SOBHAN SIR

Surveying:

Object of surveying by aim of surveying:

To prepare a map to show the thesus of the area such as town, village, agricutural land, ponds, canal, hill, river, highway, railway, tree, building

Importance of surveying:

It is vital importance in any engineering projects because →

① Without surveying not possible to select alignment of highway, railway, canal.

② Measurement of land and the fixation of bounderies.

③ Engineering feasivility impossible without surveying.

benefit
cost
} ratio > 1.5

Classification of surveying:

① Primary classification:

A. Plane : Area consider করা হয় না।
B. Geodetic : " " " হয়।

(ii) Secondary classification:

A) Based on field:

(i) Land survey

(ii) Marine survey

(iii) Astronomical survey.

B) Based on instrument used:

(i) Plane table survey

(ii) Compus survey

(iii) Theodolite survey

(iv) Chain survey.

Types of measurement:

(i) Linear measurement → horizontal, verticle

(ii) Angular measurement → horizontal, verticle

Unit of measurement:

(i) British Unit (inch, ft, yards, mile)

(ii) Mathic Unit (mm, cm, m, km...)

Methods of linear measurement:

① By pacing or stepping

② By speedmeter

③ By chaining.

Types of chain:

Sl. No.	Types of chain	length of chain	No. of links	length of one link (m)
1.	Engineer's chain	100 ft.	100	1
2.	Gounter's chain	66 ft.	100	0.6
3.	Metric chain	20 m 30 m	100 150	0.2 0.2

1 inch = 2.5 cm

1 ft = 30.48 cm

2.5 Feet 80 cm
↳ one step of walking

Types of tape:

① Cloth or linen

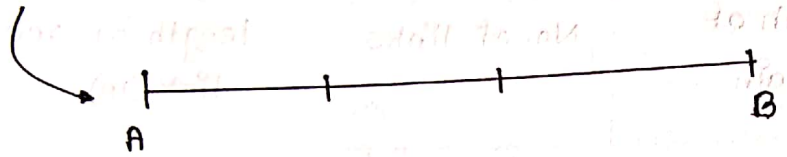
② Metalic tape

③ Steel tape

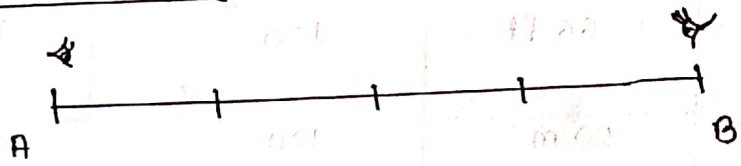
Chaining: Chaining is a process measuring the linear distance between two stations with the help of chain.

① Ranging
② Chaining

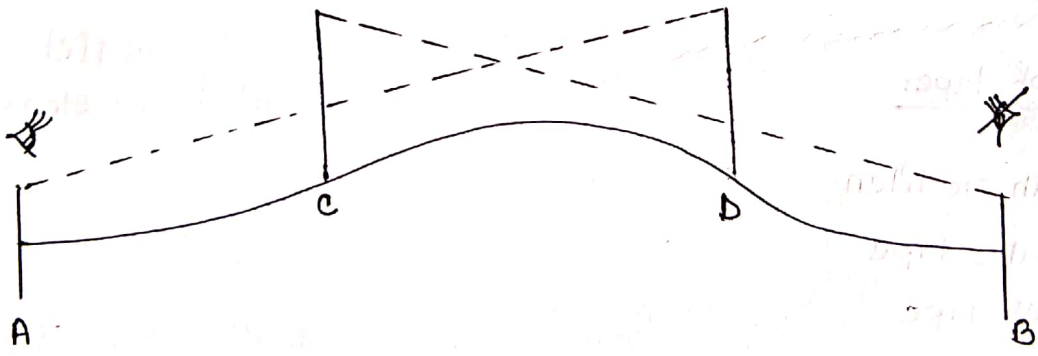
Ranging: Ranging is a process to make a line straigth.



① Direct Ranging:



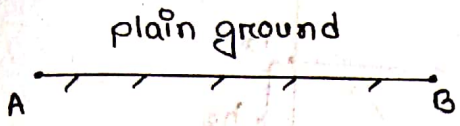
② Indirect Ranging:

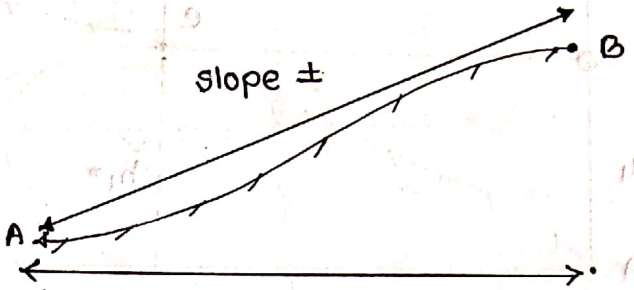


এখানে A, C ও D হলো same situation (একই line)

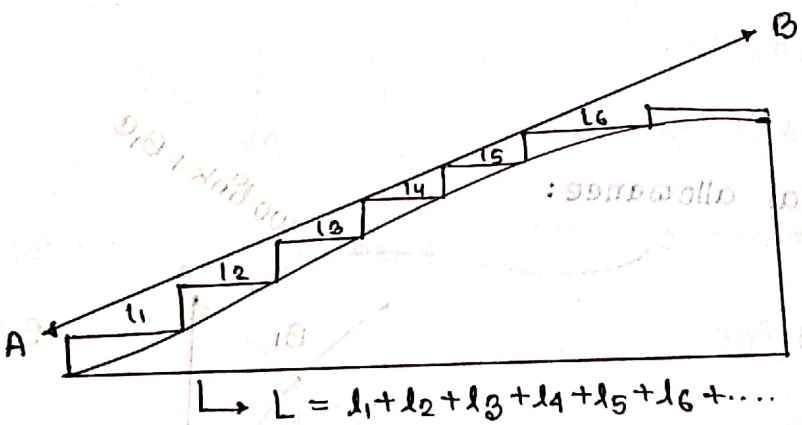
এবং B " " " " এ কারণে তাকে বলে Indirect Ranging

Methode of chaining on slopping ground:



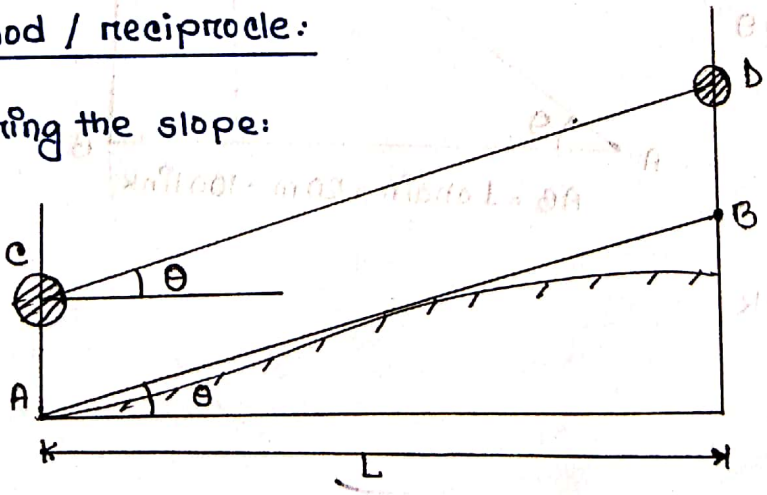


① Direct / Straight Methode:



⑪ Indirect Method / reciprocle:

A) By measuring the slope:

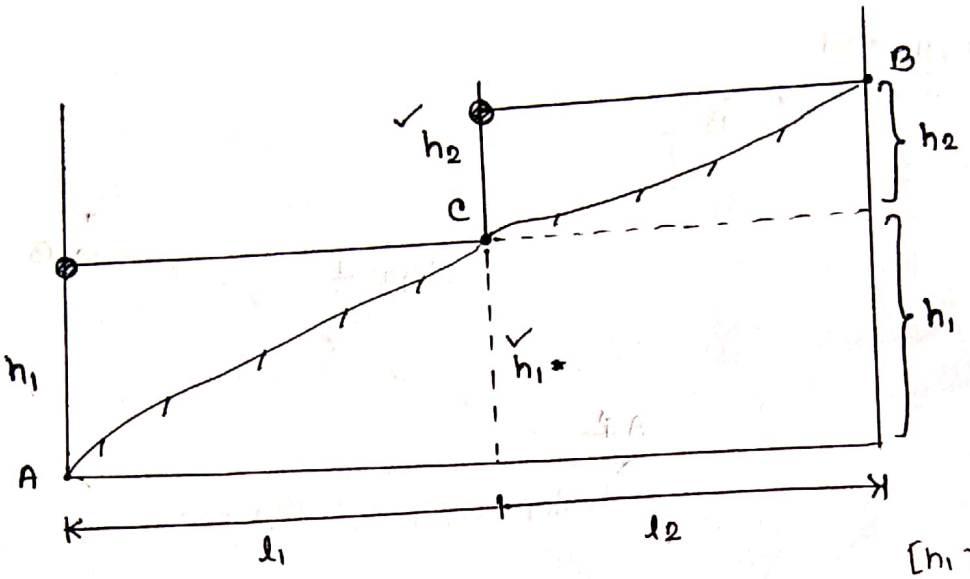


AC = BD

$\frac{L}{AB} = \cos\theta$

$\Rightarrow L = AB\cos\theta$

B) By knowing the difference of level:

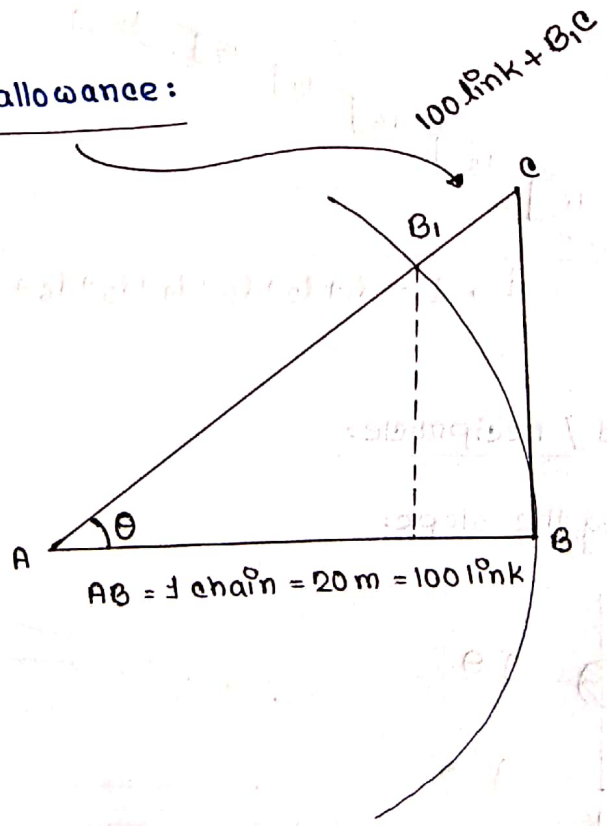


[h_1 measured by telescope

$L = l_1 + l_2$

$L_1 = \sqrt{AC^2 - h_1^2}$

C) Applying Hypotenusal allowance:



B_1C → hypotenusal allowance

$AB = AB_1$

অনুরূপ
$\Delta ABC \Rightarrow \frac{AC}{AB} = \sec\theta$

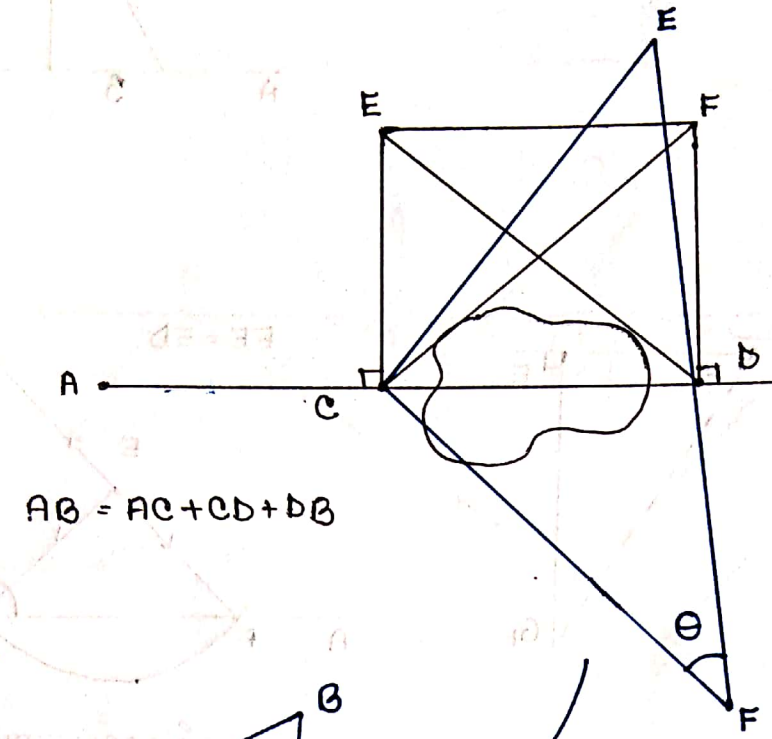
$\Rightarrow AC = 100 \sec\theta$

$B_1C = AC - AB_1$

$= 100 \sec\theta - 100$

$= 100 (\sec\theta - 1)$ link

Obstacle in chaining:



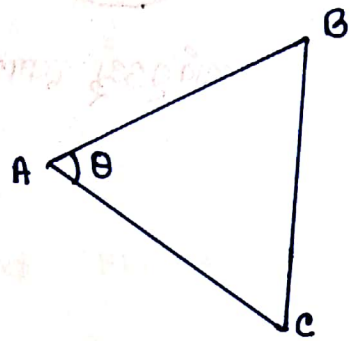
① $CD^2 = DE^2 - EC^2$

② $CD = EF$

$DE^2 = CE^2 + CD^2$

$CD = \sqrt{DE^2 - CE^2}$

$AB = AC + CD + DB$



$BC^2 = AC^2 + AB^2 - 2\,AB \cdot AC \cos\theta$

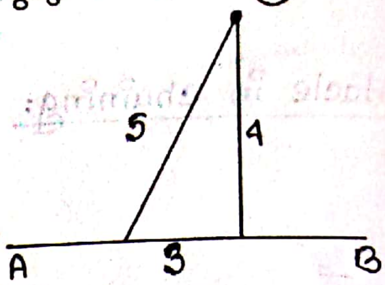
$CE^2 = CF^2 + EF^2 - 2\,CF \cdot EF \cos\theta$

$\Rightarrow \cos\theta = K$

$\Delta\, CFD \Rightarrow$

$CD^2 = CF^2 + DF^2 - 2CF \cdot DF \cos\theta$

⊞ 90° করার উপায়



River এর width যদি কম হয় →

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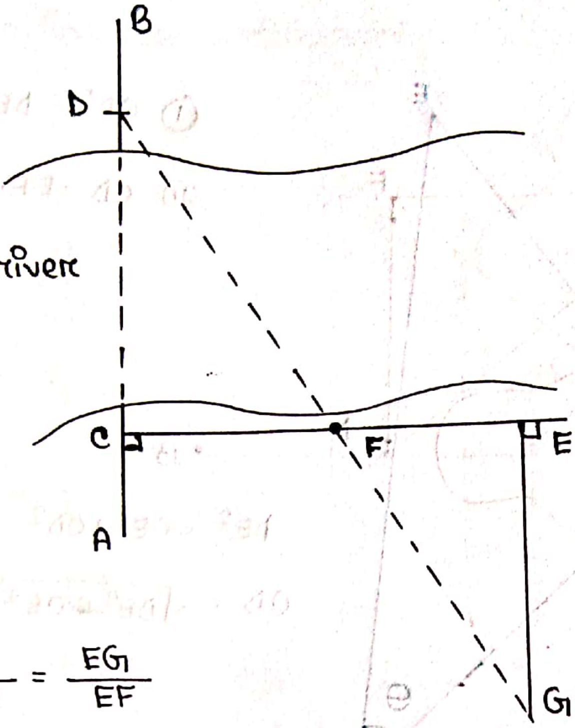
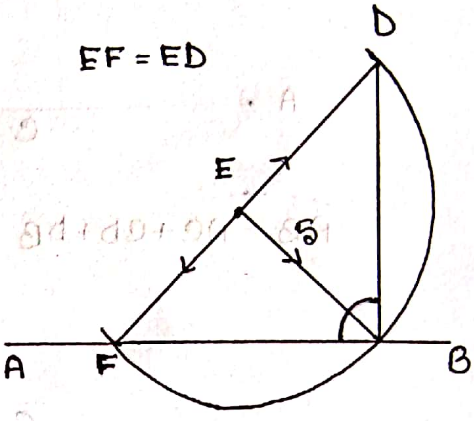
Small river



$$\frac{CD}{DF} = \frac{EG}{EF}$$

$$CD = \frac{EG}{EF} \times DF$$

EF = ED



অর্ধবৃত্তস্থ কোণ

River এর width যদি বেশি হয় →

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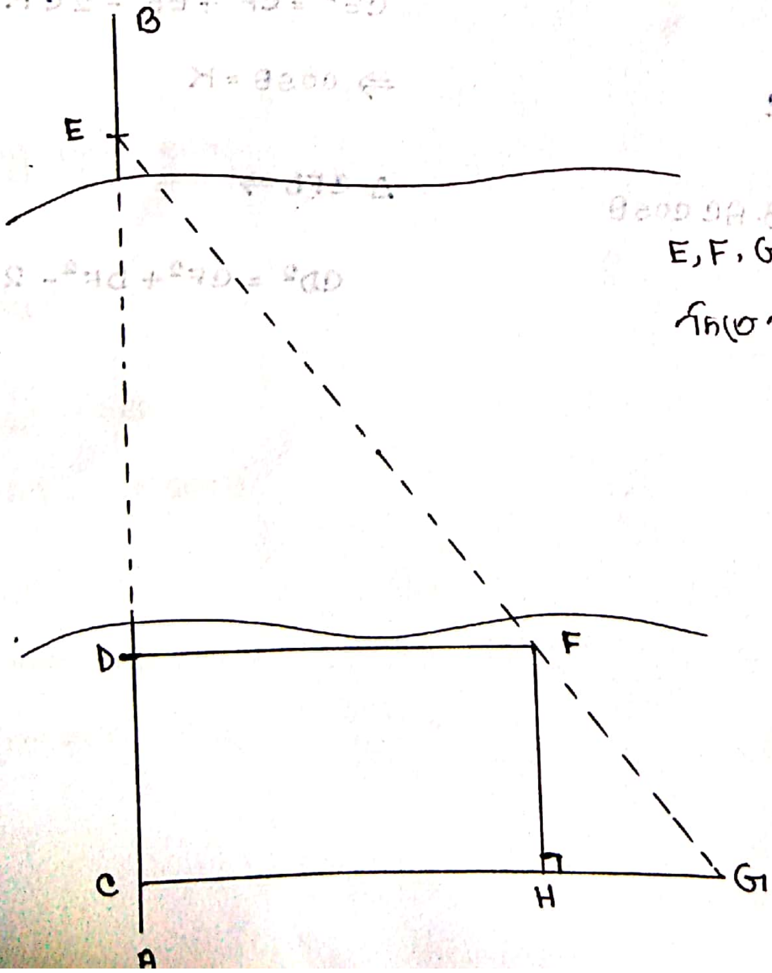
Big river

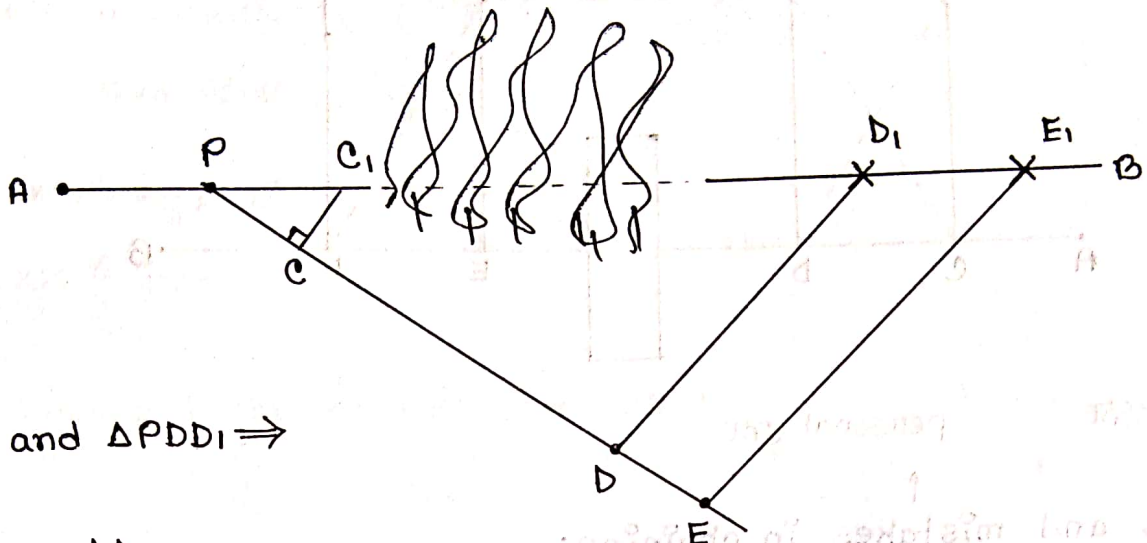


E, F, G – এ এমনভাবে Ranging Rods দিতে হবে যেন একই line – এ থাকে

$$\frac{DE}{DF} = \frac{FH}{GH}$$

$$DE = \frac{FH}{GH} \times DF$$

Obstacle in Ranging:



ΔPCC_1 and $\Delta PDD_1 \Rightarrow$

$$\frac{CC_1}{PC} = \frac{DD_1}{PD}$$

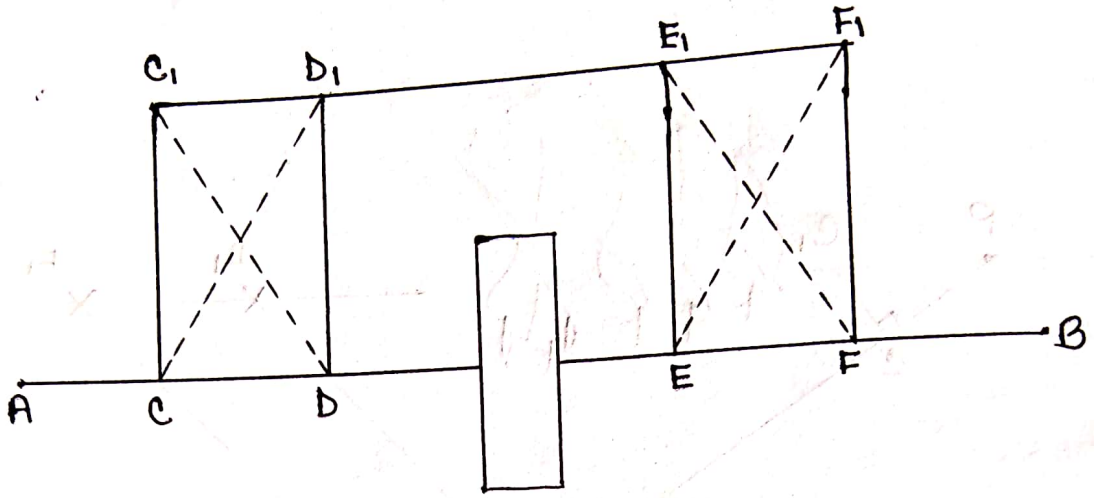
$\Rightarrow DD_1 = ?$

ΔPCC_1 and $PEE_1 \Rightarrow$

$$\frac{CC_1}{PC} = \frac{EE_1}{PE}$$

$\Rightarrow EE_1 = ?$

Obstacle in chaining and Ranging:



chaining ভুল personal ভুল

Errors and mistakes in chaining:

1. Compenseting error
2. Cumulative error → chain এ link এর কম/বেশি হওয়া
   a) Positive (when chain is too short)
   b) Negative (when chain is too long)
3. Mistake.

L = 30 m (Length of steel tape)

Weight, $\omega = 0.693$ kg

Area, $A = 0.03\ cm^2$

Standard Temperature, $T_S = 20°C$

" Pull (টান), $P_S = 10$ kg

$E = 2.1 \times 10^6\ kg/cm^2$

$\alpha = 11 \times 10^{-6}$ per °C

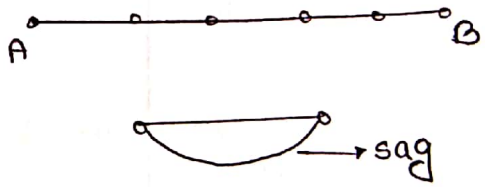
⇒ Field → Measured Temperature, $T_m = 32°C$

" Pull, $P_m = 15$ kg

$C_t = \alpha (T_m - T_S) L$ → connection for T = + 0.00396 m

$C_p = \frac{(P_m - P_S) L}{AE}$ → connection for P = + 0.00238 m

$C_s = \frac{\omega^2 L}{24 n^2 P_m^2}$ → connection for Sag = − 0.00267 m
(always (−ve))



Total error, $l = C_t + C_p + C_s$

$= 0.00396 + 0.00238 - 0.00267$

$= 0.00367$ m (+ve)

Ans.

measured Length (ML) = 780 m

Actual Length (Act. L) $= \frac{L'}{L} \times ML$

$= \left(\frac{L \pm e}{L}\right) \times ML$

$= 780.095$ m

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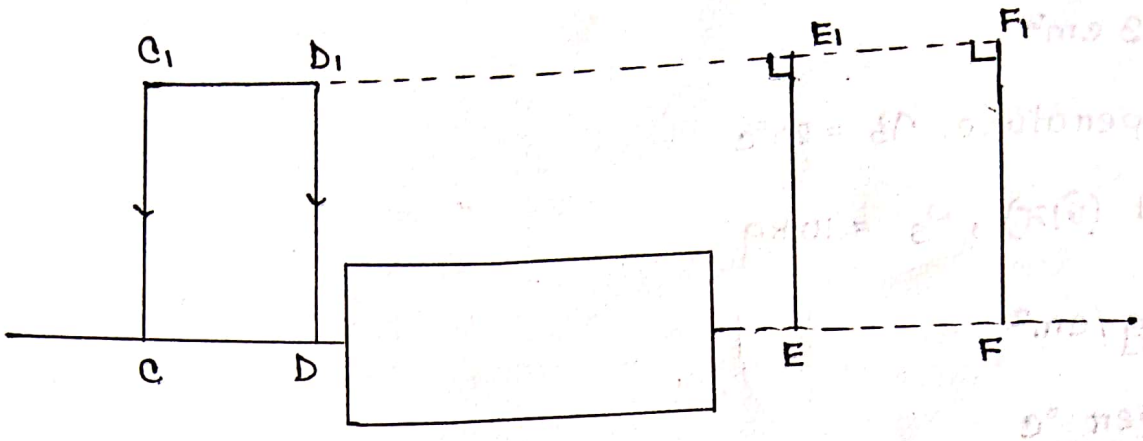
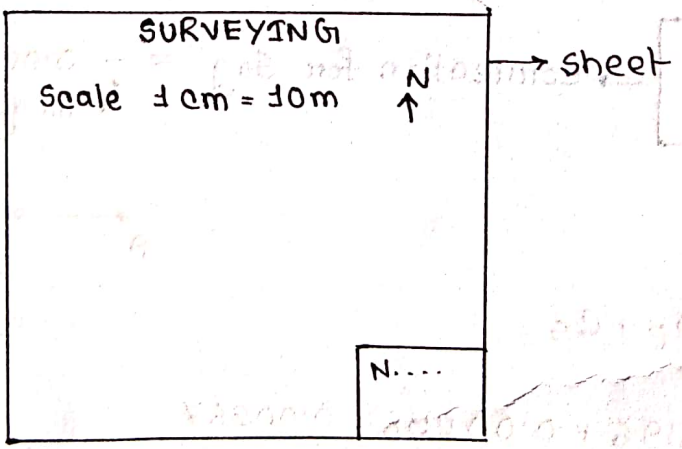


Free survey:

(i) Field work

(ii) office work

(iii) Adjustment of the instrument:



Scale is a device used to measure the size of any object.

Scale is the fixed ratio, by which the actual length of the object is reduced or increased.

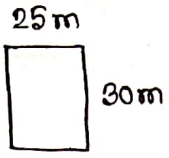
Type:

① Full size scale:

1cm = 1cm [Map ও object এর আপ same]

② Reduced scale:

1cm = 5m



③ Increased or enlarged scale:

1cm = 0·01 mm

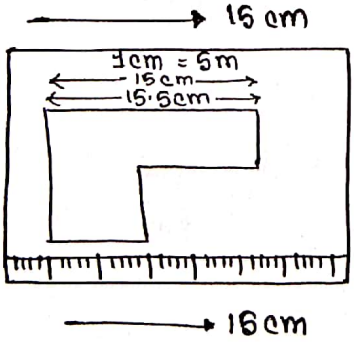
④ Engineering scale:

1cm = 5 m or, 1 cm = 10 m

Representative fraction, $RF = \frac{1cm}{5m} = \frac{1cm}{5\times100cm} = \frac{1}{500}$ [map এর 1 unit field এর 500 unit]

⑤ Graphical scale:

map 5cm → 2·5 m field



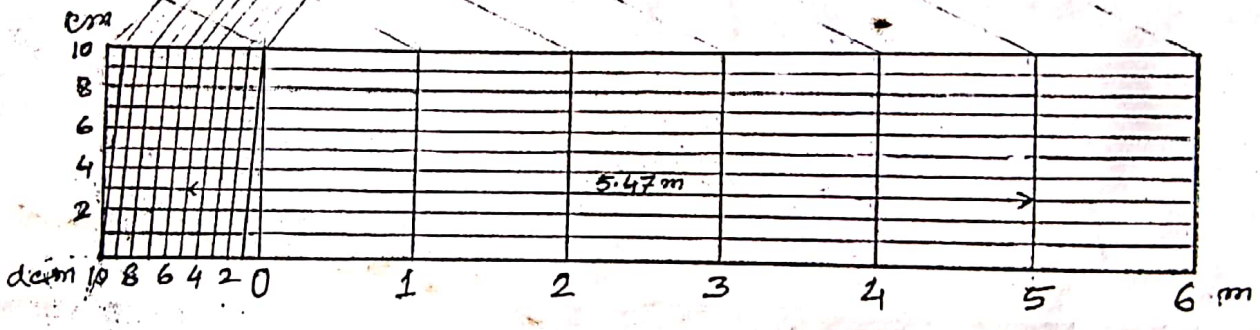
এই scale এর large এর ratio কাগজের অংশ থাকে। 10 বছর পর কাগজ যতটুকু enlarge হবে, scale ও হবে, আপ দিক থাকবে।

⑥ Plain scale: পাশাপাশি 2টা unit আপা যায়,

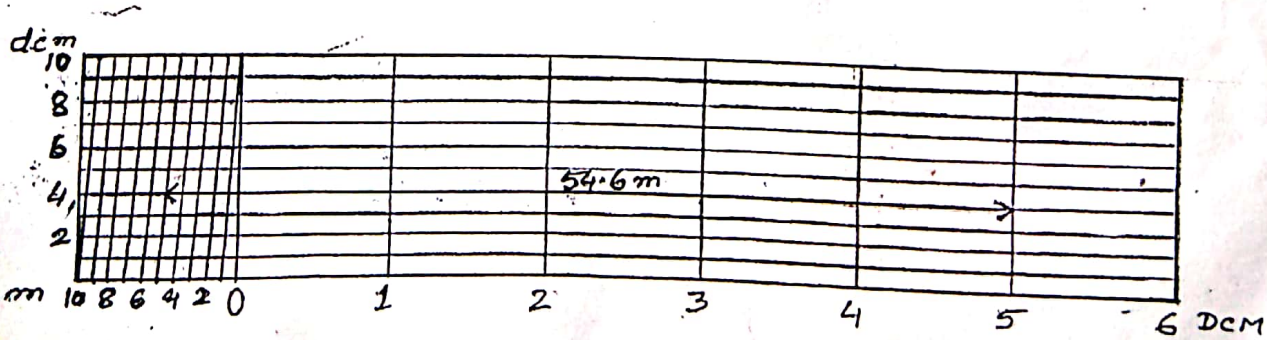
(m, inch ; dm, cm ; cm, mm) ——→ 5·2

Diagonal scale: (m, dm, cm.) → 5·74 or (dm, cm, mm)

vernier scale:



DIAGONAL SCALE



Cs নাই
n এর মান নাই

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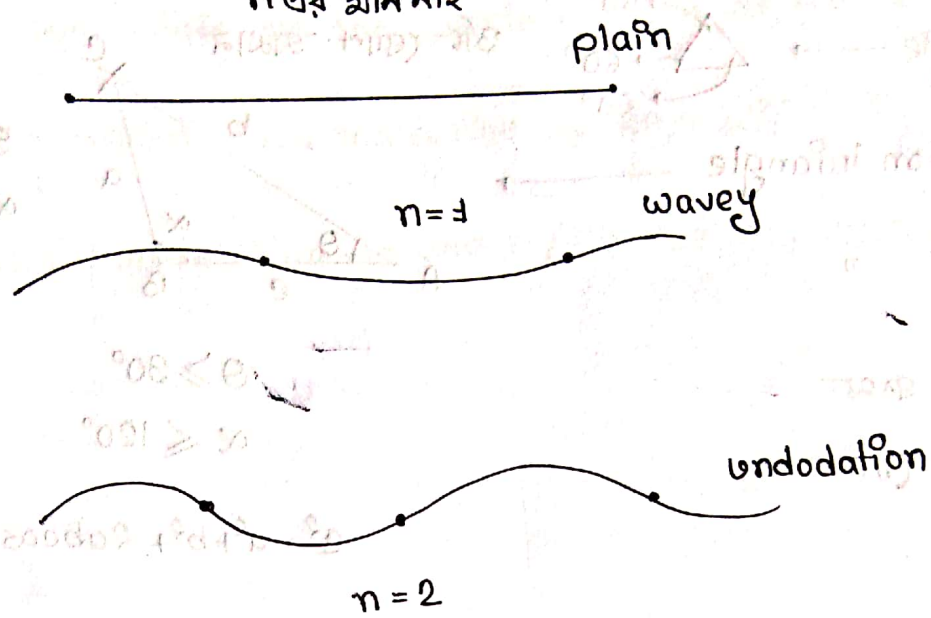
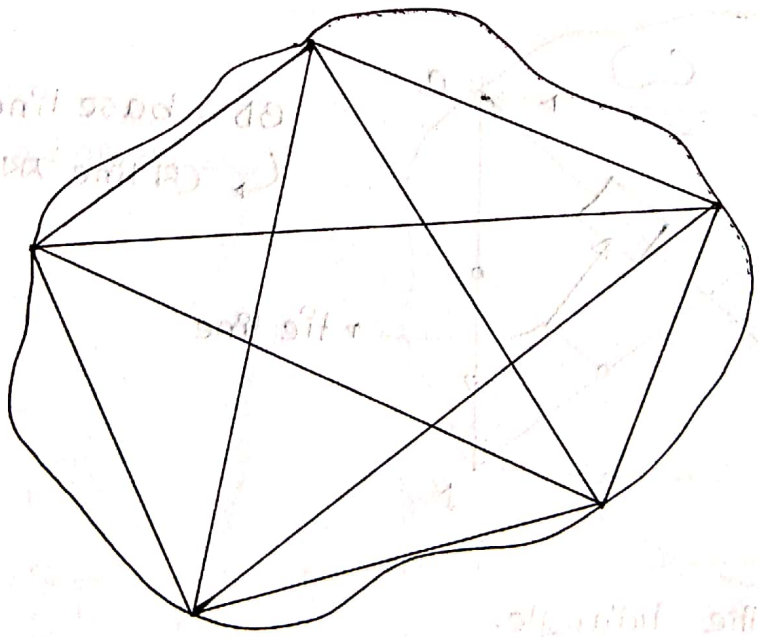
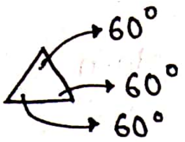


Chain surveying: (liner measurement)

Basic principle: Triangulation (area covered by three sides)

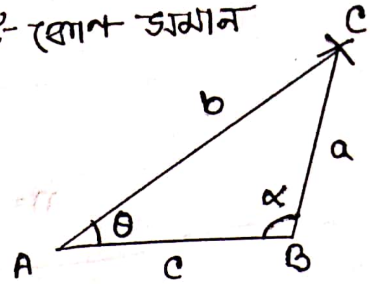


(i) Ideal triangle →



৩টি কোণ সমান

(ii) Well condition triangle →



θ = ক্ষুদ্রতম কোণ

α = স্থূলকোণ

(iii) Ill " "

θ → 30° এর কম

α → 120° এর বেশি

$\theta \geqslant 30°$

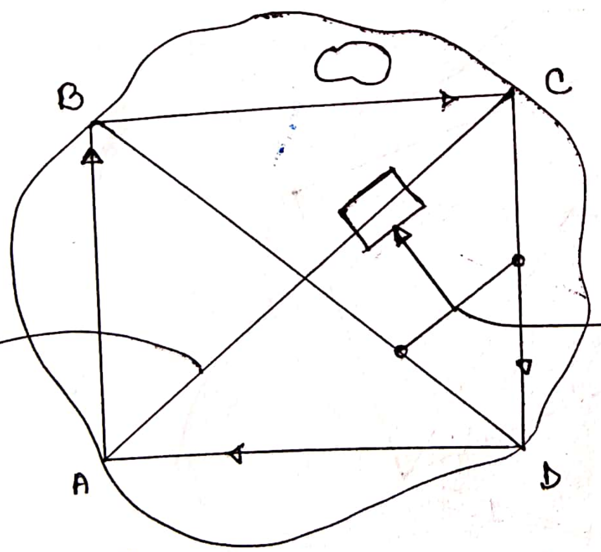
$\alpha \leqslant 120°$

$c^2 = a^2 + b^2 + 2ab\cos\theta$

Reconnissance: Primary survey

Reconnissance survey (primary survey):

(i) Index sketch (Key plan):



BD = base line

↳ যে line সবচেয়ে বড়

tie line

check line → to check the aggricy of the triangle.

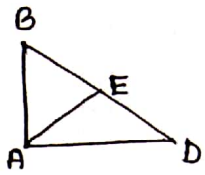
tie line is also a sag line

check line is also a tie line.

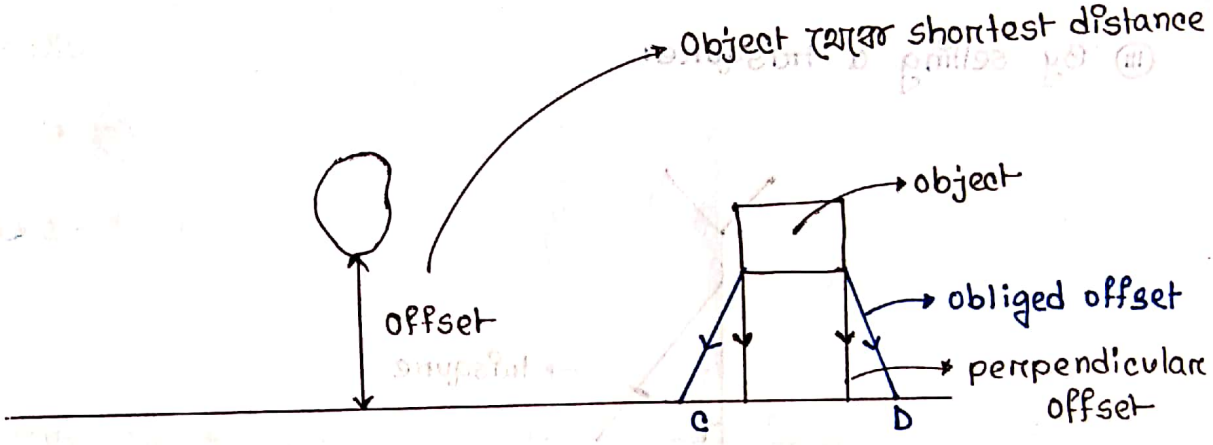
⑪ Station is the points at the begining and at the end of a chain.

a) main station ⟶ main line এর আগে থাকে △ ⊙

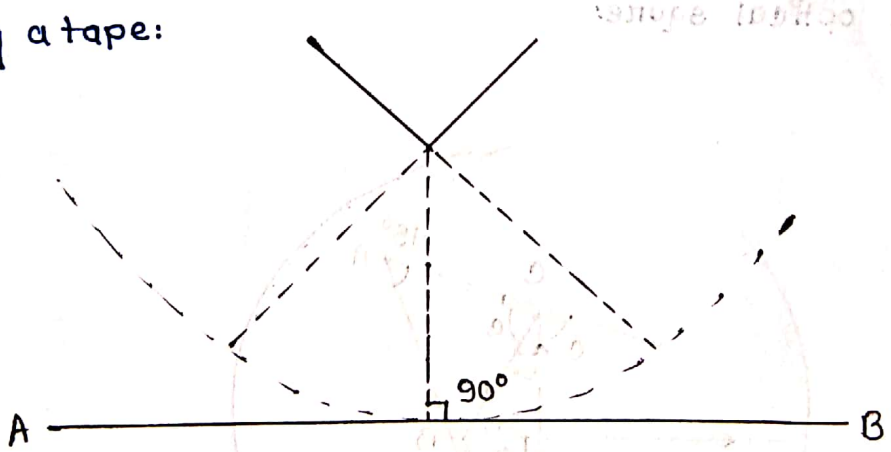
b) Subsidary station ⟶ check / tie line এর আগে থাকে



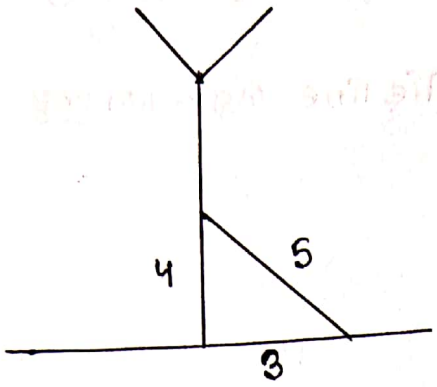
Object, offset:



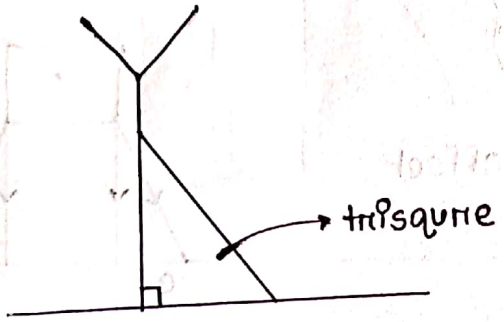
① By swinging a tape:



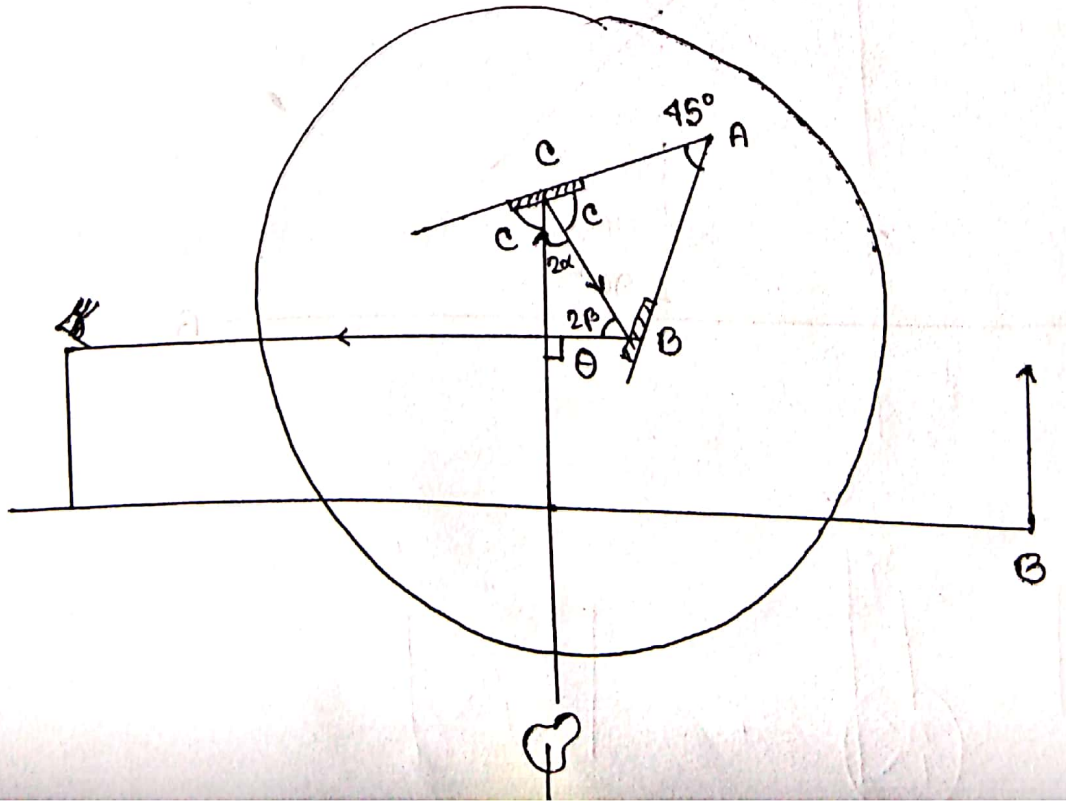
(ii) By setting a ratio 3:4:5



(iii) By setting a trisqure:



(iv) By setting a optical squre:



$$\angle A + \angle B + \angle C = 180^\circ$$

$$\theta = 2\alpha + 2\beta$$

$$\Rightarrow \alpha + \beta = \theta/2$$

$$2C + 2\alpha = 180^\circ$$

$$2B + 2\beta = 180^\circ$$

$$2C + 2\alpha + 2B + 2\beta = 360^\circ$$

$$\Rightarrow B + C + \alpha + \beta = 180^\circ$$

$$\Rightarrow \alpha + \beta = 180^\circ - (C + B)$$

$$\Rightarrow \theta/2 = A + B + C - C - B$$

$$\Rightarrow \theta/2 = A$$

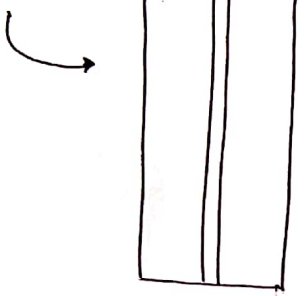
For, $\theta = 90^\circ$, $\angle A$ must be 45°.

limiting length of offset:

not more than the length of tape or 15 m.

Equepment of chain surveying:

① Engineering chain (1), tape (1), ranging rod (at least 3), arrows (5), optical squre (1), compass (North line ঠিক করা –1).

field book  , paper, pencil, eraser

Procedure of field work:

① Reconnaissalce

② Index sketch.

③ Marking the stations on the ground.

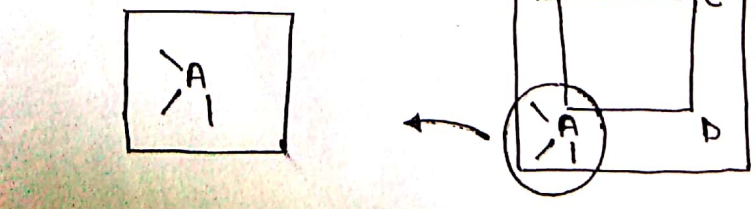
④ Reference sketches

⑤ Measurements noting in the fieldbook.

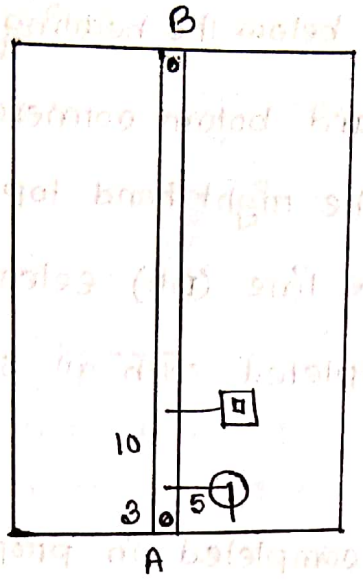
→ selection of survey station

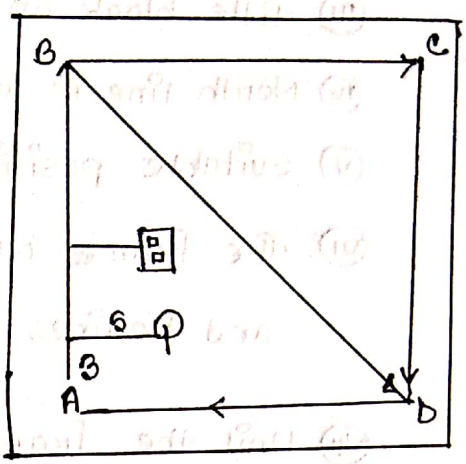
a) station should be intervisible.
b) stations should form well condition triangle.
c) Surveyline should be on level ground.
d) chaining free from obstacle

④ →

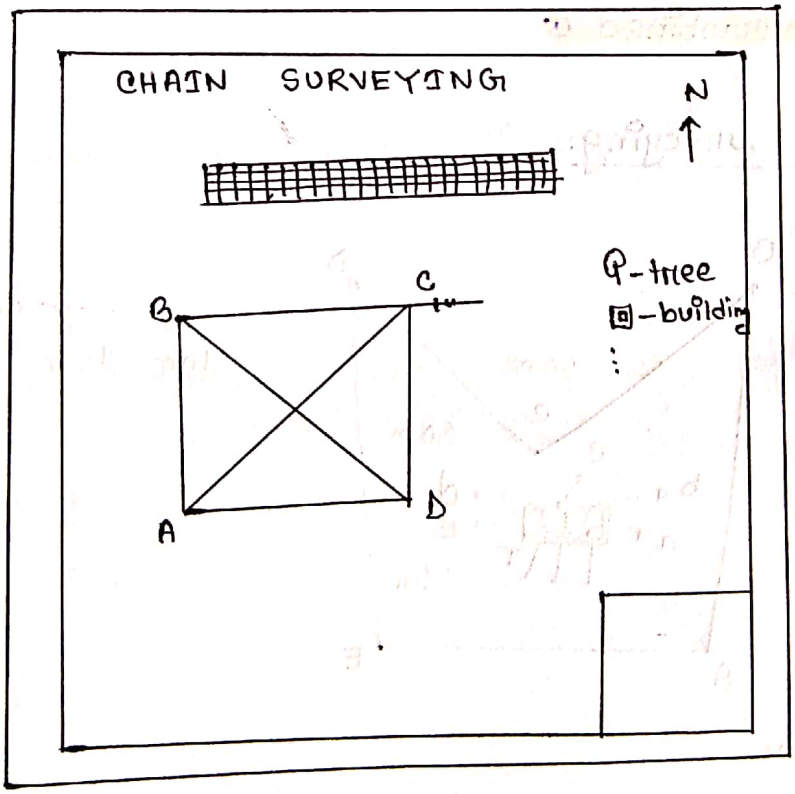


⑤ Field book recording:





Procedure of ploting:



(i) A suitable scale

(ii) Scale of the map is drawn below the heading.

(iii) Title block on the right hand botom corner.

(iv) North line is marked on the right hand top corner.

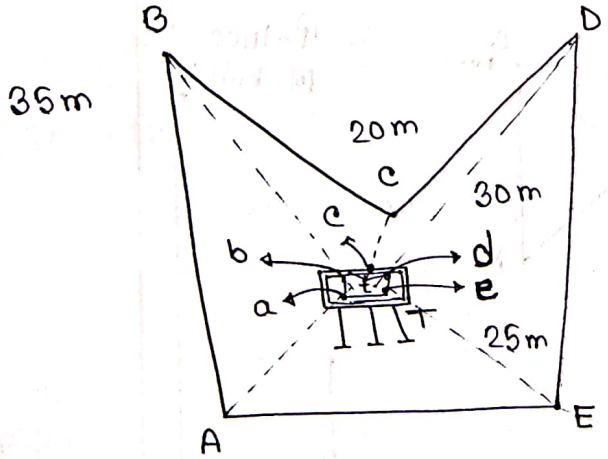
(v) Suitable position for base line (BD) selected on sheet.

(vi) The frame work is completed with all survey lines, check line and tie lines.

(vii) Unit the frame work is completed in proper form, the offsets should not be ~~bre~~ ploted.

(viii) Conventional symbols used in the map should be shown on the right hand side.

Plane table surveying:



1cm = 10 m

Principal: Parallelism (verticle)

plane tabling is a graphical method of surveying. Field work and potting are done simultaniously without the use of a fieldbook.

equipment: (i) plane table

(ii) alidade 

↳ a) plain alidade

b) teriscopic "

(iii) Spirit level

(iv) Compus

(v) U-fork  / Plumbing fork 

Orientation: Orientation is the process of putting the plane table into some fixed direction so that the line representing a certain direction of the plan is parallel to that directions on the ground.

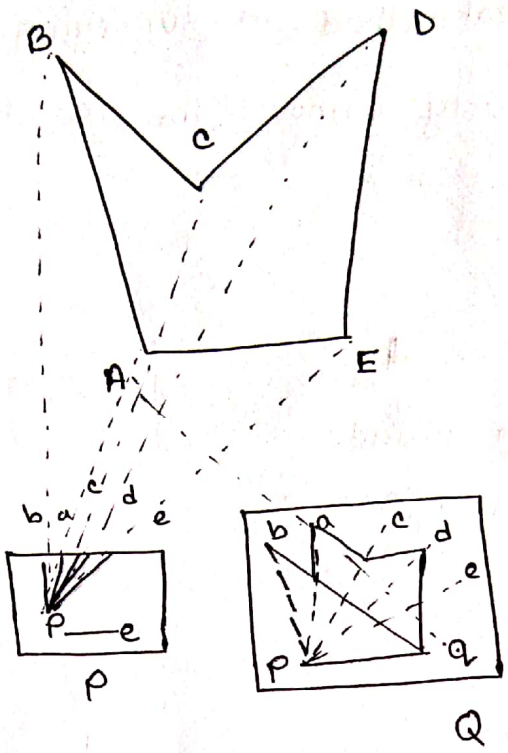
Method of Plane Tableing:

(i) Radiation → This method is morre suitable when the distances are small (within a tape length).

(ii) Intersection →

(iii) Traversing

(iv) Re-section



Intersection

1cm = 10m

Procedure:

① Fixing the table on the tripot stand.

② Leveling the table.

③ Centering the table.

④ Marking the dot line.

⑤ Orientation.

Errors:

① Instrumental

② Personal

③ Plotting

Advantage:

(i) Field work

(ii) Rapid

(iii) No field book

(iv) Suitable in magnetic area.

(v)

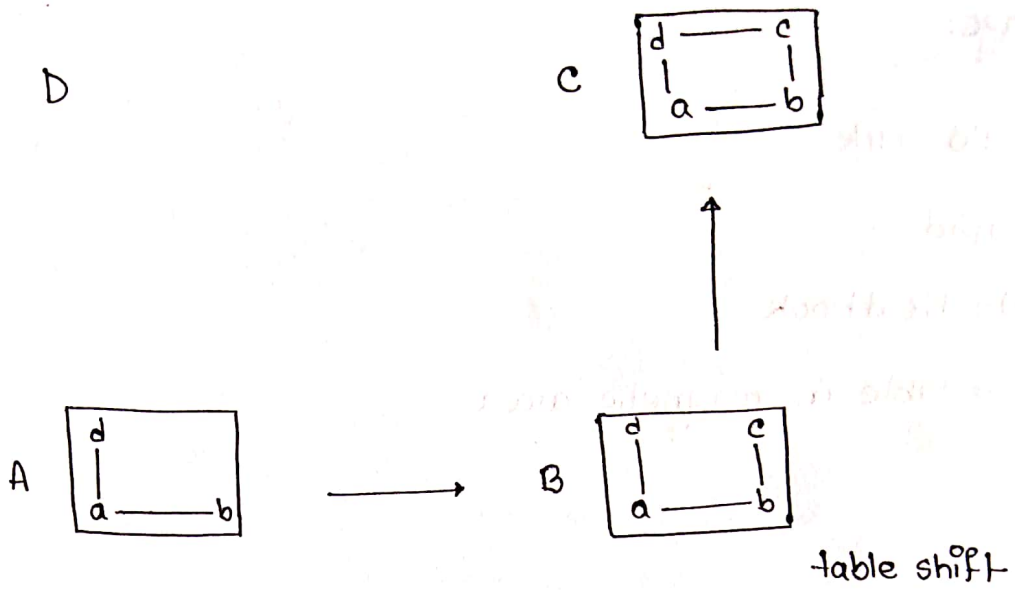
Disadvantage:

(i) Not suitable in wet ~~magnet~~ climate.

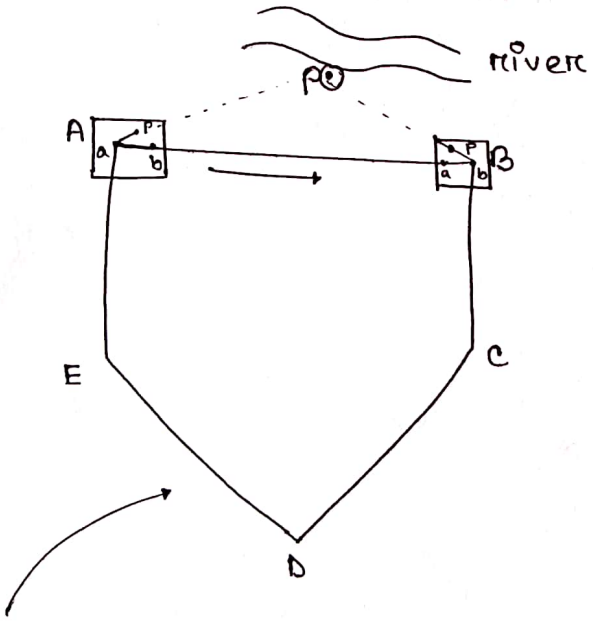
(ii) " " for accurate work.

(iii) Difficult to carry the instrument.

#



Resection: This method is suitable for stablishing new stations at a place in order to locate missing details

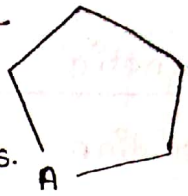


a) Two-point problem.

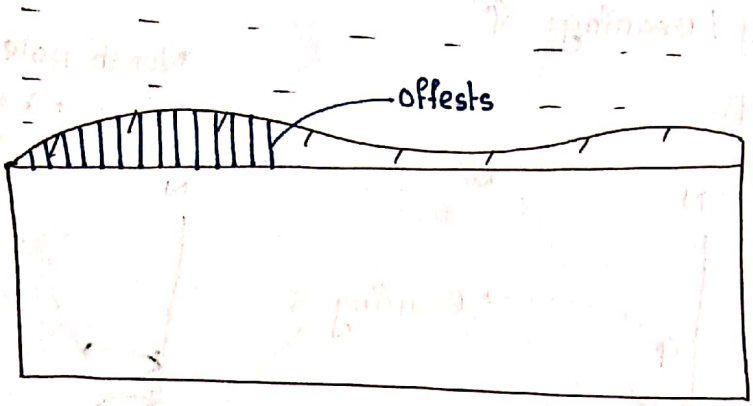
b) Three-point problem:

*

Traverse survey: A traverse is a frame work consist of a number of connected lines. Lengths are measured by chain or tape and the directions identified by compus.



① close traverse

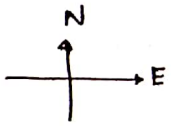


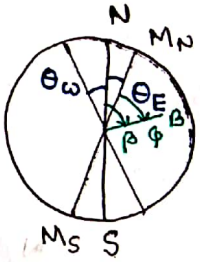
⑪ open traverse

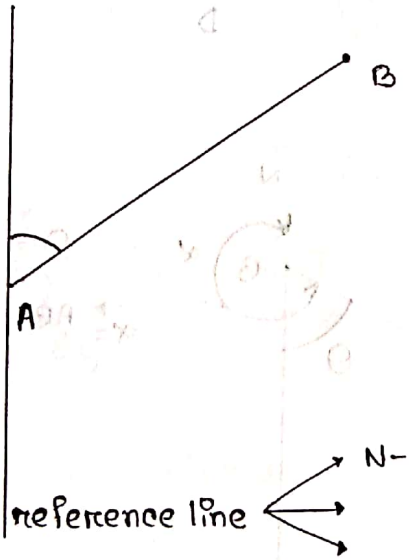
↳ last end, first end এর আগে মিলবে না।

a) compus traverse:

b) Theodolite traverse:







reference line → N-S line → true ~~too~~-meridian / Magnetic meridian

true meridian is the line passing through North pole and south pole and any point of the surface of the world.

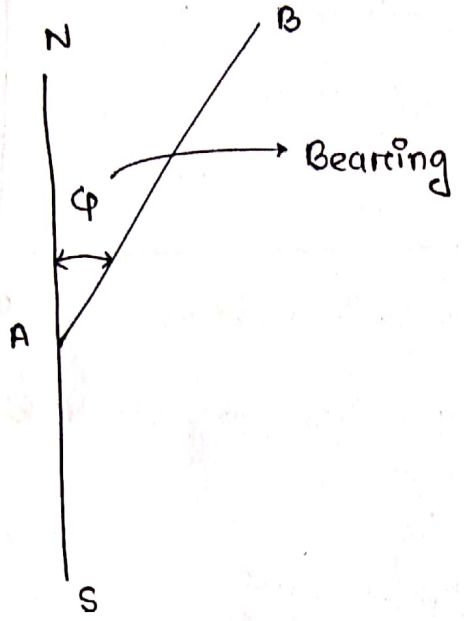
Magnetic meridian is the line indicated by a 3D suspended and balanced magnetic nidle.

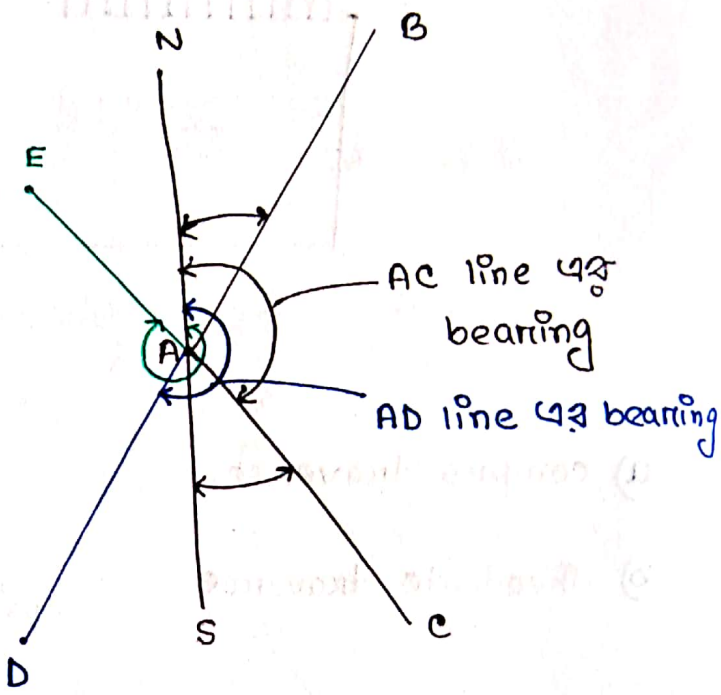
Magnetic declination: Angle between two meridian and magnetic meridian.

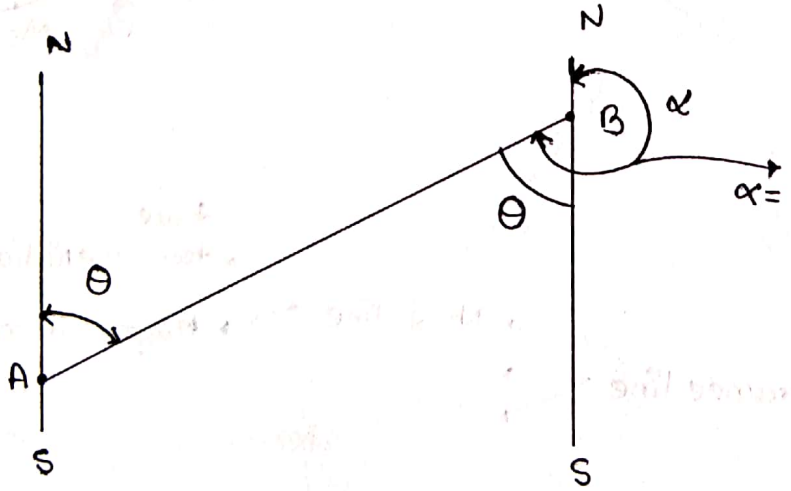
Magnetic Bearing / Bearing: ϕ

True Bearing: β

North pole থেকে clockwise → bearing







θ = AB line এর FB

α = AB line এর BB

forward bearing → FB

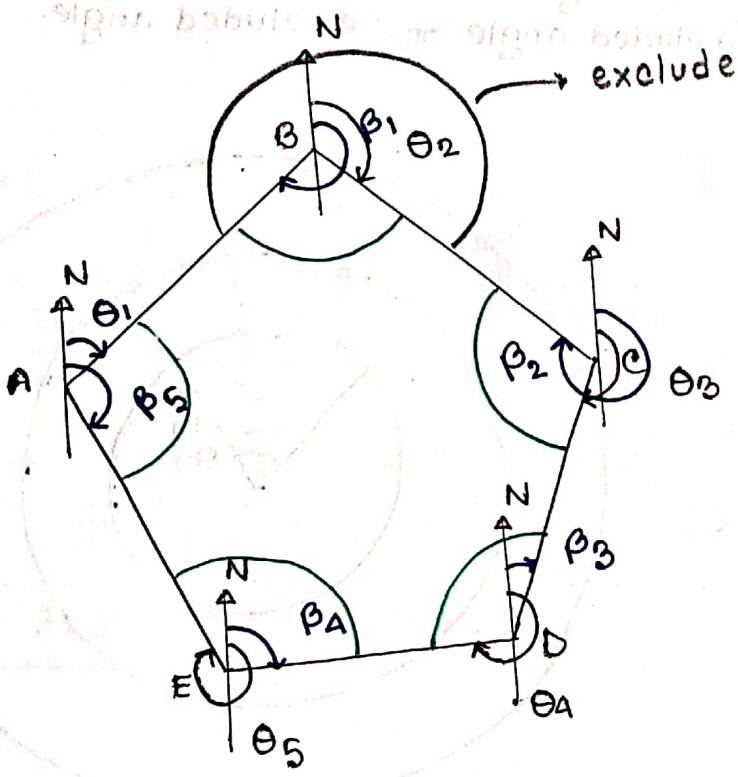
backward bearing → BB

* difference between FB and BB of any line is 180°.

$$\alpha - \theta = 180°$$

* Traverse:

Two types of traverse are → open traverse
→ closed "



$\angle A + \angle B + \angle C + \angle D + \angle E$ → included angle

Characteristics of closed Traverse:

(i) Sum of included angle = $(2N-4)\,90°$ [N = No. of side]

(ii) Sum of excluded angle = $(2N+4)\,90°$

(iii) F.B and B.B of a line must differ by 180°.

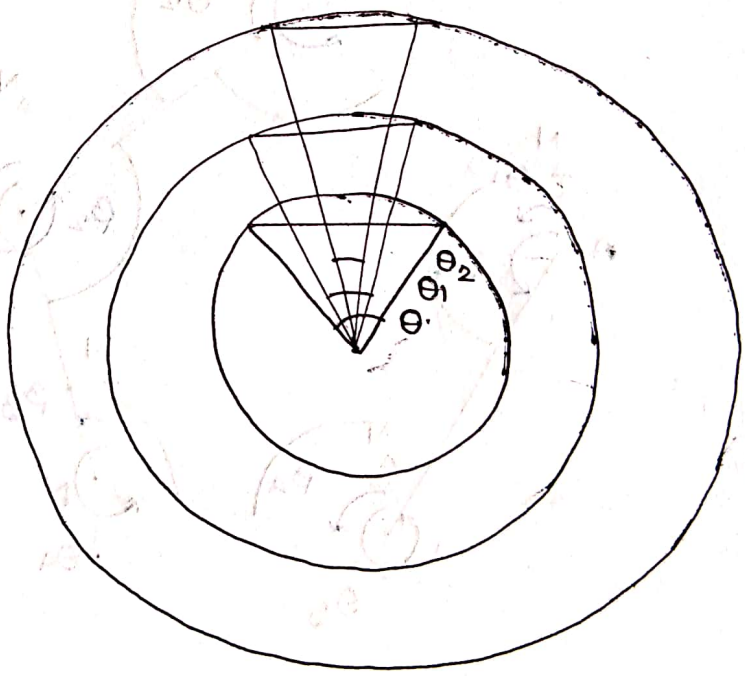
↳ F.B ~ BB = ±180°

(iv) Difference between ~~two~~ bearings of two lines at a point is — ∠included angle or, ∠excluded angle.

Designation of bearing:

$$R = \frac{1720}{b}$$

b.o.c =



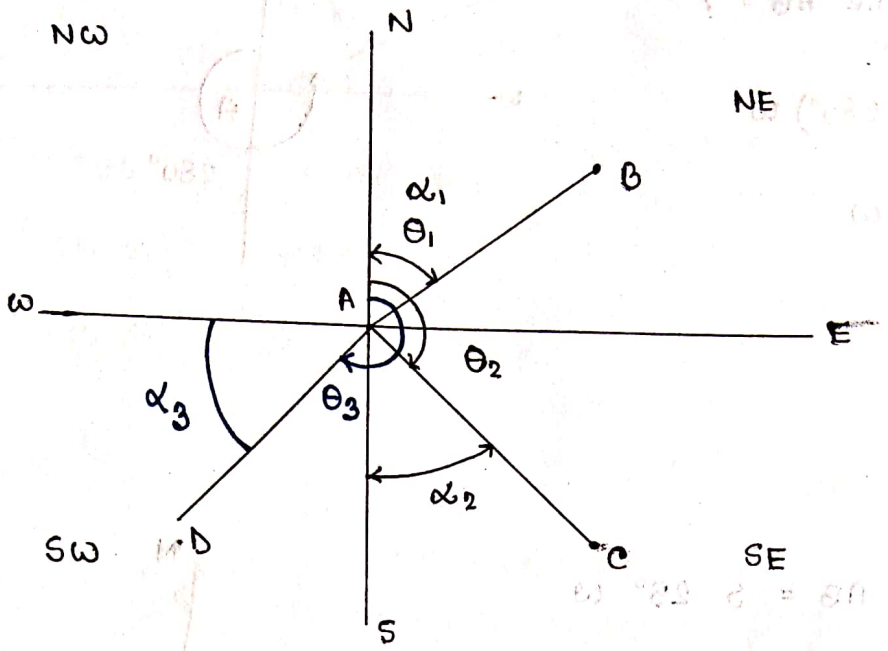
(i) Whole circle bearing (WCB)

(ii) Quadrantal bearing (QB / RB)

↳ Reduced

θ_2 = W.C.B

α_2 = Q.B



WCB = 270°

RB = N. 270° E

W.C.B	R.B	Quadrant
0°-90°	RB = WCB	NE
90°-180°	RB = 180°-WCB	SE
180°-270°	RB = WCB-180°	SW
270°-360°	RB = 360°-WCB	NW

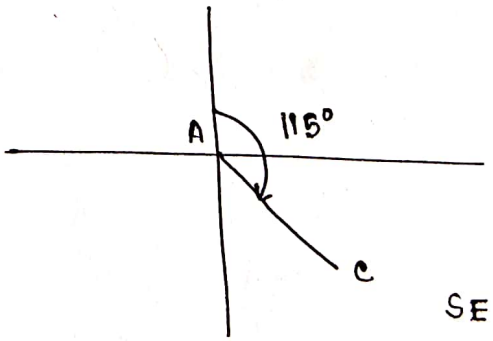
WCB of line AC = 115°.

RB of line AC = ?

↳ S (180-WCB) E

⇒ S (180-115) E

⇒ S 65° E

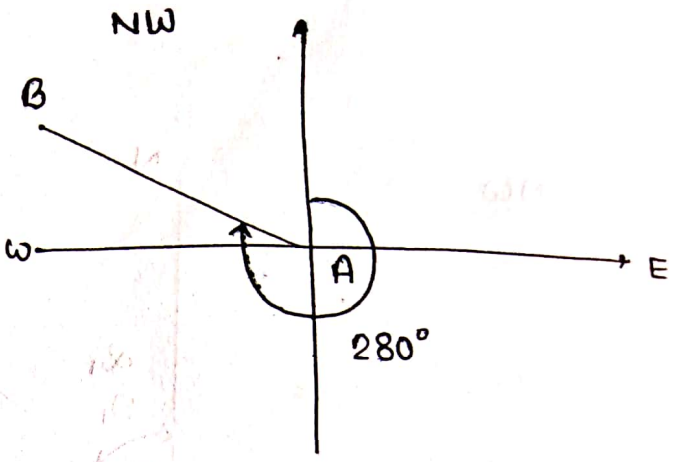


WCB of line AB = 280°

R.B of line AB = ?

↳ N (360° - 280°) W

⇒ N 80° W



RB of line AB = S 25° W

WCB = ?

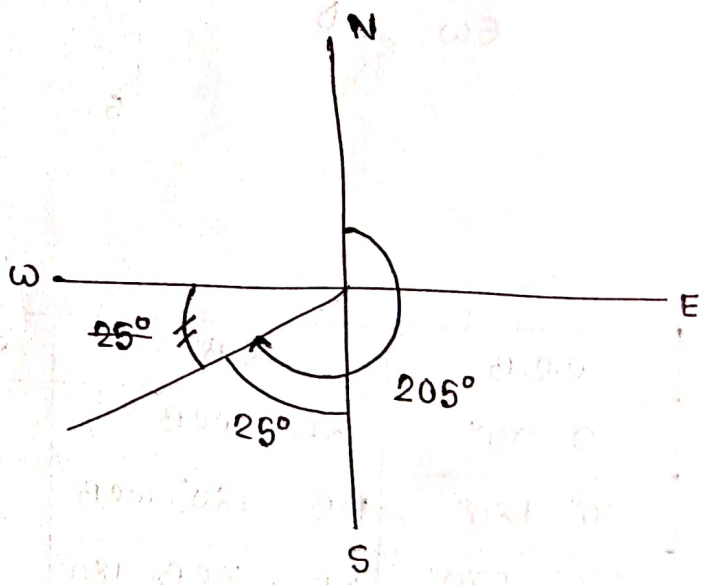
↳ 270° - 25°

⇒ ~~245°~~

WCB - 180° = 25°

⇒ WCB = 180° + 25°

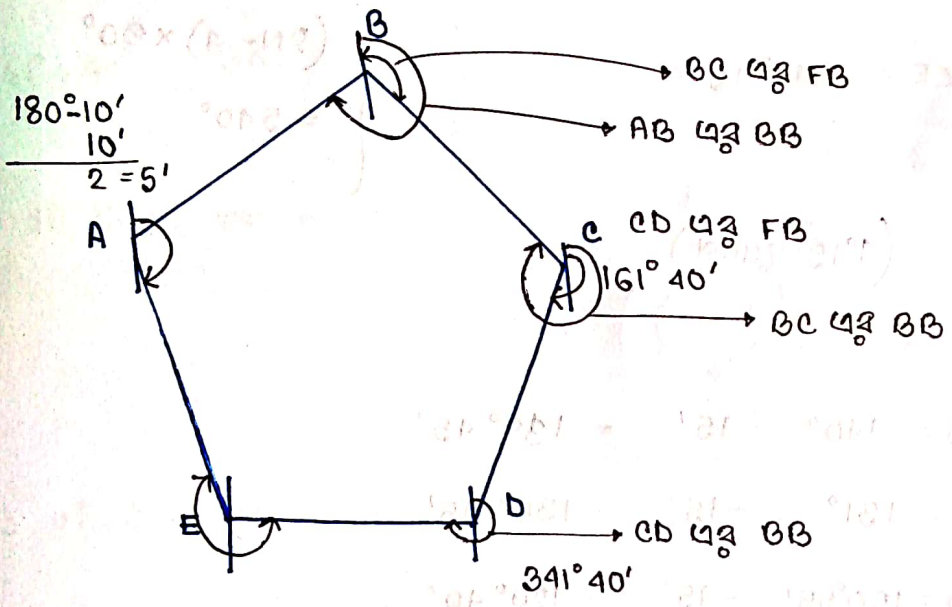
= 205°



Line | FB | BB

Line	FB	BB
AB	71°05′	250°20′
BC	110°20′	292°35′
CD	161°35′ +5′	341°45′ −5′
DE	220°50′	40°05′
EA	300°50′	221°10′

~~180~~



$\angle B = 250°20' - 110°20' = 140°$

$\angle C = 292°35' - 161°35' = 131°$

$\angle D = 341°45' + 220°50' = 120°55'$

⇒ Difference: (i) 179°15′ (ii) 182°15′ (iii) 180°10′ (iv) 180°45′ (v) 179°40′

excluded $\angle E = 300°50' + 40°05' = 260°45'$

$\therefore$ included $\angle E = 360° - 260°45' = 99°15'$

Cause:

local attraction (magnetic substance - এর উপস্থিতি.

$\angle A = 221°10' - 71°05' = 50°05'$

Determine the correct Magnetic Bearings for the lines:

$\angle A = 50°05'$

$\angle B = 140°$

$\angle C = 131°$

$\angle D = 120°55'$

$\angle E = 260°45'$

$\therefore \angle A + \angle B + \angle C + \angle D + \angle E = 541°15'$

$\left\{ (2N-4) \times 90° = 540° \right.$

$1°15' = 75'$

$\frac{75'}{5} = 15'$ (1°15' বেশি)

correct angle, $\angle B = 140° - 15' = 139°45'$

$\angle C = 131° - 15' = 130°45'$

$\angle D = 120°55' - 15' = 120°40'$

$\angle E = 260°45' - 15' = 99°$

$\angle A = 50°05' - 15' = 49°50'$

$\therefore$ Sum $= 540°$

$\therefore$ correct included angle =

$\therefore$ Corrected of Bearing:

① FB of line CD = 161°40'

BB " " CD = 341°40'

Scanned by CamScanner

(ii) FB of line DE = 341°40' − ∠D

= 241°70'

BB of line DE = 241°70' − 180°

= 41°70'

(iii) FB of line EA = 360° − ∠E

= 261°45'

= 261° + 41° = 302°

BB of line EA =

= 122°

(iv) FB of line AB = BB of line EA − ∠A

= 72°10'

BB of line AB = 72°10' + 180°

= 252°10'

(v) FB of line BC = BB of AB − ∠B

= 112°25'

BB of line BC = 112°25' + 180°

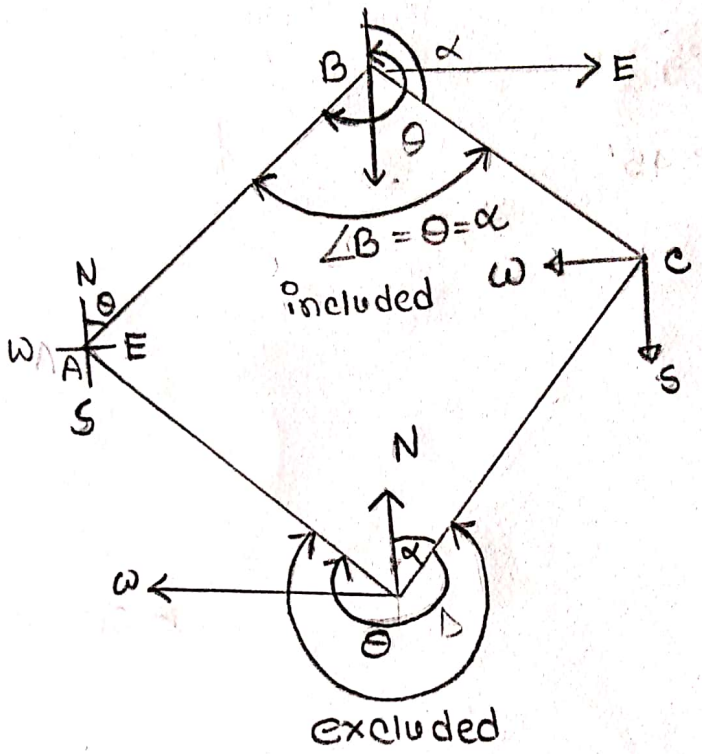
= 292°25'

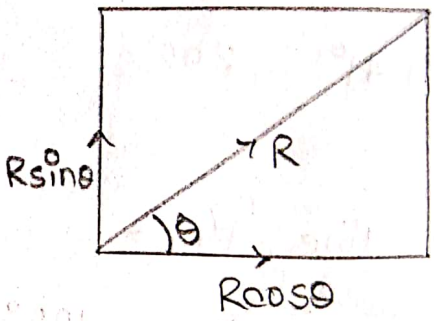
CHECK

FB of line CD = BB of line BC - ∠C

= 292°25' - 130°45'

= 161°40'





$\Sigma N = \Sigma S$
$\Sigma E = \Sigma W$

Methods of Traversing:

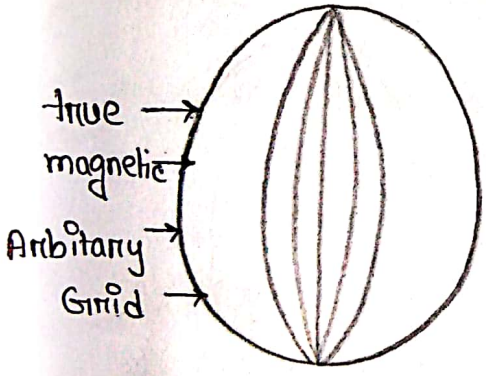
(i) Compass traverse

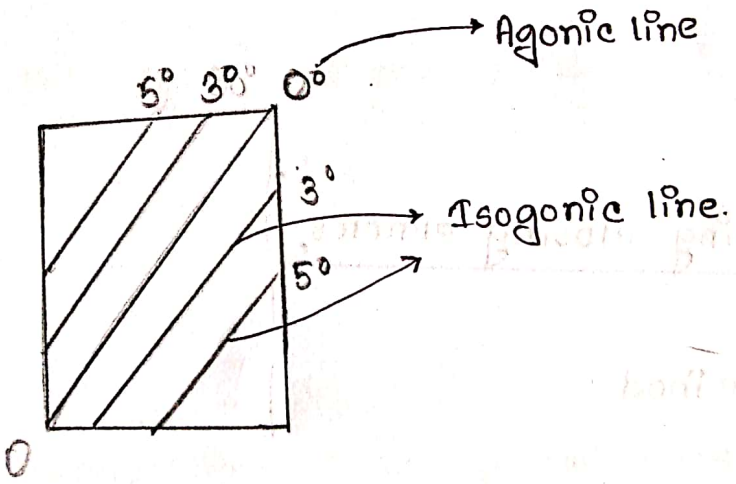
(ii) Theodolite "

(iii) Plain Table "

(iv) Chain "

Inclidation of magnetic niddle (DIP):





True meridian ⟶

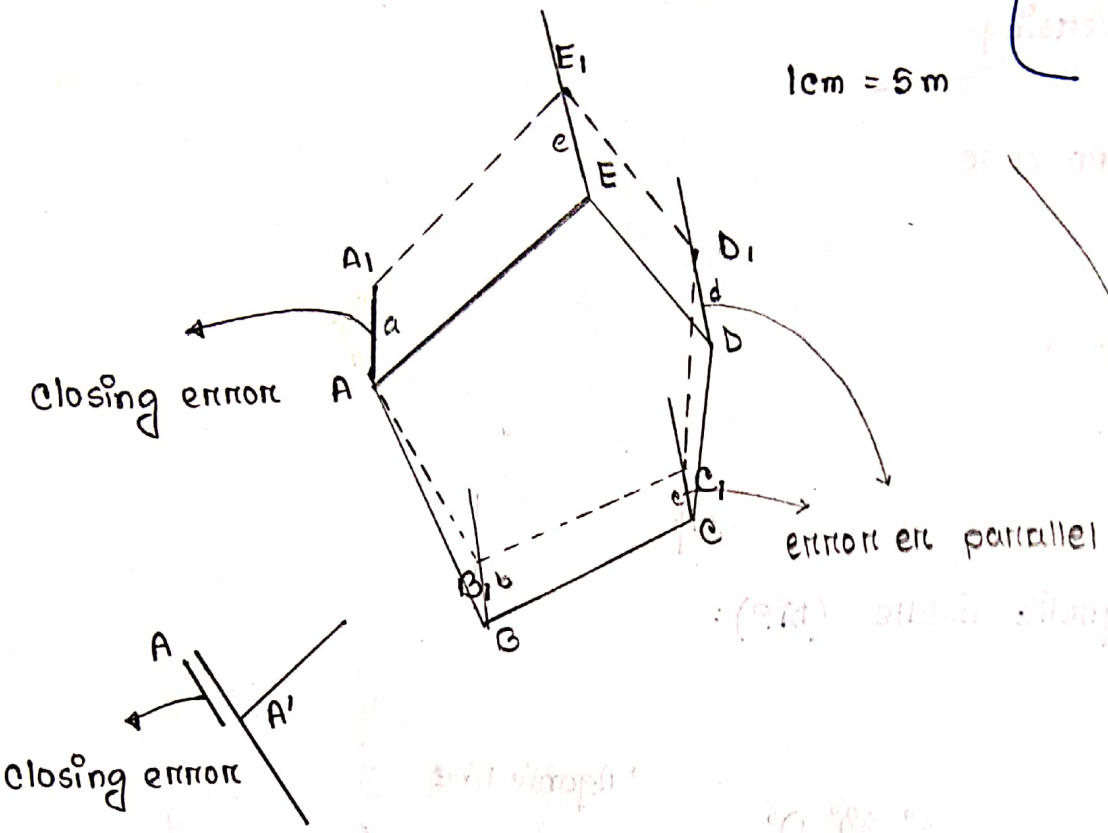
Magnetic " ⟶

Arbitary ⟶

Grid ⟶

720 sq. ft = 1 khata

435·6 sq ft = 1 decimal (ডেসিমল)



Method of correcting closing errors

① Bowditch's method

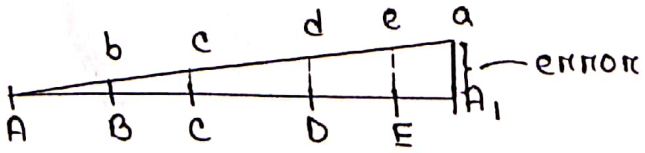
② Transid's "

③ Graphical "

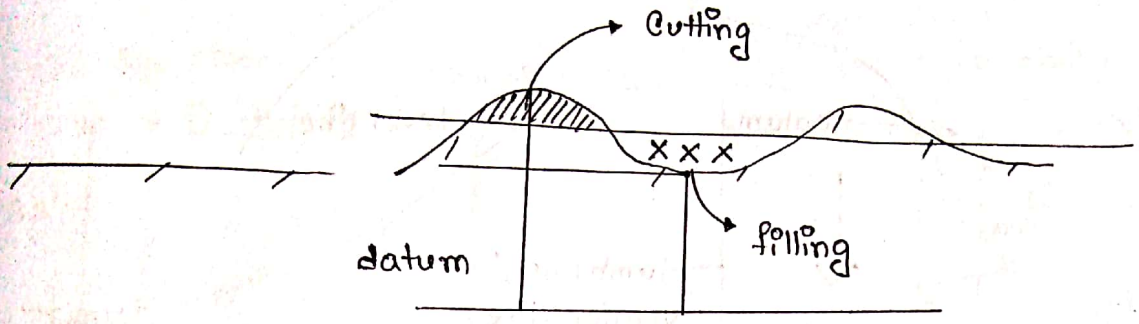
④ Axis "

Graphical method:

1cm = 10 m



LEVELLING



Levelling is the process by which the difference elevation (hight) of various points on the earth surface is determined with respect to a given datum.

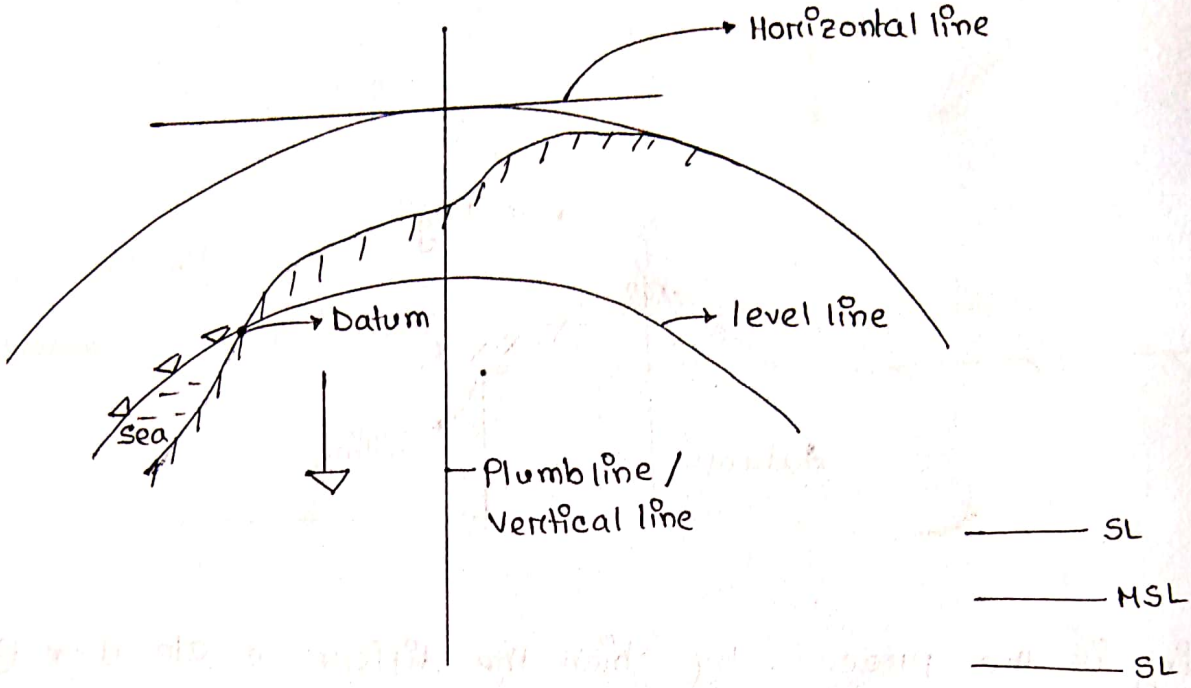
Objectives:

To determine the undulation of the ground surface by knowing the relative hight of different objects on or below the earth surface.

Uses:

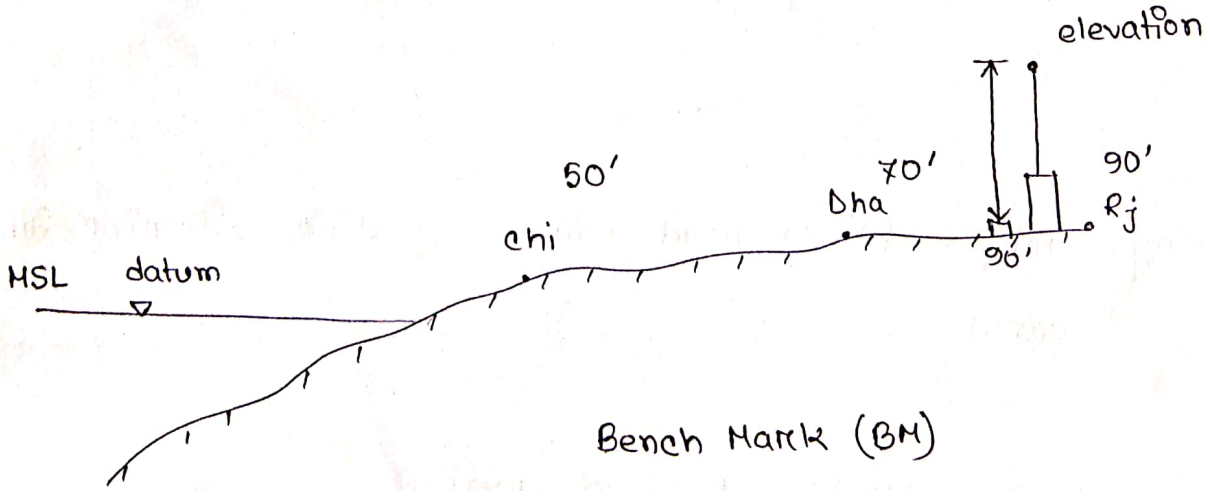
① For fixing - Alignment of road, railway, gasline, waterline, irrigation & canal

② For fixing ~~sides~~ sites for dam, reservoir

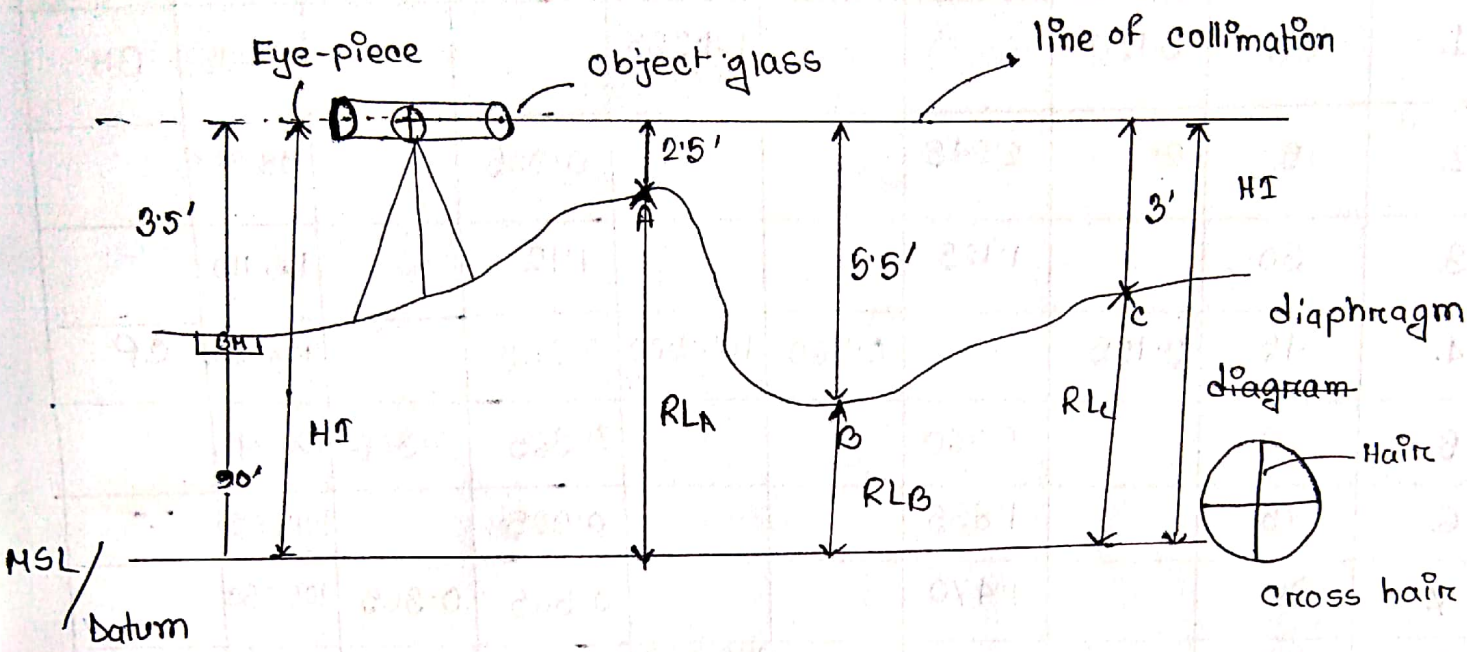


Datum is an imaginary level surface or level line from which elevation of different points are measured.

In Bangladesh **Mean Sea Level** at Cox's Bazar, is considered as datum.



Bench Mark is the fixed point or mark of known elevation determined with reference to the datum line.



levelling staff

HI = Hight of the instrument

= 90' + 3·5'

= 93·5'

RL = Recused level

RL_A = 93·5' - 2·5'

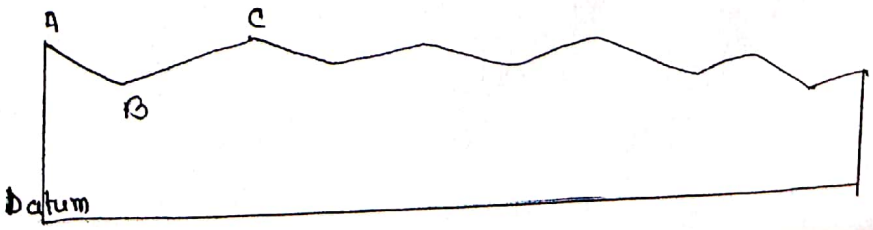
= 91·0'

RL_B = 93·5 - 5·5'

= 88'

RL_C = 93·5 - 3'

= 90·5'

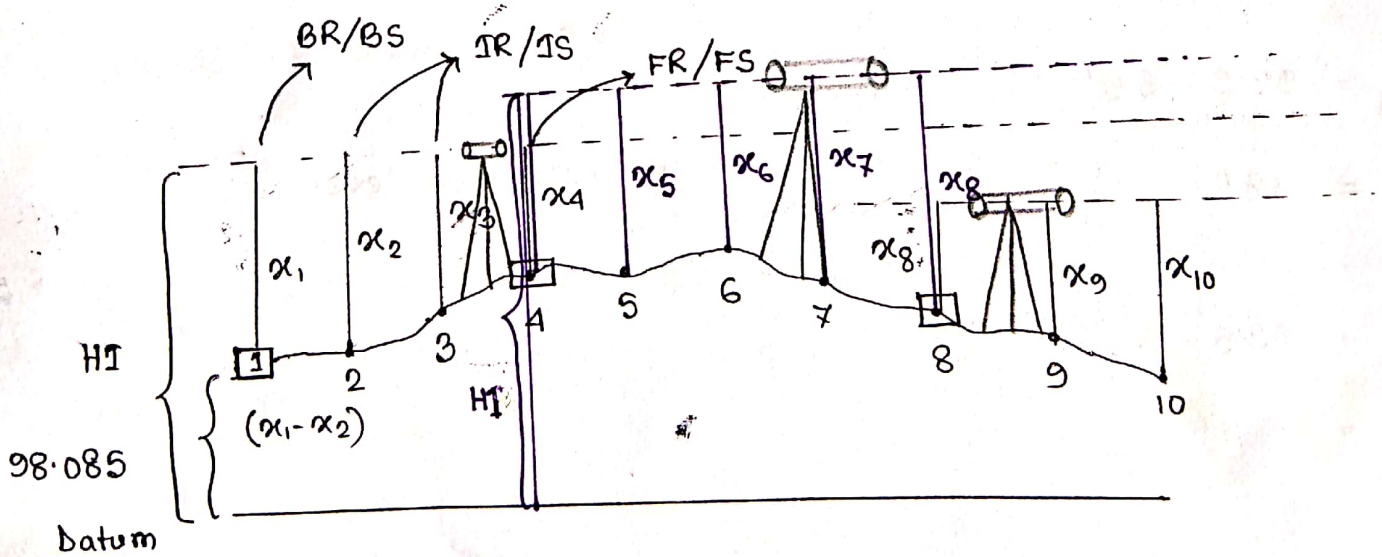


100·375 + 3·125 = 103·500

$x_1 - x_2$ → ⊕ → Rise

⊖ → Fall

Stations	Distance	Staff Readings (m)			HI	Difference		RL of Stations (m)	Remark
		BR	IR	FR		Rise	Fall		
1.	00	3·150			101·235 = 3·15 + 98·085			98·085	BM
2.	15		2·245			0·905		98·990	
3.	30		1·125			1·12		100·110	
4.	45	3·125		0·860	103·500	0·265		100·375	CP
5.	60		2·760			0·365	~~0·365~~	100·740	
6.	75		1·835			0·925		101·665	
7.	90		1·470			0·365		102·030	
8.	105	1·225		1·965	102·760 ~~98·265~~		0·495	101·535	CP
9.	120		2·390				1·165	100·37	
10.	135			3·035			0·645	99·725	



Line of collimation Method / Hight of the Instrument Method

$$\Sigma BR - \Sigma FR = \text{last RL} - 1^{st}\ \text{RL}$$

$$\Rightarrow 7.5 - 5.86 = 99.725 - 98.083$$

$$\Rightarrow 1.64 = 1.64$$

Rise and Fall Method:

$$\Sigma \text{Rise} - \Sigma \text{Fall} = \text{last RL} - 1^{st}\ \text{RL}$$

$$\Rightarrow \begin{matrix} 3.945 \\ \cancel{3.58} \end{matrix} \begin{matrix} - 2.305 \\ - \cancel{2.67} \end{matrix} = 99.725 - 98.085$$

$$\Rightarrow 1.64 = 1.64$$

Geoid: .. is the surface of the earth at mean sealevel.

Parallex: The apparent movement of the image relative to the cross hairs is known as parallex.

Types of level:

(i) Dumpy level.
(ii) Y-level (WYe level)
(iii) Reversible level
(iv) Tilting level
(v) Autometic level

Types of levelling staff:

(i) One-length staff

(ii) Folding "

(iii) Telescopic "

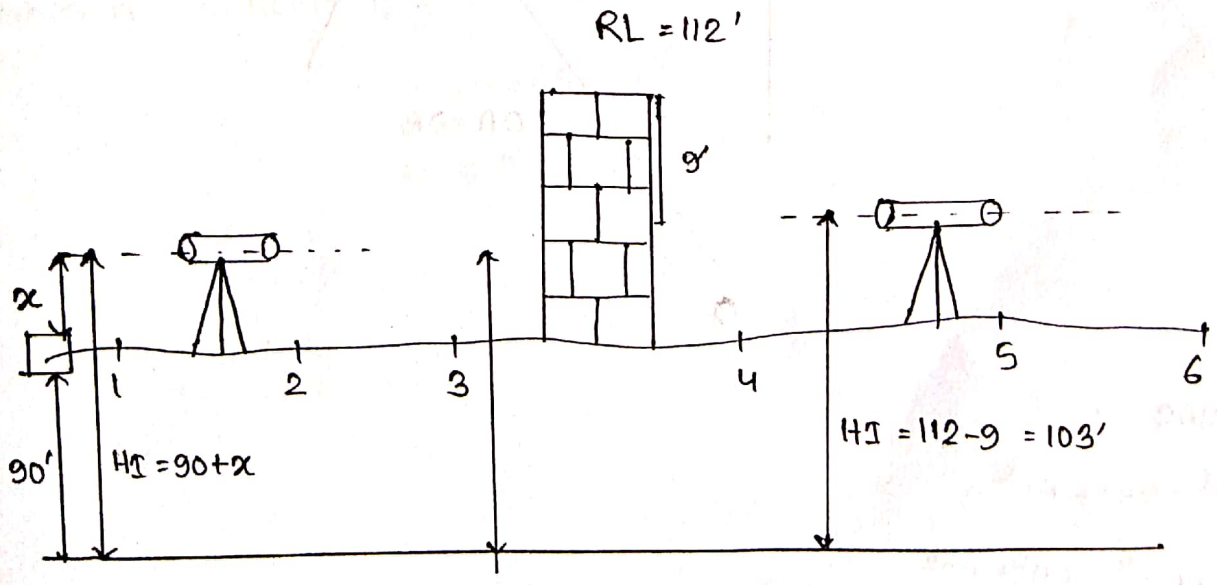
(iv) Target "

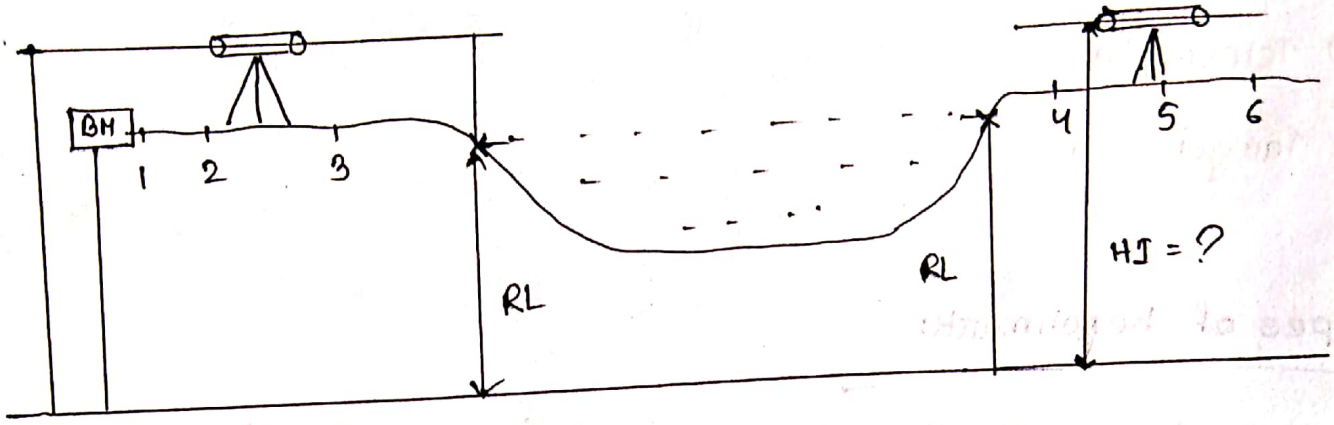
Types of benchmark:

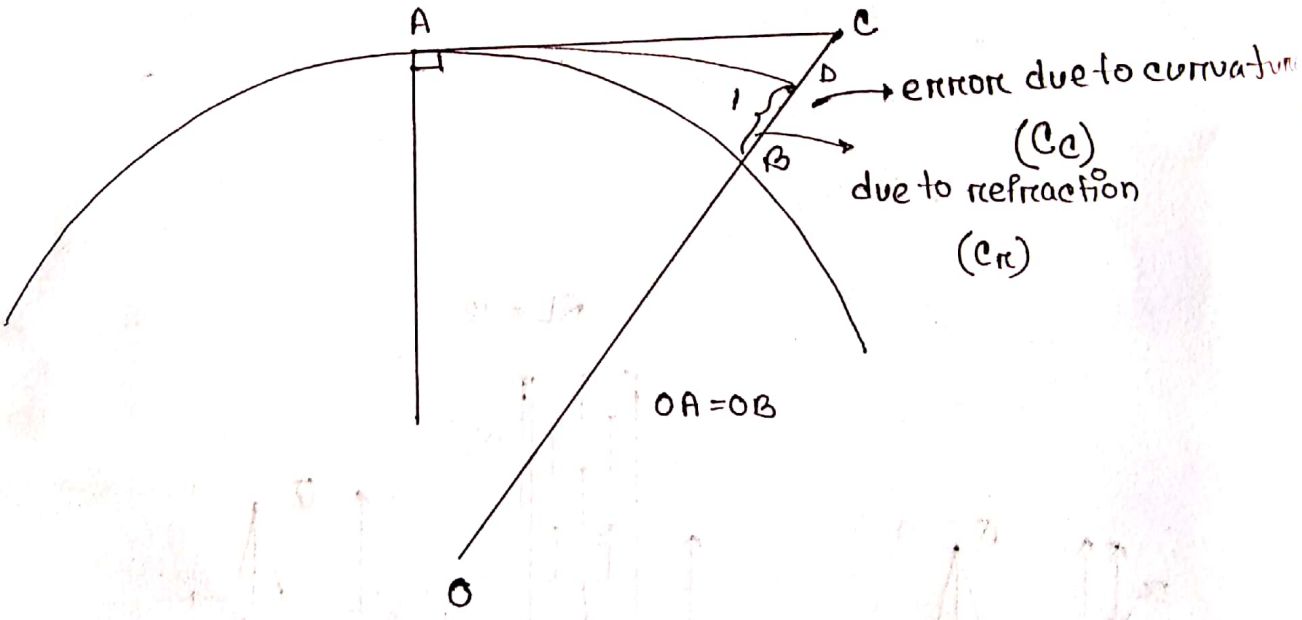
(i) GTS (Great Triconometrical survey) (mean sea level)

(ii) Permanent

(iii) Arbitrary







$\Delta OAC \rightarrow$

$OC^2 = OA^2 + AC^2$

$\Rightarrow (OB + BC)^2 = OA^2 + AC^2$

$\Rightarrow (R + C_c)^2 = R^2 + D^2$ → horizontal distance

$$\Rightarrow R^2 + 2RC_c + C_c^2 = R^2 + D^2$$

$$\Rightarrow 2RC_c = D^2$$

$$\Rightarrow \boxed{C_c = \frac{D^2}{2R}}$$

$R = 6370$ km

$$\therefore C_c = \frac{D^2}{2 \times 6370} \text{ (Km)}$$

$$= \frac{1000 \times D^2}{2 \times 6370} \text{ (m)}$$

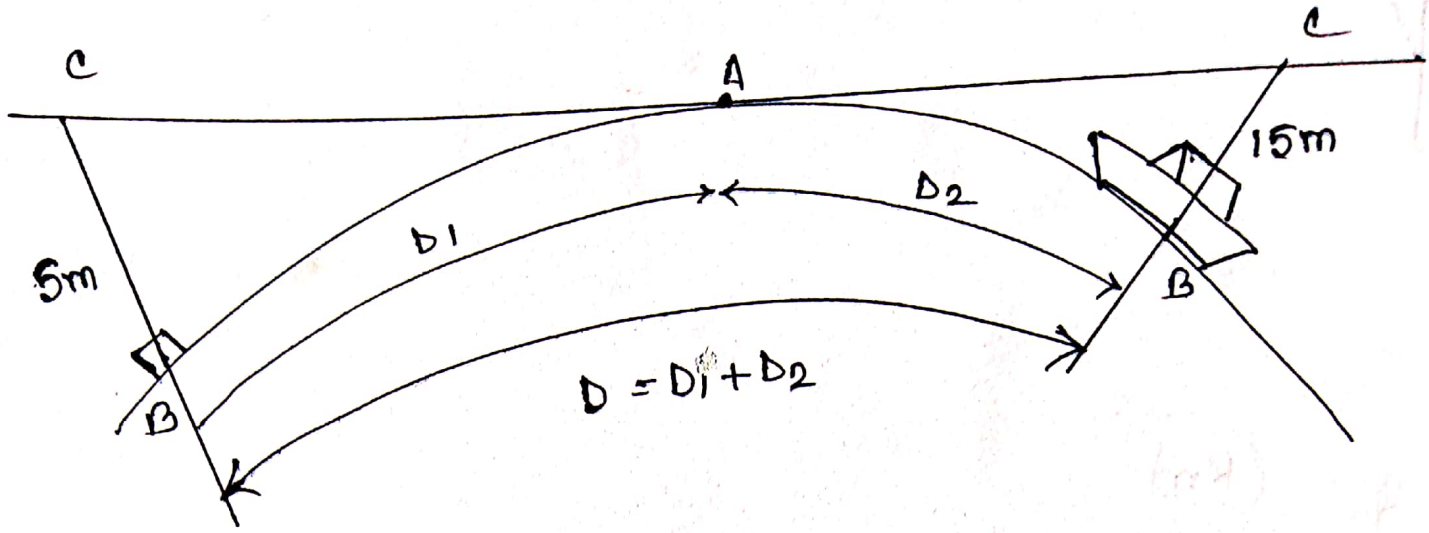
$$\boxed{C_c = 0.0785\, D^2}$$

Correction for refraction, $\boxed{C_r = \frac{1}{7} \cdot C_c}$

Combined correct, $C = C_c - C_r$

$$= 0.06728\, D^2$$

$$\approx 0.0673\, D^2$$



$$C = 0.0673\ D^2$$

$$h_1 = 0.0673\ D^2$$

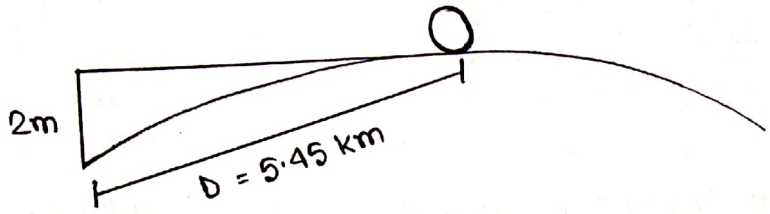
$$\Rightarrow 5 = 0.0673\ D^2$$

$$\Rightarrow D_1 =$$

$$h_1 = 0.0673\ D_2^2$$

$$\Rightarrow D_1 = \sqrt{\frac{h_1}{0.0673}}$$

Find the distance of the sun from the observer when just see the sunset. When the height of the observer is 2m.



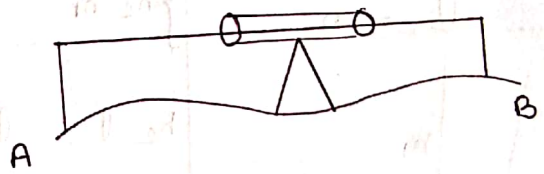
$$h = 0.0673\, D^2$$

$$D = \left(\frac{h}{0.673}\right)^2$$

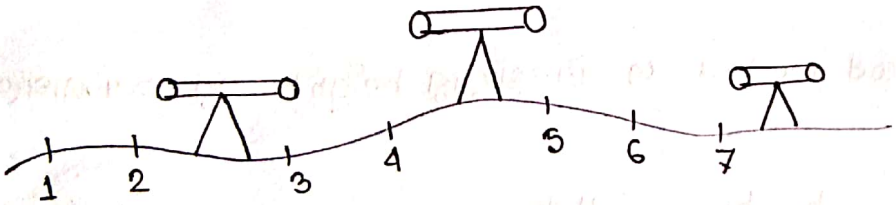
$$= 883.140 \text{ m}$$

Types of levelling operation:

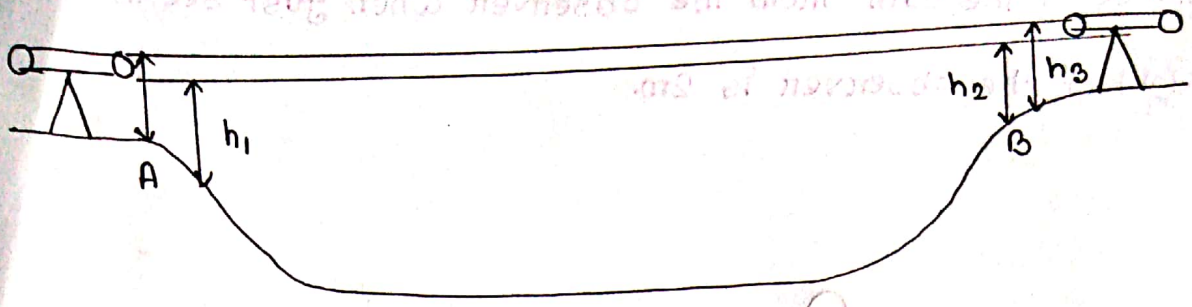
1. Simple levelling



2. Differencial levelling



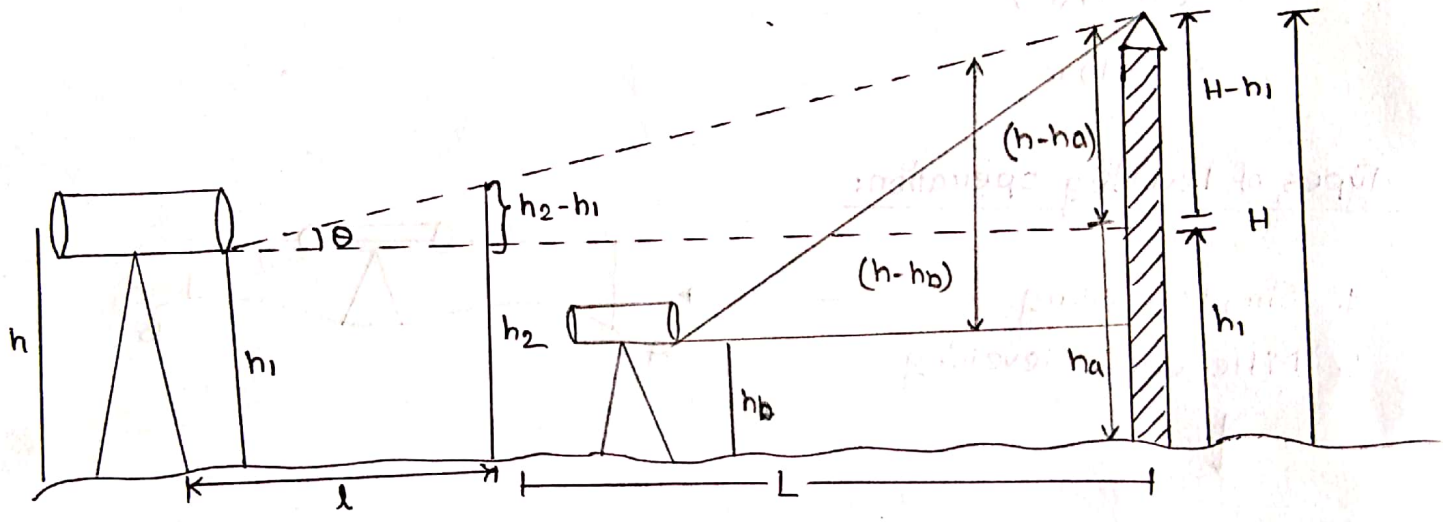
3. Fly levelling
4. longitudial / Roffle "
5. Cross sectional "
6. Reciprocal "
7. Trigonometrical "
8. Contouring levelling



$d_1 = h_1 - h_2$

$d_2 = h_3 - h_4$

$\left(\frac{d_1+d_2}{2}\right)$ ⟶ A থেকে B এর height এর তারতম্য (A ও B station কতটুকু উঁচু নাকি নীচু)



[Tape and ranging rod method -এ টাওয়ারের height বের করা লাগবে।]

$h_1 =$
$h_2 =$
$l =$
$L =$

$$\frac{h_2-h_1}{l} = \frac{H-h_1}{L+l}$$ [আনুপাতিক বিভাজন]

$\therefore H = ?$

যদি theodolite use করা হয়, তখন θ দেওয়া থাকে,

$h =$ known
$L = h$
$\theta =$

$$\tan\theta = \frac{H-h}{L}$$

যদি obstuction থাকে, তাহলে আরেকটি theodolite use করা হয়.

$h_a, h_b, l, \theta, \alpha \longrightarrow$ known

$H, L \longrightarrow$ unknown

$$\tan\theta = \frac{H-h_a}{l+L}$$

$$\tan\alpha = \frac{H-h_b}{L}$$

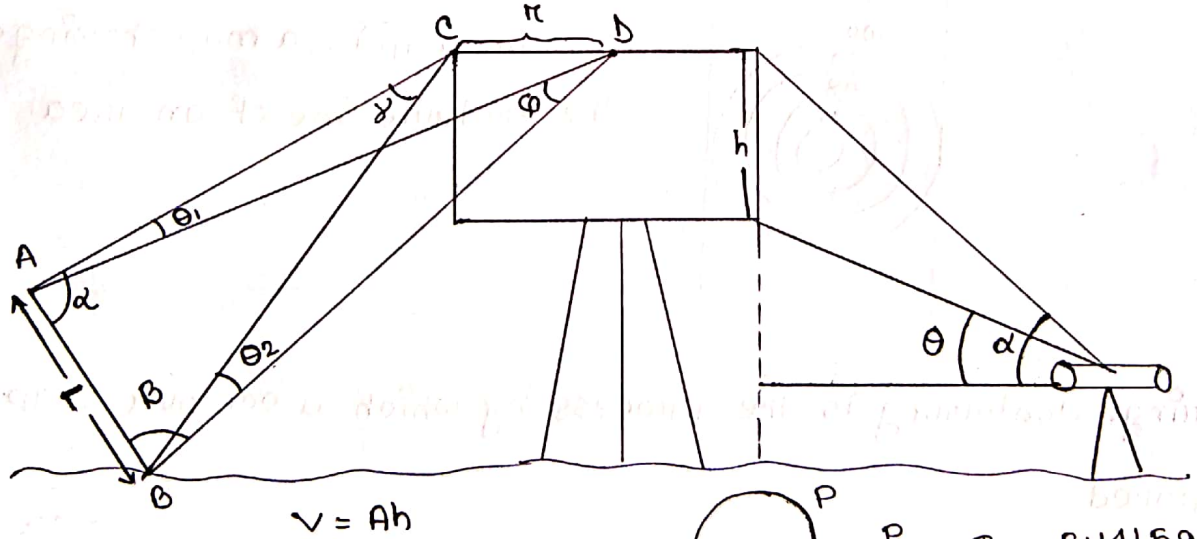
now, $L = \frac{H-h_b}{\tan\alpha}$ [L এর মান tan θ এর eqn এ বসিয়ে]

এখানে ৩টি পদ্ধতি ব্যবহার করা হয়েছে-

① Ranging rod দিয়ে (h_1, h_2)

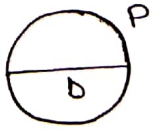
② One theodolite (h, θ)

③ Two " $(h_a, h_b, \alpha, \theta, l)$



$V = Ah$

$= \pi r^2 h$



$= \frac{P}{D} = \pi = 3.141592\,65659$

$\pi = CD$

$CD^2 = AC^2 + AD^2 - 2AC.AD.\cos\theta_1$ ——— ①

From ΔABC,

$$\frac{AC}{\sin(\beta - \theta_2)} = \frac{AB}{\sin \gamma}$$

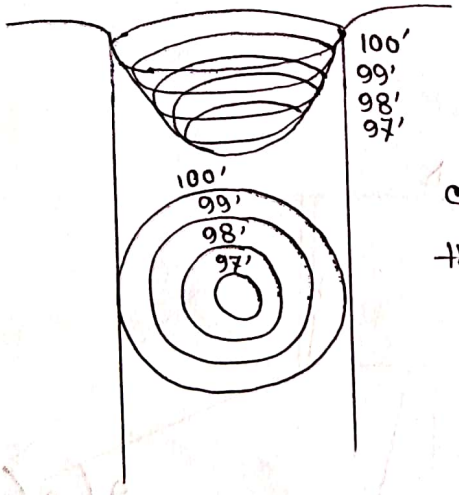
ΔADB,

$$\frac{AD}{\sin \beta} = \frac{AB}{\sin \phi}$$

$\therefore AD = ?$

① **Contour / Contour line:** Contour line is a imaginary line joining points of equal RL with respect to datum.

② **Contour map:**



contour map is a map showing only the contour line of an area.

③ **Contouring:** Contouring is the process by which a contour map is prepared.

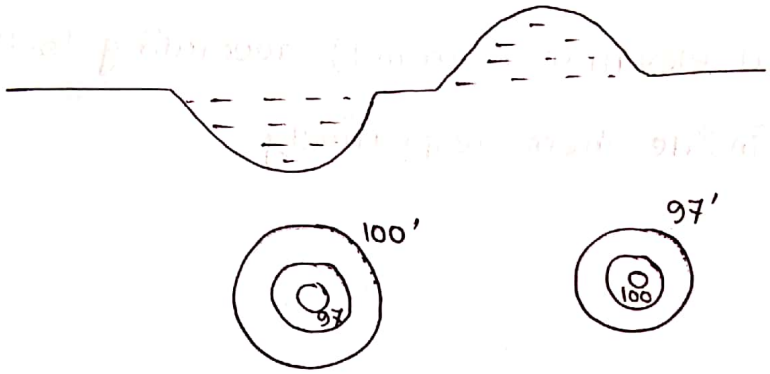
Contour interval: Vertical distance between any two consecutive contours.

Horizontal equivalent: Horizontal distance between two consequtive contours.

Objectives: ① To know the nature of the ground.

② Suitable alignments can be selected and volume of earthquak can be estimated.

Uses: ① The possible route (the direction) can be demarkcated.

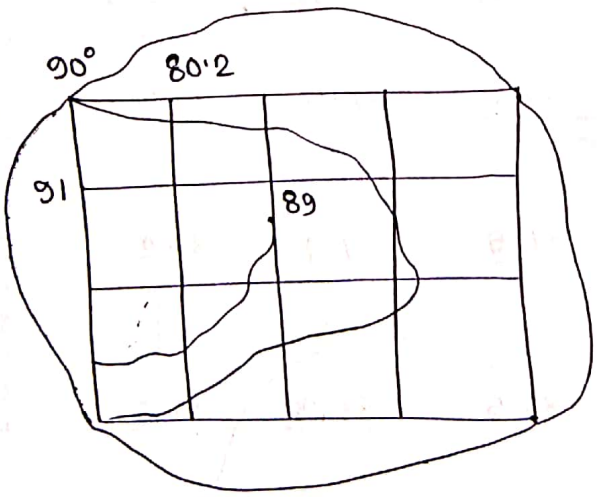


② Economical alignment can be selected.

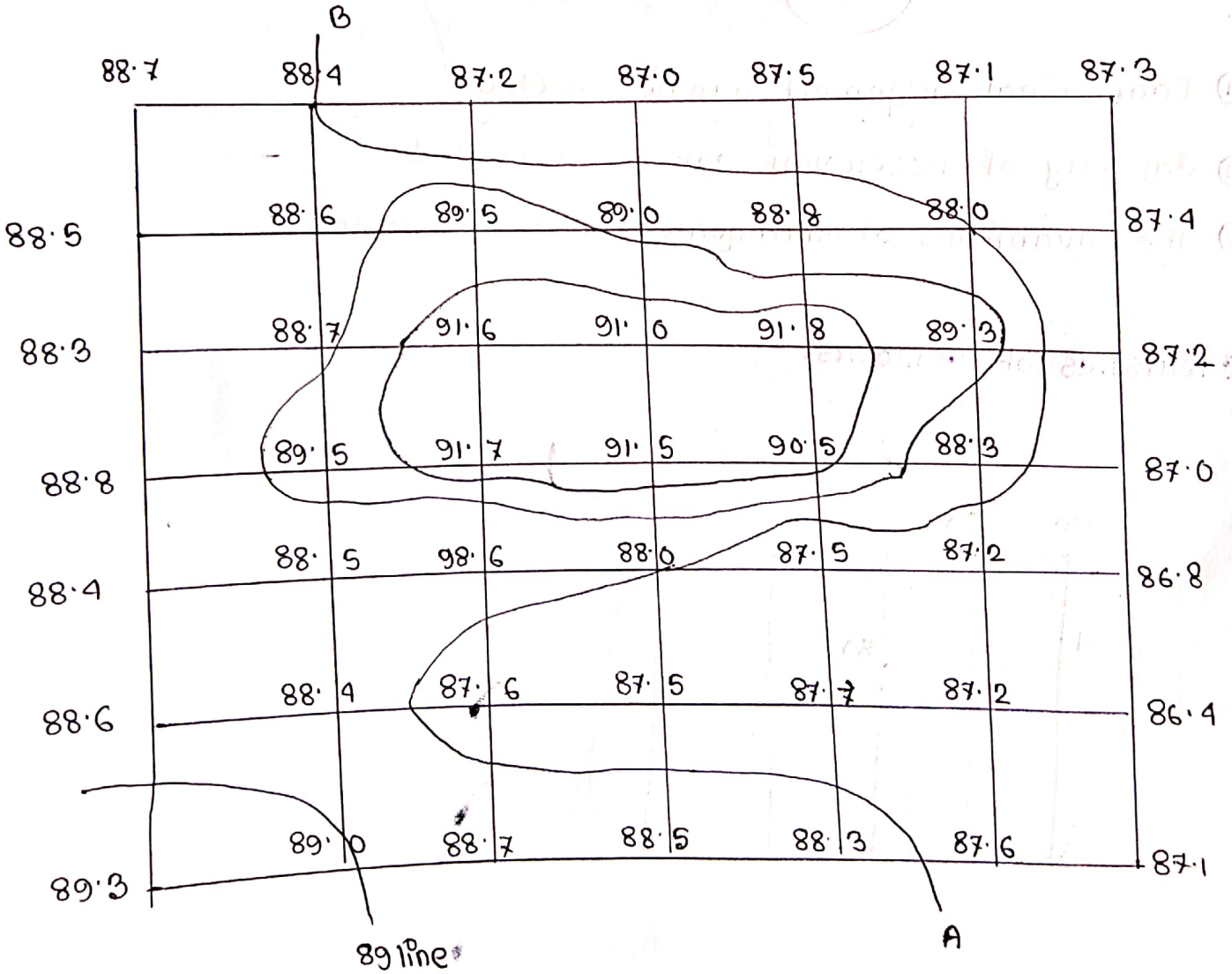
③ Capacity of reservair can be computed.

④ The quantities of earthquak can be computed.

Characteristics of contours:



1. Contour line can not cross one another except overhanging cliff.
2. Each contour lines must be a continuous, until it either goes off the map or closes itself.
3. Uniformly spaced contour lines indicate a uniform slope.
4. A series of closed contours on the map indicates a depression or and elevation (summit) according to the lower or higher values inside them respectively.



For graph;

88 AB এর contour line

89 এর contour line হলো closed line

1·2 ⟶ 10 মিমি

0·4 ⟶ $\frac{10 \times 0.4}{1.2}$ = 3·3 মিমি

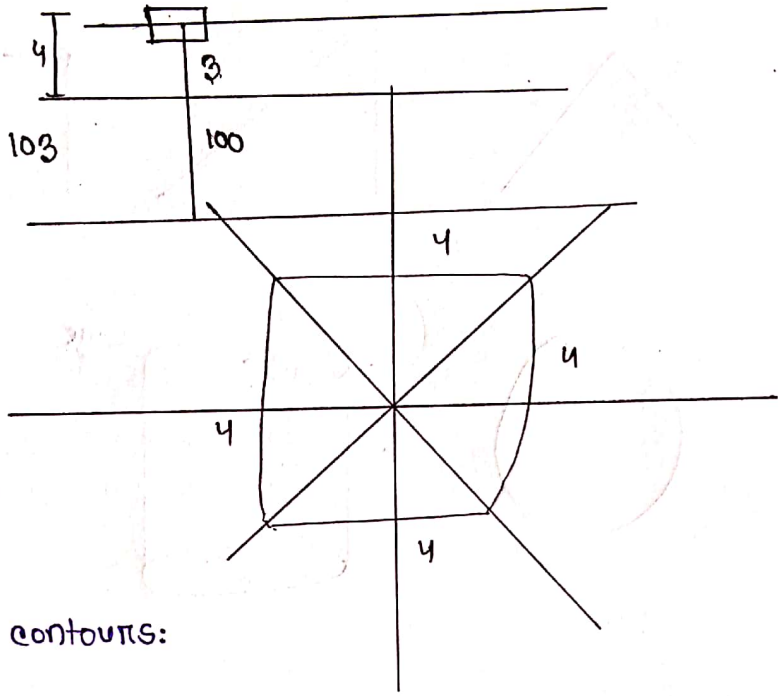
88·4 − 87·2 = 1·2

88·4 − 88 = 0·4

Method of contouring:

① Direct Method

② Indirect "

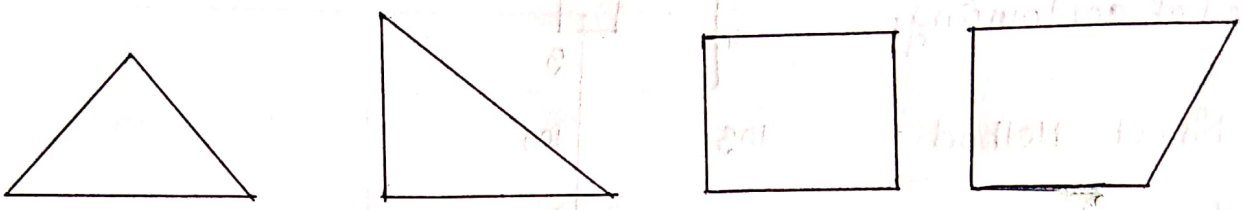


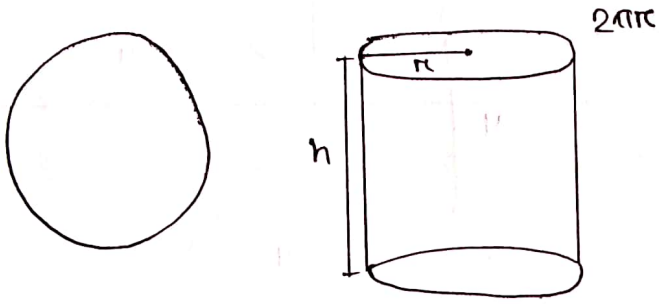
Method of interpetation of contours:

① Arithmatical method

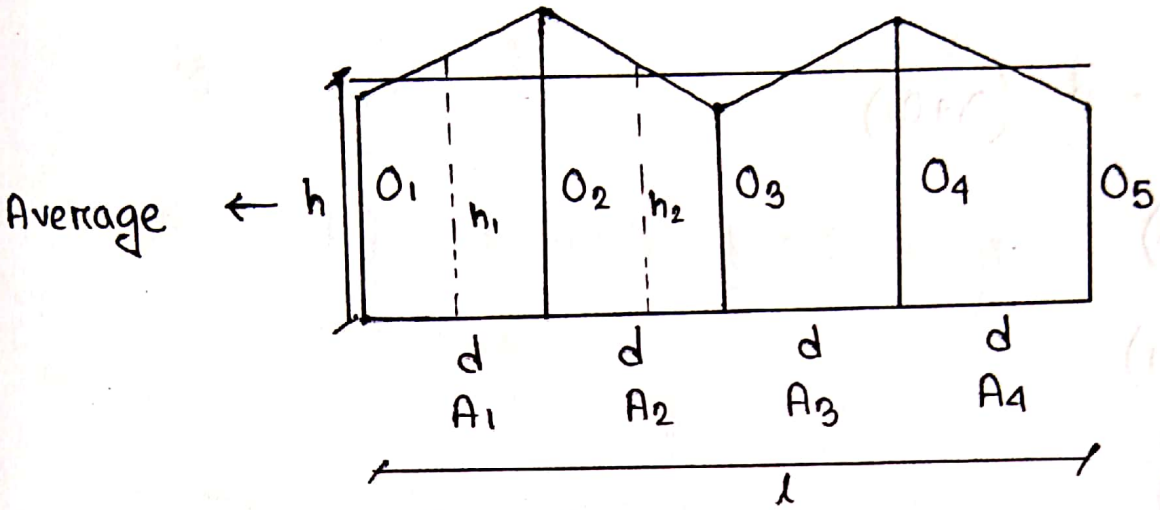
② Graphical method.

Contour gradient: During preliminary survey for roads in a hill area, the required points are first established along the gradient. The line joining these points is known as the contour gradient or grade contour.





Computation of Area



Rule: 01 → Mid-Ordinate Rule

→ $h_1, h_2, \ldots$

$A_1 = h_1 d$

$A_2 = h_2 d$

$A_3 = h_3 d$

$\therefore A = d(h_1 + h_2 + h_3 + \ldots\ldots)$

Rule: 02 → Average-Ordinate Rule

$$h = \frac{O_1 + O_2 + O_3 + O_4 + O_5}{5}$$

$A = h \times 4d$

$$\therefore A = \left(\frac{O_1 + O_2 + O_3 + \cdots + O_n}{n}\right)(n-1)\,d$$

$$\text{or, } A = \left(\frac{O_0 + O_1 + \cdots\cdots + O_n}{n+1}\right)(n-1)\,d$$

Rule → 03

Trapezoidal Rule:

$$A_1 = \frac{1}{2}(O_1+O_2)d = \frac{d}{2}(O_1+O_2)$$

$$A_2 = \frac{d}{2}(O_2+O_3)$$

$$A_3 = \frac{d}{2}(O_3+O_4)$$

$$A_4 = \frac{d}{2}(O_4+O_5)$$

$$A = \frac{d}{2}\left[O_1+O_2+O_2+O_3+O_3+O_4+O_4+O_5\right]$$

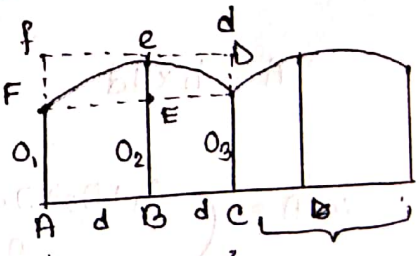
$$= \frac{d}{2}\left[O_1+O_5+2(O_2+O_3+O_4)\right]$$

$$\text{Area, } A = \frac{\text{common distance}}{2}\left[1^{st}\text{ Ordinate} + \text{last Ordinate} + 2(\text{Sum of other ordinates})\right]$$

$$A_2 = \frac{d}{3}(O_3+4O_4+O_5)$$

Rule → 04

Simpson's Rule:



$$A_1 = \frac{d}{3}(O_1+4O_2+O_3)$$

$$ACDEFA = ACDEFA + \Delta DEFED$$

$$= \frac{1}{2}(O_1+O_3)2d + \frac{2}{3}\left(O_2 - \frac{O_1+O_3}{2}\right)2d$$

$$= d(O_1+O_3) + \frac{4d}{3}\left(\frac{2O_2-O_1-O_3}{2}\right)$$

$$= d(O_1+O_3) + \frac{2d}{3}(2O_2-O_1-O_3)$$

$$= \frac{d}{3}\left[3O_1+3O_3+4O_2-2O_1-2O_3\right]$$

$$= \frac{d}{3}\left[O_1+4O_2+O_3\right]$$

$$EB = \left(\frac{O_1+O_3}{2}\right)$$

$$eE = O_2 - \left(\frac{O_1+O_3}{2}\right)$$

$A_3 = \frac{d}{3}(O_5 + 4O_6 + O_7)$

$A = A_1 + A_2 + A_3 + \cdots$

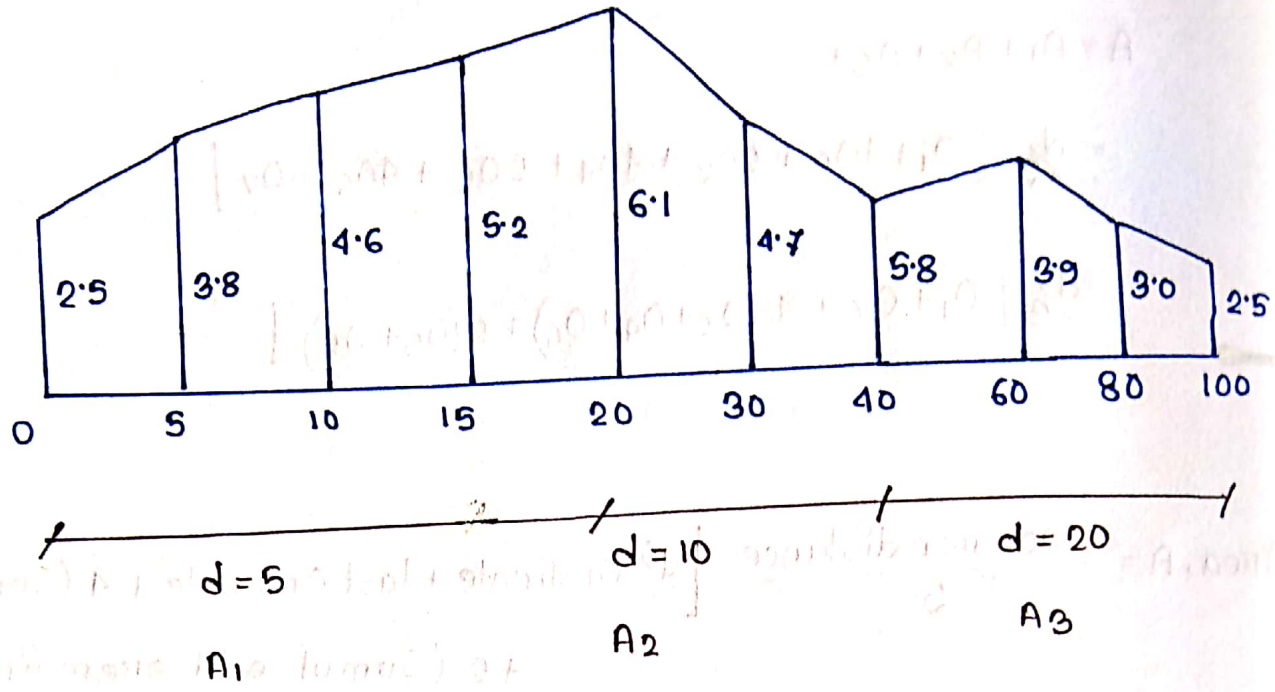
$= d/3\,[O_1 + 4O_2 + 2O_3 + 4O_4 + 2O_5 + 4O_6 + O_7]$

$= d/3\,[O_1 + O_7 + 4(O_2 + O_4 + O_6) + 2(O_3 + O_5)]$

Area, $A = \frac{\text{common distance}}{3}$ [1st Ordinate + last Ordinate + 4 (Sum of even ordinates) + 2 (Sum of ~~odd~~ remaining odd ordinates)]

[this rule is applicable for only odd ~~even~~ ordinates.]

#



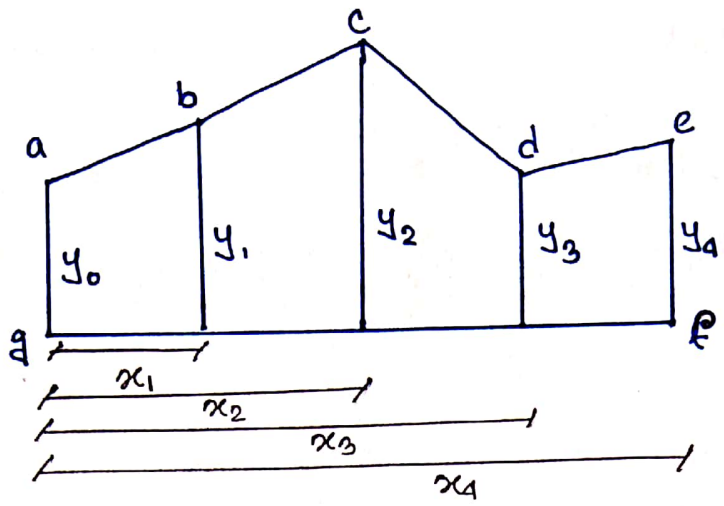
Trapezoidal Rule:

$$\text{Area} = \frac{\text{common distance}}{2}\left[1^{st}\text{ ord.} + \text{last ord} + 2\,(\text{Sum of other ord})\right]$$

$$A_1 = \frac{5}{2}\left[2{\cdot}5 + 6{\cdot}1 + 2\left(3{\cdot}8 + 4{\cdot}6 + 5{\cdot}2\right)\right] = 89{\cdot}5\ m^2$$

$$A_2 = \frac{10}{2}\left[6{\cdot}1 + 5{\cdot}8 + 2\,(4{\cdot}7)\right] = 106{\cdot}5\ m^2$$

$$A_3 = \frac{20}{2}\left[5{\cdot}8 + 2{\cdot}5 + 2\,(3{\cdot}9 + 3{\cdot}0)\right] = 221\ m^2$$



Rule → 05

Co-ordinate Rule:

Points	Co-ordinate	
	x	y
a	0	y_0
b	x_1	y_1
c	x_2	y_2
d	x_3	y_3
e	x_4	y_4
f	x_4	~~y_4~~ 0
g	0	0
a	0	y_0

a b c d e f g a

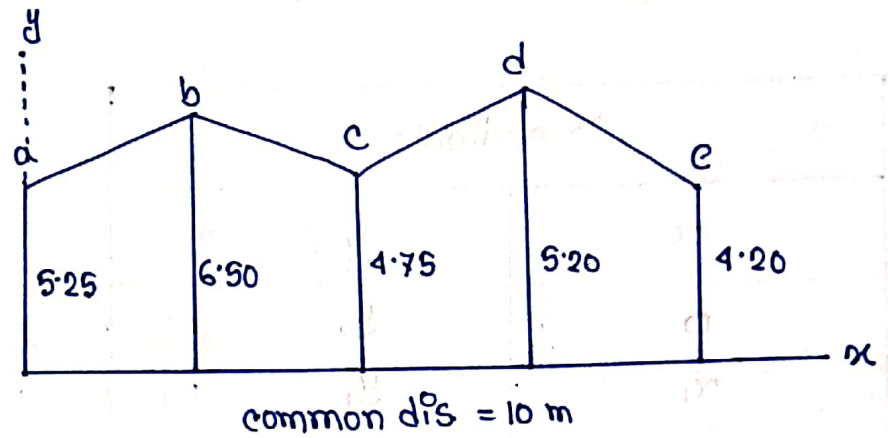
$\frac{y_0}{0}$ $\frac{y_1}{x_1}$ $\frac{y_2}{x_2}$ $\frac{y_3}{x_3}$ $\frac{y_4}{x_4}$ $\frac{0}{x_4}$ $\frac{0}{0}$ $\frac{y_0}{0}$

$$\Sigma P = y_0 x_1 + y_1 x_2 + y_2 x_3 + \cdots$$

$$\Sigma Q = 0 y_1 + x_1 y_2 + x_2 y_3 + \cdots$$

$$A = \frac{1}{2}(\Sigma P - \Sigma Q)$$

#



① Trapezoidal Rule:

$$\text{Area} = \frac{\text{common dis.}}{2}\left[1\text{st Cor.} + \text{last Cor.} + 2\,(\text{Sum of other ordinates})\right.$$

$$A = \frac{10}{2}\left[5{\cdot}25 + 4{\cdot}20 + 2\,(6{\cdot}50 + 4{\cdot}75 + 5{\cdot}20)\right]$$

$$= 211{\cdot}75\ m^2$$

⑪ Simpson's Rule:

$$\text{Area} = \frac{\text{com. dis}}{3}\left[1\text{st Cor} + \text{last Cor.} + 4\,(\text{Sum of even ord}) + 2\,(\text{Sum of remaining odd ord.})\right]$$

$$= \frac{10}{3}\left[5{\cdot}25 + 4{\cdot}20 + 4\,(6{\cdot}50 + 5{\cdot}20) + 2 \times 4{\cdot}75\right]$$

$$= 219{\cdot}17\ m^2$$

(iii) Co-ordinate Rule:.

a	b	c	d	e	f	g	a
5·25	6·50	4·75	5·20	4·20	0	0	5·25
0	10	20	30	40	4·20	0	0

$$\Sigma P = 5{\cdot}25 \times 10 + 6{\cdot}50 \times 20 + 4{\cdot}75 \times 30 + 5{\cdot}20 \times 40 + 4{\cdot}20 \times \overset{40}{\cancel{4{\cdot}20}} + 0 \times 0 + 0 \times 0$$

$$= \cancel{550{\cdot}64}\ 701$$

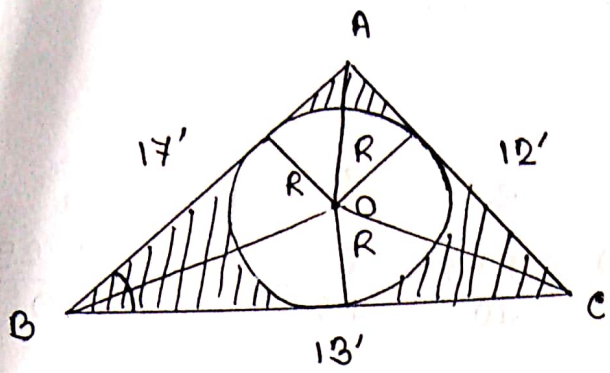
$$\Sigma Q = 0 \times 6{\cdot}50 + 10 \times 4{\cdot}75 + 20 \times 5{\cdot}20 + 30 \times 4{\cdot}20 + 40 \times 0 + 4{\cdot}20 \times 0 + 0 \times 5{\cdot}2$$

$$= 277{\cdot}5$$

$$A = \frac{1}{2}(\Sigma P - \Sigma Q)$$

$$= 211{\cdot}75\ m^2$$

TR	SR
1. 	1. 
2. any number of ordinates	2. only ~~for odd~~ ~~even~~ odd number of ordinate.
3. Approximate result	3. Accurate result



$$S = \sqrt{S(S-a)(S-b)(S-c)}$$

$$\Delta AOC = \frac{R}{2} \times 12$$

$$\Delta AOB = \frac{R}{2} \times 17$$

$$\Delta BOC = \frac{R}{2} \times 13$$

$$A = \frac{R}{2}(12+17+13)$$

[A এর মান বসালে R এর মান বের করা যাবে, তাহলে circle এর area পাওয়া যাবে।
∴ shade এর area পাওয়া যাবে]