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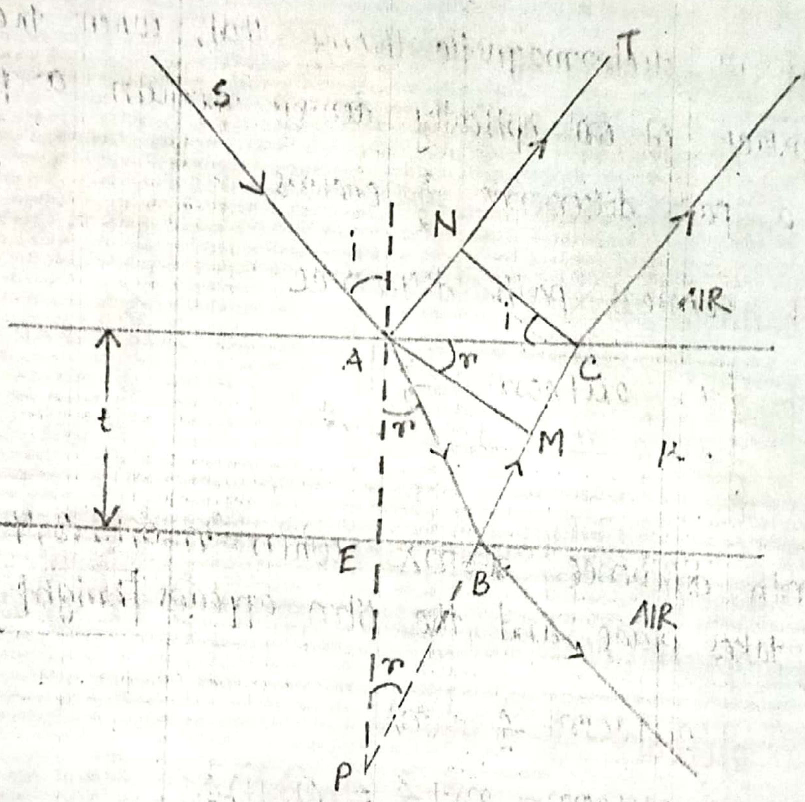
Interference due to reflected light (thin film)

Consider a transparent film of thickness t and refractive index μ. A ray SA incident on the upper surface of the film is partly reflected along AT and partly refracted along AB. At B part of it is reflected along BC and finally emerges out along CQ. The difference in path between the two rays AT and CQ can be calculated. CN is normal to AT and AM normal to BC. The angle of incidence is i and the angle of refraction is r. CB and AE meet at point P.



Here $\angle APC = r$

The optical path difference, $x = \mu(AB+BC) - AN$ —(1)

Here, $\mu = \frac{\sin i}{\sin r} = \frac{AN}{CM}$

$AN = \mu . CM$

From (1)

$x = \mu(AB+BC) - \mu . CM$

$x = \mu(AB+BC-CM)$

$= \mu(PC - CM)$

$x = \mu . PM$

$\boxed{\begin{array}{l} AB = BP \\ BP+BC = PC \end{array}}$

In the ΔAPM

$$\cos r = \frac{PM}{AP}$$

$$PM = AP\cos r = (AE + EP)\cos r \qquad \boxed{AE = EP = t}$$

$$= 2t\cos r$$

$$\boxed{x = \mu . PM = 2\mu t\cos r}$$

On the basis of electromagnetic theory that, when the light is reflected from the surface of an optically denser medium a phase change π equivalent to a path difference $\frac{\lambda}{2}$ occurs.

Therefore the correct path difference

$$\boxed{x = 2\mu t\cos r - \frac{\lambda}{2}}$$

① If the path difference $x = n\lambda$ when $n = 0,1,2,3,4$ etc., constructive interference takes place and the film appears **bright.**

$$2\mu t\cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t\cos r = n\lambda + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore \boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$$

② If the path difference $x = (2n+1)\frac{\lambda}{2}$ where $n = 0,1,2 \cdots$ etc, destructive interference takes place and the film appears **dark.**

$$2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t\cos r = (n+1)\lambda$$

Here n is an integer only, therefore $(n+1)$ can also be taken as n

$$\therefore \boxed{2\mu t\cos r = n\lambda}$$

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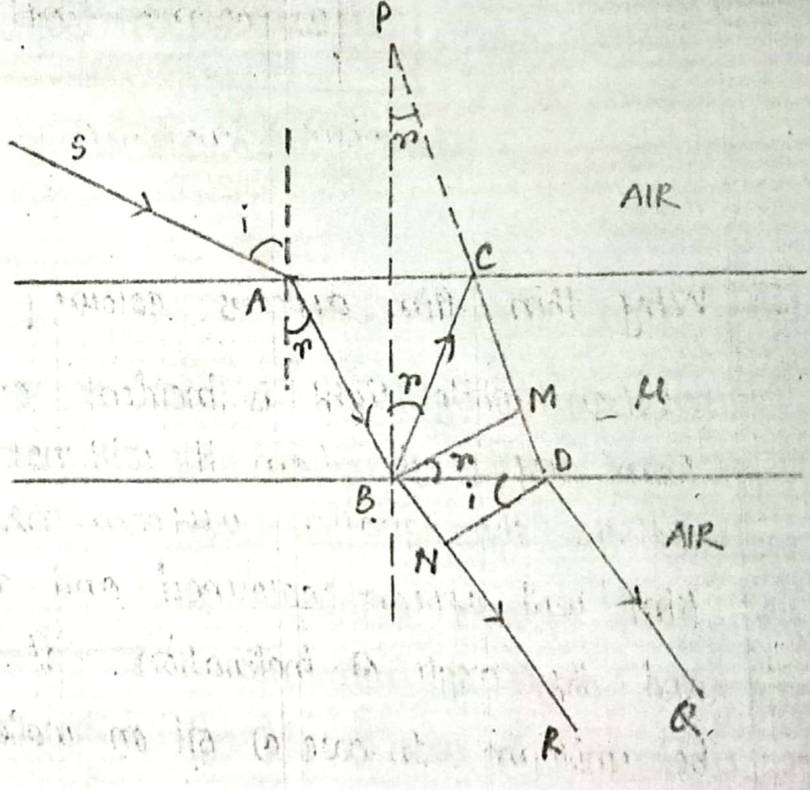
Interference due to transmitted light

Consider a thin transparent film of thickness t and refractive index μ. A ray SA after refraction goes along AB. At B it is partly reflected along BC and partly refracted along BR. The ray BC after reflection at C, finally emerges along DQ. Here at B and C reflection takes place at the rarer medium. Therefore no phase change occurs. BM is normal to CD and DN is normal to BR



The optical path difference between DQ and BR is given by,

$$x = \mu(BC+CD) - BN$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BN}{MD}$$

$$BN = \mu \cdot MD$$

$\angle BPC = r$ and $CP = BC = CD$

$$BC + CD = PD$$

$$x = \mu(PD) - \mu(MD)$$

$$= \mu(PD - MD)$$

$$= \mu \cdot PM$$

In the $\triangle BPM$, $\cos r = \frac{PM}{BP}$

$$PM = BP\cos r$$

But $BP = 2t$

$$\therefore PM = 2t\cos r$$

$$\therefore \boxed{x = \mu \cdot PM = 2\mu t\cos r}$$

(i) for bright fringes, the path difference $x = n\lambda$,

$$\boxed{2\mu t \cos r = n\lambda}$$

where, $n = 0, 1, 2, 3 \cdots$ etc.

(ii) for dark fringes, the path difference $x = (2n+1)\frac{\lambda}{2}$

$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$$

where, $n = 0, 1, 2, 3$ etc.

Why thin film appears colour?

When white light is incident on a thin film, the light which comes from any point from it will not include the colour whose wavelength satisfies the equation $2\mu t \cos r = n\lambda$, in the reflected system. Therefore the film will appear coloured and the colour will depend upon the thickness and the angle of inclination. If r and t are constant, the colour will be uniform. In case of oil on water, different colours are seen because r and t vary.

for same condition,

$$\boxed{2\mu t \cos r = n\lambda},$$

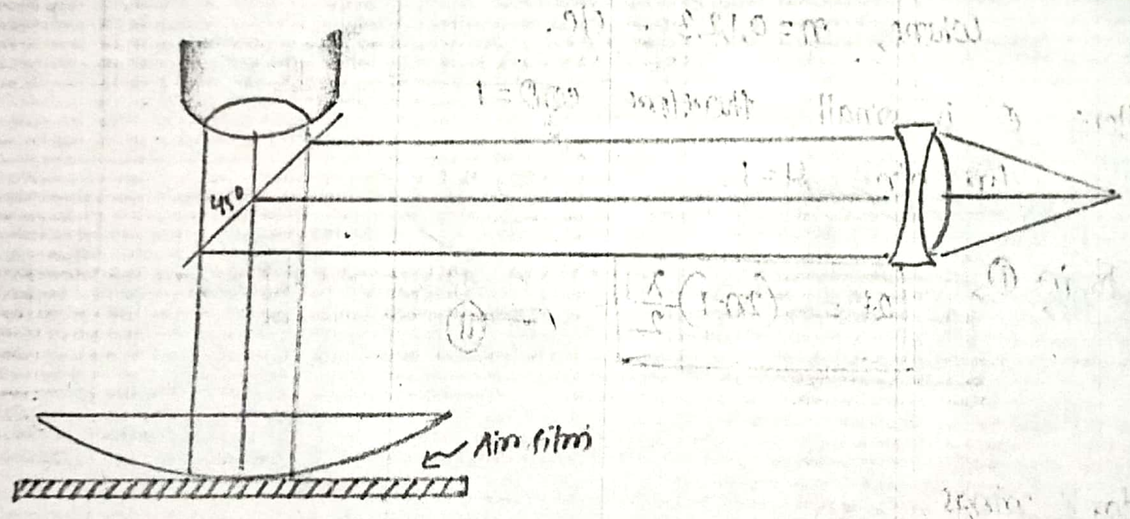
for reflected system, the film appears $\boxed{\text{dark}}$

for transmitted system, the film appears $\boxed{\text{bright}}$

...ton's ring:

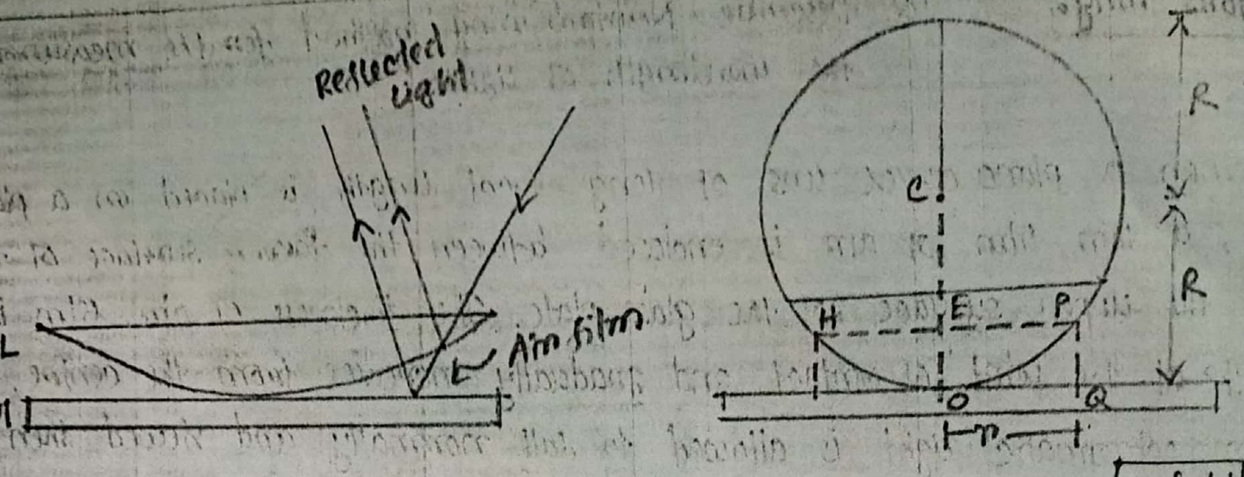
Q. Describe Newton's ring method for the measurement of the wavelength of light.

When a plano convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the glass plate. The thickness of air film is very small at the point of contact and gradually increases from the centre outwards. If monochromatic light is allowed to fall normally and viewed then alternate dark and bright circular fringes are observed. The fringes are circular because the air film has a circular symmetry. Newton's rings are formed because of the interference between waves reflected from the top and bottom surfaces of the air film formed between the plates.



Newton's ring by reflected light

Suppose the radius of the curvature of the lens is R and the air film is of thickness t at a distance $OQ = r$, from the point of contact O.



Here, interference is due to reflected light. Therefore, for the $\boxed{\text{bright}}$ rings,

$\boxed{\text{condition}}$

$$2\mu t\cos\theta + \frac{\lambda}{2} = n\lambda$$

$$2\mu t\cos\theta = (2n-1)\frac{\lambda}{2} \quad \text{—(I)}$$

where, $n = 0,1,2,3 \ldots$ etc.

Here θ is small therefore $\cos\theta = 1$

for air, $\mu = 1$.

from (I),

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad \text{—(II)}$$

for the dark rings,

$$2\mu t\cos\theta + \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$\boxed{2t = n\lambda} \quad \text{—(III)}$$

where $n = 0,1,2,3 \ldots$ etc

From figure,

$$\boxed{EP \times HE = OE\,(2R - OE)} \quad \text{—(IV)}$$

$EP = HE = r \qquad OE = PQ = t$

and

from (IV), $\quad r \times r = t\,(2R - t)$

$$\boxed{r^2 = 2Rt - t^2}$$

$r^2 = 2Rt \qquad R \gg t$

$$\boxed{t = \frac{r^2}{2R}}$$

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Substituting the value of t in equation (ii) and (iii)

For bright rings

$$2t = (2n-1)\lambda/2$$

$$2 \times \frac{r^2}{2R} = (2n-1)\lambda/2$$

$$r^2 = \frac{(2n-1)\lambda R}{2} \quad \text{--- (v)}$$

$$\boxed{r = \sqrt{\frac{(2n-1)\lambda R}{2}}} \quad *$$

In equation (v), replacing r by $\frac{D}{2}$, where D is the diameter.

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$

$$D_n^2 = 2\lambda R(2n-1)$$

$$\boxed{D_n = \sqrt{2\lambda R}\sqrt{2n-1}}$$

For dark rings

$$2t = n\lambda$$

$$2 \times \frac{r^2}{2R} = n\lambda$$

$$r^2 = n\lambda R \quad \text{--- (vi)}$$

$$\boxed{r = \sqrt{n\lambda R}} \quad *$$

From (vi), $\left(\frac{D_n}{2}\right)^2 = n\lambda R$

$$D_n^2 = 4n\lambda R$$

$$\boxed{D_n = 2\sqrt{n\lambda R}}$$

Why central Ring Dark

* When $n=0$, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore the centre ring is dark.

Thus the diameter of the ring is square root of the natural numbers.

By measuring the diameter of the Newton's rings, it is possible to calculate $\boxed{\text{wavelength}}$ of light.

We have the diameter of the nth dark fringe/ring

$$D_n^2 = 4n\lambda R$$

Similarly diameter for the $(n+p)$th dark fringe

$$D_{n+p}^2 = 4(n+p)\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

Newton's ring by transmitted light,

In case of transmitted light, the interference fringes are produced such that for bright rings,

$$2\mu t\cos\theta = n\lambda$$

and for dark rings,

$$2\mu t\cos\theta = (2n-1)\lambda/2$$

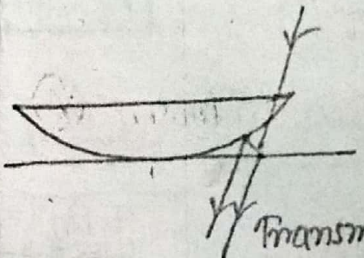
for air, $\mu = 1$,

and $\cos\theta = 1$

$\therefore$ for bright rings, $\boxed{2t = n\lambda}$

and for dark rings, $\boxed{2t = (2n-1)\lambda/2}$

due to transmitted light,
For $n=0$, $r=0$ for bright rings
The central ring is $\boxed{\text{bright}}$



$D_n^2 =$

Problem

...ple: 8.29: A parallel beam of light ($\lambda = 5890 \times 10^{-8}$ cm) is incident on a thin glass ...ate ($\mu = 1.5$). such that the angle of refraction into the plate is $60°$. calculate the smallest thickness of the glass plate which will appear dark by reflection.

Solution

$$2\mu t \cos r = n\lambda$$

$$t = \frac{n\lambda}{2\mu \cos r}$$

$$= \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$= \boxed{3.926 \times 10^{-5} \text{ cm}}$$

Here,
$\mu = 1.5$
$r = 60°$
$\cos 60° = 0.5$
$\boxed{n = 1}$
$\lambda = 5890 \times 10^{-8}$ cm

For smallest thickness, $n = 1$

Example 8.30.

A soap film 5×10^{-5} cm thick is viewed at an angle of $35°$ to the normal. Find the wavelength of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$)

Solution

Let i be the angle of incidence and r the angle of fraction.

$$\mu = \frac{\sin i}{\sin r}$$

$$1.33 = \frac{\sin 35°}{\sin r}$$

Here,
$t = 5 \times 10^{-5}$ cm

$$r = 25.55°$$

$$\cos r = 0.90$$

$$\boxed{2\mu t \cos r = n\lambda}$$

① For the first order $\boxed{n = 1}$,

$$\lambda_1 = 2 \times 1.33 \cos 25.55° \times 5 \times 10^{-5} = \boxed{12 \times 10^{-5} \text{ cm}}$$

which lies in the infra-red (invisible) region.

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(iii) For the second order, $\boxed{n=2}$

$$2\times\lambda_2 = 2\times1.33\times5\times10^{-5}\cos 25.55^\circ$$

$$\lambda_2 = \boxed{6\times10^{-5}\,cm}$$

Which lies in the visible region.

(iv) Similarly, taking $\boxed{n=3}$

$$3\times\lambda_3 = 2\times1.33\times5\times10^{-5}\cos 25.55^\circ$$

$$\lambda_3 = 4\times10^{-5}\,cm$$

which also lies in the visible region.

(v) For $\boxed{n=4}$

$$4\times\lambda_4 = 2\times1.33\times5\times10^{-5}\cos 25.55^\circ$$

$$\lambda_4 = \boxed{3\times10^{-5}\,cm}$$

which lies in the ultra violet (invisible) region.

Hence absent wavelengths in the reflected light are $\boxed{6\times10^{-5}\,cm}$ and $\boxed{4\times10^{-5}\,cm}$

7. **Example 8.34**

A soap film of refractive index 1.33 is illuminated with light of different wavelengths at an angle of 45°. There is complete destructive interference for $\lambda = 5890Å$. Find the thickness of the film.

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(ii) For the second order, $\boxed{n=2}$

$$2 \times \lambda_2 = 2 \times 1.33 \times 5 \times 10^{-5} \cos 25.55^\circ$$

$$\lambda_2 = \boxed{6 \times 10^{-5}\ cm}$$

Which lies in the visible region.

(iii) Similarly, taking $\boxed{n=3}$

$$3 \times \lambda_3 = 2 \times 1.33 \times 5 \times 10^{-5} \cos 25.55^\circ$$

$$\lambda_3 = 4 \times 10^{-5}\ cm$$

which also lies in the visible region.

(iv) For $\boxed{n=4}$

$$4 \times \lambda_4 = 2 \times 1.33 \times 5 \times 10^{-5} \cos 25.55^\circ$$

$$\lambda_4 = \boxed{3 \times 10^{-5}\ cm}$$

which lies in the ultra violet (invisible) region.

Hence absent wavelengths in the reflected light are $\boxed{6 \times 10^{-5}\ cm}$ and $\boxed{4 \times 10^{-5}\ cm}$

$\boxed{\text{Example 8.34}}$

A soap film of refractive index 1.33 is illuminated with light of different wavelengths at an angle of 45°. There is complete destructive interference for $\lambda = 5890\,Å$. Find the thickness of the film.

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For destructive interference,

$$2\mu t \cos r = n\lambda$$

$$t = \frac{n\lambda}{2\mu \cos r}$$

$$= \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.33 \cos 45^\circ}$$

$$= \boxed{3.13 \times 10^{-5}\,cm}$$

Here,
$\mu = 1.33$
$r = 45^\circ$
$\lambda = 5890 Å = 5890 \times 10^{-8} cm$
$n = 1$

Why is the centre of the rings $\boxed{\text{dark}}$ and how can we get a $\boxed{\text{bright}}$ centre?

In case of Newton's ring due to reflected light the centre ring is dark. Because there is no path and phase difference in the air. The phase difference of the reflected light is 180°. Therefore the centre ring is dark. we can get a bright centre if the phase difference is zero. Due to transmitted light the central ring is bright, that is just opposite to the ring pattern due to reflected light.

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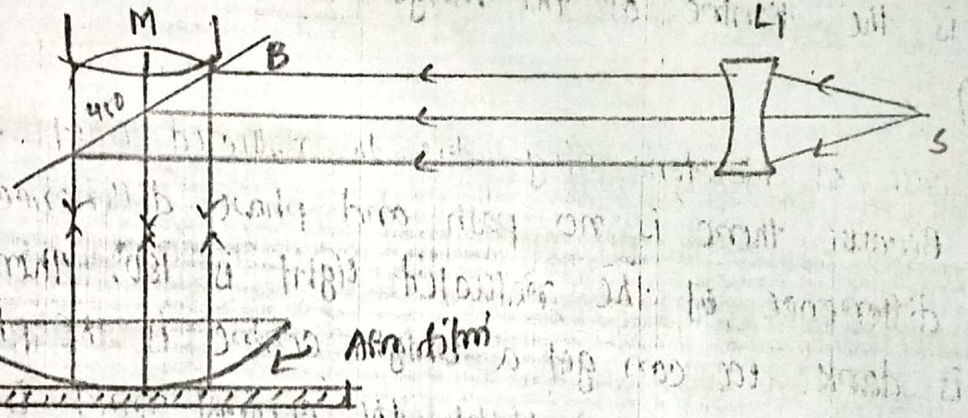
II Explain how the wavelength of light can be determined using Newton ring.

Determination of wavelength of sodium light using Newton's ring:

S is a source of sodium light. A parallel beam of light from the lens L_1 is reflected by the glass plate B inclined at an angle of 45° to the horizontal. L is a plano convex lens of large focal length. Newton's rings are viewed, through B by the travelling microscope M, focussed on the air film. With the help of a travelling microscope the diameter of the nth dark ring is measured.



Suppose, the diameter of the nth dark ring = D_n

$$r_n^2 = n\lambda R$$

But, $r_n = \frac{D_n}{2}$

$$\left(\frac{D_n}{2}\right)^2 = n\lambda R$$

$$D_n^2 = 4n\lambda R \quad \text{—(i)}$$

Measuring the diameter of (n+m) th dark ring

$$\left(\frac{D_{n+m}}{2}\right)^2 = (n+m)\lambda R$$

$$(D_{n+m})^2 = 4(n+m)\lambda R \quad \text{—(ii)}$$

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(11)

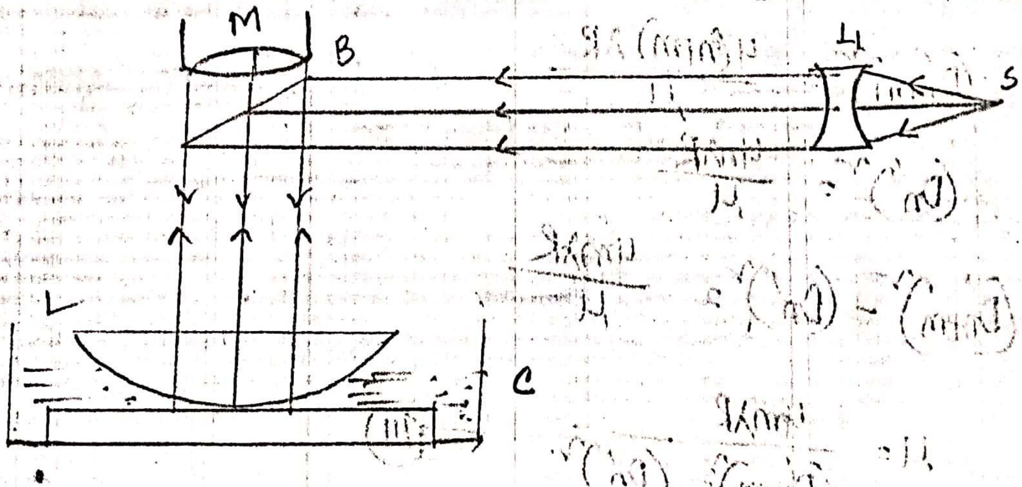
$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R$$

$$\boxed{\lambda = \frac{(D_{n+m})^2 - (D_n)^2}{4mR}}$$

λ can be calculated from above equation.

II Determination the refractive index of a liquid.

The experiment is performed, when there is an air film between the plano convex lens and the optically plane glass plate. These are kept in a metal container C. The diameter of the nth and $(n+m)$th dark rings are determined with the help of a travelling microscope.



For air,

$$(D_{n+m})^2 = 4(n+m)\lambda R$$

$$D_n^2 = 4n\lambda R$$

$$\therefore (D_{n+m})^2 - D_n^2 = 4m\lambda R \quad \text{--- (1)}$$

The liquid is poured in the container C without disturbing the arrangement. The diameters of the nth ring and $(n+m)$th rings are determined.

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for the liquid

$$\boxed{2\mu t\cos\theta = n\lambda}$$ for dark rings

$$2\mu t = n\lambda \quad \text{--- (ii)}$$

But, $t = \frac{r^2}{2R}$

from (ii)

$$2\mu \frac{r^2}{2R} = n\lambda$$

$$r^2 = \frac{n\lambda R}{\mu}$$

$$D^2 = \frac{4n\lambda R}{\mu} \quad \left[r = \frac{D}{2}\right]$$

If D_n' is the diameter of nth ring and D_{n+m}' is the diameter of the (n+m) th ring.

$$(D_{n+m}')^2 = \frac{4(n+m)\lambda R}{\mu}$$

$$(D_n')^2 = \frac{4n\lambda R}{\mu}$$

$$(D_{n+m}')^2 - (D_n')^2 = \frac{4m\lambda R}{\mu}$$

$$\mu = \frac{4m\lambda R}{(D_{n+m}')^2 - (D_n')^2} \quad \text{--- (iii)}$$

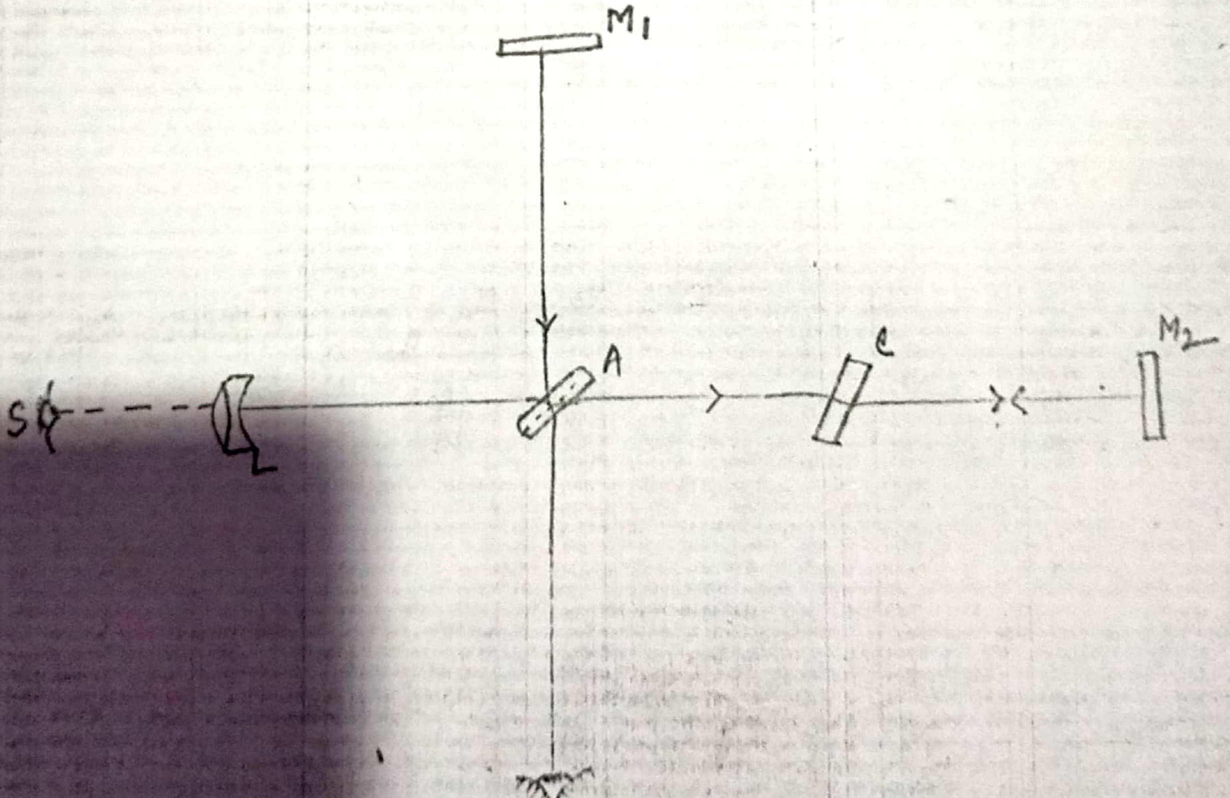
If m, λ, R, D_{n+m}' and D_n' are known μ can be calculated. If λ is not known then divide (iii) by (i)

$$\boxed{\mu = \frac{(D_{n+m})^2 - D_n^2}{(D_{n+m}')^2 - (D_n')^2}}$$

Michelson Interferometer:

[Michelson interferometer] consists of two highly polished mirrors M_1 and M_2 and two glass plates A and C parallel to each other. The rear side of the glass plate A is half [silvered] so that light coming from the source S is equally reflected and transmitted by it. Light from a monochromatic source S after passing through the lens L, falls on plate A. The lens L make the beam parallel. The plate A is inclined at an angle 45°. One half of the energy of the incident beam is [reflected] by the plate A towards the mirror M_1 and the other half is [transmitted] towards the M_2.

If the mirrors M_1 and M_2 are perfectly [perpendicular] the observer's eye will see the images of the mirrors M_1 and M_2 through A. The fringes will be perfectly [circular]. If M_1 and M_2 are [inclined], the enclosed air film will be wedge shaped and [straight line] fringes will be observed.



Applications of $\boxed{\text{michelson interferometer}}$. It can be used:

(I) To determine the wavelength of a given monochromatic source of light

(II) To determine the difference between the two neighbouring wavelengths.

(III) To determine the refractive index and thickness of various thin transparent materials.