"Waves and Oscillations"

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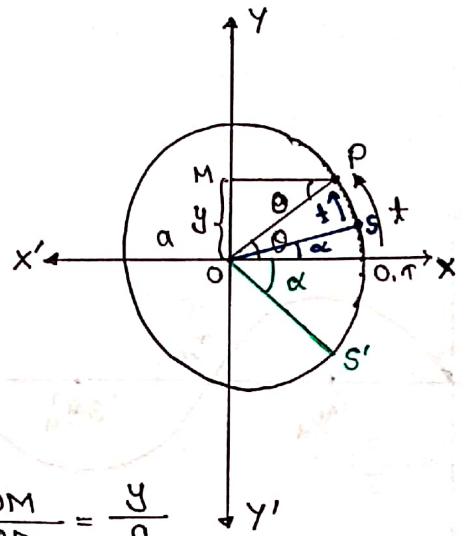
Simple Harmonic motion (SHM):

→ liner motion

→ Oscillated motion.

acc ∝ −distance.

$\Rightarrow a \propto -x$



$\theta = \omega t$

$\angle SOP = \omega t$

$\Delta OPM \rightarrow$

$\sin\theta = \sin\omega t = \frac{OM}{OP} = \frac{y}{a}$

$y = a\sin(\omega t + \alpha)$

$y = a\sin(\omega t - \delta)$

$\Rightarrow y = a\sin\theta$

$\Rightarrow \boxed{y = a\sin\omega t}$

$\alpha \rightarrow$ Epoch angle (when $t = 0$)

$\Rightarrow y = a\sin 2\pi n t$

$\Rightarrow y = a\sin\frac{2\pi}{T}.t$

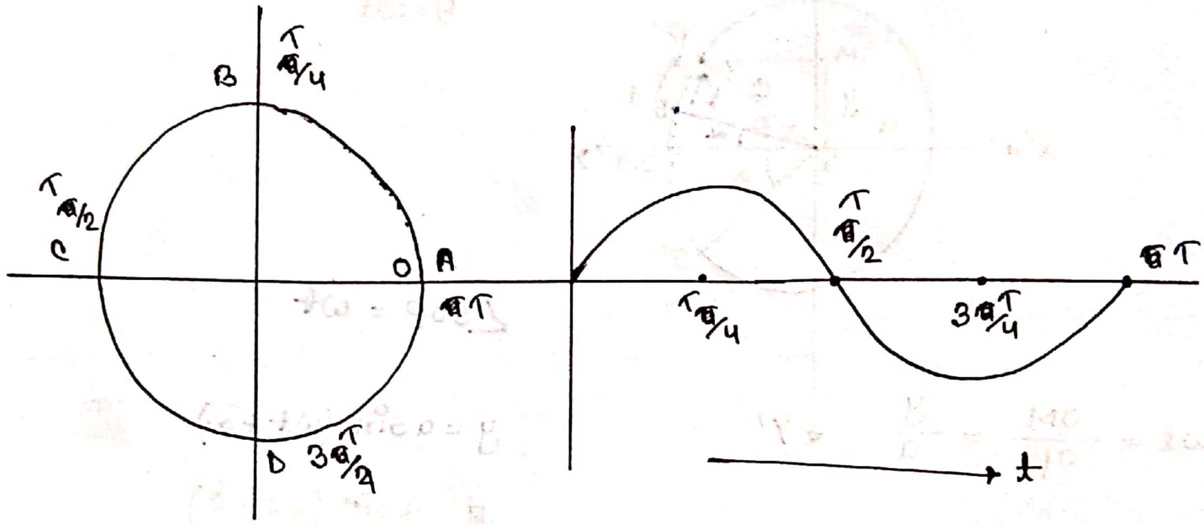
$V = \frac{d}{dt}(\text{distance}) = \frac{d}{dt}(y) = \boxed{a\omega\cos\omega t = v}$

$a = \frac{d}{dt}(v) = \frac{d^2y}{dt^2} = \boxed{-a\omega^2\sin\omega t = -\omega^2 y = a}$

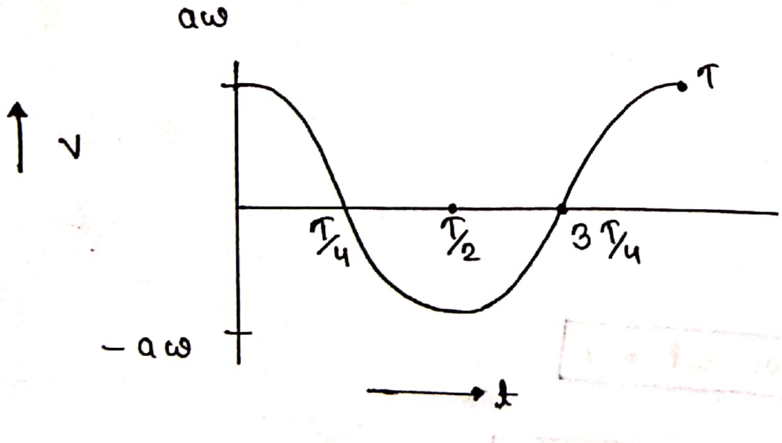
$\therefore a \propto -y$

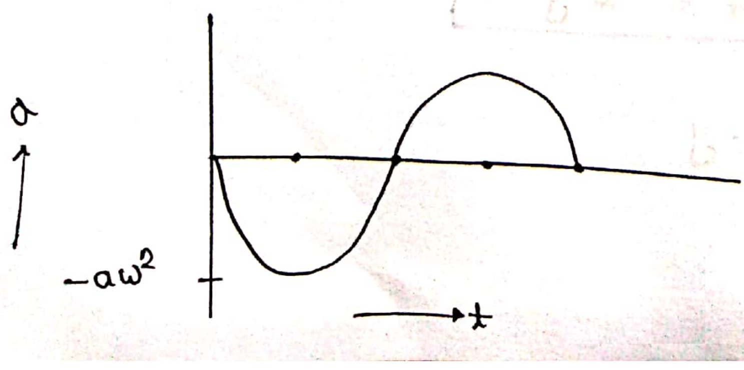
$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

θ	M	y	v	a
0	O	O	$+a\omega$	O
$\pi/2$	Y	$+a$	O	$-a\omega^2$



(y-t)





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$\frac{d^2y}{dt^2} = +\omega^2 y$ (direction বাদে)

$\Rightarrow a = \omega^2 y$

$\Rightarrow \omega = \sqrt{\frac{a}{y}}$

$\Rightarrow 2\pi n = \sqrt{\frac{a}{y}}$

$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{a}{y}}$

$\frac{d^2y}{dt^2} = -\omega^2 y$

$\Rightarrow a = -\omega^2 . y$

$\Rightarrow a \propto -y$ [y = distance]

$\Rightarrow ace = \omega^2 . dis.$

$\Rightarrow \omega^2 = \frac{ace}{dis}$

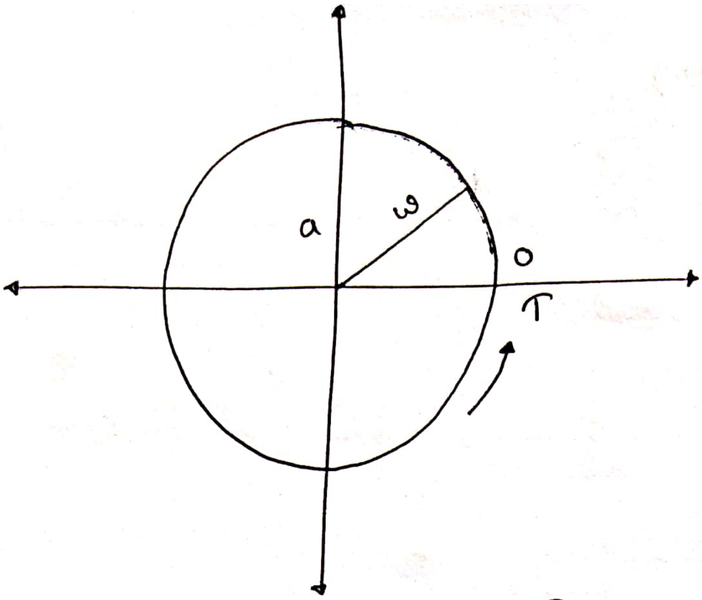
$\Rightarrow \omega = \sqrt{\frac{ace}{dis}}$

$\Rightarrow 2\pi n = \sqrt{\frac{ace}{dis}}$

$\Rightarrow n = \frac{1}{2\pi}\sqrt{\frac{ace}{dis}}$

$\Rightarrow \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{ace}{dis}}$

$\Rightarrow T = 2\pi\sqrt{\frac{dis}{ace}}$



Average kinetic Energy, AKE $= \frac{1}{T}\int_0^T \frac{1}{2} m v^2 \, dt$

$$= \frac{1}{T}\int_0^T \frac{1}{2} m a^2 \omega^2 \cos^2 \omega t . \, dt$$

$$= \frac{m a^2 \omega^2}{2T}\int_0^T \cos^2 \omega t . \, dt$$

$$= \frac{m a^2 \omega^2}{4T}\int_0^T (1 + \cos 2\omega t) \, dt$$

$$= \frac{m a^2 \omega^2}{4T}\left[\{t\}_0^T + \left[\sin 2\omega t . \frac{1}{2\omega} \right]_0^T \right]$$

$$= \frac{m a^2 \omega^2}{4T} . T$$

$$= \frac{m a^2 4 \pi^2 n^2}{4}$$

$$\boxed{AKE = m a^2 \pi^2 n^2}$$

$\therefore$ AKE $\propto a^2$

$g \rightarrow \omega^2 y$
$h \rightarrow dy$

Total Energy, $T.E = P.E + K.E$

$$P.E = \int_0^y m.\,\omega^2 y.\,dy$$

$$= \frac{m\omega^2 y^2}{2}$$

$$\boxed{P.E = \frac{1}{2} m\omega^2 y^2} \text{ --- } (1)$$

$y = a\sin\omega t$

$\Rightarrow \sin\omega t = \frac{y}{a}$

$\Rightarrow \cos\omega t = \sqrt{1-\left(\frac{y}{a}\right)^2}$

$\Rightarrow \cos\omega t = \sqrt{\frac{a^2-y^2}{a^2}}$

~~$\Rightarrow \cos\omega t$~~

$v = a\omega\cos\omega t$

$\Rightarrow v = a\omega.\sqrt{\frac{a^2-y^2}{a^2}}$

$$\boxed{v = \omega\sqrt{a^2-y^2}}$$

$K.E = \frac{1}{2} mv^2$

$$\Rightarrow \boxed{K.E = \frac{1}{2} m.\,\omega^2 (a^2-y^2)} \text{ --- } (11)$$

$\therefore T.E = \frac{1}{2} m\omega^2 a^2$

$\Rightarrow T.E = \frac{1}{2} m 4\pi^2 n^2 a^2$

$$\Rightarrow \boxed{T.E = 2m\pi^2 n^2 a^2}$$

$T.E \propto a^2$

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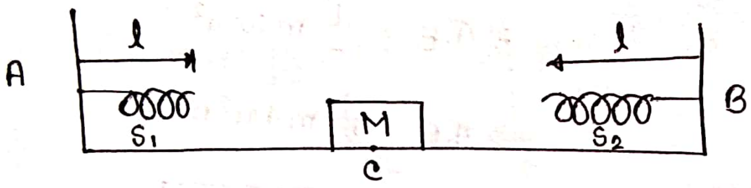
Applications of S.H.M:

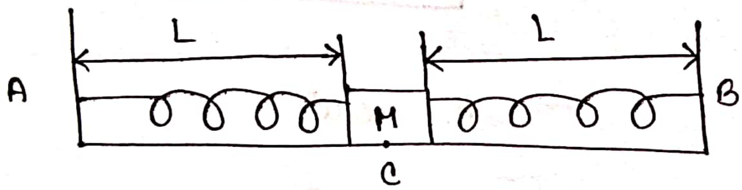
① simple

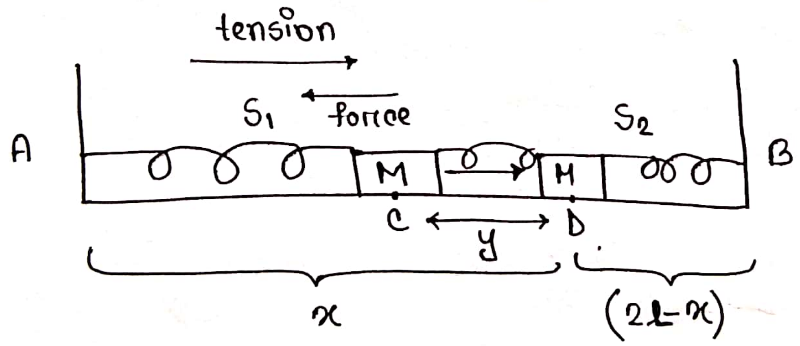
② S.H.M of a mass between two springs.

③ " of loaded spring

④ " of LC circuit.







K = tension per unit displacement

$$= \frac{T}{d} \times d$$

Tension of a spring, $S_1 = K(x - l)$

" " " $S_2 = K(2L - x - l)$

Resultant force between two springs $= K(2L-x-l) - K(x-l)$

$$= 2K(L-x)$$

$$= -2K(x-L)$$

$F = M.\frac{d^2x}{dt^2}$

$$\Rightarrow M.\frac{d^2x}{dt^2} = -2K(x-L)$$

$$\Rightarrow \frac{d^2x}{dt^2} + \frac{2K}{M}(x-L) = 0$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \left(\frac{2K}{M}\right)\cdot y = 0} \text{ --- (i)}$$

let,

$x - L = y$

$\Rightarrow \frac{dx}{dt} = \frac{dy}{dt}$

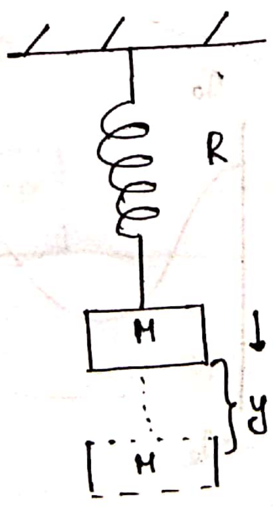
$\Rightarrow \frac{d^2x}{dt^2} = \frac{d^2y}{dt^2}$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \text{ --- (ii)}$$

$$\omega^2 = \frac{2K}{M}$$

$$\Rightarrow \omega = \sqrt{\frac{2K}{M}}$$

③ S.H.M of a cloaded spring:



Force $= -ky$

$$\Rightarrow M.\frac{d^2y}{dt^2} = -ky$$

$$\Rightarrow \frac{d^2y}{dt^2} + \left(\frac{k}{M}\right)y = 0 \text{ --- (i)}$$

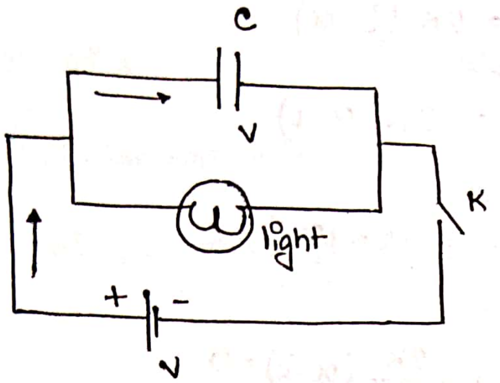
আবার,

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \text{ --- (ii)}$$

$$\omega^2 = \frac{K}{M}$$

$$\Rightarrow \omega = \sqrt{\frac{K}{M}}$$

④



$$V = \frac{Q}{C} \quad , \quad V = L \cdot \frac{dI}{dt}$$

$$\Rightarrow \frac{Q}{C} = -L \cdot \frac{dI}{dt}$$

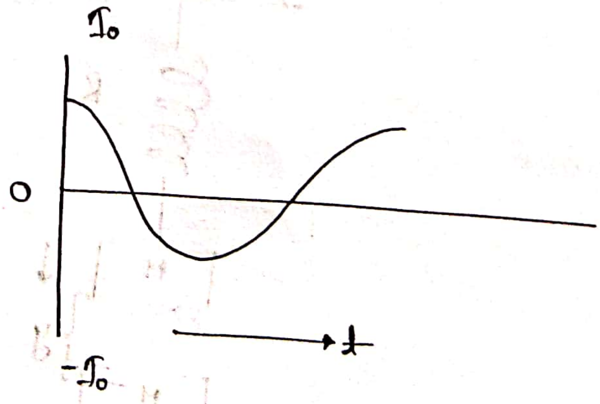
$$\Rightarrow \frac{dI}{dt} = -\frac{Q}{LC}$$

$$\Rightarrow \frac{d^2I}{dt^2} = -\frac{1}{LC} \cdot \frac{dQ}{dt}$$

$$\Rightarrow \frac{d^2I}{dt^2} + \frac{1}{LC} \cdot I = 0 \quad \text{—— (i)}$$

$$\Rightarrow \frac{d^2I}{dt^2} + \omega^2 I = 0 \quad \text{—— (ii)}$$

$$\therefore \omega = \sqrt{\frac{1}{LC}}$$



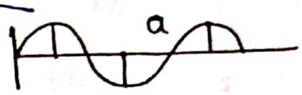
$$y = a \sin(\omega t + \alpha)$$

$$\Rightarrow I = A \sin(\omega t + \alpha)$$

$$t = 0 \longrightarrow I = I_0$$

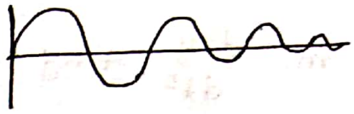
$$\alpha = \pi/2 \longrightarrow A = I_0$$

Vibration / Oscillation

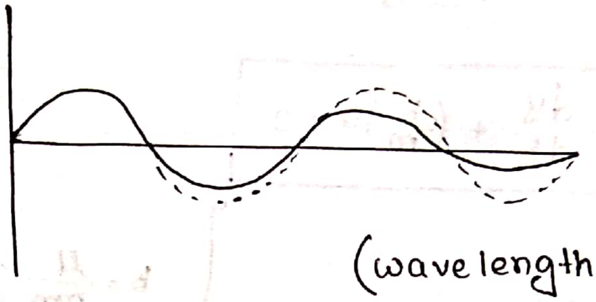


Vibration →

1. Free vibration → ⓐ Undamped Vibration (constant amplitude) (no lose of energy)
   ⓑ Damped Vibration



2. Forced vibration



(wavelength same)

Free vibration →

ⓐ Total energy = constant

$$\frac{1}{2}m\left(\frac{dy}{dt}\right)^2 + \frac{1}{2}Ky^2 = \text{constant}$$

$$\Rightarrow \frac{1}{2}m \cdot 2\left(\frac{dy}{dt}\right) \cdot \frac{d^2y}{dt^2} + \frac{1}{2} \cdot K \cdot 2y \cdot \frac{dy}{dt} = 0$$

$$\Rightarrow m \cdot \frac{d^2y}{dt^2} + Ky = 0$$

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \left(\frac{K}{m}\right)y = 0} \quad \text{--- (i)}$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{--- (ii)}$$

$$y = a\sin(\omega t - \alpha)$$

$$\therefore \omega^2 = \frac{K}{m}$$

$$\Rightarrow \omega = \sqrt{\frac{K}{m}}$$

$$y = a\sin\left(\sqrt{\frac{K}{m}}\,t - \alpha\right)$$

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(b) $F_1 \propto \frac{dy}{dt}$

$\Rightarrow F_1 = \mu . \frac{dy}{dt}$

$$m. \frac{d^2y}{dt^2} + Ky = -\mu . \frac{dy}{dt}$$

F $\quad$ F_1

$$\Rightarrow \boxed{\frac{d^2y}{dt^2} + \left(\frac{\mu}{m}\right)\frac{dy}{dt} + \left(\frac{K}{m}\right)y = 0}$$

$b = \frac{\mu}{2m}$, $k^2 = \frac{K}{m}$

\# $\frac{d^2y}{dt^2} + 2b . \frac{dy}{dt} + k^2 y = 0$

$y = a.e^{-bt} \sin(\omega t - \alpha)$

$y = A.e^{(-b+\sqrt{b^2-k^2})b} + Be^{(-b-\sqrt{b^2-k^2})t}$ —— (*)

$e^{i\omega A} = A.e^{-b} . e^{-b(\sqrt{k^2-b^2})t}$

$\omega = \sqrt{k^2-b^2}$

Trial solution:

let, $y = Ae^{xt}$

$\Rightarrow \frac{dy}{dt} = Ax.e^{xt}$

$\Rightarrow \frac{d^2y}{dt^2} = Ax^2 . e^{xt}$

$$A\alpha^2 e^{\alpha t} + \left(\frac{\mu}{m}\right) A\alpha e^{\alpha t} + \left(\frac{K}{m}\right) A e^{\alpha t} = 0$$

$$\Rightarrow \alpha^2 + \left(\frac{\mu}{m}\right)\alpha + \left(\frac{K}{m}\right) = 0$$

$$a\alpha^2 + b\alpha + c = 0$$

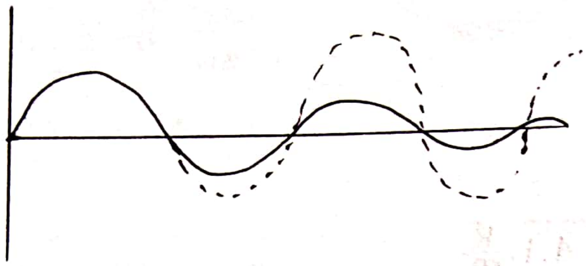
$$\alpha = \frac{-\frac{\mu}{m} \pm \sqrt{\left(\frac{\mu}{m}\right)^2 - 4.1.\frac{K}{m}}}{2.1}$$

* এ মান বসিয়ে,

$$y = A.e^{\left(\frac{-\frac{\mu}{m} + \sqrt{\left(\frac{\mu}{m}\right)^2 - \frac{4K}{m}}}{2}\right)t} + Be^{\left(\frac{-\frac{\mu}{m} - \sqrt{\left(\frac{\mu}{m}\right)^2 - \frac{4K}{m}}}{2}\right)t}$$

Forced vibration:

যে শক্তি lose হবে, external শক্তি প্রয়োগ করে a constant রাখতে হবে।



Periodic Force $= F\sin\theta$

$= F\sin pt$

$$m.\frac{d^2y}{dt^2}+\mu\frac{dy}{dt}+ky = F\sin pt \quad \text{--- (i)}$$

Trial solution of this differential equation of forced vibration:

let,

$$y = a\sin(pt-\alpha) \quad \text{--- (ii)}$$

$$\text{or, } \frac{dy}{dt} = ap\cos(pt-\alpha)$$

$$\text{or, } \frac{d^2y}{dt} = -ap^2\sin(pt-\alpha)$$

$$\therefore -map^2\sin(pt-\alpha)+\mu ap\cos(pt-\alpha)+ka\sin(pt-\alpha) = F\sin pt$$

$$\Rightarrow -map^2[\sin pt.\cos\alpha-\sin\alpha.\cos pt]+\mu ap[\cos pt.\cos\alpha+\sin pt.\sin\alpha]$$

$$+ka[\sin pt.\cos\alpha-\cos pt.\sin\alpha] = F\sin pt \quad \text{--- (iii)}$$

when,

$\sin pt = 1$

them, $\cos pt = 0$

$$\therefore -map^2 \cos\alpha + \mu ap \sin\alpha + Ka\cos\alpha = F \quad \text{---} \quad (iv)$$

when

$\sin pt = 0$

then, $\cos pt = 1$

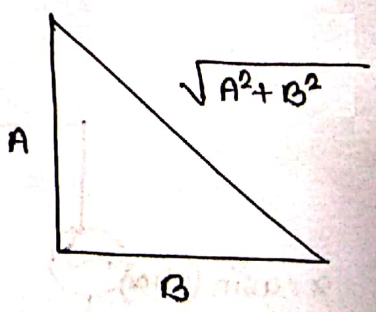
$$\therefore map^2 \sin\alpha + \mu ap \cos\alpha - Ka\sin\alpha = 0 \quad \text{---} \quad (v)$$

$$\Rightarrow mp^2 \tan\alpha + \mu p - K\tan\alpha = 0 \qquad [\cos\alpha \text{ দিয়ে ভাগ}]$$

$$\Rightarrow \tan\alpha\,(K - mp^2) = \mu p$$

$$\Rightarrow \tan\alpha = \frac{\mu p}{K - mp^2} = \frac{A}{B} \quad \text{---} \quad (vi)$$

ধরি, $\frac{\mu p}{K - mp^2} = \frac{A}{B}$



(iv) হতে,

$$-mp^2 + \mu p \tan\alpha + K = \frac{F}{a\cos\alpha}$$

$$\Rightarrow a\left[B + A \cdot \frac{A}{B}\right] = \frac{F}{\cos\alpha}$$

$\cos\alpha = \frac{B}{\sqrt{A^2+B^2}}$

$$\Rightarrow a\left[B + \frac{A^2}{B}\right] = \frac{F\sqrt{A^2+B^2}}{B}$$

$$\Rightarrow a = \frac{F}{\sqrt{A^2+B^2}} = \frac{F}{\sqrt{\mu^2p^2 + (K-mp^2)^2}} \quad \text{---} \quad (vi)$$

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Final solution:

$$y = \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}} \sin(pt - \alpha)$$

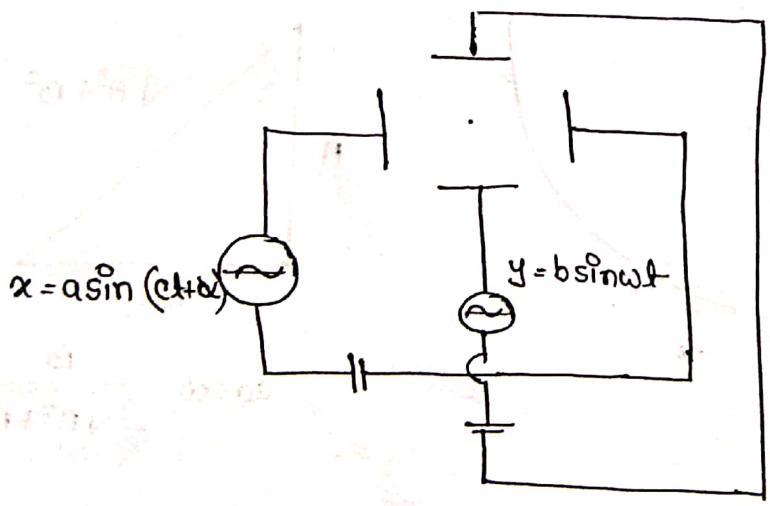
$\hookrightarrow$ particular solution

General solution:

$$y = a \cdot e^{-bt} \sin(\omega t - \alpha) + \frac{F}{\sqrt{\mu^2 p^2 + (K - mp^2)^2}} \cdot \sin(pt - \alpha)$$

Resonence:

Lissiyous Figures:



let,

$$x = a \sin(\omega t + \alpha) \quad \text{--- (1)}$$

$$y = b \sin \omega t \quad \text{--- (2)}$$

Composition of two mutually perpendicular SHM of same time period and diff. if other parametres.

Eq ② →

$$\sin \omega t = \frac{y}{b} \quad \text{or,} \quad \cos \omega t = \sqrt{1-\frac{y^2}{b^2}} \quad \text{--- ③}$$

① নং এ মান বসিয়ে,

$$\frac{x}{a} = \sin\omega t \cdot \cos\alpha + \cos\omega t \cdot \sin\alpha$$

$$\Rightarrow \frac{x}{a} = \frac{y}{b}\cos\alpha + \sqrt{1-\frac{y^2}{b^2}}\,\sin\alpha$$

$$\Rightarrow \frac{x}{a} - \frac{y}{b}\cos\alpha = \sqrt{1-\frac{y^2}{b^2}}\,\sin\alpha$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\alpha - 2\cdot\frac{xy}{ab}\cos\alpha = \left(1-\frac{y^2}{b^2}\right)\sin^2\alpha$$

$$\Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = \sin^2\alpha}$$

① $\alpha = 0$ →

$2\pi, 4\pi$ →

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\alpha = 0$$

$$\text{or,} \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\text{or,} \boxed{y = \frac{b}{a}x}$$

② $\alpha = 90° \longrightarrow$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(উপবৃত্ত)

$a = b$ হলে,

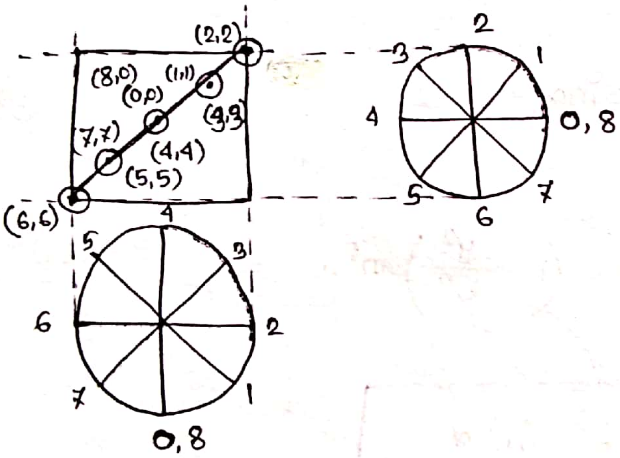
$$x^2 + y^2 = a^2 \quad \text{(বৃত্ত)}$$

$x = a \sin \omega t$

$y = b \sin t\omega t$ হলে $\longrightarrow$

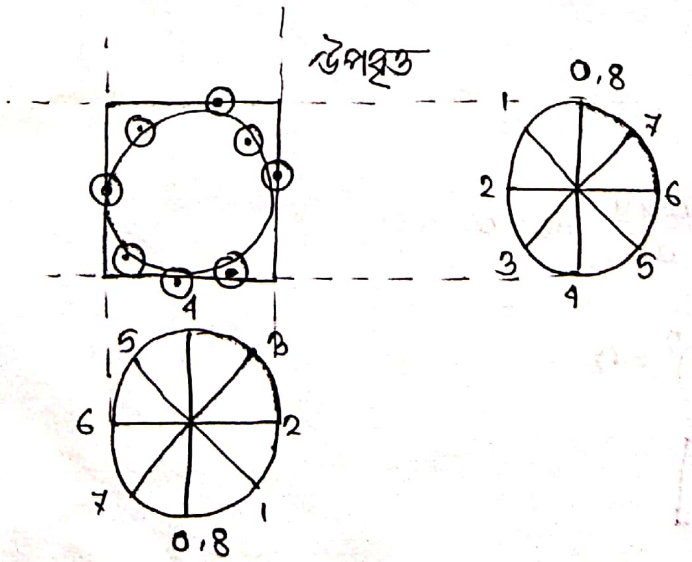
when.

$\alpha = 0$



when

$\alpha = 90°$



composition of two SHMs at a right angle to each other and having their time periods in the ratio of 1:2.

$t:2 \longrightarrow t = 2t$

$$x = a\sin(2\omega t+\alpha) \quad \text{--- (1)}$$

$$y = b\sin\omega t \quad \text{--- (2)}$$

$$\Rightarrow \sin\omega t = \frac{y}{b}$$

$$\Rightarrow \cos\omega t = \sqrt{1-\frac{y^2}{b^2}} \quad \text{--- (3)}$$

$$\frac{x}{a} = \sin 2\omega t.\cos\alpha + \cos 2\omega t.\sin\alpha$$

$$\Rightarrow \frac{x}{a} = 2.\sin\omega t.\cos\omega t.\cos\alpha + (1-2\sin^2\omega t)\sin\alpha \quad \text{--- (4)}$$

$$\Rightarrow \frac{x}{a} = 2.\frac{y}{b}.\sqrt{1-\frac{y^2}{b^2}}.\cos\alpha + \left(1-\frac{2y^2}{b^2}\right)\sin\alpha$$

$$\Rightarrow \left[\frac{x}{a} - \left(1-\frac{2y^2}{b^2}\right)\sin\alpha\right] = \frac{2y}{b}\sqrt{1-\frac{y^2}{b^2}}\cos\alpha \quad \text{--- (5)}$$

$$\Rightarrow \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^2}{b^2}\left(\frac{x}{a}-\sin\alpha\right)\sin\alpha + \frac{4y^4}{b^4}\sin^2\alpha = \frac{4y^2}{b^2}\left(1-\frac{y^2}{b^2}\right)\cos^2\alpha$$

$$\Rightarrow \left(\frac{x}{a}-\sin\alpha\right)^2 + \frac{4y^2}{b^2}.\frac{x}{a}\sin\alpha - \frac{4y^2}{b^2} + \frac{4y^4}{b^4} = 0$$

$$\Rightarrow \boxed{\left[\frac{x}{a}-\sin\alpha\right]^2 + \frac{4y^2}{b^2}\left[\frac{y^2}{b^2} + \frac{x}{a}\sin\alpha - 1\right] = 0}$$

① when $\alpha = 0, \pi, 2\pi, \ldots\ldots n\pi$

$$\boxed{\frac{x^2}{a^2} + \frac{4y^2}{b^2}\left[\frac{y^2}{b^2} - 1\right] = 0}$$

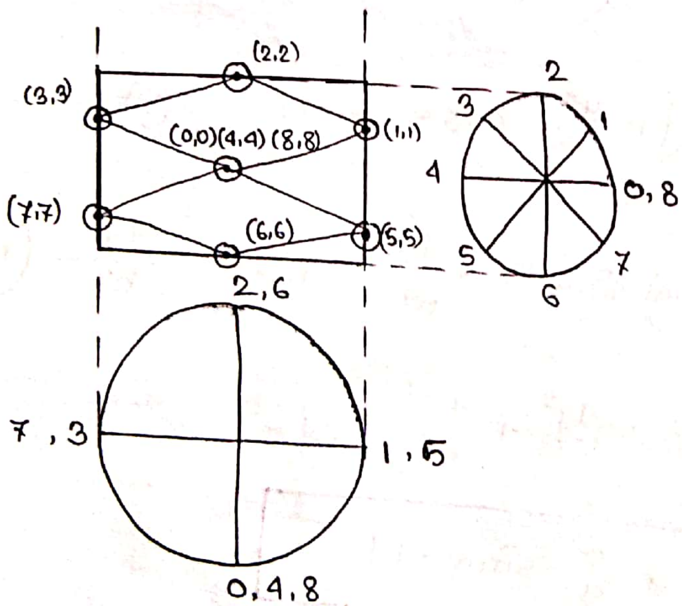
② when $\alpha = \pi/2, 3\pi/2, 5\pi/2 \ldots\ldots (2n+1)\pi/2$

$$\left(\frac{x}{a} - 1\right)^2 + \frac{4y^2}{b^2}\left[\frac{y^2}{b^2} + \left(\frac{x}{a} - 1\right)\right] = 0$$

$$\Rightarrow \left(\frac{x}{a} - 1\right)^2 + 2 \cdot \frac{2y^2}{b^2}\left(\frac{x}{a} - 1\right) + \left(\frac{2y^2}{b^2}\right)^2 = 0$$

$$\Rightarrow \left[\left(\frac{x}{a} - 1\right) + \frac{2y^2}{b^2}\right]^2 = 0$$

$$\Rightarrow \boxed{y^2 = \frac{-b^2}{2}\left(\frac{x}{a} - 1\right)}$$



phase diff = 0

Wavemotion:

[Particle যখন vibration start করে তখন wave তৈরি হয়। Particle গুলো mean position থেকে vibrate করে।]

Transferring the power of energy which is created by the vibration of particle.

(related momentum)

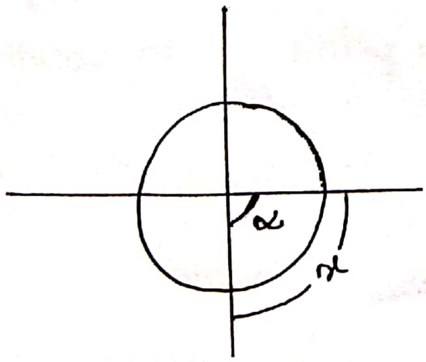
The characteristics of wave motion:

Wave এর velocity same থাকে কিন্তু particle এর velocity different থাকে

i) Transverse : Water
ii) longitudinal : sound

$\Big\}$ diff. eqn.

উভয়েরই progressive and stational wave আছে —



→ clockwise wave হলে anticlock wise particle diagram হবে and viceversa.

$y = a\sin\omega t$

$y = a\sin(\omega t - \alpha)$

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The ratio of the phase different and path difference always equal or same.

$$\frac{\alpha}{x} = \frac{2\pi}{\lambda} \qquad \omega t = 2\pi n t = \frac{2\pi}{T} t = \frac{2\pi v}{\lambda} t$$

$$\rightarrow \alpha = \frac{2\pi}{\lambda} x$$

$$\therefore y = a \sin \left(\frac{2\pi v t}{\lambda} - \frac{2\pi x}{\lambda} \right)$$

$$\boxed{y = a \sin \frac{2\pi}{\lambda} (vt - x)}$$

v → velocity of wave

$$y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

$$\frac{dy}{dt} = \frac{2\pi v}{\lambda} a \cos \frac{2\pi}{\lambda} (vt - x) \text{ ——— (i)}$$

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda} (vt - x) \text{ ——— (ii)}$$

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x) \text{ ——— (iii)}$$

$$\frac{d^2y}{dx^2} = -\frac{4a^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda} (vt - x) \text{ ——— (iv)}$$

Volume strain কি compression নাকি decompression ?

According to (i) and (iii) →

$$\frac{dy}{dt} = -\underbrace{v}_{\text{wave velocity}} \underbrace{\frac{dy}{dx}}_{\text{slope of compression / volume strain}}$$

$\frac{dy}{dt}$ → particle velocity

$v = \frac{dy}{dt}$ → particle velocity

$v_p = \frac{\omega}{K}$ → phase velocity

$K = \frac{2\pi}{\lambda}$

$$v_p = \frac{2\pi n}{\frac{2\pi}{\lambda}}$$

$$v_p = n\lambda$$

According to (ii) and (iv) →

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

Distribution of pressure and velocity in progressive wave:

$$K = \frac{\text{Change of pressure}}{\text{Vol. strain}}$$

↓

Modulas of elasticity of medium

$$K = -\frac{dp}{\frac{dy}{dx}}$$

$$dp = K\left(-\frac{dy}{dx}\right)$$

→ strain যদি negative হয়, তবে stress.

stress হওয়া মানে compression, তখন pressure positive.

→ Pressure negative হলে strain হবে, de-compression হবে।

Progressive wave:

Energy of progressive wave:

$$P.E = m.g.h$$

medium এর জন্য বিবেচনা করে

$$\frac{P.E}{V} = \frac{m}{V} gh$$

$$= \rho g h$$

$$= \rho . \frac{4\pi^2 v^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda} (vt - x) \, dy$$

$\downarrow$

dy এর জন্য P.E

$$\frac{P.E}{Vol} = \frac{4\pi^2 v^2 \rho}{\pi^2} \int y \, dy$$

$$= \frac{4\pi^2 v^2 \rho}{\pi} . \frac{y^2}{2}$$

$$= \frac{2\pi^2 v^2 \rho a^2}{\pi^2} \sin^2 \frac{2\pi}{\lambda} (vt - x) \longrightarrow *$$

$$\frac{K.E}{V} = \frac{1}{2} \rho v^2$$

$$= \frac{1}{2} \rho \left(\frac{dy}{dt}\right)^2$$

$$= \frac{1}{2} \rho \frac{4\pi^2 v^2 a^2}{\lambda^2} \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

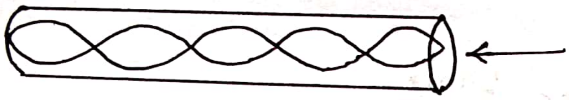
$$= \frac{2\pi^2 v^2 \rho a^2}{\lambda^2} \cos^2 \frac{2\pi}{\lambda} (vt - x)$$

$$T.E = \frac{2\pi^2 v^2 \rho a^2}{\lambda^2}$$

$$T.E = 2\pi^2 n^2 \rho a^2$$

Show that $\frac{T.E}{V}$ of progressive wave is proporsonal of square of amplitude.

Stationary wave: (definition)



$$y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \longrightarrow (i)$$

$$y_2 = a \sin \frac{2\pi}{\lambda}(vt + x) \longrightarrow (ii)$$

$$y = y_1 + y_2$$

$$= 2a \sin \frac{2\pi}{\lambda} vt \cos \frac{2\pi x}{\lambda}$$

$$Y = 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \longrightarrow (*)$$

└→ eqn of stationary wave

t এর respect এ diff

x " " " "

$$E = K = -\frac{P}{\frac{dy}{dx}}$$

↓

modulas of elasticity of medium

$$\therefore P = -E \frac{dy}{dx}$$

$$P = -v^2 \rho \frac{dy}{dx}$$

$$v = \sqrt{\frac{E}{\rho}}$$

↓

velocity

$$v^2 = \frac{E}{\rho}$$

$$\therefore E = v^2 \rho$$

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From eqn (*) $\Rightarrow$

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$P = v^2 \rho \frac{4\pi a}{\lambda} \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$P = P_0 \sin\frac{2\pi x}{\lambda} \sin\frac{2\pi v t}{\lambda}$$

$$P = P_x \sin\frac{2\pi v t}{\lambda}$$

Now,

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda}$$

$$= U_x \cos\frac{2\pi v t}{\lambda}$$

$E = P.\ \text{velocity}.\ \text{time}$

$$= \int_0^T P.\,U.\,dt$$

$$\text{Rate of } E = \frac{1}{T}\int_0^T P.\,U.\,dt$$

$$= \frac{P_x U_x}{T} \int_0^T \sin\frac{4\pi v t}{\lambda}\, dt$$

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