on Soil Mechanics and Foundation,* Engineering, Vol. 1, London.
81. Kozeny, J.S. (1927). "Uber Kapillare Leuitung des Wassers in Boden" *Ber. Wein Akad.*

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82. Kozney, J.S. (1931). "Ground wasserbewegung bei freim spiegel, Fluss and Kanalversicheringi, *Wasserkraft and Wasserwitrschaft, No. 3.*
83. Lambe, T.W. (1951). *Soil Testing,* John Wiley & Sons.
84. Lambe, T.W. (1953). "The structure of Inorgranic Soil" *Proceeding ASCE, Vol. 79, separate* No. 315.
85. Lambe, T.W. (1960). "The structure of compacted clays". *Journal Soil Mech. Found. Div. Vol 84, No. SM 2, Proc. Paper 1964;* Also *Transaction, ASCE,* Paper No. 3041, Vol. 125, Part I
86. Lambe, T.W. (1962). "Soil Stabilisation", *Foundation Engineering,* chap. 4, Ed G.A. Leonards, McGrav Hill, New York.
87. Lambe, T.W. (No. 1967). "Stress part method" *Proceedings* ASCE.
88. Lambe, T.W. and R.V. Whitman (1969). *Soil Mechanics,* John Wiley & Sons, Inc.
89. Lane, E.W. (Oct. 1935). "Security of masonry dams on earth foundations" *Transactions* ASCE.
90. Leliavsky, S. (1965), *Design of dams for percolation and erosion* Chapman and Hall.
91. Leonards G.A. (1962). *Foundation Engineering,* McGraw Hill Book Co.
92. Major, A. (1962), *Vibration Analysis and Design of Foundations for Machines and Turbines.* Collet's Holdings Ltd., London.
93. Matlock, H. and Reese, L.C. (Oct. 1960). "Generalised solutions for laterally loaded piles". *Jour Soil Mech. and Found. Division,* ASCE, Vol. 86.
94. Means, R.E. and Parcher, J.N. (1963). *Physical Properties of Soils.* Charles E. Merrill Book Co. Columbus, Ohio.
95. Mehra, S.R. and Natrajan, T.K. *Practical lesson in vertical sand drains installations.* Central Road Research Paper No. 54.
96. Meyerhof, G.G. (Dec. 1951). "The ultimate bearing capacity of Foundation." *Geotechnique,* vol. 2, No. 4, London.
97. Meyerhof. G.G. (1953). "The bearing capacity of Foundation under eccentric and inclined loads" . *Proc. Thrid Intern. Conf. Soil Mech. Found. Engg.,* Vol. 1, Zurich.
98. Meyerhof, G.G. (Jan. 1956). "Penetration tests and bearing capacity of cohensionless soils". ***Jour** Soil Mech. and Found. Div.* ASCE, Vol. 82.
99. Milan, E. (1919). "Die Druckverteilung durch eine elastische Schicht". *Beton and Eisen, H. 7*/8.
100. Mindlin, R.D. (1939). "Stress distribution around a tunnel", *Trans. Am. Soc. Civil Engers.,* Vol. 104.
101. Mohan, D.K. and Goel, R.K. (1959). "Correspondence", *Geotechnique,* Vol. 9, No. 3, London.
102. Mohr, O. (1900). "Die Elastizitatsgrenze and Bruch eines materials". (The elastic limit and refailure of a material). *Zeitschirift Veneins Deuesche Ingnieure,* Vol. 4.
103. Mohr, O. (1914). *Abhandlungen aus dem Gebeite der Technischess Mechanik* (Dissertations from the field of applied mechanics), II Edition, Wilhelm Ernst und Sohn, Berlin.
104. Muskat, M. (1946). *The Flow of Homogeneous Fluids through Porous* Media, McGraw Hill Co., 1937 ; J.W. Edward, 1946.
105. Nadai, A (1950). *Theory of Flow and Fracture of Solids* Vol. I, McGraw Hill Book Co.
106. Newmark, N.M. (1935). "Simplified computation of vertical pressures in elastic foundation". *Circular No. 24,* Engg. Expt. stations, Univ. of Illinois.
107. Newmark, N.M. (1942). "Influence charts for computation of stresses in Elastic Foundations". *Bulletin No, 338,* Engg. Expt. Station, Univ. of Illinois, Illinois.
108. Oza, H.P. (1969). *Soil Mechanics and Foundation Engineering* Charotar Book Stall, Anand.
109. Pauw, A. (1953). "A dynamic analogy for foundation-soil system". *ASTM Special Publication* No. 156.
110. Pavlovsky, N.N. (1922). *Theory of ground water beneath hydrotechnical structure.* (In Russian Petesburg.

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111. Pavlovsky, N.N . (1933), " Motion of water under dam". *First Conference on Large Dams,* Stockholm.
112. Pickett, G. and Ray, G.K. (1951). "Influence charts for rigid pavement." *Transactions ASCE.*
113. Portland Cement Association (1951). *Concrete pavement design for Roads and Streets carrying all classes of traffic,* PCA. Chicago, Illinois.
114. Portland Cement Association (1955). *Design of concrete air field pavement,* PCA. Chicago, Illinois.
115. Proctor, R.R. (Aug.-Sept. 1933). "The design and construction of rolled earth dams". *Engg. News Records,* Vol. III.
116. Punmia, B.C. (Feb. 1964). "Estimation of construction pore pressure by Laboratory test". *Fourth Annual No. of Inst, of Engrs.* Rajasthan Centre.
117. Punmia, B.C. and Pande Brij Basi Lai (1969). *Irrigation and Water Power Engineering,* Standard Publishers Distributors Delhi.
118. Punmia, B.C. and Singh A. (Jan. 1970). "Failure envelope of soils" *Journal of Inst, of Engrs., India. Rajasthan Centre, Golden Jubilee Number.*
119. Rankine, W.J.M. (1857). "On the stability of loose earth" *Philosophical Trans. Royal Soc.,* Vol. 147, London.
120. Reese, L.C. and Matlock, H. (1956). "Non-dimensional solutions for laterally loaded piles with soil modulus assumed proportional to depth" *Proceeding Eighth Texas Conf. on Soil Mech. and Found. Engg.,* Univ of Taxes, Austin.
121. Resal, J. (1910). *La Poussee des Terres, Deu xieme Portie Theorie des Terres coherantes,* Paris, Beranger.
122. Resenquist, I.T. (1959). "Physico-chemical properties of soils : Soil-water System", *Jour. Soil. Mech. Found. Div.* Vol. 85, No. SM-2 Part 1, Proc. Paper 2000, ASCE.
123. Reynolds, O. (1833). "An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous and the law of resistance in parallel channel" *Trans. Royal Soc.* Vol. 175, London.
124. Richart F.E. (1955). "Review of the theories for sand drains" *Transactions of ASCE,* Vol. 124.
125. Schaffernak, (1917). Uberidie stansicherheit durchlaes, siger ges chuetterted, Damma *Allgem, Bauzeitung.*
126. Scheidegger, A.E. (1957). *The physics of flow through Porous Media,* The MacMillan Co., New York.
127. Schofield, A. and Worth, P (1968) *Critical State Soil Mechanics.* McGraw Hill, London.
128. Schofield, R.K. (1953). "The pF of the water in Soil". *Trans. Third Intern. Congr. Soil Science,* Vol. 2, Oxford, England.
129. Scott, R.F. (1915). *Principles of Soil Mechanics.* Addison Wesley Publishing Co.
130. Scott, R.F. and Schoustra, J.J. (1968). *Soil Mechanics and Engineering,* McGraw Hill Book Co.
131. Sehgal, S. B. (1967). *Soil Mechanics, Metropolitan Book Co., Delhi.*
132. Skempton, A.W. (1951). "The bearing capacity of clays". *Building Research Congress,* Div. I, London.
133. Skempton, A.W. (Dec. 1954). "The pore pressure co-efficients A and B". *Geotechnique,* Vol. 4, No. 4, London.
134. Skempton, A.W. and Bishop, A.W. (1950). "The measurement of shear strength of soil". *Geotechnique,* Vol. 9, London.
135. Skempton. A.W. and Bjerrun, L. (1957). "A contribution to the settlement analysis of foundation of clay". *Geotechnique,* Vol. 4, 1960, London.
136. Singh, A. (April, 1962). "A rectangular plot method for slope stability analysis". *Jour. Indian National Soc. Soil Mech. Found. Engg.,* Vol. 4, No 2, New Delhi.
137. Singh, A (1965). "Instrumentation for going metric in soil engineering testing". *Jour. Indian National Sec. Soil Mech. Found. Engg.,* Vol. 4 No. 2, New Delhi.
138. Singh, A. (1967). *Soil Engineering in Theory and Practice,* Asia Publishing House, Bombay.

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139. Singh, A. and Punmia, B.C. (1962). "Critical slope stability of an earthen dam". *2nd Annual Number Inst, of Engrs. (India),* Rajasthan Centre, Jaipur.
140. Singh, A. and Punmia, B.C. (April-June, 1962). "Semiflexible pavements of soil cement". *Cement and Concrete,* Vol. 3, No. 1, New Delhi.
141. Singh, A. and Punmia, B.C. (April 1963). "Some experiments with Jodhpur Mini-compactor." *Roads and Road Construction,* Vol. 41, No. 484, London.
142. Singh, A. and Punmia B.C. (Feb. 1965). " A new laboratory compaction device and its comparison with the proctor test." *Highway Research News,* No. 17, HRB, Washington.
143. Singhal, R.P. (1967). *Soil Mechanics and Foundation Engineering,* Singhal Publications, India.
144. Sowers, G. B. and Sowers, G.F. (1961) *Introductory Soil Mechanics and Foundations,* The MacMillan Co., New York.
145. Spangler, M.G. (1963). *Soil Engineering, International Textbook Co. Scranton, Pennsylvannia.*
146. Steele, D.J. (1945). "Application of the classifications and group index in estimating desirable sub-base and total pavement thickness (Discussion)". *Proc. Highway Res. Bd.,* Vol. 25, Washington.
147. Teller, L.W. and Sutherland, E.C. (1943). "The structural design of concrete pavements, parts. An experimental study of the Westergaard analysis of stress condition in concrete pavement slabs of uniform thickness." *Public Roads,* Vol. 23, No. 8. Washington.
148. Teng, W.C. (1962). *Foundation Design,* Prentice Hall, International.
149. Terzaghi K. (1943). "Die Berechung der Durchlaessigkeitsziffer des Toneseaus dem Varlauf der hydrodynamischen spannugser schinungeen." *Sitzungsberichte de Akadennie der wissehsahaften Abt.,* II a, Vol. 132, Vienna.
150. Terzaghi, K. (1943). *Theoretical Soil Mechanics,* Chapman and Hall, London and John Wiley & Sons.
151. Terzaghi, K. and Peck, R.B. (1967). *Soil Mechanics in Engineering Practice,* John Wiley & Sons.
152. *Unified soil classification System for Roads, Airfields, Embankments and Foundations,* (1962). Military Standard ; MIL- STD-619 A, U.S. Deptt. of Defence, Washington, 25, D.C.
153. Uppal H.L. and Agarwal, H.R. (1958). "A new method of determining liquid limit of soils." *Road Research Paper No. 19* Central Road Research Institute, Delhi.
154. Uppal, H.L. and Bhalla, B.D. (1965). "A resume on consolidation of sand." *Journal, Indian Roads congress,* Vol. 29 No. 2, New Delhi.
155. Wagner, A.A. (1957). "The use of the Unified Soil Classification System by the Bureau of Reclamation." *Proc. Fourth Intern. Conf. Soil Mech. Found. Engs.,* Vol. 1, London.
156. *Well Foundations for Road Bridges, (1961).* Bridge Series, *No. 1, PWD, Uttar Pradesh, Lucknov.*
157. Westergaard, H.M. (1926). "Stressess in concrete pavements computed by theoretical analysis." *Public Road,* Vol. 7, No. 2, Washington.
158. Westergaard, H.M. (1927). "Theory of Concrete pavement design." *Proceedings, Highway Research Board,* Washington.
159. Westergaard, H.M. (1933). "Analytical tools for judging results of structural tests of concrete pavements." *Public Roads,* Vol. 14, No. 10, Washington.
160. Westergaard, H.M. (1940). "Plastic state of stress around a deep well". *J. Boston Soc. Civil. Engs.,* Vol. 27.
161. Wu, T.H., Loh, A.K. and Malvern. M.E. (Feb. 1963). "Study of failure envelope of Soils". *Journal Soil Mech. Foundations Div.,* ASCE.
162. Yoder, E.J. (1959). Principles of Pavement Design, John Wiley & Sons Inc., New York.

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