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Lateral Earth Pressure

Definition:

The word "lateral" means sideway. The lateral earth pressure means pressure to the side or sideway pressure.

Different structures such as retaining wall often subjected to lateral pressure. To design such structures we need to know the lateral force exerted on the structures. Due to this, shear failure and moment failure will occur.

Necessity of lateral Earth pressure Analysis:

1. To design Retaining wall.
2. To design sheet piles.
3. To design abutments.

Factors on which lateral earth pressure depends:-

1. Type and amount of movement of the wall.
2. Shear strength parameters of soil (c & φ)
3. Unit weight of soil.
4. Drainage condition of the backfill.

Types of lateral earth pressure:-

1. Earth pressure at rest.
2. Active earth pressure.
3. Passive earth pressure.

Lateral Earth pressure at Rest:

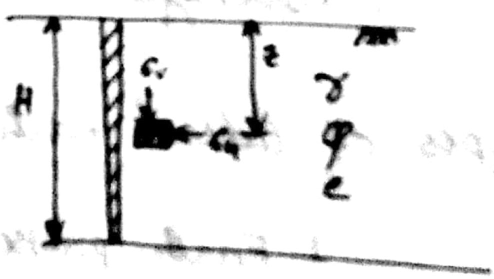
If the wall is restained from moving, the lateral earth pressure on the wall at this condition is known as lateral earth pressure at rest.

Back fill:

The material or soil retained or supported by the retaining wall is called back fill. It may have its top surface horizontal or inclined.

Co-efficient of earth pressure:-

The co-efficient of earth pressure is defined as the ratio of the horizontal stress to vertical stress is called the co-efficient of earth pressure. It is denoted by K.



Now, co-efficient of earth pressure, $K_o = \frac{\sigma_h}{\sigma_v}$

σ_v = vertical earth pressure

σ_h = lateral earth pressure

K_o = Co-efficient of earth pressure at rest.

In case of surcharge, q;

$$K_o = \frac{\sigma_h}{\sigma_v + q}$$

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Co-efficient of earth pressure at Rest (K_0):

1. For normally consolidated coarse grained soil (sand) (Jaky, 1944);

$$K_0 = 1 - \sin\varphi'$$

2. For normally consolidated fine grained soil (clay) (Massarch, 1979),

$$K_0 = 0.44 + 0.42\left[\frac{PI(\%)}{100}\right].$$

3. For normally consolidated clay, (Booker & Ireland, 1965)

$$K_0 = 0.95 - \sin\varphi'$$

4. For over consolidated clays;

$$K_{0(OC)} = K_{0(NC)}\sqrt{OCR}$$

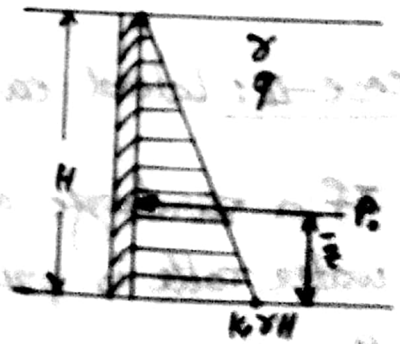
[OCR = Over Consolidation Ratio]

Question: Derive an equation for determining the magnitude of lateral earth pressure at rest condition.

Solution:

case-1: without surcharge.

Let us consider a vertical wall of height H. unit weight of soil is γ. At a depth $z = H$, vertical pressure,



$$\sigma_v = \gamma H$$

Now, co-efficient of earth pressure at rest,

$$K_0 = \frac{\sigma_h}{\sigma_v}$$

$\Rightarrow \sigma_h = K_o \sigma_v$

$\Rightarrow \sigma_h = K_o \gamma H$

$[\sigma_h = P_o = \text{Lateral earth pressure}]$

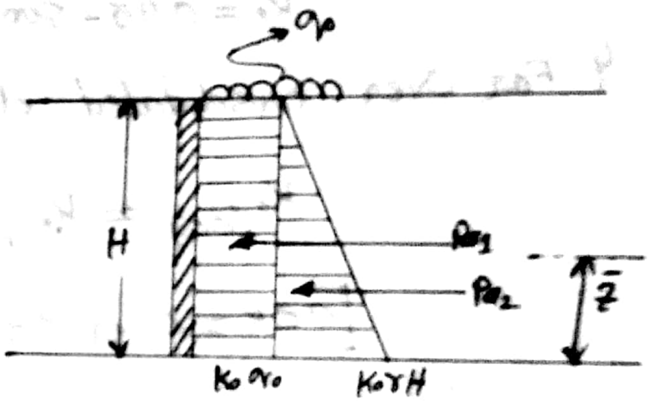
$\therefore P_o = K_o \gamma H \cdot H/2$

$P_o = \frac{1}{2} K_o \gamma H^2$.

Location, $z = H/3$ From bottom.

Case-II: With surcharge (q).

If a surcharge, q is applied then the pressure diagram is given below.



Total pressure, $P_{oT} = P_{o1} + P_{o2}$

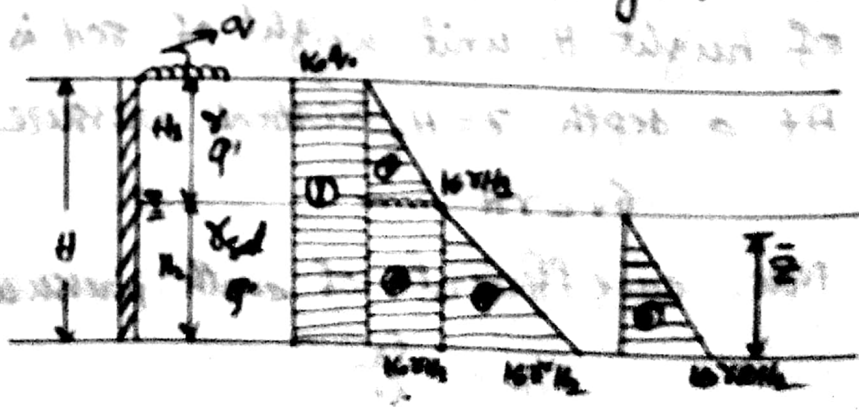
$P_{oT} = K_o q_o H + \frac{1}{2} K_o \gamma H \cdot H$

$P_{oT} = K_o q_o H + \frac{1}{2} K_o \gamma H^2$

Location, $\bar{z} = \dfrac{K_o q_o H \cdot \frac{H}{2} + \frac{1}{2} K_o \gamma H^2 \cdot H/3}{P_{o1} + P_{o2}}$.

case-III: Lateral earth pressure at rest with surcharge & water

If a surcharge and water Table is present then the pressure diagram is as follows.



where, $\gamma' = \gamma_{sat} - \gamma_w$

So, $P_{oT} = P_{o1} + P_{o2} + P_{o3} + P_{o4} + P_{o5}$

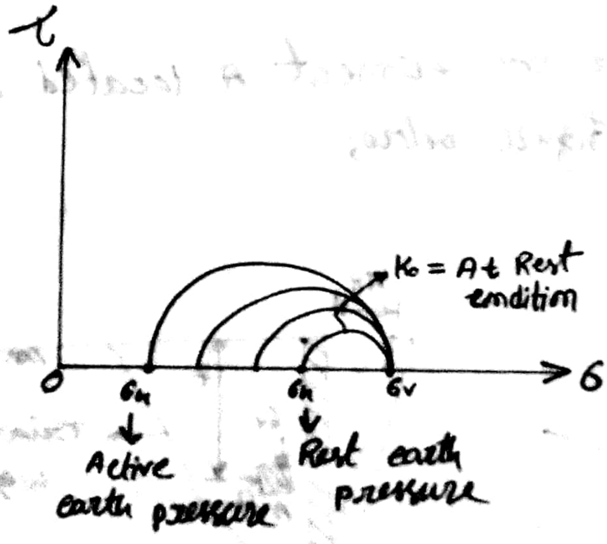
$$\therefore P_{oT} = K_0 q_0 H + \frac{1}{2} K_0 \gamma H_1^2 + K_0 \gamma H_1 H_2 + \frac{1}{2} K_0 \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

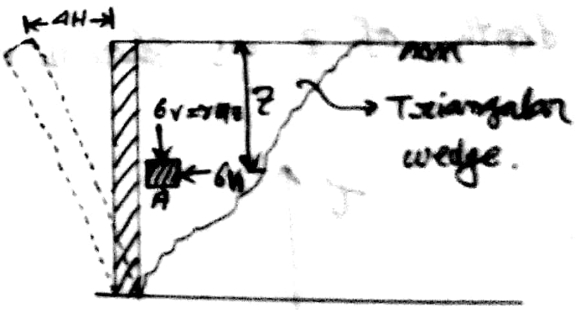
Location, $$\bar{z} = \frac{\left(P_{o1} \times \frac{H}{2}\right) + \left(P_{o2} \times \left(H_2 + \frac{H_1}{3}\right)\right) + \left(P_{o3} \times \frac{H_2}{2}\right) + \left(P_{o4} \times \frac{H_2}{3}\right) + \left(P_{o5} \times \frac{H_2}{3}\right)}{P_{o1} + P_{o2} + P_{o3} + P_{o4} + P_{o5}}$$

[NB: Taking moment about Bottom surface] $\bar{z}$ will obtained.

Active Earth pressure:

If the wall moves away or tilt away from the Soil retained, a triangular soil wedge behind the wall is failled. The lateral earth pressure at this condition is known as active earth pressure.





[Lateral Earth pressure at Rest condition এর Horizontal stress এর চেয়ে Active earth pressure এর Horizontal stress কম হয় কারণ Soil move করার কারণে Resisting force কমে যায়।]

Rankine's (1857) Active earth pressure:

Assumptions —

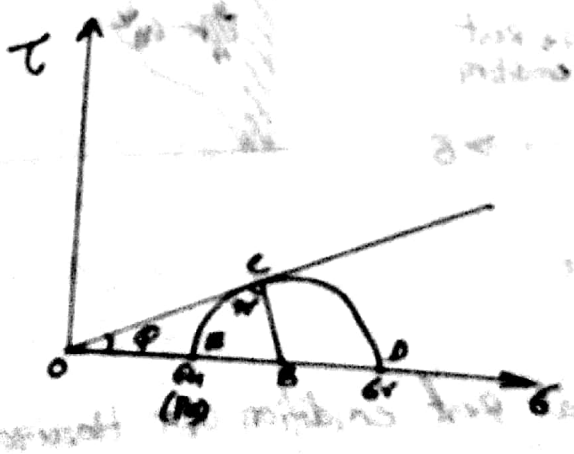
1. The soil mass is homogeneous and semi-infinite.
2. The soil mass is dry and cohesionless.
3. The ground surface is plane.
4. The back of the retaining wall is smooth and vertical.
5. The soil element is in a state of plastic equilibrium.

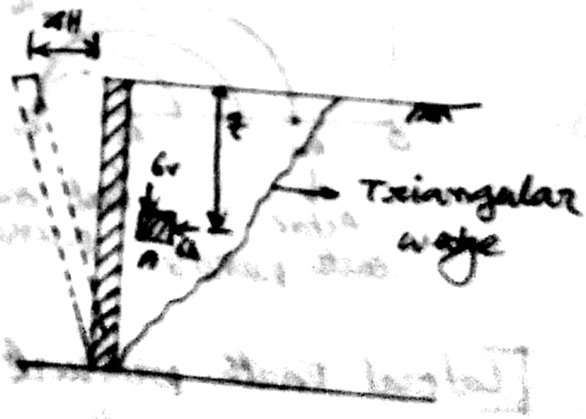
[NB: কোন soil এর সব বিভিন্ন জায়গায় element নিলে যদি সব গুলো element এর property same হয় তাহলেই ঐ soil কে Homogeneous soil বলা হয়।]

Question: Derive an expression for determining active earth pressure. or prove, $P_a = K_a \gamma z$.

Answer:

Let us consider a soil element A located at a depth of z as shown in figure below;





point E represent the active condition.

$P_a = OE = OB - BE$

$= OB - BC$ [$BE = BC =$ Radius]

$= OB - OB \sin\varphi$ $\quad$ [$\triangle OBC$; $\sin\varphi = \frac{BC}{OB}$]

$\therefore P_a = OB(1-\sin\varphi)$ —— ①

Again, $\sigma_v = OD = OB + BD$

$= OB + BC$ [$BC = BD =$ Radius]

$= OB + OB\sin\varphi$

$\therefore \sigma_v = OB(1+\sin\varphi)$ —— ②

From, ① ÷ ②; we get:

$$\frac{P_a}{\sigma_v} = \frac{OB(1-\sin\varphi)}{OB(1+\sin\varphi)}$$

$$\Rightarrow \frac{P_a}{\sigma_v} = \frac{1-\sin\varphi}{1+\sin\varphi}$$

$$\Rightarrow \frac{P_a}{\sigma_v} = K_a \quad \text{—— ③}$$

$\therefore K_a = \frac{1-\sin\varphi}{1+\sin\varphi}$ = co-efficient of active earth pressure

K_a = co-efficient of Rankine's active earth pressure.

From eq^n ③; $\frac{P_a}{\sigma_v} = K_a$

$\Rightarrow P_a = \sigma_v K_a$

$$\boxed{\therefore P_a = K_a \gamma z}$$

(proved)

Let, $\varphi = 30^\circ$

$K_o = 1 - \sin 30^\circ$

$K_o = 0.5$

$K_a = \frac{1-\sin 30^\circ}{1+\sin 30^\circ}$

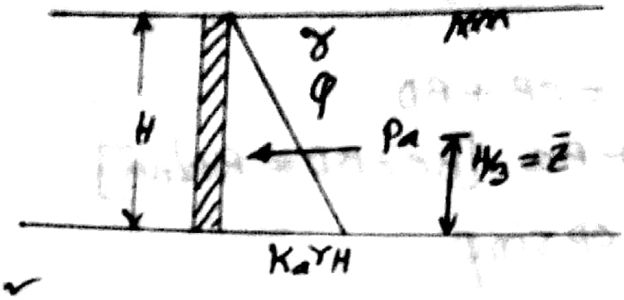
$K_a = 0.33$

$\therefore K_o > K_a$

(proved)

Several cases for pressure distribution diagram:-

⇒ Case-I: Dry Backfill



$\therefore P_a = \frac{1}{2} K_a \gamma H^2$

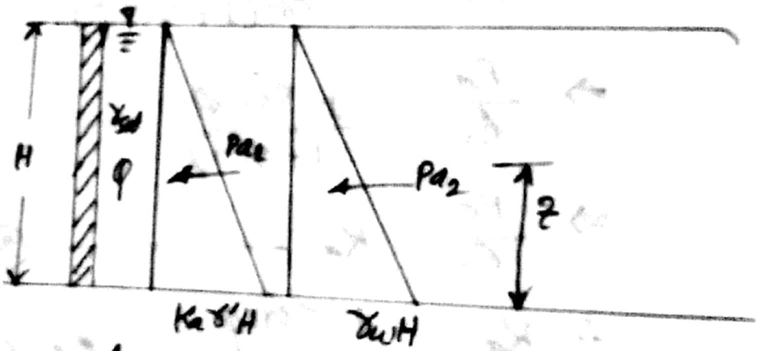
$\bar{z} = H/3$

⇒ Case-II: Submerged Backfill

$P_{aT} = P_{a1} + P_{a2}$

$P_{aT} = \frac{1}{2} K_a \gamma' H^2 + \frac{1}{2} \gamma_w H^2$

$\bar{z} = \dfrac{(P_{a1} \times H/3) + (P_{a2} \times H/3)}{P_{a1} + P_{a2}}$



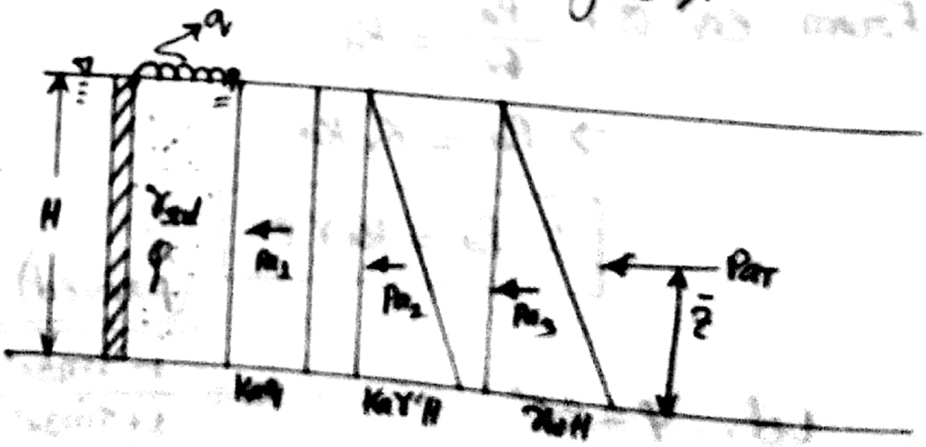
$(\gamma' = \gamma_{sat} - \gamma_w)$

⇒ Case-III: Submerged Backfill with surcharge (q)

$P_{aT} = P_{a1} + P_{a2} + P_{a3}$

$P_{aT} = K_a q H + \frac{1}{2} K_a \gamma' H^2 + \frac{1}{2} \gamma_w H^2$

$\bar{z} = \dfrac{(P_{a1} \times \frac{H}{2}) + (P_{a2} \times H/3)(P_{a3} \times H/3)}{P_{a1} + P_{a2} + P_{a3}}$



$(\gamma' = \gamma_{sat} - \gamma_w)$

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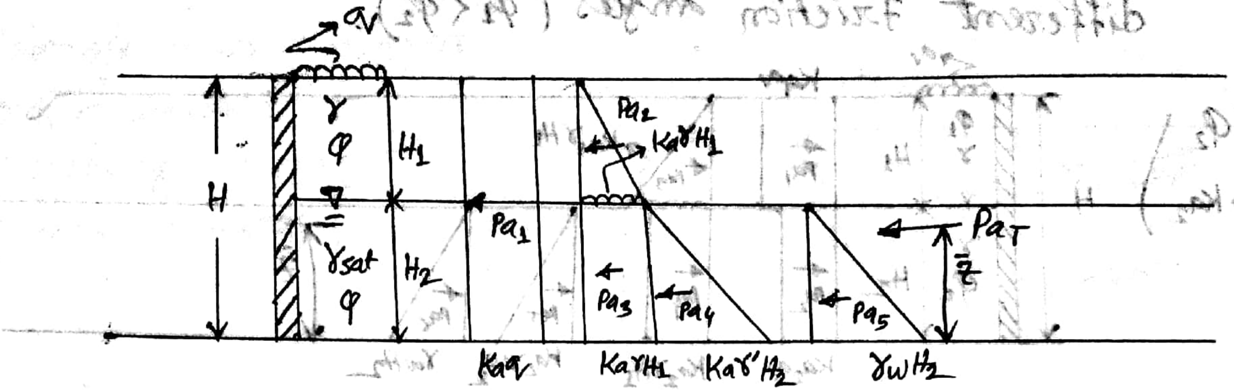
⇒ Case-IV: Partially submerged Backfill with surcharge



$\gamma' = \gamma_{sat} - \gamma_w$

$$Pa_T = Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5$$

$$Pa_T = K_a q H + \frac{1}{2} K_a \gamma H_1^2 + K_a \gamma H_1 H_2 + \frac{1}{2} K_a \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

$$\bar{z} = \frac{(Pa_1 \times H/2) + (Pa_2 \times H_1/3) + (Pa_3 \times \frac{H_2}{2}) + (Pa_4 \times H_2/3) + (Pa_5 \times H_2/3)}{Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5}$$

⇒ Case-V: Partially submerged Backfill with surcharge for different friction angles ($\varphi_1 > \varphi_2$).

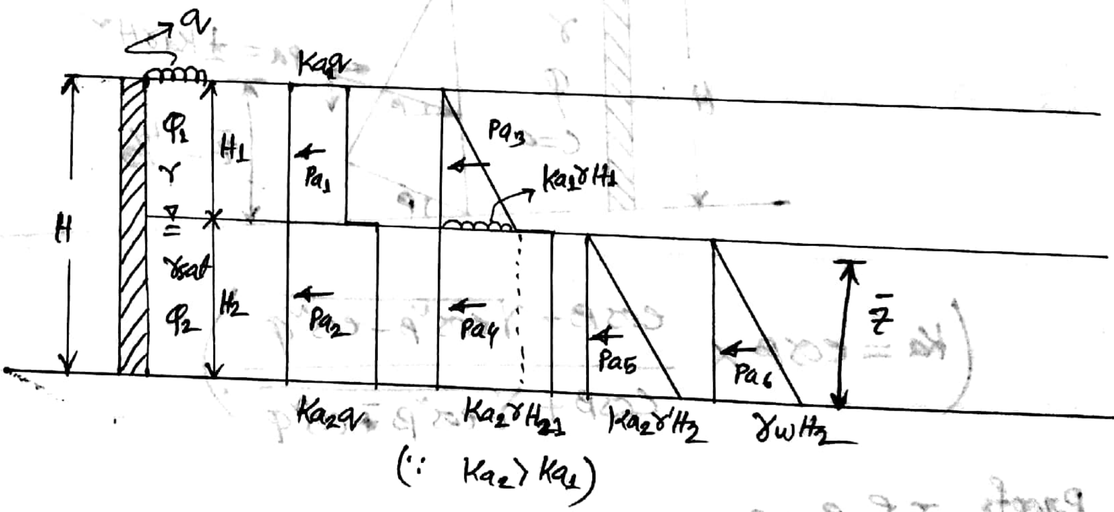
$\varphi_1 = 35°$

$\varphi_2 = 30°$

$$Ka_1 = \frac{1 - \sin 35°}{1 + \sin 35°} = 0.27$$

$$Ka_2 = \frac{1 - \sin 30°}{1 + \sin 30°} = 0.33$$

$Ka_1 < Ka_2$



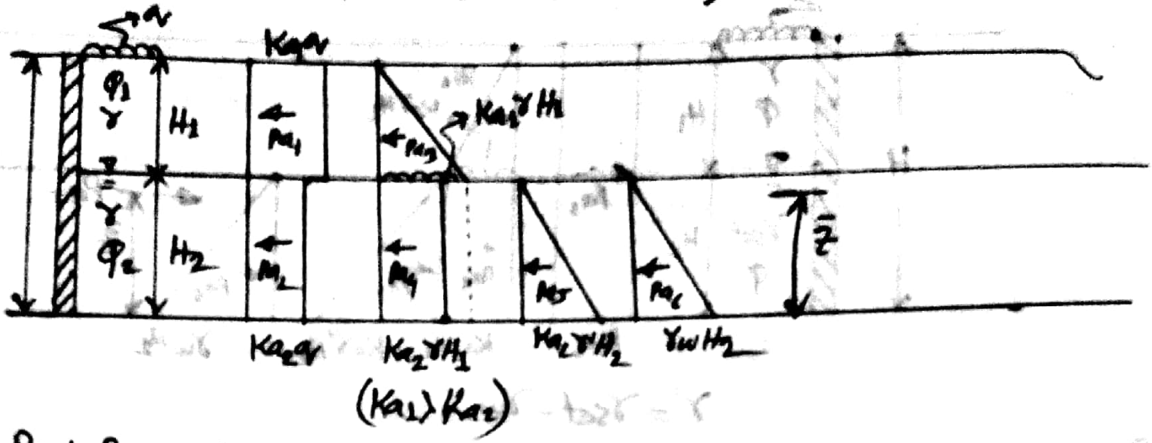
$$Pa_T = Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5 + Pa_6$$

$$\therefore Pa_T = Ka_1 q H_1 + Ka_2 q H_2 + \frac{1}{2} Ka_1 \gamma H_1^2 + Ka_2 \gamma H_1 H_2 + \frac{1}{2} Ka_2 \gamma' H_2^2 + \frac{1}{2} Ka_2 \gamma_w H_2^2$$

$$\bar{z} = \frac{(Pa_1 \times H_1/2) + (Pa_2 \times H_2/2) + (Pa_3 \times H_1/3) + (Pa_4 \times H_2/2) + (Pa_5 \times H_2/3) + (Pa_6 \times H_2/3)}{Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5 + Pa_6}$$

⇒ **Case-VI: Partially submerged Backfill with surcharge for different Friction angles ($\varphi_1 < \varphi_2$).**

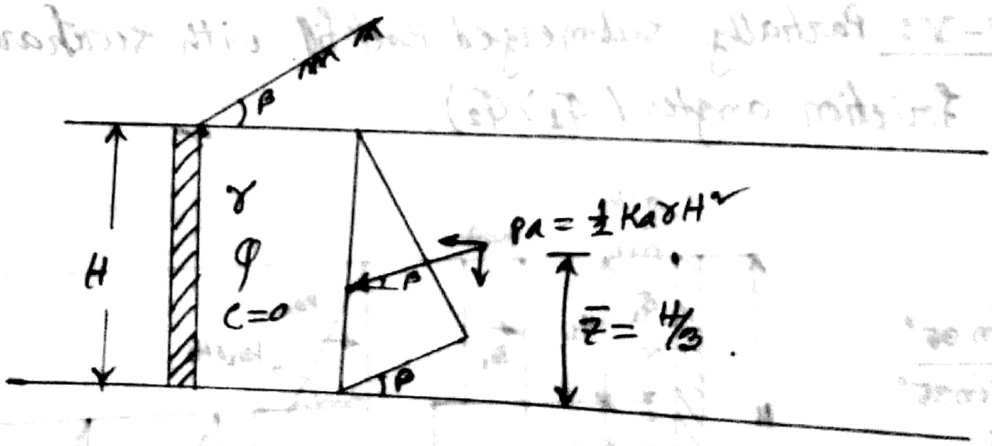
$\left(\begin{array}{l} \because \varphi_1 < \varphi_2 \\ \therefore K_{a1} > K_{a2} \end{array}\right)$



$$P_{aT} = P_{a1} + P_{a2} + P_{a3} + P_{a4} + P_{a5} + P_{a6}$$

$$\bar{z} = \left\{ \left(P_{a1} \times \frac{H_2}{2}\right) + \left(P_{a2} \times \frac{H_2}{2}\right) + \left(P_{a3} \times \frac{H_1}{3}\right) + \left(P_{a4} \times \frac{H_2}{2}\right) + \left(P_{a5} \times \frac{H_2}{3}\right) + \left(P_{a6} \times \frac{H_2}{3}\right) \right\} \div P_{aT}$$

⇒ **Case-VII: Backfill with sloping surface.**



$$\left(K_a = \cos\beta \times \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}} \right)$$

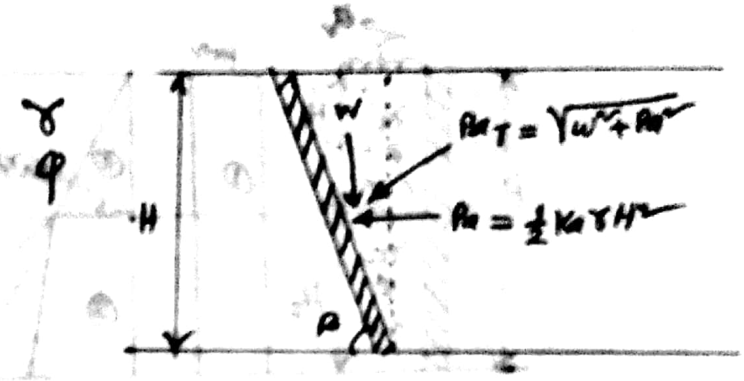
Proof: If $\beta = 0$

Then $K_a = 1 \cdot \frac{1 - \sqrt{1 - \cos^2\varphi}}{1 + \sqrt{1 - \cos^2\varphi}}$

$K_a = \frac{1 - \sin\varphi}{1 + \sin\varphi}$ (proved)

⇒ Case – VIII : Inclined Back (Retaining wall) and surcharge.

Triangle আকৃতির অংশের মাটির weight টি surcharge হিসেবে কাজ করবে।

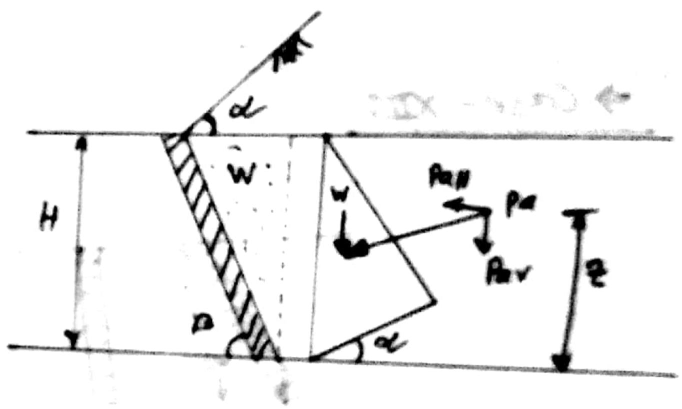


⇒ Case – IX : Inclined Back and surcharge with sloping surface.

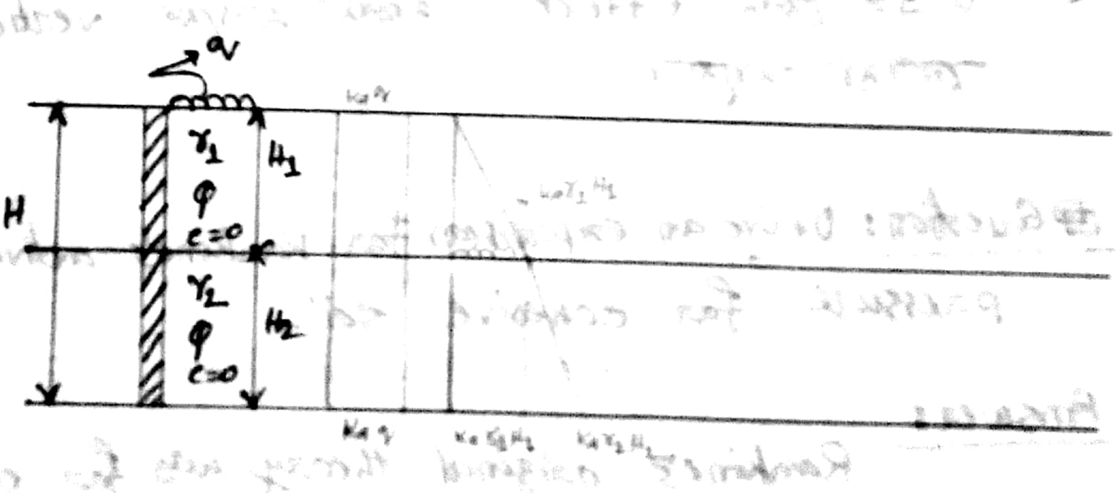
$$W_T = W + P_{av}$$

$$P_{aT} = \sqrt{W_T^2 + P_{aH}^2}$$

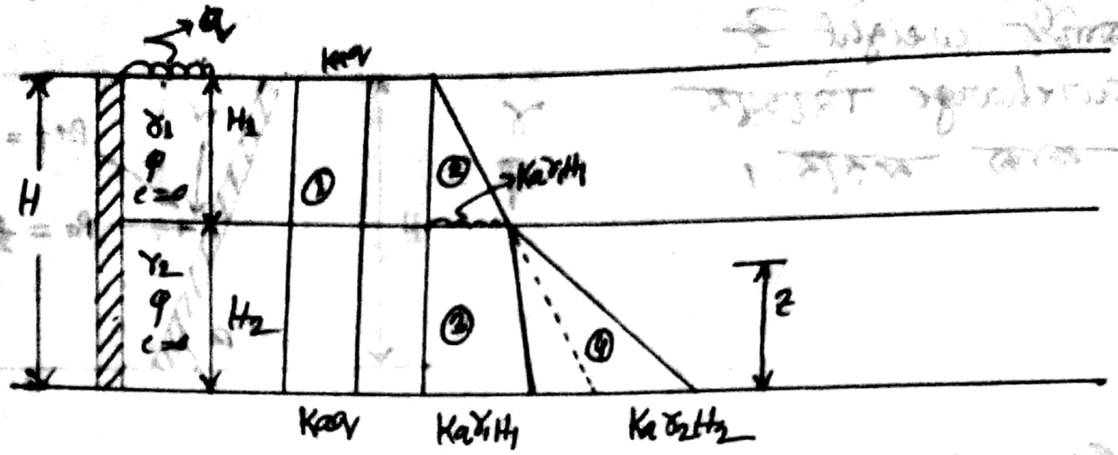
$$z = H/3$$



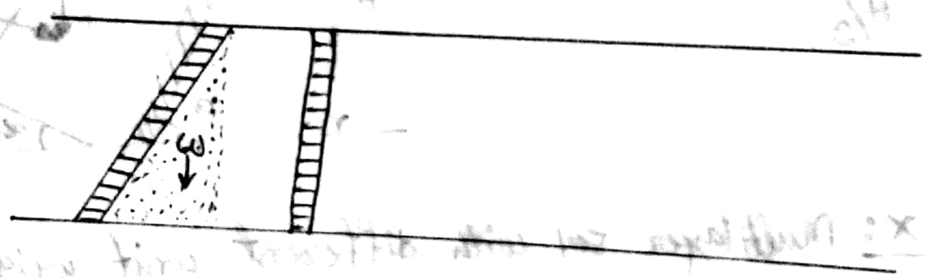
⇒ Case – X : Multilayer soil with different unit weight ($\gamma_1 > \gamma_2$)



⇒ Case-XI: multilayer soil with different unit weight ($\gamma_1 < \gamma_2$)



⇒ Case- XII:



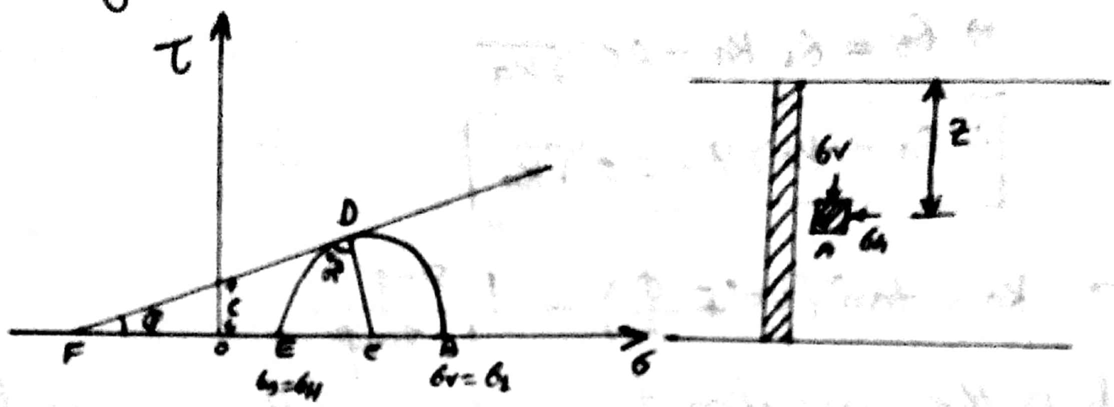
W এর কোন effect না থাকলে ওয়ালকে vertical ধরে নেওয়া যায়।

⊡ Question: Derive an expression for Rankine's active earth pressure for cohesive soil.

Answer:

Rankine's original theory was for cohesionless soil. It was extended by Resel (1910) and Bell (1915) for cohesive soil.

Let us consider a soil element located at a depth of z below the ground surface.



From triangle, AFCD;

$$\sin\varphi = \frac{CD}{FC} = \frac{CD}{OF+OC} = \frac{CE}{OF+OC} \quad \text{——— (1)} \quad [CD = CE = CB = \text{Radius}]$$

$$CE = \frac{1}{2} BE = \frac{\sigma_1 - \sigma_3}{2}$$

$$OF = c \cdot \cot\varphi$$

$$OC = OE + CE = \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2}$$

From eqn. (1), we get;

$$\sin\varphi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{c \cdot \cot\varphi + \frac{\sigma_1 + \sigma_3}{2}}$$

$$= \frac{\sigma_1 - \sigma_3}{2c \cot\varphi + \sigma_1 + \sigma_3}$$

$$\Rightarrow \sigma_1 - \sigma_3 = 2c \cot\varphi \sin\varphi + \sigma_1 \sin\varphi + \sigma_3 \sin\varphi$$

$$\Rightarrow \sigma_3 + \sigma_3 \sin\varphi = \sigma_1 - \sigma_1 \sin\varphi - 2c \cdot \cot\varphi \cdot \sin\varphi$$

$$\Rightarrow \sigma_3 (1 + \sin\varphi) = \sigma_1 (1 - \sin\varphi) - 2c \cdot \cot\varphi \cdot \sin\varphi$$

$$\Rightarrow \sigma_3 = \sigma_1 \cdot \left(\frac{1 - \sin\varphi}{1 + \sin\varphi}\right) - 2c \cdot \left(\frac{\cos\varphi}{1 + \sin\varphi}\right)$$

$$\Rightarrow \sigma_3 = \sigma_1 \tan^2(45^\circ \pm \varphi/2) - 2c.\tan(45^\circ \pm \varphi/2)$$

$$\Rightarrow \sigma_3 = \sigma_1 K_a - 2c.\sqrt{K_a}$$

$$\boxed{\therefore P_a = K_a \gamma z - 2c\sqrt{K_a}}$$

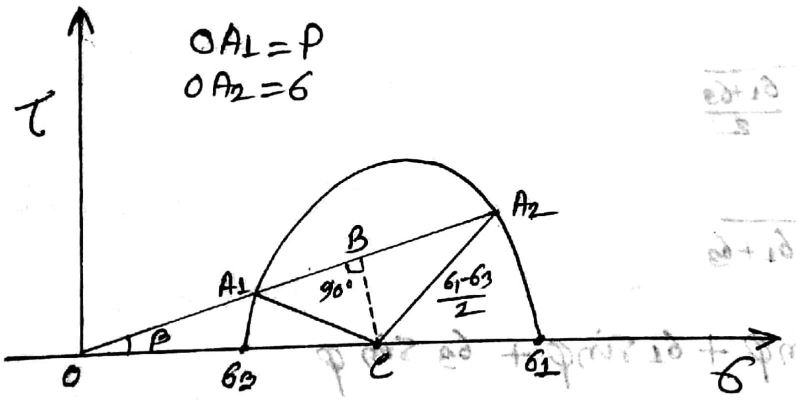
Here, $K_a = \tan^2(45^\circ \mp \frac{\varphi}{2}) = \frac{1-\sin\varphi}{1+\sin\varphi}$.

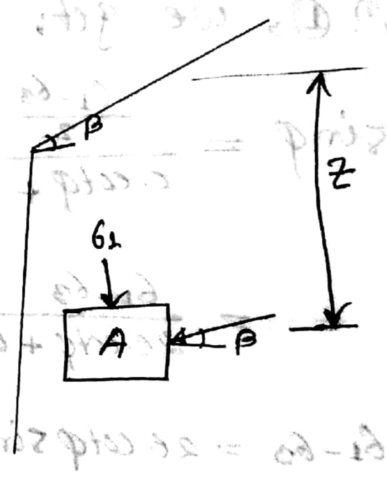
which is the equation for active earth pressure for cohesive soil.

Question: Prove that, $K_a = \cos\beta \cdot \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$.

co-efficient of backfill for sloping surface.

Answer: Consider a soil element at point A at depth of z within a backfill with a sloping surface as shown in figure below;





Here, OA_1 represent the resultant stress p and OA_2 represent the resultant stress σ. A perpendicular BC is drawn to A_1A_2

From triangle $\triangle OCB$;

Now, $OB = OC \cdot \cos\beta$

$$OB = \frac{\sigma_1+\sigma_3}{2}\cos\beta \quad \text{——— (1)}$$

and, $BC = OC \sin\beta$

$$\therefore BC = \frac{\sigma_1+\sigma_3}{2}\sin\beta \quad \text{——— (2)}$$

Now, $A_1B = A_2B = \sqrt{A_1C^2 - BC^2}$

$$= \sqrt{\left(\frac{\sigma_1-\sigma_3}{2}\right)^2 - \left(\frac{\sigma_1+\sigma_3}{2}\right)^2 \sin^2\beta}$$

But we have for cohesive soil, $\dfrac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3} = \sin\varphi$

$$\therefore (\sigma_1-\sigma_3) = (\sigma_1+\sigma_3)\sin\varphi$$

$$\therefore A_1B = A_2B = \sqrt{\left(\frac{\sigma_1+\sigma_3}{2}\right)^2 \sin^2\varphi - \left(\frac{\sigma_1+\sigma_3}{2}\right)^2 \sin^2\beta}$$

$$= \frac{\sigma_1+\sigma_3}{2}\sqrt{\sin^2\varphi - \sin^2\beta}$$

Now stress, $\sigma = OB + BA_2$

$$\therefore \sigma = \frac{\sigma_1+\sigma_3}{2}\cos\beta + \frac{\sigma_1+\sigma_3}{2}\sqrt{\sin^2\varphi - \sin^2\beta} \quad \text{——— (3)}$$

And, $P = OB - A_1B$

$$\therefore P = \frac{\sigma_1+\sigma_3}{2}\cos\beta - \frac{\sigma_1+\sigma_3}{2}\sqrt{\sin^2\varphi - \sin^2\beta} \quad \text{——— (4)}$$

Dividing eqn (4) by (3); we get;

$$\frac{P}{\sigma} = K = \frac{\cos\beta - \sqrt{\sin^2\varphi - \sin^2\beta}}{\cos\beta + \sqrt{\sin^2\varphi - \sin^2\beta}}$$

$$K = \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$$

The ratio K is called the conjugate ratio or Rankine's Lateral pressure ratio.

For present case, $\sigma = \frac{\gamma z \cdot b\cos\beta}{b} = \gamma z \cos\beta$

$p = p_a$ = Lateral earth pressure.

We have, $K = \frac{p_a}{\sigma}$

$$\Rightarrow p_a = \sigma K = \gamma z \cos\beta \cdot \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$$

$\boxed{\because p = K\gamma z}$

$$\Rightarrow K_a \gamma z = \gamma z \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$$

$$\therefore K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\varphi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\varphi}}$$

(proved)

⟹ From previous derivation;

Rankine's active earth pressure;

$$p_a = K_a \gamma z - 2c\sqrt{K_a}$$

If $z = 0$,

then, $p_a = 0 - 2c \tan(45° - \varphi/2)$

$$\therefore p_a = -2c\sqrt{K_a}$$

Negative sign indicates that the pressure is negative so it tries to cause a pull on the wall.

If $p_a = 0$

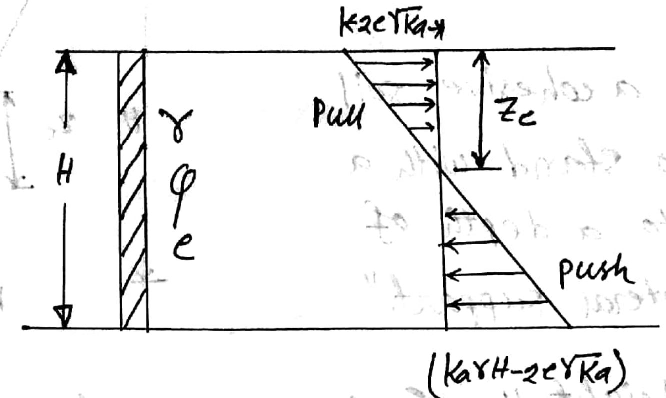
then, $0 = K_a \gamma z - 2c\sqrt{K_a}$

$$\therefore z = z_c = \frac{2c}{\gamma\sqrt{K_a}}$$

This depth, $z = z_c$ is known as the depth of tension crack.

if $z = H$

$$\therefore P_a = K_a \gamma H - 2c\sqrt{K_a}$$



Total pressure is given by;

$$P_a = \int_0^H (K_a \gamma H - 2c\sqrt{K_a})\, dz$$

$$\therefore P_a = K_a \gamma \frac{H^2}{2} - 2c\sqrt{K_a}\cdot H$$

Note that this pressure is applicable before the formation of crack.

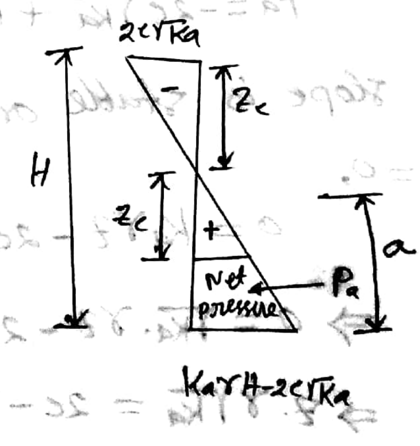
After the formation of tension crack, the force on the wall is caused only by the pressure from $z = z_c$ to $z = H$.

Thus,

$$P_a = \frac{1}{2}(H - z_c)(K_a \gamma H - 2c\sqrt{K_a})$$

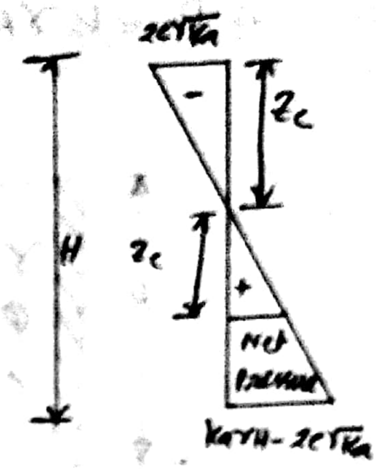
$$= \frac{1}{2} a \cdot (K_a \gamma H - 2c\sqrt{K_a})$$

$$P_a = \frac{1}{2} \times \text{Base} \times \text{Height}.$$



Critical height of unsupported vertical cut:

From the figure, it is clear that the total net pressure upon a depth of $2Z_c$ is zero.



"It reveals that a cohesive soil should be able to stand with a vertical face up to a depth of $2Z_c$ without any lateral support."

The critical height H_c of the unsupported vertical cut in cohesive soil can be given by:

$$H_c = 2Z_c = 2 \times \frac{2c}{\gamma\sqrt{K_a}}$$

$$\therefore H_c = \frac{4c}{\gamma\sqrt{K_a}}$$

Backfill with surcharge:

If the backfill carries a surcharge, q then

$$P_a = K_a \gamma Z - 2c\sqrt{K_a} + K_a q$$

At $Z = 0$ (top),

$$P_a = -2c\sqrt{K_a} + K_a q$$

Thus the slope is stable only if $2c\sqrt{K_a} > K_a q$.

when $P_a = 0$,

$$0 = K_a \gamma Z - 2c\sqrt{K_a} + K_a q$$

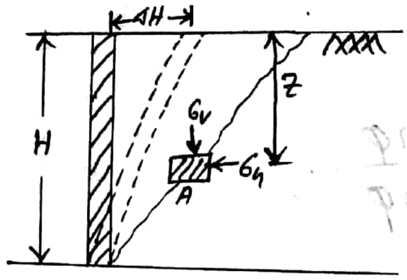
$$\Rightarrow 0 = \sqrt{K_a} \cdot \gamma Z - 2c + \sqrt{K_a} \cdot q$$

$$\Rightarrow Z \cdot \gamma\sqrt{K_a} = 2c - \sqrt{K_a} \cdot q$$

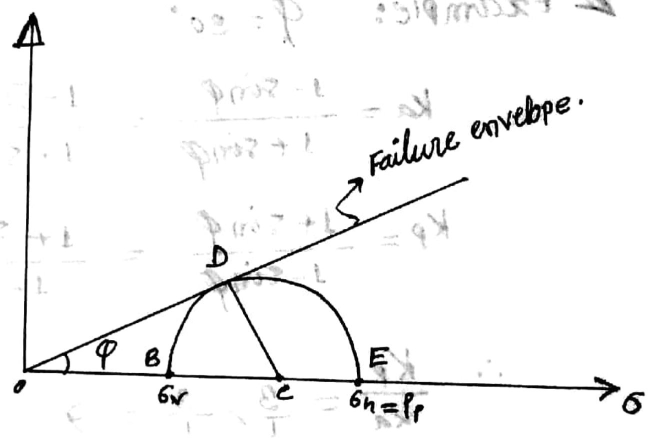
$$\therefore z = z_c = \frac{2c}{\gamma\sqrt{K_a}} - \frac{q}{\gamma}$$

Passive Earth Pressure:

Definition: If the wall moves towards or pushed into the soil retained, a triangular soil wedge behind the wall will fail. The lateral pressure at this condition is known as passive earth pressure.



For granular soil, (sand) let us consider a soil element, A, located at a depth of z below ground surface.



From figure;

$$\sigma_h = P_p = OE = OC + CE$$

$$= OC + CD \quad [\because BC = CD = CE = \text{Radius}]$$

$$= OC + OC \sin\varphi$$

$$\therefore \sigma_h = OC(1 + \sin\varphi) \quad \text{——— } (1)$$

And $\sigma_v = OB = OC - BC$

$$= OC - CD \quad [BC = CD = r]$$

$$= OC - OC \sin\varphi$$

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$$\therefore \sigma_h = (1-\sin\varphi)\, oc \quad \text{--- (2)}$$

From (1) ÷ (2), we get,

$$\frac{\sigma_h}{\sigma_v} = \frac{oc(1+\sin\varphi)}{oc(1-\sin\varphi)}$$

$$\Rightarrow \frac{P_p}{\sigma_v} = \frac{1+\sin\varphi}{1-\sin\varphi}$$

$$\Rightarrow P_p = \sigma_v . K_p$$

$$\boxed{\therefore P_p = K_p \gamma z}$$

where, $K_p = \dfrac{1+\sin\varphi}{1-\sin\varphi}$.

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Example: $\varphi = 30°$

$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$$

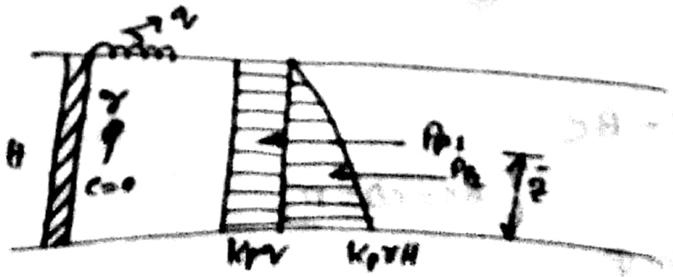
$$K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \frac{1+\sin 30°}{1-\sin 30°} = 3$$

$$\therefore \frac{K_p}{K_a} = \frac{3}{1} \times \frac{3}{1} = 9$$

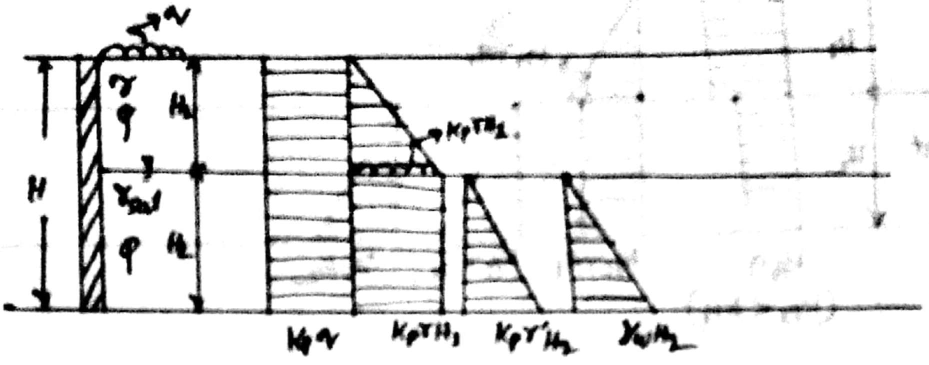
$$\therefore K_p = 9 \times K_a$$

$$\therefore K_p > K_a \quad \text{(proved)}$$

Case-I: Dry soil with surcharge.

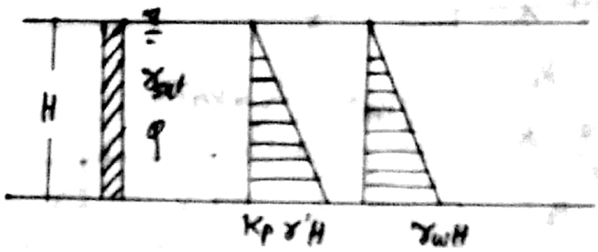


Case-II: Partially submerged soil with surcharge.



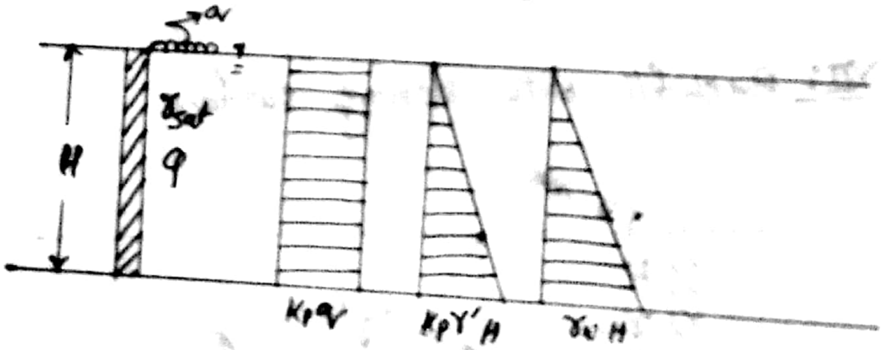
$γ' = γ_{sat} - γ_w$.

Case-III: Submerged back fill.



$γ' = (γ_{sat} - γ_w)$

Case-IV: Submerged back fill with surcharge.



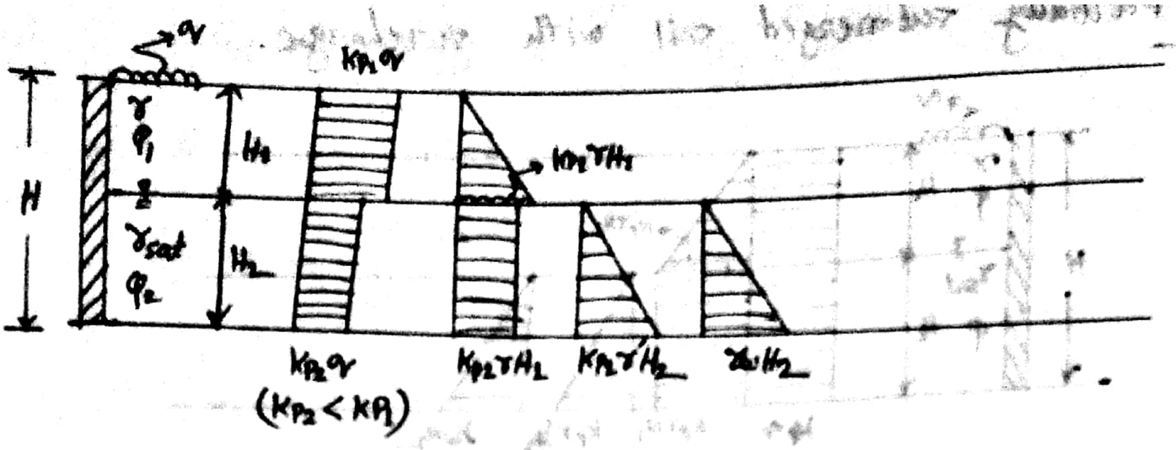
Case-V: Partially submerged backfill with surcharge for different friction angles ($φ_1 > φ_2$).

$φ_1 = 35°$ $\qquad K_{p1} = \frac{1+\sin 35°}{1-\sin 35°} = 3.69$

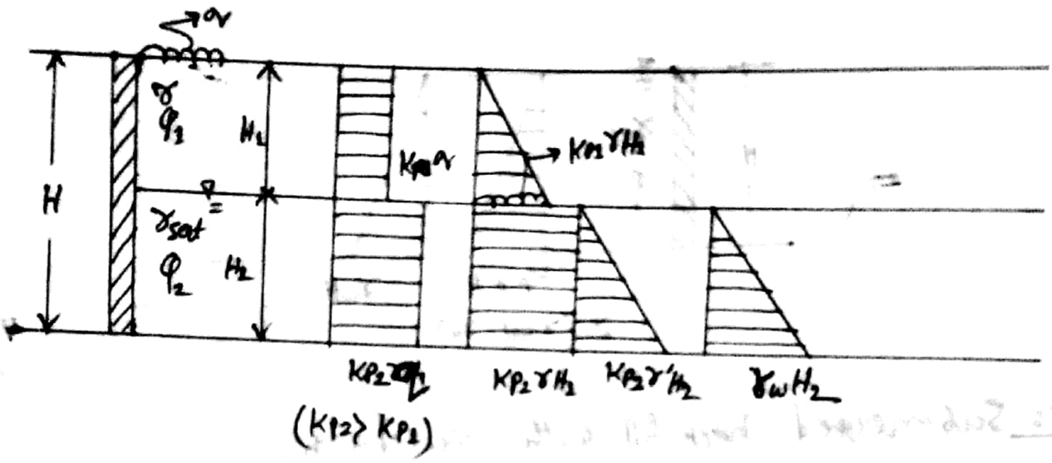
$φ_2 = 30°$

$K_{p2} = \frac{1+\sin 30°}{1-\sin 30°} = 3.00$

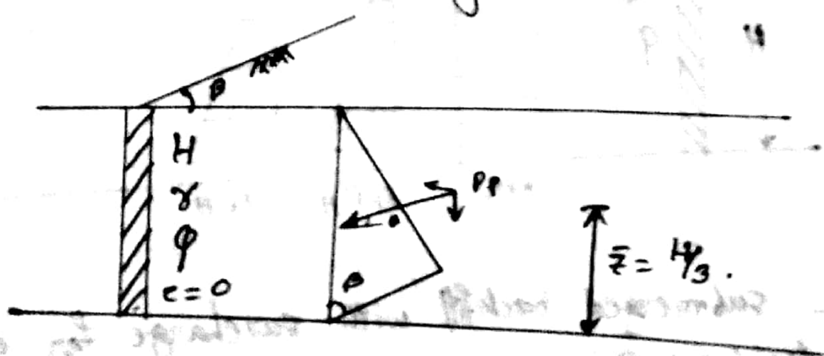
$K_{p2} < K_{p1}$.



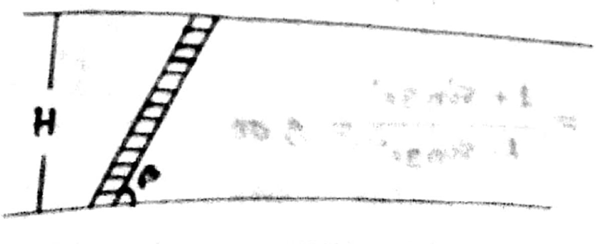
Case-VI: Partially submerged backfill with surcharge for different friction angles ($\varphi_2 > \varphi_1$). $\longrightarrow K_{P_2} > K_{P_1}$.



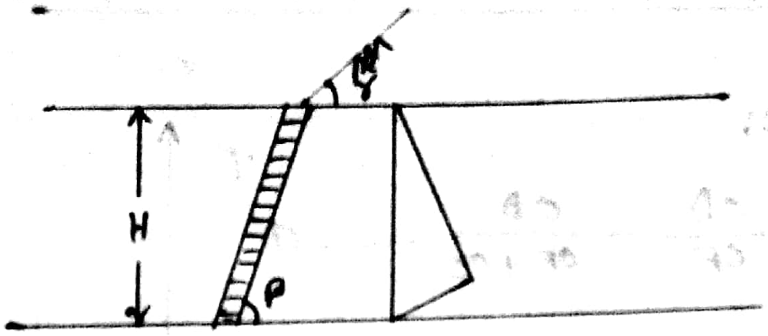
Case-VII: Backfill with sloping surface.



Case-VIII: Inclined Back (Retaining wall) and surcharge

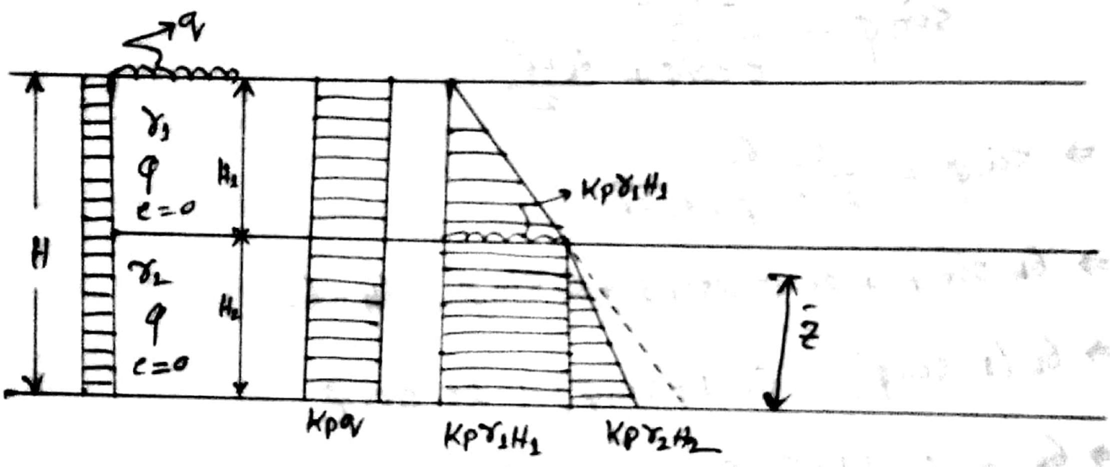


Case-IX: Inclined back and surcharge with sloping surface.



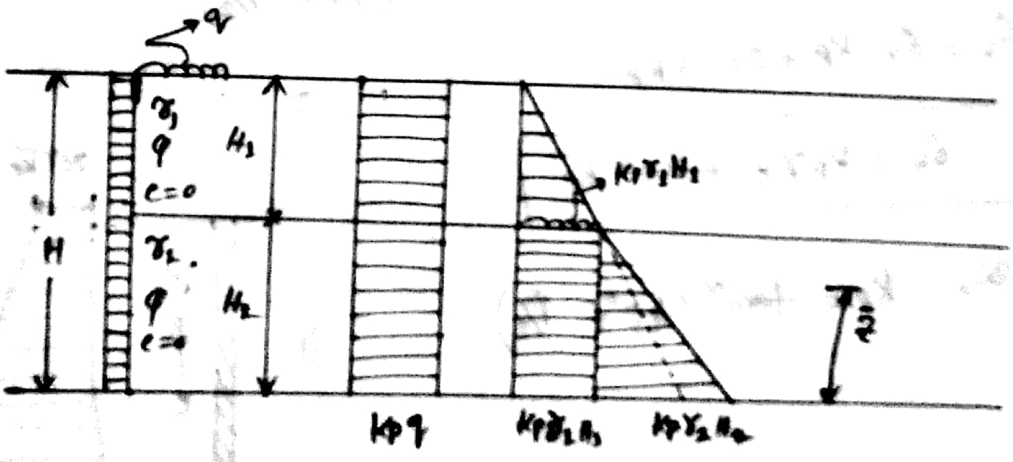
Case-X: Multilayer soil with different unit weight.

when, $\gamma_1 > \gamma_2$. ($H_1 = H_2$)



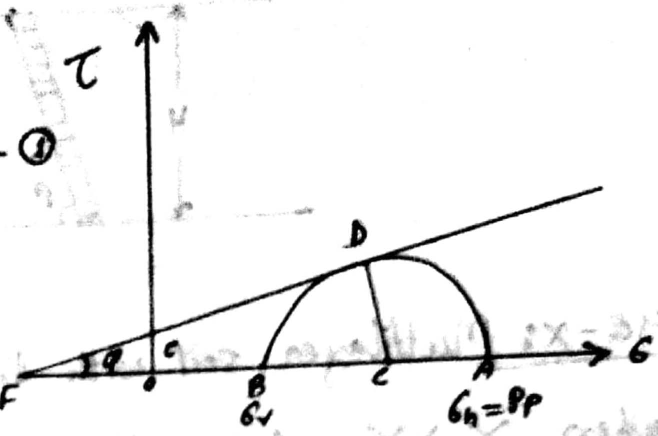
($\gamma_1 > \gamma_2$)

when $\gamma_1 < \gamma_2$.



($\gamma_2 > \gamma_1$)

Passive earth pressure for cohesive soil:-



From figure:

$$\sin\varphi = \frac{CD}{CF} = \frac{CD}{OF + OC} \qquad \text{—— ①}$$

$$\therefore OF = c.\cot\varphi$$

$$CD = \frac{\sigma_h - \sigma_v}{2}$$

$$OC = \frac{\sigma_h + \sigma_v}{2}$$

From ①;

$$\sin\varphi = \frac{\frac{\sigma_h - \sigma_v}{2}}{c.\cot\varphi + \frac{\sigma_h + \sigma_v}{2}}$$

$$\Rightarrow \sin\varphi = \frac{\sigma_h - \sigma_v}{2c.\cot\varphi + \sigma_h + \sigma_v}$$

$$\Rightarrow \sigma_h.\sin\varphi + 2c.\cot\varphi\sin\varphi + \sigma_v\sin\varphi = \sigma_h - \sigma_v$$

$$\Rightarrow \sigma_h(1 - \sin\varphi) = \sigma_v(1 + \sin\varphi) + 2c.\cos\varphi$$

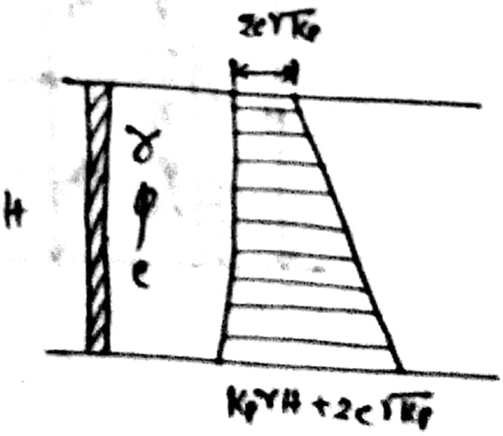
$$\Rightarrow \sigma_h = \sigma_v.\left(\frac{1+\sin\varphi}{1-\sin\varphi}\right) + 2c.\left(\frac{\cos\varphi}{1-\sin\varphi}\right)$$

$$\Rightarrow \sigma_h = \sigma_v.\tan^2(45^\circ + \varphi/2) + 2c.\tan(45^\circ + \varphi/2)$$

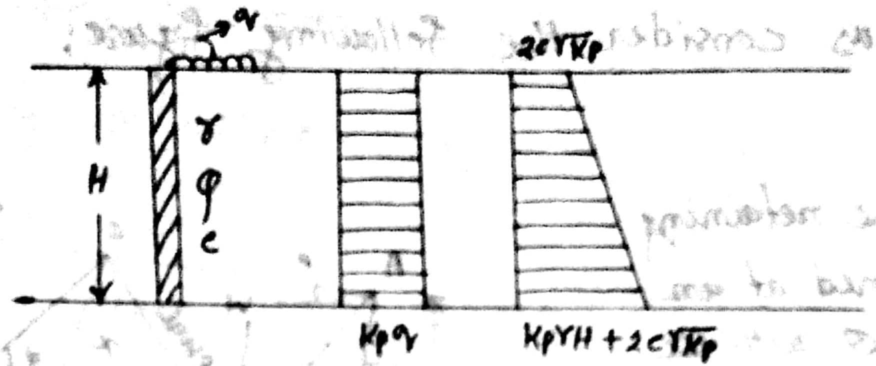
$$\Rightarrow \sigma_h = \sigma_v.K_p + 2c.\sqrt{K_p}$$

$$\boxed{\therefore \sigma_h = K_p \gamma z + 2c\sqrt{K_p}}$$

where, $K_p = \tan^2(45^\circ + \varphi/2)$



Passive earth pressure for cohesive soil with surcharge.



Coulomb's wedge Theory:-

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assumptions;

1. The backfill is dry and cohesionless.
2. The backfill is homogeneous, isotropic & plastic materials.
3. The slip surface is a plane passing through the heel of the wall.
4. The wall surface is rough.
5. The sliding wedge itself acts as a rigid body.

NB: Isotropic soil: যে soil এর সকল দিক থেকে pressure same থাকে তাকে isotropic soil বলে। $\sigma_1 = \sigma_3 = 100$ KPa.

Coulomb's Active earth pressure in cohesionless soil:-

Let us consider the following figure;

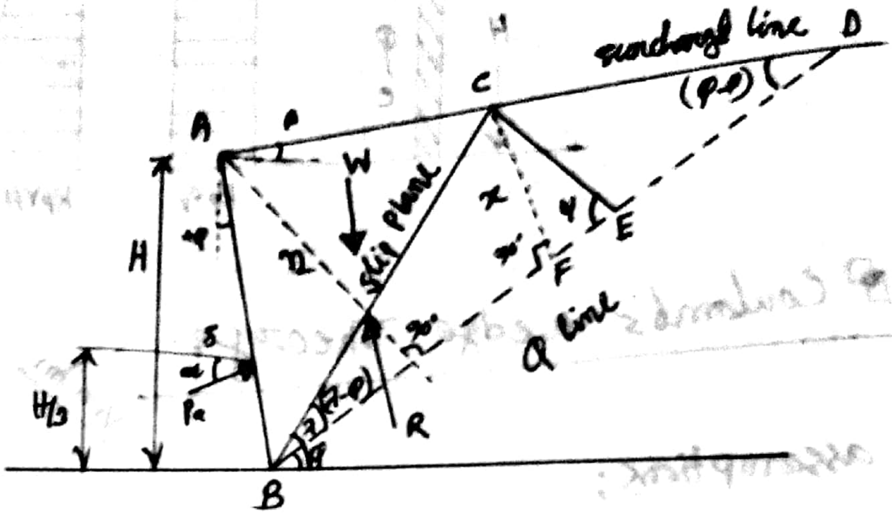
Here,

AB is the retaining wall inclined at an angle θ with the vertical.

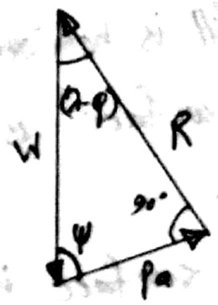
BC is the slip Plane.

BD is the φ line.

AD is the surcharge line.



From geometry, $\triangle BCE$ and $\triangle$Force triangle are similar.



$\psi = 90° - \theta - \delta$

Force triangle

Hence, $\dfrac{P_a}{W} = \dfrac{CE}{BE}$

$\therefore P_a = W \cdot \dfrac{CE}{BE}$ ——— (1)

Now from $\triangle CEF$;

$\operatorname{cosec}\psi = \dfrac{CE}{CF}$

$\Rightarrow CE = CF \cdot \operatorname{cosec}\psi$

$\Rightarrow CE = x \operatorname{cosec}\psi$

$\therefore CE = x A_1$ [let, $A_1 = \operatorname{cosec}\psi$]

Again, from $\triangle CDF$ and $\triangle CEF$;

$DE = x \cot(\varphi - \beta)$

$EF = x \cot\psi$.

Now, $BE = BD - DE$

$= BD - (DF - EF)$

$\Rightarrow BE = m - x[\cot(\varphi - \beta) - \cot\psi]$

$\therefore BE = m - A_2 x$

where, $BD = m$

$A_2 = \cot(\varphi - \beta) - \cot\psi$

Now, $W = \Delta ABC \times \gamma$

$= (\Delta ABD - \Delta BCD)\gamma$

$= \left(\frac{1}{2} n m - \frac{1}{2} x m\right)\gamma$

$\therefore W = \frac{\gamma m}{2}(n - x)$

Putting the values of CE, BE & W in eqn ①. we get:

$$P_a = W \cdot \frac{CE}{BE}$$

$$\boxed{\therefore P_a = \frac{1}{2}\gamma m (n - x)\frac{A_1 x}{m - A_2 x}}$$

which is the required equation for coulomb's Active earth pressure

For maxima, $\frac{dP_a}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left(\frac{1}{2}\gamma m (n - x) \cdot \frac{A_1 x}{m - A_2 x}\right) = 0$$

$$\Rightarrow \frac{d}{dx}\left(\frac{1}{2}\gamma m A_1 \frac{nx - x^2}{m - A_2 x}\right) = 0$$

$\Rightarrow (n-2x)(m-A_2x) = -A_2(nx-x^2) \rightarrow$

$\Rightarrow mn - mx = x(m - A_2x)$

$\Rightarrow \frac{mn}{2} - \frac{xm}{2} = \frac{x \cdot BE}{2}$ [Divided by 2]

$\Rightarrow \Delta ABD - \Delta BCD = \Delta BCE$

$$\boxed{\therefore \Delta ABC = \Delta BCE}$$

$\Rightarrow mn - A_2nx - 2mx + 2A_2x^2 + A_2nx - A_2x^2 = 0$

$\Rightarrow mn - 2mx + A_2x^2 = 0$

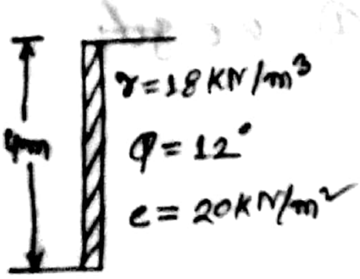
$\Rightarrow mn - mx - mx + A_2x^2 = 0$

$\Rightarrow mn - mx - x(m - A_2x) = 0$

$\therefore mn - mx = x(m - A_2x)$

The criteria for maximum active earth pressure is that the slip plane is so choosen that ΔABC and ΔBCE are equal in area.

Ex-1:



(i) Draw the earth pressure diagram.

(ii) Max. depth of potential crack.

(iii) Max. depth of unsupported excavation.

Solution: $K_a = \frac{1-\sin 12^\circ}{1+\sin 12^\circ} = 0.66$

we know, $P_a = K_a \gamma H - 2c\sqrt{K_a}$

at $H = 0$, $P_a = 0 - 2c\sqrt{K_a}$

$= -2 \times 20 \times \sqrt{0.66}$

$\therefore P_a = -32.50\ kN/m^2$

at $P_a = 0$, $0 = K_a\gamma H - 2c\sqrt{K_a}$

$\therefore H = z_c = \frac{2c\sqrt{K_a}}{K_a\gamma}$

$$z_c = \frac{2\times 20\times\sqrt{0.66}}{0.66\times 18}$$

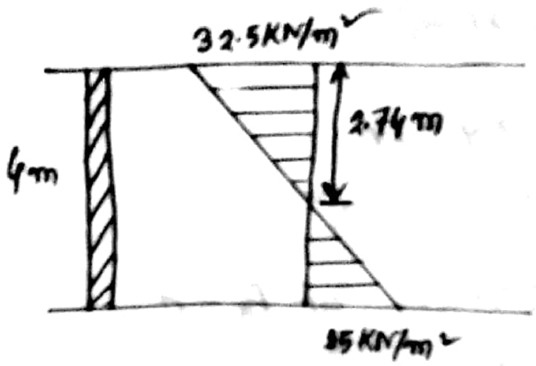
$$\therefore z_c = 2.74\,m.$$

at $H = 4m$,

$$P_a = 0.66\times 18\times 4 - 2\times 20\times\sqrt{0.66}$$

$$\therefore P_a = 15\,KN/m^2$$

(i) Earth pressure diagram.



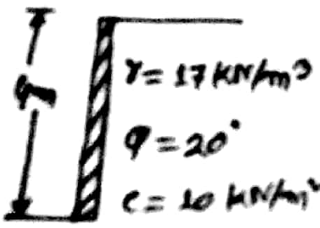
(ii) Max depth of potential crack, $z_c = 2.74\,m$.

(iii) Max depth of unsupported excavation;

$$H_c = 2z_c = 2\times 2.74 = 5.48\,m.$$

As $H = 4m < H_c = 5.48\,m$ so the unsupported cut $= 4m$ (Ans)

Ex-2:



(i) Draw the earth pressure diagram before tension crack

(ii) Active earth pressure (force) on the wall before tension crack

(iii) Location of Active pressure (force) before tension crack

(iv) Draw the earth pressure diagram after tension crack

(v) Active earth pressure (force) on the wall after tension crack.

(vi) Location of active earth pressure after tension crack.

Solution: $K_a = \dfrac{1-\sin 20^\circ}{1+\sin 20^\circ}$

$K_a = 0.49$

we know,

$$P_a = K_a \gamma z - 2c\sqrt{K_a}$$

$$= 0.49 \times 17 \times z - 2 \times 10 \times \sqrt{0.49}$$

$$\therefore P_a = 8.33z - 14 \quad \text{——} \; (1)$$

at $z = 0$,

$$P_a = -14 \ kN/m^2$$

at $z = 4$, $P_a = 8.33 \times 4 - 14$

$$\therefore P_a = 19.32 \ KN/m^2$$

Depth of tension crack, at $P_a = 0$

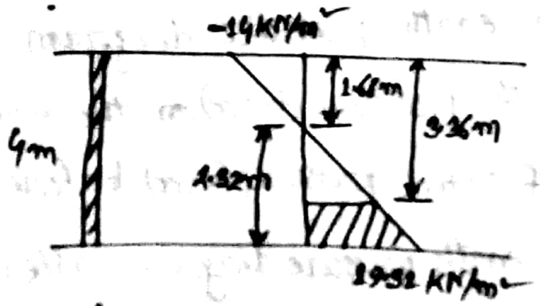
$$\therefore z_c = \frac{2c\sqrt{K_a}}{K_a \gamma} = \frac{2 \times 10 \times \sqrt{0.49}}{0.49 \times 17}$$

$$\therefore z_c = 1.68 \ m.$$

So, Max depth of unsupported cut, $H_c = 2z_c = 2 \times 1.68$

$$\therefore H_c = 3.36 \ m.$$

Earth pressure diagram before tension crack;



Before tension crack;

$$P_{aT} = \int_0^H P_a = \int_0^H (K_a \gamma z - 2c\sqrt{K_a})\, dz$$

$$P_{aT} = K_a \gamma \frac{H^2}{2} - 2cH\sqrt{K_a}$$

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$$P_{aT} = \frac{1}{2}\times 0.49\times 17\times (4)^2 - 2\times 10\times 4\times \sqrt{0.49}$$

$$P_{aT} = 66.64 - 56$$

$$\therefore P_{aT} = 10.64 \text{ kN/m}^2$$

Alternatively.

$$P_{aT} = \frac{1}{2}\times 19.32\times 2.32 - \frac{1}{2}\times 1.68\times 14$$

$$P_{aT} = 10.65 \text{ kN/m}^2\text{/m}$$

$$\text{Location, } \bar{z} = \frac{\frac{1}{2}\times 19.32\times 2.32\times \frac{2.32}{3} - \frac{1}{2}\times 14\times 1.68\times \left(2.32 + \frac{2\times 1.68}{3}\right)}{P_{aT}}$$

$$= \frac{17.33 - 40.45}{10.65}$$

$$\therefore \bar{z} = 2.17 \text{ m}$$

After tension crack;

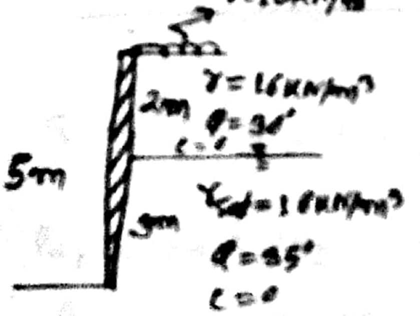
$$P_a = \frac{1}{2}\times 19.32\times 2.32$$

$$\therefore P_a = 22.41 \text{ kN/m}$$

$$\text{Location, } \bar{z} = \frac{2.32}{3} = 0.77 \text{ m}$$

After tension crack

Ex-3:



(i) Draw the earth pressure distribution diagram

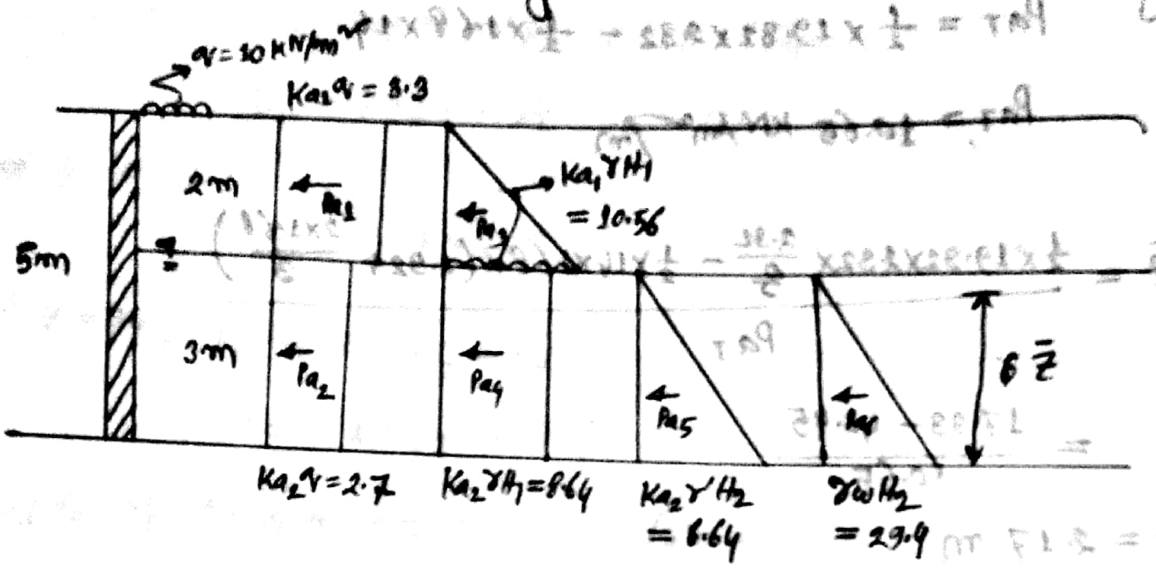
(ii) Active earth pressure (force).

(iii) Location of active earth pressure (force).

Solution: $Ka_1 = \dfrac{1-\sin 30^\circ}{1+\sin 30^\circ} = 0.33$

$Ka_2 = \dfrac{1-\sin 35^\circ}{1+\sin 35^\circ} = 0.27$

Active earth pressure diagram:



$\therefore Pa_T = Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5 + Pa_6$

$= (3.3\times 2) + (2.7\times 3) + (\frac{1}{2}\times 10.56\times 2) + (8.64\times 3) + (\frac{1}{2}\times 6.64\times 3) + (\frac{1}{2}\times 29.4\times 3)$

$= 6.6 + 8.1 + 10.56 + 25.92 + 9.96 + 44.1$

$\therefore Pa_T = 105.24$ KN/m2.

Location, $\bar{z} = \dfrac{Pa_1\times(3+\frac{2}{2}) + Pa_2\times\frac{3}{2} + Pa_3\times(3+\frac{2}{3}) + Pa_4\times\frac{3}{2} + Pa_5\times\frac{3}{3} + Pa_6\times\frac{3}{3}}{Pa_T}$

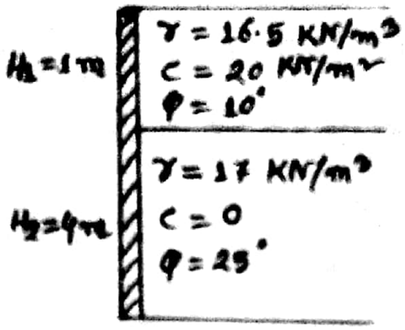
$= \dfrac{(6.6\times 4)+(8.1\times 1.5)+(10.56\times 3.67)+(25.92\times 1.5)+(9.96\times 1)+(44.1\times 1)}{105.24}$

$= \dfrac{170.25}{105.24}$

$\therefore \bar{z} = 1.62$ m (Ans)

Problem – 43

The soil profile of a multilayer soil strata is shown in figure below. Compute the active thrust and its location on the retaining wall.



Solution: For top layer:

$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 10^\circ}{1+\sin 10^\circ} = 0.704$$

we have,

$$P_a = K_a \gamma z - 2c\sqrt{K_a}$$

$$= 0.704 \times 16.5 \times z - 2 \times 20 \times \sqrt{0.704}$$

$$\therefore P_a = 11.62z - 33.56 \quad \text{——— (1)}$$

Now, $P_a = 0$

$$\Rightarrow 11.62z - 33.56 = 0$$

$$\Rightarrow z = \frac{33.56}{11.62}$$

$$\therefore z = 2.89\ m$$

if $z > H_1$ so there is no pressure for top layer.

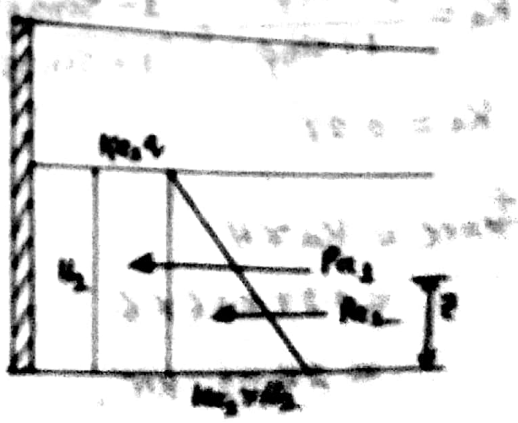
For bottom layer:

$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 25^\circ}{1+\sin 25^\circ} = 0.41$$

$$q = \gamma z = 16.5 \times 1 = 16.5\ KN/m^2$$

$$\therefore P_{a_1} = K_{a_2} q \times H_2 = 0.41 \times 16.5 \times 4$$

$$\therefore P_{a_1} = 27.06\ KN$$



$$Pa_2 = \frac{1}{2} \times Ka_2 \gamma H_2 \times H_2$$

$$= \frac{1}{2} \times 0.41 \times 17 \times 4 \times 4$$

$$\therefore Pa_2 = 55.76 \text{ KN}$$

Now, $Pa_T = Pa_1 + Pa_2 = 27.06 + 55.76$

$$\therefore Pa_T = 82.82 \text{ KN}$$

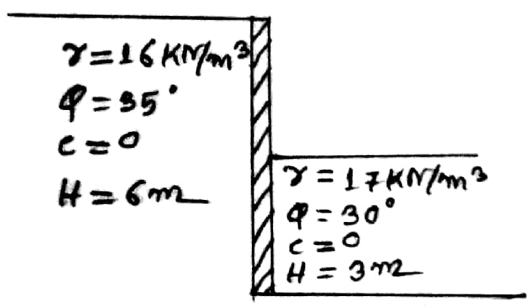
Location, $z = \dfrac{Pa_1 \times H/2 + Pa_2 \times H/3}{Pa_1 + Pa_2}$

$$= \frac{27.06 \times 4/2 + 55.76 \times 4/3}{27.06 + 55.76} = \frac{128.47}{82.82}$$

$$\therefore z = 1.55 \text{ m}.$$

(Ans:)

Problem-5:

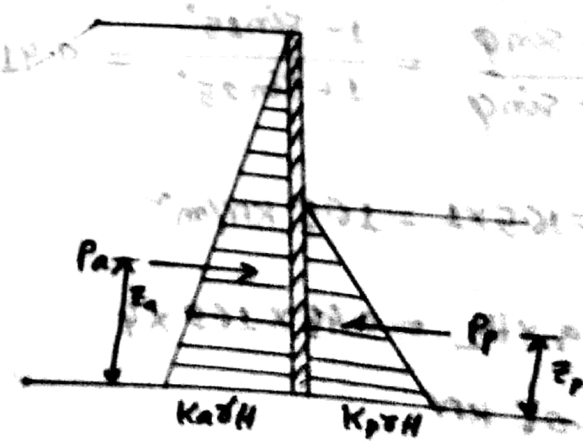


A retaining wall as shown in figure below is installed in a multilayer soil. Compute the active and passive earth pressure, active and passive earth forces and their locations.

Solution: For active earth pressure;

$$Ka = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 35^\circ}{1+\sin 35^\circ}$$

$$Ka = 0.27$$



$$\text{force} = Ka\gamma H$$

$$= 0.27 \times 16 \times 6$$

$$= 25.92 \text{ KN}.$$

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Pressure, $P_a = \frac{1}{2} \times K_a \gamma H \times H$

$= \frac{1}{2} \times 0.27 \times 16 \times 6 \times 6$

$\therefore P_a = 77.76 \ KN/m^2$

Location, $\bar{z}_a = \frac{P_a \times H/3}{P_a} = \frac{77.76 \times 6/3}{77.76}$

$\therefore \bar{z}_a = 2 \ m.$

For Passive pressure;

$K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \frac{1+\sin 30^\circ}{1-\sin 30^\circ} = 3$

Force $= K_p \gamma H = 3 \times 17 \times 3 = 153 \ KN$

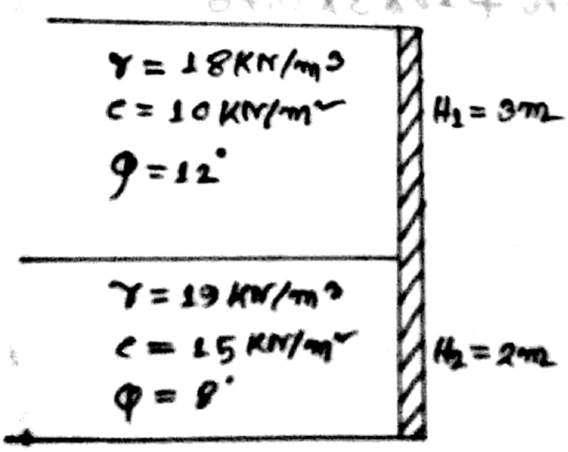
Pressure, $P_p = \frac{1}{2} \times K_p \gamma H \times H = \frac{1}{2} \times 3 \times 17 \times 3 \times 3$

$\therefore P_p = 229.5 \ KN/m^2$

Location, $\bar{z}_p = \frac{P_p \times H/3}{P_p} = \frac{229.5 \times 3/3}{229.5}$

$\therefore \bar{z}_p = 1 \ m$ (Ans)

Problem-6:



A retaining wall as shown in figure supports multilayer soil. Compute the active thrust on the retaining wall and its location.

Solution: For top layer; $K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 12^\circ}{1+\sin 12^\circ}$

$K_a = 0.66$

we have, $P_a = K_a \gamma z - 2c\sqrt{K_a}$

$= 0.66 \times 18 \times z - 2 \times 10 \times \sqrt{0.66}$

$\therefore P_a = 11.88z - 16.25$

at $H=0$, $P_a = -16.25 \ KN/m^2$

at, $P_a = 0$,

$11.88z = 16.25$

$\therefore z_c = 1.37 \ m$.

at $H = 3m$,

$P_a = 11.88 \times 3 - 16.25$

$\therefore P_a = 19.39 \ KN/m^2$

For bottom layer;

$$K_a = \frac{1 - \sin 8^\circ}{1 + \sin 8^\circ} = 0.76$$

we have, $P_a = K_a \gamma z - 2c\sqrt{K_a} + \gamma_1 H_1 K_{a_2}$

$= 0.76 \times 19 \times z - 2 \times 15 \times \sqrt{0.76} + 18 \times 3 \times 0.76$

$\therefore P_a = 14.44z - 26.15 + 41.04$

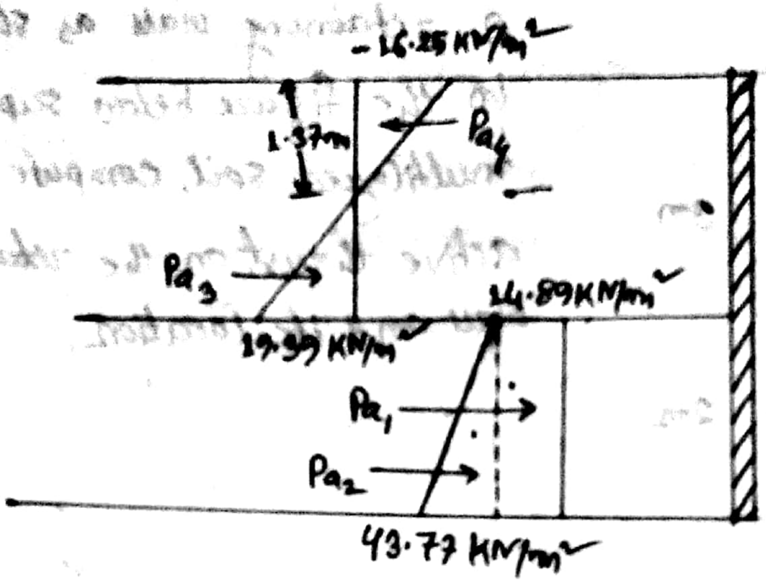
at $H=0$, $P_a = + \cancel{26.15}\ 14.89 \ KN/m^2$

at $P_a = 0$, $14.44z = \cancel{26.15} - 14.89$

$z = 1.03 \ m$.

at $H = 2m$, $P_a = 14.44 \times 2 - 26.15 + 41.04$

$\therefore P_a = \cancel{2.73}\ 43.77 \ KN/m^2$



[এই figure অনুযায়ী calculation করতে হবে]

Now, $Pa_1 = \left(\frac{1}{2} \times 19.39 \times 1.63\right) - \left(\frac{1}{2} \times 1.37 \times 16.25\right) = 4.74 \text{ KN/m}^2$

$Pa_2 = 41.04 \times 2 = 82.08 \text{ KN/m}^2$

$Pa_3 = \left(\frac{1}{2} \times 2.73 \times 0.19\right) - \left(\frac{1}{2} \times 1.81 \times 26.15\right) = -23.41 \text{ KN/m}^2$

Total thrust, $Pa_T = Pa_1 + Pa_2 + Pa_3$

$= 4.74 + 82.08 - 23.41$

$\therefore Pa_T = 63.41 \text{ KN/m}^2$

Location, $\bar{z} = \dfrac{Pa_1 \times \left(2 + \frac{1.63}{3}\right) + \left(Pa_2 \times \frac{2}{2}\right) + Pa_3 \times 0.19/3}{Pa_T}$

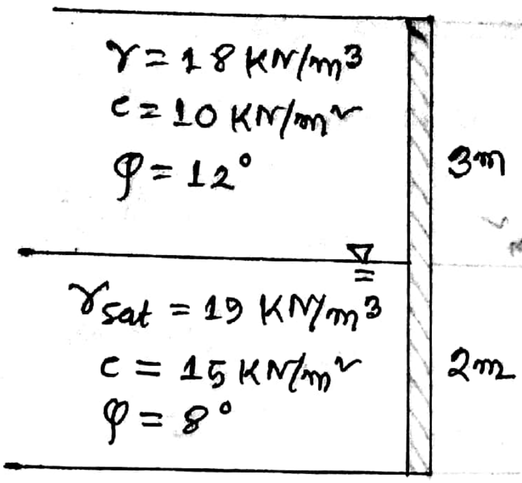
$= \dfrac{4.74 \times (2 + 0.54) + (82.08 \times 1) - 23.41 \times 0.06}{63.41}$

$= \dfrac{12.04 + 82.08 - 1.41}{63.41}$

$\therefore \bar{z} = 1.462 \text{ m}$

(Ans)

Problem-7:



A retaining wall as shown in the figure below supports multilayer soil, compute the active thrust on the retaining wall and its location.

Solution: For top layer,

$$K_a = \frac{1-\sin 12^\circ}{1+\sin 12^\circ} = 0.66$$

we have, $P_a = K_a \gamma z - 2c\sqrt{K_a}$

$$= 0.66 \times 18 \times z - 2 \times 10 \times \sqrt{0.66}$$

$$P_a = 11.88z - 16.25$$

at $H = 0$, $P_a = -16.25 \ KN/m^2$

at, $P_a = 0$, $11.88z = 16.25$

$$z = 1.37 m$$

at $H = 3m$, $P_a = 11.88 \times 3 - 16.25$

$$P_a = 19.39 \ KN/m^2$$

For bottom layer;

$$K_a = \frac{1-\sin 8^\circ}{1+\sin 8^\circ} = 0.76$$

we have, $P_a = K_a \gamma' z - 2c\sqrt{K_a} + \gamma H_1 K_a$

$$= 0.76 \times (19 - 9.81)z - 2 \times 15 \times \sqrt{0.76} + 18 \times 3 \times 0.76$$

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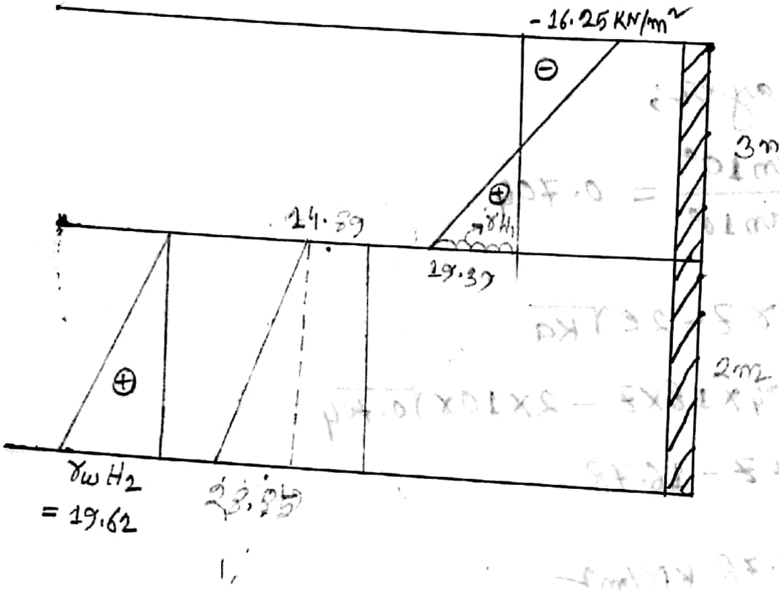
$$P_a = 6.98z - 26.15 + 41.04$$

at. $H = 0$, $P_a = \cancel{26.15}\ 14.89\ KN/m^2$

at $P_a = 0$, $6.98z = \cancel{26.15}\ 0 - 14.89$

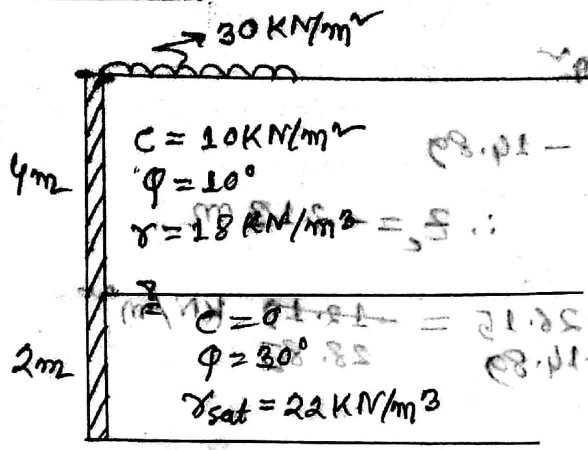
$\therefore \cancel{z = 3.74m}$. $\therefore z_c = -2.13\ m$.

at $H = 2m$, $P_a = 6.98 \times 2 - 26.15 + 14.89 = \cancel{-12.19}\ 28.85\ KN/m^2$.



Problem-8:

For the following multilayer soil profile compute the active thrust on the retaining wall and its location.



Solution: For top layer;

$$Ka_1 = \frac{1-\sin 10^\circ}{1+\sin 10^\circ} = 0.704.$$

we have,

$$Pa = Ka_1 \gamma z - 2c\sqrt{Ka}$$

$$= 0.704 \times 18 \times z - 2 \times 10 \times \sqrt{0.704}$$

$$Pa = 12.67z - 16.78$$

at $z = 0$, $Pa = -16.78 \text{ KN/m}^2$

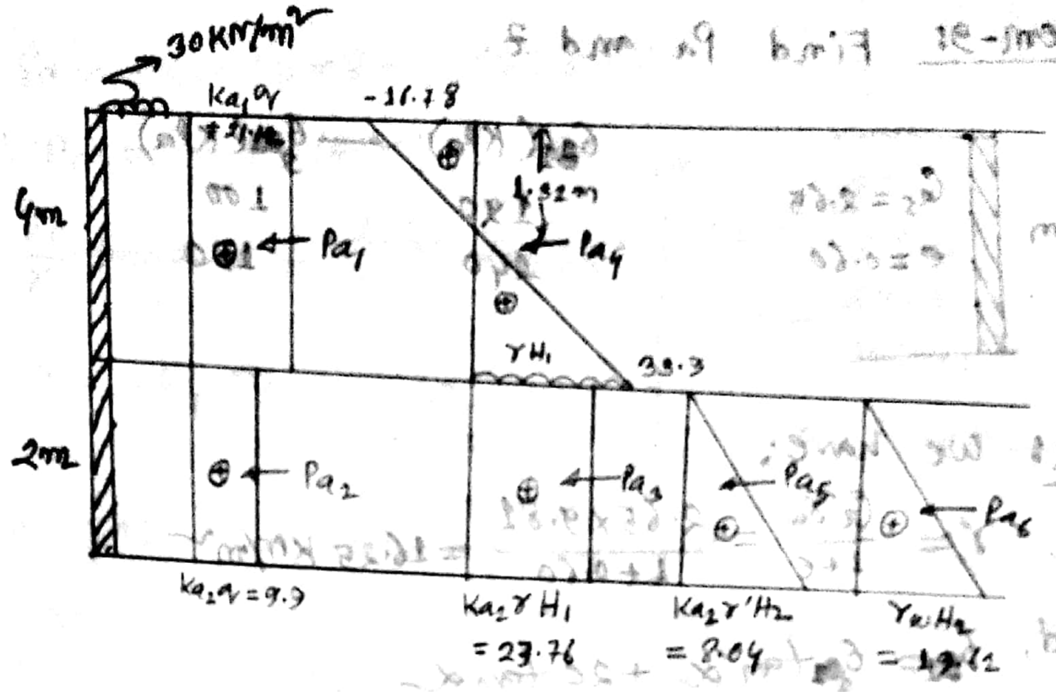
at $Pa = 0$, $z = \frac{16.78}{12.67} = 1.324 \text{ m}$

at $z = 4\text{m}$, $Pa = 12.67 \times 4 - 16.78$

$\therefore Pa = 33.9 \text{ KN/m}^2$

For bottom layer;

$$Ka_2 = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = 0.33$$



Now,

$Pa_1 = 21.12 \times 4 = 84.48\ KN/m^2$

$Pa_2 = 9.9 \times 2 = 19.8\ KN/m^2$

$Pa_3 = 23.76 \times 2 = 47.52\ KN/m^2$

$Pa_4 = (\frac{1}{2} \times 33.9 \times 2.68) - (\frac{1}{2} \times 16.78 \times 1.32) = 34.35\ KN/m^2$

$Pa_5 = \frac{1}{2} \times 8.04 \times 2 = 8.04\ KN/m^2$

$Pa_6 = \frac{1}{2} \times 19.62 \times 2 = 19.62\ KN/m^2$

active thrust, $Pa_T = Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5 + Pa_6$

$= 84.48 + 19.8 + 47.52 + 34.35 + 8.04 + 19.62$

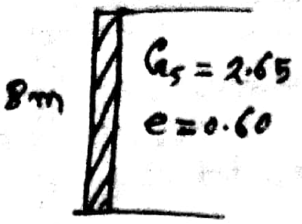
$Pa_T = 213.81\ KN/m^2$

Location, $\bar{z} = \dfrac{(84.48 \times \frac{4}{2}) + (19.8 \times \frac{2}{2}) + (47.52 \times \frac{2}{2}) + (34.35 \times 1.33) + (8.04 \times \frac{2}{3}) + (19.62 \times \frac{2}{3})}{213.81}$

$= \dfrac{168.96 + 19.8 + 47.52 + 45.69 + 5.39 + 13.15}{213.81}$

$\therefore \bar{z} = 1.41\ m$ (Ans)

Problem-9: Find P_a and $\bar{z}$.



σ_1 (KPa)	σ_3 (KPa)
190	100
240	150

Solution: we have;

$$\gamma = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.60} = 16.25 \ KN/m^2$$

and, $\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$

$$\therefore 190 = 100\tan^2\alpha + 2c\tan\alpha \quad \text{——} \ (1)$$

$$-\ 240 = 150\tan^2\alpha + 2c\tan\alpha \quad \text{——} \ (2)$$

$$-50 = -50\tan^2\alpha$$

$$\tan^2\alpha = 1$$

$$\tan\alpha = 1$$

$$\alpha = 45^\circ$$

From (1);

$$190 = 100\tan^2 45^\circ + 2c\tan 45^\circ$$

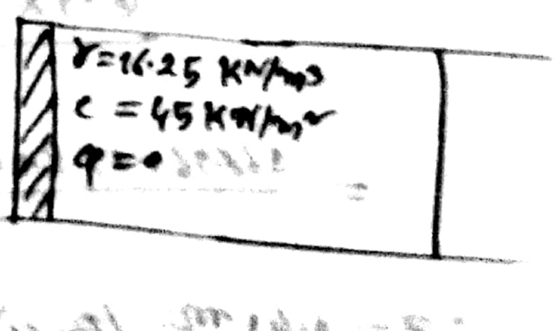
$$190 = 100\cdot(1)^2 + 2\cdot c\cdot 1$$

$$\therefore c = 45 \ KN/m^2$$

Again, $\alpha = 45^\circ + \varphi/2$

$$\Rightarrow 45^\circ = 45^\circ + \varphi/2$$

$$\therefore \varphi = 0$$



$$K_a = \frac{1-\sin 0^\circ}{1+\sin 0^\circ} = 1$$

we have, $P_a = K_a\gamma H - 2c\sqrt{K_a}$

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$$P_a = 1\times16.25\times8 - 2\times45\times\sqrt{1}$$

$$\therefore P_a = 40\ KN/m^2$$

$$z = \frac{2c}{\gamma}\tan\alpha = \frac{2\times45}{16.25}\tan45^\circ$$

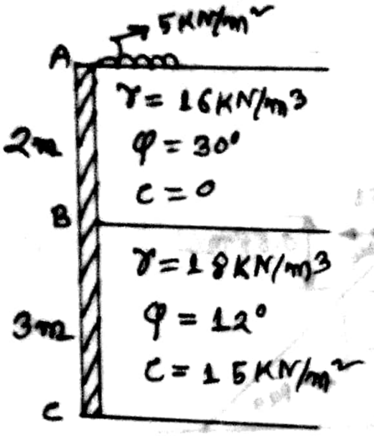
$$\therefore z = 5.54\ m. \quad \text{(Ans)}$$

or

$$Z_c = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2\times45}{16.25\times\sqrt{1}}$$

$$\therefore Z_c = 5.54\ m$$

Problem-10: A retaining wall with a stratified backfill and surcharges as shown in figure below. Draw the earth pressure diagram, detailing the values at the critical points. Also estimate the resultant thrust on the wall and its location.



Solution: For top layer; $K_{a1} = \frac{1-\sin30^\circ}{1+\sin30^\circ} = 0.33$

$$K_{a1}q = 0.33\times5 = 1.65\ KN$$

$$K_{a1}\gamma H_1 = 0.33\times2\times16 = 10.56\ KN.$$

For bottom layer; $K_{a2} = \frac{1-\sin12^\circ}{1+\sin12^\circ} = 0.66$

we have, $P_a = K_{a2}\gamma z - 2c\sqrt{K_a} + K_{a2}q + K_{a2}\gamma_1 H_1$

$$= 0.66\times18\times z - 2\times15\times\sqrt{0.66} + 0.66\times5 + 0.66\times16\times2$$

$$= 11.88z - 24.37 + 3.3 + 10.56$$

$$P_a = 11.88z - 10.51 \quad \text{—①}$$

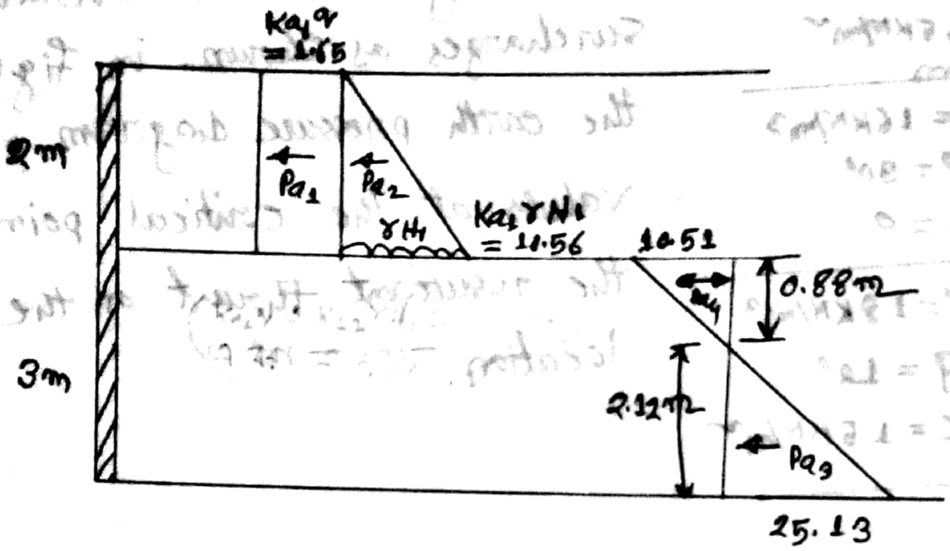
at $z = 0$, $\quad P_a = -10.51 \text{ KN/m}^2$

at $P_a = 0$, $\quad z = \frac{10.51}{11.88} = 0.88\text{ m}$

at $z = 3\text{m}$, $\quad P_a = 11.88 \times 3 - 10.51$

$\therefore P_a = 25.13 \text{ KN/m}^2$

The earth pressure diagram is as follows:



$Pa_1 = 1.65 \times 2 = 3.3 \text{ KN/m}^2$

$Pa_2 = \frac{1}{2} \times 2 \times 10.56 = 10.56 \text{ KN/m}^2$

$Pa_3 = \frac{1}{2} \times 2.12 \times 25.13 = 26.64 \text{ KN/m}^2$

$Pa_4 = \frac{1}{2} \times 0.88 \times 10.56 = 4.65 \text{ KN/m}^2$

Resultant thrust, $Pa_R = Pa_1 + Pa_2 + Pa_3 - Pa_4$

$= 3.3 + 10.56 + 26.64 - 4.65$

$\therefore Pa_R = 35.86 \text{ KN/m}^2$

location, $z = \dfrac{3.3 \times (3 + \frac{2}{2}) + 10.56 \times (3 + \frac{2}{3}) + 26.64 \times \frac{2.12}{3} - 4.65 \times (2.12 + \frac{2}{3} \times 0.88)}{Pa_R}$

$= \dfrac{13.2 + 38.76 + 18.83 + 12.59}{35.86} = 2.33 \text{ m.}$ (Ans.)

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Problem-11: A vertical excavation was made in a clay deposit having unit weight of 21 KN/m³. It caved in after the digging reached 4m depth. Assume, $\varphi = 0°$. calculate the magnitude of cohesion.

Solution: Here,

Max. depth of unsupported cut, $H_c = 4m$

unit weight, $\gamma = 21$ KN/m³

$\varphi = 0°$

$$\therefore K_a = \frac{1 - \sin 0°}{1 + \sin 0°} = 1.$$

we have,

$$H_c = \frac{4c}{\gamma \sqrt{K_a}}$$

$$\Rightarrow c = \frac{H_c \gamma \sqrt{K_a}}{4}$$

$$\Rightarrow c = \frac{4 \times 21 \times \sqrt{1}}{4}$$

$$\therefore c = 21 \text{ KN/m}^2.$$ (Ans.)

All examples from BC punmia Book have to be practiced. (V. V. Imp.)

Example — 20.1 to 20.51

page — 499 (Earth pressure).

STRESS DISTRIBUTION

Introduction:-

sub surface soil experiences two types of loads:

1. Geostatic stress → stress due to self weight of soil.
2. Excess stress → stress due to surcharges.

Sub surface stresses in road pavement and airports runways are increased by wheel load on the surface.

sub surface stresses is also increased due to surcharge from building.

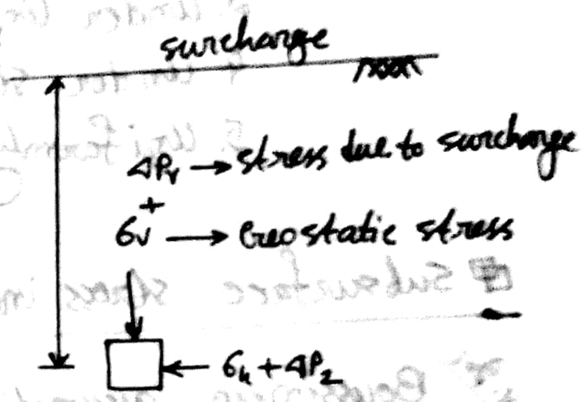


fig: Soil experience two stresses.

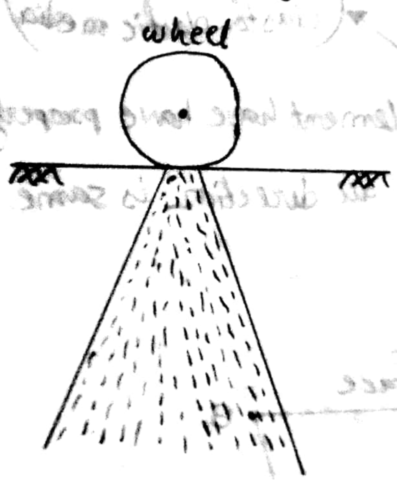


fig: stress increased and spread due to wheel load.

Embankments and landfills causes to increase sub soil stresses.

It is required to estimate the stress increase in the soil due to the applied load on the surface. It is the goal of this chapter.

Calculation of sub-surface stress increase:-

subsurface stress increase —

1. Under point/Concentrated loading.
2. Under uniformly loaded circular area.
3. Under line load.
4. Under strip load.
5. Uniformly loaded rectangular area.

Subsurface stress increase under point load:-

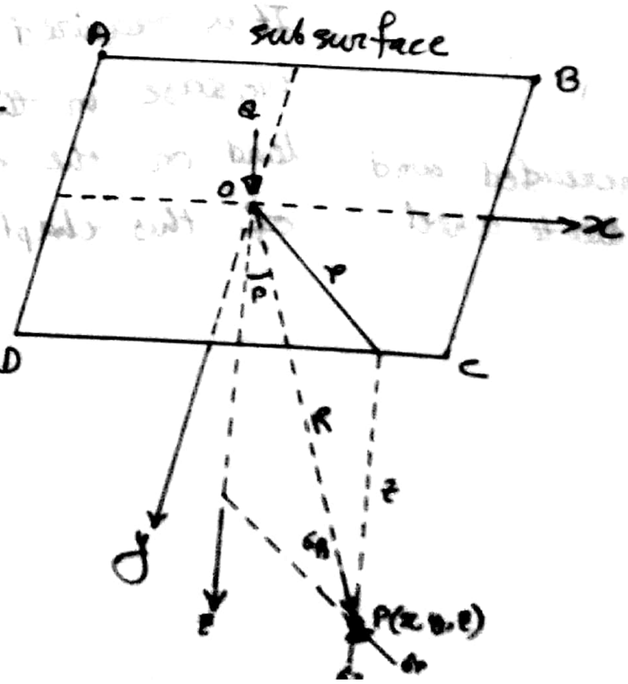
Y.V. Imp

Boussinesq assumptions:

1. The soil mass is an elastic media. → (वास्तवमा soil मा elasto-plastic media)
2. The soil is homogeneous. → All soil element have have property.
3. The soil is isotropic. → Load from all direction is same.
4. The soil mass is semi-infinite.

Derivation:

Let, a point load Q at a point o. The stress component in the soil mass at a point P having coordinate x, y & z or a radial horizontal distance r and vertical distance z from point o.



The polar radial stress;

$$\sigma_R = \frac{3Q\cos\beta}{2\pi R^2}$$

where, $r = \sqrt{\rho^2+z^2}$

$\therefore R = \sqrt{x^2+y^2+z^2}$ $[\because \rho^2 = x^2+y^2]$

$\sin\beta = \frac{r}{R}$

and $\cos\beta = \frac{z}{R}$

The vertical stress. $\sigma_z = \sigma_R \cos^2\beta$

Putting the values of σ_R:

$$\sigma_z = \frac{3Q\cos\beta}{2\pi R^2}\cos^2\beta$$

$$= \frac{3Q\cos^3\beta}{2\pi R^2}$$

$$= \frac{3Q\left(\frac{z}{R}\right)^3}{2\pi R^2}$$

$$= \frac{3Qz^3}{2\pi R^5}$$

$$= \frac{3Q}{2\pi}\frac{1}{z^2}\frac{z^5}{(r^2+z^2)^{5/2}} \quad [\because R^2 = r^2+z^2]$$

$$= \frac{3Q}{2\pi}\cdot\frac{1}{z^2}\left[\frac{z^5}{z^5\{(1+(\frac{r}{z})^2\}^{5/2}}\right]$$

$$= \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(\frac{r}{z})^2}\right]^{5/2}$$

$$\boxed{\therefore \sigma_z = \frac{Q}{z^2}K_B}$$

where, K_B = Boussinesq influence factor $= \frac{3}{2\pi}\left[\frac{1}{1+(\frac{r}{z})^2}\right]^{5/2}$.

[Note: σ_z does not depend on E and poisson ratio]

The tangential stress —

$$\tau_{rz} = \frac{1}{2}\sigma_R \sin 2\beta$$

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$$\tau_{rz} = \frac{1}{2}\,\frac{3Q\cos\beta}{2\pi R^2}\cdot 2\sin\beta\cos\beta$$

$$= \frac{3Q}{2\pi}\,\frac{\cos^2\beta\,\sin\beta}{R^2}$$

$$= \frac{3Q}{2\pi}\,\frac{1}{R^2}\left(\frac{z}{R}\right)^2\left(\frac{r}{R}\right)$$

$$= \frac{3Qz^2 r}{2\pi R^5}$$

$$= \frac{3Q}{2\pi}\,\frac{z^2 r}{(r^2+z^2)^{5/2}} \qquad \left[\because R = \sqrt{r^2+z^2}\right]$$

$$= \frac{3Q}{2\pi}\cdot\frac{1}{z^3}\,\frac{z^5 r}{z^5\left\{1+\left(\frac{r}{z}\right)^2\right\}^{5/2}}$$

$$= \frac{3Qr}{2\pi z^3}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$\boxed{\therefore \tau_{rz} = \frac{Qr}{z^3}K_B} \qquad \boxed{\tau_{rz} = \left(\frac{r}{z}\right)\sigma_z}$$

where, K_B = Boussinesqe influence factor $= \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$

[Note: τ_{rz} does not depend on E and poisson ratio]

<u>Case-I:</u> σ_z directly below the point load. $(r=0)$.

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

Putting, $r = 0$

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+0}\right]^{5/2}$$

$$\boxed{\therefore \sigma_{z(max)} = \frac{0.4775Q}{z^2}}$$

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Vertical pressure distribution diagrams:-

1. Stress isobar or isobar diagram.
2. Vertical pressure distribution on a horizontal plane.
3. Vertical pressure distribution on a vertical plane.

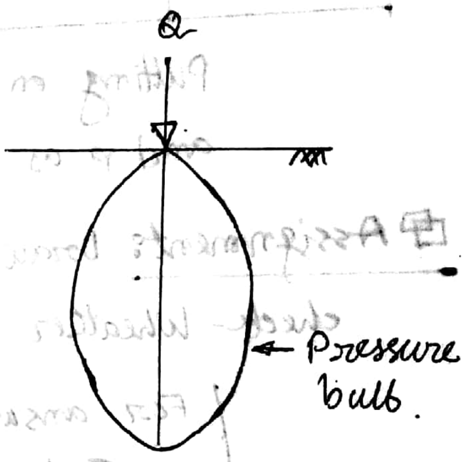
Isobar diagram :-

V.V.Im

An isobar is a curve or contour jointing the points of equal vertical stress below the ground surface.

It's shape is like an electric bulb.

The zone is bounded by an isobar is called pressure bulb.



Drawing an isobar diagram :-

Let us draw an isobar of $\sigma_z = 0.25Q$

We have, $\sigma_z = \frac{Q}{z^2} K_B$

$$\Rightarrow 0.25Q = \frac{Q}{z^2} K_B$$

$$\therefore K_B = 0.25 z^2$$

For maximum, σ_z ; $\sigma_z = \frac{0.4775Q}{z^2}$

$$\Rightarrow 0.25Q = \frac{0.4775Q}{z^2}$$

$\therefore z = 1.38$ unit depth vertically.

We know,

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

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z	K_B	p/z	p
0.2	0.01	1.992	0.384
0.4	0.04	1.30	0.52
0.6	0.09	0.97	0.58
0.8	0.16	0.74	0.59
1.0	0.25	0.54	0.54
1.2	0.36	0.34	0.41
1.38	0.48	0.00	0.00

Putting on a graph paper, z as ordinate (y axis) and p as absissa (x-axis).

Assignment: Draw isobar for $\sigma_z = 0.5Q, 0.75Q, Q, 2Q$ and check wheather the isobar is getting smaller or larger.

(For answer, please see Book. Soil mechanics and foundation BC Punmia, Page → 300)

v.v.Imp

Vertical pressure distribution on a horizontal plane:-

Let us determine the stress at a depth, z = 2 unit.

therefore, $\sigma_z = \frac{Q}{z^2} K_B$

$\sigma_z = \frac{Q}{2^2} K_B = 0.25 Q K_B$

Again, $K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$

r	r/z	K_B	σ_z	%
0.0	0	0.4775	0.1194 Q	100
0.5	0.25	0.4103	0.1026 Q	86
1.0	0.5	0.2773	0.0683 Q	57
1.5	0.75	0.1565	0.0390 Q	32.7
2.0	1.00	0.0844	0.0211 Q	17.7
2.5	1.25	0.0454	0.0113 Q	9.5
3.0	1.50	0.0251	0.0063 Q	5.2
4.0	2.00	0.0085	0.0021 Q	1.8

$\left(= \frac{0.1026}{0.1194} \times 100\%\right)$

When $r = 2z$ (depth) then the vertical pressure due to point load is negligible.

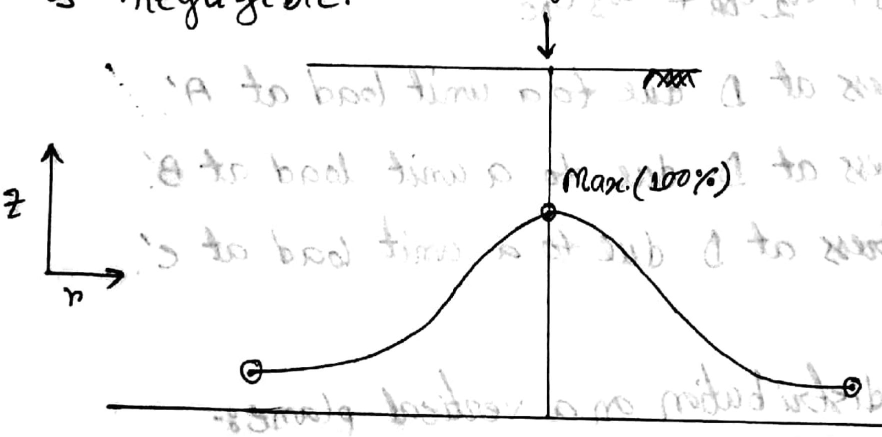

fig: vertical pressure distribution on a horizontal plane.

Influence Diagram :-

v.v.gm

An influence diagram is the vertical stress distribution diagram on a horizontal plane at a given depth due to a unit concentrated load.

It is helpful to determine the vertical stresses at any point on that horizontal plane due to a number of concentrated loads.

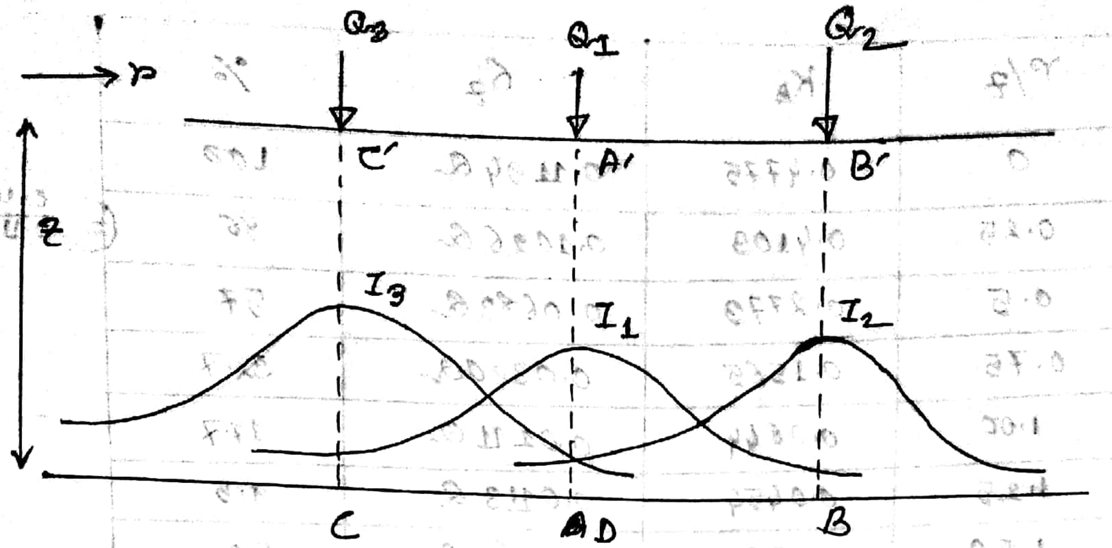

Fig: Influence diagram.

Let us determine the stress at point D.

$$(\sigma_z)_0 = Q_1 \sigma_{DD} + Q_2 \sigma_{DB} + Q_3 \sigma_{DC}$$

σ_{DD} = Vertical stress at D due to a unit load at A'.

σ_{BD} = Vertical stress at D due to a unit load at B'.

σ_{DC} = Vertical stress at D due to a unit load at C'.

Vertical stress distribution on a vertical plane:-

Let us determine the stress at a radial distance $r = 1$ unit.

Therefore;

$$\sigma_z = \frac{Q}{z^2} K_B$$

$$K_B = \frac{3}{2\pi} \left[\frac{1}{1+(r/z)^2} \right]^{5/2}$$

z	r/z	K_B	σ_z
0.25	4.00	0.0004	0.0064Q
0.50	2.00	0.0085	0.0340Q
1.00	1.00	0.0844	0.0844Q
1.50	0.667	0.1904	0.0845Q
2.00	0.50	0.2733	0.0683Q
2.50	0.40	0.3294	0.0527 Q
5.00	0.20	0.4329	0.017Q

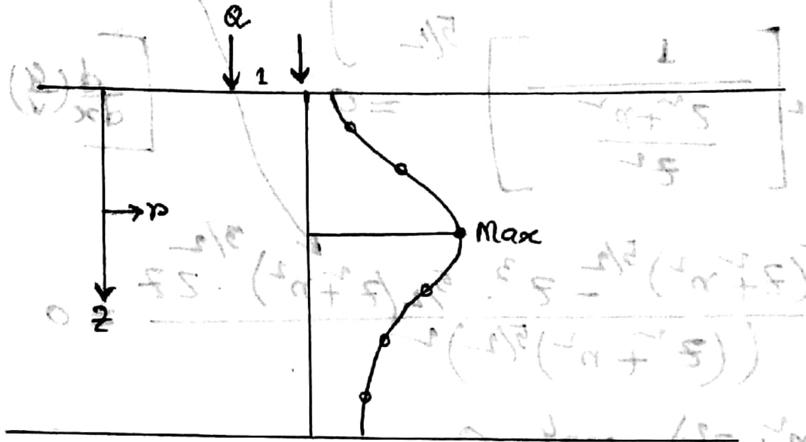

fig: Vertical pressure distribution diagram of a vertical plane.

Problem-1: v.v.imp

Prove that the maximum vertical stress on a vertical line at a constant radial distance r from the axis of a vertical load is induced at the point of intersection of the vertical line with a radial line at $\beta = 39°15'$.

Solution: We have;

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

For maximum, $\frac{d\sigma_z}{dz} = 0$

$$\frac{d}{dz}\left[\frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}\right] = 0$$

$$\Rightarrow \frac{3Q}{2\pi}\cdot\frac{d}{dz}\left(\frac{1}{z^2}\left(\frac{1}{1+\left(\frac{r}{z}\right)^2}\right)^{5/2}\right) = 0$$

$$\Rightarrow \frac{3Q}{2\pi}\,\frac{d}{dz}\left(\frac{1}{z^2}\cdot\frac{z^5}{z^5+r^5}\right) = 0$$

$$\Rightarrow \frac{3Q}{2\pi}\,\frac{d}{dz}\left(\frac{z^3}{z^5+r^5}\right) = 0$$

$$\Rightarrow$$

$$\Rightarrow \frac{d}{dz}\frac{1}{z^2}\left[\frac{1}{\frac{z^2+r^2}{z^2}}\right]^{5/2} = 0$$

$$\frac{d}{dz}\,\frac{1}{z^2}\left[\frac{z^2}{z^2+r^2}\right]^{5/2} = 0$$

$$\therefore \frac{d}{dz}\left[\frac{z^3}{(r^2+z^2)^{5/2}}\right] = 0$$

$$\left[\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\cdot\frac{du}{dx} - u\frac{dv}{dx}}{v^2}\right]$$

$$\Rightarrow \frac{3z^2(z^2+r^2)^{5/2} - z^3\cdot\frac{5}{2}(z^2+r^2)^{3/2}\cdot 2z}{\left((z^2+r^2)^{5/2}\right)^2} = 0$$

$$\Rightarrow 3z^2(r^2+z^2) - 5z^4 = 0$$

$$\Rightarrow 3r^2 + 3z^2 - 5z^2 = 0$$

$$\Rightarrow 3r^2 = 2z^2$$

$$\Rightarrow \frac{r}{z} = \sqrt{\frac{2}{3}} = 0.817$$

$$\Rightarrow \tan\theta = 0.817$$

$$\therefore \theta = 39^\circ 15'$$

(proved)

Maximum value of σ_z, $\sigma_{z(max)} = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$

$\frac{r}{z} = \sqrt{\frac{2}{3}} = 0.817$ বসালেই $\sigma_{z(max)}$ পাওয়া যাবে.

Tangential stress; $\tau_{rz} = \sigma_{z(max)}\cdot\frac{r}{z}$

(Ans)

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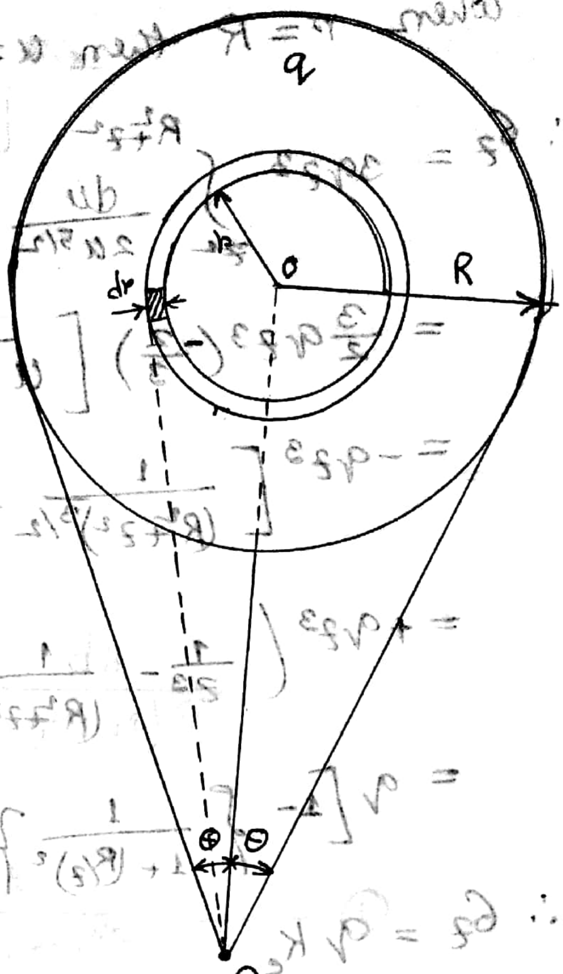
Vertical stress under uniformly loaded circular area:

Let, a uniformly circular loaded area of radius R and load intensity q per unit area.

Consider an elementary ring of radius r and width dr on the loaded area.



The load on elementary ring

$$Q = 2\pi r dr \cdot q$$

We know:

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

Following this equation, we get;

$$\Delta\sigma_z = \frac{3(2\pi r dr \cdot q)}{2\pi} \cdot \frac{1}{z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$\boxed{\text{For } \theta = 45°,\ z = R}$

$$= \frac{3qr dr}{z^2} \cdot \frac{z^5}{(r^2+z^2)^{5/2}}$$

$$= 3qz^3 \cdot \frac{r dr}{(r^2+z^2)^{5/2}}$$

Vertical load due to entire area is given by;

$$\sigma_z = 3qz^3 \int_0^R \frac{r dr}{(r^2+z^2)^{5/2}}$$

Let, $r^2 + z^2 = u$

$$r dr = du/2$$

when $p = 0$ then $u = z^2$

when $p = R$ then $u = R^2 + z^2$

$$\therefore \sigma_z = 3qz^3 \int_{z^2}^{R^2+z^2} \frac{du}{2u^{5/2}}$$

$$= \frac{3}{2} q z^3 \left(-\frac{2}{3}\right) \left[u^{-3/2}\right]_{z^2}^{R^2+z^2}$$

$$= -qz^3 \left[\frac{1}{(R^2+z^2)^{3/2}} - \frac{1}{(z^2)^{3/2}}\right]$$

$$= +qz^3 \left(\frac{1}{z^3} - \frac{1}{(R^2+z^2)^{3/2}}\right)$$

$$= q\left[1 - \left\{\frac{1}{1+(R/z)^2}\right\}^{3/2}\right]$$

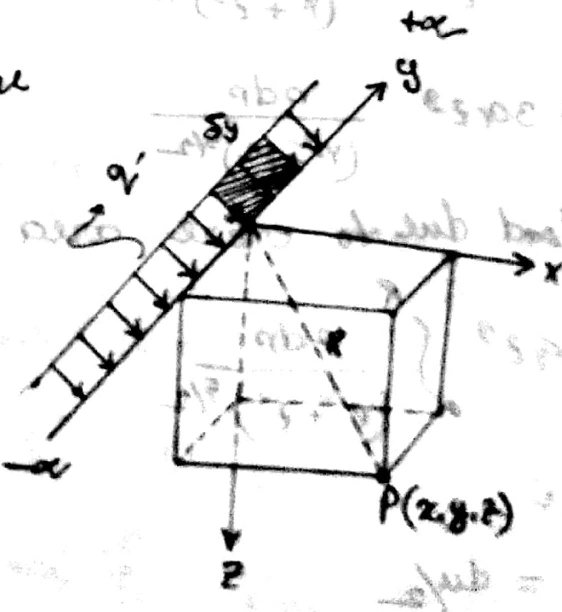
$$\therefore \sigma_z = q\,K_{c}$$

where, $K_c = \left[1 - \left\{\frac{1}{1+(R/z)^2}\right\}^{3/2}\right]$

Vertical Stress under a line Load:-

Consider an infinitely long line load of intensity q' per unit length, acting on the surface of a semi-infinite elastic medium.

Let us consider the load q' acting on a small length δy.



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$\therefore$ point load $= q' \delta y = Q$

we know, $\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$

Following this equation, we get;

$$\Delta\sigma_z = \frac{3q'\delta y}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$\Delta\sigma_z = \frac{3(q'\delta y)}{2\pi}\cdot\frac{z^3}{(r^2+z^2)^{5/2}}$$

By integrating;

$$\sigma_z = \frac{3q'z^3}{2\pi}\int_{-\alpha}^{+\alpha}\frac{dy}{(r^2+z^2)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3}{2\pi}\cdot 2\int_0^{\alpha}\frac{dy}{(r^2+z^2)^{5/2}} \quad \text{——— (1)}$$

Let, $x^2+z^2 = u^2$

$$\therefore \sigma_z = \frac{3q'z^3}{\pi}\int_0^{\alpha}\frac{dy}{(u^2+y^2)^{5/2}} \quad \text{——— (2)}$$

$[\because r^2 = x^2+y^2]$

Let, $y = u\tan\theta$

$dy = u\sec^2\theta\, d\theta$

when $y=0$ then $\theta = 0$

when $y = \alpha$ ~~tan~~ $\theta = \pi/2$

$$\therefore \sigma_z = \frac{3q'z^3}{\pi}\int_0^{\pi/2}\frac{u\sec^2\theta\, d\theta}{(u^2+u^2\tan^2\theta)^{5/2}}$$

$$= \frac{3q'z^3}{\pi}\int_0^{\pi/2}\frac{u\sec^2\theta\, d\theta}{u^5(1+\tan^2\theta)^{5/2}}$$

$$= \frac{3q'z^3}{\pi}\int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{u^4 \sec^5\theta} \quad \left[\because 1+\tan^2\theta = \sec^2\theta\right]$$

$$= \frac{3q'z^3}{\pi u^4}\int_0^{\pi/2} \cos^3\theta\, d\theta$$

$$= \frac{3q'z^3}{\pi u^4}\int_0^{\pi/2} \cos^2\theta \cos\theta\, d\theta$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4}\int_0^{\pi/2} (1-\sin^2\theta)\cos\theta\, d\theta$$

Let, $t = \sin\theta$

$dt = \cos\theta\, d\theta$

when $\theta = 0$ then $t = 0$

when $\theta = \pi/2$ then $t = 1$.

$$\therefore \sigma_z = \frac{3q'z^3}{\pi u^4}\int_0^{1} (1-t^2)\, dt$$

$$= \frac{3q'z^3}{\pi u^4}\left[t - \frac{t^3}{3}\right]_0^1$$

$$= \frac{3q'z^3}{\pi u^4}\left(2/3\right)$$

$$= \frac{2q'z^3}{\pi(x^2+z^2)^2} \quad \left[\because u^2 = x^2+z^2\right]$$

$$\therefore \sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+(x/z)^2}\right]^2$$

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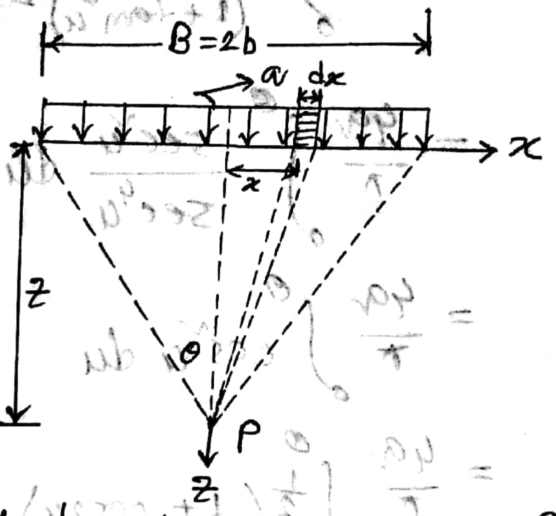
⊞ Vertical stress under a strip load:- ① (v.imp)

Consider an infinite strip of width $B = 2b$, loaded with uniformly distributed load intensity q per unit area; let, vertical axis passing through the center of the trip.



Let us consider, that load acting on a small elementary width dx at a distance x from the center.

$q' =$ line load $= q\,dx$.

We know,

$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+(x/z)^2}\right]^2$$

Following this equation;

$$\Delta\sigma_z = \frac{2q\,dx}{\pi z}\left[\frac{1}{1+(x/z)^2}\right]^2$$

The stress due to entire strip is given by;

$$\sigma_z = \frac{2q}{\pi z}\int_{-b}^{+b}\left[\frac{1}{1+(x/z)^2}\right]^2 dx$$

$$\sigma_z = 2\cdot\frac{2q}{\pi z}\int_{0}^{b}\left[\frac{1}{1+(x/z)^2}\right]^2 dx \quad \text{——— ①}$$

Let, $x/z = \tan\alpha$

$dx = z\sec^2\alpha\,d\alpha$.

when $x = 0$ then $\alpha = 0$

when $x = b$ then $\alpha = \theta$.

From equation ①; we gets

$$\sigma_z = \frac{4q}{\pi z}\int_0^{\theta} \frac{z\sec^2 u\,du}{(1+\tan^2 u)^2}\,du$$

$$= \frac{4q}{\pi}\int_0^{\theta} \frac{\sec^2 u}{\sec^4 u}\,du \qquad \left[\because 1+\tan^2\theta = \sec^2\theta\right]$$

$$= \frac{4q}{\pi}\int_0^{\theta} \cos^2 u\,du$$

$$= \frac{4q}{\pi}\int_0^{\theta} \frac{1}{2}(1+\cos 2u)\,du$$

$$= \frac{2q}{\pi}\left[u + \frac{\sin 2u}{2}\right]_0^{\theta}$$

$$= \frac{2q}{\pi}\left(\theta + \frac{\sin 2\theta}{2}\right)$$

$$= \frac{2q}{\pi}\cdot\left(\frac{2\theta + \sin 2\theta}{2}\right)$$

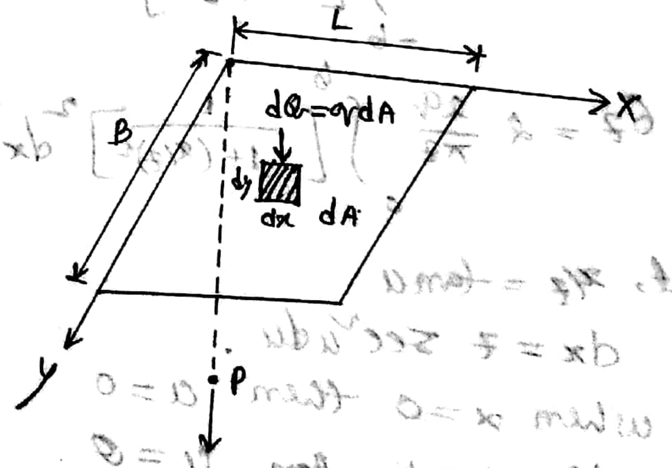
$$\therefore \sigma_z = \frac{q}{\pi}\left(2\theta + \sin 2\theta\right).$$

Vertical stress at a corner of a uniformly loaded rectangular area:-

Let, intensity of load = q

The stress at a depth z is given by taking

$dQ = q\,dA = q\,dx\,dy$.



From Boussinesque equation we have;

$$\Delta\sigma_z = \frac{3Qz^3}{2\pi}\left(\frac{1}{(x^2+y^2+z^2)^{5/2}}\right)$$

$$= \frac{3(q\,dx\,dy)z^3}{2\pi}\left(\frac{1}{(x^2+y^2+z^2)^{5/2}}\right)$$

By integrating we get;

$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^L\int_0^B \frac{dy\,dx}{(x^2+y^2+z^2)^{5/2}}$$

Newmark's solution of the above integration is as follows;

$$\sigma_z = \frac{q}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}} \times \frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1} + \sin^{-1}\left(\frac{mn}{m^2+n^2+m^2n^2+1}\right)\right]$$

$$\sigma_z = \frac{q}{2\pi}\left[\frac{mn}{\sqrt{f}} \times \frac{f+1}{f+m^2n^2} + \sin^{-1}\left(\frac{mn}{f+m^2n^2}\right)\right]$$

$$\therefore \sigma_z = q.K_N$$

where, $f = m^2+n^2+1$

$m = B/z$

$n = L/z$

$$K_N = \frac{1}{2\pi}\left[\frac{mn}{\sqrt{f}} \times \frac{f+1}{f+m^2n^2} + \sin^{-1}\left(\frac{mn}{f+m^2n^2}\right)\right]$$

Alternatively;

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\sqrt{f}}{m^2n^2+f} \times \frac{f+1}{f} + \tan^{-1}\left(\frac{2mn\sqrt{f}}{f-m^2n^2}\right)\right].$$

Case-I:

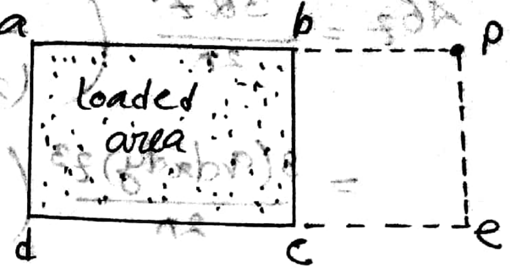
Vertical stress at depth z is, $\sigma_z = K_N q$

K_N = Newmark's influence factor for area abep.

Case-II:

Vertical stress at a depth z is;

$$\sigma_z = q\,(K_{N_1} - K_{N_2})$$

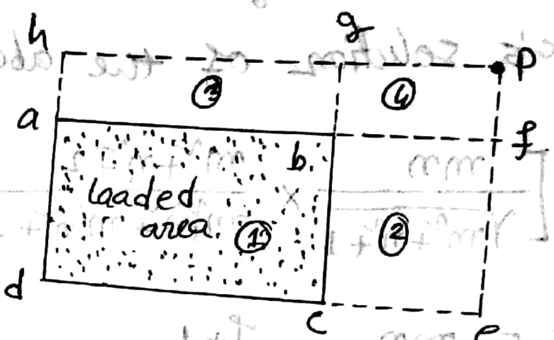


K_{N_1} = Newmark's influence factor for area adep.
K_{N_2} = Newmark's influence factor for area beep.

Case-III:

Vertical stress at depth z is;

$$\sigma_z = q\,(K_{N_1} - K_{N_2} - K_{N_3} + K_{N_4})$$

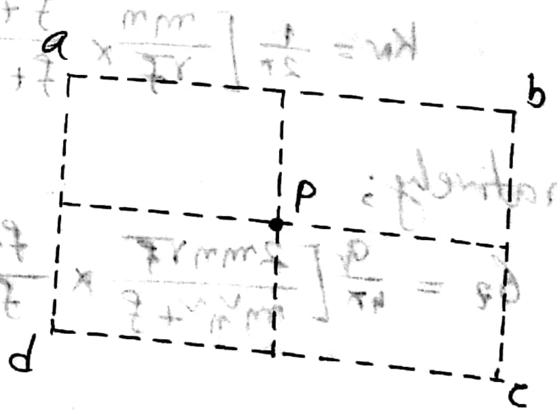


K_{N_1} = Newmark's influence factor for area hdep.
K_{N_2} = Newmark's influence factor for area hafp.
K_{N_3} = Newmark's influence factor for area geep.
K_{N_4} = Newmark's influence factor for area gbfp.

v.v.Imp

Equivalent point load method:-

1. The entire area is divided into a number of small area unit.
2. The distributed load over the unit area is replaced by a point load of the same magnitude acting at the centroid of the area unit.



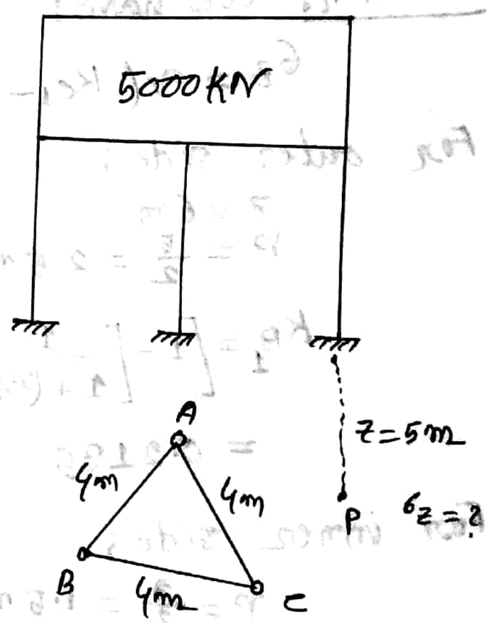
$$\sigma_z = \frac{1}{z^2}\left(Q_1 * K_{B_1} + Q_2 K_{B_2} + Q_3 K_{B_3} + \cdots + Q_n K_{B_n} \right).$$

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

→ **Limitation:** 1. Invalid for irregular shape.
2. The accuracy of the result will depend on the size of the area unit choosen. Smaller the unit higher will be the accuracy.
3. Laborious and time consuming.

Problem-1: A water tank has shown in figure carries 5000 kN of water. The water tank is supported by three leg forming an equilateral triangle at the base. Compute the vertical stress increse under any one of the legs at a depth of 5m from the ground surface.

Solution: We have;



$$\sigma_{z_{C_1}} = K_B \frac{Q}{z^2}$$

Here, $Q = \frac{5000}{3} = 1667\ kN$

$z = 5m$

$r = 0$

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$= 0.4775$

$\therefore \sigma_{z_C} = 31.83\ kN/m^2$

Effect due to A and B on C;

$$\sigma_{z_{C_2}} = K_B \cdot \frac{Q}{z^2}$$

$r = 4m$

$z = 5m$

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$K_B = 0.1386$

$\therefore \sigma_{z_{C_2}} = 9.24\ kN/m^2$

As equilateral triangle

So, $\sigma_{z_{C_3}} = 9.24\ kN/m^2$

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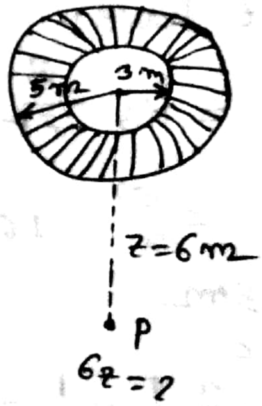
$\therefore \sigma_{zeT} = \sigma_{e1} + \sigma_{ze2} + \sigma_{ze3}$

$= 31.83 + 9.24 + 9.24$

$\therefore \sigma_{zeT} = 50.31\ KN/m^2$ (Ans)

Problem-28 A circularly loaded area as shown in figure is subjected to a uniformly distributed load of 100 KPa. If the outer diameter of the loaded area is 5m and the inner diameter is 3m, calculate the increase in vertical stress at a depth of 6m from ground surface passing through the center of the circular load.



Solution: We have;

$$\sigma_z = q(K_{e1} - K_{e2})$$

For outer side;

$z = 6m$

$r = \frac{5}{2} = 2.5m$

$$K_{B_1} = \left[1 - \left[\frac{1}{1+(r/z)^2}\right]^{3/2}\right]$$

$= 0.2135$

For inner side;

$r = \frac{3}{2} = 1.5m$

$$K_{B_2} = 1 - \left[\frac{1}{1+(r/z)^2}\right]^{3/2} = 0.0869$$

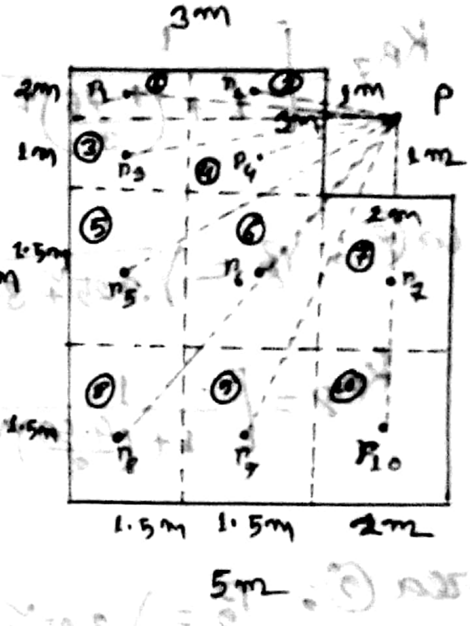
$$\therefore \sigma_z = 100(0.2135 - 0.0869)$$

$$\therefore \sigma_z = 12.66\ KN/m^2$$

(Ans)

Problem-38: A rectangular area as shown in figure is subjected to a uniformly distributed load of 80 kN/m². Calculate the stress increase at point p at a distance of 3m from the ground surface (vertically). using equivalent point load method.

Solution:



$q = 80\ kN/m^2$

For area ①; $r_1 = \sqrt{3.25^2 + 1^2} = 3.4$

$$K_{B_1} = \left[\frac{1}{1+\left(\frac{3.4}{3}\right)^2}\right]^{5/2} = 0.127$$

For area ②; $r_2 = \sqrt{1.75^2 + 1^2} = 2.02$

$$K_{B_2} = \left[\frac{1}{1+\left(\frac{2.02}{3}\right)^2}\right]^{5/2} = 0.393$$

For area ③; $r_3 = \sqrt{3.25^2 + 0.5^2} = 3.29$

$$K_{B_3} = \left[\frac{1}{1+\left(\frac{3.29}{3}\right)^2}\right]^{5/2} = 0.139$$

For area ④; $r_4 = \sqrt{1.75^2 + 0.5^2} = 1.82$

$$K_{B_4} = \left[\frac{1}{1+\left(\frac{1.82}{3}\right)^2}\right]^{5/2} = 0.457$$

For area ⑤; $r_5 = \sqrt{3.25^2 + 1.75^2} = 3.69$

$$K_{B_5} = \left[\frac{1}{1+\left(\frac{3.69}{3}\right)^2}\right]^{5/2} = 0.0999$$

For area ⑥; $r_6 = \sqrt{1.75^2 + 1.75^2} = 2.47$

$$K_{B_6} = \left[\frac{1}{1+\left(\frac{2.47}{3}\right)^2}\right]^{5/2} = 0.274$$

For area ⑦; $r_7 = \sqrt{1.75^2+0^2} = 1.75$

$$K_{B_7} = \left[\frac{1}{1+\left(\frac{1.75}{3}\right)^2}\right]^{5/2} = 0.481$$

For area ⑧; $r_8 = \sqrt{3.25^2 + 3.25^2} = 4.59$

$$K_{B_8} = \left[\frac{1}{1+\left(\frac{4.59}{3}\right)^2}\right]^{5/2} = 0.049$$

For area ⑨; $r_9 = \sqrt{3.25^2+1.75^2} = 3.69$

$$K_{B_9} = \left[\frac{1}{1+\left(\frac{3.69}{3}\right)^2}\right]^{5/2} = 0.099$$

For area ⑩; $r_{10} = \sqrt{3.25^2+0^2} = 3.25$

$$K_{B_{10}} = \left[\frac{1}{1+\left(\frac{3.25}{3}\right)^2}\right]^{5/2} = 0.144$$

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So, stress increase at point P is:

$$\sigma_{zp} = \frac{1}{z^2}\left(Q_1 K_{B1} + Q_2 K_{B2} + Q_3 K_{B3} + Q_4 K_{B4} + Q_5 K_{B5} + Q_6 K_{B6} + Q_7 K_{B7} + Q_8 K_{B8} + Q_9 K_{B9} + Q_{10} K_{B10}\right)$$

$$= \frac{1}{z^2}\big((80\times2\times1.5\times0.127) + (80\times2\times1.5\times0.393) + (80\times1\times1.5\times0.139) + (80\times1\times1.5\times0.457) + (80\times1.5\times1.5\times0.0999) + (80\times1.5\times1.5\times0.274) + (80\times2\times1.5\times0.481) + (80\times1.5\times1.5\times0.049) + (80\times1.5\times1.5\times0.099) + (80\times1.5\times2\times0.144)\big)$$

$$= \frac{1}{(3)^2}(30.48 + 94.32 + 16.68 + 54.84 + 17.98 + 49.32 + 115.44 + 8.82 + 17.82 + 34.56)$$

$$= \frac{1}{9}\times 440.26$$

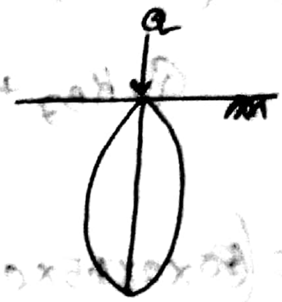
$$\therefore \sigma_{z(P)} = 48.92\ kN/m^2$$

(Ans)

Significance of Pressure Bulb:-

The zone bounded by an isobar is called pressure bulb.

Significance is as follows;

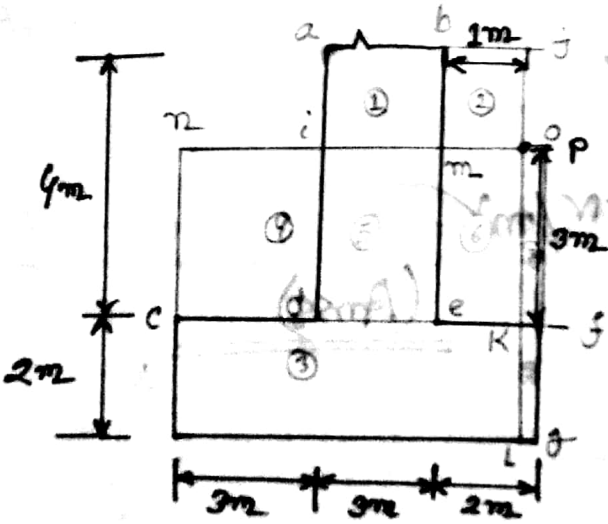


Pressure bulb.

1. The vertical stress at any point on the surface of pressure bulb is same.

2. The smaller the values of σ_z, higher the area of pressure bulb.

Problem-4: A mat foundation as shown in figure below is subjected to a uniformly distributed load of 100 KN/m². Calculate the vertical stress increase at point P at a depth of 5 m below ground surface. Use Newmark's solution.



Area ajpi, $m = \frac{B}{z} = \frac{1}{5} = 0.20$

$n = \frac{L}{z} = \frac{4}{5} = 0.80$

$f = m^2 + n^2 + 1 = (0.20)^2 + (0.80)^2 + 1 = 1.68$

$$KN_1 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{f_c}}{f+m^2n^2}\times\frac{f+1}{f} + \tan^{-1}\left(\frac{2mn\sqrt{f}}{f-m^2n^2}\right)\right.$$

$$= \frac{1}{4\pi}\left[\frac{2\times0.20\times0.80\times\sqrt{1.68}}{1.68+(0.2)^2(0.8)^2}\times\frac{1.68+1}{1.68} + \tan^{-1}\left(\frac{2\times0.20\times0.80\times\sqrt{1.68}}{1.68-(0.2)^2(0.8)^2}\right)\right]$$

$\therefore KN_1 = 0.05$

Area bjpm; $m = \frac{B}{z} = \frac{1}{5} = 0.20$ $\quad f = (0.2)^2+(0.2)^2+1$

$n = \frac{L}{z} = \frac{1}{5} = 0.20$ $\quad \therefore f = 1.08$

$$KN_2 = \frac{1}{4\pi}\left[\frac{2\times0.2\times0.2\times\sqrt{1.08}}{1.08+(0.2)^2(0.2)^2}\times\frac{1.08+1}{1.08} + \tan^{-1}\left(\frac{2\times0.2\times0.2\times\sqrt{1.08}}{1.08-(0.2)^2(0.2)^2}\right)\right]$$

$\therefore KN_2 = 0.017$

Area nplh; $m = \frac{B}{z} = \frac{5}{5} = 1$

$n = \frac{L}{z} = \frac{7}{5} = 1.4$

$f = (1)^2+(1.4)^2+1 = 3.96$

$$KN_3 = \frac{1}{4\pi}\left[\frac{2\times1\times1.4\times\sqrt{3.96}}{3.96+(1)^2(1.4)^2}\times\frac{3.96+1}{3.96} + \tan^{-1}\left(\frac{2\times1\times1.4\times\sqrt{3.96}}{3.96-(1)^2(1.4)^2}\right)\right]$$

$\therefore KN_3 = 0.19$

Area npke; $m = \frac{B}{z} = \frac{3}{5} = 0.6$

$n = \frac{L}{z} = \frac{7}{5} = 1.4$

$f = (0.6)^2+(1.4)^2+1 = 3.32$

$$KN_4 = \frac{1}{4\pi}\left[\frac{2\times0.6\times1.4\times\sqrt{3.32}}{3.32+(0.6)^2(1.4)^2}\times\frac{3.32+1}{3.32} + \tan^{-1}\left(\frac{2\times0.6\times1.4\times\sqrt{3.32}}{3.32-(0.6)^2(1.4)^2}\right)\right]$$

$\therefore KN_4 = 0.147$

Area ipKd;

$$m = \frac{B}{z} = \frac{3}{5} = 0.6$$

$$n = \frac{L}{B} = \frac{4}{5} = 0.8$$

$$f = (0.6)^2 + (0.8)^2 + 1 = 2$$

$$K_{N_5} = \frac{1}{4\pi}\left[\frac{2\times0.6\times0.8\times\sqrt{2}}{2+(0.6)^2(0.8)^2}\times\frac{2+1}{2} + \tan^{-1}\left(\frac{2\times0.6\times0.8\times\sqrt{2}}{2-(0.6)^2(0.8)^2}\right)\right]$$

$\therefore K_{N_5} = 0.12$

Area pKem:

$$m = \frac{B}{z} = \frac{3}{5} = 0.6$$

$$n = \frac{L}{z} = \frac{1}{5} = 0.2$$

$$f = (0.6)^2 + (0.2)^2 + 1 = 1.4$$

$$K_{N_6} = \frac{1}{4\pi}\left[\frac{2\times0.6\times0.2\times\sqrt{1.4}}{1.4+(0.6)^2(0.2)^2}\times\frac{1.4+1}{1.4} + \tan^{-1}\left(\frac{2\times0.6\times0.2\times\sqrt{1.4}}{1.4-(0.6)^2(0.2)^2}\right)\right]$$

$\therefore K_{N_6} = 0.04$

Area pogL;

$$m = \frac{B}{z} = \frac{1}{5} = 0.2$$

$$n = \frac{L}{z} = \frac{5}{5} = 1$$

$$f = (0.2)^2 + (1)^2 + 1 = 2.04$$

$$K_{N_7} = \frac{1}{4\pi}\left[\frac{2\times0.2\times1\times\sqrt{2.04}}{2.04+(0.2)^2(1)^2}\times\frac{2.04+1}{2.04} + \tan^{-1}\left(\frac{2\times0.2\times1\times\sqrt{2.04}}{2.04-(0.2)^2(1)^2}\right)\right]$$

$\therefore K_{N_7} = 0.05$

Area pofk;

$$m = \frac{B}{z} = \frac{1}{5} = 0.20$$

$$n = \frac{3}{5} = 0.6$$

$$f = (0.2)^2 + (0.6)^2 + 1 = 1.4$$

$$K_{N_8} = \frac{1}{4\pi}\left[\frac{2\times0.20\times0.6\times\sqrt{1.4}}{1.4+(0.2)^2(0.6)^2}\times\frac{1.4+1}{1.4} + \tan^{-1}\left(\frac{2\times0.20\times0.6\times\sqrt{1.4}}{1.4-(0.2)^2(0.6)^2}\right)\right]$$

$K_{N8} = 0.04$

$$\sigma_z = q\left(K_{N1} - K_{N2} + K_{N3} - K_{N4} + K_{N5} - K_{N6} + K_{N7} - K_{N8}\right)$$

$$= 100\left(0.05 - 0.179 + 0.19 - 0.147 + 0.12 - 0.04 + 0.05 - 0.04\right)$$

$$\therefore \sigma_z = 16.6\ kN/m^2$$

(Ans)

Newmark's influence chart:

Sometimes vertical stress (σ_z) under different shapes of foundation other than the regular shape is required. In such case, Newmark's influence chart is extremely useful.

Drawing Newmark's influence chart:-

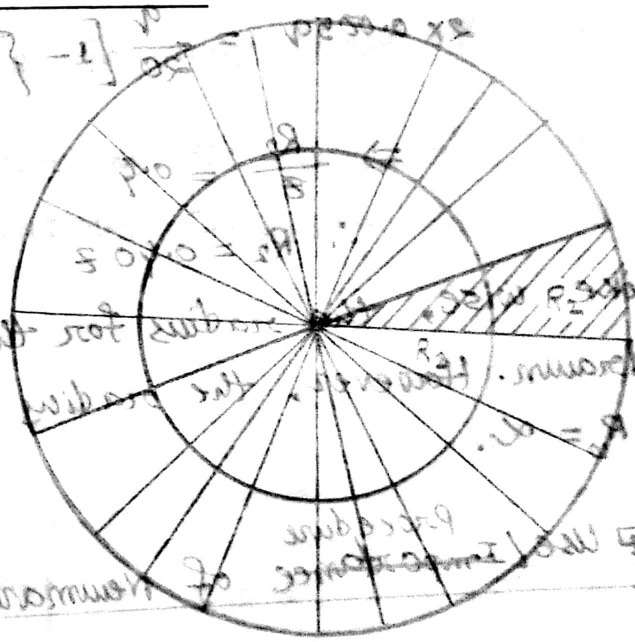


Fig: Newmark's influence chart.

Let us consider a concentric circle of radius R_1. Let the circle be divided into 20 equal sectors.

Intensity of load applied on the circular area is q.

$\therefore$ Load on each sector $= \frac{q}{20}$

vertical stress at the center is given by:

$$\sigma_z = Q n_i K_e$$

$$\therefore \sigma_z = \frac{q}{20}\left[1 - \left[\frac{1}{1+\left(\frac{R_1}{z}\right)^2}\right]^{3/2}\right] \quad \text{--- (1)}$$

Let. $\sigma_z = 0.005\,q$

Now from equation ①, we get:

$$0.005q = \frac{q}{20}\left[1-\left[\frac{1}{1+(R_1/z)^2}\right]^{3/2}\right]$$

$$\Rightarrow \frac{R_1}{z} = 0.27$$

$$\therefore R_1 = 0.27\,z$$

Let us consider a concentric circle of radius R_2. Let the circle be divided into equal 20 sectors.

Let. σ_z for additional area $= 0.005\,q$

Thus σ_z for total $(\frac{1}{20})$ sector $= 20 \times 0.005q$

$= 0.01q$

By using equation ①, we get:

$$2\times 0.005q = \frac{q}{20}\left[1-\left\{\frac{1}{1+(R_2/z)^2}\right\}^{3/2}\right]$$

$$\Rightarrow \frac{R_2}{z} = 0.4$$

$$\therefore R_2 = 0.40\,z$$

Like wise, the radius for third to nine circle can be drawn. However, the radius for the 10th circle, we get: $R_{10} = \alpha$.

Use / ~~Importance~~ Procedure of Newmark's influence chart:-

1. A plan of loaded area is drawn on a tracing paper.
2. The scale choosen such that unit length in Newmark's chart equal to depth (z) at point P below the surface.

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3. The traced plan of the loaded area is placed over Newmark's chart such that point p coincide with the center of the chart.

4. Count the number of small unit area, n covered by the traced plan. Fractions of unit area should also be counted.

Then, $\sigma_z = I \cdot n \cdot q$

Here, I = Influence co-efficient.

n = No. of small unit area.

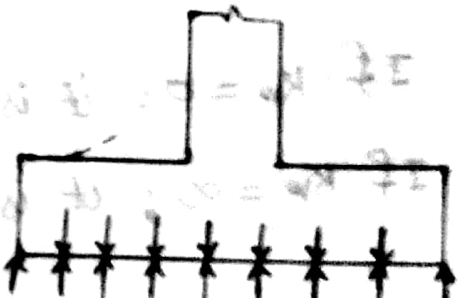
V.V.Imp

Importance of Newmark's influence chart:-

1. Vertical stress at any point under a uniformly loaded area can be determined more accurately.
2. This method can be used for any shape of loaded area.
3. Minimum chances of error as the calculation is less.
4. It is easily and less labourious compared to other method.

Contact pressure distribution:-

The upward pressure due to the soil on the under site of the footing is termed as contact pressure.



contact pressure

sofar, we assumed that, the footing is flexible and contact pressure distribution is uniform. However, actual footing are not flexible as assumed.

Factors affecting the contact pressure :-

1. Elastic properties of the footing materials.
2. Thickness of the footing.
3. Relative rigidity (K_r) of footing soil system.

Relative rigidity :-

- Relative rigidity, K_r is given by ;

$$K_r = \frac{1}{6} \cdot \frac{(1-\gamma_s)^2}{(1-\gamma_f)^2} \cdot \frac{E_f}{E_s} \cdot \frac{t}{b}$$

Where,

γ_s = Poisson ratio of soil.

γ_f = Poisson ratio of footing.

E_s = Modulus of elasticity for soil.

E_f = Modulus of elasticity for footing.

$b = \frac{B}{2}$ = Half width of the footing.

t = Thickness of the footing.

If $K_r = 0$; it is purely flexible footing.

If $K_r = \alpha$; it is perfectly rigid footing.

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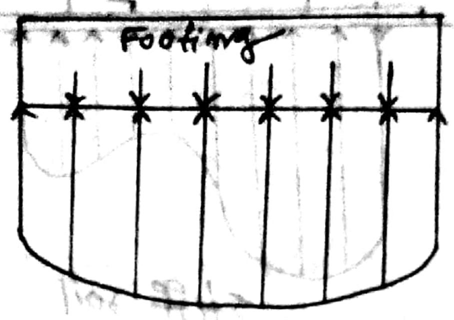
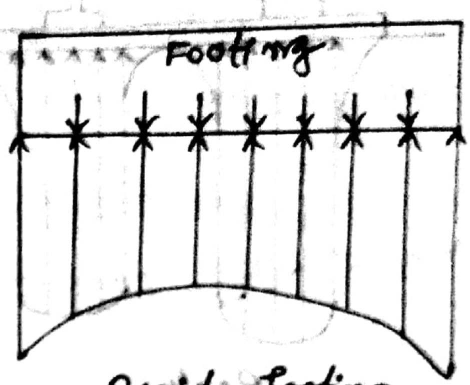
Contact pressure on saturated clay:-

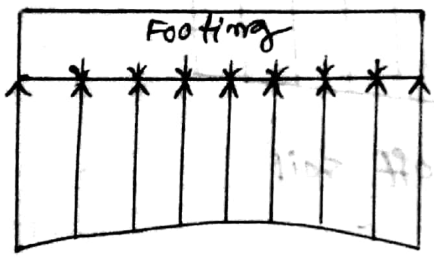


Flexible footing

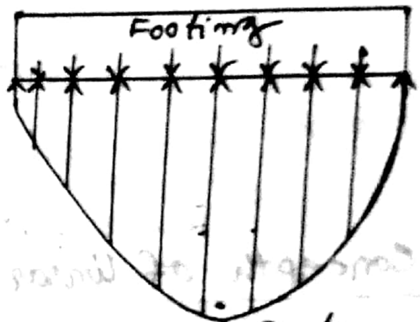


Rigid footing

Contact pressure on sand:-



Flexible footing



Rigid footing.

Contact pressure on mat foundation:-

- The contact pressure distribution for raft or mat footing are quite different from those with spread footing.
- Usually, mat foundation have a much smaller thickness to width ratio and thus more flexible than spread footing.
- Therefore, the assumptions for rigidity is no longer valid.
- Also for linear contact pressure distribution is erroneous.

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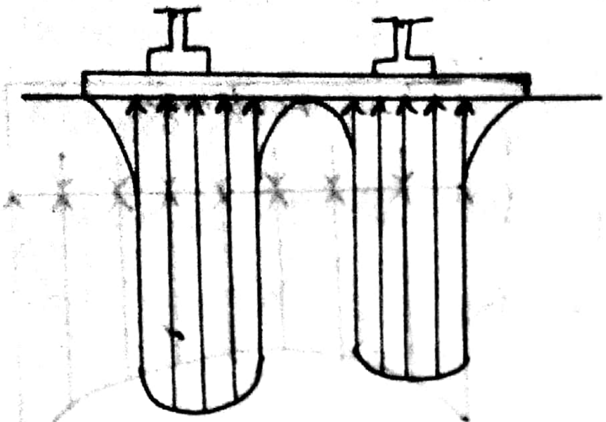
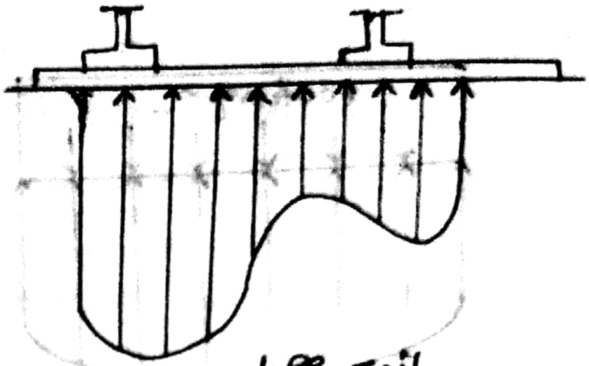
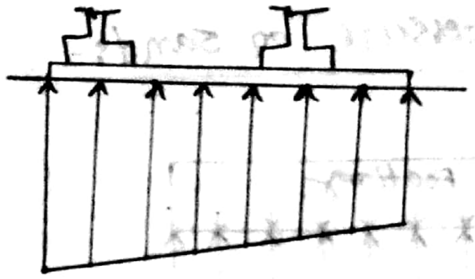


Rock



stiff soil

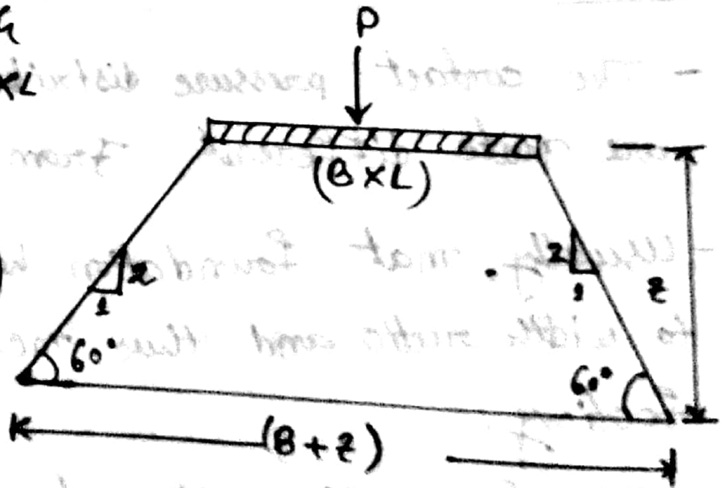


soft soil

Concepth of linear dispersion:-

1. Two-to-One load distribution Method:- (60° dispersion method)

The vertical stress at depth z for a footing of size B×L is given by;



$$\sigma_z = \frac{q(B\times L)}{(B+z)(L+z)} \quad \begin{pmatrix}\text{Rectangular} \\ B \neq L\end{pmatrix}$$

For square area, $(B = L)$

$$\sigma_z = \frac{q \cdot B^2}{(B+z)^2}$$

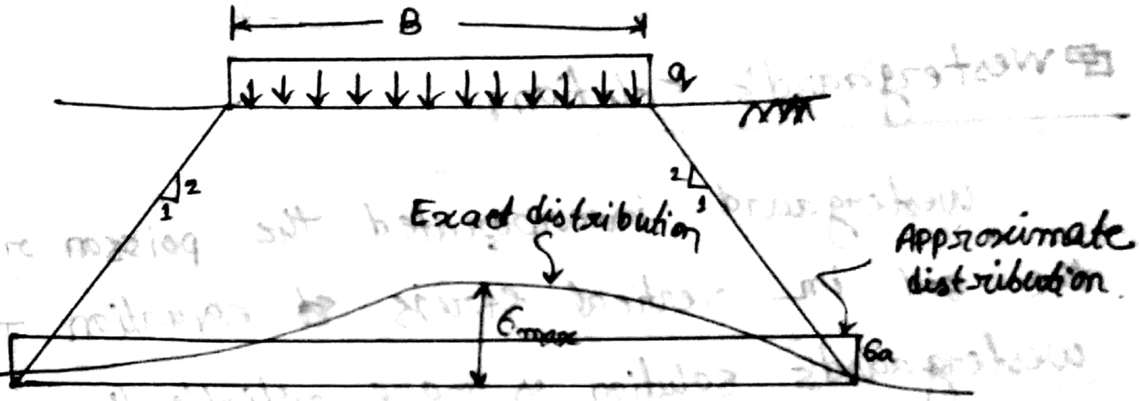
For strip area, $L \gg B$

$$\therefore \sigma_z = \frac{q \cdot B}{(B+z)}$$

For circular area; D = diameter of footing.

$$\sigma_z = \frac{qD^2}{(D+z)^2}$$

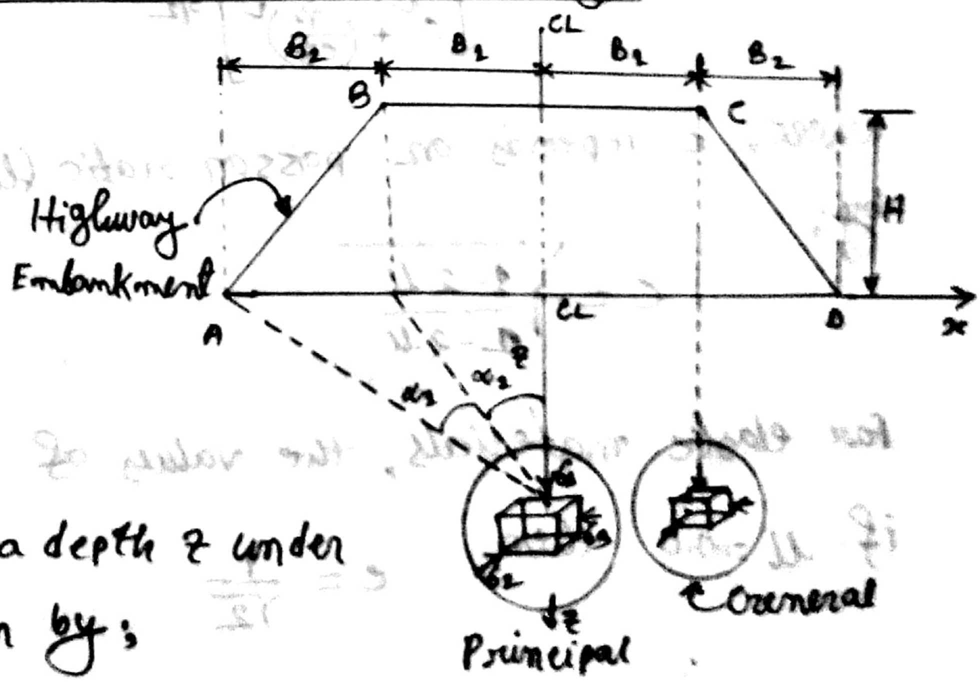
The graphical presentation of actual and approximate distribution is as follows;



This method gives fairly accurate values of the average vertical stress if the depth z is less than 2.5 times of the width of the loaded area.

Vertical stress due to Embankment loading:-

ABCD is the Highway Embankment cross section. $(B_1+B_2) = 2B_1$ is the top width. B_2 is the width of the slope portion.



So, vertical stress at a depth z under the center is given by;

$$\sigma_z = \frac{q}{\pi}\left[\frac{(B_1+B_2)}{B_2}(\alpha_1+\alpha_2) - \left(\frac{B_1}{B_2}\right)\alpha_2\right]$$

Here, $q = \gamma H$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right)$$

$$\alpha_1 = \tan^{-1}\left[\left(\frac{B_1+B_2}{z}\right)\right] - \alpha_2$$

[α_1 and α_2 are in radian.]

Westergaard's solution:-

Westergaard incorporated the poisson ratio (μ) of the soil in vertical stress ~~of~~ equation. This why, westergaard's solution is more relliable than Boussinesq's.

The vertical stress at depth z below concentrated load Q is given by;

$$\sigma_z = \frac{Q}{z^2} \times \frac{\frac{2}{2\pi}}{\left[c^2 + \left(\frac{r}{cz}\right)^2\right]^{3/2}}$$

where, c depends on poisson ratio (μ) of soil and given by;

$$c = \sqrt{\frac{1-2\mu}{2-2\mu}}$$

For elastic materials, the values of μ varies from 0.0 to 0.50

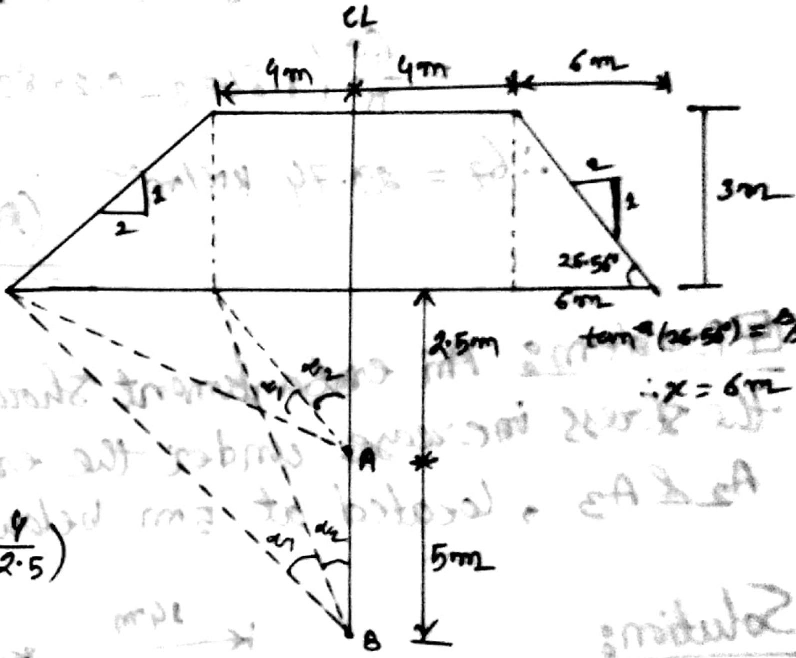
if $\mu = 0.0$ then, $c = \frac{1}{\sqrt{2}}$

then vertical stress;

$$\sigma_z = \frac{Q}{z^2} \cdot \frac{\frac{2}{2\pi}}{\left[\frac{1}{2} + \left(\frac{\sqrt{2}r}{z}\right)^2\right]^{3/2}} = \frac{Q}{z^2} \cdot K_w$$

K_w = Westergaard's influence factor.

Problem: A railway embankment is shown in figure below. Assume the unit weight of soil to be 20 KN/m^3. Compute the increase in vertical stress under the center line at a depth of 2.5m and 5m.



Solution:

$B_1 = 4m$

$B_2 = 6m$

$q = \gamma H = 20 \times 3$

$q = 60\ KN/m^2$

For depth 2.5 m:

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right) = \tan^{-1}\left(\frac{4}{2.5}\right)$$
$$= 1.0122$$

$$\alpha_1 = \tan^{-1}\left[\left(\frac{B_1+B_2}{z}\right)\right] - \alpha_2 = \tan^{-1}\left(\frac{4+6}{2.5}\right) - 1.0122$$
$$= 0.3136$$

We have,

$$\sigma_z = \frac{q}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2) - \left(\frac{B_1}{B_2}\right)\alpha_2\right]$$

$$= \frac{60}{\pi}\left[\left(\frac{6+4}{6}\right)(1.0122 + 0.3136) - \left(\frac{4}{6}\right)(0.3136)\right]$$

$$= \frac{60}{\pi}(2.2097 - 0.2091)$$

$$\therefore \sigma_z = 38.21\ KN/m^2$$

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For depth 5m:

$$\alpha_2 = \tan^{-1}\left(\frac{4}{5}\right) = 0.6747$$

$$\alpha_1 = \tan^{-1}\left(\frac{4+6}{5}\right) - 0.6747 = 0.4325$$

We have,

$$\sigma_z = \frac{q}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2) - \left(\frac{B_1}{B_2}\right)\alpha_2\right]$$

$$= \frac{60}{\pi}\left[\left(\frac{4+6}{6}\right)(0.6747+0.4325) - \left(\frac{4}{6}\right)(0.4325)\right]$$

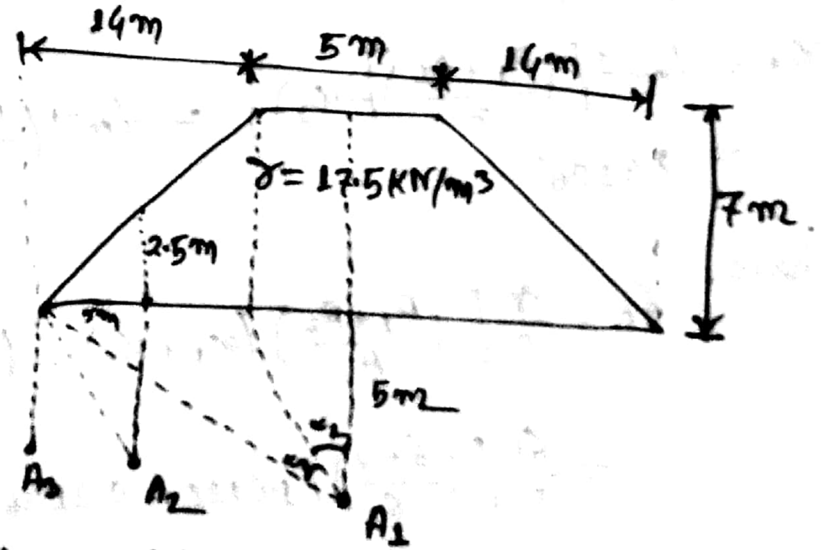
$$= \frac{60}{\pi}(1.8453 - 0.2883)$$

$$\therefore \sigma_z = 29.74 \text{ KN/m}^2$$

(Ans)

□ Problem: An embankment shown in figure below. Determine the stress increase under the embankment at point A_1 and A_2 & A_3, located at 5m below the ground surface.

Solution:



For point A_1:

$B_1 = 2.5$ m

$B_2 = 14$ m

$z = 5$ m

$q = \gamma h = 17.5 \times 7$

$= 122.5$ KN/m²

Now, $\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right) = 0.464$

$$\alpha_1 = \tan^{-1}\left(\frac{B_1+B_2}{z}\right) - \alpha_2$$

$$\alpha_1 = \tan^{-1}\left(\frac{2.5+14}{5}\right) - 0.464$$

$$= 1.277 - 0.464$$

$$\therefore \alpha_1 = 0.813$$

We have, $\sigma_z = \frac{q}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2) - \left(\frac{B_1}{B_2}\right)\alpha_2\right]$

$$\sigma_{z_{A_1}} = \frac{122.5}{\pi}\left[\left(\frac{2.5+14}{14}\right)(0.813+0.464) - \left(\frac{2.5}{14}\right)\times 0.464\right]$$

$$= \frac{122.5}{\pi}\left[(1.179\times 1.277) - (0.179\times 0.464)\right]$$

$$= \frac{122.5}{\pi}(1.506 - 0.083)$$

$$\therefore \sigma_{z_{A_1}} = 55.48 \text{ kN/m}^2$$

(Ans.)

$$\frac{14}{7} = \frac{5}{x}$$

$$x = 2.5 \text{ m}$$

For point A_2:

$q = \gamma h = 17.5 \times 2.5 = 43.75 \text{ kN/m}^2$

$B_1 = 0$ $\quad z = 5$

$B_2 = 5$

$$\therefore \cancel{\tan}\ \alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right) = 0$$

$$\alpha_1 = \tan^{-1}\left(\frac{B_1+B_2}{z}\right) - \alpha_2$$

$$= \tan^{-1}\left(\frac{0+5}{5}\right) - 0$$

$$\therefore \alpha_1 = 0.785$$

Now, $\sigma_z = \frac{q}{\pi}\left[\left(\frac{B_1+B_2}{B_2}\right)(\alpha_1+\alpha_2) - \left(\frac{B_1}{B_2}\alpha_2\right)\right]$

$$= \frac{43.75}{\pi}\left(1\times(0.785+0) - 0\right)$$

$$\sigma_z = 10.93 \text{ kN/m}^2$$

Effect of A_1 on A_2;

$r = \cancel{16.5}\ 16.5 - 5 = 11.5\ m.$

$z = 5\ m.$

$$\sigma_{z_2} = \frac{Q}{z^2} \cdot K_B \qquad K_B = \left\{ \frac{1}{1+\left(\frac{11.5}{5}\right)^2} \right\}^{5/2}$$

$$K_B = 0.0101$$

$$= \frac{122.5}{(5)^2} \times 0.0101$$

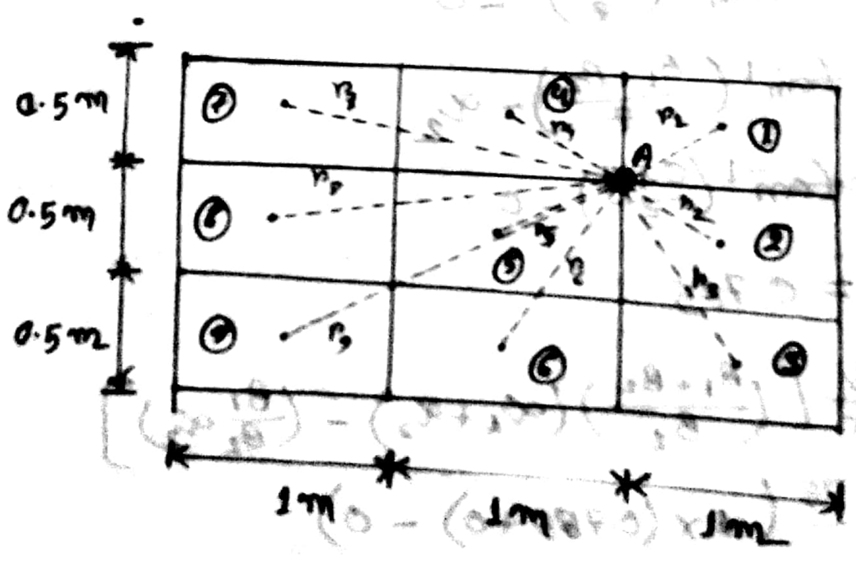
$$\sigma_{z_2} = 0.05\ kN/m^2$$

$\therefore$ Stresses at $A_2 \Rightarrow \sigma_{z_{A_2}} = 10.93 + 0.05$

$$\sigma_{z_{A_2}} = 10.98\ kN/m^2$$

Ans.

Problem: A rectangular foundation 3m × 1.5m carries a uniform load of 50KPa as shown in figure below. Determine the vertical stress at A which is 3m below the ground surface.



For area ①; $r_1 = \sqrt{0.5^2 + 0.25^2} = 0.56$

$$K_{B_1} = \left[\frac{1}{1+\left(\frac{0.56}{3}\right)^2}\right]^{5/2} = 0.918$$

For area ②; $r_2 = \sqrt{0.5^2 + 0.25^2} = 0.56$

$$K_{B_2} = \left[\frac{1}{1+\left(\frac{0.56}{3}\right)^2}\right]^{5/2} = 0.918$$

For area ③; $r_3 = \sqrt{0.75^2 + 0.5^2} = 0.90$

$$K_{B_3} = \left[\frac{1}{1+\left(\frac{0.90}{3}\right)^2}\right]^{5/2} = 0.806$$

For area ④; $r_4 = \sqrt{0.5^2 + 0.25^2} = 0.56$

$$K_{B_4} = \left[\frac{1}{1+\left(\frac{0.56}{3}\right)^2}\right]^{5/2} = 0.918$$

For area ⑤; $r_5 = \sqrt{0.5^2 + 0.25^2} = 0.56$

$$K_{B_5} = \left[\frac{1}{1+\left(\frac{0.56}{3}\right)^2}\right]^{5/2} = 0.918$$

For area, ⑥; $r_6 = \sqrt{0.75^2 + 0.5^2} = 0.90$

$$K_{B_6} = \left[\frac{1}{1+\left(\frac{0.90}{3}\right)^2}\right]^{5/2} = 0.806$$

For area ⑦; $r_7 = \sqrt{1.5^2 + 0.25^2} = 1.52$

$$K_{B_7} = \left[\frac{1}{1+\left(\frac{1.52}{3}\right)^2}\right]^{5/2} = 0.565$$

For area ⑧; $r_8 = \sqrt{1.5^2 + 0.25^2} = \cancel{1.58}\ 1.52$

$$K_{B8} = \left[\frac{1}{1+\left(\frac{1.52}{3}\right)^2}\right]^{5/2} = 0.565.$$

For area ⑨; $r_9 = \sqrt{1.5^2 + 0.75^2} = 1.68$

$$K_{B9} = \left[\frac{1}{1+\left(\frac{1.68}{3}\right)^2}\right]^{5/2} = 0.506.$$

All the small area are equal. So the equivalent point load for each area, $Q = 50 \times 0.5 \times 1$

$Q = 25\ KN$

Now stress increase at point A is,

$$\sigma_{zA} = \frac{Q}{z^2}(K_{B1} + K_{B2} + K_{B3} + K_{B4} + K_{B5} + K_{B6} + K_{B7} + K_{B8} + K_{B9}).$$

$$= \frac{25}{(3)^2}(0.918 + 0.918 + 0.806 + 0.918 + 0.918 + 0.806 + 0.565 + 0.565 + 0.506)$$

$$= \frac{25}{9} \times 6.92$$

$\therefore \sigma_{zA} = 19.22\ KN/m^2$

(Ans.)

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Subsurface Exploration
OR
Soil Investigation

Definition: (3m) v.v (100%)

The process of identifying the layers of soil deposites that underlie a proposed structure and their physical characteristics is generally reffered to as subsurface exploration or soil investigation.

Purposes of subsurface exploration: (v.v. 3m) (100%)

1. Determining the nature of soil at site and its stratification.
2. Obtaining disturbed and undisturbed soil sample for visual identification and approximate laboratory test.
3. Determining the depth and nature of bedrock if and when encountered.
4. Selecting the type and depth of foundation suitable for a given structure.
5. Evaluating the load bearing capacity of soi foundation.
6. Estimating the probable settlement of foundation.
7. Determining the potential foundation problems i.e expansive soil, collapsible soil, sanitary landfill and so on.
8. Determining the location of water Table.
9. Observing the drainage condition from and onto site.

10. Predicting lateral earth pressure for structures such as retaining walls, sheet piles etc.

11. Establishing construction method.

subsurface exploration may also be necessary when addition and alterations to existing structures are contemplated.

Planing for soil Exploration: (How to conduct ??) (Im)

1. compilation of existing information regarding structure.
2. Collection of existing information for subsoil condition; i.e
   - Geologic survey map.
   - Country soil survey map.
   - Soil manual.
   - Existing soil exploration reports prepared for construction of nearby structure
3. Reconnaissance of proposed construction site.
4. Detailed site investigation.

Approximate Minimum Depth of Boring:- (v.Imp)

To determine the approximate minimum depth of boring the ~~rose~~ rules may be used established by the American Society of civil Engineers (ASCE) in 1972.

1. Determination the net increase in the effective stress, $\Delta\sigma'$ under a foundation. As shown in fig.

2. Estimation of the variation of the vertical effective stresses σ_0' with depth.

3. Determine the Depth, $D = D_1$ at which the effective stress increase $\Delta\sigma'$ is equal to $(\frac{1}{10})q$ [q = Estimated net stress on the foundation].

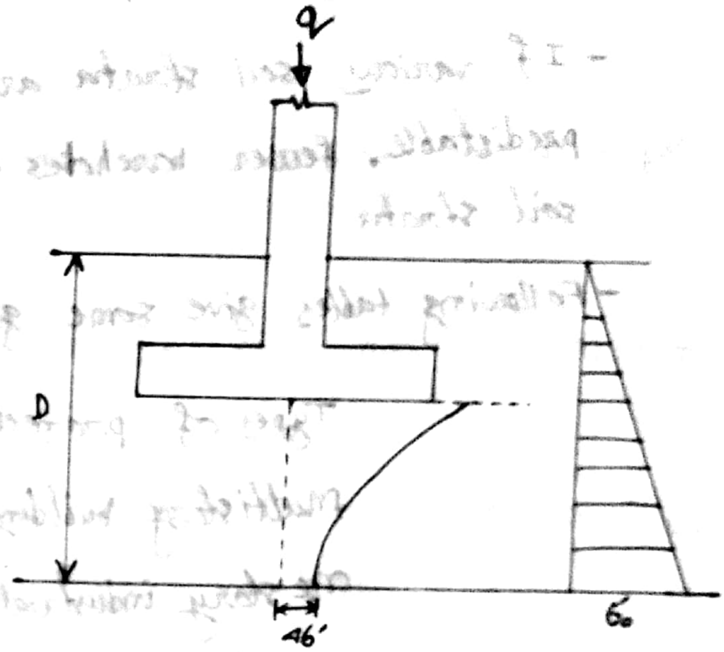

Fig: Determination of minimum depth of boring.

4. Determine the depth $D = D_2$ at which $\frac{\Delta\sigma'}{\sigma_0'} = 0.05$

5. The smaller of two depths D_1 and D_2 just determined as the approximate minimum depth of boring required unless bedrock is encountered.

→ The hole which prepared to collect the sample either disturbed or undisturbed is known as boring or borehole.

Spacing of Boring :-

- There are no hard and first rules for boreholes spacing.
- Spacing can be increased or decreased depending on the condition of sub-soil.
- If various soil strata are more or less uniform and predictable. fewer boreholes are needed than in non-homogeneous soil strata.
- Following tables give some generall guidelines —

Types of project	Spacing (m)
Multistory building	10-30
One story industrial plants	20-60
Highways	250-500
Residential subdivision	250-500
Dams and dikes	40-80

[According to sowers & sowers in 1970]

Methods of Boring :-

Following the methods of boring are generally used:

1. Auger boring
2. Wash boring.
3. Rotary drilling
4. Percusion drilling.

Auger Boring :-

- simplest method of making exploratory boreholes.
- Two types of auger;
  1. Post hole auger.
  2. Helical auger.
- Hand Auger can not be used for advancing holes to depth exceeding 3 to 5 m.
- However, they can be used for soil exploration work on some highway and small structure.
- Portable power driven helical Auger (76 mm to 305 mm in diameter) are available for making deeper bareholes.
- Soil sample are obtained from such boring are highly disturbed.
- In some non-cohesive soil or soils having low cohesion, walls of boreholes will not stand unsupported.
- In such circumstances, a metal pipe is used as a casing to prevent the soil from caving in.

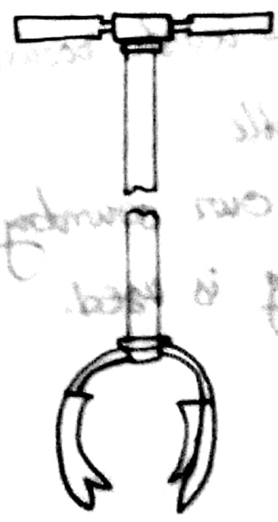

fig: post hole auger.

Wash Boring:-

- In this method, a casing about 2 to 3 m long is driven into the ground.
- Soil inside the casing is then removed by means of a choping bit attached to a drilling rod.
- Water is forced through drilling rod and exists at a very high velocity through holes at the bottom of choping bit.
- Water and chopped soils particles rise in drilled holes and overflow at the top of casing through a T-connection.
- Water is collected in a container
- The casing can be extended with additional pieces as the boreholes progresses; however that is not required if the bore holes will stay open and not cave in.
- This method is rarely used.
- If there is no hard layer then wash boring is suitable.
- In our country wash boring is used.

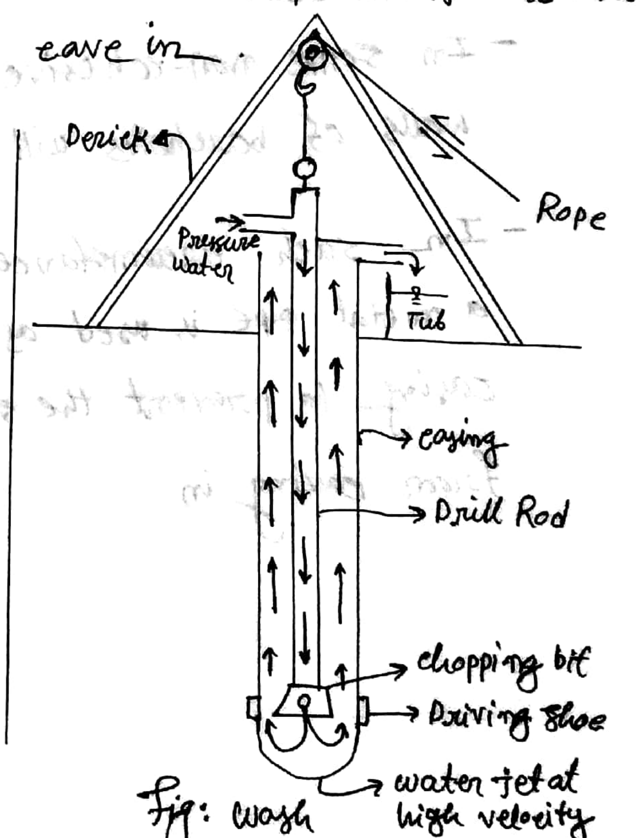

Fig: Wash Boring.

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Rotary drilling:

- By which rapidly rotating drilling bits attached with the bottom of the drilling rods cut and grinds the soil and advanced the borehole.
- Rotary drilling can be used in sand, clay and rocks.
- Water or drilling mud is forced down the drilling rods to the bites and the return flow forces the cutting to the surface.
- Boreholes with diameters of 50 to 203 mm can easily be made by this technique.
- The drilling mud is a slury of water and <u>bentonite.</u>
- Generally, it is used when the soil that is encountered is likely to cave in.
- When soil samples are needed, the drilling rod is raised and the drilling bit is replaced by a samplers.
- With the environmental drilling applications, rotary drilling with air is becoming more common.

Percussion Drilling:

- It is an alternative method of advancing a borehole, particularly through hard soil and rock.
- A heavy drilling bit is raised and lowered to chop the hard soil.
- The choped soil particles are brought up by the circulation of water. Percussion drilling may require casing.

▣ Procedures for Sampling Soil:- (2m) v.v

Two types of soil samples can be obtained during subsurface exploration. They are,

1. Disturbed sample.
2. Undisturbed sample.

Disturbed but representative samples can generally be used for the following types of laboratory test;

a. Grain size Analysis.
b. Determination of atterberg limits.
c. specific gravity of soil solids.
d. Determination of organic content.
e. classification of soil.

Undisturbed soil samples can be used for the following types of laboratory tests;

a. consolidation test.
b. Hydraulic conductivity test.
c. Shear strength test.

▣ Split-Spoon Sampling:- (2m)

Split-Spoon samplers can be used in the field to obtain soil samples that are generally disturbed but still representative. A section of a split-spoon sampler is shown in figure.

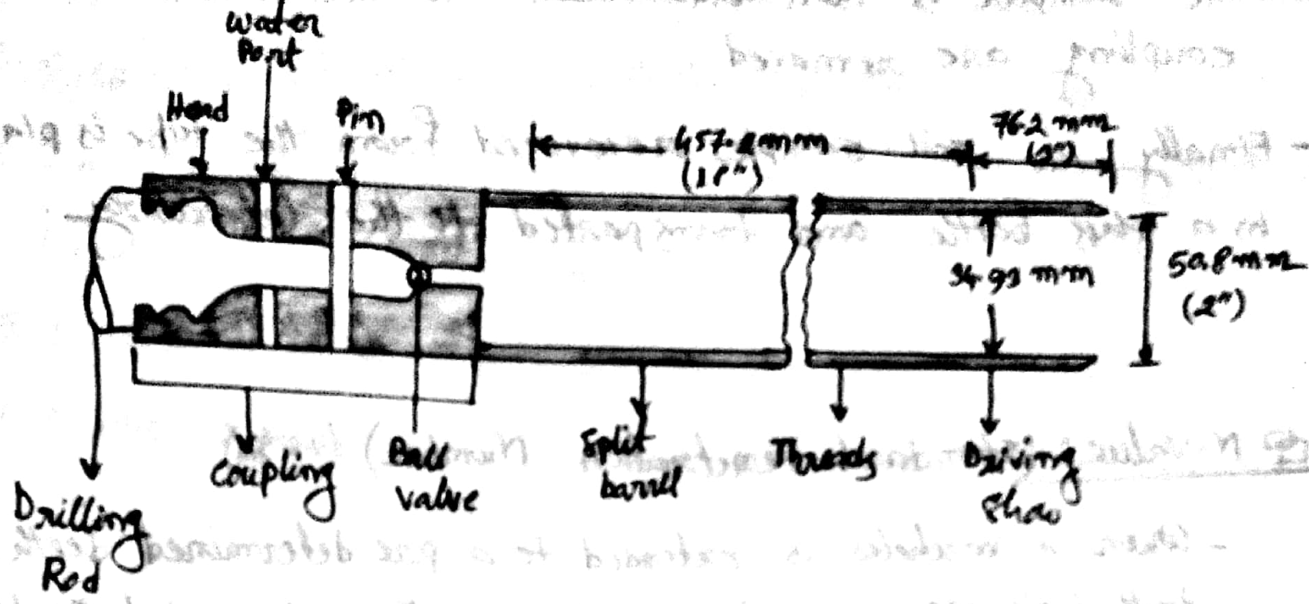

fig. Standard split spoon samplers.

- The tools consist of a steel driving shoe, a steel tube that is split longitudinally in half and a coupling at the top.
- The coupling connects the sampler to the drill rod.
- The standard split tube has an inside diameter of 34.93 mm and an outside diameter of 50.8 mm. However the inside and outside diameter of 63.5 mm and 76.2 mm respectively are also available.
- When a borehole is extended to a predetermined depth, the drills tools are removed and the samplers is lowered to the bottom of the hole
- The samplers is driven into the soils by hammer blows to the top of the drill rod.
- The standard weight of the hammer is 622.72 N and for each blow the hammer drop a distance of 0.762 m.

- The sampler is then withdrawn and the shoe and coupling are removed.
- Finally the soil sample recovered from the tube is placed in a glass bottle and transported to the laboratory.

(10%)

N-value: (Standard Penetration Number) (v.v. Im)

- When a boreholes is extended to a pre-determined depth, the drill tools are removed and the sampler is lowered to the bottom of the hole.
- The sampler is driven into the soil by hammer blows to the top of the drill rod.
- The standard weight of the hammer is 622.72 N (140 lb) and for each blow the hammer drops a distance of 0.762 m (30 inch)
- The Number of blows required for a spoon penetration of three 152.4 mm interavals are recorded.
- The number of blows required for the last two intervals are added to give the standard penetration number, N at that depth.
- This number is generally reffered to as the N value.

(10%)

Importance of N-value: (v.v. Im)

1. N-value is required to corelate the several physical parameters of soil.

N-value: In split-spoon sampling, the sum of th number of blows required for spoon penetration of th last two 6 inch intervals is reffered to as standard Petration number of that depth. This number is grally reffered to as "N-value."

2. The unconfined compression strength q_u is related to N-value.

3. Shear strength of soil is affected by the N-value.

4. In case of granular soil, N-value is highly dependent on effective overburden pressure.

5. N-value is useful guideline in subsoil exploration.

6. N-value is used in various calculations of soil mechanics problem.

7. The consistancy of clayey soil can often be estimated from N-value.

8. Net allowable bearing capacity of soil can be calculated from N-value. (Standard Penetration Number). Bearing capacity of soil is 0.15N ton/ft² (TSF) by Meyerhof.

⧉ N_{60} - Value: (Im) v.v

- Several factors that will contribute to variation of standard penetration Number N at a given depth for similar soil profile.
- These factors include;
  - a. Standard Penetration Test Hammer Efficiency.
  - b. Borehole diameter.
  - c. Sampling Method.
  - d. Rod length factors.
- Two most common types of standard Penetration Test;
  - a. Safety Hammer
  - b. Donut Hammer.

- They commonly are droped by a rope with two wraps around a pally.

- Standard Penetration Test (SPT) hammer efficiency can be expressed as;

$$E_r(\%) = \frac{\text{Actual hammer energy to the sampler}}{\text{Input energy}} \times 100\%$$

Theoritical input energy $= Wh$

where, W = weight of hammer
h = Height of hammer drop.

- In the field, the magnitude of E_r can vary from 30% to 90%.
- The standard practice now in the US is to express the N-value to an average energy ratio of 60% $\left(\approx N_{60} = \frac{30+90}{2} = 60\right)$
- Thus correcting for field procedures and on the basis of the field observation, it appears reasonable to standardize the field ~~pet~~ penetration number as a function of the input driving energy and its dissipation around the sampler into the surrounding soil or

$$N_{60} = \frac{N \cdot \eta_H \cdot \eta_B \cdot \eta_S \cdot \eta_R}{60}$$

where, N_{60} = Standard Penetration Number, corrected for field condition
N = Measured Penetration Number
η_H = Hammer efficiency (%).
η_B = correction for borehole diameter.
η_S = Sampler correction

η_R = Correction for Road length.

Collection of undisturb sample from field: (3m) v.v (90%)

→ How can you collect the undisturb sample from field? What do you mean by undisturbed sample?

- There are Two ways.

  1. Sampling by thin wall Tube.
  2. Sampling by piston sampler.

① Sampling by thin wall Tube:

- It is used for obtaining fairly undisturbed soil sample.
- Thin wall Tubes are made of seamless thin tube and commonly are reffered to as shelby tubes.
- To collect sample at a given depth in a borehole at first removed drilling tools.
- Sampler is attached to a drilling rod and lowered to the bottom of the borehole.
- After this, it is pushed hydraulically into the soil.
- The soil sample inside the tube is then pulled out.
- Sampler with soil inside is sealed and taken to the laboratory for testing.
- Outside diameter is 50.8mm (2") and inside diameter is 47.63mm ($1\frac{7}{8}$in) of a standard thin wall tube or Shelby tubes.

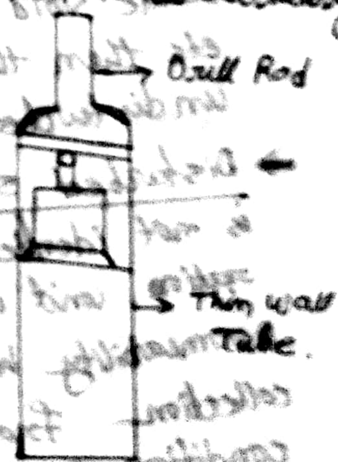

fig: Thin wall Tube.

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③ Sampling by a Piston Sampler:

- It is particularly useful when highly undisturbed soil samples are required.
- It consists of a thin wall tube with piston.
- Initially, Piston closes end of Thin Wall Tube.
- Sampler is first lowered to the bottom of the borehole.
- Then Thin wall Tube is pushed into the soil hydraulically, past the piston.
- After this, Pressure is realesed through a hole in piston rod.
- The presence of piston prevent distortion in the sample by not letting the soil squeeze into the sampling tube very fast and by not admitting excess soil.

(- Samples obtained by this manner is less disturbed than Shelby Tubes)

→ Undisturb sample: If the collected soil sample is identical to its field condition that means isotropic, same void ratio, same unit weight etc then this sample is called undisturbed sample.

→ Disturbed sample: If the soil sample is not identical that means its void ratio, unit weight, pressure and permeability have been changed after collection from field to its field condition then this sample is called disturbed sample.

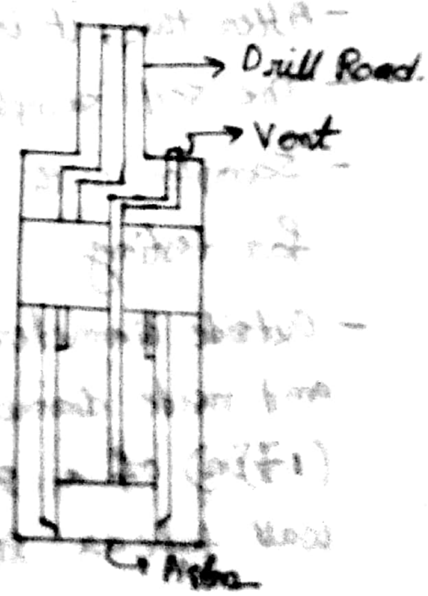

fig: Piston Sampler

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Area Ratio: (v.v. Imp)

- Degree of disturbance of sample collected by various methods can be expressed by a term called area ratio, which is given by:

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} \times 100\%$$

where, D_o = Outside diameter of the sampler.
D_i = Inside diameter of the sampler.
A_r = Area Ratio

- A soil sample is generally can be considered undisturbed if the Area ratio (A_r) is less than or equal to 10%.

→ For standard split-spoon sampler;

$D_i = 1.38''$
$D_o = 2''$

$$\therefore A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} = \frac{(2)^2 - (1.38)^2}{(1.38)^2} \times 100\% = 110\%$$

(Highly disturbed)

→ For Shelby Tube (Thin wall Tube);

$D_i = 1.875''$
$D_o = 2''$

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} = \frac{(2)^2 - (1.875)^2}{(1.875)^2} \times 100\% = 13.8\%$$ (Fairly undisturbed)

→ For Piston Sampler; $D_i = 1.875''$

$D_o = 2''$

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} = 13.8\%$$

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Correlations for N_{60} in cohesive soil:-

- The consistancy of clayey soil can be estimated from Standard Penetration Number, N_{60}.
- In order to achieve that, Szechy and Vargi (1978) calculated the consistancy index (CI) as;

$$CI = \frac{LL-\omega}{LL-PL}$$

where, ω = Moisture content
LL = liquid limit
PL = Plastic limit

- The approximate correlation between CI, N_{60} and unconfined compression strength q_u is as follows;

Standard Penetration Number, N_{60}	Consistancy	CI	unconfined compression strength, q_u, KN/m^2
<2	very soft	<0.5	<25
2-8	soft to medium	0.5-0.75	25-80
8-15	stiff	0.75-1.0	80-150
15-30	Very stiff	1.0-1.50	150-400
>30	Hard	>1.50	>400

Correction for N_{60} value in Granular soil:

- The value of N_{60} obtained from the field should be corrected for the following cases;

1. Correction for overburden pressure.
2. Correction for submergence.

① Correction for Overburden Pressures:-

Y.V.Jm

- In granular soil, the value of N is affected by the effective overburden pressure, σ_0'.
- The value of N_{60} obtained from field exploration under different effective overburden pressure should be changed to correspond to a standard value of σ_0'. That is;

$$(N_1)_{60} = C_N N_{60}$$

where,

$(N_1)_{60}$ = Value of N_{60} corrected to a standard value of σ_0'. $\boxed{100\,KN/m^2\ (2000\,lb/ft^2)}$.

N_{60} = Value of N obtained from field exploration.

C_N = Correction factor.

Emperical relations for C_N:

Y.V.Jm

σ_0' = Effective overburden pressure.

P_a = Atmospheric pressure ($\approx 100\,KN/m^2$)

According to Liao and Whitman (1986);

In SI unit, $C_N = \left(\frac{1}{\frac{\sigma_0'}{P_a}}\right)^{0.5}$

$\boxed{1\text{ US ton} = 2000\,lb}$

In English unit, $C_N = \left(\frac{1}{\sigma_0'}\right)^{0.5}$ $[\because P_a = 1\,K/m^2]$ $[\sigma_0' = \text{US ton}/ft^2]$

According to Skemption's (1986);

For SI unit $C_N = \frac{2}{1+\left(\frac{\sigma_0'}{P_a}\right)}$ (For normally consolidated fine sand)

$$C_N = \frac{3}{2+\left(\frac{\sigma_0'}{P_a}\right)}$$ (For normally consolidated coarse sand)

$$C_N = \frac{1.70}{0.70+\left(\frac{\sigma_0'}{P_a}\right)}$$ (For over consolidated sand)

For English unit; $P_a = 1\,KN/m^2$, $\sigma_0 = \square\ ton/ft^2$

$$C_N = \frac{2}{1+\sigma_0'}$$ (For normally consolidated fine sand)

$$C_N = \frac{3}{2+\sigma_0'}$$ (For normally consolidated coarse sand)

$$C_N = \frac{1.70}{0.70+\sigma_0'}$$ (For over consolidated sand)

② Correction for submergence:-

- In very fine, silty and saturated sand the effect of submergence is greater. So correction are needed.
- Tarzaghi and Peck corrected the N_{60} value for submergence, when the value of N_{60} is more than 15 than this corection is valid. Their correction is as follows;

$$(N_1)_{60} = 15 + \frac{1}{2}(N_{60} - 15)$$

$(N_1)_{60}$ = Corrected N_{60} value.

N_{60} = Values of N obtained from field exploration.

Correlation between N_{60} and Relative Density (D_r):

- Kulhawy and Mayne (1990) modified an emperical relationship for relative density that was given by Marcuson and Bieganousky (1977), which can be expressed as;

$$D_r(\%) = 12.2 + 0.75\left[222 N_{60} + 2311 - 711 OCR - 779\left(\frac{\sigma_o'}{P_a}\right) - 50 C_u^2\right]^{0.5}$$

where, D_r = Relative Density.

σ_o' = Effective overburden pressure.

C_u = Uniformity co-efficient of sand. $= \frac{D_{60}}{D_{10}}$.

P_a = Atmospheric pressure.

$OCR = \frac{\text{Preconsolidation pressure, } \sigma_c'}{\text{Effective overburden pressure, } \sigma_o'}$

Meyerhof (1957) developed a correlation between D_r and N_{60} as;

$$N_{60} = \left[17 + 24\left(\frac{\sigma_o'}{P_a}\right)\right] D_r^2$$

$$\therefore D_r = \left[\frac{N_{60}}{17 + 24\left(\frac{\sigma_o'}{P_a}\right)}\right]^{0.5}$$

[Provides a reasonable estimate only for clean medium fine sand.]

Cubrinovski and Ishihara (1999) also proposed a correlation between N_{60} and the relative density of sand (D_r) than can be expressed as;

$$D_r(\%) = \left[\frac{N_{60}\left(0.23+\frac{0.06}{D_{50}}\right)^{1.7}}{9}\left(\frac{1}{\frac{\sigma_0'}{P_a}}\right)\right]^{0.5} \times 100\%$$

Correlation between $(N_1)_{60}$ and D_r:

Relation between $(N_1)_{60}$ and D_r for sand is as follows;

Standard Penetration Number, $(N_1)_{60}$	Approximate Relative Density, $D_r(\%)$
0–5	0–5
5–10	5–30
10–30	30–60
30–50	60–95

Kulhaway and Mayne (1990) correlated $(N_1)_{60}$ and Relative density of sand in the form of;

$$D_r(\%) = \left[\frac{(N_1)_{60}}{C_p C_A C_{OCR}}\right]^{0.5} \times 100\%$$

where,

C_p = Grain size correlation factor $= 60 + 25\log D_{50}$

C_A = Correlation factor for Aging $= 1.2 + 0.05\log\left(\frac{t}{100}\right)$

t = Age of soil since deposition (year)

C_{OCR} = Correlation factor for overconsolidation $= (OCR)^{0.18}$

OCR = overconsolidation Ratio

D_{50} = Diameter through which 50% soil will pass through (mm)

Correlation between N_{60} and Angle of friction (φ'):-

1. Peck, Hanson and Thornburn (1974) give a correlation between N_{60} and φ' in a graphical form, which can be approximated as;

$$\varphi'(\text{degree}) = 27.1 + 0.3N_{60} - 0.00054[N_{60}]^2$$

$$\varphi'(\text{degree}) = 27.1 + 0.3(N_1)_{60} - 0.00054[(N_1)_{60}]^2.$$

2. Schmertmann (1975) provided the correlation between N_{60}, σ_o' and φ'. Mathematically the correlation can be expressed as;

$$\varphi' = \tan^{-1}\left[\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma_o'}{P_a}\right)}\right]^{0.34}$$

where,
N_{60} = Field standard penetration Number.
σ_o' = Effective overburden pressure.
P_a = Atm pressure.
φ' = Friction angle.

3. Hatanaka and Uchida (1996) provided a simple correlation between φ' and $(N_1)_{60}$ that can be expressed as;

$$\varphi' = \sqrt{20(N_1)_{60}} + 20$$

The following qualificantions should be noted when standard Penetration Resistance values are used in the preceeding correlation to estimate soil parameters;

1. The equations are approximate.
2. Because of the soil is not homogeneous, the ~~no~~ values of N_{60} obtained from a given bore hole vary widely.
3. In soil deposits that contain large boulders and gravel, Standard Penetration Number may be erratic and unreliable.

Correlation between N_{60} and Elasticity Modulus (E_s):

The modulus of elasticity of granular soil (E_s) is an important parameter in estimating the elastic settlement of foundation. A first order estimation for E_s was given by Kulhawy and Mayne (1990) as;

$$\frac{E_s}{P_a} = \alpha N_{60}$$

where, E_s = Modulus of Elasticity

P_a = atm pressure.

N_{60} = Field standard Penetration Number.

$$\alpha = \begin{cases} 5 \text{ for sand with fines.} \\ 10 \text{ for clean normally consolidated sand.} \\ 15 \text{ for clean over consolidated sand.} \end{cases}$$

Sources of Error in SPT:

- Although approximate, with correct interpretation the standard penetration Test provides a good evaluation of soil properties.

- The primary sources of error in SPT are as;
  - Inadequate cleaning of borehole.
  - Careless measurement of blow count.
  - Eccentric hammer strikes on drill rod.
  - Inadequate maintainance of water head in the borehole.

Vane Shear Test (VST):- v.v.Imp

- Vane shear test may be used during drilling operation to determine in situ undrained shear strength (C_u) of clay soils particularly soft clays.
- Vane shear apparatus consist of four blades on the end of a rod.
- The height, H of the vane is twice of the diameter, D.
- The vane either can be rectangular or tapered.
- The vanes of the apparatus are pushed into the soil at the bottom of a borehole without disturbing the soil appreciably.
- Torque is applied at the top of the rod to rotate the vane at a standard rate of 0.1°/sec.
- This rotation will induce failure in a soil of cylindrical shape surrounding the vane.
- Maximum torque T applied to cause failure is measured.

$$T = f(C_u, H, D)$$ where.

$$\therefore C_u = \frac{T}{K}$$

T is in Nm

C_u is in kN/m^2

K = a constant with magnitude depending on the dimension and shape of the vane.

$$\therefore K = \left(\frac{\pi}{10^6}\right)\left(\frac{D^2 H}{2}\right)\left(1 + \frac{D}{3H}\right)$$

In English unit;

T is in lb-ft

c_u is in lb/ft²

$$K = \left(\frac{\pi}{1728}\right)\left(\frac{D^2 H}{2}\right)\left(1 + \frac{D}{3H}\right)$$

if $\frac{H}{D} = 2$ then

$$K = 0.0021\, D^3 \text{ (in)}$$

$$K = 366 \times 10^{-8} D^3 \text{ (cm)}$$

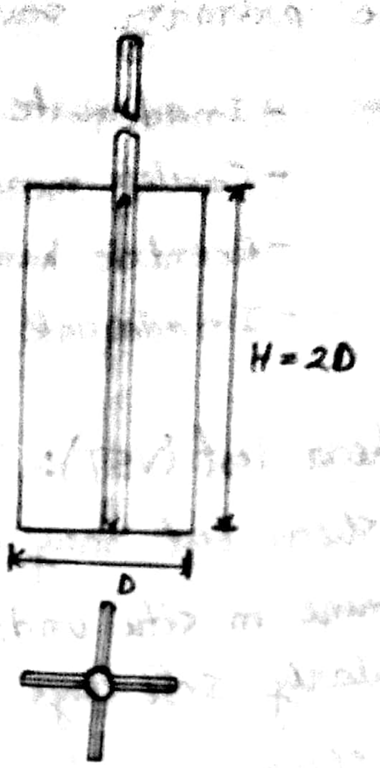

Fig: Rectangular Vane.

- Field vane shear test are moderately rapid and economical and are used extensively in field soil exploration program.
- The test gives good result in soft and medium ~~to~~ stiff clays and gives excellent results in determining the proporties of sensitive clays.
- Sources of significant error in the field vane shear test are poor calibration of torque measurement and damaged vane.
- Others error may be introduced if the rate of rotation of the vane is not properly controlled.
- For actual design purposes, the undrained shear strength values from field vane shear test [$c_{u(VST)}$] are too high and it is recommended that they be corrected according to the equation,

$$c_{u(\text{corrected})} = \lambda\, c_{u(VST)}$$

where, λ = Correction factor.

- Several correlations have been given previously for the correction factor λ. The most common used correlation for λ is that given by Bjerrum (1972) which can be expressed as;

$$\lambda = 1.7 - 0.54 \log [PI\%]$$

- Morris and Williams (1994) provided the following corelation;

$$\lambda = 1.18 e^{-0.08 PI} + 0.57 \quad (\text{for } PI > 5)$$

$$\lambda = 7.01 e^{-0.08 LL} + 0.57 \quad (LL \text{ is in } \%)$$

Cone Penetration Test (CPT):-

- The cone penetration Test (CPT) originally known as Dutch cone penetration test is a versatile sounding method.
- CPT can be used to determine the materials in a soil profile and estimate their engineering properties.
- The test is also called static penetration test and no boreholes are necessary to perform it.
- In the original version, a 60° cone with a base area of 10 cm² was pushed into the ground at a steady rate of about 20 mm/sec and resistance to penetration was measured.
- Cone penetrometer is used at present measure;

a. Cone resistance (q_c): The cone resistance to penetration developed by the cone which is equal to the vertical force applied to the cone, divided by its horizontal projected area.

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b. Frictional Resistance (f_c): which is the resistance measured by a sleeve located above the cone with the local soil surrounding it. The frictional resistance is equal to the vertical force applied to the sleeve, divided by its surface area — actually the sum of friction and adhesion.

- Generally two types of cone penetrometer is used;

1. Mechanical friction cone penetrometer:

- Tip of this penetrometer is connected to an inner set of rods.
- Tips is first advanced about 40mm giving the cone resistance.
- With further thrusting, the tip engages the friction sleeve.
- As the inner rod advances, the the rod forces is equal to the sum of the vertical force on the cone and sleeve.
- Substructing the force on the cone gives the side resistance.

fig: MF cone penetrometer.

2. Electric friction cone penetrometer:-

- The tip of this penetrometer is attached to a ~~steel~~ string of steel rod.
- The tip is pushed into the ground at the rate of 20mm/sec
- Wires from the transducers are threaded through the center of the rods

and continuously measure the cone and side resistance.

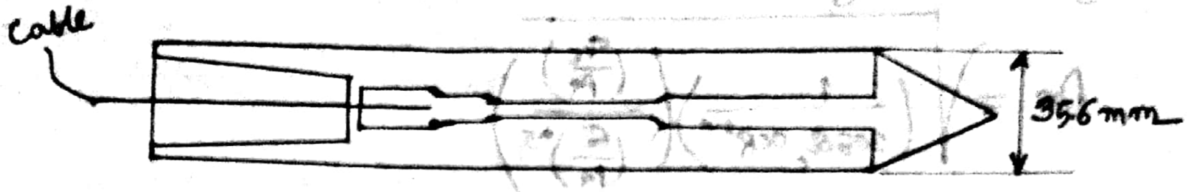


Fig: Electric friction cone penetrometer.

Friction ratio (F_r):

The friction ratio is defined as the ratio of the frictional resistance to the cone resistance. It is expressed as;

$$F_r = \frac{\text{frictional resistance}}{\text{Cone resistance}}$$

$$F_r = \frac{f_c}{q_c}$$

⇒ Several correlations:

According to Anagnostopoulos (2003);

$$F_r(\%) = 1.45 - 1.36 \log D_{50} \quad \text{(for electric Cone)}$$

$$F_r(\%) = 0.7811 - 1.611 \log D_{50} \quad \text{(for mechanical cone)}$$

where, D_{50} = Particle size through which 50% of soil will pass through (mm)

Correlation between Relative density D_r and Cone resistance q_c:

— Lancellotta and Jamiolkowski;

$$D_r(\%) = A + B \log_{10}\left(\frac{q_c}{\sqrt{\sigma'_v}}\right) = -98 + 66 \log_{10}\left(\frac{q_c}{\sqrt{\sigma'_v}}\right)$$

— Kulhawy and Mayne;

$$D_r(\%) = 68\left[\log\left(\frac{q_c}{\sqrt{p_a \cdot \sigma'_v}}\right) - 1\right]$$

P_a = Atm pressure = 100 kN/m^2

– Kulhawy and Mayane:

$$D_r = \sqrt{\left(\frac{1}{305 Q_c \, OCR^{1.8}}\right)\left(\frac{\left(\frac{q_c}{P_a}\right)}{\left(\frac{\sigma'}{P_a}\right)^{0.5}}\right)}$$

Q_c = Compressibility factor.

The recommended values of Q_c are as follows;

Highly compressible sand = 0.91
Moderately compressible sand = 1.0
Low compressible sand = 1.09

Correlation between q_c and φ':-

q_c = Cone resistance
φ' = friction angle.

– Robertson and Campanella;

$$\varphi' = \tan^{-1}\left[0.1 + 0.38 \log\left(\frac{q_c}{\sigma_0'}\right)\right]$$

– Venice Lagoon and Ricceri;

$$\varphi' = \tan^{-1}\left[0.38 + 0.27 \log\left(\frac{q_c}{\sigma_0'}\right)\right]$$

– More recent, Lee at al.;

$$\varphi' = 15.575 \left(\frac{q_c}{\sigma_h'}\right)^{0.1714}$$

σ_h' = Effective horizontal stress.

Correlation between c_u, σ_c', OCR;

$$C_u = \frac{q_c - \sigma_0}{N_K}$$

σ_0 = Total vertical pressure.
N_K = Bearing capacity factor.
q_c = Cone resistance

→ Mayne and Kemper:

$$\sigma_c' = 0.243\,(q_c)^{0.96}$$

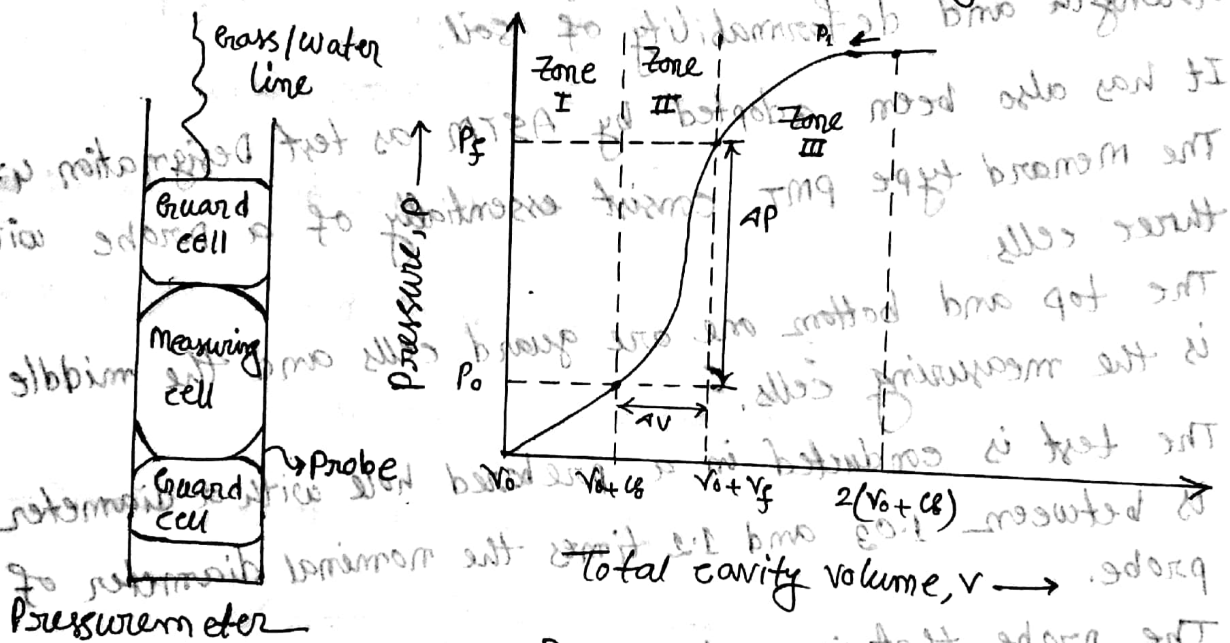
σ_c' ↓ MN/m², q_c ↓ MN/m²

$$OCR = 0.37\left(\frac{q_c - \sigma_o}{\sigma_o'}\right)^{1.01}$$

⊕ Pressuremeter Test (PMT):

- PMT is an in situ test conducted in a borehole.
- It was originally developed by Menard (1956) to measure the strength and deformability of soil.
- It has also been adopted by ASTM as test Designation 4719.
- The Menard type PMT consist essentially of a probe with three cells.
- The top and bottom one are guard cells and the middle one is the measuring cells.
- The test is conducted in a prebored hole with a diameter that is between 1.03 and 1.2 times the nominal diameter of the probe.
- The probe that is most commonly used as a diameter of 58 mm and a length of 420 mm.
- The probe cells can be expanded by either liquid or gas.
- The guard cells are expanded to reduce the end condition effect on the measuring cells, which has a volume of 535 cm³.
- In order to conduct the test, measuring cell volume V_o is measured and the probe is inserted into the borehole.
- Pressure is applied in increaments and the new volume of the cell is measured.

- The process continues untill the soil fails or ~~untill~~ the pressure limit of the device is reached.
- The soil is considered to have failed when the total volume of the expanded cavity is about twice the volume of the original cavity.
- After the completion of the test, the probe is deflated and advanced for testing at another depth.
- The result of the pressuremeter test are expressed in the graphical form of pressure versus cavity volume plot;



Pressure versus total cavity volume plot.

Pressure meter modulus;

$$E_P = 2(1+\mu_s)(V_o+v_m)\left(\frac{\Delta P}{\Delta V}\right) \qquad \left[v_m = \frac{v_o+v_f}{2}\right]$$

where, $v_m = \dfrac{v_o+v_f}{2}$

$\Delta P = P_f - P_o$

$\Delta V = v_f - v_o$

μ_s = Poisson's ratio (0.33)

Limit pressure is usually obtained by extrapolation.

– Kulhawy and Mayane;

$$\sigma_c' = 0.45 P_l$$

– Baguelin et. al.;

$$C_u = \frac{P_l - P_0}{N_p}$$

$$N_p = 1 + \ln\left(\frac{E_p}{3C_u}\right)$$

– Ohya;

$$\text{Clay: } E_p (kN/m^2) = 1930\, N_{60}^{0.63}$$

$$\text{Sand: } E_p (kN/m^2) = 908\, N_{60}^{0.66}$$

Preparation of Boring Logs: (v.v.Im) 100%

– (The detailed information gathered from each borehole is presented in a graphical form called the boring log.)

– A standard boring log should provide;

1. Name and address of the drilling company.
2. Driller's Name.
3. Job description and number.
4. Number, type and location of boring.
5. Date of boring.
7. Sub-surface stratification which can be obtained by visual observation.
6. Elevation of water table and date observed use of casing and mud losses and so on.
9. Standard penetration resistance and the depth of SPT.
10. Number, type and depth of soil sample collected.
11. In case of rock coring, type of core barrel used and for each run the actual length of coring, length of core recovery.

Sub-soil Exploration Report:- (Im):V

- At the end of all soil exproration program, the soil and rock specimens collected in the field are subjected to visual observation and appropiate laboratory testing.
- After all the required information have been compiled, a soil exploration report is prepared for use by the design office and for reference during construction work.
- Although the detailed and sequence of information in such reports may vary to some degree, depending on the structure ~~after~~ under consideration and the person compiling the report.

Features 90%

- Each report should be include the following items;

1. A description of the scope of investigation.
2. A description of the proposed structure for which the soil exploration has been conducted.
3. A description of the location of the site.
4. A description of the geological setting of the site.
5. Details of the field exploration.
6. A general description of the sub soil condition.
7. A description of the water table condition.
8. Recommendation regarding the foundation.
9. Conclusions and limitations of the investigations.

⇒ Factors Affecting N-value :-

1. Homoginity of soil helps widely to get N-value.
2. Effective overburden pressure - granular soil.
3. Types of soil — boulders, gravel due to them N-value will be unreliable.

⇒ Why N-value Needed to be corrected :-

1. Sandy soil resist the penetration.
2. Silty soil for it, the N-value will be varied for its physical significant.
3. Water table affects the penetration.

That's why the correction of N-value is needed.

(All the examples and Exercise problems
should be practiced
Book: BM Dass)

Bearing Capacity of Shallow Foundation

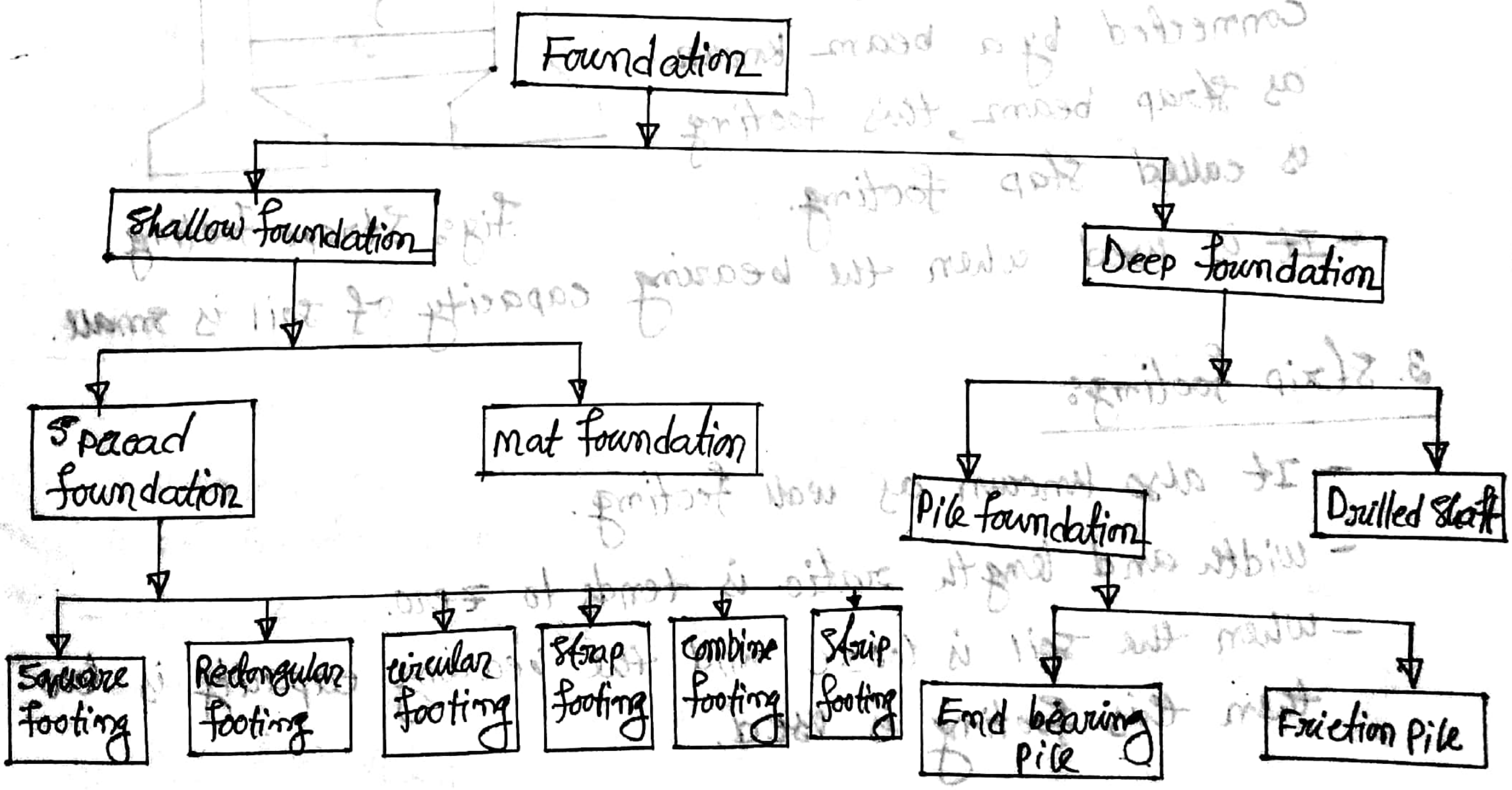
Foundation :-

- The lower part of a structure is generally reffered to as foundation.
- It transfer load from superstructure to footing.

Footing :

- A footing is a foundation unit constructed in brick work masonry or concrete which transfer the loads coming from foundation directly to the hard strata of soil mass.
- In simple words, foundation means leg and footing means foot of leg.

Types of foundation :-



Suitability of foundations :- (7m)

1. Shallow foundation :-

- Depth and width ratio is less than or equal to 4 ($\frac{D}{W} \le 4$).
- A spread footing is simply an enlargement of a load bearing wall or column that makes it possible to spread.
- It is the most economical type of footing.
- When the bearing capacity of the soil is large but the axial load on the column is small then it is used.

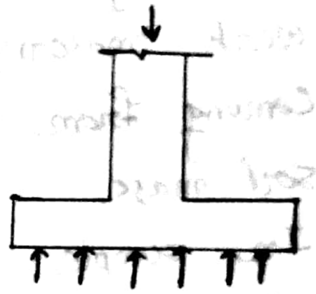


Fig: Spread footing

2. Strap footing :-

- When two columns are connected by a beam known as strap beam, this footing is called stap footing.

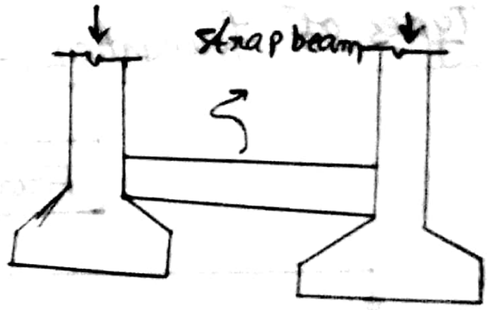


Fig: Strape footing.

- It is used when the bearing capacity of soil is small.

3. Strip footing :

- It also known as wall footing.
- Width and length ratio is tends to zero.
- When the soil is loose and the bearing capacity is low then this footing is used.

4. Combined footings:-

- Construction cost is higher than spread footing.
- It is used when the foundation can not be extended due to the end of the property line and the bearing capacity of soil is relatively low and the interior columns are heavily loaded.

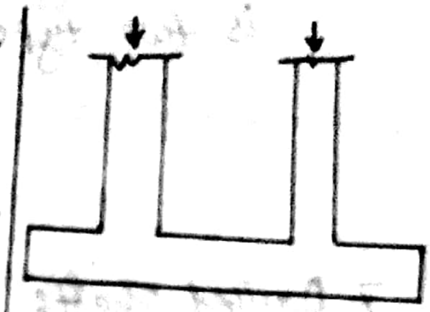


fig: Combined footing

5. Mat or Raft footings:-

- If the indivisual footing covers more than 50% area then Mat foundation is prefferable.
- If the soil is very weak and/or Columns loads are great, the required footing area becomes so large as to be uneconomical. Then Mat / Raft footing is used.

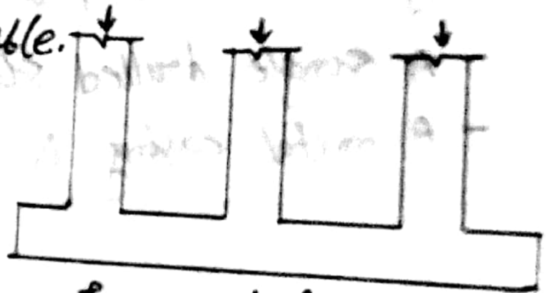


fig: mat foundation

- It is useful in preventing the differential settlement.
- When the depth of suitable bearing capacity strata for strip footing becomes too deep then Mat footing is used.

6. Pile foundation:

- Piles are structural member which transfer the structure load to a hard strata of soil to a greater depth.
- When the columns loads is so large and the bearing capacity of the soil is very low then it is used.

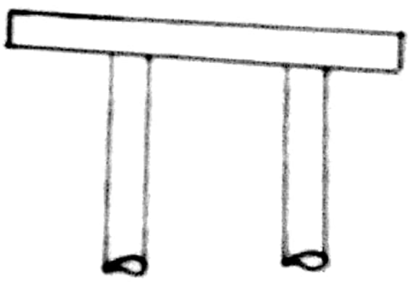


fig: Pile footing.

- where soils are affected by seasonal change i.e expansive soil.
- According to how they carry load, the pile foundation is two types:
  a. End bearing pile
  b. Friction pile.

7. Drilled shaft:

- Drilled shaft reffers to cast in situ or cast inplace pile generally having a diameter of about 2.5 ft or more ususally greater than pile with ~~out~~ or without reinforcement.
- A single drilled shaft may be used instead of group of piles.
- A metal casing is used.

fig: Drilled shaft

8. Well foundation or Caisson:

⊞ characteristics of shallow foundation: (v.v.imp)

- To perform satisfactorily, shallow foundations must have two main characteristics;

1. They have to be safe against overall shear failure in soil that support them.
2. The foundation should not go under excessive settlement or displacement. (the term excessive is relative because degree of settlement allowed for a structure depends on several considerations)

⊞ Ultimate bearing capacity:- (v.v.imp)

The load per unit area of the foundation at which shear failure in soil will occur is called the ultimate bearing capacity of the soil.

⊞ Modes of Shear Failure: v.v.imp 100%

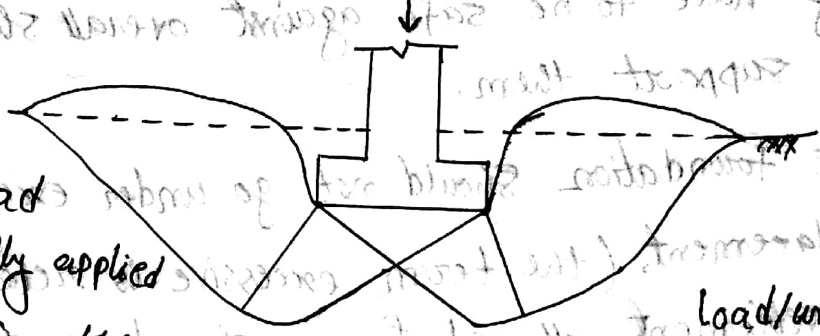
→ Discuss with neat sketches the nature of bearing capacity failure of soil.

- When a footing fails due to insufficient bearing capacity, distinct failure patterns are developed depending upon the type of failure mechanism. Vesic (1963) observed three types of bearing capacity failure.

1. General shear failure.
2. Local shear failure.
3. Punching shear failure.

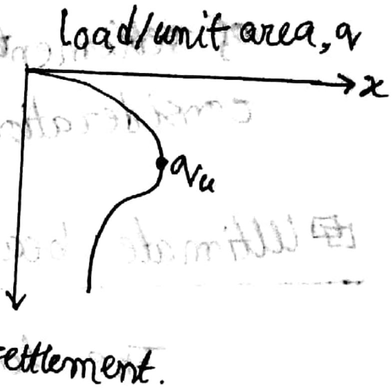
General shear failure:- (Imp)

- Failure surface in soil will extended to ground surface.
- It occurs in a dense sand or stiff cohesive soil.



- If the load is gradually applied to foundation the settlement will also increased.

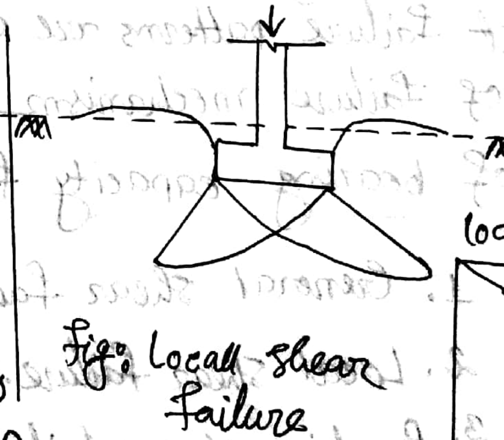
fig: General shear failure



- At a certain point, settlement will increase without increase in load.
- This load is known as ultimate bearing capacity, q_u.

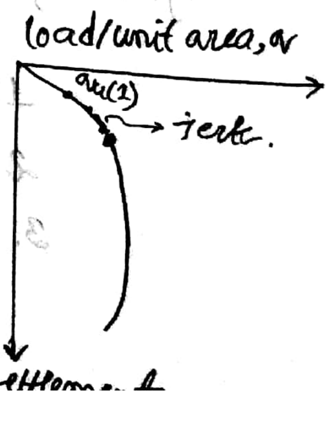
Local shear failure: (Imp)

- Failure surface of the soil will gradually extend outward from foundation and do not or little extend to ground surface.



- It occurs in sand or clayey soil of medium compaction.
- When load per unit area of foundation equals to $q_{u(1)}$ movement of foundation will be accompanied by a sudden jerk.

fig: Local shear failure



- A considerable movement of foundation is the required for local shear failure.
- The load per unit area at which the movement of the foundation happens is called ultimate bearing capacity.
- Beyond this point, settlement will increase with the increase in load.
- A peak value of q is not realized.

Punching Shear Failure:- (Im)

- Failure surface of the soil will not extend to ground surface.
- It occurs in fairly loose soil.
- Relatively large settlement occurs in this failure.
- The ultimate bearing capacity is not well defined.

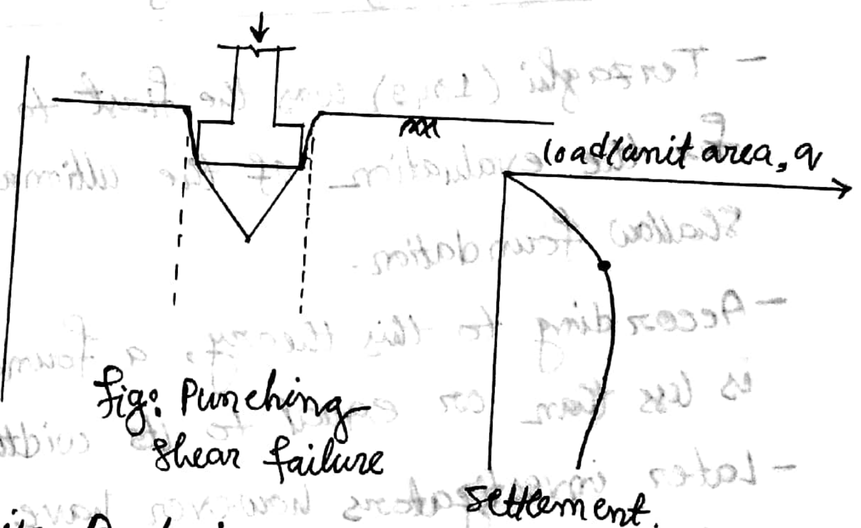


Fig: Punching shear failure

Assumption in Terzaghi's Analysis:- (v.v. Im)

1. The soil is homogeneous and isotropic and its shear strength is represented by coulomb's equation.
2. The strip footing has a rough base and the problem is essentially two dimensional.

3. The elastic zone has straight boundaries inclined at $\psi = \phi$ to the horizontal and the plastic zone is fully develop.

4. Passive pressure consists of three component can be calculated separately and added, although the critical surface for these components are not identical.

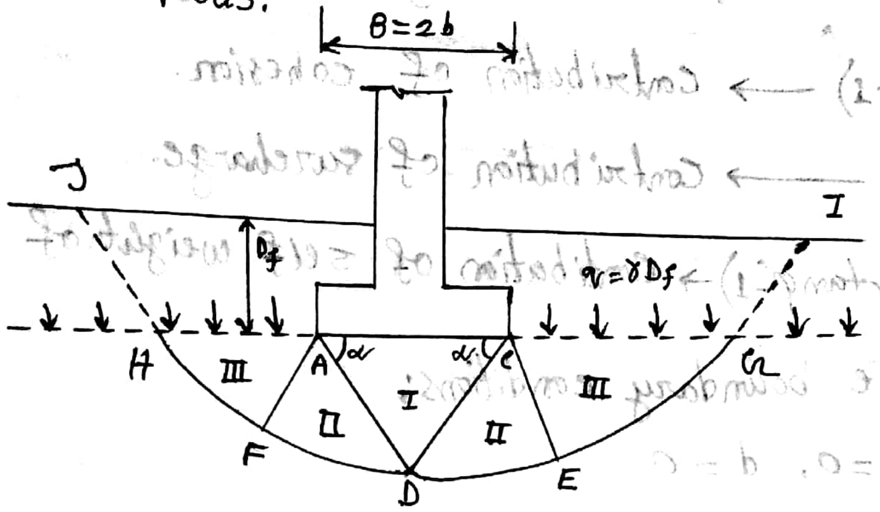
5. Failure zone do not extend above the horizontal plane through the base of the footing. i.e the shear resistance of soil above the base is neglected and the effect of soil around the footing is considered equivalent to a surcharge, $\sigma = \gamma D$.

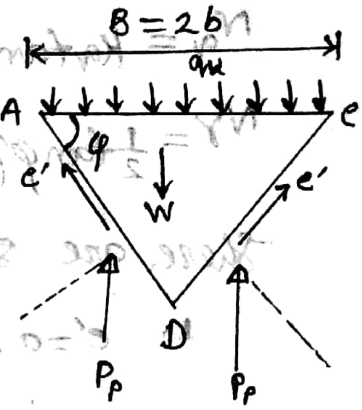
④ Terzaghi's Bearing capacity Theory: (Im)

- Terzaghi (1943) was the first to present a comprehensive theory for the evaluation of the ultimate bearing capacity of rough shallow foundation.
- According to this theory, a foundation is shallow if its depth is less than or equal to its width. $\left(\frac{D_f}{B} \leq 1\right)$
- Later investigators however have suggested that foundation ~~widt~~ Depth equal to 3 to 4 times of their width may be defined as shallow foundation. $\left(\frac{D_f}{B} \leq 4\right)$
- Terzaghi suggested a continuous strip foundation, the failure surface in soil at ultimate load may be assumed to be similar to the general shear failure.
- Effect of soil above the bottom of the foundation may also

be assumed to be replaced by an equivalent surcharge, $q = \gamma D_f$.

- The failure zone under the foundation can be ~~spe~~ separated into three parts.





Freebody diagram.

Zone-I: Triangular zone ACD immediately under the foundation. It is elastic zone.

Zone-II: The radial shear zone ADF and CDE with the curves DE and DF being arcs of a logarithmic spiral.

Zone-III: Arcs equation, $r = r_0 \, e^{\theta \tan\varphi'}$

Two triangular Rankine passive zone. AFH and CEG

- From Freebody diagram; we get,

$\Sigma F_y = 0$.

$$q_u \cdot 2b \cdot 1 = -W + 2c \sin\varphi' + 2P_p$$

Here, $W = \gamma b^2 \tan\varphi'$

$b = B/2$

$c = \dfrac{c'b}{\cos\varphi'}$

$$\therefore P_p = \tfrac{1}{2}\gamma (b\tan\varphi')^2 K_\gamma + c'(b\tan\varphi') K_c + q(b\tan\varphi') K_q$$

Here, K_γ, K_c and K_q are the earth pressure co-efficient.

$$\therefore q_u = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma$$ → strip foundation only.

where, N_c, N_q and N_γ are Terzaghi's bearing capacity factor.

$N_c = \tan\phi' (K_c + 1)$ → contribution of cohesion.

$N_q = K_q \tan\phi'$ → contribution of surcharge.

$N_\gamma = \frac{1}{2} \tan\phi' (K_\gamma \tan\phi' - 1)$ → contribution of self weight of soil.

There are some boundary conditions;

1. $c' = 0$, $q = 0$, $d = 0$

$$\therefore q_u = q_\gamma = \frac{1}{2} \gamma B N_\gamma$$

2. $\gamma = 0$, $q = 0$, $d = 0$

$$\therefore q_u = q_c$$

3. $\gamma' = 0$, $c' = 0$

$$\therefore q_u = q_a = q N_q$$

$$\therefore q_a = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma$$

This is the bearing capacity equation given by Terzaghi. It is only for strip foundation and general shear failure.

→ For square footing;

$$q_u = 1.3 c' N_c + q N_q + 0.4 \gamma B N_\gamma$$

→ For circular footing;

$$q_u = 1.3 c' N_c + q N_q + 0.3 \gamma B N_\gamma$$

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Here, $N_c = \cot\varphi' \left[\dfrac{e^{2(3\pi/4 - \varphi'/2)\tan\varphi'}}{2\cos^2\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)} - 1 \right] = \cot\varphi' (N_q - 1)$

$$N_q = \frac{e^{2(3\pi/4 - \varphi'/2)\tan\varphi'}}{2\cos^2\left(45 + \frac{\varphi'}{2}\right)}$$

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2\varphi'} - 1\right)\tan\varphi'$$

where, $K_{p\gamma}$ = Passive pressure co-efficient

➡ In case of local shear failure:-

we have to assume that;

$$\bar{c}' = \frac{2}{3} c'$$

$$\tan\bar{\varphi}' = \frac{2}{3}\tan\varphi'$$

For strip footing:

$$q_u' = \bar{c}' N_c' + q N_q' + \frac{1}{2}\gamma B N_\gamma'$$

For square footing:

$$q_u = 1.3\, \bar{c}' N_c' + q N_q' + 0.4 \gamma B N_\gamma'$$

For circular footing:

$$q_u = 1.3\, \bar{c}' N_c' + q N_q' + 0.3 \gamma B N_\gamma'$$

Here, $\bar{\varphi}' = \tan^{-1}\left(\frac{2}{3}\tan\varphi'\right)$

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Limitations of Terzaghi's Bearing capacity theory:- (v.v.Imp) 100%

1. He considered, the depth of foundation is less than or equal to width of foundation.
2. No sliding between the footing and the wall.
3. Soil is a homogeneous semi-infinite mass.
4. Failure plane angle is equal to ϕ.
5. No applicable for inclined load and rectangular foundation
6. No resistance of soil above the level of the base of the foundation.

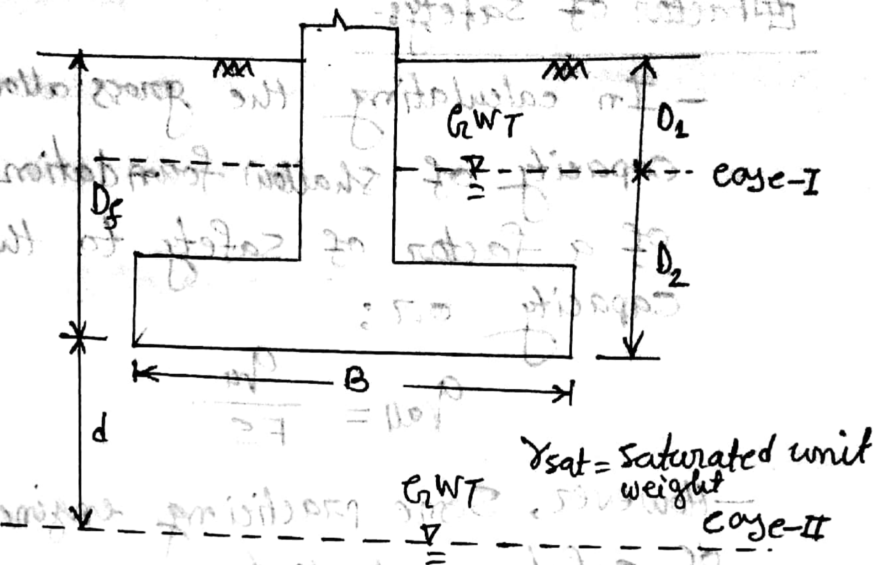
(v.v.Imp) (100%)

Effects of Ground water Table: (on bearing capacity)

- In developing the bearing capacity equation it is assumed that the ground water table is located at a depth much greater than the width (B) of the footing.
- However, if ground water table is close to footing, some changes are required in second and third terms of the bearing capacity equation.
- Three different conditions can arise regarding the location of ground water table. They are discussed below;

Case-I: If the water table is located so that $0 \leq D_1 \leq D_f$, the factor q in the bearing capacity equation takes the form;

$$q = D_1\gamma + D_2(\gamma_{sat} - \gamma_w)$$



where,

q = Effective surcharge.

γ_{sat} = saturated unit weight.

γ_w = unit weight of water.

fig: Effect of GW on BCE.

Also the value of γ in the last term of the equation has to be replaced by $\gamma' = \gamma_{sat} - \gamma_w$.

Case-II: For a water table located so that $0 \leq d \leq B$.

$\therefore$ Effective surcharge, $q = \gamma D_f$.

In this case, the factor γ in the last term of the bearing capacity equation must be replaced by the factor; Average, $\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$

Case-III: When the water table is located so that $d \geq B$, the water will have no effect on the ultimate bearing capacity.

Factor of Safety:-

- In calculating the gross allowable load bearing capacity of shallow foundation requires the application of a factor of safety to the gross ultimate bearing capacity or;

$$q_{all} = \frac{q_u}{FS}$$

- However, some practicing engineers prefers to use a factor of safety such that

$$\text{Net stress increase in soil} = \frac{\text{Net ultimate bearing capacity}}{\text{Factor of Safety (FS)}}$$

→ The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the foundation that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level.

- If the difference unit weight of concrete used in the foundation and the unit weight of soil surrounding is assumed to be negligible, then

$$q_{net(u)} = q_u - q$$

where, $q_{net(u)}$ = Net ultimate bearing capacity.
$q = \gamma D_f$

$$\therefore q_{all(net)} = \frac{q_u - q}{FS}$$

(The Factor of safety should be atleast 3 in all cases)

General Bearing Capacity Equation:-

Meyerhof (1963) suggested the following form of the general bearing capacity equation;

$$q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

where,
- c' = Cohesion
- q = Effective stress at the level of bottom of the foundation.
- γ = unit weight of soil.
- B = Width of foundation. (= diameter of a circular foundation)

$F_{cs}, F_{qs}, F_{\gamma s}$ = Shape factor.

$F_{cd}, F_{qd}, F_{\gamma d}$ = Depth factor.

$F_{ci}, F_{qi}, F_{\gamma i}$ = Load inclination factor.

N_c, N_q, N_γ = Bearing capacity factor.

Bearing capacity factors:-

$$N_q = \tan^2\left(45^\circ + \frac{\phi'}{2}\right) e^{\pi \tan\phi'} \quad \text{(Reissner, 1924)}$$

$$N_c = (N_q - 1)\cot\phi' \quad \text{(Prandtl, 1921)}$$

$$N_\gamma = 2(N_q + 1)\tan\phi' \quad \text{(Vesic, 1973)}$$

Shape factors:- (De Beer, 1970)

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi'$$

$$F_{\gamma s} = 1 - 0.4\,(B/L)$$

where, B = width of footing.
L = length of footing.

(Eccentricity থাকলে B ও L এর পরিবর্তে B' ও L' দিয়ে replace করতে)

Depth factors:- (Hanson, 1970)

Case-I: $\frac{D_f}{B} \leq 1$.

For, $\varphi' = 0°$;

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$
$$F_{qd} = 1$$
$$F_{\gamma d} = 1$$

For, $\varphi' > 0°$;

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\varphi'}$$
$$F_{qd} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2\left(\frac{D_f}{B}\right).$$
$$F_{\gamma d} = 1.$$

Case-II: $\frac{D_f}{B} > 1$.

For, $\varphi' = 0$;

$$F_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B}\right) \rightarrow \text{in radian}$$
$$F_{qd} = 1$$
$$F_{\gamma d} = 1$$

For, $\varphi' > 0°$;

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\varphi'}$$
$$F_{qd} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2\tan^{-1}\left(\frac{D_f}{B}\right) \rightarrow \text{in radian}$$
$$F_{\gamma d} = 1.$$

Load inclination factors:- (Meyerhof, 1963 & Hanna and Meyerhof, 1981)

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\varphi'}\right)^2$$

where, β = Inclination of the load on the foundation with respect to the vertical.

Problem-1:-

A square foundation $2m \times 2m$ in plan. The soil supporting the foundation has a friction angle, $\varphi' = 25^\circ$ and $c' = 20\ KN/m^2$. The unit weight of soil is, $\gamma = 16.5\ KN/m^3$. Determine the allowable gross load on the foundation with a factor of safety of 3. Assume, that the depth of the foundation D_f is 1.5m and that general shear failure occure in the soil.

Solution: we have;

$$q_u = c' N_c F_{cs} F_{ci} F_{cd} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \quad \text{--- (1)}$$

Since the load is vertical;

$$F_{ci} = F_{qi} = F_{\gamma i} = 1$$

$$N_q = \tan^2\left(45^\circ + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'}$$

$$= \tan^2\left(45^\circ + \frac{25^\circ}{2}\right) e^{\pi \tan 25^\circ}$$

$$\therefore N_q = 10.66$$

$$N_c = (N_q - 1)\cot\varphi' = (10.66 - 1)\cot 25^\circ = 20.72$$

$$N_\gamma = 2(N_q + 1)\tan\varphi' = 2(10.66 + 1)\tan 25^\circ = 10.87.$$

Now,

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{2}{2}\right)\left(\frac{10.66}{20.72}\right)$$

$$= 1.514$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\varphi' = 1 + \left(\frac{2}{2}\right)\tan 25^\circ$$

$$= 1.466$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{2}{2}\right)$$

$$= 0.60$$

Assuming; $\frac{D_f}{B} \le 1$.

$$F_{qd} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 25^\circ(1 - \sin 25^\circ)^2\left(\frac{1.5}{2}\right)$$

$$= 1.233$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\varphi'} = 1.23 - \frac{1 - 1.233}{20.72\tan 25^\circ}$$

$$= 1.257$$

$$F_{\gamma d} = 1.$$

Now, From equation ①; we get;

$$① — q_u = (20.72 \times 20 \times 1.514 \times 1.257 \times 1) + ((1.5 \times 16.5) \times 10.66 \times 1.466 \times 1.233 \times 1)$$
$$+ \left(\tfrac{1}{2} \times 16.5 \times 2 \times 10.87 \times 0.6 \times 1 \times 1\right)$$

$$= 788.64 + 476.90 + 107.61$$

$$\therefore q_u = 1373.2 \text{ kN/m}^2$$

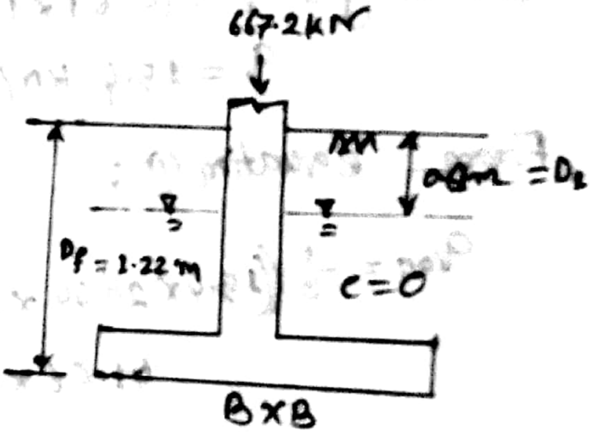
$\therefore q_{all} = \frac{q_u}{FS} = \frac{1373.2}{3} = 457.73\ KN/m^2$

So, Gross allowable load, $Q = q_{all} \times Area$

$= 457.73 \times (2 \times 2)$

$\therefore Q = 1830.93\ KN.$ (Ans)

Problem-2:- A square foundation $(B \times B)$ has to be construct as shown in figure. Assume that $\gamma = 16.5\ KN/m^3$, $\gamma_{sat} = 18.55\ KN/m^3$, $\varphi' = 34°$; $D_f = 1.22m$ and $D_1 = 0.61m$. The gross allowable load Q_{all} with FS 3 is 667.2 KN. Determine the size of the footing.



Solution:

We have;

$$q_{all} = \frac{Q_{all}}{B \times B} = \frac{667.2}{B^2} \quad \text{--- (1)}$$

Now, $q_{all} = \frac{q_u}{FS} = \frac{1}{3}[q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma' B N_\gamma F_{\gamma d} F_{\gamma s} F_{\gamma i})$ --- (2)

Since, $c = 0$ so;

$$N_c = F_{cs} = F_{cd} = F_{ci} = 0$$

And the load is vertical so; $F_{qi} = F_{\gamma i} = 1$.

Assume, $D/B \leq 1$.

For $\varphi' = 34°$;

$$N_q = 1 + 2\tan\varphi'(1 - \sin\varphi')^2 \frac{D_f}{B}$$

$$N_q = \tan^2(45° + \frac{\varphi}{2}) e^{\pi\tan\varphi} = \tan^2(45° + \frac{34°}{2}) e^{\pi\tan 34°}$$

$\therefore N_q = 29.44$

$N_\gamma = 2(N_q+1)\tan\phi' = 2(29.44+1)\tan 34^\circ$
$= 41.06$

$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1+\left(\frac{B}{B}\right)\tan 34^\circ = 1.67$

$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1-0.4\left(\frac{B}{B}\right) = 0.6$

$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \frac{D_f}{B}$

$= 1 + 2\tan 34^\circ (1-\sin 34^\circ)^2 \frac{1.022}{B}$

$= 1 + \frac{0.32}{B}$

$F_{\gamma d} = 1$

surcharge, $q = (0.61\times 16.5) + (1.22-0.61)(18.55-9.81)$
$q = 15.4 \text{ KN/m}^2$

From equation (1);

$q_{all} = \frac{1}{3}\left(\left(15.4\times 29.44\times 1.67\times 1\left(1+\frac{0.32}{B}\right)\right) + \left(\frac{1}{2}\times(18.55-9.81)B\times 41.06\times 0.6\times 1\right)\right)$

$q_{all} = 252.38 + \frac{80.76}{B} + 107.66B$ —— (3)

Now, equating equation (1) and (3); we get;

$\frac{667.2}{B^2} = 252.38 + \frac{80.76}{B} + 107.66B$

$\Rightarrow 667.2 = 252.38B^2 + 80.76B + 107.66B^3$

$\Rightarrow 107.66B^3 + 252.38B^2 + 80.76B - 667.2 = 0$

$\therefore B = 1.22 \text{ m}$

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check, $\frac{D_f}{B} = \frac{1.22}{1.22} = 1 \leq 1$ (OK)

$\therefore$ Size of the footing ($1.22m \times 1.22m$). (Ans)

Footing with one way eccentricity:

- In several instances, as with the base of retaining wall, foundations are subjected to moments in addition to the vertical load.

- In such cases, the distribution of pressure by the foundation on the soil is not uniform. The nominal distribution of pressure is;

$$q_{max} = \frac{Q}{BL} + \frac{6M}{B^2L}$$

$$q_{min} = \frac{Q}{BL} - \frac{6M}{B^2L}$$

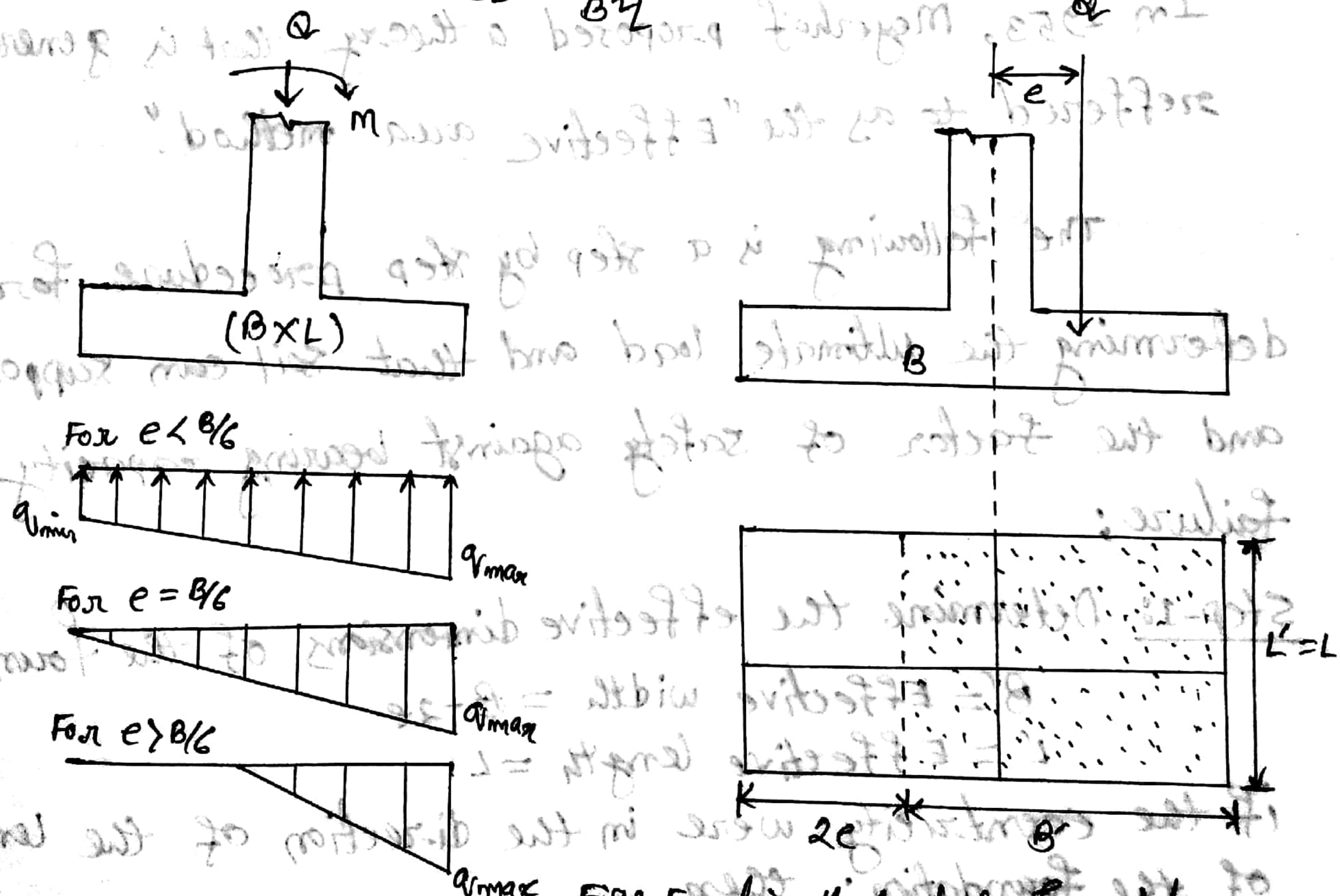

Fig: Eccentrically loaded foundation.

Q = Total vertical load.
m = Moment of the foundation.

$\therefore$ Eccentricity, $e = \frac{M}{Q}$

$$\therefore q_{max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right)$$

$$q_{min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right).$$

For $e > B/6$;

$$q_{max} = \frac{4Q}{3L(B-2e)}.$$

Factor of safety, $FS = \frac{Q_{ult}}{Q} = \frac{q_u}{q_{max}}$

One way eccentrically foundation solution:-

In 1953, Meyerhof proposed a theory that is generally reffered to as the "Effective area method".

The following is a step by step procedure for deforming the ultimate load and that soil can support and the factor of safety against bearing capacity failure;

Step-1: Determine the effective dimensions of the foundation.

$B' =$ Effective width $= B - 2e$
$L' =$ Effective length $= L$

if the eccentricity were in the direction of the length of the foundation; then

$L' =$ Effective length $= L - 2e$

$B' =$ Effective width $= B'$

The smaller of two dimensions (L' and B') is the effective width of the foundation.

Step-2: Use ultimate bearing capacity equation.

$$q'_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}.$$

In determination of F_{cs}, F_{qs} and $F_{\gamma s}$, the B and L must be replace by B' and L'.

In determination of F_{cd}, F_{qd} and $F_{\gamma d}$, B and L is used not B' and L'.

Step-3: The total ultimate load that the foundation can sustain is;

$$Q_{ult} = q' B' L'$$

Step-4: The factor of safety against bearing capacity failure is;

$$FS = \frac{Q_{ult}}{Q}$$

(Example and Problem from Book and given by MAH sir are done later)

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Plate load Test : (ya)

- In some cases, conducting field load test to determine the soil bearing capacity of foundation is desirable.
- Standard method for field load test is given by ASTM under designation D-1194 (1997).
- Circular steel bearing plate 162 to 760 mm (6~30") in diameter and 305 mm x 305 mm (1' x 1') square are used for this test.
- To conduct this test, one must have a pit of depth D_f excavated width of test pit should be at least four times width of bearing plate to be used for this test.
- Bearing plate is placed on soil at the bottom of the pit and an incremental load on Bearing plate is applied.
- After application of incremental load each time, some time is given the soil for settlement and when it reach equilibrium then the next load is applied.
- When the settlement of bearing plate becomes negligible another incremental load is applied.
- In this manner, a load ~ settlement plot can be obtained.

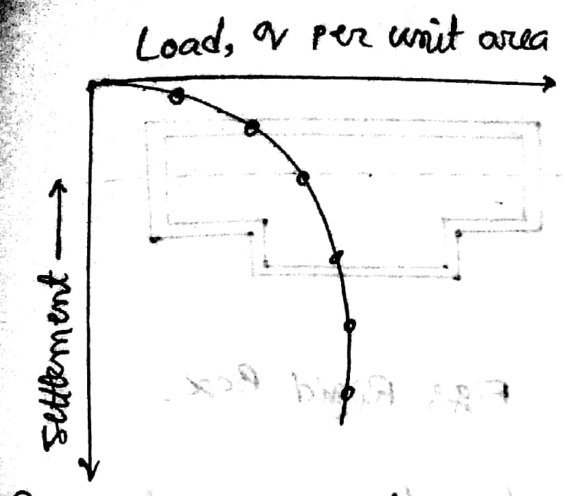


fig: load – settlement plot.

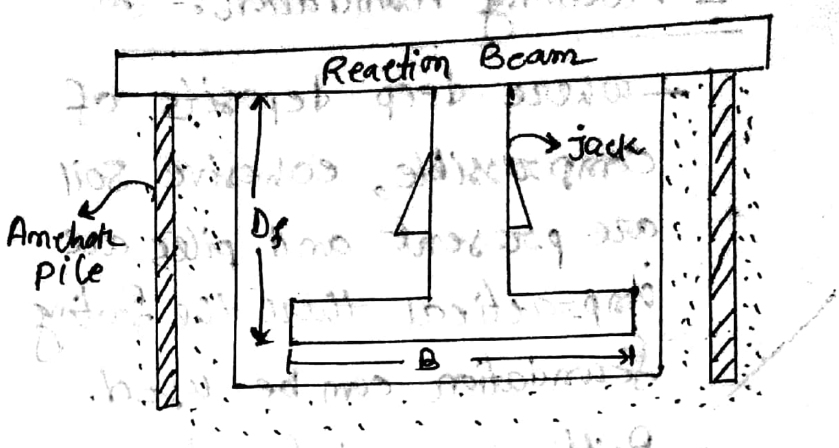


fig: Plate load test.

- From the result of field load test, the ultimate soil bearing capacity of actual footing can be approximated as follows;

For clay soil; $q_{u(footing)} = q_{u(plate)}$

For sandy soil; $q_{u(footing)} = q_{u(plate)} \cdot \dfrac{B_{(footing)}}{B_{(plate)}}$

- For a given load q, settlement of actual footing is as follows;

For clay soil, $S_{e(footing)} = S_{e(plate)} \cdot \dfrac{B_{(footing)}}{B_{(plate)}}$

For sandy soil,

$$S_{e(footing)} = S_{e(plate)} \cdot \left[\frac{2B_{(footing)}}{B_{(footing)} + B_{(plates)}} \right]^2$$

Where, S_e = Elastic / immediate settlement.

☐ Floating Foundation :- (Imp)

- where deep deposits of compressible, cohesive soil are present and piles are impractical there the floating foundation can be used.

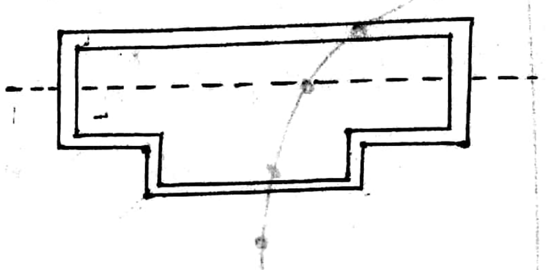
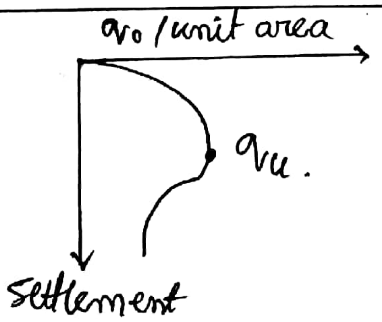
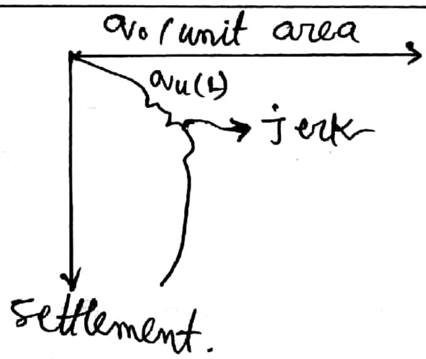


Fig: Rigid Box.

- Building's substructure is a combination of mat and caisson to create a rigid box.
- The weight of the earth displaced by foundation is equal to the total weight of structure, thereby minimising settlement from consolidation.

> Foundation করার জন্য খননকৃত মাটির ওজন এবং সমগ্র structure এর ওজন সমান হলে তাকে Floating foundation বলে। এক্ষেত্রে foundation এর নিচের pressure zero (শূন্য) হওয়ায় consolidation settlement minimise হয়।

Question: Differentiate between general shear failure and Local shear failure. V.V.Imp

Sr	General Shear Failure	Local Shear Failure
1.	It occurs in dense sand or stiff cohesive soil.	It occurs in sand or clayey soil of medium compaction.
2.	Failure surface extends up to the ground surface.	Failure surface donot or little extend to the ground surface
3.	settlement of the foundation is smooth.	settlement of the foundation accompanied with sudden jerk.
4.	A peak ultimate bearing capacity is defined.	A peake ultimate bearing capacity do not reelized.
5.	In this case, N > 30 N = SPT value.	In this case, N < 5.
6.		

Settlement of Shallow Foundation

Settlement: (v.v.imp)

Settlement is defined as the total vertical deformation of the surface with respect to its original position resulting from external load and dewatering.

Types of settlement:

1. Differential settlement.
2. Uniform settlement.

Uniform settlement again classified as;

a. Elastic or immediate settlement.
b. Primary consolidation settlement.
c. Secondary consolidation settlement.

Differential Settlement: (v.v.imp 100%)

When the settlement is non-uniform or the difference in settlement between two adjacent columns is called the differential settlement. It is the great causes of structural failure.

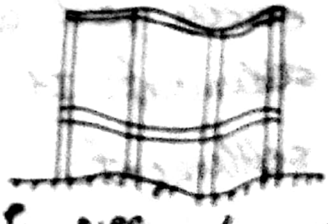


fig: Differential settlement

Causes of differential settlement:

1. Compaction is not uniform.
2. Fluctuation of water table.

3. Fluctuation of loads on different columns.
4. Soil profile if different.

Preventive measures:

1. To prevent the differential settlement of the structure Raft or mat foundation is provided.
2. Evenly and uniformly compaction have to be done.
3. The structure should be symmetrical so that all the columns can carry same loads.

Immediate/Elastic Settlement: (S_e) (v.v.Imp)

The settlement which occurs immediately after the application of load usually for sand, it is called elastic or immediate settlement. For clay soil, elastic settlement is negligible. Elastic settlement occurs due to—

a. Elastic deformation of soil grains.
b. Escape of pore air pressure from voids.

Primary consolidation settlement (S_c): (v.v.Imp)

The settlement which is caused by the escape of excess pore water pressure is called primary consolidation settlement. It occurs for clay soil. It is time depended settlement.

Secondary consolidation settlement (S_s):- (Creep). .v.Imp

The settlement which is caused by the rearrangement of soil particals on soil profile due to sustained load is called Secondary consolidation settlement.

It is also known as creep. After the completion of primary consolidation settlement, then the secondary consolidation settlement will start. It will take very long time. to start.

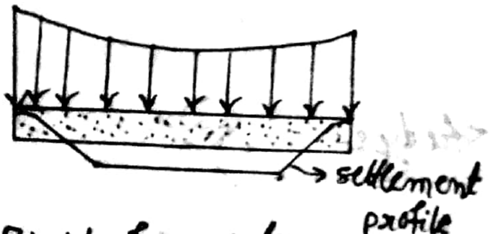
Purposes of settlement study:- (Imp)

1. To study / know the settlement behavior.
2. To determine the settlement value and time.
3. To study the settlement influence to the structure stability.

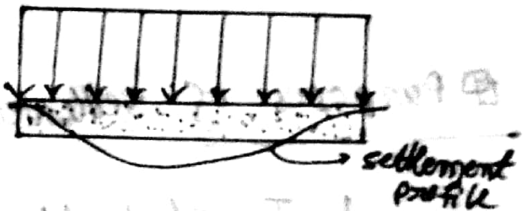
Immediate or Elastic settlement (S_e):- (v.Imp)

- It is defined as the settlement which occured directly after the application of load without a change in moisture content.
- It caused by the soil elastic behavior.
- Magnitude of contact settlement will depend on the flexibility of foundation and type of material on which it is resting.

- For clay soil, the elastic settlement (S_e) is generally very small comparing to primary consolidation settlement. Therefore this elastic settlement (S_e) mostly ignored.
- Usually considered at sand or sandy soil.
- Elastic settlement calculation is generally based on the theory of elasticity.
- For clay soil;

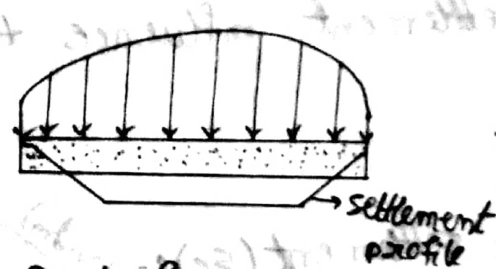


Rigid foundation

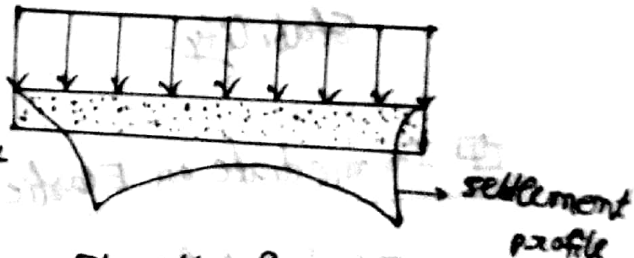


Flexible foundation

- For Sand;



Rigid foundation



Flexible foundation

Elastic settlement (S_e) of foundation on saturated clay. ($\mu_s = 0.5$)

- Jambu et al. (et al. means gang or group of people working together) (1956) proposed an equation for evaluating the average settlement of flexible foundation on saturated clay soils (Poisson ratio $\mu_s = 0.5$)
- The equation is given;

$$S_e = A_1 A_2 \frac{q_0 B}{E_s}$$

where, A_1 is the function of H/B and L/B.

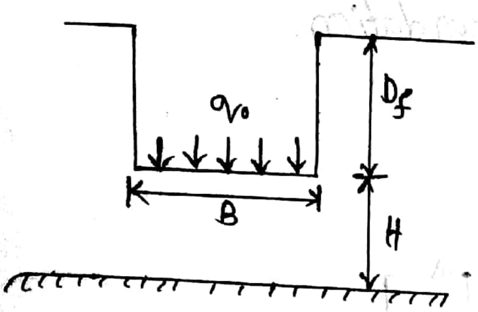
A_2 is the function of D_f/B.

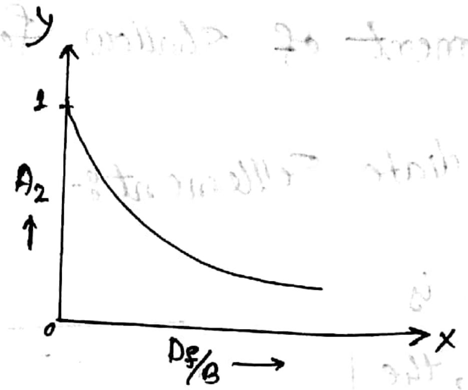
H = Thickness of compressible layer.

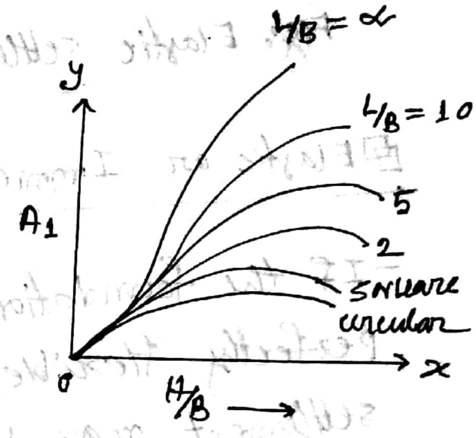
B = Width of foundation.

L = Length of foundation.

D_f = Depth of foundation.







– The modulus of elasticity (E_s) for clays can be given as;

$$E_s = \beta\, C_u$$

where, C_u = Undrained shear strength.

β = Function of plasticity index and overconsolidation ratio

Settlement based on the Theory of Elasticity :-

– The elastic settlement of a shallow foundation can be estimated by the theory of elasticity. From Hook's law, we get;

$$S_e = \int_0^H \epsilon_z\, dz = \frac{1}{E_s} \int_0^H (\Delta\sigma_z - \mu_s \Delta\sigma_x - \mu_s \Delta\sigma_y)\, dz$$

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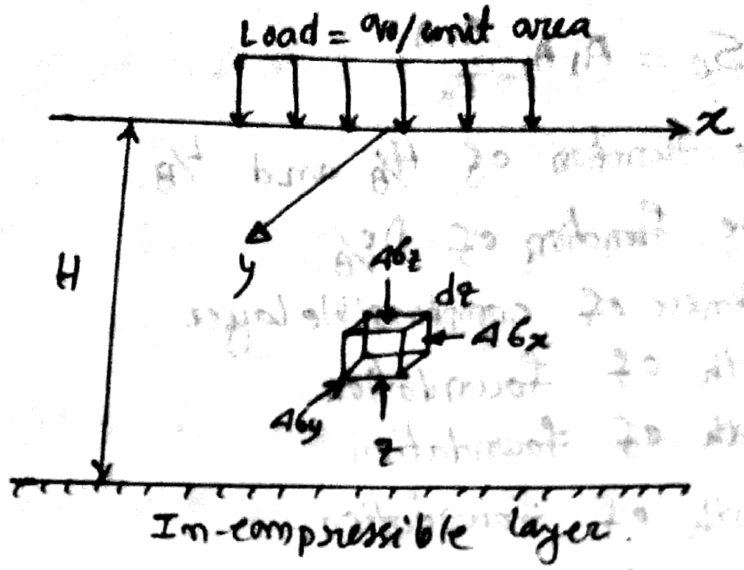


Fig: Elastic settlement of shallow foundation.

Elastic or Immediate Settlement:-

- If the foundation is perfectly flexible, the settlement may be expressed as;

$$S_e = q_0(\alpha B')\left(\frac{1-\mu_s^2}{E_s}\right) I_f I_s$$

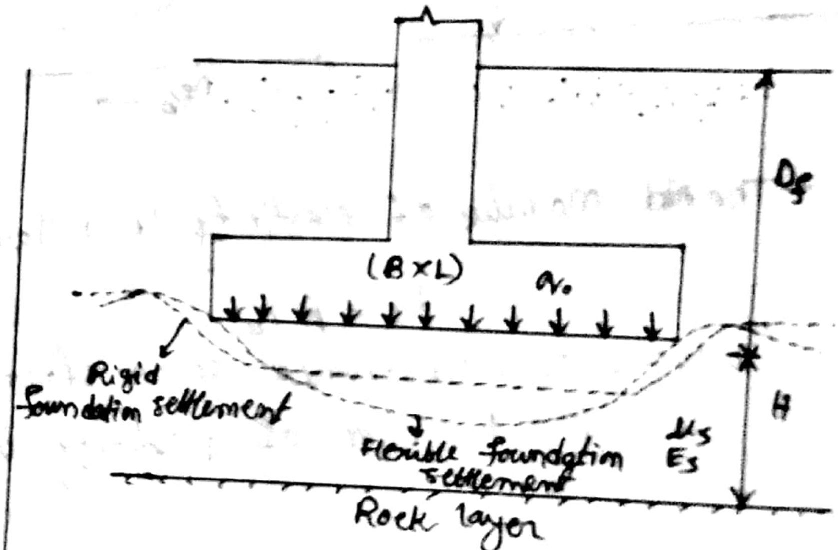


Fig: Elastic settlement of rigid and flexible foundation.

where,

q_0 = Net applid load on the foundation

μ_s = Poisson ratio of soil

E_s = Average modulus of elasticity of the soil under the foundation, measured from $z = 0$ to about $z = 5B$

$B' = B/2$ for center of foundation

$= B$ for corner of foundation.

I_s = Shape factor $= F_1 + \left(\frac{1-2\mu_s}{1-\mu_s}\right) F_2$

$$F_1 = \frac{1}{\pi}(A_0 + A_1)$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}(A_2)$$

$$A_0 = m' \ln\left[\frac{\left(1+\sqrt{m'^2+1}\right)\left(\sqrt{m'^2+n'^2}\right)}{m'\left(1+\sqrt{m'^2+n'^2+1}\right)}\right]$$

$$A_1 = \ln\left[\frac{\left(m'+\sqrt{m'^2+1}\right)\left(\sqrt{1+n'^2}\right)}{\left(m'+\sqrt{m'^2+n'^2+1}\right)}\right]$$

$$A_2 = \frac{m'}{n'\sqrt{m'^2+n'^2+1}}$$

For the center of the foundation;

$\alpha = 4$ = a factor that depends on the location on the foundation where settlement is being calculated

$$m' = \frac{L}{B}$$

$$n' = \frac{H}{\left(\frac{B}{2}\right)}$$

For the corner of the foundation;

$$\alpha = 1$$

$$m' = \frac{L}{B}$$

$$n' = \frac{H}{B}$$

and I_f = Depth factor $= f\left(\frac{D_f}{B}, \mu_s \text{ and } \frac{L}{B}\right)$

(The value of I_f can be obtained by table-5.10)

The elastic settlement of rigid foundation can be estimated as:

$$S_{e\,(rigid)} \simeq 0.93\, S_{e\,(flexible,\ center)}$$

- Due to the non-homogeneous nature of soil deposits, the magnitude of E_s may vary with depth. For that reason, Bowles (1987) recommended using a weighted average of E_s, which is given by:

$$E_s = \frac{\sum E_{s(i)} \cdot \Delta z}{\bar{z}} \quad [i = 1, 2, 3 - - -]$$

where,

$E_{s(i)}$ = Soil modulus of elasticity within a depth Δz.

$\bar{z}$ = H or $5B$, whichever is smaller

Improved equation of Elastic settlement:-

- In 1999, Mayne and Poulos presented an improved formula for calculating the elastic settlement of the foundation.
- The formula takes into account of rigidity of the foundation, depth of the embankment of the foundation, the increase in modulus of elasticity of soil with depth and the location of rigid layers at a limited depth.
- To use the Mayne and Poulos equation, one needs to determine the equivalent diameter B_e of a rectangular foundation;

$$\therefore B_e = \sqrt{\frac{4BL}{\pi}}$$

where, B = width of the foundation

L = length of the foundation

- For circular foundations;

$$B_e = B$$

where, B = Diameter of the foundation.

- The figure shows a foundation with an equivalent diameter B_e located at a depth D_f below the ground surface. The thickness of the foundation be t and the modulus of elasticity of the foundation material be E_f.

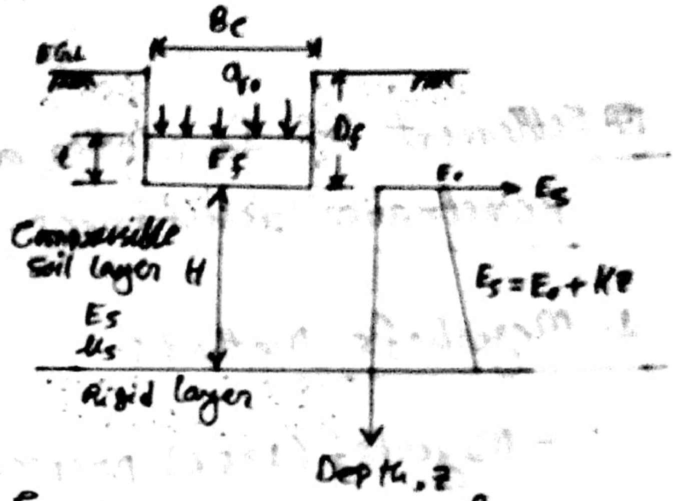


Fig: Improved equation for calculating elastic settlement.

A rigid layer is located at a depth H below the bottom of the foundation. The modulus of elasticity of the compressible soil layer can be given as;

$$E_s = E_0 + kz$$

- The elastic settlement below the center of the foundation is;

$$S_e = \frac{q_0 B_e I_G I_F I_E}{E_0}(1-\mu_s^2)$$

where,

I_G = Influence factor for the variation of E_s with depth

$$= f\left(\beta = \frac{E_0}{kB_e}, \frac{H}{B_e}\right) \rightarrow \text{(obtained from graph)}$$

I_F = Foundation rigidity correction factor.

$$= \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\frac{E_f}{E_0 + \frac{B_e}{2}k}\right)\left(\frac{2t}{B_e}\right)^3}$$

I_E = Foundation embedment correction factor.

$$= 1 - \frac{1}{3.5\,\exp(1.22\mu_s - 0.4)\left(\frac{B_e}{D_f} + 1.6\right)}$$

④ Settlement of foundation on sand based on standard Penetration resistance:-

1. Meyerhof's Method :-

- Meyerhof (1956) proposed a correlation for the net bearing pressure for foundation with the standard penetration resistance, N_{60}. The net pressure has been defined as;

$$q_{net} = \bar{q} - \gamma D_f$$

where, $\bar{q}$ = stress at the level of the foundation.

- According to meyerhof's theory, for 25 mm (1 inch) of estimated maximum settlement ;

$$q_{net}\,(kN/m^2) = \frac{N_{60}}{0.08} \quad (\text{For } B \leq 1.22\,m)$$

$$q_{net}\,(kN/m^2) = \frac{N_{60}}{0.125}\left(\frac{B+0.3}{B}\right)^2 \quad (\text{for } B > 1.22\,m)$$

- Meyerhof (1965) suggested that the net allowable pressure should be increased by about 50%.

- Bowles (1977) proposed the modified form of bearing equation, expressed as;

$$q_{net}\,(kN/m^2) = \frac{N_{60}}{2.5} \cdot F_d \left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22\,m)$$

$$q_{net}\,(kN/m^2) = \frac{N_{60}}{0.08} \left(\frac{B+0.3}{B}\right)^2 \cdot F_d \left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22\,m)$$

where,

F_d = Depth factor $= 1 + 0.33 \left(\frac{D_f}{B}\right)$

B = Foundation width (m)

S_e = Settlement (mm).

Hence,

$$S_e\,(mm) = \frac{1.25\, q_{net}\,(kN/m^2)}{F_d \cdot N_{60}} \quad (\text{for } B \leq 1.22\,m)$$

$$S_e\,(mm) = \frac{2\, q_{net}\,(kN/m^2)}{N_{60} \cdot F_d} \left(\frac{B}{B+0.3}\right)^2 \quad (\text{for } B > 1.22\,m)$$

where, N_{60} = Standardard penetration resistance between the bottom of the foundation and 2B below the bottom.

2. Burland and Burbidge's method.

(Self study
Book: BM Das. (7th.e)
page-264)

Consolidation settlement:-

1. Primary consolidation settlement:-

- Consolidation settlement occurs over time in saturated clayey soils subjected to an increased load caused by the construction of the foundation.

- On the basis of the one-dimensional consolidation settlement equations is given by:

$$S_{c(p)} = \int \epsilon_z \, dz$$

where,

ϵ_z = vertical strain

$$= \frac{\Delta e}{1+e_0}$$

Δe = change in void ratio

$= f(\sigma_0', \sigma_c' \text{ and } \Delta\sigma')$

e_0 = Initial void ratio.

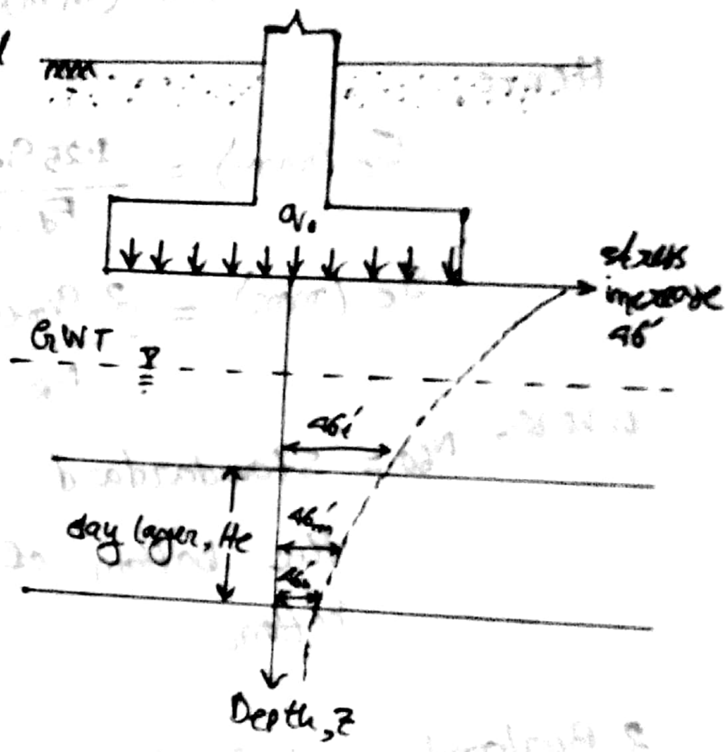


Fig: Consolidation settlement.

So, For normally consolidated clays;

$$S_{c(p)} = \frac{C_c H_e}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma'_{ave}}{\sigma_0'}\right)$$

- For over consolidated clays;

$$S_{c(p)} = \frac{C_s H_e}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma'_{ave}}{\sigma_0'}\right)$$

- For over consolidated clays with $\sigma_0' < \sigma_c' < (\sigma_0' + \Delta\sigma'_{ave})$;

$$S_{c(p)} = \frac{C_s H_e}{1+e_0} \log\left(\frac{\sigma_c'}{\sigma_0'}\right) + \frac{C_c H_e}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma'_{ave}}{\sigma_c'}\right)$$

where; σ_0' = Present effective overburden pressure.

$\Delta\sigma'_{ave}$ = Average increase in effective pressure on the clay layer.

σ_c' = Preconsolidation pressure.

e_0 = Initial void ratio of the clay layer.

C_c = Compression index.

C_s = Swelling index.

H_c = Thickness of the clay layer.

— Note that the increase in effective pressure, $\Delta\sigma'$ on the clay layer is not constant with depth. Magnitude of $\Delta\sigma'$ will decrease with the increase in depth measured from the bottom of the foundation. However, the average increase in pressure may be approximated by;

$$\Delta\sigma'_{ave} = \frac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

where, $\Delta\sigma'_t$ = Effective pressure increase at the top.

$\Delta\sigma'_m$ = Effective pressure increase at the middle.

$\Delta\sigma'_b$ = Effective pressure increase at the bottom.

Secondary consolidation settlement :-

— At the end of primary consolidation (i.e after the complete dissipation of excess pore water pressure) some settlement is observed that is due to plastic adjustment of soil fabrics. This stage of consolidation is called secondary consolidation.

- A plot of deformation against the logarithm of time during secondary consolidation is practically linear

- From the plot, the secondary compression index can be defined as;

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1}$$

$$\therefore C_\alpha = \frac{\Delta e}{\log(t_2/t_1)}.$$

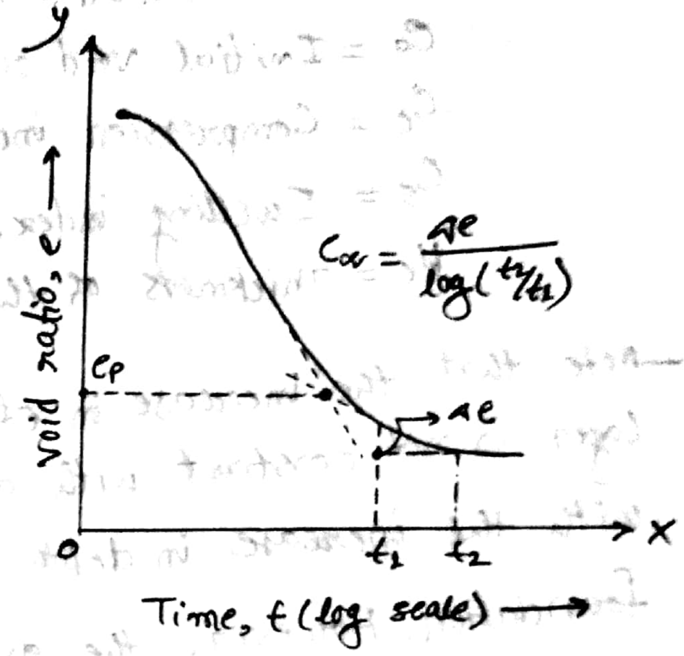


fig: variation of e with log t.

where,

C_α = Secondary compression index.

Δe = change in void ratio.

t_1, t_2 = time.

- The magnitude of secondary consolidation settlement can be calculated as;

$$S_{c(s)} = C'_\alpha H_c \log\left(\frac{t_2}{t_1}\right).$$

where, $C'_\alpha = \frac{C_\alpha}{1+e_p}$

e_p = Void ratio at the end of primary consolidation.

H_c = Thickness of the clay layer.

- Mesri (1973) correlated C'_α with the natural moisture content (w) of several soils, from which it appears that

$$C'_\alpha \simeq 0.0001\,w$$

where, w = Moisture content in percentage (%).

For most overconsolidated soils C'_α varies between 0.0005 to 0.001.

- Mesri and Godlewski (1977) compiled the magnitude of $\left(\frac{C_\alpha}{C_c}\right)$ (C_c = Compression index) for a number of soils. Based on their compilation, it can be summarized that;

→ For inorganic clays and silts;

$$\frac{C_\alpha}{C_c} = 0.04 \pm 0.01$$

→ For organic clays and silts;

$$\frac{C_\alpha}{C_c} = 0.05 \pm 0.01$$

→ For peats;

$$\frac{C_\alpha}{C_c} = 0.075 \pm 0.01$$

- The secondary consolidation settlement is more important in case of all organic and highly compressible inorganic soils.

- In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

⇒ The ratio of secondary to primary compression index for a given thickness of soil layer is dependent on the ratio of the stress increment ($\Delta\sigma'$), to the initial effective overburden stress (σ_0'). For small $\left(\frac{\Delta\sigma'}{\sigma_0'}\right)$ ratios, the secondary-to-primary compression ratio is larger.

SLOPE STABILITY

Slope:

Chapter – 15

Slope stability

Book: Principles of Geotechnical Engineering (7th Edition)

— B. M. Das.

(হুবহু Lecture)

Geotech (3233)

Math Problem

Ganay

Bearing Capacity of Shallow Foundation

⇒ **Problem-1:** A square foundation 2m × 2m in plan. The soil supporting the foundation has a friction angle $\varphi' = 25°$ and $c' = 20\ KN/m^2$. The unit weight of soil, $\gamma = 16.5\ KN/m^3$. Determine the allowable gross load on the foundation with a factor of safety of 3. Assume that the depth of the foundation D_f is 1.5m and that general shear failure occur in the soil.

Solution: we have;

$$q_u = c' N_c F_{ci} F_{cs} F_{cd} + q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2} \gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d} \text{ —ⓐ}$$

As the load is vertical:

$$F_{ci} = F_{qi} = F_{\gamma i} = 1.$$

Now; Bearing capacity factors:

$$N_q = \tan^2\left(45 + \frac{\varphi'}{2}\right) e^{\pi \tan \varphi'}$$

$$= \tan^2\left(45 + \frac{25}{2}\right) e^{\pi \tan 25°}$$

$$\therefore N_q = 10.66$$

$$N_c = (N_q - 1) \cot \varphi' = (10.66 - 1) \cot 25° = 20.72$$

$$N_\gamma = 2(N_q + 1) \tan \varphi' = 2(10.66 + 1) \tan 25° = 10.87$$

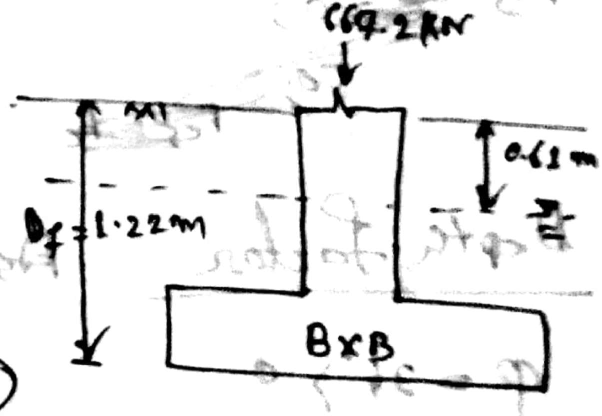
$\therefore q_{all} = \frac{q_u}{F_S} = \frac{1374.45}{3} = 458.15 \ kN/m^2$

So, Gross allowable Load, $Q = q_{all} \times Area$

$= 458.15 \times (2 \times 2)$

$Q = 1832.61 \ kN$. Ans.

⇒ Problem-28 A square foundation (B×B) has to be constructed as shown in figure. Assume that $\gamma = 16.5 \ kN/m^3$, $\gamma_{sat} = 18.55 \ kN/m^3$, $\phi' = 34°$, $D_f = 1.22 m$ and $D_1 = 0.61 m$. The gross allowable load Q_{all} with factor of safety of 3 is 667.2 kN. Determine the size of the footing.



Solution:

We have;

$$q_{all} = \frac{Q_{all}}{B \times B} = \frac{667.2}{B^2} \quad \text{--- (1)}$$

Now; $q_{all} = \frac{q_u}{F_S}$

$$= \frac{1}{3}\left[c' N_c F_{ci} F_{cs} F_{cd} + q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2}\gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}\right] \quad \text{--- (2)}$$

Bearing capacity factors;

$$N_q = \tan^2\left(45° + \frac{\phi'}{2}\right) e^{\pi \tan\phi'}$$

$$= \tan^2\left(45° + \frac{34}{2}\right) e^{\pi \tan 34°} = 29.44$$

$N_c = (N_q - 1)\cot\varphi' = (29.44-1)\cot 34^\circ = 42.16$

$N_\gamma = 2(N_q+1)\tan\varphi' = 2(29.44+1)\tan 34^\circ = 41.06$

As the load is vertical:

$F_{ci} = F_{qi} = F_{\gamma i} = 1$

Shape factors;

$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{B}{B}\right)\left(\frac{29.44}{42.16}\right) = 1.698$

$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\varphi' = 1 + \left(\frac{B}{B}\right)\tan 34^\circ = 1.67$

$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{B}{B}\right) = 0.60$

~~As B = 0;~~

~~So, $F_{cs} = F_c$~~

Depth factor

Assume $\frac{D_f}{B} \le 1$.

$\varphi = 34^\circ > 0$

$F_{qd} = 1 + 2\tan\varphi'(1-\sin\varphi')^2\left(\frac{D_f}{B}\right)$

$= 1 + 2\tan 34^\circ(1-\sin 34^\circ)^2\left(\frac{1.22}{B}\right)$

$= 1 + \frac{0.3198}{B}$

$F_{\gamma d} = 1.$

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Surcharge: $q = \gamma D_{f1} + D_2 (\gamma_{sat} - \gamma_w)$

$= (16.5 \times 0.61) + (1.22 - 0.61)(18.55 - 9.81)$

$q = 15.39 \ kN/m^2$

From eqn ②, we have:

$$q_{all} = \frac{1}{3}\left(0 + 15.39 \times 29.44 \times 1 \times 1.67 \left(1 + \frac{0.3198}{B}\right) + \frac{1}{2} \times (18.55 - 9.81) B \times 1 \times 41.06 \times 0.60\right) \ [c' = 0]$$

$$q_{all} = \frac{1}{3}\left[756.65 \left(1 + \frac{0.32}{B}\right) + 107.66 B\right] \quad \text{——— ③}$$

From equation ① and ③: we get:

$$\frac{667.2}{B^2} = \frac{1}{3}\left(756.65 + \frac{242.13}{B} + 107.66 B\right)$$

$$\therefore 2001.6 = B^2 \times 756.65 + 242.13 B + 107.66 B^3$$

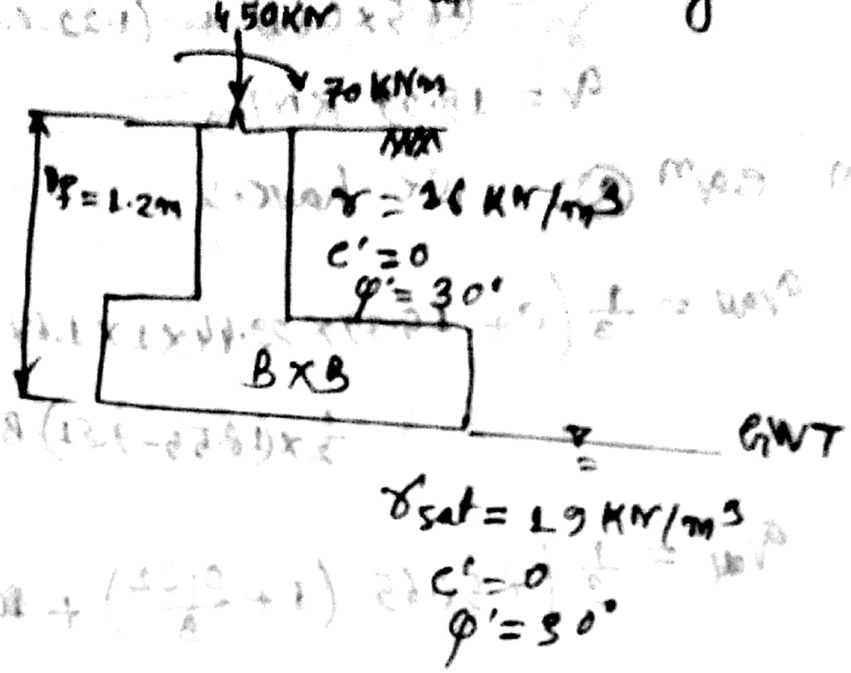
By trial and error:

$B = 1.36 \ m$.

check: $\frac{D_f}{B} = \frac{1.22}{1.36} = 0.897 < 1$.

$\therefore$ Size of the footing $(1.36 m \times 1.36 m)$

⇒ **Problem-3:** A square footing is shown in figure below. use $FS = 6$. Determine the size of the footing.



Solution:

we have;

Eccentricity, $e = \frac{M}{Q} = \frac{70}{450} = 0.16\ m.$

Now, effective width, $B' = B - 2e = B - (2 \times 0.16)$

$\therefore B' = B - 0.32$

effective length, $L' = L = B$ [As square footing]

$$\therefore q_{all} = \frac{Q_{all}}{Area} = \frac{Q_{all}}{B' . B} = \frac{450}{B(B-0.32)} \quad \text{——— (1)}$$

We know; [$\because c' = 0$]

$$\therefore q_{all} = \frac{q_u}{F_S} = \frac{1}{6}\left[q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2}\gamma' B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}\right] \quad \text{——— (2)}$$

Bearing capacity factors;

$$N_q = \tan^2\left(45 + \frac{\varphi'}{2}\right) e^{\pi \tan \varphi'}$$

$$= \tan^2\left(45° + \frac{30°}{2}\right) e^{\pi \tan 30°} = 18.40$$

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$N_\gamma = 2(N_q+1)\tan\varphi' = 2(18.40+1)\tan 30°$

$= 22.40$

Shape factors;

$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\varphi'$

$= 1 + \left(\frac{B-0.32}{B}\right)\tan 30° = 1 + 0.58\left(\frac{B-0.32}{B}\right)$

$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{B-0.32}{B}\right)$.

Depth factors; Assume $\frac{D_f}{B} \leq 1$

$\because \varphi = 30° > 0$

$\therefore F_{qd} = 1 + 2\tan\varphi'(1-\sin\varphi')^2\left(\frac{D_f}{B}\right)$

$= 1 + 2\tan 30°(1-\sin 30°)^2\left(\frac{1.2}{B}\right)$

$F_{qd} = 1 + \frac{0.35}{B}$

$F_{\gamma d} = 1$.

Surcharge, $q = 1.2 \times 16 = 19.2\ kN/m^2$

unit weight, $\gamma' = \gamma_{sat} - \gamma_w$

$= 19 - 9.81$

$\gamma' = 9.19\ kN/m^3$

From eq^n (2);

$q_{all} = \frac{1}{6}\Big[19.2 \times 18.4 \times 1 \times \left(1 + 0.58\left(\frac{B-0.32}{B}\right)\right) \times \left(1 + \frac{0.35}{B}\right)$

$+ \frac{1}{2} \times 9.19 \times B \times 22.40 \times 1 \times \left(1 - 0.4\left(\frac{B-0.32}{B}\right)\right) \times 1\Big]$

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$$q_{all} = \frac{1}{6}\left[353.28\left(1+\frac{0.58B-0.19}{B}\right)\left(1+\frac{0.35}{B}\right) + 102.93B\left(1-\frac{0.4B-0.13}{B}\right)\right] \quad \text{--- (3)}$$

From eq^n (1) and (3): we get,

$$\frac{450}{B(B-0.32)} = \frac{1}{6}\left[353.28\left(1+\frac{0.58B-0.19}{B}\right)\left(1+\frac{0.35}{B}\right) + 102.93B\left(1-\frac{0.4B-0.13}{B}\right.\right.$$

$$\therefore 2700 = B\,(B-0.32)\left[353.28\left(1+\frac{0.58B-0.19}{B}\right)\left(1+\frac{0.35}{B}\right) + 102.93B\left(1-\frac{0.4B-0.13}{B}\right)\right.$$

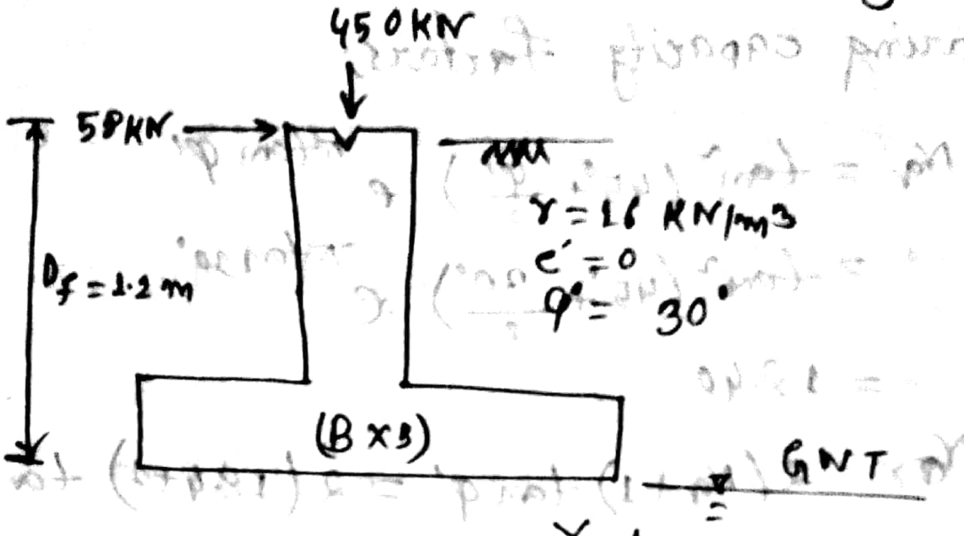
By trial and error:

$$B = 208m \approx 2.1\ m.$$

check: $\frac{D_f}{B} = \frac{1.2}{2.1} = 0.57 < 1.$ (OK)

$\therefore$ Size of the footing $(2.1\,m \times 2.1\,m)$.

⇒ **Problem-48** A square footing is shown in figure below. Use $F_S = 6$. Determine the size of the footing.



Solution:

Moment, $M = 58 \times 1.2 = \cancel{69.6}$ KNm.
69.6

Eccentricity, $e = \frac{M}{Q} = \frac{69.6}{450} = 0.15$ m

effective width, $B' = B - 2e = B - 2 \times 0.15 = B - 0.3$

effective length, $L' = \cancel{L} = B$ [∵ square footing]

Given; $q_{all} = \frac{Q_{all}}{Area} = \frac{450}{B'L'}$

$\therefore q_{all} = \frac{450}{B(B-0.3)}$ ——— (1)

As $c' = 0$;

$\therefore q_{all} = \frac{1}{6}\left[q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2}\gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}\right]$ —— (2)

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As the load is vertical

$F_{qi} = F_{\gamma i} = 1.$

Bearing capacity factors;

$$N_q = \tan^2\left(45^\circ + \frac{\varphi'}{2}\right) e^{\pi \tan \varphi'}$$

$$= \tan^2\left(45^\circ + \frac{30^\circ}{2}\right) e^{\pi \tan 30^\circ}$$

$$= 18.40$$

$$N_\gamma = 2(N_q + 1)\tan\varphi' = 2(18.4 + 1)\tan 30^\circ = 22.40$$

Shape factors;

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\varphi' = 1 + \left(\frac{B-0.3}{B}\right)\tan 30^\circ$$

$$= 1 + 0.58\left(\frac{B-0.3}{B}\right)$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{B-0.3}{B}\right)$$

Depth factor; Assume $\frac{D_f}{B} \leq 1$.

$\because \varphi' = 30^\circ > 0^\circ$

$$\therefore F_{qd} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2\left(\frac{D_f}{B}\right)$$

$$= 1 + 2\tan 30^\circ(1 - \sin 30^\circ)^2\left(\frac{1.2}{B}\right)$$

$$= 1 + \frac{0.35}{B}$$

$F_{\gamma d} = 1.$

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Surcharge, $q = 1.2 \times 16 = 19.2 \; kN/m^2$

From eq^n ②;

$$q_{nu} = \frac{1}{6}\left[19.2 \times 18.40 \times 1 \times \left(1 + \frac{0.5B - 0.17}{B}\right)\left(1 + \frac{0.35}{B}\right) + \frac{1}{2} \times (19-9.81) \times B \times 1 \times 22.40 \times \left(1 - \left(\frac{0.4B - 0.12}{B}\right)\right)\right] \quad \text{— ③}$$

From eq^n ① and ③, we get;

$$\frac{450}{B(B-0.30)} = \frac{1}{6}\left[353.28\left(1 + \frac{0.58B - 0.17}{B}\right)\left(1 + \frac{0.35}{B}\right) + 102.99B\left(1 - \frac{0.4B - 0.12}{B}\right)\right]$$

$$\therefore 2700 = B(B-0.3)\left[353.28\left(1 + \frac{0.58B - 0.17}{B}\right)\left(1 + \frac{0.35}{B}\right) + 102.99B\left(1 - \frac{0.4B - 0.12}{B}\right)\right]$$

By trial and error;

$$B = 2.045 \; m.$$

Check: $\frac{D_f}{B} = \frac{1.2}{2.045} = 0.586 < 1$. (OK)

$\therefore$ Size of the footing $(2.05 \, m \times 2.05 \, m)$. (Ans)

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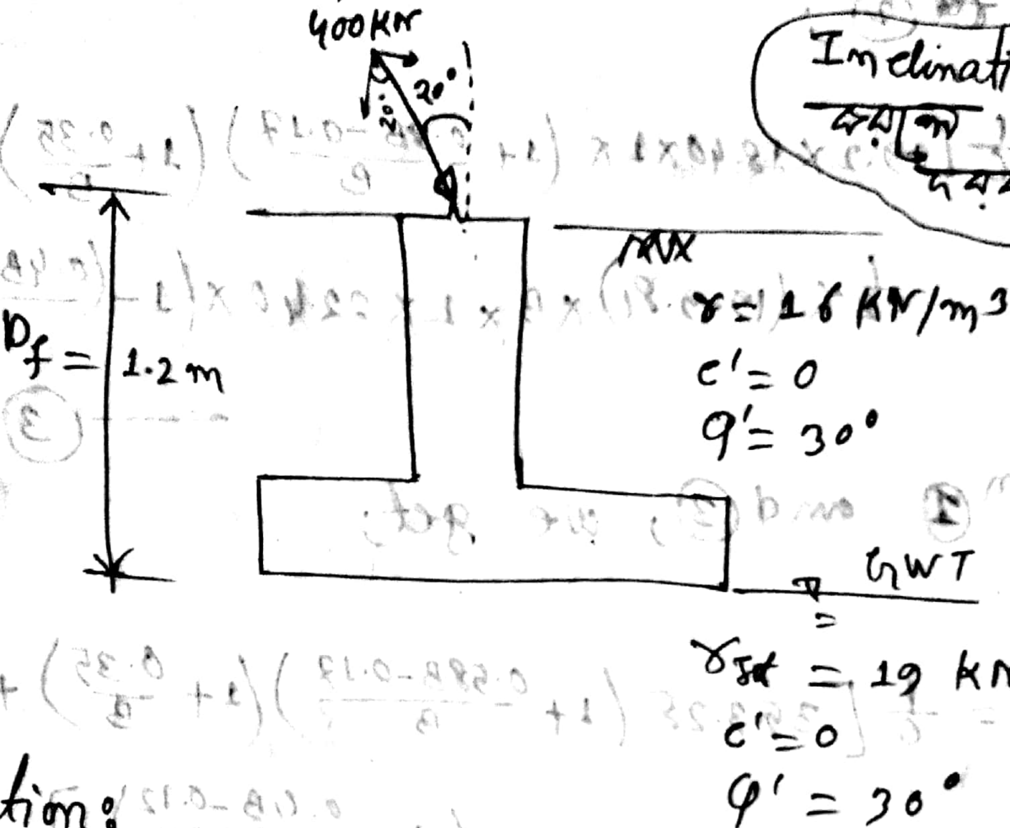
→ **Problem-5:** A footing is shown in figure below. Use FS = 6. Determine the size of the footing.



Inclination factor এখানে ব্যবহার করার দরকার নাই-

Solution:

Consider the footing is square of size $(B \times B)$.

$$Q_y = 400 \times \cos 20° = 375.88 \text{ KN}$$

$$Q_x = 400 \times \sin 20° = 136.81 \text{ KN}.$$

Now, Moment, $M = 136.81 \times 1.2 = 164.17$ KNm

Eccentricity, $e = \frac{M}{Q_y} = \frac{164.17}{357.88} = 0.44$ m.

∴ Effective width; $B' = B - 2e = B - 2 \times 0.44 = B - 0.88$

effective length, $L' = L = B$ [∵ square footing]

Bearing capacity factors;

$N_q = \tan^2\left(45° + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'}$

$= \tan^2\left(45° + \frac{30°}{2}\right) e^{\pi \tan 30°}$

$= 18.40$

$N_\gamma = 2(N_q + 1)\tan\varphi' = 2(18.40 + 1)\tan 30°$

$= 22.40.$

Shape factors;

$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\varphi' = 1 + \left(\frac{B - 0.88}{B}\right) 0.58$

$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{B - 0.88}{B}\right)$

Depth factors; Assume $\frac{D_f}{B} \leq 1$.

$\because \varphi' = 30° > 0°$

$\therefore F_{qd} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2 \left(\frac{D_f}{B}\right)$

$= 1 + 2\tan 30°(1 - \sin 30°)^2 \left(\frac{1.2}{B}\right)$

$= 1 + \frac{0.35}{B}$

$F_{\gamma d} = 1.$

Load inclination factors;

$F_{ci} = F_{qi} = \left(1 - \frac{\beta°}{90°}\right)^2 = \left(1 - \frac{20}{90}\right)^2 = 0.61$

Now;

$$F_{ri} = \left(1 - \frac{\beta}{\varphi'}\right)^2 = \left(1 - \frac{20}{30}\right)^2 = 0.11 .$$

$$q_{all} = \frac{Q_y}{\cancel{Fs}\ \text{area}} = \frac{375.88}{B'L'} = \frac{375.88}{B(B-0.88)} \quad \text{—— (1)}$$

whe have;

$$q_{all} = \frac{q_u}{F_s} = \frac{1}{6}\left[q_r N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2}\gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}\right]$$

$$= \frac{1}{6}\left[(1.2\times 21)\times 18.4\times 0.61\times\left(1+\frac{0.58B-0.51}{B}\right)\left(1+\frac{0.35}{B}\right)\right.$$

$$\left. + \frac{1}{2}\times(19-9.81)\times B\times 22.4\times 0.11\times\left(1-\frac{0.4B-0.35}{B}\right)\times 1\right)$$

$$q_{all} = \frac{1}{6}\left[215.5\left(1+\frac{0.58B-0.51}{B}\right)\left(1+\frac{0.35}{B}\right) + 11.33B\left(1-\frac{0.4B-0.35}{B}\right)\right.$$

—— (2)

From eqn (1) and (2); we get;

$$\frac{375.88}{B(B-0.88)} = \frac{1}{6}\left[215.5\left(1+\frac{0.58B-0.51}{B}\right)\left(1+\frac{0.35}{B}\right) + 11.33B\left(1-\frac{0.4B-0.35}{B}\right)\right]$$

$$\therefore 2255.28 = B(B-0.88)\left[215.5\left(1+\frac{0.58B-0.51}{B}\right)\left(1+\frac{0.35}{B}\right) + 11.33B\left(1-\frac{0.4B-0.35}{B}\right)\right]$$

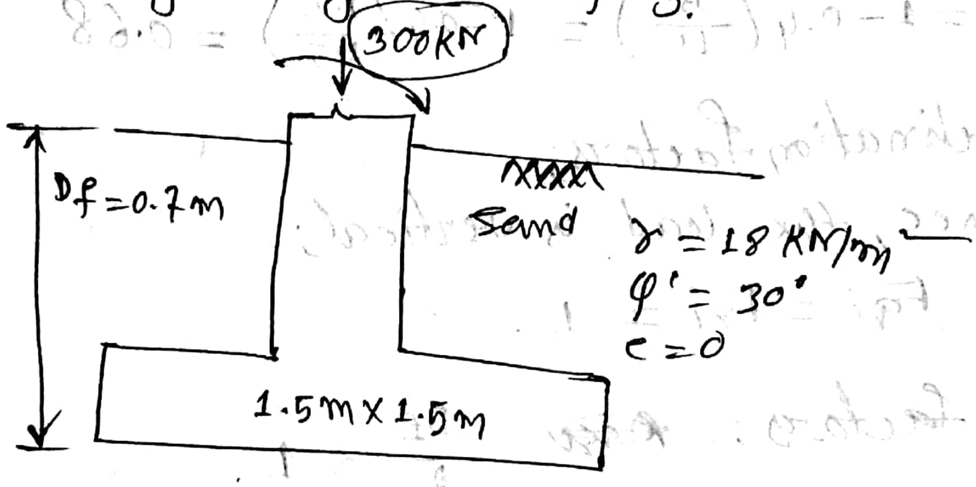
By trial and error;

$$B = 3m.$$

check: $\frac{D_f}{B} = \frac{1.2}{3} = 0.4 < 1$. OK.

$\therefore$ Size of the footing $(3m \times 3m)$. (Ans)

⇒ Problem-6: A square footing of $1.5m \times 1.5m$ is shown in figure below. Assume that the one way load eccentricity $e = 0.15$ m. Determine the allowable bearing capacity of the footing using a FS of 3.



Solution: Given, eccentricity, $e = 0.15$ m.

$\therefore$ effective width, $B' = B - 2e = 1.5 - 2 \times 0.15 = 1.2$ m.

effective length, $L' = L = B = 1.5$ m. [square]

Since $e' = 0$;

$\therefore q_u = q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2} \gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}$ —— (1).

Bearing capacity factors;

$N_q = \tan^2(45^\circ + \frac{\varphi'}{2}) e^{\pi \tan\varphi'} = \tan^2(45^\circ + \frac{30^\circ}{2}) e^{\pi \tan 30^\circ}$

$= 18.40$

$N_\gamma = 2(N_q + 1)\tan\varphi' = 22.40$

Shape factors;

$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\varphi' = 1 + \left(\frac{1.2}{1.5}\right)\tan 30^\circ$

$= 1.46$

$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.2}{1.5}\right) = 0.68$

Load inclination factors;

Since, the load is vertical;

$F_{qi} = F_{\gamma i} = 1.$

Depth factors; ~~Assu~~ $\frac{D_f}{B} \leq 1.$

$\because \varphi' = 30^\circ > 0$

$\therefore F_{qd} = 1 + 2\tan\varphi'(1 - \sin\varphi')^2\left(\frac{D_f}{B}\right)$

$= 1 + 2\tan 30^\circ(1 - \sin 30^\circ)^2\left(\frac{0.7}{1.5}\right)$

$= 1.14$

$F_{\gamma d} = 1$

From eq^n (1), we get,

$q_u = (0.7 \times 18) \times 18.40 \times 1 \times 1.46 \times 1.14 + \frac{1}{2} \times 18 \times 1.5 \times 1 \times 22.4 \times 0.68 \times 1$

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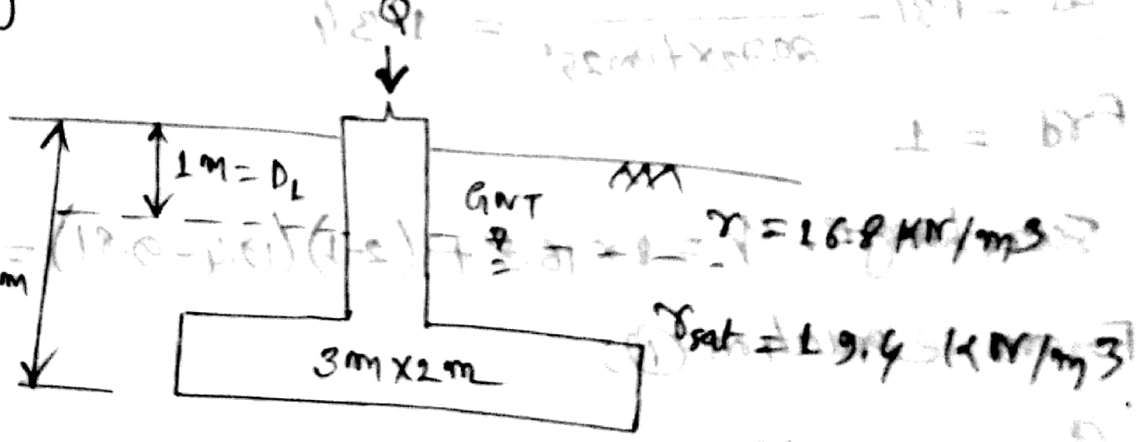
$$q_u = 385.87 + 205.63$$
$$= 591.5 \text{ KN/m}^2$$

∴ Allowable bearing capacity;

$$q_{all} = \frac{q_u}{FS} = \frac{591.5}{3} = 197.17 \text{ KN/m}^2$$

(Ans)

⇒ **Problem-7:** A column foundation is 3m×2m in plan. Given, $D_f = \cancel{1.5}\ 2$ m, $\varphi = 25°$, $c = 50$ KN/m² using FS=4. Determine the net allowable load that the foundation could carry.



Solution: $q_u = c N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$

$c = 50$ KN/m²

Bearing capacity factor;

$$N_q = \tan^2\left(45° + \frac{25°}{2}\right) e^{\pi \tan 25°} = 10.66$$

$$N_c = (N_q - 1) \cot\varphi' = (10.66 - 1) \cot 25° = 20.72$$

$$N_\gamma = 2(10.66 + 1) \tan 25° = 10.87$$

Load inclination factor;

$F_{ci} = F_{qi} = F_{\gamma i} = 1$

Shape factor;

$F_{qs} = 1 + \left(\frac{B}{L}\right)\left(\tan\phi'\right) = 1 + \frac{2}{3} \times \tan 25^\circ = 1.31$

$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \frac{2}{3} \times \frac{10.66}{20.72} = 1.34$

$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\frac{2}{3} = 0.73$

Depth factor;

$F_{qd} = 1 + 2\tan 25^\circ (1-\sin 25^\circ)^2 \cdot \frac{0.72}{2} = 1.31$

$F_{cd} = 1.31 - \frac{1-1.31}{20.72 \times \tan 25^\circ} = 1.34$

$F_{\gamma d} = 1.$

Surcharge, $q = 1 \times 16.8 + (2-1)(19.4 - 9.81) = 26.39$ KN.

From equation (1);

$q_u = 50 \times \cancel{10.66}\,20.72 \times 1.34 \times 1.34 \times 1 + 26.39 \times \cancel{20}\,10.66 \times 1.31 \times 1.31 \times 1$

$+ \frac{1}{2} \times (19.4 - 9.81) \times 2 \times 10.87 \times 0.73 \times 1 \times 1$

$= 1860.24 + 482.77 + 76.097$

$\Rightarrow q_u = 2419.11\ KN/m^2$

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$$q_{all} = \frac{q_u}{F.S} = \frac{2419.4}{4} = 604.78 \ KN/m^2$$

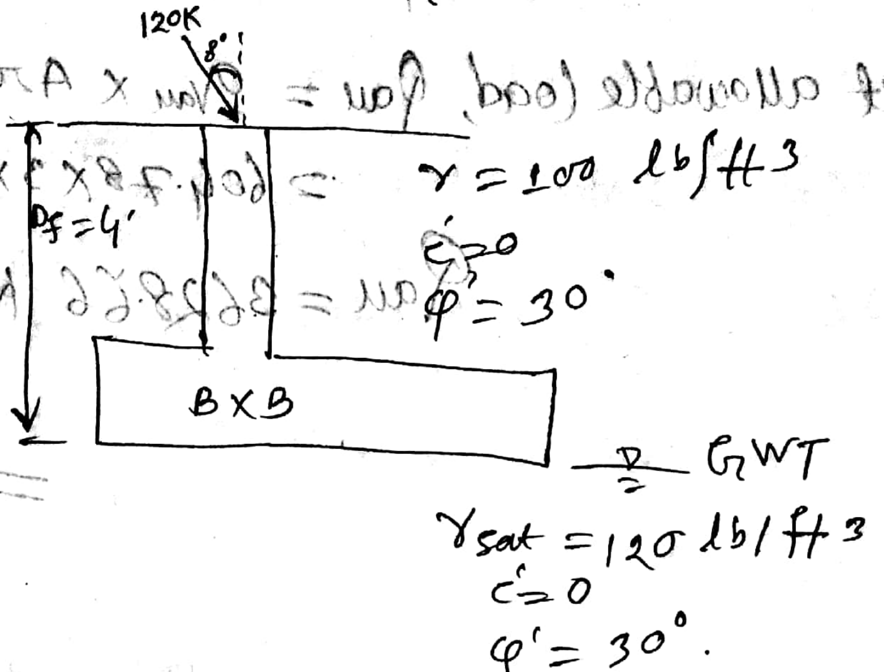
$\therefore$ Net allowable load, $Q_{all} = q_{all} \times Area$

$$= 604.78 \times 3 \times 2$$

$$Q_{all} = 3628.66 \ KN.$$

(Ans.)

Problem - 8: A square footing is shown in figure below. Using FS = 6. Determine the size of footing.



Solution:

$$q_{all} = \frac{Q_{all}}{Area} = \frac{120}{B^2} \longrightarrow (1)$$

As $c' = 0$ so;

$$q_{all} = \frac{1}{6}\left[q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2}\gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}\right] \longrightarrow (2)$$

Surcharge, $q = 4 \times 100 = 400\ lb/ft^2$

Bearing capacity factor;

$N_q = \tan^2(45° + \frac{30}{2})\, e^{\pi \tan 30°}$

$= 18.40$

$N_\gamma = 2(N_q + 1)\tan 30° = 22.40$

~~Shape factors;~~

$\cancel{F_{qd} = 1 + 2\tan 30°(1-\sin 30°)^2 \left(\frac{4}{B}\right) = 1 + \frac{1.155}{B}}$

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$F_{\gamma d} = 1 - 0.4\left(\frac{B}{B}\right) = 0.6$

~~Depth factors; Assume $\frac{D_f}{B} \leq 1$.~~

~~As $\varphi' \neq 30°$~~

Shape factors;

$F_{qs} = 1 + \left(\frac{B}{B}\right)\tan 30° = \cancel{0.42} \quad 1.58$

$F_{\gamma s} = 1 - \left(\frac{B}{B}\right)(0.4) = 0.6$

Depth factors; Assume $\frac{D_f}{B} \leq 1$.

As $\varphi' \neq 30°$

$F_{qd} = 1 + 2\tan 30°(1 - \sin 30°)^2 \frac{4}{B} = 1 + \frac{1.155}{B}$

$F_{\gamma d} = 1$.

Load inclination factor;

$F_{qi} = \left(1 - \frac{\beta°}{90°}\right)^2 = \left(1 - \frac{8}{90}\right)^2 = 0.83$

$F_{\gamma i} = \left(1 - \frac{8}{30}\right)^2 = 0.54$

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From ②;

$$q_{all} = \frac{1}{6}\left[400 \times 18.4 \times 0.83 \times 1.58 \times \left(1 + \frac{1.155}{B}\right) + \frac{1}{2}(120 - 62.4) \times B \times 22.4 \times 0.58 \times 0.6 \times 1\right]$$

$$q_{all} = \frac{1}{6}\left(9651\left(1 + \frac{1.155}{B}\right) + 224.5B\right) \quad \text{--- ③}$$

From eq^n ① & ③; we get;

$$\frac{120000}{B^2} = \frac{1}{6}\left(9651\left(1 + \frac{1.156}{B}\right) + 224.5B\right)$$

$$\Rightarrow 720000 = 9651B + 11146.91B + 224.5B^3$$

By trial and error;

$$B = 7.49' \approx 7.5'$$

check: $\frac{D_f}{B} = \frac{4}{7.5} = 0.53 < 1$ (OK)

∴ Size of the footing $(7.5' \times 7.5')$

Formulas

$$q_{all} = \frac{Q_{all}}{Area}$$

$$q_{all} = \frac{q_u}{FS}$$

$$q_u = c' N_c F_{ci} F_{cs} F_{cd} + q N_q F_{qi} F_{qs} F_{qd} + \frac{1}{2} \gamma B N_\gamma F_{\gamma i} F_{\gamma s} F_{\gamma d}.$$

Bearing capacity factors;

$$N_q = \tan^2\left(45^\circ + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'}$$

$$N_c = (N_q - 1) \cot\varphi'$$

$$N_\gamma = 2(N_q + 1) \tan\varphi'$$

Shape factors;

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right) \tan\varphi'$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

Depth factors;

For $D_f/B \leq 1$.

$\varphi > 0^\circ$

$$F_{qd} = 1 + 2\tan\varphi' (1 - \sin\varphi')^2 \left(\frac{D_f}{B}\right)$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\varphi'}$$

$$F_{\gamma d} = 1.$$

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Math Problem

⇒ Subsurface Exploration or soil investigation:

Problem-17

1. $(N_1)_{60} = C_N \cdot N_{60}$

Liao and Whitman; $C_N = \left(\dfrac{1}{\frac{\sigma_o'}{P_a}}\right)^{0.5}$ $\left[\begin{array}{l}\sigma_o' \text{ in } KN/m^2 \\ P_a = 100\ KN/m^2\end{array}\right.$

Skempton; $C_N = \dfrac{2}{1+\left(\frac{\sigma_o'}{P_a}\right)}$ $\dfrac{\sigma_o' = \text{in } KN/m^2}{P_a = 100\ KN/m^2}$ (fine sand)

$C_N = \dfrac{3}{2+\left(\frac{\sigma_o'}{P_a}\right)} \longrightarrow$ NC coarse sand.

2. $\phi'(\text{degree}) = 27.1 + 0.3\, N_{60} - 0.00054\, (N_{60})^2$

3. $\phi'(\text{deg}) = \tan^{-1}\left[\dfrac{N_{60}}{12.2 + 20.3\left(\frac{\sigma_o'}{P_a}\right)}\right]^{0.34}$

4. $D_r = \left[\dfrac{N_{60}}{17 + 24\left(\frac{\sigma_o'}{P_a}\right)}\right]^{0.5}$

5. Degree of disturbance, $A_R(\%) = \dfrac{D_o^2 - D_i^2}{D_i^2} \times 100\%$

⇒ σ_o' যদি KN/m^2 হয়, তাহলে $\frac{1}{95.6}$ দ্বারা গুন করে ton/ft^2 করতে হয়,

⇒ σ_o' যদি lb/ft^2 হয়, তাহলে $\frac{1}{200}$ দ্বারা গুন করে ton/ft^2 করতে হয়।

6. $\dfrac{E_s}{P_a} = \alpha \cdot N_{60}$

7. Average friction angle. $= \dfrac{\Sigma \phi'}{n}$

8. Average relative density $= \dfrac{\Sigma D_r \%}{n}$

Problem-1: Find $(N_1)_{60}$ at various depth. water table at a depth of 10.5 m below ground. $\gamma = 17.3\ KN/m^2$

Solution:

Depth (z)(m)	Effective pressure $\sigma_0' = \gamma \cdot h\ (KN/m^2)$	correction factor $C_N = \left(\frac{1}{\frac{\sigma_0'}{P_a}}\right)^{0.5}$	N_{60}	$(N_1)_{60} = C_N \cdot N_{60}$
1.5	25.95	1.96	8	16
3.0	51.9	1.39	7	10
4.5	77.85	1.13	12	14
6.0	103.8	0.98	14	14
7.5	129.75	0.88	13	11

100% (Ans.)

Problem-2: Calculate the corrected standard penetration number using Skempton correction factor. The GWT is located at a depth of 5.5 m and the dry unit weight of sand and saturated unit weight of sand are 18 KN/m² and 19.5 KN/m² respectively.

Depth (m) →	1.5	3.0	4.5	6.0	7.5	9.0	10.5
N_{60} →	5	7	9	8	13	12	14

Solution:

$\gamma' = \gamma_{sat} - \gamma_w$

$\gamma_w = 9.81\ KN/m^3$

$\gamma_w = 62.4\ lb/ft^3$

$\gamma_w = 1000\ Kg/m^3$

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Solution: $P_a = 100\ kN/m^2$

Depth (m)	Effective stress $\sigma' = \gamma \cdot h$ kN/m^2	N_{60}	Correction factor $C_N = \frac{2}{1+(\frac{\sigma'}{P_a})}$	Corrected N_{60} $(N_1)_{60} = C_N \cdot N_{60}$
1.5	$1.5 \times 18 = 27$	5	1.57	≈ 8
3.0	$3 \times 18 = 54$	7	1.30	≈ 9
4.5	$4.5 \times 18 = 81$	9	1.10	≈ 10
6.0	$(5.5 \times 18) + \{(6.0-5.5) \times (19.5-9.81)\}$ $= 99 + (0.5 \times 9.69)$ $= 103.85$	8	0.98	≈ 8
7.5	$(5.5 \times 18) + (2 \times 9.69)$ $= 118.38$	13	0.92	≈ 12
9.0	$(5.5 \times 18) + (3.5 \times 9.69)$ $= 132.92$	12	0.86	≈ 10
10.5	$(5.5 \times 18) + (5 \times 9.69)$ $= 147.45$	14	0.81	≈ 11

(Ans.)

Problem-3: Find $(N_1)_{60}$ using Liao and Whitman corelation factor. GWT is at a depth of 5.5 m, $\gamma_{dry} = 18\ KN/m^3$, $\gamma_{sat} = 19.5\ KN/m^3$.

$P_a = 100\ KN/m^2$

Depth (m)	Effective stress $\sigma_0' = \gamma \cdot h$ KN/m^2	N_{60}	Correction factor $C_N = \left(\frac{1}{\sigma_0'/P_a}\right)^{0.5}$	Corrected No. $(N_1)_{60} = C_N \cdot N_{60}$
1.5	$1.5 \times 18 = 27$	5	1.92	≈ 10
3.0	$3 \times 18 = 54$	7	1.36	≈ 10
4.5	$4.5 \times 18 = 81$	9	1.11	≈ 10
6.0	$(5.5 \times 18) + (6-5.5)(19.5-9.81) = 103.85$	8	0.98	≈ 8
7.5	$99 + (2.0 \times 9.69) = 118.38$	13	0.92	≈ 12
9.0	$99 + (3.5 \times 9.69) = 132.92$	12	0.87	≈ 10
10.5	$99 + (5 \times 9.69) = 147.45$	14	0.82	≈ 11

Problem-4:

Depth	N_{60}
4.5 m	10

$\therefore \varphi' = ?$ (Ans)

$\sigma_0' = \gamma = 4.5 \times 18 = 81\ KN/m^2$

Skempton corelation factor, $C_N = \frac{2}{1 + \frac{\sigma_0'}{P_a}}$

$= \frac{2}{1 + \frac{81}{100}}$

$C_N = 1.1$

$(N_1)_{60} = C_N N_{60} = 1.1 \times 10 = \underline{11}$

$\therefore \varphi = 27.1 + 0.3 (N_1)_{60} - 0.00054 (N_1)_{60}^2$

$= 27.1 + (0.3 \times 11) - 0.00054 (11)^2$

$= 27.1 + 3.3 - 0.065$

$\therefore \varphi' = 30.34°$ (Ans)

Problem-5: Estimate average D_r (%) if $C_u = 3.0$.

Depth (m)	E. ov. pressure σ_v' (lb/ft^2)	σ_v' (lb/in^2)	S.P.N N_F	D_r (%) $D_r = 11.7 + 0.76(222 N_F + 1600 - 53\sigma_v' - 50 C_u^2)^{0.6}$
5	1100	7.64	8	49.86
10	1250	8.68	10	52.70
15	1650	11.46	11	53.22
20	2050	14.24	13	55.23
25	2500	17.36	15	57.05
n=5				$\Sigma D_r = 268.06$ %

$\therefore$ Average relative density $D_r(\%)_{ave} = \dfrac{\Sigma D_r(\%)}{n}$

$= \dfrac{268.08}{5}$

$D_r(\%)_{ave} = 53.61$ % (Ans.)

Bearing capacity of Shallow foundation

Problem – 3: A square footing is shown in fig. below use $F_S = 6$. Determinge the size of the footing.

▷ Settlement ◁

Formula:

1. Elastic or Immediate settlement, $S_e = \frac{Bq_o}{E_s}(1-\mu_s^2)\alpha I_s I_f$

Here, $q_o = \frac{Q}{A} = \gamma h$

$$I_s = F_1 + \left(\frac{1-2\mu_s}{1-\mu_s}\right)F_2$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1)$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}(A_2)$$

$$A_0 = m'\ln\left[\frac{(1+\sqrt{m'^2+1})\sqrt{m'^2+n'^2}}{m'(1+\sqrt{m'^2+n'^2+1})}\right]$$

$$A_1 = \ln\left[\frac{(m'+\sqrt{m'^2+1})\sqrt{1+n'^2}}{m'+\sqrt{m'^2+n'^2+1}}\right]$$

$$A_2 = \frac{m'}{n'\sqrt{m'^2+n'^2+1}}$$

Settlement at the center of the foundation;	Settlement at the corner of the foundation;
$\alpha = 4$	$\alpha = 1$
$m' = L/B$	$m' = L/B$
$n' = \frac{H}{B/2}$	$n' = \frac{H}{B}$

$$E_s = \frac{\sum E_{s(i)}\Delta z}{\bar{z}}$$

$S_{e\,(rigid)} = 0.93\,S_{e\,(flexible),\,center}$

$I_f = 1$ or given.

2. Primary consolidation settlement;

$$S_{c(P)} = \frac{C_c \cdot H_c}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma_{ave}'}{\sigma_0'}\right) \rightarrow \sigma_c' > \sigma_0'$$

Here, $C_c = 0.009(LL-10)$

$C_c = 0.0001w$

$$\Delta\sigma_{ave}' = \frac{1}{6}(\Delta\sigma_t' + 4 \cdot \Delta\sigma_m' + \Delta\sigma_b')$$

$$\Delta\sigma' = I \times q_0$$

$$I = \frac{2}{\pi}\left(\frac{mn\sqrt{1+m^2+2n^2}}{\sqrt{1+m^2+n^2}\,(m^2+n^2)(1+n^2)} + \sin^{-1}\frac{m}{\sqrt{m^2+n^2}\sqrt{1+n^2}}\right)$$

$$m = \frac{L}{B}$$

$$n = \frac{2z}{B}$$

$$b = \frac{B}{2}$$

3. Secondary consolidation settlement;

$$S_{c(S)} = C_\alpha' H_c \cdot \log(t_2/t_1) \qquad \left(\begin{aligned} C_\alpha &= 0.0001w \\ \frac{C_\alpha}{C_c} &= 0.05 \end{aligned}\right)$$

$$C_\alpha' = \frac{C_\alpha}{1+e_p}$$

For over consolidated soil;

$$S_{c(P)} = \frac{C_s H_c}{1+e} \log\left(\frac{\sigma_0' + \Delta\sigma_{ave}'}{\sigma_0'}\right) \rightarrow \Delta\sigma_{ave}' \leq \sigma_c'$$

when, $\sigma_c' > \Delta\sigma_{ave}'$ and $\sigma_c' < (\sigma_0' + \Delta\sigma_{ave}')$ then:

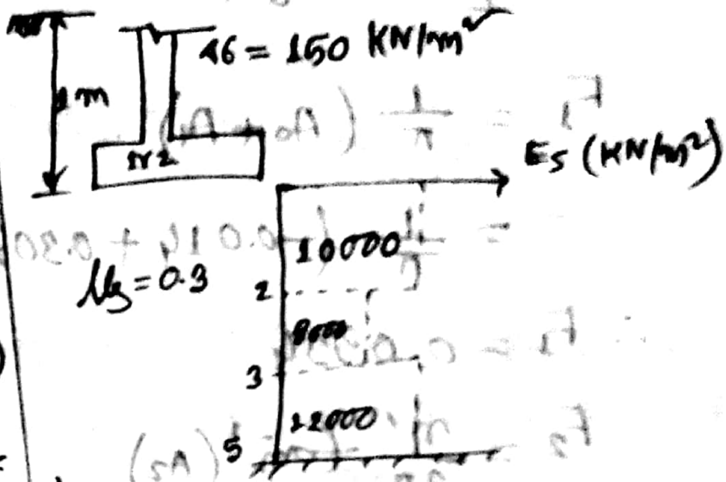
$$S_c = \frac{C_s H}{1+e_0} \log\left(\frac{\sigma_c'}{\sigma_0'}\right) + \frac{C_c H}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma_{ave}'}{\sigma_c'}\right).$$

Total settlement, $S = S_e + S_{c(P)} + S_{c(S)}$.

Problem-11.1: A rigid shallow foundation 1m x 2m is shown in figure below. Calculate the elastic settlement at the center of the foundation.

Solution:

Here;



For center;

$\alpha = 4$

$m = \frac{L}{B} = \frac{2}{1} = 2$

$n = \frac{2H}{B} = \frac{2\times1}{2} = 1 \quad (H = 5 \text{ ?})$

$\therefore A_0 = m \ln\left(\frac{(1+\sqrt{1+m^2})\sqrt{m^2+n^2}}{m(1+\sqrt{m^2+n^2+1})}\right)$

$= 2\ln\left(\frac{(1+\sqrt{5})(\sqrt{5})}{2(1+\sqrt{7})}\right)$

$= 2\ln\left(\frac{7.24}{7.29}\right)$

$\therefore A_0 = -0.014$

$A_1 = \ln\frac{(m+\sqrt{1+m^2})(\sqrt{1+n^2})}{m+\sqrt{m^2+n^2+1}}$

$= \ln\frac{(2+\sqrt{5})\sqrt{2}}{2+\sqrt{7}}$

$= \ln\left(\frac{6.324}{4.65}\right)$

$A_1 = 0.3075.$

$A_2 = \frac{m}{m\sqrt{m^2+n^2+1}}$

$A_2 = 0.756$.

$$F_1 = \frac{1}{\pi}(A_0 + A_1)$$

$$= \frac{1}{\pi}(-0.014 + 0.3075)$$

$$\therefore F_1 = 0.0934$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}(A_2)$$

$$= \frac{1}{2\pi}\tan^{-1}(0.756)$$

$$F_2 = 0.103$$

$$\therefore I_s = F_1 + \left(\frac{1-2\mu_s}{1-\mu_s}\right)F_2$$

$$= 0.0934 + \left(\frac{1-0.6}{1-0.3}\right)\times 0.103$$

$$\therefore I_s = 0.1523$$

Now; $E_s = \frac{\sum E_{s(i)}\,\Delta z}{\bar{z}}$

$$= \frac{(10000\times 2)+(8000\times 1)+(12000\times 2)}{5}$$

$$\therefore E_s = 10400 \text{ kN/m}^2$$

$\therefore$ Elastic settlement, $S_e = \frac{Bq_0}{E_s}(1-\mu_s^2)\,\alpha n I_s I_f$.

$$S_e = \frac{1\times 150}{10400}\left(1-0.3^2\right)\times 4\times 0.1523\times 1$$

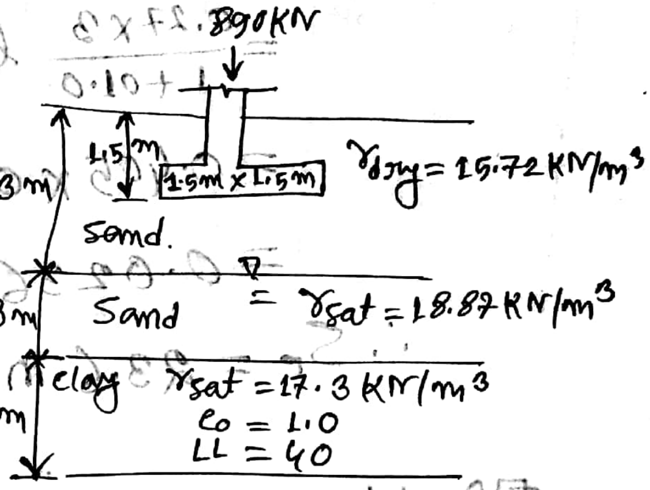
$$S_e = 0.0008 \text{ m}$$

$$S_e = 0.8 \text{ mm}$$

$$\therefore S_{e(rigid)} = 0.93\times 0.8 = 0.744 \text{ mm}$$

(Ans)

Problem-11.12: Calculate the consolidation settlement. The clay is normally consolidated. Use weighted average method.



Solution:

$$C_c = 0.009(LL-10)$$
$$= 0.009(40-10)$$
$$\therefore C_c = 0.27$$

$$\sigma_0' = (3\times 15.72) + 3\times(18.87-9.81) + \frac{3}{2}(17.3-9.81)$$
$$= 47.16 + 27.18 + 11.09$$
$$\sigma_0' = 85.43 \text{ KN/m}^2$$

For $\Delta\sigma'_{ave}$:

$$I_4 = \frac{2}{\pi}\left(\frac{mn(1+m^2+2n^2)}{\sqrt{1+m^2+n^2}\,(m^2+n^2)(1+n^2)} + \sin^{-1}\frac{m}{\sqrt{m^2+n^2}\sqrt{1+n^2}}\right)$$

$m = \frac{L}{B}$	z (m)	$b = \frac{B}{2}$ (m)	$n = \frac{z}{b}$	$q_0 = \frac{Q}{A}$	I_4	$\Delta\sigma' = I_4\times q_0$
1	4.5	0.75	6	395.56	0.051	20.17
1	6.0	0.75	8	395.56	0.029	11.47
1	7.5	0.75	10	395.56	0.019	7.52

$$\therefore \Delta\sigma'_{ave} = \frac{1}{6}\left(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b\right)$$

$$= \frac{1}{6}\left(20.17 + (4\times 11.47) + 7.52\right)$$

$$= 12.26 \text{ kN/m}^2$$

∴ Consolidation settlement;

$$S_c = \frac{C_c \cdot H}{1+e_0}\log\left(\frac{\sigma'_0 + \Delta\sigma'_{ave}}{\sigma'_0}\right)$$

$$= \frac{0.27\times 3}{1+01.0}\log\left(\frac{85.43+12.26}{85.43}\right)$$

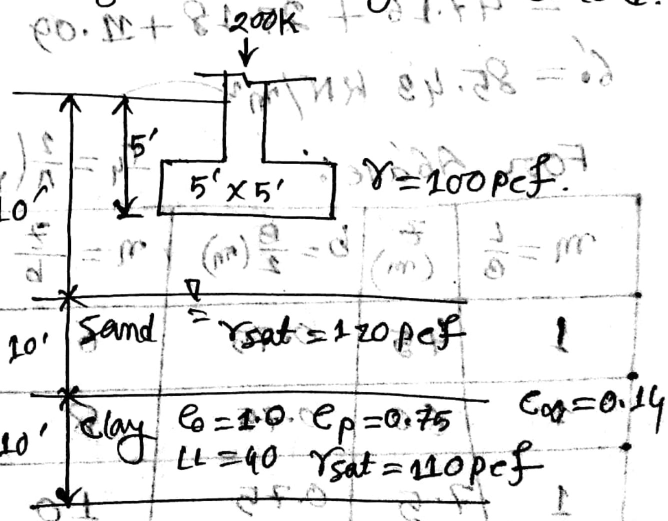
$$= 0.405 \times 0.0582$$

$$= 0.0236 \text{ m}$$

$$\therefore S_c = 23.6 \text{ mm}$$ (Ans.)

Problem-2: Calculate consolidation settlement. clay is normally consolidated. use weighted average method.

Solution:



$$C_c = 0.009(40-10)$$

$$= 0.27$$

$$C'_\alpha = \frac{C_\alpha}{1+e_p} = \frac{0.14}{1+0.75}$$

$$C_\alpha = 0.08$$

$$\sigma_0' = (10 \times 100) + 10 \times (120 - 62.4) + \frac{10}{2}(110 - 62.4)$$

$$= 1000 + 576 + 238$$

$$\therefore \sigma_0' = 1814 \text{ lb/ft}^2$$

$$q_0 = \frac{Q}{A} = \frac{200}{5 \times 5} = 8 \text{ Kip/ft}^2 = 8000 \text{ lb/ft}^2$$

For $\Delta\sigma'_{ave}$: $I_4 = \frac{2}{\pi}\left[\frac{mn(1+m^2+2n^2)}{\sqrt{1+m^2+n^2}\,(m^2+n^2)(1+n^2)} + \sin^{-1}\frac{m}{\sqrt{m^2+n^2}\sqrt{1+n^2}}\right]$

$m = \frac{L}{B}$	z (m)	$b = \frac{B}{2}$	$n = \frac{z}{b}$	q_0	I_4	$\Delta\sigma' = I_4 q_0$
1	15	2.5	6	8000	0.051	408
1	20	2.5	8	8000	0.029	232
1	25	2.5	10	8000	0.019	152

$$\therefore \Delta\sigma'_{ave} = \frac{1}{6}(\Delta\sigma'_t + 4\Delta\sigma'_m + \Delta\sigma'_b)$$

$$= \frac{1}{6}(408 + (4 \times 232) + 152)$$

$$= 248 \text{ lb/ft}^2$$

$\therefore$ Primary consolidation settlement;

$$S_{c(p)} = \frac{c_c \cdot H}{1+e} \log\left(\frac{\sigma_0' + \Delta\sigma'_{ave}}{\sigma_0'}\right)$$

$$= \frac{0.27 \times 10}{1 + 1.0} \log\left(\frac{1814 + 248}{1814}\right)$$

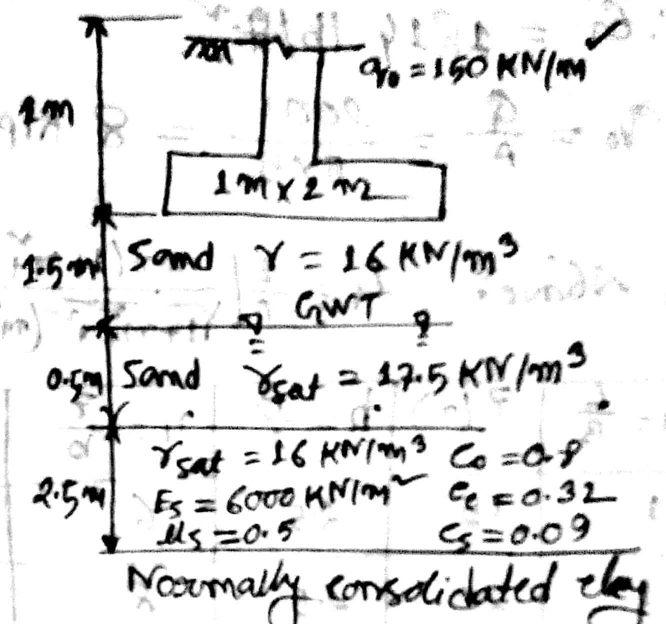
$$= 1.35 \times 0.0557$$

$$= 0.075 \text{ ft}$$

$\therefore S_{c(p)} = 0.075'$ Ans.

Problem-38: A plan of foundation 1m x 2m is shown in figure below. Estimate (i) $S_{c(p)}$ (ii) $S_{c(s)}$. (Use $C_\alpha = 0.0005$ and $t_2/t_1 = 10000$).

Solution:



$$\sigma_0' = 2.5\times16 + 0.5(17.5-9.81) + \frac{2.5}{2}(16-9.81)$$

$$= 40 + 3.85 + 7.74$$

$$\sigma_0' = 51.59 \text{ KN/m}^2$$

For $\Delta\sigma_{ave}$:

$$I_4 = \frac{2}{\pi}\left[\frac{mn(1+m^2+2n^2)}{\sqrt{1+m^2+n^2}\,(m^2+n^2)(1+n^2)} + \sin^{-1}\left(\frac{m}{\sqrt{m^2+n^2}\sqrt{1+n^2}}\right)\right]$$

$m = \frac{L}{B}$	z (m)	$b = \frac{B}{2}$	$n = \frac{z}{b}$	I_4	$\Delta\sigma' = I_4 q_0$
2	2	0.5	4	0.19	28.5
2	3.25	0.5	6.5	0.082	12.3
2	4.5	0.5	9	0.045	6.75

$$\Delta\sigma_{ave} = \frac{1}{6}(28.5 + (4\times12.3) + 6.75)$$

$$= 14.08 \text{ KN/m}^2$$

∴ Primary consolidation settlement:

$$S_{c(p)} = \frac{C_c \cdot H}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma_{ave}'}{\sigma_0'}\right)$$

$$= \frac{0.32 \times 2.5}{1+0.80} \log\left(\frac{51.59 + 14.08}{51.59}\right)$$

$$= 0.444 \times 0.1048$$

$$S_{c(p)} = 0.0465 \text{ m}$$ (Ans.)

Secondary consolidation settlement:

$$S_{c(s)} = C_\alpha' H \cdot \log(t_2/t_1)$$

$$= 0.0005 \times 2.5 \times \log(10000)$$

$$S_{c(s)} = 0.005 \text{ m.}$$ (Ans.)

Problem-4: Estimate the immediate settlement of a concrete footing, 1m × 2m, founded at a depth of 1m in a soil with $E = 10^4$ KN/m², $\mu_s = 0.3$. The footing is subjected to a pressure of 200 KN/m². Assume the footing to be rigid. Given $L/B = 2$, $I = 1.20$.

Solution: Given:

$B = 1$ m

$q_0 = 200$ KN/m²

$E_s = 10000$ KN/m²

$\mu_s = 0.3$

$I = 1.20$

$$\therefore \alpha = \frac{1}{\pi}\left[\ln\frac{\sqrt{m^2+1}+m}{\sqrt{1+m^2}-m} + m\ln\frac{\sqrt{1+m^2}+1}{\sqrt{1+m^2}-1}\right]$$

Here, $m = \frac{L}{B} = \frac{2}{1} = 2.$

$$\therefore \alpha = \frac{1}{\pi}\left[\ln\left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right) + 2\ln\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right)\right]$$

$$= \frac{1}{\pi}\left[\ln\left(\frac{4.24}{0.24}\right) + 2\ln\left(\frac{3.24}{1.24}\right)\right]$$

$$= \frac{1}{\pi}(2.872 + 1.921)$$

$$\therefore \alpha = 1.526$$

Now elastic settlement:

$$S_e = \frac{Bq_0}{E_s}(1-\mu_s^2)\alpha I_s$$

$$= \frac{1\times 200}{10000}(1-0.3^2)\times 1.526\times 1.20$$

$$= 0.0333 \text{ m}$$

$$\therefore S_e = 33.3 \text{ mm}$$

$$\therefore S_{e(\text{rigid})} = 0.93 S_{e(\text{flexible, center})}$$

$$= 0.93\times 33.3$$

$$\therefore S_{e(\text{rigid})} = 31 \text{ mm}$$

(Ans.)

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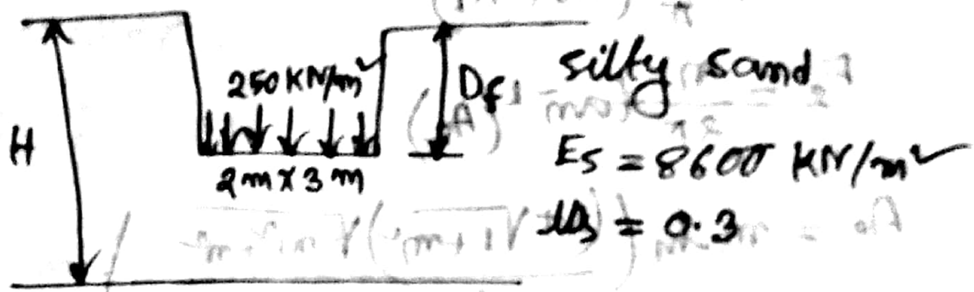
Problem-5: A flexible area in figure below is 2m×3m in plan and carries a uniformly distributed load of 250 KN/m². Estimate the elastic settlement below the center of the load area. assume (i) $H = \alpha$, $D_f = 0$
(ii) $H = 6\,m$, $D_f = 0$.



Solution: Given,

$q_0 = 250\ KN/m^2$
$E_s = 8600\ KN/m^2$
$\mu_s = 0.3$
$L = 3\,m$
$B = 2\,m$

(i) $H = \alpha$, $D_f = 0$.

Elastic settlement, $S_e = \dfrac{Bq_0}{E_s}(1-\mu_s^2)\alpha$.

For center of the foundation; $\alpha = 4$

$m = \dfrac{L}{B} = \dfrac{3}{2} = 1.5$

$$\therefore S_e = \frac{2\times 250}{86000}(1-0.3^2)\times 4$$

$$S_e = 0.0212\ m.$$ (Ans.)

(iii) $H = 6\,m$, $D_f = 0$

$$n = \frac{2H}{B} = \frac{2\times 6}{2} = 6$$

$$S_e = \frac{B q_0}{E_s}(1-\mu_s^2)\left[(1-\mu_s^2)F_1 + (1-\mu_s-2\mu_s^2)F_2\right]$$

Here, $F_1 = \frac{1}{\pi}(A_0 + A_1)$

$$F_2 = \frac{n}{2\pi}\tan^{-1}(A_2)$$

$$A_0 = m \ln\left(\frac{(1+\sqrt{1+m^2})\sqrt{m^2+n^2}}{m(1+\sqrt{m^2+n^2+1})}\right)$$

$$= 1.5 \ln\left(\frac{(1+\sqrt{3.25})\sqrt{38.25}}{1.5(1+\sqrt{39.25})}\right)$$

$$= 1.5 \ln\left(\frac{17.33}{10.89}\right)$$

$$A_0 = 0.6969$$

$$A_1 = \ln\left(\frac{(m+\sqrt{1+m^2})\sqrt{1+m^2}}{m+\sqrt{m^2+n^2+1}}\right)$$

$$= \ln\left(\frac{(1.5+\sqrt{3.25})\sqrt{3.25}}{1.5+\sqrt{39.25}}\right)$$

$$= \ln\left(\frac{5.954}{7.765}\right)$$

$$A_1 = -0.266$$

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$$A_2 = \frac{m}{n\sqrt{m^2+n^2+1}} = \frac{1.5}{6\times\sqrt{39.25}}$$

$\therefore A_2 = 0.0399.$

$\therefore F_1 = \frac{1}{\pi}(A_0 + A_1)$

$= \frac{1}{\pi}(0.6969 - 0.266)$

$F_1 = 0.137$

$F_2 = \frac{6}{2\pi}\tan^{-1}(0.0399)$

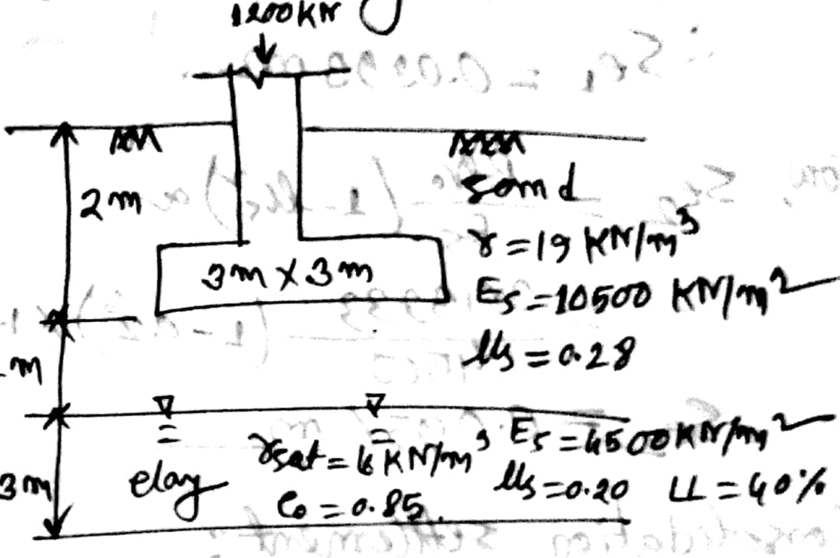
$F_2 = 0.0381$

$\therefore$ Settlement; $S_e = \frac{2\times250}{86000}(1-0.3^2)\left[(1-0.3^2)\times 0.137 + (1-0.3-\cancel{2}2\times(0.3)^2)\times 0.038\right]$

$= 0.00529(0.1247 + 0.0198)$

$S_e = 7.64\times10^{-4}$ m. (Ans)

Problem-1: A square footing 3m x 3m carries a column load of 1200 KN as shown in fig below. Determine the total settlement of the footing.



Solution:

Immediate settlement:

$$S_{e_1} = \frac{B q_0}{E_s}(1-\mu_s^2)\alpha.$$

$$q_0 = \frac{1200}{3\times 3} = 133.33\ KN/m^2$$

$$m = \frac{L}{B} = 1$$

$$\alpha = \frac{1}{\pi}\left[\ln\frac{\sqrt{1+m^2}+m}{\sqrt{1+m^2}-m} + m\ln\frac{\sqrt{1+m^2}+1}{\sqrt{1+m^2}-1}\right]$$

$$= \frac{1}{\pi}\left[\ln\frac{\sqrt{1+1}+1}{\sqrt{1+1}-1} + 1.\ln\frac{\sqrt{1+1}+1}{\sqrt{1+1}-1}\right]$$

$$= \frac{1}{\pi}\left[\ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\frac{2.414}{0.414}\right]$$

$$= \frac{1}{\pi}\left(0.\cancel{1.763} + 1.763\right)$$

$$\therefore \alpha = 1.12$$

$$\therefore S_{e_1} = \frac{3 \times 133.33}{10500} \times (1 - 0.28^2) \times 1.12$$

$$\therefore S_{e_1} = 0.0393 \, m.$$

Now, $$S_{e_2} = \frac{B q_0}{E_s}(1 - \mu_s^2)\alpha$$

$$= \frac{3 \times 133.33}{4500}(1 - 0.2^2) \times 1.12$$

$$\therefore S_{e_2} = 0.0956 \, m$$

Consolidation settlement:

$$S_c = \frac{C_c . H_c}{1 + e_0} \cdot \log\left(\frac{\sigma_0' + \Delta\sigma_{ave}}{\sigma_0'}\right)$$

Here, $H = 3m$

$$C_c = (LL - 10) \times 0.009 = 0.009(40 - 10) = 0.27$$

$$e_0 = 0.85$$

$$\sigma_0' = 3 \times 19 + 1.5 \times (16 - 9.81) = 66.29 \, KN/m^2$$

$$I = \frac{2}{\pi}\left[\frac{mn(1 + m^2 + 2n^2)}{\sqrt{1 + m^2 + n^2}(m^2 + n^2)(1 + n^2)} + \sin^{-1}\frac{m_1}{\sqrt{m_1^2 + n^2}\sqrt{1 + n^2}}\right]$$

$L = 3m$, $B = 3m$, $q_0 = 133.33 \, KN/m^2$

$m = \frac{L}{B}$	z	$b = \frac{B}{2}$	$n = \frac{2z}{B}$	I	$\Delta\sigma = I \times q_0$
1	1	1.5	0.67	0.86	114.66
1	2.5	1.5	1.67	0.43	57.33
1	4	1.5	2.67	0.22	29.33

$$\therefore \Delta\sigma'_{ave} = \frac{1}{6}\left(\Delta\sigma'_t + 4\,\Delta\sigma'_m + \Delta\sigma'_b\right)$$

$$= \frac{114.66 + 4 \times 57.33 + 29.33}{6}$$

$$\therefore \Delta\sigma'_{ave} = 62.22 \text{ KN/m}^2$$

$$\therefore S_c = \frac{0.27 \times 3}{1 + 0.85} \log\left(\frac{66.29 + 62.22}{66.29}\right)$$

$$= 0.4378 \times 0.2875 = 0.126 \text{ m}$$

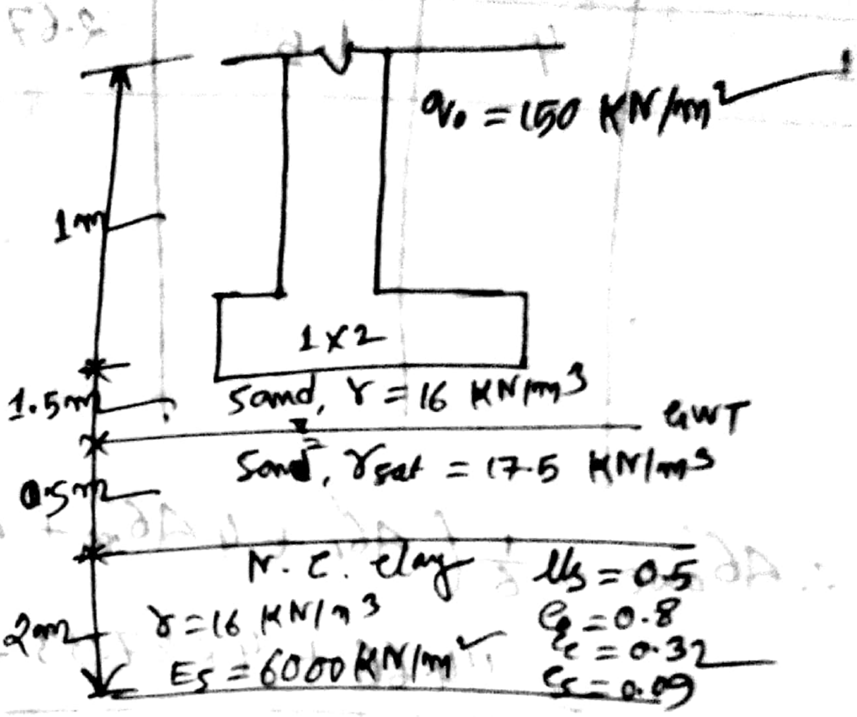
$$\therefore \text{Total settlement } S = S_{e_1} + S_{e_2} + S_c$$

$$= 0.0393 + 0.0956 + 0.126$$

$$S = 0.261 \text{ m}$$

(Ans.)

Problem: A plan of a foundation 1m x 2m is shown in figure below. Estimate (i) primary consolidation settlement (ii) secondary consolidation settlement of the foundation. Use $C_\alpha = 0.005$ and $t_2/t_1 = 10000$.



Sol^n: Primary consolidation settlement for normally consolidated clay:

$$S_c = \frac{C_c \cdot H_c}{1+e_0} \log\left(\frac{\sigma_0' + \Delta\sigma'_{ave}}{\sigma_0'}\right)$$

Here, $C_c = 0.32$ | $\sigma_0' = (2.5 \times 16) + (5-2.5) \times (17.5 - 9.81)$

$H_c = 2m$ | $= 59.23$ KN/m²

$e_0 = 0.80$

$m = \frac{L}{B} = \frac{2}{1} = 2$

$n = \frac{2z}{B}$

$$I = \frac{2}{\pi}\left[\frac{mn(1+m^2+2n^2)}{\sqrt{1+m^2+n^2}\,(m^2+n^2)(1+n^2)} + \sin^{-1}\frac{m}{\sqrt{m^2+n^2}\,\sqrt{1+n^2}}\right]$$

m	z	$b=\frac{B}{2}$	$n=\frac{2z}{B}$	I	$\Delta\sigma' = I\times q_0$
2	1.0	0.5	2	0.481	72.15
2	2.5	0.5	5	0.131	19.65
2	4.0	0.5	8	0.056	8.4

$$\therefore \Delta\sigma'_{ave} = \frac{72.15 + 4\times 19.65 + 8.4}{6} = 26.53\ KN/m^2$$

$$\therefore S_{c(p)} = \frac{C_c \cdot H_c}{1+e_0}\log\left(\frac{\sigma_0' + \Delta\sigma'_{ave}}{\sigma_0'}\right)$$

$$= \frac{0.32\times 2}{1+0.8}\log\left(\frac{59.23+26.53}{59.23}\right)$$

$$= 0.356 \times 0.1607$$

$$S_{c(p)} = 0.0572\ m$$

$$S_s = C_\alpha H_c \cdot \log(t_2/t_1)$$

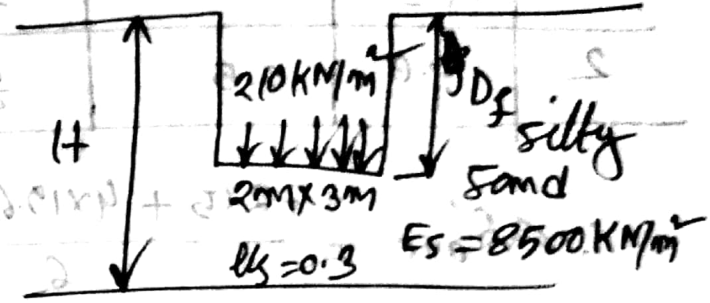
$$= 0.0005\times 2\times\log(10000)$$

$$= \cancel{0.0004} = 0.004\ m.$$

Problem: A flexible area shown in fig below is 2m x 3m in plan and carries a uniformly distributed load of 210 KN/m². Estimate the elastic settlement below the center of the loaded area.

Assume (i) $H = \infty$, $D_f = 0$ (ii) $H = 9\,m$, $D_f = 0$

Solution:



$$S_e = \frac{B q_o}{E_s}(1-\mu_s^2)\,\alpha \quad \text{--- (1)}$$

$q_o = 210\ KN/m^2$

$B = 2\,m$, $L = 3\,m$

$E_s = 8500\ KN/m^2$, $\mu_s = 0.3$

$$m = \frac{L}{B} = \frac{3}{2} = 1.5$$

$$\alpha = \frac{1}{\pi}\left[\ln\frac{\sqrt{1+m^2}+m}{\sqrt{1+m^2}-m} + m\ln\frac{\sqrt{1+m^2}+1}{\sqrt{1+m^2}-1}\right]$$

$$= \frac{1}{\pi}\left[\ln\left(\frac{\overset{3.3}{\cancel{2.80}}}{0.83}\right) + 1.5\times\ln\left(\frac{2.8}{0.8}\right)\right]$$

$$= \frac{1}{\pi}\left(2.378 + \overset{1.879}{\cancel{2.839}}\right)$$

$$\alpha = 1.\cancel{4}355$$

$$\therefore S_e = \frac{2\times 210}{8500}(1-0.3^2)\times 1.355$$

$$= 0.06115\ m.$$

(ii) $H = 5m$, $D_f = 0$.

$$S_e = \frac{B.q_0}{E_s}(1-\mu_s^2)\left[(1-\mu_s^2)F_1 + (1-\mu_s-2\mu_s^2)F_2\right]$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1) = \frac{1}{\pi}(0.59 + 0.758) = 0.429$$

$$F_2 = \frac{n}{2\pi}\tan^{-1} A_2 = \frac{5}{2\pi}\tan^{-1}(0.056) = 0.0445.$$

$$n = \frac{H}{B/2} = \frac{5}{2/2} = 5.$$

$$A_0 = m' \ln\left[\frac{1+\sqrt{m'^2+1}\sqrt{m'^2+n'^2}}{m'(1+\sqrt{m'^2+n'^2+1})}\right] = 1.5\ln\left(\frac{1+13.05}{1.5(6.32)}\right)$$

$$= 0.59$$

$$A_1 = \ln\left[\frac{m'+\sqrt{m'^2+1}\sqrt{1+n'^2}}{m'+\sqrt{m'^2+n'^2+1}}\right] = \ln\left(\frac{1.5+13.05}{1.5+5.32}\right)$$

$$= 0.758$$

$$A_2 = \frac{m_1}{n'\sqrt{m'^2+n'^2+1}} = \frac{1.5}{5 \times 5.32} = 0.056.$$

$$\therefore S_e = \frac{2 \times 210}{8500}(1-0.3^2)\left[(1-0.3^2) \times 0.429 + (1-0.3-2\times 0.3^2) \times 0.0445\right]$$

$$= 0.04496 \times (0.39 + 0.023)$$

$$= 0.0186 \text{ m}$$

(Ans.)

Bearing Capacity of Shallow Foundation

Formulas:

1. General (Meyerhof) bearing capacity equation;

$$q_u = cN_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

c = Cohesion

$q = \gamma h$

γ = Below the bottom of foundation

⇒ Bearing capacity factors;

$$N_q = \tan^2\left(45^\circ + \frac{\varphi'}{2}\right) e^{\pi \tan\varphi'}$$

$$N_c = (N_q - 1)\cot\varphi'$$

$$N_\gamma = 2(N_q + 1)\tan\varphi'$$

⇒ Shape factors;

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\varphi'$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$$

[Eccentricity থাকে শুধুমাত্র F_{cs}, F_{qs} ও $F_{\gamma s}$ এর B ও L যথাক্রমে B' ও L' দ্বারা Replace হবে।]

⇒ Load inclination factor;

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta^\circ}{\varphi'}\right)^2$$ [Angle β with respect to vertical]

⇒ **Depth factors:**

(i) $\frac{D_f}{B} < 1$

if $\phi' = 0°$

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

if $\phi' > 0°$

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \cdot \left(\frac{D_f}{B}\right)$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$$

$$F_{\gamma d} = 1.$$

(ii) $\frac{D_f}{B} > 1$

if $\phi' = 0°$

$$F_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B}\right).$$

$$F_{\gamma d} = F_{qd} = 1.$$

if $\phi' > 0°$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$$

$$F_{qd} = 1 + 2\tan\phi'(1-\sin\phi')^2 \tan^{-1}\left(\frac{D_f}{B}\right)$$

$$F_{\gamma d} = 1.$$

2. Terzaghi's Bearing capacity equation:

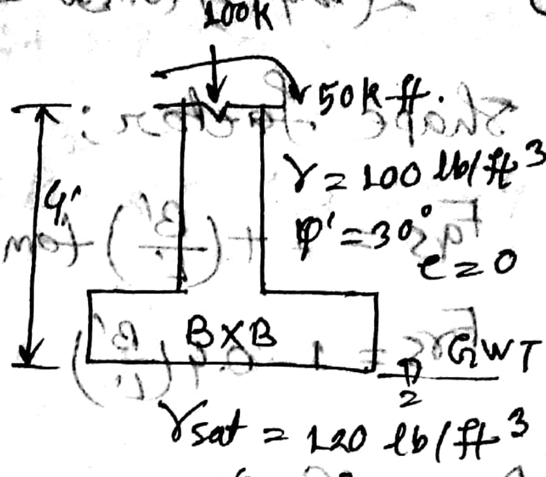
$$q_u = c'N_c + qN_q + \frac{1}{2}\gamma B N_\gamma$$

Here, $N_q = \dfrac{e^{2\left(\frac{3\pi}{4} - \frac{\phi'}{2}\right)\cdot\tan\phi'}}{2\cos^2\left(45° + \frac{\phi'}{2}\right)}$

$$N_c = (N_q - 1)\cot\phi'$$

$$N_\gamma = \frac{1}{2}\cdot\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)\tan\phi'$$

Problem-18: A square footing is shown in figure below. use an FS of 3 and determine the size of the footing.



Solution: $Q = 100K = 100000\, lb.$

$M = 50k\text{-}ft = 50000\ lb\text{-}ft$

Eccentricity, $e = \frac{M}{Q} = \frac{50000}{100000}$

$\therefore e = 0.5\,ft.$

$\therefore$ Effective width, $B' = B - 2e = B - 1.$

Effective length, $L' = L = B$

$\therefore q_{all} = \frac{q_u}{FS} = \frac{100000}{B \times B}$

$\therefore q_u = \frac{300000}{B^2}$ —— (1)

We have; As $c = 0$;

$q_u = q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$ —— (2)

As the load is vertical so;

$F_{qi} = F_{\gamma i} = 1.$

Bearing capacity factor:

$N_q = \tan^2\left(45 + \frac{30}{2}\right) e^{\pi \tan 30^\circ} = 18.40$

$N_c = (N_q - 1)\cot\phi = (18.4 - 1)\cot 30° = 30.14$

$N_\gamma = 2(18.4 + 1)\tan 30° = 22.4$

Shape factor;

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{B-1}{B}\right)0.58$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\,\frac{B-1}{B}.$$

Depth factor; Assume, $\frac{D_f}{B} < 1$.

$$\therefore F_{qd} = 1 + 2\tan 30°(1 - \sin 30°)^2\,\frac{4}{B}$$

$$= 1 + \frac{1.15}{B}$$

$$F_{\gamma d} = 1.$$

From (2); we have;

$$q_u = (4 \times 100) \times 18.4 \times \left\{1 + \frac{0.58B - 0.58}{B}\right\}\left\{1 + \frac{0.15}{B}\right\} \times 1 + \frac{1}{2} \times (120 - 62.4)\,B \times 22.4 \times \left(1 - \frac{0.4B - 0.4}{B}\right) \times 1 \times 1$$

$= \cancel{7360B + 4268.8B}$

$$= 7360\left(\frac{1.58B - 0.58}{B}\right)\left(\frac{B + 0.15}{B}\right) + 645.12B\left(\frac{0.6B - 0.4}{B}\right)$$

$$= 7360\left(\frac{1.58B^2 + 0.24B - 0.58B - 0.24}{B^2}\right) + \frac{387.07B - 258.05}{B}$$

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$$\Rightarrow \frac{300000}{B^2} = \frac{11628.8B^2 - 2502.4B - 2944}{B^2} + 387.07B - 258.06$$

$$\Rightarrow 300000 = 11628.8B^2 - 2502.4B - 2944 + 387.07B^3 - 258.05B^2$$

$$\therefore 300000 = 387.07B^3 + 11370.75B^2 - 2502.4B - 2944$$

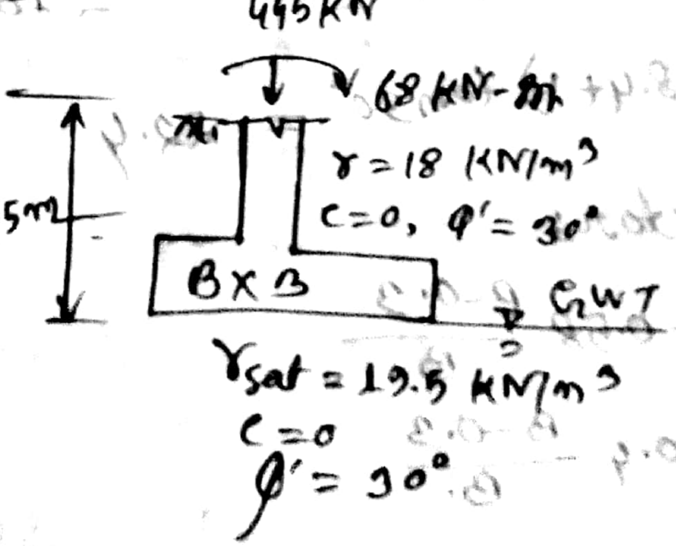
By Trial and error;

$$B = 4.88' \approx 5'$$

check: $\frac{D_f}{B} = \frac{4}{5} = 0.8 < 1$, (OK).

Size of footing $(5' \times 5')$ (Ans)

Problem-2: A square footing is shown in figure below. Using an FS of 3, determine the size of the footing.



Solution:- Eccentricity, $e = \frac{M}{Q} = \frac{68}{445} = 0.15\,m.$

Effective width, $B' = B - 2e = B - 0.3$

Effective length, $L' = L = B.$

Now; $q_{all} = \frac{q_u}{FS} = \frac{445}{B \times B}$

$\therefore q_u = \frac{1335}{B^2}$ —— (1)

As $c = 0$;

$\therefore q_u = q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$ —— (2).

As the load is vertical so;

$F_{qi} = F_{\gamma i} = 1.$

Bearing capacity factor;

$N_q = \tan^2\left(45^\circ + \frac{30^\circ}{2}\right) e^{\pi \tan 30^\circ} = 18.4$

$N_\gamma = 2(18.4 + 1) \tan 30^\circ = 22.4$

Shape factor;

$F_{qs} = 1 + 0.58 \frac{B - 0.3}{B}$

$F_{\gamma s} = 1 - 0.4 \frac{B - 0.3}{B}$

Depth factor: Assume, $\frac{D_f}{B} < 1$

$$F_{qd} = 1 + 2\tan 30^\circ (1 - \sin\phi)^2 \frac{1.5}{B} = 1 + \frac{0.43}{B}.$$

$F_{\gamma d} = 1.$

From equation ①, we get;

$$q_u = 18 \times 1.5 \times 18.4 \times \left(1 + \frac{0.58B - 0.12}{B}\right)\left(1 + \frac{0.43}{B}\right) \cdot 1 + \frac{1}{2}(19.5 - 9.81)$$
$$\times B \times 22.40 \times \left(1 - \frac{0.4B - 0.12}{B}\right) \times 1 \times 1.$$

$$= 496.8 \left(\frac{1.58B - 0.12}{B} \times \frac{B + 0.43}{B}\right) + 108.53B\left(\frac{0.6B - 0.12}{B}\right).$$

$$\Rightarrow \frac{1335}{B^2} = 496.8 \frac{1.58B^2 + 0.51B - 0.073}{B^2} + 65.12B - 13.03$$

$$\Rightarrow 1335 = 784.94B^2 + 253.37B - 36.27 + 65.12B^3 - 13.03B^2$$

$$\therefore 1335 = 65.12B^3 + 771.91B^2 + 253.37B - 36.27.$$

By trial and error;

$B = 1.15$ m.

check: $\frac{D_f}{B} = \frac{1.5}{1.15} = 1.304 > 1.$

(Not OK)

So assume, $\frac{D_f}{B} > 1.$

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$\therefore F_{qd} = 1 + 2\tan 30^\circ (1-\sin 30^\circ)^2 \tan^{-1}\left(\frac{1.5}{B}\right)$

$= 1 + 0.29 \tan^{-1}\left(\frac{1.5}{B}\right)$

$F_{\gamma d} = F_{\gamma d} = 1$

$\therefore q_u = 496.8 \left\{1 + 0.29\tan^{-1}\left(\frac{1.5}{B}\right)\right\}\left\{\frac{1.58B - 0.17}{B}\right\} \times 1 + \left(\frac{65.12B - 13.03}{B}\right)$

$\frac{1335}{B^2} = 496.8\left(1 + 0.29\tan^{-1}\left(\frac{1.5}{B}\right)\right)\left(\frac{1.58B - 0.17}{B}\right) + 65.12B - 13.03$

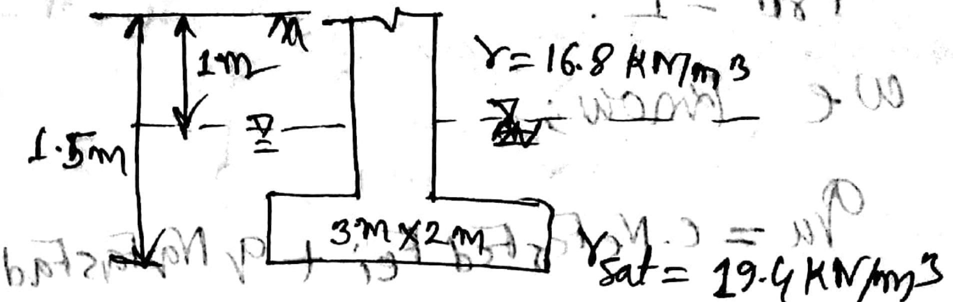
By Trial and error:

$B = 1.18\ m$

check: $\frac{D_f}{B} = \frac{1.5}{1.18} = 1.27 > 1$ (Ok)

$\therefore$ Size of footing $(1.2m \times 1.2m)$ (Ans)

Problem-4: A column foundation is 3m x 2m in plan. Given $D_f = 1.5$ m, $\varphi' = 25°$, $c = 50$ KN/m², using FS = 4 determine the net allowable load that the foundation would carry.



Solution:

$q = 1 \times 16.8 + (1.5 - 1)(19.4 - 9.81)$

$= 16.8 + 4.80$

$q = 21.6$ KN/m²

$N_q = \tan^2\left(45° + \frac{25}{2}\right) e^{\pi \tan 25°} = 10.66$

$N_c = (10.66 - 1) \cancel{\tan} \cot 25° = 20.72$

$N_\gamma = 2(10.66 + 1) \tan 25° = 10.87$

$F_{cs} = 1 + \left(\frac{2}{3}\right)\left(\frac{10.66}{20.72}\right) = 1.34$

$F_{qs} = 1 + \left(\frac{2}{3}\right)\tan 25° = 1.31$

$F_{\gamma s} = 1 - 0.4 \frac{2}{3} = 0.73$

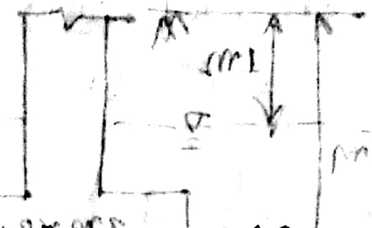
$F_{ci} = F_{qi} = F_{\gamma i} = 1.$

As $\frac{D_f}{B} < 1.$

$F_{qd} = 1 + 2\tan 25^\circ (1-\sin 25^\circ)^2 \frac{1.5}{2} = 1.23$

$F_{cd} = F_{qd} - \frac{1-F_{qd}}{N_c \tan 25^\circ} = 1.21$

$F_{\gamma d} = 1.$



We know;

$q_u = c \cdot N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$

$= (50 \times 20.72 \times 1.34 \times 1.21 \times 1) + (21.6 \times 10.66 \times 1.31 \times 1.23 \times 1)$

$+ 0.5 \times (19.4 - 9.81) \times 2 \times 10.82 \times 0.73 \times 1 \times 1.$

$= 1679.77 + 371.01 + 38.05$

$\therefore q_u = 2088.83 \text{ KN/m}^2$

Now; $q_{all} = \frac{q_u}{FS} = \frac{2088.83}{4} = 522.21 \text{ KN/m}^2$

$\therefore$ Net allowable load;

$Q_{all} = q_{all} \times Area = 522.21 \times 3 \times 2$

$\therefore Q_{all} = 3133.24$ KN. (Ans.)

Sub surface Exploration

1. Degree of disturbance: $A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} \times 100\%$

For slit-spoon sampler:
$D_o = 2$ inch
$D_i = 1.37$ inch

For thin wall tube/shelby tube:
$D_o = 2''$
$D_i = 1.87''$

2. Correction of N-value:

$$(N_1)_{60} = C_N \cdot N_{60}$$

- Liao and whitman, $C_N = \left(\frac{1}{\frac{\sigma_o'}{P_a}}\right)^{0.5} \rightarrow \left(\begin{array}{l} P_a = 100\ kN/m^2 \\ \sigma_o' \text{ is in } kN/m^2 \end{array}\right)$

- Skempton, $C_N = \frac{2}{1+\left(\frac{\sigma_o'}{P_a}\right)}$ $\left(\begin{array}{l} \sigma_o' \text{ is in } kN/m^2 \\ P_a = 100\ kN/m^2 \end{array}\right)$

⇒ If σ_o' is in ton/ft² (TSF) then $P_a = 1$ ton/ft²

lb/ft² থেকে ton/ft² করতে $\frac{1}{200}$ দ্বারা গুণ
kN/m² থেকে ton/ft² করতে $\frac{1}{95.6}$ দ্বারা গুণ

3. $$D_r = \left[\frac{N_{60}}{17 + 24\left(\frac{\sigma_o'}{P_a}\right)}\right]^{0.5}$$

$$D_r(\%) = 11.7 + 0.76\left(222 N_{60} + 1600 - 53\sigma_v' - 50 C_u^2\right)^{0.5}$$

4. $$\phi'(\text{degree}) = 27.1 + 0.3(N_1)_{60} - 0.00054\left((N_1)_{60}\right)^2$$

Problem-1: Calculate corrected N-value using Skempton's correction factor, GWT is at 5.5 m below the ground.

$\gamma_d = 18\ KN/m^3$, $\gamma_{sat} = 19.5\ KN/m^3$

Depth (m) →	1.5	3.0	4.5	6.0	7.5	9.0	10.5
N_{60} →	5	7	9	8	13	12	14

Solution:

Depth (m)	Effective stress σ_0' (KN/m²) $=\gamma h$	Correction factor $C_N = \frac{2}{1+\left(\frac{\sigma_0'}{P_a}\right)}$	N_{60}	$(N_1)_{60} = C_N \cdot N_{60}$
1.5	1.5×18 = 27	1.575	5	≈ 8
3	3×18 = 54	1.299	7	≈ 9
4.5	4.5×18 = 81	1.105	9	≈ 10
6.0	(5.5×18) + (6−5.5) × (19.5−9.81) = 103.85	0.981	8	≈ 8
7.5	99 + (7.5−5.5)×9.69 = 118.38	0.916	13	≈ 12
9.0	99 + (3.5× 9.69) = 132.92	0.859	12	≈ 10
10.5	99 + 5×9.69 = 147.45	0.808	14	≈ 11

Problem-2: Estimate the average value of φ and D_r.

Depth (m): 2 3 4 5 7 9

γ kN/m^3: 16 16 16 18 18 18

N_{60}: 6 8 10 15 18 20

Solution: $\sigma_0' = \gamma h$. $C_N = \frac{2}{1+\left(\frac{\sigma_0'}{P_a}\right)}$. $(N_1)_{60} = C_N N_{60}$

$\varphi(\text{degree}) = 27.1 \cancel{+0.76(} + 0.3(N_1)_{60} - 0.00054(N_1)_{60}^2$

$D_r(\%) = 11.7 + 0.76\left(222(N_1)_{60} + 1600 - 53\sigma_v' - 50C_u^2\right)^{0.5}$

$C_u = 3$

Depth (m)	γ (kN/m^2)	σ_0' (kN/m^2)	C_N	N_{60}	$(N_1)_{60}$	$\varphi°$	D_r(%)
2	16	32	1.52	6	9	29.76	40.66
3	16	48	1.35	8	11	30.33	36.30
4	16	64	1.22	10	12	30.62	27.91
5	18	64+(1×18) = 82	1.09	15	16	31.76	26.04
7	18	64+(3×18) = 118	0.92	18	17	32.04	
9	18	64+(5×18) = 154	0.79	20	16	31.76	

$\Sigma\varphi' = 186.27°$

$\therefore$ Average friction angle, $\varphi'_{ave} = \frac{\Sigma\varphi'}{n}$

$= \frac{186.27}{6}$

$\therefore \varphi'_{ave} = 31.05°$ (Ans)