Heaven's Light is Our Guide

DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering FIRST year **ODD SEMESTER** Examination, 2017

Math 1101

Mathematics - I

Full Marks: 72 | **Time: 3 Hours**

N.B.:-
(i) Answer SIX questions, taking THREE from each section.
(ii) Figure in the margin indicates full marks.
(iii) Use separate answer script for each section.

SECTION-A

Q.1(a) What is meant by continuity of a function? A function $f(x)$ is given as 5.0

$$f(x) = \begin{cases} 3 + 2x \ for -\frac{3}{2} \leq x < 0 \\ 3 - 2x \ for \ 0 \leq x < 3/2 \\ -3 - 2x \ for \ x \geq \frac{3}{2}, \quad x = 0 \end{cases}$$

Draw the graph. Test its continuity at x =0.

(b) Find from first principles the derivative of $x^2 tanx$. 4.0

(c) Differentiate $x^{\sin^{-1}x}$ with respect to $sin^{-1}x$. 3.0

Q.2(a) State and prove Leibnitz's theorem. 5.0

(b) Apply Leibnitz's theorem to find y_n where $y = e^{ax}\sin bx$. 3.0

(c) If $u = 3(ax+by+cz)^2 - (x^2+y^2+z^2)$ and $a^2+ b^2 + c^2 = 1$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. 4.0

Q.3(a) State Rolle's theorem. Verify the Rolle's theorem for the function $f(x) = e^x \sin x$ in the interval $(0,\pi)$. 4.0

(b) Show that the largest rectangle of a given perimeter is a square. 4.00

(c) Find the tangent and normal to the curve $xy^2 = 16-4x$ at the point where it is cut by the line $y = x$. 4.00

Q.4(a) Find the pedal equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with regard to the center. 7.0

(b) Find the radius of curvature of $y = xe^{-x}$ at its maximum point. 5.0

SECTION-B

Q.5(a) Integrate (any three) 12.0

(i) $\int \frac{x^5}{\sqrt{1-x^2}} dx$ (ii) $\int \frac{x^2-1}{x^4+1} dx$ (iii) $\sqrt{(x-\alpha)(\beta-x)}\, dx$ (iv) $\int_0^{\pi/2} \frac{dx}{1+\cos\theta \cos x}$

Q.6(a) Determine $\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n-1^2}} + \frac{1}{\sqrt{4n-2^2}} + \cdots \ldots\ldots + \frac{1}{n}\right]$ 4.0

(b) Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x+\cos x} dx$ 5.0

(c) If $U_n = \int_0^{\frac{\pi}{2}} x \sin^n x \, dx$, $n > 1$, then show that $U_n = \frac{1}{n^2} + \frac{n-1}{n} u_{n-2}$. 3.0

Q.7(a) Show that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$ 4.0

(b) Show that $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx = \frac{\Gamma\frac{m+1}{2}\Gamma\frac{n+1}{2}}{2\Gamma\frac{m+n+2}{2}}$ $m, n > -1$. 4.0

(c) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ 4.0

Q.8(a) Find the area bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and x-axis between the ordinates $x=c$ and $x=d$. 4.0

(b) What is the area included between the parabolas $y = 6x - x^2$ and $y = x^2 - 2x$. 4.0

(c) Find the area enclosed by $r = 2- \cos\theta$. 4.0

DEPARTMENT OF CIVIL ENGINEERING
RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering First Year Odd Semester Examination, 2016

Math 1101
Mathematics-I

Full marks: 72 **Time: 3 Hours**

N.B:-
(i) Answer any SIX questions, taking THREE from each section.
(ii) Figures in the margin indicate full marks.
(iii) Use separate answer script for each section.
(iv) Assume reasonable value for any data missing.

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SECTION-A

Q.1(a) Discuss about the continuity and differentiability of the function $f(x)=|x|+|x-1|$ at $x=1$. 4.0

(b) Find $\frac{dy}{dx}$, when $y=(\sin x)^{\cos x}+(\cos x)^{\sin x}$ 4.0

(c) Evaluate $Lt_{x\to 0}\left\{\frac{(e^x-1)\tan^2 x}{x^3}\right\}$ 4.0

Q.2 (a) If $y=\frac{1}{x^2+a^2}$, find y_n, when y_n represents the n-th difference. 5.0

(b) If $y=\sin(n\sin^{-1}x)$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(x^2-n^2)y_n=0$ and also find the value of $(y_n)_0$. 5.0

(c) State mean value theorem. Consider a function $f(x)=\sqrt{x}$. Find a point between (1, 1) and (4, 2) where the tangent line on the curve is parallel to the chord joining (1, 1) and (4, 2). 2.0

Q.3 (a) State Euler's theorem for homogeneous functions of two variables. If $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{\tan u}{2}$. 4.0

(b) If $u=F(x^2+y^2+z^2)f(xy+yz+zx)$, prove that $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$ 4.0

(c) Find the maximum and minimum values of $x+\frac{1}{x}$. 4.0

Q.4 (a) Show that the maximum rectangle inscribed in a circle is a square. 3.0

(b) Find the altitude of the right circular cylinder of maximum volume that can be inscribed in a given right circular cone of height h. 3.0

(c) If $lx+my=1$ is normal to the parabola $y^2=4ax$ then $al^3+2alm^2=n^2$. 3.0

(d) If V is a function of x and y then prove that $\frac{\partial}{\partial x}\equiv\left(\cos\theta\frac{\partial}{\partial r}-\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\right)$, where $x=r\cos\theta,\ y=r\sin\theta$. 3.0

SECTION-B

Q.5 Integrate 12.0

(i) $\int\frac{dx}{(2+x)\sqrt{1+x}}$, (ii) $\int\sqrt{\frac{a+x}{a-x}}\,dx$ (iii) $\int\frac{dx}{a+b\cos x}$ (a>b>0)

Q.6 (i) Evaluate $Lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots\ldots+\frac{n}{n^2+n^2}\right]$ (ii) Integrate $\int_0^{\frac{\pi}{2}}\log(\sin x)dx$ 12.0

Q.7(a) Define gamma and beta function. Establish the relation between them. 6.0

(b) Show $\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=2^{1-2n}\Gamma(2n)\sqrt{\pi}$ 6.0

Q.8(a) Find the area bounded by the curve $y^2(2a-x)=x^3$ and its asymptote. 4.0

(b) Find the total length of the curve $r^2=a^2\cos 2\theta$ 4.0

(c) Find the volume of the solid generated by revolving the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ about its base. 4.0

DEPARTMENT OF CIVIL ENGINEERING
RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering First Year Odd Semester Examination, 2014

Math 1101
Mathematics- I

Full marks: 70 **Time: 3 Hours**

N.B:- (i) Answer any SIX questions, taking THREE from each section.
(ii) Figure in the margin indicate full marks.
(iii) Use separate answer script for each section.

SECTION-A

Q.1(a) A man is 5 ft. tall walks away from a lamp-post 12.5 ft high at the rate of 3 miles per hour. How fast is the further end of his shadow moving on the pavement? 4.00

(b) Discuss the continuity and differentiability of the function f(x) at x = 2. $f(x) = |x-2|$ 4.00

(c) Find the first derivative of $y = x^{\ln(\sin x)}$ 3.67

Q.2(a) If $y = e^{ax}\sin bx$, find y_n. 4.00

(b) If $y = e^{a\sin^{-1}x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. 3.67

(c) State and prove mean value theorem. 4.00

Q.3(a) Find the maximum and /or minimum values of u, where $u = \frac{4}{x} + \frac{36}{y}$ and $x+y = 2$. 4.67

(b) Find the limit $Lt_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x}$. 4.00

(c) If $u = e^{xyz}$, find $\frac{\partial^3 u}{\partial x \partial y \partial z}$. 3.00

Q.4(a) If $u = F(y-z, z-x, x-y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 4.00

(b) Find the equation of tangent at (a,b) to the curve $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 2$. 4.00

(c) Find the radius of curvature at the point (r,0) on the cardioide $r = a(1-\cos\theta)$. 3.67

SECTION-B

Q.5 Integrate (i) $\int e^x \frac{x^2+1}{(x+1)^2}dx$ (ii) $\int \frac{e^x}{x}(1+x\log x)dx$ (iii) $\int_0^{\pi/2} \frac{dx}{1+\cos x\cos\theta}$ 11.67

Q.6(a) Evaluate $Lt_{n\to\infty}\left[\frac{1}{\sqrt{2n-1^2}} + \frac{1}{\sqrt{4n-2^2}} + \dots + \frac{1}{n}\right]$ 3.67

(b) Find the value of $\int_0^{\pi/2} \ln(\tan x + \cot x)dx$ 4.00

(c) If $U_n = \int_0^1 x^n \tan^{-1} x\,dx$, then show that $(n+1)U_n + (n-1)U_{n-2} = \frac{\pi}{2} - \frac{1}{n}$ 4.00

Q.7(a) Show that (i) $\int_0^1 x^{m-1}(1-x^a)^n dx = \frac{\Gamma(n+1)\Gamma(m/a)}{a\Gamma(m/a+n+1)}$ 3.67

(ii) $\int_0^{\pi/2} \sin^m x\,dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\sqrt{\pi}}{2\Gamma\left(\frac{m+2}{2}\right)}; m > -1$ 4.00

(b) Evaluate $\int_0^\infty e^{-x^4}x^2dx \times \int_0^\infty e^{-x^4}dx$ 4.00

Q.8(a) Find the area common to the cardioide $r = a(1+\cos\theta)$ and the circle $r = \frac{3}{2}a$, and also the area of the remainder of the cardioide. 5.67

(b) A quadrant of a circle of radius a, revolves round its chord. Find the volume and the surface area of the solid spindle thus generated. 6.0

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Heaven's Light is Our Guide

DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering FIRST year FIRST Semester Examination, 2013

Math 101

Mathematics-I

Full Marks: 70 **Time: 3 Hours**

N.B.:-
(i) Answer SIX questions, taking THREE from each section.
(ii) Figure in the margin indicates full marks.
(iii) Use separate answer script for each section.

SECTION-A

Q.1(a) Define limit of a function. Does the limit $Lt_{x\to\pi/2}\dfrac{e^{\tan x}-1}{e^{\tan x}+1}$ exist? (না) 4.00

(b) Find the differential coefficient of $(\operatorname{Sin}x)^{\operatorname{Cos}x}+(\operatorname{Cos}x)^{\operatorname{Sin}x}$. 3.67

(c) A ladder AB, 25 ft long, leans against a vertical wall. If the lower end A, which is at a distance of 7 ft from the bottom of the wall, is being moved away on the ground from the wall at the rate of 2 ft per second, find how fast is the top B descending on the wall. 7/12 4.00

Q.2(a) If $y=e^{x}\operatorname{Sin}x\operatorname{Sin}2x$, find y_n. 4.00

(b) If $y=\operatorname{Sin}(m\operatorname{Sin}^{-1}x)$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$. 3.67

(c) Prove that $(x+h)^{3/2}=x^{3/2}+\dfrac{3}{2}x^{1/2}h+\dfrac{3}{2}\cdot\dfrac{1}{2}\cdot\dfrac{h^2}{2!}\dfrac{1}{\sqrt{x+\theta h}}$, when $0<\theta<1$ and hence find θ when $x=0$. 4.00

Q.3(a) Find the maximum and minimum value of $1+2\operatorname{Sin}x+3\operatorname{Cos}^2x$; $(0\le x\le\pi/2)$. $\frac{13}{3}$, 3 5.00

(b) Evaluate $Lt_{x\to 1}\left[\dfrac{x}{x-1}-\dfrac{1}{\log x}\right]$. 0 3.00

(c) If $u=\operatorname{Cos}^{-1}\{(x+y)/(\sqrt{x}+\sqrt{y})\}$, then show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+\dfrac{1}{2}\operatorname{Cot}u=0$. 3.67

Q.4(a) The radius of right circular cone is measured as 5 inches with a possible error of 0.01 inch, and altitude as 8 inches with a possible error of 0.024 inch. Find the possible relative error and percentage error in the volume as calculated from these measurements. .007, 0.7% 3.67

(b) Find the condition that the conics $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ shall cut orthogonally. $\frac{1}{a_1}-\frac{1}{b_1}=\frac{1}{a}-\frac{1}{b}$. 4.00

(c) Prove that all point of the curve $y^2=4a\left(x+a\operatorname{Sin}\left(\dfrac{x}{a}\right)\right)$, at which the tangent is parallel to the x-axis lie on a parabola. 4.00

SECTION-B

Q.5 Integrate the following integrals (any three) 11.67

i) $\displaystyle\int\frac{dx}{\sqrt{(x-\alpha)(x-\beta)}}$ ii) $\displaystyle\int e^{x}\frac{x^2+1}{(x+1)^2}dx$ iii) $\displaystyle\int\frac{x^2+1}{x^4+1}dx$ iv) $\displaystyle\int\frac{e^{x}}{x}(1+x\log x)\,dx$

Q.6(a) Evaluate $Lt_{n\to\infty}\left\{\left(1+\dfrac{1}{n^2}\right)^{2/n^2}\left(1+\dfrac{2^2}{n^2}\right)^{4/n^2}\left(1+\dfrac{3^2}{n^2}\right)^{6/n^2}\cdots\left(1+\dfrac{n^2}{n^2}\right)^{2n/n^2}\right\}$. $\frac{4}{e}$ 3.67

(b) Evaluate $\displaystyle\int_0^{\pi}\frac{x\tan x}{\operatorname{Sec}x+\operatorname{Cos}x}dx$. $\pi/4$ 4.00

(c) If $U_n=\displaystyle\int_0^{\pi/2}\theta\operatorname{Sin}^n\theta\,d\theta$, $n>1$ prove that $U_n=\dfrac{n-1}{n}U_{n-2}+\dfrac{1}{n^2}$. 4.00

Q.7(a) Show that $\Gamma\left(\dfrac{1}{2}\right)\Gamma(2n)=2^{2n-1}\Gamma(n)\Gamma\left(n+\dfrac{1}{2}\right)$. 5.67

(b) Show that (i) $\Gamma\left(\tfrac{1}{2}\right)=\sqrt{\pi}$, (ii) $\displaystyle\int_0^{\infty}e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}$. 6.00

Q.8(a) Find the area bounded by the curve $y^2=x^3$ and the line $y=x$. 3.67

(b) Find the perimeter of the curve, $r=a(1+\operatorname{Cos}\theta)$. 4.00

(c) Find the volume of the solid generated by revolving the cycloid, $x=a(\theta+\operatorname{Sin}\theta)$, $y=a(1+\operatorname{Cos}\theta)$ about its base. 4.00

B.Sc. Engineering First Year First Semester Examination, 2012

Math 161

Mathematics - I

Full Marks: 70 Time: 3 Hours

N.B.:
(i) Answer SIX questions, taking THREE from each section.
(ii) Figure in the margin indicates full marks.
(iii) Use separate answer script for each section.
(iv) Assume reasonable value for any data not given.

SECTION-A

59 Q.1(a) Define continuity and differentiability of a function. Show that every differentiable function is continuous, but the converse is not always true. 6.00

(b) Find the differential co-efficient of $y = x^{x^{x^{\cdots}}}$. 3.00

126 (c) If $y = x^{2n}$, where n is a positive integer, show that $y_n = 2^n\{1.3.5\ldots\ldots(2n-1)\}x^n$. 2.67

Q.2(a) State and prove mean value theorem. Also write down its geometrical interpretation. 5.67

58 (b) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!}f''(\theta h)$, $0<\theta<1$, find θ when $h = 7$ and $f(x) = \frac{1}{(1+x)}$. 3.00

203 (c) Evaluate $\lim \left[\frac{(e^x-1)\tan^2 x}{x^3}\right]$ 3.00

Q.3(a) Given that $\frac{x}{2}+\frac{y}{3}=1$, find the ... 4.00

231 (b) If $v = f(u)$, u being a homogeneous function of degree n, show that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=nu\frac{dv}{du}$ 3.67

(c) If $u = f(x^2+y^2+z^2, \ldots)$, prove that $(y-z)\frac{\partial u}{\partial x}+(z-x)\frac{\partial u}{\partial y}+(x-y)\frac{\partial u}{\partial z}=0$. 4.00

Q.4(a) Prove that the curve $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{a'}+\frac{y^2}{b'}=1$ will cut orthogonally if $a-b=a'-b'$. 6.00

(b) Show that each of radius of curvature of $x^3+y^3=3axy$ at origin is $\frac{3a}{2}$. 5.67

SECTION-B

Q.5 Integrate the following integrals – (any three) 11.67

i) $\int\frac{dx}{(1+x)\sqrt{1-x^2}}$

ii) $\int e^x\frac{x^2+1}{(x+1)^2}\,dx$

iii) $\int\frac{dx}{\sin x+\cos x}$

iv) $\int\log(\sin x)\,dx$

Q.6(a) Show that $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}\,dx = \pi\left(\frac{\pi}{2}-1\right)$ 4.00

(b) If $U_n = \int_0^{\pi/2}\theta\sin^n\theta\,d\theta$ and $n>1$, then prove that $U_n = \frac{n-1}{n}U_{n-2}+\frac{1}{n^2}$. 4.00

(c) Evaluate $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots\ldots\ldots\ldots+\frac{1}{3n}\right]$. 3.67

Q.7(a) Define gamma and beta function. Establish the relation between them. 6.00

(b) Show that $\int_0^\infty e^{-x^4}x^2\,dx\times\int_0^\infty e^{-x^4}\,dx = \frac{\pi}{8\sqrt{2}}$ 5.67

Q.8(a) Evaluate $\iint_A (x^2+y^2)\,dx\,dy$, where A is bounded by $y=x^2$, $x=2$, $y=1$ in the xy plane. 4.00

(b) Find the area bounded by the curve $a^2y^2 = x^3(2a-x)$. 4.00

(c) Find the perimeter of the curve $r = a(1+\cos\theta)$. 3.67

Math 101
Mathematics – I

Full marks: 70 **Time: 3 Hours**

N.B:-
(i) Answer any SIX questions, taking THREE from each section.
(ii) Figure in the margin indicate full marks.
(iii) Use separate answer script for each section.
(iv) Assume reasonable value for any data missing.

SECTION-A

Q.1(a) Find differential coefficient of the following: 6.00

(i) $(\sin x)^{\cos x} + (\cos x)^{\sin x}$ (ii) $\tan^{-1}\left\{\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right\}$

(b) A man 5 ft tall walks away from a lamp post 12.50 ft high at the rate of 3 miles per hour. (i) How fast is the further end of his shadow moving on the pavement? (ii) How fast is his shadow lengthening? 5.67

Q.2(a) If $y = e^x \sin x \sin 2x$, find y_n. 3.67

(b) $y = e^{a\sin^{-1}x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ 4.00

(c) Prove that $(x+h)^{\frac{1}{2}} = x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}h + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{h^2}{2!}\frac{1}{\sqrt{x+\theta h}}$, when $0<\theta<1$ and hence find θ when $x=0$. 4.00

Q.3(a) Find the minimum value of $4e^{2x}+9e^{-2x}$. 3.67

(b) Evaluate: $\lim_{x\to 0}(\tan x/x)^{1/x}$. 4.00

(c) If $u=\sin^{-1}(x/y)+\tan^{-1}(y/x)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ 4.00

Q.4(a) Define radius of curvature. Find the radius of curvature for the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point where $y=x$ cuts it. 4.00

(b) If $u=\log(x^3+y^3+z^3-3xyz)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$ 3.67

(c) Find the tangent and the normal to the curve $xy^2=4(4-x)$ at the point where it is cut by the line $y=x$. 4.00

SECTION-B

Q.5 Integrate the following integrals (any three) 11.67

(i) $\int_\alpha^\beta\sqrt{(x-\alpha)(\beta-x)}\,dx$ (ii) $\int e^x\frac{x^2+1}{(x+1)^2}dx$ (iii) $\int_0^1 x^6\sqrt{1-x^2}\,dx$ (iv) $\int\frac{dx}{(1+x)\sqrt{1+2x-x^2}}$

Q.6(a) Evaluate $\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n-1^2}}+\frac{1}{\sqrt{4n-2^2}}+\cdots\cdots+\frac{1}{n}\right]$ 4.00

(b) Evaluate $\int\frac{x\tan x}{\sec x+\tan x}dx$ 4.00

(c) Obtain a reduction formula for $\int\sec^n x\,dx$. 3.67

Q.7(a) Show that $\int_0^\infty e^{-x^4}x^2dx\times\int_0^\infty e^{-x^4}dx=\frac{\pi}{8\sqrt{2}}$ 4.00

(b) Show that $\beta(m,1-m)=\int_0^\infty\frac{x^{m-1}}{1+x}dx$ 4.00

(c) Show that $\Gamma\left(\frac{2n+1}{2}\right)=\frac{1.3.5\cdots\cdots(2n-1)}{2^n}\sqrt{\pi}$ 3.67

Q.8(a) Find the area common to the cardioide $r=a(1+\cos\theta)$ and the circle $r=3a/2$, and also area of the remainder of the cardioide. 5.67

(b) Find the volume and the surface area of the solid generated by revolving the cycloid $x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ about its base. 6.00

DEPARTMENT OF CIVIL ENGINEERING
RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY
B.Sc. Engineering FIRST Year FIRST Semester Examination, 2010

MATH-101
Mathematics - I

5

Full Marks: 70 Time: 3 hours

N.B:
(i) Answer Six questions taking Three from each Section
(ii) Figures in the margin indicate full marks
(iii) Use separate answer script for each section

SECTION-A

Q.1(a) Does the following limit exist? $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$? 5.67

(b) Find from first principles the derivative of x^x, $(x>0)$. 6.00

Q.2(a) Find the n-th derivative of $y=\frac{1}{x^2+a^2}$ 5.67

(b) If $y=\sin(m\sin^{-1}x)$, then show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$ and expand y in ascending power of x. 6.00

Q.3(a) Find the maxima and minima of $1+2\sin x+3\cos^2 x$ $\left(0\le x\le\frac{\pi}{2}\right)$ 5.67

(b) If $V=ax^2+2hxy+by^2$, then show that $V_x^2V_{yy}-2V_xV_yV_{xy}+V_y^2V_{xx}=8(ab-h^2)V$ 6.00

Q.4(a) Evaluate $\lim_{x\to 1}\left[\frac{x}{x-1}-\frac{1}{\log x}\right]$ 3.67

(b) Find the point on the curve $y^2=x^2+3x+4$, the tangents at which pass through the origin. 4.00

(c) Find the radius of curvature at any point (r,θ) for the curve $r=a(1-\cos\theta)$. 4.00

SECTION-B

Q.5 Integrate the following integrals (any three) 11.67

i) $\int\frac{x^2+1}{x^4+x^2+1}dx$

ii) $\int\frac{dx}{(1+x)\sqrt{1+x+x^2}}$

iii) $\int_0^{\pi/2}\log(\sin x)\,dx$

iv) $\int x^3\sqrt{\frac{1+x^2}{1-x^2}}\,dx$

Q.6(a) Evaluate $\lim_{n\to\infty}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+\cdots\cdots+\frac{1}{nb}\right]$ 3.67

(b) Evaluate $\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}$ 4.00

(c) If $I_n=\int_0^{\pi/2}x\sin^n x\,dx$, $n>1$, then show that $I_n=\frac{1}{n^2}+\frac{n-1}{n}I_{n-2}$ 4.00

Q.7(a) Define Beta and Gamma function. Show that $\beta(m,n)=\frac{m!n!}{(m+n)!}$

(b) Evaluate (i) $\int_0^{\infty}\left(\frac{\sin x}{x}\right)^3dx$ (ii) $\int_0^1\frac{\log x}{1+x}dx$ 6.00

Q.8(a) Find the area of a loop of the curve $r^2=a^2\cos 2\theta$. 3.67

(b) Find the perimeter of the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$ 4.00

(c) Show that the volume of a right circular cone of height h base of radius a is $\frac{1}{3}\pi a^2h$. 4.00

$x=\alpha\cos^2\theta+\beta\sin^2\theta$

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DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

B. Sc. Engineering First Year First Semester Examination, 2009

Math 101

Mathematics -I

Full Marks: 70 **Time: 3 Hours**

N.B. : (i) Answer SIX questions, taking THREE from each section
(ii) Figures in the margin indicate the full marks
(iii) Use separate answer script for each section.
(iv) Assume reasonable value for any data missing.

SECTION -A

Q.1.(a) Define limit and continuity of a function. Show that the function 5.67

$f(x) = 3 + 2x$ for $-\frac{3}{2} < x \le 0$

$f(x) = 3 - 2x$ for $0 < x < \frac{3}{2}$

is continuous at $x = 0$; but its derivate does not exist at this point.

(b) Find $\frac{dy}{dx}$ if $(\cos x)^y = (\sin y)^x$ 3.00

(c) If $f(x) = \left(\frac{a+x}{b+x}\right)^{a+b+2x}$, then show that $f'(0) = \left(2\log\frac{a}{b} + \frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$. 3.00

Q.2.(a) Find n-th derivative of $y = \frac{1}{x^2+a^2}$, when $n = 1, 2, \ldots$ 5.67

(b) Expand $(\sin^{-1}x)^2$ in a series of ascending powers of x. 6.00

Q.3.(a) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. 3.00

(b) State and prove Euler's theorem for the homogeneous functions. 3.00

(c) If $u = x\varphi(y/x) + \psi(y/x)$, show that (i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\varphi(y/x)$, (ii) $x^2u_{xx} + 2xy\,u_{xy} + y^2u_{yy} = 0$ 5.67

Q.4.(a) Find that the maximum or minimum values of $\left(\frac{1}{x}\right)^x$. 4.00

(b) Prove that the curves $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{a'} + \frac{y^2}{b'} = 1$ cut orthogonally if $a-b = a'-b'$. 4.00

(c) Find the radius of curvature of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (i) $(a, 0)$ and (ii) $(0, b)$ 3.67

SECTION-B

Q.5. Integrate (i) $\int \frac{xe^x}{(1+x)^2}dx$ (ii) $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\,dx$ (iii) $\int_0^1 x^2(1-x)^{\frac{3}{2}}dx$ 11.67

Q.6.(a) Evaluate, $\lim_{n\to\infty}\left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \cdots + \frac{1}{8n}\right]$ 3.67

(b) Integrate (i) $\int_0^{\pi/2}\log\sin x\,dx$, (ii) $\int_0^\infty \frac{\sin bx}{x}dx$ 8.00

Q.7 (a) Obtain the reduction formula for $\int \sec^n x\,dx$, $n = 1, 2, 3\ldots$ 5.67

Hence find the value of $\int \sec^n x\,dx$

(b) Prove that $\Gamma n\,\Gamma(n+\frac{1}{2}) = 2^{1-2n}\sqrt{\pi}\,\Gamma 2n$ 6.00

Q.8.(a) Find the common area to the cardioide $r = a(1+\cos\theta)$ and the circle $r = \frac{3}{2}a$, and also the area of the remainder of the cardioide. 5.67

(b) Find the volume and the surface-area of the solid generated by revolving the cardioide 6.00