Structural Analysis & Design

CE-4111

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Never give up hope of Allah’s Mercy (Quran: 12:87)

Special THANKS to-
My friend
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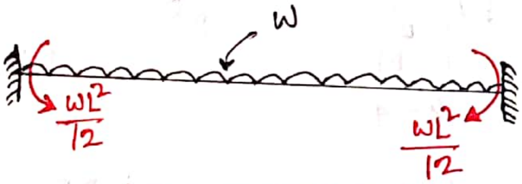
MOMENT DISTRIBUTION METHOD

⊞ Mainly, There are two methods for analysing a Indeterminate Structure.

1. Displacement Method
2. Force Method. (also called method of flexibility or, consistent deformation method)
   → Double Integration, Area moment, conjugate-beam method, unit load method etc.

⊞ Moment Distribution Method:

In order to apply the moment distribution method to analysis a structure, the following things must be considered:



(i) Fixed End Moment.
(ii) Flexural Stiffness.
(iii) Distribution Factor.
(iv) carry over Factor.
(v) Sign convention.

Prismatic Beam → stiffness or cross section through all is same.

* In Moment Distribution Method, Anticlockwise moment is considered to be positive (+ve) and clockwise moment is considered to be negative (-ve).

⊞ Distribution Factor:

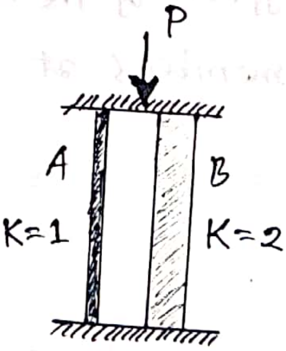
We Know,

$$\Delta = \frac{PL}{AE}$$

$$\Rightarrow P = \frac{AE\Delta}{L}$$

Axial Stiffness

if $\Delta = 1$, $P = \frac{AE}{L} = K$ (= Stiffness)



Hence, The load distribution between the members depends on their cross sectional area.

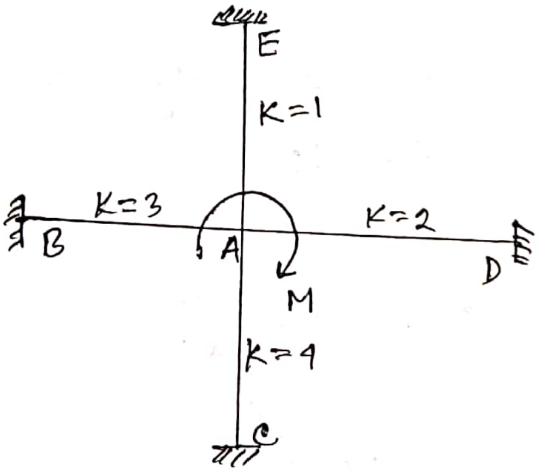
* Deformations a equal.
* Material property is same
* Length is same.

Similarly, The load distribution depends on their stiffness.

Given, $K_1 = 1$ and $K_2 = 2$

$\therefore \frac{P_1}{P_2} = \frac{K_1}{K_2} = \frac{1}{2}$ Hence, $\boxed{P_1 = \frac{1}{3} \text{ and } P_2 = \frac{2}{3}}$

$\boxed{\text{Distribution Factor}}$



Here, Moment distribution depends on the flexural stiffness.

$$DF = \frac{\text{Member Stiffness}}{\text{Total stiffness}}$$

$\therefore DF_{AE} = \frac{1}{10}$, $DF_{AD} = \frac{2}{10}$, $DF_{AB} = \frac{3}{10}$

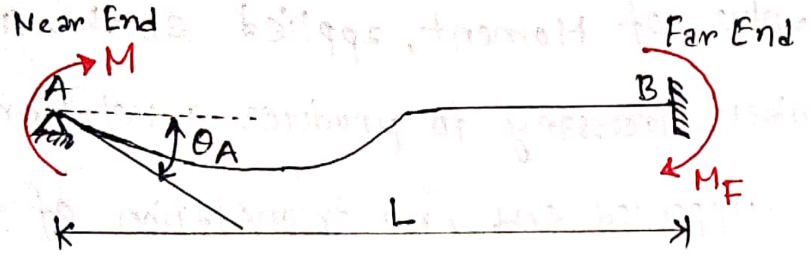
and, $DF_{AC} = \frac{4}{10}$

* Distribution factor is the ratio according to which an externally applied unbalanced Moment M at a joint is apportioned/distributed to the various members m ting at the joint.

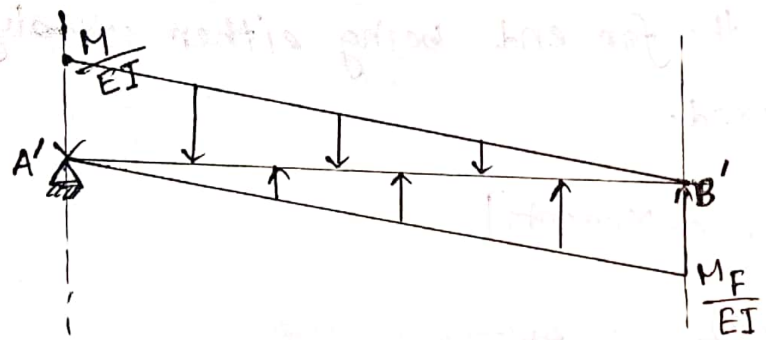
The distribution factor for any member at a joint is equal to the stiffness of the member divided by the sum of the stiffness of all members at the joint.

⊞ Stiffness and Carry-over Factor:

Carry over Factor:



* (Prismatic Beam)



(conjugate Beam) $\frac{M}{EI}$ Diagram

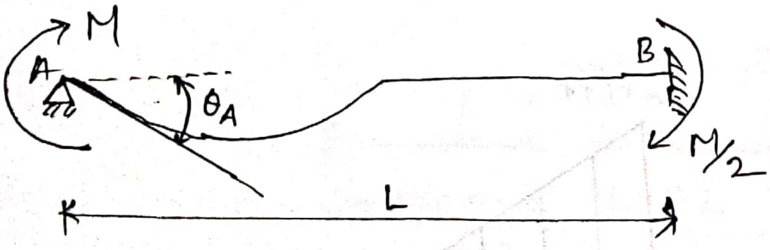
$\Sigma M_{A'} = 0$

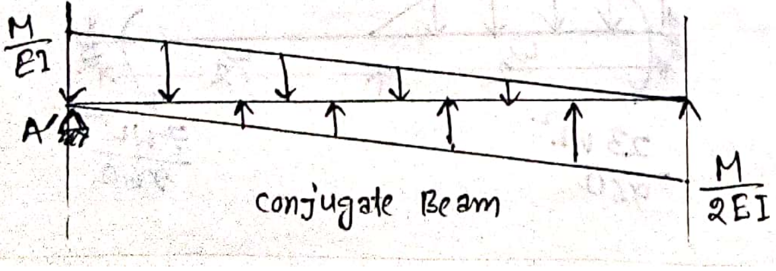
$$\frac{1}{2} \times L \times \frac{M}{EI} \times \left(\frac{1}{3} \times L\right) - \frac{1}{2} \times L \times \frac{M_F}{EI} \times \left(\frac{2}{3} \times L\right) = 0$$

$\Rightarrow \boxed{M_F = \frac{1}{2} M}$ ∴ carry over factor $= \frac{1}{2}$

The carry over factor is that factor by which developed moment at the rotated end of a member may be multiplied to give the induced moment at the fixed or restrained end.

Stiffness:





Here, $R_{A'} = \theta_A = \frac{1}{2} \times L \times \frac{M}{EI} - \frac{1}{2} \times L \times \frac{M}{2EI}$

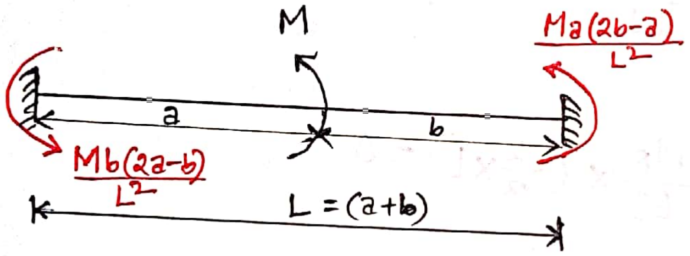
$\Rightarrow \theta_A = \frac{ML}{4EI}$

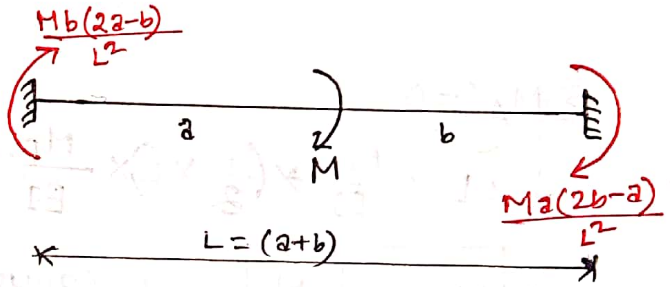
$\therefore M = \frac{4EI\theta_A}{L}$

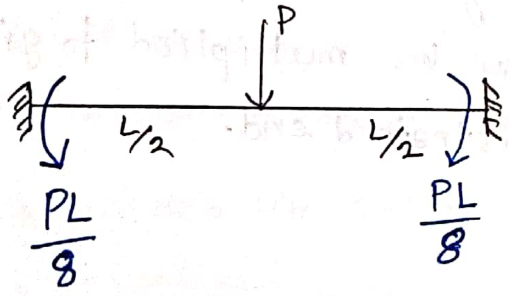
Now, if $\theta_A = 1$ Then $M = \frac{4EI}{L} = K$ (= Flexural Stiffness)

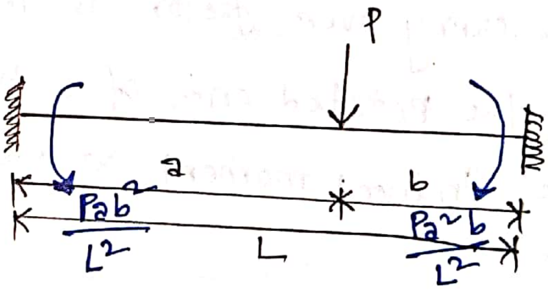
* Absolute stiffness is the value of Moment, applied at the simply supported end of a member, necessary to produce a rotation of 1 radian of this simple supported end, no translation of either end being permitted and the far end being either simply supported, restrained or Fixed.

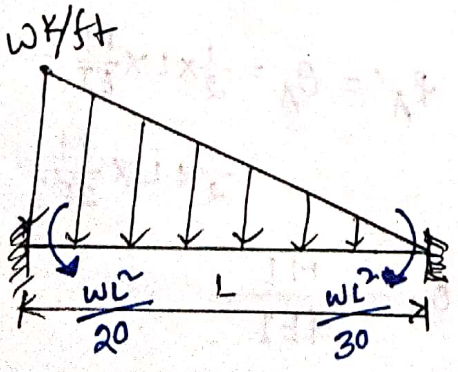
Fixed End Moments

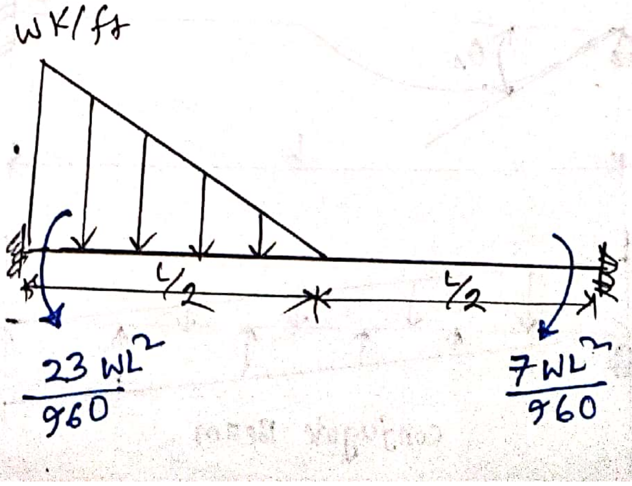


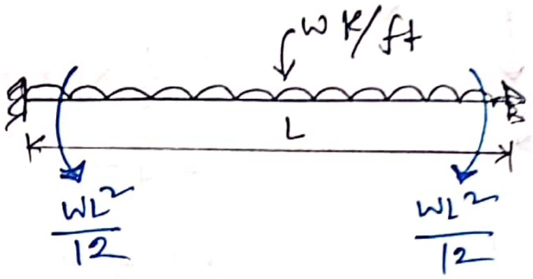


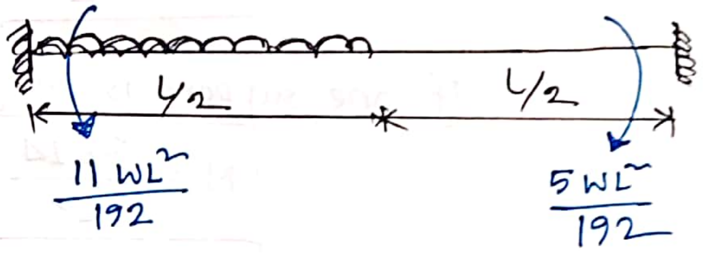


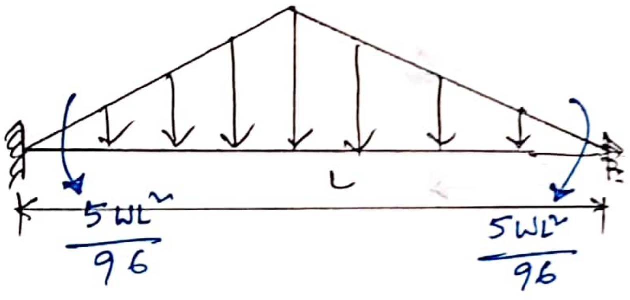


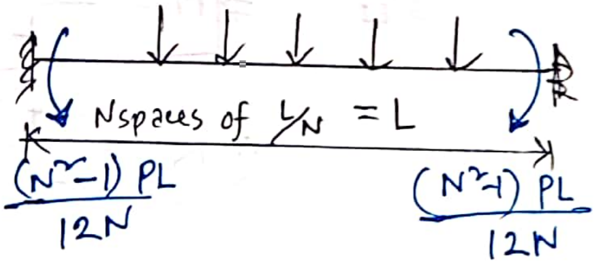




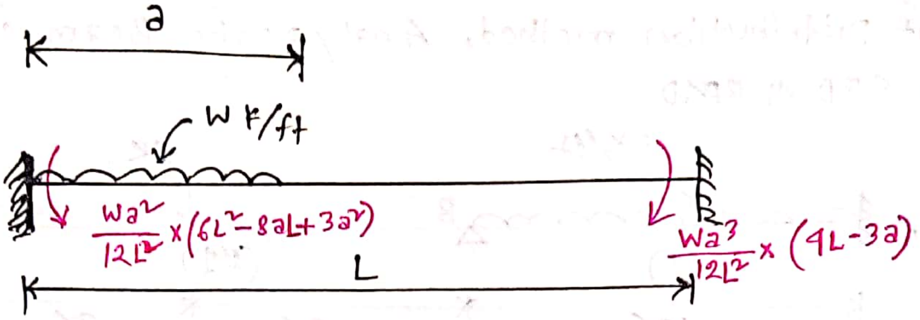

w k/ft
L
WL^2/12
WL^2/12

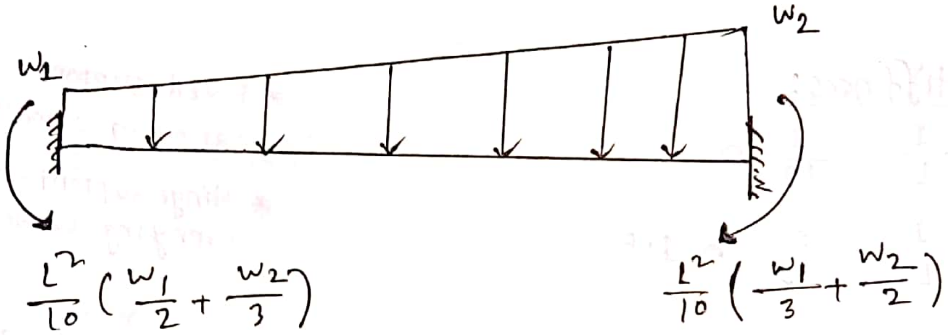

L/2
L/2
11 WL^2/192
5 WL^2/192

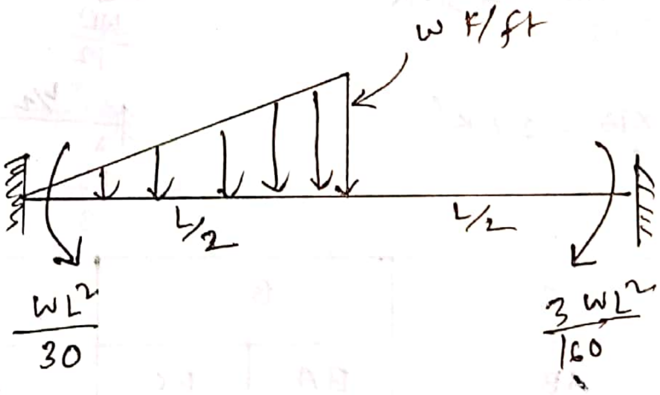

L
5 WL^2/96
5 WL^2/96

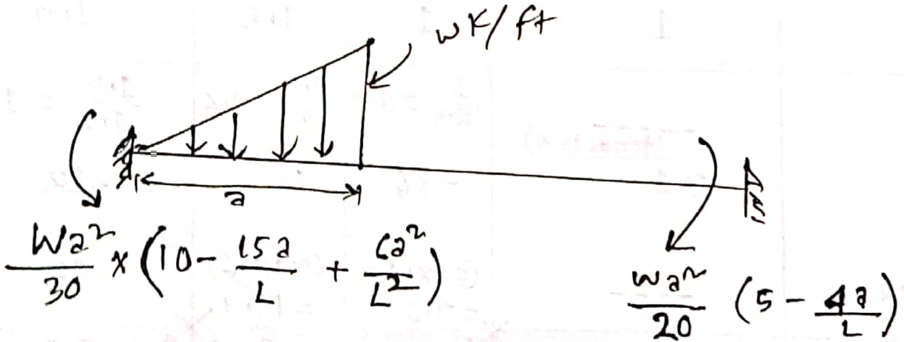

N spaces of L/N = L
(N^2-1) PL/12N
(N^2-1) PL/12N

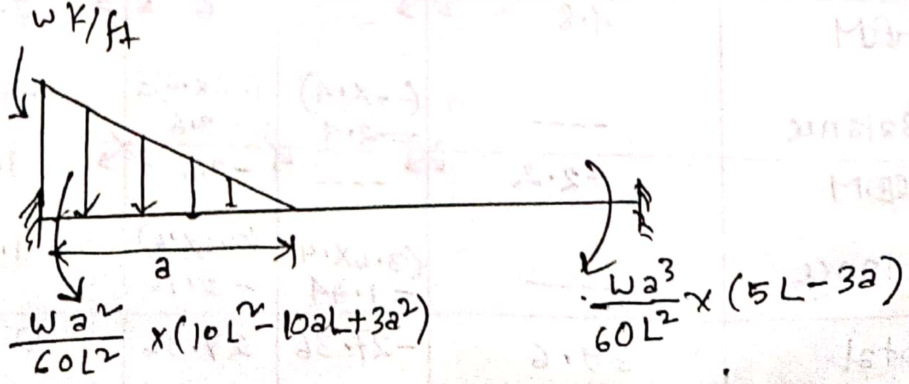
FEM:


a
w K/ft
$\frac{wa^2}{12L^2}\times(6L^2-8aL+3a^2)$
$\frac{wa^3}{12L^2}\times(4L-3a)$
L

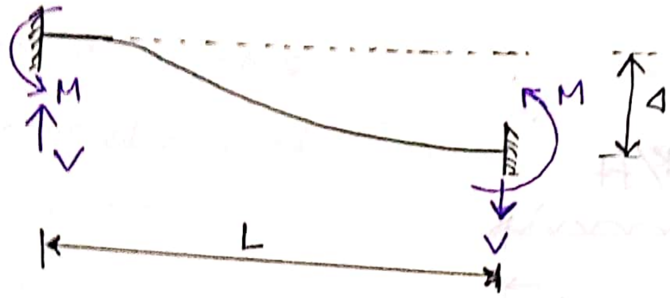

w_2
w_1
$\frac{L^2}{10}\left(\frac{w_1}{2}+\frac{w_2}{3}\right)$
$\frac{L^2}{10}\left(\frac{w_1}{3}+\frac{w_2}{2}\right)$


w K/ft
L/2
L/2
$\frac{wL^2}{30}$
$\frac{3wL^2}{160}$


wK/ft
a
$\frac{Wa^2}{30}\times\left(10-\frac{15a}{L}+\frac{6a^2}{L^2}\right)$
$\frac{wa^2}{20}\left(5-\frac{4a}{L}\right)$

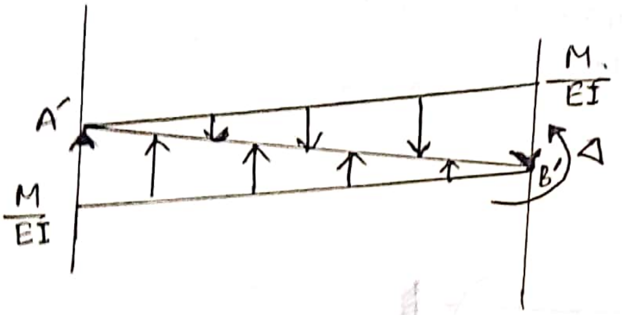

w K/ft
a
$\frac{wa^2}{60L^2}\times(10L^2-10aL+3a^2)$
$\frac{wa^3}{60L^2}\times(5L-3a)$

ii) Support Settlement:



$$\Sigma M_{B'} = 0$$

$$\frac{1}{2}\times\frac{M}{EI}\times L\times\frac{2}{3}L - \frac{1}{2}\times\frac{M}{EI}\times L\times\frac{1}{3}L - \Delta = 0$$

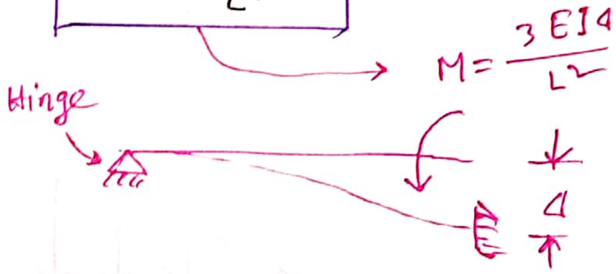


$$\Rightarrow \frac{ML^2}{EI}\times\left(\frac{1}{3}-\frac{1}{6}\right) = \Delta$$

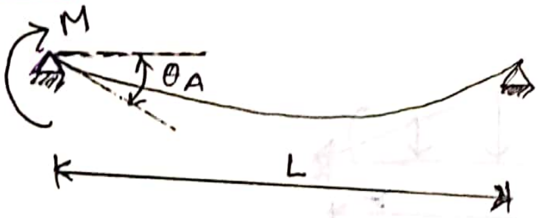
$$\Rightarrow \boxed{M = \frac{6EI\Delta}{L^2}}$$

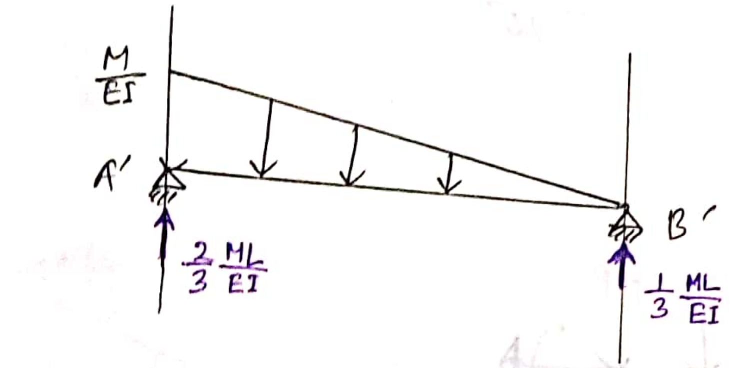
If one support is hinge, then,

$$\boxed{M = \frac{3EI\Delta}{L^2}}$$



Stiffness:





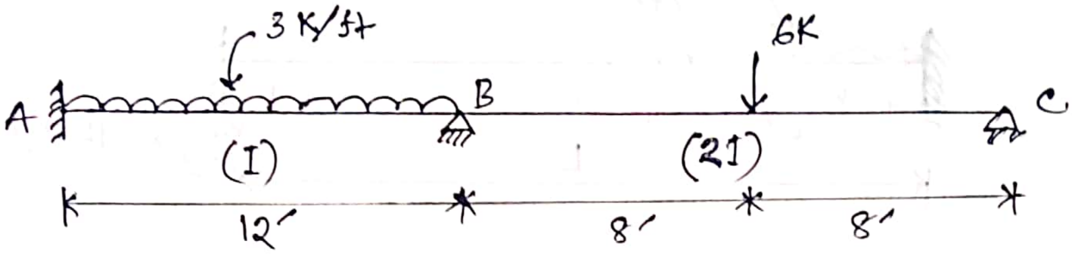
$$\theta_A = \frac{1}{2}\times\frac{M}{EI}\times L\times\frac{2}{3}L\times\frac{1}{L}$$

$$\Rightarrow \boxed{M = \frac{3EI\theta_A}{L}}$$

$$\therefore K = \frac{3EI}{L}$$

Problem: 01

Using Moment Distribution method, Analyze the Beam shown in Fig. below. Draw SFD & BMD



Solution:

Relative Stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{I}{L} = \frac{2}{16} \simeq 1.5$

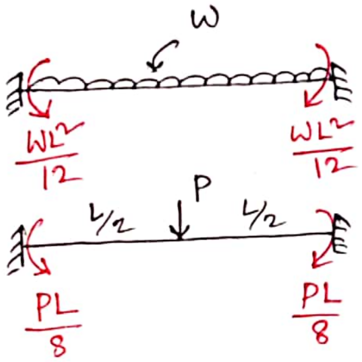
* Fixed support-এর moment carrying capacity tends to $\propto$

* Hinge support-এর moment carrying capacity zero.

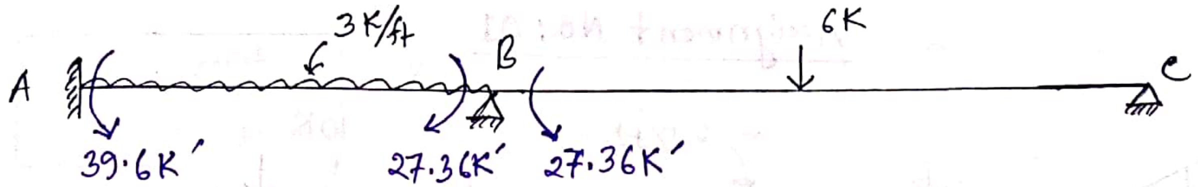
Fixed End Moments:

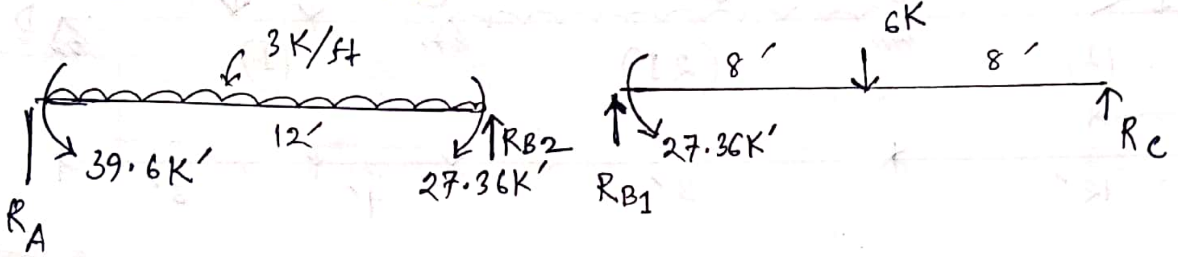
$F_{AB} = -F_{BA} = \frac{WL^2}{12} = \frac{3\times12^2}{12} = 36K'$

$F_{BC} = -F_{CB} = \frac{PL}{8} = \frac{6\times16}{8} = 12K'$



	Joint	A	B		C
	Member	AB	BA	BC	CB
	K	1	1	1.5	1.5
	D.F	----- (tends to 0)	$\frac{1}{2.5}=0.4$	$\frac{1.5}{2.5}=0.6$	$\frac{1.5}{1.5}=1$
1st cycle	FEM	36	-36	12	-12
	Balance	----	(24×.4) =9.6	(24×.6) =14.4	12
2nd cycle	CO	4.8	----	6	7.2
	Balance	----	(-6×.4) =-2.4	(-6×.6) =-3.6	-7.2
3rd cycle	CO	-1.2	----	-3.6	-1.8
	Balance	-----	(3.6×.4) =1.44	(3.6×.6) =2.16	1.8
	Total	39.6	-27.36	27.36	0





$\Sigma M_A = 0$

$-39.6 + 3\times 12\times 6 + 27.36K' - R_{B_2}\times 12 = 0$

$\Rightarrow R_{B_2} = 16.98\ K$

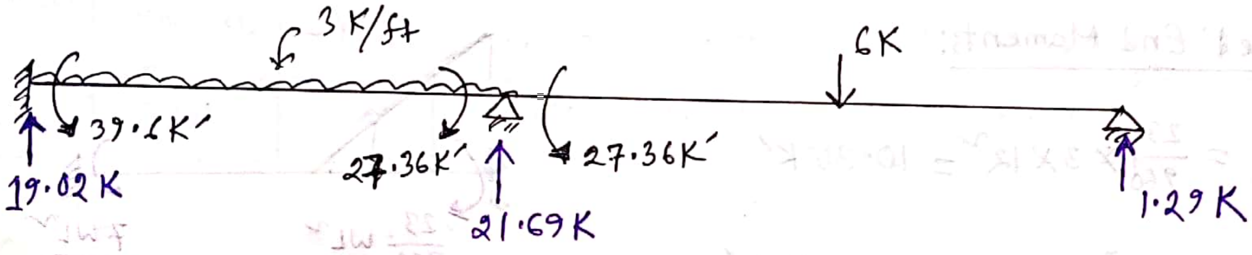
$R_A = 19.02\ K$

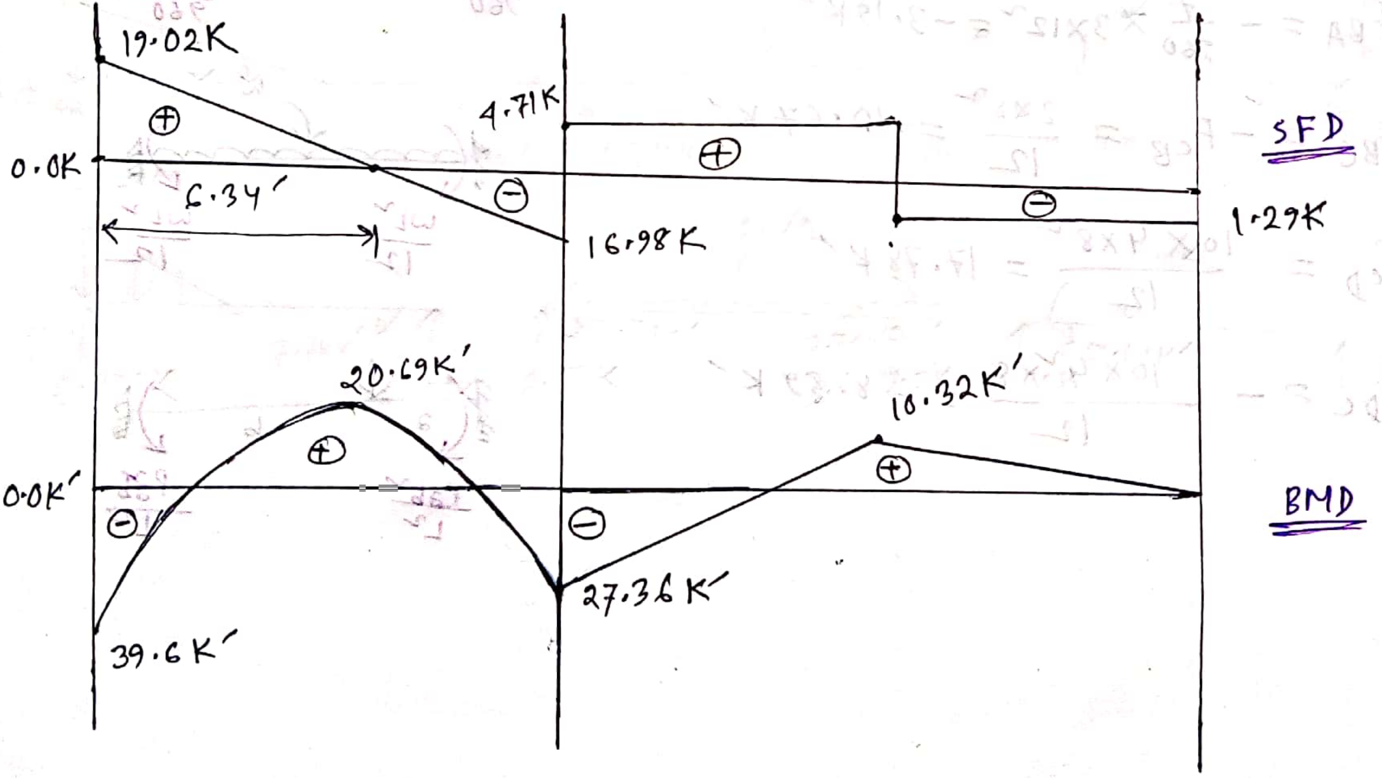
$\Sigma M_c = 0$

$-27.36 - 6\times 8 + R_{B_1}\times 16 = 0$

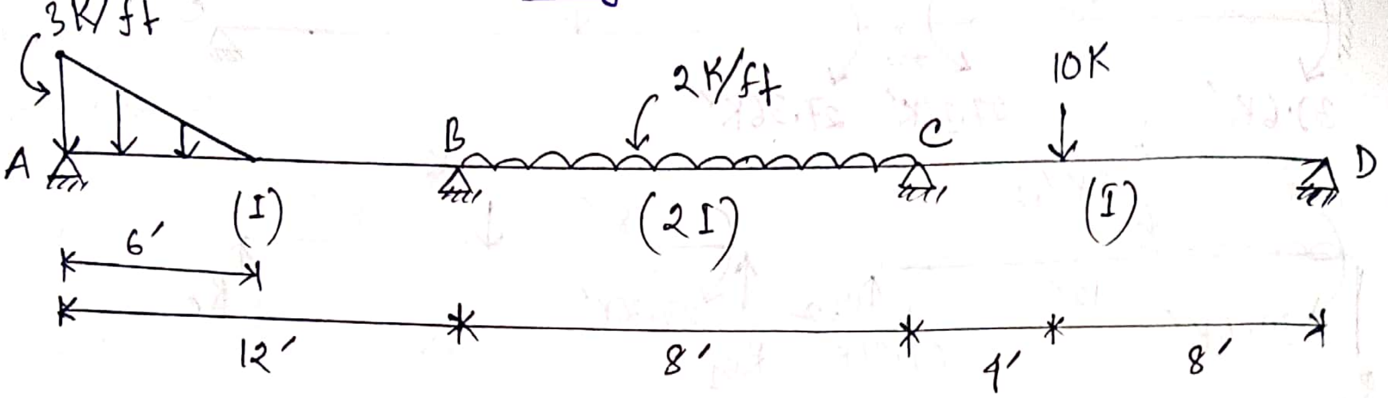
$\Rightarrow R_{B_1} = 4.71\ K \quad \therefore R_c = (6-4.71) = 1.29\ K$

$\therefore R_B = R_{B_1} + R_{B_2} = (16.98+4.71) = 21.69\ K$





Assignment No: 01



Solution:

Relative Stiffness:

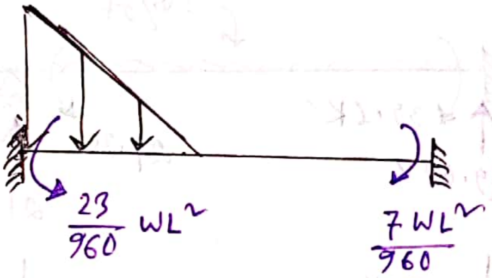
$$K_{AB} = \frac{I}{L} = \frac{1}{12} \simeq 1$$

$$K_{Be} = \frac{I}{L} = \frac{2}{8} \simeq 3$$

$$K_{eD} = \frac{I}{L} = \frac{1}{12} \simeq 12$$

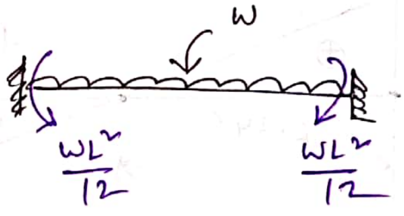
Fixed End Moments:

$$F_{AB} = \frac{23}{960} \times 3 \times 12^2 = 10.35 K'$$



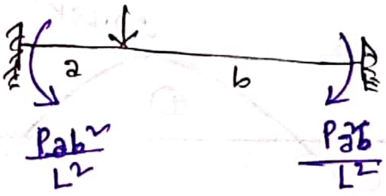
$$F_{BA} = -\frac{7}{960} \times 3 \times 12^2 = -3.15 K'$$

$$F_{BC} = -F_{CB} = \frac{2 \times 8^2}{12} = 10.67 K'$$

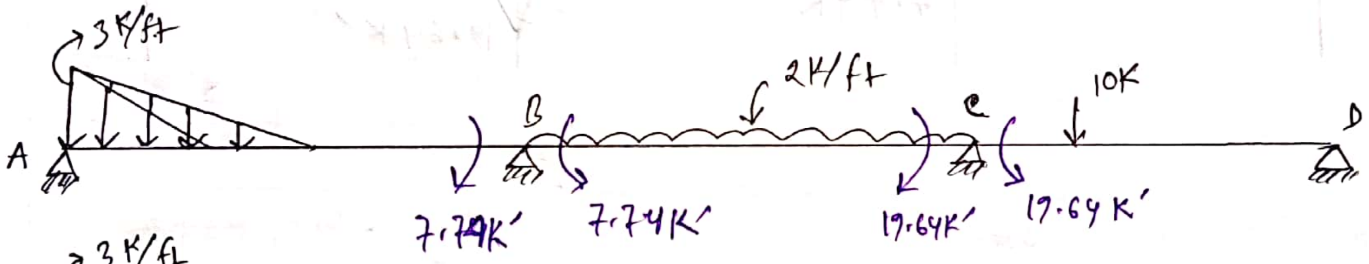


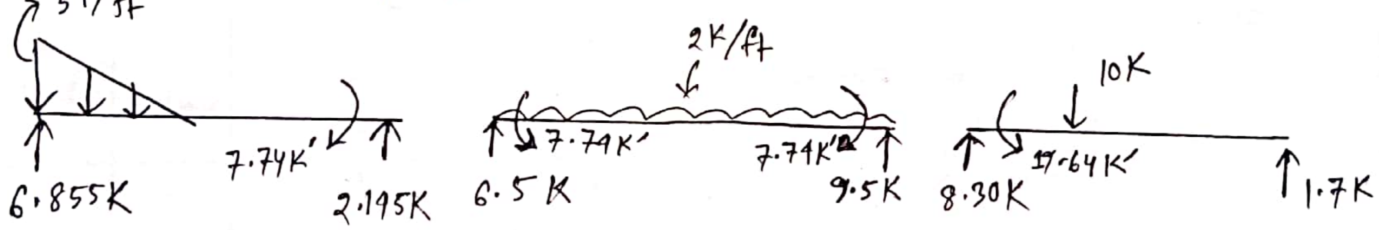
$$F_{CD} = \frac{10 \times 4 \times 8^2}{12} = 17.78 K'$$

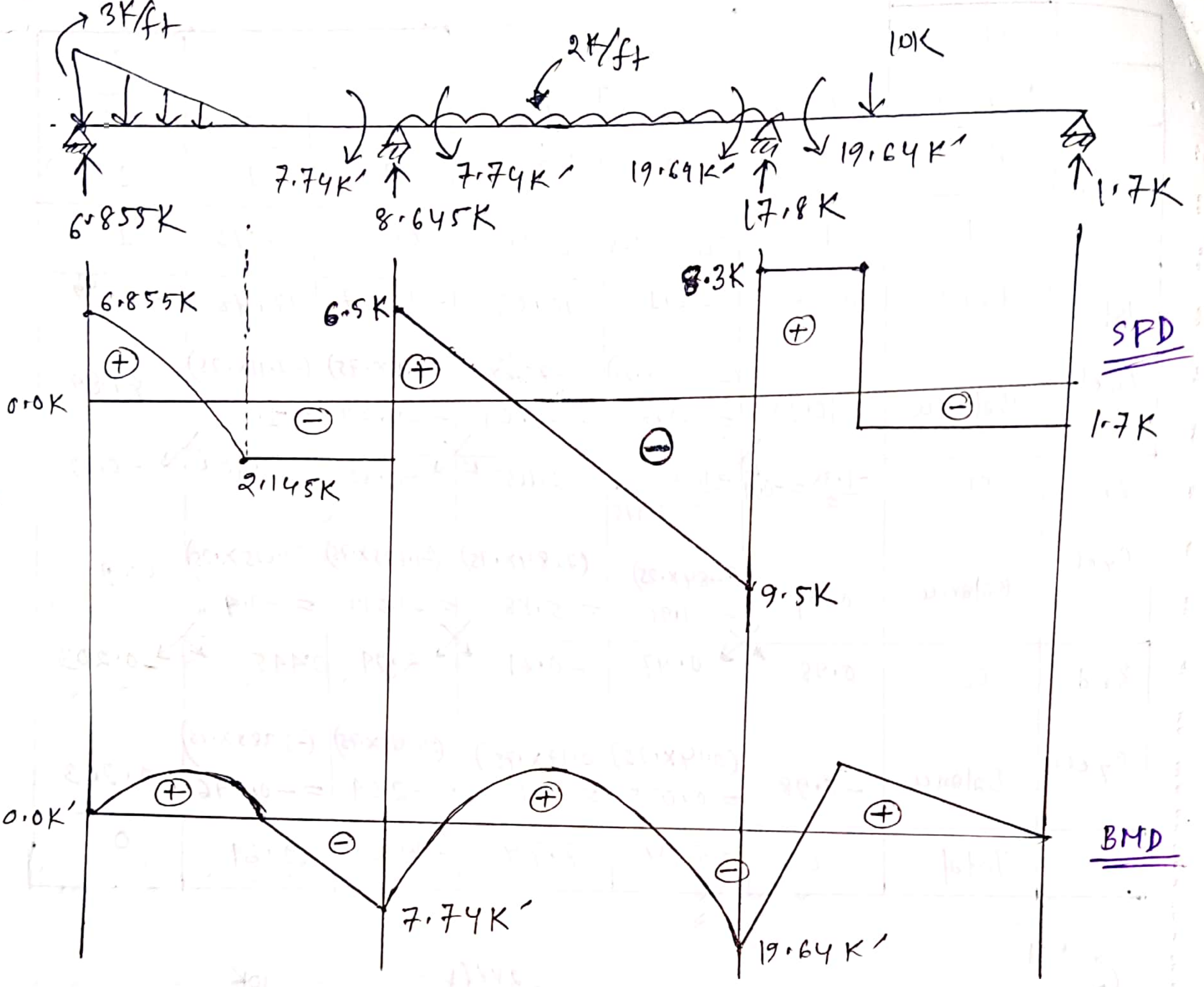
$$F_{DC} = -\frac{10 \times 4^2 \times 8}{12} = -8.89 K'$$



	joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	1	1	3	3	1	1
	D.F	1	$\frac{1}{4}$ =0.25	0.75	0.75	0.25	1
1st	FEM	10.35	−3.15	10.67	−10.67	17.78	−8.89
Cycle	Balance	−10.35	(−7.52×.25) = −1.88	(−7.52×.75) = −5.64	(−7.11×.75) = −5.33	(−7.11×.25) = −1.78	8.89
2nd	CO	$\frac{-1.88}{2}$ = −0.94	$-\frac{10.35}{2}$ = −5.175	−2.665	−2.82	4.445	−0.89
Cycle	Balance	0.94	(7.84×.25) = 1.96	(7.84×.75) = 5.88	(−1.625×.75) = −1.219	(−1.625×.25) = −0.406	0.89
3rd	CO	0.98	0.47	−0.61	2.94	0.445	−0.203
Cycle	Balance	−0.98	(0.14×.25) = 0.035	(0.14×.75) = 0.105	(−3.385×.75) = −2.54	(−3.385×.25) = −0.846	0.203
	Total	0	−7.74	7.74	−19.64	19.64	0

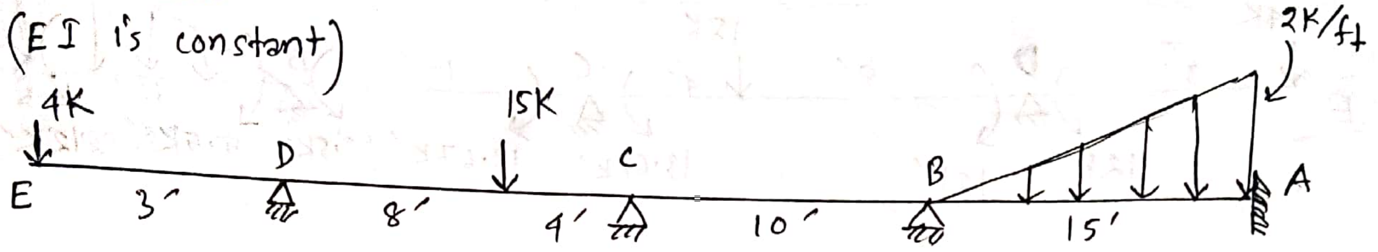





3K/ft
2K/ft
10K
7.74K'
7.74K'
19.64K'
19.64K'
6.855K
8.645K
17.8K
1.7K
6.855K
6.5K
8.3K
SPD
0.0K
2.145K
9.5K
1.7K
0.0K'
7.74K'
19.64K'
BMD

Problem: 02

(EI is constant)



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{15} \simeq 1$

$K_{BC} = \frac{1}{10} \simeq 1.5$

$K_{CD} = \frac{1}{12} \simeq 1.25$

Fixed End Moments:

$F_{AB} = -\frac{WL^2}{20} = \frac{2 \times 15^2}{20} = -22.5$

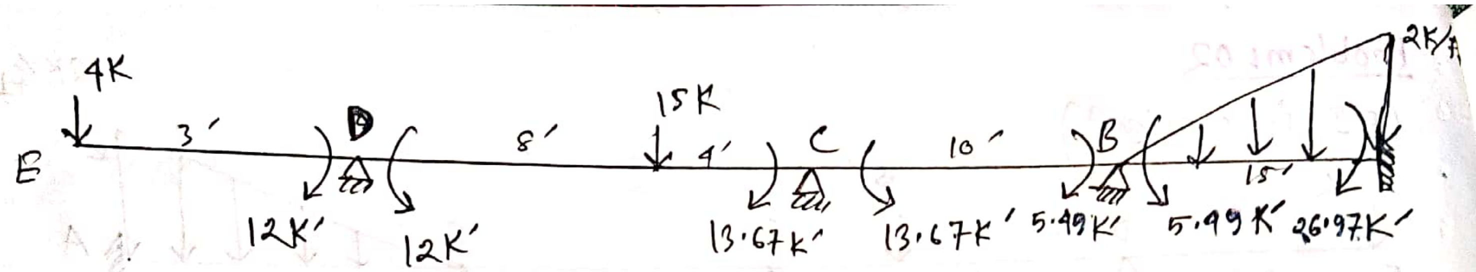
$F_{BA} = \frac{WL^2}{30} = \frac{2 \times 15^2}{30} = 15$

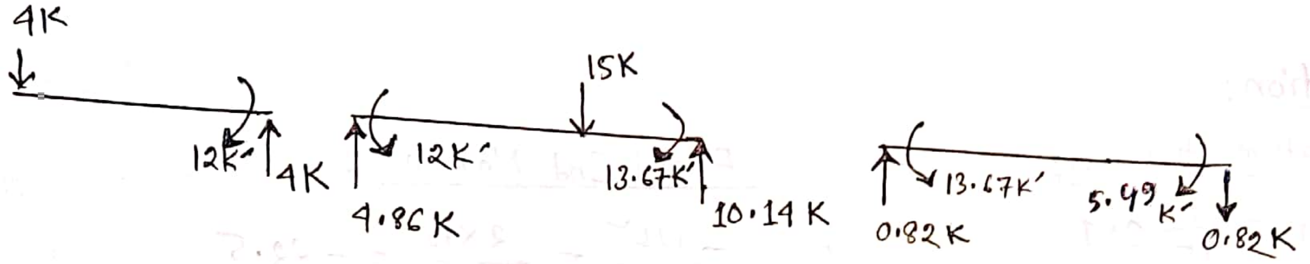
$F_{CD} = -\frac{Pab^2}{L^2} = \frac{15 \times 4 \times 8^2}{12^2} = -26.67$

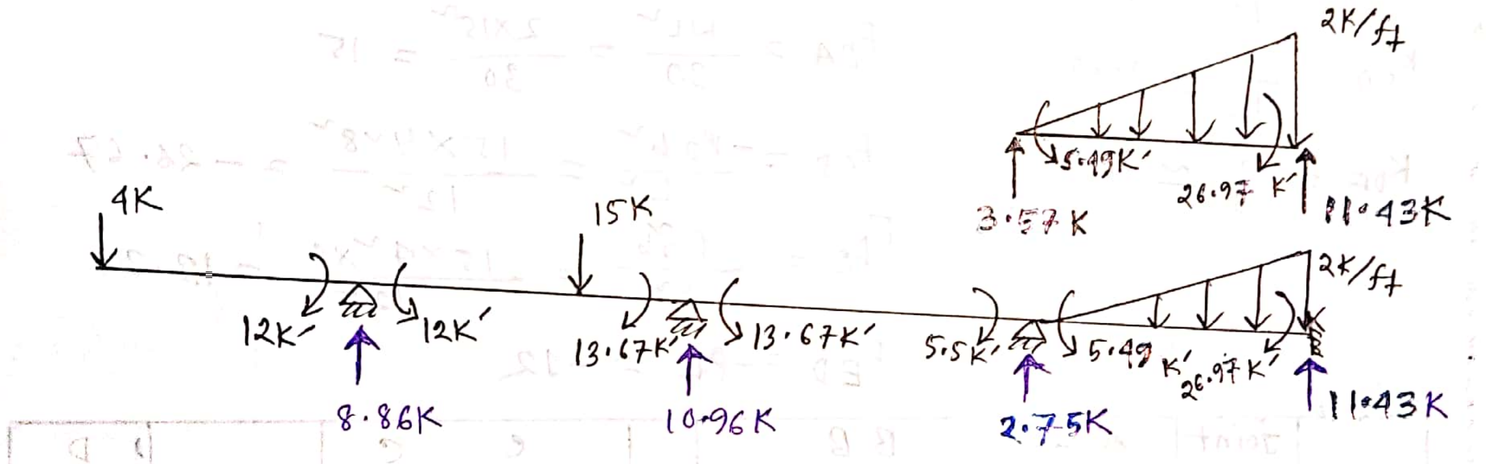
$F_{DC} = \frac{Pa^2b}{L^2} = \frac{15 \times 4^2 \times 8}{12^2} = 13.33$

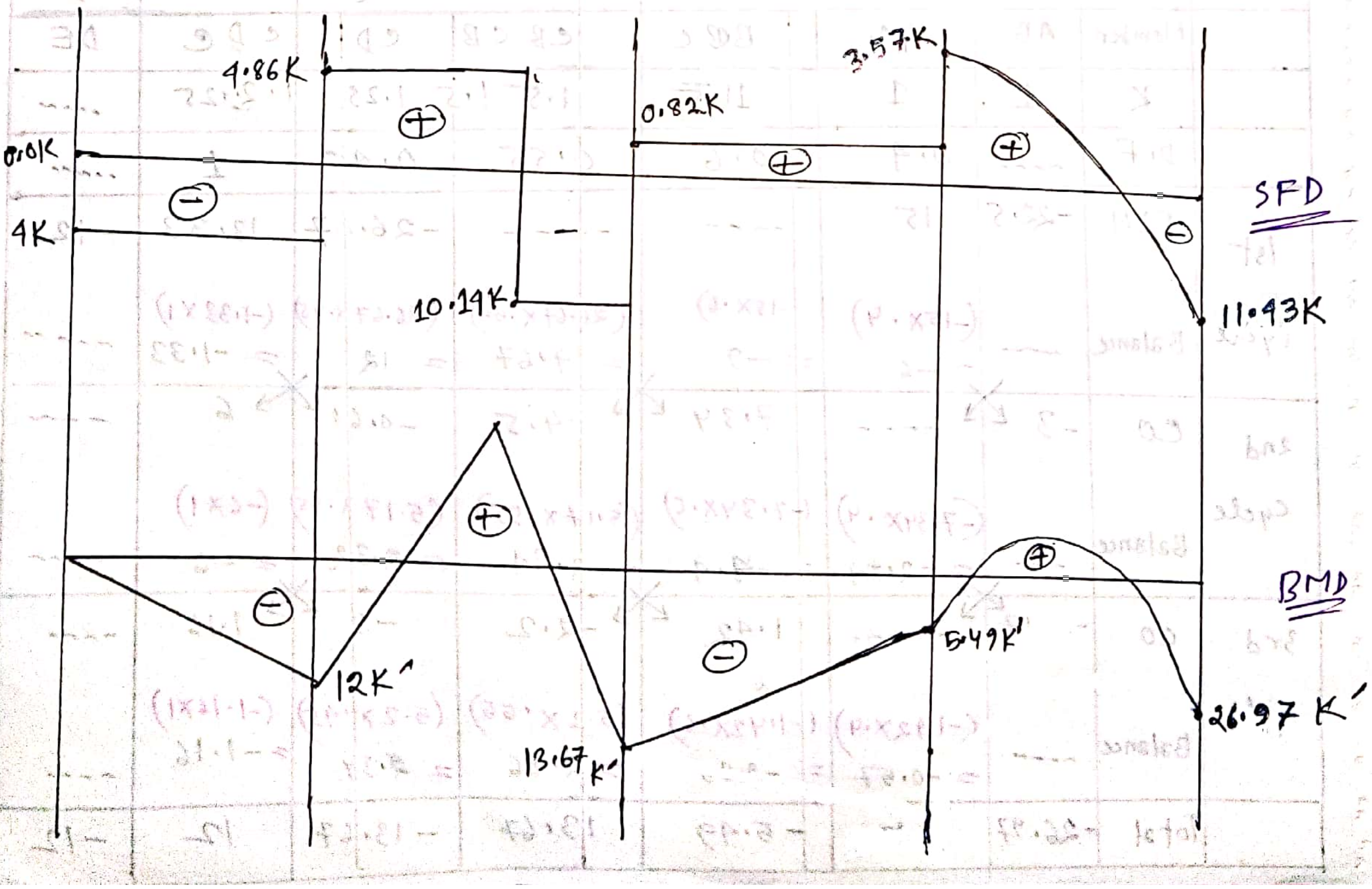
$F_{DE} = -P.L = -12$

	Joint	A	B		C		D	
	Member	AB	BA	BC	CB	CD	DC	DE
	K	1	1	1.5	1.5	1.25	1.25	----
	D.F	----	0.4	0.6	0.55	0.45	1	----
1st Cycle	FEM	-22.5	15	----	----	-26.67	13.33	-12
	Balance	----	(-15×.4) = -6	(-15×.6) = -9	(26.67×.55) = 14.67	(26.67×.45) = 12	(-1.33×1) = -1.33	----
2nd Cycle	CO	-3	----	7.34	-4.5	-0.67	6	----
	Balance	----	(-7.34×.4) = -2.94	(-7.34×.6) = -4.4	(5.17×.55) = 2.84	(5.17×.45) = 2.32	(-6×1) = -6	----
3rd Cycle	CO	-1.47	----	1.42	-2.2	-3	1.16	----
	Balance	----	(-1.42×.4) = -0.57	(-1.42×.6) = -0.85	(5.2×.55) = 2.86	(5.2×.45) = 2.34	(-1.16×1) = -1.16	----
	Total	-26.97	5.49	-5.49	13.67	-13.67	12	-12

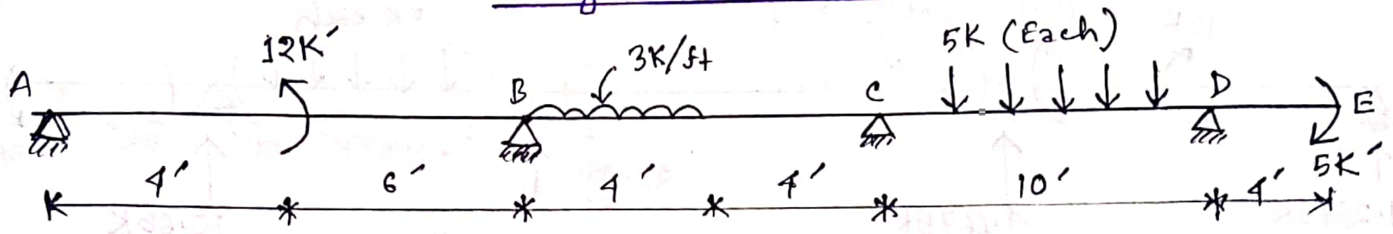

4K
E
3'
D
12K'
12K'
15K
8'
4'
C
13.67K'
13.67K'
10'
B
5.49K'
5.49K'
15'
26.97K'
2K/ft


4K
12K'
4K
15K
12K'
13.67K'
4.86K
10.14K
13.67K'
5.49K'
0.82K
0.82K


2K/ft
5.49K'
26.97K'
3.57K
11.43K
4K
15K
2K/ft
12K'
12K'
13.67K'
13.67K'
5.5K'
5.49K'
26.97K'
8.86K
10.96K
2.75K
11.43K


4.86K
0.82K
3.57K
0.0K
4K
10.14K
11.43K
+
+
+
−
−
−
SFD
+
+
−
−
12K'
13.67K'
5.49K'
26.97K'
BMD

Assignment No:02



Solution:

(i) Relative Stiffness Calculation:

$K_{AB} = \frac{I}{L} = \frac{1}{10} \simeq 1$

$K_{BC} = \frac{1}{8} \simeq 1.25$

$K_{CD} = \frac{1}{10} \simeq 1$

(ii) Fixed End Moment:

$F_{AB} = \frac{Mb(2a-b)}{L^2} = \frac{12 \times 6 \times (8-6)}{10^2} = 1.44 \ K'$

$F_{BA} = \frac{Ma(2b-a)}{L^2} = \frac{12 \times 4 \times (12-4)}{10^2} = 3.84 K'$

$F_{BC} = \frac{11 WL^2}{192} = \frac{11 \times 3 \times 8^2}{192} = 11 K'$

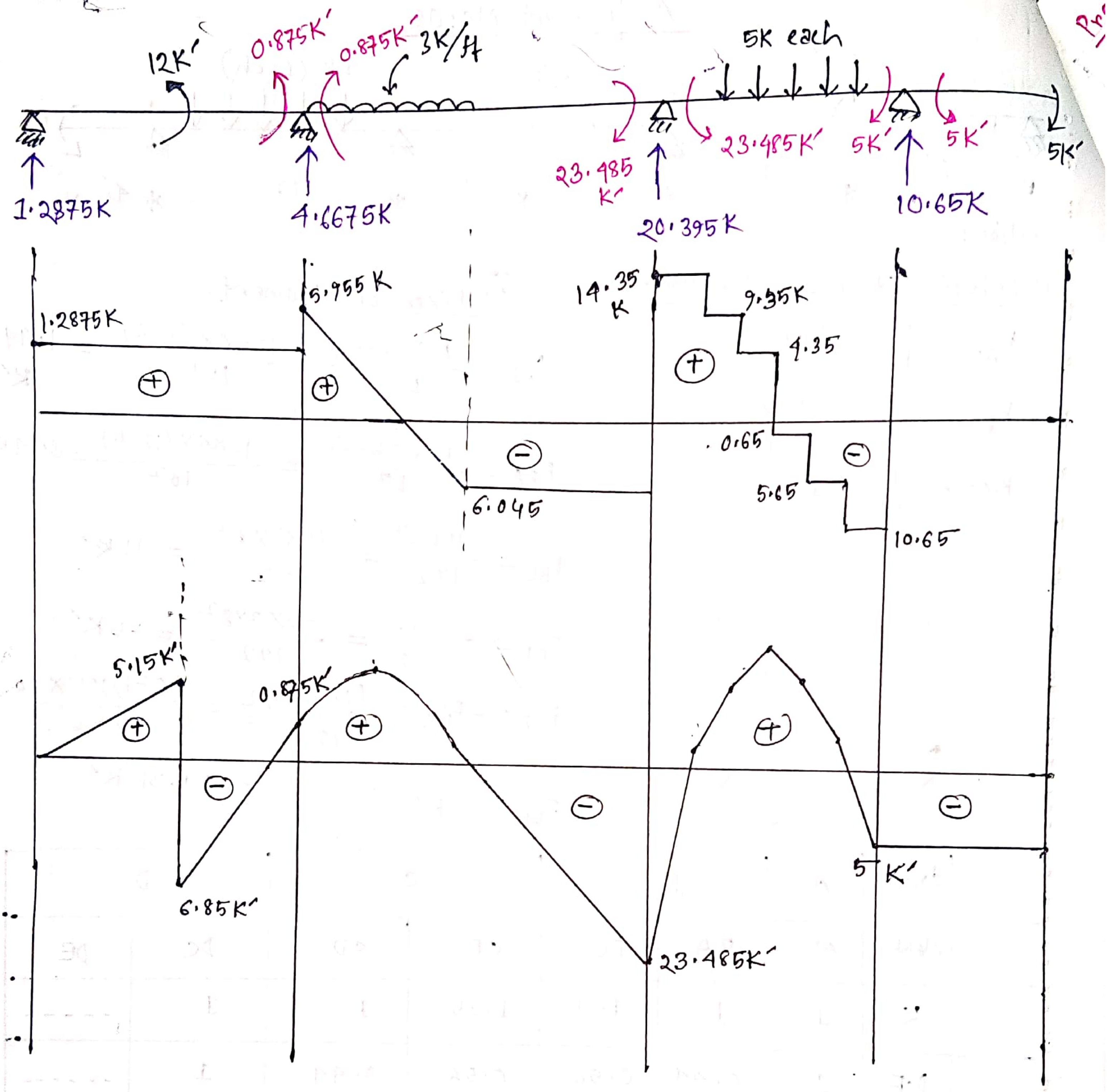
$F_{CB} = -\frac{5WL^2}{192} = \frac{-5 \times 3 \times 8^2}{192} = -5K'$

$F_{CD} = -F_{DC} = \frac{(N^2-1)PL}{12N} = \frac{(6^2-1) \times 5 \times 10}{12 \times 6}$

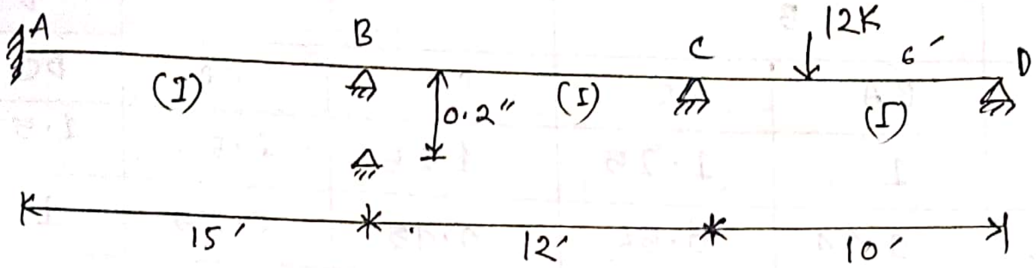
$= 24.31 \ K'$

$F_{DE} = 5K'$

	Joint	A	B		C		D	
	Member	AB	BA	BC	CB	CD	DC	DE
	K	1	1	1.25	1.25	1	1	-----
	D.F	1	0.44	0.56	0.56	0.44	1	------
1st Cycle	FEM	1.44	3.84	11	-5	24.31	-24.31	5
	Balance	-1.44	-6.53	-8.31	-10.81	-8.5	19.31	-----
2nd Cycle	CO	-3.265	-0.72	-5.405	-4.155	9.655	-4.25	-----
	Balance	3.265	2.695	3.43	-3.08	-2.42	4.25	-----
3rd Cycle	CO	1.35	1.63	-1.54	1.72	2.13	-1.21	------
	Balance	-1.35	-0.04	-0.05	-2.16	-1.69	1.25	-----
	Total	0	+0.875	-0.875	-23.485	23.485	-5	5


12K'
0.875K'
0.875K'
3K/ft
5K each
1.2875K
4.6675K
23.485 K'
20.395K
23.485K'
5K'
5K'
10.65K
5K'
1.2875K
5.955 K
14.35 K
9.35K
4.35
0.65
5.65
10.65
6.045
+
+
+
−
−
5.15K'
0.875K'
6.85K'
23.485K'
5 K'
+
+
+
−
−
−

Problem-03:



E = 30 000 ksi

I = 200 in^4

$\Delta_B = 0.2''$

$\theta_A = 0.0015$ rad (cw)

Solution:

(i) Relative stiffness:

$$K_{AB} = \frac{I}{L} = \frac{1}{15} \simeq 1$$

$$K_{BC} = \frac{I}{L} = \frac{1}{12} \simeq 1.25$$

$$K_{CD} = \frac{I}{L} = \frac{1}{10} \simeq 1.5$$

(ii) Fixed End Moment:

FEM due to support settlement:

$$F_{AB} = F_{BA} = \frac{6EI\Delta}{L^2}$$

$$= \frac{6 \times 30000 \times 200 \times 0.2}{15^2 \times 1728} = 18.52K'$$

$$F_{BC} = F_{CB} = -\frac{6EI\Delta}{L^2}$$

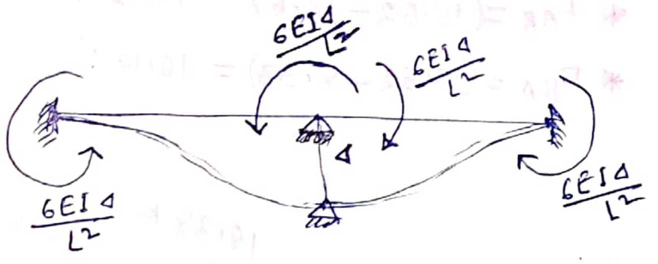
$$= -\frac{6 \times 30000 \times 200 \times 0.2}{12^2 \times 1728} = -28.935 K'$$

FEM due to rotation:

$$F_{AB} = -\frac{4EI\theta}{L} = -\frac{4 \times 30000 \times 200 \times 0.0015}{15 \times 144}$$

$$= -16.67 K'$$

$$\therefore F_{BA} = -\frac{2EI\theta}{L} = -8.33K'$$



clockwise — (-ve)

Anticlock — (+ve)

FEM due to load:

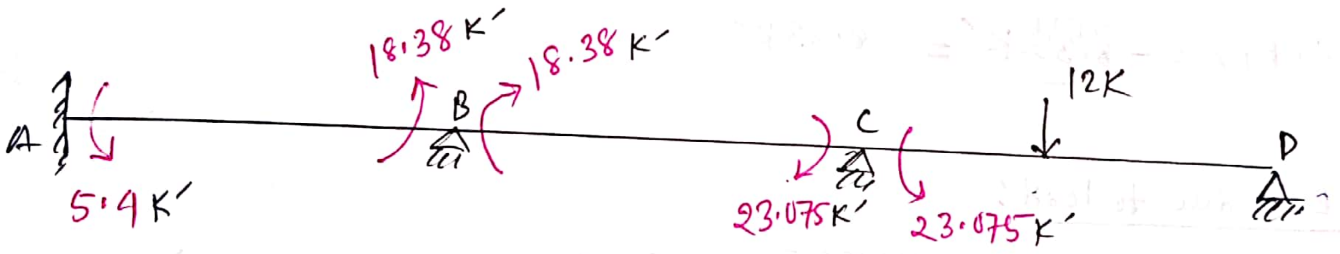
$$F_{CD} = \frac{Pab^2}{L^2} = \frac{12 \times 4 \times 6^2}{10^2} = 17.28 K'$$

$$F_{DC} = -\frac{Pba^2}{L^2} = -\frac{12 \times 6 \times 4^2}{10^2} = -11.52K'$$

	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	1	1	1.25	1.25	1.5	1.5
	D.F	----	0.44	0.56	0.45	0.55	1
1st Cycle	FEM	*1.85	*10.19	-28.935	-28.935	17.78	-11.52
	Balance	----	8.25	10.495	5.245	6.41	11.52
2nd Cycle	CO	4.125	----	2.62	5.25	5.76	3.205
	Balance	----	- 1.15	- 1.47	- 4.95	- 6.06	-3.205
3rd Cycle	CO	-0.575	----	-2.475	-0.735	-1.60	- 3.63
	Balance	----	1.09	1.385	1.05	1.285	3.03
	Total	5.4	18.38	-18.38	- 23.075	+ 23.075	0

* $F_{AB} = (18.52 - 16.57) = 1.85$ K′

* $F_{BA} = (18.52 - 8.33) = 10.19$ K′

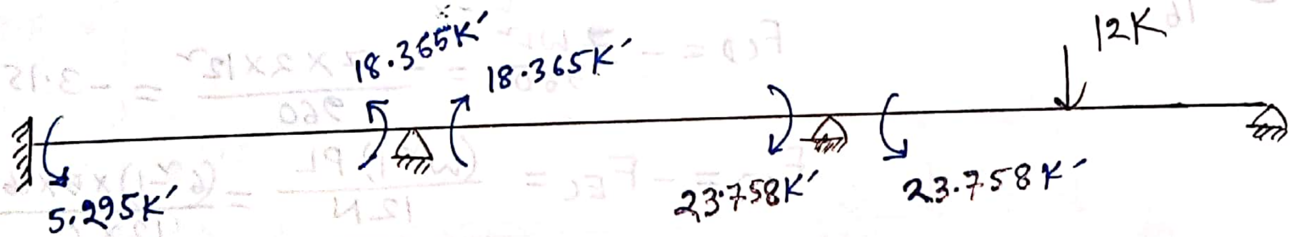


same problem:

* Far end Hinge stiffness will be 75%

Modified Stiffness Method

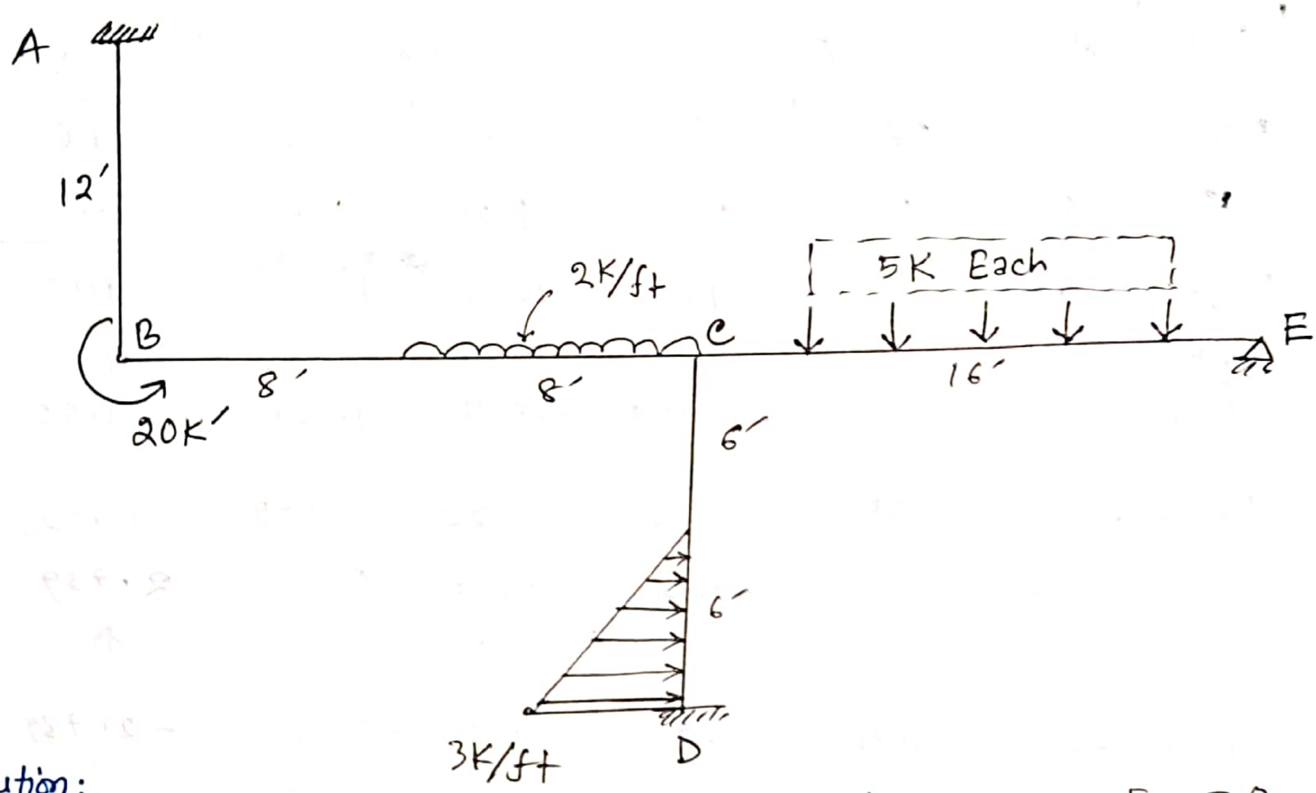
	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	1	1	1.25	1.25	1.5	1.5
	Modified K	1	1	1.25	1.25	**$(\frac{3}{4} \times 1.5) =$ 1.125	1.5
	D.F	....	0.41	0.56	0.53	0.47	1
1st Cycle	FEM	1.85	10.19	-28.935	-28.935	17.28	-11.52
	Balance	.....	8.25	10.495	6.177	5.478	11.52
2nd cycle	CO	4.125	-----	3.09	5.25	5.76	
	Balance	----	-1.36	-1.73	-5.84	-5.17	
3rd cycle	CO	-0.68	----	-2.92	-0.87	-	
	Balance	----	1.285	1.635	0.46	0.41	
	Total	5.295	18.365	-18.365	-23.758	+23.758	0



** যদি কোনো Member-এর একটি support Hinge হয় তাহলে Modified Stiffness 'K' এর 75% বা $\frac{3}{4}$K হবে।

support → end support

Problem - 04



Solution:

Relative Stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{12} \simeq 2$

$K_{BC} = \frac{1}{16} \simeq 1.5$

$K_{CD} = \frac{1}{12} \simeq 2$

$K_{CE} = \frac{1}{16} \simeq 1.5$

Fixed End Moment: $F_{AB} = F_{BA} = 0$

$F_{BC} = \frac{5WL^2}{192} = \frac{5 \times 2 \times 16^2}{192} = 13.33 K'$

$F_{CB} = -\frac{11WL^2}{192} = \frac{-11 \times 2 \times 16^2}{192} = -29.33 K'$

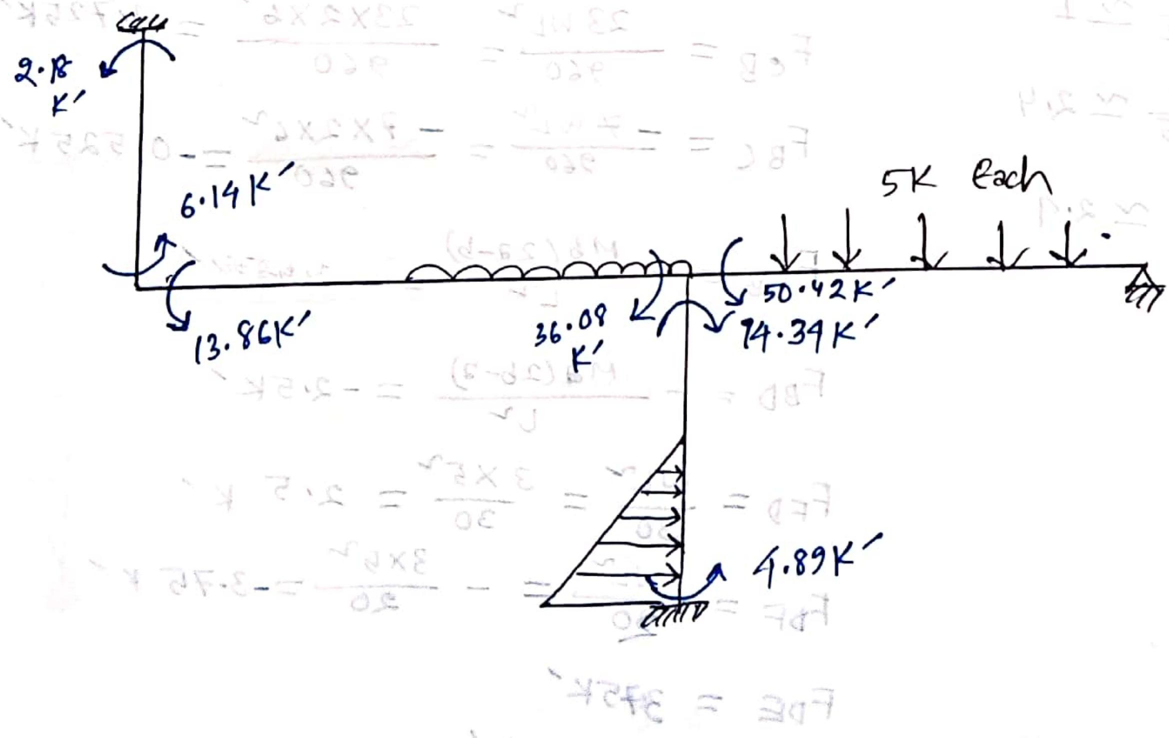
$F_{DC} = \frac{23WL^2}{960} = \frac{23 \times 2 \times 12^2}{960} = 10.35 K'$

$F_{CD} = -\frac{7WL^2}{960} = -\frac{7 \times 2 \times 12^2}{960} = -3.15 K'$

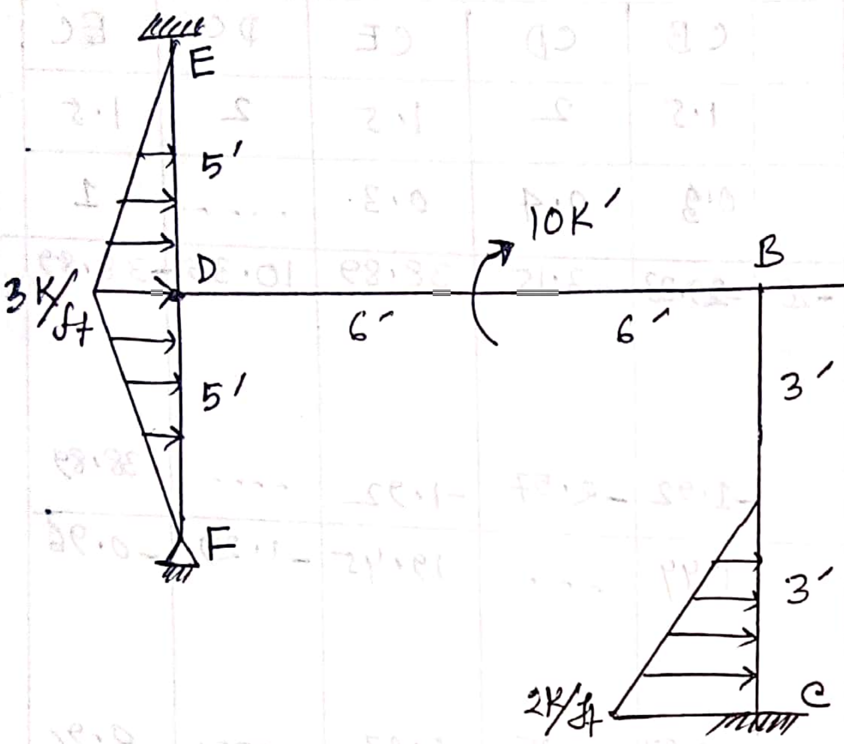
$F_{CE} = -F_{EC} = \frac{(w^2-1)PL}{12N} = \frac{(6^2-1) \times 5 \times 16}{12 \times 6}$

$\therefore F_{CE} = -F_{EC} = 38.89 K'$

	Joint	A	B			C			D	E
	Member	AB	BA	BC		CB	CD	CE	DC	EC
	K	2	2	1.5		1.5	2	1.5	2	1.5
	D.F	----	0.57	0.43		0.3	0.4	0.3	-----	1
1st	FEM	---	----	13.33	-20	-29.33	-3.15	38.89	10.35	-38.89
Cycle	Balance	-----	3.8	2.87		-1.92	-2.57	-1.92	----	38.89
2nd	CO	1.9	---	-0.96		1.44	---	19.45	-1.29	-0.96
Cycle	Balance	---	0.55	0.41		-6.27	-8.35	-6.27	---	0.96
3rd	CO	0.28	---	-3.14		0.21	---	0.48	-4.18	-3.14
Cycle	Balance	---	1.79	1.35		-0.21	-0.27	-0.21	---	3.14
		2.18	6.14	13.86	-20	-36.08	-14.39	50.42	4.88	0



Assignment : 04



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{1}{6} \simeq 2$

$K_{BD} = \frac{1}{12} \simeq 1$

$K_{DE} = \frac{1}{5} \simeq 2.4$

$K_{DF} = \frac{1}{5} \simeq 2.4$

Fixed End Moment:

$F_{AB} = \frac{Ma(2b-a)}{L^2} = \frac{10 \times 6 \times (12-6)}{12^2} = 2.5 K'$

$F_{BA} = \frac{Mb(2a-b)}{L^2} = \frac{10 \times 6 \times (12-6)}{12^2} = 2.5 K'$

$F_{CB} = \frac{23 WL^2}{960} = \frac{23 \times 2 \times 6^2}{960} = 1.725 K'$

$F_{BC} = \frac{-7 WL^2}{960} = \frac{-7 \times 2 \times 6^2}{960} = -0.525 K'$

$F_{DB} = - \frac{Mb(2a-b)}{L^2} = - 2.5 K'$

$F_{BD} = - \frac{Ma(2b-a)}{L^2} = -2.5 K'$

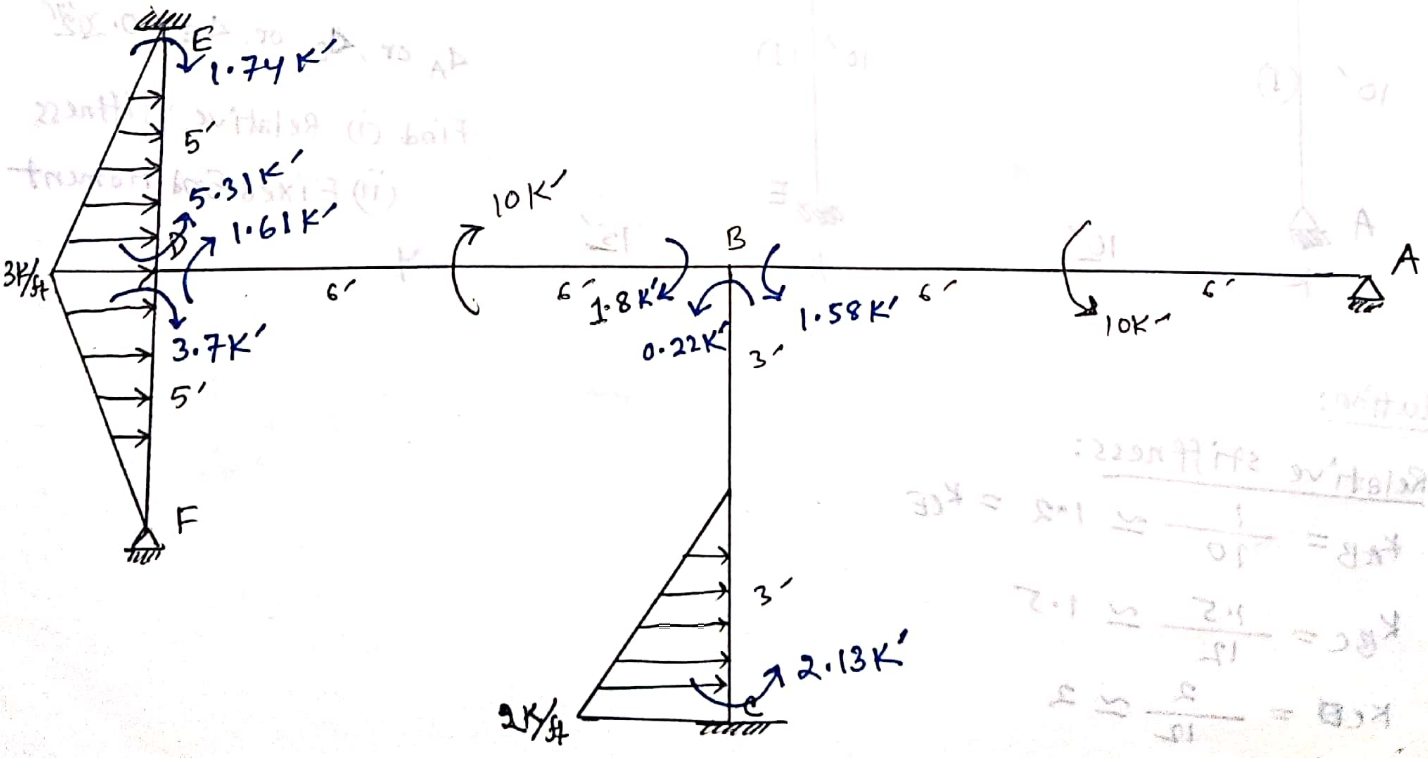
$F_{FD} = \frac{WL^2}{30} = \frac{3 \times 5^2}{30} = 2.5 K'$

$F_{DF} = \frac{-WL^2}{20} = - \frac{3 \times 5^2}{20} = -3.75 K'$

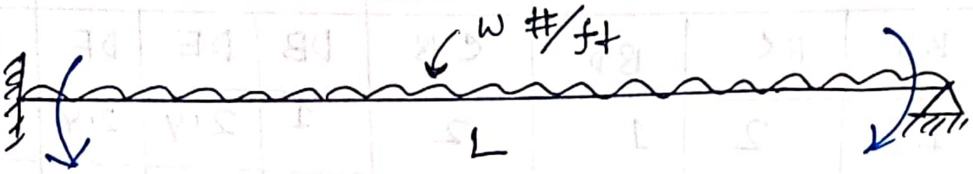
$F_{DE} = 3.75 K'$

$F_{ED} = - 2.5 K'$

	Joint	A	B			C	D			E	F
	Member	AB	BA	BC	BD	CB	DB	DE	DF	ED	FD
	K	1	1	2	1	2	1	2.4	2.4	2.4	2.4
	D.F.	1	0.25	0.50	0.25	-----	0.18	0.41	0.41	----	1
1st	FEM	2.5	2.5	-0.53	-2.5	1.73	-2.5	3.75	-3.75	-2.5	2.5
Cycle	Balance	-2.5	0.13	0.27	0.13	----	0.49	1.03	1.03	----	-2.5
2nd	CO	0.07	-1.25	----	0.22	0.14	0.07	---	-1.25	0.52	0.52
Cycle	Balance	-0.07	0.25	0.51	0.26	----	0.22	0.48	0.48	----	-0.52
3rd	CO	0.13	-0.04	----	0.11	0.26	0.13	---	-0.26	0.24	0.24
Cycle	Balance	-0.13	-0.02	-0.03	-0.02	----	0.03	0.05	0.05	----	-0.24
	Total	0	1.58	0.22	-1.8	2.13	-1.61	5.31	-3.7	-1.74	0

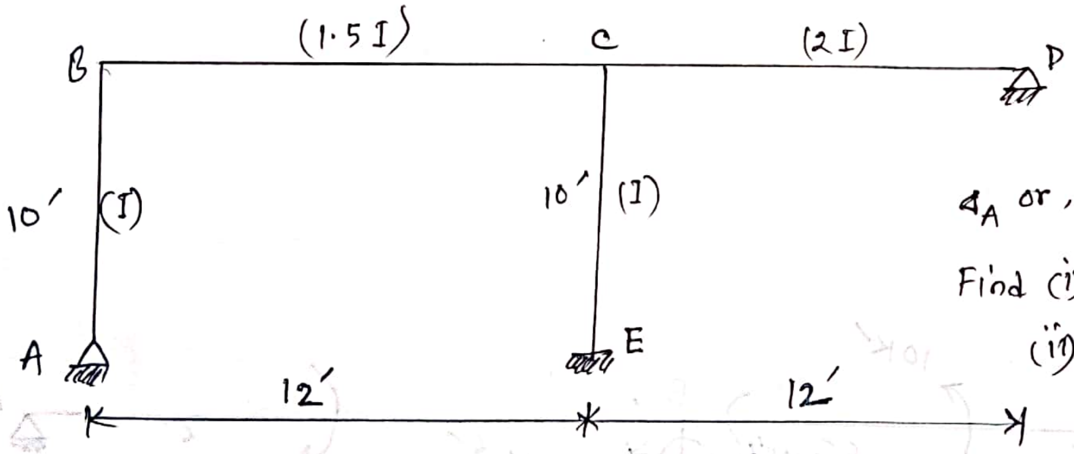


BCS



FEM:	$\frac{WL^2}{12}$	$-\frac{WL^2}{12}$
Balance:	∼∼∼∼	$\frac{WL^2}{12}$
CO:	$\frac{WL^2}{24}$	∼∼∼∼
Balance:	− ∼∼∼∼	∼∼∼∼
Total:	$\frac{WL^2}{8}$	0

Assignment: 05



$E = 30000$ ksi

$I = 200$ in^4

Δ_A or, Δ_E or, $\Delta_E = 0.2''$

Find (i) Relative stiffness

(ii) Fixed End Moment

Solution:

Relative stiffness:

$K_{AB} = \frac{1}{10} \simeq 1.2 = K_{CE}$

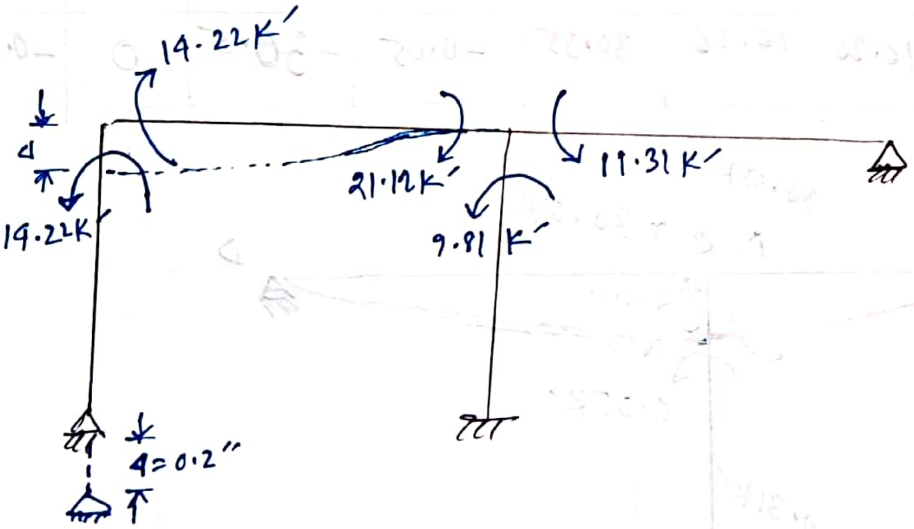
$K_{BC} = \frac{1.5}{12} \simeq 1.5$

$K_{CD} = \frac{2}{12} \simeq 2$

For $\Delta_A = 0.2''$: FEM:

$$F_{BC} = F_{CB} = \frac{-6E \times 1.5I \times \Delta}{L^2} = \frac{-6 \times 30000 \times 1.5 \times 200 \times 0.2}{12^2 \times 1728} = -43.40K'$$

	Joint	A	B		C			D	E
	Member	AB	BA	BC	CB	CE	CD	DC	EC
	K	1.2	1.2	1.5	1.5	1.2	2.0	2.0	1.2
	D.F	1	0.49	0.56	0.32	0.26	0.42	1	---
1st	FEM	---	---	-43.40	-43.40	---	---	---	---
Cycle	Balance	---	19.1	24.3	13.89	11.28	18.23	---	---
2nd	CO	9.55	---	6.95	12.15	---	---	9.12	5.64
Cycle	Balance	-9.55	-3.06	-3.89	-3.89	-3.16	-5.1	-9.12	---
3rd	CO	-1.53	-4.78	-1.95	-1.95	---	-4.56	-2.55	-1.58
Cycle	Balance	1.53	2.96	3.77	2.08	1.69	2.74	2.55	---
	Total	0	14.22	-14.22	-21.12	9.81	11.31	0	4.06

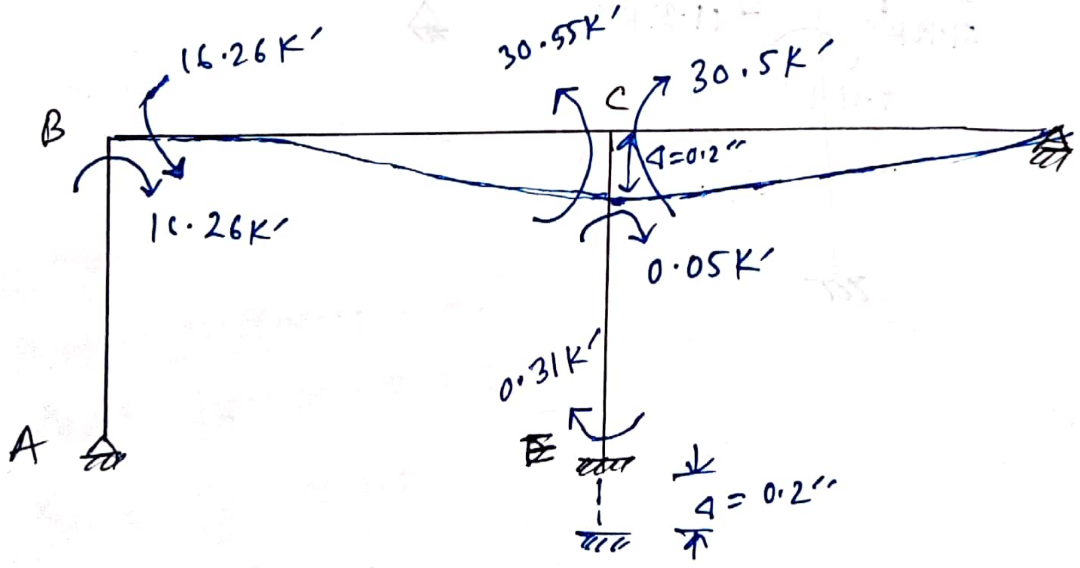


if $\Delta_E = 0.2''$:

FEM: $F_{BC} = F_{CB} = \frac{6\times30000\times1.5\times200\times0.2}{12^2\times1728} = 43.40K'$

$F_{CD} = F_{DC} = -\frac{6\times30000\times2\times200\times0.2}{12^2\times1728} = -57.87K'$

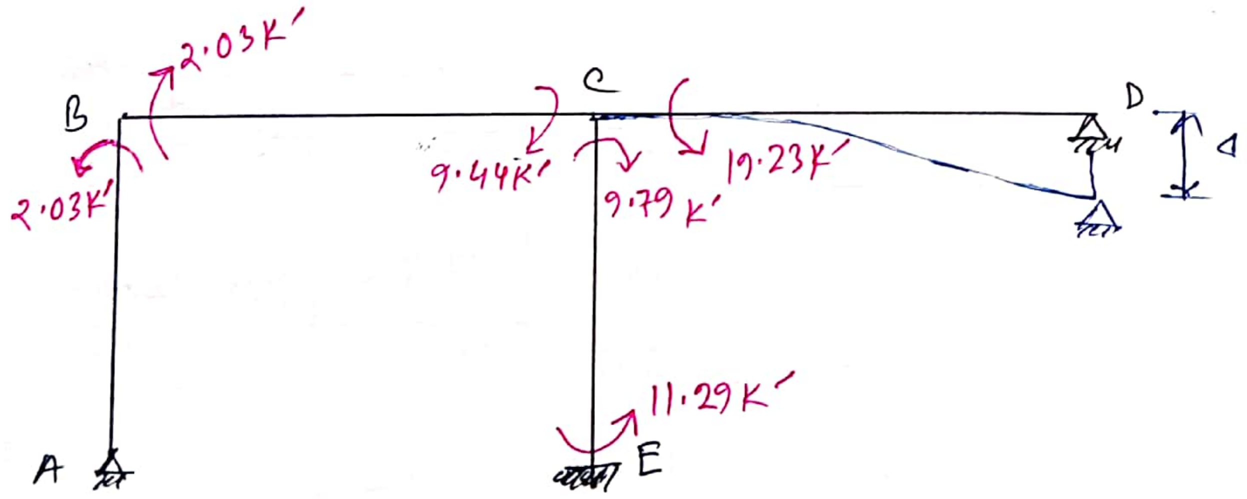
	Joint	A	B		C			D	E
	Member	AB	BA	BC	CB	CE	CD	DC	EC
	K	1.2	1.2	1.5	1.5	1.2	2	2	1.2
	D.F.	1	0.44	0.56	0.32	0.26	0.42	1	---
1st Cycle	FEM	---	---	43.40	43.4	-----	-57.87	-57.87	---
	Balance	---	-19.1	-24.3	4.63	3.76	6.08	57.87	---
2nd Cycle	CO	-9.55	---	2.32	-12.15	---	28.94	3.04	1.88
	Balance	9.55	-1.02	-1.3	-5.37	-4.37	-7.05	-3.04	----
3rd Cycle	CO	-0.51	4.78	-2.69	-0.65	---	-1.52	-3.53	-2.19
	Balance	0.51	-0.92	-1.17	0.69	0.56	0.92	3.53	---
	Total	0	-16.26	16.26	30.55	-0.05	-30.5	0	-0.31



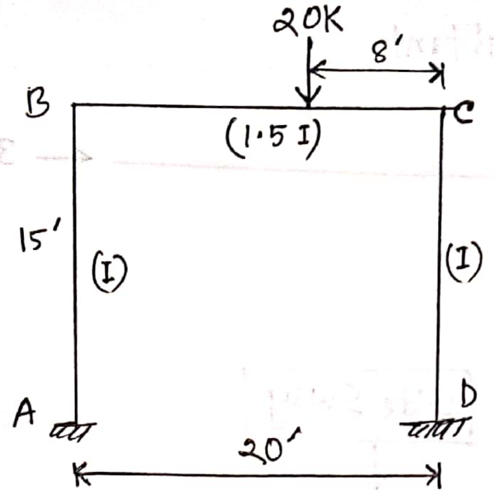
if $\Delta_E = 0.2''$:

$$F_{CD} = F_{DC} = \frac{6 \times 30000 \times (2 \times 200) \times 0.2}{12^2 \times 1728} = 57.87\ K'$$

	Joint	A	B		C			D	E
	Member	AB	BA	BC	CB	CE	CD	DC	EC
	K	1.2	1.2	1.5	1.5	1.2	2	2	1.2
	D.F.	1	0.44	0.56	0.32	0.26	0.42	1	....
1st cycle	FEM	...	....	....	....	....	57.87	57.87	....
	Balance	....	....	....	−18.52	−15.05	−24.3	−57.87	...
2nd cycle	CO	...	...	−9.26	...	....	−28.94	−12.15	7.53
	Balance	....	4.07	5.19	9.26	7.52	12.16	12.15	...
3rd cycle	CO	2.04	...	4.63	2.60	....	6.08	6.08	3.76
	Balance	−2.04	−2.04	−2.59	−2.78	−2.26	−3.64	−6.08	...
	Total	0	2.03	−203	−9.44	−9.79	19.23	0	11.29



Problem: 06



Calculate:

1. Relative stiffness
2. Fixed End Moment

Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{15} \simeq 2$

$K_{BC} = \frac{1.5}{20} \simeq 2.25$

$K_{CD} = \frac{1}{15} \simeq 2$

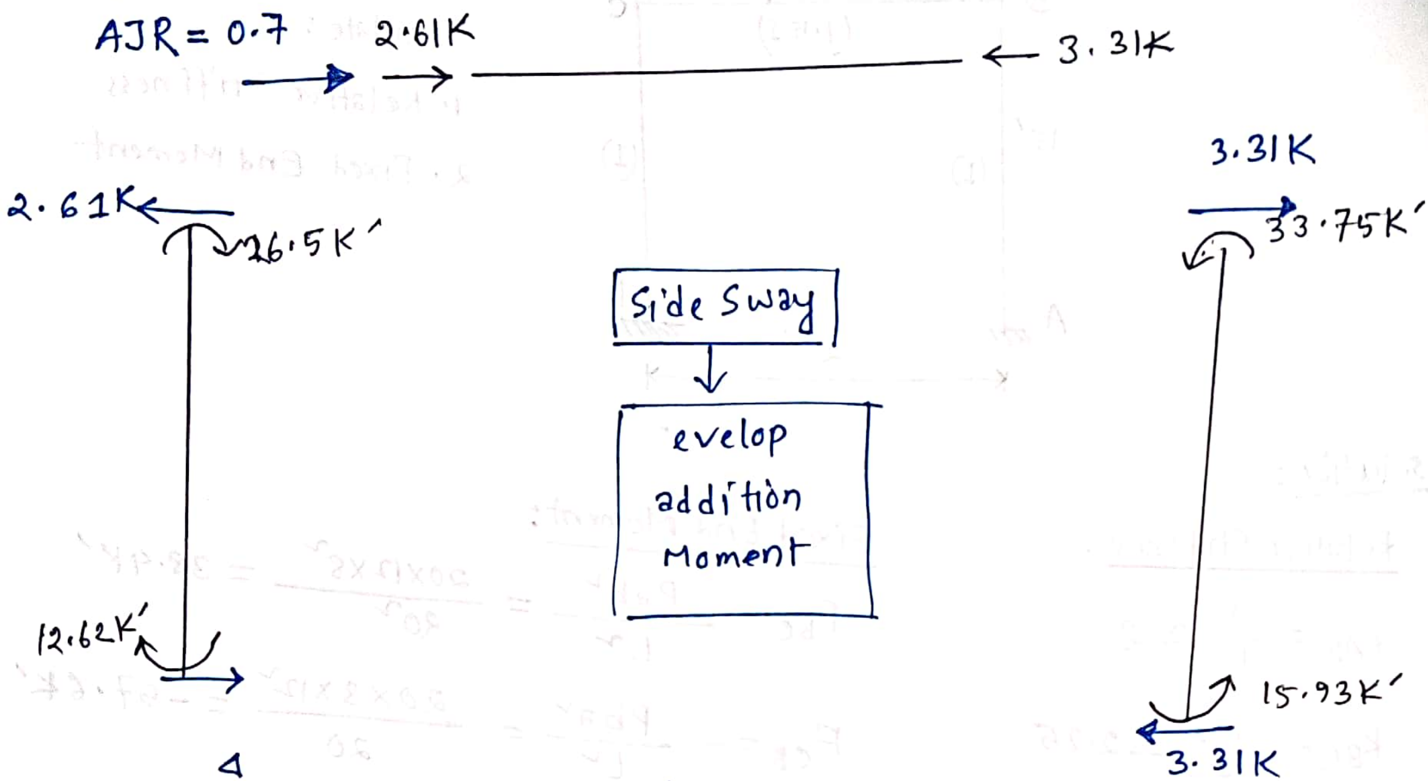
Fixed End Moment:

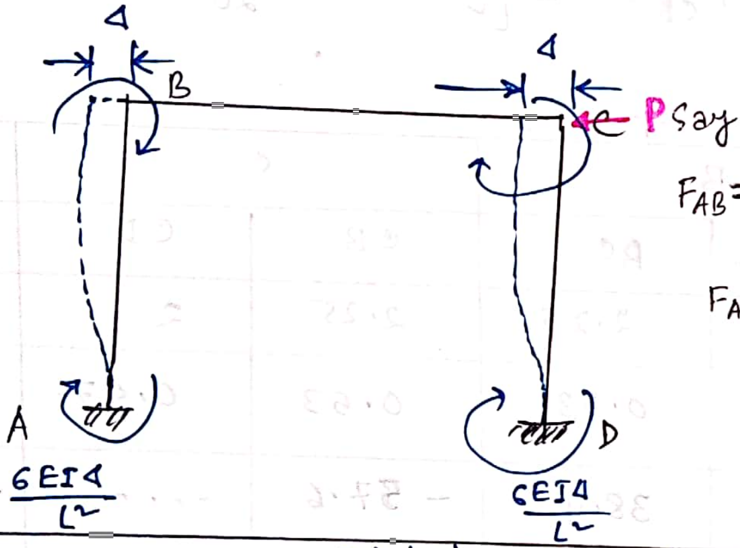
$F_{BC} = \frac{Pab^2}{L^2} = \frac{20 \times 12 \times 8^2}{20^2} = 38.4K'$

$F_{CB} = -\frac{Pba^2}{L^2} = \frac{20 \times 8 \times 12^2}{20} = -57.6K'$

	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	2	2	2.25	2.25	2	2
	D.F	-----	0.47	0.53	0.53	0.47	-----
1st	FEM	-----	-----	38.4	-57.6	-----	-----
Cycle	Balance	-----	-18.05	-20.35	30.53	27.07	-----
2nd	CO	-9.03	-----	15.27	-10.18	-----	13.54
Cycle	Balance	-----	-7.18	-8.09	5.40	4.78	-----
3rd	CO	-3.59	-----	2.7	-4.05	-----	-2.39
Cycle	Balance	-----	-1.27	-1.43	2.15	1.9	-----
		-12.62	-26.5	+26.5	-33.75	33.75	15.93

→ Artificial Joint Resistant



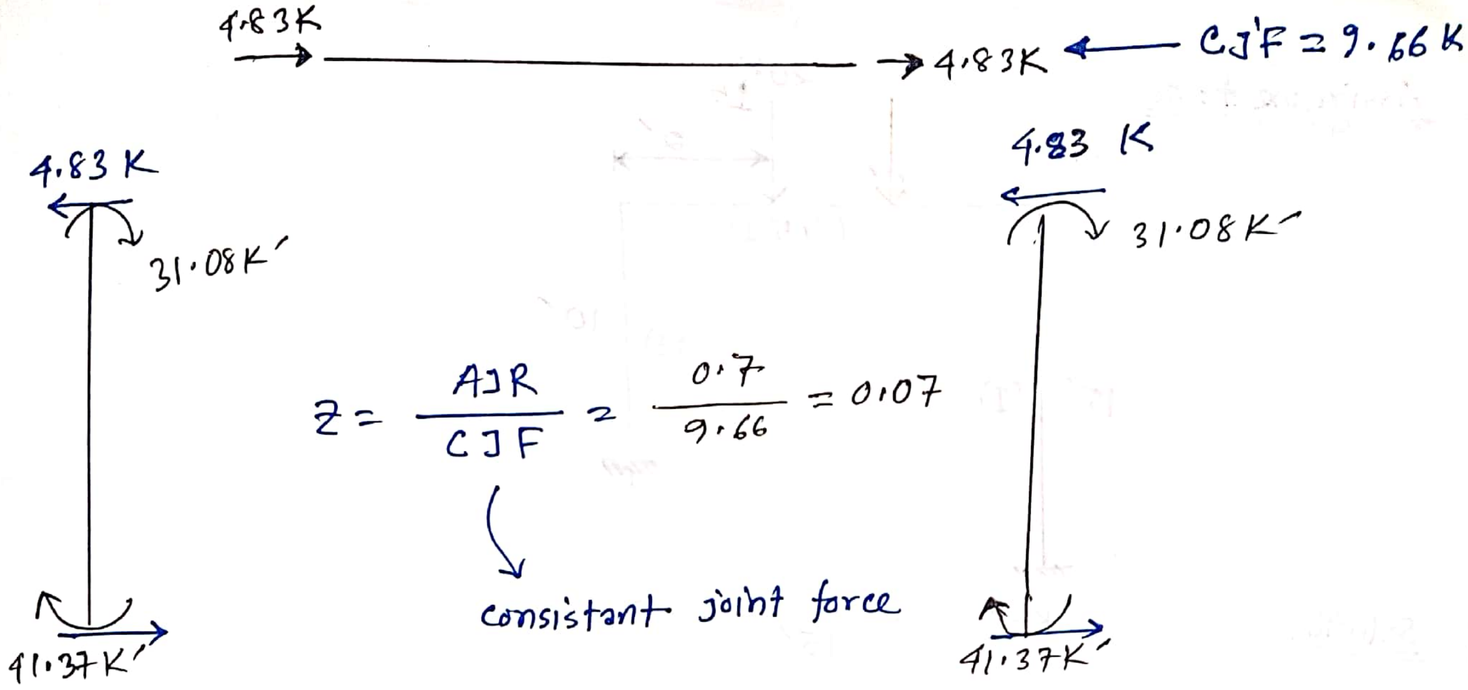


$F_{AB} = F_{DC} = \frac{6EI\Delta}{L^2} = -50$ K ft

$F_{AB} = F_{BA}$ and $F_{DC} = F_{CD}$

Distribution for Side way

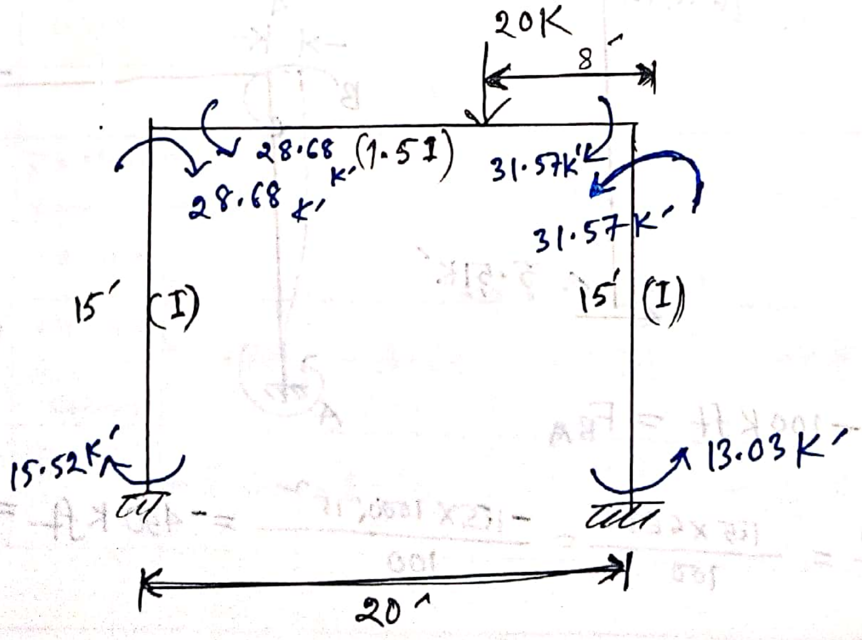
	Member	AB	BA	BC	CB	CD	DC
1st cycle	FEM	−50	−50	----	----	−50	−50
	Balance	----	23.5	26.5	26.5	23.5	----
2nd cycle	FEM	11.75	----	13.25	13.25	----	11.75
	Balance	----	−6.23	−7.02	−7.02	−6.23	----
3rd cycle	FEM	−3.12	----	−3.51	−3.51	----	−3.12
	Balance	----	1.65	1.86	1.86	1.65	----
	Total	−41.37	−31.08	31.08	31.08	−31.08	−41.37



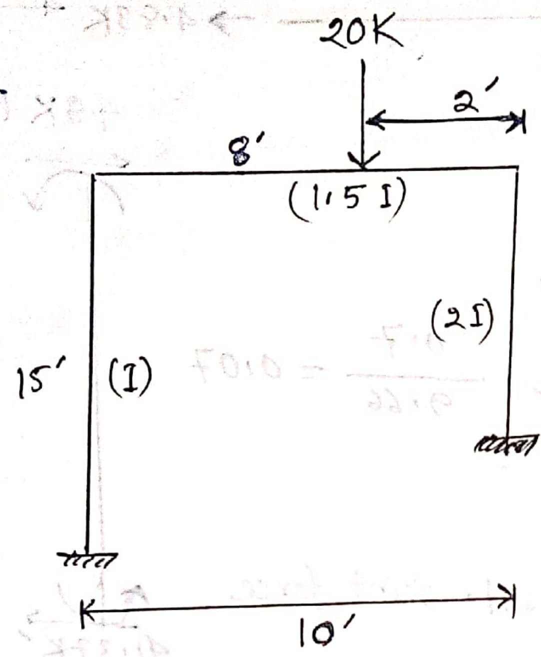
$$Z = \frac{AJR}{CJF} = \frac{0.7}{9.66} = 0.07$$

CJF → consistant joint force

$Z = 0.07$						
Member	AB	BA	BC	CB	CD	DC
Z× Moment from Second Balance	(0.07×-41.37) = -2.90	-2.18	2.18	2.18	-2.18	-2.90
Moment from First Balance	-12.62	-26.5	26.5	-33.75	33.75	15.93
Total	-15.52	-28.68	28.68	-31.57	31.57	13.03



Assignment: 06



Solution :

Relative Stiffness:

$K_{AB} = \frac{1}{15} \simeq 1$

$K_{BC} = \frac{1.5}{10} \simeq 2.25$

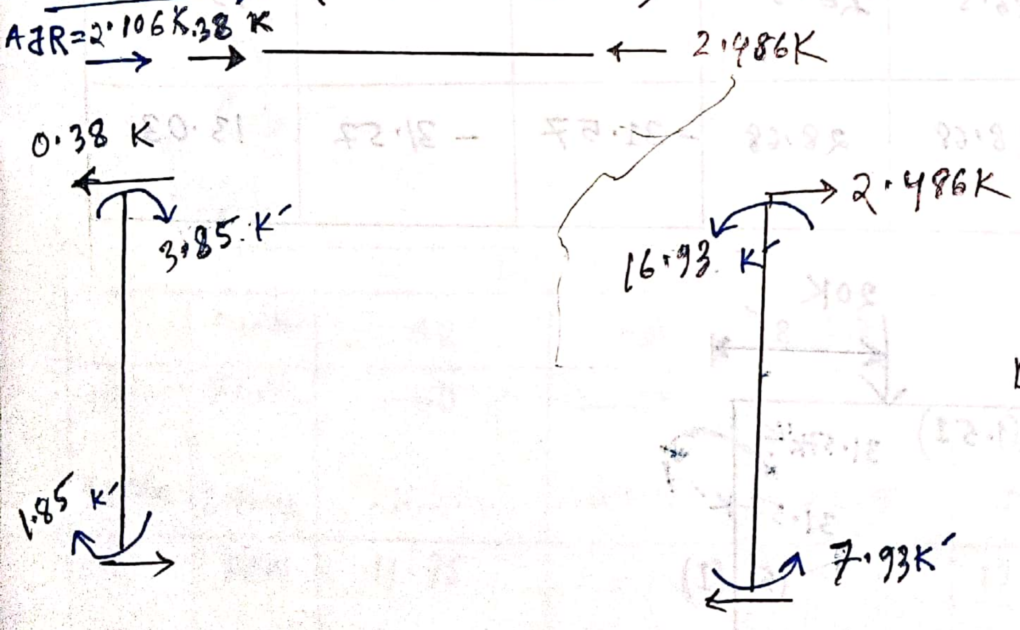
$K_{CD} = \frac{2}{10} \simeq 3$

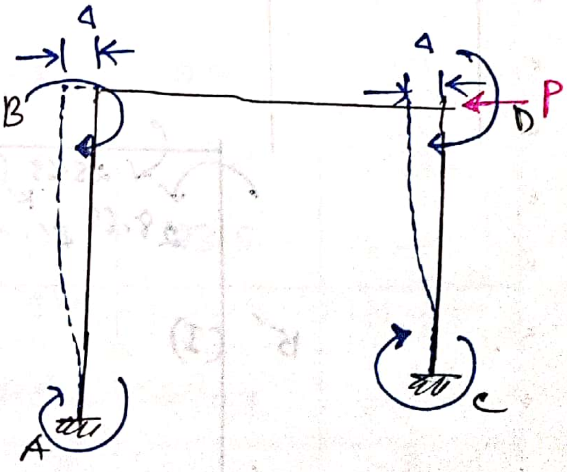
Fixed End Moment :

$F_{BC} = \frac{Pab^2}{L^2} = \frac{20 \times 8 \times 2^2}{1^2} = 6.4\ K'$

$F_{CB} = -\frac{Pa^2b}{L^2} = -\frac{20 \times 8^2 \times 2}{1^2} = -25.6\ K'$

For side sway: (After 1st Balance)



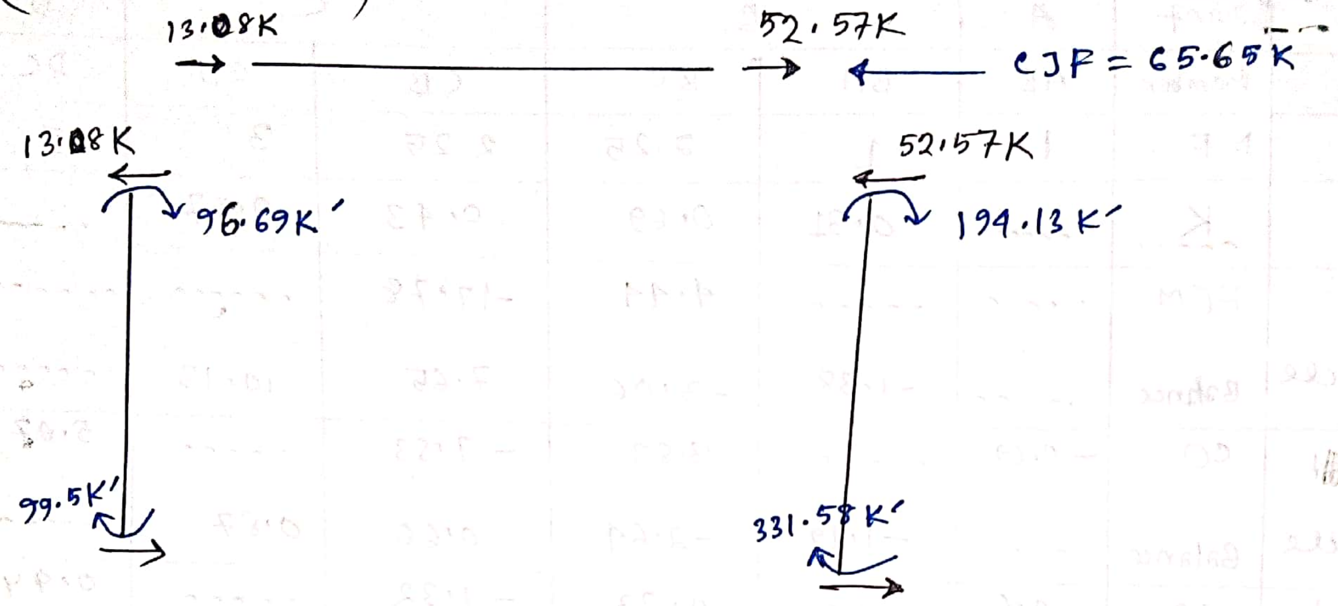


Say, $F_{AB} = \frac{6EI\Delta}{L^2} = -100\ K\,ft = F_{BA}$

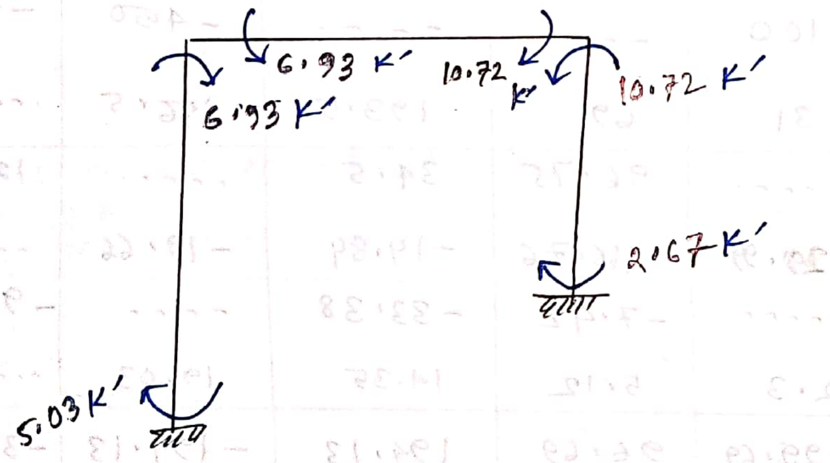
$\therefore F_{DC} = \frac{6E \times (2I) \times \Delta}{(10)^2} = \frac{2 \times 6EI\Delta}{100} = \frac{-2 \times 100 \times 15^2}{100} = -450\ K\,ft = F_{CD}$

—	joint	A	B		C		D
	member	AB	BA	BC	CB	CD	DC
	K	1	1	2.25	2.25	3	3
	DF	---	0.31	0.69	0.43	0.57	---
1st	FEM	---	---	6.4	-25.6	---	---
cycle	Balance	---	-1.98	-4.42	11.01	14.59	---
2nd	CO	-0.99	---	5.51	-2.21	---	7.3
cycle	Balance	---	-1.71	-3.8	0.95	1.26	---
3rd	CO	-0.86	---	0.48	-1.9	---	0.63
cycle	Balance	---	-0.15	-0.33	0.82	1.08	---
	Total	-1.85	-3.84	3.84	-16.93	16.93	7.93
			Distribution for side-Sway				
1st	FEM	-100	-100	----	----	-450	-450
Cycle	Balance	----	31	69	193.5	256.5	----
2nd	CO	15.5	----	96.75	34.5	----	128.25
Cycle	Balance	----	-29.99	-66.76	-14.84	-19.66	----
3rd	CO	-15	----	-7.42	-33.38	----	-9.83
Cycle	Balance	----	2.3	5.12	14.35	19.03	----
	Total	-99.5	-96.69	96.69	194.13	-194.13	-331.58
			z = 0.032				
	Z× Moment from 2nd balance	-3.18	-3.09	3.09	6.21	-6.21	-10.61
	Moment from 1st Balance	-1.85 ✓	-3.84	3.84	-16.93	16.93	7.93
	Total	-5.03	-6.93	6.93	-10.72	10.72	-2.67

(After 2nd Balance)



$$Z = \frac{AJR}{CJF} = \frac{2.106}{65.65} = 0.032$$



*** Side-sway হবে কিনা তা নির্ভর করে দুইটি বিষয়ের উপরে:

(i) Structure symmetrical আছে কিনা

(ii) load symmetrical আছে কিনা।

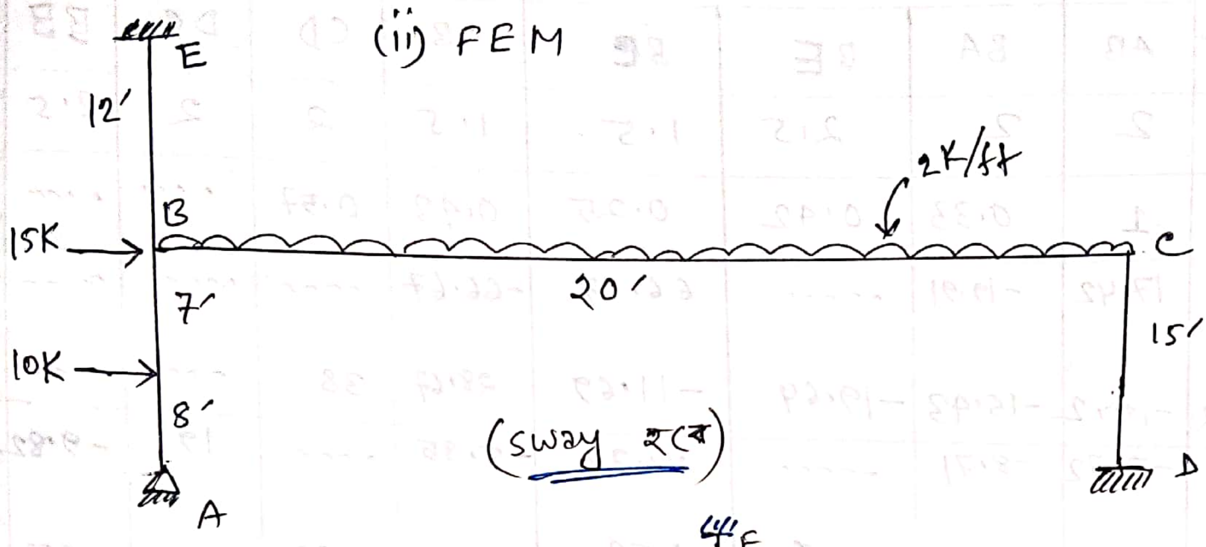
যদি structure and load symmetrical থাকে side sway হবে না।

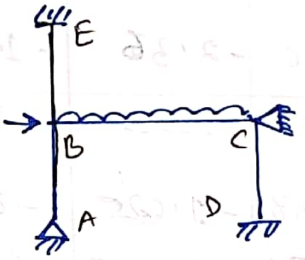
Problem: 07

Calculate:

(i) Relative Stiffness

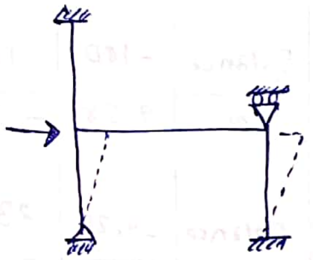
(ii) FEM





(sway হবে না)
কারণ C point-এ Hinge support আছে যা sway হতে দিবে না।

Rollar support থাকলে sway হবে।



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{15} \simeq 2$

$K_{BC} = \frac{1}{20} \simeq 1.5$

$K_{BE} = \frac{1}{12} \simeq 2.5$

$K_{CD} = \frac{1}{15} \simeq 2$

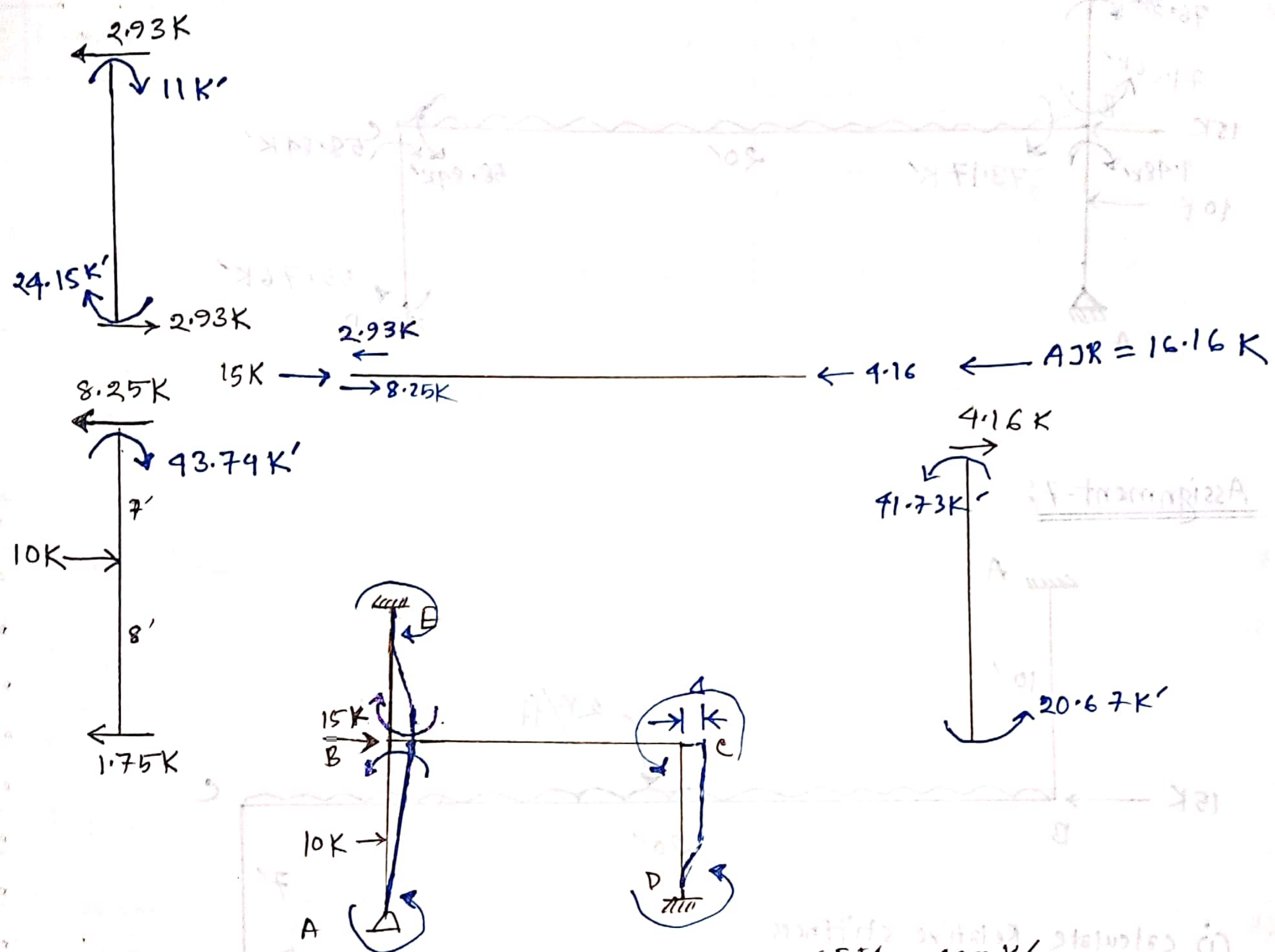
Fixed End Moment:

$$F_{AB} = \frac{Pab^2}{L^2} = \frac{10 \times 8 \times 7^2}{15^2} = 17.42 K'$$

$$F_{BA} = -\frac{Pa^2b}{L^2} = \frac{10 \times 8^2 \times 7}{15^2} = -19.91 K'$$

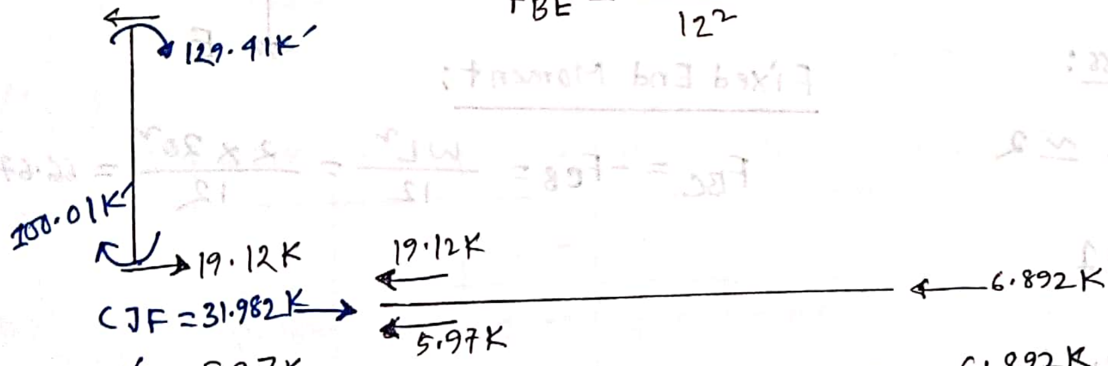
$$F_{BC} = -F_{CB} = \frac{WL^2}{12} = \frac{2 \times 20^2}{12} = 66.67 K'$$

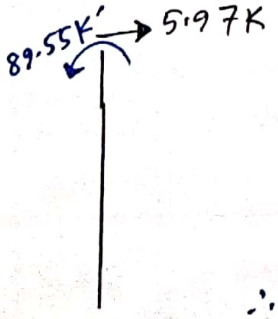
	Joint	A	B			C		D	E
	Member	AB	BA	BE	BC	CB	CD	DC	EB
	K	2	2	2.5	1.5	1.5	2	2	2.5
	D.F	1	0.33	0.42	0.25	0.43	0.57	----	----
1st	FEM	17.42	-19.91	-----	66.67	-66.67	----	----	---
Cycle	Balance	-17.42	-15.43	-19.64	-11.69	28.67	38	---	---
2nd	CO	-7.72	-8.71	-----	14.34	-5.85	----	19	-9.82
cycle	Balance	7.72	-1.86	-2.36	-1.41	2.52	3.33	----	---
3rd	CO	-0.93	3.86	-----	1.26	-0.71	----	1.67	-1.18
Cycle	Balance	0.93	-1.69	-2.15	-1.28	0.31	0.40	---	----
	Total	0	-43.79	-24.15	67.89	-41.73	41.73	20.67	-11
		Distribution for Sidesway							
1st	FEM	100	100	-156.25	----	----	100	100	-156.25
cycle	Balance	-100	18.56	23.63	14.06	-43	-57	---	---
2nd	CO	9.28	-50	----	-21.5	7.03	----	-28.5	11.82
Cycle	Balance	-9.28	23.6	30.03	17.87	-3.02	-4.01	----	---
3rd	CO	11.8	-4.64	----	-1.51	8.94	----	-2.01	15.02
Cycle	Balance	-11.8	2.03	2.58	1.54	-3.84	-5.1	---	---
	Total	0	89.55	-100.01	10.46	-33.89	33.89	69.49	-129.41
		$z = 0.505$							
Z X Moment from 2nd Balance		0	45.22	-50.51	5.28	-17.11	17.11	35.09	-65.35
Moment from 1st Balance		0	-43.79	-24.15	67.89	-41.73	41.73	20.67	-11
	Total		1.48	-74.66	73.17	-58.89	58.89	55.76	-76.35

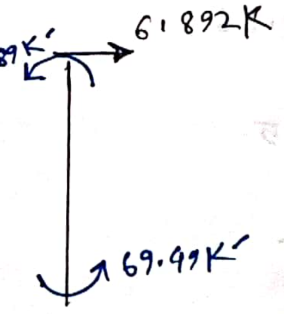


Say, $F_{AB} = F_{BA} = F_{DC} = F_{CD} = \frac{6EI\Delta}{15^2} = 100K'$

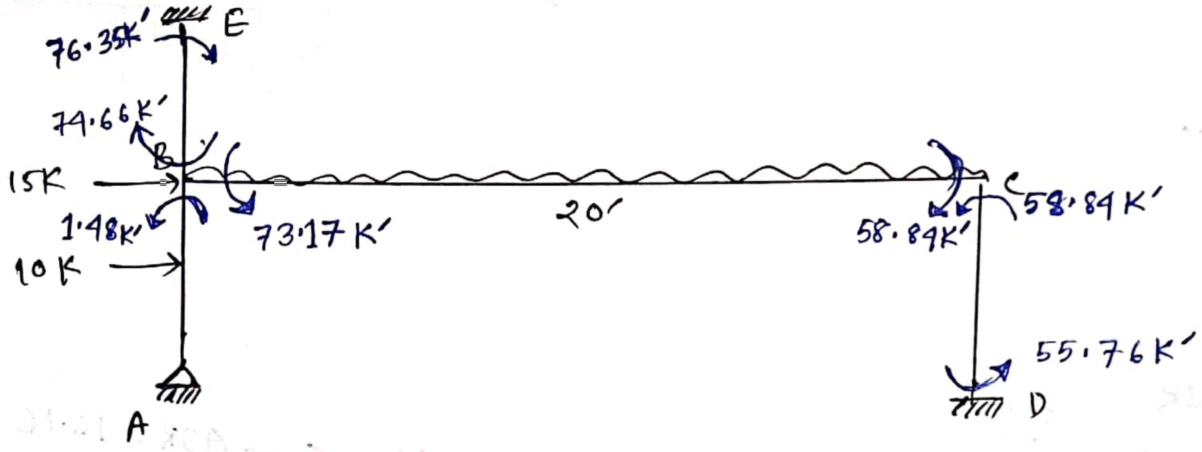
$$F_{BE} = \frac{-100 \times 15^2}{12^2} = -156.25K'$$



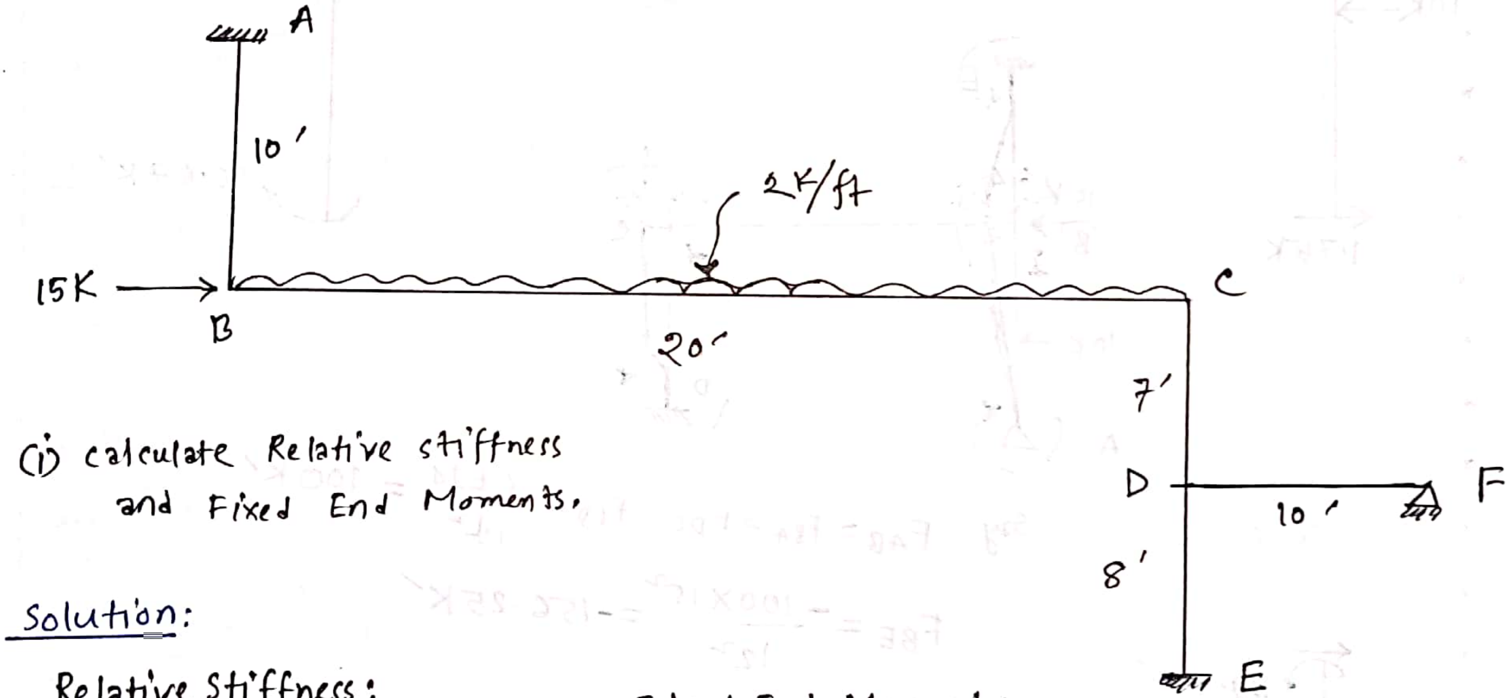




$$\therefore z = \frac{AJR}{CJF} = \frac{16.16}{31.982} = 0.505$$



Assignment-7:



(i) calculate Relative stiffness and Fixed End Moments.

Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{L} = \frac{1}{10} \simeq 2$

$K_{BC} = \frac{1}{20} \simeq 1$

$K_{CD} = \frac{1}{7} \simeq 2.86$

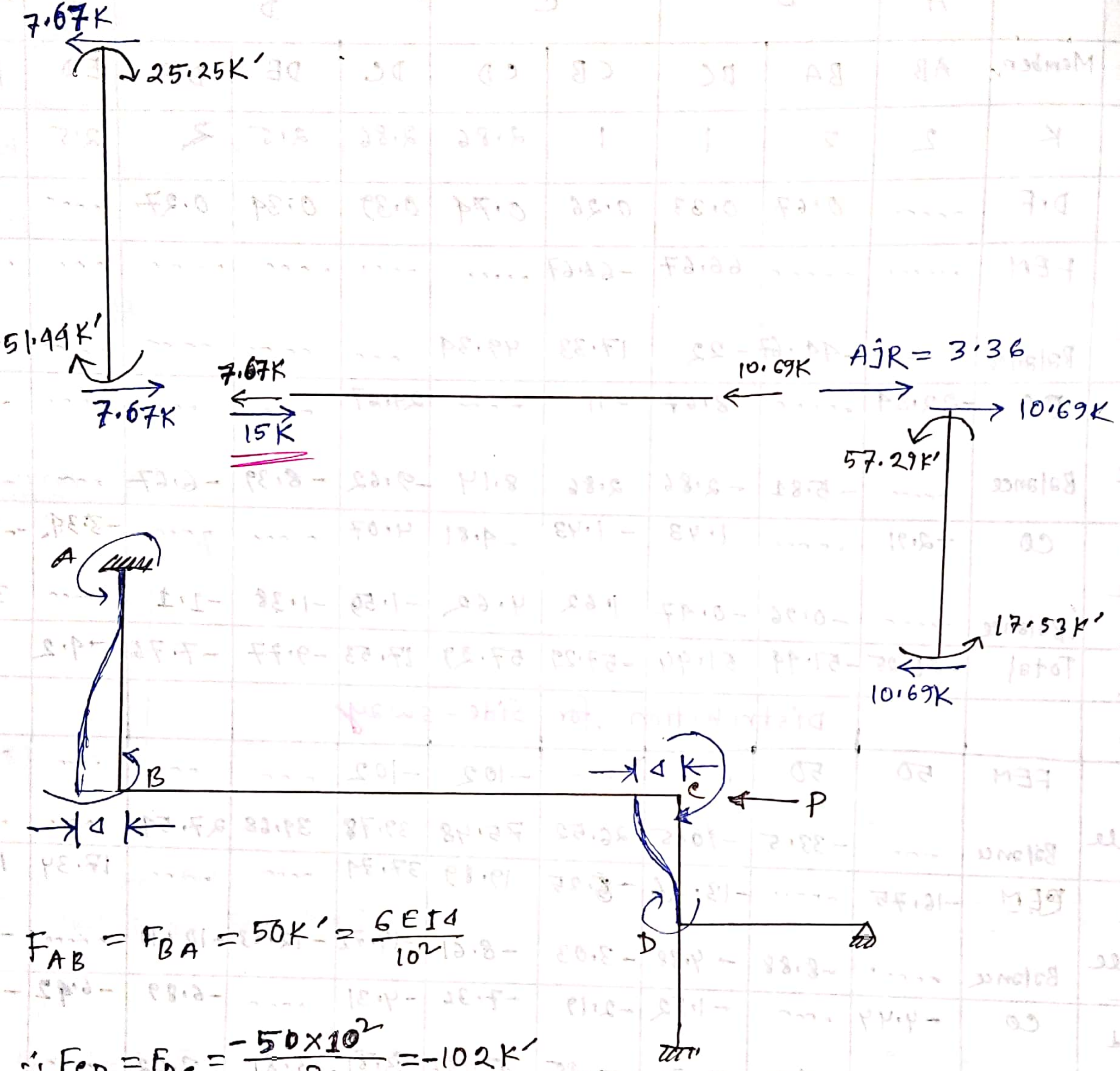
$K_{DE} = \frac{1}{8} \simeq 2.5$

$K_{DF} = \frac{1}{10} \simeq 2$

Fixed End Moment:

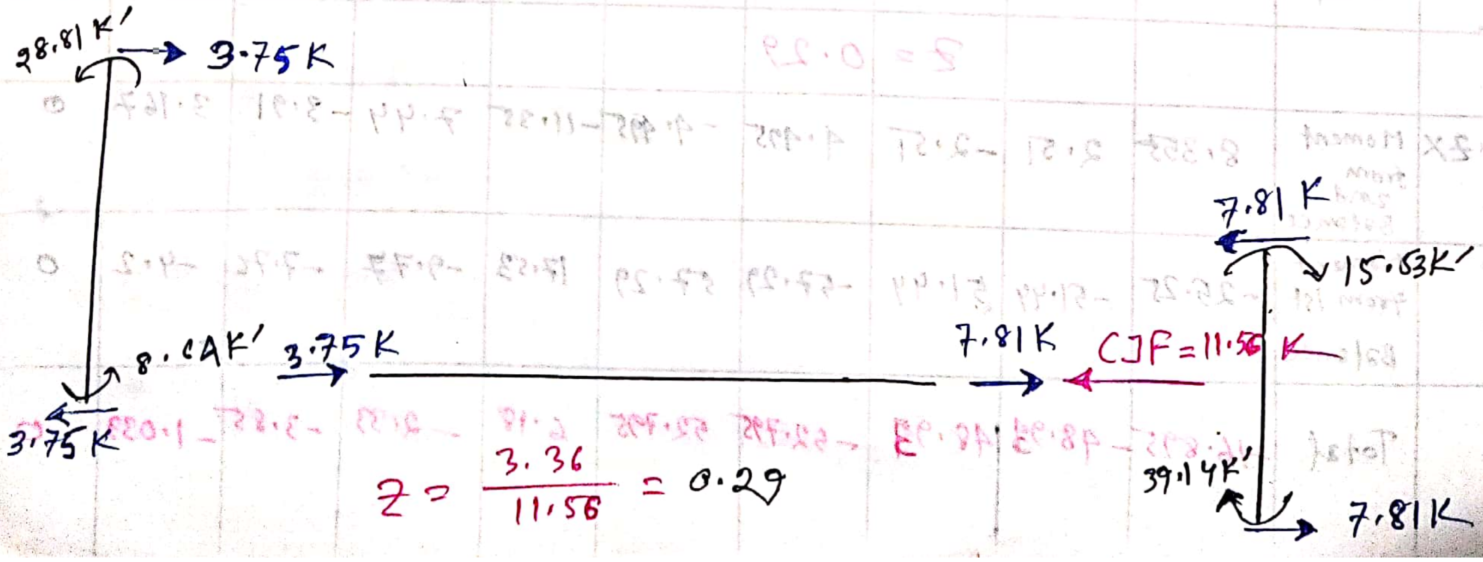
$F_{BC} = -F_{CB} = \frac{WL^2}{12} = \frac{2 \times 20^2}{12} = 66.67K'$

		A	B		C		D			E	F
	Member	AB	BA	BC	CB	CD	DC	DE	DF	ED	FD
	K	2	2	1	1	2.86	2.86	2.5	2	2.5	2
	D.F	----	0.67	0.33	0.26	0.74	0.39	0.34	0.27	----	1
1st cycle	FEM	------	-----	66.67	-66.67	-----	----	----	----	---	----
	Balance	------	-44.67	-22	17.33	49.34	---	---	---	---	---
2nd cycle	CO	-22.34	-----	8.67	-11	---	24.67	---	---	----	----
	Balance	----	-5.81	-2.86	2.86	8.14	-9.62	-8.39	-6.66	---	---
3rd cycle	CO	-2.91	-----	1.43	-1.43	-4.81	4.07	----	----	-4.2	-3.33
	Balance	-----	-0.96	-0.47	1.62	4.62	-1.59	-1.38	-1.1	-----	3.33
	Total	-25.25	-51.44	51.44	-57.29	57.29	17.53	-9.77	-7.76	-4.2	0
			Distribution for Side-sway								
1st cycle	FEM	50	50	----	---	-102	-102	---	---	----	---
	Balance	---	-33.5	-16.5	26.52	75.48	39.78	34.68	27.54	----	----
2nd cycle	CO	-16.75	-----	13.26	-8.25	19.89	37.74	----	----	17.34	13.77
	Balance	-----	-8.88	-4.38	-3.03	-8.61	-14.72	-12.83	-10.19	-----	-13.77
3rd cycle	CO	-4.44	----	-1.52	-2.19	-7.36	-4.31	----	-6.89	-6.42	-5.1
	Balance	----	1.02	0.50	2.48	7.07	4.37	3.81	3.02	---	5.1
	Total	28.81	8.64	-8.64	15.53	-15.53	-39.14	25.66	13.48	10.92	0
				Z =	0.29						
ZX	Moment from 2nd Balance	8.355	2.51	-2.51	4.5	-4.5	-11.35	7.44	3.91	3.167	0
	Moment from 1st Balance	-25.25	-51.44	51.44	-57.29	57.29	17.53	-9.77	-7.76	-4.2	0
	Total	-16.895	-48.93	48.93	-52.79	52.79	6.18	-2.33	-3.85	-1.033	0

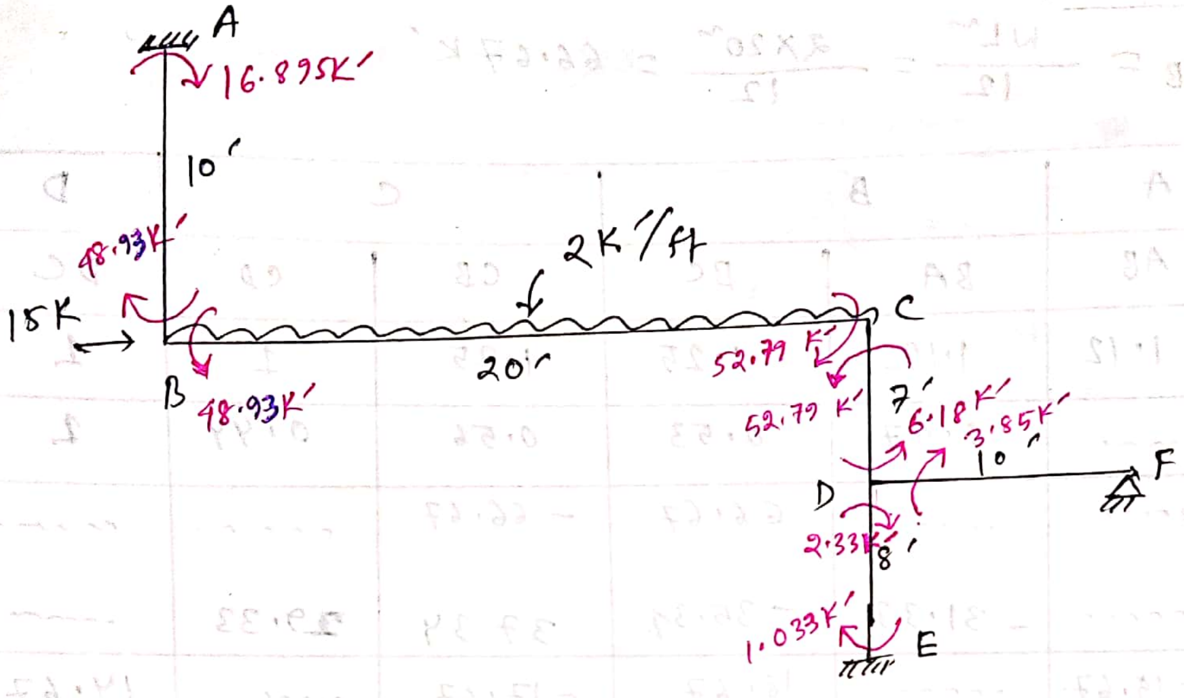


$$F_{AB} = F_{BA} = 50K' = \frac{6EI\Delta}{10^2}$$

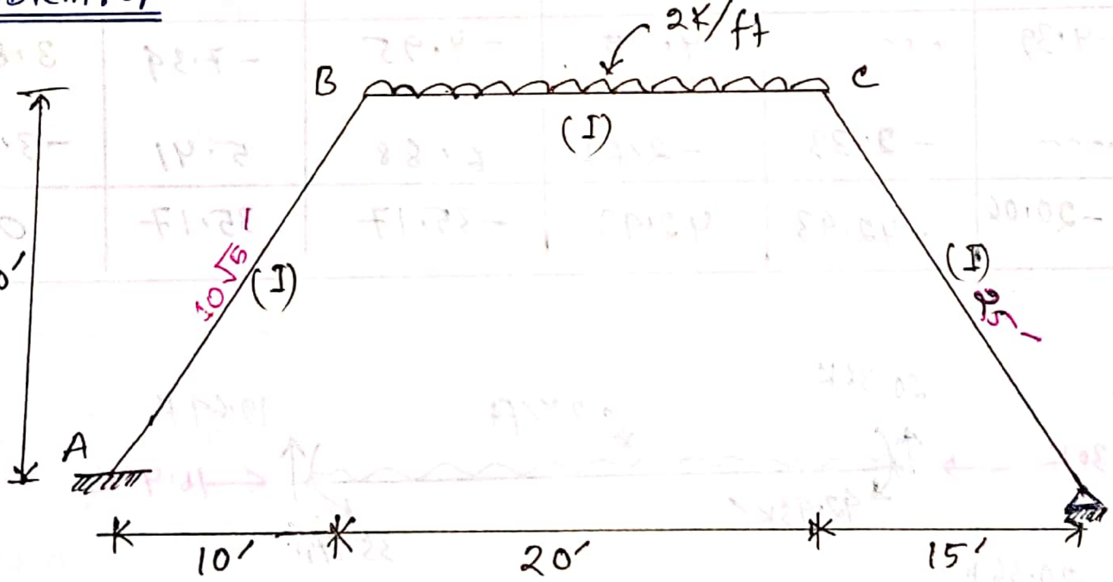
$$\therefore F_{CD} = F_{DC} = \frac{-50\times10^2}{7^2} = -102K'$$



$$Z = \frac{3.36}{11.56} = 0.29$$



Problem: 07



Calculate : (i) Relative stiffness.

(ii) Fixed End Moments.

Solution:

Relative Stiffness:

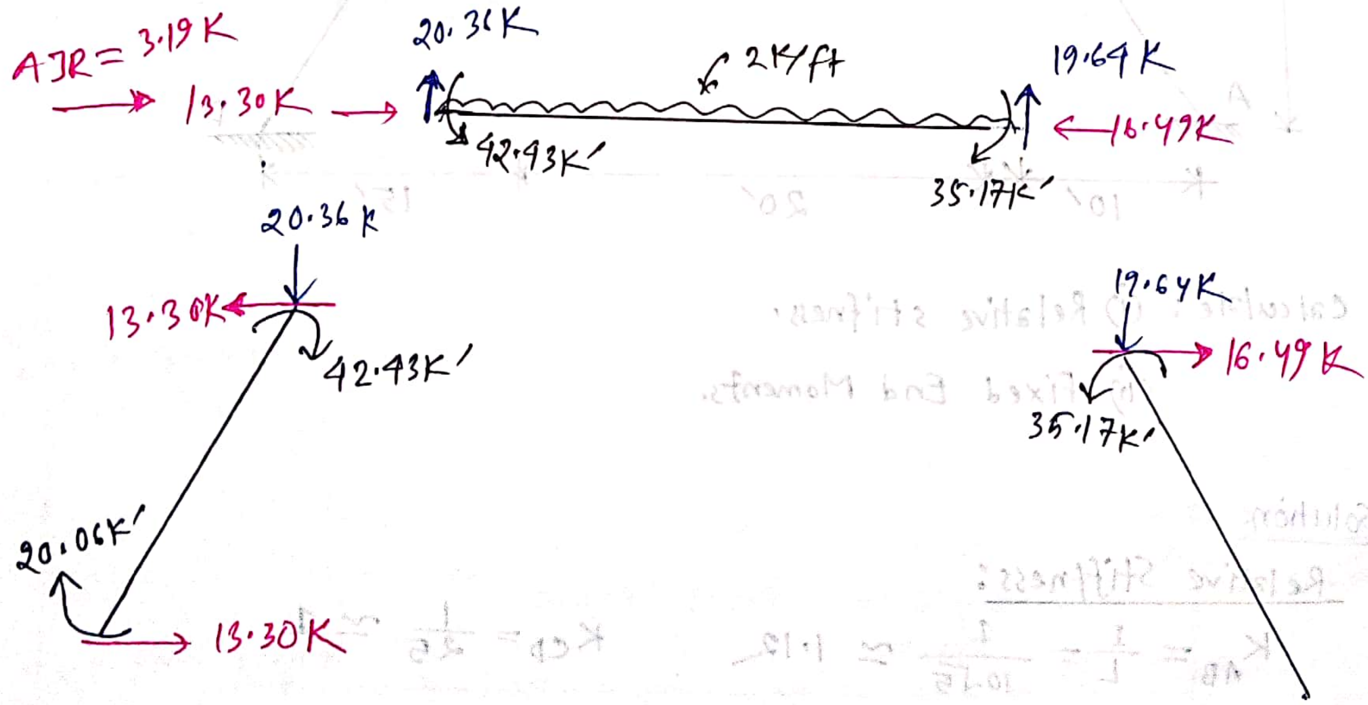
$$K_{AB} = \frac{I}{L} = \frac{1}{10\sqrt{5}} \simeq 1.12 \qquad K_{CD} = \frac{1}{25} \simeq 1$$

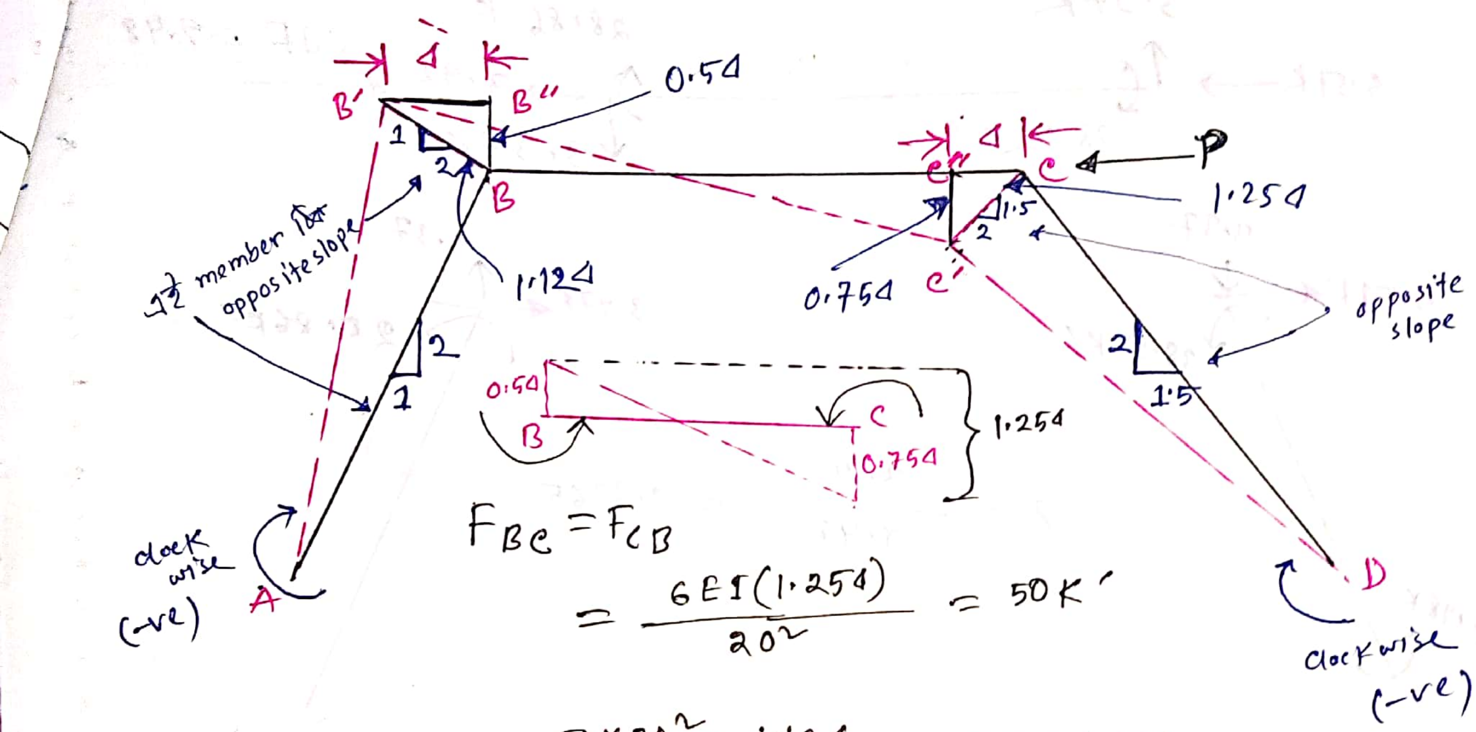
$$K_{BC} = \frac{1}{20} \simeq 1.25$$

Fixed End Moments:

$$F_{BC} = -F_{CB} = \frac{WL^2}{12} = \frac{2\times 20^2}{12} = 66.67K'$$

	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	1.12	1.12	1.25	1.25	1	1
	D.F	-----	0.47	0.53	0.56	0.44	1
1st cycle	FEM	-----	-----	66.67	-66.67	-----	-----
	Balance	-----	-31.33	-35.34	37.34	29.33	-----
2nd cycle	CO	-15.67	-----	18.67	-17.67	-----	14.67
	Balance	-----	-8.77	-9.9	9.9	7.77	-14.67
3rd cycle	CO	-4.39	-----	4.95	-4.95	-7.34	3.89
	Balance	-----	-2.33	-2.62	6.88	5.41	-3.89
	Total	-20.06	-42.43	42.43	-35.17	35.17	0





$F_{BC} = F_{CB}$

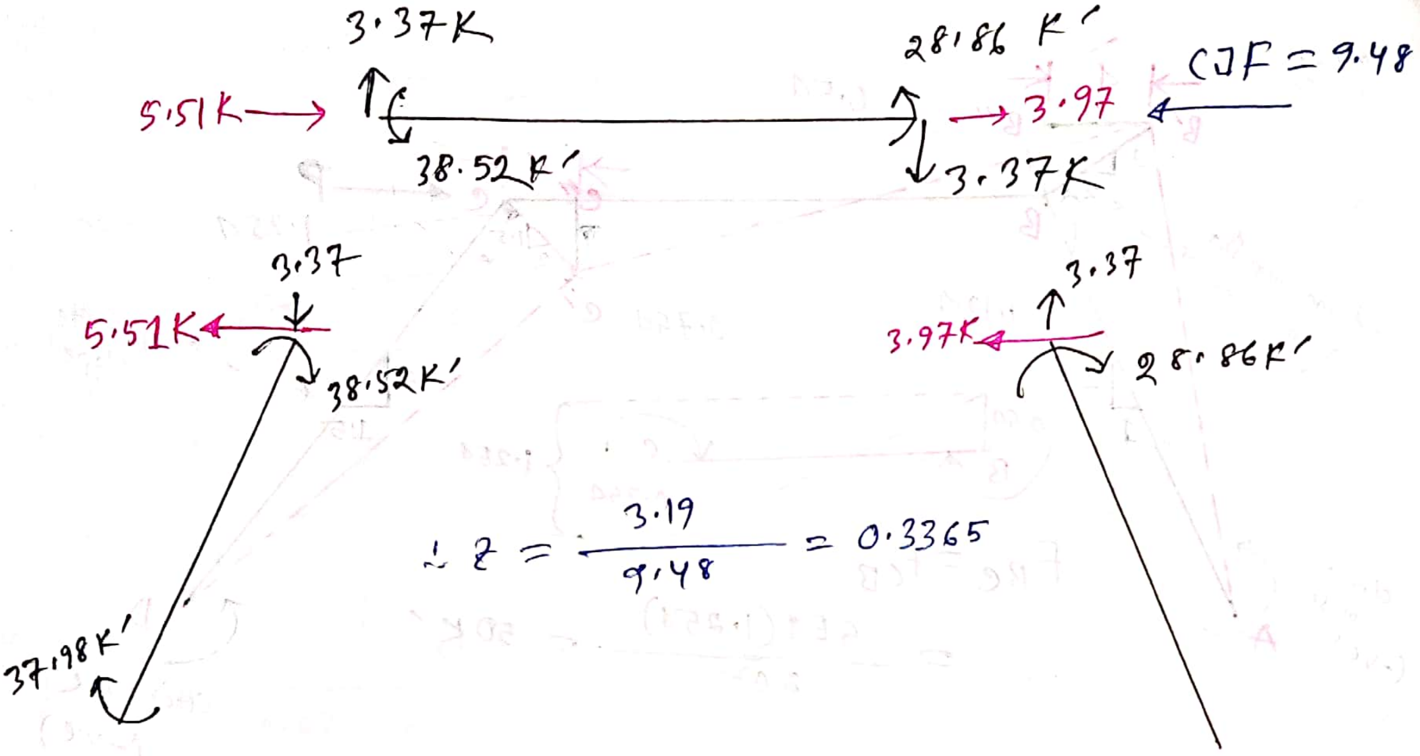
$= \frac{6EI(1.25\Delta)}{20^2} = 50K'$

$F_{AB} = F_{BA} = \frac{50 \times 20^2 \times 1.12\Delta}{1.25\Delta \times (10\sqrt{5})^2} = -35.84K'$

$F_{CD} = F_{DC} = \frac{50 \times 20^2 \times 1.25\Delta}{25^2 \times 1.25\Delta} = -32K'$

Distribution for side sway

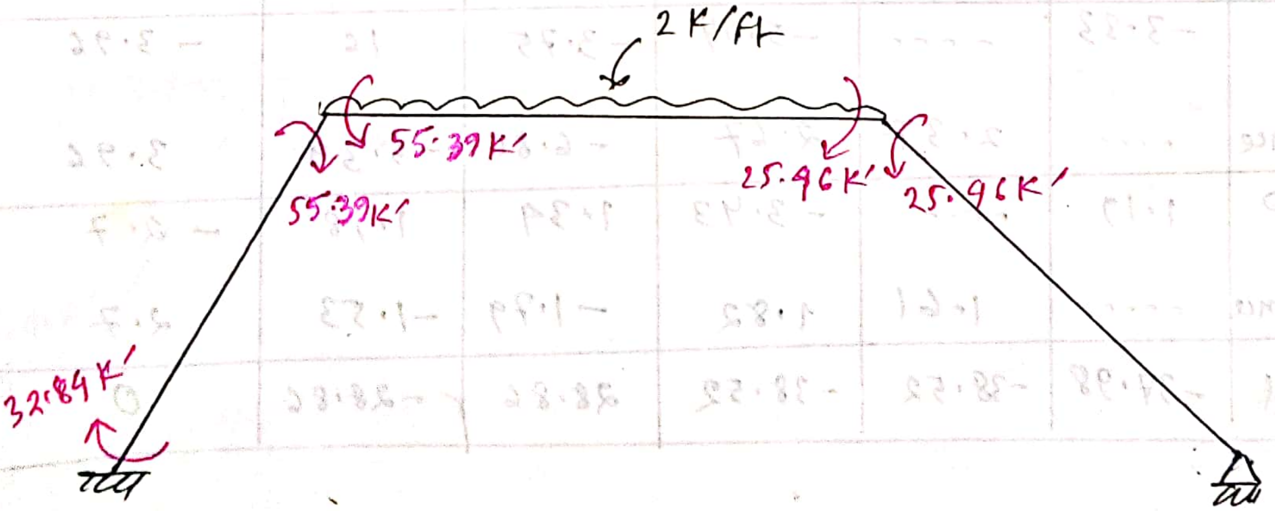
		AB	BA	BC	CB	CD	DC
1st cycle	FEM	-35.84	-35.84	50	50	-32	-32
	Balance	-----	-6.66	-7.5	-10.08	-7.92	32
2nd cycle	CO	-3.33	----	-5.04	-3.75	16	-3.96
	Balance	.....	2.37	2.67	-6.86	-5.39	3.96
3rd cycle	CO	1.19	----	-3.43	1.34	1.98	-2.7
	Balance	----	1.61	1.82	-1.79	-1.53	2.7
Total		-37.98	-38.52	38.52	28.86	-28.86	0



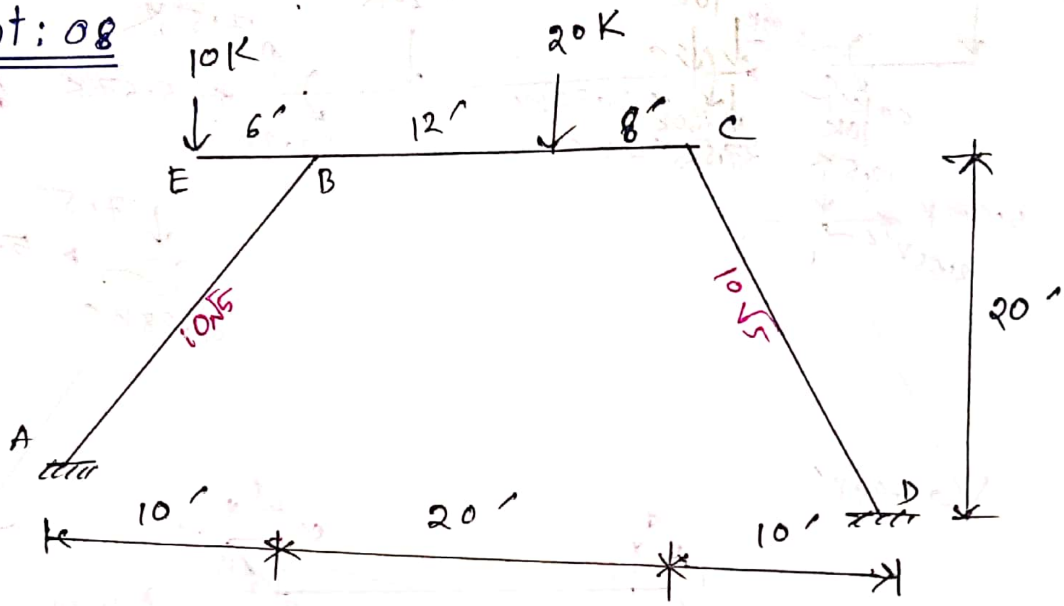
$$\therefore z = \frac{3.19}{9.48} = 0.3365$$

$z = 0.3365$

	AB	BA	BC	CB	CD	DC
Z× Moment from 2nd Balance	-12.78	-12.96	12.96	9.71	-9.71	0
Moment from 1st Balance	-20.06	-42.43	42.43	-35.17	35.17	0
Total	-32.84	-55.39	55.39	-25.46	25.46	0



Assignment: 08



Solution:

Relative stiffness:

$$K_{AB} = \frac{I}{L} = \frac{1}{10\sqrt{5}} \simeq 4.47$$

$$K_{BC} = \frac{1}{20} \simeq 5$$

$$K_{CD} = \frac{1}{10\sqrt{5}} \simeq 4.47$$

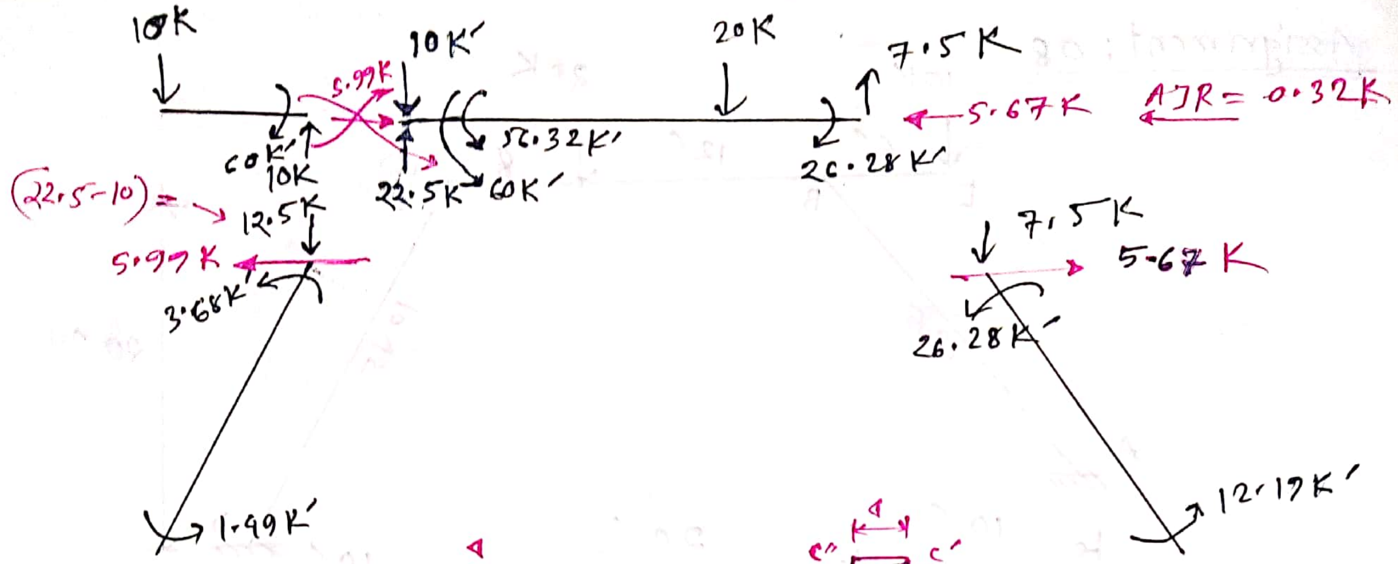
FEM:

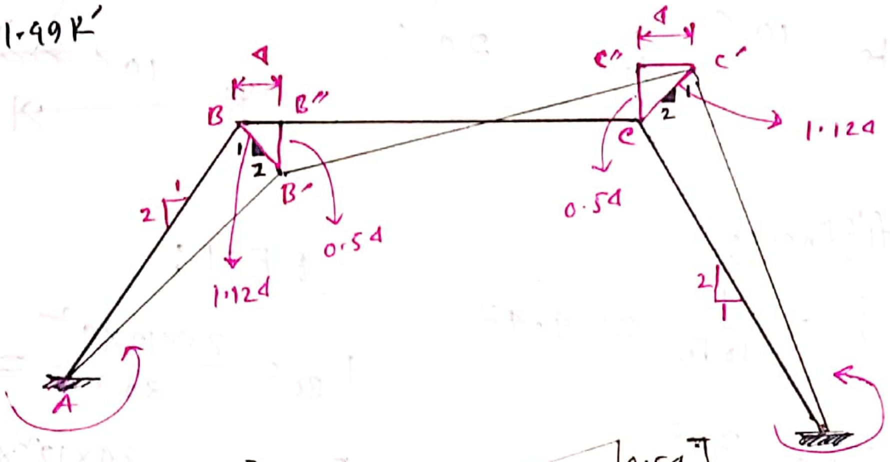
$$F_{BC} = \frac{20 \times 12 \times 8^2}{20^2} = 38.4K'$$

$$F_{CB} = -\frac{20 \times 12^2 \times 8}{20^2} = -57.6K'$$

$$F_{BE} = -10 \times 6 = -60K'$$

	Joint	A	B			C		D
		AB	BA	BE	BC	CB	CD	DC
	K	4.47	4.47	—	5	5	4.47	4.47
	D.F	—	0.47	—	0.53	0.53	0.47	—
1st cycle	FEM	—	—	-60	38.4	-57.6	—	—
	Balance	—	10.15	—	11.45	30.53	27.07	—
2nd Cycle	CO	5.08	—	—	15.27	5.72	—	13.54
	Balance	—	-7.18	—	-8.69	-3.03	-2.69	—
3rd Cycle	CO	-3.59	—	—	-1.52	-4.05	—	-1.35
	Balance	—	0.71	—	0.81	2.15	1.9	—
	Total	1.49	3.68	-60	56.32	-26.28	26.28	12.15





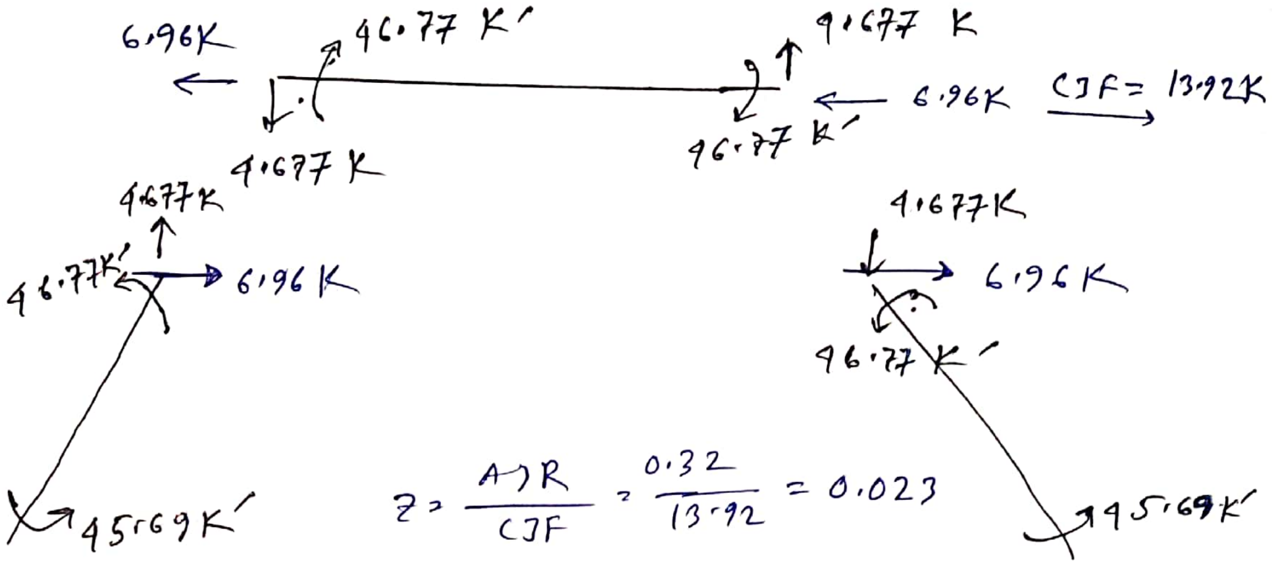
Let, $F_{BC} = F_{CB} = \frac{6EI \times 14}{20^2} = -50$ K'

$F_{AB} = F_{BA} = \frac{20^2 \times 50 \times 1.124}{(10\sqrt{5})^2} = 44.8$ K'

$F_{CD} = F_{DC} = 44.8$ K'

Distribution for side sway

Member	AB	BA	BE	BC	CB	CD	DC
FEM	44.8	44.8	—	−50	−50	44.8	44.8
Balance	—	2.44	—	2.76	2.76	2.44	—
CO	1.22	—	-	1.38	1.38	—	1.22
Balance	—	−0.65	—	−0.75	−0.75	−0.65	—
CO	−0.33	—	—	−0.38	−0.38	—	−0.33
Balance	—	0.18	—	0.20	0.20	0.18	—
Total	45.69	46.77	—	−46.77	−46.77	46.77	45.69



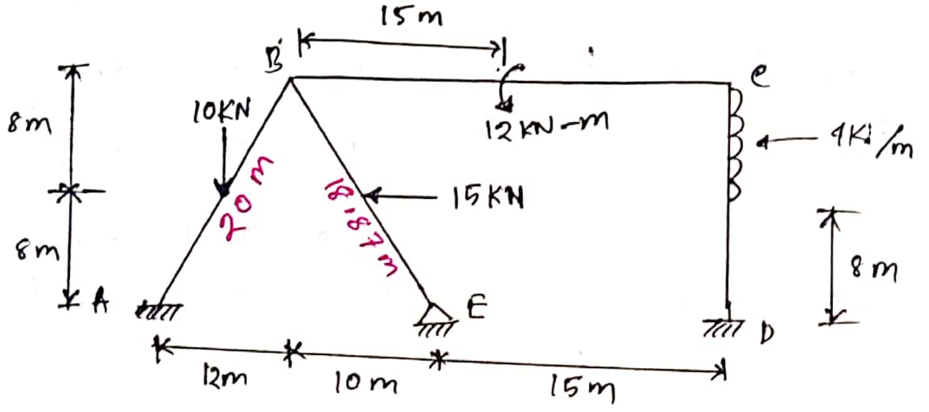
$$Z = \frac{AJR}{CJF} = \frac{0.32}{13.92} = 0.023$$

Member	AB	BA	BE	BC	CB	CD	DC
1st Balance	1.49	3.68	-60	56.32	-26.28	26.28	12.19
2X 2nd Balance	1.05	1.075	—	-1.075	-0.604	0.604	0.28
Total	2.54	4.755	-60	55.245	26.884	26.884	12.47

Moment Distribution Method

2018

#



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{L} = \frac{1}{20} \simeq 5$

$K_{BC} = \frac{1}{25} \simeq 4$

$K_{CD} = \frac{1}{16} \simeq 6.25$

$K_{BE} = \frac{1}{18.87} \simeq 5.3$

FEM:

$F_{AB} = \frac{10 \times 12}{8} = 15 KN.m$

$F_{BA} = -15 KNm$

$F_{BE} = \frac{15 \times 16}{8} = 30 KN-m$

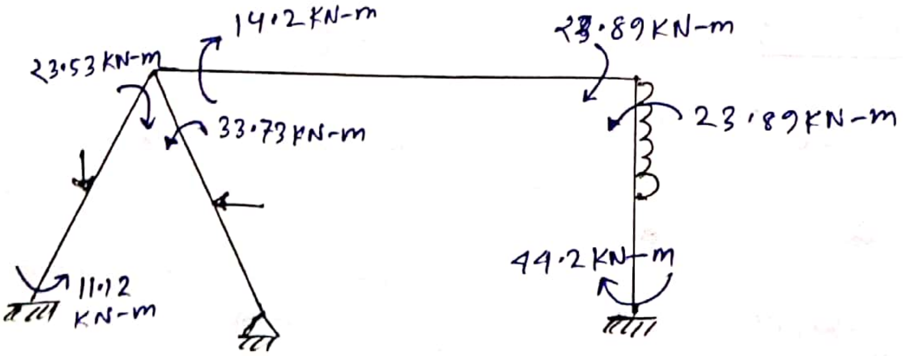
$F_{EB} = -30 KN m$

$F_{BC} = \frac{M b(2a-b)}{L^2} = \frac{12 \times 10 \times (2 \times 15 - 10)}{25^2} = 3.84 KN-m$

$F_{CB} = \frac{M a(2b-a)}{L^2} = \frac{12 \times 15 \times (2 \times 10 - 15)}{25^2} = 1.44 KN-m$

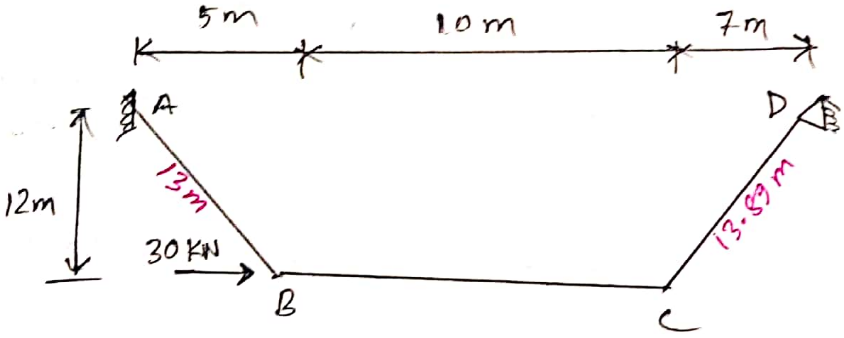
$F_{CD} = \frac{11}{192} wL^2 = \frac{11}{192} \times 4 \times 16^2 = 58.67 KN-m$

$F_{DC} = \frac{-5}{192} wL^2 = -\frac{5}{192} \times 4 \times 16^2 = -26.67 KN-m$



	Joint	A	B			C		D	E
	Member	AB	BA	BE	BC	CB	CD	DC	EB
	K	5	5	5.3	4	1	6.25	6.25	5.3
	D.F	—	0.35	0.37	0.28	0.39	0.61	—	1
1st cycle	FEM	15	−15	30	3.84	1.44	58.67	−26.67	−30
	Balance	—	−6.59	−6.97	−5.98	−23.44	−36.67	—	30
2nd cycle	CO	−3.30	—	15	−11.72	−2.64	—	−18.34	−3.49
	Balance	—	−1.15	−1.21	−0.92	1.03	1.61	—	3.49
3rd cycle	CO	−0.58	—	1.75	0.52	−0.46	—	0.81	−0.61
	Balance	—	−0.79	−0.84	−0.64	0.18	0.28	—	0.61
	Total	11.12	−23.53	37.73	−14.2	−23.89	23.89	−44.2	0

2017
#



Solution:

Relative Stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{13} \simeq 7.69$

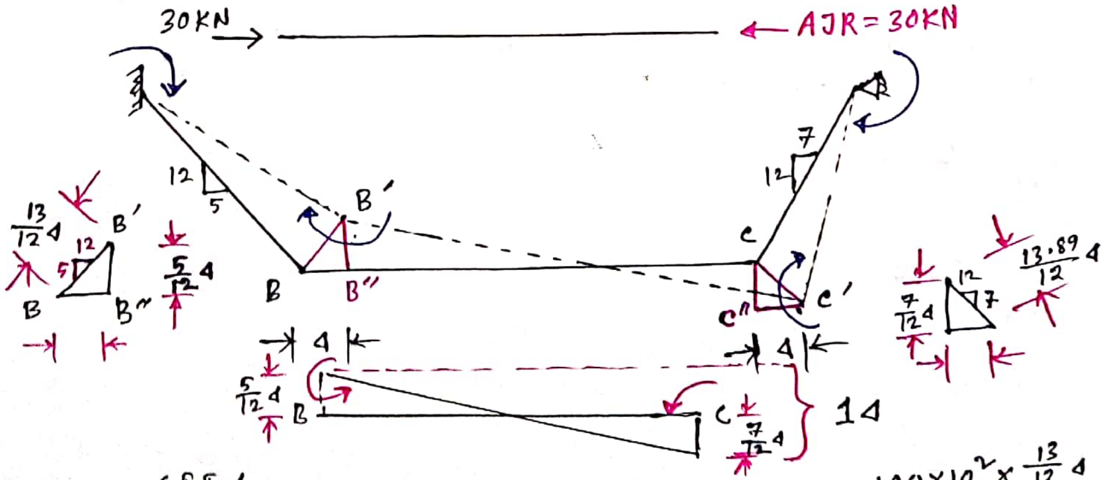
$K_{BC} = \frac{1}{10} \simeq 10$

$K_{CD} = \frac{1}{13.89} \simeq 7.2$

FEM:

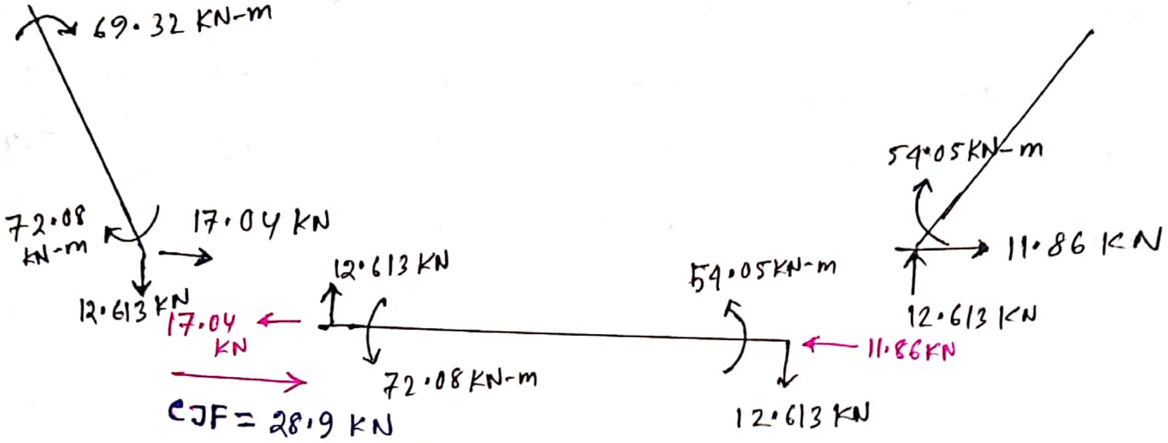
All are zero

	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	7.69	7.69	10	10	7.2	7.2
	D.F	—	0.43	0.57	0.58	0.42	1
1st cycle	FEM	—	—	—	—	—	—
	Balance	—	—	—	—	—	—
	Total	0	0	0	0	0	0
				Distribution for side sway			
1st cycle	FEM	-64.1	-64.1	100	100	-60	-60
	Balance	—	-15.44	-20.46	-23.2	-16.8	60
2nd cycle	CO	-7.72	—	-11.6	-10.23	30	-8.4
	Balance	—	4.99	6.61	-11.47	-8.3	8.4
3rd cycle	CO	2.5	—	5.74	3.31	4.2	-4.15
	Balance	—	2.47	3.27	-4.36	-3.15	4.15
	Total	-69.32	-72.08	72.08	54.05	-54.05	0
				Z = 1.04			
2 × 2nd Balance		-72.09	-74.96	74.96	56.21	-56.21	0
1st Balance		—	—	—	—	—	—
Total		-72.09	-74.96	74.96	56.21	-56.21	0



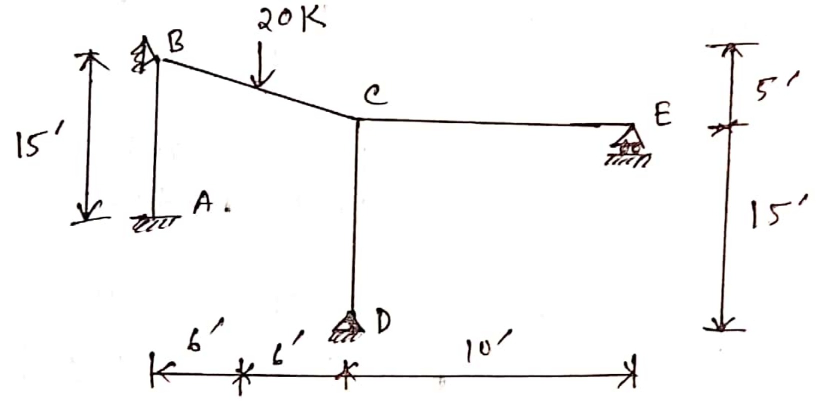
$$F_{BC} = F_{CB} = \frac{6EI\Delta}{10^2} = 100 \text{ KN-m} \quad ; \quad F_{AB} = F_{BA} = \frac{-100\times10^2 \times \frac{13}{12}\Delta}{4\times13^2} \simeq -64.1 \text{ KN-m}$$

$$\text{and, } F_{CD} = F_{DC} = \frac{-100\times10^2 \times \frac{13.89}{12}\Delta}{4\times13.89^2} = -60 \text{ KN-m}$$



$$\boxed{z = \frac{30}{28.9} = 1.04}$$

2016
#



Solution:

Relati stiffness:

$K_{AB} = \frac{1}{15} \simeq 2$

$K_{BC} = \frac{1}{13} \simeq 2.31$

$K_{CD} = \frac{1}{15} \simeq 2$

$K_{CE} = \frac{1}{10} \simeq 3$

FEM:

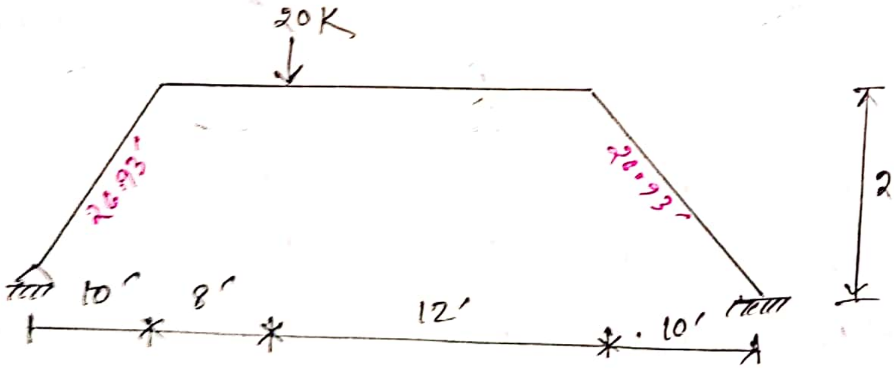
$F_{BC} = \frac{PL}{8} = \frac{20 \times 12}{8} = 30 KN$

$F_{CB} = -30 KN$

	Joint	A	B		C			D	E
	Member	AB	BA	BC	CB	CE	CD	DC	EC
	K	2	2	2.31	2.31	3	2	2	3
	D.F	—	0.46	0.54	0.32	0.41	0.27	1	1
1st	FEM	—	—	30	-30	—	—	—	—
Cycle	Balance	—	-13.8	-16.2	9.6	12.3	8.1	—	—
2nd	CO	-6.9	—	4.8	-8.1	—	—	4.05	6.15
Cycle	Balance	—	-2.21	-2.59	2.59	3.32	2.19	-4.05	-6.15
3rd	CO	-1.1	—	1.3	-1.3	—	-2.03	1.1	1.66
Cycle	Balance	—	-0.6	-0.7	1.07	1.37	0.89	-1.1	-1.66
	Total	-8	-16.61	16.61	-26.14	16.99	9.15	0	0

2015

#



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{26.93} \simeq 3.7$

$K_{BC} = \frac{1}{20} \simeq 5$

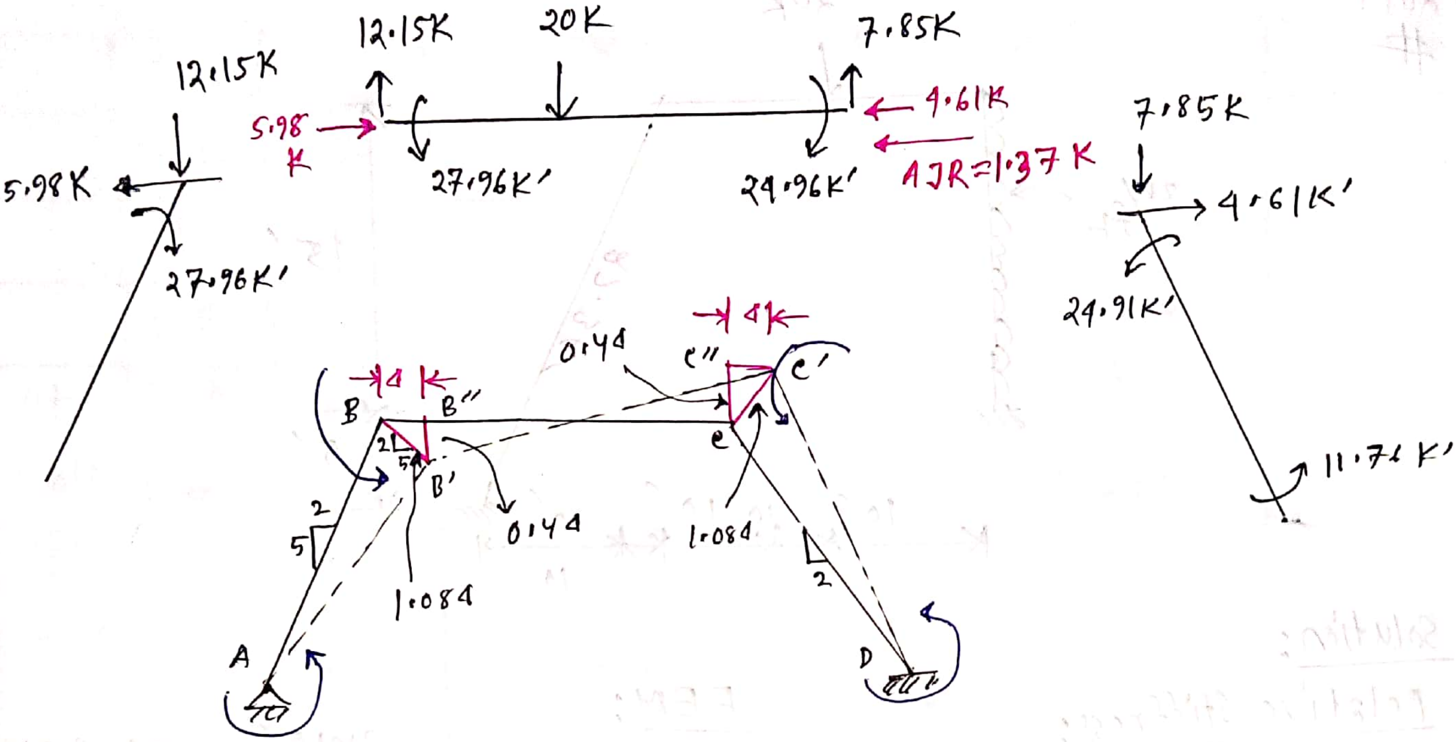
$K_{CD} = \frac{1}{26.93} \simeq 3.7$

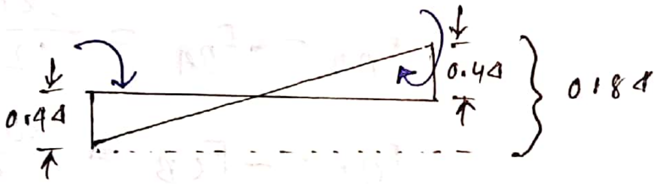
FEM:

$F_{BC} = \frac{20 \times 8 \times 12^2}{20^2} = 57.6K'$

$F_{CB} = \frac{-20 \times 8^2 \times 12}{20^2} = -38.4K'$

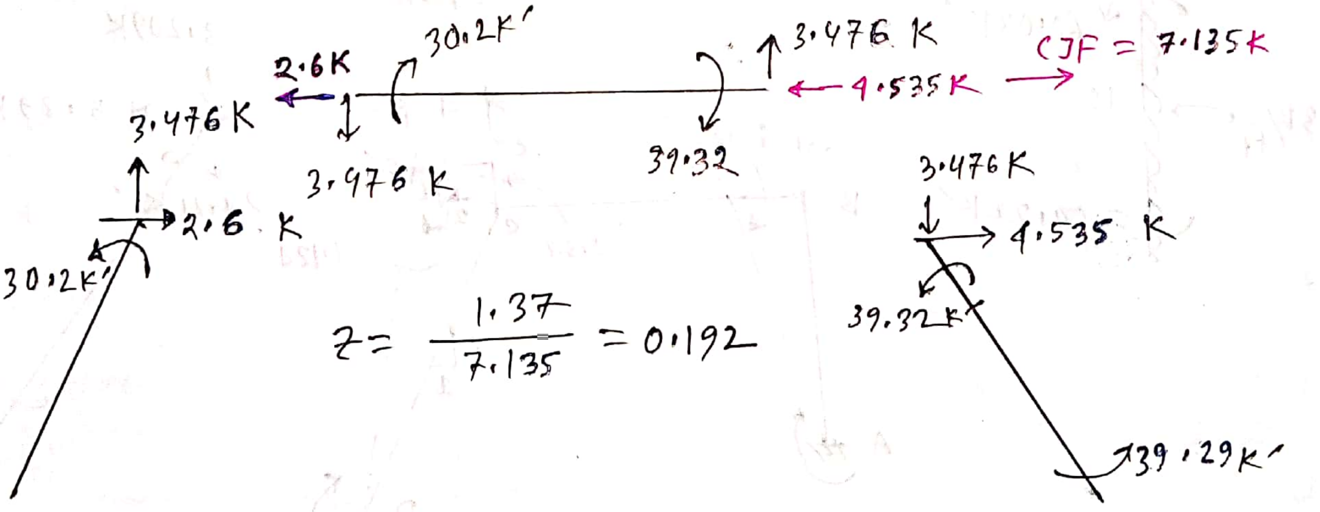
	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	3.7	3.7	5	5	3.7	3.7
	D.F	1	0.43	0.57	0.57	0.43	—
1st cycle	FEM	—	—	57.6	-38.4	—	—
	Balance	—	-29.77	-32.83	21.89	16.57	—
2nd cycle	CO	-12.39	—	10.95	-16.42	—	8.26
	Balance	12.39	-4.71	-6.24	9.36	7.06	—
3rd cycle	CO	-2.36	6.2	4.68	-3.12	—	3.53
	Balance	2.36	-4.68	-6.2	1.78	1.34	—
	Total	0	-27.96	27.96	-24.91	24.91	11.76
Distribution for side sway							
1st cycle	FEM	37.23	37.23	-50	-50	37.23	37.23
	Balance	-37.23	5.49	7.28	7.28	5.49	—
2nd cycle	CO	2.75	-18.62	3.64	3.64	—	2.75
	Balance	-2.75	6.44	8.54	-2.07	-1.57	—
3rd cycle	CO	3.22	-1.38	-1.04	4.27	—	-0.79
	Balance	-3.22	1.04	1.38	-2.44	-1.83	—
	Total	0	30.2	-30.2	-39.32	39.32	39.29
Z = 0.192							
Z × 2nd Balance		0	5.8	-5.8	-7.55	7.55	7.544
1st Balance		0	-27.96	27.96	-24.91	24.91	11.76
Total		0	-22.16	22.16	-32.46	32.46	19.304





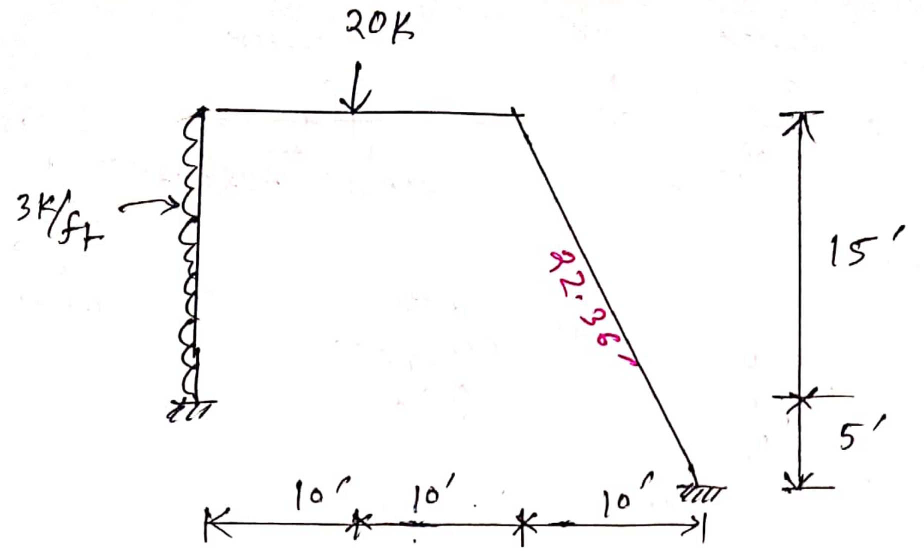
$$F_{BC} = F_{CB} = \frac{6EI \times 0.84}{20^2} = -50\ K'$$

$$\therefore F_{AB} = F_{BA} = F_{CD} = F_{DC} = \frac{50 \times 20^2 \times 1.084}{0.84 \times 26.93^2} = 37.23\ K'$$



$$z = \frac{1.37}{7.135} = 0.192$$

2014
#



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{15} \simeq 2$

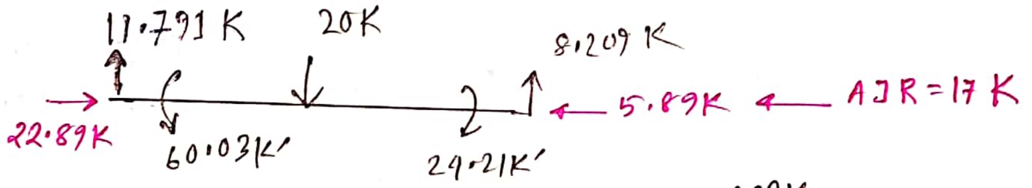
$K_{BC} = \frac{1}{20} \simeq 1.5$

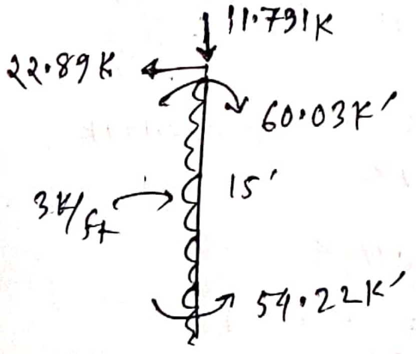
$K_{CD} = \frac{1}{22.36} \simeq 1.34$

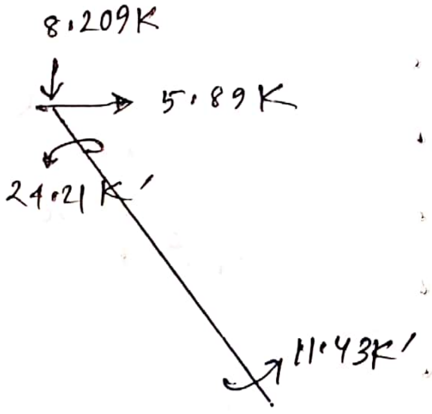
FEM:

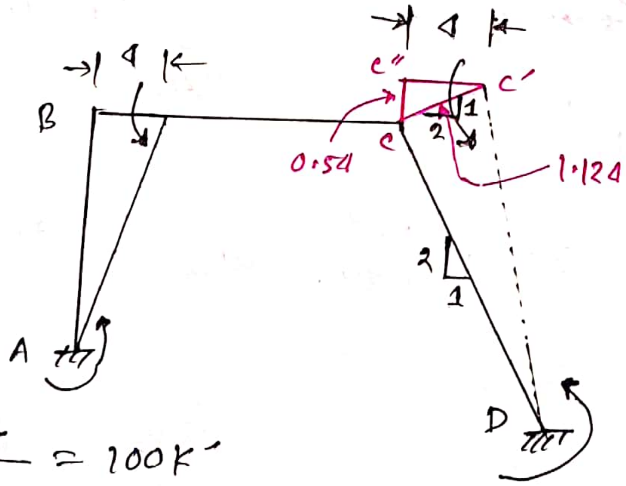
$F_{AB} = -F_{BA} = \frac{3 \times 15^2}{12} = 56.25K'$

$F_{BC} = -F_{CB} = \frac{20 \times 20}{8} = 50K'$



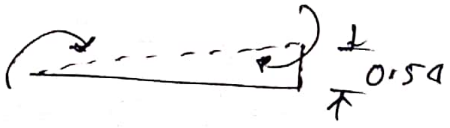






$F_{AB} = F_{BA} = \frac{6EI\Delta}{15^2} = 100K'$

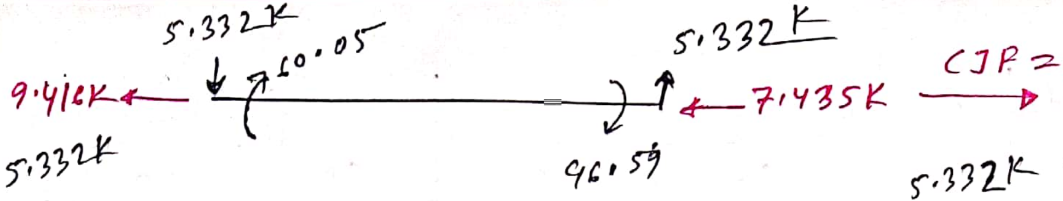
$F_{CD} = F_{DC} = \frac{100 \times 15^2 \times 1.124}{4 \times 22.36^2} = 50.4K'$



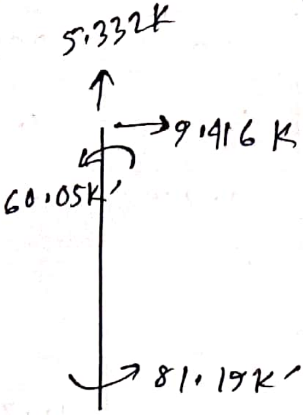
$F_{BC} = F_{CB} = \frac{-100 \times 15^2 \times 0.54}{4 \times 20^2} = -28.13K'$

Scanned by CamScanner

	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	2	2	1.5	1.5	1.34	1.34
	D.F	—	0.57	0.43	0.53	0.47	—
1st	FEM	56.25	-56.25	50	-50	—	—
cycle	Balance	—	3.56	2.69	26.5	23.5	—
2nd	CO	1.78	—	13.25	1.35	—	11.75
cycle	Balance	—	-7.55	-5.7	-0.72	-0.63	—
3rd	CO	-3.78	—	-0.36	-2.85	—	-0.32
cycle	Balance	—	0.21	0.15	1.51	1.34	
	Total	54.22	-60.03	60.03	-24.21	24.21	11.43
				Distribution for Side sway			
	FEM	100	100	-28.13	-28.13	50.4	50.4
	Balance	—	-40.97	-30.90	-11.8	-10.47	—
	CO	-20.49	—	-5.19	-15.45	—	-5.24
	Balance	—	3.36	2.54	8.19	7.26	—
	CO	1.68	—	4.1	1.27	—	3.63
	Balance	—	-2.34	-1.76	-0.67	-0.6	—
	Total	81.19	60.05	-60.05	-46.59	46.59	48.79
				Z = 1.009			
Z x 2nd	Balance	81.92	60.59	-60.59	-47.01	47.01	49.23
1st	Balance	54.22	-60.03	60.03	-24.21	24.21	11.43
Total		136.14	0.56	-0.56	-71.22	71.22	60.66

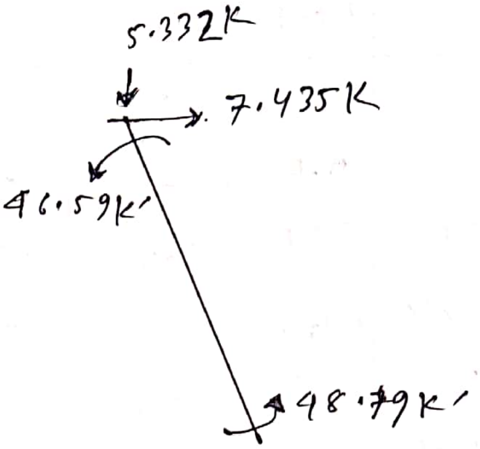


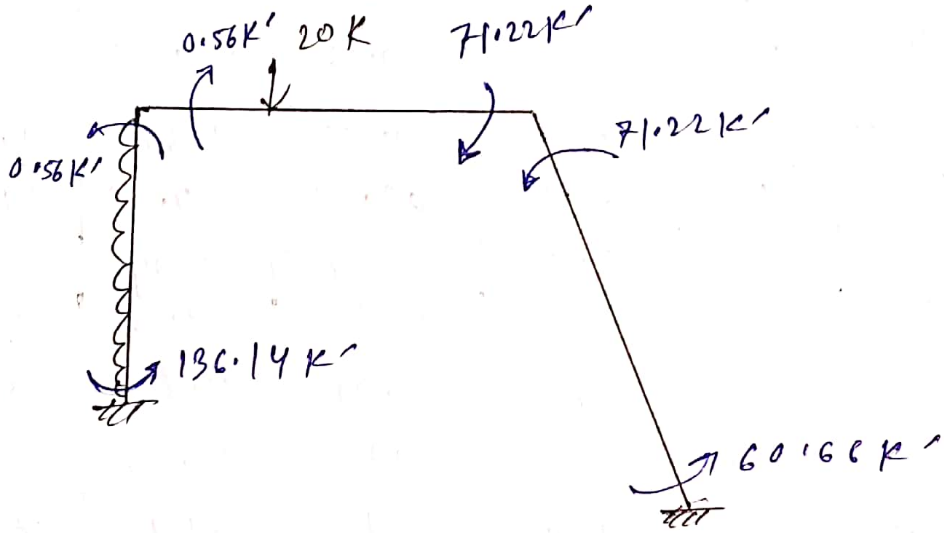
CJF = 16.851



$$z = \frac{AJR}{CJF} = \frac{17}{16.851}$$

$$\therefore z = 1.009$$





2013

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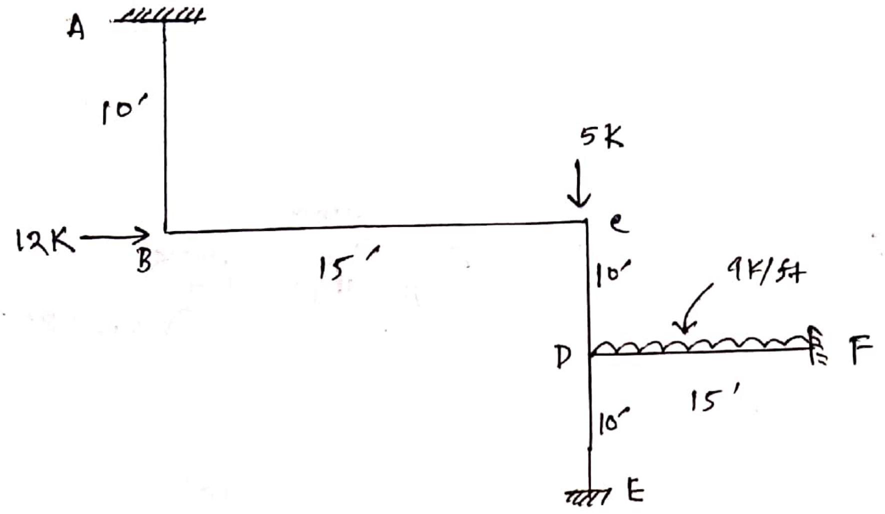


Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{10} \simeq 1.5$

$K_{BC} = \frac{1}{15} \simeq 1.0$

$K_{CD} = \frac{1}{10} \simeq 1.5$

$K_{DE} = \frac{1}{10} \simeq 1.5$

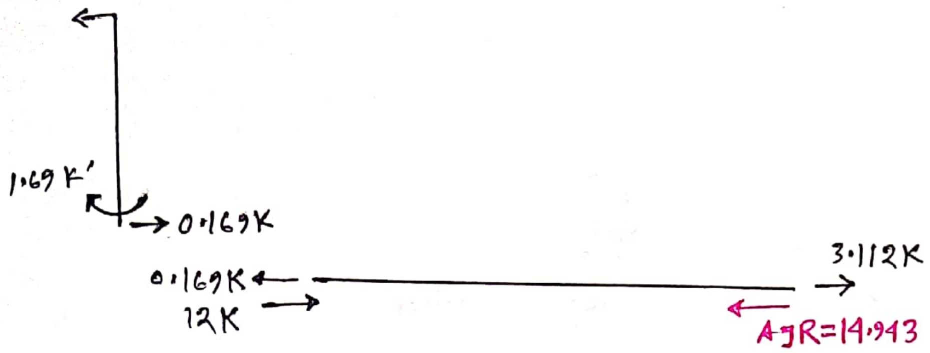
$K_{DF} = \frac{1}{15} \simeq 1.0$

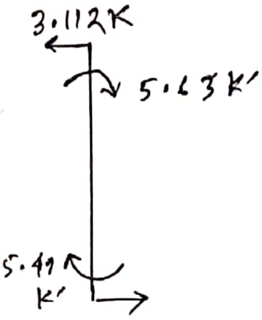
Fixed End Moment:

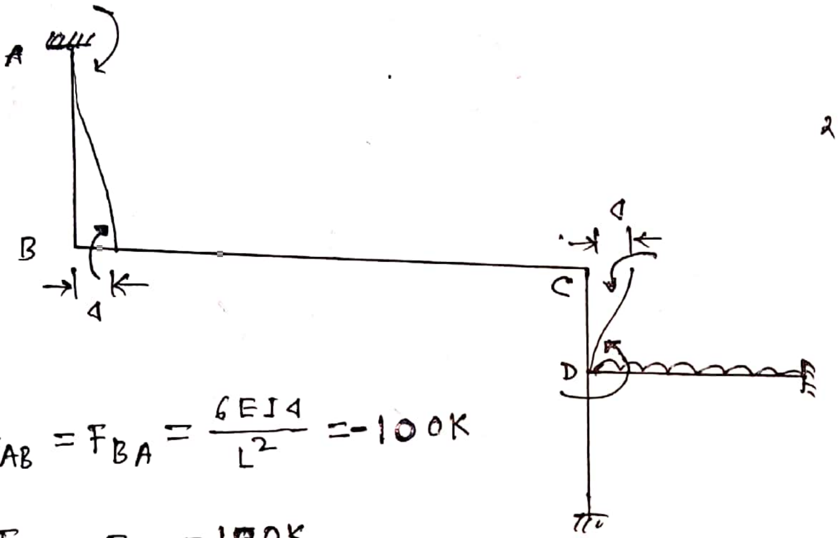
$F_{DF} = \frac{4 \times 15^2}{12} = 75\,K'$

$F_{FD} = -75\,K'$

	Joint	A	B		C		D			E	F
	Member	AB	BA	BC	CB	CD	DC	DF	DE	ED	FD
	K	1.5	1.5	1	1	1.5	1.5	1	1.5	1.5	1
	D.F	–	0.6	0.4	0.4	0.6	0.375	0.25	0.375	–	–
1st cycle	FEM	–	–	–	–	–	–	75	–	–	-75
	Balance	–	–	–	–	–	-28.13	-18.75	-28.12	–	–
2nd Cycle	CO	–	–	–	–	-14.07	–	–	–	-14.06	-9.38
	Balance	–	–	–	5.63	8.44	–	–	–	–	–
3rd cycle	CO	–	–	2.82	–	–	9.22	–	–	–	–
	Balance	–	-1.69	-1.13	–	–	-1.58	-1.06	-1.58	–	–
	Total	–	-1.69	1.69	5.63	-5.63	-25.49	55.19	-29.7	-14.06	-84.38



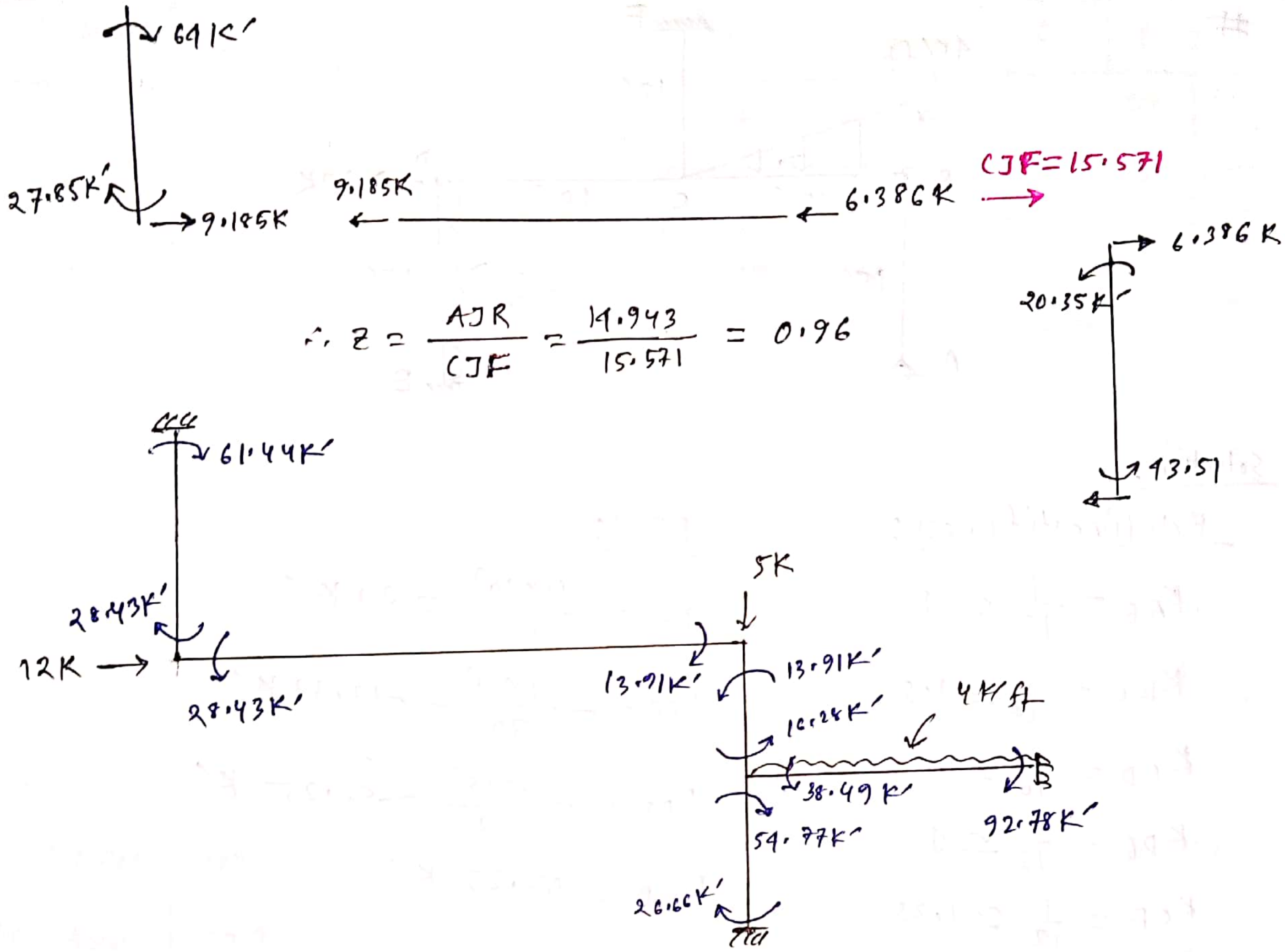




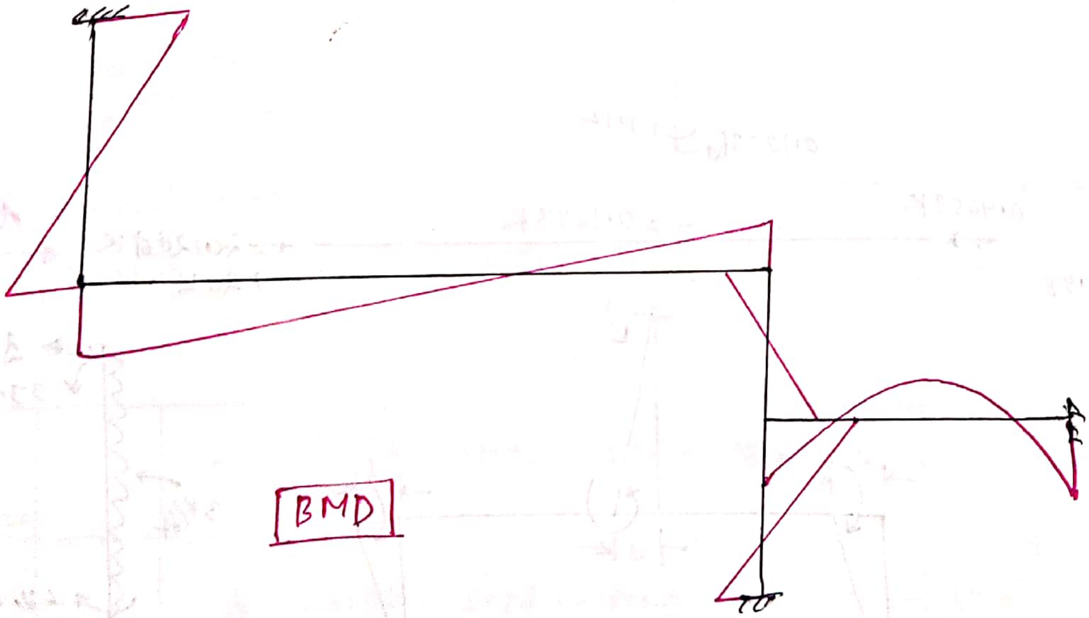
$$F_{AB} = F_{BA} = \frac{6EIΔ}{L^2} = -100K$$

$$F_{CD} = F_{DC} = 100K$$

	Distribution for sidesway								
Member	AB	BA	BC	CB	CD	DC	DF	DE	ED	FD
D.F	–	0.6	0.4	0.4	0.6	0.375	0.25	0.375	–	–
FEM	-100	-100	–	–	100	100	–	–	–	–
Balance	–	60	40	-40	-60	-37.5	-25	-37.5	–	–
CO	30	–	-20	20	-18.75	-30	–	–	-18.75	-12.5
Balance	–	12	8	-0.5	-0.75	11.25	7.5	11.25	–	–
Co	6	–	-0.25	4	5.63	-0.38	–	–	5.63	3.75
Balance	–	0.15	0.10	-3.85	-5.78	0.14	0.1	0.14	–	–
Total	-64	-27.85	27.85	-20.35	20.35	43.51	-17.4	-26.11	-13.12	-8.75
				Z = 0.96						
2nd balance × Z	-61.44	-26.74	26.74	-19.54	19.54	41.77	-16.7	-25.07	-12.6	-8.4
1st Balance	–	-1.69	1.69	5.63	-5.63	-25.49	55.19	-29.7	-14.06	-84.38
Total	-61.44	-28.43	28.43	-13.91	13.91	16.28	38.49	-54.77	-26.66	-92.78



$$\therefore z = \frac{AJR}{CJF} = \frac{14.943}{15.571} = 0.96$$



2012

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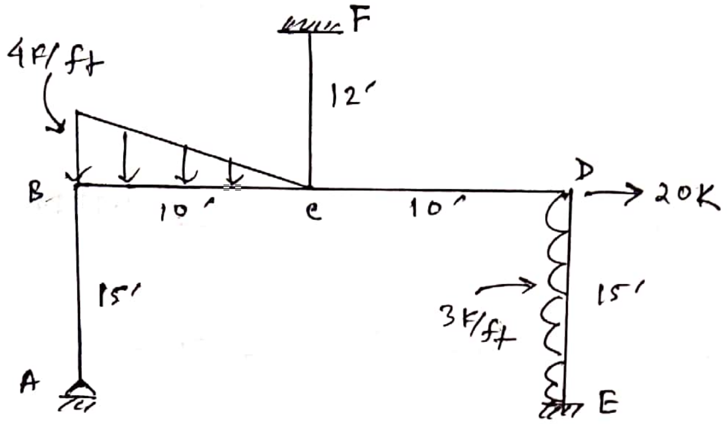


Solution:

Relative stiffness:

$K_{AB} = \frac{1}{15} \simeq 1$

$K_{BC} = \frac{1}{10} \simeq 1.5$

$K_{CD} = \frac{1}{10} \simeq 1.5$

$K_{DE} = \frac{1}{15} \simeq 1$

$K_{CF} = \frac{1}{12} \simeq 1.25$

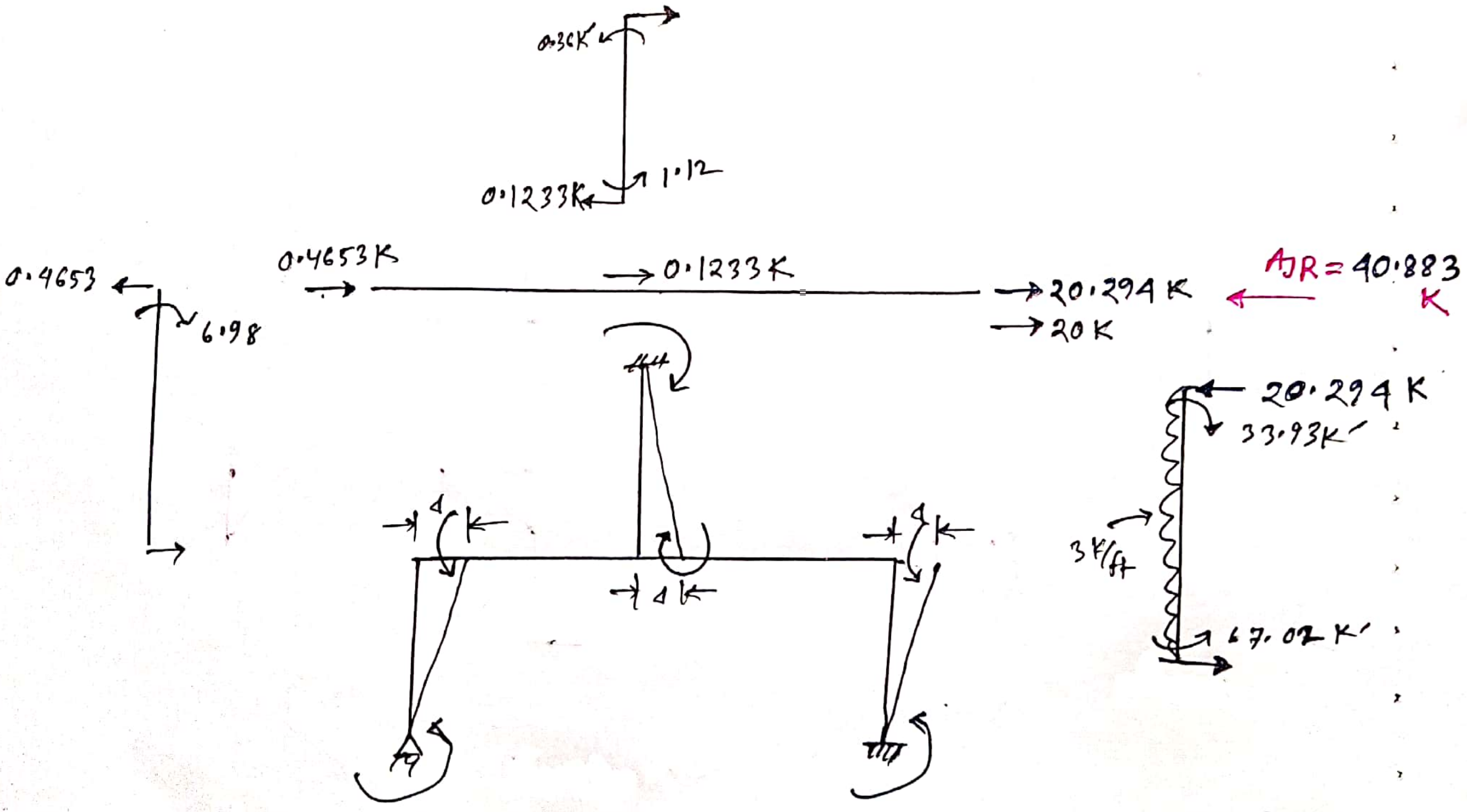
FEM:

$F_{BC} = \frac{4 \times 10^2}{20} = 20 K'$

$F_{CB} = -\frac{4 \times 10^2}{30} = -13.33 K'$

$F_{DE} = -\frac{3 \times 15^2}{12} = -56.25 K'$

$F_{ED} = 56.25 K'$



$F_{AB} = F_{BA} = F_{DE} = F_{ED} = \frac{6EI\Delta}{L^2} = 100 K'$ $\therefore F_{CF} = F_{FC} = \frac{-100 \times 15^2 \times \Delta}{\Delta \times 12^2}$

$= -156.25 K'$

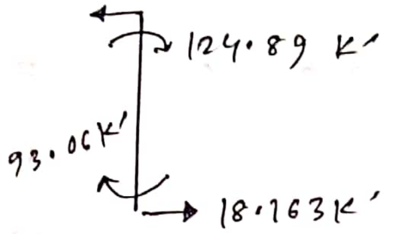
	Joint	A	B		C			D		E	F
	Member	AB	BA	BC	CB	CF	CD	DC	DE	ED	FC
	K	1	1	1.5	1.5	1.25	1.5	1.5	1	1	1.25
	D.F	1	0.4	0.6	0.35	0.29	0.36	0.6	0.4	—	—
1st cycle	FEM	—	—	20	-13.33	—	—	—	-56.25	56.25	—
	Balance	-	-8	-12	4.67	3.87	4.79	33.75	22.5	—	—
2nd cycle	CO	-4	—	2.34	-6	—	16.88	2.4	—	11.25	1.94
	Balance	4	-0.94	-1.4	-3.81	-3.16	-3.91	-1.44	-0.96	—	—
3rd cycle	CO	-0.47	2	-1.91	-0.17	—	-0.72	-1.96	—	-0.48	-1.58
	Balance	0.47	-0.04	-0.05	0.5	0.41	0.51	1.18	0.78	—	—
	Total	0	-6.98	6.98	-18.67	1.12	17.55	33.93	-33.93	67.02	0.36

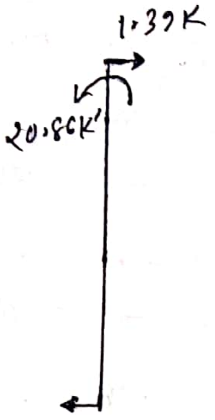
Distribution for side sway

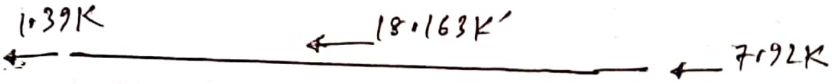
1st cycle	FEM	100	100	—	—	-156.25	—	—	100	100	-156.25
	Balance	-100	-40	-60	54.69	45.31	56.25	-60	-40	—	—
2nd cycle	CO	-20	-50	27.35	-30	—	-30	28.13	—	-20	22.66
	Balance	20	9.06	13.59	21	17.4	21.6	-16.88	-11.25	—	—
3rd cycle	CO	4.53	10	10.5	6.8	—	-8.44	10.8	—	-5.63	8.7
	Balance	-4.53	-8.2	-12.3	0.57	0.48	0.59	-6.48	-4.32	—	—
	Total	0	20.86	-20.86	53.06	-93.06	40	-44.43	44.43	74.37	-124.89

z = 1.49

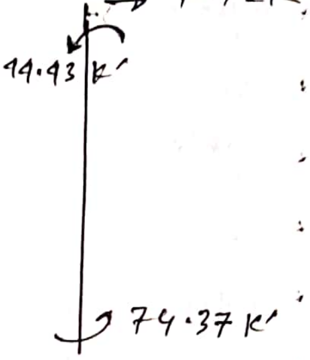
Z× 2nd Balance		0	31.08	-31.08	79.06	-138.66	59.6	-66.2	66.2	110.81	-186.1
1st Balance		0	-6.98	6.98	-18.67	1.12	17.55	33.93	-33.93	67.02	0.36
Total	Total	0	24.1	-24.1	60.39	-137.54	77.15	-32.27	32.27	177.83	-185.74



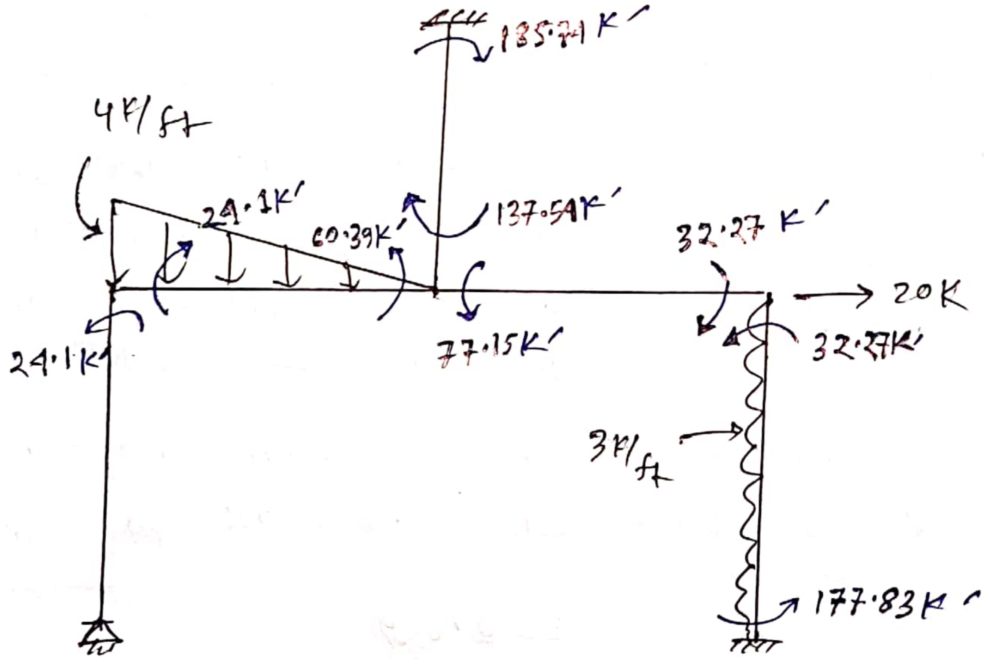




CJF = 27.473 →



$$Z = \frac{AJR}{CJF} = \frac{40.883}{27.473} = 1.49$$



2011
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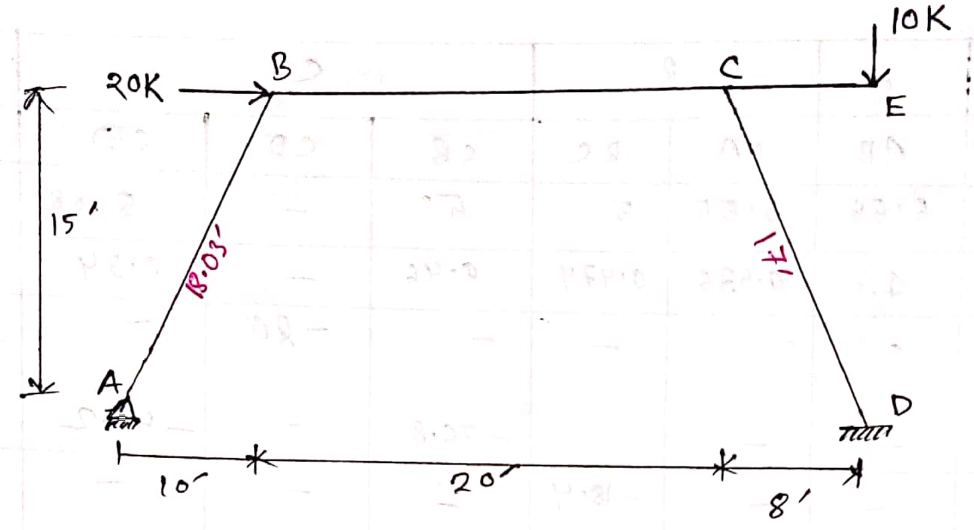


Solution:

Relative stiffness:

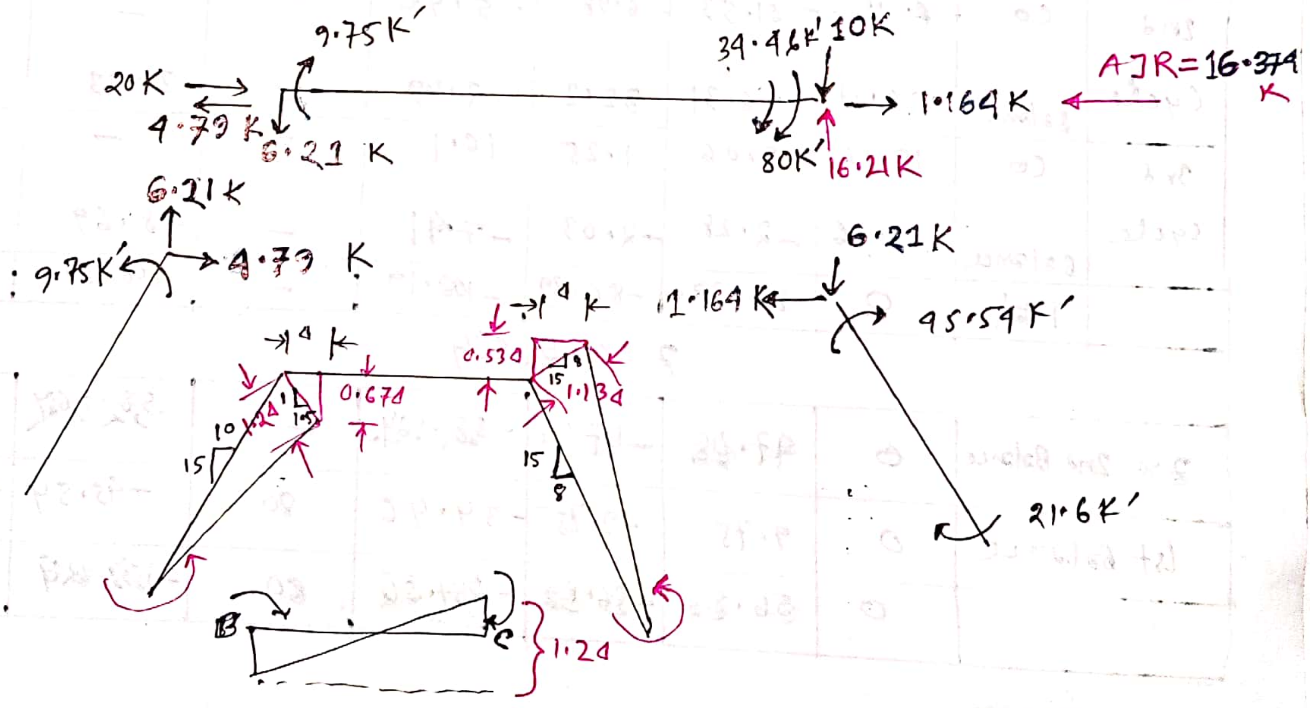
$K_{AB} = \frac{1}{18.03} \simeq 5.55$

$K_{BC} = \frac{1}{20} \simeq 5$

$K_{CD} = \frac{1}{17} \simeq 5.88$

Fixed End Moment:

$F_{CE} = 80K'$

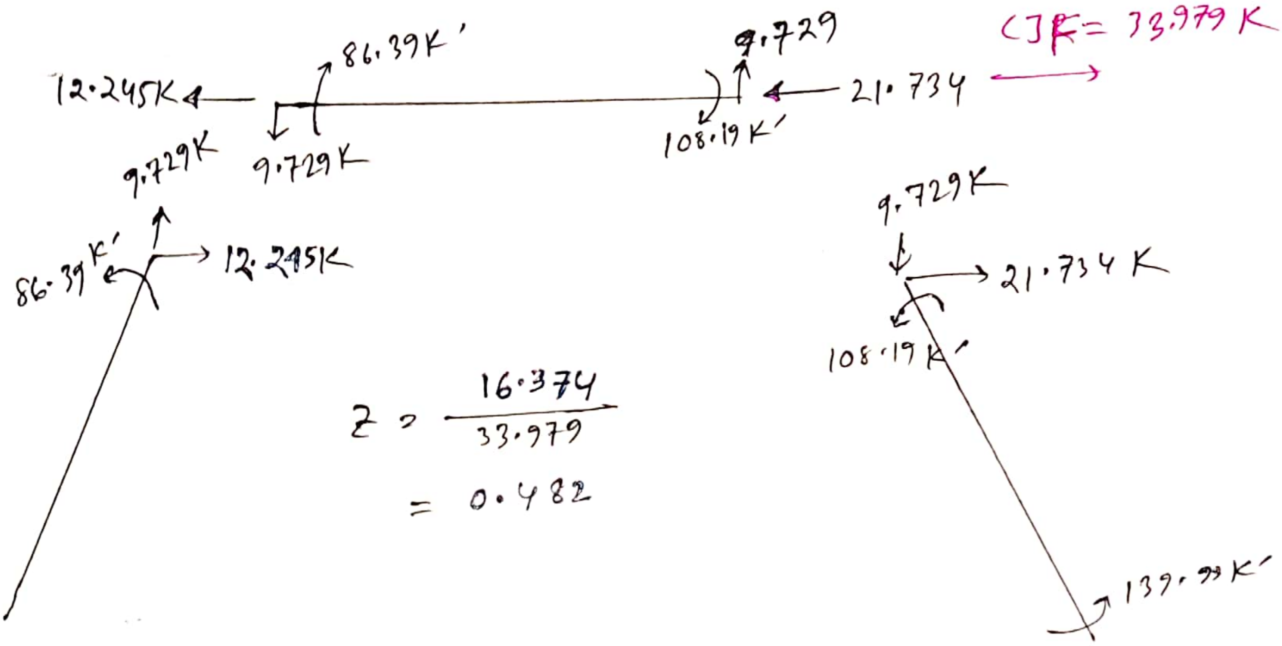


$F_{BC} = F_{CB} = \frac{6EI \times 1.24}{20^2} = -100K'$

$\therefore F_{CD} = F_{DC} = \frac{100 \times 20^2 \times 1.130}{1.24 \times (17)^2} = 130.33 K'$

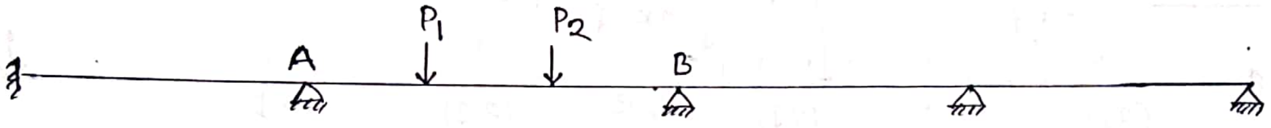
$F_{AB} = F_{BA} = \frac{100 \times 20^2 \times 1.24}{1.24 \times (18.03)^2} = 123.05K'$

	Joint	A	B		C			D
	Member	AB	BA	BC	CB	CE	CD	DC
	K	5.55	5.55	5	5	—	5.88	5.88
	D.F	1	0.53	0.47	0.46	—	0.54	—
1st cycle	FEM	—	—	—	—	80	—	—
	Balance	—	—	—	-36.8	—	-43.2	—
2nd cycle		—	—	-18.4	—	—	—	-21.6
	Balance	—	9.75	8.65	—	—	—	—
3rd cycle	CO	4.88	—	—	4.33	—	—	—
	Balance	-4.88	—	—	-1.99	—	-2.34	
	Total	0	9.75	-9.75	-34.46	80	-45.54	-21.6
Distribution for sidesway								
1st cycle	FEM	123.05	123.05	-100	-100	—	130.33	130.33
	Balance	-123.05	-12.22	-10.83	-13.95	—	-16.38	—
2nd cycle	CO	-6.11	-61.53	-6.98	-5.42	—	—	8.19
	Balance	6.11	36.31	32.2	2.49	—	2.93	—
3rd cycle	CO	18.18	3.06	1.25	16.1	—	—	1.47
	Balance	-18.16	-2.28	-2.03	-7.41	—	-8.69	—
	Total	0	86.39	-86.39	-108.19	—	108.19	139.99
Z = 0.482								
Z× 2nd Balance		0	41.64	-41.64	-52.15	—	52.15	67.48
1st Balance		0	9.75	-9.75	-34.46	80	-45.54	-21.6
		0	51.39	-51.39	-86.61	80	6.61	4.88

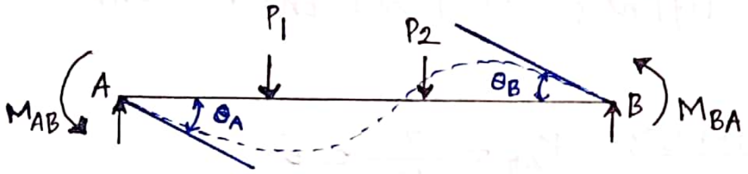


$$Z = \frac{16.374}{33.979}$$
$$= 0.482$$

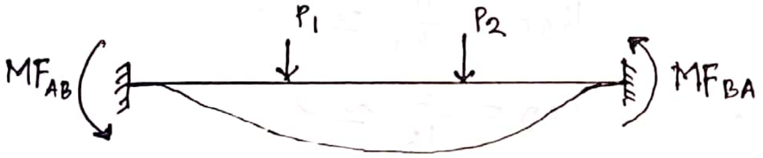
SLOPE DEFLECTION METHOD



considering AB member,



S| (Equivalent To)



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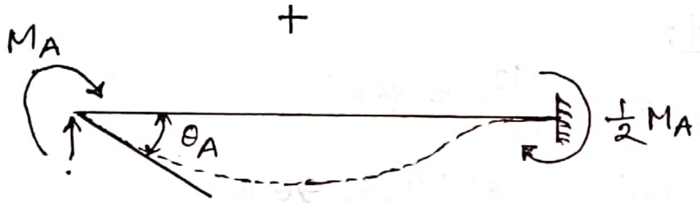


+

$$M_{AB} = MF_{AB} - M_A - \frac{1}{2} M_B$$

$$= MF_{AB} - \frac{4EI\theta_A}{L} - \frac{1}{2} \times \frac{4EI\theta_B}{L}$$

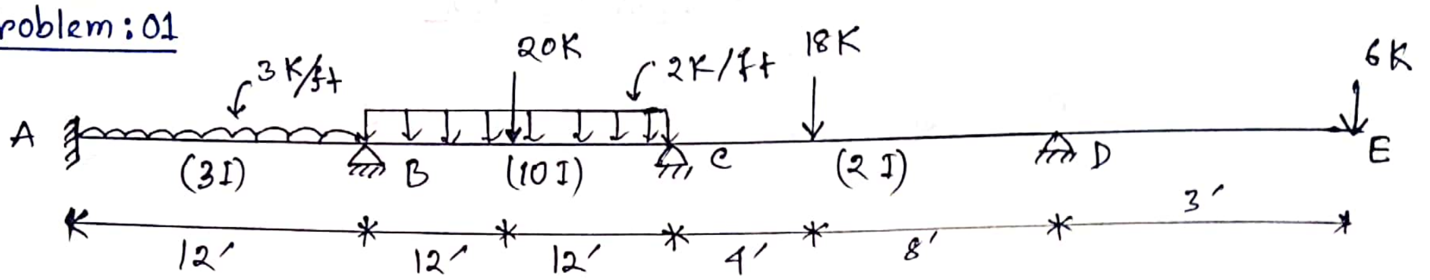
$$\therefore M_{AB} = MF_{AB} + \frac{2EI}{L} \left\{ \underbrace{-2\theta_A}_{\text{Near End }(2\theta)} - \overbrace{\theta_B}^{\text{Far End }(\theta)} \right\} \quad \ldots\ldots ①$$

Similarly,

$$M_{BA} = MF_{BA} + \frac{2EI}{L} \left\{ -\theta_A - 2\theta_B \right\} \quad \ldots\ldots ⑪$$

Equations ① & ⑪ are the slope deflection equations which express the end moments in terms of the end rotations and the applied loading.

Problem: 01



Calculate (i) Relative stiffness (ii) Fixed End Moment

Solution: Relative stiffness:

$$K_{AB} = \frac{3}{12} \simeq 3$$

$$K_{BC} = \frac{10}{24} \simeq 5$$

$$K_{CD} = \frac{2}{12} \simeq 2$$

Fixed End Moment:

$$F_{AB} = -F_{BA} = \frac{WL^2}{12} = \frac{3 \times 12^2}{12} = 36K'$$

$$(F_{BC})_1 = -(F_{CB})_1 = \frac{WL^2}{12} = \frac{2 \times 24^2}{12} = 96K'$$

$$(F_{BC})_2 = -(F_{CB})_2 = \frac{PL}{8} = \frac{20 \times 24}{8} = 60K'$$

$$\therefore F_{BC} = -F_{CB} = 156K'$$

$$F_{CD} = \frac{Pab^2}{L^2} = \frac{18 \times 4 \times 8^2}{12^2} = 32K'$$

$$F_{DC} = -\frac{Pa^2b}{L^2} = \frac{18 \times 4^2 \times 8}{12^2} = -16K'$$

$$F_{DE} = (6 \times 3) = 18K'$$

Slope deflection equations:

$$M_{AB} = 36 + 3(-2\overset{0}{\cancel{\theta_A}} - \theta_B) = 36 - 3\theta_B$$

$$M_{BA} = -36 + 3(-\overset{0}{\cancel{\theta_A}} - 2\theta_B) = -36 - 6\theta_B$$

$M_{BC} = 156 + 5(-2\theta_B - \theta_C) = 156 - 10\theta_B - 5\theta_C$

$M_{CB} = -156 + 5(-\theta_B - 2\theta_C) = -156 - 5\theta_B - 10\theta_C$

$M_{CD} = 32 + 2(-2\theta_C - \theta_D) = 32 - 4\theta_C - 2\theta_D$

$M_{DC} = -16 + 2(-\theta_C - 2\theta_D) = -16 - 2\theta_C - 4\theta_D$

$M_{DE} = 18\ K'$

joint Condition:

joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -36 - 6\theta_B + 156 - 10\theta_B - 5\theta_C = 0$

$\Rightarrow 16\theta_B + 5\theta_C = 120$ .... ①

joint C: $M_{CB} + M_{CD} = 0$

$\Rightarrow -156 - 5\theta_B - 10\theta_C + 32 - 4\theta_C - 2\theta_D = 0$

$\Rightarrow 5\theta_B + 14\theta_C + 2\theta_D = -124$ .... (ii)

joint D: $M_{DC} + 18 = 0$

$\Rightarrow -16 - 2\theta_C - 4\theta_D + 18 = 0$

$\Rightarrow 2\theta_C + 4\theta_D = 2$ .... (iii)

Check:

$$\begin{array}{cccc} \theta_B & \theta_C & \theta_D & C \\ 16 & 5 & 0 & 120 \\ 5 & 14 & 2 & -124 \\ 0 & 2 & 4 & 2 \end{array}$$

* upper triangle = lower triangle

(Exam-এ করার প্রয়োজন নেই)

By solving Eqn ①, Ⓘⓘ & ⓘⓘⓘ . we obtain,

$\theta_B = 11.94$ rad.

$\theta_C = -14.21$ rad.

$\theta_D = 7.60$ rad.

Now, $M_{AB} = 36 - 3 \times 11.94 = 0.18$ K′

$M_{BA} = -36 - 6 \times 11.94 = -107.64$ K′

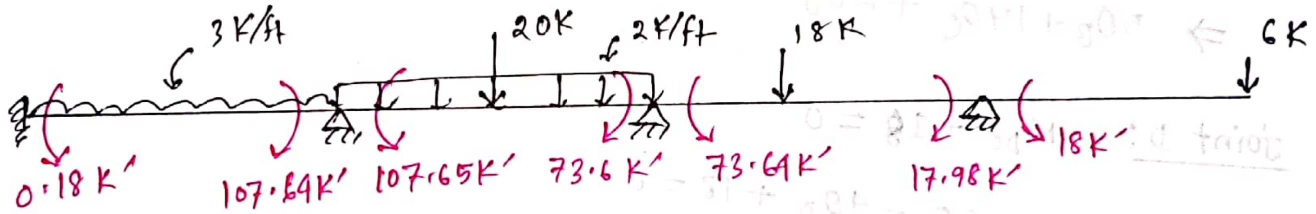
$M_{BC} = 156 - 10 \times 11.94 - 5 \times (-14.21) = 107.65$ K′

$M_{CB} = -156 - 5 \times 11.94 - 10 \times (-14.21) = -73.6$ K′

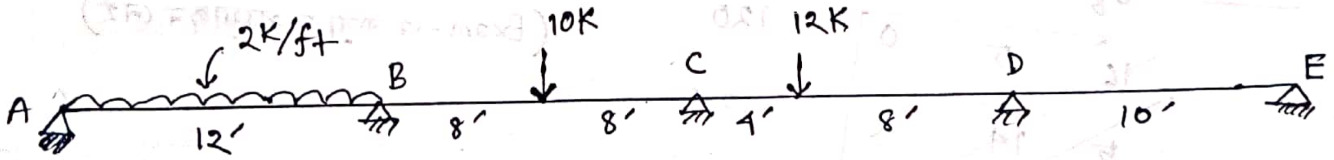
$M_{CD} = 32 - 4 \times (-14.21) - 2 \times 7.6 = 73.64$ K′

$M_{DC} = -16 - 2 \times (-14.21) - 4 \times 7.6 = -17.98$ K′

$M_{DE} = 18$ K′



Assignment : 01



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{1}{16} \simeq 0.75$

$K_{CD} = \frac{1}{12} \simeq 1$

$K_{DE} = \frac{1}{10} \simeq 1.2$

Fixed End Moment:

$F_{AB} = \frac{WL^2}{12} = \frac{2\times 12^2}{12} = 24K' = -F_{BA}$

$F_{BC} = \frac{PL}{8} = \frac{10\times 16}{8} = 20K' = -F_{CB}$

$F_{CD} = \frac{Pab^2}{L^2} = \frac{12\times 4\times 8^2}{12^2} = 21.33K'$

$F_{DC} = -\frac{Pa^2b}{L^2} = \frac{-12\times 4^2\times 8}{12^2} = -10.67K'$

Slope deflection Equations:

$M_{AB} = 24+1\times(-2\theta_A - \theta_B) = 24-2\theta_A-\theta_B$

$M_{BA} = -24+1\times(-\theta_A-2\theta_B) = -24-\theta_A-2\theta_B$

$M_{BC} = 20+0.75\times(-2\theta_B-\theta_C) = 20-1.5\theta_B-0.75\theta_C$

$M_{CB} = -20+0.75\times(-\theta_B-2\theta_C) = -20-0.75\theta_B-1.5\theta_C$

$M_{CD} = 21.33+1\times(-2\theta_C-\theta_D) = 21.33-2\theta_C-\theta_D$

$M_{DC} = -10.67+1\times(-\theta_C-2\theta_D) = -10.67-\theta_C-2\theta_D$

$M_{DE} = 0+1.2\times(-2\theta_D-\theta_E) = -2.4\theta_D-1.2\theta_E$

$M_{ED} = 0+1.2\times(-\theta_D-2\theta_E) = -1.2\theta_D-2.4\theta_E$

Joint conditions:

Joint A: $M_{AB} = 0$

$\Rightarrow 24-2\theta_A-\theta_B = 0 \quad \Rightarrow 2\theta_A+\theta_B = 24$ .... ①

Joint B: $M_{BA}+M_{BC} = 0$

$\Rightarrow -24-\theta_A-2\theta_B+20-1.5\theta_B-0.75\theta_C = 0$

$\Rightarrow \theta_A+3.5\theta_B+0.75\theta_C = -4$ .... (ii)

Joint C: $M_{CB} + M_{CD} = 0$

$$\Rightarrow -20 - 0.75\theta_B - 1.5\theta_C + 21.33 - 2\theta_C - \theta_D = 0$$

$$\Rightarrow 0.75\theta_B + 3.5\theta_C + \theta_D = 1.33 \;\ldots\ldots\; (iii)$$

Joint D: $M_{DC} + M_{DE} = 0$

$$\Rightarrow -10.67 - \theta_C - 2\theta_D - 2.4\theta_D - 1.2\theta_E = 0$$

$$\Rightarrow \theta_C + 4.4\theta_D + 1.2\theta_E = -10.67 \;\ldots\ldots\; (iv)$$

Joint E: $M_{ED} = 0$

$$\Rightarrow -1.2\theta_D - 2.4\theta_E = 0 \quad \Rightarrow 1.2\theta_D + 2.4\theta_E = 0 \;\ldots\ldots\; (v)$$

Check:

θ_A	θ_B	θ_C	θ_D	θ_E	c
2	1	0	0	0	24
1	3.5	0.75	0	0	-4
0	0.75	3.5	1	0	1.33
0	0	1	4.4	1.2	-10.67
0	0	0	1.2	2.4	0

From eqn ①, $2\theta_A + \theta_B = 24$

$$\Rightarrow \theta_A = 12 - 0.5\theta_B$$

Now, Putting the value of θ_A in eqn (ii) we obtain,

$$12 - 0.5\theta_B + 3.5\theta_B + 0.75\theta_C = -4$$

$$\Rightarrow 3\theta_B + 0.75\theta_C = -16 \;\ldots\ldots\; (vi)$$

Again from eqn (v), $1.2\theta_D + 2.4\theta_E = 0$

$$\Rightarrow \theta_E = -0.5\theta_D$$

Putting the value of θ_E in eqn (iv) we obtain,

$$\theta_C + 4.4\theta_D + 1.2\times(-0.5\theta_D) = -10.67$$

$$\Rightarrow \theta_C + 4.4\theta_D - 0.6\theta_D = -10.67$$

$$\Rightarrow \theta_C + 3.8\theta_D = -10.67 \text{ .... (vii)}$$

Now, From eqn (iii), (vi) & (vii), we get,

$\theta_B = -6$ rad.

$\theta_C = 2.67$ rad.

$\theta_D = -3.51$ rad.

Therefore, $\theta_A = 12 - 0.5\times(-6) = 15$ rad.

and, $\theta_E = -0.5\times-3.51$ rad $= 1.755$ rad

Final End Moments:

$M_{AB} = 24 - 2\times15 - (-6) = 0$ K′

$M_{BA} = -24 - 15 - 2\times(-6) = -27$ K′

$M_{BC} = 20 - 1.5\times(-6) - 0.75\times2.67 = 27$ K′

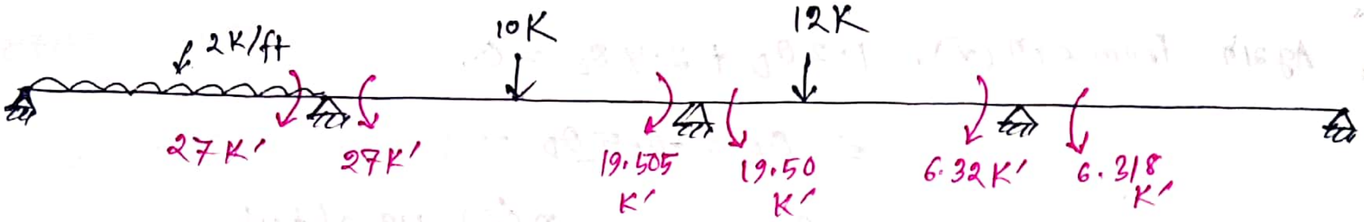
$M_{CB} = -20 - 0.75\times(-6) - 1.5\times2.67 = -19.505$ K′

$M_{CD} = 21.33 - 2\times2.67 - (-3.51) = 19.5$ K′

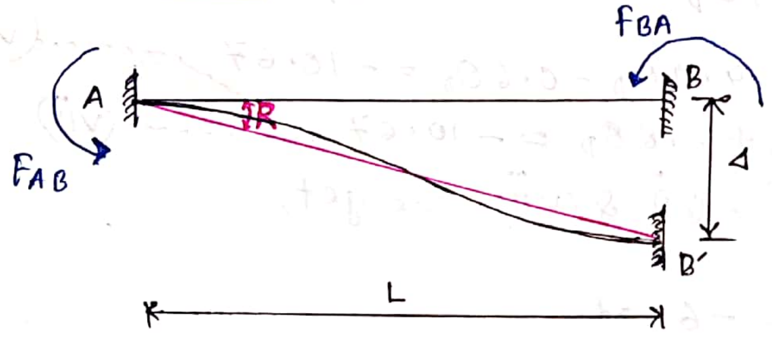
$M_{DC} = -10.67 - 2.67 - 2\times(-3.51) = -6.32$ K′

$M_{DE} = -2.4\times(-3.51) - 1.2\times(1.755) = 6.318$ K′

$M_{ED} = -1.2\times(-3.51) - 2.4\times(1.755) = 0$ K′



◫ Support settlement:



$$\tan R = \frac{\Delta}{L}$$

$$\Rightarrow R = \tan^{-1}\left(\frac{\Delta}{L}\right)$$

when R is very small, then $\tan^{-1}\left(\frac{\Delta}{L}\right) = \frac{\Delta}{L}$

$$\therefore R = \frac{\Delta}{L}$$

Due to settlement, $F_{AB} = F_{BA} = \dfrac{6EI\Delta}{L^2}$

Therefore,

Slope deflection equations,

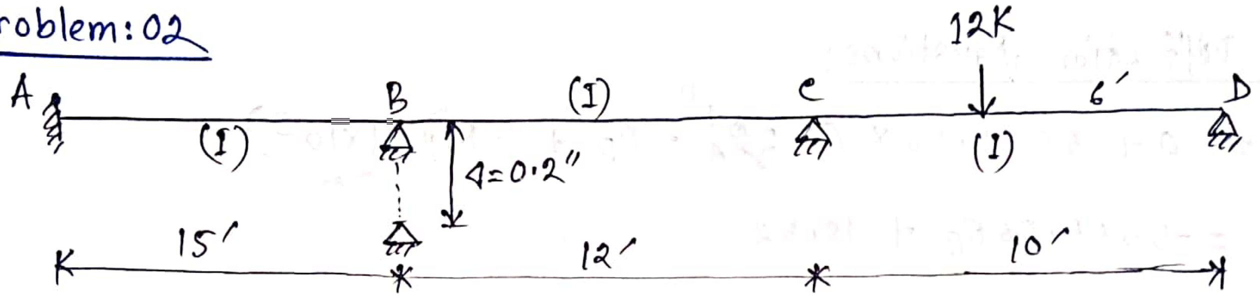
$$M_{AB} = MF_{AB} + \frac{2EI}{L}\{-2\theta_A - \theta_B\} + \frac{6EI\Delta}{L^2}$$

$$= MF_{AB} + \frac{2EI}{L}\left\{-2\theta_A - \theta_B + \frac{3\Delta}{L}\right\}$$

$$\therefore M_{AB} = MF_{AB} + \frac{2EI}{L}\{-2\theta_A - \theta_B + 3R\}$$

Similarly, $M_{BA} = MF_{BA} + \dfrac{2EI}{L}\{-\theta_A - 2\theta_B + 3R\}$

Problem: 02



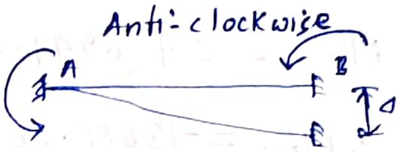
Given, $E = 30000$ ksi

$I = 200$ in^4

$\Delta_B = 0.2$ in.

calculate Relative stiffness and Final End Moments.

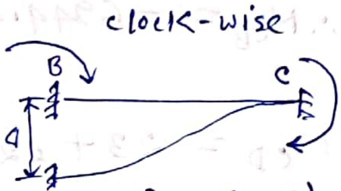
Solution: Values of R and $\frac{2EI}{L}$:



$R \rightarrow (+ve)$

Span AB: $R_{AB} = \frac{\Delta}{L} = \frac{0.2}{15\times12} = 1.111\times10^{-3}$ rad.

$K_{AB} = \frac{2EI}{L} = \frac{2\times30000\times200}{15\times144} = 5555.56$ K'



$R \rightarrow (-ve)$

Span BC: $R_{BC} = \frac{-\Delta}{L} = -\frac{0.2}{12\times12} = -1.389\times10^{-3}$ rad.

$K_{BC} = \frac{2EI}{L} = \frac{2\times30000\times200}{12\times144} = 6944.44$ K'

Span CD: $K_{CD} = \frac{2EI}{L} = \frac{2\times30000\times200}{10\times144}\times\frac{3}{4} = (8333.33\times\frac{3}{4}) = 6250$ K'

(Modified Stiffness Method)

* Far end Hinge stiffness will be 75%.

Fixed End Moments:

$F_{CD} = \frac{Pab^2}{L^2} + \frac{1}{2}\times\frac{Pa^2b}{L^2} = \frac{12\times4\times6^2}{10^2} + \frac{1}{2}\times\frac{12\times4^2\times6}{10^2} = 23$ K'

carry-over

(Modified Stiffness Method)

$F_{DC} = 0$

Slope Deflection Equations:

$$M_{AB} = 0 + 5555.56 \times (-2\overset{0}{\cancel{\theta_A}} - \theta_B + 3 \times \underbrace{1.111\times10^{-3}}_{R_{AB}})$$

$$\therefore M_{AB} = -5555.56\theta_B + 18.52$$

$$M_{BA} = 0 + 5555.56 \times (-\overset{0}{\cancel{\theta_A}} - 2\theta_B + 3 \times 1.111 \times 10^{-3})$$

$$\therefore M_{BA} = -11111.12\theta_B + 18.52$$

$$M_{BC} = 0 + 6944.44 \times \{-2\theta_B - \theta_C + 3 \times (\underbrace{-1.389\times10^{-3}}_{R_{BC}})\}$$

$$\therefore M_{BC} = -13888.88\theta_B - 6944.44\theta_C - 28.94$$

$$M_{CB} = 0 + 6944.44 \times \{-\theta_B - 2\theta_C + 3 \times (-1.389\times10^{-3})\}$$

$$\therefore M_{CB} = -6944.44\theta_B - 13888.88\theta_C - 28.94$$

$$M_{CD} = 23 + 6250 \times \{-2\theta_C\} \quad = 23 - 12500\theta_C$$

joint conditions;

joint B: $M_{BA} + M_{BC} = 0$

$$\Rightarrow -11111.12\theta_B + 18.52 - 13888.88\theta_B - 6944.44\theta_C - 28.94 = 0$$

$$\Rightarrow 25000\theta_B + 6944.44\theta_C = -10.42 \quad \text{......} \; ①$$

joint C: $M_{CB} + M_{CD} = 0$

$$\Rightarrow -6944.44\theta_B - 13888.88\theta_C - 28.94 + 23 - 12500\theta_C = 0$$

$$\Rightarrow 6944.44\theta_B + 26388.88\theta_C = -5.94 \quad \text{......} \; (ii)$$

From ① & ⑪ we obtain,

$$\theta_B = -3.822 \times 10^{-4} \text{ rad.}$$

$$\theta_C = -1.245 \times 10^{-4} \text{ rad.}$$

Now, Final End Moments,

$$M_{AB} = -5555.56 \times (-3.822 \times 10^{-4}) + 18.52$$
$$= 20.64 \text{ K}'$$

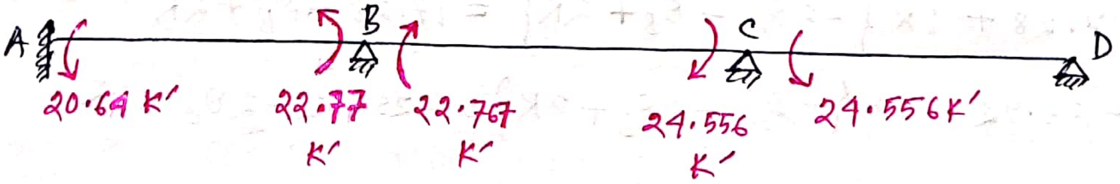
$$M_{BA} = -11111.12 \times (-3.822 \times 10^{-4}) + 18.52$$
$$= 22.77 \text{ K}'$$

$$M_{BC} = -13888.88 \times (-3.822 \times 10^{-4}) - 6944.44 \times (-1.245 \times 10^{-4}) - 28.94$$
$$= -22.767 \text{ K}'$$

$$M_{CD} = 23 - 12500 \times (-1.245 \times 10^{-4})$$
$$= 24.556 \text{ K}'$$

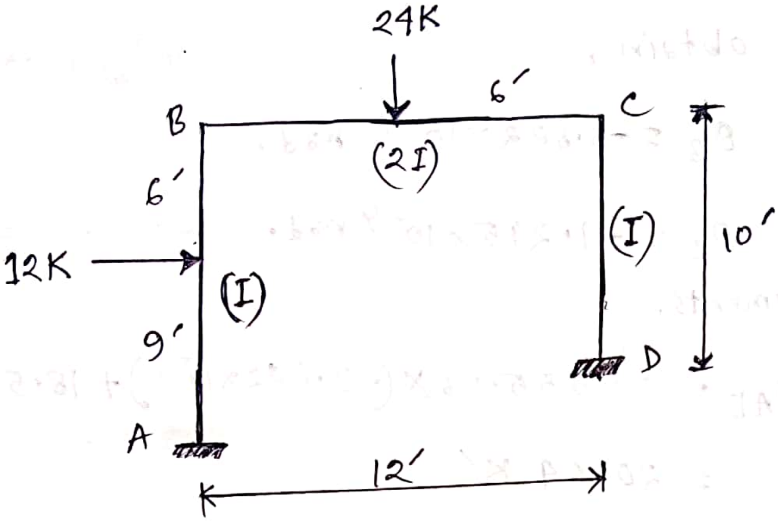
$$M_{CB} = -6944.44 \times (-3.822 \times 10^{-4}) - 13888.88 \times (-1.245 \times 10^{-4}) - 28.94$$
$$= -24.556 \text{ K}'$$

$$M_{DC} = 0$$



* support settlement হলে, member-এ উপরে Tension & support-এ নিচে Tension creat হয়।

Problem: 03



Solution:

Relative Stiffness:

$K_{AB} = \frac{1}{15} \simeq 1$

$K_{BC} = \frac{2}{12} \simeq 2.5$

$K_{CD} = \frac{1}{10} \simeq 1.5$

Fixed End Moments;

$F_{AB} = \frac{12 \times 9 \times 6^2}{15^2} = 17.28 K'$

$F_{BA} = \frac{-12 \times 9^2 \times 6}{15^2} = -25.92 K'$

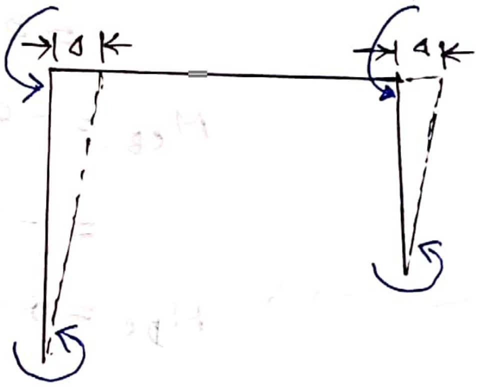
$F_{BC} = \frac{24 \times 12}{8} = 36K' = -F_{CB}$

Relative values of R:

$R_{AB} = \frac{\Delta}{L} = \frac{\Delta}{15} \simeq 2R$

$R_{BC} = 0$

$R_{CD} = \frac{\Delta}{10} \simeq 3R$



Slope deflection equations:

$M_{AB} = 17.28 + 1 \times \{-2\overset{0}{\cancel{\theta_A}} - \theta_B + 2R\} = 17.28 - \theta_B + 2R$

$M_{BA} = -25.92 + 1 \times \{-\overset{0}{\cancel{\theta_A}} - 2\theta_B + 2R\} = -25.92 - 2\theta_B + 2R$

$M_{BC} = 36 + 2.5\{-2\theta_B - \theta_C + 0\} = 36 - 5\theta_B - 2.5\theta_C$

$M_{CB} = -36 + 2.5\{\theta_B - 2\theta_C + 0\} = -36 - 2.5\theta_B - 5\theta_C$

$M_{CD} = 0 + 1.5\{-2\theta_C - \overset{0}{\cancel{\theta_D}} + 3R\} = -3\theta_C + 4.5R$

$M_{DC} = 0 + 1.5\{-\theta_C - 2\overset{0}{\cancel{\theta_D}} + 3R\} = -1.5\theta_C + 4.5R$

joint conditions:

joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -25.92 - 2\theta_B + 2R + 36 - 5\theta_B - 2.5\theta_C = 0$

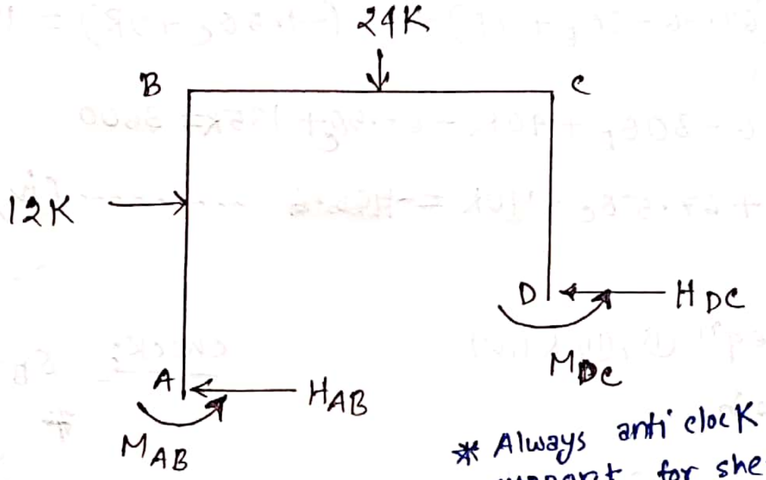
$\Rightarrow 7\theta_B + 2.5\theta_C - 2R = 10.08$ ....①

joint C: $M_{CB} + M_{CD} = 0$

$\Rightarrow -36 - 2.5\theta_B - 5\theta_C - 3\theta_C + 4.5R = 0$

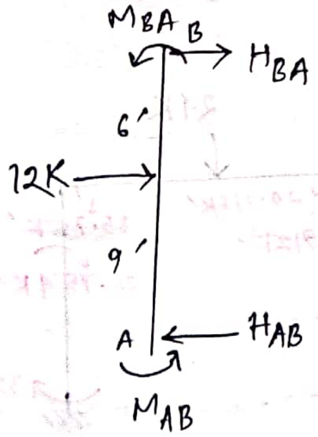
$\Rightarrow 2.5\theta_B + 8\theta_C - 4.5R = -36$ ....⑪

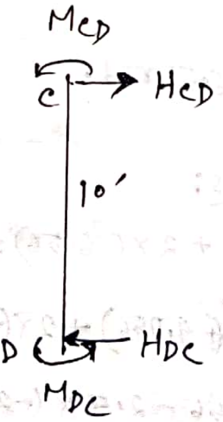
shear conditions:



* Always anti clock moment at support for shear condition

$H_{AB} + H_{DC} = 12$ ....Ⓘⅲ





$\Sigma M_B = 0$

$M_{BA} + M_{AB} + 12 \times 6 - H_{AB} \times 15 = 0$

$\Rightarrow 15 H_{AB} = -25.92 - 2\theta_B + 2R + 17.28 - \theta_B + 2R + 72$

$\Rightarrow H_{AB} = \dfrac{63.36 - 3\theta_B + 4R}{15}$

$\Sigma M_c = 0$

$M_{CD} + M_{DC} - 10 \times H_{DC} = 0$

$\Rightarrow 10 H_{DC} = -3\theta_c + 4.5R - 1.5\theta_c + 4.5R$

$\Rightarrow H_{DC} = \dfrac{-4.5\theta_c + 9R}{10}$

Putting the values of H_{AB} and H_{DC} in eqⁿ (iii) we obtain,

$$\frac{63.36 - 3\theta_B + 4R}{15} + \frac{-4.5\theta_c + 9R}{10} = 12$$

$\Rightarrow 10(63.36 - 3\theta_B + 4R) + 15(-4.5\theta_c + 9R) = 12 \times 150$

$\Rightarrow 633.6 - 30\theta_B + 40R - 67.5\theta_c + 135R = 1800$

$\Rightarrow 30\theta_B + 67.5\theta_c - 175R = -1166.4$ ------- (iv)

Now, from eqⁿ ①, ② & (iv) we obtain,

$\theta_B = 4.056$ rad.

$\theta_c = -2.078$ rad.

$R = +6.56$ rad.

check:

θ_B	θ_c	R	c
7	2.5	−2	10.08
2.5	8	−4.5	−36
$\frac{30}{-15}$	$\frac{67.5}{-15}$	$\frac{-175}{-15}$	$\frac{-1166.4}{-15}$

Final End Moments:

$M_{AB} = 17.28 - 4.056 + 2 \times (6.56) = 26.344K'$

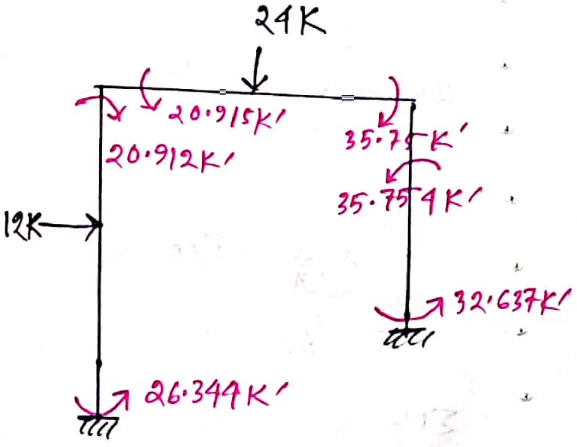
$M_{BA} = -25.92 - 2 \times (4.056) + 2 \times (6.56) = -20.912K'$

$M_{BC} = 36 - 5 \times 4.056 - 2.5 \times (-2.078) = 20.915K'$

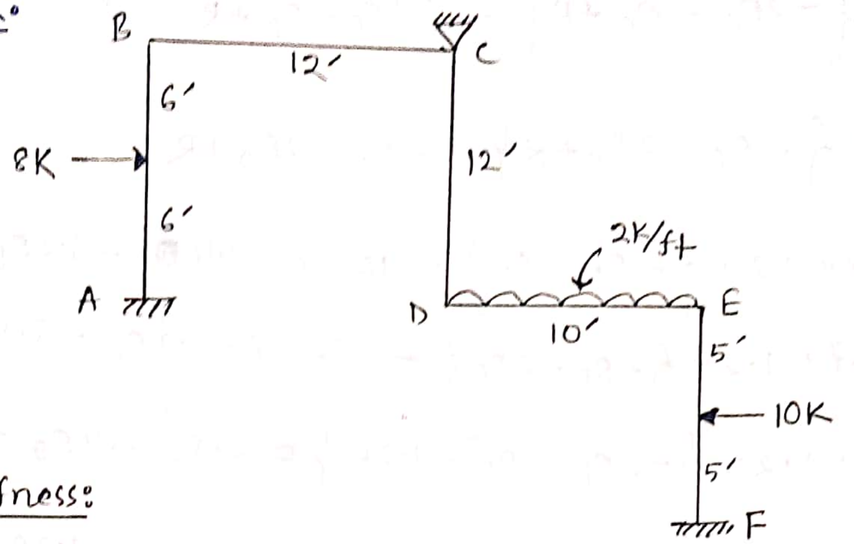
$M_{CB} = -36 - 2.5 \times 4.056 - 5 \times (-2.078) = -35.75K'$

$M_{CD} = -3 \times (-2.078) + 4.5 \times 6.56 = 35.754K'$

$M_{DC} = -1.5 \times (-2.078) + 4.5 \times 6.56 = 32.637K'$



Assignment-02:



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{1}{12} \simeq 1$

$K_{CD} = \frac{1}{12} \simeq 1$

$K_{DE} = \frac{1}{10} \simeq 1.2$

$K_{EF} = \frac{1}{10} \simeq 1.2$

Fixed End Moments:

$F_{AB} = \frac{8 \times 12}{8} = 12K' = -F_{BA}$

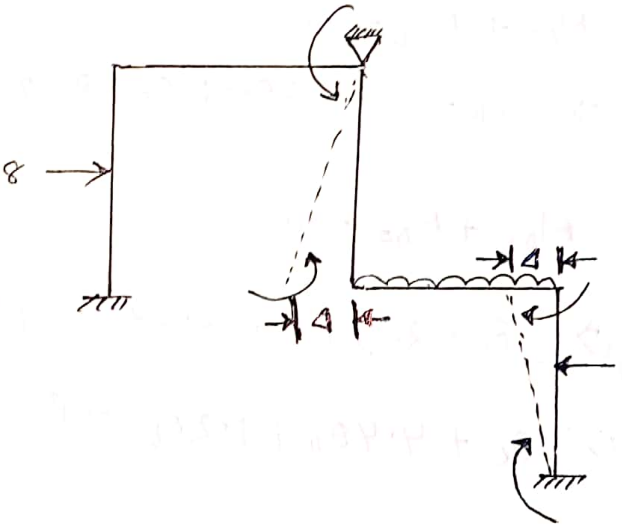
$F_{DE} = -F_{ED} = \frac{2 \times 10^2}{12} = 16.67K'$

$F_{EF} = -F_{FE} = \frac{10 \times 10}{8} = 12.5K'$

Relative values of R:

$R_{CD} = \frac{\Delta}{L} = \frac{\Delta}{12} \simeq R$

$R_{EF} = \frac{-\Delta}{10} \simeq -1.2R$



Slope Deflection equations:

$M_{AB} = 12 + 1\{-2\cancel{\theta_A}^{0} - \theta_B\} = 12 - \theta_B$

$M_{BA} = -12 + 1\{-\cancel{\theta_A}^{0} - 2\theta_B\} = -12 - 2\theta_B$

$M_{BC} = 0 + 1\{-2\theta_B - \theta_C\} = -2\theta_B - \theta_C$

$M_{CB} = 0 + 1\{-\theta_B - 2\theta_C\} = -\theta_B - 2\theta_C$

$M_{CD} = 0 + 1\{-2\theta_C - \theta_D + R\} = -2\theta_C - \theta_D + R$

$M_{DC} = 0 + 1\{-\theta_C - 2\theta_D + R\} = -\theta_C - 2\theta_D + R$

$M_{DE} = 16.67 + 1.2 \times \{-2\theta_D - \theta_E\} = 16.67 - 2.4\theta_D - 1.2\theta_E$

$M_{ED} = -16.67 + 1.2 \times \{-\theta_D - 2\theta_E\} = -16.67 - 1.2\theta_D - 2.4\theta_E$

$M_{EF} = 12.5 + 1.2 \times \{-2\theta_E - \cancel{\theta_F}^{0} - 1.2R\} = 12.5 - 2.4\theta_E - 1.44R$

$M_{FE} = -12.5 + 1.2 \times \{-\theta_E - \cancel{2\theta_F}^{0} - 1.2R\} = -12.5 - 1.2\theta_E - 1.44R$

joint conditions:

joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -12 - 2\theta_B - 2\theta_B - \theta_C = 0 \Rightarrow 4\theta_B + \theta_C = -12$ ..... ①

joint C: $M_{CB} + M_{CD} = 0$

$\Rightarrow -\theta_B - 2\theta_C - 2\theta_C - \theta_D + R = 0 \Rightarrow \theta_B + 4\theta_C + \theta_D - R = 0$ .... (ii)

joint D: $M_{DC} + M_{DE} = 0$

$\Rightarrow -\theta_C - 2\theta_D + R + 16.67 - 2.4\theta_D - 1.2\theta_E = 0$

$\Rightarrow \theta_C + 4.4\theta_D + 1.2\theta_E - R = 16.67$ ...... (iii)

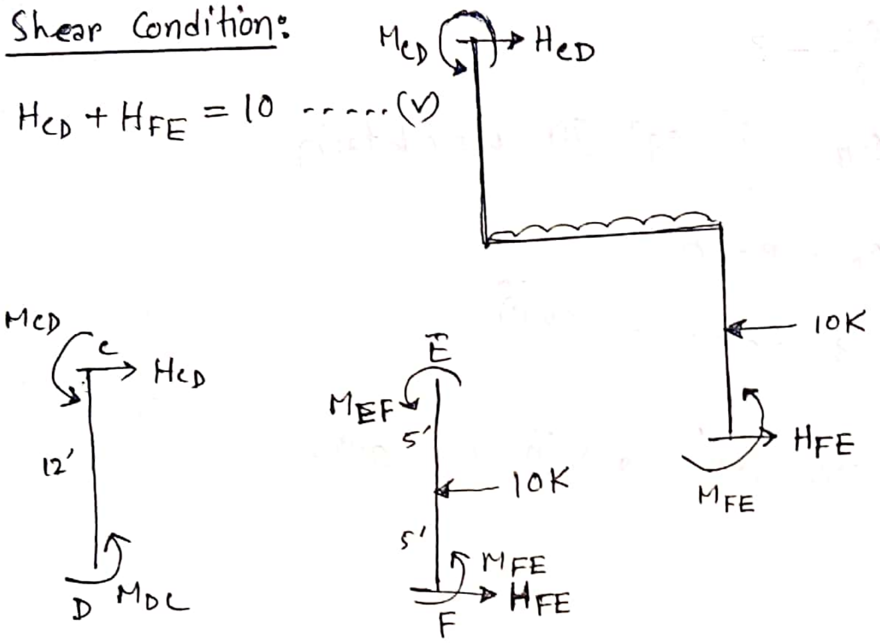
joint E: $M_{ED} + M_{EF} = 0$

$\Rightarrow -16.67 - 1.2\theta_D - 2.4\theta_E + 12.5 - 2.4\theta_E - 1.44R = 0$

$\Rightarrow 1.2\theta_D + 4.8\theta_E + 1.44R = -4.17$ ..... (iv)

Shear Condition:

$H_{CD} + H_{FE} = 10$ ......(V)



$\Sigma M_D = 0$

$M_{DC} + M_{CD} - H_{CD} \times 12 = 0$

$\Rightarrow H_{CD} = \dfrac{M_{DC} + M_{CD}}{12}$

$\Sigma M_E = 0$

$M_{EF} + M_{FE} + H_{FE} \times 10 - 10 \times 5 = 0$

$\Rightarrow H_{FE} = \dfrac{50 - M_{EF} - M_{FE}}{10}$

From eqn (V) we obtain,

$$\frac{M_{DC} + M_{CD}}{12} + \frac{50 - M_{EF} - M_{FE}}{10} = 10$$

$\Rightarrow 10 M_{DC} + 10 M_{CD} + 600 - 12 M_{EF} - 12 M_{FE} = 1200$

$\Rightarrow -10\theta_C - 20\theta_D + 10R - 20\theta_C - 10\theta_D + 10R - \cancel{150} + 28.8\theta_E + 17.28R$
$+ \cancel{150} + 14.4\theta_E + 17.28R = 600$

$\Rightarrow -30\theta_C - 30\theta_D + 43.2\theta_E + 54.56R = 600$

$\Rightarrow -\theta_C - \theta_D + 1.44\theta_E + 1.82R = 20$ .... (VI)

Check:

θ_B	θ_C	θ_D	θ_E	R
4	1	0	0	0
1	4	1	0	-1
0	1	4.4	1.2	-1
0	0	1.2	4.8	1.44
0	-1	-1	1.44	1.82

From eqn ①, $\theta_B = \frac{-\theta_c}{4} - 3$

putting the value of θ_B in eqn ⑪ we obtain

$-0.25\theta_c - 3 + 4\theta_c + \theta_D - R = 0$

$\Rightarrow 3.75\theta_c + \theta_D - R = 3$ ------- (vii)

Now, From eqn (iii), (iv), (vi) & (vii) we get,

$\theta_c = 5.95$

$\theta_D = 13.88$

$\theta_E = -14.3$

$R = 33.2$

$\therefore \theta_B = -0.25 \times 5.95 - 3 = -4.4875$

Now,

$M_{AB} = 12 + 4.4875 = 16.4875$ K'

$M_{BA} = -12 + 2 \times 4.4875 = -3.025$ K'

$M_{BC} = 2 \times 4.4875 - 5.95 = 3.025$ K'

$M_{CB} = 4.4875 - 2 \times 5.95 = -7.4125$ K'

$M_{CD} = -2 \times 5.95 - 13.88 + 33.2 = 7.42$ K'

$M_{DC} = -5.95 - 2 \times 13.88 + 33.2 = -0.51$ K'

$M_{DE} = 16.67 - 2.4 \times 13.88 + 1.2 \times 14.3 = 0.518$ K'

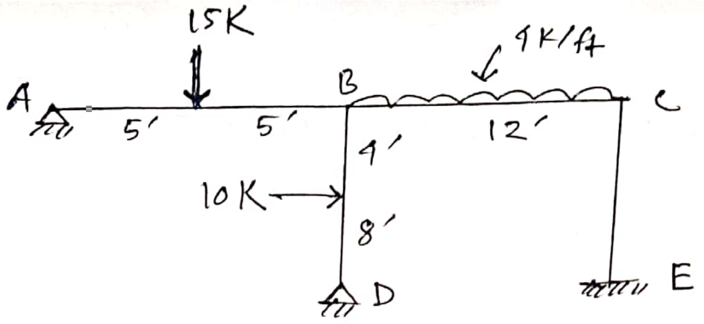
$M_{ED} = -16.67 - 1.2 \times 13.88 + 2.4 \times 14.33 = 1.066$ K' $\simeq$ 1K'

$M_{EF} = 12.5 + 2.4 \times 14.3 - 1.44 \times 33.2 = -0.988$ K' $\simeq$ 1K'

$M_{FE} = -12.5 + 1.2 \times 14.3 - 1.44 \times 33.2 = -43.148$ K'

(Ans.)

Assignment-03:



Solution:

Relative stiffness:

$K_{AB} = \frac{1}{10} \simeq 1.2$

$K_{BD} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{1}{12} \simeq 1$

$K_{CE} = \frac{1}{12} \simeq 1$

FEM:

$F_{AB} = -F_{BA} = \frac{15 \times 10}{8} = 18.75K'$

$F_{BD} = \frac{-10 \times 4 \times 8^2}{12^2} = -17.78\ K'$

$F_{DB} = \frac{10 \times 8 \times 4^2}{12^2} = 8.89\ K'$

$F_{BC} = -F_{CB} = \frac{4 \times 12^2}{12} = 48\ K'$

Slope deflection equations:

$M_{AB} = 18.75 + 1.2 \times \{-2\theta_A - \theta_B\} = 18.75 - 2.4\theta_A - 1.2\theta_B$

$M_{BA} = -18.75 + 1.2 \times \{-\theta_A - 2\theta_B\} = -18.75 - 1.2\theta_A - 2.4\theta_B$

$M_{DB} = 8.89 + 1 \times \{-2\theta_D - \theta_B\} = 8.89 - 2\theta_D - \theta_B$

$M_{BD} = -17.78 + 1 \times \{-\theta_D - 2\theta_B\} = -17.78 - \theta_D - 2\theta_B$

$M_{BC} = 48 + 1 \times \{-2\theta_B - \theta_C\} = 48 - 2\theta_B - \theta_C$

$M_{CB} = -48 + 1 \times \{-\theta_B - 2\theta_C\} = -48 - \theta_B - 2\theta_C$

$M_{CE} = 0 + 1 \times \{-2\theta_C - \overset{0}{\cancel{\theta_E}}\} = -2\theta_C$

$M_{EC} = 0 + 1 \times \{-\theta_C - \overset{0}{\cancel{2\theta_E}}\} = -\theta_C$

joint condition:

joint A:

$M_{AB} = 0 \Rightarrow 18.75 - 2.4\theta_A - 1.2\theta_B = 0$

$\Rightarrow 2.4\theta_A + 1.2\theta_B = 18.75$ ...... ①

joint B: $M_{BA} + M_{BD} + M_{BC} = 0$

$\Rightarrow -18.75 - 1.2\theta_A - 2.4\theta_B - 17.78 - \theta_D - 2\theta_B + 48 - 2\theta_B - \theta_C = 0$

$\Rightarrow 1.2\theta_A + 6.4\theta_B + \theta_C + \theta_D = 11.47$ ...... (ii)

joint D: $M_{DB} = 0 \Rightarrow 8.89 - 2\theta_D - \theta_B = 0$

$\Rightarrow \theta_B + 2\theta_D = 8.89$ ...... (iii)

joint E: $M_{CB} + M_{CE} = 0$

$\Rightarrow -48 - \theta_B - 2\theta_C - 2\theta_C = 0$

$\Rightarrow \theta_B + 4\theta_C = -48$ .... (iv)

From eqn ①, (ii), (iii) & (iv) we obtain,

$\theta_A = 6.857$

$\theta_B = 1.911$

$\theta_C = -12.478$

$\theta_D = 3.49$

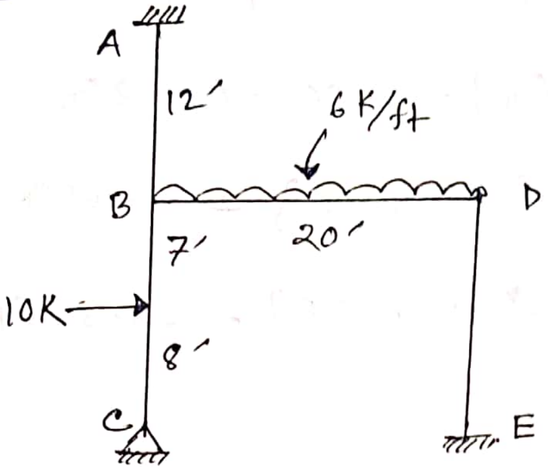
Check:

θ_A	θ_B	θ_C	θ_D	
2.4	1.2	0	0	— joint A
1.2	6.4	1	1	— joint B
0	1	4	0	— joint C
0	1	0	2	— joint D

Then, (do yourself)

Find out Moments ($M_{AB}, M_{BA}, M_{BD}, M_{DB}, M_{BC}, M_{CB}, M_{CE}, M_{EC}$)

Assignment - 04:



Solution:

Relative stiffness:

$K_{AB} = \frac{1}{12} \simeq 1.67$

$K_{BC} = \frac{1}{15} \simeq 1.33$

$K_{BD} = \frac{1}{20} \simeq 1$

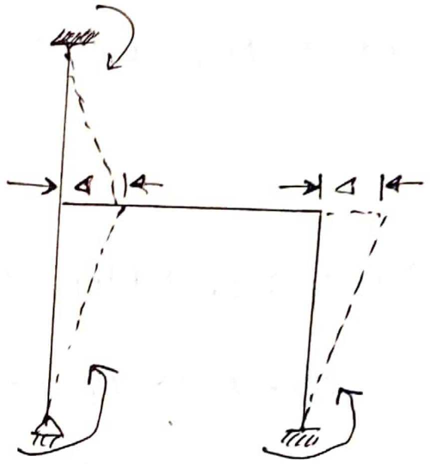
$K_{DE} = \frac{1}{15} \simeq 1.33$

FEM:

$F_{BC} = \frac{-10 \times 7 \times 8^2}{15^2} = -19.91K'$

$F_{CB} = \frac{10 \times 8 \times 7^2}{15^2} = 17.42 K'$

$F_{BD} = -F_{DB} = \frac{6 \times 20^2}{12} = 200K'$



Relative values of R:

$R_{AB} = \frac{-\Delta}{12} \simeq -1.67R$

$R_{BC} = \frac{\Delta}{15} \simeq 1.33R$

$R_{DE} = \frac{\Delta}{15} \simeq 1.33R$

Slope deflection equations:

$M_{AB} = 0 + 1.67 \times \{-2\cancelto{0}{\theta_A} - \theta_B - 1.67R\} = \dashrightarrow -1.67\theta_B - 2.79R$

$M_{BA} = 0 + 1.67 \times \{-\cancelto{0}{\theta_A} - 2\theta_B - 1.67R\} = \dashrightarrow -3.34\theta_B - 2.79R$

$M_{BC} = -19.91 + 1.33 \times \{-2\theta_B - \theta_C + 1.33R\} = -19.91 - 2.66\theta_B - 1.33\theta_C + 1.77R$

$M_{CB} = 17.42 + 1.33 \times \{-\theta_B - 2\theta_C + 1.33R\} = 17.42 - 1.33\theta_B - 2.66\theta_C + 1.77R$

$$M_{BD} = 200 + 1\times\{-2\theta_B - \theta_D\} = 200 - 2\theta_B - \theta_D$$

$$M_{DB} = -200 + 1\times\{-\theta_B - 2\theta_D\} = -200 - \theta_B - 2\theta_D$$

$$M_{DE} = 0 + 1.33\times\{-2\theta_D - \cancel{\theta_E}^{0} + 1.33R\} = -2.66\theta_D + 1.77R$$

$$M_{ED} = 0 + 1.33\times\{-\theta_D - \cancel{2\theta_E}^{0} + 1.33R\} = -1.33\theta_D + 1.77R$$

joint condition:

Joint B: $M_{BA} + M_{BC} + M_{BD} = 0$

$$\Rightarrow -3.34\theta_B - 2.79R - 19.91 - 2.66\theta_B - 1.33\theta_C + 1.77R$$

$$+200 - 2\theta_B - \theta_D = 0$$

$$\Rightarrow 8\theta_B + 1.33\theta_C + \theta_D + R = 180.09 \quad \text{......} ①$$

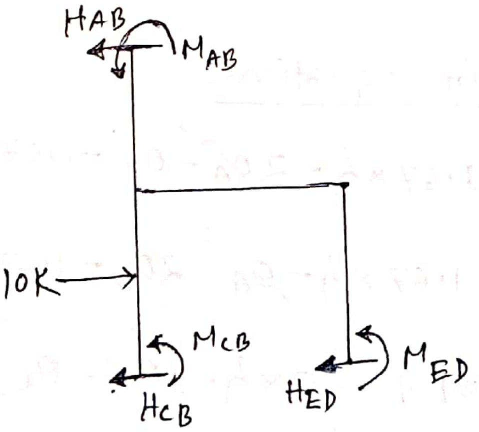
joint C: $M_{CB} = 0 \Rightarrow 17.42 - 1.33\theta_B - 2.66\theta_C + 1.77R = 0$

$$\Rightarrow 1.33\theta_B + 2.66\theta_C - 1.77R = 17.42 \quad \text{......(ii)}$$

joint D: $M_{DB} + M_{DE} = 0$

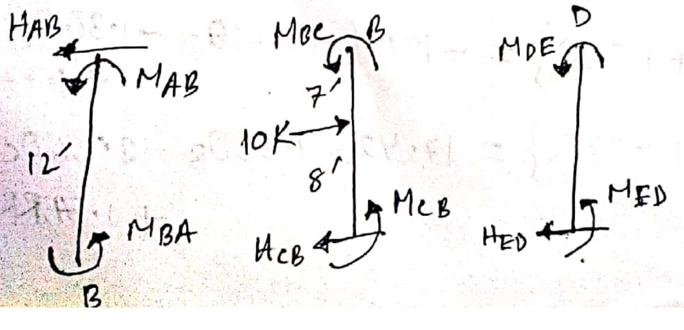
$$\Rightarrow -200 - \theta_B - 2\theta_D - 2.66\theta_D + 1.77R = 0$$

$$\Rightarrow \theta_B + 4.66\theta_D - 1.77R = -200 \quad \text{......(iii)}$$



Shear condition:

$$H_{AB} + H_{CB} + H_{ED} = 10 \quad \text{......(iv)}$$



$\Sigma M_B = 0$

$\Rightarrow M_{AB} + M_{BA} + H_{AB} \times 12 = 0$

$\Rightarrow H_{AB} = \dfrac{-M_{AB} - M_{BA}}{12}$

Again, $\Sigma M_B = 0$

$\Rightarrow M_{BC} + M_{CB} + 10 \times 7 - 15 \times H_{CB} = 0$

$\Rightarrow H_{CB} = \dfrac{M_{BC} + M_{CB} + 70}{15}$

and,

$\Sigma M_D = 0$

$\Rightarrow M_{ED} + M_{DE} - 15 \times H_{ED} = 0$

$\Rightarrow H_{ED} = \dfrac{M_{ED} + M_{DE}}{15}$

Now, From eq^n (iv), we obtain,

$$\frac{-M_{AB} - M_{BA}}{12} + \frac{M_{BC} + M_{CB} + 70}{15} + \frac{M_{ED} + M_{DE}}{15} = 10$$

$\Rightarrow -15M_{AB} - 15M_{BA} + 12M_{BC} + 12M_{CB} + 840 + 12M_{ED} + 12M_{DE} = 1800$

$\Rightarrow 25.05\theta_B + 41.85R + 50.10\theta_B + 41.85R - 238.92 - 31.92\theta_B - 15.96\theta_C + 21.24R + 209.04 - 15.96\theta_B - 31.92\theta_C + 21.24R - 31.92\theta_D + 21.24R - 15.96\theta_D + 21.24R = 960$

$\Rightarrow 27.27\theta_B - 47.88\theta_C - 47.88\theta_D + 168.66R = 989.88$

$\Rightarrow \theta_B - 1.76\theta_C - 1.76\theta_D + 6.185 = 36.3$ ...... (v)

check:

θ_B	θ_C	θ_D	R
8	1.33	1	1
1.33	2.66	0	−1.77
1	0	4.66	−1.77
1	−1.76	−1.76	6.185

From eqⁿ ①, ⑪, ⑪ & ⓥ we obtain,

$\theta_B = 38.485$

$\theta_C = -30.242$

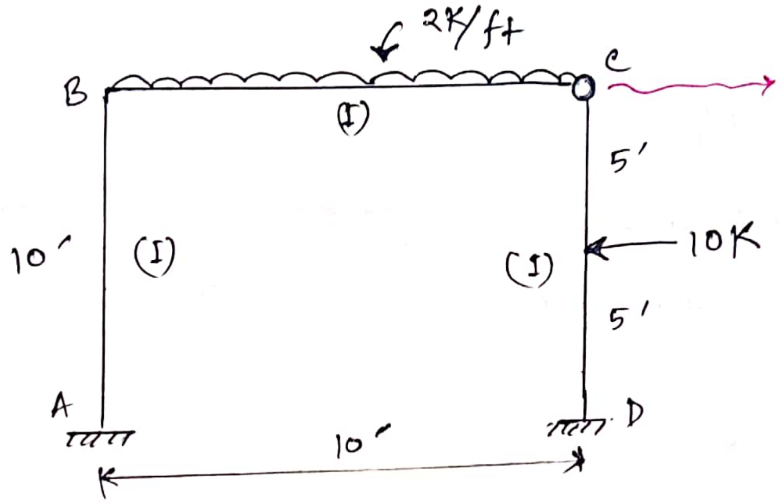
$\theta_D = -61.194$

$R = -26.372$

Then, Find Moments (M_{AB}, M_{BA}, M_{BC}, M_{CB}, M_{BD}, M_{DB}, M_{DE}, M_{ED})

[Do yourself]

Problem :04



* C joint-এ দুইটা আলাদা Rotation হবে।

① θ_{CB}

② θ_{CD}

Solution:

Relative Stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{10} \simeq 1$

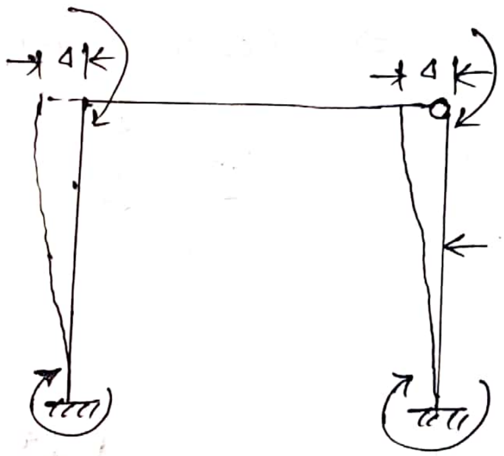
$K_{BC} = \frac{1}{10} \simeq 1$

$K_{CD} = \frac{1}{10} \simeq 1$

Fixed End Moment:

$F_{BC} = -F_{BC} = \frac{WL^2}{12} = \frac{2 \times 10^2}{12} = 16.67K'$

$F_{CD} = -F_{DC} = \frac{PL}{8} = \frac{10 \times 10}{8} = 12.5K'$



Relative values of R:

$R_{AB} = \frac{-\Delta}{L} = \frac{-\Delta}{10} \simeq -R$

$R_{CD} \simeq -R$

Slope deflection Equations:

$M_{AB} = 0 + 1 \times \{-2\cancel{\theta_A}^{\,0} - \theta_B - R\} = -\theta_B - R$

$M_{BA} = 0 + 1 \times \{-\cancel{\theta_A}^{\,0} - 2\theta_B - R\} = -2\theta_B - R$

$M_{BC} = 16.67 + 1 \times \{-2\theta_B - \theta_{CB}\} = 16.67 - 2\theta_B - \theta_{CB}$

$M_{CB} = -16.67 + 1 \times \{-\theta_B - 2\theta_{CB}\} = -16.67 - \theta_B - 2\theta_{CB}$

$M_{CD} = 12.5 + 1 \times \{-2\theta_{CD} - \cancel{\theta_D}^{\,0} - R\} = 12.5 - 2\theta_{CD} - R$

$M_{DC} = -12.5 + 1 \times \{-\theta_{CD} - 2\cancel{\theta_D}^{\,0} - R\} = -12.5 - \theta_{CD} - R$

joint conditions:

joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -2\theta_B - R + 16.67 - 2\theta_B - \theta_{CB} = 0$

$\Rightarrow 4\theta_B + \theta_{CB} + R = 16.67$ .... ①

joint C: $M_{CB} = 0$

$\Rightarrow -16.67 - \theta_B - 2\theta_{CB} = 0$

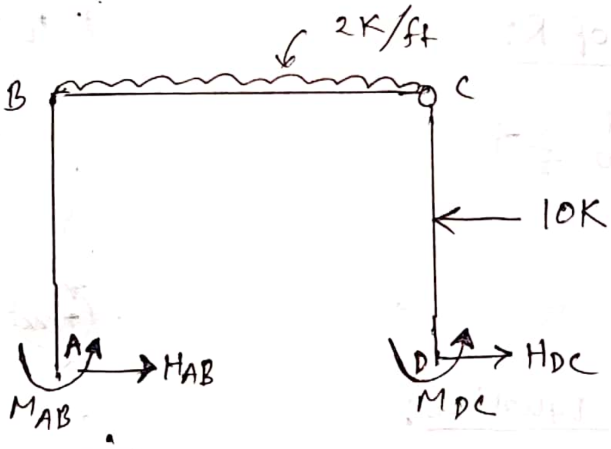
$\Rightarrow \theta_B + 2\theta_{CB} = -16.67$ .... (ii)

And, $M_{CD} = 0$

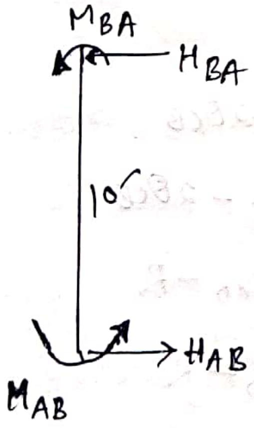
$\Rightarrow 12.5 - 2\theta_{CD} - R = 0$

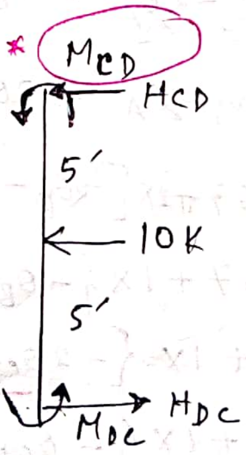
$\Rightarrow 2\theta_{CD} + R = 12.5$ ...... (iii)

shear conditions:



$H_{AB} + H_{DC} = 10$ ...... (iv)





$\Sigma M_B = 0$

$M_{AB} + M_{BA} + 10 \times H_{AB} = 0$

$\Rightarrow 10 H_{AB} = -M_{AB} - M_{BA}$

$$\Rightarrow H_{AB} = \frac{\theta_B + R + 2\theta_B + R}{10}$$

$$\therefore H_{AB} = \frac{3\theta_B + 2R}{10}$$

$\Sigma M_C = 0$

$M_{DC} + M_{CD} + 10 \times H_{DC} - 10 \times 5 = 0$

$\Rightarrow 10 H_{DC} = 50 - M_{DC} - M_{CD}$

$$\Rightarrow H_{DC} = \frac{50 + 12.5 + \theta_{CD} + R - 12.5 + 2\theta_{CD} + R}{10}$$

$$\therefore H_{DC} = \frac{50 + 3\theta_{CD} + 2R}{10}$$

From eq^n (iv) we obtain,

$$\frac{3\theta_B + 2R}{10} + \frac{50 + 3\theta_{CD} + 2R}{10} = 10$$

$\Rightarrow 10(3\theta_B + 2R) + 10(50 + 3\theta_{CD} + 2R) = 100$

$\Rightarrow 30\theta_B + 20R + 500 + 30\theta_{CD} + 20R = 100$

$\Rightarrow 30\theta_B + 30\theta_{CD} + 40R = -400$

$\Rightarrow \theta_B + \theta_{CD} + 1.33R = -13.33$

From eq^n (i), (ii), (iii) & (iv)

we obtain, (Use FX-991 EX calculator)

$\theta_B = 21.173$ rad.

$\theta_{CB} = -18.92$ rad.

$\theta_{CD} = 30.8$ rad.

$R = -49.01$ rad.

*(Exam -এ না করালেও হবে)

Check:

θ_B	θ_{CB}	θ_{CD}	R	C
4	1	0	1	76.67
1	2	0	0	-16.67
0	0	2	1	12.5
1	0	1	1.33	-13.33

Final End Moment:

$M_{AB} = -21.173 - (-49.01) = 27.837\ K'$

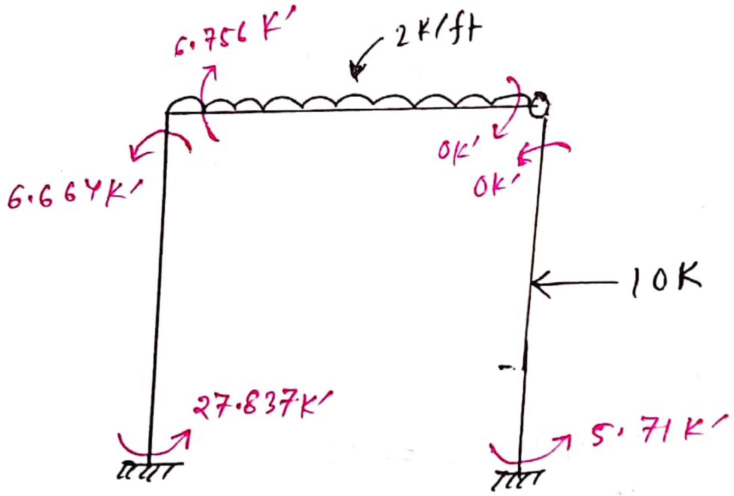
$M_{BA} = -2 \times 21.173 - (-49.01) = 6.664\ K'$

$M_{BC} = 16.67 - 2 \times 21.173 - (-18.92) = -6.756\ K'$

$M_{CB} = -16.67 - 21.173 - 2 \times (-18.92) = 0\ K'$

$M_{CD} = 12.5 - 2 \times 30.8 - (-49.01) = -0.09 K'$

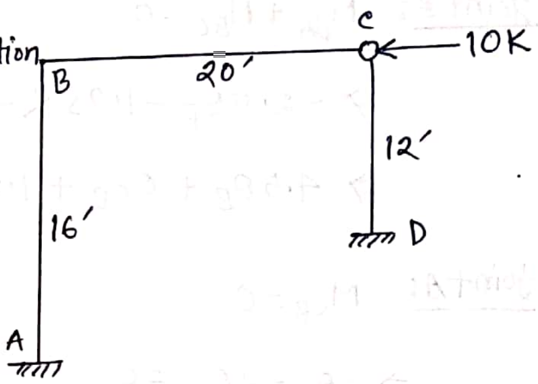
$M_{DC} = -12.5 - 30.8 - (-49.01) = 5.71\ K'\ K'$



(Class Test -13 Series) & (Class Test -15 Series)

A frame ABCD is loaded as shown in figure:

Find out the ends moment using slope-deflection Method. Draw BMD and deflected shape for the frame. EI is constant.



Solution: Relative Stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{16} \simeq 1.25$

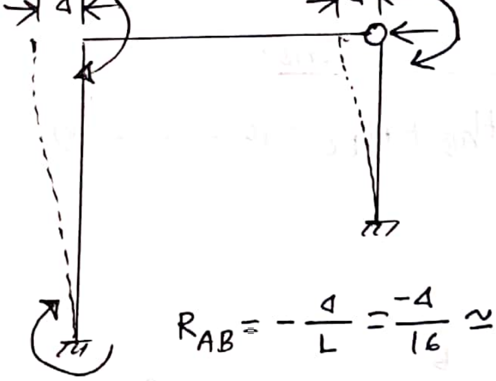
$K_{BC} = \frac{1}{20} \simeq 1$

$K_{CD} = \frac{1}{12} \simeq 1.67$

Fixed End Moments:

$F_{AB} = F_{BA} = F_{BC} = F_{CB} = F_{CD} = F_{DC} = 0$

Relative values of R:



$R_{AB} = -\frac{\Delta}{L} = \frac{-\Delta}{16} \simeq -R$

$R_{CD} = -\frac{\Delta}{12} \simeq -1.33R$

Slope Deflection Equations:

$M_{AB} = 0 + 1.25 \times \{-2\cancelto{0}{\theta_A} - \theta_B - R\}$

$= -1.25\theta_B - 1.25R$

$M_{BA} = 0 + 1.25 \times \{-\cancelto{0}{\theta_A} - 2\theta_B - R\} = -2.5\theta_B - 1.25R$

$M_{BC} = 0 + 1 \times \{-2\theta_B - \theta_{CB}\} = -2\theta_B - \theta_{CB}$

$M_{CB} = 0 + 1 \times \{-\theta_B - 2\theta_{CB}\} = -\theta_B - 2\theta_{CB}$

$M_{CD} = 0 + 1.67 \times \{-2\theta_{CD} - \cancelto{0}{\theta_D} - 1.33R\} = -3.34\theta_{CD} - 2.22R$

$M_{DC} = 0 + 1.67 \times \{-\theta_{CD} - 2\cancelto{0}{\theta_D} - 1.33R\} = -1.67\theta_{CD} - 2.22R$

joint conditions:

joint B: $M_{BA} + M_{Bc} = 0$

$\Rightarrow -2.5\theta_B - 1.25R - 2\theta_B - \theta_{cB} = 0$

$\Rightarrow 4.5\theta_B + \theta_{cB} + 1.25R = 0$ ..... ①

joint c: $M_{cB} = 0$ and, $M_{cD} = 0$

$\Rightarrow -\theta_B - 2\theta_{cB} = 0$

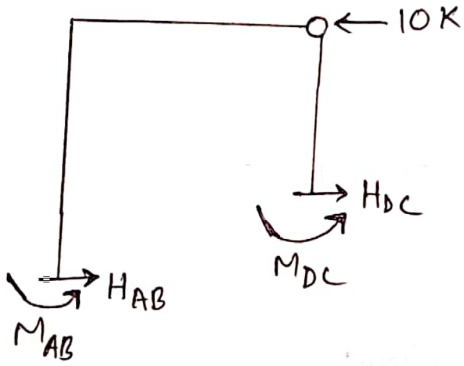
$\Rightarrow \theta_B + 2\theta_{cB} = 0$ ...... (ii)

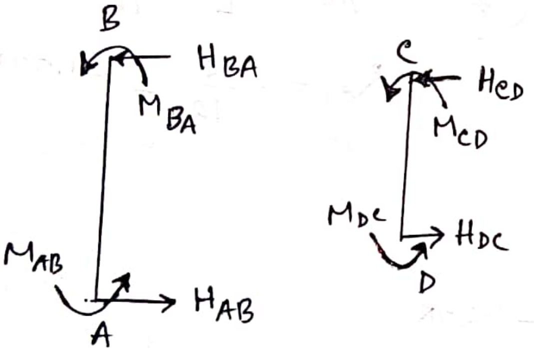
$\Rightarrow -3.34\theta_{cD} - 2.22R = 0$

$\Rightarrow 3.34\theta_{cD} + 2.22R = 0$ .... (iii)

Shear conditions:

$H_{AB} + H_{Dc} = 10$ ..... (iv)





$\Sigma M_B = 0$

$M_{AB} + M_{BA} + 16 \times H_{AB} = 0$

$\Rightarrow 16H_{AB} = 1.25\theta_B + 1.25R + 2.5\theta_B + 1.25R$

$\Rightarrow H_{AB} = \dfrac{3.75\theta_B + 2.5R}{16}$

$\Sigma M_c = 0$

$M_{Dc} + M_{cD} + H_{Dc} \times 12 = 0$

$\Rightarrow 12H_{Dc} = (3.34\theta_{cD} + 2.22R) + (1.67\theta_{cD} + 2.22R)$

$\Rightarrow H_{Dc} = \dfrac{5\theta_{cD} + 4.44R}{12}$

putting the value of H_{AB} and H_{DC} in eq^n (iv) we obtain,

$$\frac{3.75\theta_B + 2.5R}{16} + \frac{5\theta_{CD} + 4.44R}{12} = 10$$

$$\Rightarrow 12 \times (3.75\theta_B + 2.5R) + 16(5\theta_{CD} + 4.44R) = 10 \times 16 \times 12$$

$$\Rightarrow 45\theta_B + 30R + 80\theta_{CD} + 71.04R = 1920$$

$$\Rightarrow 45\theta_B + 80\theta_{CD} + 101.04R = +1920 \quad \text{......(v)}$$

From eq^n (i), (ii), (iii) & (v) we obtain,

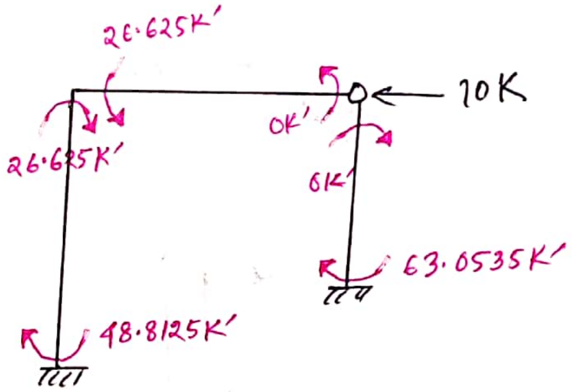
$\theta_B = -17.75$ rad.

$\theta_{CB} = 8.875$ rad

$\theta_{CD} = -37.75$ rad.

$R = 56.8$ rad.

Final End Moments:



$M_{AB} = -1.25 \times (-17.75) - 1.25 \times (+56.8)$
$= -48.8125\ K'$

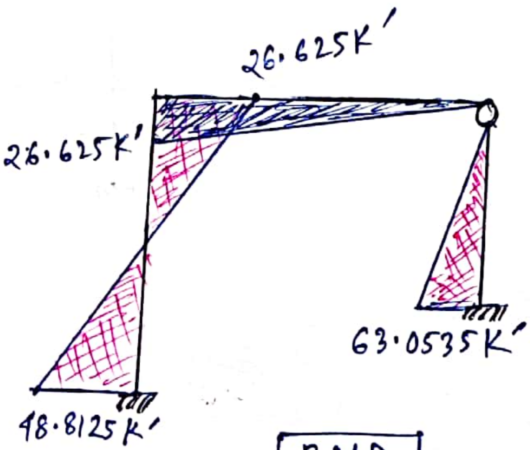
$M_{BA} = -2.5 \times (-17.75) - 1.25 \times (+56.8)$
$= -26.625\ K'$

$M_{BC} = -2 \times (-17.75) - (+8.875)$
$= 26.625\ K'$

$M_{CB} = +17.75 - 2 \times (+8.875)$
$= 0K'$

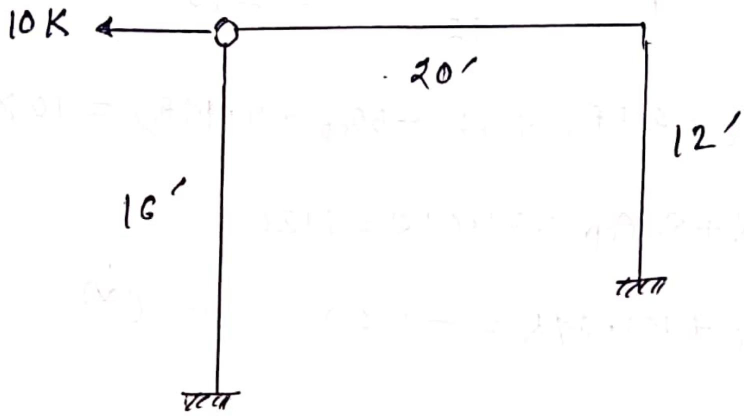
$M_{CD} = -3.34 \times (-37.75) - 2.22 \times (+56.8)$
$= -0.011\ K'$

$M_{DC} = -1.67 \times (-37.75) - 2.22 \times (+56.8)$
$= -63.0535K'$



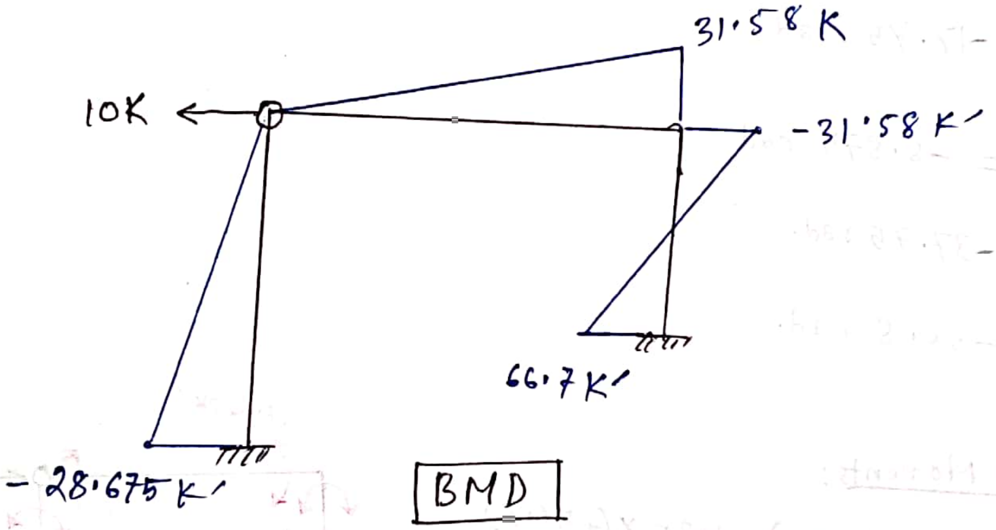
BMD

Class Test - 15 Series

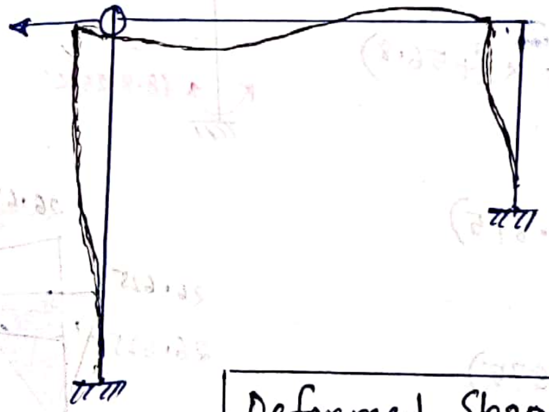


নিজে কর - Same concept & Same procedure

Answer:

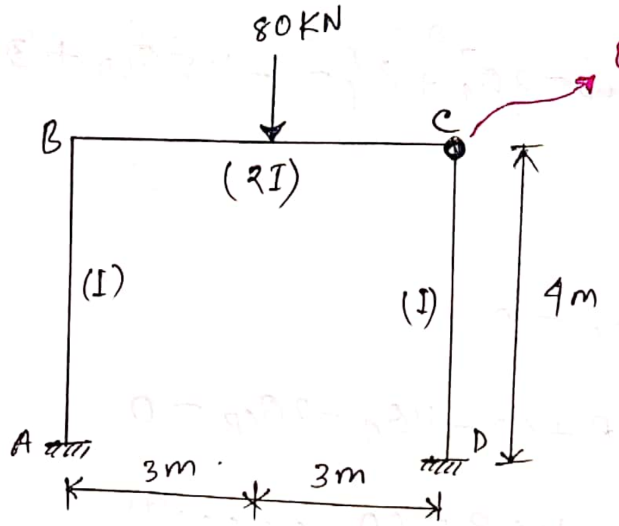


BMD



Deformed Shape

Problem: Analysis the frame and draw BMD.



θ_{CB} & θ_{CD}

* (sway হবে)

[একপাশের Link থাকে, এজন্য Frame is unsymmetrical]

Solution:

Relative Stiffness:

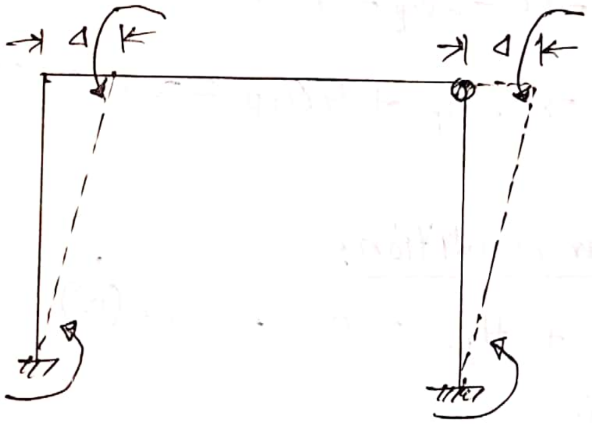
$K_{AB} = \frac{I}{L} = \frac{1}{4} \simeq 1.5$

$K_{BC} = \frac{2}{6} \simeq 2$

$K_{CD} = \frac{1}{4} \simeq 1.5$

Fixed End Moment:

$F_{BC} = -F_{CB} = 60 KN\text{-}m$



Relative values of R:

$R_{AB} = \frac{\Delta}{L} = \frac{\Delta}{4} \simeq R$

$R_{CD} = \frac{\Delta}{4} \simeq R$

Slope Deflection Equation:

$$M_{AB} = 0 + 1.5 \times \{-\cancel{2\theta_A}^{0} - \theta_B + R\} = -1.5\theta_B + 1.5R$$

$$M_{BA} = 0 + 1.5 \times \{-\cancel{\theta_A}^{0} - 2\theta_B + R\} = -3\theta_B + 1.5R$$

$$M_{BC} = 60 + 2 \times \{-2\theta_B - \theta_{CB}\} = 60 - 4\theta_B - 2\theta_{CB}$$

$$M_{CB} = -60 + 2 \times \{-\theta_B - 2\theta_{CB}\} = -60 - 2\theta_B - 4\theta_{CB}$$

$$M_{CD} = 0 + 1.5 \times \{-2\theta_{CD} - \overset{0}{\cancel{\theta_D}} + R\} = -3\theta_{CD} + 1.5R$$

$$M_{DC} = 0 + 1.5 \times \{-\theta_{CD} - \overset{0}{\cancel{2\theta_D}} + R\} = -1.5\theta_{CD} + 1.5R$$

Joint condition:

Joint B: $M_{BA} + M_{BC} = 0$

$$\Rightarrow -3\theta_B + 1.5R + 60 - 4\theta_B - 2\theta_{CB} = 0$$

$$\Rightarrow 7\theta_B + 2\theta_{CB} - 1.5R = 60 \quad \ldots\ldots ①$$

Joint C: $M_{CB} = 0$

$$-60 - 2\theta_B - 4\theta_{CB} = 0$$

$$\Rightarrow 2\theta_B + 4\theta_{CB} = -60 \quad \ldots\ldots (ii)$$

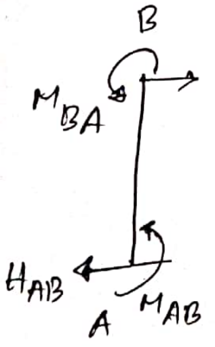
$M_{CD} = 0$

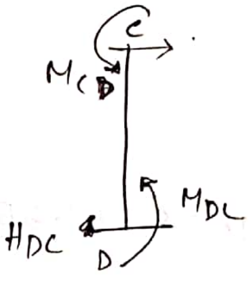
$$\Rightarrow -3\theta_{CD} + 1.5R = 0$$

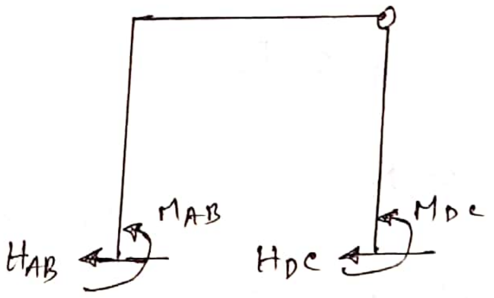
$$\Rightarrow 3\theta_{CD} - 1.5R = 0 \quad \ldots\ldots (iii)$$

Shear condition:

$$H_{AB} + H_{DC} = 0 \quad \ldots\ldots (iv)$$







$\Sigma M_B = 0$

$$M_{AB} + M_{BA} - 4H_{AB} = 0$$

$$\therefore H_{AB} = \frac{M_{AB} + M_{BA}}{4}$$

And, $\Sigma M_D = 0$

$$M_{DC} + M_{CD} - 4 \times H_{DC} = 0$$

$$\Rightarrow H_{DC} = \frac{M_{DC} + M_{CD}}{4}$$

From eq^n (iv) we obtain,

$$\frac{M_{AB}+M_{BA}}{4}+\frac{M_{DC}+M_{CD}}{4}=0$$

$$\Rightarrow M_{AB}+M_{BA}+M_{DC}+M_{CD}=0$$

$$\Rightarrow -4.5\theta_B+3R-4.5\theta_{CD}+3R=0$$

$$\Rightarrow -1.5\theta_B-1.5\theta_{CD}+2=0 \;\ldots\ldots (v)$$

Now, From eq^n ①, ⑪, ⑪⑪ & ⓥ

we obtain,

check:

$$\begin{array}{cccc} \theta_B & \theta_{CB} & \theta_{CD} & R \\ 7 & 2 & 0 & -1.5 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 3 & -1.5 \\ -1.5 & 0 & -1.5 & 2 \end{array}$$

$\theta_B = 21.4286$

$\theta_{CB} = -25.7143$

$\theta_{CD} = 12.857$

$R = 25.7143$

$\therefore M_{AB} = -1.5\times 21.4286+1.5\times 25.7143 = 6.43$ KN-m

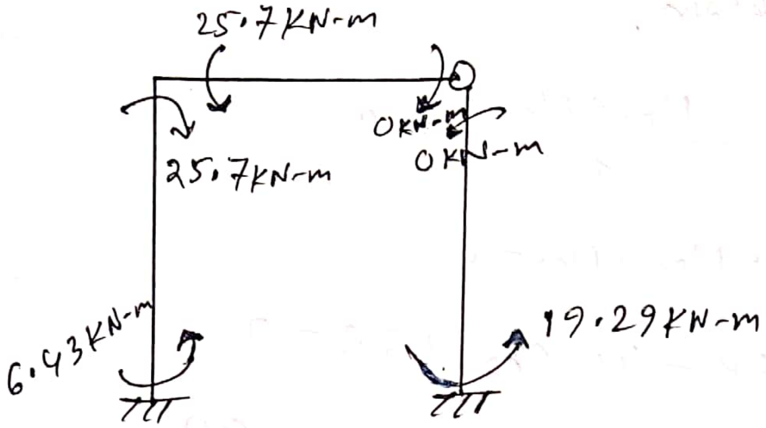
$M_{BA} = -3\times 21.4286+1.5\times 25.7143 = -25.7$ KN-m

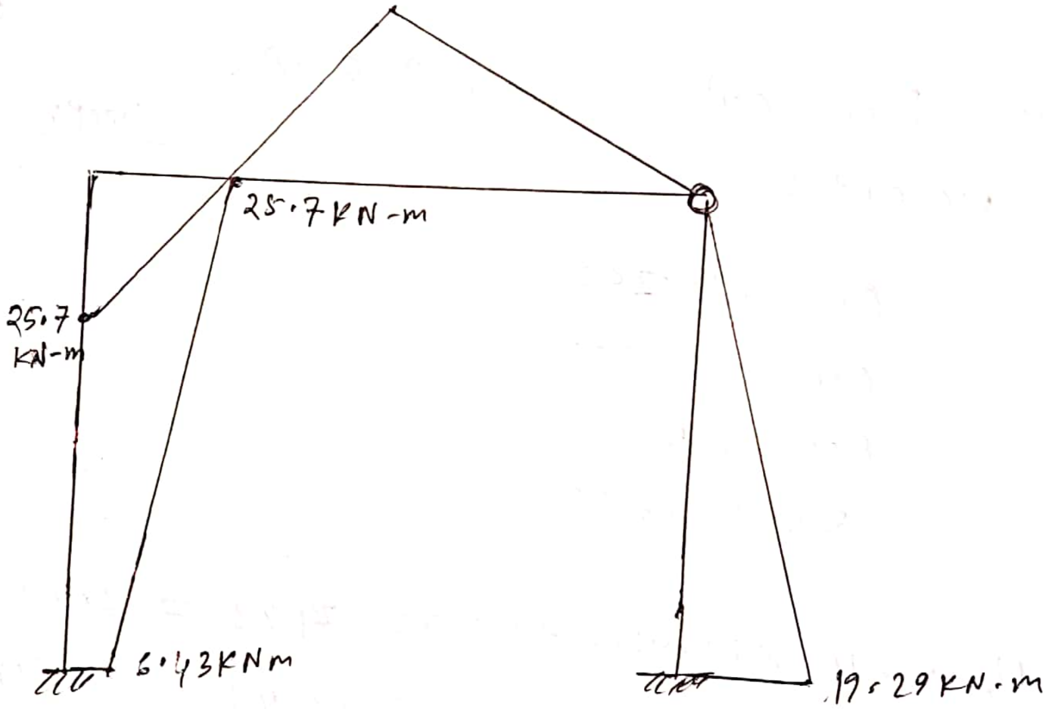
$M_{BC} = 60-4\times 21.4286-2\times(-25.7143) = 25.7$ KN-m

$M_{CB} = -60-2\times 21.4286-4\times(-25.7143) = 0$

$M_{CD} = -3\times 12.857+1.5\times 25.7143 = 0$

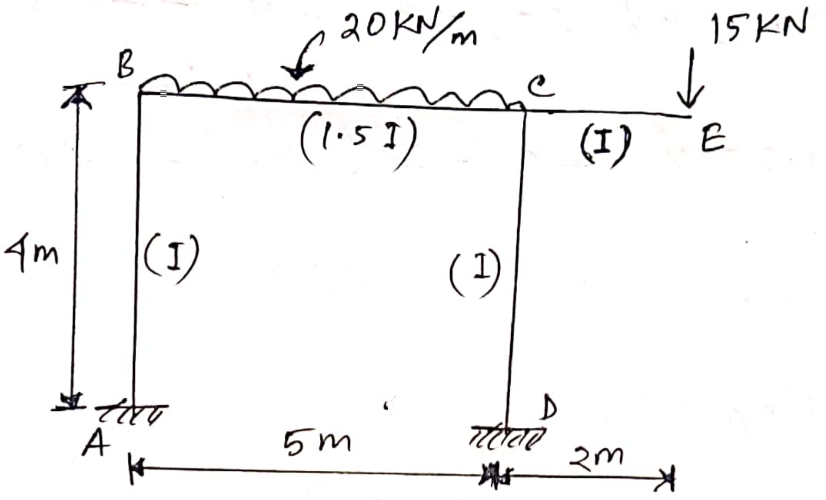
$M_{DC} = -1.5\times 12.857+1.5\times 25.7143 = 19.29$ KN-m


25.7 KN-m
25.7 KN-m
0 KN-m
0 KN-m
6.43 KN-m
19.29 KN-m


25.7 KN-m
25.7 KN-m
6.43 KNm
19.29 KN.m

BMD

Problem:



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{4} \simeq 1.25$

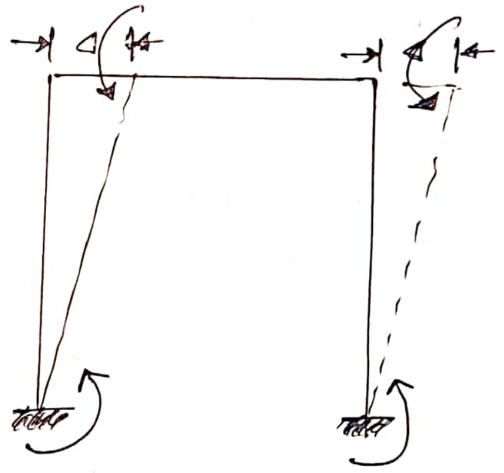
$K_{BC} = \frac{1.5}{5} \simeq 1.5$

$K_{CD} = \frac{1}{4} \simeq 1.25$

FEM:

$F_{BC} = -F_{CB} = \frac{20 \times 5^2}{12} = 41.67$ KN-m

$F_{CE} = 30$ KN-m



Relative values of R:

$R_{AB} = \frac{\Delta}{L} = \frac{\Delta}{4} \simeq R$

$R_{CD} = \frac{\Delta}{L} = \frac{\Delta}{4} \simeq R$

Slope deflection equations:

$M_{AB} = 0 + 1.25\{-\cancelto{0}{2\theta_A} - \theta_B + R\} = -1.25\theta_B + 1.25R$

$M_{BA} = 0 + 1.25\{-\cancelto{0}{\theta_A} - 2\theta_B + R\} = -2.5\theta_B + 1.25R$

$M_{BC} = 41.67 + 1.5 \times \{-2\theta_B - \theta_C\} = 41.67 - 3 \quad -1.5\theta_C$

$M_{CB} = -41.67 + 1.5 \times \{-\theta_B - 2\theta_C\} = -41.67 - 1.5\theta_B - 3\theta_C$

$M_{CE} = 30$ KN-m

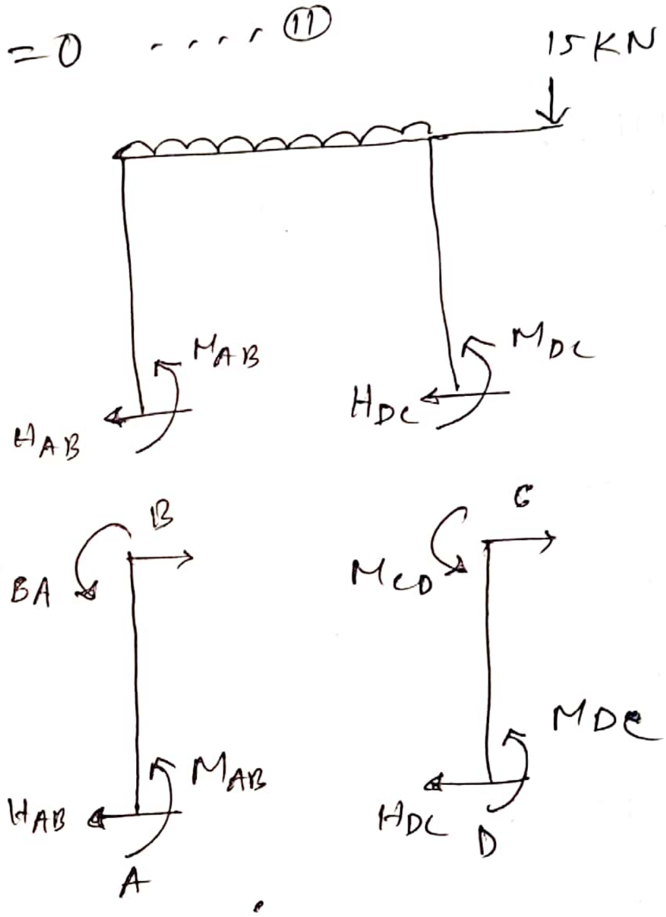
$M_{CD} = 0 + 1.25 \times \{-2\theta_C - \cancel{\theta_D}^{0} + R\} = -2.5\theta_C + 1.25R$

$M_{DC} = 0 + 1.25 \times \{-\theta_C - 2\cancel{\theta_D}^{0} + R\} = -1.25\theta_C + 1.25R$

joint conditions: (do your self)

joint B: $M_{BA} + M_{BC} = 0$ ..... ①

joint C: $M_{CB} + M_{CD} + M_{CE} = 0$ ..... ⑪



shear condition:

$H_{AB} + H_{DC} = 0$ ... (111)

$\Sigma M_B = 0$

$$H_{AB} = \frac{M_{AB} + M_{BA}}{4}$$

$\Sigma M_C = 0$

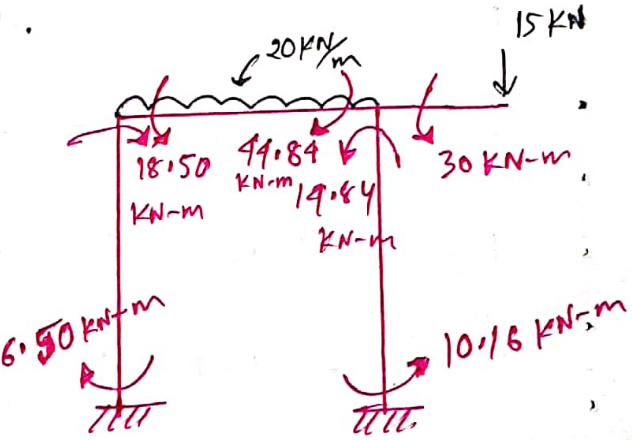
$$H_{DC} = \frac{M_{CD} + M_{DC}}{4}$$

From eqn (111) we obtain,

$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$ .... (iv)

Check Answer:

From eqn ①, ⑪, & (iv) we obtain.



$\theta_B =$ (do your self)

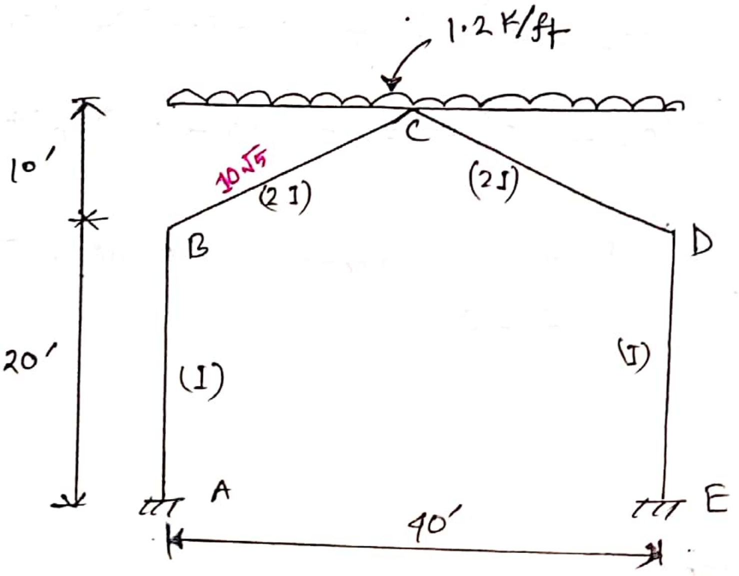
$\theta_C =$ Then find Moments

$R =$

Analysis of Gable Frame by the Slope-Deflection Method

2016, 2010, 2008, 2013

Problem: 05 Analyze the gable frame as shown in Figure. Draw SFD and BMD. Sketch deformed structure.



Solution:

Relative stiffness:

$$K_{AB} = \frac{I}{L} = \frac{1}{20} \simeq 1 = K_{DE}$$

$$K_{BC} = \frac{2}{\sqrt{20^2+10^2}} \simeq 1.79 = K_{CD}$$

Fixed End Moment:

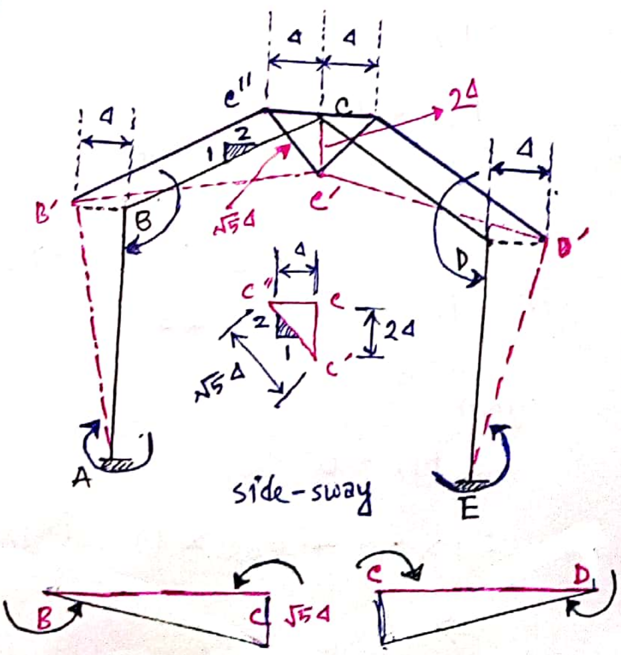
$$F_{AB} = F_{BA} = F_{DE} = F_{ED} = 0$$

$$F_{BC} = \frac{1.2 \times 20^2}{12} = 40\ K'$$

$$F_{CB} = -40\ K'$$

$$F_{CD} = 40K'$$

$$F_{DC} = -40K'$$



Relative values of R:

$$R_{AB} = \frac{-4}{20} \simeq -R \ , \ R_{DE} = \frac{4}{20} \simeq R$$

$$R_{BC} = \frac{\sqrt{54}}{10\sqrt{5}} \simeq 2R \ , \ R_{CD} = -\frac{\sqrt{54}}{10\sqrt{5}} \simeq -2R$$

Slope Deflection Equations:

(i) $\theta_A = \theta_E = 0$ (Given)
(ii) $\theta_C = 0$ (because of symmetry)
(iii) $\theta_D = -\theta_B$ (")

$$M_{AB} = 0 + 1\left(-2\cancelto{0}{\theta_A} - \theta_B - R\right) = -\theta_B - R$$

$$M_{BA} = 0 + 1\left(-2\theta_B - \cancelto{0}{\theta_A} - R\right) = -2\theta_B - R$$

$$M_{BC} = 40 + 1.79\left(-2\theta_B - \cancelto{0}{\theta_C} + 2R\right) = 40 - 3.58\theta_B + 3.58R$$

$$M_{CB} = -40 + 1.79\left(-\theta_B - 2\cancelto{0}{\theta_C} + 2R\right) = -40 - 1.79\theta_B + 3.58R$$

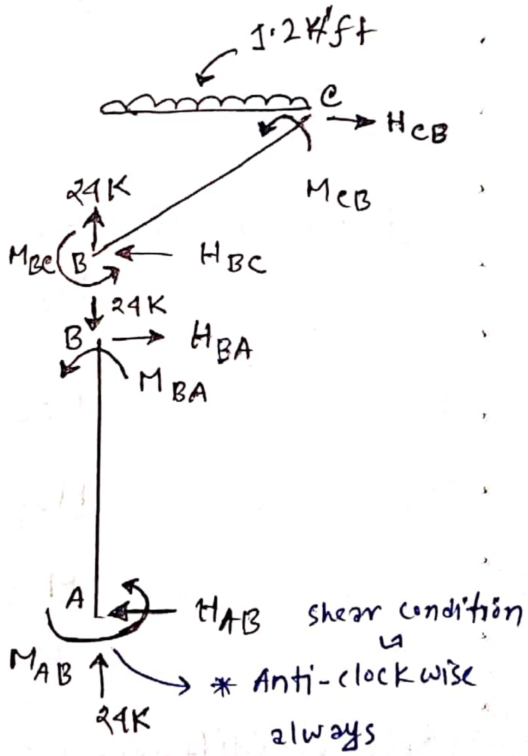
(* দুই পাশেই symmetric load & Symmetric structure, তাই যেকোনো এক পাশে analysis করলেই হবে)

Joint Conditions:

Joint B: $M_{BA} + M_{BC} = 0$

$$\Rightarrow -2\theta_B - R + 40 - 3.58\theta_B + 3.58R = 0$$

$$\Rightarrow 5.58\theta_B - 2.58R = 40 \quad \cdots\cdots\cdots\cdots ①$$



shear condition ↳ * Anti-clockwise always

Shear Conditions:

$$H_{BA} = H_{BC} \quad \cdots\cdots ⑪$$

$\Sigma M_A = 0$

$$\frac{M_{AB} + M_{BA}}{20} = H_{BA}$$

$$\Rightarrow H_{BA} = \frac{-\theta_B - R - 2\theta_B - R}{20} = \frac{-3\theta_B - 2R}{20}$$

Again,

$\Sigma M_C = 0$

$$24 \times 20 + H_{BC} \times 10 - M_{BC} - M_{CB} - 1.2 \times \frac{20^2}{2} = 0$$

$$\Rightarrow H_{BC} = \frac{M_{BC} + M_{CB} - 240}{10}$$

$$\Rightarrow H_{BC} = \frac{40 - 3.58\theta_B + 3.58R - 40 - 1.79\theta_B + 3.58R - 240}{10}$$

$$\therefore H_{BC} = \frac{-5.37\theta_B + 7.16R - 240}{10}$$

From equation, (ii)

$$\frac{-\theta_B - R - 2\theta_B - R}{20} = \frac{-5.37\theta_B + 7.16R - 240}{10}$$

$$\Rightarrow -\theta_B - R - 2\theta_B - R = -10.74\theta_B + 14.32R - 480$$

$$\Rightarrow 7.74\theta_B - 16.32R + 480 = 0 \quad \ldots\ldots \text{(iii)}$$

solving eqn ① & (iii) we obtain,

$$\theta_B = 26.6$$

$$R = 42.03$$

Now,

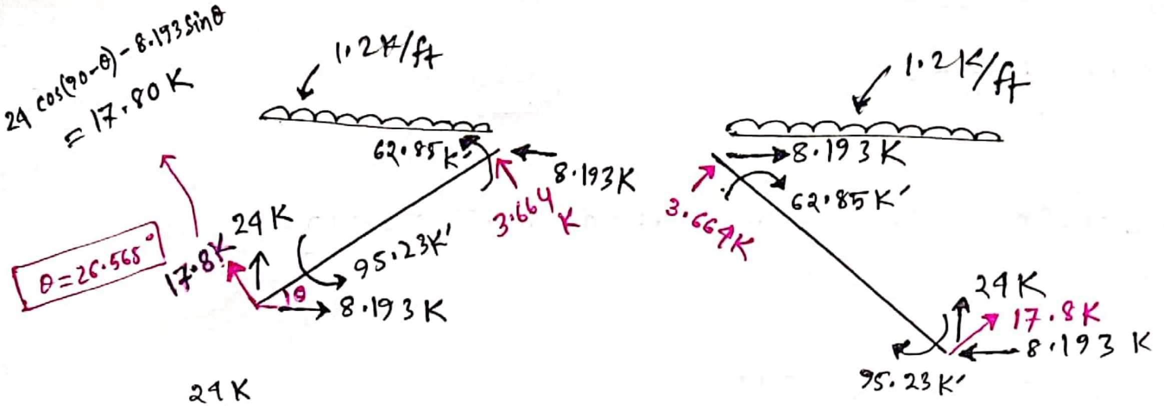
$$M_{AB} = -26.6 - 42.03 = -68.63 K'$$

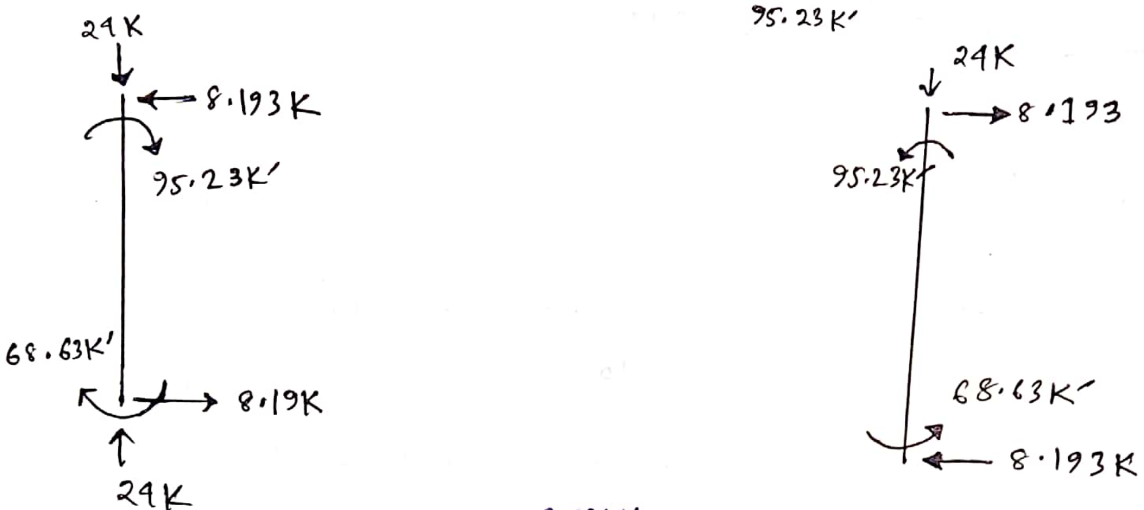
$$M_{BA} = -2 \times 26.6 - 42.03 = -95.23 K'$$

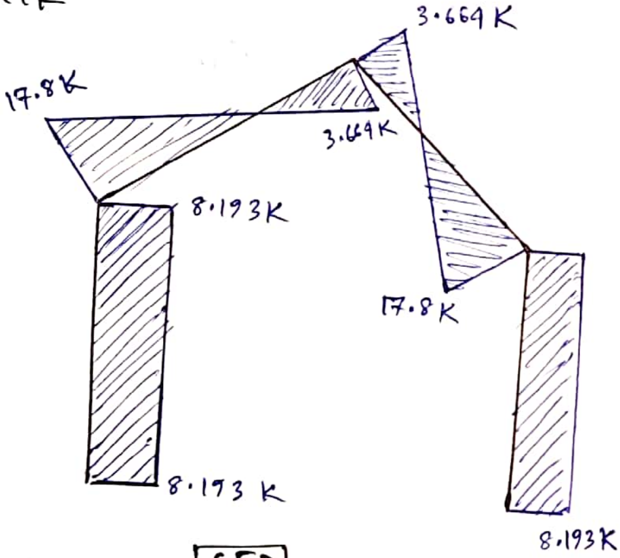
$$M_{BC} = 40 - 3.58 \times 26.6 + 3.58 \times 42.03 = 95.24 K'$$

$$M_{CB} = -40 - 1.79 \times 26.6 + 3.58 \times 42.03 = 62.85 K'$$

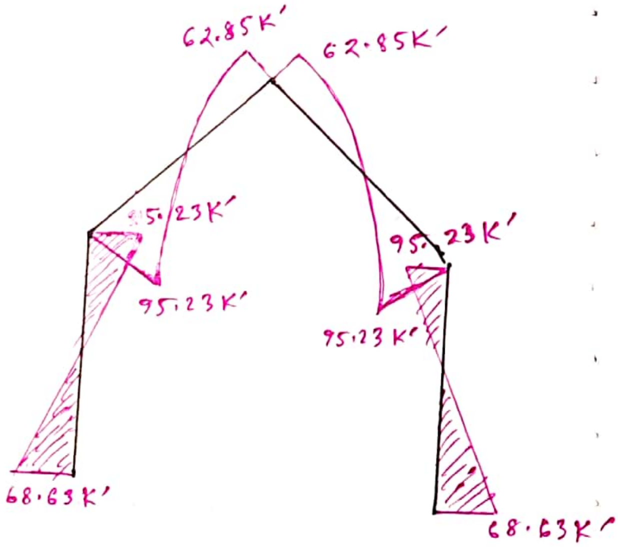
Now, $$H_{BA} = \frac{-3 \times 26.6 - 2 \times 42.03}{20} = -8.193K = H_{BC}$$



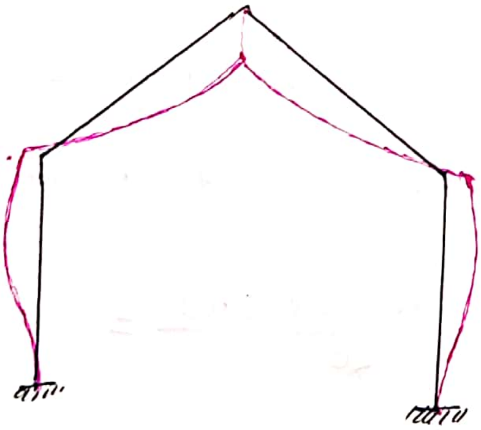




SFD

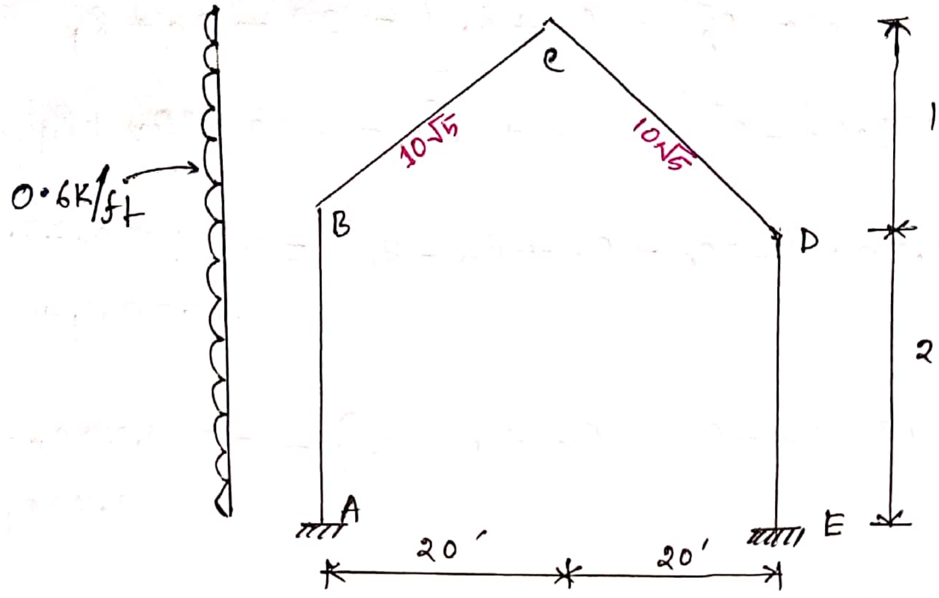


BMD



Deformed Shape

Problem: 06



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{20} \simeq 1$

$K_{BC} = \frac{2}{10\sqrt{5}} \simeq 1.79$

$K_{DE} = \frac{1}{20} \simeq 1$

$K_{CD} = \frac{2}{10\sqrt{5}} \simeq 1.79$

FEM:

$F_{AB} = \frac{0.6 \times 20^2}{12} = 20K'$

$F_{BA} = -20K'$

$F_{BC} = \frac{0.6 \times 10^2}{12} = 5K'$

$F_{CB} = -5K'$

Relative Values of R:

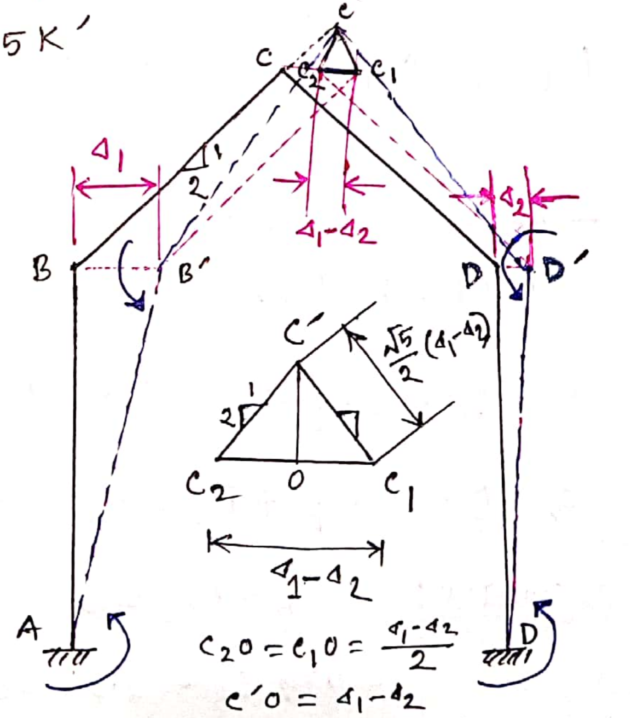
$R_{AB} = \frac{\Delta}{L} = \frac{\Delta_1}{20} \simeq R_1$

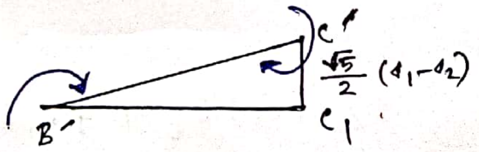
$R_{DE} = \frac{\Delta_2}{20} \simeq R_2$

$R_{BC} = \frac{c'c_1}{L} = -\frac{\sqrt{5}(\Delta_1-\Delta_2)}{2 \times 10\sqrt{5}} = -\left(\frac{\Delta_1-\Delta_2}{20}\right)$

$\therefore R_{BC} = -R_1 + R_2$

$R_{CD} = R_1 - R_2$





Slope deflection equations:

$$M_{AB} = 20 + 1 \times \{-\cancelto{0}{2\theta_A} - \theta_B + R_1\} = 20 - \theta_B + R_1$$

$$M_{BA} = -20 + 1 \times \{-\cancelto{0}{\theta_A} - 2\theta_B + R_1\} = -20 - 2\theta_B + R_1$$

$$M_{BC} = 5 + 1.79 \times \{-2\theta_B - \theta_C + R_2 - R_1\} = 5 - 3.58\theta_B - 1.79\theta_C - 1.79R_1 + 1.79R_2$$

$$M_{CB} = -5 + 1.79 \times \{-\theta_B - 2\theta_C + R_2 - R_1\} = -5 - 1.79\theta_B - 3.58\theta_C - 1.79R_1 + 1.79R_2$$

$$M_{CD} = 0 + 1.79 \times \{-2\theta_C - \theta_D + R_1 - R_2\} = -3.58\theta_C - 1.79\theta_D + 1.79R_1 - 1.79R_2$$

$$M_{DC} = 0 + 1.79 \times \{-\theta_C - 2\theta_D + R_1 - R_2\} = -1.79\theta_C - 3.58\theta_D + 1.79R_1 - 1.79R_2$$

$$M_{DE} = 0 + 1 \times \{-2\theta_D - \cancelto{0}{\theta_E} + R_2\} = -2\theta_D + R_2$$

$$M_{ED} = 0 + 1 \times \{-\theta_D - \cancelto{0}{2\theta_E} + R_2\} = -\theta_D + R_2$$

Joint conditions:

Joint B: $M_{BA} + M_{BC} = 0$

$$\Rightarrow -20 - 2\theta_B + R_1 + 5 - 3.58\theta_B - 1.79\theta_C - 1.79R_1 + 1.79R_2 = 0$$

$$\Rightarrow 5.58\theta_B + 1.79\theta_C + 0.79R_1 - 1.79R_2 = -15 \ldots\ldots ①$$

Joint C: $M_{CB} + M_{CD} = 0$

$$\Rightarrow -5 - 1.79\theta_B - 3.58\theta_C - 1.79R_1 + 1.79R_2 - 3.58\theta_C - 1.79\theta_D + 1.79R_1 - 1.79R_2 = 0$$

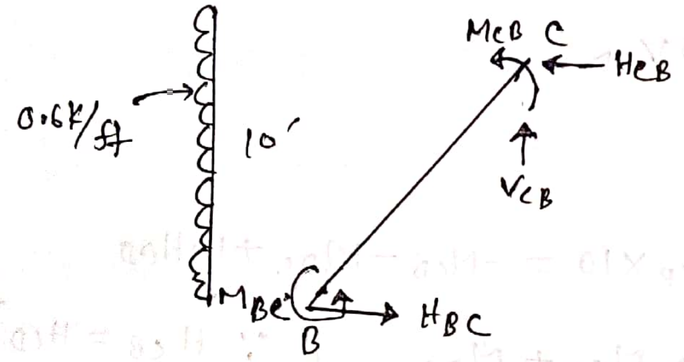
$$\Rightarrow 1.79\theta_B + 7.16\theta_C + 1.79\theta_D = -5 \ldots\ldots (ii)$$

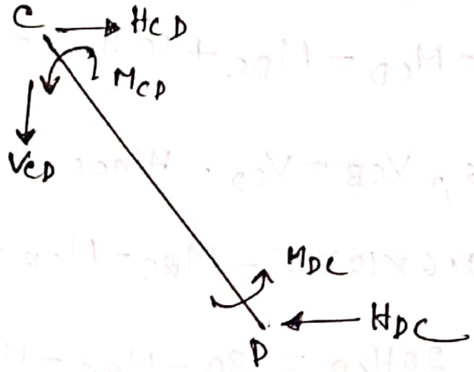
Joint D: $M_{DC} + M_{DE} = 0$

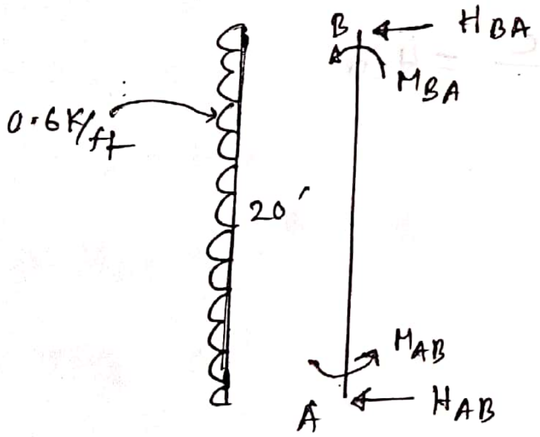
$\Rightarrow -1.79\,\theta_C - 3.58\theta_D + 1.79 R_1 - 1.79 R_2 - 2\theta_D + R_2 = 0$

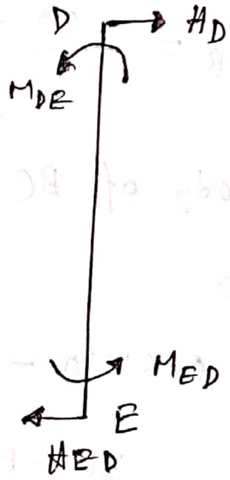
$\Rightarrow 1.79\,\theta_C + 5.58\theta_D - 1.79 R_1 + 0.79 R_2 = 0$ ......(iii)

Shear conditions:









Shear conditions:

$H_{BA} = H_{BC}$ ......(iv)

$H_{DC} = H_{DE}$ ......(v)

$H_{CB} = H_{CD}$ ......(vi)

$V_{CB} = V_{CD}$ ....(vii)

free body of BC,

$\Sigma M_B = 0$

$0.6 \times 10 \times 5 - M_{BC} - M_{CB} - H_{CB} \times 10 = 20 V_{CB}$

And, free body of CD

$\Sigma M_D = 0$

$- M_{CD} - M_{DC} + 10 H_{CD} = 20 V_{CD}$

As, $V_{CB} = V_{CD}$. Hence,

$0.6 \times 10 \times 5 - M_{BC} - M_{CB} - H_{CB} \times 10 = - M_{CD} - M_{DC} + 10 H_{CD}$

$\Rightarrow 20 H_{CB} = 30 - M_{BC} - M_{CB} + M_{CD} + M_{DC}$ $[\because H_{CB} = H_{CD}]$

$$\Rightarrow H_{CB} = \frac{30 - M_{BC} - M_{CB} + M_{CD} + M_{DC}}{20} = H_{CD}$$

free body of BC,

$\Sigma F_x = 0$

$H_{BC} + 0.6 \times 10 - H_{CB} = 0$

$$\Rightarrow H_{BC} = \frac{30 - M_{BC} - M_{CB} + M_{CD} + M_{DC} - 120}{20}$$

free body of AB,

$\Sigma M_A = 0$

$- M_{AB} - M_{BA} + 0.6 \times 20 \times 10 - 20 H_{BA} = 0$

$$\Rightarrow H_{BA} = \frac{- M_{AB} - M_{BA} + 120}{20}$$

From equation (iv) we obtain, $H_{BA} = H_{BC}$

$$\frac{-M_{AB} - M_{BA} + 120}{20} = \frac{30 - M_{BC} - M_{CB} + M_{CD} + M_{DC} - 120}{20}$$

$$\Rightarrow M_{AB} + M_{BA} - M_{BC} - M_{CB} + M_{CD} + M_{DC} = 210$$

$$\Rightarrow -3\theta_B + 2R_1 + 5.37\theta_B + 5.\cancel{37}\theta_C + 3.58R_1 - 3.58R_2 - 5.\cancel{37}\theta_C$$
$$- 5.37\theta_D + 3.58R_1 - 3.58R_2 = 210$$

$$\Rightarrow 2.37\theta_B - 5.37\theta_D + 9.16R_1 - 7.16R_2 = 210$$

$$\Rightarrow 0.79\theta_B - 1.79\theta_D + 3.0533R_1 - 2.3867R_2 = 70 \quad \text{.....(viii)}$$

Free body of CD,

$\Sigma F_x = 0$

$H_{CD} = H_{DC}$

$$\Rightarrow H_{DC} = \frac{30 - M_{BC} - M_{CB} + M_{CD} + M_{DC}}{20}$$

$\Sigma M_E = 0$

$20 \times H_{DE} - M_{DE} - M_{ED} = 0$

$$\Rightarrow H_{DE} = \frac{M_{DE} + M_{ED}}{20}$$

From equation (v) we obtain, $H_{DE} = D_{DC}$

$$\Rightarrow \frac{30 - M_{BC} - M_{CB} + M_{CD} + M_{DC}}{20} = \frac{M_{DE} + M_{ED}}{20}$$

$$\Rightarrow M_{BC} + M_{CB} - M_{CD} - M_{DC} + M_{DE} + M_{DE} = 30$$

$$\Rightarrow -5.37\theta_B - 5.\cancel{37}\theta_C - 3.58R_1 + 3.58R_2 + 5.\cancel{37}\theta_C + 5.37\theta_D$$
$$- 3.58R_1 + 3.58R_2 - 3\theta_D + 2R_2 = 0$$

$\Rightarrow -5.37\theta_B + 2.37\theta_D - 7.16R_1 + 9.16R_2 = 30$

$\Rightarrow -1.79\theta_B + 0.79\theta_D - 2.3867R_1 + 3.0533R_2 = 10$ ......(ix)

Check:

θ_B	θ_C	θ_D	R_1	R_2	C
5.58	1.79	0	0.79	-1.79	-15
1.79	7.16	1.79	0	0	-5
0	1.79	5.58	-1.79	0.79	0
0.79	0	-1.79	3.0533	-2.3867	70
-1.79	0	0.79	-2.3867	3.0533	10

From equation (iii) we get,

$1.79\theta_C = -5.58\theta_D + 1.79R_1 - 0.79R_2$

putting this value in equation (i) & (ii) we obtain,

From (i),

$5.58\theta_B - 5.58\theta_D + 1.79R_1 - 0.79R_2 + 0.79R_1 - 1.79R_2 = -15$

$\Rightarrow 5.58\theta_B - 5.58\theta_D + 2.58R_1 - 2.58R_1 = -15$ ......(x)

From (ii),

$1.79\theta_B + 4 \times (-5.58\theta_D + 1.79R_1 - 0.79R_2) + 1.79\theta_D = -5$

$\Rightarrow 1.79\theta_B - 20.53\theta_D + 7.16R_1 - 3.16R_2 = -5$ ......(xi)

From eqⁿ (viii), (ix), (X), (xi) we obtain,

$\theta_B = 11.891$

$\theta_D = 21.866$

$R_1 = 93.206$

$R_2 = 77.446$

Now, $\theta_C = \dfrac{-5.58\times 21.866 + 1.79\times 93.206 - 0.79\times 77.446}{1.79} = -9.137$

Thus,

$M_{AB} = 20 - 11.891 + 93.206 = 101.315\,K'$

$M_{BA} = -20 - 2\times 11.891 + 93.206 = 49.424\,K'$

$M_{BC} = 5 - 3.58\times 11.891 - 1.79\times(-9.137) - 1.79\times 93.206 + 1.79\times 77.446$

$= -49.425\,K'$

$M_{CB} = -5 - 1.79\times 11.891 - 3.58\times(-9.137) - 1.79\times 93.206 + 1.79\times 77.446$

$= -21.785\,K'$

$M_{CD} = -3.58\times(-9.137) - 1.79\times 21.866 + 1.79\times 93.206 - 1.79\times 77.446$

$= 21.781\,K'$

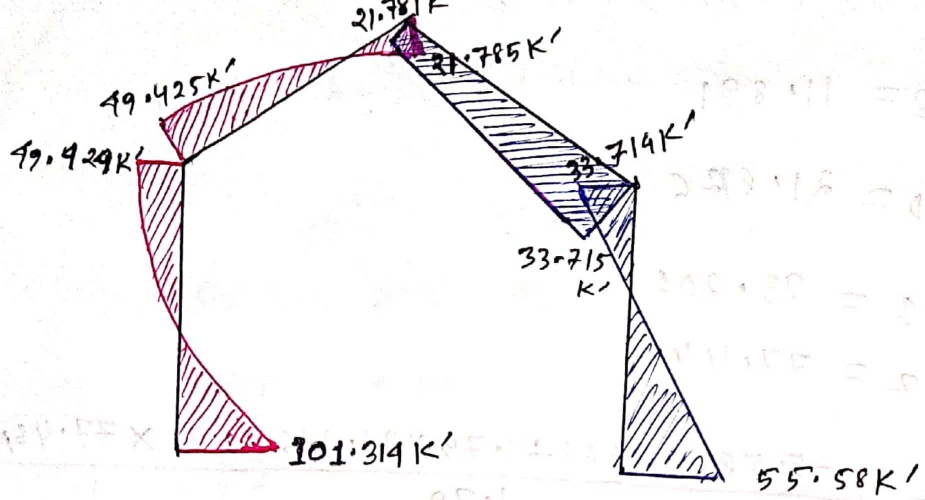
$M_{DC} = -1.79\times(-9.137) - 3.58\times 21.866 + 1.79\times 93.206 - 1.79\times 77.446$

$= -33.715\,K'$

$M_{DE} = -2\times 21.866 + 77.446 = 33.714\,K'$

$M_{ED} = -21.866 + 77.446 = 55.58\,K'$

(Ans.)

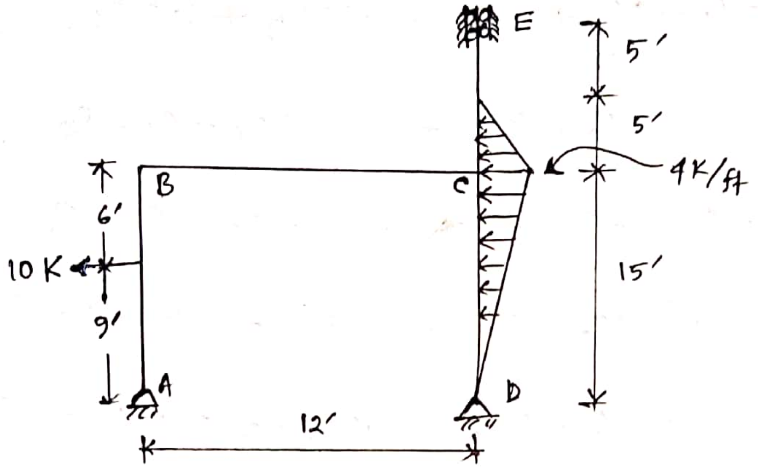


BMD

Slope Deflection Method

2018

\# Analyze the frame shown in figure. Draw BMD of the frame. EI is constant.



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{15} \simeq 1$

$K_{BC} = \frac{1}{12} \simeq 1.25$

$K_{CD} = \frac{1}{15} \simeq 1$

$K_{CE} = \frac{1}{10} \simeq 1.5$

Fixed End Moments:

$F_{AB} = -\frac{10 \times 6^2 \times 9}{15^2} = -14.4\,K'$

$F_{BA} = \frac{10 \times 6 \times 9^2}{15^2} = 21.6\,K'$

$F_{CD} = \frac{4 \times 15^2}{20} = 45\,K'$

$F_{DC} = \frac{-4 \times 15^2}{30} = -30\,K'$

$F_{CE} = -\frac{23}{960} \times 4 \times 10^2 = -9.5833\,K'$

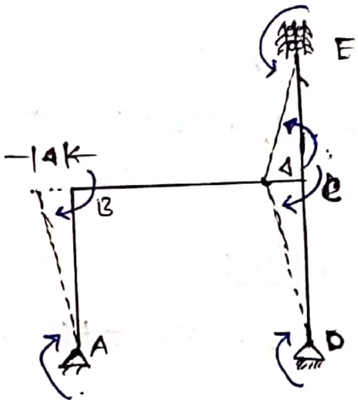
$F_{EC} = \frac{7}{960} \times 4 \times 10^2 = 2.9167\,K'$

Relative values of R:

$R_{AB} = \frac{\Delta}{L} = \frac{-\Delta}{15} \simeq -R$

$R_{CD} = \frac{-\Delta}{15} \simeq -R$

$R_{CE} = \frac{\Delta}{10} \simeq 1.5R$



Slope deflection Equations:

$M_{AB} = -14.4 + 1 \times \{-2\theta_A - \theta_B - R\}$

$= -14.4 - 2\theta_A - \theta_B - R$

$M_{BA} = 21.6 + 1 \times \{-\theta_A - 2\theta_B - R\} = 21.6 - \theta_A - 2\theta_B - R$

$M_{BC} = 0 + 1.25 \times \{-2\theta_B - \theta_C + 0\} = -2.5\theta_B - 1.25\theta_C$

$M_{CB} = 0 + 1.25 \times \{-\theta_B - 2\theta_C + 0\} = -1.25\theta_B - 2.5\theta_C$

$M_{CD} = 45 + 1 \times \{-2\theta_C - \theta_D - R\} = 45 - 2\theta_C - \theta_D - R$

$M_{DC} = -30 + 1 \times \{-\theta_C - 2\theta_D - R\} = -30 - \theta_C - 2\theta_D - R$

$M_{CE} = -9.5833 + 1.5 \times \{-2\theta_C - \cancel{\theta_E}^{\,0} + 1.5R\} = -9.5833 - 3\theta_C + 2.25R$

$M_{EC} = 2.9167 + 1.5 \times \{-\theta_C - \cancel{2\theta_E}^{\,0} + 1.5R\} = 2.9167 - 1.5\theta_C + 2.25R$

<u>joint conditions:</u> <u>joint A:</u> $M_{AB} = 0$ $\Rightarrow 2\theta_A + \theta_B + R = -19.4$ ...... (I)

<u>joint B:</u> $M_{BA} + M_{BC} = 0$

$\Rightarrow 21.6 - \theta_A - 2\theta_B - R - 2.5\theta_B - 1.25\theta_C = 0$

$\Rightarrow \theta_A + 4.5\theta_B + 1.25\theta_C + R = 21.6$ ...... (II)

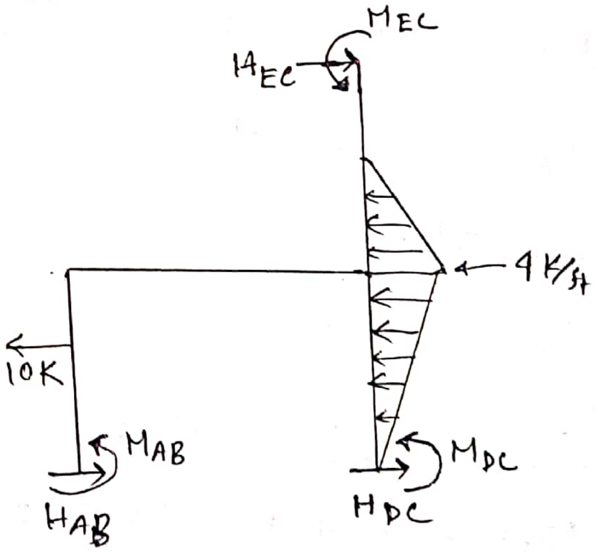
<u>joint C:</u> $M_{CB} + M_{CD} + M_{CE} = 0$

$\Rightarrow -1.25\theta_B - 2.5\theta_C + 45 - 2\theta_C - \theta_D - R - 9.5833 - 3\theta_C + 2.25R = 0$

$\Rightarrow 1.25\theta_B + 7.5\theta_C + \theta_D - 1.25R = 35.4167$ .... (III)

<u>joint D:</u> $M_{DC} = 0$

$\Rightarrow \theta_C + 2\theta_D + R = -30.0$ ..... (IV)



<u>Shear Conditions</u>

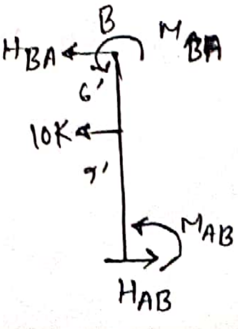
$H_{AB} + H_{DC} + H_{EC} = 50$ ...... (V)

$\Sigma M_B = 0$

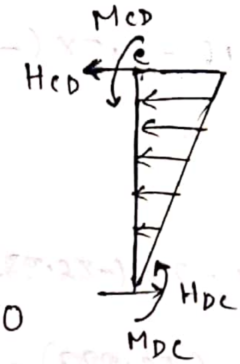
$M_{AB} + M_{BA} - 10 \times 6 + H_{AB} \times 15 = 0$

$\Rightarrow -14.4 - 2\theta_A - \theta_B - R + 21.6 - \theta_A - 2\theta_B - R$

$-60 + H_{AB} \times 15 = 0$



$$\Rightarrow H_{AB} = \frac{3\theta_A + 3\theta_B + 2R + 52.8}{15}.$$

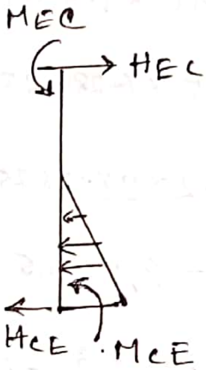


$$\Sigma M_C = 0$$

$$M_{CD} + M_{DC} + H_{DC} \times 15 - \tfrac{1}{2} \times 15 \times 4 \times (\tfrac{1}{3} \times 15) = 0$$

$$\Rightarrow 45 - 2\theta_C - \theta_D - R - 30 - \theta_C - 2\theta_D R - 150 + H_{DC} \times 15 = 0$$

$$\Rightarrow H_{DC} = \frac{3\theta_C + 3\theta_D + 2R + 135}{15}$$



$$\Sigma M_C = 0$$

$$M_{CE} + M_{EC} + \tfrac{1}{2} \times 4 \times 5 \times \tfrac{1}{3} \times 5 - H_{EC} \times 10 = 0$$

$$\Rightarrow H_{EC} = \frac{-4.5\theta_C + 4.5R + 10.0}{10}$$

Now, putting the values of H_{AB}, H_{DC} & H_{EC} in to the equation ⓥ

$$\frac{3\theta_A + 3\theta_B + 2R + 52.8}{15} + \frac{3\theta_C + 3\theta_D + 2R + 135}{15} + \frac{-4.5\theta_C + 4.5R + 10.0}{10} = 50$$

$$\Rightarrow 30\theta_A + 30\theta_B + 20R + 528 + 30\theta_C + 30\theta_D + 20R + 1350 - 67.5\theta_C + 67.5R + 150 = 150 \times 50$$

$$\Rightarrow 30\theta_A + 30\theta_B - 37.5\theta_C + 30\theta_D + 107.5R = 5472$$

$$\Rightarrow \theta_A + \theta_B - 1.25\theta_C + \theta_D + 3.5833R = 182.4 \quad \ldots\ldots\ldots(vi)$$

From eqn (ii) we obtain, $\theta_A = 21.6 - 4.5\theta_B - 1.25\theta_C - R$. putting the value of θ_A in eqn ① & (vi) we obtain,

$$-8\theta_B - 2.5\theta_C - R = -57.6 \quad \ldots\ldots (vii)$$

and, $-3.5\theta_B - 2.5\theta_C + \theta_D + 2.5833R = 160.8 \quad \ldots\ldots(viii)$

From eqn (iii), (iv), (vii), (viii), we get,

$\theta_B = -18.00$
$\theta_C = 37.4265$
$\theta_D = -87.733$
$R = 108.04$

$\therefore \theta_A = 21.6 - 4.5 \times (-18.00) - 1.25 \times (37.4265) - 108.04 = -52.223$

Now,

$M_{AB} = -14.4 - 2 \times (-52.223) - (-18.00) - (108.04) = 0 \, K'$

$M_{BA} = 21.6 - (-52.223) - 2 \times (-18.00) - 108.04 = 1.783 \, K'$

$M_{BC} = -2.5 \times (-18.00) - 1.25 \times 37.4265 = -1.783 \, K'$

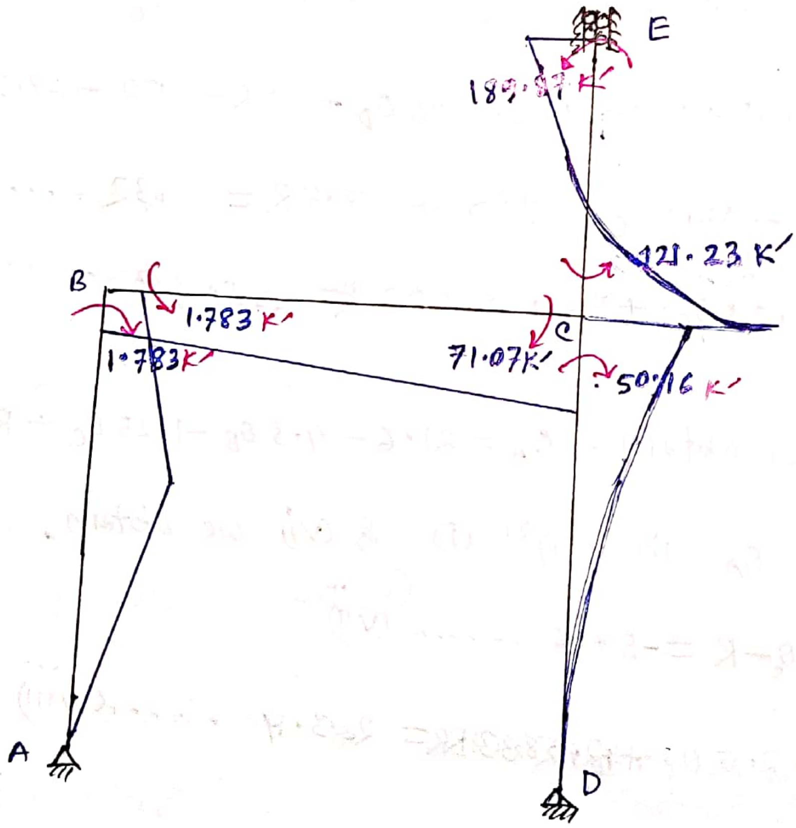
$M_{CB} = -1.25 \times (-18.00) - 2.5 \times 37.4265 = -71.07 \, K'$

$M_{CD} = 45 - 2 \times 37.4265 - (-87.733) - 108.04 = -50.16 \, K'$

$M_{DC} = -30 - 37.4265 - 2 \times (-87.733) - 108.04 = 0 \, K'$

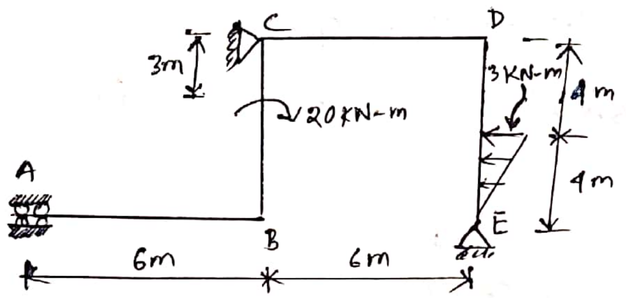
$M_{CE} = -9.5833 - 3 \times 37.4265 + 2.25 \times 108.04 = 121.23 \, K'$

$M_{EC} = 2.9167 - 1.5 \times 37.4265 + 2.25 \times 108.04 = 189.87 \, K'$



2017

#



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{6} \simeq 2$

$K_{BC} = \frac{1}{8} \simeq 1.5$

$K_{CD} = \frac{1}{6} \simeq 2$

$K_{DE} = \frac{1}{8} \simeq 1.5$

Fixed End Moment:

$F_{CB} = -\frac{Mb(2a-b)}{L^2} = -\frac{20 \times 5 \times (2\times3-5)}{8^2} = -1.5625$ KN-m

$F_{BC} = -\frac{Ma(2b-a)}{L^2} = -\frac{20\times3\times(2\times5-3)}{8^2} = -6.5625$ KN-m

$F_{DE} = \frac{3}{160} wL^2 = \frac{3}{160} \times 3 \times 8^2 = 3.6$ KN-m

$F_{ED} = \frac{-1}{30} WL^2 = \frac{1}{30} \times 3 \times 8^2 = -6.4$ KN-m

Slope deflection equations:

$M_{AB} = 0 + 2\times\{-2\overset{0}{\cancel{\theta_A}} - \theta_B + 0\} = -2\theta_B$

$M_{BA} = 0 + 2\times\{-\overset{0}{\cancel{\theta_A}} - 2\theta_B + 0\} = -4\theta_B$

$M_{BC} = -6.5625 + 1.5\times\{-2\theta_B - \theta_C - R\} = -6.5625 - 3\theta_B - 1.5\theta_C - 1.5R$

$M_{CB} = -1.5625 + 1.5\times\{-\theta_B - 2\theta_C - R\} = -1.5625 - 1.5\theta_B - 3\theta_C - 1.5R$

$M_{CD} = 0 + 2\times\{-2\theta_C - \theta_D\} = -4\theta_C - 2\theta_D$

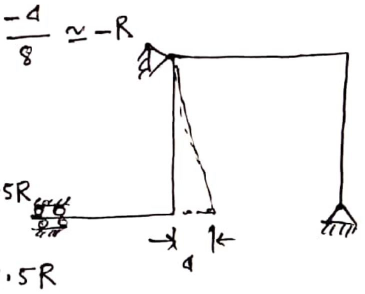
$M_{DC} = 0 + 2\times\{-\theta_C - 2\theta_D\} = -2\theta_C - 4\theta_D$

$M_{DE} = 3.6 + 1.5\times\{-2\theta_D - \theta_E\} = 3.6 - 3\theta_D - 1.5\theta_E$

$M_{ED} = -6.4 + 1.5\times\{-\theta_D - 2\theta_E\} = -6.4 - 1.5\theta_D - 3\theta_E$

Relative values of R:

$R_{BC} = \frac{\Delta}{L} = \frac{-\Delta}{8} \simeq -R$



Joint conditions:

Joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -4\theta_B - 6.5625 - 3\theta_B - 1.5\theta_C - 1.5R = 0$

$\Rightarrow 7\theta_B + 1.5\theta_C + 1.5R = -6.5625$ ...... ①

joint C: $M_{CB} + M_{CD} = 0$

$\Rightarrow -1.5625 - 1.5\theta_B - 3\theta_C - 1.5R - 4\theta_C - 2\theta_D = 0$

$\Rightarrow 1.5\theta_B + 7\theta_C + 2\theta_D + 1.5R = -1.5625$ ...... (ii)

joint D: $M_{DC} + M_{DE} = 0$

$\Rightarrow -2\theta_C - 4\theta_D + 3.6 - 3\theta_D - 1.5\theta_E = 0$

$\Rightarrow 2\theta_C + 7\theta_D + 1.5\theta_E = 3.6$ ...... (iii)

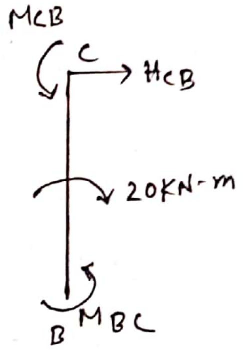
joint E: $M_{ED} = 0$

$\Rightarrow -6.4 - 1.5\theta_D - 3\theta_E = 0$

$\Rightarrow 1.5\theta_D + 3\theta_E = -6.4$ ...... (iv)

shear condition:

$H_{CB} = 0$



$\Sigma M_B = 0$

$M_{BC} + M_{CB} - 20 - 8 \times H_{CB} = 0$

$\Rightarrow -6.5625 - 3\theta_B - 1.5\theta_C - 1.5R - 1.5625 - 1.5\theta_B$

$- 3\theta_C - 1.5R - 20 = 0$ $[\because H_{CB} = 0]$

$\Rightarrow -4.5\theta_B - 4.5\theta_C - 3R - 28.125 = 0$

$\Rightarrow 1.5\theta_B + 1.5\theta_C + R = -28.125$ ...... (v)

Check:

θ_B	θ_C	θ_D	θ_E	R
7	1.5	0	0	1.5
1.5	7	2	0	1.5
0	2	7	1.5	0
0	0	1.5	3	0
1.5	1.5	0	0	1

From eqn (iv) we obtain,

$3\theta_E = -6.4 - 1.5\theta_D$

$\Rightarrow 1.5\theta_E = -3.2 - 0.75\theta_D$. putting

this value in the eqn of (iii) we obtain,

$2\theta_C + 7\theta_D - 3.2 - 0.75\theta_D = 3.6$

$\Rightarrow 2\theta_C + 6.25\theta_D = 6.8$ .... (vi)

Now, From equation ①, (ii), (vi) & (v) we obtain,

$\theta_B = 9.243$

$\theta_C = 11.042$

$\theta_D = -2.445$

$R = -58.553$

Hence,

$$\theta_E = \frac{-3.2 - 0.75\times(-2.445)}{1.5} = -0.911$$

$M_{AB} = -2\times 9.243 = -18.486$ K'

$M_{BA} = -4\times 9.243 = -36.972$ K'

$M_{BC} = -6.5625 - 3\times 9.243 - 1.5\times 11.042 - 1.5\times(-58.553) = 36.975$ K'

$M_{CB} = -1.5625 - 1.5\times 9.243 - 3\times 11.042 - 1.5\times(-58.553) = 39.2765$ K'

$M_{CD} = -4\times(11.042) - 2\times(-2.445) = -39.278$ K'

$M_{DC} = -2\times(11.042) - 4\times(-2.445) = -12.304$ K'

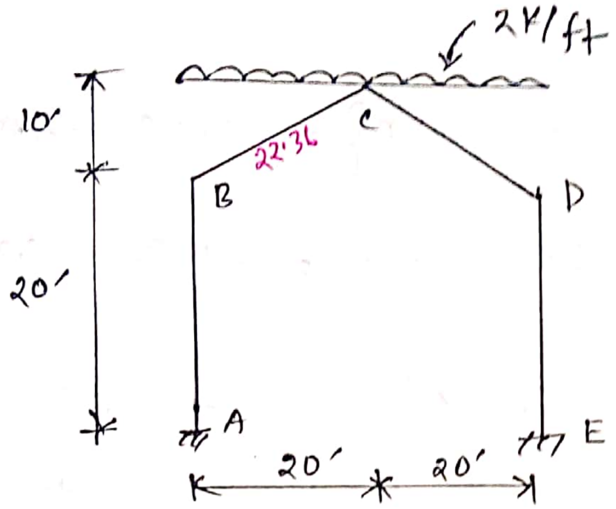
$M_{DC} = 3.6 - 3\times(-2.445) - 1.5\times(-0.911) = 12.3015$ K'

$M_{ED} = -6.4 - 1.5\times(-2.445) - 3\times(-0.911) = 0$ K'

(Ans.)

2016

#



Solution:

Relative stiffness:

$K_{AB} = \frac{1}{20} \simeq 2.5$

$K_{BC} = \frac{1}{22.36} \simeq 2.236$

$K_{CD} = \frac{1}{22.36} \simeq 2.236$

$K_{DE} = \frac{1}{20} \simeq 2.5$

Relative values of R:

$R_{AB} = \frac{\Delta}{L} = \frac{-\Delta}{20} \simeq -2.5R$

$R_{DE} = \frac{\Delta}{20} \simeq 2.5R$

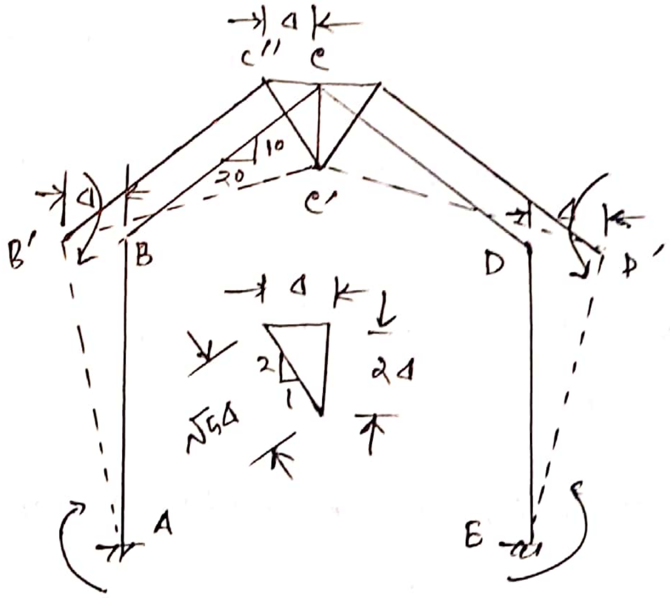
$R_{BC} = \frac{\sqrt{5}\Delta}{10\sqrt{5}} \simeq 5R$

$R_{CB} = -5R$

Fixed End Moments:

$F_{BC} = -F_{BC} = \frac{2 \times 20^2}{12} = 66.67K'$

$F_{CD} = -F_{DC} = 66.67K'$



Slope Deflection Equations:

$$M_{AB} = 0 + 2.5 \times \left\{ -\cancel{2\theta_A}^{0} - \theta_B - 2.5R \right\} = -2.5\theta_B - 6.25R$$

$$M_{BA} = 0 + 2.5 \times \left\{ -\cancel{\theta_A}^{0} - 2\theta_B - 2.5R \right\} = -5\theta_B - 6.25R$$

$$M_{BC} = 66.67 + 2.236 \times \left\{ -2\theta_B - \cancel{\theta_C}^{0} + 5R \right\} = 66.67 - 4.472\theta_B + 11.18R$$

$$M_{CB} = -66.67 + 2.236 \times \left\{ -\theta_B - \cancel{2\theta_C}^{0} + 5R \right\} = -66.67 - 2.236\theta_B + 11.18R$$

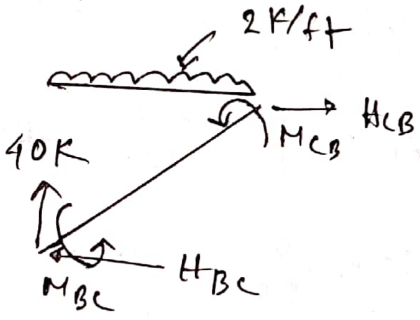
Joint Condition:

Joint B: $M_{BA} + M_{BC} = 0$

$$66.67 - 9.472\theta_B + 4.93R = 0$$

$$\Rightarrow 9.472\theta_B - 4.93R = 66.67 \quad \ldots\ldots ①$$

Shear conditions: $H_{BC} = H_{BA} \quad \ldots\ldots (ii)$

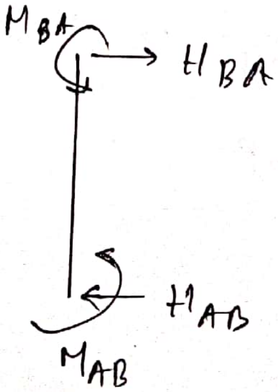


$\Sigma M_C = 0$

$$40 \times 20 - M_{BC} - M_{CB} - 2 \times 20 \times 10 + H_{BC} \times 10 = 0$$

$$\Rightarrow H_{BC} \times 10 = -6.708\theta_B + 22.36R - 400$$

$$H_{BC} = \frac{-6.708\theta_B + 22.36R - 400}{10}$$



And, $\Sigma M_A = 0$

$$H_{AB} = \frac{M_{AB} + M_{BA}}{20}$$

$$\therefore H_{AB} = \frac{-7.5\theta_B - 12.5R}{20}$$

Hence, From equation (ii)

$$\frac{-6.708\theta_B + 22.36R - 400}{10} = \frac{-7.5\theta_B - 12.5R}{20}$$

$$\Rightarrow -13.708\theta_B + 44.72R - 800 = -7.5\theta_B - 12.5R$$

$$\Rightarrow -6.208\theta_B + 32.22R = 800$$

$$\Rightarrow -4.93\theta_B + 25.59R = 635.425 \quad \ldots\ldots (iii).$$

Now, from (ii) & (iii) we obtain, $\theta_B = 22.1875°$

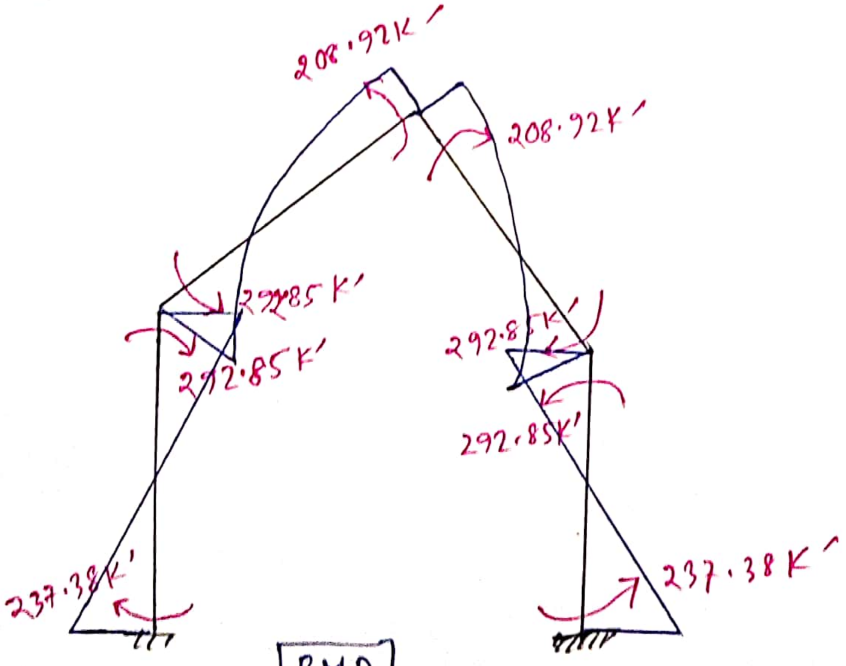
$R = 29.1055$

$\therefore M_{AB} = -2.5 \times 22.1875 - 6.25 \times 29.1055 = -237.38K'$

$M_{BA} = -5 \times 22.1875 - 6.25 \times 29.1055 = -292.85 K'$

$M_{BC} = 66.67 - 4.472 \times 22.1875 + 11.18 \times 29.1055 = 292.85K'$

$M_{CB} = -66.67 - 2.236 \times 22.1875 + 11.18 \times 29.1055 = 208.92K'$



BMD

2015

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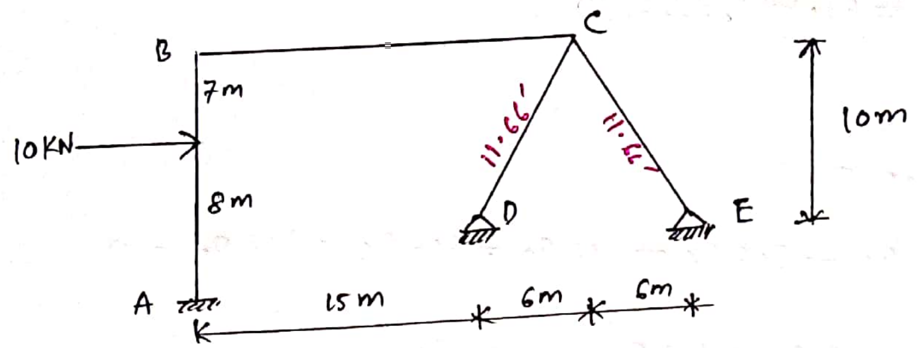


Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{15} \simeq 1.4$

$K_{BC} = \frac{1}{21} \simeq 1$

$K_{CD} = \frac{1}{11.66} \simeq 1.8$

$K_{CE} = \frac{1}{11.66} \simeq 1.8$

Fixed End Moment:

$F_{AB} = \frac{10 \times 8 \times 7^2}{15^2} = 17.42$ KN-m

$F_{BA} = \frac{-10 \times 8^2 \times 7}{15^2} = -19.91$ KN-m

$F_{BC} = \frac{2 \times 21^2}{12} = 73.5$ KN-m

$F_{CB} = -73.5$ KNm

Slope Deflection Equations:

$M_{AB} = 17.42 + 1.4 \times \{-2\overset{0}{\cancel{\theta_A}} - \theta_B\} = 17.42 - 1.4\theta_B$

$M_{BA} = -19.91 + 1.4 \times \{-\overset{0}{\cancel{\theta_A}} - 2\theta_B) = -19.91 - 2.8\theta_B$

$M_{BC} = 73.5 + 1 \times \{-2\theta_B - \theta_C\} = 73.5 - 2\theta_B - \theta_C$

$M_{CB} = -73.5 + 1 \times \{-\theta_B - 2\theta_C\} = -73.5 - \theta_B - 2\theta_C$

$M_{CD} = 0 + 1.8 \times \{-2\theta_C - \theta_D\} = -3.6\theta_C - 1.8\theta_D$

$M_{DC} = 0 + 1.8 \times \{-\theta_C - 2\theta_D\} = -1.8\theta_C - 3.6\theta_D$

$M_{CE} = 0 + 1.8 \times \{-2\theta_C - \theta_E\} = -3.6\theta_C - 1.8\theta_E$

$M_{EC} = 0 + 1.8 \times \{-\theta_C - 2\theta_E\} = -1.8\theta_C - 3.6\theta_E$

joint condition:

Joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -19.91 - 2.8\theta_B + 73.5 - 2\theta_B - \theta_C = 0$

$\Rightarrow 4.8\theta_B + \theta_C = 53.59$ ......①

joint C:

$M_{CB} + M_{CD} + M_{CE} = 0$

$\Rightarrow -73.5 - \theta_B - 2\theta_C - 3.6\theta_C - 1.8\theta_D - 3.6\theta_C - 1.8\theta_E = 0$

$\Rightarrow \theta_B + 9.2\theta_C + 1.8\theta_D + 1.8\theta_E = -73.5$ ......(II)

joint D:

$M_{DC} = 0$

$\Rightarrow -1.8\theta_C - 3.6\theta_D = 0$ ......(III)

$\Rightarrow 1.8\theta_C + 3.6\theta_D = 0$

joint E:

$M_{EC} = 0$

$\Rightarrow -1.8\theta_C - 3.6\theta_E = 0$ ......(IV)

$\Rightarrow 1.8\theta_C + 3.6\theta_E = 0$

From eqn (I), (II), (III) & (IV) we obtain,

$\theta_B = 13.617$

$\theta_C = -11.773$

$\theta_D = 5.886$

$\theta_E = 5.886$

$M_{AB} = 17.72 - 1.4 \times 13.617 = -1.344$ KN-m

$M_{BA} = -19.91 - 2.8 \times 13.617 = -58.038$ KN-m

$M_{BC} = 73.5 - 2 \times 13.617 - (-11.773) = 58.039$ KN-m

$M_{CB} = -73.5 - 13.617 - 2 \times (-11.773) = -63.571$ KN-m

$M_{CD} = -3.6 \times (-11.773) - 1.8 \times 5.886 = 31.788$ KN-m

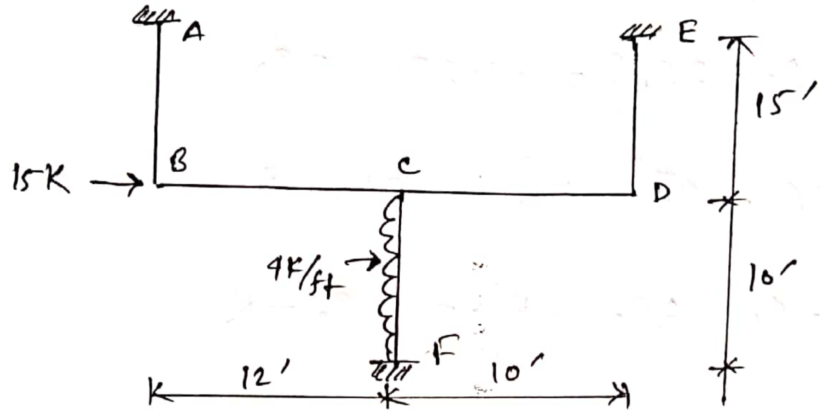
$M_{DC} = -1.8 \times (-11.773) - 3.6 \times 5.886 = 0$ KN-m

$M_{CE} = -3.6 \times (-11.773) - 1.8 \times 5.886 = 31.788$ KN-m

$M_{EC} = -1.8 \times (-11.773) - 3.6 \times 5.886 = 0$ KN-m

(Ans.)

2014
#



Solution:

Relative stiffness:

$K_{AB} = \frac{1}{15} \simeq 1$

$K_{BC} = \frac{1}{12} \simeq 1.25$

$K_{CF} = \frac{1}{10} \simeq 1.5$

$K_{CD} = \frac{1}{10} \simeq 1.5$

$K_{DE} = \frac{1}{15} \simeq 1$

FEM:

$F_{CF} = -\frac{4\times10^2}{12} = -33.33$ kft

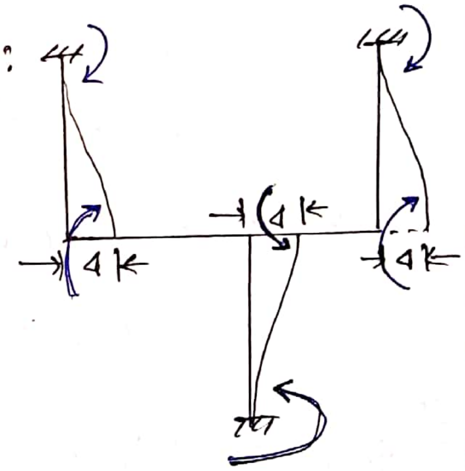
$F_{FC} = \frac{4\times10^2}{12} = 33.33$ k-ft

Relative values of R:

$R_{AB} = \frac{-\Delta}{15} \simeq -R$

$R_{CF} = \frac{\Delta}{10} \simeq 1.5R$

$R_{DE} = -\frac{\Delta}{15} \simeq R$



Slope Deflection Equations:

$M_{AB} = 0 + 1\times\{-\cancel{2\theta_A}^{0} - \theta_B - R\} = -\theta_B - R$

$M_{BA} = 0 + 1\times\{-\cancel{\theta_A}^{0} - 2\theta_B - R\} = -2\theta_B - R$

$M_{BC} = 0 + 1.25\times\{-2\theta_B - \theta_C\} = -2.5\theta_B - 1.25\theta_C$

$M_{CB} = 0 + 1.25\times\{-\theta_B - 2\theta_C\} = -1.25\theta_B - 2.5\theta_C$

$M_{CF} = -33.33 + 1.5\times\{-2\theta_C - \cancel{\theta_D}^{0} + 1.5R\} = -33.33 - 3\theta_C + 2.25R$

$M_{FC} = 33.33 + 1.5\times\{-\theta_C - \cancel{2\theta_D}^{0} + 1.5R\} = 33.33 - 1.5\theta_C + 2.25R$

$M_{CD} = 0 + 1.5 \times \{-2\theta_C - \theta_D\} = -3\theta_C - 1.5\theta_D$

$M_{DC} = 0 + 1.5 \times \{-\theta_C - 2\theta_D\} = -1.5\theta_C - 3\theta_D$

$M_{DE} = 0 + 1 \times \{-2\theta_D - \cancel{\theta_E}^{0} - R\} = -2\theta_D - R$

$M_{ED} = 0 + 1 \times \{-\theta_D - \cancel{2\theta_E}^{0} - R\} = -\theta_D - R$

Joint conditions:

joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -2\theta_B - R - 2.5\theta_B - 1.25\theta_C = 0 \Rightarrow 4.5\theta_B + 1.25\theta_C + R = 0$ …… (i)

joint C: $M_{CB} + M_{CF} + M_{CD} = 0$

$\Rightarrow -1.25\theta_B - 2.5\theta_C - 33.33 - 3\theta_C + 2.25R - 3\theta_C - 1.5\theta_D = 0$

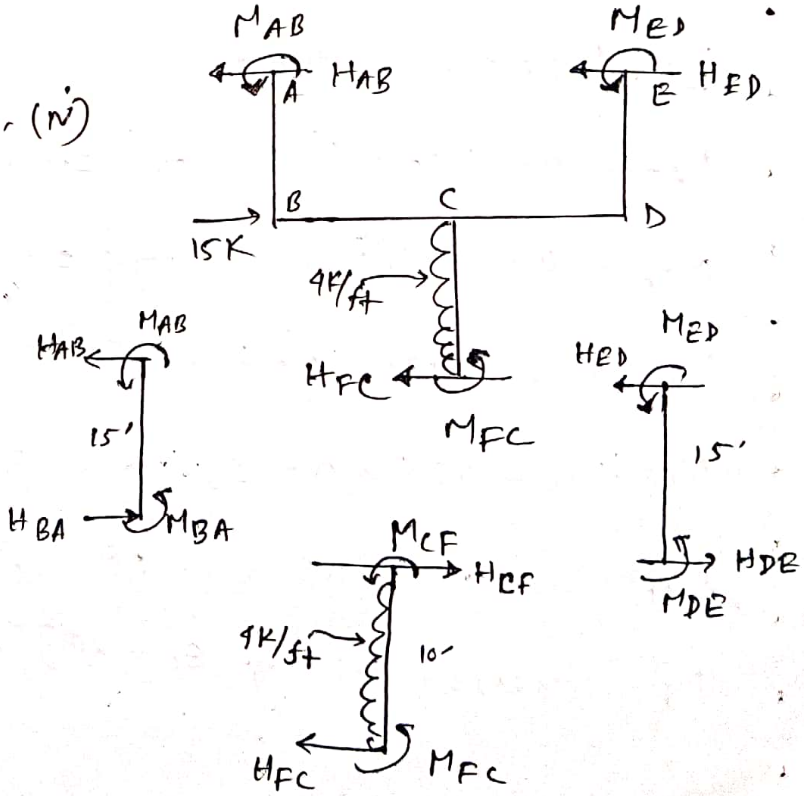
$\Rightarrow 1.25\theta_B + 8.5\theta_C + 1.5\theta_D - 2.25R = -33.33$ …… (ii)

joint D: $M_{DC} + M_{DE} = 0$

$\Rightarrow -1.5\theta_C - 3\theta_D - 2\theta_D - R = 0 \Rightarrow 1.5\theta_C + 5\theta_D + R = 0$ …… (iii)

Shear Conditions:

$H_{AB} + H_{FC} + H_{ED} = 55$ …… (iv)



$\Sigma M_B = 0$

$M_{BA} + M_{AB} + H_{AB} \times 15 = 0$

$\Rightarrow -2\theta_B - R - \theta_B - R + H_{AB} \times 15 = 0$

$\Rightarrow H_{AB} = \dfrac{3\theta_B + 2R}{15}$

$\Sigma M_C = 0$

$M_{CF} + M_{FC} + 4 \times 10 \times 5 - H_{FC} \times 10 = 0$

$\Rightarrow -33.33 - 3\theta_C + 2.25R + 33.33 - 1.5\theta_C + 2.25R + 200 = H_{FC} \times 10$

$$\Rightarrow \quad H_{FC} = \frac{-4.5\theta_C + 4.5R + 200}{10}$$

$\Sigma M_D = 0$

$M_{ED} + M_{DE} + H_{ED} \times 15 = 0$

$\Rightarrow -2\theta_D - R - \theta_D - R + 15 \times H_{ED} = 0$

$$\Rightarrow H_{ED} = \frac{3\theta_D + 2R}{15}$$

putting the values of H_{AB}, H_{FG} & H_{ED} in to the equation (iv)

$$\frac{3\theta_B + 2R}{15} + \frac{-4.5\theta_C + 4.5R + 200}{10} + \frac{3\theta_D + 2R}{15} = 55$$

$\Rightarrow 30\theta_B + 20R - 67.5\theta_C + 67.5R + 3000 + 30\theta_D + 20R = 55 \times 150$

$\Rightarrow 30\theta_B - 67.5\theta_C + 30\theta_D + 107.5 = 5250$

$\Rightarrow \theta_B - 2.25\theta_C + \theta_D + 3.5833 = 175$ ...... (v)

Now, From eq^n ①, (ii), (iii) & (v) we obtain,

$\theta_B = -23.653$

$\theta_C = 23.818$

$\theta_D = -22.48$

$R = 76.67$

$\therefore M_{AB} = -(-23.653) - 76.67 = -53.017 K'$

$M_{BA} = -(2 \times -23.653) - 76.67 = -29.364 K'$

$M_{BC} = -2.5 \times (-23.653) - 1.25 \times 23.818 = 29.36 K'$

$M_{CB} = -1.25 \times (-23.653) - 2.5 \times (23.818) = -29.98 K'$

$M_{CF} = -33.33 - 3 \times 23.818 + 2.25 \times 76.67 = 67.724 K'$

$M_{FC} = 33.33 - 1.5 \times 23.818 + 2.25 \times 76.67 = 170.11 K'$

$M_{CD} = -3 \times 23.818 - 1.5 \times (-22.48) = -37.734 K'$

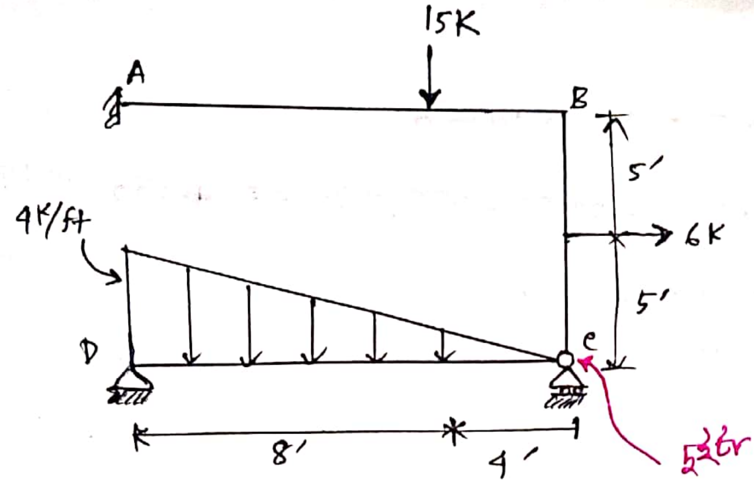
$M_{DC} = -1.5 \times 23.818 - 3 \times (-22.48) = 31.713 K'$

$M_{DE} = -2 \times (-22.48) - 76.67 = -31.71 K'$

$M_{ED} = -(22.48) - 76.67 = -54.19 K'$

(Ans.)

2012
#



দুইটা আলাদা θcB & θcD

Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{1}{10} \simeq 1.2$

$K_{CD} = \frac{1}{12} \simeq 1$

Fixed End Moment:

$F_{AB} = \frac{15 \times 8 \times 4^2}{12^2} = 13.33 K'$

$F_{BA} = -\frac{15 \times 8^2 \times 4}{12^2} = -26.67 K'$

$F_{BC} = -\frac{6 \times 10}{8} = -7.5 K'$

$F_{CB} = 7.5 K'$

$F_{CD} = -\frac{4 \times 12^2}{30} = -19.2 K'$

$F_{DC} = \frac{4 \times 12^2}{20} = 28.8 K'$

Slope Deflection Equation:

$M_{AB} = 13.33 + 1 \times \{-\overset{0}{\cancel{2\theta_A}} - \theta_B\}$

$= 13.33 - \theta_B$

$M_{BA} = -26.67 + 1 \times \{-\overset{0}{\cancel{\theta_A}} - 2\theta_B\}$

$= -26.67 - 2\theta_B$

$M_{BC} = -7.5 + 1.2 \times \{-2\theta_B - \theta_{\underline{CB}}\}$

$= -7.5 - 2.4\theta_B - 1.2\theta_{\underline{CB}}$

$M_{CB} = 7.5 + 1.2 \times \{-\theta_B - 2\theta_{\underline{CB}}\}$

$= 7.5 - 1.2\theta_B - 2.4\theta_{\underline{CB}}$

$M_{CD} = -19.2 + 1 \times \{-2\theta_{\underline{CD}} - \theta_D\}$

$= -19.2 - 2\theta_{\underline{CD}} - \theta_D$

and, $M_{DC} = 28.8 + 1 \times \{-\theta_{\underline{CD}} - 2\theta_D\}$

$= 28.8 - \theta_{\underline{CD}} - 2\theta_D$

Joint conditions:

Joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -26.67 - 2\theta_B - 7.5 - 2.4\theta_B - 1.2\theta_{CB} = 0$

$\Rightarrow 4.4\theta_B + 1.2\theta_{CB} = -34.17$ .... (i)

Joint C: $M_{CB} = 0$

$\Rightarrow 7.5 - 1.2\theta_B - 2.4\theta_{CB} = 0$

$\Rightarrow 1.2\theta_B + 2.4\theta_{CB} = 7.5$ .... (ii)

$M_{CD} = 0$

$\Rightarrow -19.2 - 2\theta_{CD} - \theta_D = 0$

$\Rightarrow 2\theta_{CD} + \theta_D = -19.2$ .... (iii)

Joint D: $M_{DC} = 0$

$\Rightarrow 28.8 - \theta_{CD} - 2\theta_D = 0$

$\Rightarrow \theta_{CD} + 2\theta_D = 28.8$ .... (iv)

From eqn (i), (ii), (iii) & (iv) we obtain,

$\theta_B = -9.98$

$\theta_{CB} = 8.114$

$\theta_{CD} = -22.4$

$\theta_D = 25.6$

$M_{AB} = 13.33 - (-9.98) = 23.31$ K'

$M_{BA} = -26.67 - 2\times(-9.98) = -6.71$ K'

$M_{BC} = -7.5 - 2.4\times(-9.98) - 1.2\times(8.114) = 6.72$K'

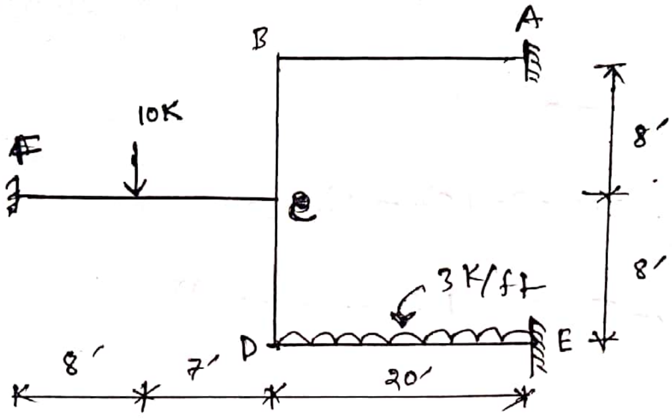
$M_{CB} = 7.5 - 1.2\times(-9.98) - 2.4\times(8.114) = 0$

$M_{CD} = -19.2 - 2\times(-22.4) - 25.6 = 0$

$M_{DC} = 28.8 - (-22.4) - 2\times 25.6 = 0$

(Ans.)

2011
#



Solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{20} \simeq 1$

$K_{BC} = \frac{1}{8} \simeq 2.5$

$K_{CD} = \frac{1}{8} \simeq 2.5$

$K_{DE} = \frac{1}{20} \simeq 1$

$K_{CF} = \frac{1}{15} \simeq 1.33$

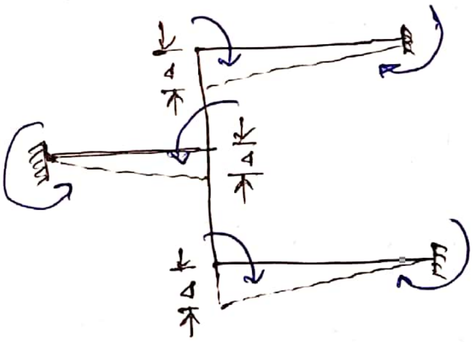
FEM:

$F_{DE} = \frac{3\times 20^2}{12} = 100\ K'$

$F_{ED} = -100\ K'$

$F_{FC} = \frac{10\times 8\times 7^2}{15^2} = 17.42\ K'$

$F_{CF} = -\frac{10\times 8^2\times 7}{15^2} = -19.91\ K'$



Relative values of R:

$R_{AB} = \frac{\Delta}{L} = \frac{-\Delta}{20} \simeq -R$

$R_{CF} = \frac{\Delta}{15} \simeq 1.33R$

$R_{ED} = -R$

Slope deflection equations:

$M_{AB} = 0 + 1\times\{-2\cancel{\theta_A}^{0} - \theta_B - R\} = -\theta_B - R$

$M_{BA} = 0 + 1\times\{-\cancel{\theta_A}^{0} - 2\theta_B - R\} = -2\theta_B - R$

$M_{BC} = 0 + 2.5\times\{-2\theta_B - \theta_C\} = -5\theta_B - 2.5\theta_C$

$M_{CB} = 0 + 2.5\times\{-\theta_B - 2\theta_C\} = -2.5\theta_B - 5\theta_C$

$M_{CD} = 0 + 2.5 \times \{-2\theta_C - \theta_D\} = -5\theta_C - 2.5\theta_D$

$M_{DC} = 0 + 2.5 \times \{-\theta_C - 2\theta_D\} = -2.5\theta_C - 5\theta_D$

$M_{DE} = 100 + 1 \times \{-2\theta_D - \cancelto{0}{\theta_E} - R\} = 100 - 2\theta_D - R$

$M_{ED} = -100 + 1 \times \{-\theta_D - 2\cancelto{0}{\theta_E} - R\} = -100 - \theta_D - R$

$M_{CF} = -19.91 + 1.33 \times \{-2\theta_C - \cancelto{0}{\theta_F} + 1.33R\} = -19.91 - 2.66\theta_C + 1.77R$

$M_{FC} = 17.42 + 1.33 \times \{-\theta_C - 2\cancelto{0}{\theta_F} + 1.33R\} = 17.92 - 1.33\theta_C + 1.77R$

joint conditions:

Joint B: $M_{BA} + M_{BC} = 0$

$\Rightarrow -2\theta_B - R - 5\theta_B - 2.5\theta_C = 0 \Rightarrow 7\theta_B + 2.5\theta_C + R = 0$ ...... ①

joint C: $M_{CB} + M_{CF} + M_{CD} = 0$

$\Rightarrow -2.5\theta_B - 5\theta_C - 19.91 - 2.66\theta_C + 1.77R - 5\theta_C - 2.5\theta_D = 0$

$\Rightarrow 2.5\theta_B + 12.66\theta_C + 2.5\theta_D - 1.77R = -19.91$ ...... (ii)

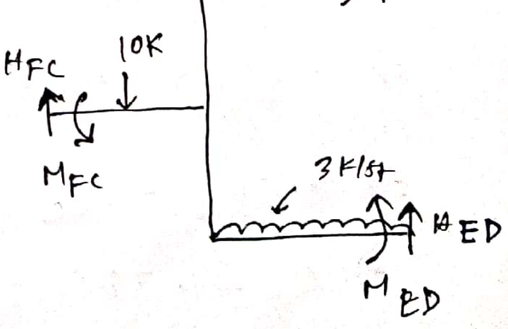
joint D: $M_{DC} + M_{DE} = 0$

$\Rightarrow -2.5\theta_C - 5\theta_D + 100 - 2\theta_D - R = 0$

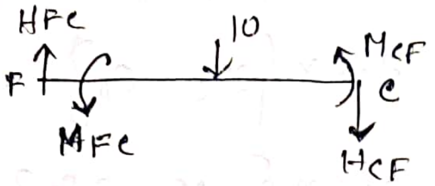
$\Rightarrow 2.5\theta_C + 7\theta_D + R = 100$ .... (iii)

Shear condition:

$H_{FC} + H_{AB} + H_{ED} = 70$ ...... (iv)



$\Sigma M_C = 0$

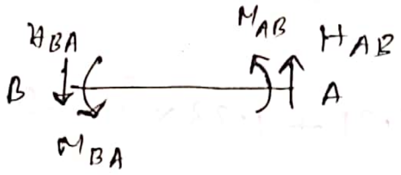


$M_{FC} + M_{CF} + 10 \times 7 - H_{FC} \times 15 = 0$

$\Rightarrow 17.42 - 1.33\theta_C + 1.77R - 19.91 - 2.66\theta_C + 1.77R + 70 = H_{FC} \times 15$

$\Rightarrow H_{PC} = \dfrac{-4\theta_C + 3.54R + 67.51}{15}$

$\Sigma M_B = 0$

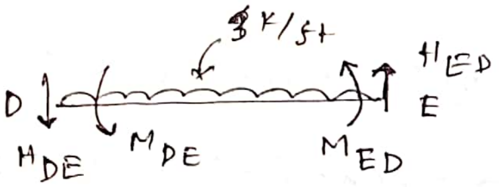


$M_{BA} + M_{AB} + 20 \times H_{AB} = 0$

$\Rightarrow -2\theta_B - R - \theta_B - R + 20 \times H_{AB} = 0$

$\Rightarrow H_{AB} = \dfrac{3\theta_B + 2R}{20}$

$\Sigma M_D = 0$



$M_{DE} + M_{ED} - 3 \times 20 \times 10 + H_{ED} \times 20 = 0$

$\Rightarrow 100 - 2\theta_D - R - 100 - \theta_D - R - 600 + 20H_{ED} = 0$

$\Rightarrow H_{ED} = \dfrac{3\theta_D + 2R + 600}{20}$

putting The values of H_{FC}, H_{AB} & H_{ED} in the equation (iv),

we obtain,

$$\frac{-4\theta_C + 3.54R + 67.51}{15} + \frac{3\theta_B + 2R}{20} + \frac{3\theta_D + 2R + 600}{20} = 70$$

$\Rightarrow -8\theta_C + 7.08R + 135.02 + 4.5\theta_B + 3R + 4.5\theta_D + 3R + 900 = 70 \times 30$

$\Rightarrow 4.5\theta_B - 8\theta_C + 4.5\theta_D + 13.08R = 1064.98$

$\Rightarrow \theta_B - 1.78\theta_C + \theta_D + 2.9067R = 236.66$ ~~~~~ (v)

From equation (i), (ii), (iii) & (v) we obtain,

$\theta_B = -21.11$

$\theta_C = 18.23$

$\theta_D = -6.82$

$R = 102.194$

$M_A = -(-21.11) - 102.194 = -81.084\ K'$

$M_{BA} = -(2 \times -21.11) - 102.194 = -59.974\ K'$

$M_{BC} = -5 \times (-21.11) - 2.5 \times 18.23 = 59.975 K'$

$M_{CB} = -2.5 \times (-21.11) - 5 \times 18.23 = -38.375\ K'$

$M_{CD} = -5 \times 18.23 - 2.5 \times (-6.82) = -74.1\ K'$

$M_{DC} = -2.5 \times 18.23 - 5 \times (-6.82) = -11.475\ K'$

$M_{DE} = 100 - 2 \times (-6.82) - 102.194 = 11.446\ K'$

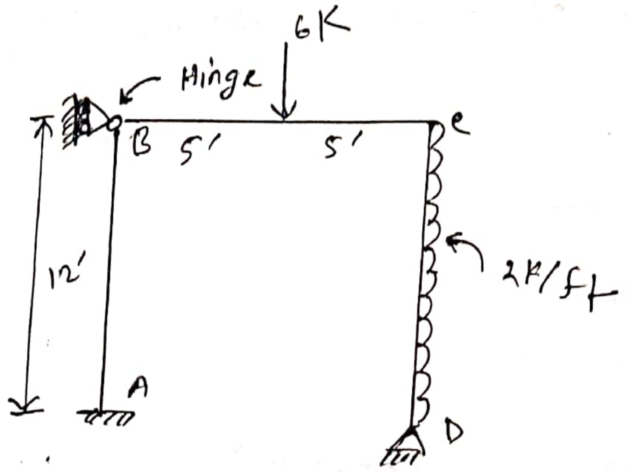
$M_{ED} = -100 - (-6.82) - 102.194 = -195.379\ K'$

$M_{CF} = -19.91 - 2.66 \times (78.23) + 1.77 \times 102.194 = 112.48\ K'$

$M_{FC} = 17.42 - 1.33 \times (18.23) + 1.77 \times 102.194 = 174.06\ K'$

2009

#



Solution:

Relative stiffness:

$K_{AB} = \frac{1}{12} \simeq 1$

$K_{BC} = \frac{1}{10} \simeq 1{\cdot}2$

$K_{CD} = \frac{1}{12} \simeq 1$

FEM:

$F_{BC} = \frac{6 \times 10}{8} = 7{\cdot}5K'$

$F_{CB} = -7{\cdot}5K'$

$F_{CD} = \frac{2 \times 12^2}{12} = 24\ K'$

$F_{DC} = -24K'$

Slope deflection Equations:

$M_{AB} = 0 + 1 \times \{-\cancelto{0}{2\theta_A} - \theta_{BA}\} = -\theta_{BA}$

$M_{BA} = 0 + 1 \times \{-\cancelto{0}{\theta_A} - 2\theta_{BA}\} = -2\theta_{BA}$

$M_{BC} = 7{\cdot}5 + 1{\cdot}2 \times \{-2\theta_{BC} - \theta_C\} = 7{\cdot}5 - 2{\cdot}4\theta_{BC} - 1{\cdot}2\theta_C$

$M_{CB} = -7{\cdot}5 + 1{\cdot}2 \times \{-\theta_{BC} - 2\theta_C\} = -7{\cdot}5 - 1{\cdot}2\theta_{BC} - 2{\cdot}4\theta_C$

$M_{CD} = 24 + 1 \times \{-2\theta_C - \theta_D\} = 24 - 2\theta_C - \theta_D$

$M_{DC} = -24 + 1 \times \{-\theta_C - 2\theta_D\} = -24 - \theta_C - 2\theta_D$

Joint conditions:

Joint B: $M_{BA} = 0 \Rightarrow -2\theta_{BA} = 0 \quad \therefore \theta_{BA} = 0$ ....①

$M_{BC} = 0$

$\Rightarrow 7.5 - 2.4\,\theta_{BC} - 1.2\,\theta_C = 0$

$\Rightarrow 2.4\,\theta_{BC} + 1.2\,\theta_C = 7.5$ ...... (ii)

<u>joint C:</u> $M_{CB} + M_{CD} = 0$

$\Rightarrow -7.5 - 1.2\,\theta_{BC} - 2.4\,\theta_C + 24 - 2\theta_C - \theta_D = 0$

$\Rightarrow 1.2\,\theta_{BC} + 4.4\,\theta_C + \theta_D = 16.5$ ...... (iii)

<u>joint D:</u> $M_{DC} = 0$

$\Rightarrow -24 - \theta_C - 2\theta_D = 0$

$\Rightarrow \theta_C + 2\theta_D = 24$ ...... (iv)

From eqn (ii), (iii) & (iv), we obtain,

$\theta_{BC} = -8.875$

$\theta_C = 24$

$\theta_D = -78.45$

Now,

$M_{AB} = 0$

$M_{BA} = 0$

$M_{BC} = 7.5 - 2.4 \times (-8.875) - 1.2 \times 24 = 0$

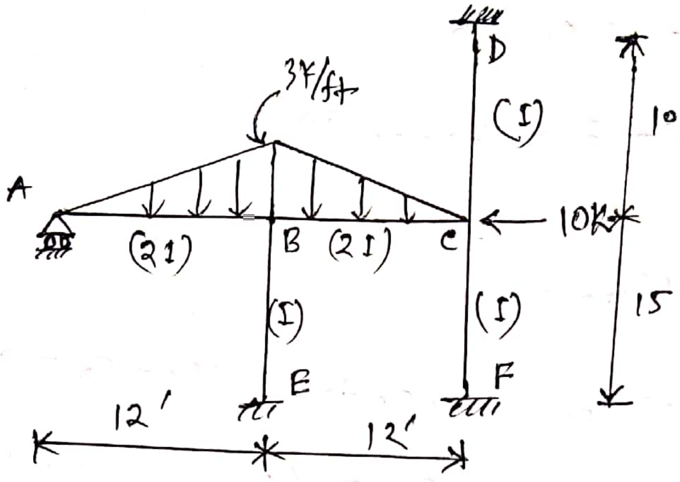
$M_{CB} = -7.5 - 1.2 \times (-8.875) - 2.4 \times 24 = -54.45$ K'

$M_{CD} = 24 - 2 \times 24 + 78.45 = 54.45$ K'

$M_{DC} = -24 - 24 - 2 \times (-78.45) = 108.9$ K'

2008

#



Solution:

Relative Stiffness:

$K_{AB} = \frac{I}{L} = \frac{2}{12} \simeq 2.5$

$K_{BC} = \frac{2}{12} \simeq 2.5$

$K_{BE} = \frac{1}{15} \simeq 1$

$K_{CF} = \frac{1}{15} \simeq 1$

$K_{CD} = \frac{1}{10} \simeq 1.5$

FEM:

$F_{AB} = \frac{3 \times 12^2}{30} = 14.4\, K'$

$F_{BA} = -\frac{3 \times 12^2}{20} = -21.6\, K'$

$F_{BC} = 21.6\, K'$

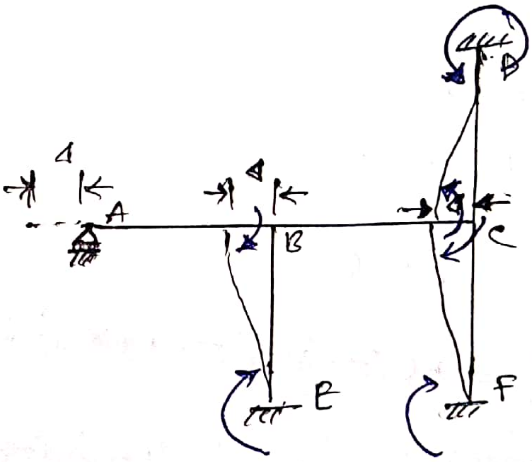
$F_{CB} = -14.4\, K'$

Relative Value of R:

$R_{BE} = \frac{\Delta}{L} = \frac{-\Delta}{1} \simeq -R$

$R_{CF} = -R$

$R_{CD} = \frac{\Delta}{10} \simeq 1.5R$



Slope deflection Equations:

$M_{AB} = 14.4 + 2.5 \times \{-2\theta_A - \theta_B\} = 14.4 - 5\theta_A - 2.5\theta_B$

$M_{BA} = -21.6 + 2.5 \times \{-\theta_A - 2\theta_B\} = -21.6 - 2.5\theta_A - 5\theta_B$

$M_{BC} = 21.6 + 2.5 \times \{-2\theta_B - \theta_C\} = 21.6 - 5\theta_B - 2.5\theta_C$

$M_{CB} = -14.4 + 2.5 \times \{-\theta_B - 2\theta_C\} = -14.4 - 2.5\theta_B - 5\theta_C$

$M_{BE} = 0 + 1 \times \{-2\theta_B - \cancelto{0}{\theta_E} - R\} = -2\theta_B - R$

$M_{EB} = 0 + 1 \times \{-\theta_B - 2\cancelto{0}{\theta_E} - R\} = -\theta_B - R$

$M_{CF} = 0 + 1 \times \{-2\theta_C - \cancelto{0}{\theta_F} - R\} = -2\theta_C - R$

$M_{FC} = 0 + 1 \times \{-\theta_C - 2\cancelto{0}{\theta_F} - R\} = -\theta_C - R$

$M_{CD} = 0 + 1.5 \times \{-2\theta_C - \cancelto{0}{\theta_D} + 1.5R\} = -3\theta_C + 2.25R$

$M_{DC} = 0 + 1.5 \times \{-\theta_C - 2\cancelto{0}{\theta_D} + 1.5R\} = -1.5\theta_C + 2.25R$

joint conditions:

joint A: $M_{AB} = 0$

$\Rightarrow 14.4 - 5\theta_A - 2.5\theta_B = 0 \Rightarrow 5\theta_A + 2.5\theta_B = 14.4$ ...... ①

joint B: $M_{BA} + M_{BE} + M_{BC} = 0$

$\Rightarrow -21.6 - 2.5\theta_A - 5\theta_B - 2\theta_B - R + 21.6 - 5\theta_B - 2.5\theta_C = 0$

$\Rightarrow 2.5\theta_A + 12\theta_B + 2.5\theta_C + R = 0$ ...... (II)

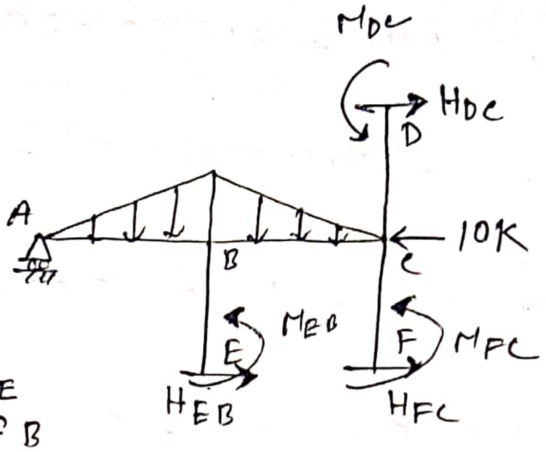
joint C: $M_{CB} + M_{CF} + M_{CD} = 0$

$\Rightarrow -14.4 - 2.5\theta_B - 5\theta_C - 2\theta_C - R - 3\theta_C + 2.25R = 0$

$\Rightarrow 2.5\theta_B + 10\theta_C - 1.25R = -14.4$ ...... (III)

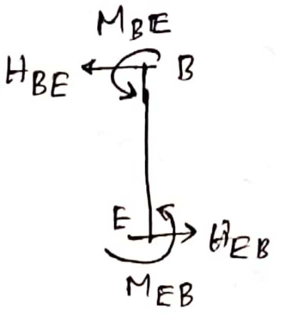
Shear conditions:

$$H_{EB} + H_{FC} + H_{DC} = 10 \quad \ldots\ldots (iv)$$



$\Sigma M_B = 0$

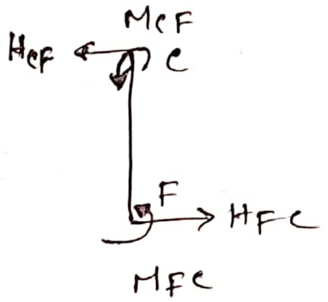
$$M_{BE} + M_{EB} + H_{EB} \times 15 = 0$$



$$\Rightarrow -2\theta_B - R - \theta_B - R + H_{EB} \times 15 = 0$$

$$\Rightarrow H_{EB} = \frac{3\theta_B + 2R}{15}$$

$\Sigma M_C = 0$ (for CF)

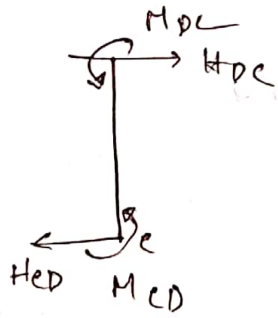


$$\Rightarrow M_{CF} + M_{FC} + H_{FC} \times 15 = 0$$

$$\Rightarrow -2\theta_C - R - \theta_C - R + H_{FC} \times 15 = 0$$

$$\Rightarrow H_{FC} = \frac{3\theta_C + 2R}{15}$$

$\Sigma M_C = 0$ (for CD)



$$M_{DC} + M_{CD} - H_{DC} \times 10 = 0$$

$$\Rightarrow -3\theta_C + 2.25R - 1.5\theta_C + 2.25 = 10H_{DC}$$

$$\Rightarrow H_{DC} = \frac{-4.5\theta_C + 4.5R}{10}$$

Putting the value of H_{EB}, H_{FC} & H_{DC} in the equation (iv) we obtain,

$$\frac{3\theta_B + 2R}{15} + \frac{3\theta_C + 2R}{15} + \frac{-4.5\theta_C + 4.5R}{10} = 10$$

$$\Rightarrow 30\theta_B + 20R + 30\theta_C + 20R - 67.5\theta_C + 67.5R = 10 \times 150$$

$\Rightarrow 30\theta_B - 37.5\theta_C + 107.5R = 1500$

$\Rightarrow \theta_B - 1.25\theta_C + 3.5833R = 50$ ..... (v)

From eqn (i), (ii), (iii) & (v) we obtain,

$\theta_A = 4.026$

$\theta_B = -2.293$

$\theta_C = 1$

$R = 14.943$

$M_{AB} = 14.4 - 5 \times 4.026 - 2.5 \times (-2.293)$
$= 0$

$M_{BA} = -21.6 - 2.5 \times 4.026 - 5 \times (-2.293)$
$= -20.2\ K'$

$M_{BC} = 21.6 - 5 \times (-2.293) - 2.5 \times 1$
$= 30.565\ K'$

$M_{CB} = -14.4 - 2.5 \times (-2.293) - 5 \times 1 = -13.667\ K'$

$M_{BE} = -2 \times (-2.293) - 14.943 = -10.357\ K'$

$M_{EB} = -(-2.293) - 14.943 = -12.65\ K'$

$M_{CF} = -2 \times 1 - 14.943 = -16.943\ K'$

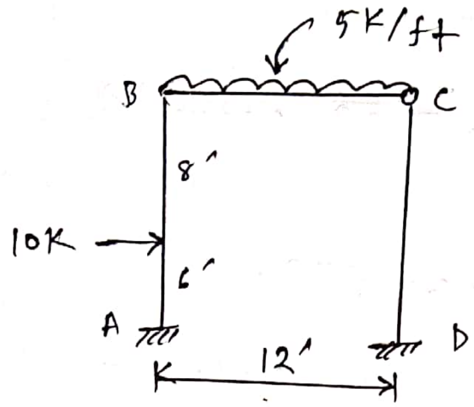
$M_{FC} = -1 - 14.943 = -15.943\ K'$

$M_{CD} = -3 \times 1 + 2.25 \times 14.943 = 30.62\ K'$

$M_{DC} = -1.5 \times 1 + 2.25 \times 14.943 = 32.12\ K'$

(Ans.)

2007
#



solution:

Relative stiffness:

$K_{AB} = \frac{I}{L} = \frac{1}{14} \simeq 1$

$K_{BC} = \frac{1}{12} \simeq 1.67$

$K_{CD} = \frac{1}{14} \simeq 1$

FEM:

$F_{AB} = \frac{10\times6\times8^2}{14^2} = 19.59\ K'$

$F_{BA} = \frac{-10\times6^2\times8}{14^2} = -14.69\ K'$

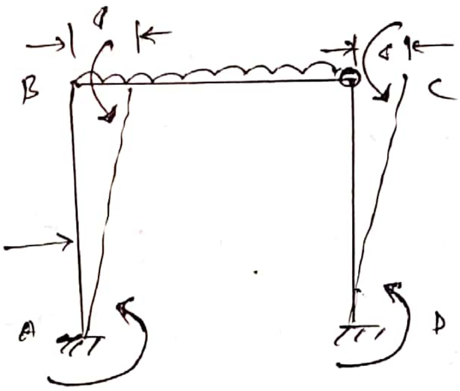
$F_{BC} = \frac{5\times12^2}{12} = 60\ K'$

$F_{CB} = -\frac{5\times12^2}{12} = -60K'$

Relative value of R:

$R_{AB} = \frac{\Delta}{L} = \frac{\Delta}{14} \simeq R$

$R_{CD} = \frac{4}{14} \simeq R$



Slope deflection Equations:

$M_{AB} = 19.59 + 1\times\{-\overset{0}{\cancel{2\theta_A}} - \theta_B + R\} = 19.59 - \theta_B + R$

$M_{BA} = -14.69 + 1\times\{-\overset{0}{\cancel{\theta_A}} - 2\theta_B + R\} = -14.69 - 2\theta_B + R$

$M_{BC} = 60 + 1.67\times\{-2\theta_B - \theta_{CB}\} = 60 - 3.34\theta_B - 1.67\theta_{CB}$

$M_{CB} = -60 + 1.67\times\{-\theta_B - 2\theta_{CB}\} = -60 - 1.67\theta_B - 3.34\theta_{CB}$

$M_{CD} = 0 + 1\times\{-2\theta_{CD} - \overset{0}{\cancel{\theta_D}} + R\} = -2\theta_{CD} + R$

$M_{DC} = 0 + 1\times\{-\theta_{CD} - \overset{0}{\cancel{2\theta_4}} + R\} = -\theta_{CD} + R$

joint conditions:

joint B: $M_{BA} + M_{BC} = 0$

$$\Rightarrow -14.69 - 2\theta_B + R + 60 - 3.34\theta_B - 1.67\theta_{CB} = 0$$

$$\Rightarrow 5.34\theta_B + 1.67\theta_{CB} - R = 45.31 \cdots\cdots ①$$

joint C: $M_{CB} = 0$

$$\Rightarrow -60 - 1.67\theta_B - 3.34\theta_{CB} = 0$$

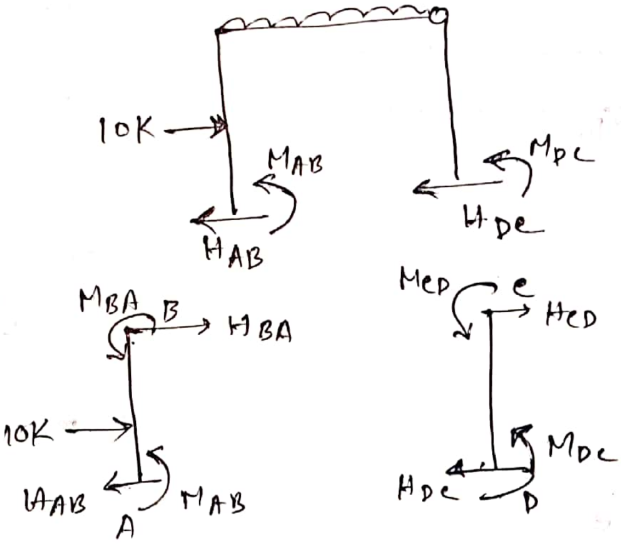
$$\Rightarrow 1.67\theta_B + 3.34\theta_{CB} = -60 \cdots\cdots (ii)$$

$M_{CD} = 0$

$$\Rightarrow -2\theta_{CD} + R = 0 \qquad \Rightarrow 2\theta_{CD} - R = 0 \cdots\cdots (iii)$$

Shear condition:

$$H_{AB} + H_{DC} = 10 \cdots\cdots (iv)$$



$\Sigma M_B = 0$

$$M_{BA} + M_{AB} - H_{AB} \times 14 + 10 \times 8 = 0$$

$$\Rightarrow -14.69 - 2\theta_B + R + 19.59 - \theta_B + R$$

$$80 = 14 \times H_{AB}$$

$$\Rightarrow H_{AB} = \frac{-3\theta_B + 2R + 84.9}{14}$$

$\Sigma M_C = 0$

$$M_{CD} + M_{DC} - 14 \times H_{DC} = 0$$

$$\Rightarrow -2\theta_{CD} + R - \theta_{CD} + R = 14 \times H_{DC}$$

$$\Rightarrow H_{DC} = \frac{-3\theta_{CD} + 2R}{14}$$

Putting the values of H_{AB} & H_{DC} in the equation (iv) we obtain,.

$$\frac{-3\theta_B + 2R + 84.9}{14} + \frac{-3\theta_{CD} + 2R}{14} = 10$$

$$\Rightarrow -30\theta_B + 20R + 849 - 30\theta_{CD} + 20R = 10 \times 140$$

$$\Rightarrow -30\theta_B - 30\theta_{CD} + 40R = 551$$

$$\Rightarrow -\theta_B - \theta_{CD} + 1.33R = 18.37 \quad \text{......} \text{(v)}$$

From eqn (i), (ii), (iii) & (v) we obtain,

$\theta_B = 29.526$

$\theta_{CB} = -32.727$

$\theta_{CD} = 28.853$

$R = 57.707$

$\therefore M_{AB} = 19.59 - 29.526 + 57.707 = 47.77 K'$

$M_{BA} = -14.59 - 2 \times 29.526 + 57.707 = -15.935 K'$

$M_{BC} = 60 - 3.34 \times 29.526 - 1.67 \times (-32.727)$

$= 16.037 K'$

$M_{CB} = -60 - 1.67 \times 29.526 - 3.34 \times (-32.727)$

$= 0 K'$

$M_{CD} = -2 \times 28.853 + 57.707$

$= 0 K'$

$M_{DC} = -28.853 + 57.707$

$= 28.854 K'$

(Ans.)

Composite Structure

⊞ Most structures fall in to one of the following three classifications:

1. Beams
2. Trusses
3. Frames.

Beams:

* A beam is subjected to bending only.

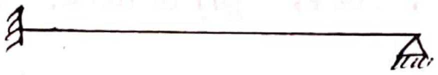


Fig. Beam

Trusses:

* The members of a truss with smooth hinged joints are subjected to direct stresses only.

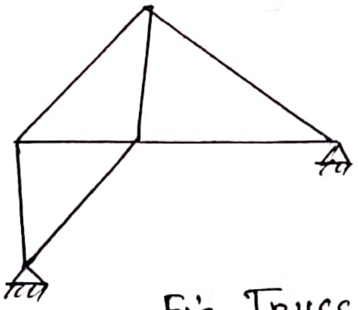


Fig. Truss

Frames:

* The members in a frame with rigid joints, however are usually subjected to both direct and bending stresses.

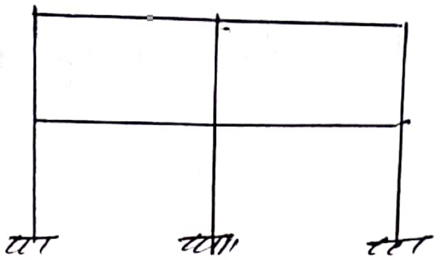


Fig. Frame

Composite Structure:

There are structures, however, in which some members are primarily subjected to direct stresses and others to bending stress. Such structures can be called "Truss Beams" or "Beam-Trusses" but generally known as Composite structures.

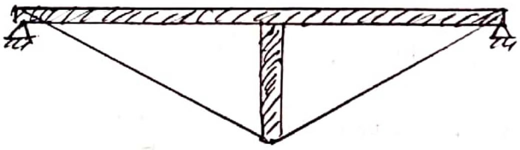
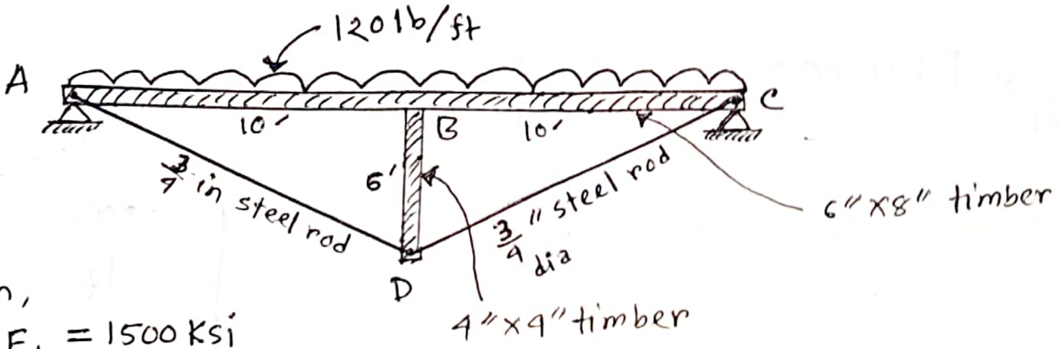


Fig. Composite structure

2012

 # Problem-01: Analyze the King post Truss as shown in figure:

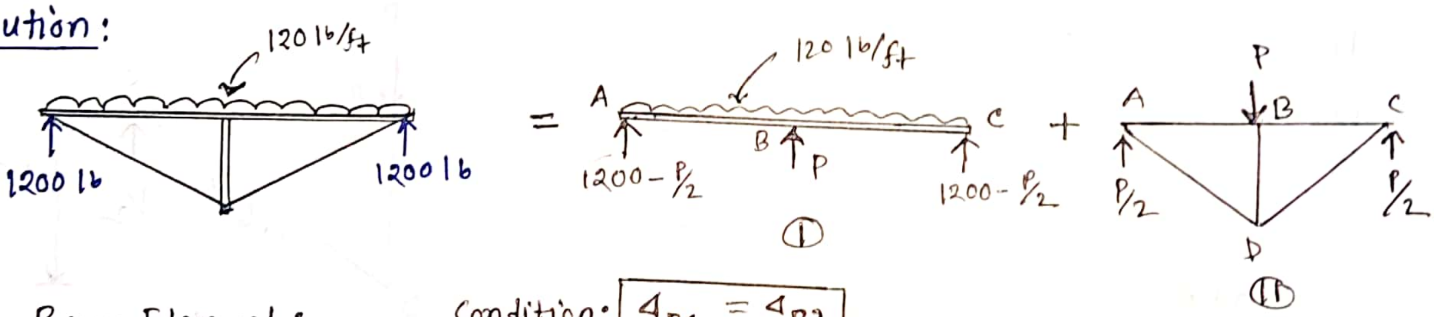


Given,

$E_t = 1500$ ksi

$E_s = 30000$ ksi

Solution:



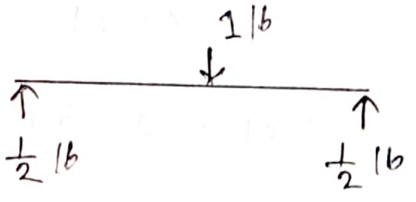
Beam Element:

Condition: $\boxed{\Delta_{B1} = \Delta_{B2}}$

We know, in unit load method,

$$\Delta_B = \int_0^L \frac{Mm\,dx}{EI}$$

Portion –	AB	BC
origin –	A	C
Limit –	0–10	0–10
M –	$(1200-\frac{P}{2})x - 120\times\frac{x^2}{2}$	$(1200-\frac{P}{2})x - 120\times\frac{x^2}{2}$
m –	$\frac{1}{2}x$	$\frac{1}{2}x$



For unit load

both portion - symmetric

Now,
$$\Delta_B = \frac{②}{E_tI}\int_0^{10}\left[\left(1200x - \frac{Px}{2}\right) - 60x^2\right]\times\frac{x}{2}\,dx$$

$$= \frac{2}{E_tI}\times\int_0^{10}\left(600x^2 - \frac{P}{4}x^2 - 30x^3\right)dx$$

$$= \frac{2}{E_t I_t} \times \left[600 \times \frac{10^3}{3} - \frac{P}{4} \times \frac{10^3}{3} - 30 \times \frac{10^4}{4} \right]$$

$$= \frac{2}{E_t I_t} \times \left[125000 - \frac{250}{3} P \right]$$

$$= \frac{250000 - 166.67 P}{1500 \times 1000 \times 256} \times 12^3$$

$$\Delta_B = 1.125 - 7.5 \times 10^{-4} P$$

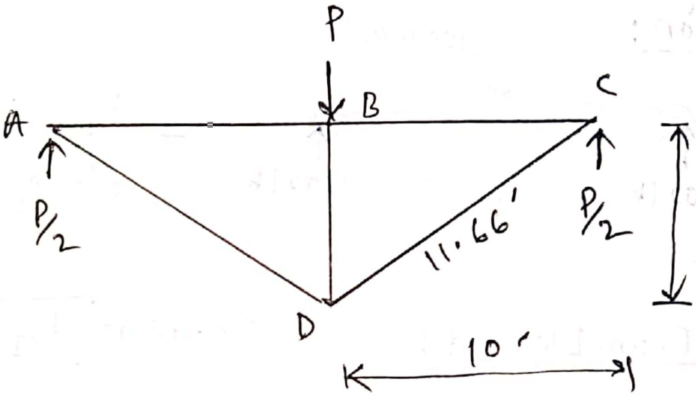
Here,

$$I_t = \frac{6 \times 8^3}{12} \text{ in}^4$$

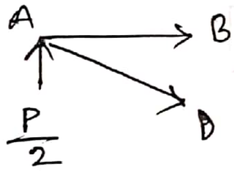
$$= 256 \text{ in}^4$$

Truss Element:

we know, $\Delta_B = \Sigma \frac{SuL}{AE}$



joint A:



$$AD = \frac{0.5P}{6} \times 11.66$$

$$= 0.97P$$

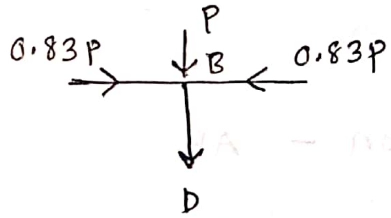
$$\therefore CD = 0.97P$$

$$AB = \frac{-0.97P}{11.66} \times 10$$

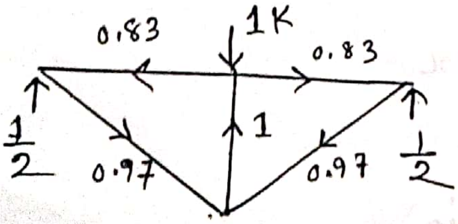
$$= -0.83P$$

$$\therefore CB = -0.83P$$

joint B:



$$\therefore BD = -P$$



For unit load * P=1

Member	S (lb)	U (lb)	L (in)	A (in²)	$\frac{SuL}{AE}$ (in)
AB (t)	$-0.83P$	-0.83	120	48	$1.15 \times 10^{-6} P$
BC (t)	$-0.83P$	-0.83	120	48	$1.15 \times 10^{-6} P$
AD (s)	$0.97P$	0.97	139.92	0.44	$9.97 \times 10^{-6} P$
CD (s)	$0.97P$	0.97	139.92	0.44	$9.97 \times 10^{-5} P$
BD (t)	$-P$	-1	72	16	$3 \times 10^{-6} P$

$$\Delta_B = 2.524 \times 10^{-5} P$$

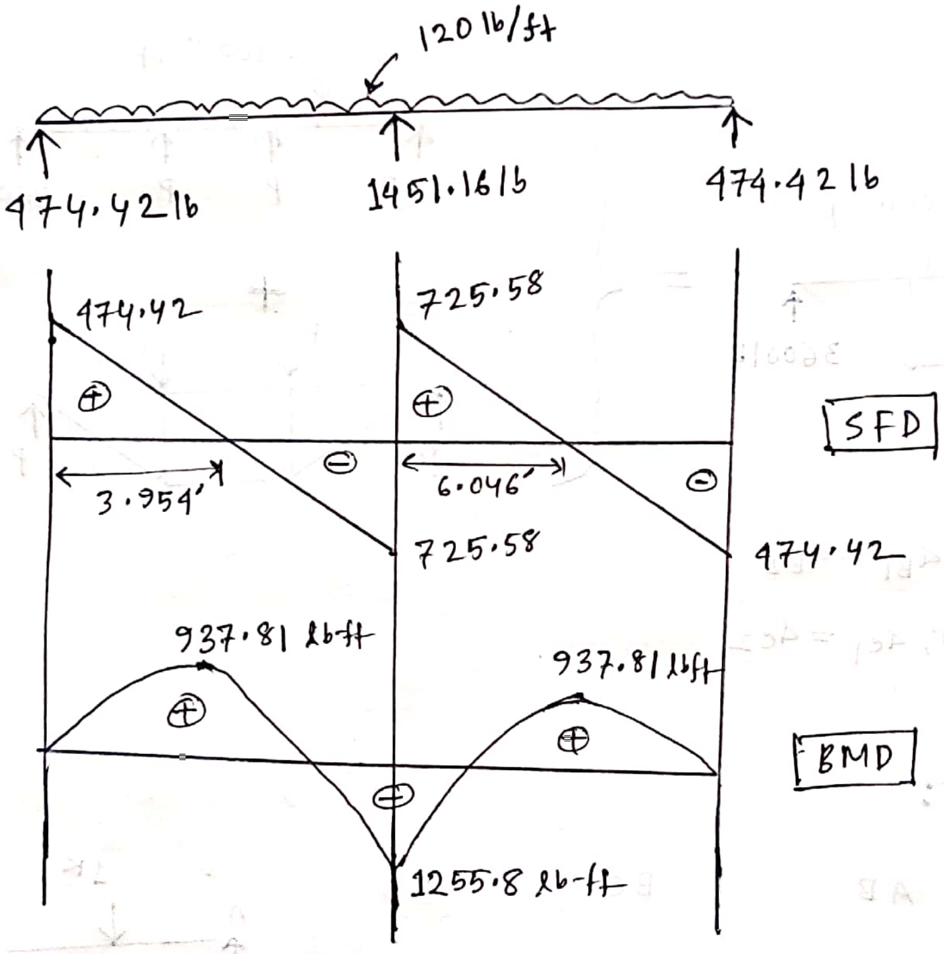
Thus,

$$\Delta_{B1} = \Delta_{B2}$$

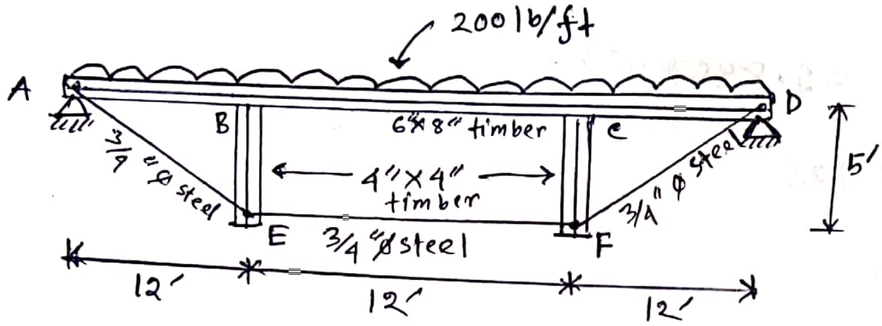
$$\Rightarrow 1.125 - 7.5\times10^{-4}P = 2.524\times10^{-5}P$$

$$\Rightarrow 7.7524\times10^{-4}P = 1.125$$

$$\therefore P = 1451.16 \text{ lb}$$



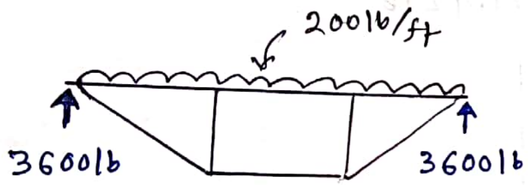
 # Problem-02: Analyze the queen-post truss as shown in figure.

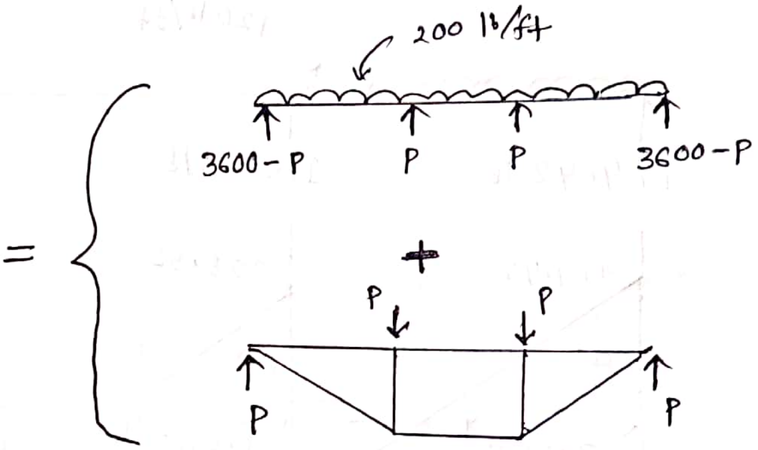


$E_t = 1500$ ksi

$E_s = 30000$ ksi

Solution:





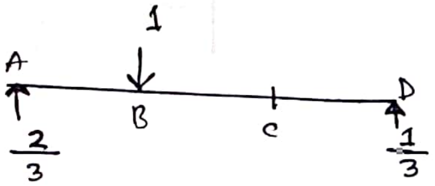
condition:

$\Delta_{B1} = \Delta_{B2}$

or, $\Delta_{C1} = \Delta_{C2}$

Beam Element:

Portion –	AB	BC	CD
origin –	A	A	D
Limit –	0–12	12–24	0–12
M –	$(3600-P)x - 100x^2$	$(3600-P)x - 100x^2 + P(x-12)$	$(3600-P)x - 100x^2$
m –	$\frac{2x}{3}$	$\frac{2}{3}x - (x-12)$	$\frac{x}{3}$



$$\Delta_B \cdot E_t I_t = \int_0^{12} (3600x - Px - 100x^2) \times \frac{2x}{3}\, dx + \int_{12}^{24} (3600x - Px - 100x^2 + Px - 12P) \times$$

$$\left(\frac{2x}{3} - x + 12\right) dx + \int_0^{12} (3600x - px - 100x^2) \frac{x}{3}\, dx$$

$$= \int_0^{12} \left(2400x^2 - \frac{2P}{3}x^2 - \frac{200}{3}x^3\right) dx + \int_{12}^{24} (3600x - 100x^2 - 12P) \times$$

$$\left(12 - \frac{x}{3}\right) dx + \int_0^{12} \left(1200x^2 - \frac{Px^2}{3} - \frac{100}{3}x^3\right) dx$$

$$= 1036800 - \int_0^{12} \frac{2P}{3} x^2\, dx + 2246400 - \int_{12}^{24} 12P\left(12 - \frac{x}{3}\right) dx$$

$$+ 518400 - \int_0^{12} \frac{P}{3} x^2\, dx$$

$$= 3801600 - \frac{2P}{3} \times \frac{12^3}{3} - \int_{12}^{24} (144P - 4Px)\, dx - \frac{P}{3} \times \frac{12^3}{3}$$

$$= 3801600 - 384P - 144P(24-12) + 4P \times \left(\frac{24^2}{2} - \frac{12^2}{2}\right) - 192P$$

$$= 3801600 - 576P - 1728P + 864P$$

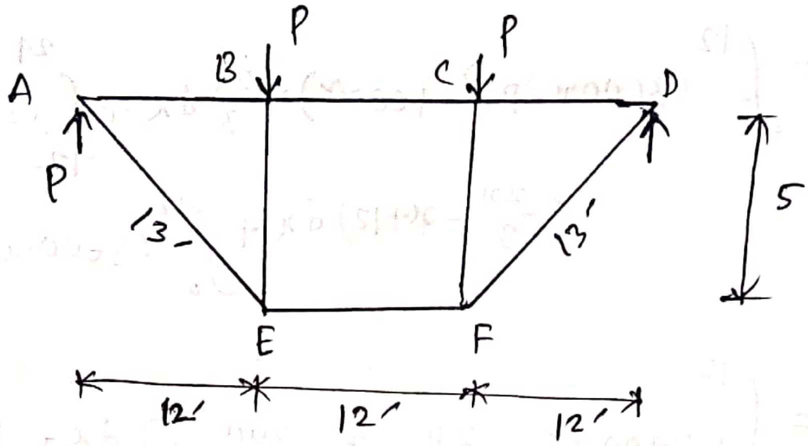
$$\Rightarrow \Delta_B \cdot E_t \cdot I_t = 3801600 - 1440P$$

$$\Rightarrow \Delta_B = \frac{3801600 - 1440P}{1500 \times 1000 \times 256} \times 12^3$$

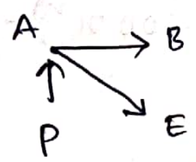
Here, $I = \frac{6 \times 8^3}{12}$

$$\therefore \Delta_B = 17{\cdot}1072 - 6{\cdot}48 \times 10^{-3} P$$

Truss Element:



joint A:



$AE = \frac{P}{5} \times 13$

$= 2.6\,P$

$\therefore DF = 2.6\,P$

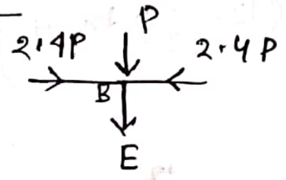
$AB = \frac{-2.6P}{13} \times 12$

$= -2.4\,P$

$\therefore CD = -2.4P$

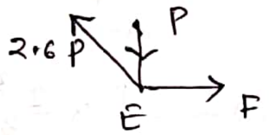
$BC = -2.4\,P$

joint B:



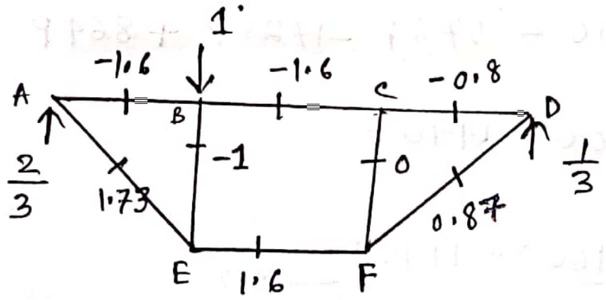
$\therefore BE = -P$ and, $CF = -P$

joint E:



$EF = \frac{2.6\,P}{13} \times 12 = 2.4\,P$

For unit load:



Member	S (lb)	U (lb)	L (in)	Area (in²)	E (lb/in²)	$\frac{SUL}{AE}$ (in)
AB	$-2.4P$	-1.6	144	48	1500×10^3	$7.68\times10^{-6}P$
DC	$-2.4P$	-0.8	144	48	1500×10^3	$3.84\times10^{-6}P$
BC	$-2.4P$	-1.6	144	48	1500×10^3	$7.68\times10^{-6}P$
EF	$2.4P$	1.6	144	0.44	30000×10^3	$4.19\times10^{-5}P$
AE	$2.6P$	1.73	156	0.44	30000×10^3	$5.32\times10^{-5}P$
DF	$2.6P$	0.87	156	0.44	30000×10^3	$2.67\times10^{-5}P$
BE	$-P$	-1	60	16	1500×10^3	$2.5\times10^{-6}P$
CF	$-P$	0	60	16	1500×10^3	0

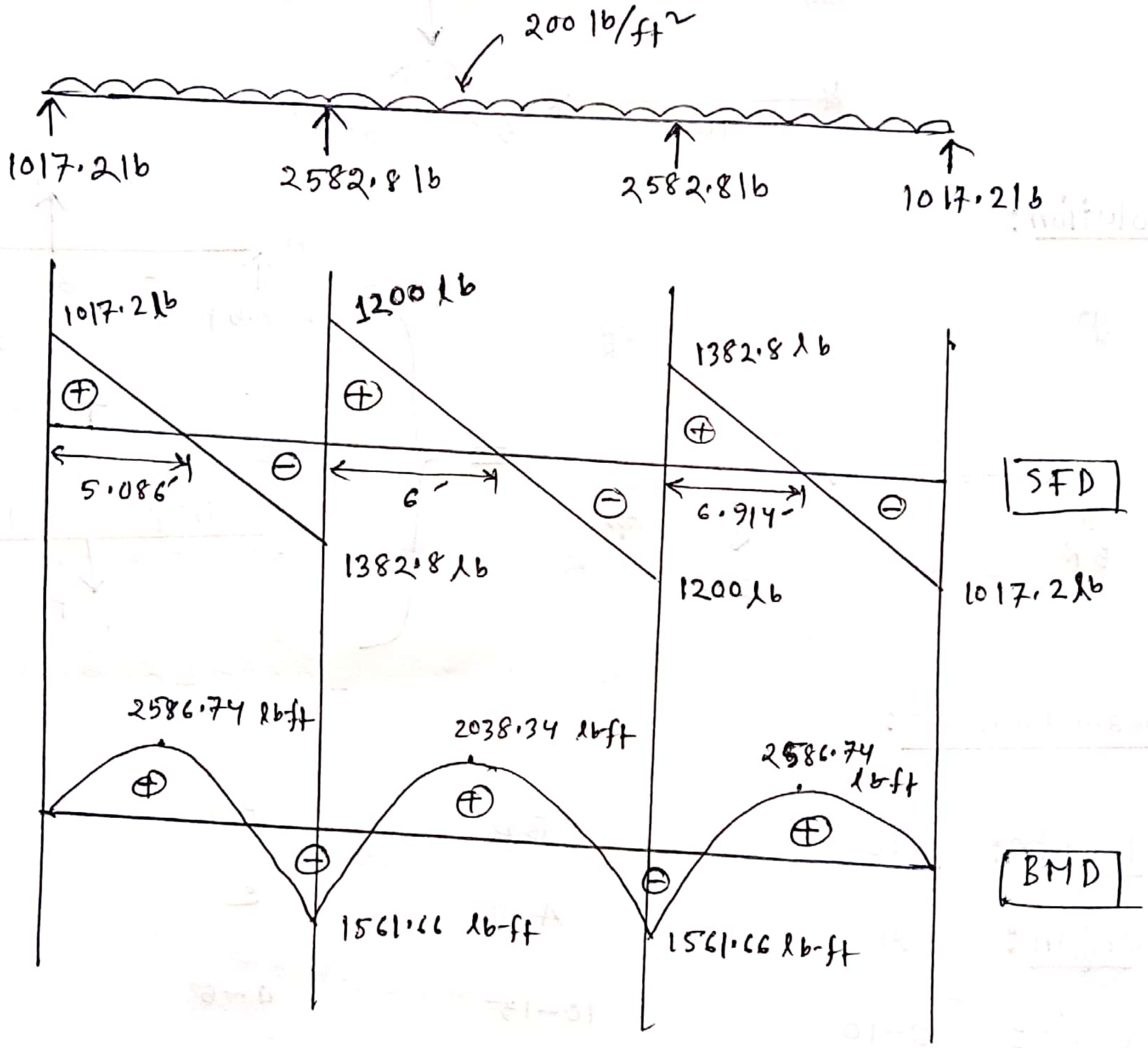
$\Delta B = 1.435\times10^{-4}\,P$

Thus, $\Delta_{B_1} = \Delta_{B_2}$

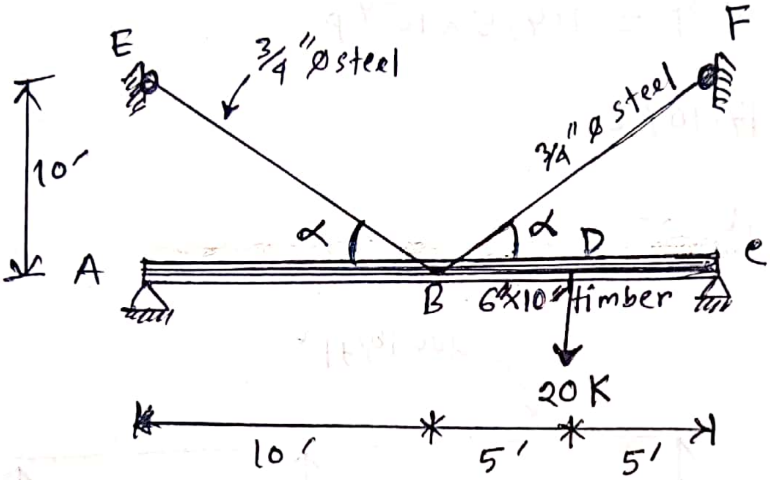
$\Rightarrow 17.1072 - 6.48 \times 10^{-3} P = 1.435 \times 10^{-4} P$

$\Rightarrow 6.6235 \times 10^{-3} P = 17.1072$

$\therefore P = 2582.8$ lb



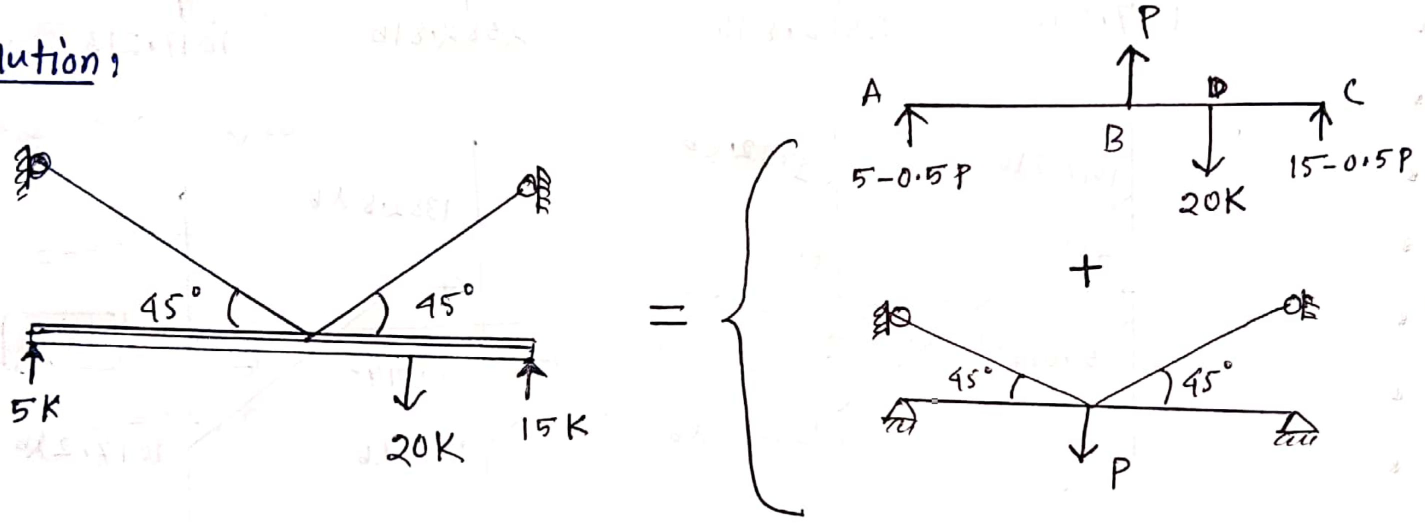
Problem-03: Draw SFD & BFD of the Beam 'AB'.



$E_t = 1500$ Ksi

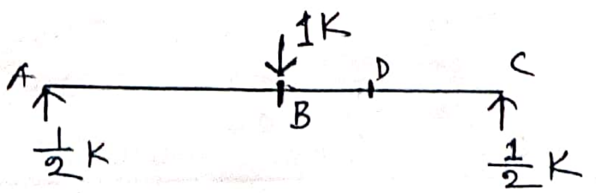
$E_s = 30000$ Ksi

Solution:



Beam Element:

Portion:	AB	BD	eD
Origin:	A	A	e
Limit:	0-10	10-15	0-5
M:	$(5-0.5P)x$	$(5-0.5P)x + P(x-10)$	$(15-0.5P)x$
m:	$\frac{x}{2}$	$\frac{x}{2} - (x-10)$	$\frac{x}{2}$



$$\Delta_B \cdot E_t \cdot I_t = \int_0^{10} \left(5x - \frac{Px}{2}\right) \times \frac{x}{2}\,dx + \int_{10}^{15} \left(5x - \frac{Px}{2} + Px - 10P\right) \times \left(10 - \frac{x}{2}\right) dx$$

$$+ \int_0^{5} \left(15x - \frac{Px}{2}\right) \times \frac{x}{2}\,dx$$

$$= \int_0^{10} (2.5x^2 - 0.25Px^2)\,dx + \int_{10}^{15} (50x + 5Px - 100P - 2.5x^2 - 0.25Px^2 + 5Px)\,dx$$

$$+ \int_0^{5} (7.5x^2 - 0.25Px^2)\,dx^2$$

$$= \frac{2500}{3} - 0.25P \times \frac{10^3}{3} + 50 \times \left[\frac{15^2}{2} - \frac{10^2}{2}\right] - 100P \times (15-10)$$

$$- 2.5 \times \left(\frac{15^3}{3} - \frac{10^3}{3}\right) - 0.25P \times \left[\frac{15^3}{3} - \frac{10^3}{3}\right] + 10P \times \left[\frac{15^2}{2} - \frac{10^2}{2}\right] + 312.5 - 0.25P \times \frac{5^3}{3}$$

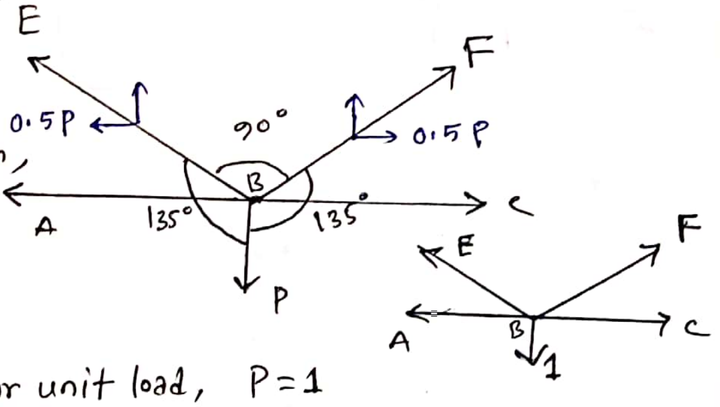
$$= 2291.67 - 83.3333P - 500P - 197.9167P + 625P - 10.4167P$$

$$\Rightarrow \Delta_B \cdot E_t \cdot I_t = 2291.67 - 166.67P$$

$$\therefore \Delta_B = \frac{2291.67 - 166.67P}{1500 \times 500} \times 12^3 = 5.28 - 0.384P$$

$$\frac{0.707}{14.14} \times 10 = 0.5P$$

Truss Element: From Lamy's theorem,



$$\frac{P}{\sin 90^\circ} = \frac{BE}{\sin 135} = \frac{BF}{\sin 135}$$

$$\Rightarrow BE = BF = 0.707P$$

For unit load, $P = 1$

$\therefore$ $BE = BF = 0.707$

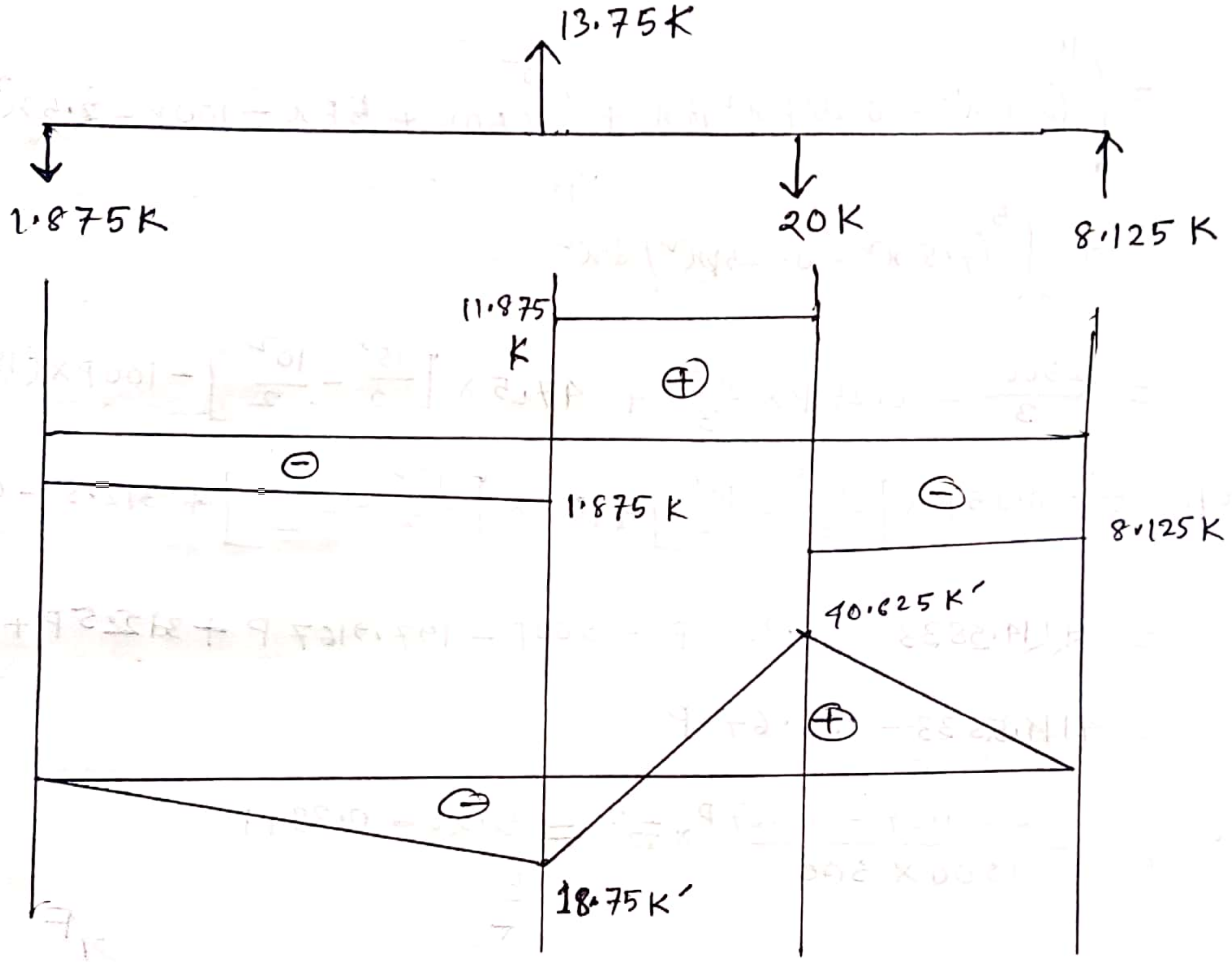
Member	Length(in)	S (K)	U (K)	A (in²)	$\frac{SuL}{AE}$
AB	120	0	0	60	0
BD	120	0	0	60	0
BE	169.7	0.707P	0.707	0.44	6.426×10^{-6} P
BF	169.7	0.707P	0.707	0.44	6.426×10^{-6} P

$$\Delta_B = 1.2852 \times 10^{-5} P$$

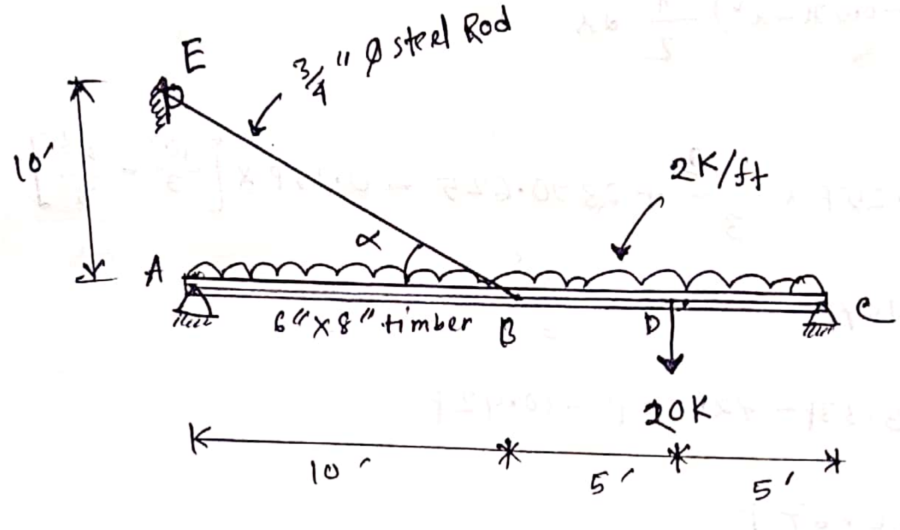
Now, $\Delta_{B_1} = \Delta_{B_2}$

$\Rightarrow 5.28 - 0.384P = 1.2852 \times 10^{-5} P$

$\Rightarrow \quad P = 13.75K$



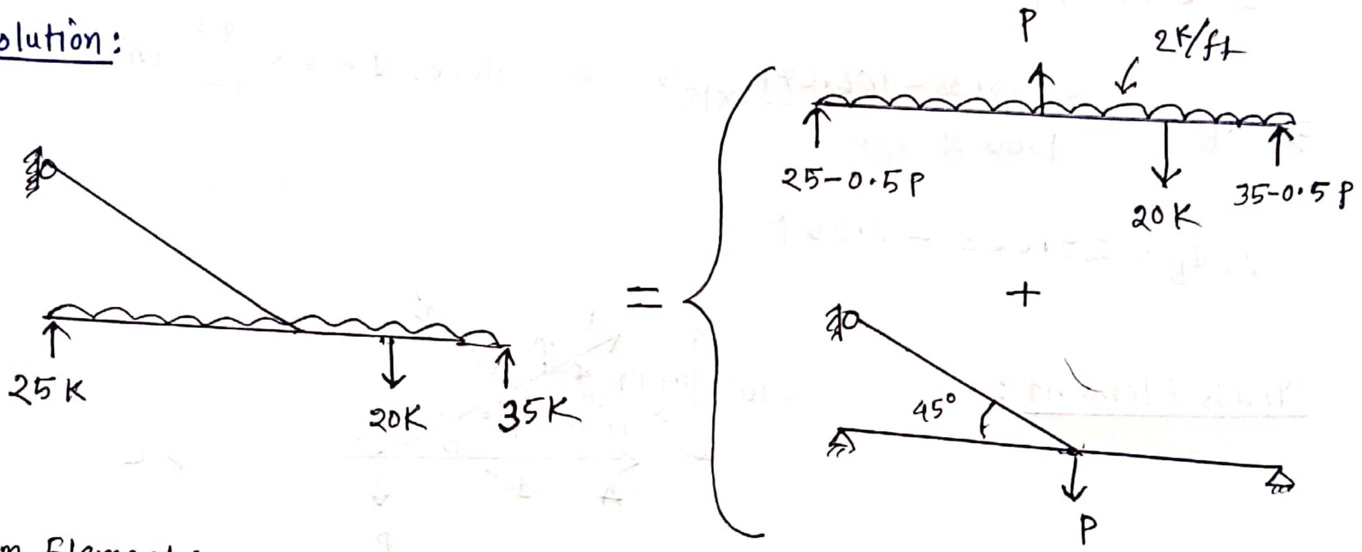
Problem-04: Draw the SFD & BMD of Member 'AB':



$E_t = 1500$ Ksi

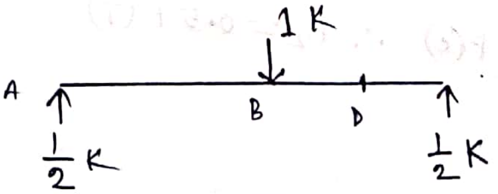
$E_s = 30000$ Ksi

Solution:



Beam Element:

Portion	—	AB	BD	CD
origin	—	A	D	D
Limit	—	0−10	5−10	0−5
M	—	$(25-0.5P)x - x^2$	$(35-0.5P)x - x^2$ $-20(x-5)$	$(35-0.5P)x$
m	—	$\frac{x}{2}$	$\frac{x}{2}$	$\frac{x}{2}$



$$\Delta_B EI = \int_0^{10} (25x - 0.5px - x^2) \times \frac{x}{2}\, dx + \int_5^{10} (35x - 0.5px - x^2 - 20x + 100) \times \frac{x}{2}\, dx$$

$$+ \int_0^{5} (35x - 0.5x - x^2)\, \frac{x}{2}\, dx$$

$$= 2916.67 - 0.25P \times \frac{10^3}{3} + 2890.625 - 0.25P \times \left[\frac{10^3}{3} - \frac{5^3}{3}\right]$$

$$+ 651.04167 - 0.25P \times \frac{5^3}{3}$$

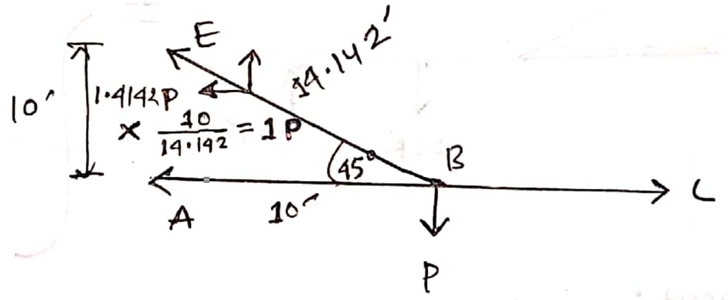
$$= 6458.34 - 83.33P - 72.92P - 10.42P$$

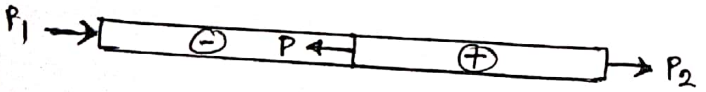
$$= 6458.34 - 166.67P$$

$$\Rightarrow \Delta_B = \frac{6458.38 - 166.67P}{1500 \times 256} \times 12^3 \qquad \text{Here, } I = 6 \times \frac{8^3}{12} \text{ in}^4$$

$$\therefore \Delta_B = 29.063 - 0.75P$$

Truss Element:





$$\Sigma F_y = 0$$

$$BE = \frac{P}{10} \times 14.142$$

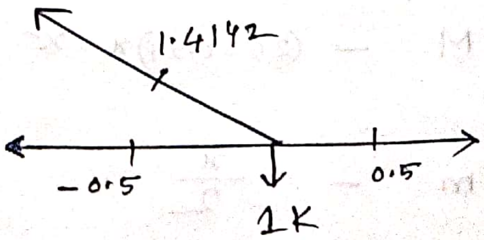
$$= 1.4142P$$

We know,

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$$

$$\Rightarrow P_1 \times 10 = P_2 \times 10 \qquad [\text{Here, } A_1E_1 = A_2E_2]$$

$$\Rightarrow P_1 = P_2$$



Again, $P_1 + P_2 = P$

$$\Rightarrow 2P_1 = P$$

$$\Rightarrow P_1 = 0.5P(C) \quad \therefore P_2 = 0.5P\,(T)$$

* For unit load
$P = 1K$

Member	Length (in)	S(K)	U(K)	A (in^2)	$\frac{SUL}{AE}$ (in)
AB	120	$-0.5P$	-0.5	48	$4.167\times10^{-4}P$
BC	120	$0.5P$	0.5	48	$4.167\times10^{-4}P$
BE	169.7	$1.4142P$	1.4142	0.44	$0.0257P$

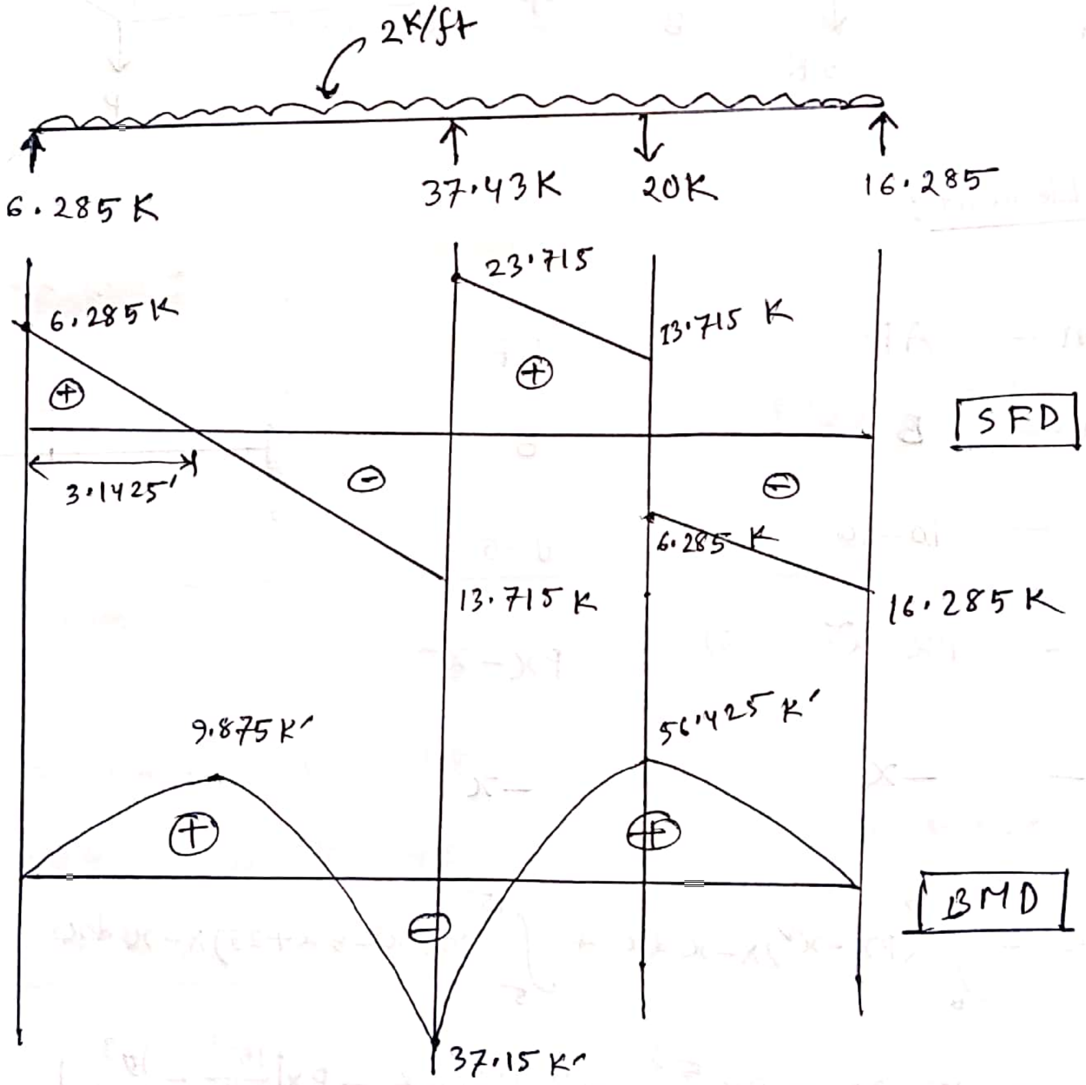
$\Delta_B = 0.02655P$

Thus, $\Delta_{B_1} = \Delta_{B_2}$

$\Rightarrow 29.063 - 0.75P = 0.02655$

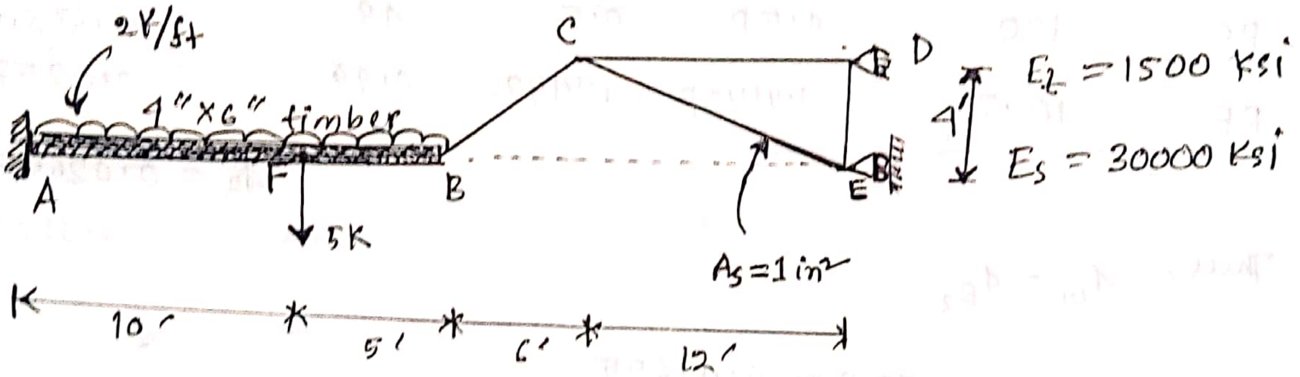
$\Rightarrow 0.77655P = 29.063$

$\therefore P = 37.43$ K

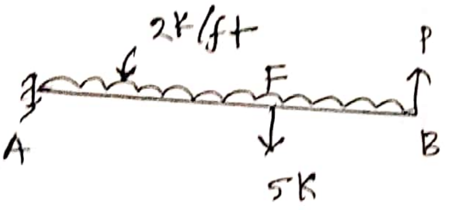


2007

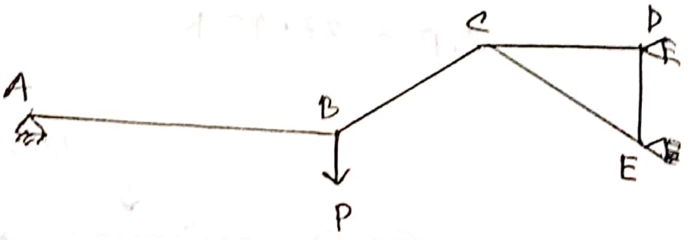
Problem-5: Draw SFD & BMD of Member 'AB'



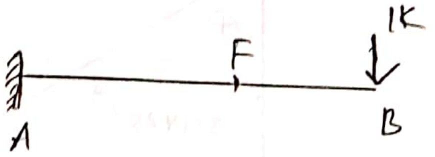
Solution:



+



Beam Element:



Portion —	AF	BF
origin —	B	B
Limit —	5 − 15	0 − 5
M —	$Px - x^2 - 5(x-5)$	$Px - x^2$
m —	$-x$	$-x$

$$\Delta_B \cdot E_t \cdot I = \int_0^5 (Px - x^2)\, x - x \, dx + \int_5^{15} (Px - x^2 - 5x + 25)\, x - x \, dx$$

$$= 156.25 - P \times \frac{5^3}{3} + 15416.67 - P \times \left[\frac{15^3}{3} - \frac{5^3}{3}\right]$$

$$= 15572.92 - 1125P$$

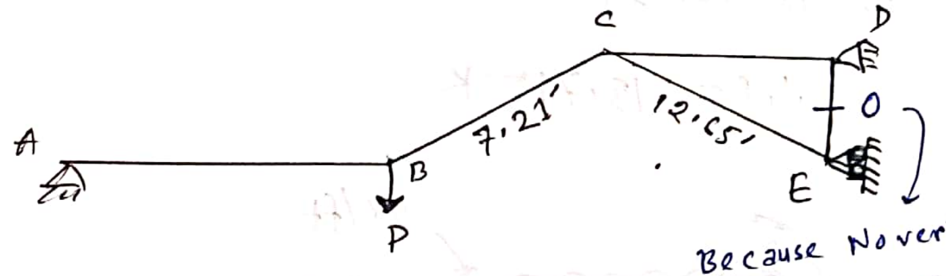
$$\Delta_B = \frac{15572.92 - 1125P}{1500 \times 72} \times 12^3$$

Here, $I = 4 \times \frac{6^3}{12}$

$= 72 \text{ in}^4$

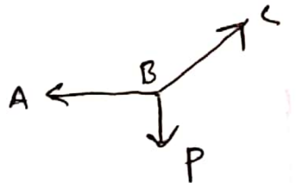
$$= 249.17 - 18P$$

Truss Element:



Because No vertical member in Truss

Joint B



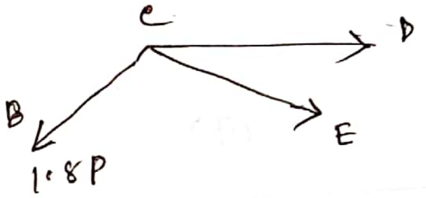
$BC = \frac{P}{4} \times 7.21$

$BC = 1.8P$

$AB = \frac{1.8P}{7.21} \times 6$

$= 1.5P$

Joint C



$CE = \frac{-1.8P}{7.21} \times 12.65$

$= -3.16P$

$\Sigma F_x = 0$

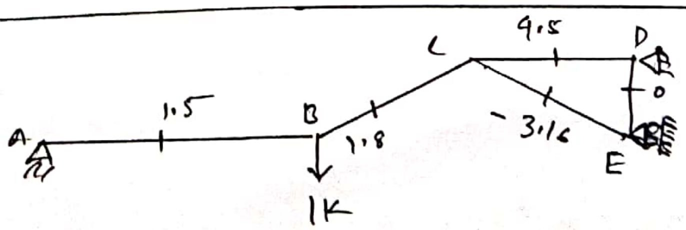
$CD = \frac{1.8P}{7.21} \times 6 + \frac{3.16P}{12.65} \times 12$

$= 4.5P$

Member	L(in)	S(K)	U(K)	Area(in²)	$\frac{SUL}{AE}$ (in)
AB	180	1.5P	1.5	24	0.01125P
BC	86.52	1.8P	1.8	1	9.34916×10^{-3}
CD	144	4.5P	4.5	1	0.0972P
CE	151.8	-3.16P	-3.16	1	0.05053P

$\Delta_B = 0.1683P$

For unit load:

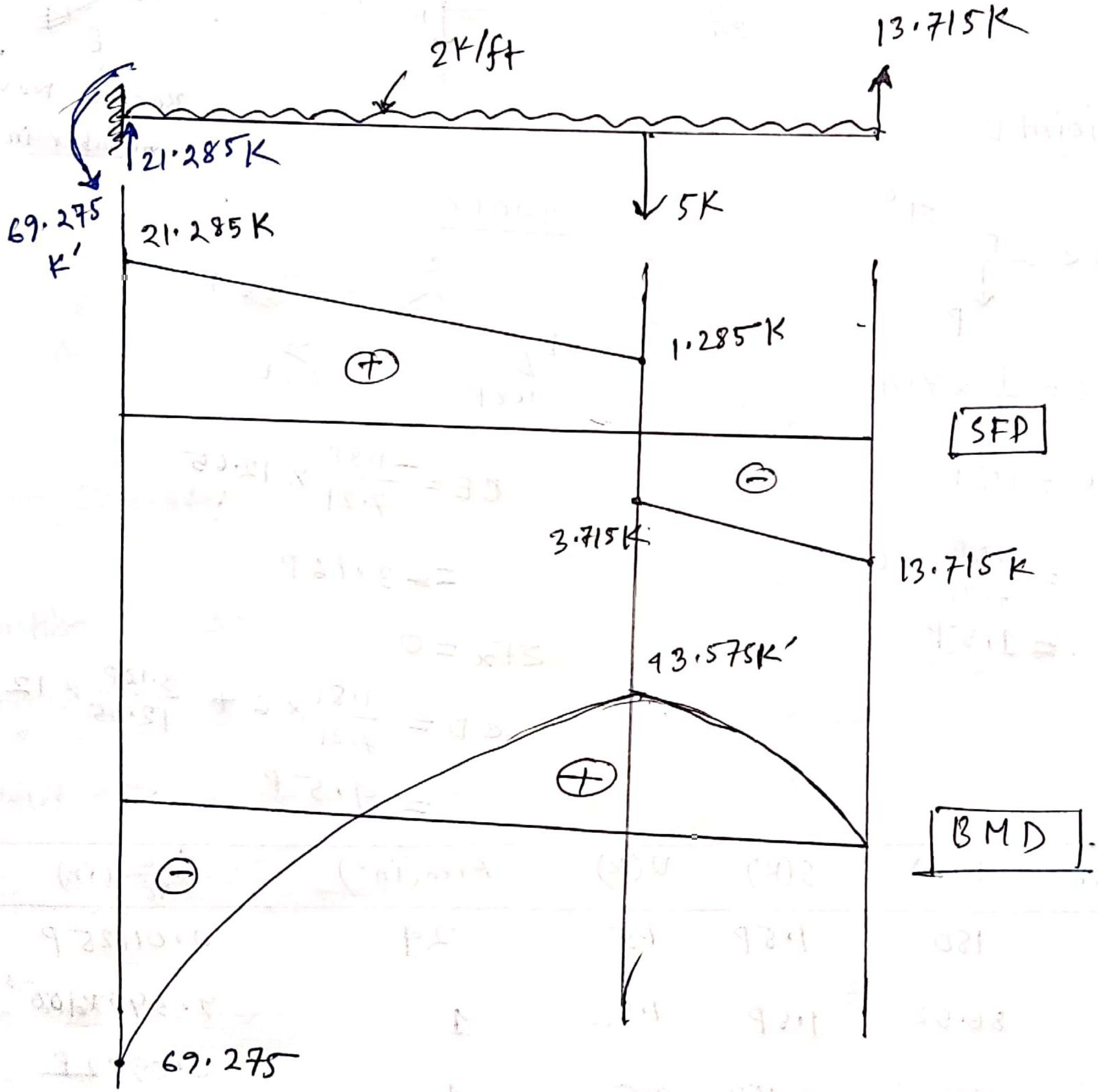


Thus, $\Delta_{B_1} = \Delta_{B_2}$

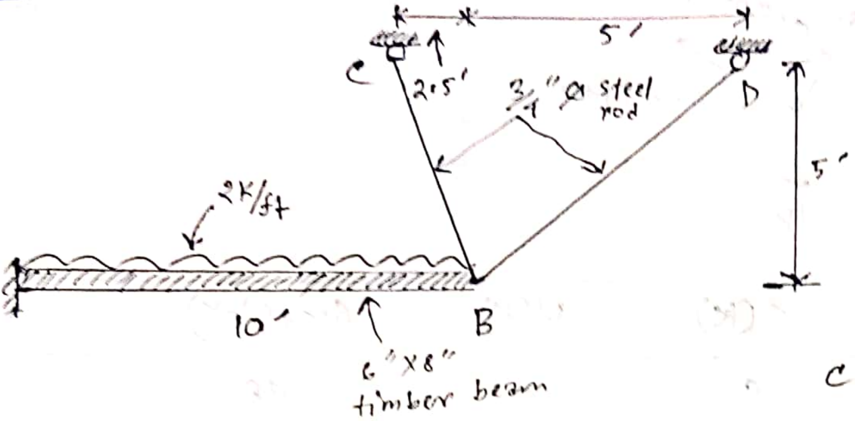
$\Rightarrow 249.17 - 18P = 0.1683$

$\Rightarrow 18.1683P = 249.17$

$\therefore P = 13.715\,K$



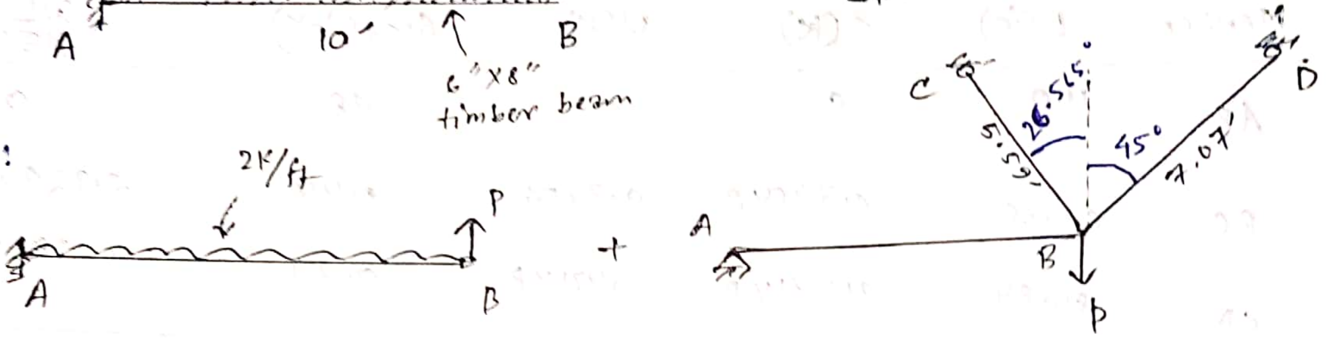
Problem-06: Draw SFD & BMD of Member AB.



$E_t = 1500$ ksi

$E_s = 30000$ ksi

Solution:



Beam Element:

Portion — AB

origin — B

Limit — 0-10'

M — $(px - x^2)$

m — $(-x)$

$$E_t \cdot I_t \cdot d_B = \int_0^{10} (px - x^2) \times (-x)\, dx$$

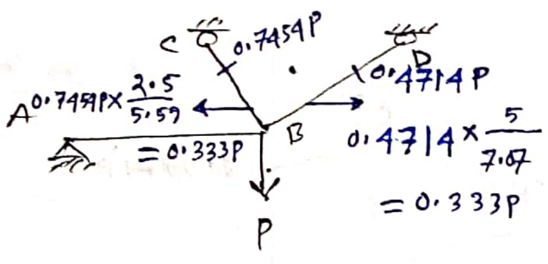
$$= \int_0^{10} - px^2\, dx + \int_0^{10} x^3\, dx$$

$$= 2500 - 333.33P$$

$$\therefore d_B = \frac{2500 - 333.33P}{1500 \times 6 \times \frac{8^3}{12}} \times 12^3$$

$$= 11.25 - 1.25P$$

Truss Element:



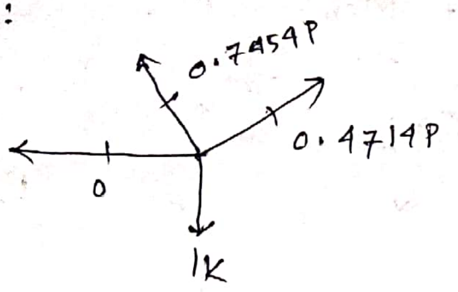
using Lamy's theorem,

$$\frac{P}{\sin 71.565} = \frac{CB}{\sin 135} = \frac{BD}{\sin 153.435}$$

$\Rightarrow CB = 0.7454P$

and, $BD = 0.4714P$

For unit load:



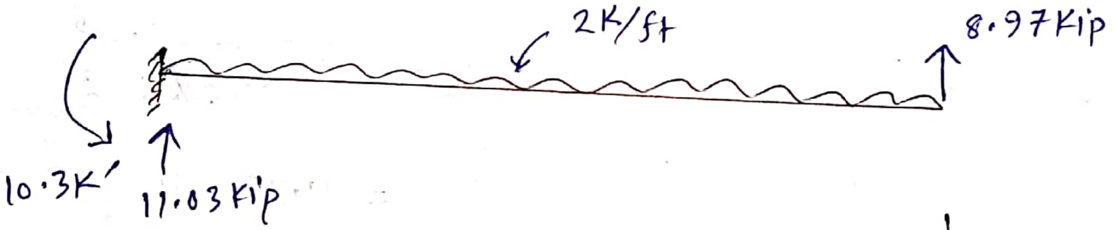
Member	l (in)	S (K)	U(K)	Area (in²)	$\frac{SUL}{AE}$
AB	120	0	0	48	0
BC	67.08	0.7454P	0.7454	0.44	2.824×10^{-3} P
CD	84.84	0.4714P	0.4714	0.44	1.4283×10^{-3} p

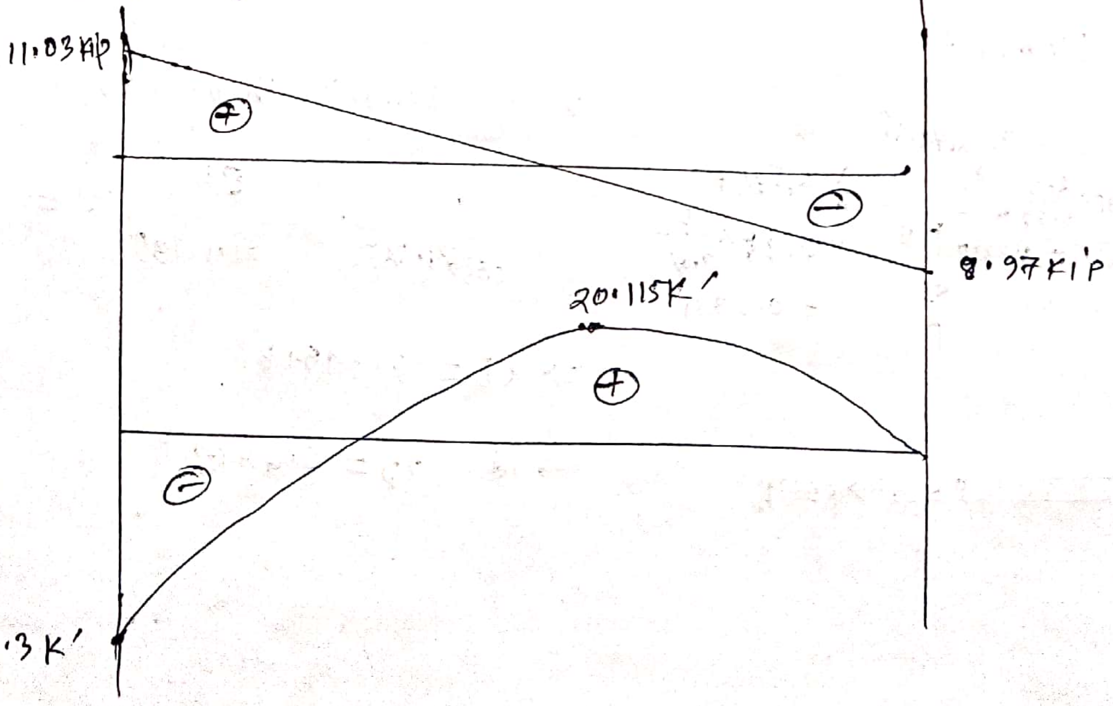
$\Delta_B = 4.2523 \times 10^{-3}$ P

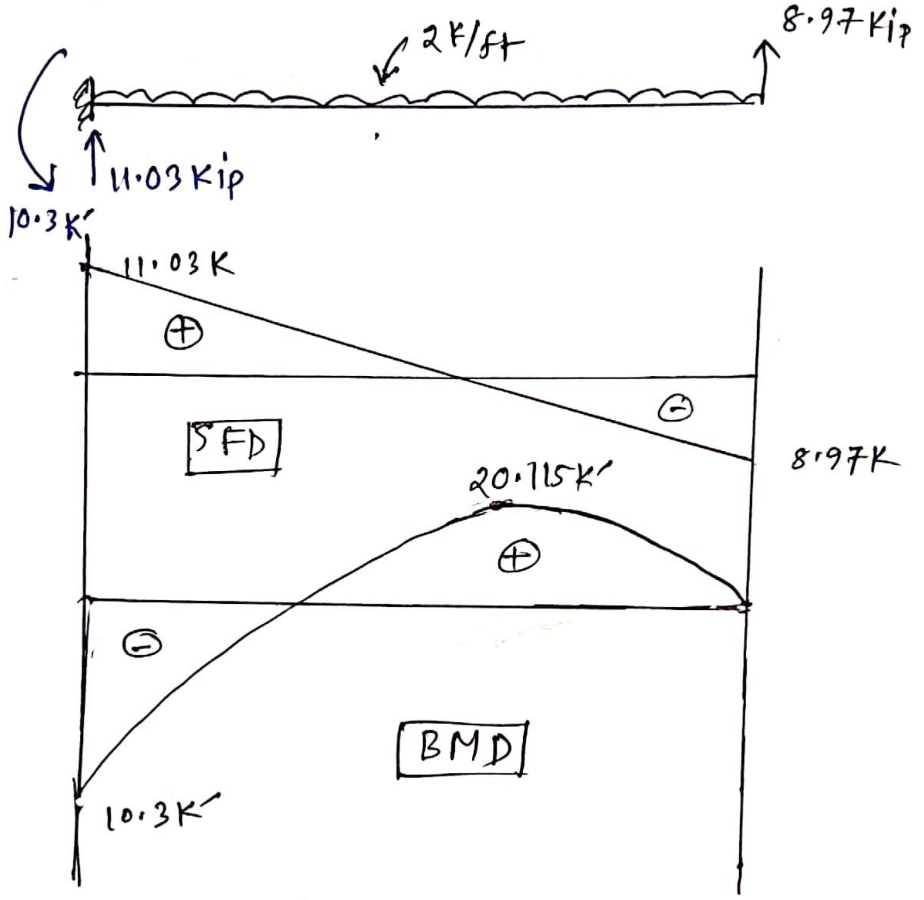
$\Delta_{B_1} = \Delta_{B_2}$

$\Rightarrow 11.25 - 1.25P = 4.2523 \times 10^{-3} p$

$\Rightarrow P = 8.97$ kip.



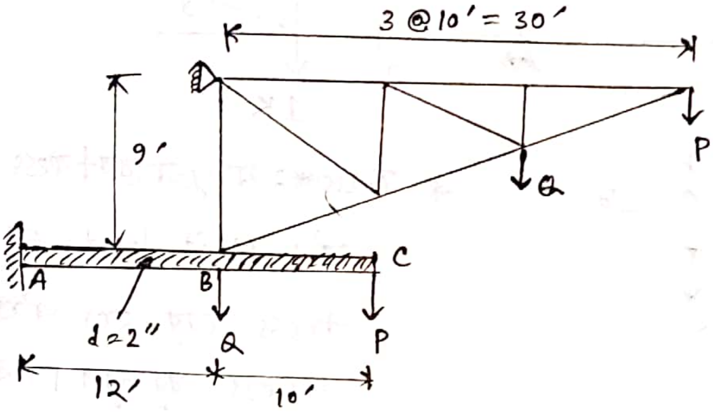



2K/ft
8.97 Kip
11.03 Kip
10.3K'
11.03K
⊕
⊖
SFD
8.97K
20.715K'
⊕
⊖
BMD
10.3K'

Composite Structure

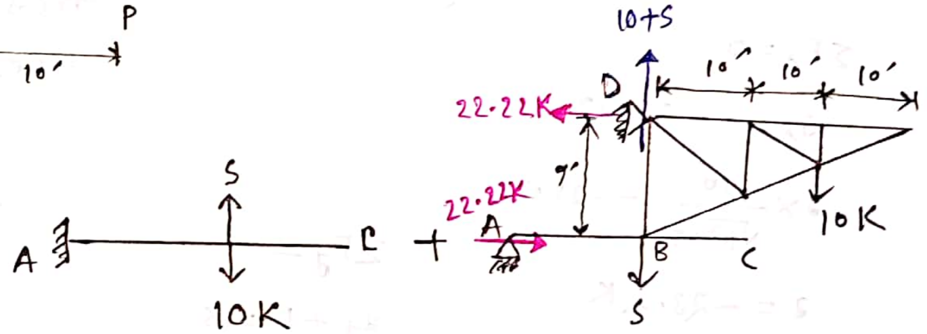
2017

A part of an electric transmission tower, made of steel is supported by a steel beam ABC and carried concentrated load P and Q as shown in figure below. Calculate vertical deflection at B if the value of P is zero and Q = 10 kips. Also draw SFD and BMD of the Beam ABC. Cross sectional area of each member of the truss is 0.15 in².



Solution:

Equivalent structure:



$\Sigma M = 0$

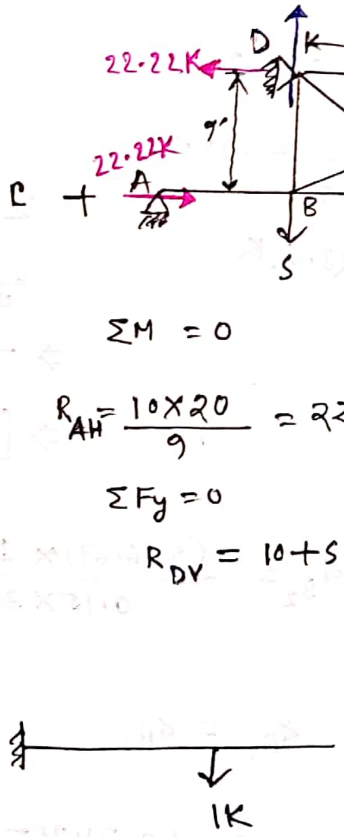
$R_{AH} = \frac{10 \times 20}{9} = 22.22K$

$\Sigma F_y = 0$

$R_{DV} = 10 + S$

Beam Element:

Portion –	AB	BC
origin –	c	c
Limit –	(10 – 22)	(0 – 10)
M –	$-(10-S) \times (x-10)$	0
m –	$-1 \times (x-10)$	0

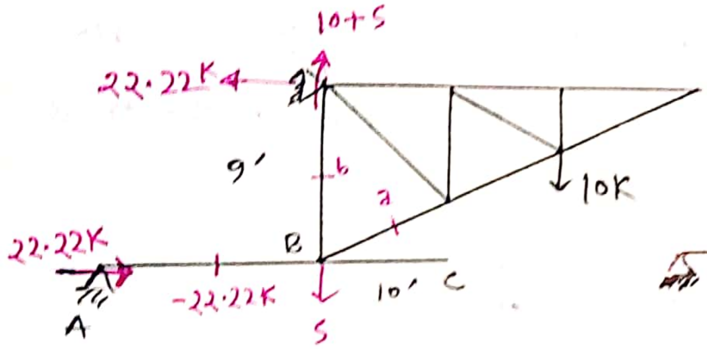


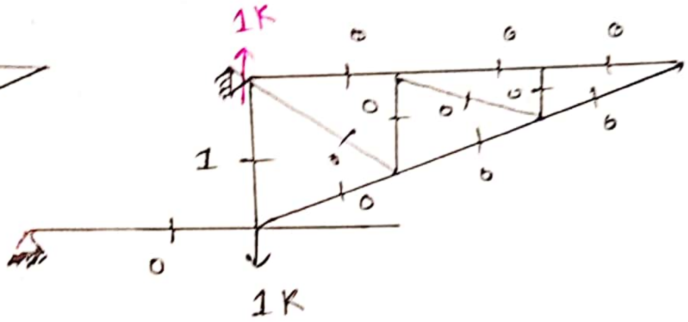
$$\Delta_B \cdot EI = \int_{10}^{22} \{-(10-S) \times (x-10)\} \times \{-(x-10)\}\, dx$$

$$= \int_{10}^{22} 10\,(x-10)^2\, dx - \int_{10}^{22} S\,(x-10)^2\, dx = 5760 - 576S$$

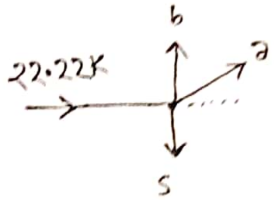
$$\Delta_{B_1} = \frac{5760 - 576S}{30000 \times \frac{\pi}{64} \times (2)^4} \times 17.28 = 422.43 - 42.243S$$

Truss Element:





Joint B:



* Truss-এ সবগুলার stress বের করার প্রয়োজন নেই। Unit load-এর জন্য যেগুলোতে stress বের হবে ঐগুলা member-এর stress বের করব। কারণ –

$$\Delta = \frac{SUL}{AE} \cdot \boxed{U = 0 \text{ হলে}, \Delta = 0}$$

$\Sigma F_x = 0$

$a_x = -22.22$

$a \times \frac{10}{\sqrt{10^2+3^2}} = -22.22$

$a = -23.2K$

$\Sigma F_y = 0$

$a_y + b = S$

$\Rightarrow -23.2 \times \frac{3}{\sqrt{10^2+3^2}} + b = S$

$\Rightarrow \boxed{b = S + 6.67}$

$$\therefore \Delta_{B_2} = \frac{(S+6.67) \times 1 \times (9 \times 12)}{0.15 \times 30000} = 0.024S + 0.16$$

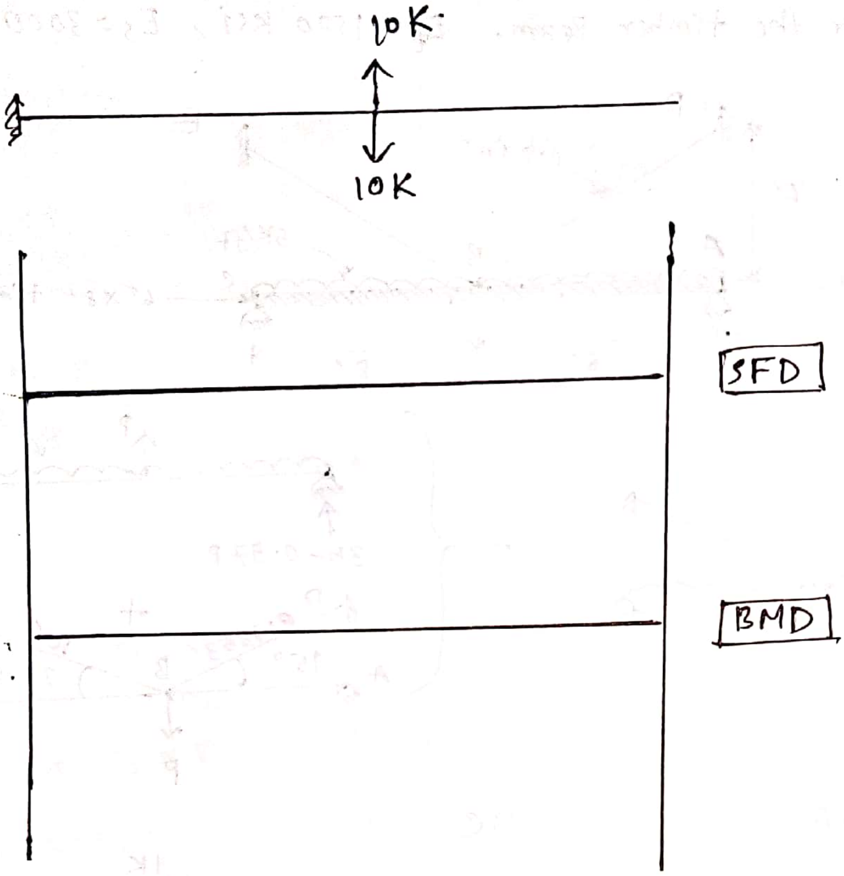
Hence, $\Delta_{B_1} = \Delta_{B_2}$

$\Rightarrow 422.43 - 42.243S = 0.024S + 0.16$

$\Rightarrow S = 9.99K \simeq 10K$

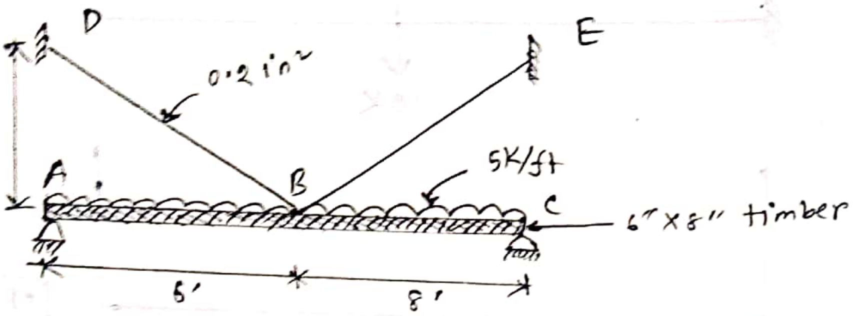
Now, $\Delta_B = 0.024 \times 10 + 0.16 = 0.4$ in. (↓)

(Ans)

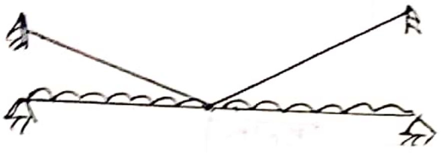

10K
10K
SFD
BMD

2016

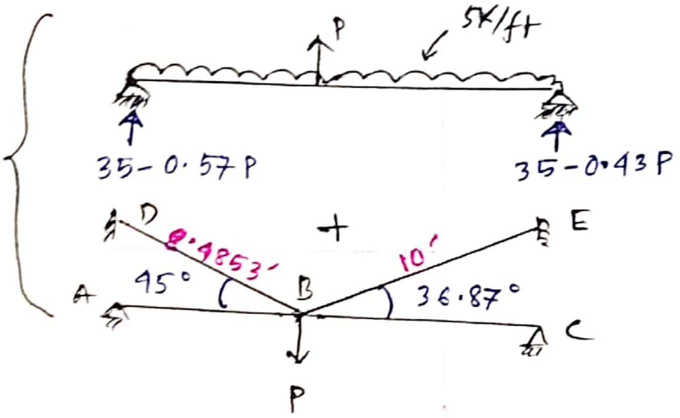
Draw BMD for the timber Beam. $E_t = 1500$ ksi, $E_s = 30000$ ksi



Solution:



=



Beam Element:

Portion –	AB	BC
origin –	A	B
Limit –	0 − 6	0 − 8
M	— (35 − 0.57P)x − 2.5x²	(35 − 0.43P)x − 2.5x²
m	— 0.57x	0.43x

$$\Delta_B \cdot E_t \cdot I = \int_0^6 (35x - 0.57Px - 2.5x^2) \times 0.57x\,dx + \int_0^8 (35x - 0.43Px - 2.5x^2) \times 0.43x\,dx$$

$$= 974.7 - 0.57^2 P \int_0^6 x^2 dx + 1467.73 - 0.43^2 P \int_0^8 x^2 dx$$

$$= 2442.433 - 23.3928P - 31.5563P$$

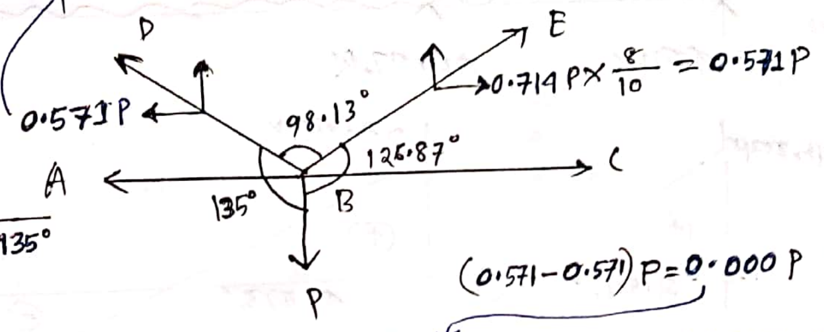
$$\Rightarrow \Delta_B = \frac{2442.433 - 54.9491P}{1500 \times 6 \times \frac{8^3}{12}} \times 12^3$$

$$\therefore \Delta_B = 10.991 - 0.2473P$$

Truss element:

$$\boxed{0.808P\times\frac{6}{8.4853} = 0.571P}$$

From Lamy's theorem,



$$\frac{P}{\sin 98.13^\circ} = \frac{BD}{\sin 126.87} = \frac{BE}{\sin 135^\circ}$$

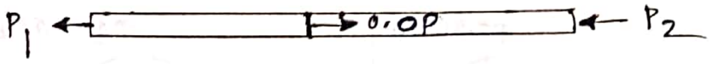
Hence,

BD = 0.808 P

and BE = 0.714P

$(0.571-0.571)P = 0.000 P$ (→)

* যদি difference থাকত, তাহলে এই procedure-এ করতাম

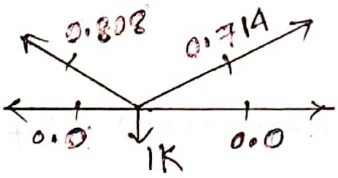


We know,

$$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2} \quad [\because A_1E_1 = A_2E_2]$$

$\Rightarrow P_1 \times 6 = P_2 \times 8$

$\Rightarrow P_1 = 1.333P_2$



And,

$P_1 + P_2 = 0.000 P$

$\Rightarrow 1.333P_2 + P_2 = 0.000 P$

$\Rightarrow P_2 = 0.000P$ (BC)

$\therefore P_1 = (1.333 \times 0.000P)$

$= 0.000P$ (AB)

Member	Length (in)	S (K)	U (K)	A (in²)	$\frac{SUL}{AE}$
AB	72	0	0	48	0
BC	96	0	0	48	0
BD	101.8236	0.808P	0.808	0.2	0.01108P
BE	120	0.714P	0.714	0.2	0.0102P

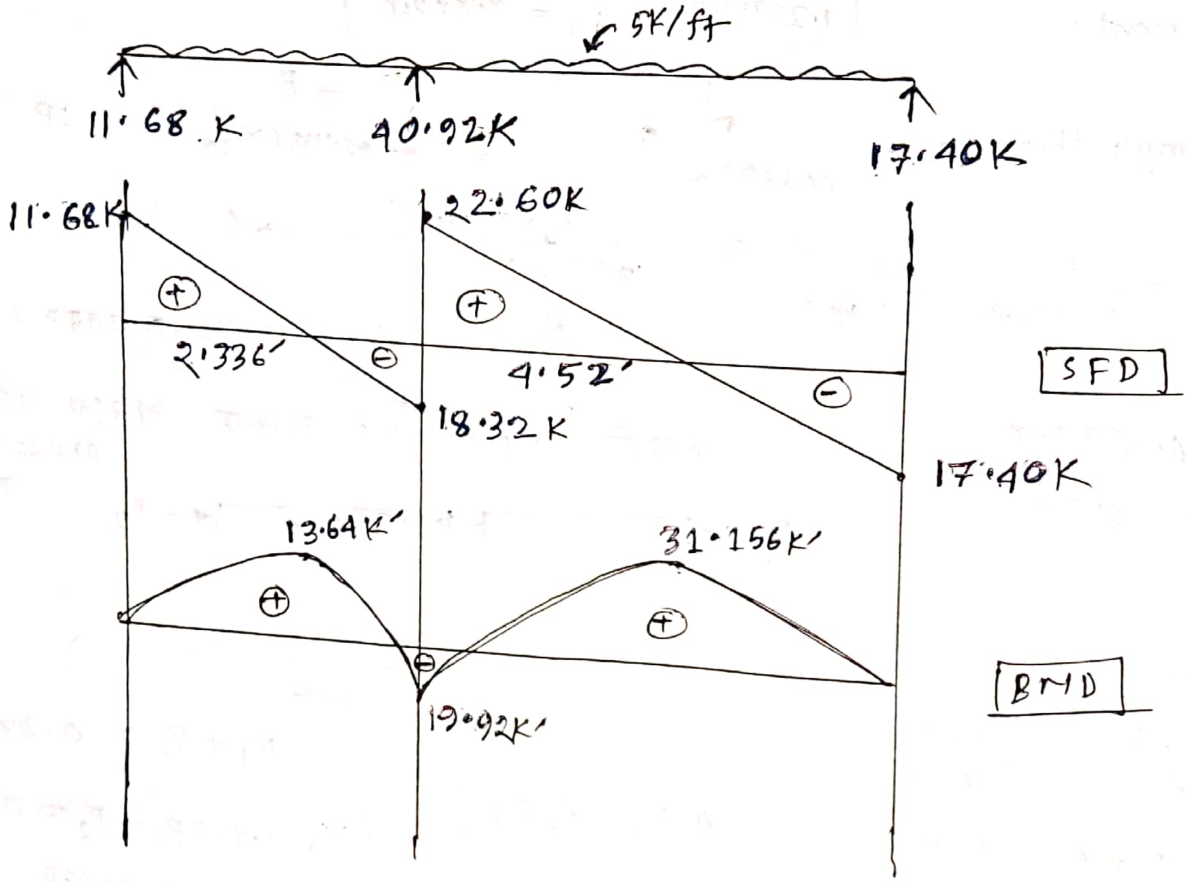
$\Delta_B = 0.02128P$

Thus,

$\Delta_{B_1} = \Delta_{B_2}$

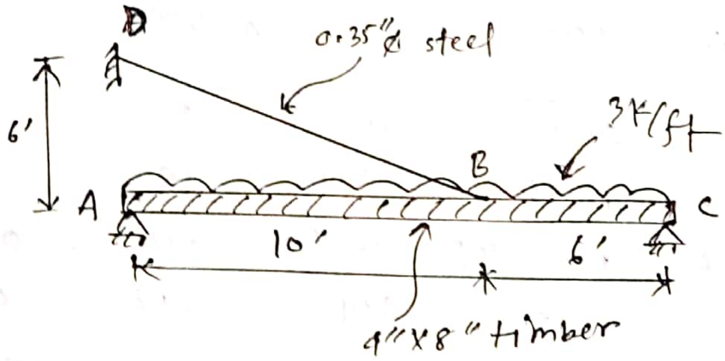
$\Rightarrow 10.991 - 0.2473P = 0.02128P$

$\Rightarrow P = 40.92K$

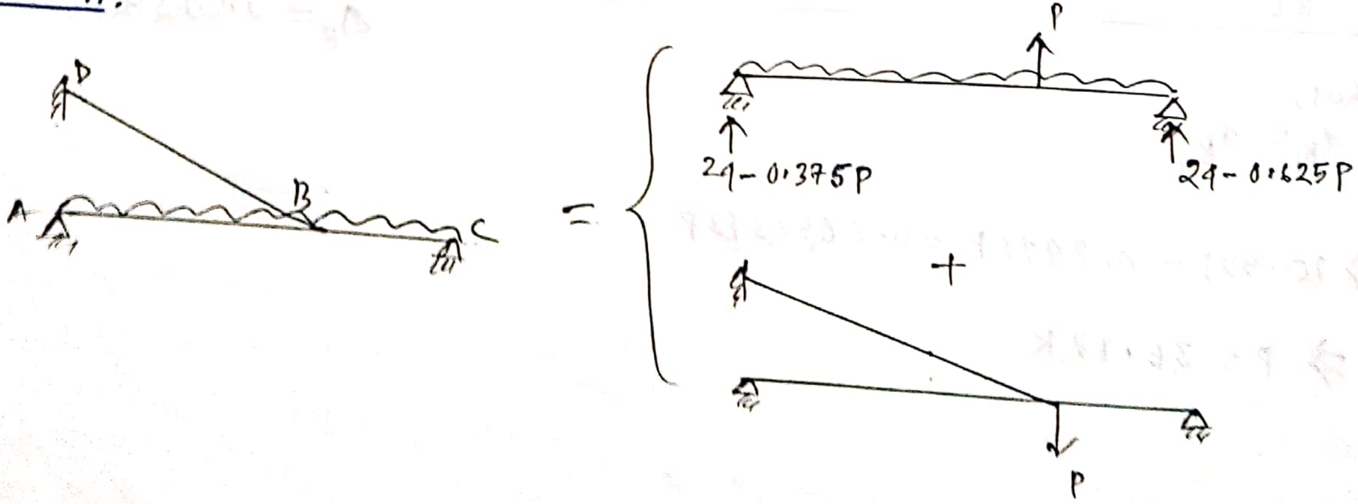


2015

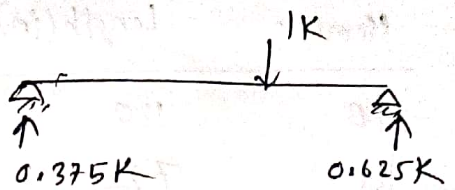
Draw BMD for the timber beam. $E_t = 1500$ ksi , $E_s = 30000$ ksi



Solution:



Beam element:



Portion -	AB	BC
origin -	A	C
Limit -	0-10'	0-6'
M -	$(24-0.375P)x-1.5x^2$	$(24-0.625P)x-1.5x^2$
m -	$0.375x$	$0.625x$

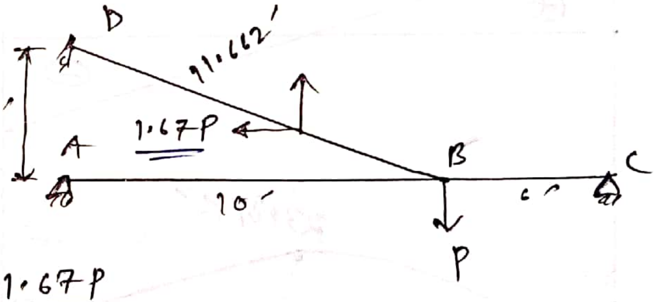
$$\Delta_B \cdot E_t I = \int_0^{10} (24x - 0.375px - 1.5x^2)\times 0.375x\,dx + \int_0^{6} (24x-0.625px-1.5x^2)\times 0.625x\,dx$$

$$= 1593.75 - 0.375^2 p \int_0^{10} x^2\,dx + 776.25 - 0.625^2 p \int_0^{6} x^2\,dx$$

$$= 2370 - 46.875P - 28.125P$$

$$\Rightarrow \Delta_B = \frac{2370 - 75P}{1500 \times 4 \times \frac{83}{12}} \times 12^3 = 15.9975 - 0.50625P$$

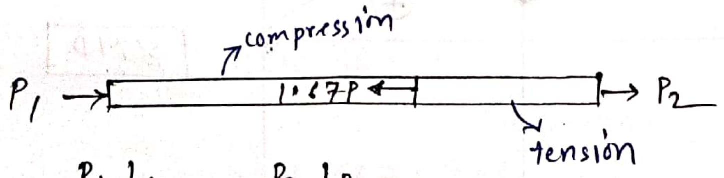
Truss Element:

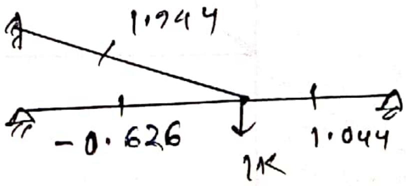


$BD = \frac{P}{6} \times 11.662$

$= 1.944P$

Now, $1.944 \times \frac{10}{11.662} = 1.67P$





$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$ $[\because A_1 E_1 = A_2 E_2]$

$\Rightarrow P_1 \times 10 = P_2 \times 6$

$\Rightarrow P_1 = 0.6 P_2$

And, $P_1 + P_2 = 1.67P$

$\Rightarrow 1.6 P_2 = 1.67P$

$\Rightarrow P_2 = 1.044P$

$\therefore P_1 = (0.6 \times 1.044)P$
$= 0.626P$

Member	Length(in)	S(K)	v(K)	Area(in²)	$\frac{SuL}{AE}$ (in)
AB	120	−0.626P	−0.626	32	9.8×10^{-4} P
BC	72	1.044P	1.044	32	1.635×10^{-3}P
BD	139.944	1.944P	1.944	0.0962	0.1833P

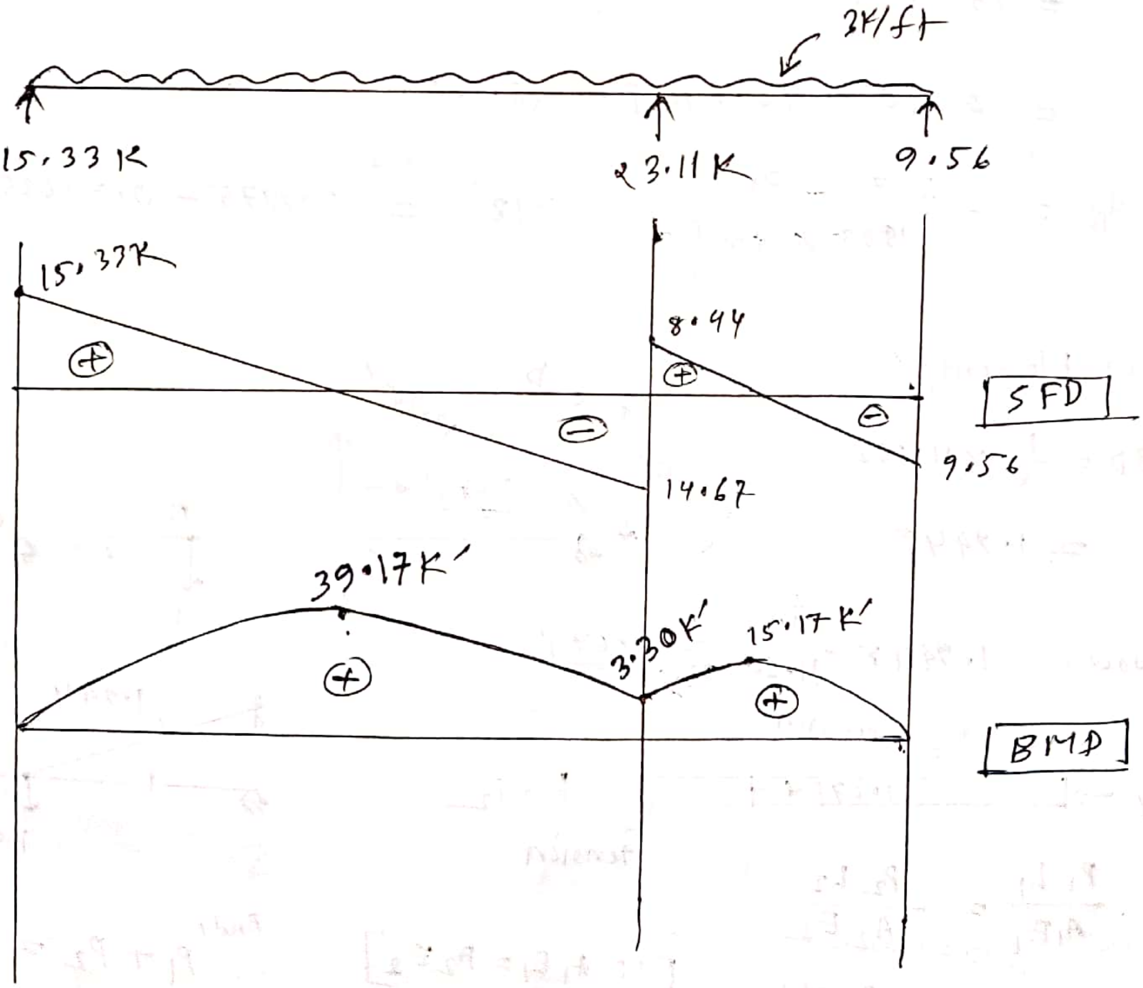
$\Delta_B = 0.18592\ P$

Thus,

$\Delta_{B_1} = \Delta_{B_2}$

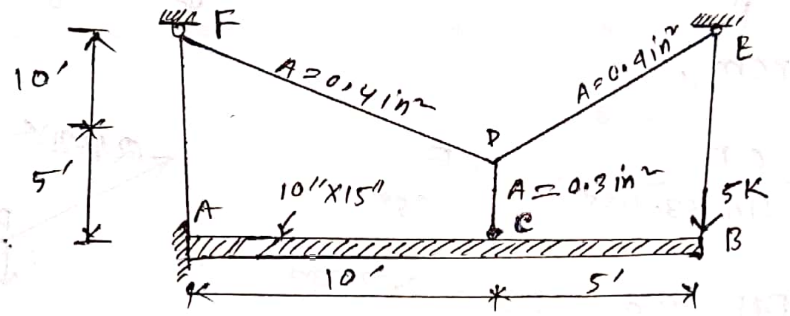
$\Rightarrow 15.9975 - 0.50625P = 0.18592\ P$

$\Rightarrow P = 23.11\ K$



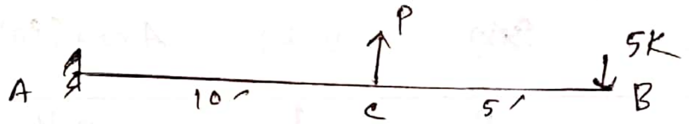
2014

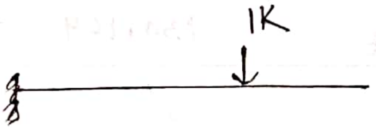
A concrete beam AB is supported by a simple steel truss as shown in figure below. Draw SFD & BMD of the beam. $E_s = 30 \times 10^3$ Ksi & $E_c = 3200$ Ksi



Solution:

Beam portion:





portion —	AC	CB
origin —	B	B
Limit —	(5-15)	(0-5)
M —	$-5x+P(x-5)$	$-5x$
m —	$-1(x-5)$	0

$$\Delta_C \cdot E_c I = \int_5^{15} (-5x + Px - 5P)[-(x-5)]\,dx + \int_0^5 (-5x) \times 0\,dx$$

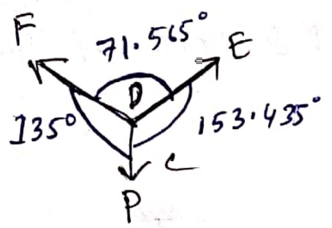
$$= 2916.667 - P\int_5^{15} (x-5)^2\,dx + 0$$

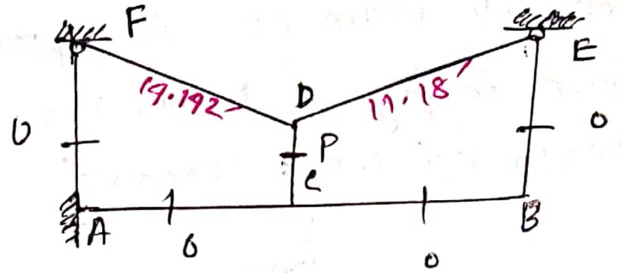
$$= 2916.667 - 333.333\,P$$

$$\Rightarrow \Delta_C = \frac{2916.667 - 333.33\,P}{3200 \times 10 \times \frac{15^3}{12}} \times 12^3 = 0.56 - 0.064\,P$$

Truss portion:

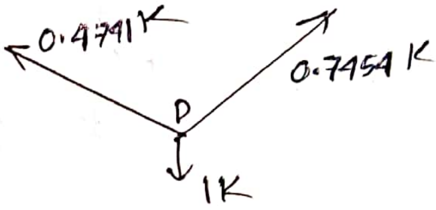
joint D:





From Lamy's theorem,

$$\frac{P}{\sin 71.565} = \frac{CF}{\sin 153.435} = \frac{DE}{\sin 135°}$$



$\Rightarrow$ DF = 0.4714P and, DE = 0.7454P

member	Length (in)	S (K)	U (K)	Area (in²)	$\frac{SUL}{AE}$ (in)
CD	60	P	1	0.3	$6.67\times10^{-3}P$
DF	169.7	0.4714P	0.4714	0.4	$3.143\times10^{-3}P$
DE	134.164	0.7454 P	0.7454	0.4	$6.212\times10^{-3}P$

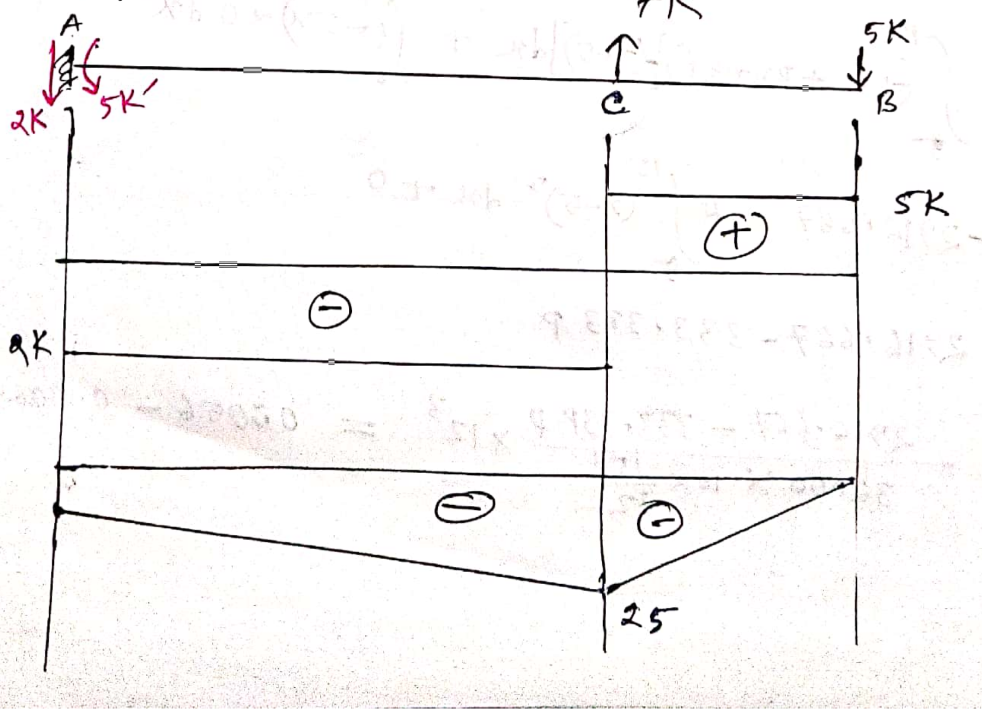
$\Delta_C = 0.016022P$

Thus,

$$\Delta_{c1} = \Delta_{c2}$$

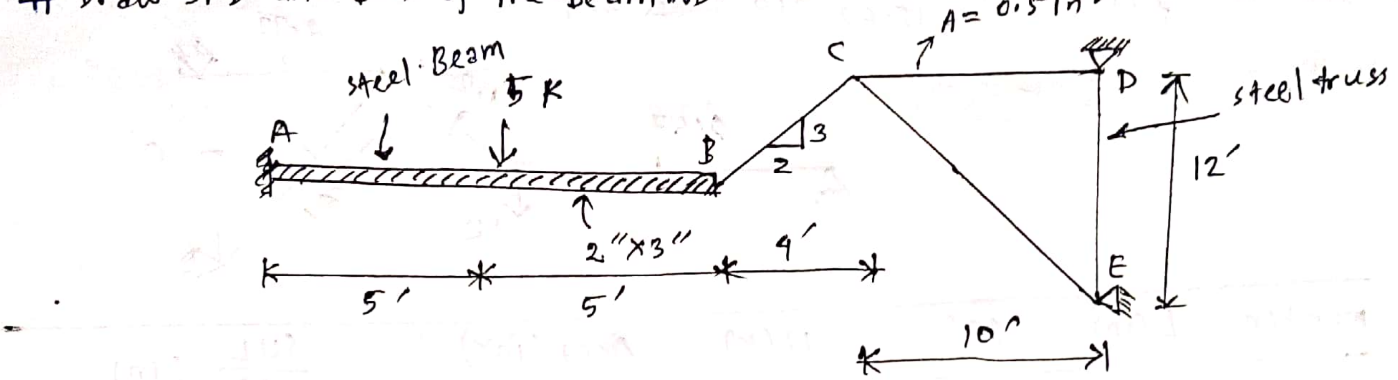
$$0.56 - 0.064P = 0.016022P$$

$\Rightarrow$ P = 7K



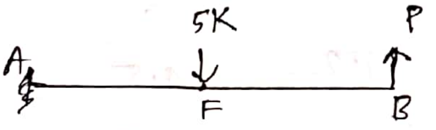
2013

Draw SFD and BMD of the Beam AB.

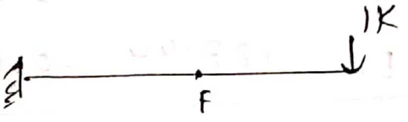


Solution:

Beam Element:



Portion —	A F	FB
origin —	B	B
Limit —	(5 – 10´)	(0 – 5´)
M —	$-5(x-5)+Px$	Px
m —	$-x$	$-x$



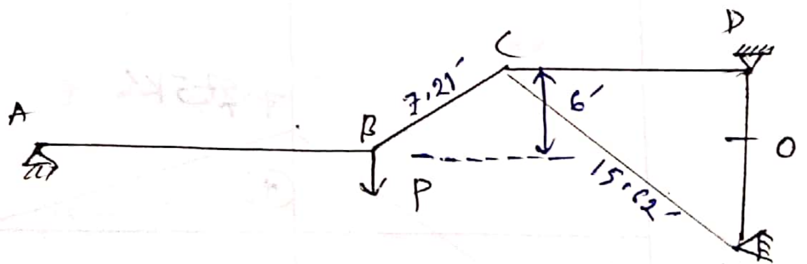
$$\Delta_B EI = \int_0^5 [Px\times(-x)]\,dx + \int_5^{10} [5(x-5)]\cdot x\,dx + \int_5^{10} Px\times(-x)\,dx$$

$$= -41.667P + 520.83 - 291.667P$$

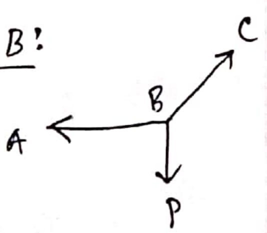
$$= 520.83 - 333.334P$$

$$\Delta_B = \frac{520.83 - 333.334P}{30000\times 2\times \frac{3^3}{12}}\times 12^3 = 6.67 - 4.27P$$

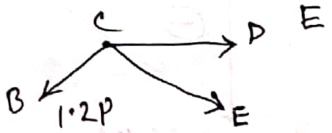
Truss element:



Joint B:



Joint C:

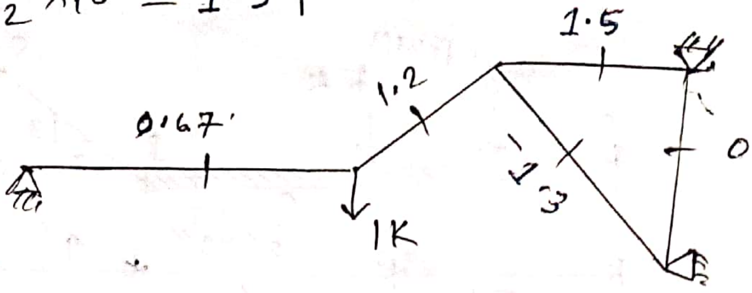


$BC = \frac{P}{6}\times 7.21 = 1.2P$

$\therefore BA = \frac{1.2P}{7.21}\times 4 = 0.67P$

$CE = \frac{-1.2P}{7.21}\times 6\times \frac{15.62}{12} = -1.3P$

$CD = \frac{1.2P}{7.21} \times 4 + \frac{1.3P}{15.62} \times 10 = 1.5P$



member	L (in)	S (K)	U (K)	Area (in²)	$\frac{SUL}{AE}$ (in)
AB	120	0.67P	0.67	6	$2.993 \times 10^{-4} P$
BC	86.52	1.2P	1.2	0.5	$8.306 \times 10^{-3} P$
CD	120	1.5P	1.5	0.5	0.018 P
CE	187.44	-1.3P	-1.3	0.5	0.02112 P

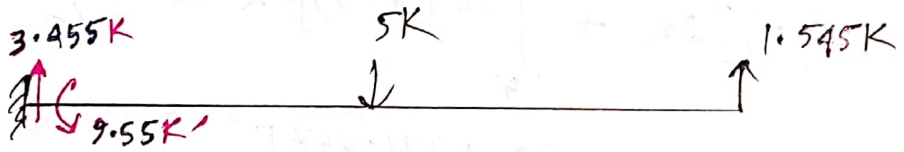
$\Delta_B = 0.04773P$

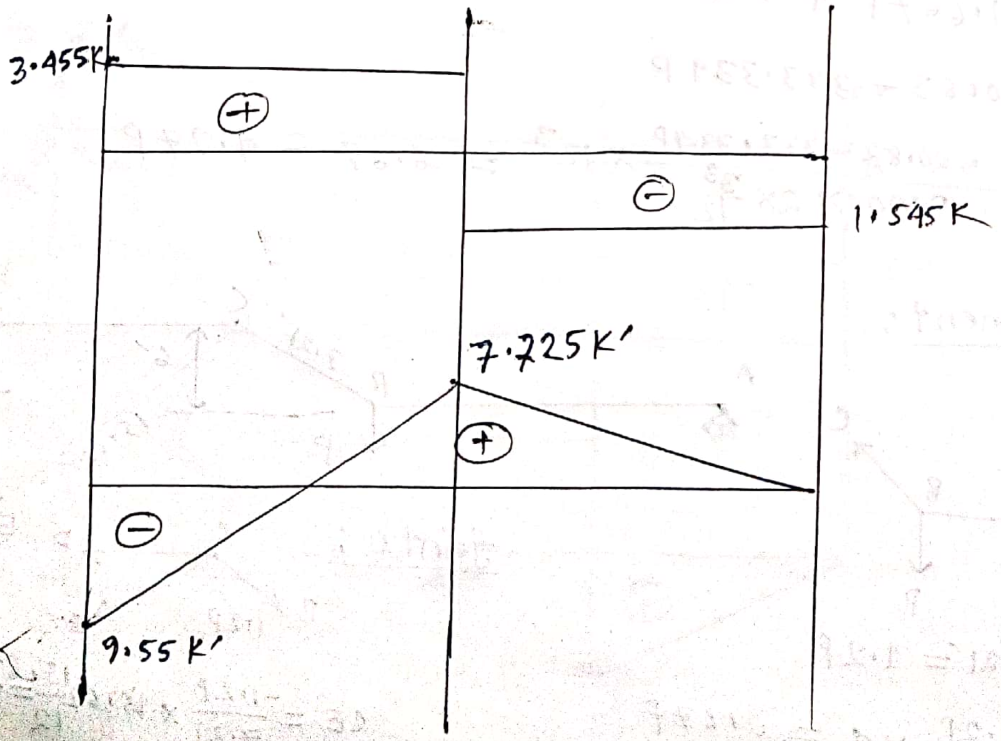
Thus,

$\Delta_{B_1} = \Delta_{B_2}$

$\Rightarrow 6.67 - 4.27P = 0.04773P$

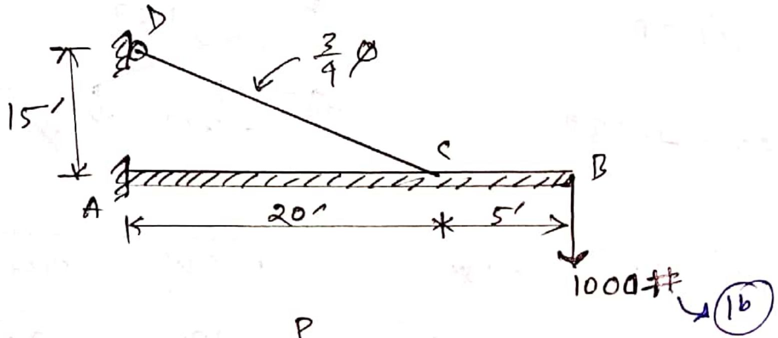
$\Rightarrow P = 1.545K$



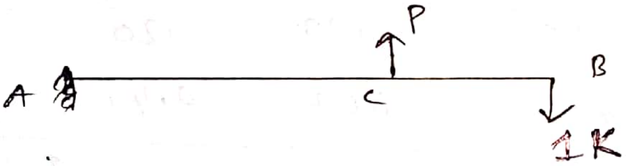


2011

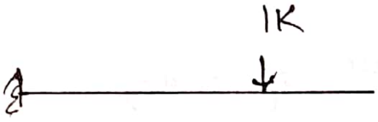
The timber beam shown in figure of 15 in deep and 8 in wide with $E_t = 1500$ ksi. Find the stress in the steel if its diameter is $\frac{3}{4}$ in. $E_s = 29\times10^3$ ksi



Solution:



Beam Element:



Portion -	AC	BC
origin -	B	B
Limit -	(5 - 25)	(0 - 5)
M -	$P(x-5) - x$	$-x$
m -	$-(x-5)$	0

$$4_c \cdot EI = \int_0^5 (-x)\times 0\ dx + \int_5^{25} [P(x-5) - x]\ x\, -(x-5)\, dx$$

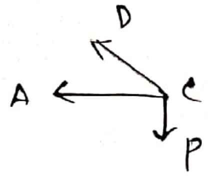
$$= 0 + 3666.67 - P\int_5^{25} (x-5)^2\ dx$$

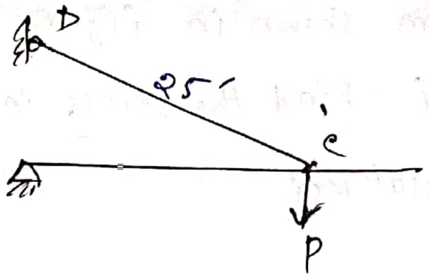
$$= 3666.67 - 2666.67P$$

$$4_c = \frac{3666.67 - 666.67P}{1500\times 8\times\frac{15^3}{12}} \times 12^3 = 1.88 - 1.37P$$

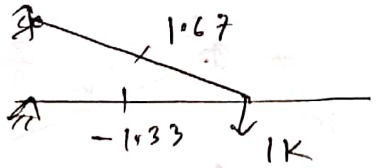
Truss Element:

Joints:





$CD = \frac{P}{15} \times 25 = 1.67P$



$AC = \frac{-1.67P}{25} \times 20 = -1.33P$

Member	Length (in)	S (K)	U (K)	Area (in²)	$\frac{SUL}{AE}$ (in)
AC	240	-1.33P	1.33	120	$2.36 \times 10^{-3}P$
CD	300	1.67P	1.67	0.44	0.0656P

$\Delta_c = 0.06796P$

Thus, $\Delta_{C_1} = \Delta_{C_2}$

$\Rightarrow 1.88 - 1.37P = 0.06796P$

$\Rightarrow P = 1.31K$

Stress:

$CD = 1.67 \times 1.31$

$= 2.1877K$

$AC = -1.33 \times 1.31$

$= -1.7423K$

(Ans.)

Deflection:

$\Delta AC = (2.36 \times 10^{-3} \times 1.31)$

$= 3.0916 \times 10^{-3} K$

$= 3.0916 \text{ lb}$

$\Delta CD = (0.0656 \times 1.31)$

$= 0.08594 K$

$= 85.94 \text{ lb}$

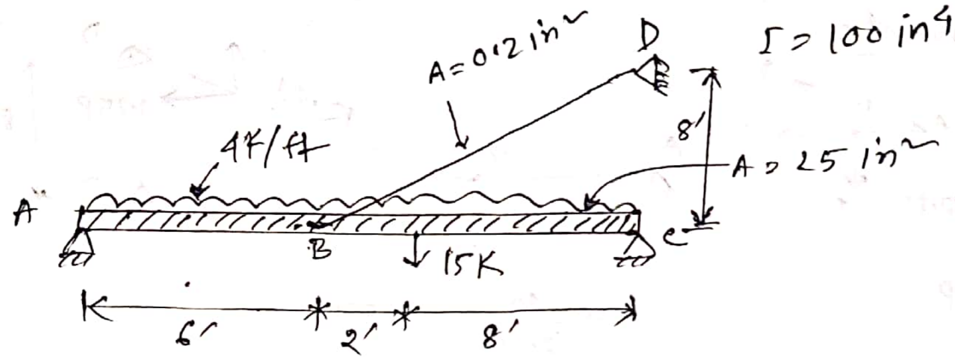
(Ans.)

2010

Draw SFD & BMD of the timber beam. $E_t = 1500$ ksi

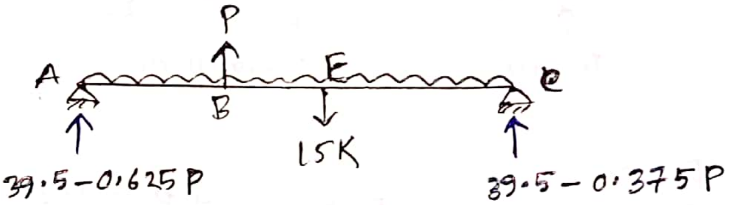
$E_s = 30000$ ksi

$I = 100$ in⁴

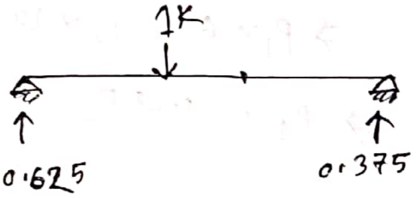


Solution:

Beam Element:



Portion −	AB	BE	CE
origin −	A	A	C
Limit −	(0−6')	(6'−8')	(0'−8')
M −	$(39.5-0.625P)x - 2x^2$	$(39.5-0.625P)x - 2x^2 + P(x-6)$	$(39.5-0.375P)x - 2x^2$
m −	$0.625x$	$0.625x-(x-6)$	$0.375x$



$$\Delta_B EI = \int_0^6 (39.5x - 0.625Px - 2x^2) \times 0.625x\,dx + \int_6^8 (39.5x - 0.625Px - 2x^2 + Px - 6P) \times [0.625x - (x-6)]\,dx + \int_0^8 (39.5x - 0.375Px - 2x^2) \times 0.375x\,dx$$

$$= 1372.5 - 28.125P + 1197.5 - \int_6^8 P(0.625x - x + 6) \times [0.625x - (x-6)]\,dx$$

$$+ 1760 - \int_0^8 0.375^2 x^2\,dx$$

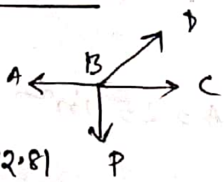
$$= 4330 - 28.125P - 22.875P - 24P$$

$$= 4330 - 75P$$

$$\Delta_B = \frac{(4330 - 75P) \times 12^3}{1500 \times 100} = 49.8816 - 0.864P$$

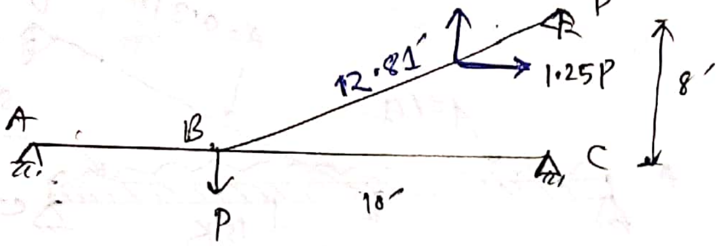
Truss Element:

Joint B:

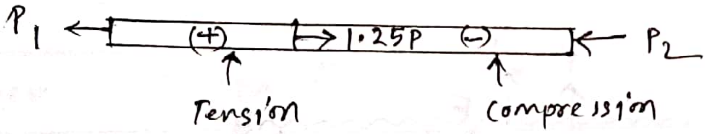


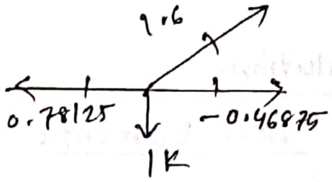
$BD = \frac{P}{8} \times 12.81$

$\therefore BD = 1.6P$



$1.6P \times \frac{10}{12.81} = 1.25P$





$\frac{P_1 L_1}{A_1 E_1} = \frac{P_2 L_2}{A_2 E_2}$

$\Rightarrow P_1 \times 6 = P_2 \times 10$

$\Rightarrow P_1 = 1.67 P_2$

Now, $P_1 + P_2 = 1.25P$

$\Rightarrow 1.67P_2 + P_2 = 1.25P$

$\Rightarrow P_2 = 0.46875P$

$\therefore P_1 = (1.67 \times 0.46875)P$

$= 0.78125P$

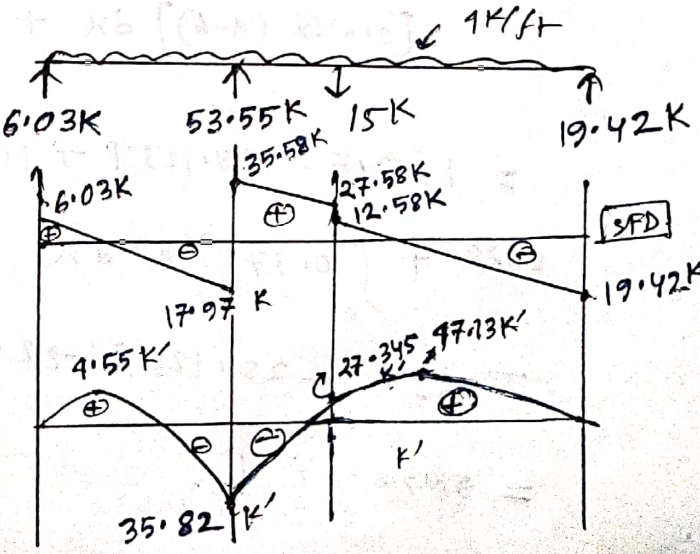
member	length (in)	S(K)	U(K)	Area (in²)	$\frac{SUL}{AE}$
BC	120	$-0.46875P$	-0.46875	25	$7.03 \times 10^{-4}P$
AB	72	$0.78125P$	$0.78125P$	25	$1.172 \times 10^{-3}P$
BD	153.72	$1.6P$	1.6	0.2	$0.0656P$

$\Delta_B = 0.0675P$

Thus, $\Delta_{B_1} = \Delta_{B_2}$

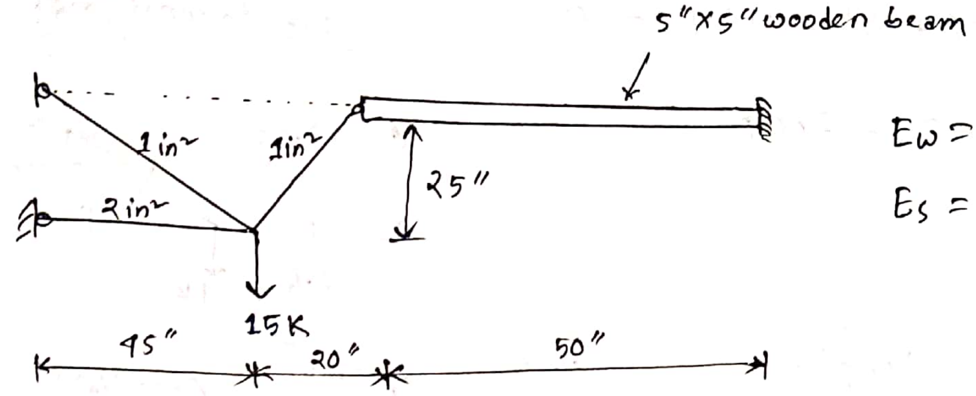
$\Rightarrow 49.8816 - 0.864P = 0.0675P$

$\Rightarrow P = 53.55K$



2008

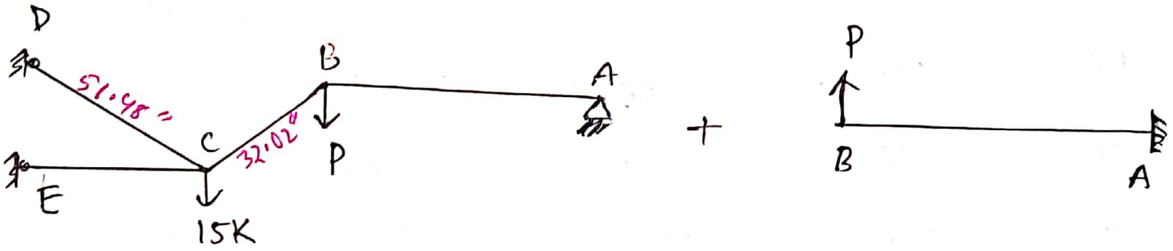
#



$E_w = 2000$ ksi

$E_s = 30000$ ksi

Solution:



Beam Portion:

Portion — AB

Origin — B

Limit — 0-50"

M — Px

m — -x

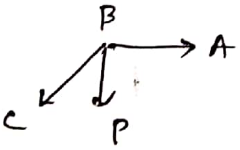
$$\Delta_B EI = \int_0^{50} (Px)\times(-x)\,dx$$

$$= -41666.67P$$

$$\Delta_B = \frac{-41666.67P}{2000\times 5\times\frac{5^3}{12}} = -0.4P$$

Truss Element:

joint B:

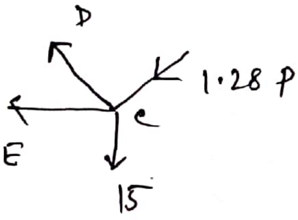


$$BC = -\frac{P}{25}\times 32.02$$

$$= -1.28P$$

$$AB = -1.28P\times\frac{20}{32.02} = -0.8P$$

joint C:



$$(CD)_y = 15 + (1.28P)_y$$

$$\Rightarrow CD\times\frac{25}{51.48} = 15 + 1.28P\times\frac{25}{32.02}$$

$$\Rightarrow CD = 30.89 + 2.06P$$

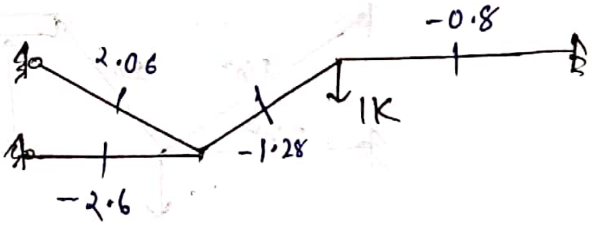
$$CE + (1.28P)_x + (CD)_x = 0$$

$$\Rightarrow CE + 1.28P\times\frac{20}{32.02} + (30.89+2.06P)\times\frac{45}{51.48}$$

$\Rightarrow CE + 0.8P + 27 + 1.8P = 0$

$\Rightarrow CE = -27 - 2.6P$

due to unit load:



Member	Length (in)	S (K)	U (K)	Area (in²)	$\frac{SUL}{AE}$
AB	50	$-0.8P$	-0.8	25	$0.00064P$
BC	32.02	$-1.28P$	-1.28	1	$0.00175P$
CE	45	$-27-2.6P$	-2.6	2	$0.05265+0.00507P$
CD	51.48	$30.89+2.06P$	2.06	1	$0.1092+0.0073P$

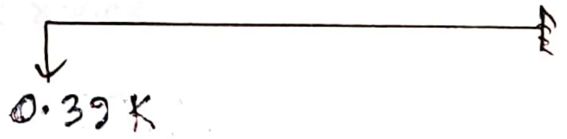
$\Delta_{B_2} = 0.16185 + 0.01476P$

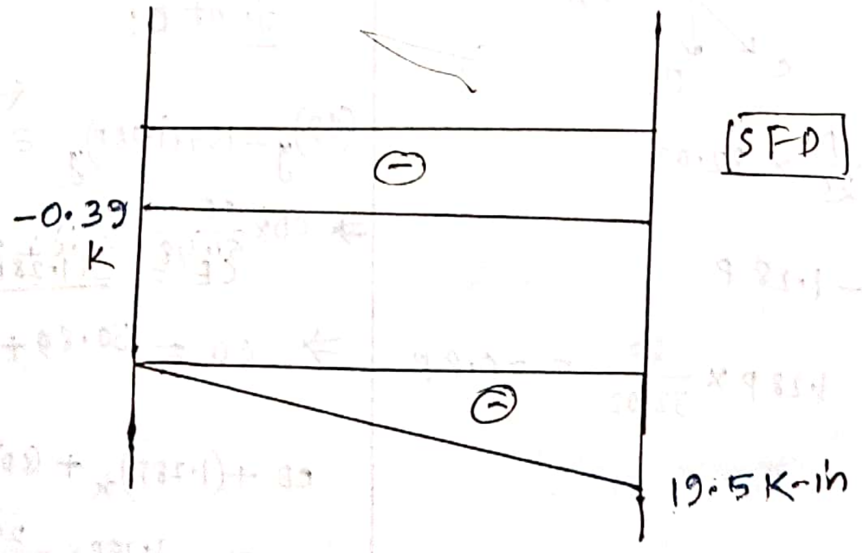
Thus,

$\therefore \Delta_{B_1} = \Delta_{B_2}$

$\Rightarrow -0.4P = 0.16185 + 0.01476P$

$\Rightarrow P = \frac{-0.16185}{0.41476} = -0.39\ K$





Stiffness Matrix Method [BEAM]

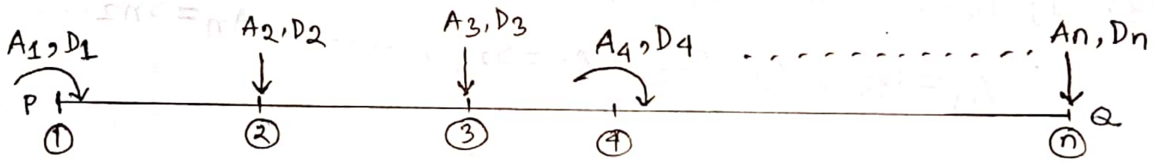
Mainly, There are two methods for analysing a Indeterminate Structure.

1. Displacement Method or, Stiffness Matrix Method.
2. Force Method or, Flexibility Matrix Method.

Stiffness: Stiffness may be defined as the force required to have a unit deformation.

Flexibility: Flexibility may be defined as the displacement due to application of unit Load.

Derivation of the relation $A = SD$: 2012, 2013



Let us, consider a beam PQ where 1, 2, 3, ...n are the joints.

$A_1, A_2, A_3, \ldots\ldots, A_n$ = Forces applied on the nodes ①, ② ③, ...ⓝ respectively.

$D_1, D_2, D_3, \ldots\ldots, D_n$ = Displacements at the nodes ①, ②, ③, ...ⓝ respectively.

D_1 is not only the effect of A_1 but also take the effect from $A_2, A_3, A_4, \ldots\ldots A_n$. And,

A_1 contributes to $D_1, D_2, D_3, \ldots\ldots D_n$

Therefore,

$$A_1 = A_{11} + A_{12} + A_{13} + \ldots\ldots + A_{1n}$$

$$= S_{11}D_1 + S_{12}D_2 + S_{13}D_3 + \ldots\ldots + S_{1n}D_n$$

Deri

$$A_2 = A_{21} + A_{22} + A_{23} + \cdots\cdots + A_{2n}$$

$$= S_{21}D_1 + S_{22}D_2 + S_{23}D_3 + \cdots\cdots + S_{2n}D_n$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$A_n = A_{n1} + A_{n2} + A_{n3} + \cdots\cdots + A_{nn}$$

$$= S_{n1}D_1 + S_{n2}D_2 + S_{n3}D_3 + \cdots\cdots + S_{nn}D_n$$

Case-1: If $D_1 = 1$, and $D_2 = D_3 = D_4 \cdots\cdots = D_n = 0$

Then, $A_1 = S_{11}$, $A_2 = S_{21}$, $A_3 = S_{31}$, $\cdots\cdots$ $A_n = S_{n1}$

Case-2: If $D_2 = 1$, and, $D_1 = D_3 = D_4 = \cdots\cdots = D_n = 0$

$A_1 = S_{12}$, $A_2 = S_{22}$; $A_3 = S_{32}$, $\cdots\cdots$ $A_n = S_{n2}$

- - - - - - - - - - - - -

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Case-n: if $D_n = 1$, $D_1 = D_2 = D_3 = \cdots\cdots = D_{n-1} = 0$

Then, $A_1 = S_{1n}$, $A_2 = S_{2n}$, $\cdots\cdots$ $A_n = S_{nn}$

Now, In Matrix Form,

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ \vdots \\ A_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots\cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots\cdots & S_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ S_{n1} & S_{n2} & S_{n3} & \cdots\cdots & S_{nn} \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ \vdots \\ D_n \end{bmatrix}$$

$$\Rightarrow [A] = [S][D] \qquad \Rightarrow [D] = [S]^{-1}[A]$$

Where, $[A]$ = Load Matrix:

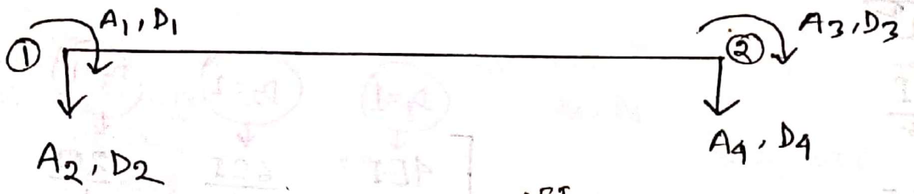
$[S]$ = Stiffness matrix.

$[D]$ = Deformation Matrix.

2014, 2012

Derivation of stiffness matrix of a single Beam:

Assume, a single beam of length 'L', Modulus of elasticity = E and Moment of Inertia = I



Case-I: $D_1 = 1$, $D_2 = D_3 = D_4 = 0$

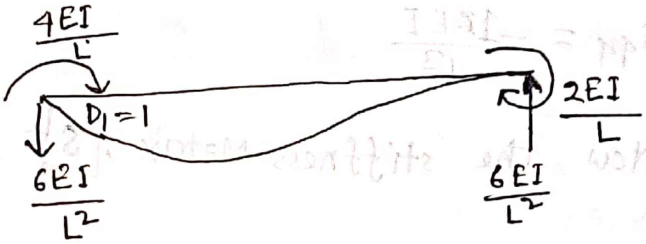
$S_{11} = \frac{4EI}{L}$

$S_{21} = \frac{6EI}{L^2}$

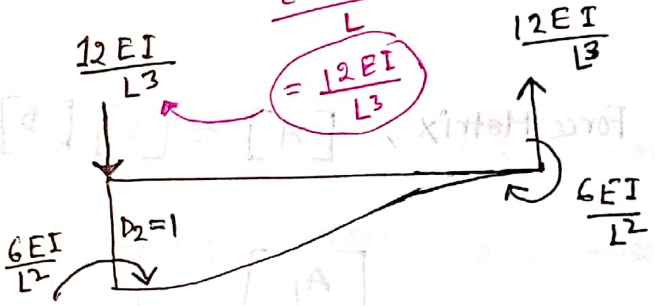
$S_{31} = \frac{2EI}{L}$

$S_{41} = -\frac{6EI}{L^2}$

$$\frac{\left(\frac{4EI}{L} + \frac{2EI}{L}\right)}{L} = \frac{6EI}{L^2}$$



$$\frac{\frac{6EI}{L^2} + \frac{6EI}{L^2}}{L} = \frac{12EI}{L^3}$$



Case-II: $D_2 = 1$, $D_1 = D_3 = D_4 = 0$

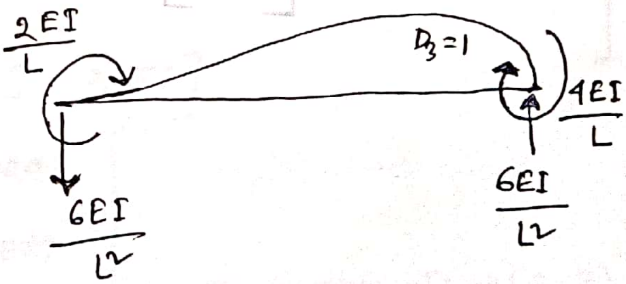
$S_{12} = \frac{12EI}{L^3}$

$S_{22} = \frac{6EI}{L^2}$

$S_{32} = \frac{-12EI}{L^3}$

$S_{42} = \frac{12EI}{L^3}$

Case-III: $D_3 = 1$, $D_1 = D_2 = D_4 = 0$



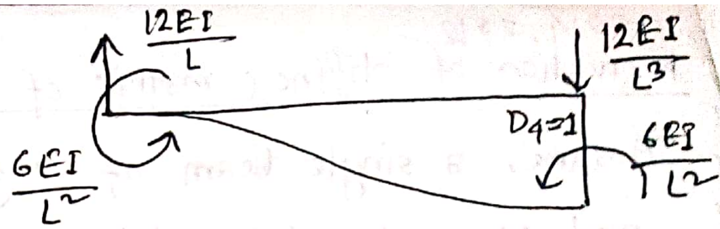
$S_{13} = \frac{2EI}{L}$

$S_{23} = \frac{6EI}{L^2}$

$S_{33} = \frac{4EI}{L^2}$

$S_{43} = -\frac{6EI}{L^2}$

case-IV : $D_4 = 1$, $D_1 = D_2 = D_3 = 0$



$$S_{14} = -\frac{6EI}{L^2}$$

$$S_{24} = -\frac{12EI}{L^3}$$

$$S_{34} = \frac{-6EI}{L^2}$$

$$S_{44} = \frac{12EI}{L^3}$$

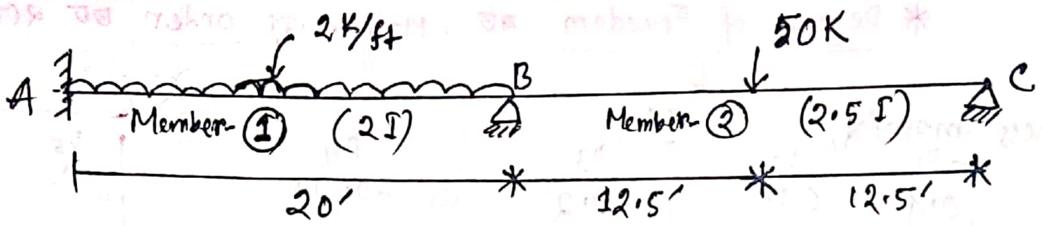
Now , the stiffness Matrix , $[S] =$

$$\begin{array}{cccc} D_1=1 & D_2=1 & D_3=1 & D_4=1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \left[\begin{matrix} \frac{4EI}{L} \\ \frac{6EI}{L^2} \\ \frac{2EI}{L} \\ -\frac{6EI}{L^2} \end{matrix}\right. & \begin{matrix} \frac{6EI}{L^2} \\ \frac{12EI}{L^3} \\ \frac{6EI}{L^2} \\ -\frac{12EI}{L^3} \end{matrix} & \begin{matrix} \frac{2EI}{L} \\ \frac{6EI}{L^2} \\ \frac{4EI}{L} \\ -\frac{6EI}{L^2} \end{matrix} & \left.\begin{matrix} -\frac{6EI}{L^2} \\ -\frac{12EI}{L^3} \\ -\frac{6EI}{L^2} \\ \frac{12EI}{L^3} \end{matrix}\right] \end{array}$$

Force Matrix , $[A] = [S][D]$

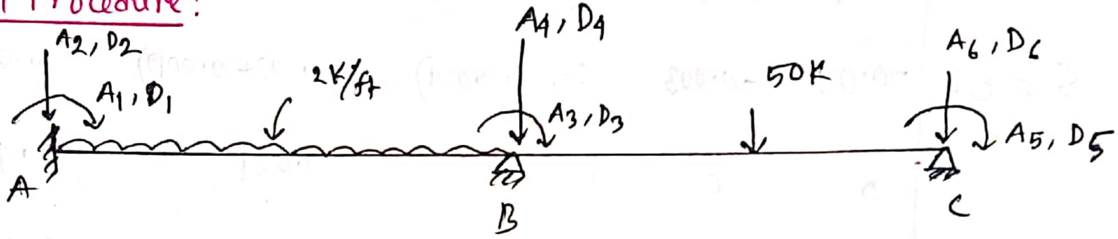
$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix}$$

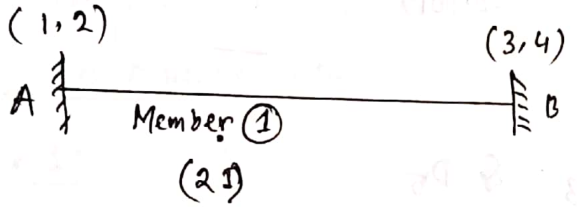
Problem:1

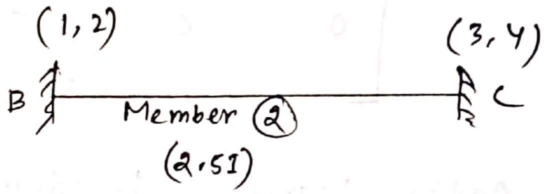


Calculate Fixed End Moment.

Basic Solution Procedure:







Stiffness matrix for Member ① [I=2I]

$$S_1 = \begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} \\ \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix} = EI \begin{bmatrix} 0.4 & 0.03 & 0.2 & -0.03 \\ 0.03 & 0.003 & 0.03 & -0.003 \\ 0.2 & 0.03 & 0.4 & -0.03 \\ -0.03 & -0.003 & -0.03 & 0.003 \end{bmatrix}$$

Similarly

Stiffness matrix for member ② [I = 2.5I]

$$S_2 = \begin{bmatrix} 0.4 & 0.024 & 0.2 & -0.024 \\ 0.024 & 0.0019 & 0.024 & -0.0019 \\ 0.2 & 0.024 & 0.4 & -0.024 \\ -0.024 & -0.0019 & -0.024 & 0.0019 \end{bmatrix}$$

* Member-① → (3, 4)
& Member-② → (1, 2)
একই point-এ . তাই
super impose করব।

* Degree of Freedom যত, Matrix এর order তত হবে

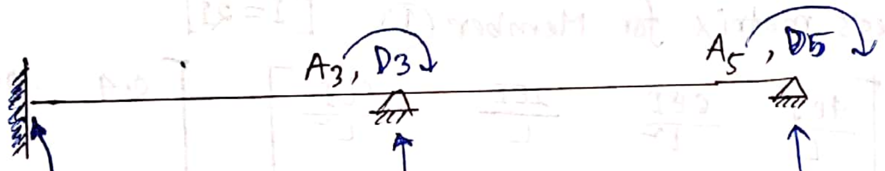
$\downarrow$ ⑥ $\qquad\qquad \downarrow$ 6×6

Now,

stiffness matrix,

$$S = EI\begin{array}{c|cccccc|c} & D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & \\ \hline & 0.4 & 0.03 & 0.2 & -0.03 & 0 & 0 & A_1 \\ & 0.03 & 0.003 & 0.03 & -0.003 & 0 & 0 & A_2 \\ & 0.2 & 0.03 & *(0.4+0.4) & (-0.03+0.024) & *0.2 & -0.024 & A_3 \\ & -0.03 & -0.003 & (-0.03+0.024) & (0.003+0.0019) & 0.024 & -0.0019 & A_4 \\ & 0 & 0 & *0.2 & 0.024 & *0.4 & -0.024 & A_5 \\ & 0 & 0 & -0.024 & -0.0019 & -0.024 & 0.0019 & A_6 \end{array}$$

Active Degree of Freedom : D_3 & D_5



rotation
&
vertical
deflection
কোনটাই হবে না

$\boxed{D_1=0}, \boxed{D_2=0}$

শুধু rotation
হবে কিন্তু
vertical
deflection
হবে না

$\boxed{D_4 = 0}$

similarly
শুধু rotation
হবে

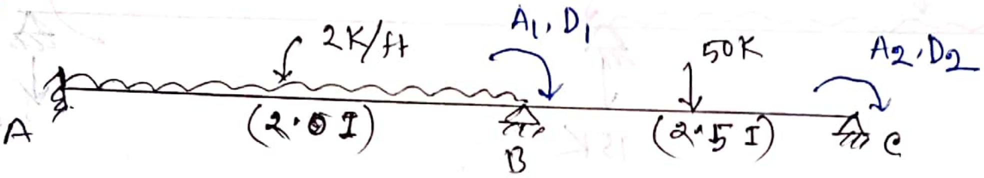
$\boxed{D_6=0}$

There fore, stiffness Matrix, $S = EI\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$

We know, $[A] = [S][D]$

$$\begin{bmatrix} A_3 \\ A_5 \end{bmatrix} = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} D_3 \\ D_5 \end{bmatrix}$$

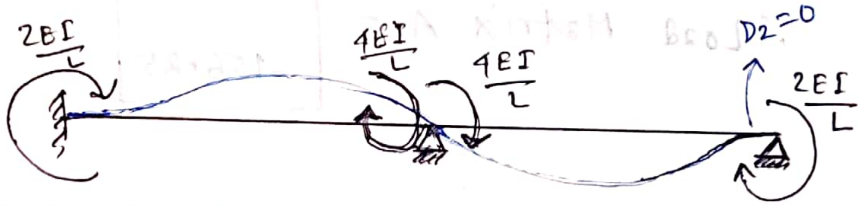
General Solution:



* Active degree of freedom = 2Nos (B & C - (5 rotation))

Stiffness Matrix Calculation:

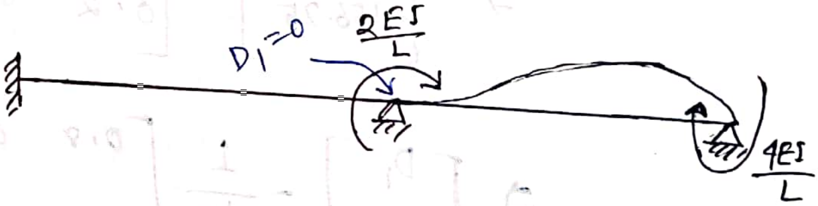
Case 1: $D_1 = 1$, $D_2 = 0$



$$S_{11} = \frac{4E\times 2I}{20} + \frac{4E\times 2.5I}{25} = 0.8\,EI$$

$$S_{21} = \frac{2E\times 2.5}{25} = 0.2\,EI$$

Case 2: $D_2 = 1$, $D_1 = 0$

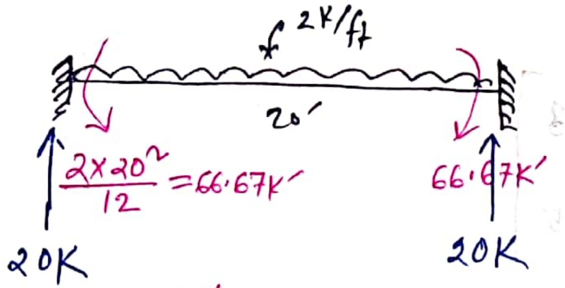


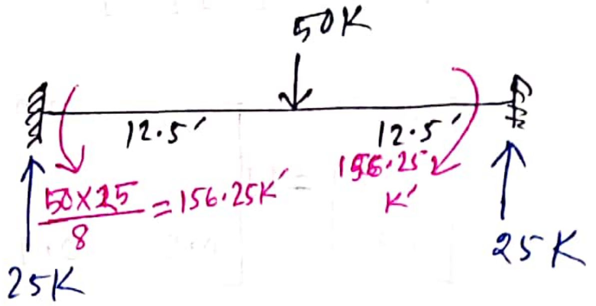
$$S_{12} = \frac{2E\times 2.5I}{25} = 0.2\,EI$$

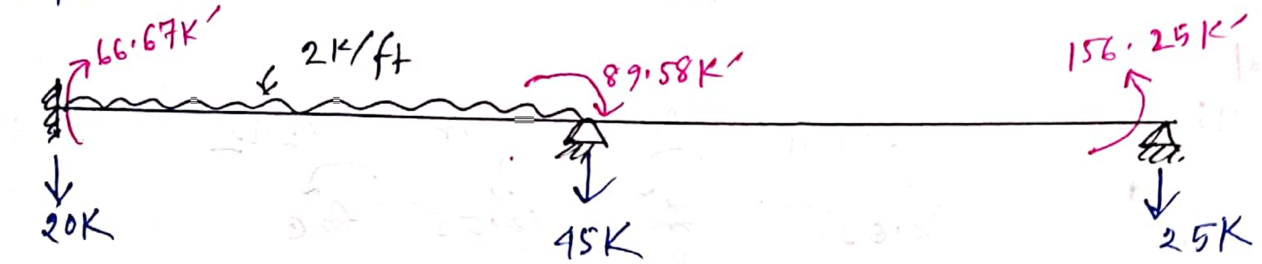
$$S_{22} = \frac{4E\times 2.5I}{25} = 0.4\,EI$$

$$\text{Stiffness Matrix, } S = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

Load Matrix calculation:







$$\therefore \text{Load Matrix } A = \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

Now, we know, $[A]=[S][D]$

$$\Rightarrow \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}^{-1}\begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \times \frac{\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.8 \end{bmatrix}}{(0.8\times0.4)-(0.2\times0.2)} \times \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \times \frac{1}{0.28} \times \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.8 \end{bmatrix} \times \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{0.28EI} \begin{bmatrix} (0.4 \times 89.58) + (-0.2 \times -156.25) \\ (-0.2 \times 89.58) + (0.8 \times -156.25) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{0.28\,EI} \times \begin{bmatrix} 67.082 \\ -142.916 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \times \begin{bmatrix} 239.58 \\ -510.4 \end{bmatrix}$$

We know,

$$[M] = FEM + [S][D]$$

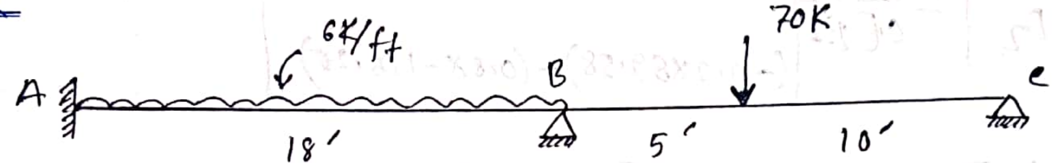
$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -66.67 \\ 66.67 \\ -156.25 \\ 156.25 \end{bmatrix} + EI \begin{array}{cc} D_1 & D_2 \\ \left[\begin{array}{c} 0.2 \\ 0.4 \\ 0.4 \\ 0.2 \end{array}\right. & \left.\begin{array}{c} 0 \\ 0 \\ 0.2 \\ 0.4 \end{array}\right] \end{array} \times \frac{1}{EI} \begin{bmatrix} 239.58 \\ -510.4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -66.67 \\ 66.67 \\ -156.25 \\ 156.25 \end{bmatrix} + \begin{bmatrix} (0.2 \times 239.58) \\ (0.4 \times 239.58) \\ (.4 \times 239.58) + (0.2 \times -510.4) \\ (0.2 \times 239.58) + (0.4 \times -510.4) \end{bmatrix}$$

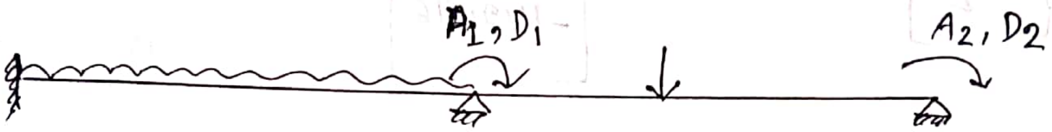
$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -66.67 \\ 66.67 \\ -156.25 \\ 156.25 \end{bmatrix} + \begin{bmatrix} 47.92 \\ 95.83 \\ -6.25 \\ -156.25 \end{bmatrix} = \begin{bmatrix} -18.75 \\ 162.5 \\ -162.5 \\ 0 \end{bmatrix}$$

(Ans.)

Problem:02 Calculate Fixed End Moments by stiffness matrix method

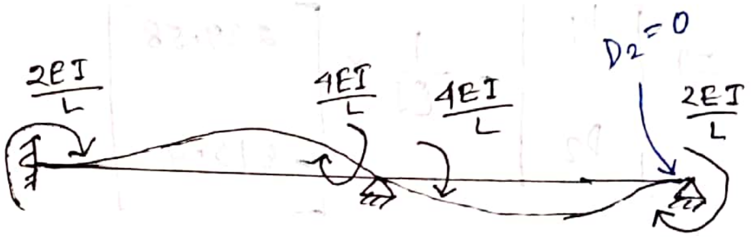


Solution: Active Degree of freedom = 2 Nos.



Stiffness Matrix Calculation:

Case-1: $D_1 = 1, D_2 = 0$

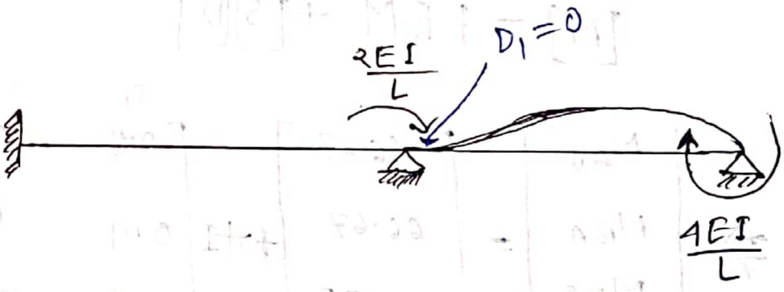


$$S_{11} = \frac{4EI}{18} + \frac{4EI}{15}$$

$$= 0.49EI$$

$$S_{21} = \frac{2EI}{15} = 0.13EI$$

Case-2: $D_2 = 1, D_1 = 0$

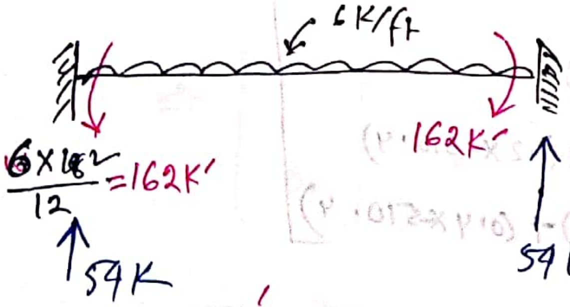


$$S_{12} = \frac{2EI}{15} = 0.13EI$$

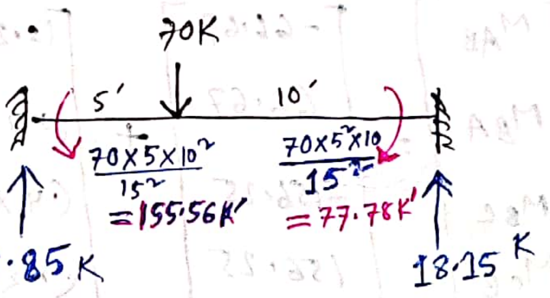
$$S_{22} = \frac{4EI}{15} = 0.27EI$$

$\therefore$ Stiffness Matrix, $S = EI \begin{bmatrix} 0.49 & 0.13 \\ 0.13 & 0.27 \end{bmatrix}$

Load Matrix Calculation:

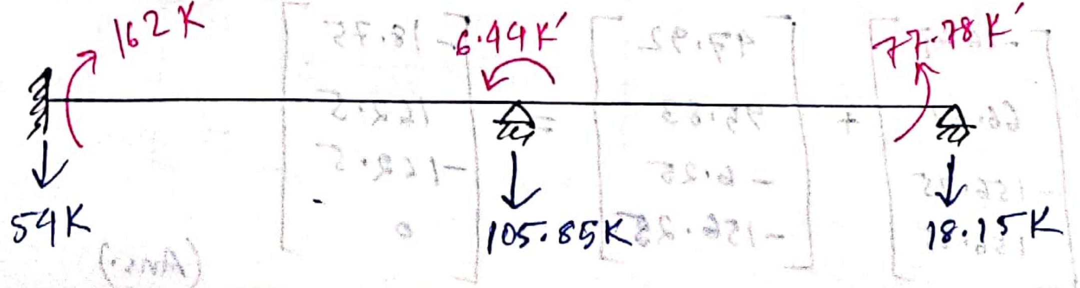


$\frac{6 \times 18^2}{12} = 162K'$



$\frac{70 \times 5 \times 10^2}{15^2} = 155.56K'$

$\frac{70 \times 5^2 \times 10}{15^2} = 77.78K'$



∴ Load Matrix, $A = \begin{bmatrix} -6.44 \\ -77.78 \end{bmatrix}$

Now, we know, $[A] = [S][D]$

$$\Rightarrow \begin{bmatrix} -6.44 \\ -77.78 \end{bmatrix} = \begin{bmatrix} 0.49 & 0.13 \\ 0.13 & 0.27 \end{bmatrix} \times EI \times \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.49 & 0.13 \\ 0.13 & 0.27 \end{bmatrix}^{-1} \times \begin{bmatrix} -6.44 \\ -77.78 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \times \frac{1}{(0.49 \times 0.27) - (0.13 \times 0.13)} \times \begin{bmatrix} 0.27 & -0.13 \\ -0.13 & 0.49 \end{bmatrix} \times \begin{bmatrix} -6.44 \\ -77.78 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{0.1154EI} \times \begin{bmatrix} (0.27 \times -6.44) + (-0.13 \times -77.78) \\ (-0.13 \times -6.44) + (0.49 \times -77.78) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 72.55 \\ -323 \end{bmatrix}$$

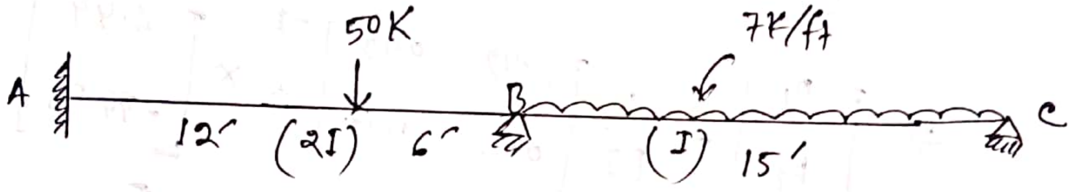
Then, we know, $M = FEM + [S][D]$

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -162 \\ 162 \\ -155.56 \\ 77.78 \end{bmatrix} + EI \begin{array}{c} \begin{array}{cc} D_1 & D_2 \end{array} \\ \begin{bmatrix} 0.11 & 0 \\ 0.22 & 0 \\ 0.27 & 0.13 \\ 0.13 & 0.27 \end{bmatrix} \end{array} \times \frac{1}{EI} \begin{bmatrix} 72.55 \\ -323 \end{bmatrix}$$

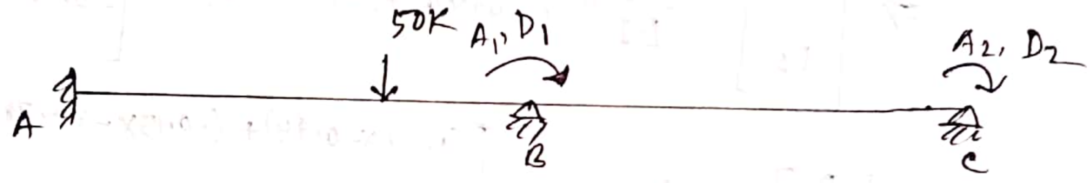
$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -162 \\ 162 \\ -155.56 \\ 77.78 \end{bmatrix} + \begin{bmatrix} 7.98 \\ 15.96 \\ -22.40 \\ -77.78 \end{bmatrix} = \begin{bmatrix} -154.02 \\ 177.96 \\ -177.96 \\ 0 \end{bmatrix}$$

(Ans)

Problem: 03 Calculate Fixed End Moments by stiffness matrix method.



Solution: Active degree of free dome = 02 Nos.

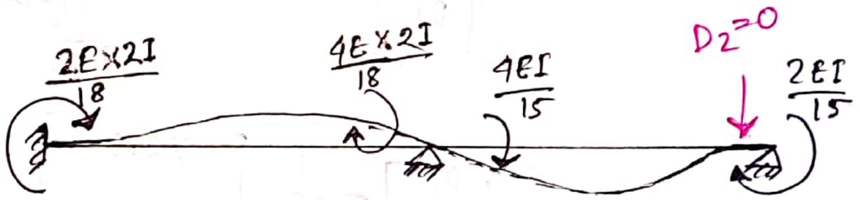


Stiffness Matrix calculation:

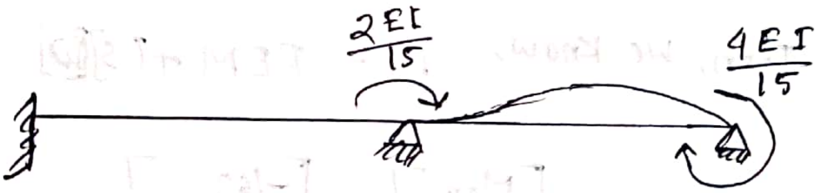
Case-1: $D_1 = 1$, $D_2 = 0$

$S_{11} = \frac{8EI}{18} + \frac{4EI}{15} = 0.71\,EI$

$S_{21} = \frac{2EI}{15} = 0.13\,EI$



Case-2: $D_2 = 1$, $D_1 = 0$

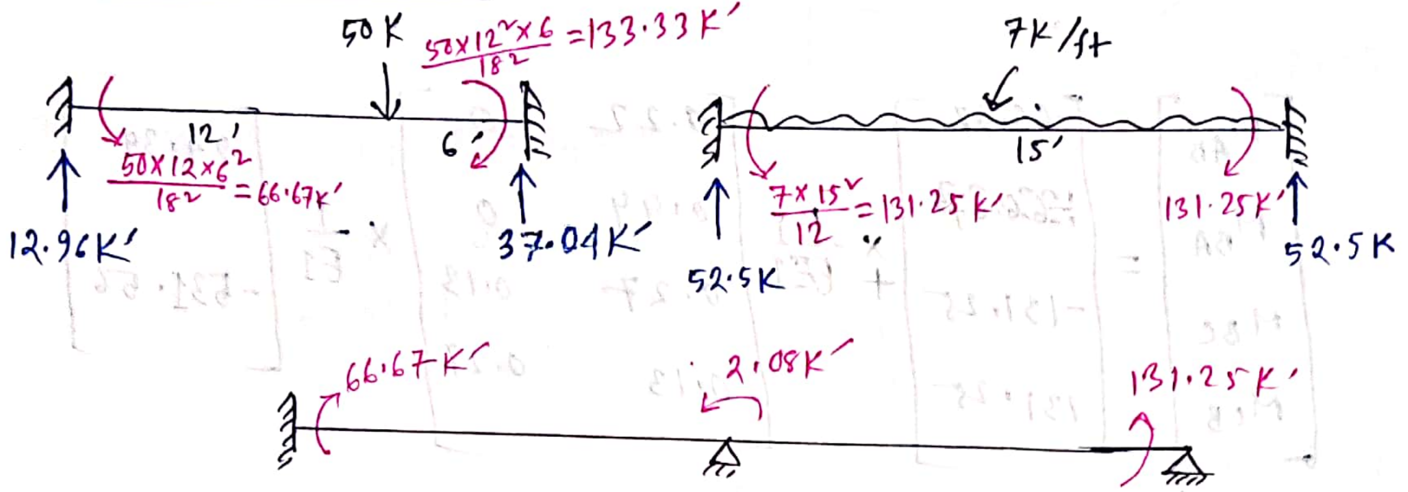


$S_{12} = \frac{2EI}{15} = 0.13\,EI$

$S_{22} = \frac{4EI}{15} = 0.27\,EI$

$\therefore$ Stiffness matrix, $S = EI \begin{bmatrix} 0.71 & 0.13 \\ 0.13 & 0.27 \end{bmatrix}$

Load Matrix Calculation:



$\therefore$ Load matrix, $A = \begin{bmatrix} -2.08 \\ -131.25 \end{bmatrix}$

Now, $[A] = [S]\,[D]$

$$\Rightarrow \begin{bmatrix} -2.08 \\ -131.25 \end{bmatrix} = EI \begin{bmatrix} 0.71 & 0.13 \\ 0.13 & 0.7 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.71 & -0.13 \\ -0.13 & 0.27 \end{bmatrix}^{-1} \times \begin{bmatrix} -2.08 \\ -131.25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \times \frac{1}{(0.71\times0.27)-(0.13\times0.13)} \times \begin{bmatrix} 0.7 & -0.13 \\ -0.13 & 0.71 \end{bmatrix} \times \begin{bmatrix} -2.08 \\ -131.25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{0.1748\,EI} \times \begin{bmatrix} 16.50 \\ -92.917 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 94.39 \\ -531.56 \end{bmatrix}$$

Then, $M = FEM + [S][D]$

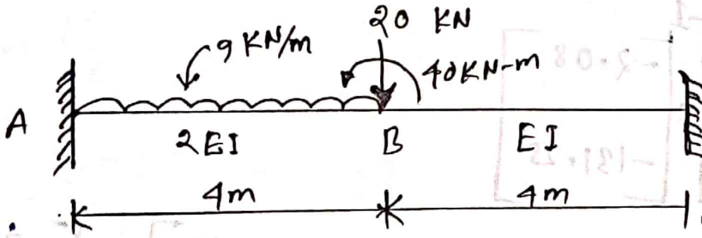
$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -66.67 \\ 133.33 \\ -131.25 \\ 131.25 \end{bmatrix} + EI \begin{bmatrix} 0.22 & 0 \\ 0.44 & 0 \\ 0.27 & 0.13 \\ 0.13 & 0.27 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 94.39 \\ -531.56 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -66.67 \\ 133.33 \\ -131.25 \\ 131.25 \end{bmatrix} + \begin{bmatrix} 20.77 \\ 41.54 \\ -43.62 \\ -131.25 \end{bmatrix} = \begin{bmatrix} -45.9 \\ 174.87 \\ 174.87 \\ 0 \end{bmatrix}$$

(Ans.)

Problem: 04

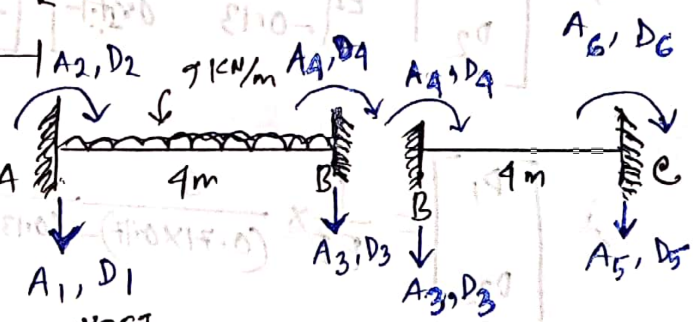
Calculate Fixed End Moment & Draw SFD and BMD.



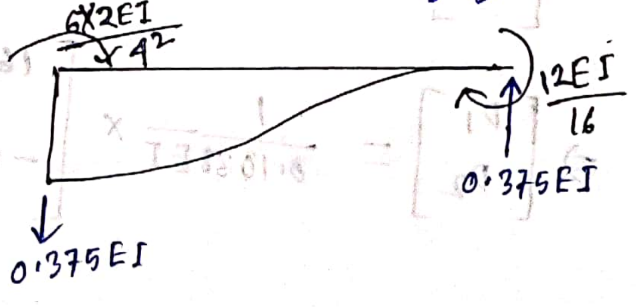
Solution:

By local stiffness Matrix Method:

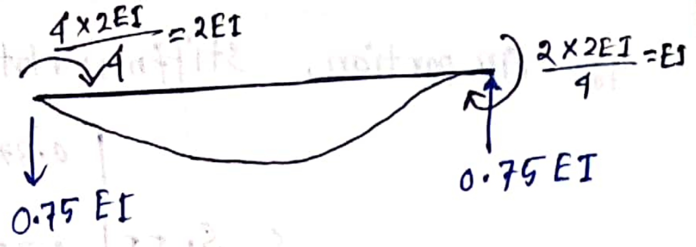
Taking AB portion,



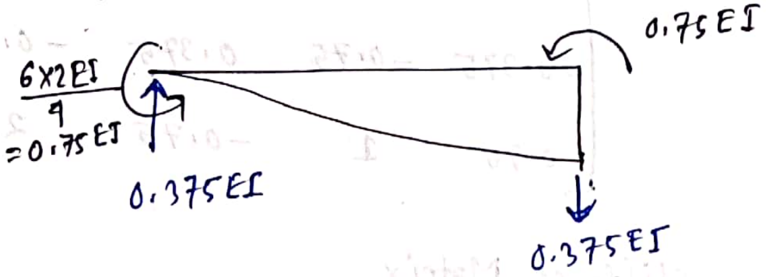
case: 1: $D_1 = 1$, $D_2 = D_3 = D_4 = 0$



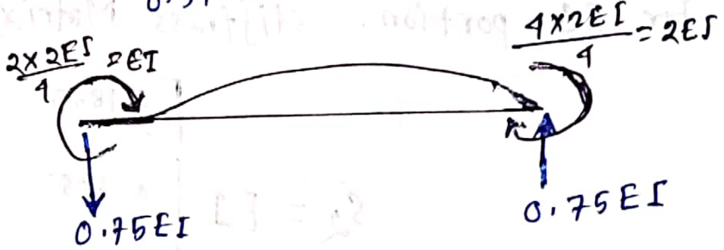
Case-2: $D_2 = 1$, $D_1 = D_3 = D_4 = 0$



Case-3: $D_3 = 1$, $D_1 = D_2 = D_4 = 0$

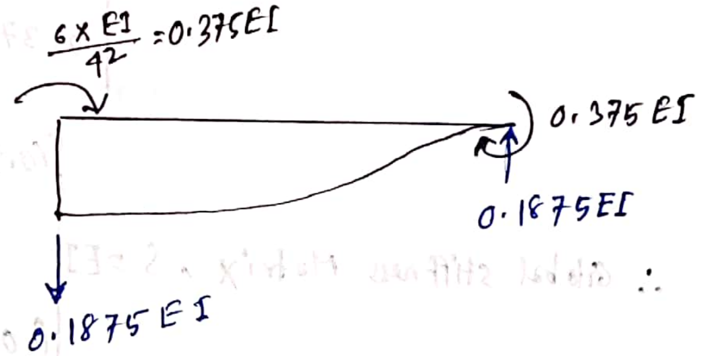


Case-4: $D_4 = 1$, $D_1 = D_2 = D_3 = 0$

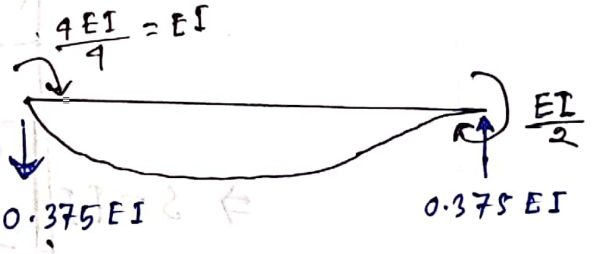


Similary for BC portion:

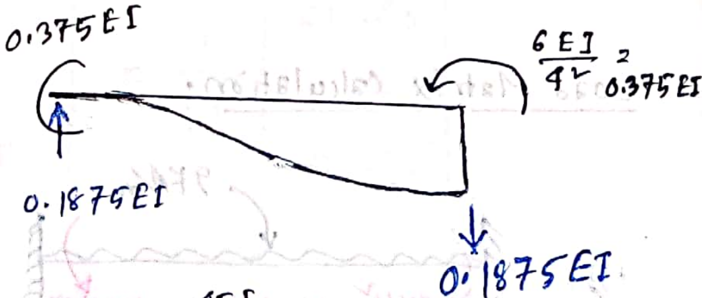
case-1: $D_3 = 1$, $D_4 = D_5 = D_6 = 0$



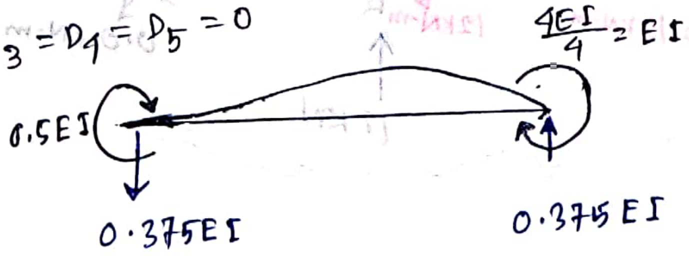
case-2: $D_4 = 1$, $D_3 = D_5 = D_6 = 0$



case-3: $D_5 = 1$, $D_3 = D_4 = D_6 = 0$



case-4: $D_6 = 1$, $D_3 = D_4 = D_5 = 0$



For AB portion, stiffness Matrix,

$$S_1 = EI \begin{bmatrix} 0.375 & 0.75 & -0.375 & 0.75 \\ 0.75 & 2 & -0.75 & 1 \\ -0.375 & -0.75 & 0.375 & -0.75 \\ 0.75 & 1 & -0.75 & 2 \end{bmatrix}$$

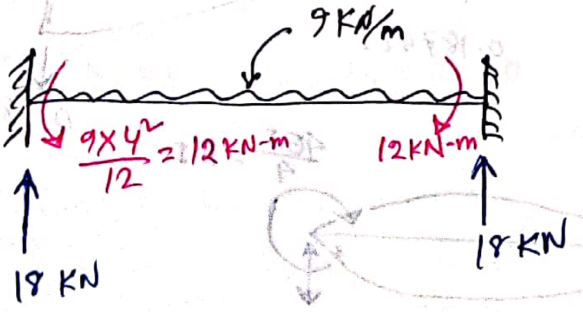
For BC portion, stiffness Matrix,

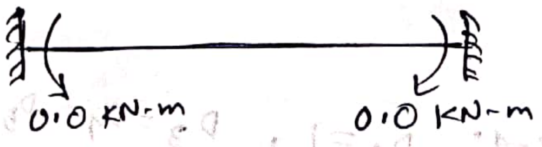
$$S_2 = EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{bmatrix}$$

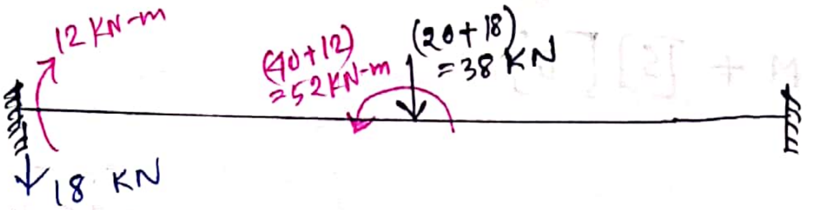
$$\text{stiffness Matrix, } S = EI \begin{bmatrix} (0.375+0.1875) & (-0.75+0.375) \\ (-0.75+0.375) & (2+1) \end{bmatrix}$$

$$\Rightarrow S = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix}$$

<u>Load Matrix Calculation:</u>







Load Matrix,

$$\begin{bmatrix} A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} 38 \\ -52 \end{bmatrix}$$

Now, we know, $[A] = [S]\,[D]$

$$\begin{bmatrix} 38 \\ -52 \end{bmatrix} = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix} \times \begin{bmatrix} D_3 \\ D_4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} 38 \\ -52 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{EI} \times \frac{1}{1.547} \begin{bmatrix} 3 & 0.375 \\ 0.375 & 0.5625 \end{bmatrix} \times \begin{bmatrix} 38 \\ -52 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{1.547EI} \begin{bmatrix} 94.5 \\ -15 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 61.086 \\ -9.696 \end{bmatrix}$$

Then, $M = FEM + [S][D]$

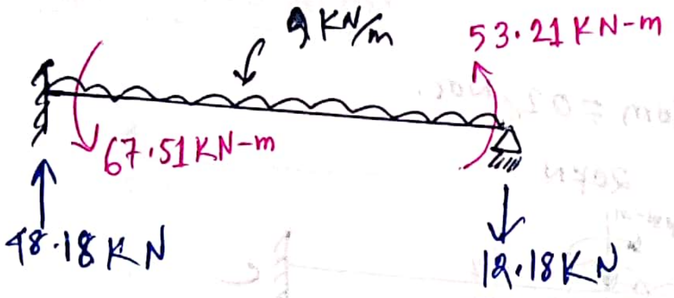
$$\begin{bmatrix} F_{AB} \\ M_{AB} \\ F_{BA} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -18 \\ -12 \\ -18 \\ 12 \end{bmatrix} + EI \begin{bmatrix} 0.375 & 0.75 & -0.375 & 0.75 \\ 0.75 & 2 & -0.75 & 1 \\ -0.375 & -0.75 & 0.375 & -0.75 \\ 0.75 & 1 & -0.75 & 2 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 0.0 \\ 0.0 \\ 61.086 \\ -9.696 \end{bmatrix}$$

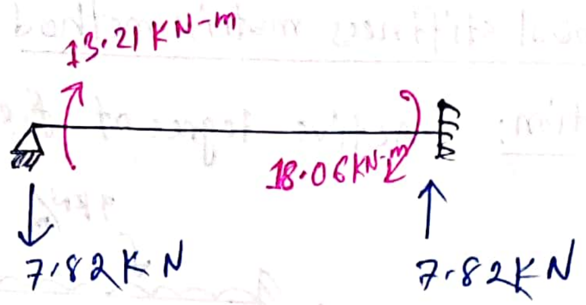
$$\begin{bmatrix} F_{AB} \\ M_{AB} \\ F_{BA} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -18 \\ -12 \\ -18 \\ 12 \end{bmatrix} + \begin{bmatrix} -30.18 \\ -55.51 \\ 30.18 \\ -65.21 \end{bmatrix} = \begin{bmatrix} -48.18 \\ -67.51 \\ 12.18 \\ -53.21 \end{bmatrix}$$

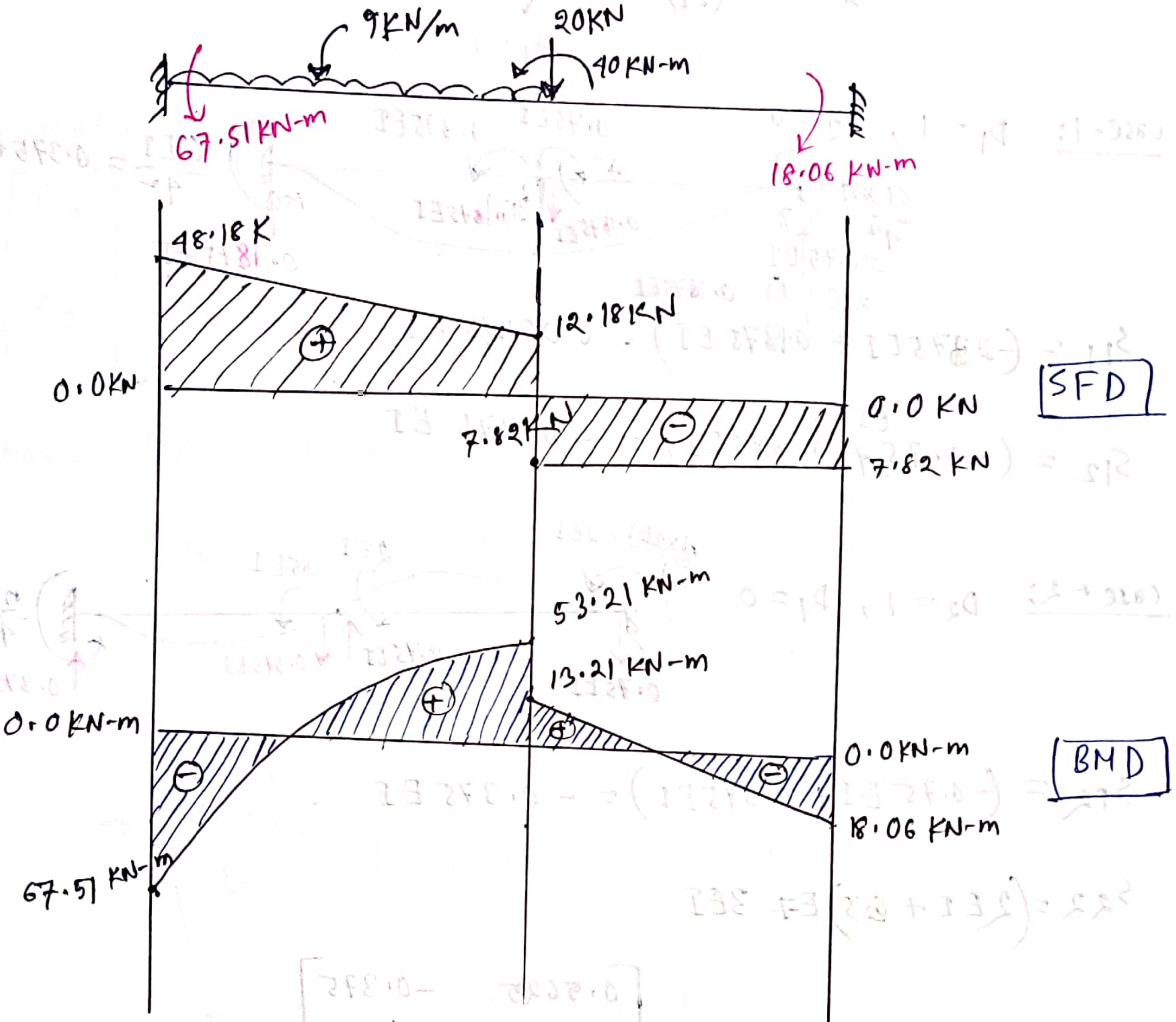
Again,

$$\begin{bmatrix} F_{BC} \\ M_{BC} \\ F_{CB} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 61.086 \\ -9.696 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} F_{BC} \\ M_{BC} \\ F_{CB} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} 7.82 \\ 13.21 \\ -7.82 \\ 18.06 \end{bmatrix}$$

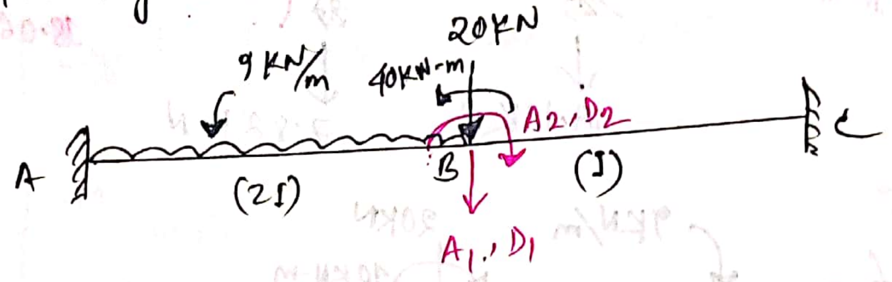

9 KN/m
53.21 KN-m
67.51 KN-m
48.18 KN
12.18 KN


13.21 KN-m
18.06 KN-m
7.82 K N
7.82 KN

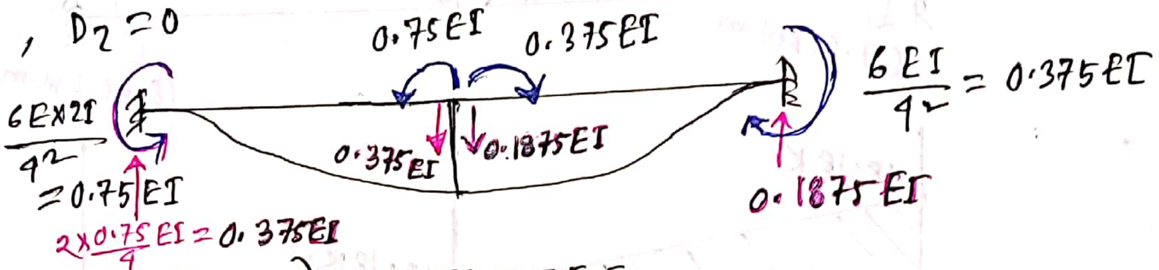

9 KN/m
20KN
40 KN-m
67.51 KN-m
18.06 KN-m
48.18 K
12.18 KN
+
0.0 KN
0.0 KN
7.82 KN
−
7.82 KN
SFD
53.21 KN-m
13.21 KN-m
0.0 KN-m
+
+
−
−
0.0 KN-m
18.06 KN-m
67.51 KN-m
BMD

by Global stiffness matrix method:

solution: Active degree of freedom = 02 Nos.



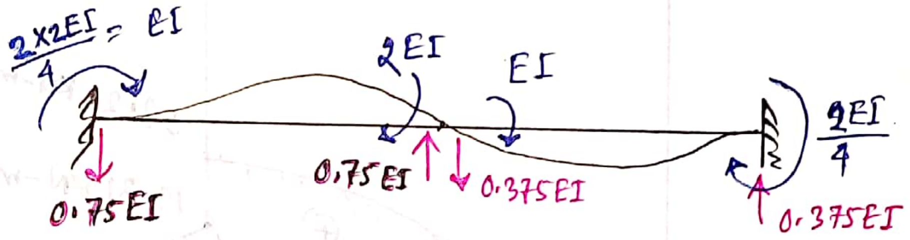
case-1: $D_1 = 1$, $D_2 = 0$



$S_{11} = (0.375EI + 0.1875\,EI) = 0.5625\,EI$

$S_{21} = (-0.75\,EI + 0.375EI) = -0.375\,EI$

case-2: $D_2 = 1$, $D_1 = 0$

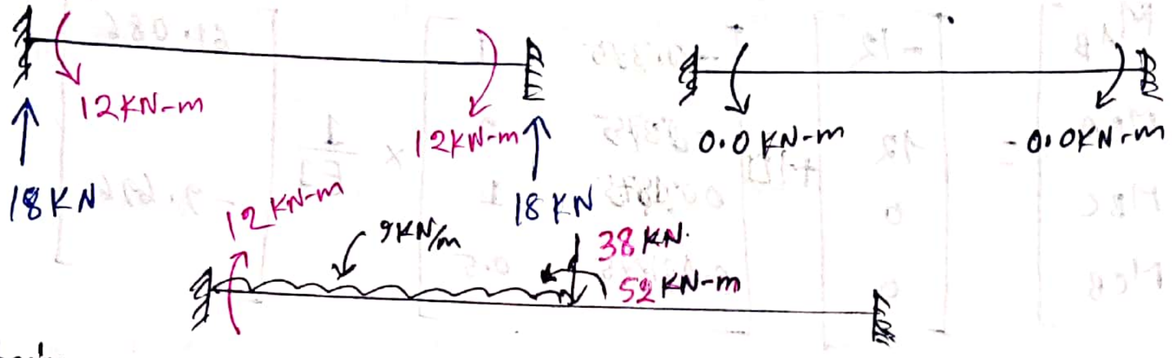


$S_{12} = (-0.75\,EI + 0.375\,EI) = -0.375\,EI$

$S_{22} = (2EI + EI) = 3EI$

$$\therefore \text{stiffness matrix, } S = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix}$$

Load Matrix Calculation:



load matrix,

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 38 \\ -52 \end{bmatrix}$$

Now, $[A] = [S][D]$

$$\Rightarrow \begin{bmatrix} 38 \\ -52 \end{bmatrix} = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix} \times \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \times \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix}^{-1} \times \begin{bmatrix} 38 \\ -52 \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 61.086 \\ -9.696 \end{bmatrix}$$

Then, $M = FEM + [S][D]$

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \\ 0 \\ 0 \end{bmatrix} + EI \begin{bmatrix} -0.75 & 1 \\ -0.75 & 2 \\ 0.375 & 1 \\ 0.375 & 0.5 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 61.086 \\ -9.696 \end{bmatrix}$$

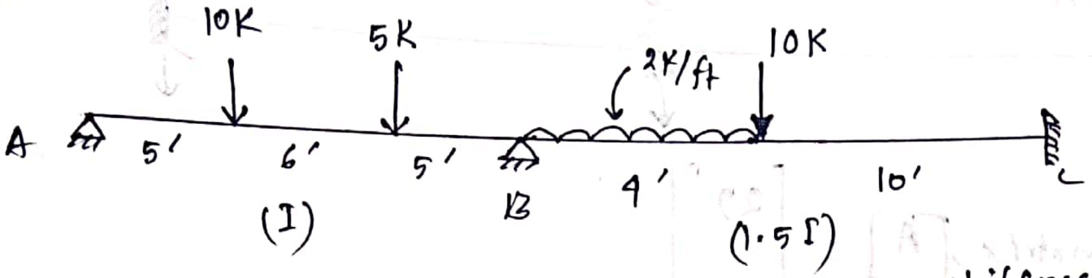
$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -55.51 \\ -65.21 \\ 13.21 \\ 18.06 \end{bmatrix} = \begin{bmatrix} -67.51 \\ -53.21 \\ 13.21 \\ 18.06 \end{bmatrix}$$

Again, (যদি SFD আঁকতে বলে তাহলে Member shear বের করতে হবে)

$$\begin{bmatrix} F_{AB} \\ F_{BA} \\ F_{BC} \\ F_{CB} \end{bmatrix} = \begin{bmatrix} -18 \\ -18 \\ 0 \\ 0 \end{bmatrix} + EI \begin{bmatrix} -0.375 & 0.75 \\ 0.375 & -0.75 \\ 0.1875 & 0.375 \\ -0.1875 & -0.375 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 61.086 \\ -9.696 \end{bmatrix}$$

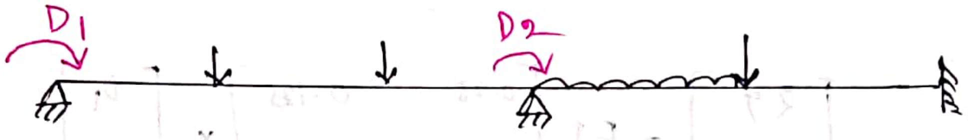
$$\Rightarrow \begin{bmatrix} F_{AB} \\ F_{BA} \\ F_{BC} \\ F_{CB} \end{bmatrix} = \begin{bmatrix} -18 \\ -18 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -30.18 \\ 30.18 \\ 7.82 \\ -7.82 \end{bmatrix} = \begin{bmatrix} -48.18 \\ 12.18 \\ 7.82 \\ -7.82 \end{bmatrix}$$

Problem: 05



Find out Fixed End moments of the beam by stiffness matrix method.

Solution: Active degree of Freedom = 02 Nos.



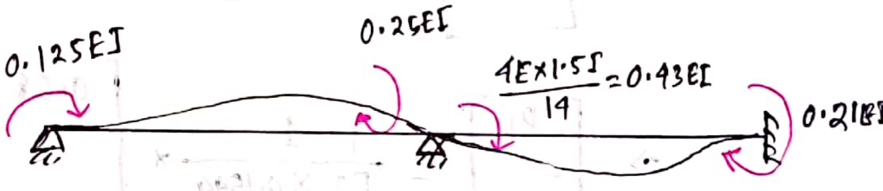
stiffness matrix:

Case-1: $D_1 = 1$, $D_2 = 0$

$$S_{11} = \frac{4EI}{16} = 0.25EI$$

$$S_{21} = 0.125EI$$

Case-2: $D_2 = 1$, $D_1 = 0$

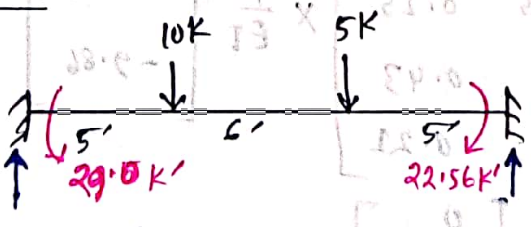


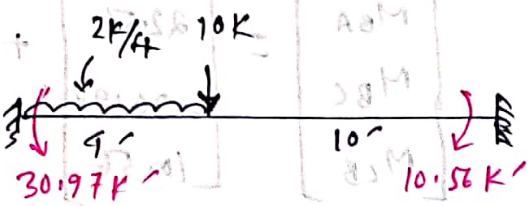
$$S_{12} = 0.125EI$$

$$S_{22} = (0.25EI + 0.43EI) = 0.68EI$$

$$\therefore \text{stiffness matrix, } S = EI\begin{bmatrix} 0.25 & 0.125 \\ 0.125 & 0.68 \end{bmatrix}$$

Load Matrix:





$$\frac{10\times5\times11^2}{16^2} + \frac{5\times11\times5^2}{16^2} = 29\,K'$$

$$\frac{10\times5^2\times11}{16^2} + \frac{5\times11^2\times5}{16^2} = 22.56\,K'$$

$$\frac{2\times4^2}{12\times14^2}\times(6\times14^2 - 8\times4\times14 + 3\times4^2) + \frac{10\times4\times10^2}{14^2} = 30.97\,K'$$

$$\frac{2\times4^3}{12\times14^2}\times(4\times14 - 3\times4) + \frac{10\times4^2\times10}{14^2} = 10.56\,K'$$

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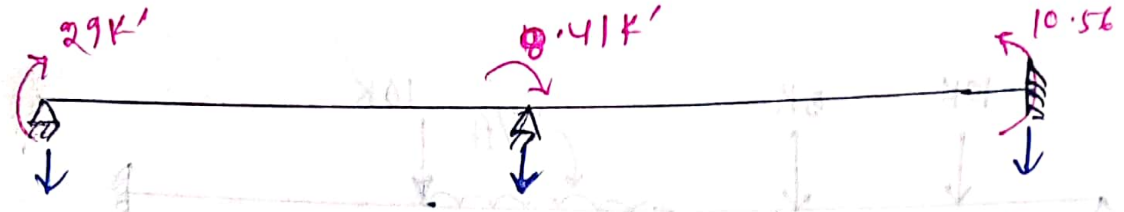


$\therefore$ Load matrix, $[A] = \begin{bmatrix} 29 \\ 8.41 \end{bmatrix}$

Now, we know, $[A] = [S][D]$

$$\begin{bmatrix} 29 \\ 8.41 \end{bmatrix} = EI \begin{bmatrix} 0.25 & 0.125 \\ 0.125 & 0.68 \end{bmatrix} \times \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.25 & 0.125 \\ 0.125 & 0.68 \end{bmatrix}^{-1} \times \begin{bmatrix} 29 \\ 8.41 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI \times 0.1544} \times \begin{bmatrix} 0.68 & -0.125 \\ -0.125 & 0.25 \end{bmatrix} \times \begin{bmatrix} 29 \\ 8.41 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 120.91 \\ -9.86 \end{bmatrix}$$

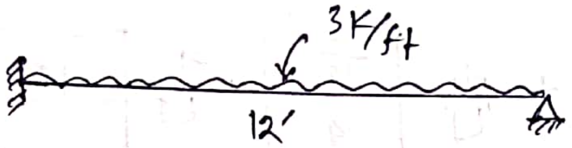
Now,

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -29 \\ 22.56 \\ -30.97 \\ 10.56 \end{bmatrix} + EI \begin{bmatrix} 0.25 & 0.125 \\ 0.125 & 0.25 \\ 0 & 0.43 \\ 0 & 0.21 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 120.91 \\ -9.86 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -29 \\ 22.56 \\ -30.97 \\ 10.56 \end{bmatrix} + \begin{bmatrix} 29 \\ 12.65 \\ -4.2 \\ -2.07 \end{bmatrix} = \begin{bmatrix} 0 \\ 35.21 \\ -35.21 \\ 8.49 \end{bmatrix}$$ (Ans.)

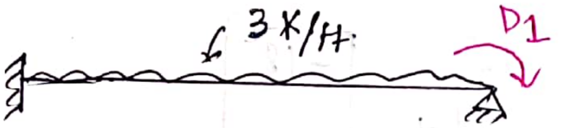
CT (13 Series)

Find out the moments for the member by stiffness Matrix method. EI is constant.



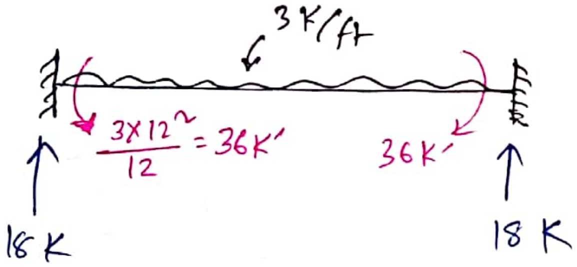
Solution:

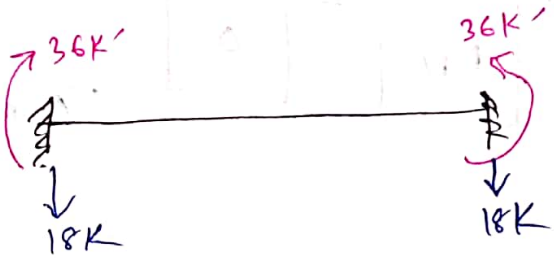
Active degree of freedom = 01



when, $D_1 = 1$

$\therefore s_{11} = EI\left[\frac{4}{12}\right]$





$\therefore$ Load matrix, $A = [-36]$

Now,

we know. $[A] = [S][D]$

$\Rightarrow [-36] = EI \times \left[\frac{4}{12}\right] \times [D]$

$\Rightarrow [D] = \frac{1}{EI} \times \left[\frac{4}{12}\right]^{-1} \times [-36]$

$\Rightarrow [D] = \frac{1}{EI} \times \left[\frac{12}{4}\right] \times [-36]$

$\therefore [D] = \frac{1}{EI} \times [-108]$

Then,

We know,

$$M = FEM + [S][D]$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -36 \\ 36 \end{bmatrix} + EI \times \begin{bmatrix} \frac{2}{12} \\ \frac{4}{12} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -108 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -36 \\ 36 \end{bmatrix} + \begin{bmatrix} -18 \\ -36 \end{bmatrix}$$

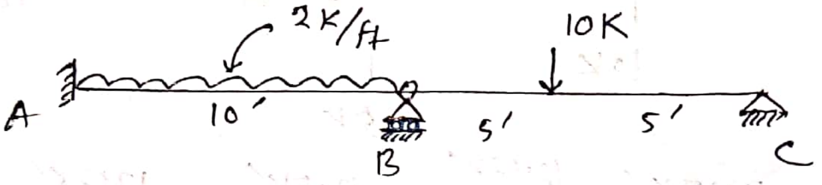
$$\therefore \begin{bmatrix} M_{AB} \\ M_{BA} \end{bmatrix} = \begin{bmatrix} -54 \\ 0 \end{bmatrix}$$

(Ans.)

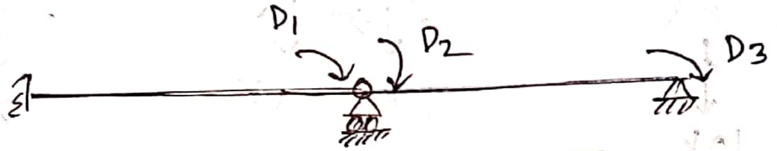
Stiffness Matrix (BEAM)

2018

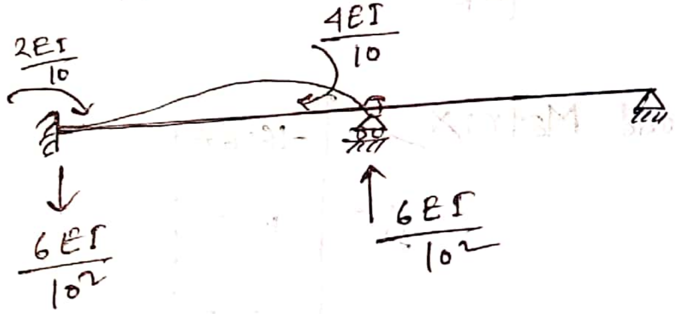
Analyze the beam by stiffness matrix method. EI is constant.



Solution: Active degree of freedom = 03



when $D_1 = 1$, $D_2 = D_3 = 0$:

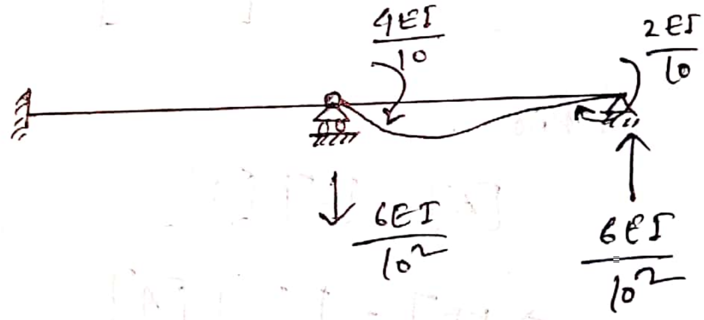


$S_{11} = 0.4EI$

$S_{21} = 0$

$S_{31} = 0$

when $D_2 = 1$, $D_1 = D_3 = 0$:

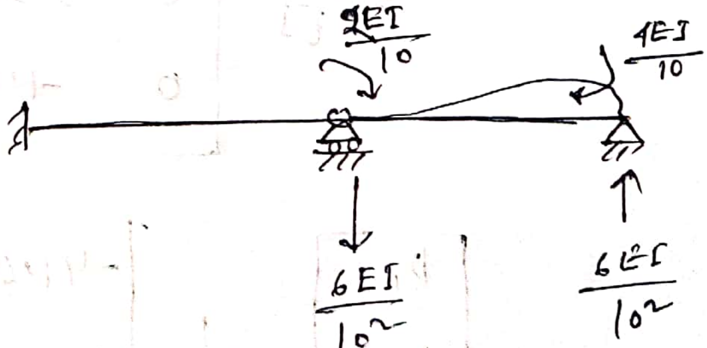


$S_{12} = 0$

$S_{22} = 0.4EI$

$S_{32} = 0.2EI$

when $D_3 = 1$, $D_1 = D_2 = 0$:



$S_{13} = 0$

$S_{23} = 0.2EI$

$S_{33} = 0.4EI$

∴ Stiffness Matrix,

$$S = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.4 & 0.2 \\ 0 & 0.2 & 0.4 \end{bmatrix}$$

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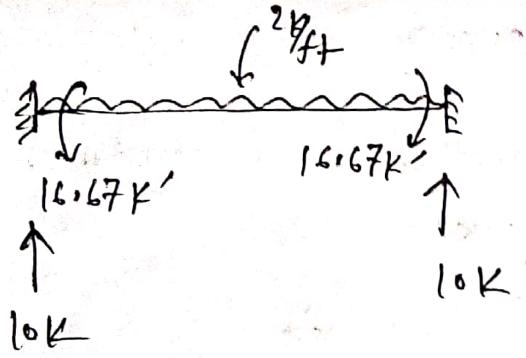
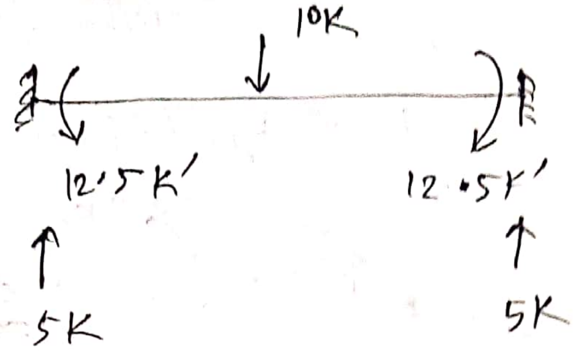
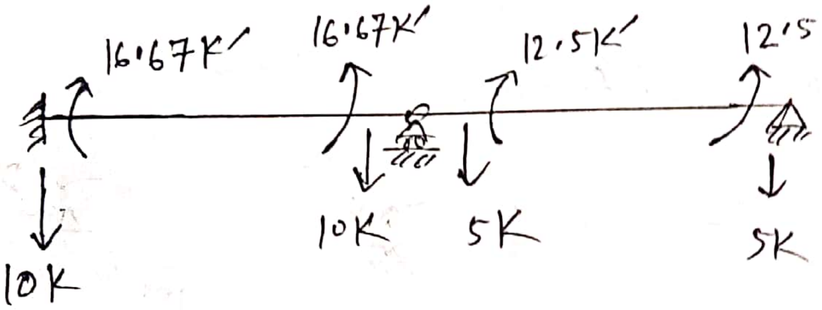






Load Matrix

$$A = \begin{bmatrix} -16.67 \\ 12.5 \\ -12.5 \end{bmatrix}$$

we know,

$$[A] = [S][D]$$

$$\Rightarrow [D] = [S^{-1}][A]$$

$$= \frac{1}{EI} \times \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 3.333 & -1.667 \\ 0 & -1.667 & 3.333 \end{bmatrix} \times \begin{bmatrix} -16.67 \\ 12.5 \\ -12.5 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -41.67 \\ 62.5 \\ -62.5 \end{bmatrix}$$

Now, $M = FEM + [S][D]$

$$\begin{bmatrix} M_{AB}\\ M_{BA}\\ M_{BC}\\ M_{CB}\end{bmatrix} = \begin{bmatrix}-16.67\\16.67\\-12.5\\12.5\end{bmatrix} + EI\begin{bmatrix}0.2&0&0\\0.4&0&0\\0&0.4&0.2\\0&0.2&0.4\end{bmatrix} \times \frac{1}{EI}\begin{bmatrix}-41.67\\62.5\\-62.5\end{bmatrix}$$

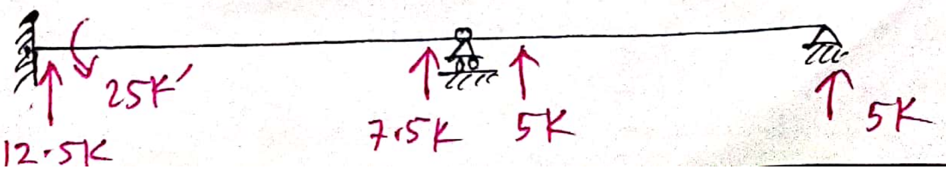
$$\begin{bmatrix} M_{AB}\\ M_{BA}\\ M_{BC}\\ M_{CB}\end{bmatrix} = \begin{bmatrix}-16.67\\16.67\\-12.5\\12.5\end{bmatrix} + \begin{bmatrix}-8.33\\-16.67\\12.5\\-12.5\end{bmatrix} = \begin{bmatrix}-25\\0\\0\\0\end{bmatrix}$$

(Ans.)

Again,

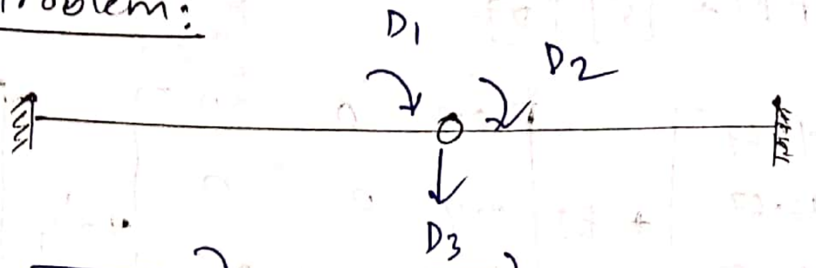
$$\begin{bmatrix} F_{AB}\\ F_{BA}\\ F_{BC}\\ F_{CB}\end{bmatrix} = \begin{bmatrix}10\\10\\5\\5\end{bmatrix} + EI \times \begin{bmatrix}-0.06&0&0\\0.06&0&0\\0&-0.06&-0.06\\0&0.06&0.06\end{bmatrix} \times \frac{1}{EI}\begin{bmatrix}-41.67\\62.5\\-62.5\end{bmatrix}$$

$$\begin{bmatrix} F_{AB}\\ F_{BA}\\ F_{BC}\\ F_{CB}\end{bmatrix} = \begin{bmatrix}10\\10\\5\\5\end{bmatrix} + \begin{bmatrix}2.5\\-2.5\\0\\0\end{bmatrix} = \begin{bmatrix}12.5\\7.5\\5\\5\end{bmatrix}$$

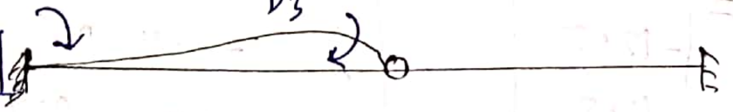


Possible Problem:

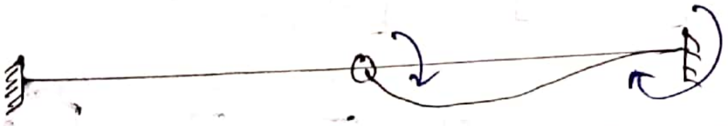
#



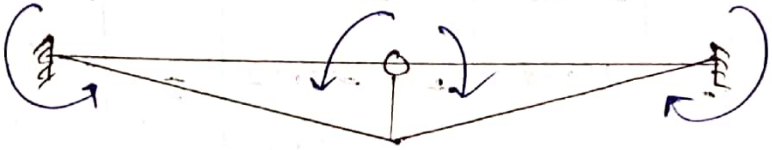
Hints: $\boxed{D_1=1}$



$\boxed{D_2=1}$



$\boxed{D_3=1}$



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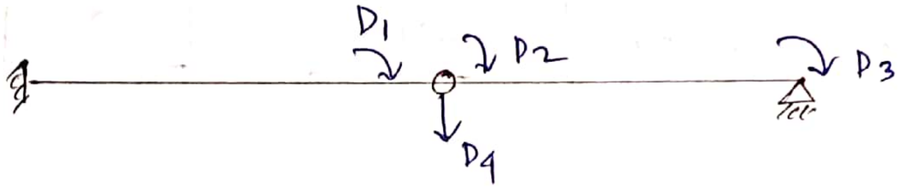
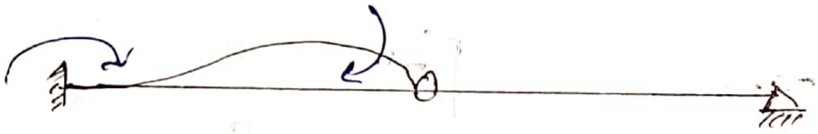
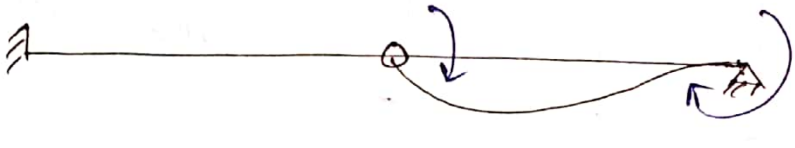


Hints:

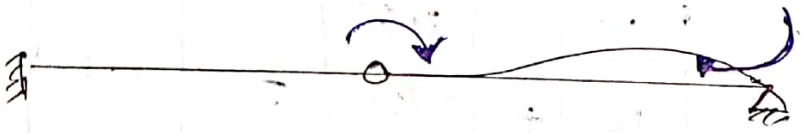
$\boxed{D_1=1}$



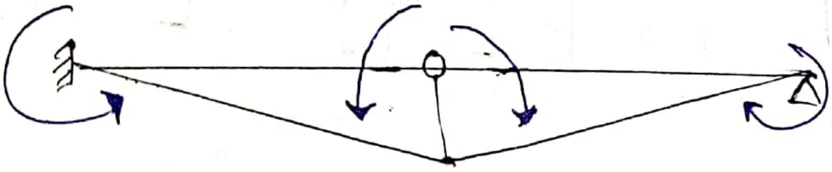
$\boxed{D_2=1}$



$\boxed{D_3=1}$



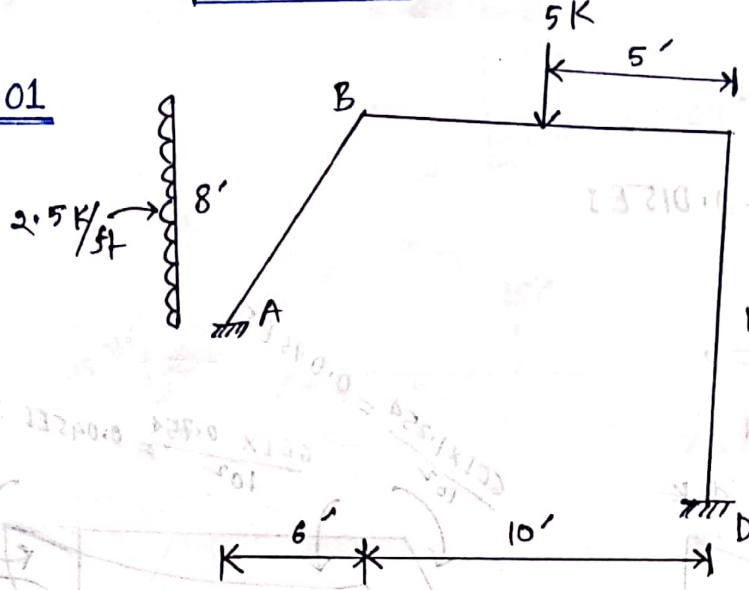
$\boxed{D_4=1}$

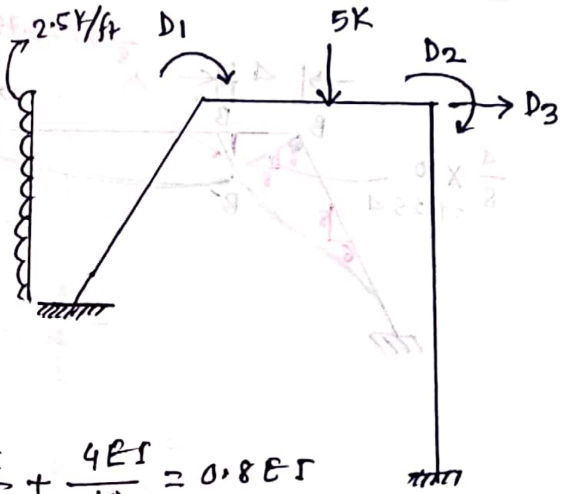


1

FRAME (Stiffness Matrix Method)

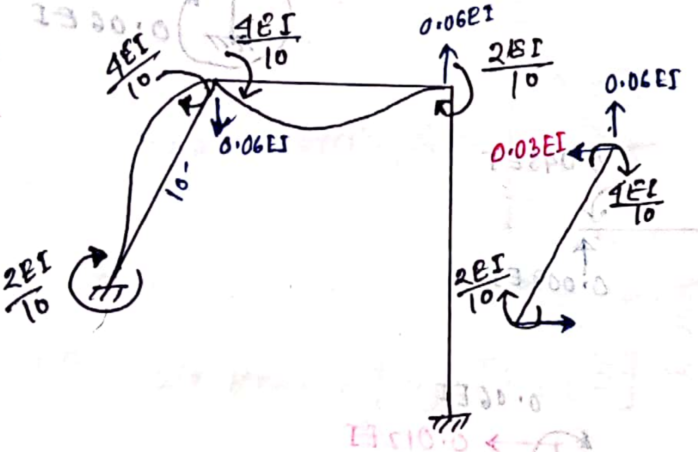
Problem-01





Solution: Active degree of freedom = 03 Nos.

Case-1: $D_1 = 1$, $D_2 = D_3 = 0$

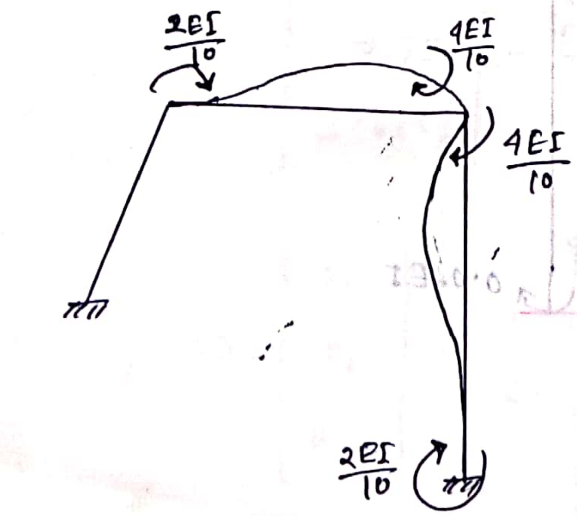


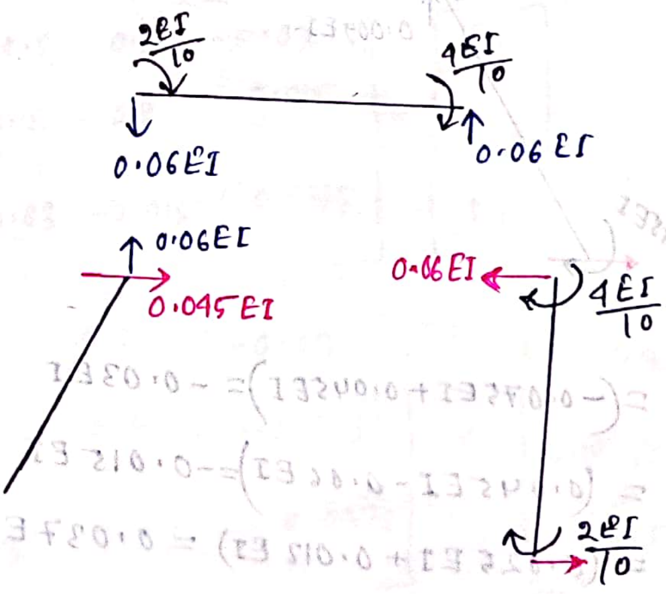
$$S_{11} = \frac{4EI}{10} + \frac{4EI}{10} = 0.8EI$$

$$S_{21} = \frac{2EI}{10} = 0.2EI$$

$$S_{31} = -0.03EI$$

Case-2: $D_2 = 1$, $D_1 = D_3 = 0$



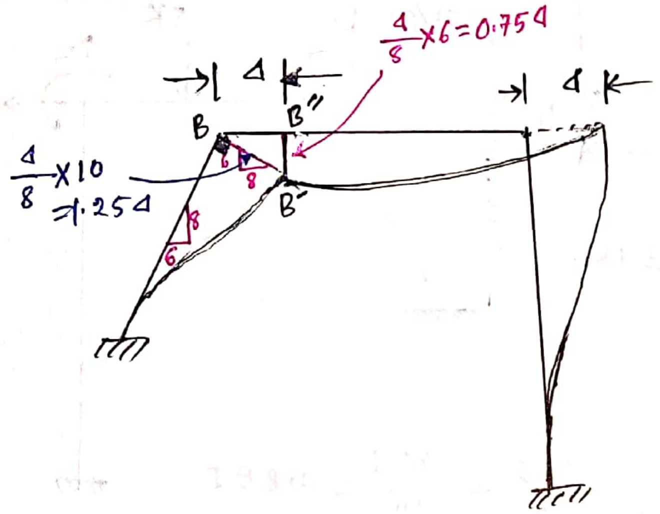


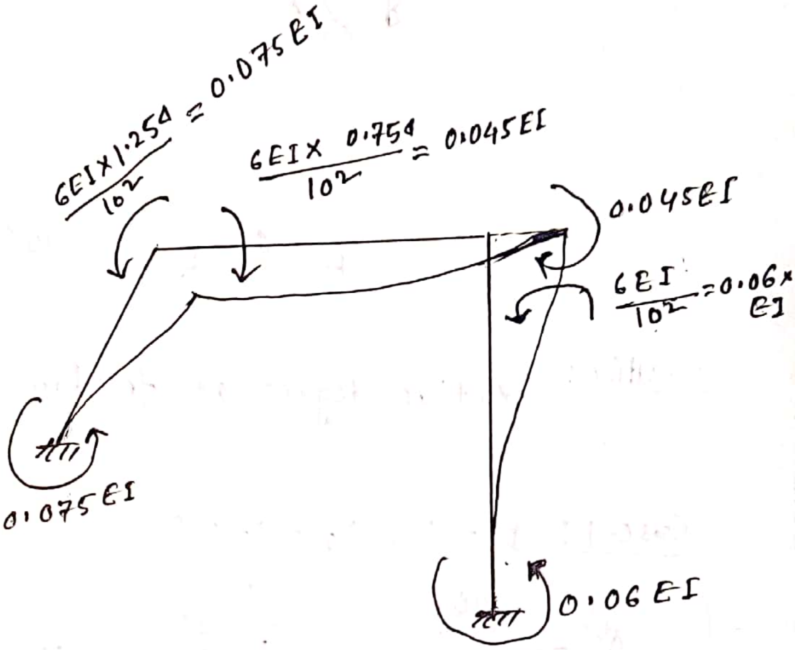
$$S_{12} = \frac{2EI}{10} = 0.2EI$$

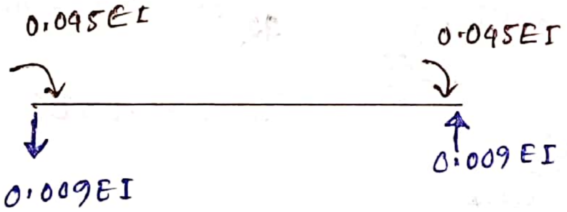
$$S_{22} = \frac{4EI}{10} + \frac{4EI}{10} = 0.8EI$$

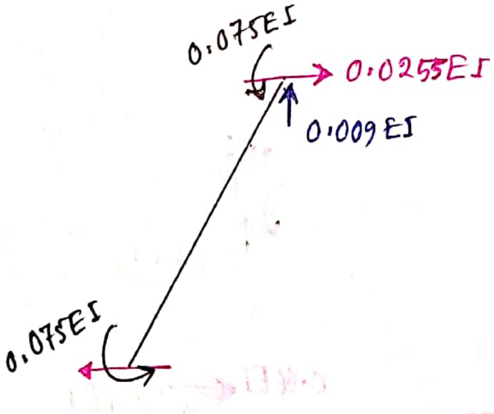
$$S_{32} = (0.045 - 0.06)EI = -0.015EI$$

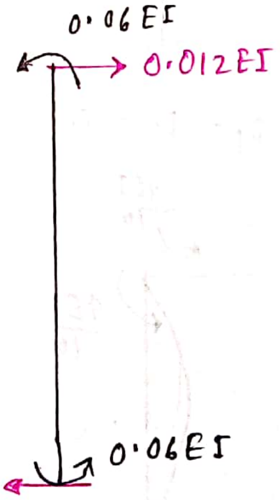
Case-3: $D_3 = 1$, $D_1 = D_2 = 0$









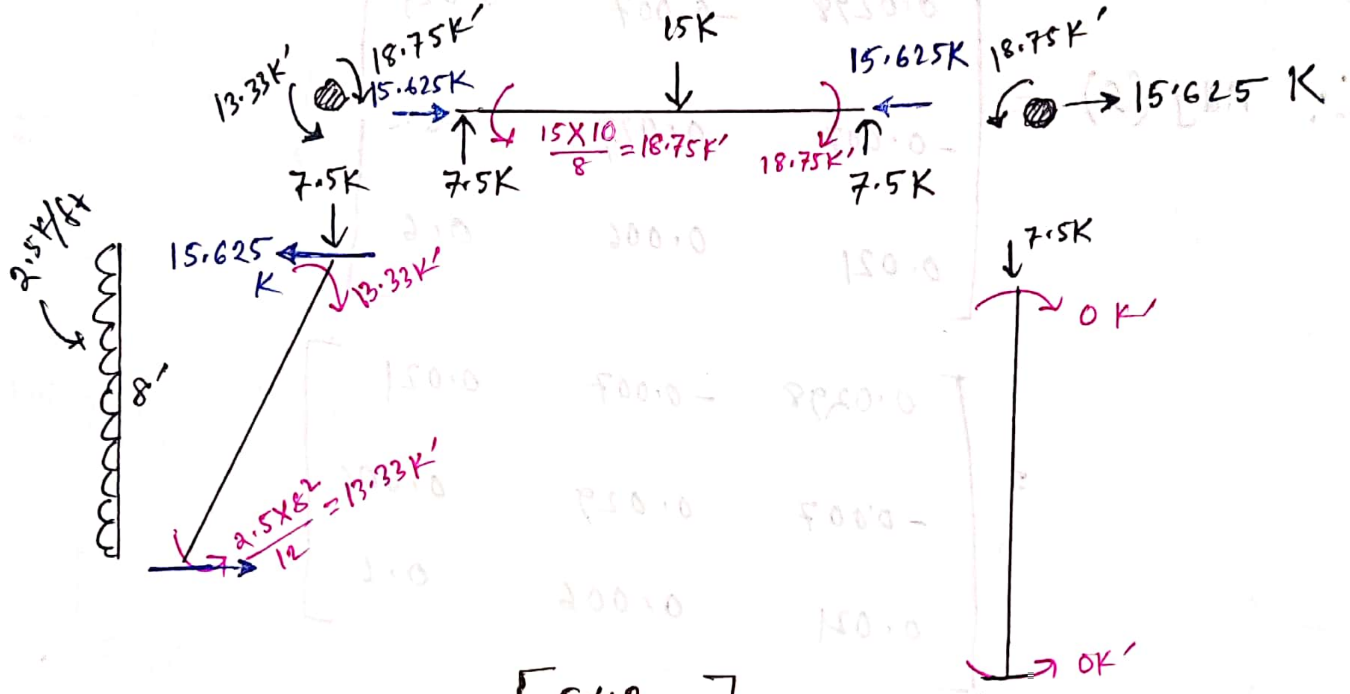


$$S_{13} = (-0.075EI + 0.045EI) = -0.03EI$$

$$S_{23} = (0.045EI - 0.06EI) = -0.015EI$$

$$S_{33} = (0.0255EI + 0.012EI) = 0.0375EI$$

∴ Stiffness Matrix , $S = EI \begin{bmatrix} 0.8 & 0.2 & -0.03 \\ 0.2 & 0.8 & -0.015 \\ -0.03 & -0.015 & 0.0375 \end{bmatrix}$



∴ Load Matrix, $A = \begin{bmatrix} 5.42 \\ -18.75 \\ 15.625 \end{bmatrix}$

Now, we know, $[A] = [S][D]$

$$\begin{bmatrix} 5.42 \\ -18.75 \\ 15.625 \end{bmatrix} = EI \begin{bmatrix} 0.8 & 0.2 & -0.03 \\ 0.2 & 0.8 & -0.015 \\ -0.03 & -0.015 & 0.0375 \end{bmatrix} \times \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.8 & 0.2 & -0.03 \\ 0.2 & 0.8 & -0.015 \\ -0.03 & -0.015 & 0.0375 \end{bmatrix}^{-1} \times \begin{bmatrix} 5.42 \\ -18.75 \\ 15.625 \end{bmatrix}$$

$$\text{co-factor matrix of } s = \begin{bmatrix} 0.0298 & -0.007 & 0.021 \\ -0.007 & 0.029 & 0.006 \\ 0.021 & 0.006 & 0.6 \end{bmatrix}$$

$$\therefore \text{Adj}(S) = \begin{bmatrix} 0.0298 & -0.007 & 0.021 \\ -0.007 & 0.029 & 0.006 \\ 0.021 & 0.006 & 0.6 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0.0298 & -0.007 & 0.021 \\ -0.007 & 0.029 & 0.006 \\ 0.021 & 0.006 & 0.6 \end{bmatrix}$$

$$\text{Det}|s| = 0.8\times[(0.8\times 0.0375)-(-0.015\times -0.015)] - 0.2\times[(0.2\times 0.0375)-(-0.015\times -0.03)] - 0.03\times[(0.2\times -0.015)-(0.8\times -0.03)]$$

$$\therefore \text{Det}|s| = 0.02178$$

$$\therefore s^{-1} = \frac{\text{Adj}(s)}{\text{Det}|s|} = \frac{1}{0.02178} \times \begin{bmatrix} 0.0298 & -0.007 & 0.021 \\ -0.007 & 0.029 & 0.006 \\ 0.021 & 0.006 & 0.6 \end{bmatrix}$$

$$\therefore s^{-1} = \begin{bmatrix} 1.388 & -0.324 & 0.964 \\ -0.324 & 1.336 & 0.2754 \\ 0.964 & 0.2754 & 27.548 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \times \begin{bmatrix} 1.368 & -0.324 & 0.964 \\ -0.324 & 1.336 & 0.2754 \\ 0.964 & 0.2754 & 27.548 \end{bmatrix} \times \begin{bmatrix} 5.42 \\ -18.75 \\ 15.625 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 28.55 \\ -22.5 \\ 430.5 \end{bmatrix}$$

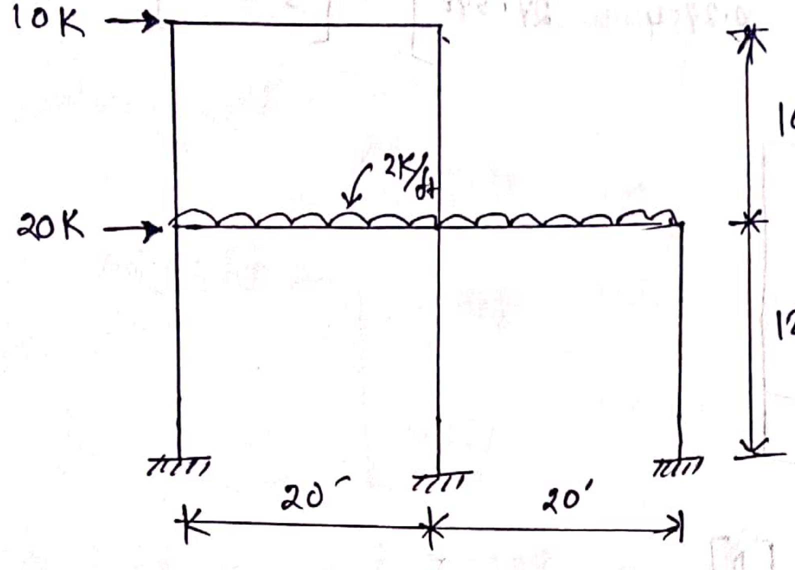
Now, $[M] = FEM + [S][D]$

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} -13.33 \\ 13.33 \\ -18.75 \\ 18.75 \\ 0 \\ 0 \end{bmatrix} + EI \begin{bmatrix} 0.2 & 0 & -0.075 \\ 0.4 & 0 & -0.075 \\ 0.4 & 0.2 & 0.045 \\ 0.2 & 0.4 & 0.045 \\ 0 & 0.4 & -0.06 \\ 0 & 0.2 & -0.06 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 28.55 \\ -22.5 \\ 430.5 \end{bmatrix}$$

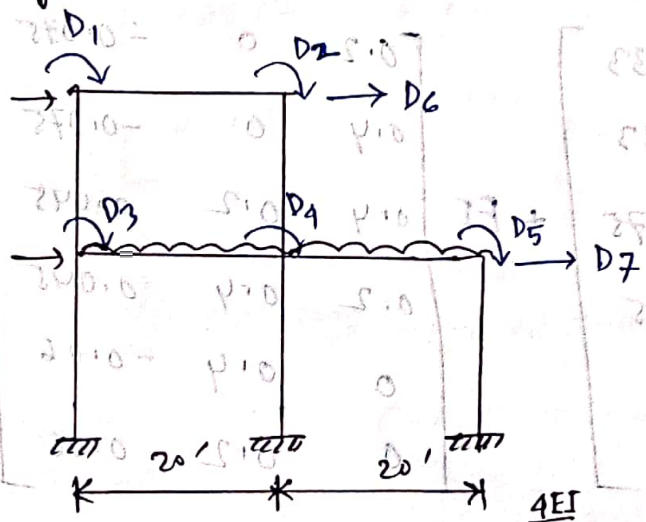
$$\therefore \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} -13.33 \\ 13.33 \\ -18.75 \\ 18.75 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -26.58 \\ -20.87 \\ 26.29 \\ 16.08 \\ -34.83 \\ -30.33 \end{bmatrix} = \begin{bmatrix} 39.91 \\ -7.54 \\ 7.54 \\ 34.83 \\ -34.83 \\ -30.33 \end{bmatrix}$$

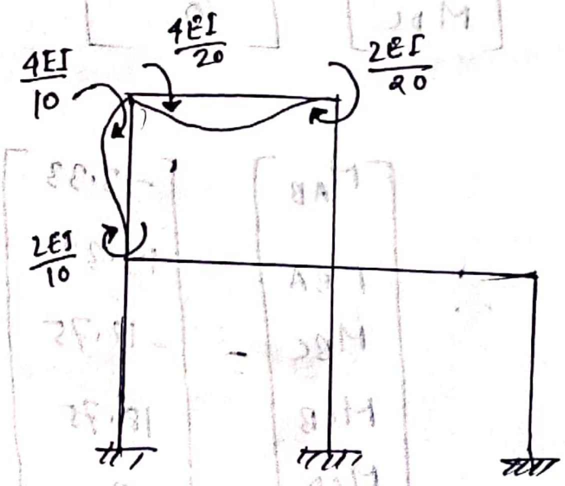
(Ans.)

Problem: 02 Find out the stiffness matrix and load Matrix.



Solution: Active degree of freedom = 07 Nos.





Case-1: $D_1 = 0$, $D_2 = D_3 = \cdots = D_7 = 0$

$S_{11} = \frac{4EI}{10} + \frac{4EI}{20} = 0.6\,EI$

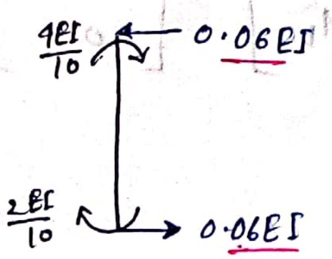
$S_{21} = \frac{2EI}{20} = 0.1\,EI$

$S_{31} = \frac{2EI}{10} = 0.2\,EI$

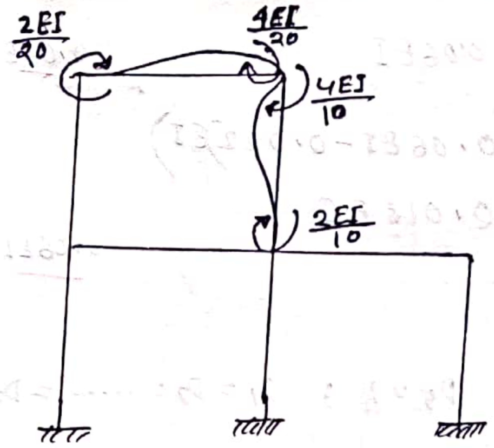
$S_{41} = 0$

$S_{51} = 0$; $S_{61} = -0.06\,EI$

$S_{71} = 0.06\,EI$



Case-2 ; $D_2 = 1$, $D_1 = D_3 = \cdots = D_7 = 0$



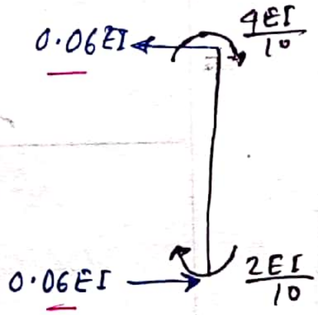
$s_{12} = \frac{2EI}{20} = 0.1EI$

$s_{22} = \frac{4EI}{20} + \frac{4EI}{10} = 0.6EI$; $s_{32} = 0$

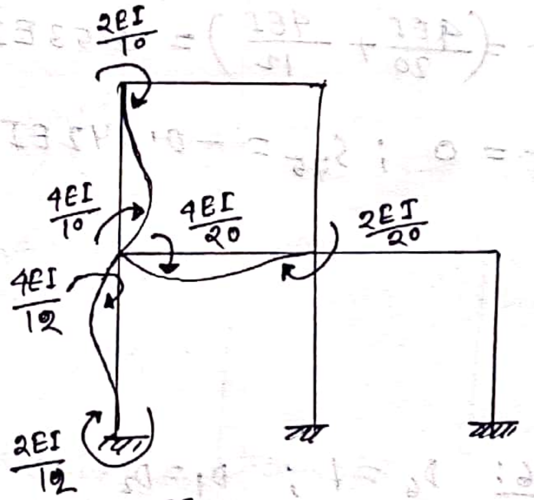
$s_{42} = \frac{2EI}{10} = 0.2EI$; $s_{52} = 0$

$s_{62} = -0.06EI$

$s_{72} = 0.06EI$



Case-3 : $D_3 = 1$, $D_1 = D_2 = \cdots = D_7 = 0$



$s_{13} = \frac{2EI}{10} = 0.2EI$

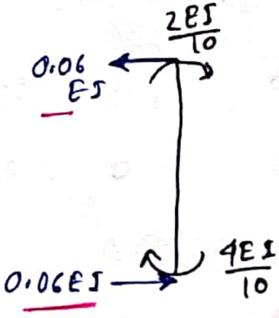
$s_{23} = 0$

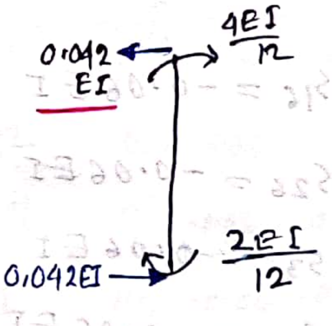
$s_{33} = \frac{4EI}{10} + \frac{4EI}{12} + \frac{4EI}{20} = 0.93EI$

$s_{43} = \frac{2EI}{20} = 0.1EI$

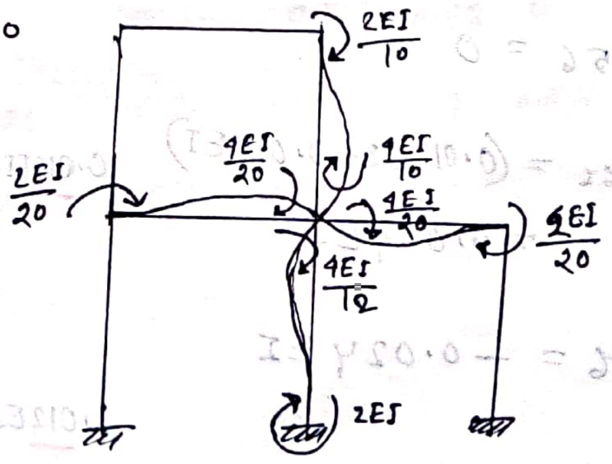
$s_{53} = 0$; $s_{63} = -0.06EI$

$s_{73} = (0.06EI - 0.042EI) = +0.018EI$





CASE-4 : $D_4 = 1$, $D_1 = D_2 = \cdots = D_7 = 0$



$s_{14} = 0$; $s_{24} = 0.2EI$

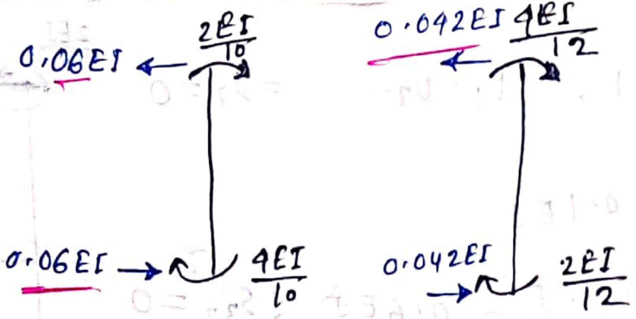
$s_{34} = 0.1EI$; $s_{44} = \left(\frac{4EI}{20} + \frac{4EI}{10} + \frac{4EI}{20} + \frac{4EI}{12}\right)$

$\therefore s_{44} = 1.13EI$

$s_{54} = 0.1EI$

$s_{64} = -0.06EI$

$s_{74} = (0.06EI - 0.042EI)$
$= 0.018EI$



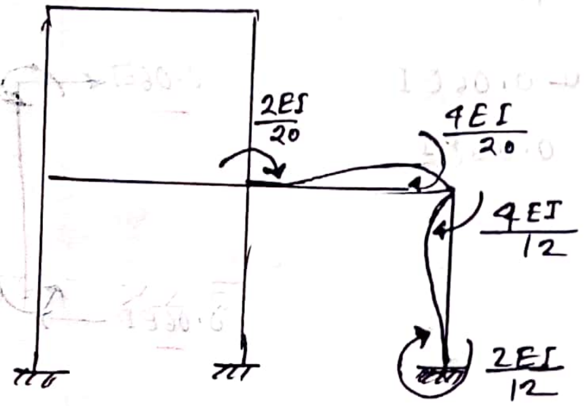
Case-5: $D_5 = 1$; $D_1 = D_2 = \cdots\cdots = D_7 = 0$

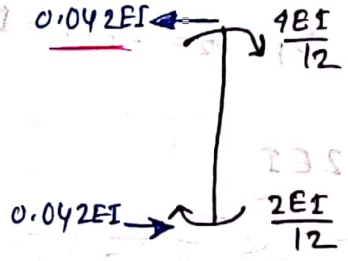
$s_{15} = 0$; $s_{25} = 0$

$s_{35} = 0$; $s_{45} = 0.1EI$

$s_{55} = \left(\frac{4EI}{20} + \frac{4EI}{12}\right) = 0.53EI$

$s_{65} = 0$; $s_{75} = -0.042EI$





Case-6: $D_6 = 1$; $D_1 = D_2 = \cdots\cdots = D_7 = 0$

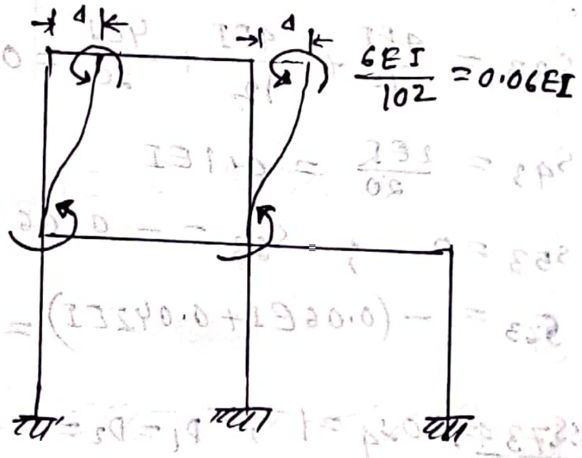
$s_{16} = -0.06EI$

$s_{26} = -0.06EI$

$s_{36} = -0.06EI$

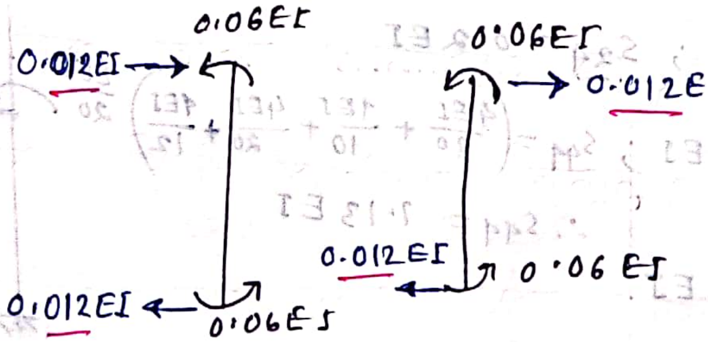
$s_{46} = -0.06EI$

$s_{56} = 0$

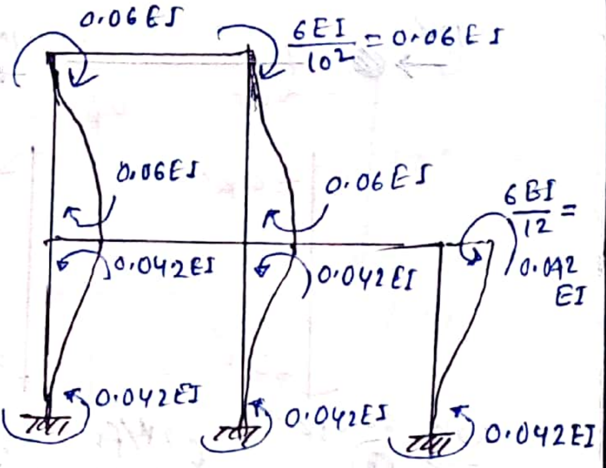


$s_{66} = (0.012EI + 0.012EI)$
$= 0.024EI$

$s_{76} = -0.024EI$



Case-7: $D_7=1$, $D_1=D_2=------=D_6=0$



$S_{17} = 0.06EI$

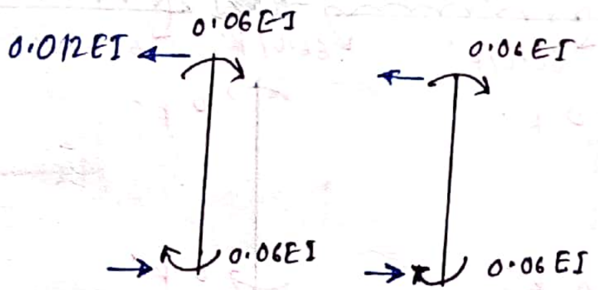
$S_{27} = 0.06EI$

$S_{37} = (0.06EI - 0.042EI) = 0.018EI$

$S_{47} = 0.018EI$

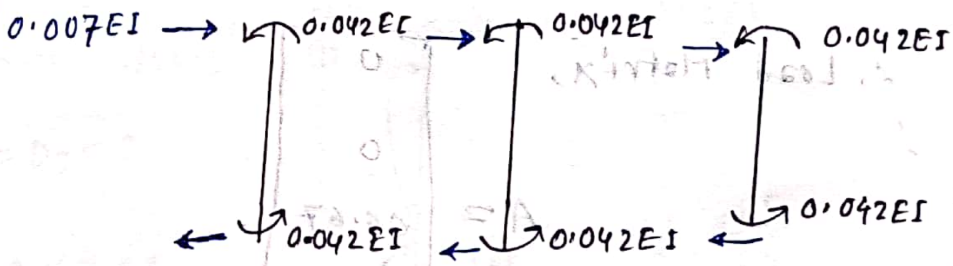
$S_{57} = -0.042EI$

$S_{67} = -(0.012EI + 0.012EI)$
$= -0.024EI$



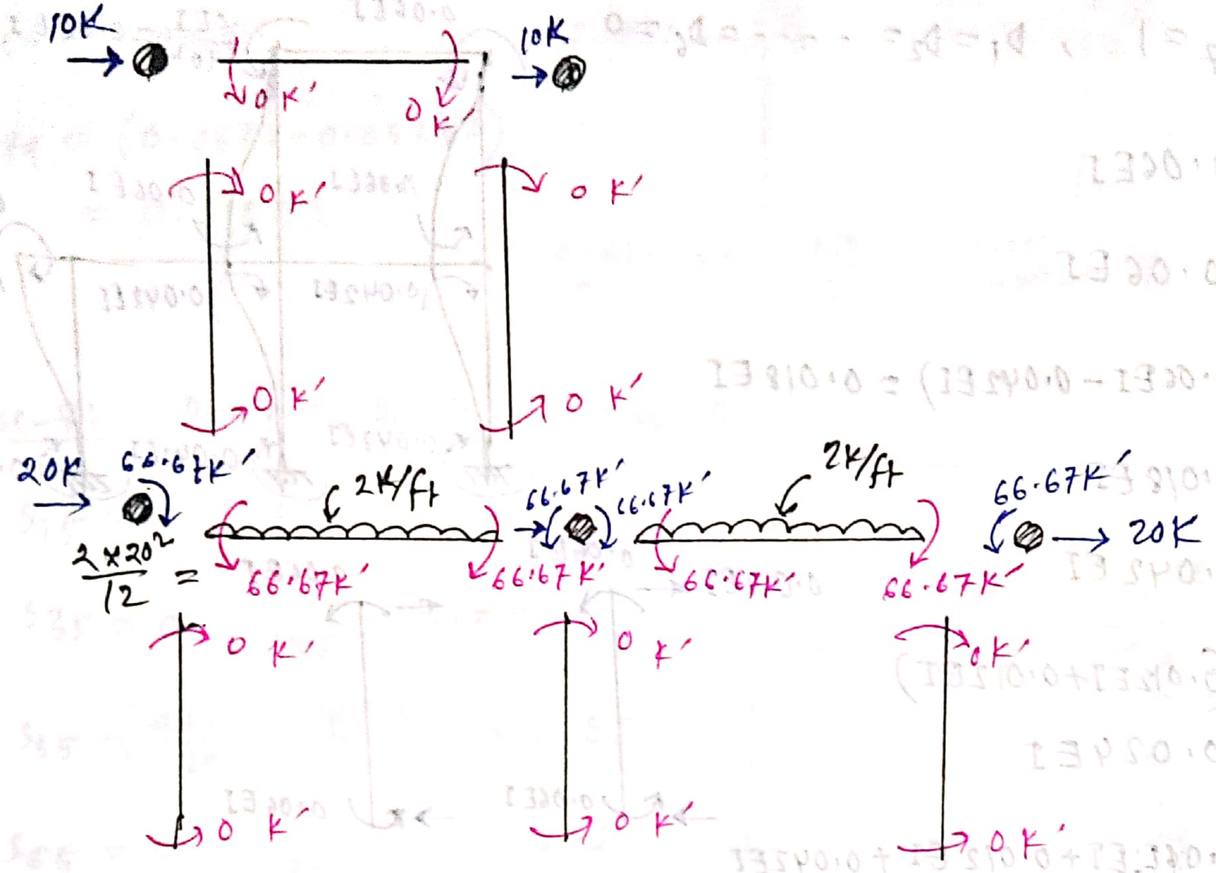
$S_{77} = (0.012EI + 0.012EI + 0.007EI$
$+ 0.007EI \quad 0.007EI)$

$= 0.045EI$



∴ Stiffness Matrix, $S = EI$

$$S = EI\begin{bmatrix} 0.6 & 0.1 & 0.2 & 0 & 0 & -0.06 & 0.06 \\ 0.1 & 0.6 & 0 & 0.2 & 0 & -0.06 & 0.06 \\ 0.2 & 0 & 0.93 & 0.1 & 0 & -0.06 & 0.018 \\ 0 & 0.2 & 0.1 & 1.13 & 0.1 & -0.06 & 0.018 \\ 0 & 0 & 0 & 0.1 & 0.53 & 0 & -0.042 \\ -0.06 & -0.06 & -0.06 & -0.06 & 0 & 0.024 & -0.024 \\ 0.06 & 0.06 & 0.018 & 0.018 & -0.042 & -0.024 & 0.045 \end{bmatrix}$$



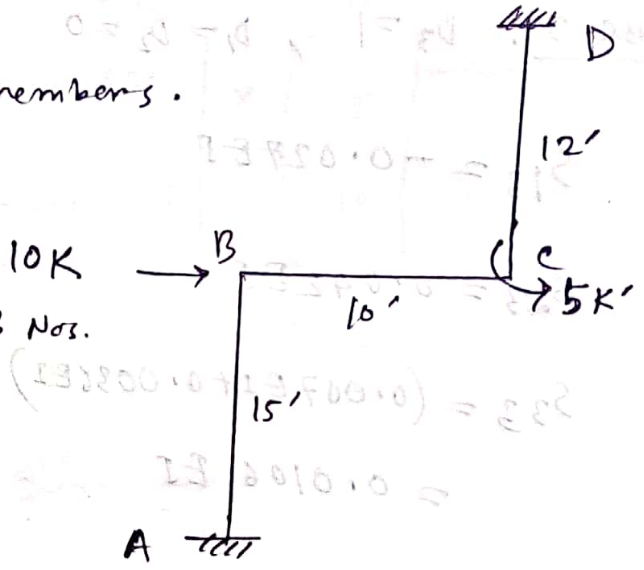
∴ Load Matrix,

$$A = \begin{bmatrix} 0 \\ 0 \\ 66.67 \\ 0 \\ -66.67 \\ 10 \\ 20 \end{bmatrix}$$

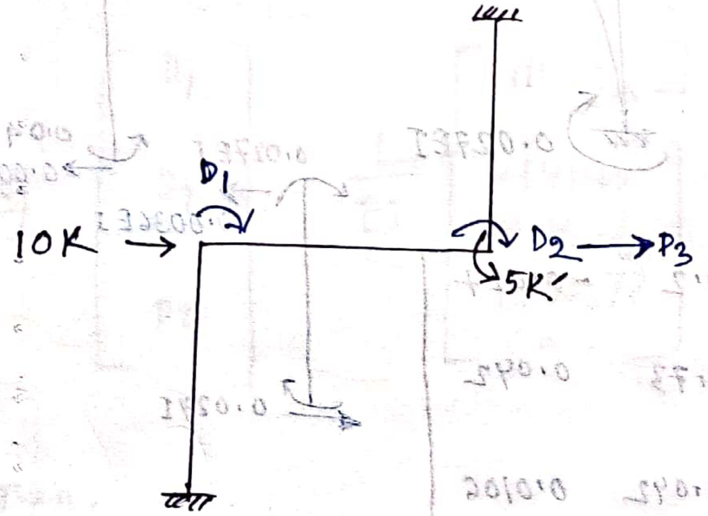
(Ans.)

Problem: 03

Find out Fixed End Moment for members.



Solution: Active degree of freedom = 03 Nos.



case-1: $D_1 = 1$, $D_2 = D_3 = 0$

$$S_{11} = \frac{4EI}{15} + \frac{4EI}{10} = 0.67\,EI$$

$$S_{21} = \frac{2EI}{10} = 0.2\,EI$$

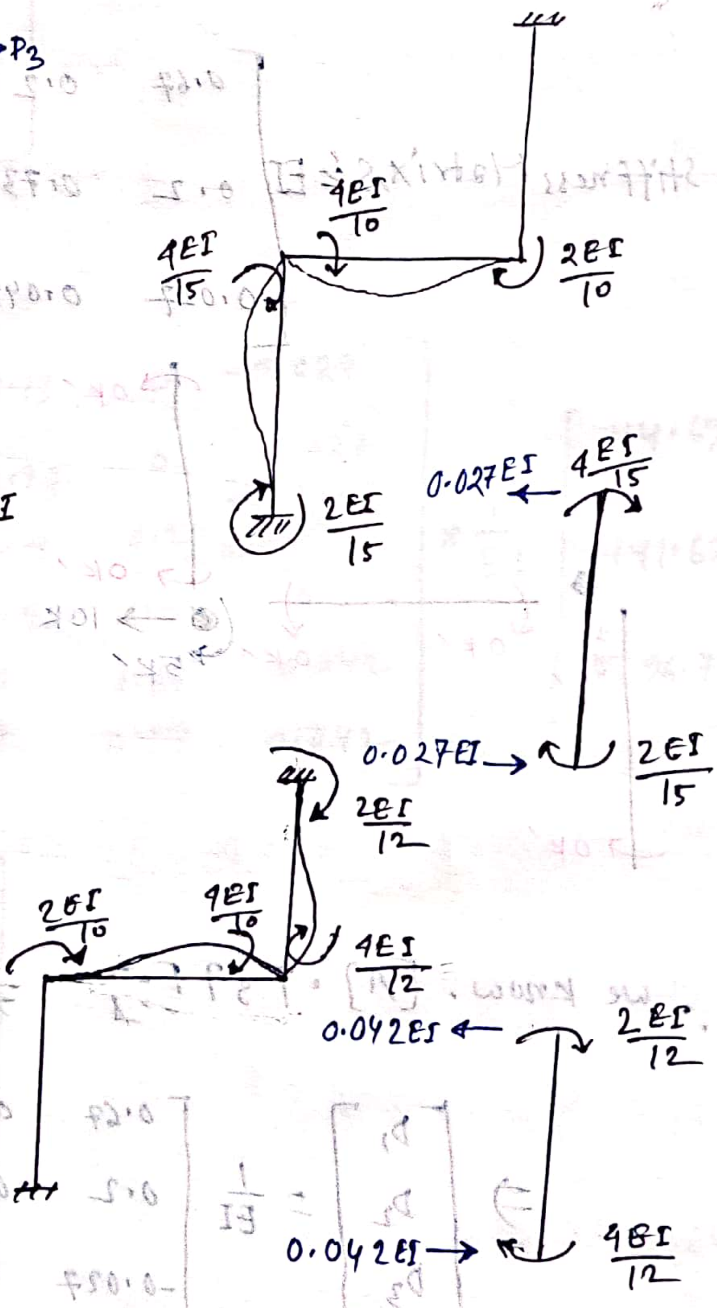
$$S_{31} = -0.027\,EI$$

case-2: $D_2 = 0$, $D_1 = D_3 = 0$

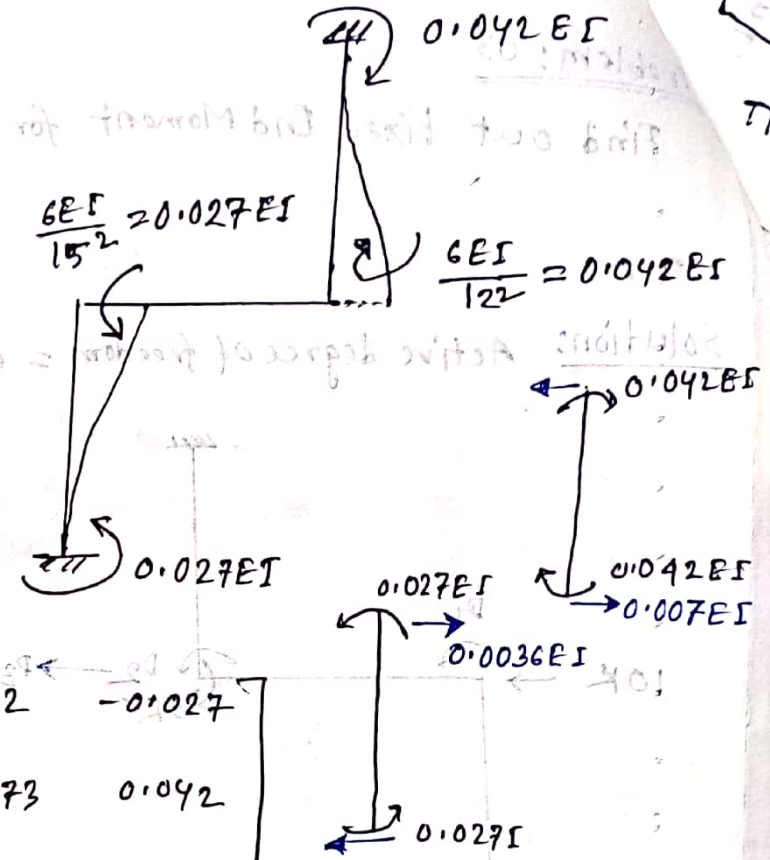
$$S_{12} = 0.2\,EI$$

$$S_{22} = \left(\frac{4EI}{10} + \frac{4EI}{12}\right) = 0.73\,EI$$

$$S_{32} = 0.042\,EI$$



Case-3 : $D_3 = 1$, $D_1 = D_2 = 0$



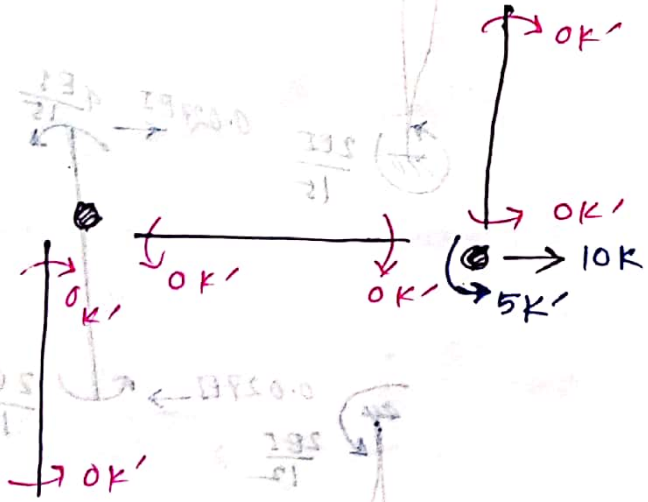
$S_{13} = -0.027EI$

$S_{23} = 0.042EI$

$S_{33} = (0.007EI + 0.0036EI)$

$= 0.0106\ EI$

∴ Stiffness Matrix, $S = EI \begin{bmatrix} 0.67 & 0.2 & -0.027 \\ 0.2 & 0.73 & 0.042 \\ -0.027 & 0.042 & 0.0106 \end{bmatrix}$



∴ Load Matrix,

$A = \begin{bmatrix} 0 \\ -5 \\ 10 \end{bmatrix}$

∴ we know, $[A] = [S][D] \Rightarrow [D] = [S]^{-1}[A]$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.67 & 0.2 & -0.027 \\ 0.2 & 0.73 & 0.042 \\ -0.027 & 0.042 & 0.0106 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ -5 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 2.304 & -1.255 & 10.842 \\ -1.255 & 2.458 & -12.936 \\ 10.842 & -12.936 & 173.21 \end{bmatrix} \times \begin{bmatrix} 0 \\ -5 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 114.69 \\ -141.65 \\ 1796.77 \end{bmatrix}$$

Then,

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + EI \begin{bmatrix} 0.13 & 0 & -0.027 \\ 0.27 & 0 & -0.027 \\ 0.4 & 0.2 & 0 \\ 0.2 & 0.4 & 0 \\ 0 & 0.33 & 0.042 \\ 0 & 0.17 & 0.042 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 114.69 \\ -141.65 \\ 1796.77 \end{bmatrix}$$

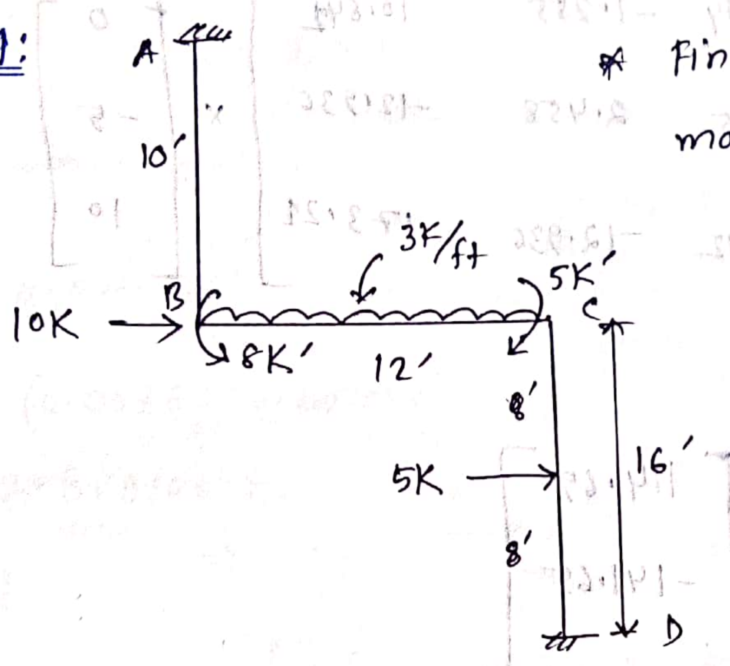
$$\therefore \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} -33.60 \\ -17.55 \\ 17.55 \\ -33.72 \\ 28.72 \\ 55.4 \end{bmatrix}$$

(Ans.)

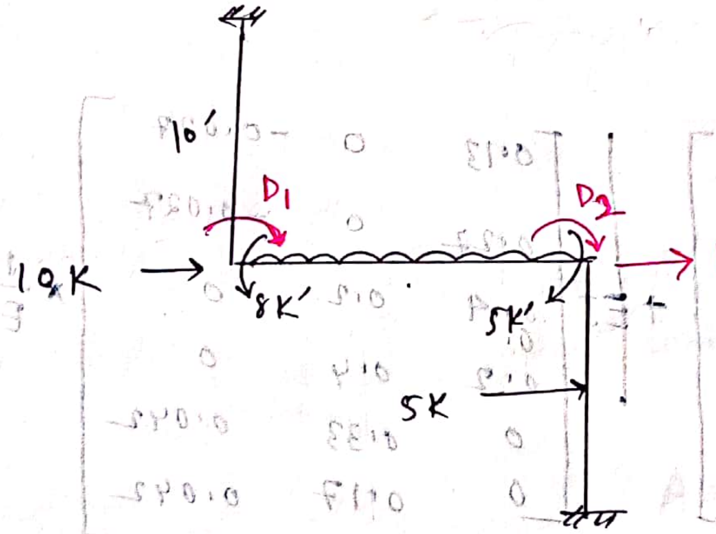
Problem: 04:

* Find out the fixed end moment for members

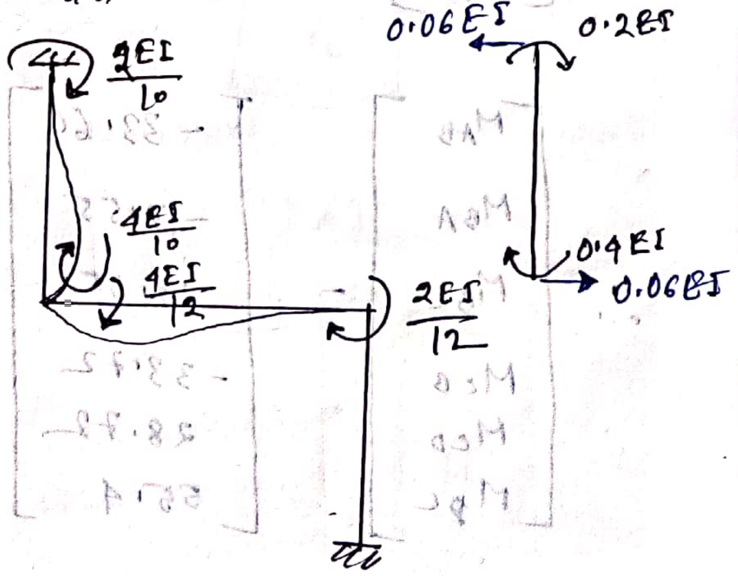
Case-



Solution: Active degree of freedom = 03 Nos.



Case-1: $D_1 = 1$, $D_2 = D_3 = 0$



$$S_{11} = \frac{4EI}{10} + \frac{4EI}{12} = 0.73\,EI$$

$$S_{21} = \frac{2EI}{12} = 0.17\,EI$$

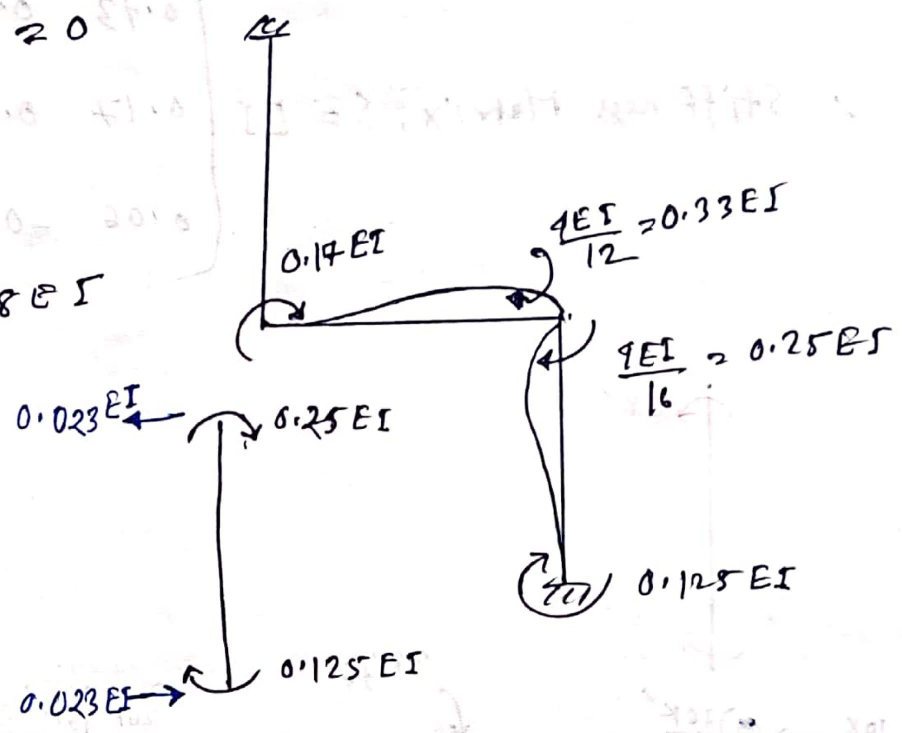
$$S_{31} = 0.06\,EI$$

Case-2 : $D_2 = 1$, $D_1 = D_3 = 0$

$S_{12} = 0.17 EI$

$S_{22} = \frac{4EI}{12} + \frac{4EI}{16} = 0.58 EI$

$S_{32} = -0.023 EI$

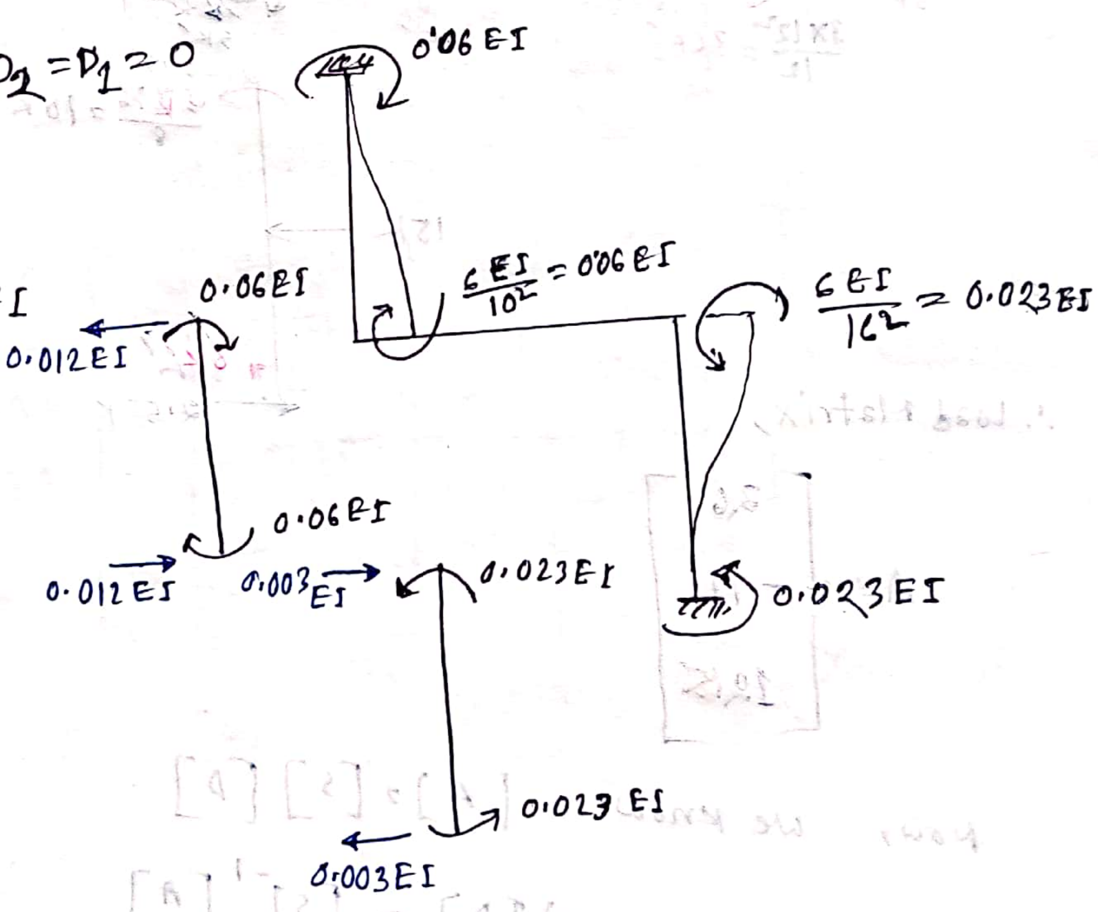


Case-3: $D_3 = 1$, $D_2 = D_1 = 0$

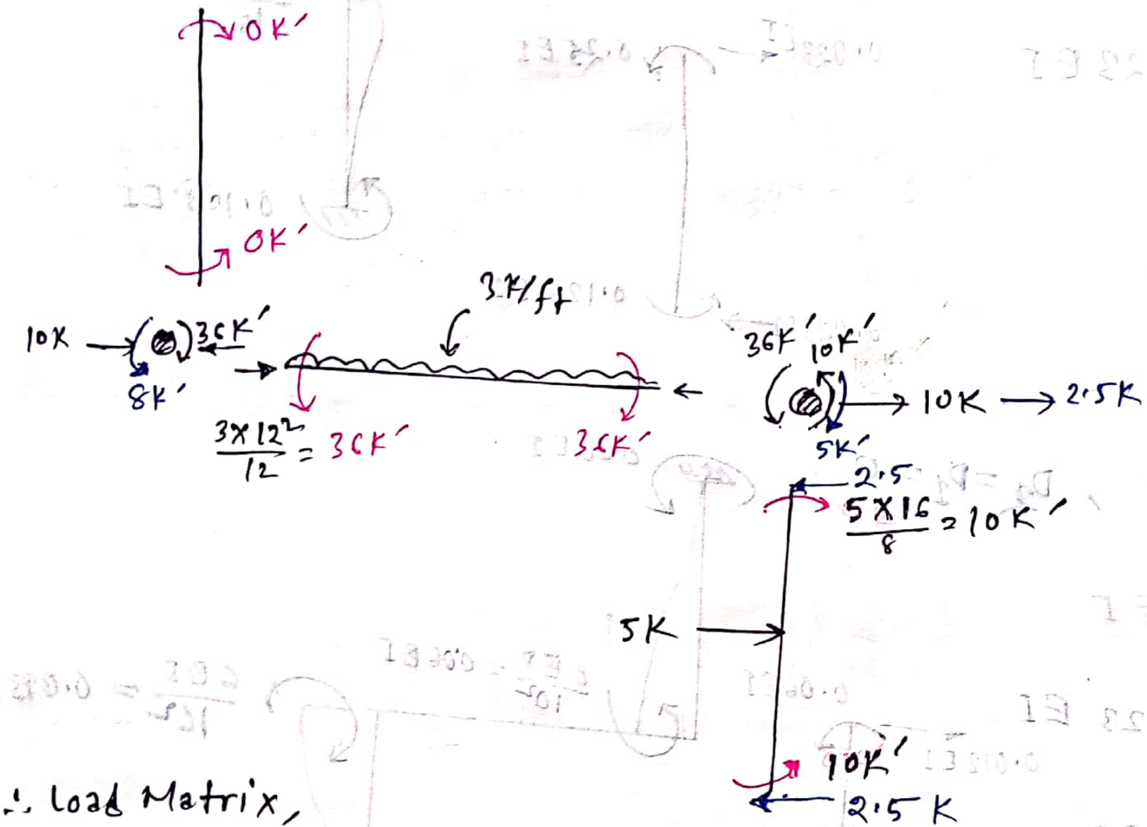
$S_{13} = +0.06 EI$

$S_{23} = -0.023 EI$

$S_{33} = 0.015 EI$



∴ Stiffness Matrix, $S = EI \begin{bmatrix} 0.73 & 0.17 & 0.06 \\ 0.17 & 0.58 & -0.023 \\ 0.06 & -0.023 & 0.015 \end{bmatrix}$



∴ Load Matrix,

$$A = \begin{bmatrix} 28 \\ -41 \\ 12.5 \end{bmatrix}$$

Now, we know, $[A] = [S][D]$

$$\Rightarrow [D] = [S]^{-1}[A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.73 & 0.17 & 0.06 \\ 0.17 & 0.58 & -0.023 \\ 0.06 & -0.023 & 0.015 \end{bmatrix}^{-1} \times \begin{bmatrix} 28 \\ -41 \\ 12.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -31.59 \\ -24.89 \\ 921.54 \end{bmatrix}$$

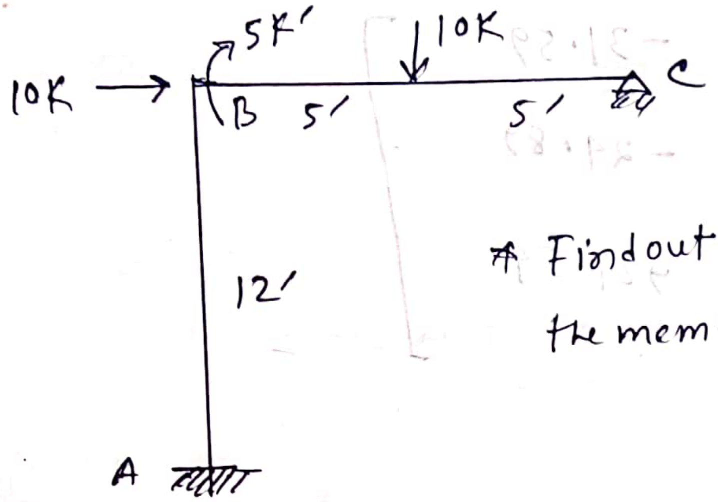
Then,

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -36 \\ 36 \\ 10 \\ -10 \end{bmatrix} + EI \begin{bmatrix} 0.2 & 0 & 0.06 \\ 0.4 & 0 & 0.06 \\ 0.33 & 0.17 & 0 \\ 0.17 & 0.33 & 0 \\ 0 & 0.25 & -0.023 \\ 0 & 0.125 & -0.023 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -31.59 \\ -24.89 \\ 921.54 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -36 \\ 36 \\ 10 \\ -10 \end{bmatrix} + \begin{bmatrix} 48.97 \\ 42.66 \\ -14.66 \\ -13.58 \\ -27.42 \\ -24.31 \end{bmatrix} = \begin{bmatrix} 48.97 \\ 42.66 \\ -50.66 \\ 22.42 \\ -17.42 \\ -34.31 \end{bmatrix}$$

(Ans.)

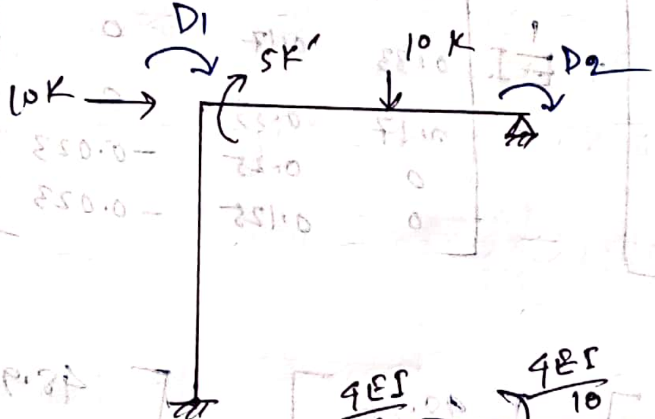
CT-(14 series)



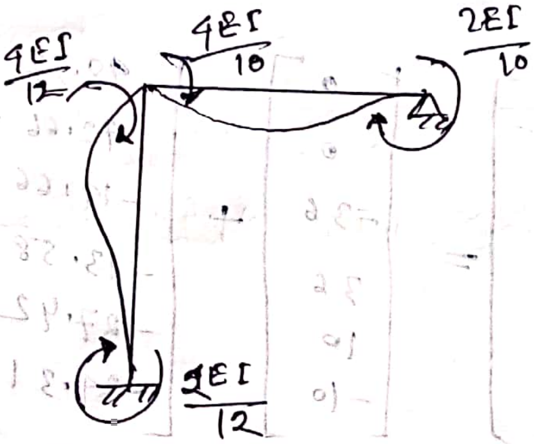
* Findout the moments for the members.

Solution:

Active degree of freedom = 02 Nos.

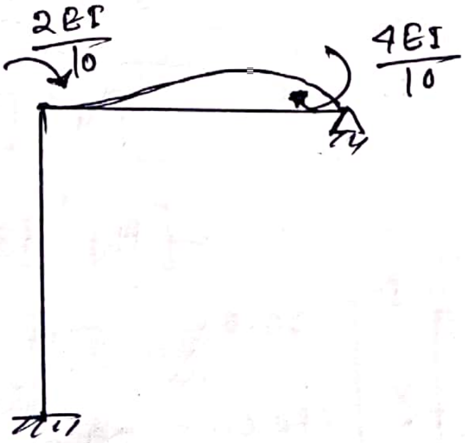


Case-1: $D_1 = 1,\ D_2 = 0$



$$S_{11} = \frac{4EI}{12} + \frac{4EI}{10} = 0.73EI$$

$$S_{21} = 0.2EI$$



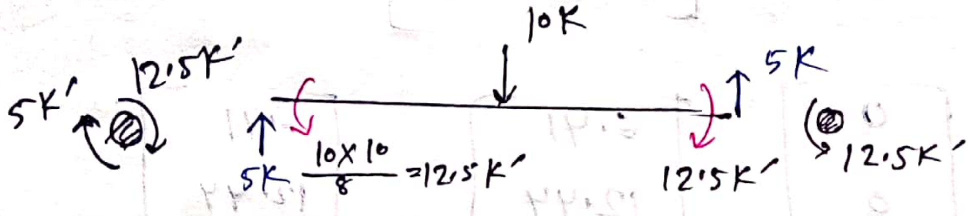
Case-02: $D_2 = 1,\ D_1 = 0$

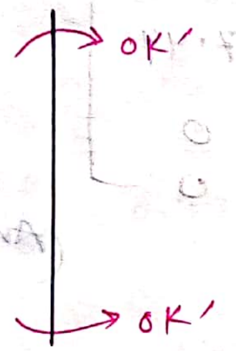
$$S_{12} = 0.2EI$$

$$S_{22} = 0.4EI$$

∴ Stiffness Matrix,

$$S = EI \begin{bmatrix} 0.73 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$





∴ Load Matrix, A = $\begin{bmatrix} 17.5 \\ -12.5 \end{bmatrix}$

we know, $[A] = [S][D] \Rightarrow [D] = [S]^{-1}[A]$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.73 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}^{-1} \times \begin{bmatrix} 17.5 \\ -12.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{0.252EI} \times \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.73 \end{bmatrix} \times \begin{bmatrix} 17.5 \\ -12.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{0.252EI} \times \begin{bmatrix} 9.5 \\ -12.625 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 37.7 \\ -50.1 \end{bmatrix}$$

Now, $[M] = FEM + [S][D]$

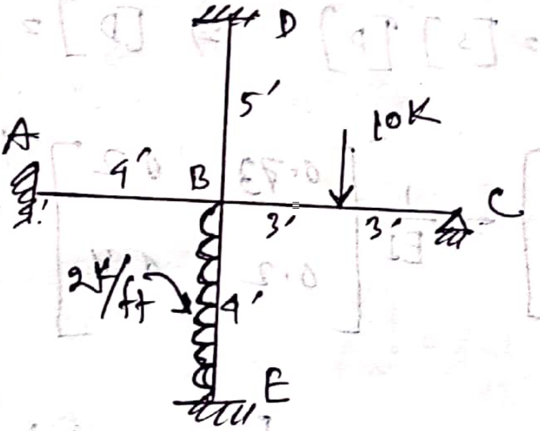
$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -12.5 \\ 12.5 \end{bmatrix} + EI \begin{bmatrix} 0.17 & 0 \\ 0.33 & 0 \\ 0.4 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 37.7 \\ -50.1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -12.5 \\ 12.5 \end{bmatrix} + \begin{bmatrix} 6.41 \\ 12.44 \\ 5.06 \\ -12.5 \end{bmatrix} = \begin{bmatrix} 6.41 \\ 12.44 \\ -7.44 \\ 0 \end{bmatrix}$$

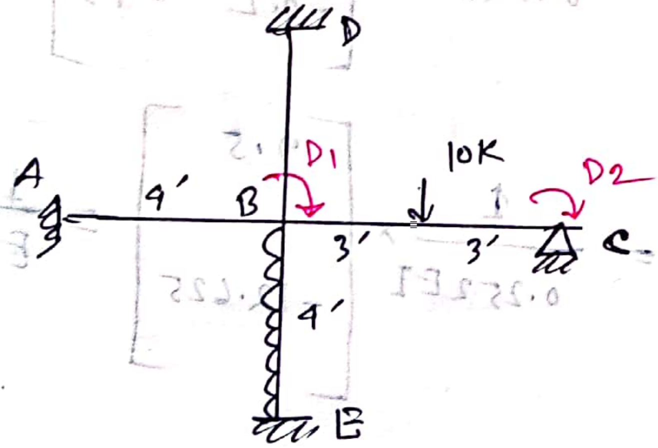
(Ans.)

CT-(13 series)

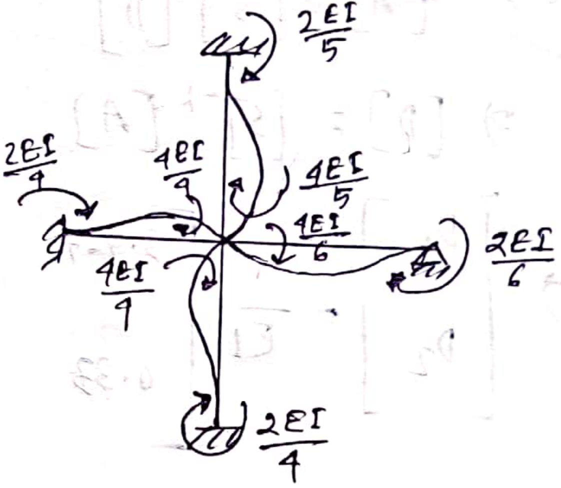
Find out the moments for the members.



Solution: Active degree of freedom = 02 Nos.



Case-1: $D_1 = 1$, $D_2 = 0$

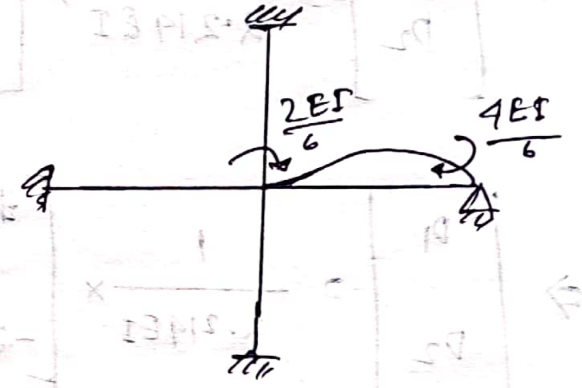


$$S_{11} = \left(\frac{4EI}{4} + \frac{4EI}{5} + \frac{4EI}{6} + \frac{4EI}{4}\right)$$

$$= 3.467\,EI$$

$$S_{21} = \frac{2EI}{6} = 0.33\,EI$$

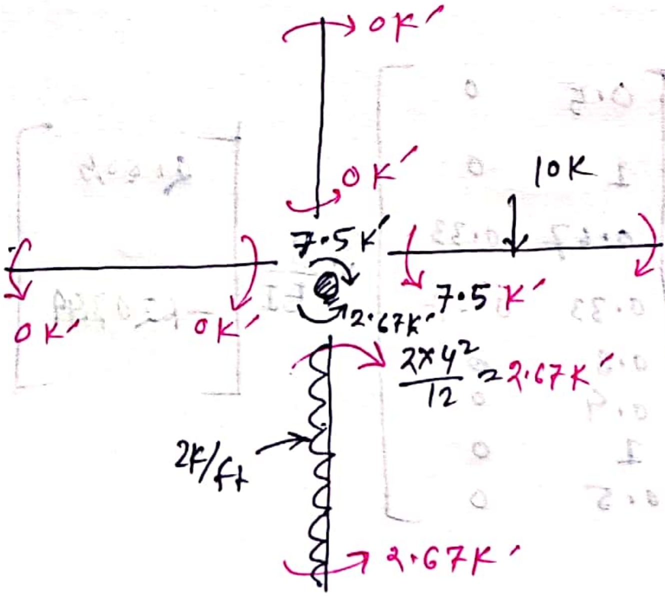
Case-2: $D_2 = 1$, $D_1 = 0$



$$S_{12} = \frac{2EI}{6} = 0.33\,EI$$

$$S_{22} = \frac{4EI}{6} = 0167\,EI$$

$$\therefore \text{stiffness Matrix, } S = \begin{bmatrix} 3.467 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$



$$\therefore \text{Load Matrix, } A = \begin{bmatrix} 4.83 \\ -7.5 \end{bmatrix}$$

We know, $[A] = [S][D]$

$$\Rightarrow [D] = [S]^{-1}[A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 3.467 & 0.33 \\ 0.33 & 6.67 \end{bmatrix}^{-1} \times \begin{bmatrix} 4.83 \\ -7.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{2.214EI} \times \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 3.467 \end{bmatrix} \times \begin{bmatrix} 4.83 \\ -7.5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{2.214EI} \times \begin{bmatrix} 5.691 \\ -27.5964 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 2.57 \\ -12.464 \end{bmatrix}$$

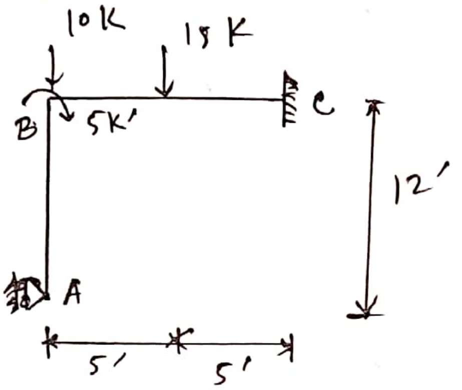
Then, $[M] = FEM + [S][D]$

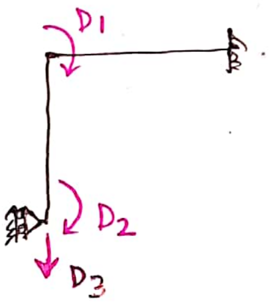
$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{BD} \\ M_{DB} \\ M_{BE} \\ M_{EB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -7.5 \\ 7.5 \\ 0 \\ 0 \\ 2.67 \\ -2.67 \end{bmatrix} + EI \begin{bmatrix} 0.5 & 0 \\ 1 & 0 \\ 0.67 & 0.33 \\ 0.33 & 0.67 \\ 0.8 & 0 \\ 0.4 & 0 \\ 1 & 0 \\ 0.5 & 0 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} 2.57 \\ -12.464 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{BD} \\ M_{DB} \\ M_{BE} \\ M_{EB} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -7.5 \\ 7.5 \\ 0 \\ 0 \\ 2.67 \\ -2.67 \end{bmatrix} + \begin{bmatrix} 1.285 \\ 2.57 \\ -2.39 \\ -7.5 \\ 2.056 \\ 1.028 \\ 2.57 \\ 1.285 \end{bmatrix} = \begin{bmatrix} 1.285 \\ 2.57 \\ -9.89 \\ 0 \\ 2.056 \\ 1.028 \\ 5.24 \\ -1.385 \end{bmatrix}$$

CT- 2015 series:

Findout Stiffness matrix and Load Matrix.





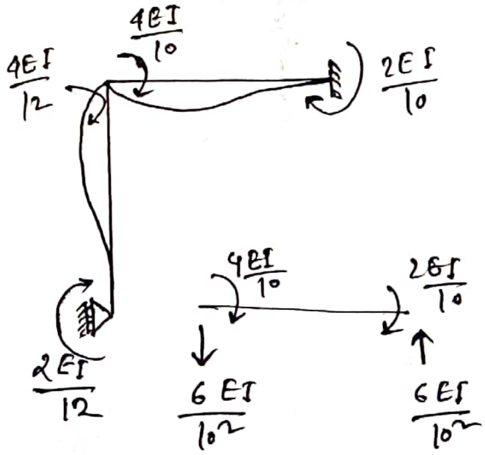
Solution: degree of freedom = 03

When $D_1 = 1$, $D_2 = D_3 = 0$:

$$S_{11} = \frac{4EI}{10} + \frac{4EI}{12} = 0.733\,EI$$

$$S_{21} = \frac{2EI}{12} = 0.167\,EI$$

$$S_{31} = \frac{6EI}{10^2} = 0.06\,EI$$

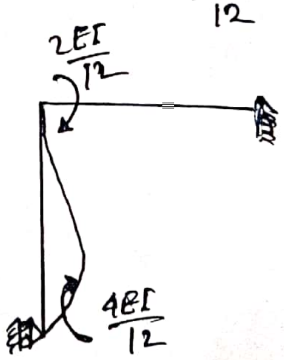


When $D_2 = 1$, $D_1 = D_3 = 0$:

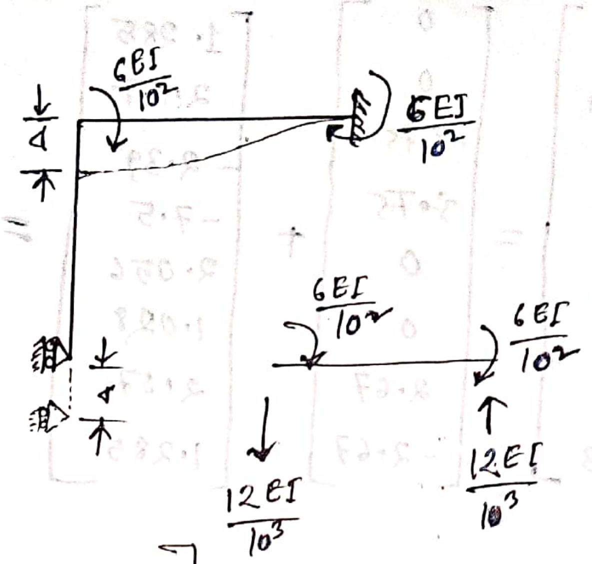
$$S_{12} = \frac{2EI}{12} = 0.167\,EI$$

$$S_{22} = \frac{4EI}{12} = 0.333\,EI$$

$$S_{32} = 0$$



When $D_3 = 1, \; D_1 = D_2 = 0$:



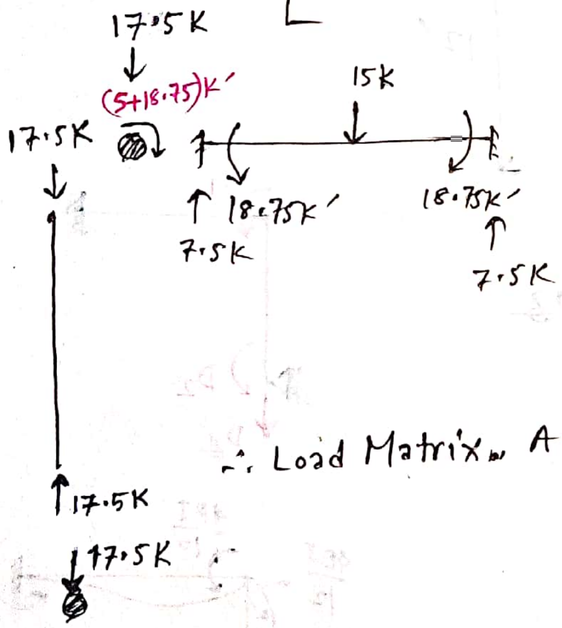
$S_{13} = 0.06EI$

$S_{23} = 0$

$S_{33} = 0.012EI$

Stiffness Matrix,

$$S = EI \begin{bmatrix} 0.733 & 0.167 & 0.06 \\ 0.167 & 0.333 & 0 \\ 0.06 & 0 & 0.012 \end{bmatrix}$$

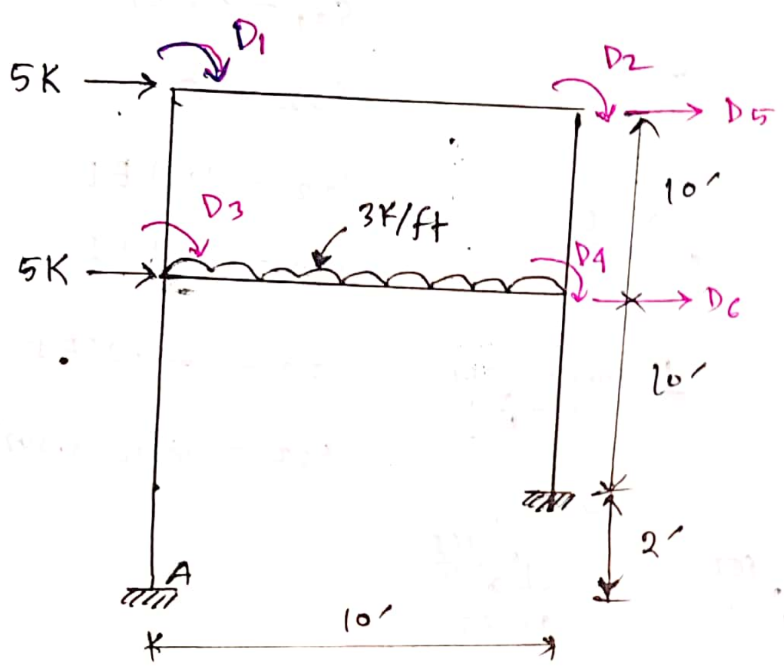


$$\therefore \text{Load Matrix, } A = \begin{bmatrix} 23.75 \\ 0 \\ 17.5 \end{bmatrix}$$

Stiffness Matrix Method (FRAME)

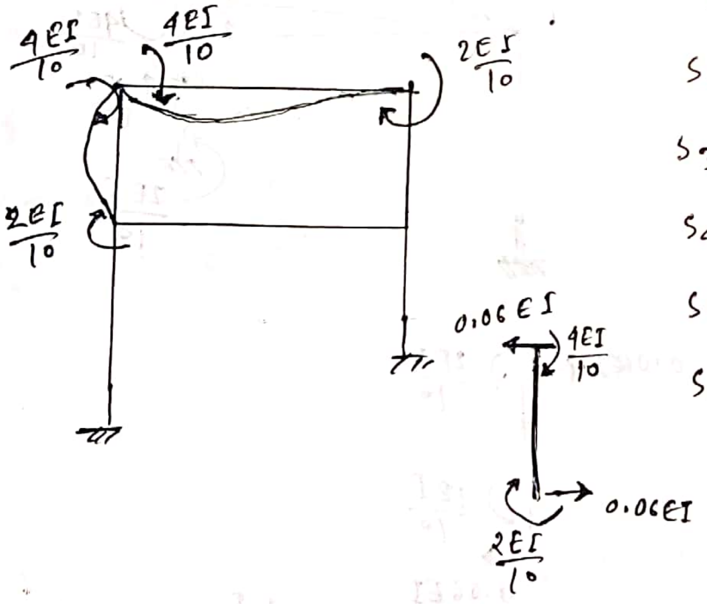
2016

#



Solution: Active digree of freedom = 06

when, $D_1 = 1$, all are zero.



$S_{11} = 0.8 EI$

$S_{21} = 0.2 EI$

$S_{31} = 0.2 EI$

$S_{41} = 0$

$S_{51} = -0.06 EI$

$S_{61} = 0.06 EI$

when, $D_2 = 1$, all are zero:

$S_{12} = 0.2 EI$

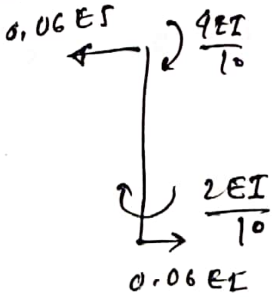
$S_{22} = 0.8 EI$

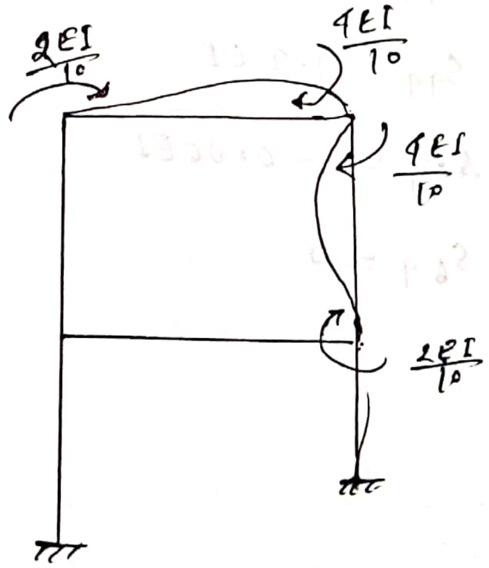
$S_{32} = 0$

$S_{42} = 0.2 EI$

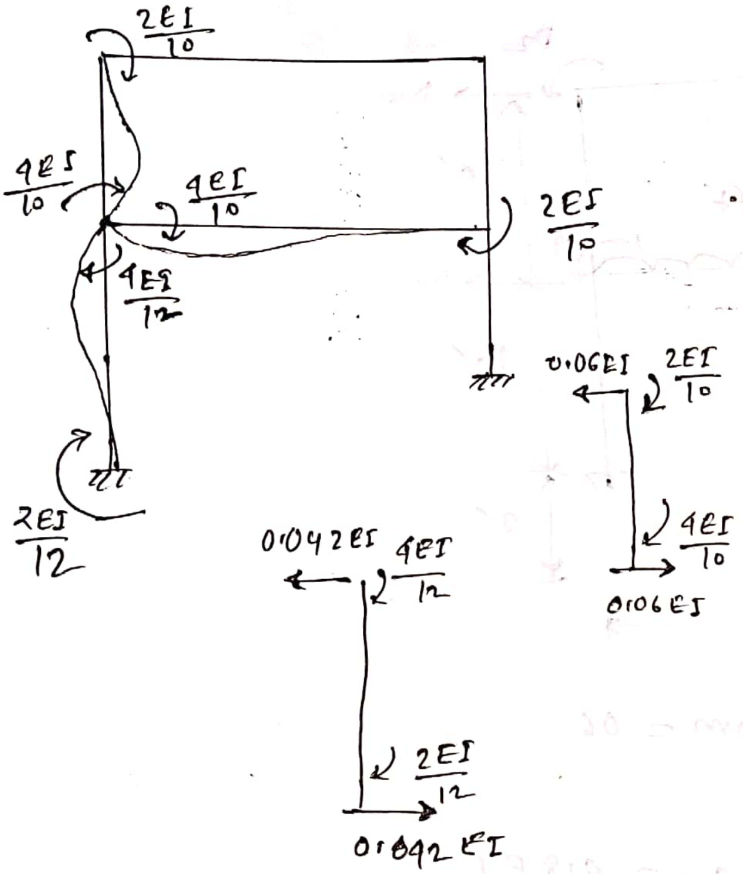
$S_{52} = -0.06 EI$

$S_{62} = 0.06 EI$





when $D_3 = 1$, all are zero:



$S_{31} = 0.2 EI$

$S_{32} = 0$

$S_{33} = 1.13 EI$

$S_{43} = 0.2 EI$

$S_{53} = -0.06 EI$

$S_{63} = (0.06 - 0.042) EI = 0.018 EI$;

when $D_4 = 1$, all are zero:

$S_{41} = 0$

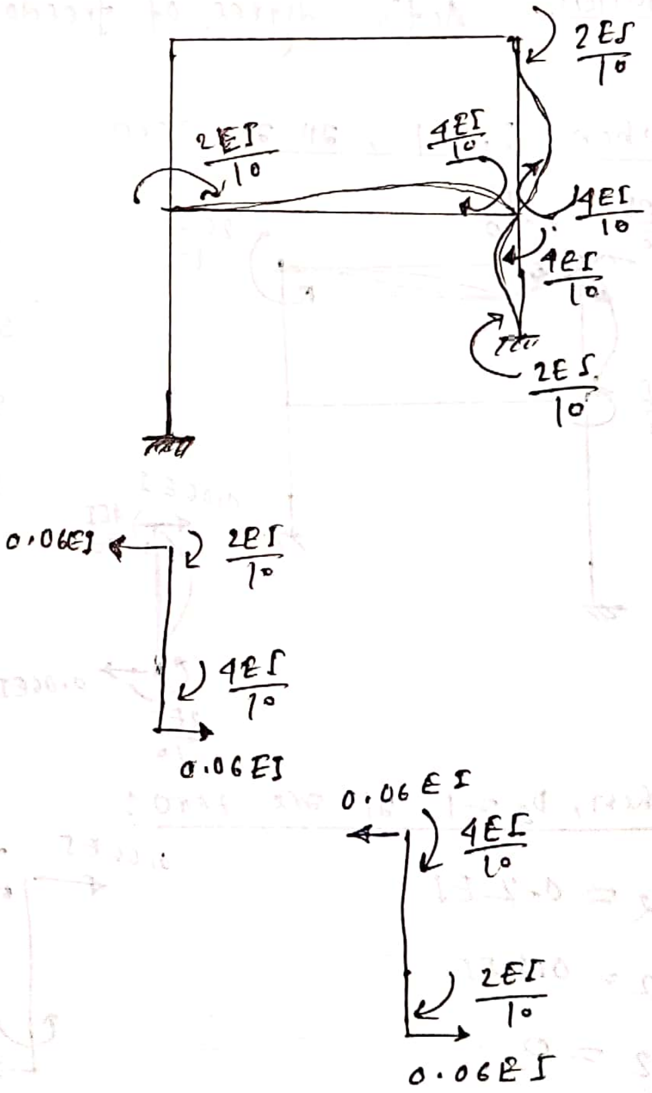
$S_{42} = 0.2 EI$

$S_{43} = 0.2 EI$

$S_{44} = 1.2 EI$

$S_{54} = -0.06 EI$

$S_{64} = 0$



When $D_5 = 1$, all are zero:

$S_{15} = -0.06\ EI$

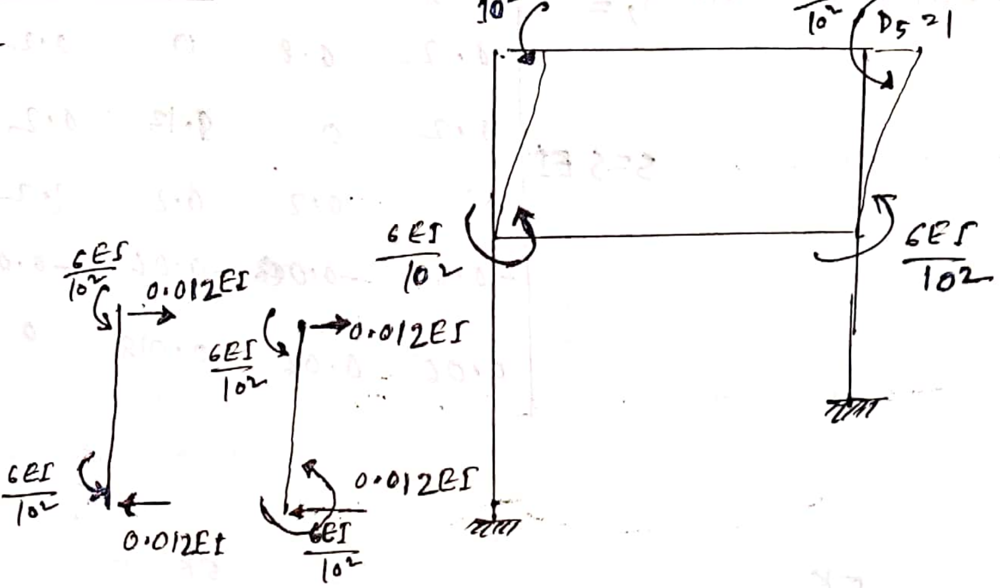
$S_{25} = -0.06EI$

$S_{35} = -0.06EI$

$S_{45} = -0.06\ EI$

$S_{55} = 0.024\ EI$

$S_{65} = -0.024\ EI$



When $D_6 = 1$, all are zero:

$S_{16} = 0.06\ EI$

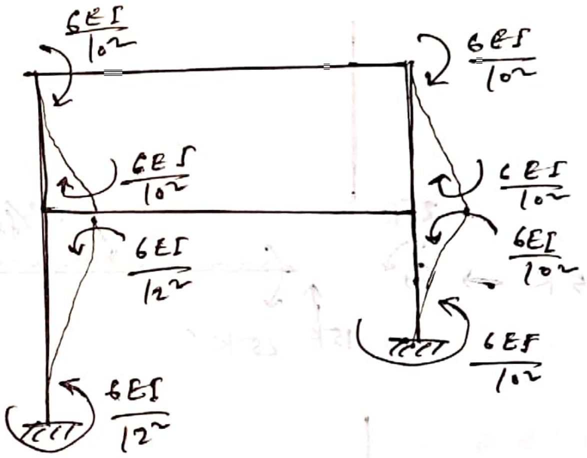
$S_{26} = 0.06\ EI$

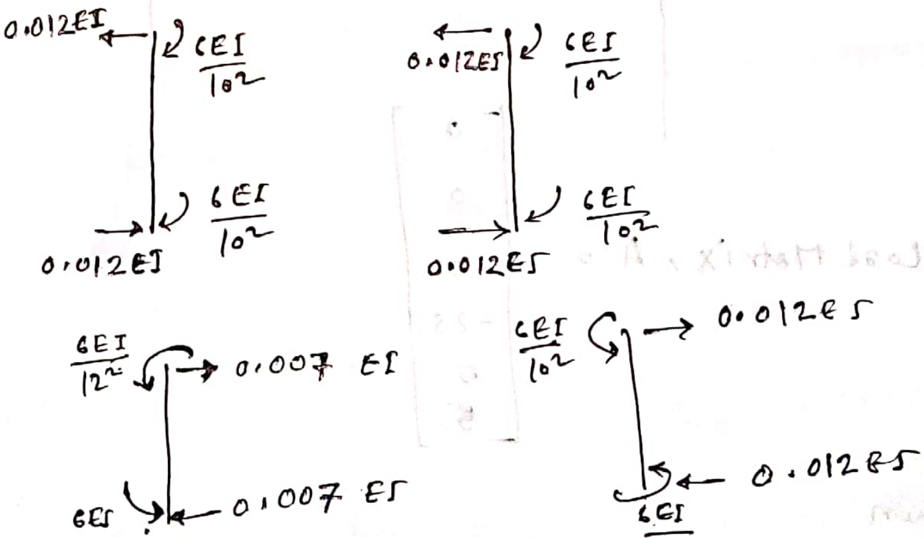
$S_{36} = 0.018\ EI$

$S_{46} = 0$

$S_{56} = -0.024\ EI$

$S_{66} = 0.043\ EI$

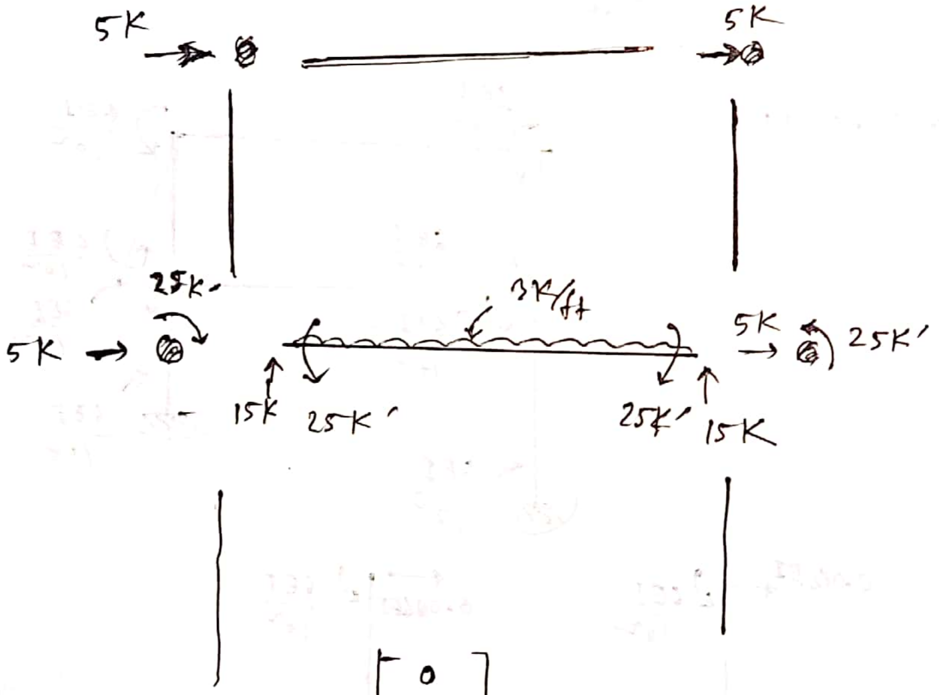




2017 #

∴ Stiffness Matrix,

$$S = EI \begin{bmatrix} 0.8 & 0.2 & 0.2 & 0 & -0.06 & 0.06 \\ 0.2 & 0.8 & 0 & 0.2 & -0.06 & -0.06 \\ 0.2 & 0 & 1.13 & 0.2 & -0.06 & 0.018 \\ 0 & 0.2 & 0.2 & 1.2 & -0.06 & 0 \\ -0.06 & -0.06 & -0.06 & -0.06 & 0.024 & -0.024 \\ 0.06 & 0.06 & 0.018 & 0 & -0.024 & 0.043 \end{bmatrix}$$



Load Matrix, $A = \begin{bmatrix} 0 \\ 0 \\ 25 \\ -25 \\ 5 \\ 5 \end{bmatrix}$

Then

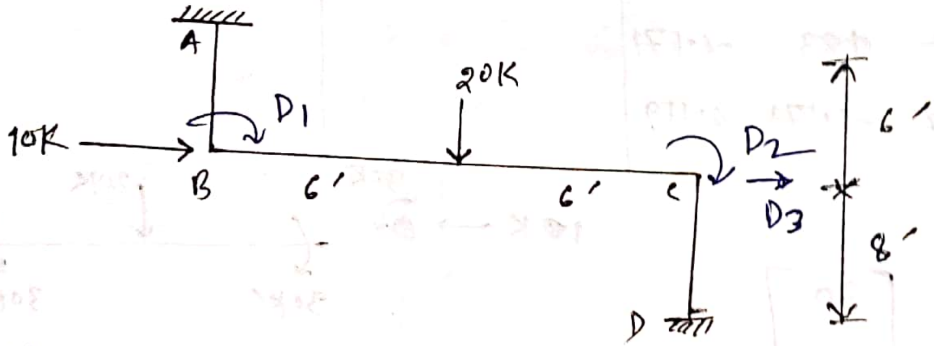
$[A] = [S][D]$

$\Rightarrow [D] = [S^{-1}][A]$

And, $M = FEM + [S][D]$

2017

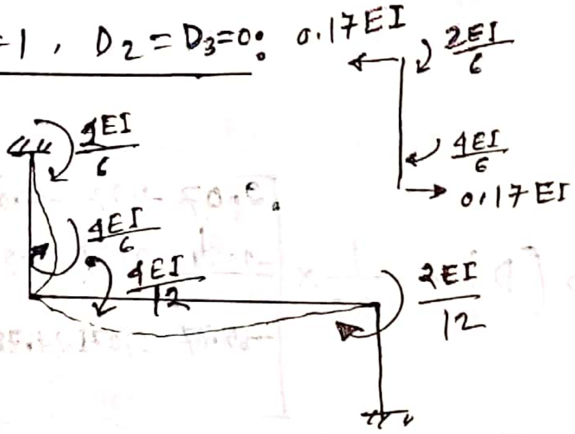
\#



Solution:

Active degree of freedom = 03

When $D_1 = 1$, $D_2 = D_3 = 0$:

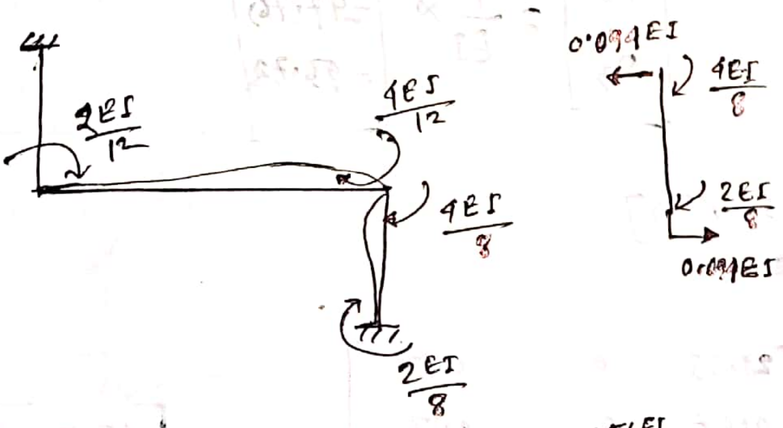


$S_{11} = EI$

$S_{21} = 0.17\ EI$

$S_{31} = 0.17\ EI$

When $D_2 = 1$, $D_1 = D_3 = 0$:

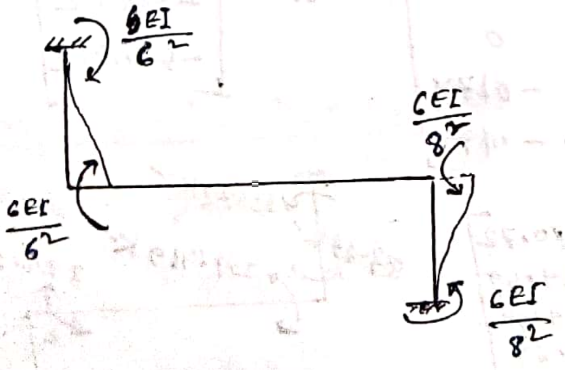


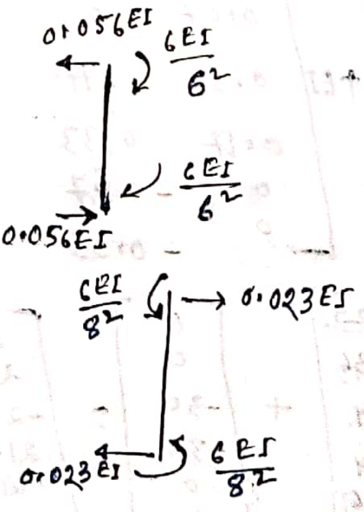
$S_{12} = 0.17\ EI$

$S_{22} = 0.83\ EI$

$S_{32} = -0.094\ EI$

When $D_3 = 1$, $D_1 = D_2 = 0$





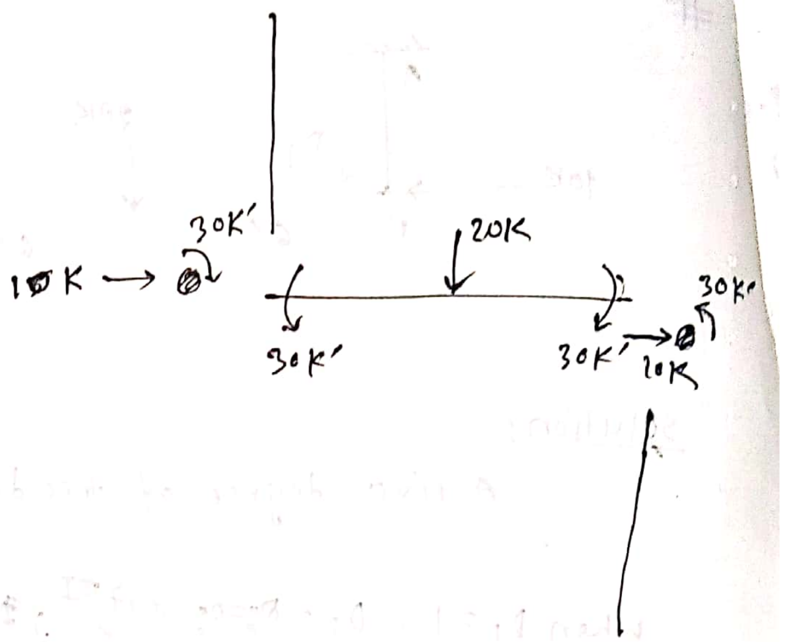
$\therefore S_{13} = 0.17\ EI$

$S_{23} = -0.094\ EI$

$S_{33} = 0879\ EI$

∴ Stiffness matrix.

$$S = EI\begin{bmatrix} I & 0.17 & 0.17 \\ 0.17 & 0.83 & -0.094 \\ 0.17 & -0.094 & 0.079 \end{bmatrix}$$



load Matrix, $A = \begin{bmatrix} 30 \\ -30 \\ 10 \end{bmatrix}$

we know,

$$[A] = [S][D]$$

$$\Rightarrow [D] = [S^{-1}][A] \Rightarrow [D] = \frac{1}{EI} \times \begin{bmatrix} 2.27 & -1.17 & -6.28 \\ -1.17 & 2 & 4.91 \\ -6.28 & 4.91 & 32 \end{bmatrix} \times \begin{bmatrix} 30 \\ -30 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI} \times \begin{bmatrix} 40.49 \\ -46.2 \\ -15.52 \end{bmatrix}$$

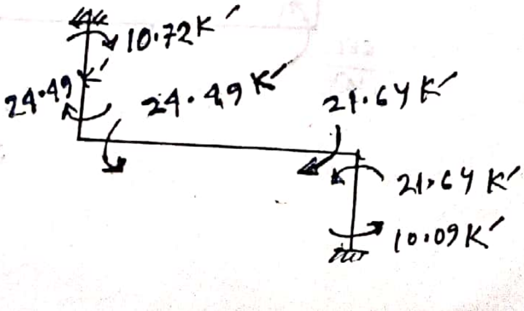
Then,

we know,

$$[M] = FEM + [S] \times [D]$$

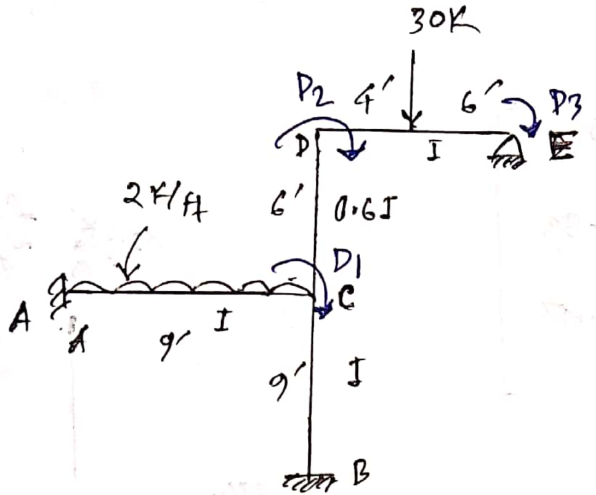
$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -30 \\ 30 \\ 0 \\ 0 \end{bmatrix} + EI\begin{bmatrix} 0.33 & 0 & 0.17 \\ 0.67 & 0 & 0.17 \\ 0.33 & 0.17 & 0 \\ 0.17 & 0.33 & 0 \\ 0 & 0.5 & -0.094 \\ 0 & 0.25 & -.094 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} 40.49 \\ -46.2 \\ -15.52 \end{bmatrix}$$

$$[M] = \begin{bmatrix} 10.72 \\ 24.49 \\ 5.51 \\ -8.36 \\ -21.64 \\ -10.09 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -30 \\ 30 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10.72 \\ 24.49 \\ -24.49 \\ 21.64 \\ -21.64 \\ -10.09 \end{bmatrix}$$



2015

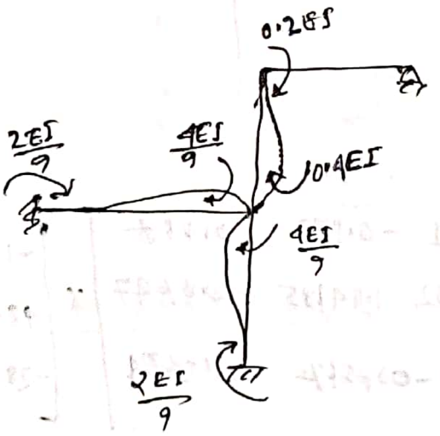
#



Solution:

Active degree of freedom = 03 Nos.

when $D_1 = 1$, $D_2 = D_3 = 0$:

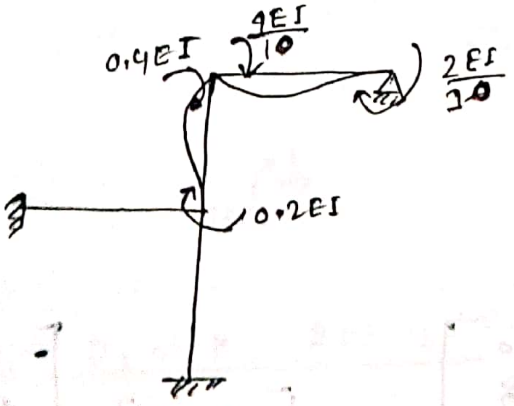


$s_{11} = 1.29$ EI

$s_{21} = 0.2$ EI

$s_{31} = 0$ EI

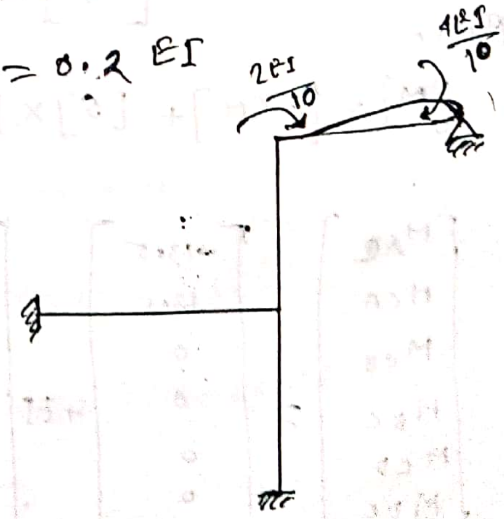
when $D_2 = 1$, $D_1 = D_3 = 0$:



$s_{12} = 0.2$ EI

$s_{22} = 0.8$ EI

$s_{32} = 0.2$ EI



when $D_3 = 1$, $D_1 = D_2 = 0$:

$s_{3} = 0$ EI

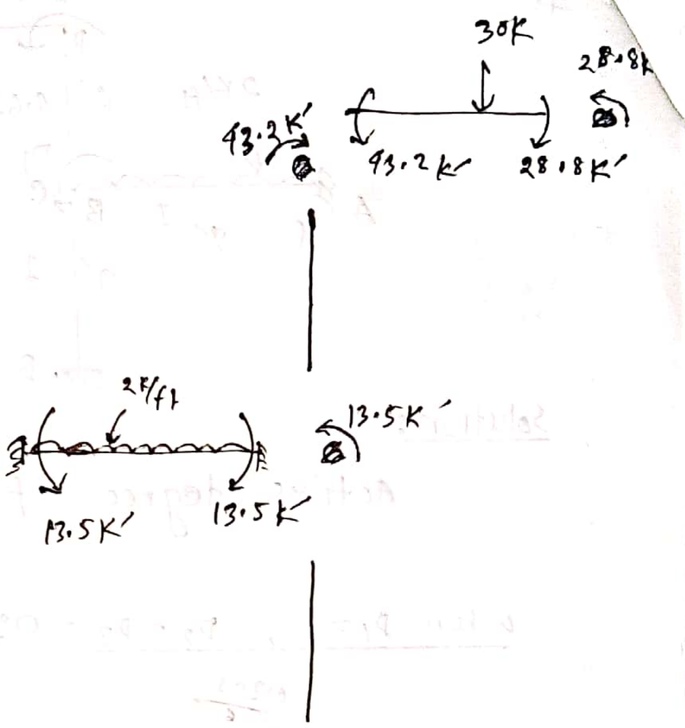
$s_{23} = 0.2$ EI

$s_{33} = 0.4$ EI

2014 #

Stiffness Matrix,

$$S = EI\begin{bmatrix} 1.29 & 0.2 & 0 \\ 0.2 & 0.8 & 0.2 \\ 0 & 0.2 & 0.4 \end{bmatrix}$$



load Matrix,

$$A = \begin{bmatrix} -13.5 \\ 43.2 \\ -28.8 \end{bmatrix}$$

Now,

$$[A] = [S]\,[D]$$

$$[D] = [S^{-1}]\,[A] \Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0.811 & -0.232 & 0.116 \\ -0.232 & 1.495 & -0.747 \\ 0.116 & -0.747 & 2.874 \end{bmatrix} \times \begin{bmatrix} -13.5 \\ 43.2 \\ -28.8 \end{bmatrix}$$

$$\therefore \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -24.3 \\ 89.23 \\ -116.6 \end{bmatrix}$$

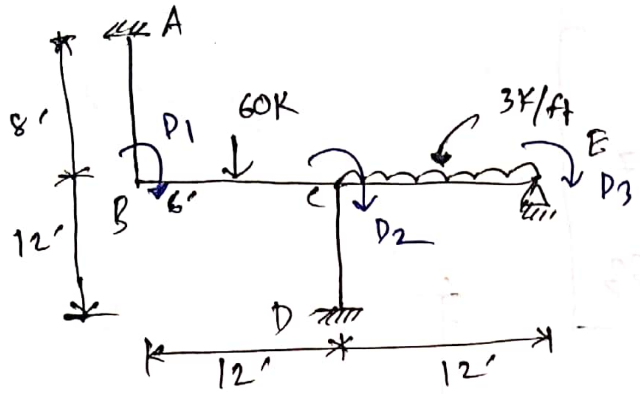
Then,

$$[M] = [FEM] + [S] \times [D]$$

$$\begin{bmatrix} M_{AC} \\ M_{CA} \\ M_{CB} \\ M_{BC} \\ M_{CD} \\ M_{DC} \\ M_{DE} \\ M_{ED} \end{bmatrix} = \begin{bmatrix} -13.5 \\ 13.5 \\ 0 \\ 0 \\ 0 \\ 0 \\ -43.2 \\ 28.8 \end{bmatrix} + EI\begin{bmatrix} 0.22 & 0 & 0 \\ 0.44 & 0 & 0 \\ 0.44 & 0 & 0 \\ 0.22 & 0 & 0 \\ 0.4 & 0.2 & 0 \\ 0.2 & 0.4 & 0 \\ 0 & 0.4 & 0.2 \\ 0 & 0.2 & 0.4 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} -24.3 \\ 89.23 \\ -116.6 \end{bmatrix} = \begin{bmatrix} -18.85 \\ 2.81 \\ -10.69 \\ -5.35 \\ 8.12 \\ 30.82 \\ -30.84 \\ 0 \end{bmatrix}$$

2014

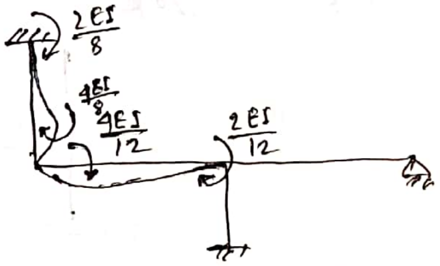
#



Solution:

Active degree of freedom = 03 Nos.

When $D_1 = 1K$, $D_2 = D_3 = 0$:

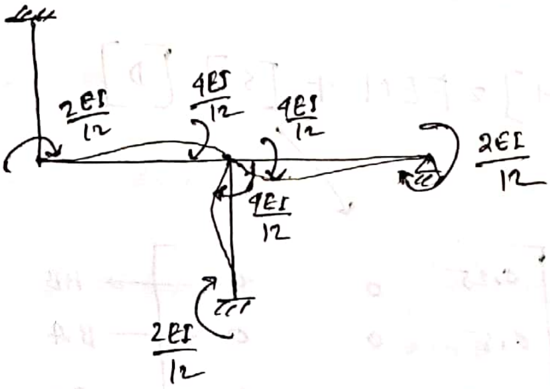


$S_{11} = 0.83\ EI$

$S_{21} = 0.17\ EI$

$S_{31} = 0$

When $D_2 = 1K$, $D_1 = D_3 = 0$:

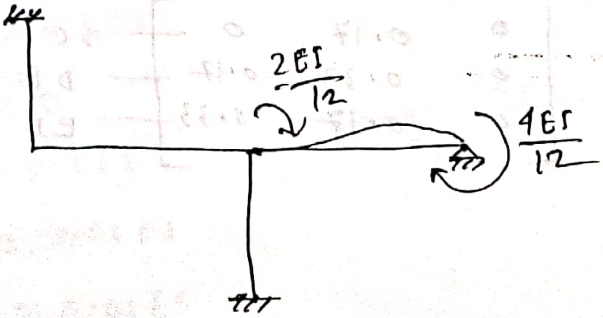


$S_{12} = 0.17\ EI$

$S_{22} = EI$

$S_{32} = 0.17 EI$

When $D_3 = 1K$, $D_1 = D_2 = 0$:



$S_{13} = 0$

$S_{23} = 0.17EI$

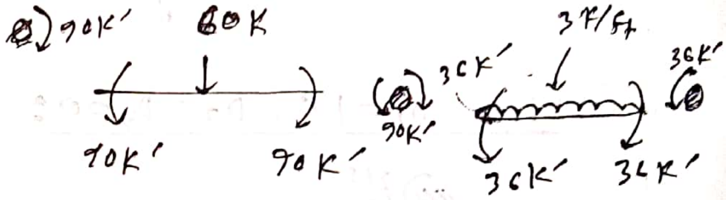
$S_{33} = 0.33\ EI$

∴ Stiffness Matrix,

$$S = EI\begin{bmatrix} 0.83 & 0.17 & 0 \\ 0.17 & 1 & 0.17 \\ 0 & 0.17 & 0.33 \end{bmatrix}$$

∴ Load Matrix,

$$A = \begin{bmatrix} 90 \\ -54 \\ -36 \end{bmatrix}$$



Then, (do yourself) $[A] = [S][D]$

$$[D] = [S]^{-1}[A]$$

And then, $[M] = FEM + [S][D]$

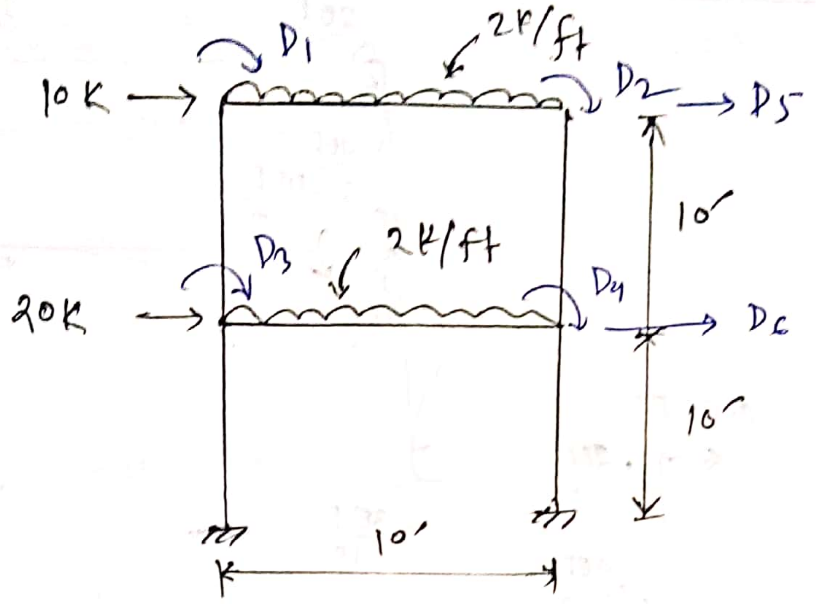
$$S = EI\begin{array}{c|l} \begin{bmatrix} 0.25 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0.33 & 0.17 & 0 \\ 0.17 & 0.33 & 0 \\ 0 & 0.33 & 0 \\ 0 & 0.17 & 0 \\ 0 & 0.33 & 0.17 \\ 0 & 0.17 & 0.33 \end{bmatrix} & \begin{array}{l} \rightarrow AB \\ BA \\ BC \\ CB \\ CD \\ DC \\ DE \\ ED \end{array} \end{array}$$

Answer

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \\ M_{DE} \\ M_{ED} \end{bmatrix} = \begin{bmatrix} 30.25 \\ 60.51 \\ -60.51 \\ 90.03 \\ -20.26 \\ -10.43 \\ -69.42 \\ 0 \end{bmatrix}$$

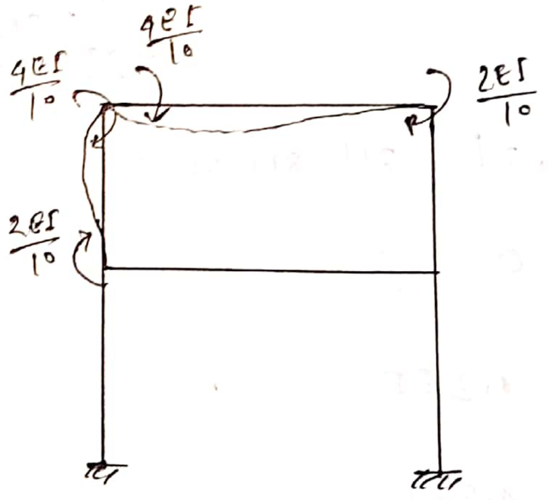
2014, 2013

#



Solution: Active degree of freedom = 06 Nos.

when $D_1 = 1$, all are zero:



$S_{11} = 0.8EI$

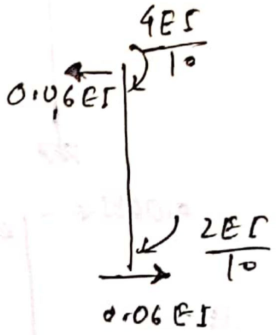
$S_{21} = 0.2EI$

$S_{31} = 0.2EI$

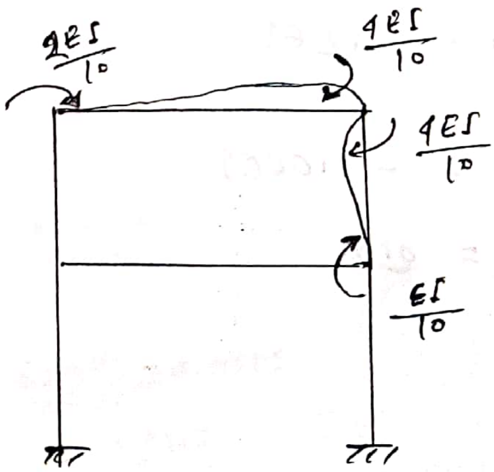
$S_{41} = 0$

$S_{51} = -0.06EI$

$S_{61} = 0.06EI$



when $D_2 = 1$, all are zero:



$S_{21} = 0.2EI$

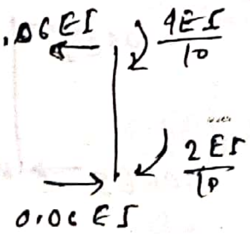
$S_{22} = 0.8EI$

$S_{32} = 0$

$S_{42} = 0.2EI$

$S_{52} = -0.06EI$

$S_{62} = 0.06EI$



When $D_3 = 1$, all are zero;

$S_{13} = 0.2EI$

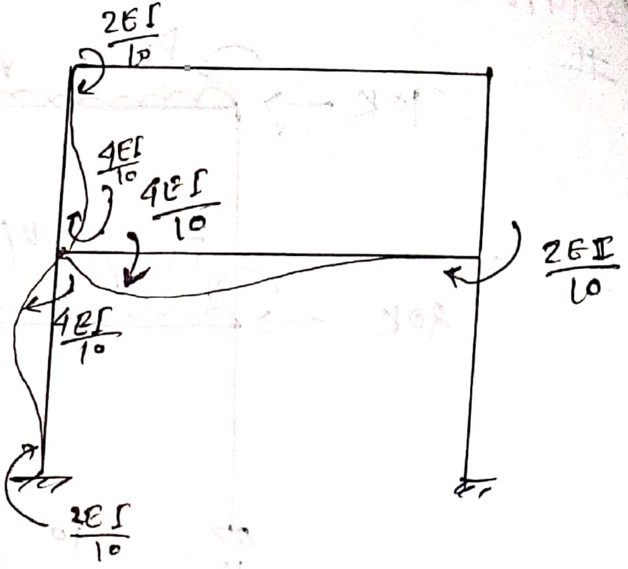
$S_{23} = 0$

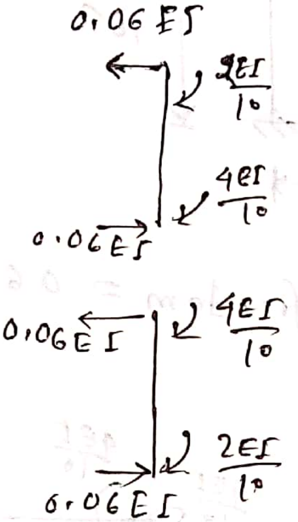
$S_{33} = 1.2EI$

$S_{43} = 0.2EI$

$S_{53} = -0.06EI$

$S_{63} = 0$





When $D_4 = 1$, all are zero;

$S_{14} = 0$

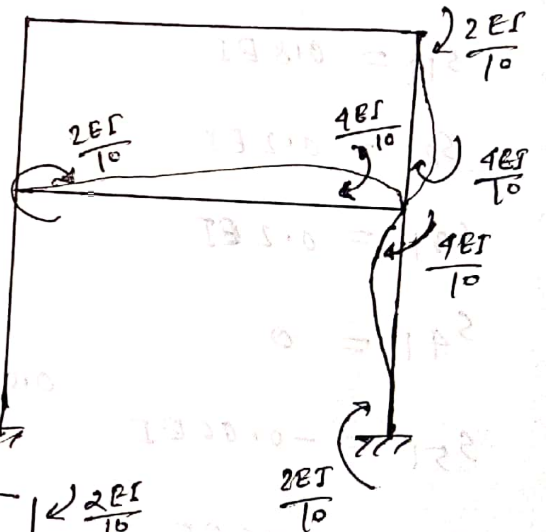
$S_{24} = 0.2EI$

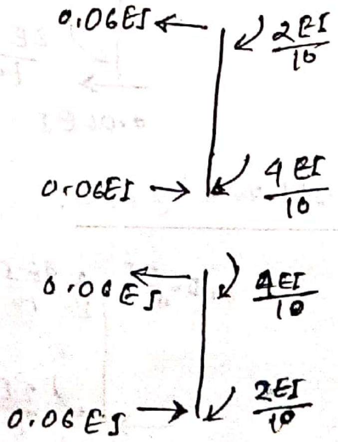
$S_{34} = 0.2EI$

$S_{44} = 1.2EI$

$S_{54} = -0.06EI$

$S_{64} = 0$





when $D_5 = 1$, all are zero:

$S_{15} = -0.06EI$

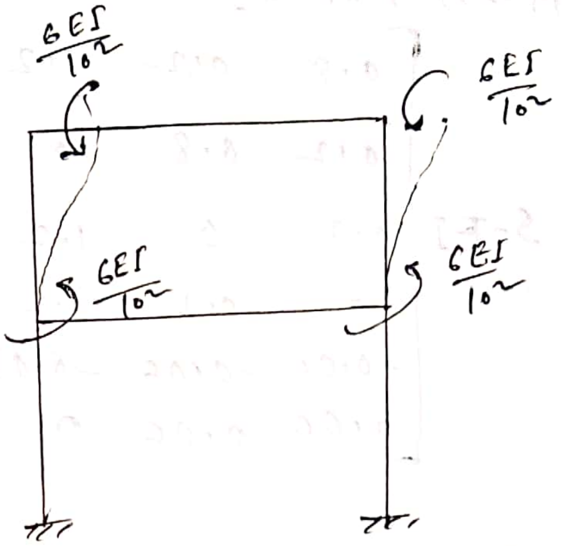
$S_{25} = -0.06EI$

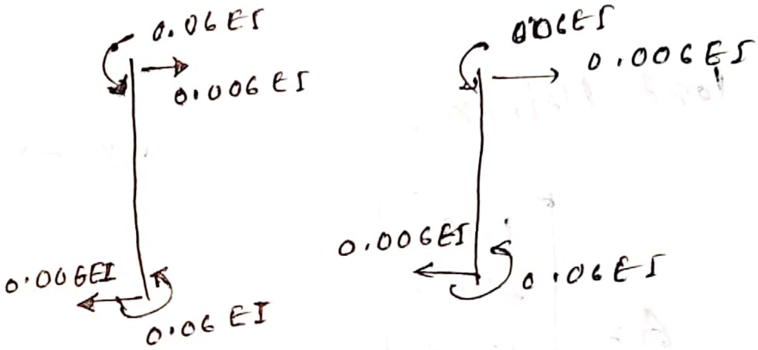
$S_{35} = -0.06EI$

$S_{45} = -0.06EI$

$S_{55} = 0.012EI$

$S_{65} = -0.012EI$





when $D_6 = 1$, all are zero:

$S_{16} = 0.06EI$

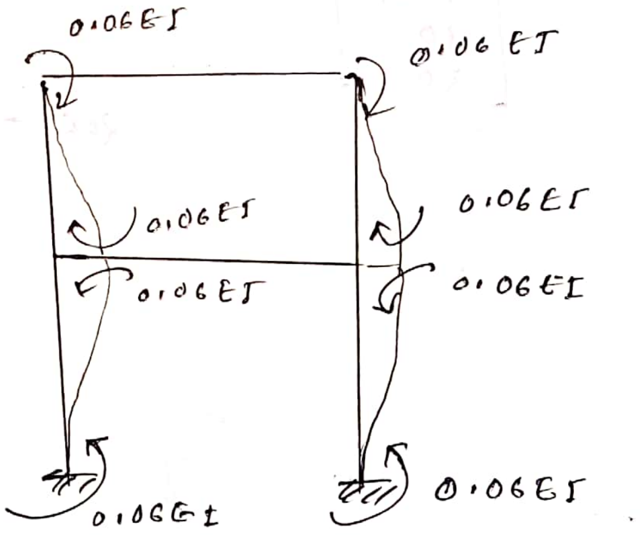
$S_{26} = 0.06EI$

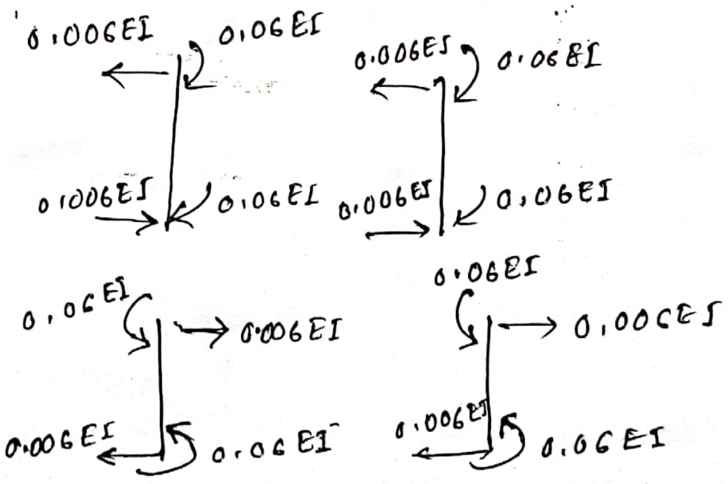
$S_{36} = 0$

$S_{46} = 0$

$S_{56} = -0.012EI$

$S_{66} = 0.024EI$





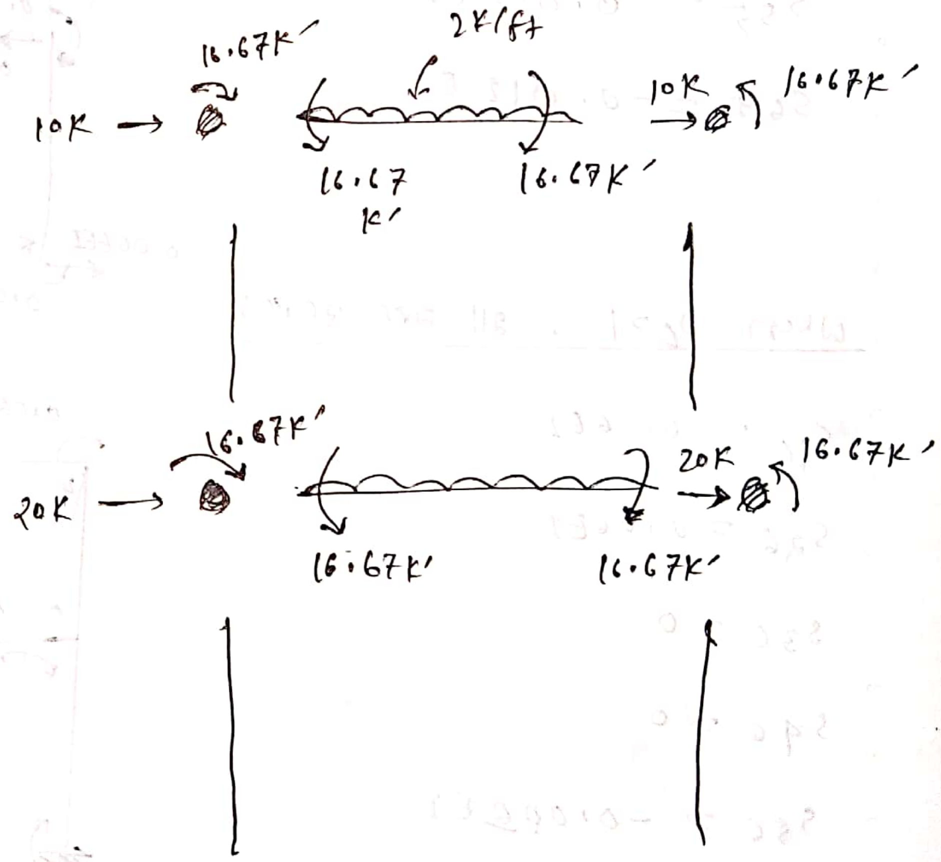
20. #

stiffness Matrix,

$$S = EI\begin{bmatrix} 0.8 & 0.2 & 0.2 & 0 & -0.06 & 0.06 \\ 0.2 & 0.8 & 0 & 0.2 & -0.06 & 0.06 \\ 0.2 & 0 & 1.2 & 0.2 & -0.06 & 0 \\ 0 & 0.2 & 0.2 & 1.2 & -0.06 & 0 \\ -0.06 & -0.06 & -0.06 & -0.06 & 0.012 & -0.012 \\ 0.06 & 0.06 & 0 & 0 & -0.012 & 0.024 \end{bmatrix}$$

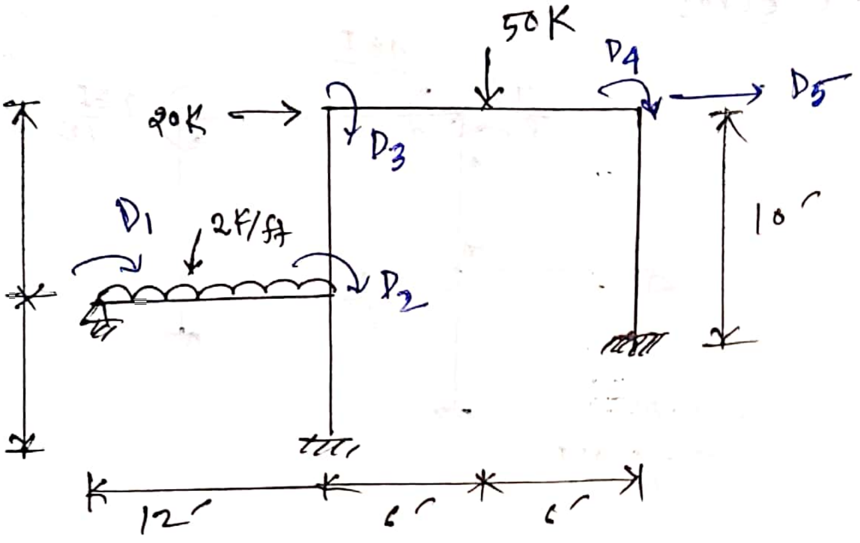
load Matrix,

$$A = \begin{bmatrix} 16.67 \\ -16.67 \\ 16.67 \\ -16.67 \\ 10 \\ 20 \end{bmatrix}$$



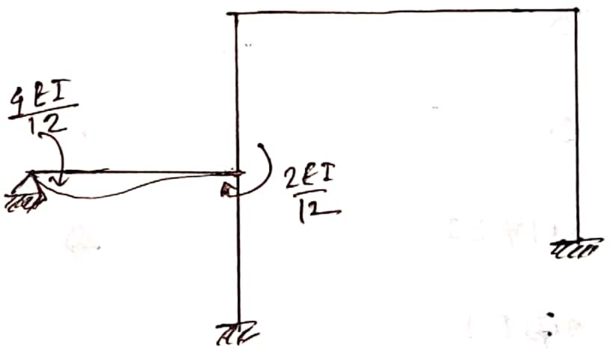
2012

#



Solution: Active degree of freedom = 04 Nos.

when, $D_1 = 1$, all are zero:



$S_{11} = 0.33\ EI$

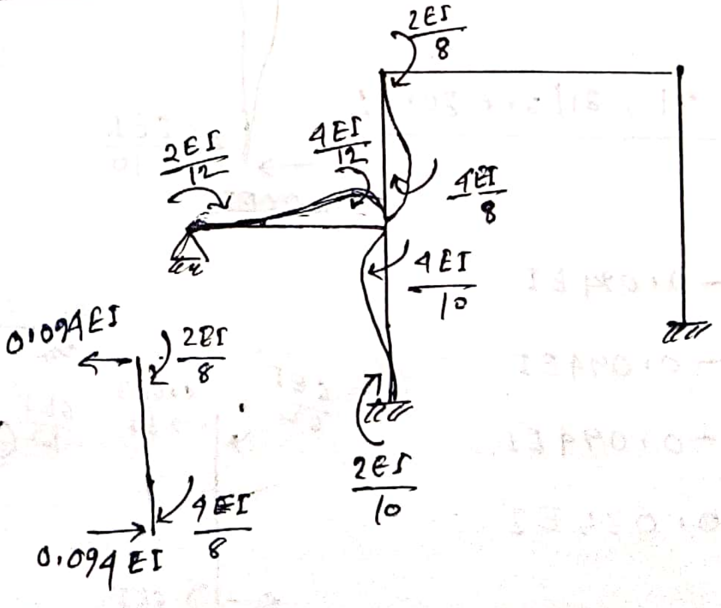
$S_{21} = 0.17\ EI$

$S_{31} = 0$

$S_{41} = 0$

$S_{51} = 0$

when $D_2 = 1$, all are zero:



$S_{12} = 0.17 EI$

$S_{22} = 1.23\ EI$

$S_{32} = 0.25\ EI$

$S_{42} = 0$

$S_{52} = -0.094\ EI$

when $D_3 = 1$, all are zero:

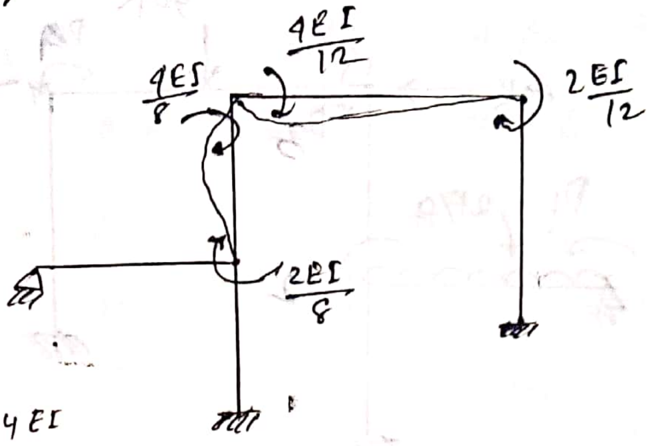
$S_{13} = 0$

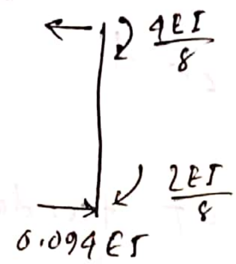
$S_{23} = 0.125 EI$

$S_{33} = 0.83 EI$

$S_{43} = 0.17 EI$

$S_{53} = -0.094 EI$





when $D_4 = 1$, all are zero:

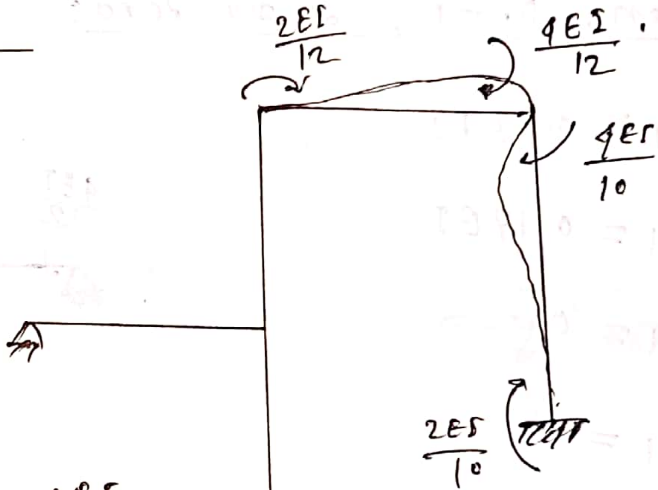
$S_{14} = 0$

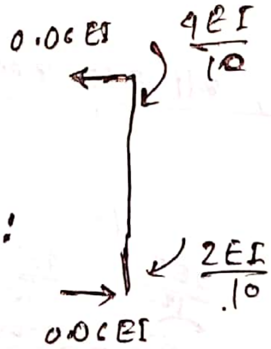
$S_{24} = 0$

$S_{34} = 0.17 EI$

$S_{44} = 0.73 EI$

$S_{54} = -0.06 EI$





when $D_5 = 1$, all are zero:

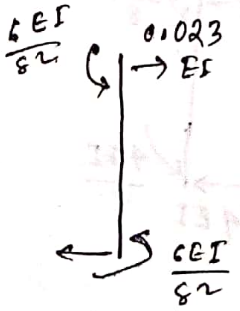
$S_{15} = 0$

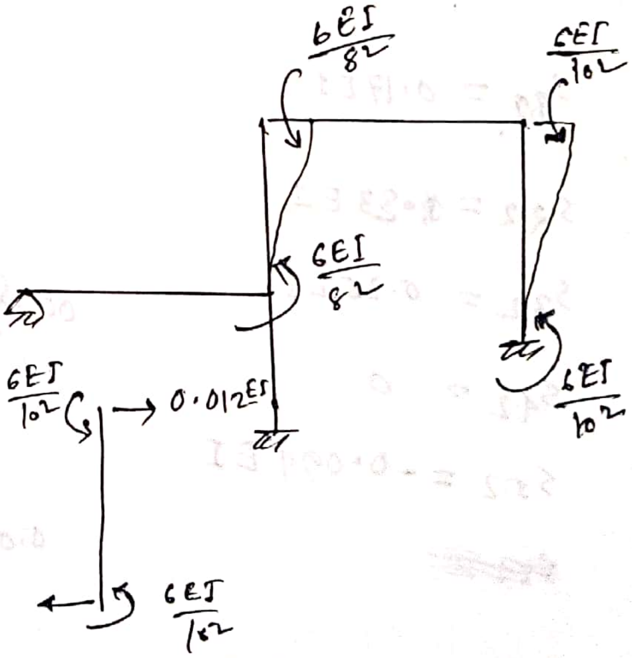
$S_{25} = -0.094 EI$

$S_{35} = -0.094 EI$

$S_{45} = -0.06 EI$

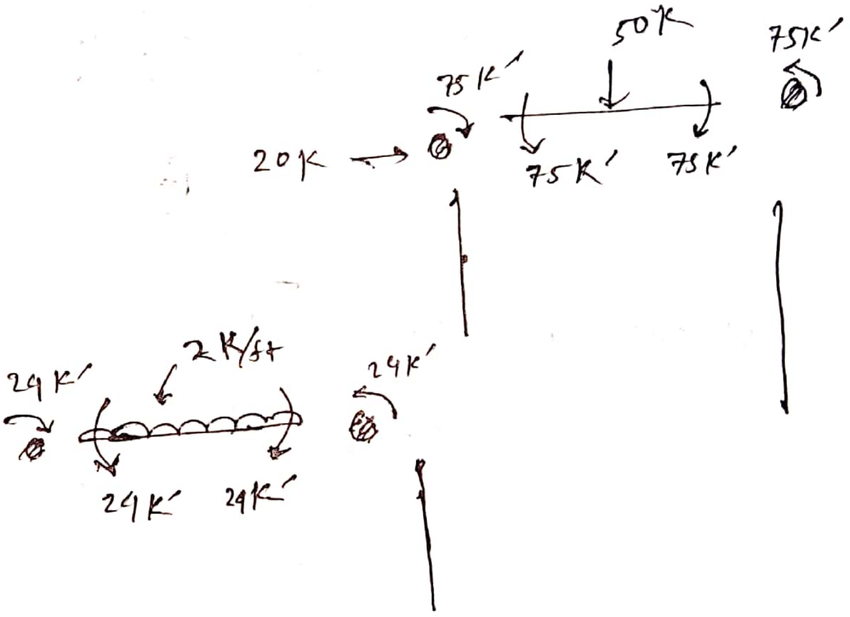
$S_{55} = 0.035 EI$





stiffness matrix,

$$S = EI\begin{bmatrix} 0.33 & 0.17 & 0 & 0 & 0 \\ 0.17 & 1.23 & 0.25 & 0 & -0.094 \\ 0 & 0.25 & 0.83 & 0.17 & -0.094 \\ 0 & 0 & 0.17 & 0.73 & -0.06 \\ 0 & -0.094 & -0.094 & -0.06 & 0.035 \end{bmatrix}$$

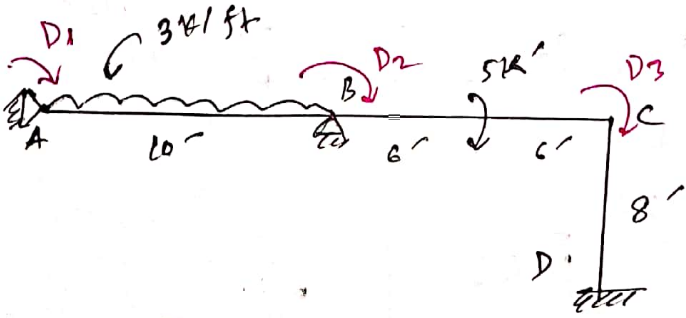


load Matrix,

$$A = \begin{bmatrix} 24 \\ -24 \\ 75 \\ -75 \\ 20 \end{bmatrix}$$

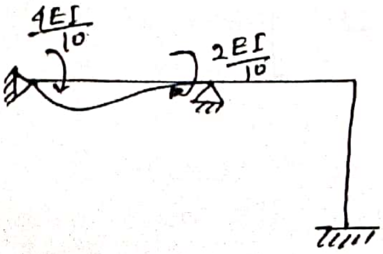
2016

#



Solution: degree of Freedom = 03

when $D_1 = 1$:



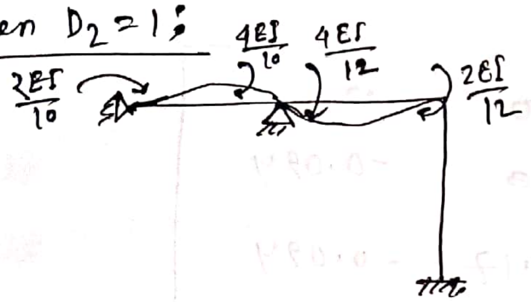
$S_{11} = 0.4EI$

$S_{21} = 0.2EI$

$S_{31} = 0$

Scanned by CamScanner

When $D_2 = 1$:

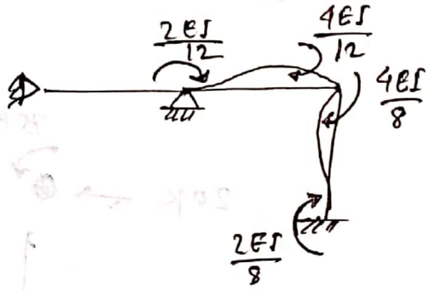


$S_{12} = 0.2EI$

$S_{22} = 0.733\ EI$

$S_{32} = 0.167\ EI$

When $D_3 = 1$:



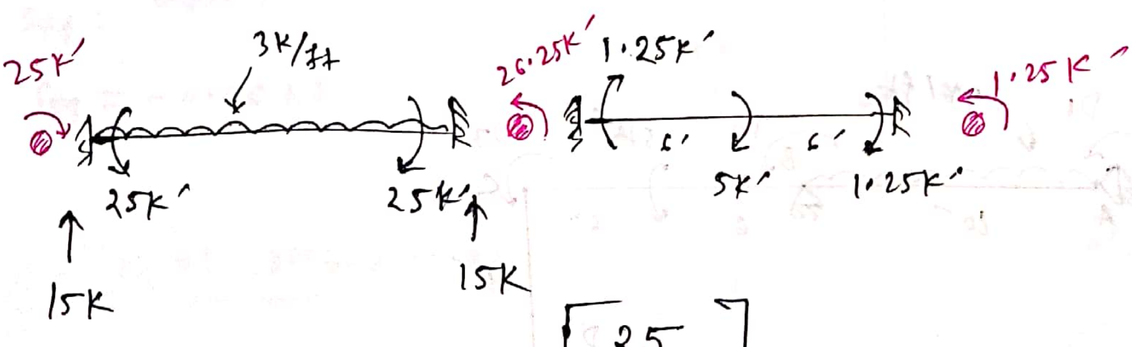
$S_{13} = 0$

$S_{23} = 0.167\ EI$

$S_{33} = 0.833\ EI$

Stiffness Matrix,

$$S = EI \begin{bmatrix} 0.4 & 0.2 & 0 \\ 0.2 & 0.733 & 0.167 \\ 0 & 0.167 & 0.833 \end{bmatrix}$$

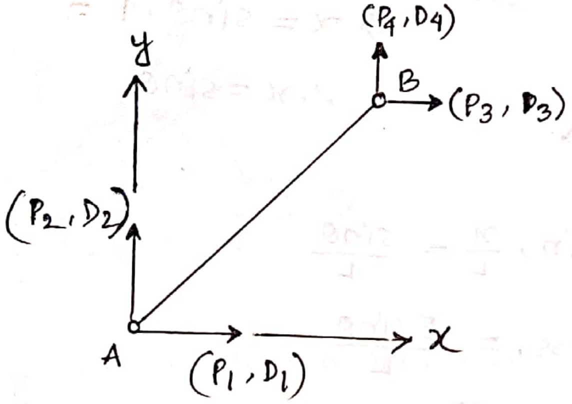


Load Matrix, $A = \begin{bmatrix} 25 \\ -26.25 \\ -1.25 \end{bmatrix}$

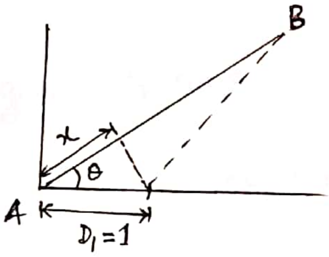
TRUSS (Stiffness Matrix Method)

2015

Stiffness Matrix for a truss element. having inclination angle θ with the x- axis.



Case-1: $D_1 = 1$ & $D_2 = D_3 = D_4 = 0$



$$\cos\theta = \frac{x}{D_1}$$

$$\Rightarrow x = \cos\theta \cdot 1$$

$$\therefore x = \cos\theta$$

Along the member,

$$\text{Strain} = \frac{x}{L} = \frac{\cos\theta}{L}$$

$$\text{Stress} = \text{Strain} \times E = \frac{E\cos\theta}{L}$$

$$\text{Force} = \text{Area} \times \text{stress}$$

$$= A \cdot \frac{E\cos\theta}{L} = \frac{AE\cos\theta}{L}$$

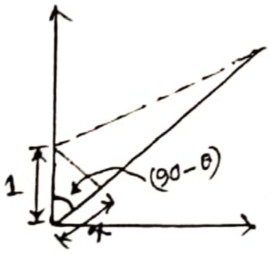
$$S_{11} = \left(\frac{AE\cos\theta}{L}\right)\cos\theta = \frac{AE}{L}\cos^2\theta$$

$$S_{21} = \frac{AE}{L}\cos\theta \cdot \sin\theta$$

$$S_{31} = -\frac{AE}{L}\cos^2\theta$$

$$S_{41} = -\frac{AE}{L}\cos\theta \cdot \sin\theta$$

Case-2: $D_2 = 1$ & $D_1 = D_3 = D_4 = 0$



$$\cos(90-\theta) = \frac{x}{D_2}$$

$$\Rightarrow x = \sin\theta \cdot 1$$

$$\therefore x = \sin\theta$$

Along the member, Strain, $\frac{x}{L} = \frac{\sin\theta}{L}$

$$\text{Stress} = \frac{E\sin\theta}{L}$$

$$\text{Force} = \frac{AE}{L}\sin\theta$$

$$S_{12} = \frac{AE}{L}\sin\theta \cdot \cos\theta$$

$$S_{22} = \frac{AE}{L}\sin^2\theta$$

$$S_{32} = -\frac{AE}{L}\sin\theta \cdot \cos\theta$$

$$S_{42} = -\frac{AE}{L}\sin^2\theta$$

Similarly, $\boxed{\text{Case-3 \& Case-4}}$ Here, $P_1 = -P_3$ & $P_2 = -P_4$

Thus, we obtain,

Case-3:

$$S_{13} = -\frac{AE}{L}\cos^2\theta$$

$$S_{23} = -\frac{AE}{L}\cos\theta \cdot \sin\theta$$

$$S_{33} = \frac{AE}{L}\cos^2\theta$$

$$S_{44} = \frac{AE}{L}\cos\theta \cdot \sin\theta$$

Case-1:

$$S_{14} = -\frac{AE}{L}\sin\theta \cdot \cos\theta$$

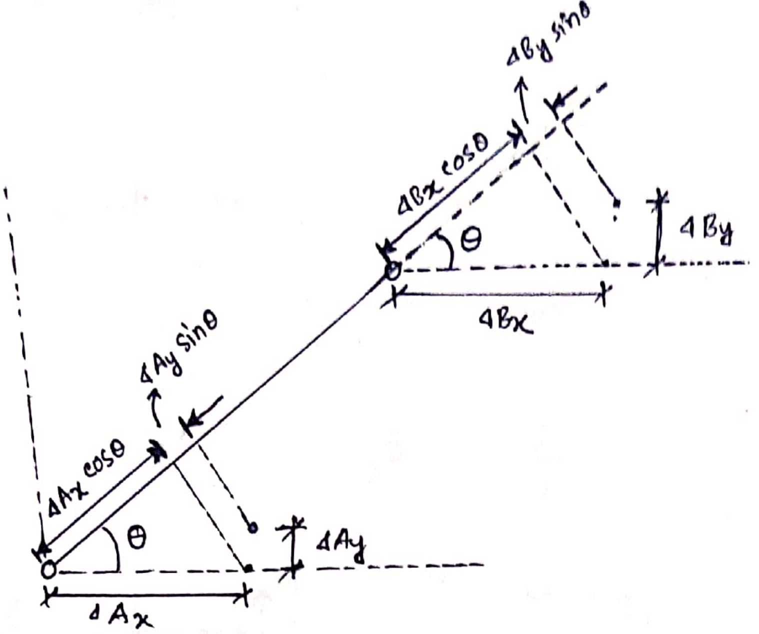
$$S_{24} = -\frac{AE}{L}\sin^2\theta$$

$$S_{34} = \frac{AE}{L}\sin\theta \cdot \cos\theta$$

$$S_{44} = \frac{AE}{L}\sin^2\theta$$

$\therefore$ Stiffness Matrix

$$S = \frac{AE}{L}\begin{bmatrix} \cos^2\theta & \sin\theta\cdot\cos\theta & -\cos^2\theta & -\sin\theta\cdot\cos\theta \\ \cos\theta\cdot\sin\theta & \sin^2\theta & -\cos\theta\cdot\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\sin\theta\cdot\cos\theta & \cos^2\theta & \sin\theta\cdot\cos\theta \\ -\cos\theta\cdot\sin\theta & -\sin^2\theta & \cos\theta\cdot\sin\theta & \sin^2\theta \end{bmatrix}$$



Shortening due to the displacement of A = $\Delta A_x \cos\theta + \Delta A_y \sin\theta$

Elongation due to the displacement of B = $\Delta B_x \cos\theta + \Delta B_y \sin\theta$

Net shortening = $\Delta A_x \cos\theta + \Delta A_y \sin\theta - \Delta B_x \cos\theta - \Delta B_y \sin\theta$

$= (\Delta A_x - \Delta B_x) \cos\theta + (\Delta A_y - \Delta B_y) \sin\theta$

Net force on the member = stress × Area

$= (\text{Strain} \times E) \times A$

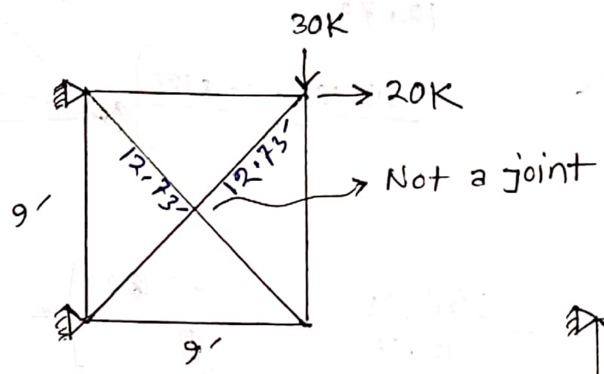
$= AE \times \left[- \frac{\text{shortening}}{L} \right]$

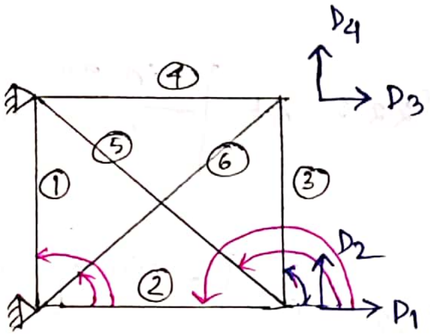
$= -\frac{AE}{L} \left[(\Delta A_x - \Delta B_x)\cos\theta + (\Delta A_y - \Delta B_y)\sin\theta \right]$

where, (−ve) sig indicates the shortening.

Problem: 01

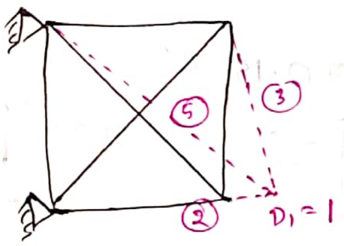
Calculate the member force for the truss. EA is constant





Solution: Degree of freedom = 04

Member	Length	Angle
①	9'	90
②	9'	180
③	9'	90
④	9'	180
⑤	12.73'	135°
⑥	12.73'	45°



when $D_1 = 1K$, all are zero:

$$S_{11} = AE \times \left[\frac{\cos^2 90}{9} + \frac{\cos^2 135}{12.73} + \frac{\cos^2 180}{9} \right]$$

$$= 0.15\ AE$$

$$S_{21} = AE \times \left[\frac{\cos 90 \cdot \sin 90}{9} + \frac{\cos 135 \cdot \sin 135}{12.73} + \frac{\cos 180 \cdot \sin 180}{9} \right]$$

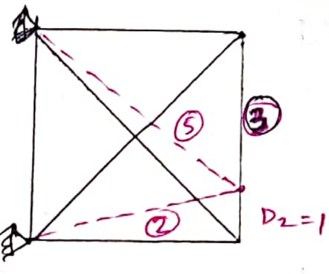
$$= -0.04\ AE$$

$$S_{31} = AE \times \left[-\frac{\cos^2 90}{9} \right] = 0$$

$$S_{41} = AE \times \left[-\frac{\cos 90 \cdot \sin 90}{9} \right] = 0$$

$D_2 = 1$, all are zero:

$$S_{12} = AE\left[\frac{\sin 90 \cdot \cos 90}{9} + \frac{\cos 135^\circ \cdot \sin 135^\circ}{12.73} + \frac{\sin 180 \cdot \cos 180}{9}\right]$$

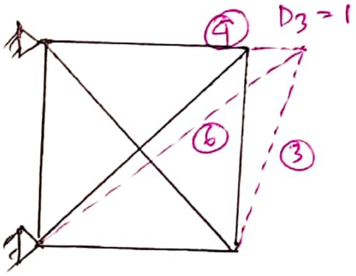


$$= -0.04 AE$$

$$S_{22} = AE\left[\frac{\sin^2 90}{9} + \frac{\sin^2 135}{12.73} + \frac{\sin^2 180}{9}\right] = 0.15 AE$$

$$S_{32} = AE\left[\frac{-\sin 90 \cdot \cos 90}{9}\right] = 0$$

$$S_{42} = AE\left[\frac{-\sin^2 90}{9}\right] = -0.11 AE$$



$D_3 = 1$, all are zero;

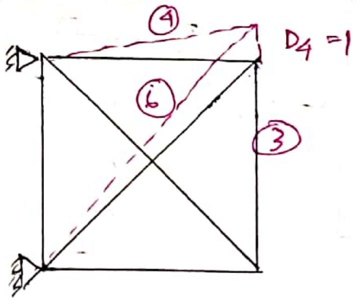
$$S_{13} = \left[\frac{-\cos^2 90}{9}\right] = 0$$

$$S_{23} = \left[\frac{-\cos 90 \cdot \sin 90}{9}\right] = 0$$

$$S_{33} = \left[\frac{\cos^2 90}{9}\right] + \left[\frac{\cos^2 45}{12.73}\right] + \left[\frac{\cos^2 180}{9}\right] = 0.15 AE$$

$$S_{43} = \left[\frac{\cos 90 \cdot \sin 90}{9} + \frac{\cos 45 \cdot \sin 45}{12.73} + \frac{\cos 180 \cdot \sin 180}{9}\right] = 0.04 AE$$

$D_4 = 1$, all are zero:



$$S_{14} = \left[\frac{-\sin 90 \cdot \cos 90}{9} \right] = 0$$

$$S_{24} = \left[-\frac{\sin^2 90}{9} \right] = -0.11AE$$

$$S_{34} = \left[\frac{\sin 90 \cdot \cos 90}{9} + \frac{\sin 45 \cdot \cos 45}{12.73} + \frac{\sin 180 \cdot \cos 180}{9} \right]$$

$$= 0.04AE$$

$$S_{44} = \left[\frac{\sin^2 90}{9} + \frac{\sin^2 45}{12.73} + \frac{\sin^2 180}{9} \right] = 0.15AE$$

Stiffness Matrix,

$$S = AE \begin{bmatrix} 0.15 & -0.04 & 0 & 0 \\ -0.04 & 0.15 & 0 & -0.11 \\ 0 & 0 & 0.15 & 0.04 \\ 0 & -0.11 & 0.04 & 0.15 \end{bmatrix}$$

Force Matrix,

$$A = \begin{bmatrix} 0 \\ 0 \\ 20 \\ -30 \end{bmatrix}$$

$$S^{-1} = \frac{1}{AE} \begin{bmatrix} 8.02 & 5.08 & -1.07 & 4.01 \\ 5.08 & 19.05 & -4.01 & 15.04 \\ -1.07 & -4.01 & 8.02 & -5.08 \\ 4.01 & 15.04 & -5.08 & 19.05 \end{bmatrix}$$

(using calculator)

We know, $[A] = [S][D]$

$$\Rightarrow [D] = [S]^{-1} \times [A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \frac{1}{AE} \times \begin{bmatrix} 8.02 & 5.08 & -1.07 & 4.01 \\ 5.08 & 19.05 & -4.01 & 15.04 \\ -1.07 & -4.01 & 8.02 & -5.08 \\ 4.01 & 15.04 & -5.08 & 19.05 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 20 \\ -30 \end{bmatrix}$$

$$\therefore \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{bmatrix} = \frac{1}{AE}\begin{bmatrix} -141.7 \\ -531.4 \\ 312.8 \\ -673.1 \end{bmatrix}$$

Now, $[F] = [S] \times [D]$

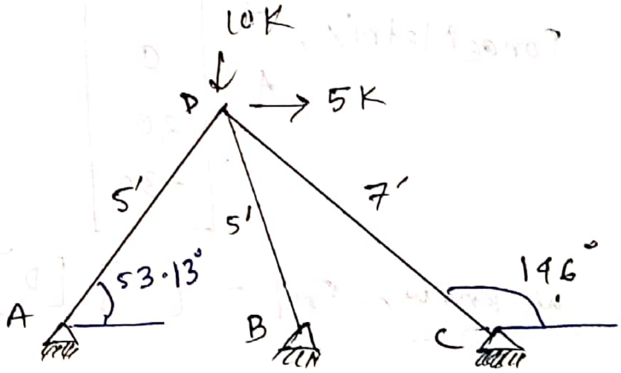
$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = AE \left[\begin{array}{cccc} D_1 = 1 & D_2 = 1 & D_3 = 1 & D_4 = 1 \\ -\frac{\cos\theta}{L} & -\frac{\sin\theta}{L} & -\frac{\cos\theta}{L} & -\frac{\sin\theta}{L} \\ {}^{*}0 & {}^{*}0 & {}^{*}0 & {}^{*}0 \\ 0.111 & 0 & {}^{*}0 & {}^{*}0 \\ 0 & -0.111 & 0 & -0.111 \\ {}^{*}0 & {}^{*}0 & 0.111 & 0 \\ 0.056 & -0.056 & {}^{*}0 & {}^{*}0 \\ {}^{*}0 & {}^{*}0 & -0.056 & -0.056 \end{array}\right] \times \frac{1}{AE}\begin{bmatrix} -141.7 \\ -531.4 \\ 312.4 \\ -673.1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} 0 \\ -15.73 \\ 133.7 \\ 34.68 \\ 21.82 \\ 20.2 \end{bmatrix}$$

(Ans.)

Note:

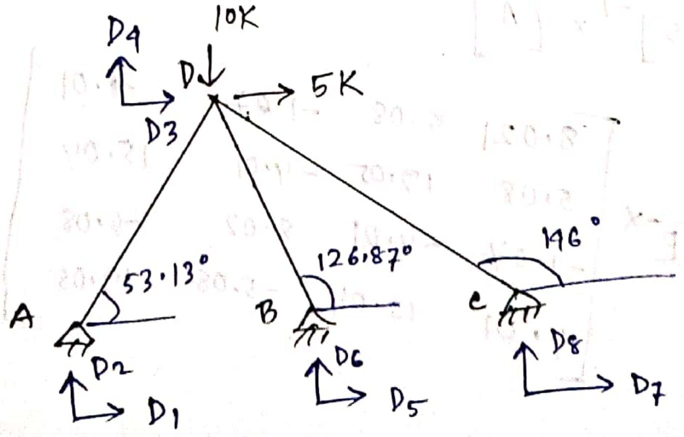
* ↳ যেগুলো Deflected হবে না সেগুলো আগেই zero .



Problem: 02

calculate the member force for the truss, EA is constant. (By local stiffness Matrix)

Solution:



For Member AD, $L = 5'$, $\theta = 53.13^\circ$

$$S_{AD} = AE \begin{array}{c|cccc|c} & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} & \\ \hline & 0.072 & 0.096 & -0.072 & -0.096 & 1 \\ & 0.096 & 0.128 & -0.096 & -0.128 & 2 \\ & -0.072 & -0.096 & 0.072 & 0.096 & 3 \\ & -0.096 & -0.128 & 0.096 & 0.128 & 4 \end{array}$$

For Member BD, $L = 5'$, $\theta = 126.87^\circ$

$$S_{BD} = AE \begin{array}{c|cccc|c} & \textcircled{5} & \textcircled{6} & \textcircled{3} & \textcircled{4} & \\ \hline & 0.072 & -0.096 & -0.072 & 0.096 & \textcircled{5} \\ & -0.096 & 0.128 & 0.096 & -0.128 & \textcircled{6} \\ & -0.072 & 0.096 & 0.072 & -0.096 & \textcircled{3} \\ & 0.096 & -0.128 & -0.096 & 0.128 & \textcircled{4} \end{array}$$

For Member CD, $L = 7'$, $\theta = 145^\circ$

$$S_{CD} = AE \begin{array}{c|cccc|c} & \textcircled{7} & \textcircled{8} & \textcircled{3} & \textcircled{4} & \\ \hline & 0.098 & -0.066 & -0.098 & 0.066 & \textcircled{7} \\ & -0.066 & 0.045 & 0.066 & -0.045 & \textcircled{8} \\ & -0.098 & 0.066 & 0.098 & -0.066 & \textcircled{3} \\ & 0.066 & -0.045 & -0.066 & 0.045 & \textcircled{4} \end{array}$$

Active Degree of Freedom = D_3, D_4

$\therefore$ Global Stiffness Matrix, $S = AE \begin{bmatrix} 0.242 & -0.066 \\ -0.066 & 0.301 \end{bmatrix}$

load Matrix,

$$A = \begin{bmatrix} 5 \\ -10 \end{bmatrix}.$$

We know, $A = SD$

$$\Rightarrow D = [S]^{-1} \times A$$

$$\Rightarrow \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} 0.242 & -0.066 \\ -0.066 & 0.301 \end{bmatrix}^{-1} \times \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{AE} \times \frac{1}{0.0685} \times \begin{bmatrix} 0.301 & 0.066 \\ 0.066 & 0.242 \end{bmatrix} \times \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{AE} \times \begin{bmatrix} 4.394 & 0.964 \\ 0.964 & 3.533 \end{bmatrix} \times \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} 12.33 \\ -30.51 \end{bmatrix}$$

Now, $[F] = [S][D]$

For AD,

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = AE \begin{bmatrix} 0.072 & 0.096 & -0.072 & -0.096 \\ 0.096 & 0.128 & -0.096 & -0.128 \\ -0.072 & -0.096 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0.096 & 0.128 \end{bmatrix} \times \frac{1}{AE} \begin{bmatrix} 0 \\ 0 \\ 12.33 \\ -30.51 \end{bmatrix}$$

$$\therefore \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 2.04 \\ 2.72 \\ -2.04 \\ 2.72 \end{bmatrix}$$

For BD,

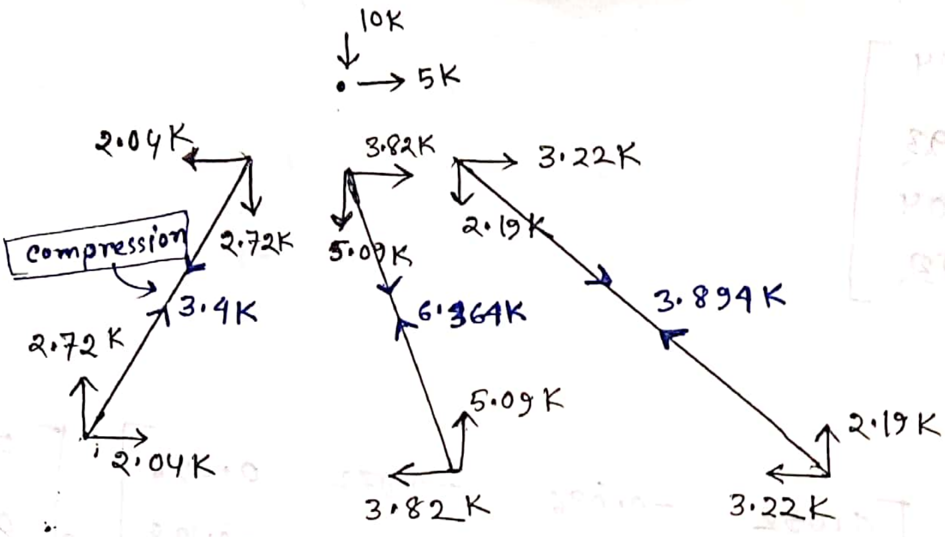
$$\begin{bmatrix} F_5 \\ F_6 \\ F_3 \\ F_4 \end{bmatrix} = AE \begin{bmatrix} 0.072 & -0.096 & -0.072 & 0.096 \\ -0.096 & 0.128 & 0.096 & -0.128 \\ -0.072 & 0.096 & 0.072 & -0.096 \\ 0.096 & -0.128 & -0.096 & 0.128 \end{bmatrix} \times \frac{1}{AE} \begin{bmatrix} 0 \\ 0 \\ 12.33 \\ -30.51 \end{bmatrix}$$

$$\therefore \begin{bmatrix} F_5 \\ F_6 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} -3.82 \\ 5.09 \\ 3.82 \\ -5.09 \end{bmatrix}$$

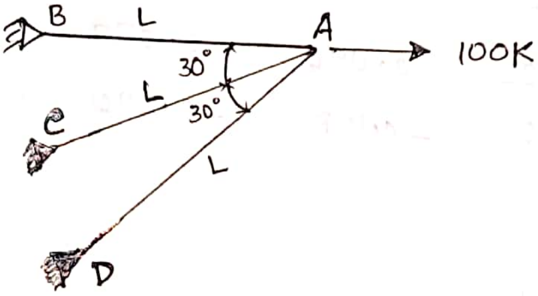
For CD,

$$\begin{bmatrix} F_7 \\ F_8 \\ F_3 \\ F_4 \end{bmatrix} = AE \begin{bmatrix} 0.098 & -0.066 & -0.098 & 0.066 \\ -0.066 & 0.045 & 0.066 & -0.045 \\ -0.098 & 0.066 & 0.098 & -0.066 \\ 0.066 & -0.045 & -0.066 & 0.045 \end{bmatrix} \times \frac{1}{AE} \begin{bmatrix} 0 \\ 0 \\ 12.33 \\ -30.51 \end{bmatrix}$$

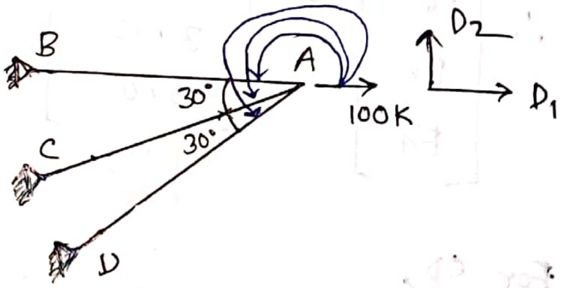
$$\therefore \begin{bmatrix} F_7 \\ F_8 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} -3.22 \\ 2.19 \\ 3.22 \\ -2.19 \end{bmatrix}$$



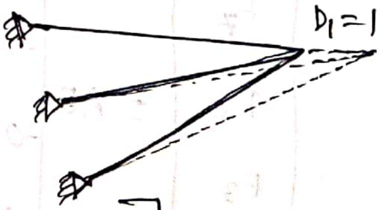
Problem -03: calculate Member forces for the truss. EA is constant.



Solution: Degree of freedom = 2



Case-1: $D_1 = 1$, $D_2 = 0$



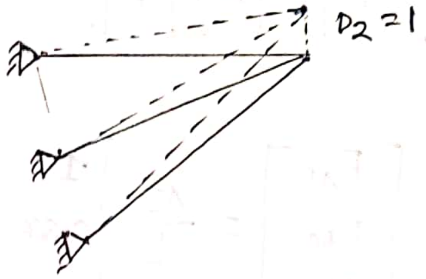
$$S_{11} = \frac{AE}{L}\left[\cos^2 180 + \cos^2 210 + \cos^2 240\right]$$

$$= 2\frac{AE}{L}$$

$$S_{21} = \frac{AE}{L}\left[\sin 180 \cdot \cos 180 + \sin 210 \cdot \cos 210 + \sin 240 \cdot \cos 240\right]$$

$$= 0.87\frac{AE}{L}$$

case-2: $D_2 = 1,\ D_1 = 0$



$$S_{21} = \frac{AE}{L}\left[\sin 180 \cdot \cos 180 + \sin 210 \cdot \cos 210 + \sin 240 \cdot \sin 240\right]$$

$$= 0.87\,\frac{AE}{L}$$

$$S_{22} = \frac{AE}{L}\left[\sin^2 180 + \sin^2 210 + \sin^2 240\right]$$

$$= \frac{AE}{L}$$

$\therefore$ Stiffness Matrix, $S = \frac{AE}{L}\begin{bmatrix} 2 & 0.87 \\ 0.87 & 1 \end{bmatrix}$

Load Matrix, $A = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$

We know, $[A] = [S][D]$

$$\Rightarrow [D] = [S]^{-1}[A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{L}{AE}\begin{bmatrix} 2 & 0.87 \\ 0.87 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{L}{AE} \times \frac{1}{1.2431} \times \begin{bmatrix} 1 & -0.87 \\ -0.87 & 2 \end{bmatrix} \times \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{L}{AE}\begin{bmatrix} 0.8044 & -0.7 \\ -0.7 & 1.61 \end{bmatrix} \times \begin{bmatrix} 100 \\ 0 \end{bmatrix} = \frac{L}{AE}\begin{bmatrix} 80.44 \\ -70 \end{bmatrix}$$

Now, $[F] = [S][D]$

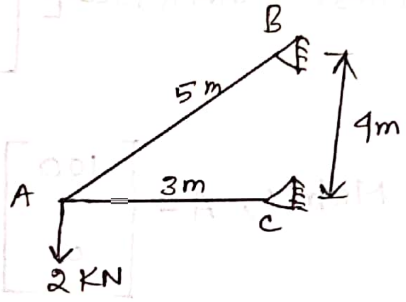
$$\begin{bmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & 0 \\ 0.866 & 0.5 \\ 0.5 & 0.866 \end{bmatrix} \times \frac{L}{AE} \begin{bmatrix} 80.44 \\ -70 \end{bmatrix}$$

$$\begin{bmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{bmatrix} = \begin{bmatrix} 80.44 \\ 34.66 \\ -20.4 \end{bmatrix}$$

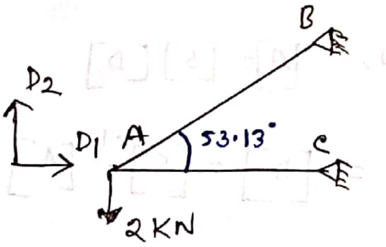
(Ans.)

Problem: 04

calculate the Member force for the truss. EA is constant.



Solution: Degree of Freedom = 2



Case-1: $D_1 = 1$, $D_2 = 0$

$$S_{11} = AE \left[\frac{\cos^2 53.13^\circ}{5} + \frac{\cos^2 0}{3} \right]$$

$$= AE \times 0.4053$$

$$S_{21} = AE \left[\frac{\sin 53.13 \cdot \cos 53.13}{5} + \frac{\sin 0 \cdot \cos 0}{3} \right] = 0.096\, AE$$

case-2: $D_2 = 1\ ,\ D_1 = 0$

$$S_{12} = AE\left[\frac{\sin 53.13 \cdot \cos 53.13}{5} + \frac{\sin 0 \cdot \cos 0}{5}\right] = 0.096\,AE$$

$$S_{22} = AE\left[\frac{\sin^2 53.13 \cdot}{5} + \frac{\sin^2 0 \cdot}{3}\right] = 0.128\,AE$$

$\therefore$ Stiffness Matrix,

$$S = AE\begin{bmatrix} 0.4053 & 0.096 \\ 0.096 & 0.128 \end{bmatrix}$$

Load Matrix, $A = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

we know, $[A] = [S][D]$

$$\Rightarrow [D] = [S]^{-1}[A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE}\begin{bmatrix} 0.4053 & 0.096 \\ 0.096 & 0.128 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE} \times \frac{1}{0.043}\begin{bmatrix} 0.128 & -0.096 \\ -0.096 & 0.4053 \end{bmatrix} \times \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE} \times \frac{1}{0.043} \times \begin{bmatrix} 0.192 \\ -0.8106 \end{bmatrix} = \frac{1}{AE}\begin{bmatrix} 4.465 \\ -18.85 \end{bmatrix}$$

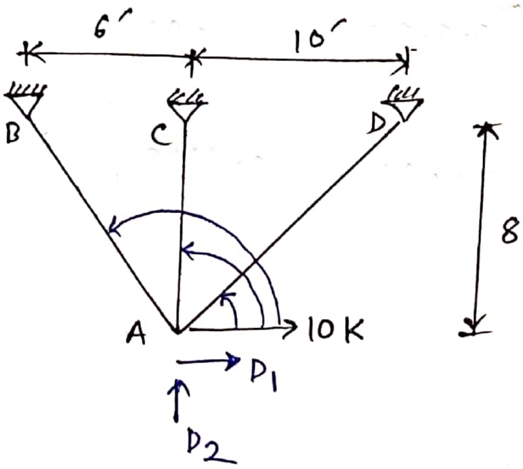
Now,

$$\begin{bmatrix} F_{AB} \\ F_{AC} \end{bmatrix} = AE \begin{bmatrix} -0.12 & -0.16 \\ -0.12 & 0 \end{bmatrix} \times \frac{1}{AE} \begin{bmatrix} 4.465 \\ -18.85 \end{bmatrix} = \begin{bmatrix} 2.48 \\ -0.893 \end{bmatrix}$$

(Ans.)

Stiffness Matrix Method (Truss)

2018
#



Solution:

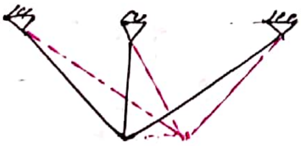
degree of freedom = 02

Member	Length	Angle
AD	12.81	38.66
AC	8	90
AB	10	126.87

Load Matrix,

$$A = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

When $D_1 = 1$, $D_2 = 0$:

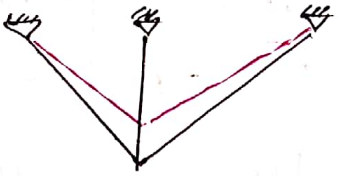


$$S_{11} = AE \times \left[\frac{\cos^2 38.66}{12.81} + \frac{\cos^2 90}{8} + \frac{\cos^2 126.87}{10} \right]$$

$$= 0.0836\ AE$$

$$S_{21} = AE \times \left[\frac{\cos 38.66 \times \sin 38.66}{12.81} + \frac{\cos 90 \times \sin 90}{8} + \frac{\cos 126.87 \times \sin 126.87}{10} \right]$$

$$= -0.01\ AE$$



W en $D_2 = 1$, $D_1 = 0$:

$$S_{12} = AE \times \left[\frac{\sin 38.66 \cdot \cos 38.66}{12.81} + \frac{\cos 90 \cdot \sin 90}{8} + \frac{\cos 126.87 \times \sin 126.87}{10} \right]$$

$$= -0.01\ AE$$

$$S_{22} = AE \times \left[\frac{\sin^2 38.66}{12.81} + \frac{\sin^2 90}{8} + \frac{\sin^2 126.87}{10} \right]$$

$$= 0.2195\ AE$$

Stiffness Matrix,

$$S = AE \begin{bmatrix} 0.0836 & -0.01 \\ -0.01 & 0.2195 \end{bmatrix}$$

We know,

$$[A] = [S]\,[D]$$

$$\Rightarrow [D] = [S^{-1}] \times [A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE} \times \frac{1}{0.0182502} \times \begin{bmatrix} 0.2195 & 0.01 \\ 0.01 & 0.0836 \end{bmatrix} \times \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE} \times \begin{bmatrix} 120.3 \\ 5.5 \end{bmatrix}$$

Then, $[F] = [S]\,[D]$

$$\begin{bmatrix} F_{AD} \\ F_{AC} \\ F_{AB} \end{bmatrix} = AE \begin{array}{c} \begin{array}{cc} -\frac{\cos\theta}{L} & -\frac{\sin\theta}{L} \end{array} \\ \begin{bmatrix} -0.06 & -0.05 \\ 0 & -0.125 \\ 0.06 & -0.08 \end{bmatrix} \end{array} \times \frac{1}{AE} \begin{bmatrix} 120.3 \\ 5.5 \end{bmatrix}$$

$$\therefore \begin{bmatrix} F_{AD} \\ F_{AC} \\ F_{AB} \end{bmatrix} = \begin{bmatrix} -6.94 \\ -0.68 \\ 6.78 \end{bmatrix}$$

(Ans.)

2017

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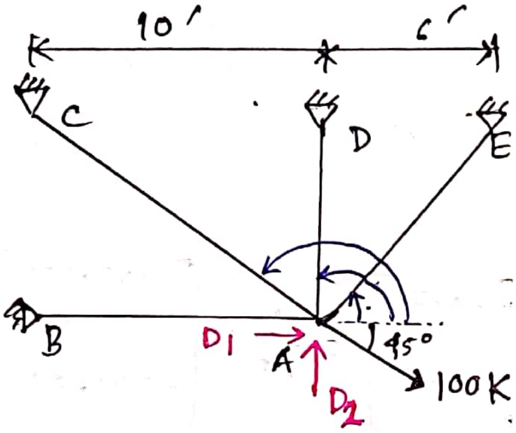
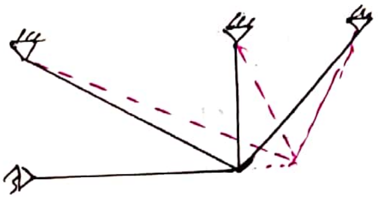


Solution: degree of freedom = 02

Member	length	Angle
AE	10	53.13
AD	8	90
AC	12.81	141.34
AB	10	180

Load Matrix,

$$A = \begin{bmatrix} 70.71 \\ -70.71 \end{bmatrix}$$



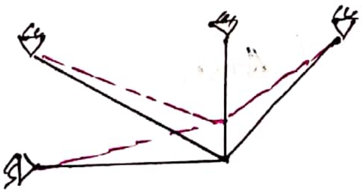
when $D_1 = 1$, $D_2 = 0$:

$$S_{11} = AE \times \left[\frac{\cos^2 53.13}{10} + \frac{\cos^2 90}{8} + \frac{\cos^2 141.39}{12.81} + \frac{\cos^2 180}{10} \right]$$

$$= 0.1835 AE$$

$$S_{21} = AE \times \left[\frac{\cos 53.13 \times \sin 53.13}{10} + \frac{\cos 90 \times \sin 90^\circ}{8} + \frac{\cos 141.34 \times \sin 141.34}{12.81} + \frac{\sin 180 \times \cos 180}{10} \right]$$

$$= 0.01 AE$$



when $D_2 = 1$, $D_1 = 0$:

$$S_{12} = S_{21} = 0.01 AE$$

$$S_{22} = AE \times \left[\frac{\sin^2 53.13}{10} + \frac{\sin^2 90}{8} + \frac{\sin^2 141.34}{12.81} + \frac{\sin^2 180}{10} \right]$$

$$= 0.22 AE$$

$$\text{stiffness Matrix, } S = AE \begin{bmatrix} 0.1835 & 0.01 \\ 0.01 & 0.22 \end{bmatrix}$$

$[A] = [S][D]$

$[D] = [S^{-1}][A]$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE} \times \begin{bmatrix} 5.45 & -0.25 \\ -0.25 & 4.58 \end{bmatrix} \times \begin{bmatrix} 70.71 \\ -70.71 \end{bmatrix} = \frac{1}{AE} \times \begin{bmatrix} 403.05 \\ -341.53 \end{bmatrix}$$

Then, $[F] = [S][D]$

$$\begin{bmatrix} F_{AE} \\ F_{AD} \\ F_{AC} \\ F_{AB} \end{bmatrix} = AE \begin{array}{c} \begin{array}{cc} -\frac{\cos\theta}{L} & -\frac{\sin\theta}{L} \end{array} \\ \begin{bmatrix} -0.06 & -0.08 \\ 0 & -0.125 \\ 0.06 & -0.049 \\ 0.1 & 0 \end{bmatrix} \end{array} \times \frac{1}{AE} \begin{bmatrix} 403.05 \\ -341.53 \end{bmatrix}$$

$$\begin{bmatrix} F_{AE} \\ F_{AD} \\ F_{AC} \\ F_{AB} \end{bmatrix} = \begin{bmatrix} 3.14 \\ 42.69 \\ 40.92 \\ 40.31 \end{bmatrix}$$

(Ans.)

2015

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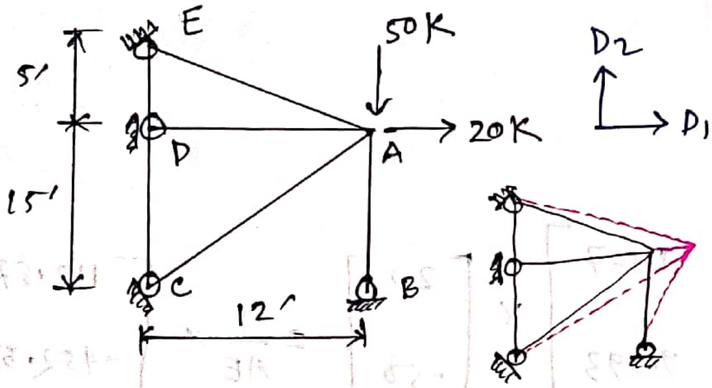


Member	Length	Angle
AE	13'	157.38
AD	12'	180
AC	19.21'	231.34
AB	15'	270

solution: degree of freedom = 02

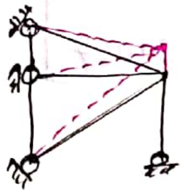
When $D_1 = 1$:

$$S_{11} = AE \times \left[\frac{\cos^2 157.38}{13} + \frac{\cos^2 180}{12} + \frac{\cos^2 231.34}{19.21} + \frac{\cos^2 270}{15} \right]$$

$$= 0.1692 \, EI$$

$$S_{21} = AE \times \left[\frac{\cos 157.38 \times \sin 157.38}{13} + \frac{\sin 180 \times \cos 180}{12} + \frac{\sin 231.34 \times \cos 231.34}{19.21} + \frac{\sin 270 \times \cos 270}{15} \right]$$

$$= -0.002 \, EI$$



When $D_2 = 2$:

$$S_{12} = S_{21} = -0.002 \, EI$$

$$S_{22} = AE \times \left[\frac{\sin^2 157.38}{13} + \frac{\sin^2 180}{12} + \frac{\sin^2 231.34}{19.21} + \frac{\sin^2 270}{15} \right]$$

$$= 0.11 \, EI$$

Stiffness Matrix, $S = AE \begin{bmatrix} 0.1692 & -0.002 \\ -0.002 & 0.11 \end{bmatrix}$

load Matrix, $A = \begin{bmatrix} 20 \\ -50 \end{bmatrix}$

$$[A] = [S][D]$$

$$\Rightarrow [D] = [S^{-1}][A]$$

$$\Rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} 5.911 & 0.107 \\ 0.107 & 9.093 \end{bmatrix} \times \begin{bmatrix} 20 \\ -50 \end{bmatrix} = \frac{1}{AE} \times \begin{bmatrix} 112.87 \\ -452.51 \end{bmatrix}$$

$$[F] = [S][D]$$

$$\Rightarrow \begin{bmatrix} F_{AE} \\ F_{AD} \\ F_{AC} \\ F_{AB} \end{bmatrix} = AE \begin{array}{cc} -\frac{\cos\theta}{L} & -\frac{\sin\theta}{L} \\ \hline 0.071 & -0.03 \\ 0.083 & 0 \\ 0.042 & 0.041 \\ 0 & 0.067 \end{array} \times \frac{1}{AE} \begin{bmatrix} 112.87 \\ -452.51 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} F_{AE} \\ F_{AD} \\ F_{AC} \\ F_{AB} \end{bmatrix} = \begin{bmatrix} 21.59 \\ 9.37 \\ -13.81 \\ -30.32 \end{bmatrix}$$

(Ans.)

2013
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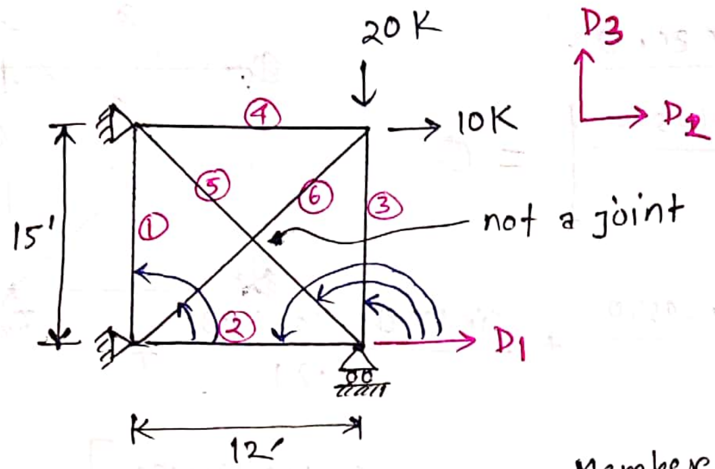
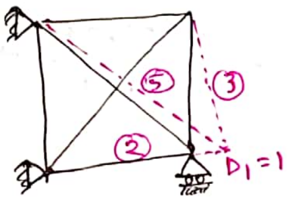


Solution: Degree of freedom = 03

Member	Length	Angle
①	15'	90
②	12'	180
③	15'	90
④	12'	180
⑤	19.21'	128.66°
⑥	19.21'	51.34°

When, $D_1 = 1$, all are zero:

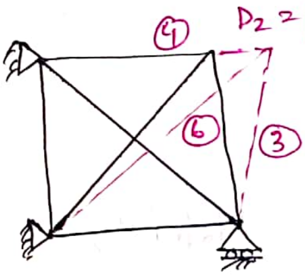


$$s_{11} = AE \times \left[\frac{\cos^2 180}{12} + \frac{\cos^2 90}{15} + \frac{\cos^2 128.66}{19.21} \right]$$

$$= 0.10365\,AE$$

$$s_{21} = AE \times \left[\frac{-\cos^2 90}{15} \right] = 0$$

$$s_{31} = AE \times \left[\frac{-\cos 90 \cdot \sin 90}{15} \right] = 0$$



When $D_2 = 1$, all are zero:

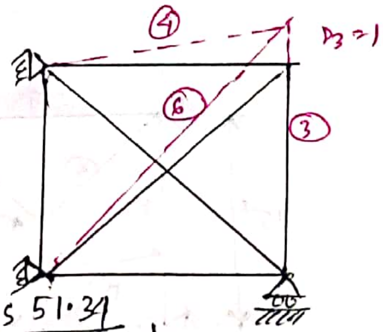
$$s_{12} = \left[\frac{-\cos^2 90}{15} \right] = 0$$

$$s_{22} = \left[\frac{\cos^2 90}{15} + \frac{\cos^2 51.34}{19.21} + \frac{\cos^2 180}{12} \right] = 0.10365\,AE$$

$$s_{32} = \left[\frac{\cos 90 \cdot \sin 90}{15} + \frac{\cos 51.34 \cdot \sin 51.34}{19.21} + \frac{\cos 180 \cdot \sin 180}{12} \right]$$

$$= 0.0254\,AE$$

When $D_3 = 1$, all are zero.



$$S_{13} = AE \times \left[\frac{-\sin 90 \cdot \cos 90}{15} \right] = 0$$

$$S_{23} = AE \times \left[\frac{\sin 90 \cdot \cos 90}{15} + \frac{\sin 51.34 \cdot \cos 51.34}{19.21} + \frac{\sin 180 \cdot \cos 180}{12} \right]$$

$$= 0.0254\,AE$$

$$S_{33} = AE \times \left[\frac{\sin^2 90}{15} + \frac{\sin^2 51.34}{19.21} + \frac{\sin^2 180}{12} \right] = 0.0984\,AE$$

∴ Stiffness Matrix,

$$S = AE \begin{bmatrix} 0.10365 & 0 & 0 \\ 0 & 0.10365 & 0.0254 \\ 0 & 0.0254 & 0.0984 \end{bmatrix}$$

Then,

Do yourself,

$$(D] = [S^{-1}] \times [A]$$

Now,

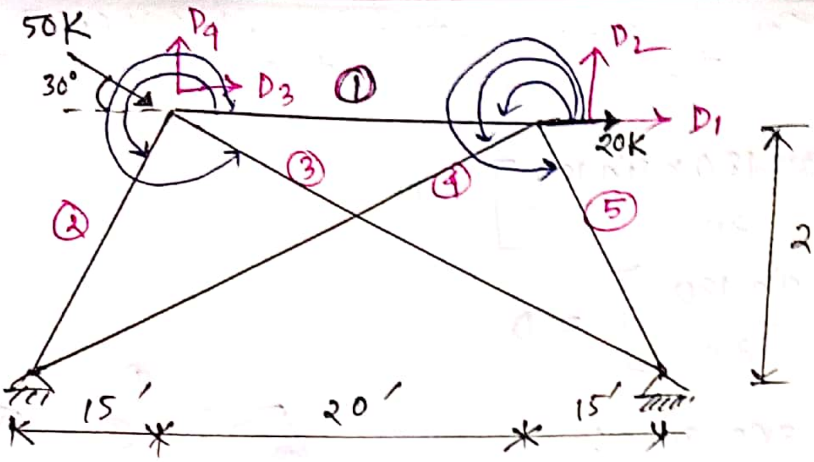
$$F = [S]\,[D]$$

$$\begin{bmatrix} -\frac{\cos\theta}{L} & -\frac{\cos\theta}{L} & -\frac{\sin\theta}{L} \\ \vdots & \vdots & \vdots \end{bmatrix}$$

Load Matrix,

$$A = \begin{bmatrix} 0 \\ 10 \\ -20 \end{bmatrix}$$

2012
#

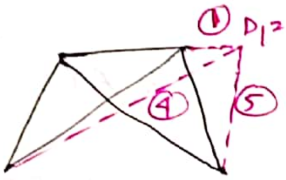


Solution: Degree of freedom = 04

Member	Length	Angle
①	20'	180°
②	25'	233.13°
③	40.31'	330.26°
④	40.31'	209.745°
⑤	25'	306.87°

Load Matrix

$$A = \begin{bmatrix} 20 \\ 0 \\ 43.3 \\ -25 \end{bmatrix} \begin{matrix} \\ \\ \rightarrow 50\cos(30) \\ \rightarrow 50\sin(30) \end{matrix}$$



When $D_1 = 1$ all are zero:

$$S_{11} = AE\left[\frac{\cos^2 180}{20} + \frac{\cos^2 209.745}{40.31} + \frac{\cos^2 306.87}{25}\right]$$

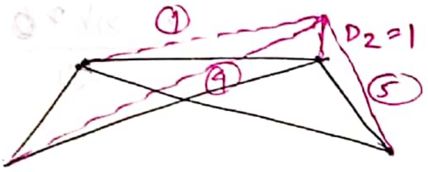
$$= 0.0831\,AE$$

$$S_{21} = AE \times \left[\frac{\cos 180 \times \sin 180}{20} + \frac{\cos 209.745 \times \sin 209.745}{40.31} + \frac{\cos 306.87 \times \sin 306.87}{25}\right]$$

$$= -0.0085AE$$

$$S_{31} = AE \times \left[\frac{-\cos^2 180}{20}\right] = -0.05AE$$

$$S_{41} = AE \times \left[\frac{-\cos 180 \times \sin 180}{20}\right] = 0$$



When $D_2 = 1$; All are zero:

$$S_{12} = AE \times \left[\frac{\sin 180 \times \cos 180}{20} + \frac{\sin 209.745 \times \cos 209.745}{40.31} + \frac{\sin 306.87 \times \overset{\cos 306.87}{\wedge}}{25}\right]$$

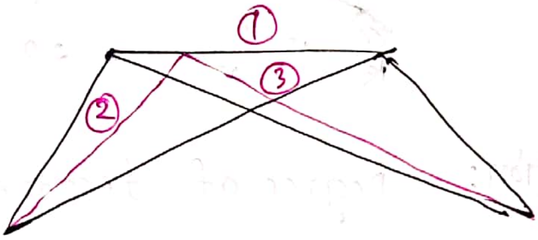
$$= -0.0085\,AE$$

Scanned by CamScanner

$$S_{22} = AE \times \left[\frac{\sin^2 180}{20} + \frac{\sin^2 209.745}{40.31} + \frac{\sin^2 306.87}{25} \right] = 0.0317\,AE$$

$$S_{32} = AE \times \left[\frac{-\cos 180 \times \sin 180}{20} \right] = 0$$

$$S_{42} = AE \times \left[\frac{-\sin 180}{20} \right] = 0$$



when $D_3 = 1$, all are zero:

$$S_{13} = AE \times \left[\frac{-\cos^2 180}{20} \right] = -0.05AE$$

$$S_{23} = AE \times \left[\frac{-\sin 180 \times \cos 180}{20} \right] = 0$$

$$S_{33} = AE \times \left[\frac{\cos^2 180}{20} + \frac{\cos^2 233.13}{25} + \frac{\cos^2 330.26}{40.31} \right] = 0.0831\,AE$$

$$S_{43} = AE \times \left[\frac{\cos 180 \times \sin 180}{20} + \frac{\cos 233.13 \times \sin 233.13}{25} + \frac{\cos 330.26 \times \sin 330.26}{40.31} \right]$$

$$= 0.0085\,AE$$

when $D_4 = 1$, all are zero:

$$S_{14} = AE \times \left[\frac{-\sin 180 \times \cos 180}{20} \right] = 0$$

$$S_{24} = AE \times \left[\frac{-\sin^2 180}{20} \right] = 0$$

$$S_{34} = S_{43} = 0.0085\,AE$$

$$S_{44} = AE \times \left[\frac{\sin^2 0}{20} + \frac{\sin^2 233.13}{25} + \frac{\sin^2 330.26}{40.31} \right] = 0.0317\,AE$$

Stiffness Matrix, $S = AE \begin{bmatrix} 0.0831 & -0.0085 & -0.05 & 0 \\ -0.0085 & 0.0317 & 0 & 0 \\ -0.05 & 0 & 0.0831 & 0.0085 \\ 0 & 0 & 0.0085 & 0.0317 \end{bmatrix}$

I.

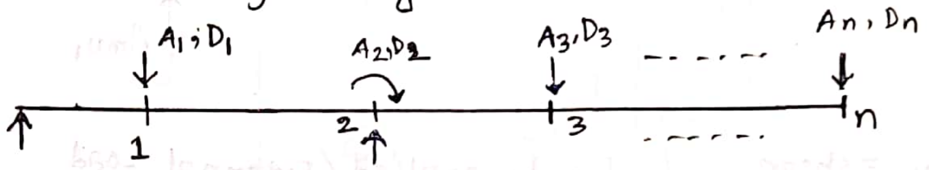
Flexibility Matrix Method

BEAM

2017, 2014

⊞ flexibility matrix of a beam:

Let Us condiser the following beam:



Here, 1, 2, 3 ----- n are the nodal points.

$A_1, A_2, A_3 \ldots\ldots A_n$ are the forces applied at the nodal points.

$D_1, D_2, D_3 \ldots\ldots D_n$ are the displacement caused by the forces at nodal points.

$$D_1 = D_{11} + D_{12} + D_{13} + \cdots\cdots + D_{1n}$$

$$= F_{11}A_1 + F_{12}A_2 + F_{13}A_3 + \cdots\cdots + F_{1n}A_n$$

$$D_2 = F_{21}A_1 + F_{22}A_2 + F_{23}A_3 + \cdots\cdots + F_{2n}A_n$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

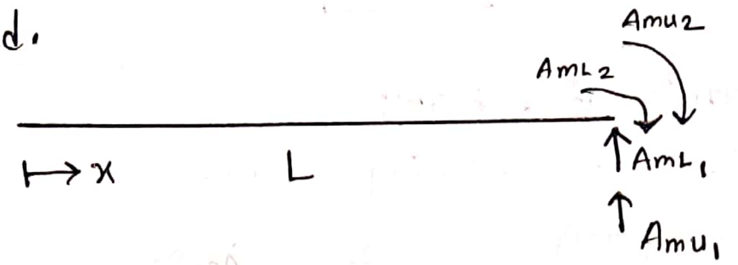
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$D_n = F_{n1}A_1 + F_{n2}A_2 + F_{n3}A_3 + \cdots\cdots + F_{nn}A_n$$

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & \cdots & F_{1n} \\ F_{21} & F_{22} & F_{23} & \cdots & F_{2n} \\ F_{31} & F_{32} & F_{33} & \cdots & F_{3n} \\ \vdots & \vdots & \vdots & & \\ F_{n1} & F_{n2} & F_{n3} & \cdots & F_{nn} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_n \end{bmatrix}$$

$$[D] = [F][A]$$

consider a beam of length L. A moment and a vertical shear are applied at one end.



Here,

A_{mL_1} = shear
A_{mL_2} = Moment
} due to applied/External Load

A_{mu_1} – shear
A_{mu_2} – Moment
} due to unit load

We know,

$$D = u = \int_0^L \frac{Mm}{EI}\, dx$$

Moment due to applied load,

$$M = A_{mL_2} - A_{mL_1}(L-x)$$

$$m = A_{mu_2} - A_{mu_1}(L-x)$$

Now,

$$D = \frac{1}{EI}\int_0^L \left\{A_{mL_2} - A_{mL_1}(L-x)\right\}\left\{A_{mu_2} - A_{mu_1}(L-x)\right\} dx$$

$$= \frac{1}{EI}\int_0^L \Big[A_{mL_2} A_{mu_2} - A_{mL_2} A_{mu_1}(L-x) - A_{mu_2} A_{mL_1}(L-x) + A_{mL_1} A_{mu_1}(L-x)^2\Big] dx$$

$$= \frac{1}{EI}\Big[A_{mL_2} A_{mu_2}\, x - A_{mL_2} A_{mu_1}\left(Lx - \frac{x^2}{2}\right) - A_{m2} A_{mL_1}\left(Lx - \frac{x^2}{2}\right) + A_{mL_1} A_{mu_1}\left(L^2 x - 2L\frac{x^2}{2} + \frac{x^3}{3}\right)\Big]_0^L$$

$$= \frac{1}{EI} \times \left[A_{ML_2} A_{MU_2} L - A_{ML_2} A_{MU_1} \left(\frac{L^2}{2}\right) - A_{MU_2} A_{ML_1} \left(\frac{L^2}{2}\right) + A_{ML_1} A_{MU_1} \left(\frac{L^3}{3}\right) \right]$$

$$D = \begin{bmatrix} A_{MU_1} & A_{MU_2} \end{bmatrix} \times \begin{bmatrix} \frac{L^3}{3EI} & -\frac{L^2}{2EI} \\ \frac{-L^2}{2EI} & \frac{L}{EI} \end{bmatrix} \begin{bmatrix} A_{ML_1} \\ A_{ML_2} \end{bmatrix}$$

$$D = \sum_{m=1}^{n} [A_{MU}]^t [F_m] [A_{ML}]$$

Equilibrium Equation :

$$D_{QL} + FQ = 0$$

Here , Q = Redundant force

F = Flexibity

$$\Rightarrow FQ = -D_{QL}$$

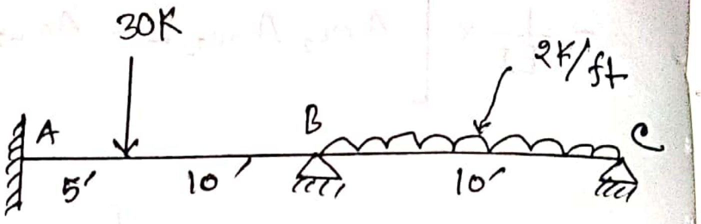
$$\Rightarrow Q = -[F^{-1}] \times [D_{QL}]$$

Solving Procedure:

1. Find the degree of indeterminacy
2. Select the redundant force , Q
3. Displacement of equivalent Joint and Find A_{ML}
4. calculation of A_{MU}
5. calculation of F_m
6. calculation of D_{QL}
7. calculation of F
8. calculation of Q
9. Calculation of A_m

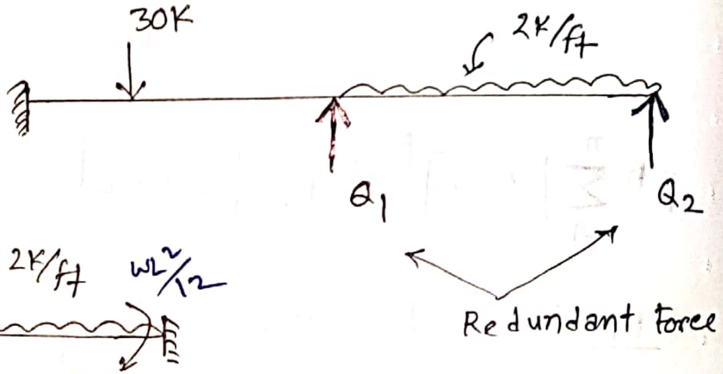
Problem-01:

Analyze the beam by flexibility matrix method. EI is constant.

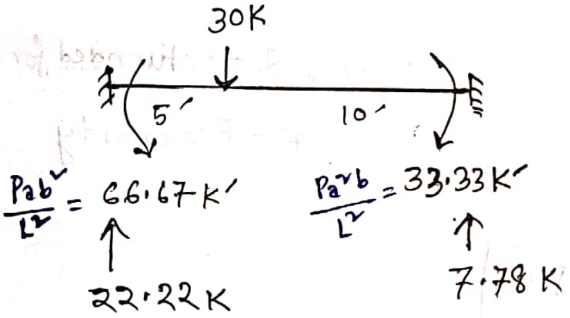


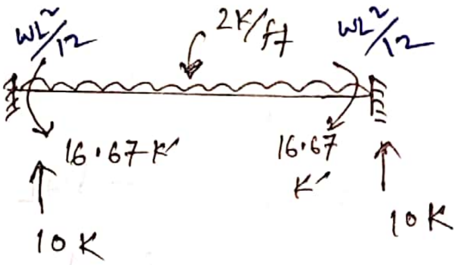
Solution:

No. of indeterminacy = 4-2 = 2 (without axial force)

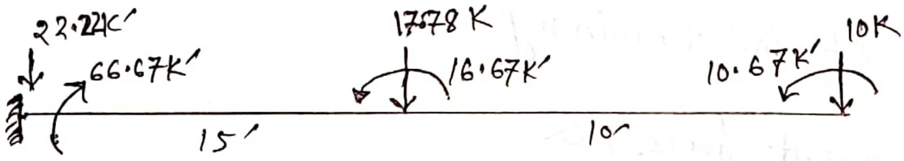


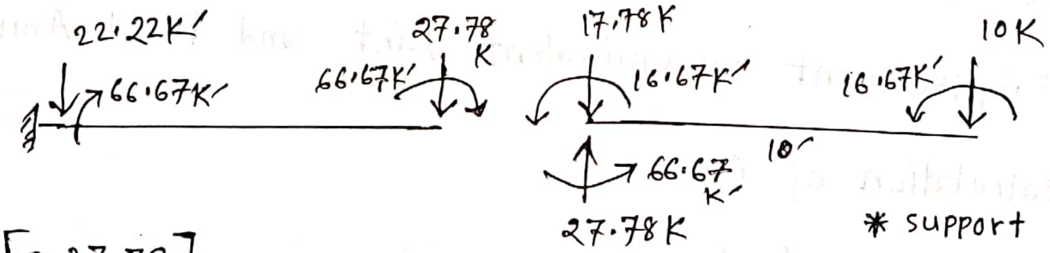
Fixed End Moment:





Replacement of equivalent joint force:



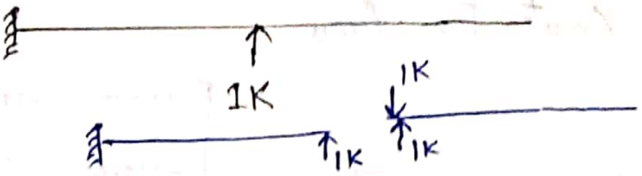


* support point-এর just বাদে কাটবো

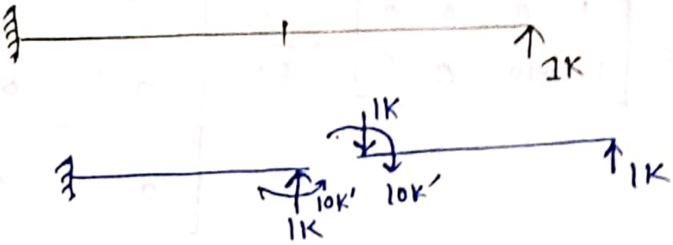
$$\therefore A_{ML} = \begin{bmatrix} -27.78 \\ 66.67 \\ -10 \\ -16.67 \end{bmatrix}$$

Due to unit Load:

1st case: $Q_1 = 1$, $Q_2 = 0$



2nd case: $Q_2 = 1$, $Q_1 = 0$



$$A_{mu} = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cc} Q_1=1 & Q_2=1 \\ \end{array}$$

$$A_{mu} = \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$\frac{L^3}{3EI}$

$$F_m = \frac{1}{EI}\begin{bmatrix} 1125 & -112.5 & 0 & 0 \\ -112.5 & 15 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix}$$

(Diagonally 2×2 matrix বসবে, বাকি ঘর শূন্য দিয়ে Fill up হবে)

We know,

$$D_{QL} = A_{mu}^{t} . F_m . A_{mL}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} 1125 & -112.5 & 0 & 0 \\ -112.5 & 15 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix} \times \begin{bmatrix} -27.78 \\ 66.67 \\ -10 \\ -16.67 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} -38752.875 \\ 4125.3 \\ -2499.8 \\ 333.33 \end{bmatrix}$$

$$\therefore D_{QL} = \frac{1}{EI} \times \begin{bmatrix} -38752.875 \\ -82505.675 \end{bmatrix}$$

Again,

$$F = A_{mu}^{t} \cdot F_m \cdot A_{mu}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} 1125 & -112.5 & 0 & 0 \\ -112.5 & 15 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} 1125 & 2250 \\ -112.5 & -262.5 \\ 0 & 333.33 \\ 0 & -50 \end{bmatrix}$$

$$F = \frac{1}{EI} \times \begin{bmatrix} 1125 & 2250 \\ 2250 & 5208.33 \end{bmatrix}$$

We know,

$$D_{QL} + FQ = 0$$

$$\Rightarrow Q = -[F]^{-1} \times [D_{QL}]$$

$$= -EI \times \begin{bmatrix} 1125 & 2250 \\ 2250 & 5208.33 \end{bmatrix}^{-1} \times \frac{1}{EI} \begin{bmatrix} -38752.875 \\ -82505.675 \end{bmatrix}$$

$$= \frac{1}{796871.25} \times \begin{bmatrix} 5208.33 & -2250 \\ -2250 & 1125 \end{bmatrix} \times \begin{bmatrix} 38752.875 \\ 82505.675 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 20.33 \\ 7.06 \end{bmatrix}$$

Then,

$$A_m = A_{mL} + A_{mu} \cdot Q + A_{mR} \longrightarrow$$ FEM এর মত পাশের load & Moment

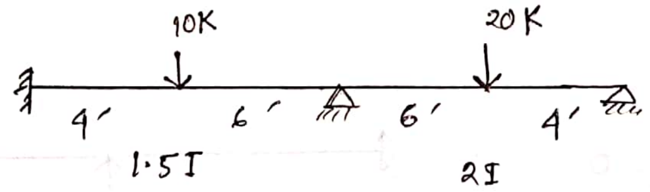
$$= \begin{bmatrix} -27.78 \\ 66.67 \\ -10 \\ -16.67 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 20.33 \\ 7.06 \end{bmatrix} + \begin{bmatrix} 7.78 \\ 33.33 \\ 10 \\ 16.67 \end{bmatrix}$$

$$= \begin{bmatrix} -27.78 \\ 66.67 \\ -10 \\ -16.67 \end{bmatrix} + \begin{bmatrix} 27.39 \\ -70.6 \\ 7.06 \\ 0 \end{bmatrix} + \begin{bmatrix} 7.78 \\ 33.33 \\ 10 \\ 16.67 \end{bmatrix}$$

$$\therefore A_m = \begin{bmatrix} 7.39 \\ 29.4 \\ 7.06 \\ 0 \end{bmatrix}$$

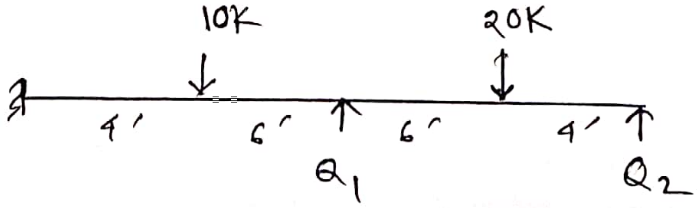
(Ans.)

Problem-02:



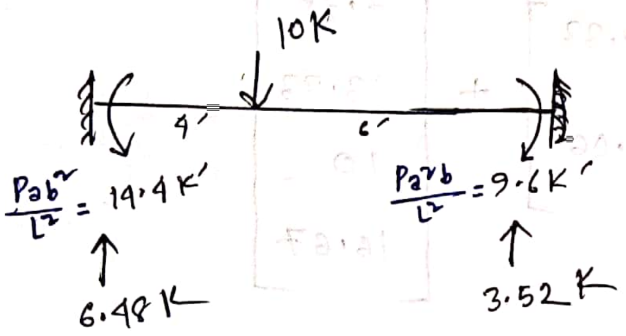
Analyze the Beam by flexibility matrix.

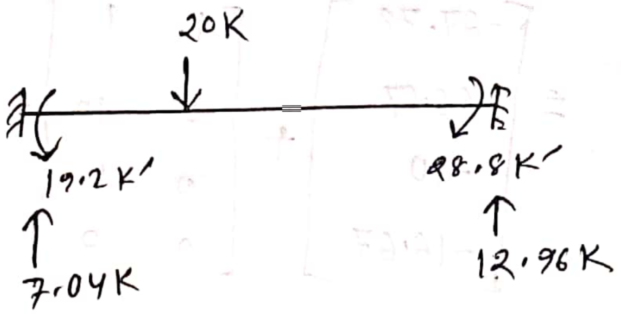
Solution: No. of indeterminacy = 4-2 = 2



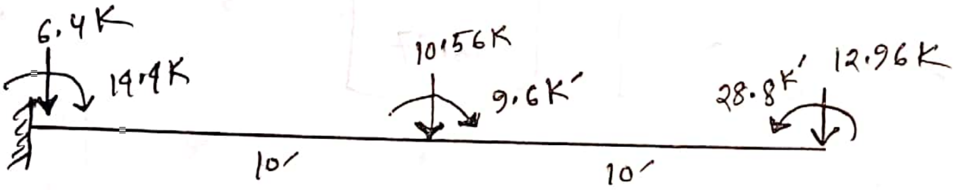
Redundant forces.

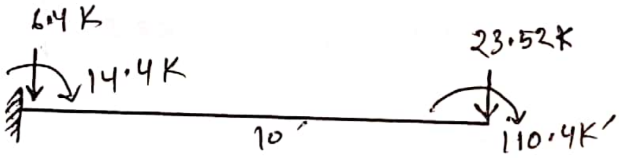
Fixed End Moment:

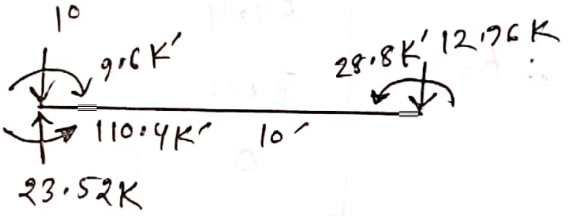




Replacement of equivalent joint force:



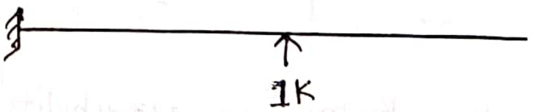




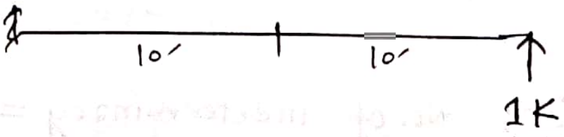
$$\therefore A_{ML} = \begin{bmatrix} -23.52 \\ 110.4 \\ -12.96 \\ -28.8 \end{bmatrix}$$

Due to unit load:

1st case: $Q_1 = K,\ Q_2 = 0$



2nd Case: $Q_1 = 0,\ Q_2 = 1K$



$$A_{mu} = \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 222.22 & -33.33 & 0 & 0 \\ -33.33 & 6.67 & 0 & 0 \\ 0 & 0 & 166.67 & -25 \\ 0 & 0 & -25 & 5 \end{bmatrix}$$

$$D_{QL} = [A_{mu}]^t\,[F_m]\,[A_{mL}]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 222.22 & -33.33 & 0 & 0 \\ -33.33 & 6.67 & 0 & 0 \\ 0 & 0 & 166.67 & -25 \\ 0 & 0 & -25 & 5 \end{bmatrix} \begin{bmatrix} -23.52 \\ 110.4 \\ -12.96 \\ -28.8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \frac{1}{EI} \times \begin{bmatrix} -8906.2464 \\ 1520.2896 \\ -1440.0432 \\ 180 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} -8906.2464 \\ -25549.1856 \end{bmatrix}$$

$$F = [A_{mu}]^t\,[F_m]\,[A_{mu}]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 222.22 & -33.33 & 0 & 0 \\ -33.33 & 6.67 & 0 & 0 \\ 0 & 0 & 166.67 & -25 \\ 0 & 0 & -25 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 222.22 & -33.33 & 0 & 0 \\ 555.52 & -100.03 & 166.67 & -25 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 7 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{EI}\begin{bmatrix} 222.22 & 555.52 \\ 555.52 & 1722.49 \end{bmatrix}$$

$$\Rightarrow F^{-1} = \frac{EI}{74169.2574}\begin{bmatrix} 1722.49 & -555.52 \\ -555.52 & 222.22 \end{bmatrix}$$

Now,

$$Q = -F^{-1} D_{QL}$$

$$= \frac{-EI}{74169.2574}\begin{bmatrix} 1722.49 & -555.52 \\ -555.52 & 222.22 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} -8906.2464 \\ -25549.1856 \end{bmatrix}$$

$$= \frac{-1}{74169.2574} \times \begin{bmatrix} -1147836.777 \\ -729942.0239 \end{bmatrix}$$

$$Q = \begin{bmatrix} 15.476 \\ 9.84 \end{bmatrix}$$

Then,

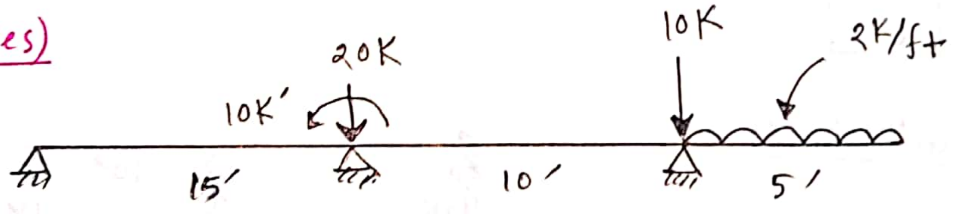
$$A_m = A_{mL} + A_{mu} \cdot Q + A_{mR}$$

$$= \begin{bmatrix} -23.52 \\ 110.4 \\ -12.96 \\ -28.8 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 15.476 \\ \\ 9.84 \end{bmatrix} + \begin{bmatrix} 3.52 \\ 9.6 \\ 12.96 \\ 28.8 \end{bmatrix}$$

$$= \begin{bmatrix} -23.52 \\ 110.4 \\ -12.96 \\ -28.8 \end{bmatrix} + \begin{bmatrix} 25.316 \\ -98.4 \\ 9.84 \\ 0 \end{bmatrix} + \begin{bmatrix} 3.52 \\ 9.6 \\ 12.96 \\ 28.8 \end{bmatrix}$$

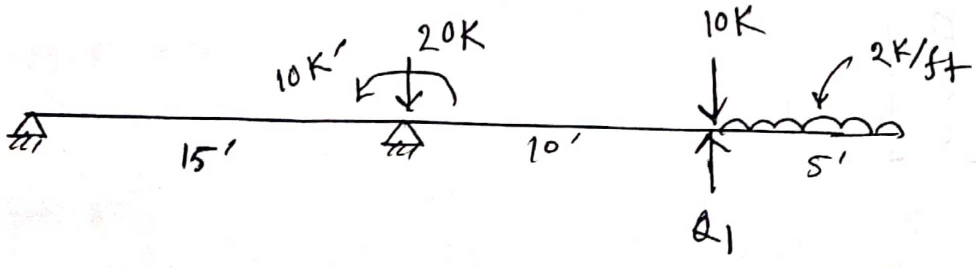
$$= \begin{bmatrix} 5.316 \\ 21.6 \\ 9.84 \\ 0 \end{bmatrix}$$

CT (15 Series)

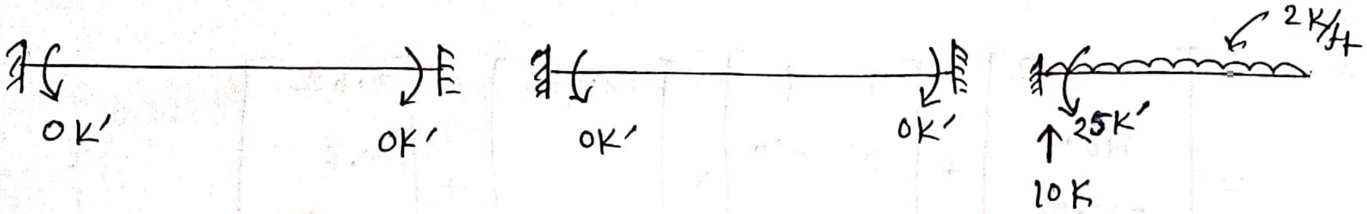


Analyze the beam by flexibility matrix method. EI is constant.

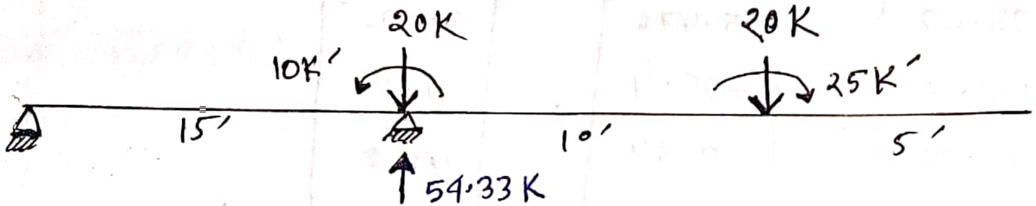
Solution: No. of Indeterminacy $=(3-2)=1$

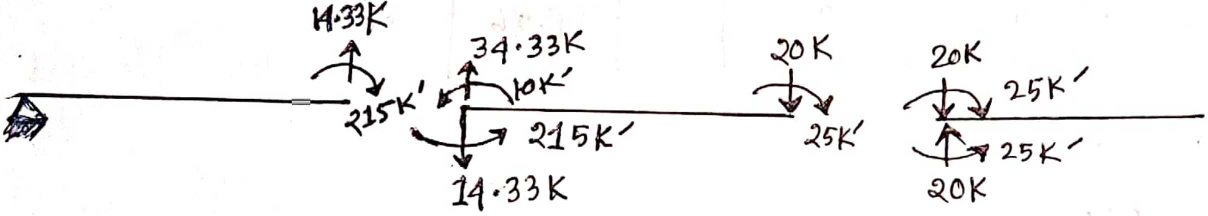


Fixed End Moment:



Replacement of Equivalent joint Force:

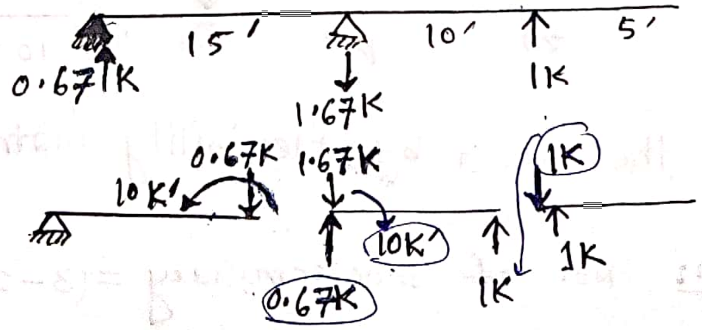




$$A_{ML} = \begin{bmatrix} 14.33 \\ 215 \\ -20 \\ 25 \\ 0 \\ 0 \end{bmatrix}$$

Due to unit Load:

When $Q_1 = 1$



$$A_{MU} = \begin{bmatrix} -0.67 \\ -10 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D_{QL} = [A_{mu}]^t \times [F_m] \times [A_{mL}]$$

$$= \begin{bmatrix} -0.67 & -10 & 1 & 0 & 0 & 0 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 333.33 & -50 & 0 & 0 \\ 0 & 0 & -50 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 41.67 & -12.5 \\ 0 & 0 & 0 & 0 & -12.5 & 5 \end{bmatrix} \times \begin{bmatrix} 14.33 \\ 215 \\ -20 \\ 25 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 371.25 & -74.625 & 333.33 & -50 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 14.33 \\ 215 \\ -20 \\ 25 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} -18640.9625 \end{bmatrix}$$

$$[F] = [A_{mu}]^t \times [F_m] \times [A_{mu}]$$

$$= \frac{1}{EI} \times \begin{bmatrix} 371.25 & -74.625 & 333.33 & -50 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} -0.67 \\ -10 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 830.8425 \end{bmatrix}$$

$$\therefore F^{-1} = EI \begin{bmatrix} \frac{1}{830.8425} \end{bmatrix}$$

$$Q = -[F^{-1}] \times [D_{QL}] = EI \begin{bmatrix} \frac{1}{830.8425} \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} -18640.9625 \end{bmatrix}$$

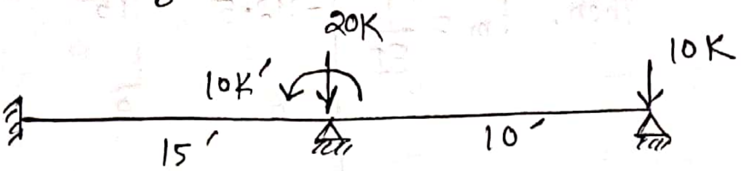
$$[Q_1] = \begin{bmatrix} 22.436 \end{bmatrix}$$

Now, $A_m = [A_{mL}] + [A_{mu}] \times [Q] + [A_{mR}]$

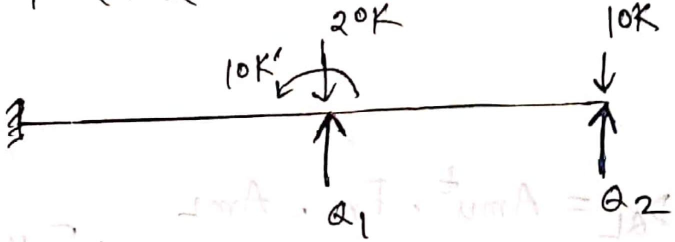
$$= \begin{bmatrix} 14.33 \\ 215 \\ -20 \\ 25 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.67 \\ -10 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 22.436 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.702 \\ -9.36 \\ 2.436 \\ 25 \\ 0 \\ 0 \end{bmatrix}$$

CT (2014 Series)

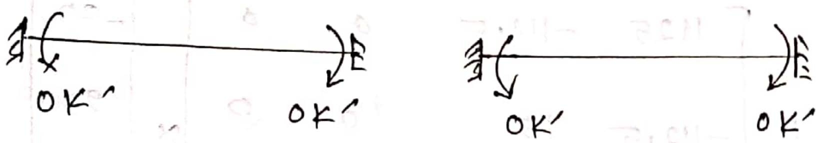
\# Analyze the beam by flexibility Matrix method. EI is constant.



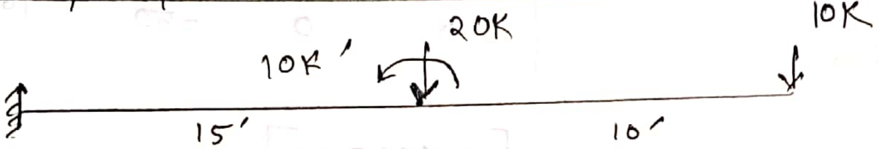
Solution: No. of Indeterminacy = 4 − 2 = 2

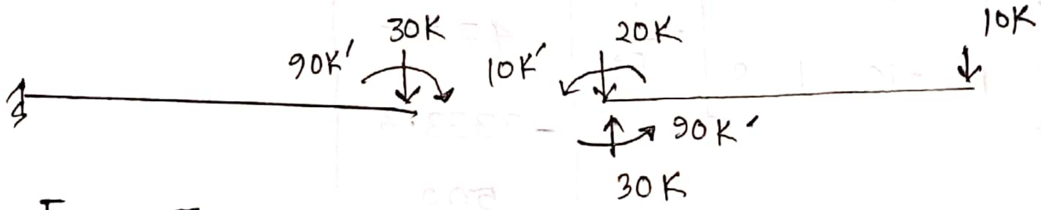


Fixed End Moment:



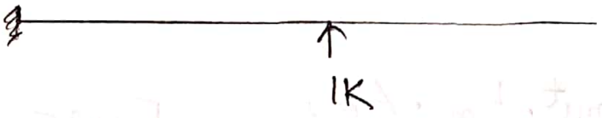
Replacement of Equivalent joint Force:



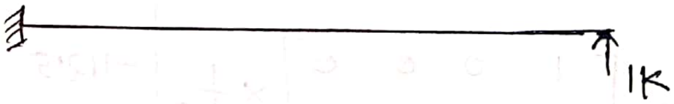


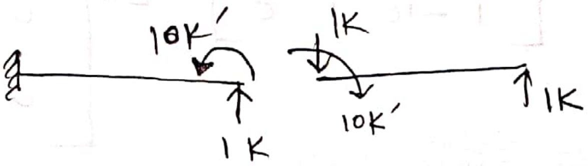
$$\therefore A_{mL} = \begin{bmatrix} -30 \\ 90 \\ -10 \\ 0 \end{bmatrix}$$

Case-1: $Q_1 = 1$, $Q_2 = 0$



Case-2: $Q_2 = 1$, $Q_1 = 0$





$$A_{mu} = \begin{array}{cc} Q_1=1 & Q_2=1 \\ \left[\begin{matrix} 1 \\ 0 \\ 0 \\ 0 \end{matrix}\right. & \left.\begin{matrix} 1 \\ -10 \\ 1 \\ 0 \end{matrix}\right] \end{array}$$

Then, $F_m = \frac{1}{EI}\begin{bmatrix} 1125 & -112.5 & 0 & 0 \\ -112.5 & 15 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix}$

$$D_{QL} = A_{mu}^t \cdot F_m \cdot A_{mL}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} 1125 & -112.5 & 0 & 0 \\ -112.5 & 15 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix} \times \begin{bmatrix} -30 \\ 90 \\ -10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} -43875 \\ 4725 \\ -3333.3 \\ 500 \end{bmatrix}$$

$$\therefore D_{QL} = \frac{1}{EI} \times \begin{bmatrix} -43875 \\ -94458.3 \end{bmatrix}$$

$$F = A_{mu}^t \cdot F_m \cdot A_{mu}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \cdot \begin{bmatrix} 1125 & -112.5 & 0 & 0 \\ -112.5 & 15 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -10 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} 1125 & 2250 \\ -112.5 & -262.5 \\ 0 & 333.33 \\ 0 & -50 \end{bmatrix}$$

$$\therefore F = \frac{1}{EI} \times \begin{bmatrix} 1125 & 2250 \\ 2250 & 5208.33 \end{bmatrix}$$

We know, $D_{QL} + FQ = 0$

$$\Rightarrow Q = [F]^{-1} \times [D_{QL}]$$

$$= -EI \times \begin{bmatrix} 1125 & 2250 \\ 2250 & 5208.33 \end{bmatrix}^{-1} \times \frac{1}{EI} \begin{bmatrix} -43875 \\ -94458.3 \end{bmatrix}$$

$$= \frac{1}{796871.25} \times \begin{bmatrix} 5208.33 & -2250 \\ -2250 & 1125 \end{bmatrix} \times \begin{bmatrix} 43875 \\ 94458.3 \end{bmatrix}$$

$$= \frac{1}{796871.25} \times \begin{bmatrix} 15984303.75 \\ 7546837.5 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 20.06 \\ 9.47 \end{bmatrix}$$

Then, $A_m = A_{mL} + A_{mu} \cdot Q + A_{mR}$

$$= \begin{bmatrix} -30 \\ 90 \\ -10 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 20.06 \\ 9.47 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

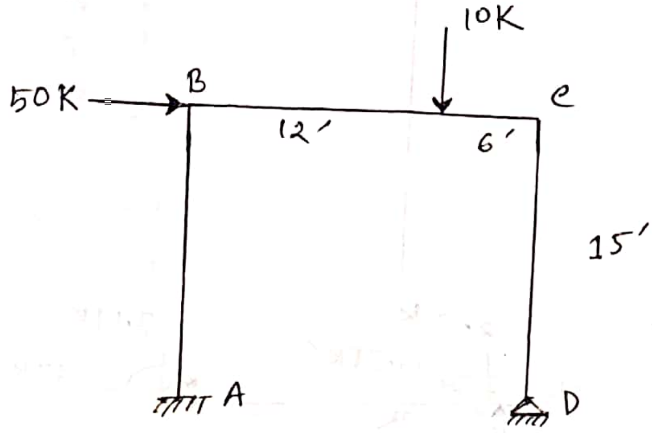
$$= \begin{bmatrix} -30 \\ 90 \\ -10 \\ 0 \end{bmatrix} + \begin{bmatrix} 29.53 \\ -94.7 \\ 9.47 \\ 0 \end{bmatrix}$$

$$\therefore A_m = \begin{bmatrix} -0.47 \\ -4.7 \\ -0.53 \\ 0 \end{bmatrix}$$

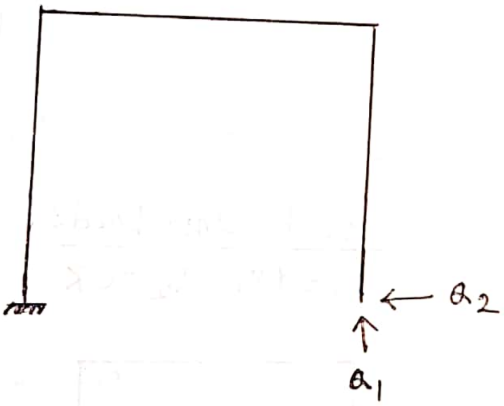
(Ans.)

FRAME (Flexibility Method)

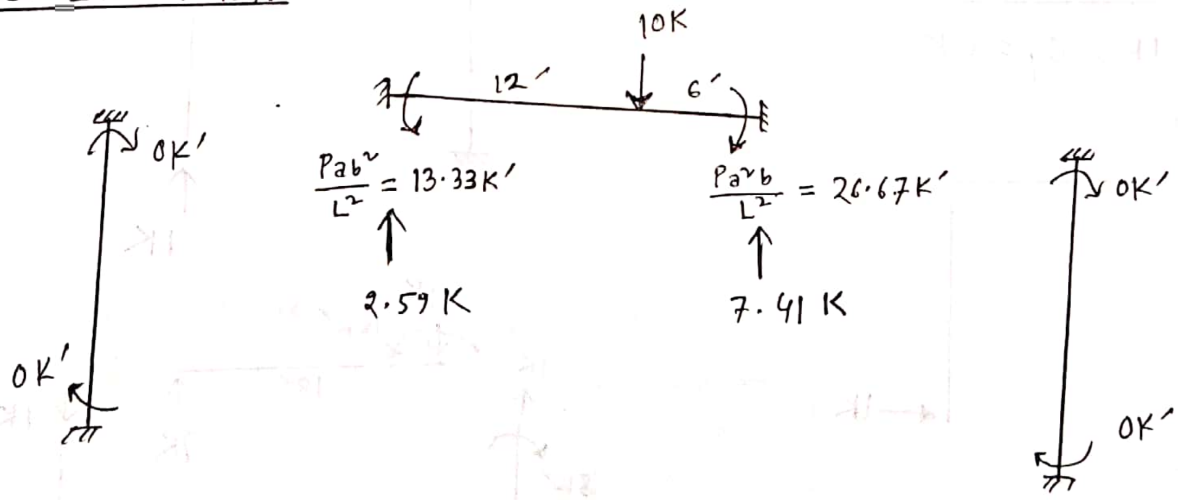
Problem-1: Analyze the frame by flexibility Matrix Method



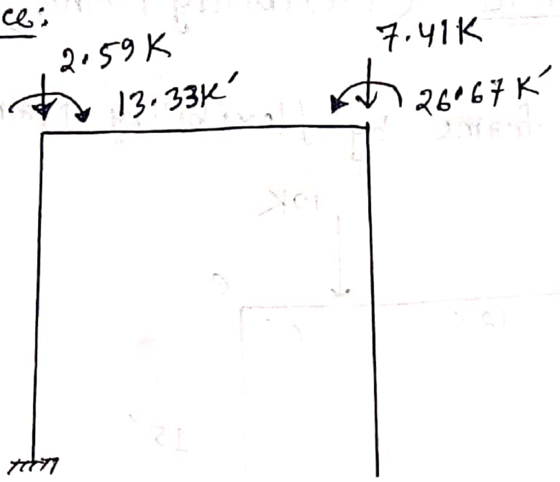
Solution: No. of Indeterminacy = 5 - 3 = 2

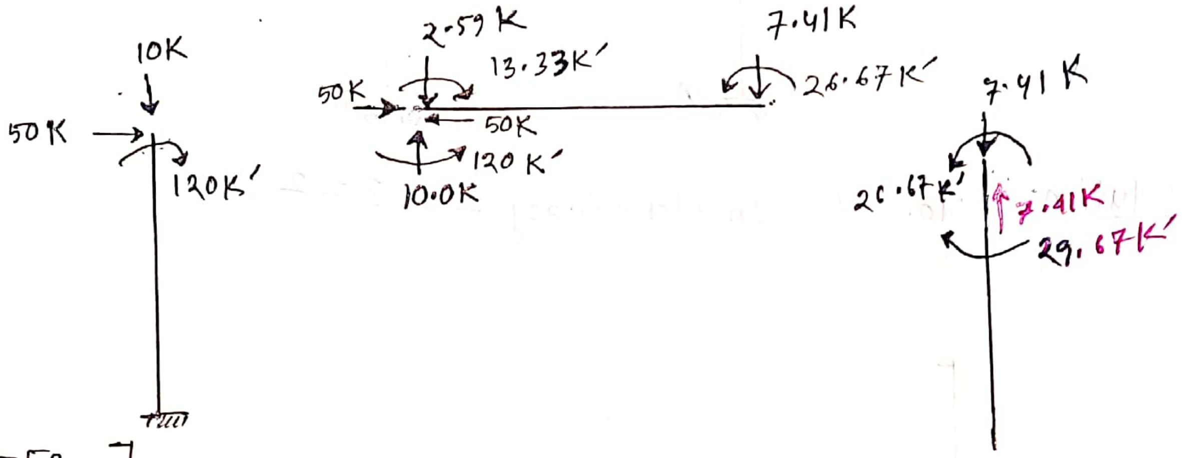


Fixed End Moment:



Equivalent joint Force:

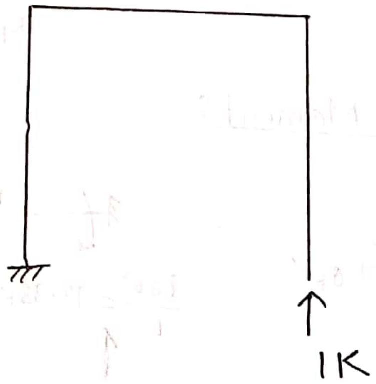


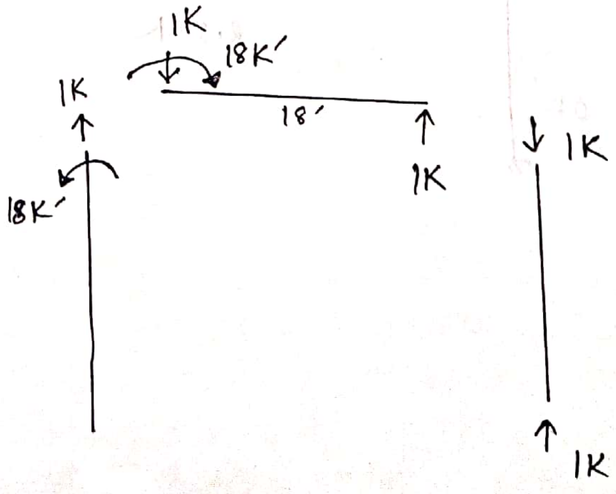


$$\therefore A_{mL} = \begin{bmatrix} -50 \\ 120 \\ -7.41 \\ -26.67 \\ 0 \\ 0 \end{bmatrix}$$

Due to Unit Load:

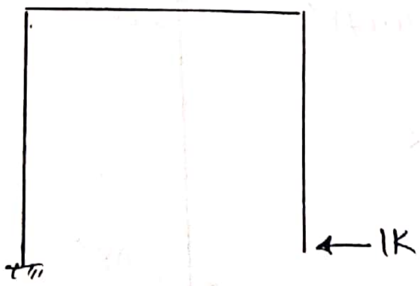
$Q_1 = 1K, \; Q_2 = 0K$

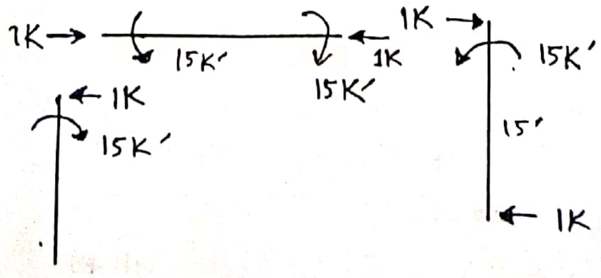




Due to Unit Load:

$Q_2 = 1K, \; Q_1 = 0K$





$$\therefore A_{mu} = \begin{array}{c} \begin{array}{cc} Q_1=1 & Q_2=1 \end{array} \\ \begin{bmatrix} 0 & 1 \\ -18 & 15 \\ 1 & 0 \\ 0 & 15 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$$

$$F_m = \frac{1}{EI} \begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1944 & -162.0 & 0 & 0 \\ 0 & 0 & -162.0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1125 & -112.5 \\ 0 & 0 & 0 & 0 & -112.5 & 15 \end{bmatrix}$$

$$D_{QL} = [A_{mu}]^t \, [F_m] \, [A_{mL}]$$

$$= \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 15 & 0 & 15 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1944 & -162.0 & 0 & 0 \\ 0 & 0 & -162.0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1125 & -112.5 \\ 0 & 0 & 0 & 0 & -112.5 & 15 \end{bmatrix} \times \begin{bmatrix} -50 \\ 120 \\ -7.41 \\ -26.67 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 15 & 0 & 15 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} -69750 \\ 7425 \\ -10084.5 \\ 720.36 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} -143734.5 \\ 52430.4 \end{bmatrix}$$

$$F = [A_{mu}]^{t} \times F_m \times A_{mu}$$

$$= \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 15 & 0 & 15 & 1 & 0 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1944 & -162 & 0 & 0 \\ 0 & 0 & -162 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1125 & -112.5 \\ 0 & 0 & 0 & 0 & -112.5 & 15 \end{bmatrix} \times$$

$$= \frac{1}{EI} \times \begin{bmatrix} 2025 & -270 & 1944 & -162 & 0 & 0 \\ -562.5 & 112.5 & -2430 & 270 & 1125 & -112.5 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -18 & 15 \\ 1 & 0 \\ 0 & 15 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ -18 & 15 \\ 1 & 0 \\ 0 & 15 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 6804 & -4455 \\ -4455 & 6300 \end{bmatrix}$$

$$Q = -[F^{-1}][D_{QL}]$$

$$= -EI \times \frac{1}{23018175} \times \begin{bmatrix} 6300 & 4455 \\ 4455 & 6804 \end{bmatrix} \times \frac{1}{EI} \times \begin{bmatrix} -143734.5 \\ 52430.4 \end{bmatrix}$$

$$= \frac{-1}{23018175} \times \begin{bmatrix} -671949918 \\ -283600755.9 \end{bmatrix}$$

$$= \begin{bmatrix} 29.19 \\ 12.32 \end{bmatrix}$$

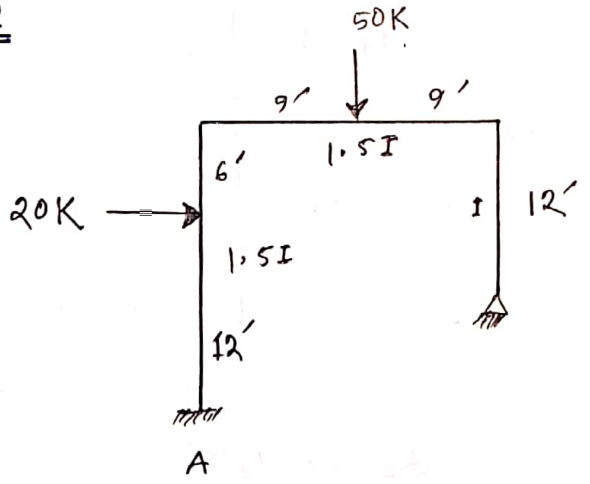
$$A_m = [A_{mL}] + [A_{mu}] \cdot [Q] + [A_{mR}]$$

$$= \begin{bmatrix} -50 \\ 120 \\ -7.41 \\ -26.67 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -18 & 15 \\ 1 & 0 \\ 0 & 15 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 29.19 \\ 12.32 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 7.41 \\ 26.67 \\ 0 \\ 0 \end{bmatrix}$$

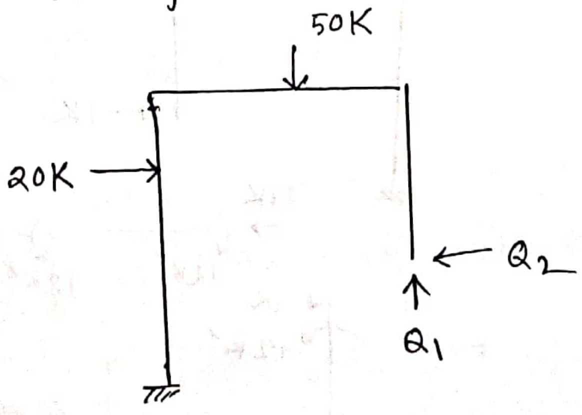
$$= \begin{bmatrix} -50 \\ 120 \\ -7.41 \\ -26.67 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 12.32 \\ -340.62 \\ 29.19 \\ 184.8 \\ 12.32 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 7.41 \\ 26.67 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 37.68 \\ -220.62 \\ 29.19 \\ 184.8 \\ 12.32 \\ 0 \end{bmatrix}$$

(Ans.)

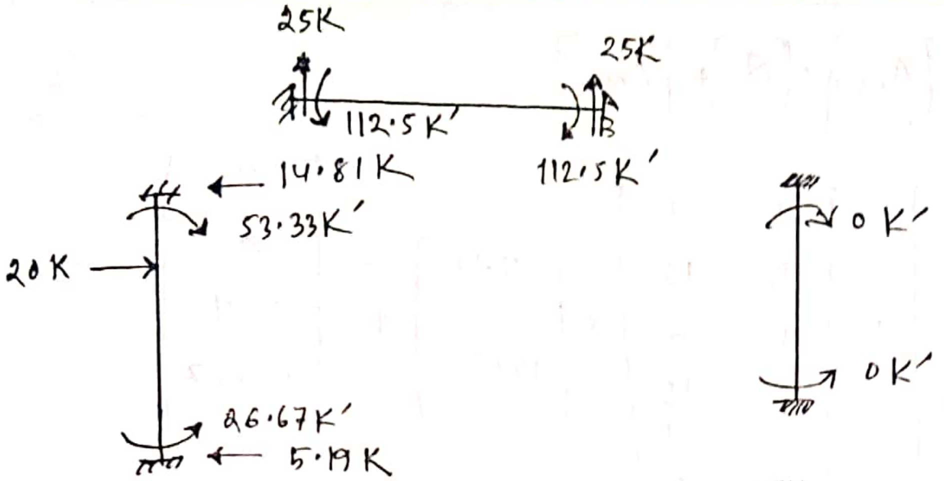
Problem: 02



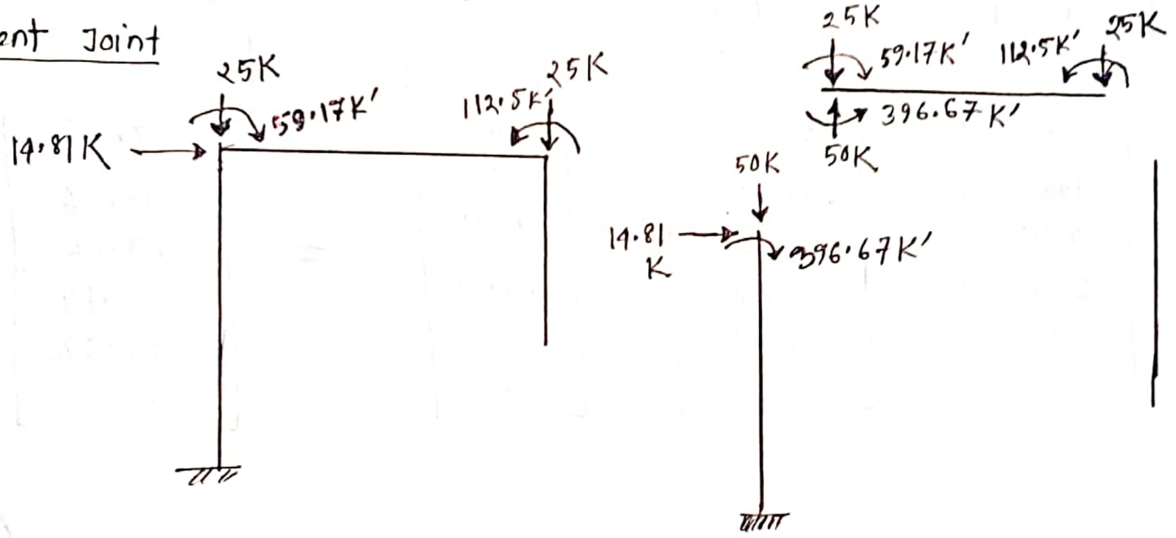
Solution: No. of Indeterminacy = 5-3 = 2



F.E.M:



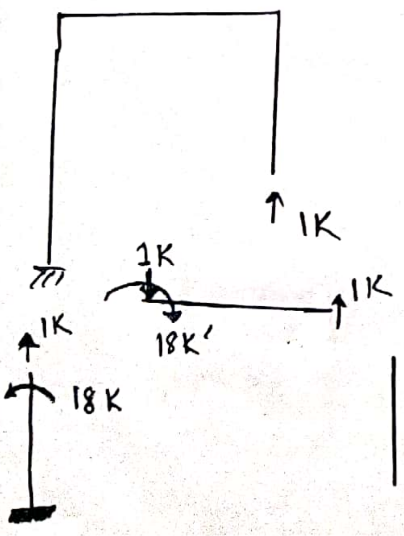
Equivalent Joint force:



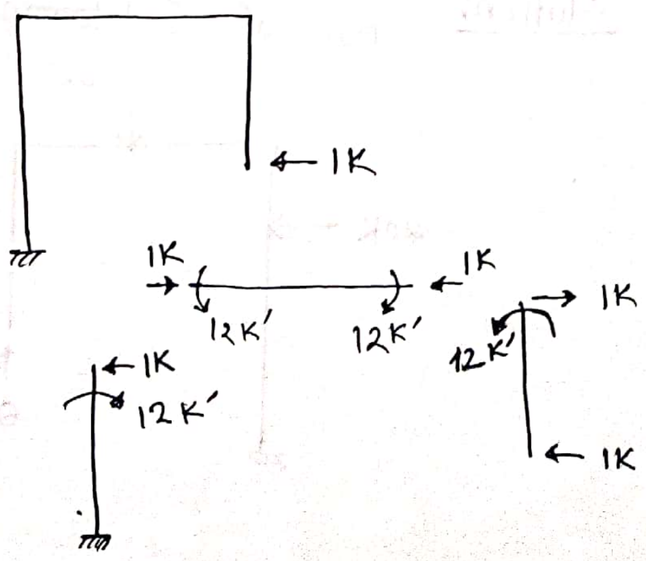
$$A_{mL} = \begin{bmatrix} -14.81 \\ 396.67 \\ -25 \\ -112.5 \\ 0 \\ 0 \end{bmatrix} \qquad A_{mR} = \begin{bmatrix} 14.81 \\ 53.33 \\ 25 \\ 112.5 \\ 0 \\ 0 \end{bmatrix}$$

Due to unit load:

$Q_1 = 1$, $Q_2 = 0$



$Q_2 = 1$, $Q_1 = 0$



$$A_{mu} = \begin{array}{cc} Q_1=1 & Q_2=1 \\ \left[\begin{matrix} 0 \\ -18 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix}\right. & \left.\begin{matrix} 1 \\ 12 \\ 0 \\ 12 \\ 1 \\ 0 \end{matrix}\right] \end{array}$$

$$F_m = \frac{1}{EI} \times \begin{bmatrix} 1296 & -108 & 0 & 0 & 0 & 0 \\ -108 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1296 & -108 & 0 & 0 \\ 0 & 0 & -108 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 576 & -72 \\ 0 & 0 & 0 & 0 & -72 & 12 \end{bmatrix}$$

We know,

$$D_{QL} = [A_{mu}]^t . [F_m] \, [A_m]$$

$$= \frac{1}{EI} \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 12 & 0 & 12 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1296 & -108 & 0 & 0 & 0 & 0 \\ -108 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1276 & -108 & 0 & 0 \\ 0 & 0 & -108 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 576 & -72 \\ 0 & 0 & 0 & 0 & -72 & 12 \end{bmatrix} \times \begin{bmatrix} -14.81 \\ 396.67 \\ -25 \\ -112.5 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 1944 & -216 & 1296 & -108 & 0 & 0 \\ 0 & 36 & -1296 & 144 & 576 & 72 \end{bmatrix} \times \begin{bmatrix} -14.81 \\ 396.67 \\ -25 \\ -112.5 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} -134721.36 \\ 30980.12 \end{bmatrix}$$

$$F = [A_{mu}]^t . F_m . [A_{mu}]$$

$$= \frac{1}{EI} \times \begin{bmatrix} 1944 & -216 & 1296 & -108 & 0 & 0 \\ 0 & 36 & -1296 & 144 & 576 & 72 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -18 & 12 \\ 1 & 0 \\ 0 & 12 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 5184 & -1944 \\ -1944 & 2736 \end{bmatrix}$$

Now,

$$Q = -\begin{bmatrix} F^{-1} \end{bmatrix} \times \begin{bmatrix} D_{QL} \end{bmatrix}$$

$$= -EI \times \frac{1}{10404288} \times \begin{bmatrix} 2736 & 1944 \\ 1944 & 5184 \end{bmatrix} \times \begin{bmatrix} -134721.36 \\ 30480.12 \end{bmatrix}$$

$$= \begin{bmatrix} 29.73 \\ 9.99 \end{bmatrix}$$

Then,

$$A_m = \begin{bmatrix} A_{mL} \end{bmatrix} + \begin{bmatrix} A_{mu} \end{bmatrix}\begin{bmatrix} Q \end{bmatrix} + \begin{bmatrix} A_{mR} \end{bmatrix}$$

$$= \begin{bmatrix} -14.81 \\ 396.67 \\ -25 \\ -112.5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 9.99 \\ -415.26 \\ 29.73 \\ 119.88 \\ 9.99 \\ 0 \end{bmatrix} + \begin{bmatrix} 14.81 \\ 53.33 \\ 25 \\ 112.5 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9.99 \\ 34.74 \\ 29.73 \\ 119.88 \\ 9.99 \\ 0 \end{bmatrix}$$

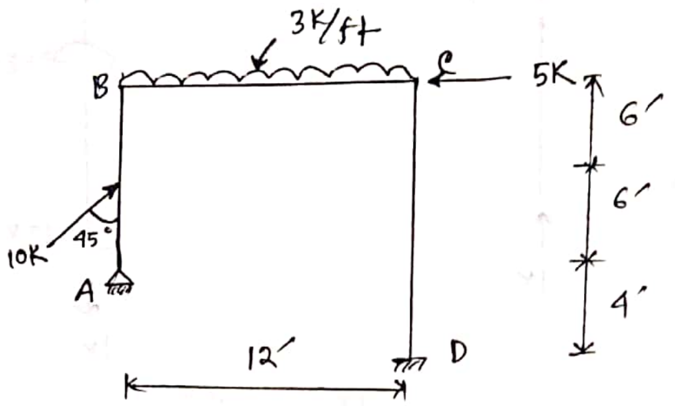
(Ans)

Flexibility Matrix Method

FRAME

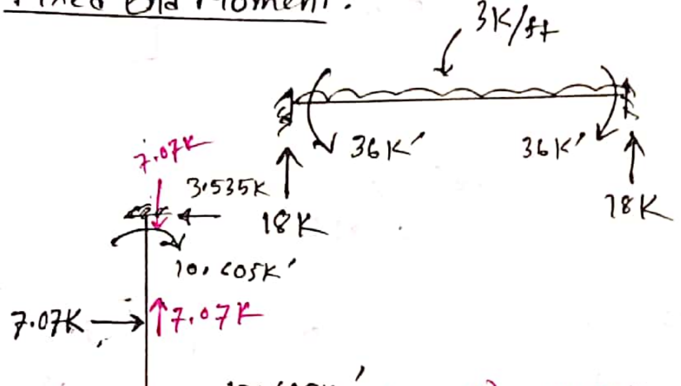
2018

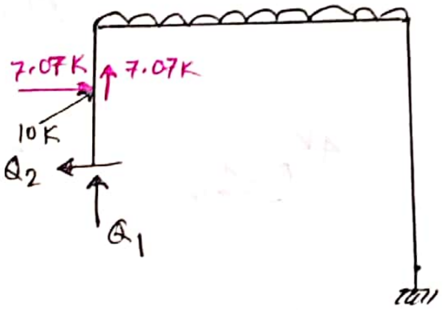
Analyze the frame. EI is constant



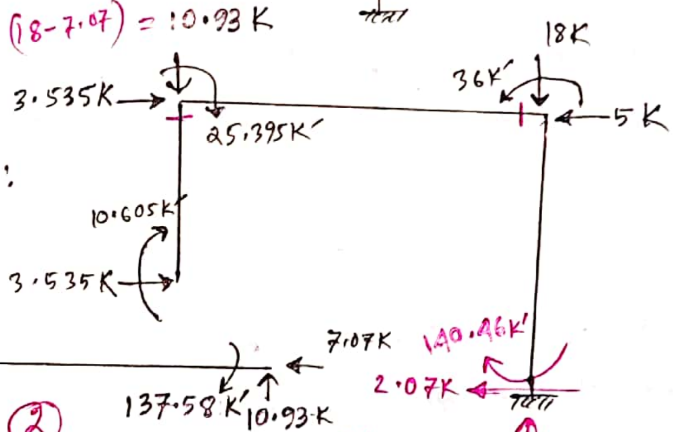
Solution: No. of Indeterminacy = 5-3=2

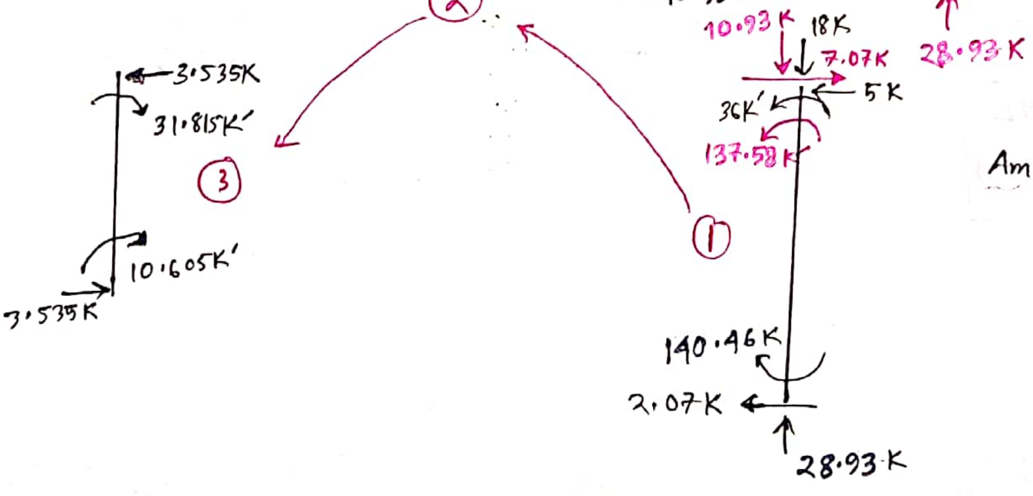
Fixed End Moment:





Equivalent Joint force:

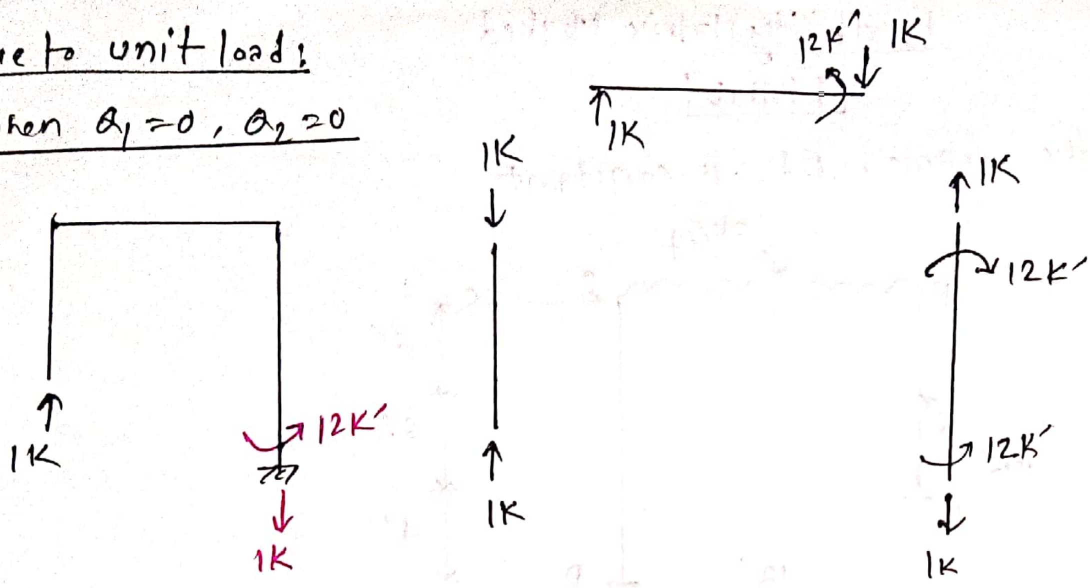




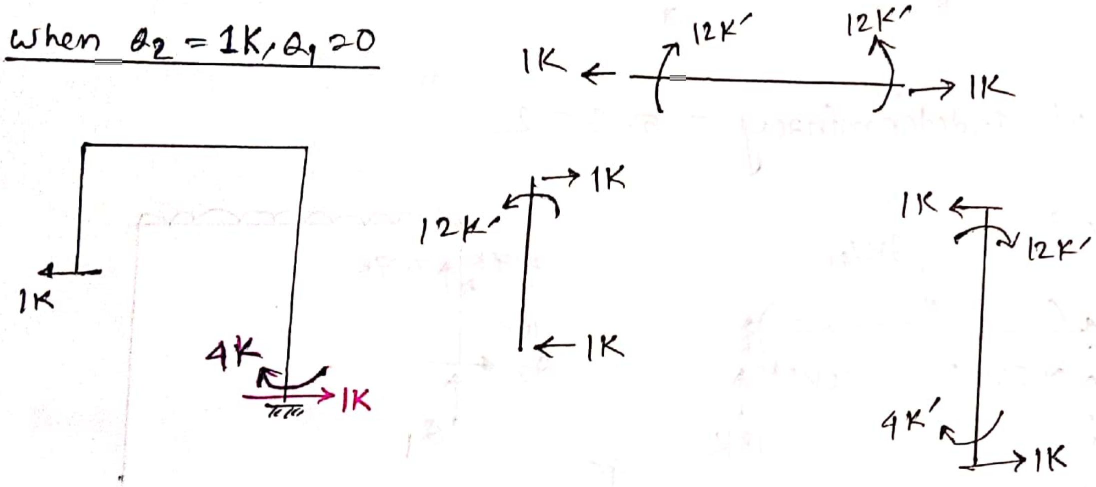
$$A_{mL} = \begin{bmatrix} 3.535 \\ 31.815 \\ 10.93 \\ 137.58 \\ 2.07 \\ 140.46 \end{bmatrix}$$

Due to unit load:

when $Q_1 = 0, Q_2 = 0$



when $Q_2 = 1K, Q_1 = 0$



$$A_{mu} = \begin{array}{cc} Q_1 = 1 & Q_2 = 1 \\ \left[\begin{matrix} 0 & -1 \\ 0 & -12 \\ -1 & 0 \\ -12 & -12 \\ 0 & -1 \\ -12 & 4 \end{matrix}\right] \end{array}$$

$$D_{QL} = [A_{mu}]^t \times [F_m] \times [A_{mL}]$$

$$= \begin{bmatrix} 0 & 0 & -1 & -12 & 0 & -12 \\ -1 & -12 & 0 & -12 & -1 & 4 \end{bmatrix} \times \frac{1}{EI} \begin{bmatrix} 576 & -72 & 0 & 0 & 0 & 0 \\ -72 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 576 & -72 & 0 & 0 \\ 0 & 0 & -72 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1365.33 & -128 \\ 0 & 0 & 0 & 0 & -128 & 16 \end{bmatrix} \times \begin{bmatrix} 3.535 \\ 31.815 \\ 10.93 \\ 137.58 \\ 2.07 \\ 140.46 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 0 & 0 & 288 & -72 & 1536 & -192 \\ 288 & -72 & 864 & -144 & -18\cancel{77}.33 & 192 \end{bmatrix} \times [A_{mL}]$$

$$= \frac{1}{EI} \times \begin{bmatrix} -30546.72 \\ 11441.6469 \end{bmatrix}$$

8.

$$F = [A_{mu}]^t \times [F_m] \times [A_{mu}]$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & 0 & 288 & -72 & 1536 & -192 \\ 288 & -72 & 864 & -144 & -1877.33 & 192 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 0 & -12 \\ -1 & 0 \\ -12 & -12 \\ 0 & -1 \\ -12 & 9 \end{bmatrix}$$

$$= \frac{1}{EI} \times \begin{bmatrix} 2880 & -1440 \\ -1440 & 4949.33 \end{bmatrix}$$

$$Q = -[F^{-1}][D_{QL}] = \frac{-EI}{12160470.4} \times \begin{bmatrix} 4949.33 & 1440 \\ 1440 & 2880 \end{bmatrix} \times \frac{1}{EI}\begin{bmatrix} -30546.72 \\ 11441.6469 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 11.06 \\ 0.906 \end{bmatrix}$$

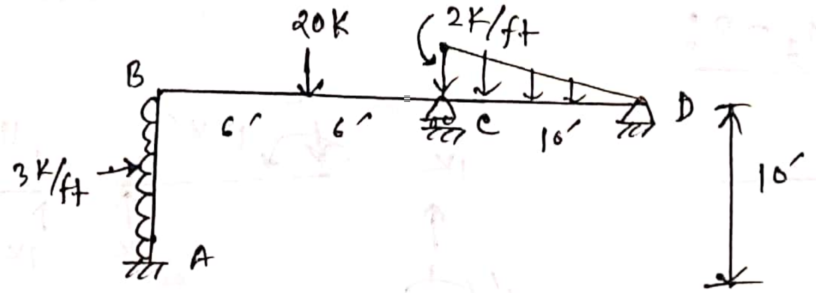
$$A_m = [A_{mL}] + [A_{mu}] \times Q + [A_{mR}]$$

$$= \begin{bmatrix} 3.535 \\ 31.815 \\ 10.93 \\ 137.58 \\ 2.07 \\ 140.46 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & -12 \\ -1 & 0 \\ -12 & -12 \\ 0 & -1 \\ -12 & 4 \end{bmatrix} \times \begin{bmatrix} 11.06 \\ 0.906 \end{bmatrix} + \begin{bmatrix} 3.535 \\ 10.605 \\ 18 \\ 36 \\ 0 \\ 0 \end{bmatrix}$$

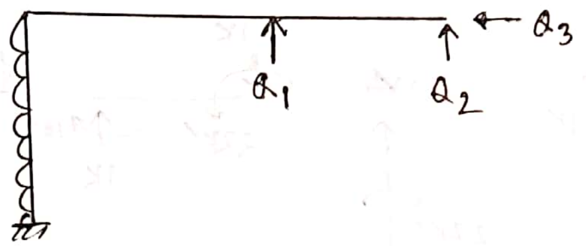
$$= \begin{bmatrix} 7.07 \\ 42.42 \\ 28.93 \\ 173.38 \\ 2.07 \\ 140.46 \end{bmatrix} + \begin{bmatrix} -0.906 \\ -10.872 \\ -11.06 \\ -143.592 \\ -0.906 \\ -129.096 \end{bmatrix} = \begin{bmatrix} 6.164 \\ 31.548 \\ 17.87 \\ 29.408 \\ 1.164 \\ 11.364 \end{bmatrix}$$

2017

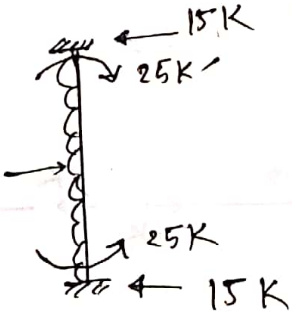
Analyze the frame. $E = 30\times10^3$ ksi, $I = 1000$ in^4

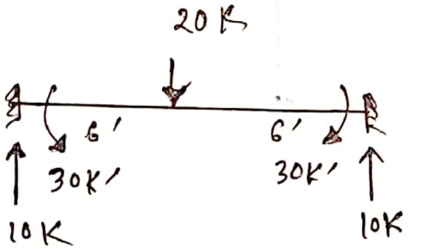


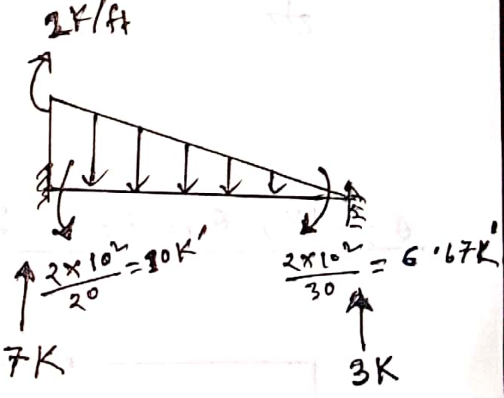
Solution: No. of indeterminacy = 6 − 3 = 3



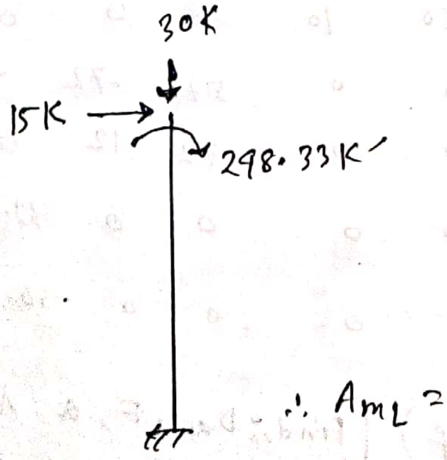
FEM:

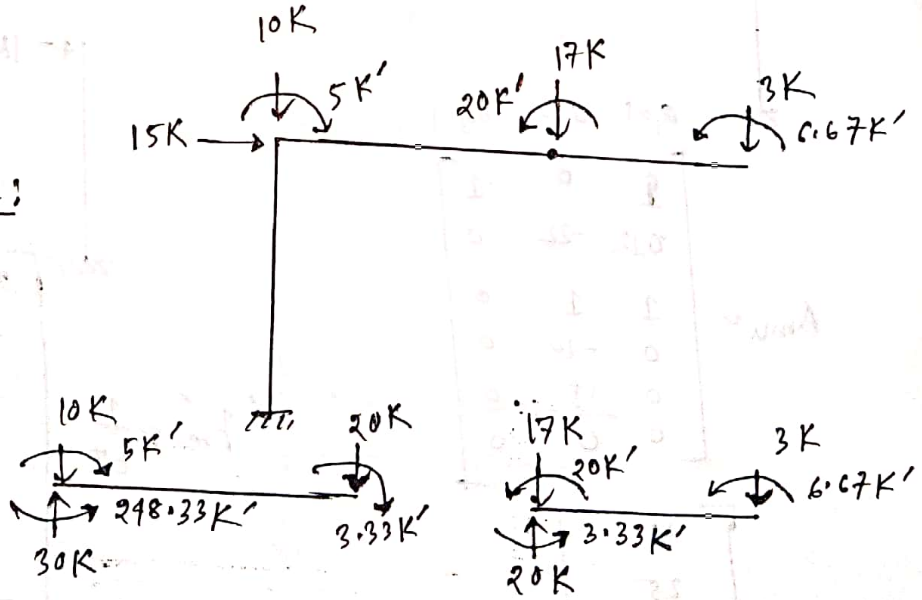






Equivalent Joint Force:

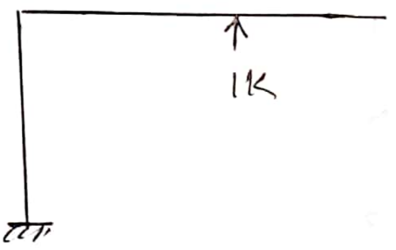


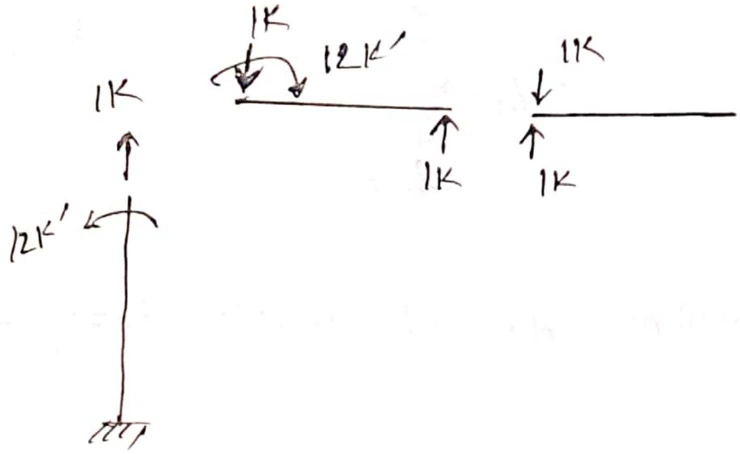


$$\therefore A_{ML} = \begin{bmatrix} -15 \\ 298.33 \\ -20 \\ 3.33 \\ -3 \\ -6.67 \end{bmatrix}$$

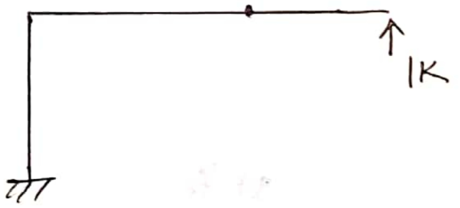
Due to unit Load:

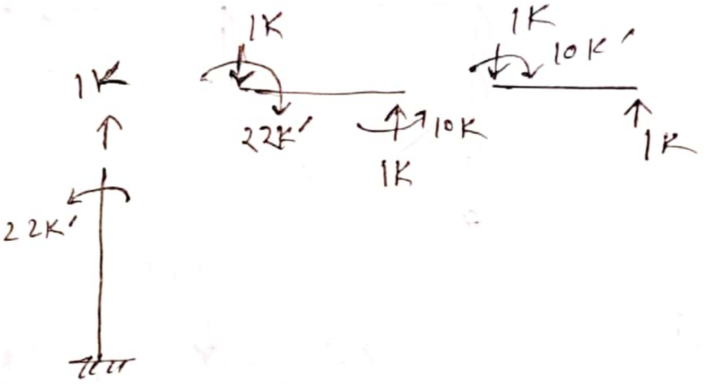
$Q_1 = 1$, $Q_2 = 0$, $Q_3 = 0$:



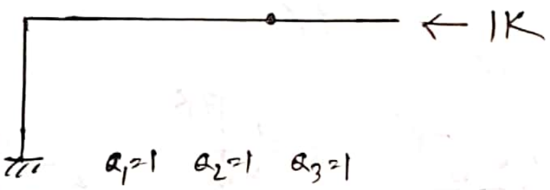


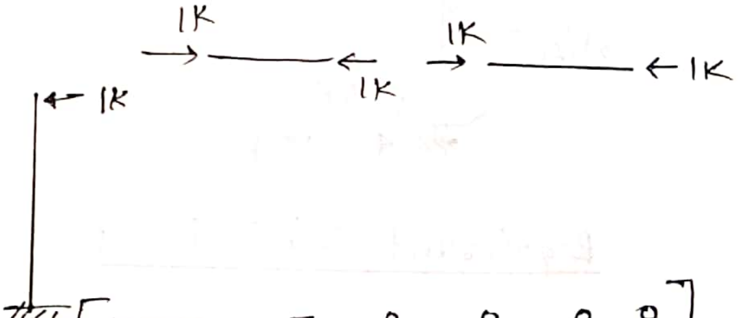
$Q_2 = 1K$, $Q_1 = Q_3 = 0$:





$Q_3 = 1K$, $Q_1 = Q_2 = 0$:





$$A_{mu} = \begin{array}{c} \begin{array}{ccc} Q_1=1 & Q_2=1 & Q_3=1 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ -12 & -22 & 0 \\ 1 & 1 & 0 \\ 0 & -10 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

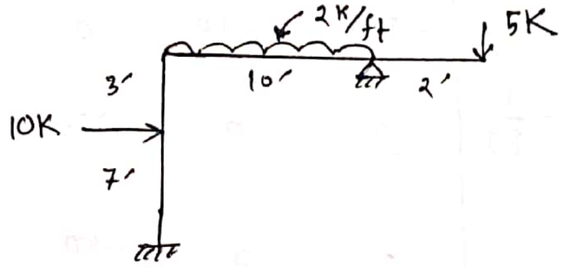
$$F_m = \frac{1}{EI}\begin{bmatrix} 333.33 & -50 & 0 & 0 & 0 & 0 \\ -50 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 576 & -72 & 0 & 0 \\ 0 & 0 & -72 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 333.33 & -50 \\ 0 & 0 & 0 & 0 & -50 & 10 \end{bmatrix}$$

$$A_{mR} = \begin{bmatrix} 15 \\ 25 \\ 10 \\ 30 \\ 3 \\ 6.67 \end{bmatrix}$$

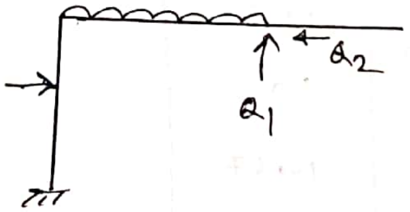
Then, (do yourself) [Find: D_{RL}, F, Q, Am]

2016

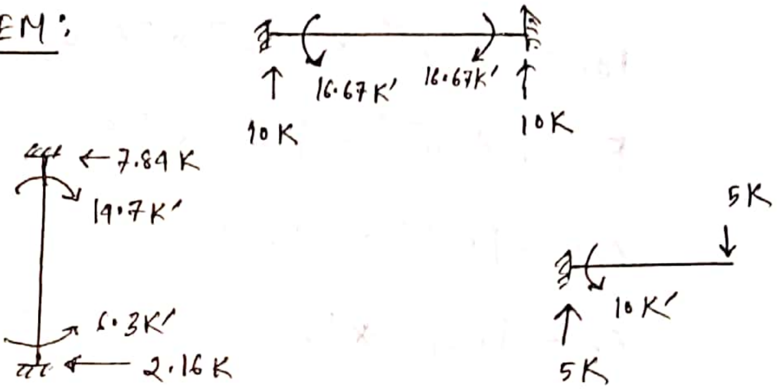
Analyze the frame & Draw BMD . EI is constant



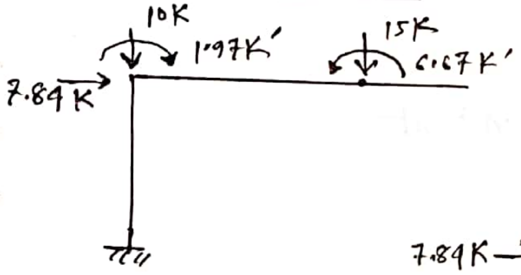
Solution: No. of Indeterminacy = 5 − 3 = 2

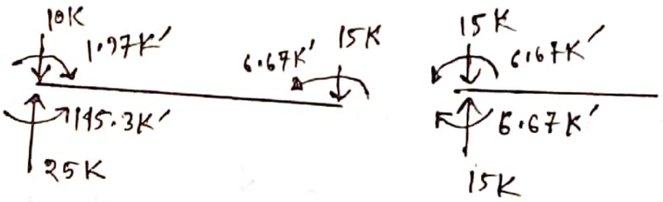


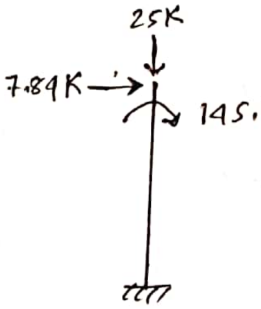
FEM:



Equivalent Joint Force:



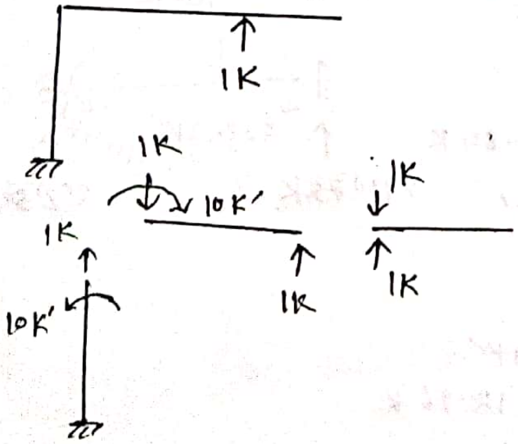




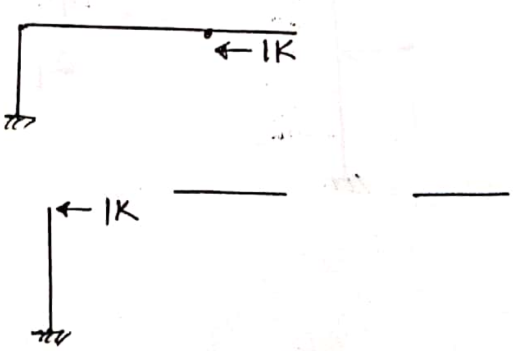
$$\therefore A_{mL} = \begin{bmatrix} -7.84 \\ 145.3 \\ -15 \\ -6.67 \end{bmatrix}$$

Due to Unit Load:

$Q_1 = 1K$, $Q_2 = 0$



$Q_2 = 1K$, $Q_1 = 1K$:



$$A_{mu} = \begin{array}{c} \begin{array}{cc} Q_1=1 & Q_2=1 \end{array} \\ \begin{bmatrix} 0 & 1 \\ -10 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 333.33 & -50 & 0 & 0 \\ -50 & 10 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 10 \end{bmatrix}$$

Then,

(do yourself)

$$D_{QL} = [A_{mu}]^t \times [F_m] \times [A_{mL}]$$

$$F = [A_{mu}]^t \times [F_m] \times [A_{mu}]$$

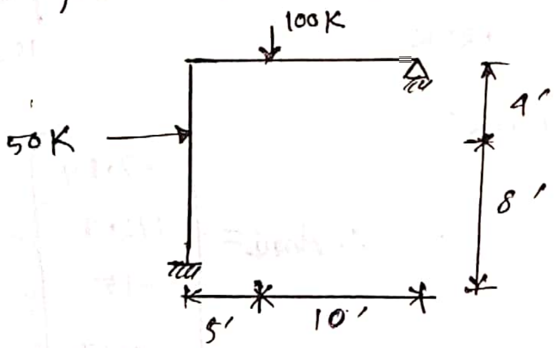
$$Q = -[F^{-1}] \times [D_{QL}]$$

$$A_m = [A_{mL}] + [A_{mu}] \times [Q] + A_{mR}$$

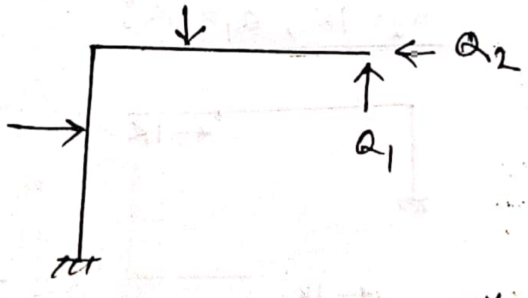
$$A_{mR} = \begin{bmatrix} 7.84 \\ 14.7 \\ 10 \\ 16.67 \end{bmatrix}$$

2015

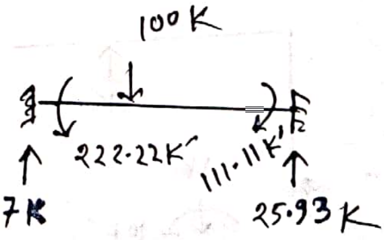
\# Analyze the frame & Draw BMD. EI is constant

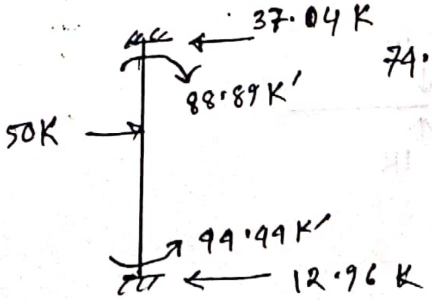


Solution: No. of Indeterminacy = 5 - 3 = 2



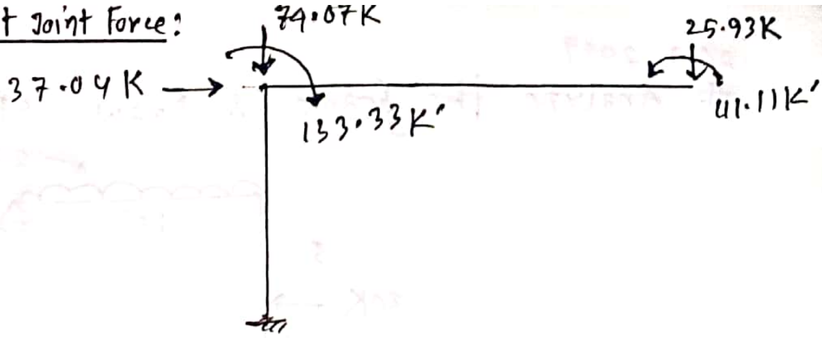
FEM:

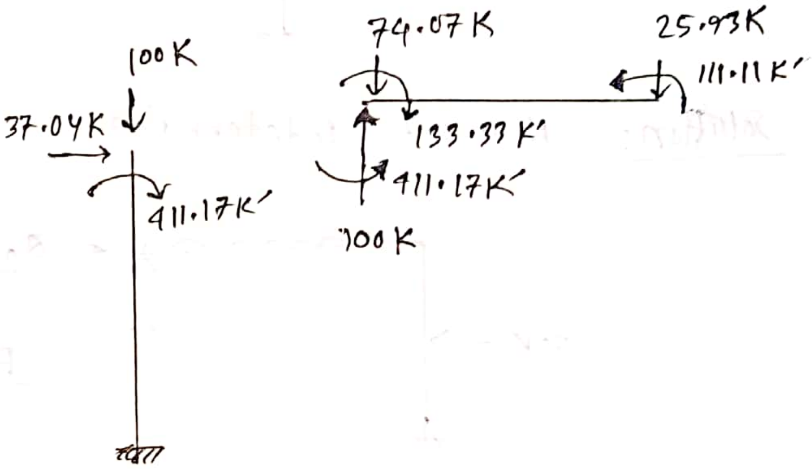




$$\therefore A_{ML} = \begin{bmatrix} -37.04 \\ 411.17' \\ -25.93 \\ -111.11 \end{bmatrix}$$

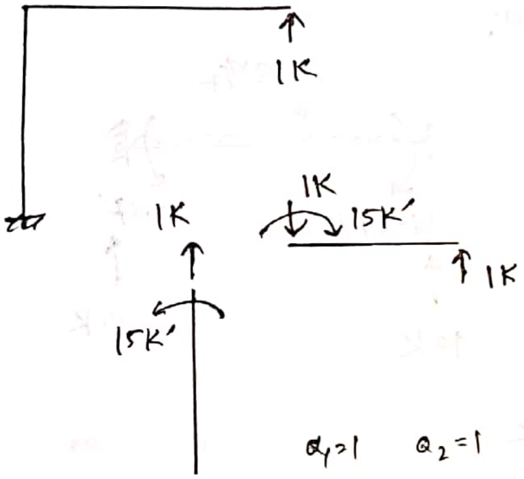
Equivalent Joint Force:



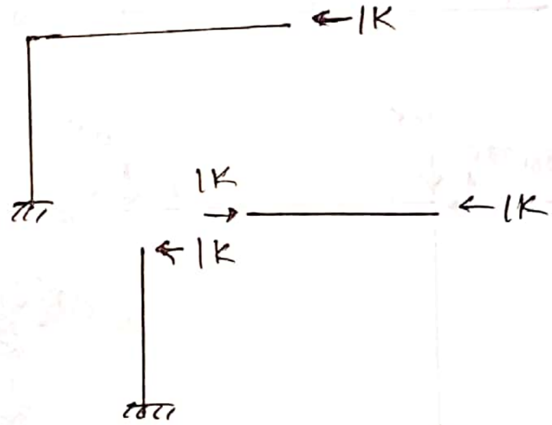


Due to Unit Load:

$Q_1 = 1K,\ Q_2 = 0$:



$Q_1 = 0K,\ Q_2 = 1K$!



$$\therefore A_{mu} = \begin{array}{cc} Q_1=1 & Q_2=1 \\ \left[\begin{matrix} 0 & 1 \\ -15 & 0 \\ 1 & 0 \\ 0 & 0 \end{matrix}\right] \end{array}$$

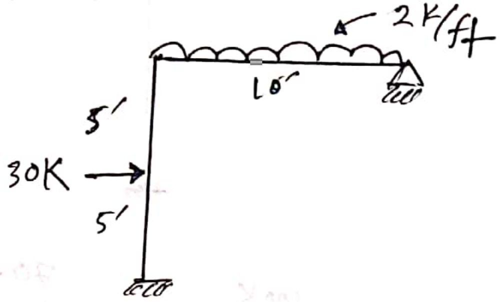
$$F_m = \frac{1}{EI} \times \begin{bmatrix} 576 & -72 & 0 & 0 \\ -72 & 12 & 0 & 0 \\ 0 & 0 & 1125 & -112.5 \\ 0 & 0 & -112.5 & 15 \end{bmatrix}$$

$$A_{mR} = \begin{bmatrix} 37.04 \\ 88.89 \\ 25.93 \\ 111.11 \end{bmatrix}$$

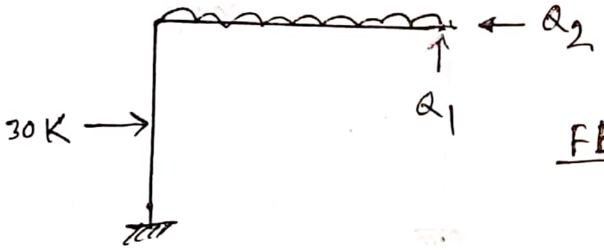
Then, (do yourself)

2014, 2009

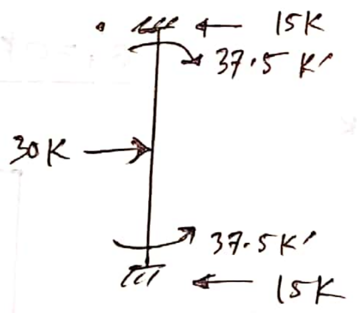
\# Analyze the frame & Draw SFD and BMD. EI is constant.

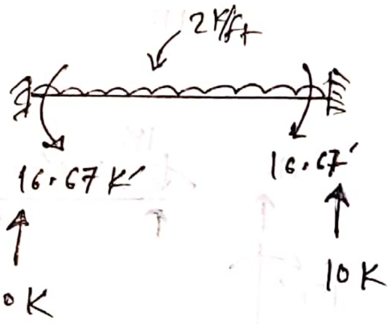


Solution: No. of Indeterminacy = 5 − 3 = 2

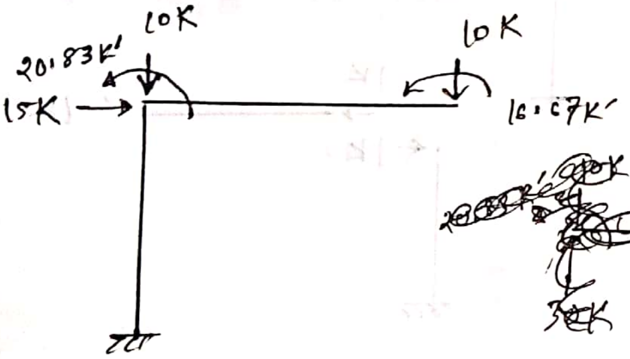


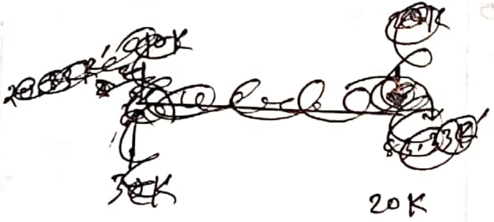
FEM:

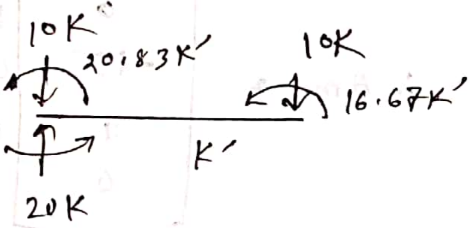


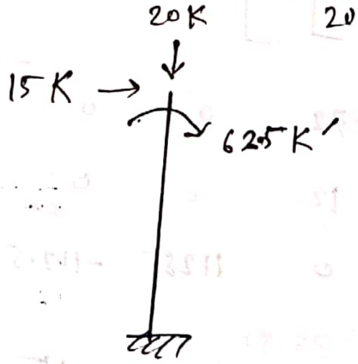


Equivalent joint forces





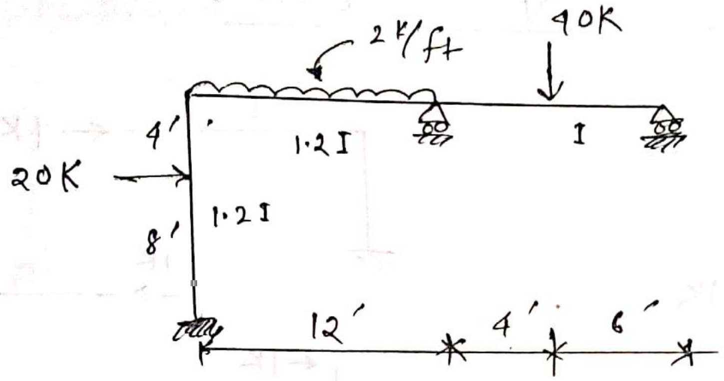




$$A_{ML} = \begin{bmatrix} -15 \\ 12.5 \\ -10 \\ -16.67 \end{bmatrix}$$

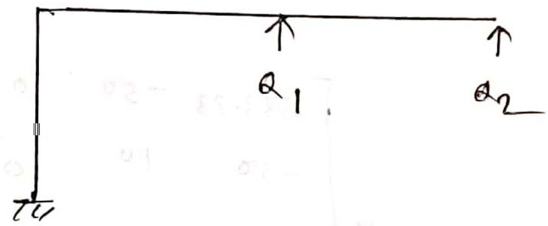
2013

Analyze the frame. Draw SF and BM diagrams.

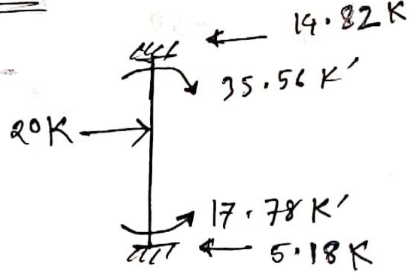


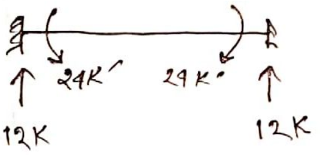
Solution:

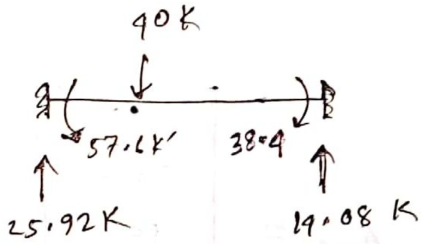
No. of Indeterminacy = 5-3 = 2

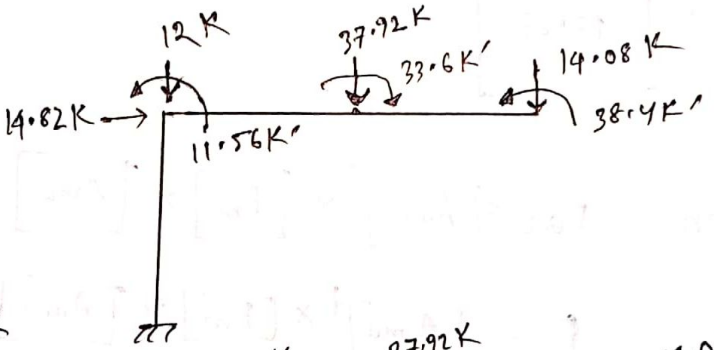


FEM

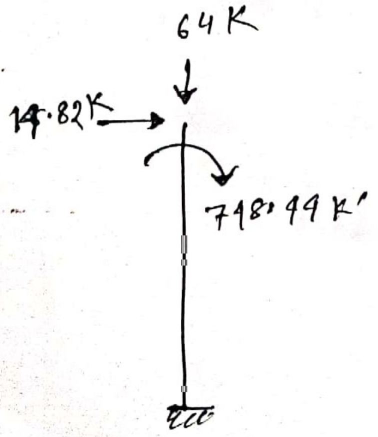


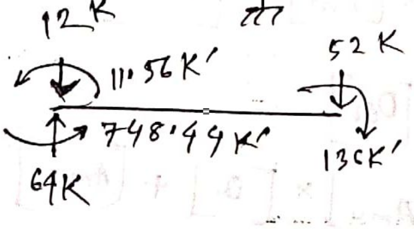


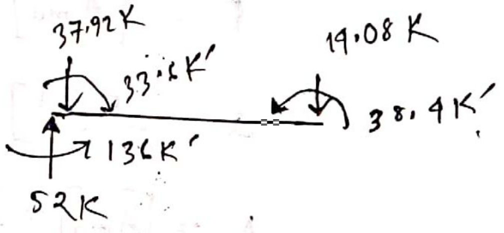




Equivalent Joint Force:





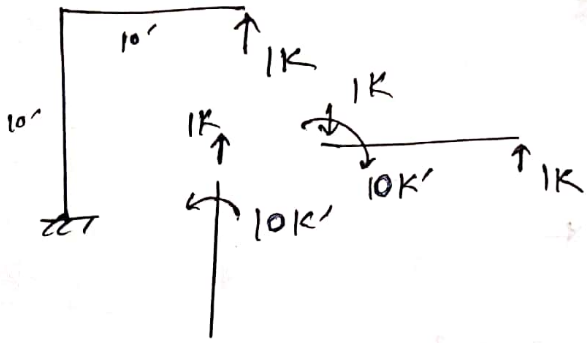


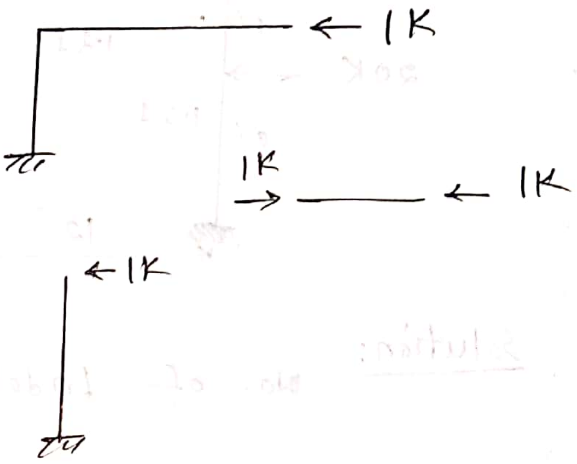
$$A_{ML} = \begin{bmatrix} -14.82 \\ 748.44 \\ -52 \\ 136 \\ -14.08 \\ -38.4 \end{bmatrix}.$$

Due to unit load:

$Q_1 = 1K,\ Q_2 = 0$:

$Q_1 = 0,\ Q_2 = 1K$:





$$A_{mu} = \left[\begin{array}{cc} Q_1=1, & Q_2=1 \\ 0 & 1 \\ -10 & 0 \\ 1 & 0 \\ 0 & 0 \end{array}\right]$$

$$F_m = \frac{1}{EI} \times \begin{bmatrix} 333.33 & -50 & 0 & 0 \\ -50 & 10 & 0 & 0 \\ 0 & 0 & 333.33 & -50 \\ 0 & 0 & -50 & 0 \end{bmatrix}$$

$$A_{mR} = \begin{bmatrix} 15 \\ 37.5 \\ 10 \\ 16.67 \end{bmatrix}$$

Then, $D_{QL} = [A_{mu}]^t \times [F_m] \times [A_{mL}]$

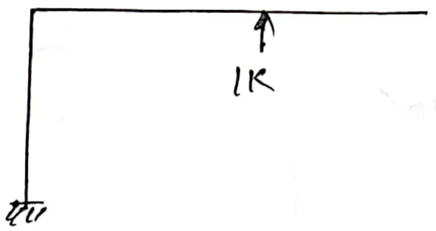
$F = [A_{mu}]^t \times [F_m] \times [A_{mu}]$

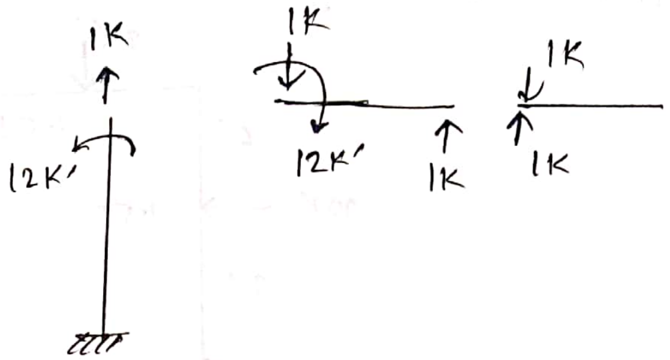
$Q = -[F^{-1}] \times [D_{QL}]$

$A_m = [A_{mL}] + [A_{mu}] \times [Q] + [A_{mR}]$

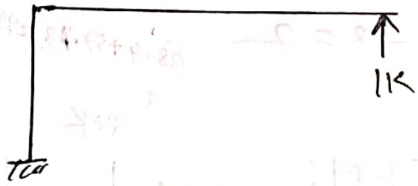
Due to unit load:

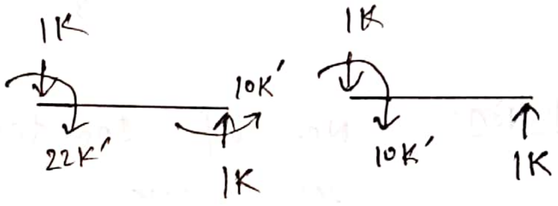
$Q_1 = 1K,\ Q_2 = 0K$:





$Q_2 = 1K,\ Q_1 = 0K$:





$$A_{mu} = \begin{array}{c} \begin{array}{cc} Q_1=1 & Q_2=1 \end{array} \\ \begin{bmatrix} 0 & 0 \\ -12 & -22 \\ 1 & 1 \\ 0 & -10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$$

$\frac{L^3}{3 \times 1.2EI}$ $\frac{L^2}{2 \times 1.2EI}$

$$F_m = \frac{1}{EI} \begin{bmatrix} \boxed{480} & \boxed{-60} & 0 & 0 & 0 & 0 \\ -60 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 480 & -60 & 0 & 0 \\ 0 & 0 & -60 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 333.33 & -50 \\ 0 & 0 & 0 & 0 & -50 & 10 \end{bmatrix}$$

$$A_{mR} = \begin{bmatrix} 14.82 \\ 35.56 \\ 12 \\ 24 \\ 14.08 \\ 38.4 \end{bmatrix}$$

Then, (do yourself)

Find (i) D_{QL}

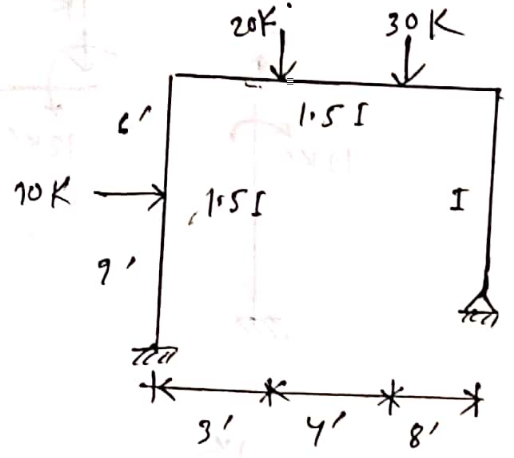
(ii) F

(iii) Q

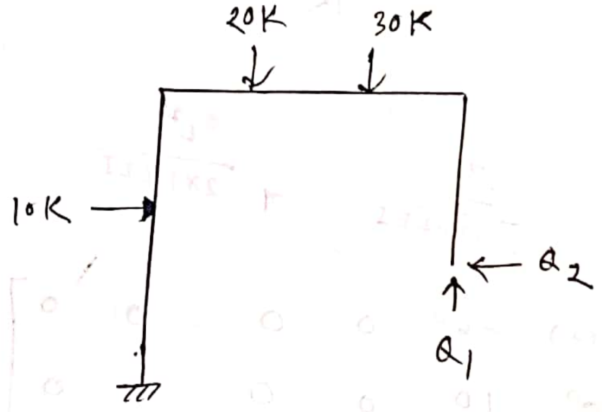
(iv) Am

2012

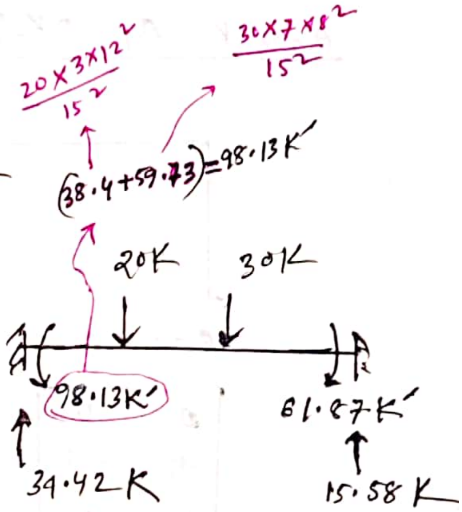
Analyze the frame. Draw SFD & BMD.

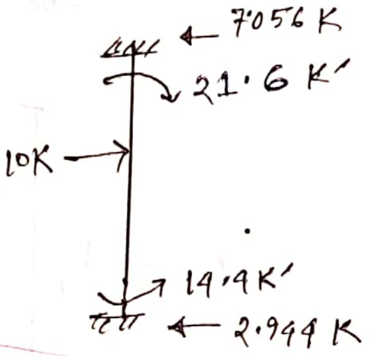


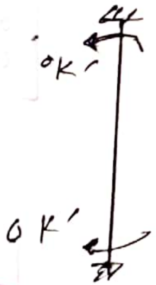
Solution: No. of Indeterminacy = 5 − 3 = 2



FEM:

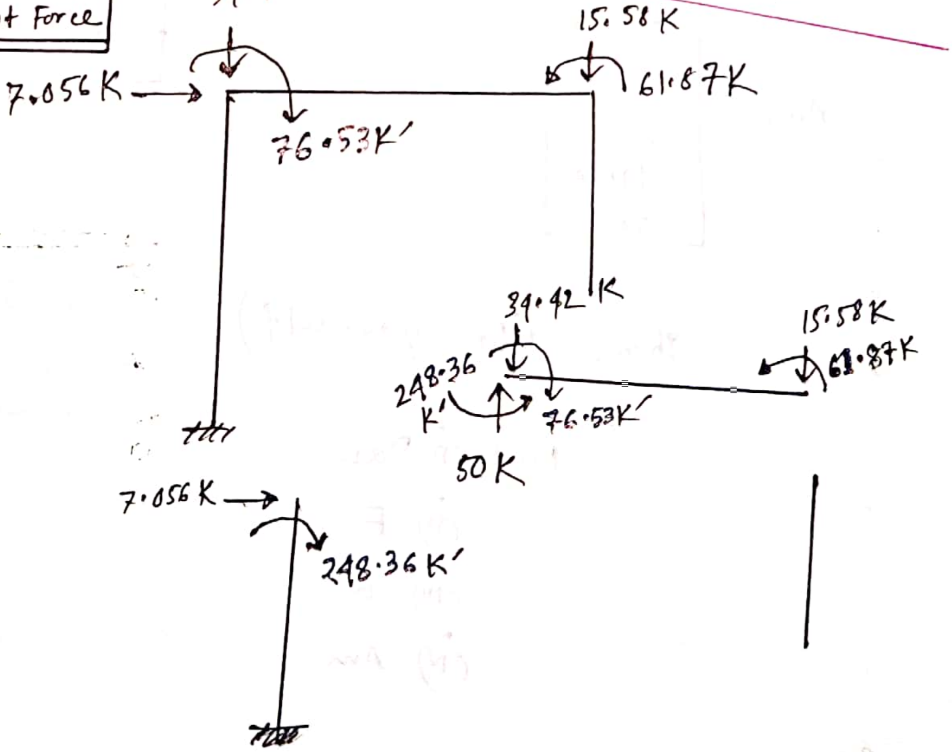






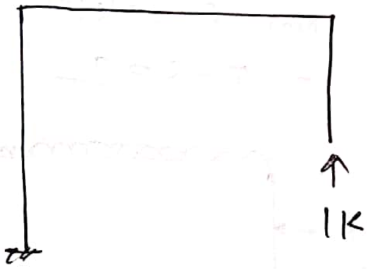
$$Am_L = \begin{bmatrix} -7.056 \\ 248.36 \\ -15.58 \\ -61.87 \\ 0 \\ 0 \end{bmatrix}$$

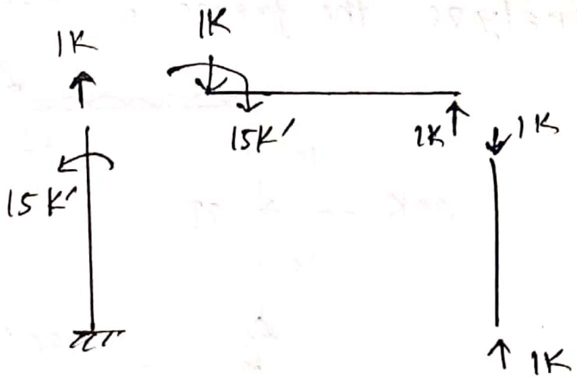
Equivalent joint Force



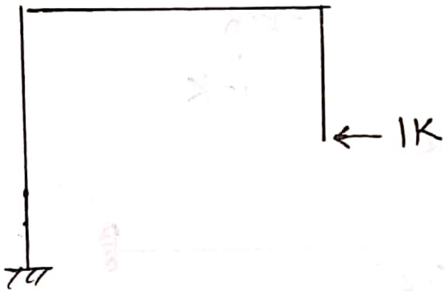
Due to unit load:

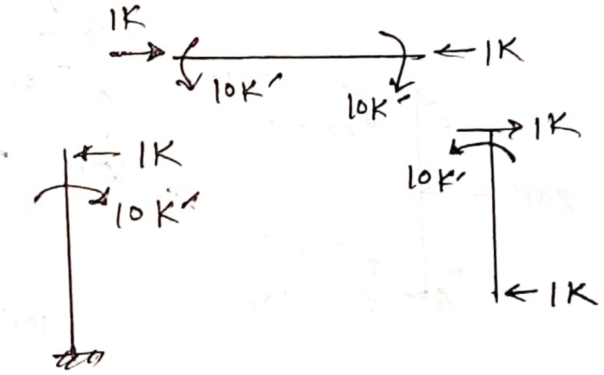
$Q_1 = 1K, \; Q_2 = 0$:





$Q_2 = 1K, \; Q_2 = 0$:





$$A_{MU} = \begin{array}{c} Q_1=1 \quad Q_2=1 \\ \begin{bmatrix} 0 & 1 \\ -15 & 10 \\ 1 & 0 \\ 0 & 10 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array}$$

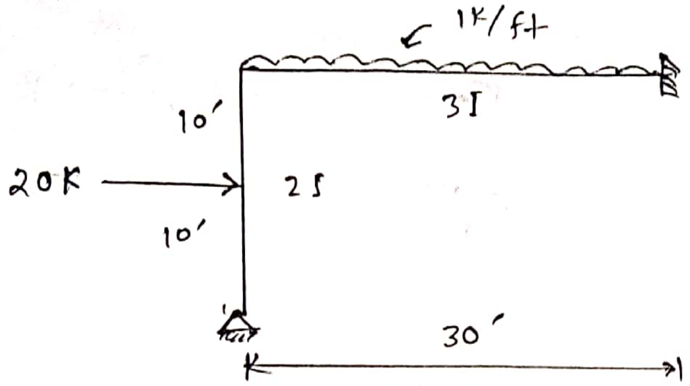
$$F_m = \frac{1}{EI} \begin{bmatrix} 750 & -75 & 0 & 0 & 0 & 0 \\ -75 & 12.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 750 & -75 & 0 & 0 \\ 0 & 0 & -75 & 12.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 333.33 & -50 \\ 0 & 0 & 0 & 0 & -50 & 10 \end{bmatrix}$$

$$A_{MR} = \begin{bmatrix} 7.056 \\ 30.24 \\ 15.58 \\ 61.87 \\ 0 \\ 0 \end{bmatrix}$$

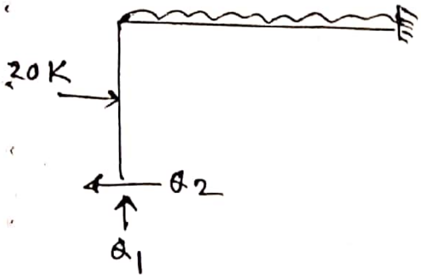
Then, (do yourself)

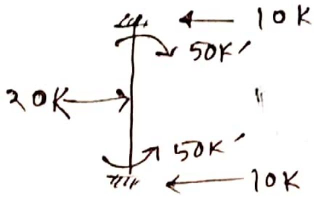
2006

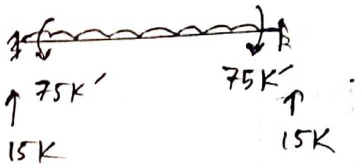
#



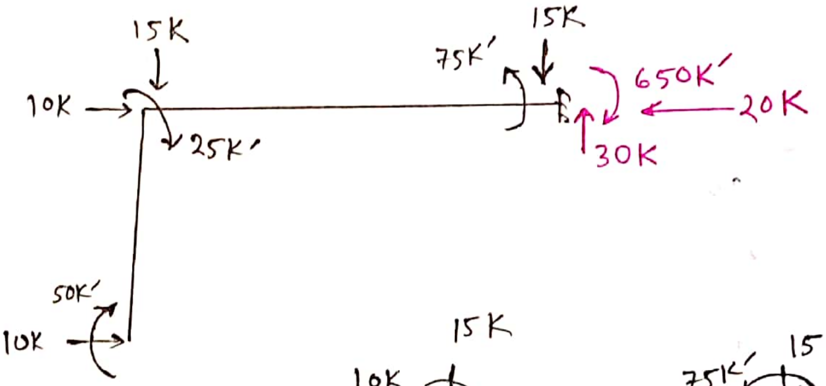
Solution: Degree of indeterminacy = 5 - 3 = 2



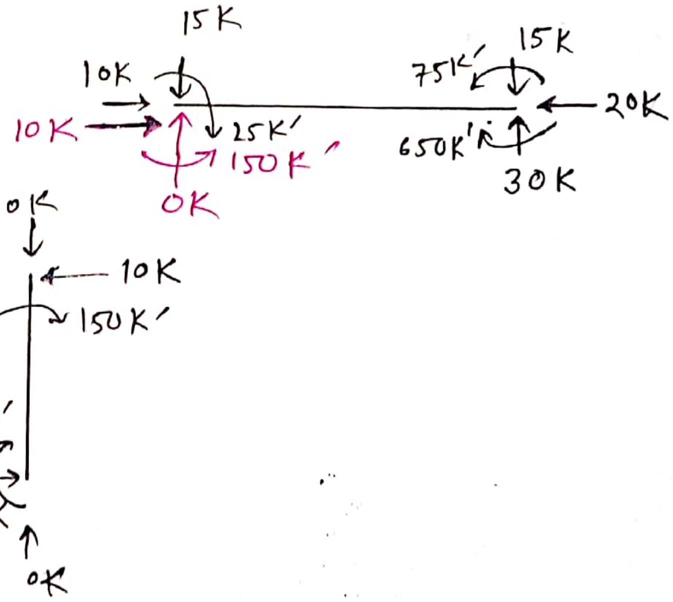




Equivalent joint force :

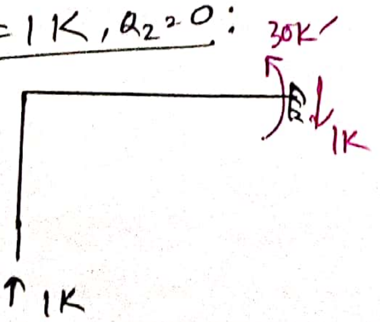


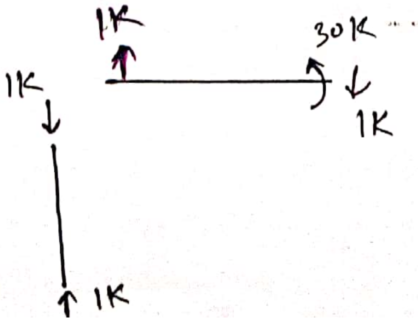
$$A_{ML} = \begin{bmatrix} 10 \\ 150 \\ 15 \\ 575 \end{bmatrix}$$



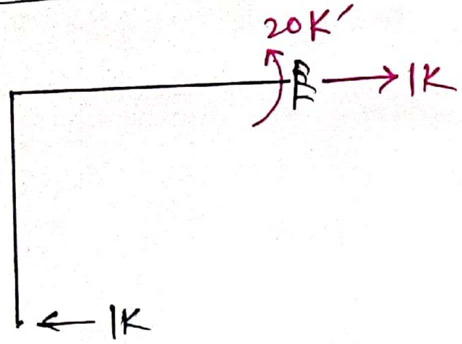
Due to unit load :

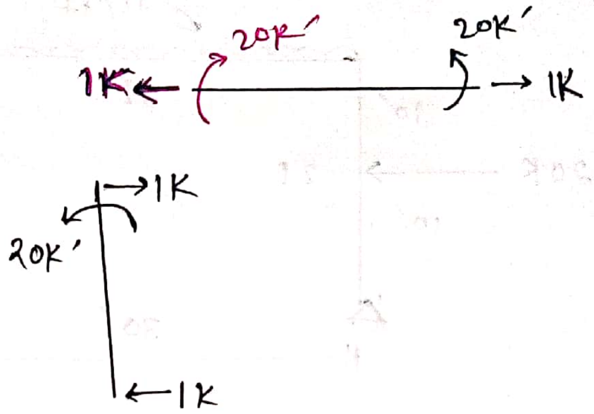
$Q_1 = 1K, Q_2 = 0$:





$Q_2 = 1K,\ Q_1 = 0$:





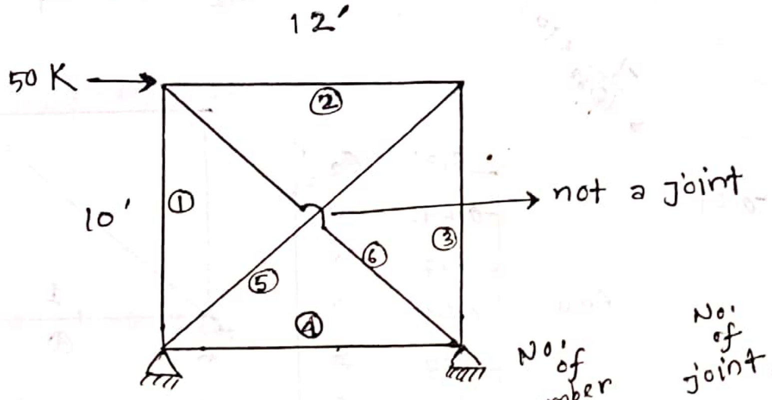
$$A_{mu} = \begin{bmatrix} 0 & -1 \\ 0 & -20 \\ -1 & 0 \\ -30 & -20 \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 1333.33 & -100 & 0 & 0 \\ -100 & 10 & 0 & 0 \\ 0 & 0 & 3000 & -150 \\ 0 & 0 & -150 & 10 \end{bmatrix}$$

$$A_{mR} = \begin{bmatrix} 10 \\ 50 \\ 15 \\ 75 \end{bmatrix}$$

Truss (Flexibility Matrix Method)

Problem-01:



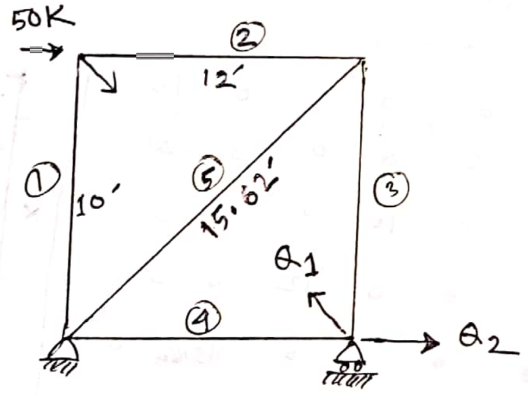
Solution:

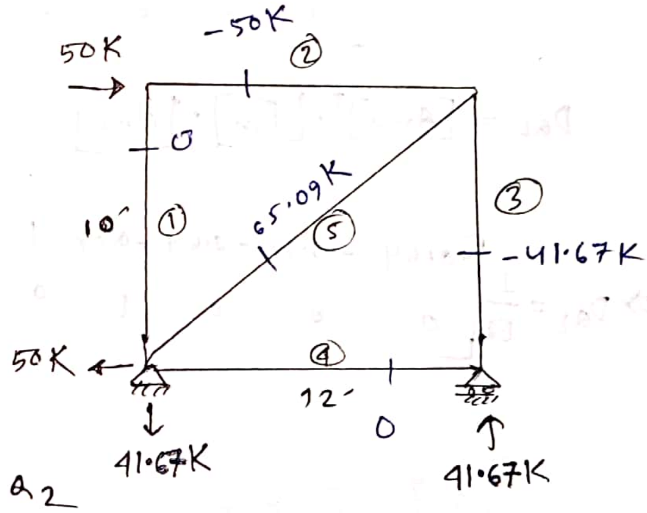
$$\text{No. of Internal Indeterminacy} = \left[\overset{\text{No. of member}}{m} - (2\overset{\text{No. of joint}}{j} - 3)\right] = \left[6 - (2\times4 - 3)\right]$$

$$= (6-5) = 1$$

$$\text{No. of External Indeterminacy} = \overset{\text{reaction}}{r} - 3$$

$$= 4 - 3 = 1$$

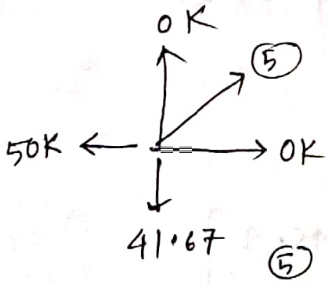




* Member বাদ দিলে internal Redundant Q_1

* Hinge-কে Rollar করে External Redundant Q_2

$$A_{ML} = \begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array}\begin{bmatrix} 0 \\ -50 \\ -41.67 \\ 0 \\ 65.09 \\ 0 \end{bmatrix}\begin{array}{l} \to \text{member ①} \\ \to \text{member ②} \\ - - - - \\ - - - - \\ - - - - \\ \to \text{member ⑥} \end{array}$$



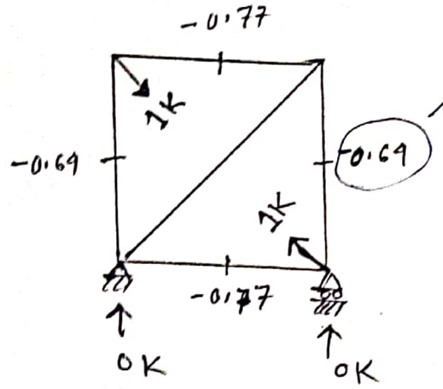
$$⑤ = \frac{41.67}{10} \times 15.62$$

$$= 65.09\ K$$

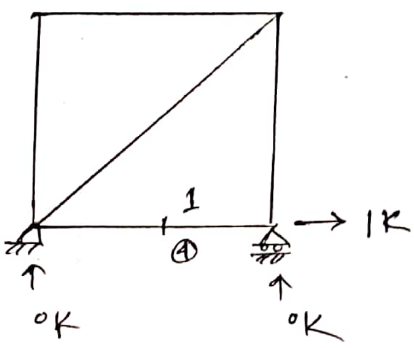
Due to unit load :

$Q_1 = 1K,\ Q_2 = 0$ $\qquad\qquad$ $Q_2 = 1K,\ Q_1 = 0$

$\frac{-1}{15.62} \times 10 = -0.64$



$$A_{mu} = \begin{array}{cc} Q_1=1 & Q_2=1 \\ \left[\begin{matrix} -0.64 & 0 \\ -0.77 & 0 \\ -0.64 & 0 \\ -0.77 & 1 \\ 1 & 0 \\ 1 & 0 \end{matrix}\right] \end{array}$$



member length

$$F_m = \frac{1}{EA} \times \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15.62 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.62 \end{bmatrix}$$

$$D_{QL} = [A_{mu}]^t \cdot [F_m] \cdot [A_{mL}]$$

$$\Rightarrow D_{QL} = \frac{1}{EA} \begin{bmatrix} -0.64 & -0.77 & -0.64 & -0.77 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15.62 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.62 \end{bmatrix} \times \begin{bmatrix} 0 \\ -50 \\ -41.67 \\ 0 \\ 65.09 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EA} \begin{bmatrix} -6.4 & -9.24 & -6.4 & -9.24 & 15.62 & 15.62 \\ 0 & 0 & 0 & 12 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -50 \\ -41.67 \\ 0 \\ 65.09 \\ 0 \end{bmatrix}$$

$$= \frac{1}{EA} \begin{bmatrix} 1745.4 \\ 0 \end{bmatrix}$$

$$F = [A_{mu}]^{t} \cdot [F_m] \cdot [A_{mu}]$$

$$= \frac{1}{EA}\begin{bmatrix} -6.4 & -9.24 & -6.4 & -9.24 & 15.62 & 15.62 \\ 0 & 0 & 0 & 12 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} -0.64 & 0 \\ -0.77 & 0 \\ -0.64 & 0 \\ -0.77 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{EA} \times \begin{bmatrix} 53.66 & -9.24 \\ -9.24 & 12 \end{bmatrix}$$

$$Q = -[F^{-1}] \times [D_{QL}]$$

$$= -\frac{EA}{558.5424} \times \begin{bmatrix} 12 & 9.24 \\ 9.24 & 53.66 \end{bmatrix} \times \frac{1}{EA}\begin{bmatrix} 1745.4 \\ 0 \end{bmatrix}$$

$$= -\frac{1}{558.5424} \times \begin{bmatrix} 20940 \\ 16127.5 \end{bmatrix}$$

$$= \begin{bmatrix} -37.5 \\ -28.875 \end{bmatrix}$$

$$A_m = [A_{mL}] + [A_{mu}] \times [Q]$$

$$\therefore A_m = \begin{bmatrix} 0 \\ -50 \\ -41.67 \\ 0 \\ 65.09 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.64 & 0 \\ -0.77 & 0 \\ -0.64 & 0 \\ -0.77 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} -37.5 \\ -28.875 \end{bmatrix} = \begin{bmatrix} 0 \\ -50 \\ -41.67 \\ 0 \\ 65.09 \\ 0 \end{bmatrix} + \begin{bmatrix} 24 \\ 28.875 \\ 24 \\ 0 \\ -37.5 \\ -37.5 \end{bmatrix}$$

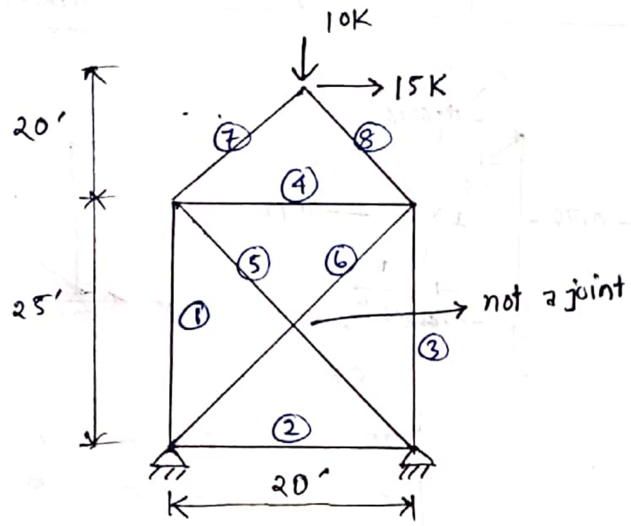
$$\therefore A_m = \begin{bmatrix} 24 \\ -21.125 \\ -17.67 \\ 0 \\ 27.59 \\ -37.5 \end{bmatrix}$$

(Ans)

Flexibility Matrix Method (TRUSS)

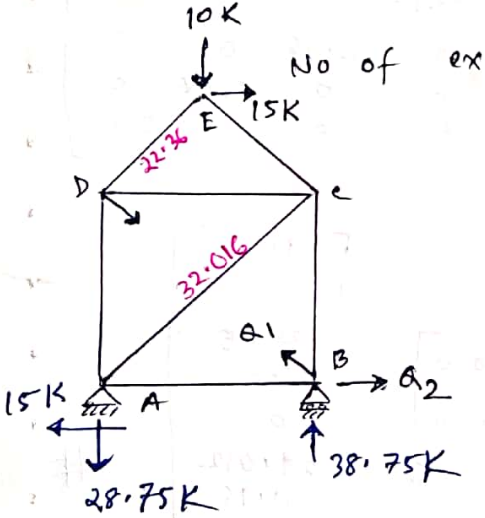
2018

Analyze the truss. EA is constant.



Solution: No. of internal indeterminacy $= [m - (2j - 3)] = 8 - (2 \times 5 - 3)$
$= 1$

No of external inter minaey $= r - 3$
$= (4 - 3) = 1$

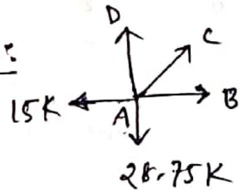


joint B:

③ $BC + 38.75 = 0$ and, ② $BA = 0$ K.

$\therefore BC = -38.75$ K

Joint A:



$\Sigma F_x = 0$

$(AC)_x + AB = 15$

$\Rightarrow AC \times \frac{20}{32.016} = 15$

⑥ $\therefore AC = 24.012$ K

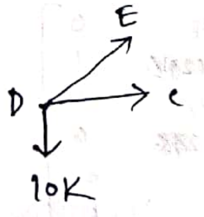
$\Sigma F_y = 0$

$(AC)_y + AD = 28.75$

$\Rightarrow 24.012 \times \frac{25}{32.016} + AD = 28.75$

① $\therefore AD = 10$ K

joint D:

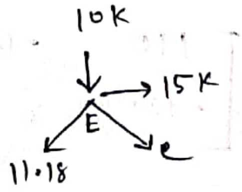


$(DE)_y = 10$

⑦ $\Rightarrow DE = \frac{10}{20} \times 22.36 = 11.18$K

And, ④ $DC = \frac{-11.18}{22.36} \times 10 = 5$K

Joint E:



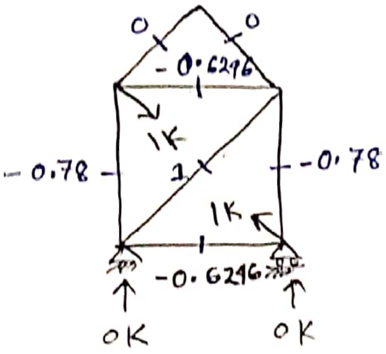
$(EC)_x + 15 = \frac{11.18}{22.36} \times 10$

⑧ $\Rightarrow EC \times \frac{10}{22.36} = -10$ $\therefore EC = -22.36$ K

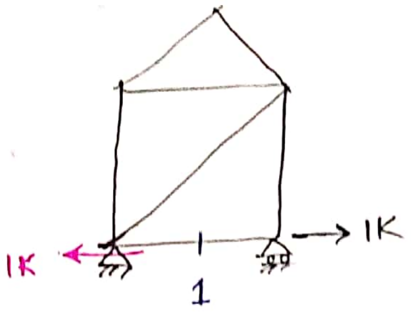
$$\therefore A_{mL} = \begin{bmatrix} 10 \\ 0 \\ -38.75 \\ -5 \\ 0 \\ 21.012 \\ 11.18 \\ -22.36 \end{bmatrix}$$

Due to unit load:

when $Q_1 = 1K,\ Q_2 = 0$:



when $Q_2 = 1K,\ Q_1 = 0$:



$$A_{mu} = \begin{array}{cc} \underline{Q_1=1} & \underline{Q_2=1} \\ \left[\begin{matrix} -0.78 & 0 \\ -0.6246 & 1 \\ -0.78 & 0 \\ -0.6246 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix}\right] \end{array}$$

$$F_m = \frac{1}{EA}\begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 32.016 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 32.016 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 22.36 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.3C \end{bmatrix}$$

$$D_{QL} = [A_{mu}]^t \times [F_m] \times [A_{mL}]$$

$$= \frac{1}{EA}\begin{bmatrix} \overset{-19.5}{\cancel{-0.78}} & -12.492 & -19.5 & -12.492 & 32.016 & 32.016 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 10 \\ 0 \\ -38.75 \\ -5 \\ 0 \\ 21.012 \\ 11.18 \\ -22.36 \end{bmatrix}$$

$$= \frac{1}{EA} \times \begin{bmatrix} 1391.853 \\ 0 \end{bmatrix}$$

$$F = [A_{mu}]^t \times [F_m] \times [A_{mu}]$$

$$= \frac{1}{EA} \times \begin{bmatrix} -19.5 & -12.492 & -19.5 & -12.492 & 32.016 & 32.016 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} -0.78 & 0 \\ -0.6246 & 1 \\ -0.78 & 0 \\ -0.6246 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{1}{EA} \times \begin{bmatrix} 110 & -12.492 \\ -12.492 & 20 \end{bmatrix}$$

$$Q = -\left[F^{-1}\right] \times \left[D_{QL}\right]$$

$$= -EA \times \frac{1}{2043.95} \times \begin{bmatrix} 20 & 12.492 \\ 12.492 & 110 \end{bmatrix} \times \frac{1}{EA} \times \begin{bmatrix} 1391.853 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} -13.62 \\ -8.51 \end{bmatrix}$$

$$Am = \left[Am_L\right] + \left[Am_u\right] \times \left[Q\right]$$

$$\begin{bmatrix} AD \\ AB \\ BC \\ CD \\ DB \\ AC \\ DE \\ EC \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -38.75 \\ 5 \\ 0 \\ 24.012 \\ 11.18 \\ -22.36 \end{bmatrix} + \begin{bmatrix} -0.78 & 0 \\ -0.6246 & 1 \\ -0.78 & 0 \\ -0.6246 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} -13.62 \\ -8.51 \end{bmatrix} = \begin{bmatrix} 20.62 \\ 0 \\ -28.13 \\ 13.51 \\ -13.62 \\ 10.39 \\ 11.18 \\ -22.36 \end{bmatrix} (Am_1)$$

2017

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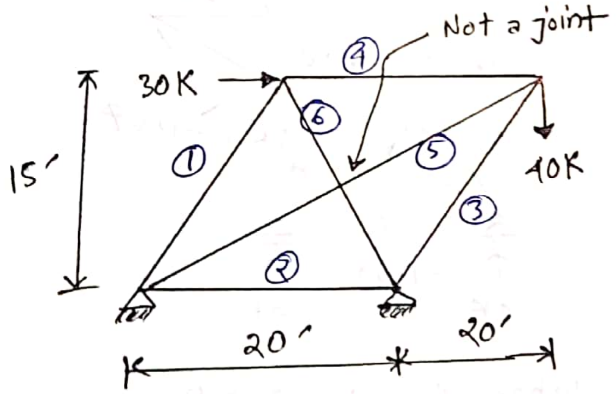
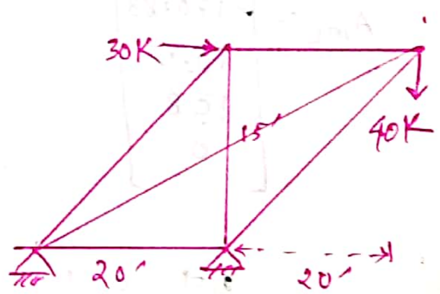
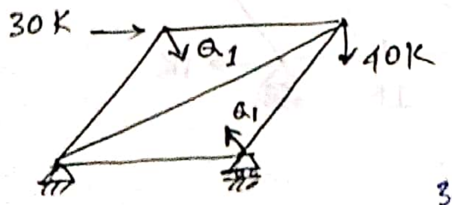


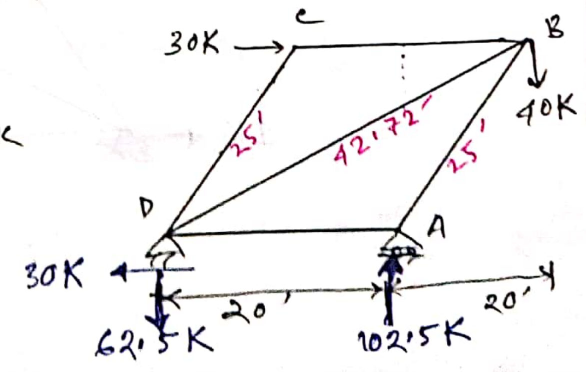
This figure actually is:



Solution: No. of internal indeterminacy $= [6-(2\times4-3)] = 1$.

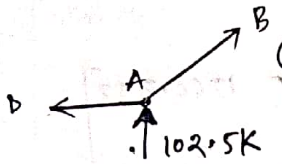
" " external " $= r-3 = (4-3) = 1$





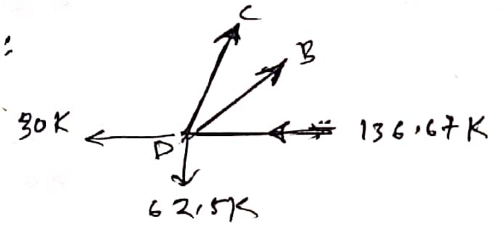
joint A:

③ $AB = \frac{-102.5}{15} \times 25 = -170.83$ K



② $AD = -\frac{170.83}{25} \times 20 = -136.67$ K

joint D:



$\Sigma F_x = 0$

$(DB)_x + (DC)_x = 136.67 + 30$

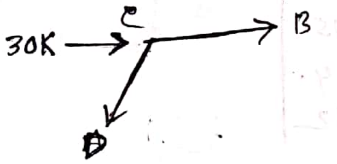
$\Rightarrow DB \times \frac{40}{42.72} + DC \times \frac{20}{25} = 166.67 \ldots\ldots (I)$

$\Sigma F_y = 0$

$(DB)_y + (DC)_y = 62.5$

$\Rightarrow DB \times \frac{15}{42.72} + DC \times \frac{15}{25} = 62.5 \ldots\ldots (II)$

joint C:



From (I) & (II), ⑤ DB = 178 K

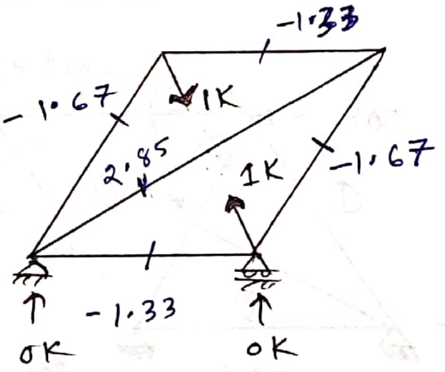
① DC = 0 K

④ (CB) = −30 K

$$A_{ML} = \begin{bmatrix} 0 \\ -136.67 \\ -170.83 \\ -30 \\ 178 \\ 0 \end{bmatrix}$$

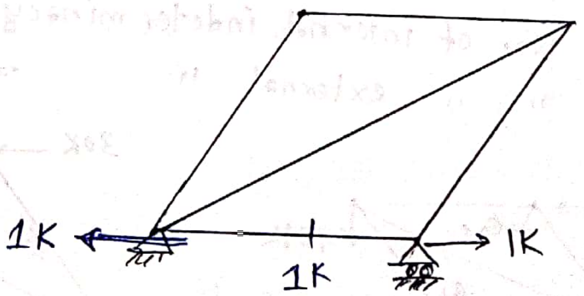
Due to unit load:

when $Q_1 = 1K$, $Q_2 = 0$:



$$A_{MU} = \begin{array}{cc} Q_1=1 & Q_2=1 \\ \left[\begin{matrix} -1.67 & 0 \\ -1.33 & 1 \\ -1.67 & 0 \\ -1.33 & 0 \\ 1 & 0 \\ 2.85 & 0 \end{matrix}\right] \end{array}$$

when $Q_2 = 1$, $Q_1 = 0$



$$F_m = \frac{1}{EA}\begin{bmatrix} 25 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 42.72 \end{bmatrix}$$

Then, (do yourself)

$D_{QL} = [A_{mu}]^t \times [f_m] \times [A_{mL}]$

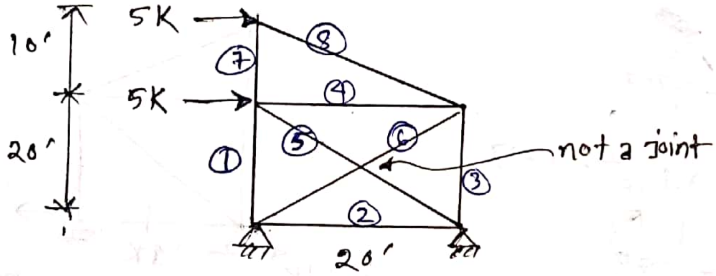
$F = [A_{mu}]^t \times [F_m] \times [A_{mu}]$

$Q = -[F^{-1}]\,[D_{QL}]$

$A_m = [A_{mL}] + [A_{mu}] \times [Q]$

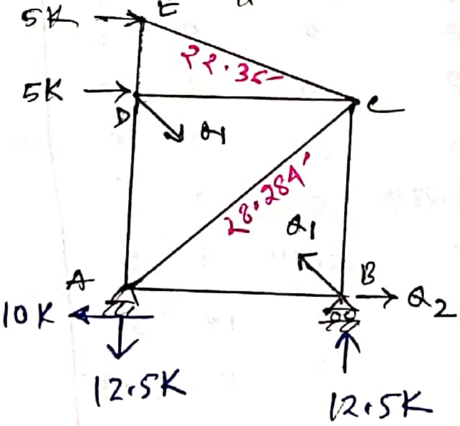
2016

\#

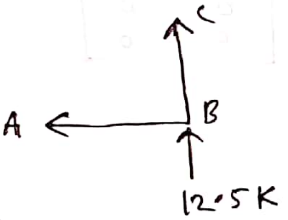


Solution: No. of internal indeterminacy $= 8-(2\times 5-3) = 1$

" " External " $= r-3 = 1$



joint B:

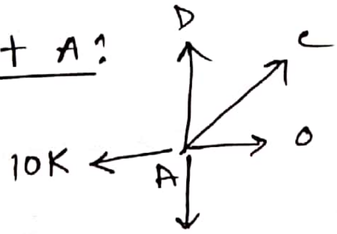


$BC + 12.5 = 0$

③ $\Rightarrow BC = -12.5K$

② $AB = 0$

joint A:



$\Sigma F_x = 0$

$(AC)_x = 10$

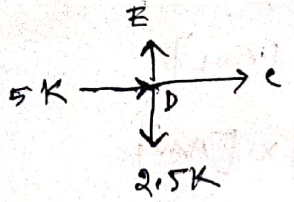
⑥ $\Rightarrow AC = \frac{10}{20} \times 28.284 = 14.14$ K

$(AD) + (AC)_y = 12.5$

① $\Rightarrow AD = 12.5 - 14.14 \times \frac{20}{28.284} = 2.5K$

Scanned by CamScanner

joint D:

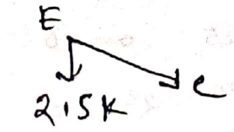


DC + 5 = 0

④ DC = -5K

⑦ DE = 2.5 K

joint E:



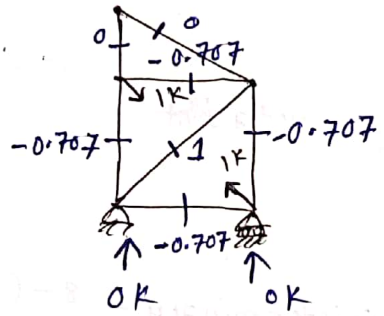
$(EC)_y = -2.5$

⑧ $\Rightarrow EC = \frac{-2.5}{10} \times 22.36 = -5.59\ K$

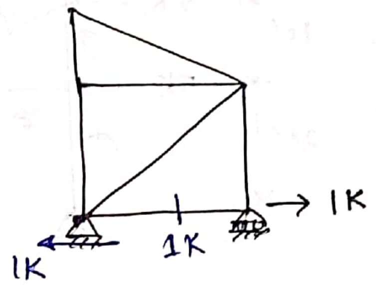
$$A_{mL} = \begin{bmatrix} 2.5 \\ 0 \\ -12.5 \\ -5 \\ 0 \\ 14.14 \\ 2.5 \\ -5.59 \end{bmatrix}$$

Due to unit Load:

$Q_1 = 1,\ Q_2 = 0$



$Q_2 = 1,\ Q_1 = 0$



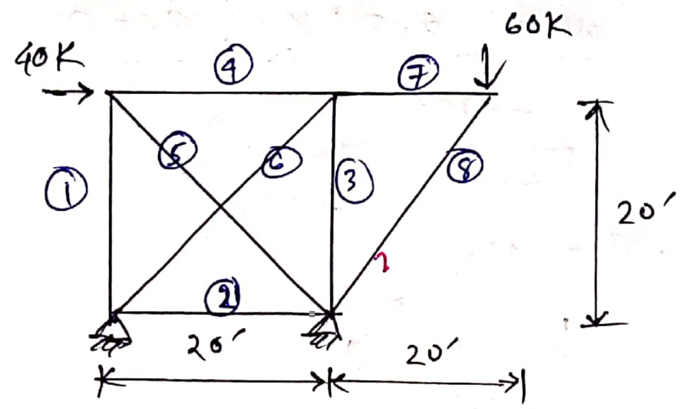
$$A_{mu} = \begin{array}{c} Q_1=1 \quad Q_2=1 \\ \begin{bmatrix} -0.707 & 0 \\ -0.707 & 1 \\ -0.707 & 0 \\ -0.707 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

$$F_m = \frac{1}{EA} \begin{bmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 28.284 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 28.284 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 22.36 \end{bmatrix}$$

Then (do yourself)

2015

#

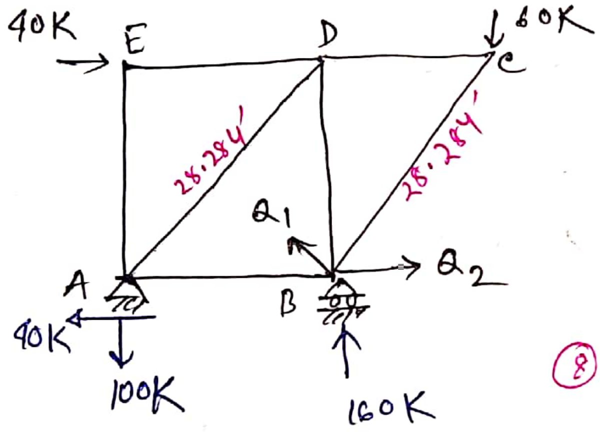


<u>Solution:</u> No. of External Indeterminacy = R − 3 = 4 − 3 = 1

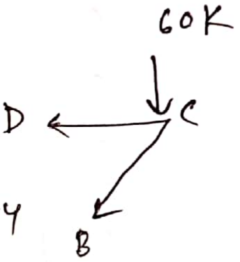
No of Internal " = m − (2j − 3)

= 8 − (2×5 − 3)

= 1



<u>Joint C:</u>

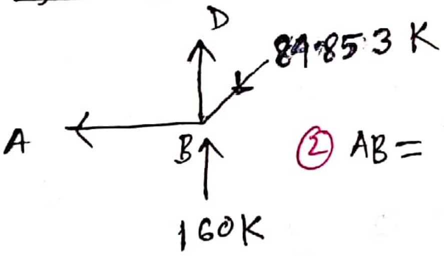


⑧ CB = $-\frac{60}{20} \times 28.284$

$= -84.853K$

⑦ CD = $\frac{84.853}{28.284} \times 20 = 60K$

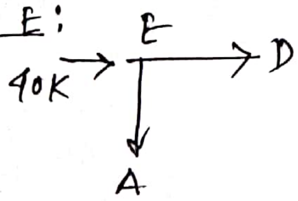
<u>Joint B:</u>



② AB = $\frac{-84.853}{28.284} \times 20 = -60.00$ K

③ BD = $-160 + \frac{84.853}{28.284} \times 20 = -100.00$ K

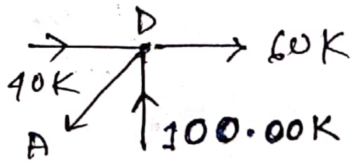
<u>Joint E:</u>



④ ED = −40K

① AE = 0

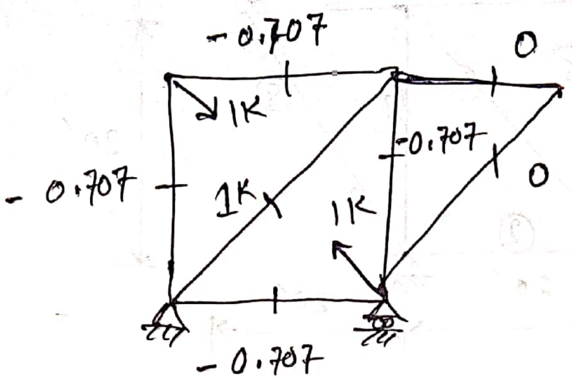
<u>Joint D:</u>



⑥ AD = $\frac{100.00}{20} \times 28.284 = 141.42$ K

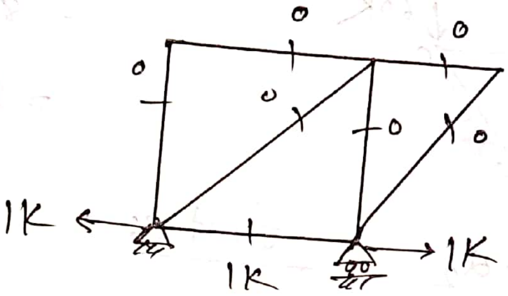
$$A_{ML} = \begin{bmatrix} 0 \\ -60 \\ -100 \\ -40 \\ 0 \\ 141.42 \\ 60 \\ -84.853 \end{bmatrix}$$

when, $Q_1 = 1K$, $Q_2 = 0$



when $Q_1 = 0K$, $Q_2 = 1K$

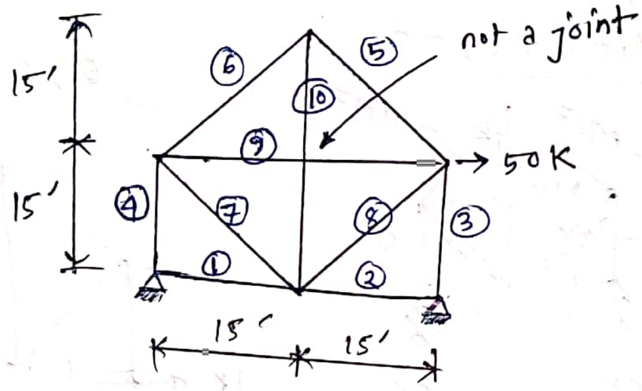
$$A_{MU} = \begin{bmatrix} -0.707 & 0 \\ -0.707 & 1 \\ -0.707 & 0 \\ -0.707 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$



$$F_m = \frac{1}{EA}\begin{bmatrix} 20 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 28.284 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 28.284 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 28.284 \end{bmatrix}$$

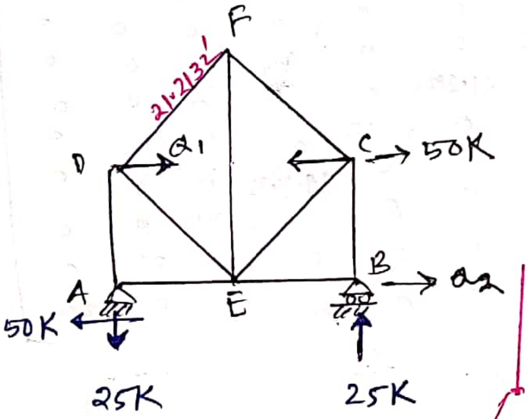
2019

#



Solution: Internal indeterminacy = 10 − (2×6−3) = 1

External indeterminacy = (4 − 3) = 1



Joint A:

④ AD = 25 K

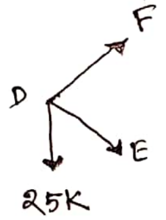
① AE = 50 K

Joint B:

③ BC = −25 K

② BE = 0 K

Joint D:



$(DF)_x + (DE)_x = 0$ ...

$\Rightarrow DF \times \frac{15}{21.2132} + DE \times \frac{15}{21.2132} = 0$ ..... ①

Again, $(DF)_y - (DE)_y = 25$

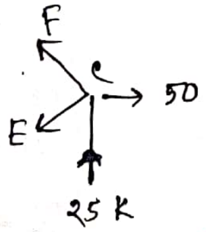
$\Rightarrow DF \times \frac{15}{21.2132} - \frac{15 \times DE}{21.2132} = 25$ ..... (ii)

From ① & (ii) we obtain,

⑥ DF = 17.67 K

⑦ DE = −17.67 K

Joint C:



$CF \times \frac{15}{21.2132} + \frac{CE \times 15}{21.2132} = 50$ .....(iii)

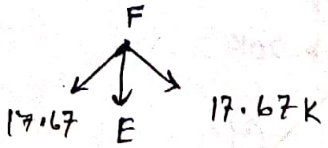
$CF \times \frac{15}{21.2132} - CE \times \frac{15}{21.2132} = -25$ .....(iv)

From (iii) & (iv) we obtain,

⑤ CF = 17.67 K

⑧ CE = 53.03 K

Joint F:



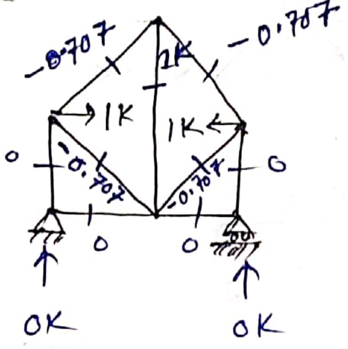
$$⑩\ FE = -\left(17.67 \times \frac{15}{21.2132} + 17.67 \times \frac{15}{21.2132}\right)$$

$$= -25\text{ K}$$

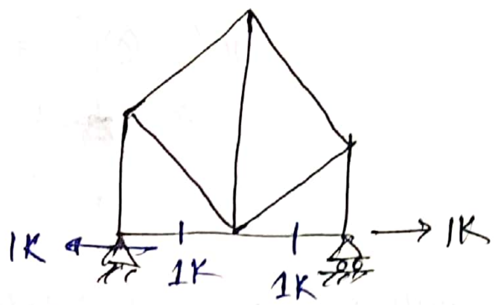
$$A_{ML} = \begin{bmatrix} 50 \\ 0 \\ -25 \\ 25 \\ 17.67 \\ 17.67 \\ -17.67 \\ -53.03 \\ 0 \\ -25 \end{bmatrix}$$

Due to unit load:

$Q_1 = 1$, $Q_2 = 0$:



$Q_2 = 1$, $Q_1 = 0$:

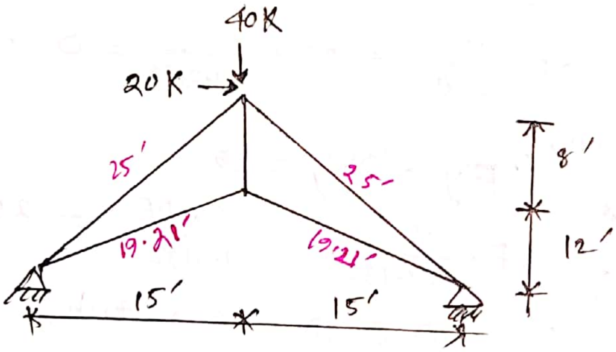


$$A_{MU} = \begin{array}{c} \begin{array}{cc} Q_1=1 & Q_2=0 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ -0.707 & 0 \\ -0.707 & 0 \\ -0.707 & 0 \\ -0.707 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \end{array}$$

$$F = \frac{1}{AE}\begin{bmatrix} 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 15 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 21.2132 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 21.2132 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 21.2132 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21.2132 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 30 \end{bmatrix}$$

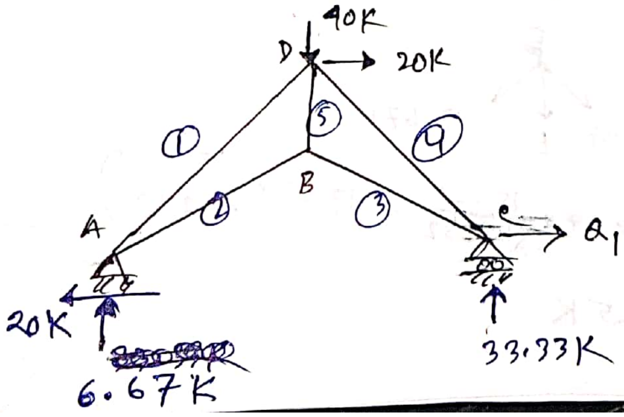
Then, (do yourself)

2013
#

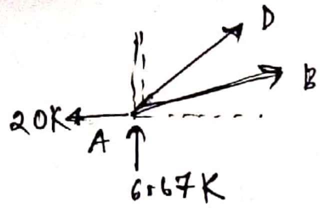


solution: internal indeterminacy = 5-(2×4-3) = 0

External indeterminacy = 4-3 = 1



joint A:



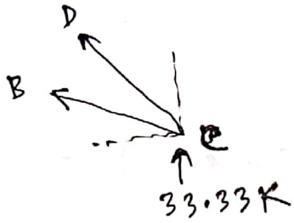
$$AB \times \frac{15}{19.21} + AD \times \frac{15}{25} = 20 \cdots ①$$

$$AB \times \frac{12}{19.21} + AD \times \frac{20}{25} = -6.67 \cdots (ii)$$

From ① & (ii), we obtain, ② $AB = 80.05$ K

① $AD = -70.844$ K

joint C:



$$CB \times \frac{15}{19.21} + CD \times \frac{15}{25} = 0 \cdots (iii)$$

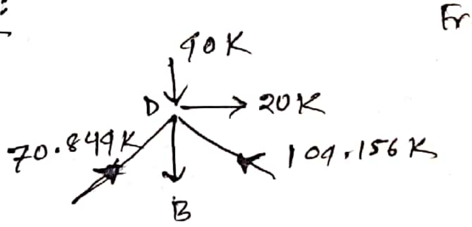
$$CB \times \frac{12}{19.21} + CD \times \frac{20}{25} = -33.33 \cdots (iv)$$

joint E:

From (iii) & (iv),

③ $CB = 80.033$ K

④ $CD = -104.156$ K



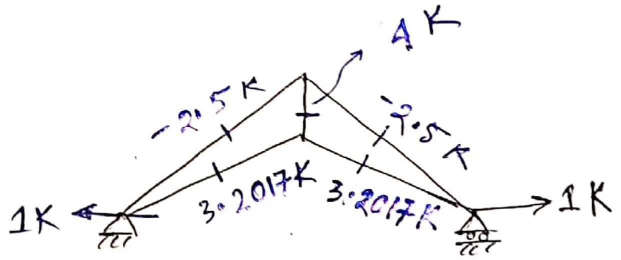
$$⑤\ BD = 104.156 \times \frac{20}{25} + 74.844 \times \frac{20}{25} - 40 = 100.0\text{ K}$$

check: $104.156 \times \frac{15}{25} - 70.844 \times \frac{15}{25} = 19.99\ (\simeq 20\text{K})$

$$A_{ML} = \begin{bmatrix} -70.844 \\ 80.05 \\ 80.033 \\ -104.156 \\ 100.0 \end{bmatrix}$$

Due to unit Load:

when $Q_1 = 1$:

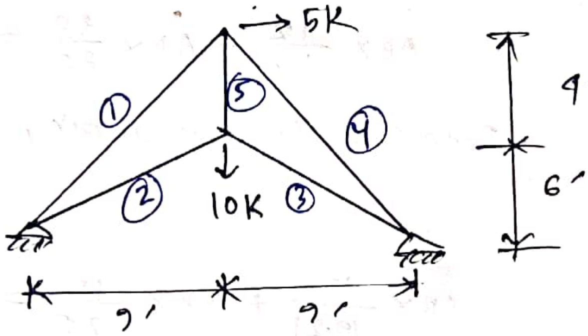


$$\therefore A_{MU} = \begin{bmatrix} -2.5 \\ 3.2017 \\ 3.2017 \\ -2.5 \\ 4 \end{bmatrix}$$

$$F_M = \frac{1}{EA} \times \begin{bmatrix} 25 & 0 & 0 & 0 & 0 \\ 0 & 19.21 & 0 & 0 & 0 \\ 0 & 0 & 19.21 & 0 & 0 \\ 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$$

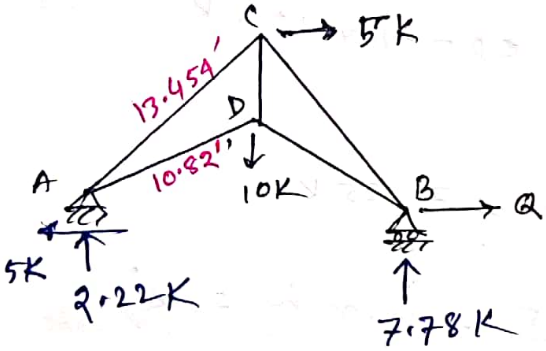
2011

#



solution: Internal indeterminacy $= 5 - (2\times 4 - 3) = 0$

External " $= (4-3) = 1$



Joint A:

$$AD\times\frac{9}{10.82} + AC\times\frac{9}{13.454} = 5 \cdots ①$$

$$AD\times\frac{6}{10.82} + AC\times\frac{10}{13.454} = -2.22 \text{ — } (ii)$$

From ① & (ii)

② AD = 21.033 K

① AC = −18.68 K

Joint B:

$$BD\times\frac{9}{10.82} + BC\times\frac{9}{13.454} = 0 \cdots (iii)$$

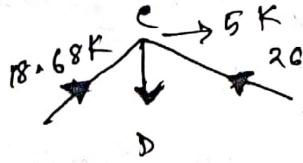
$$BD\times\frac{6}{10.82} + BC\times\frac{10}{13.454} = -7.78 \cdots (iv)$$

From (iii) & (iv)

③ BD = 21.05 K

④ BC = −26.17 K

joint C:



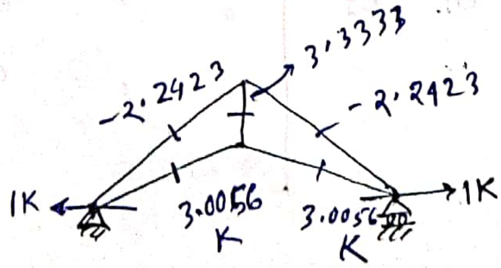
$$⑤\ CD = 26.17\times\frac{10}{13.454} + 18.68\times\frac{10}{13.454}$$

$$= 33.34\text{ K}$$

(check:)

$$26.17\times\frac{9}{13.454} - 18.68\times\frac{9}{13.454} = 5.01\text{ K} (\simeq 5K)$$

$$A_{ML} = \begin{bmatrix} -18.68 \\ 21.033 \\ 21.05 \\ -26.17 \\ 33.34 \end{bmatrix}$$



$$A_{MU} = \begin{bmatrix} -2.2423 \\ 3.0056 \\ 3.0056 \\ -2.2423 \\ 3.3333 \end{bmatrix}$$

SHELL STRUCTURES

INTRODUCTION:

- A shell structure is a thin, curved membrane or slab, usually of reinforced concrete, that functions both as structure and covering, the structure deriving its strength and rigidity from the curved shell forms.
- Shell structures predominantly resist loads on them by direct compression. That is without bending or flexure.
- Since most materials are more effective in compression than in bending, shell structures result in lesser thickness than flat structures.

MATERIAL:

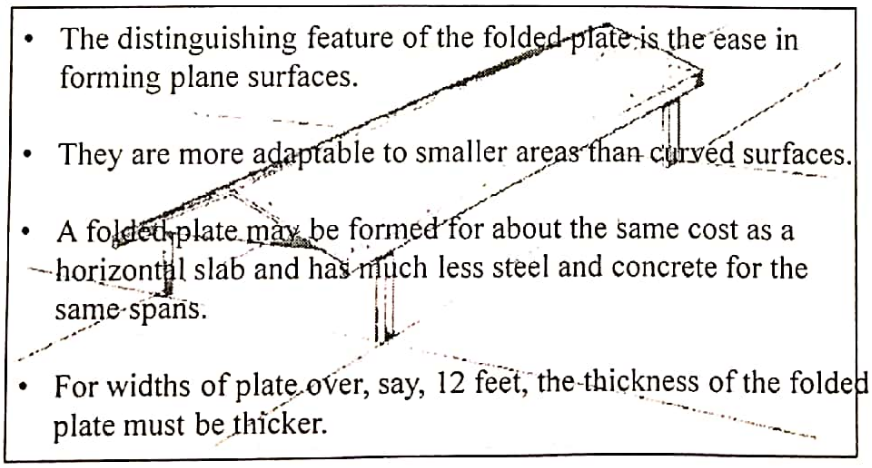
- The material most suited to the construction of a shell structure is concrete.

TYPES:

- Shell structures are sometimes described as single or double curvature shells.
- Single curvature shells, curved on one linear axis, are part of cylindrical or cone in the form of barrel vaults and conoid shells.
- Double curvature shells are either part of a sphere, as a dome, or a hyperboloid of revolution.

FOLDED PLATES:

- Folded plates are the simplest of the shell structures.



- The distinguishing feature of the folded plate is the ease in forming plane surfaces.
- They are more adaptable to smaller areas than curved surfaces.
- A folded plate may be formed for about the same cost as a horizontal slab and has much less steel and concrete for the same spans.
- For widths of plate over, say, 12 feet, the thickness of the folded plate must be thicker.
- Some advantage may be gained by increasing the thickness of the slab just at the valleys so it will act as a haunched beam and as an I section plate girder.

BASIC ELEMENTS:

1) The inclined plates

2) edge plates which must be used to stiffen the wide plates

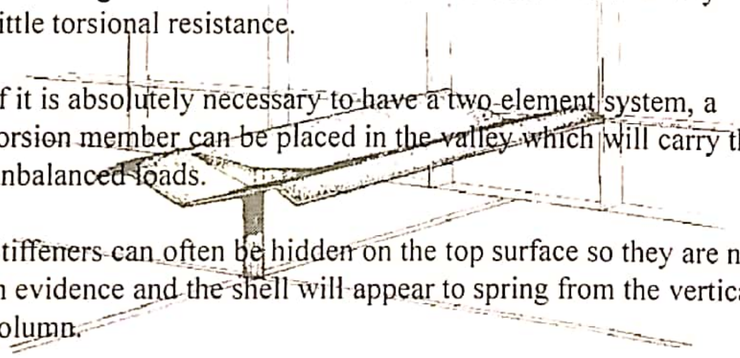
3) stiffeners to carry the loads to the supports and to hold the plates in line, and

4) columns to support the structure in the air. Are the main 4 elements of folded plates.

- If several units were placed side by side, the edge plates should be omitted except for the first and last plate.
- If the edge plate is not omitted on inside edges, the form should be called a two segment folded plate with a common edge plate.

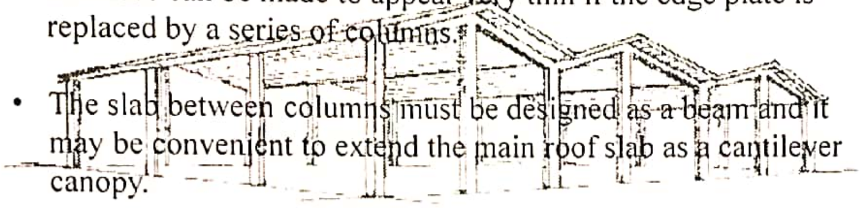
CANOPIES:

- This folded plate has four segments.



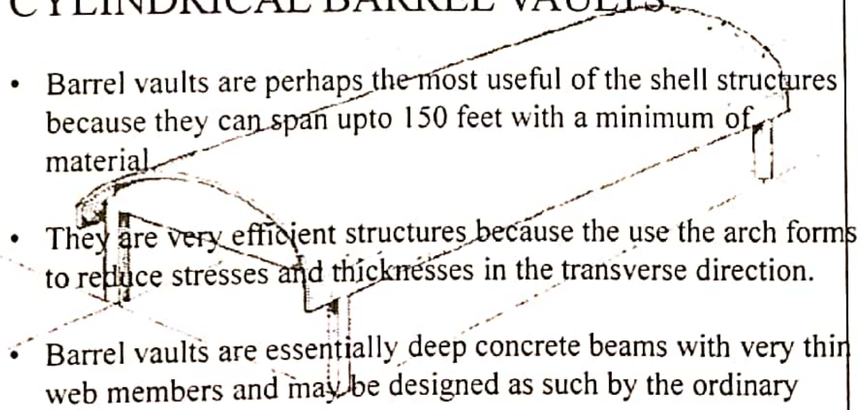
- A two segment structure is not desirable because it has very little torsional resistance.
- If it is absolutely necessary to have a two-element system, a torsion member can be placed in the valley which will carry the unbalanced loads.
- Stiffeners can often be hidden on the top surface so they are not in evidence and the shell will appear to spring from the vertical column.
- At the wall of the building there should also be a stiffener hidden in the wall construction.
- Provision should be made for drainage of the centre valley.

EDGE SUPPORTED FOLDED PLATES:



- The usual upturned edge plate can be eliminated and the roof structure can be made to appear very thin if the edge plate is replaced by a series of columns.
- The slab between columns must be designed as a beam and it may be convenient to extend the main roof slab as a cantilever canopy.
- The vertical columns in the end walls at the crown of the gable takes the reactions of the plates and the horizontal ties may be eliminated.
- Wind loads are taken by rigid frame action in the columns and stiffeners.

CYLINDRICAL BARREL VAULTS:



- Barrel vaults are perhaps the most useful of the shell structures because they can span upto 150 feet with a minimum of material.
- They are very efficient structures because the use the arch forms to reduce stresses and thicknesses in the transverse direction.
- Barrel vaults are essentially deep concrete beams with very thin web members and may be designed as such by the ordinary methods of reinforced concrete.

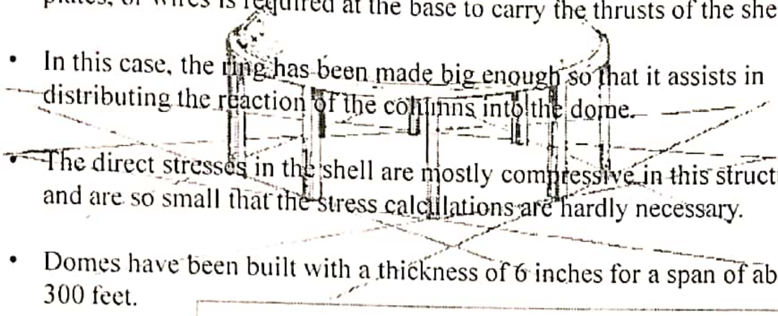
ELEMENTS OF BARREL VAULTS:

- The shell has been allowed to project beyond the edge of the stiffener in order to show the shape of the shell.
- Stiffeners are required at columns.
- In contrast to folded plates where the thickness is based on the design of a slab element, the thickness of the barrel shell is usually based on the minimum thickness required for covering the steel for fireproofing, plus the space required for three layers of bars, plus some space for tolerance.
- Near the supports the thickness may be greater for containing the larger longitudinal bars.
- If more than one barrel is placed side by side, the structure is a multiple barrel structure and if more than one span, it is called a multiple span structure.

DOMES OF REVOLUTION:

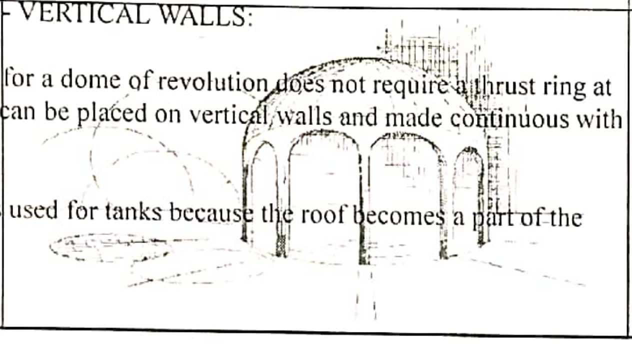
- A dome is a space structure covering a more or less square or circular area.
- They are formed by a surface generated by a curve of any form revolving about a vertical line.
- This surface has double curvature and the resulting structure is much stiffer and stronger than a single curved surface, such as a cylindrical shell.
- The simple dome of revolution is a portion of a sphere.
- However, other curves are also satisfactory, such as the ellipse, the parabola, other conic sections, or random curves.

SPHERE SEGMENT - COLUMN SUPPORTS:



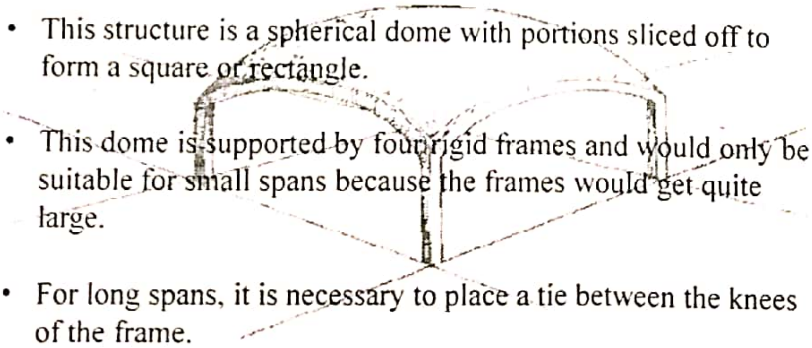
- If a dome is built as less than a half sphere, a tension ring of steel bars, plates, or wires is required at the base to carry the thrusts of the shell.
- In this case, the ring has been made big enough so that it assists in distributing the reaction of the columns into the dome.
- The direct stresses in the shell are mostly compressive in this structure and are so small that the stress calculations are hardly necessary.
- Domes have been built with a thickness of 6 inches for a span of about 300 feet.

HALF SPHERE - VERTICAL WALLS:



- A half sphere for a dome of revolution does not require a thrust ring at the base so it can be placed on vertical walls and made continuous with the walls.
- This design is used for tanks because the roof becomes a part of the tank.

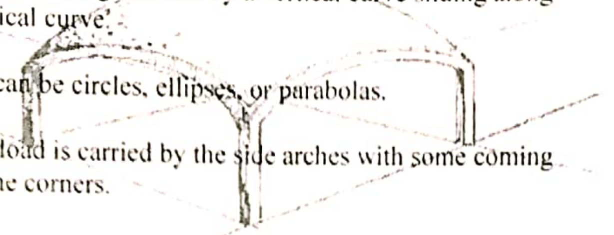
DOMES - SQUARE IN PLAN:



- This structure is a spherical dome with portions sliced off to form a square or rectangle.
- This dome is supported by four rigid frames and would only be suitable for small spans because the frames would get quite large.
- For long spans, it is necessary to place a tie between the knees of the frame.
- Stresses in the shell are direct compression (membrane) stresses except across the corner where there are direct tensile forces due to the outward spread of the forces.

TRANSLATION DOMES:

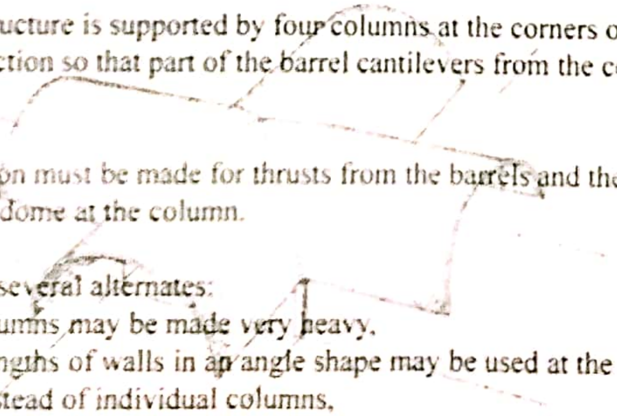
- This structure looks very much like the Square Dome except the shape is generated by an entirely different method.
- A translation shell is generated by a vertical curve sliding along another vertical curve.
- The curves can be circles, ellipses, or parabolas.
- Most of the load is carried by the side arches with some coming directly to the corners.
- Such shells are suitable for quite long spans with some interior lighting furnished by skylights in the shell.
- Barrel shells, folded plates, and shell arches are all special cases of translation shells.



INTERSECTION SHELL - CROSS FORM:

- Four cylindrical barrels intersect to form a central dome.
- The structure is supported by four columns at the corners of the intersection so that part of the barrel cantilevers from the central dome.
- Provision must be made for thrusts from the barrels and the central dome at the column.

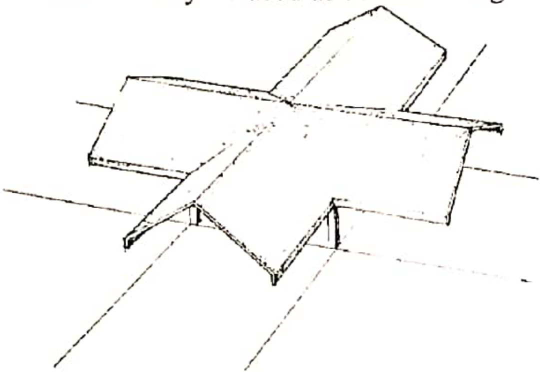
There are several alternates:
1) the columns may be made very heavy,
2) short lengths of walls in an angle shape may be used at the corners instead of individual columns,
3) diagonal members may be placed in each of the walls, or
4) ties may be place between tops of columns.



- The architectural advantage of this structure is that it appears to float in the air.

INTERSECTION SHELL - FOLDED PLATE:

- Almost all the combinations used for curved shells may be used for folded plates, the resulting forms are almost unlimited.
- The columns may be place so that there is no column at the corner and the central dome is suspended from four cantilevers.
- However, it is better to put the column in the corner so that the central intersection may be used as the stiffening element.



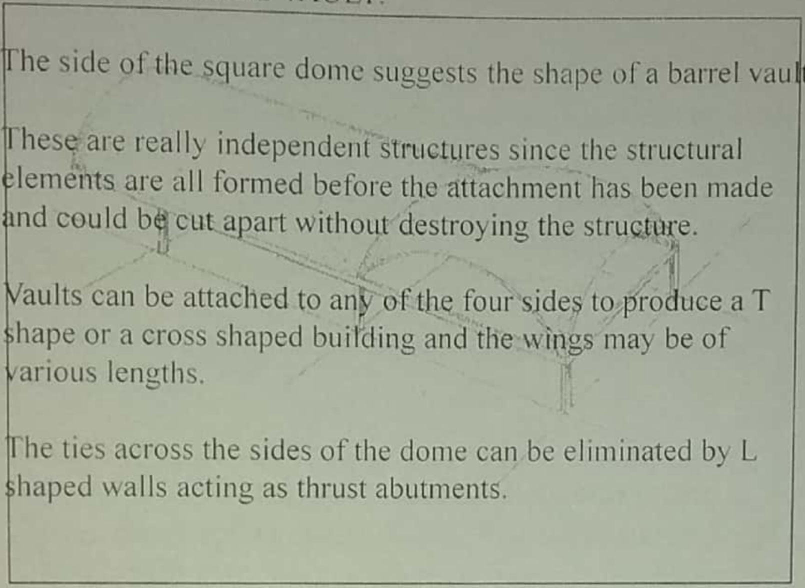
COMBINATIONS OF SHELLS:

- Combinations of shells are useful and lend variety to the other shapes and forms.

The number of combinations is practically unlimited.
The combinations possible are:

1) barrel shells and folded plates,
2) barrel shells and short shells,
3) barrel shells and domes of revolution,
4) barrel shells and conoids,
5) folded plates and short shells,
6) folded plates and domes of revolution,
7) folded plates and conoids,
8) short shells and domes of revolution,
9) short shells and conoids, and
10) domes of revolution and conoids.

DOME AND BARREL VAULT:

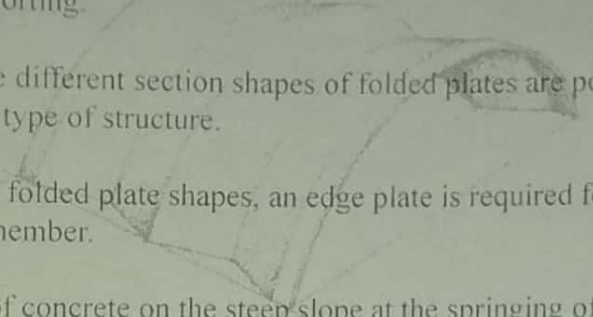


- The side of the square dome suggests the shape of a barrel vault.
- These are really independent structures since the structural elements are all formed before the attachment has been made and could be cut apart without destroying the structure.
- Vaults can be attached to any of the four sides to produce a T shape or a cross shaped building and the wings may be of various lengths.
- The ties across the sides of the dome can be eliminated by L shaped walls acting as thrust abutments.

SHELL ARCHES:

- Folded plates and cylindrical barrel shells are essentially beams.
- The same cross sectional shapes can be used for arches and a new set of forms, having different structural properties, is obtained.
- There are types of shells that fit in several categories.
- The hyperbolic paraboloidal dome is really a shell arch.

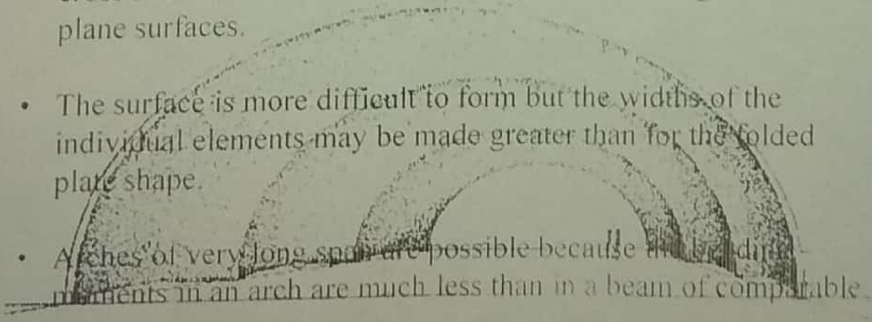
FOLDED PLATE ARCH:



- This structure is suitable for quite long spans and forms for the concrete can be used many times because each unit can be made self-supporting.
- All of the different section shapes of folded plates are possible with this type of structure.
- As in the folded plate shapes, an edge plate is required for the outside member.
- Placing of concrete on the steep slope at the springing of the arches may be a problem unless blown-on concrete is used or the lower portion of the shell may be precast on the ground and lifted into place.

BARREL ARCH:

- This shape is similar to the folded plate shell arch except that cross sectional elements are curved instead of being made with plane surfaces.



- The surface is more difficult to form but the widths of the individual elements may be made greater than for the folded plate shape.
- Arches of very long span are possible because the bending moments in an arch are much less than in a beam of comparable span.

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