Arif ce17

Grating: Equally spaced more than one slit arranged is called grating.

Two types of grating
- Transmitted grating
- Reflection grating

Dispersive power of grating: The dispersive power of the grating is defined as the change in the angle of diffraction corresponding in the wavelength of light. The diffraction of the nth order principal maximum for a wavelength λ, is given by the equation,

$$(a+b)\sin\theta = n\lambda$$

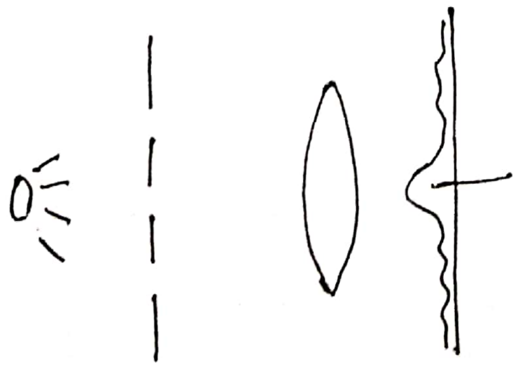
Differentiating this equation w.r.to θ and λ, where $(a+b)$ and n is constant

$$(a+b)\cos\theta\, d\theta = n\, d\lambda$$

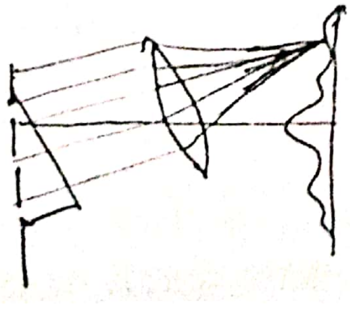
$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

$$\frac{d\theta}{d\lambda} = \frac{nN'}{\cos\theta}$$

$\frac{d\theta}{d\lambda}$ is the **dispersive power**, n is the order of the spectrum, θ is the angle of diffraction, N' is the number of lines per cm



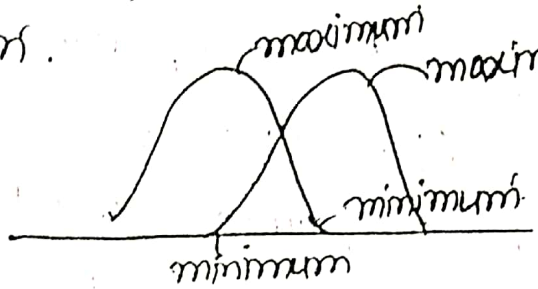
central maximum
white হবে
slit থেকে convex lens এর ray এর distance সমান। সুতরাং Path difference **zero**
তাই colour white.



Path difference changable
তাই coloured হবে।

Resolving power: The ability of an optical instrument, expressed in numerical measure, to resolve the images of two nearby points is termed as its resolving power.

Ray leighs limit of Resulation: The two objects may be regarded as separated or resolved if the central maxima of one falls upon the first subsidiary maximum of the other. This is called Ray leighs limiting of resulation.



Problem
(Grating)

Problem 9.21 Light is incident normally on a grating 0.5 cm wide with 2500 lines. Find the angles of diffraction for the principal maxima of the two sodium lines in the first order spectrum. Are the two lines resolved?
$\lambda_1 = 5890\,\text{Å}$ and $\lambda_2 = 5896\,\text{Å}$

Solution

width = 0.5 cm
Number of line = 2500
Number of line per cm = $\frac{2500}{0.5} = 5000$ lines/cm

Grating element $(a+b) = \frac{1}{5000}$ cm

$$(a+b)\sin\theta_n = n\lambda$$

for, $\lambda_1 = 5890\,\text{Å} = 5890 \times 10^{-8}$ cm,

$n = 1$.

$$(a+b)\sin\theta_1 = 1 \times \lambda_1$$

$$\frac{1}{5000}\sin\theta_1 = 5890\times10^{-8}$$

$$\theta_1 = \boxed{17.1^\circ}$$

for $\lambda_2 = 5896\times10^{-8}$ cm, $n = 1$

$$(a+b)\sin\theta_2 = 1\times\lambda_2$$

$$\frac{1}{5000}\sin\theta_2 = 1\times5896\times10^{-8}$$

$$\theta_2 = \boxed{17.2^\circ}$$

The condition for just resalation is

$\lambda = 5890\times10^{-8}$ cm
$d\lambda = 6\times10^{-8}$ cm
$n = 1$
$N = ?$

$$\frac{\lambda}{d\lambda} = nN$$

$$\frac{5890\times10^{-8}}{6\times10^{-8}} = 1\times N$$

$$N = 982$$

As the total number of lines on the grating is 2500, the two lines will appear well resolved.

Scanned by CamScanner

Problem: 9.22 A parallel beam of monochromatic light is allowed to be incident normally on a plane grating having 1250 lines per cm and a second order spectral line is observed to be deviated through 30°. Calculate the wavelength of the spectral line.

Solution

$(a+b)\sin\theta_n = n\lambda$

$a+b = \frac{1}{1250}$ $\theta = 30°$, $\sin 30° = \frac{1}{2}$, $n=2$

$(a+b)\sin\theta = n\lambda$

$$\lambda = \frac{(a+b)\sin\theta}{n} = \frac{1\times\sin 30°}{1250\times 2}$$

$$= \frac{1}{1250\times2\times2}.$$

$$= 2\times10^{-4}\,cm$$

9.23 What is the highest order spectrum, which may be seen with monochromatic light of wavelength 6000Å by means of a diffraction grating with 5000 lines/cm?

Solution For the maximum possible values of

$\sin\theta_n = 1$

$(a+b)\sin\theta_n = n\lambda$ $a+b = \frac{1}{5000}\,cm$

$(a+b) = n\lambda$ $\lambda = 6000\times10^{-8}\,cm$

$$n = \frac{a+b}{\lambda} = \frac{1\times10^{8}}{5000\times6000}$$

$= 3.33$

... can be seen is ...

A plane grating has 15000 lines per inch. Find the angle of separation of the 5048 Å and 5016 Å lines of helium in the second order spectrum.

Solution

$\lambda_1 = 5048 \times 10^{-8}$ cm

$\lambda_2 = 5016 \times 10^{-8}$ cm

$n = 2$

$$(a+b) = \frac{1}{\frac{15000}{2.54}} = \frac{2.54}{15000} \text{ cm} = 1.69 \times 10^{-4} \text{ cm}$$

Let θ_1 and θ_2 be the angle of diffraction for the second order for the wavelengths λ_1 and λ_2 respectively.

$$(a+b)\sin\theta_2 = 2\lambda_2$$

$$\sin\theta_2 = \frac{2\lambda_2}{a+b} = \frac{2 \times 5016 \times 10^{-8}}{1.69 \times 10^{-4}} = 0.593$$

$$\theta_2 = 36^\circ 24'$$

$$\sin\theta_1 = \frac{2\lambda_1}{a+b} = \frac{2 \times 5048 \times 10^{-8}}{1.69 \times 10^{-4}} = 0.597$$

$$\theta_1 = 36^\circ 41'$$

Angle of separation $= \theta_1 - \theta_2 = 36^\circ 41' - 36^\circ 24'$

$= 17'$

9.25 same of 9.24

light of wavelength 5000 Å is incident normally on a plane transmission grating. Find the difference in the angle of deviation in the first and third order spectra. The number of lines per cm on the grating surface is 6000.

Solution

$\lambda = 5000 \times 10^{-8}$ cm.

$(a+b) = \frac{1}{6000}$

For first order, $n = 1$

$(a+b) \sin\theta_1 = 1 \times \lambda$

$\sin\theta_1 = \frac{5000 \times 10^{-8}}{\frac{1}{6000}} = 0.3$

$\theta_1 = 17.45^\circ$

For 3rd order, $n = 3$.

$(a+b) \sin\theta_3 = 3 \times \lambda$

$\sin\theta_3 = \frac{3 \times 5000 \times 10^{-8}}{\frac{1}{6000}} = 0.9$

$\theta_3 = 64.15^\circ$

$\theta_3 - \theta_1 = 64.15^\circ - 17.45^\circ = 46^\circ 42'$

How many orders will be visible if the wavelength of the incident radiation is 5000 Å and the number of lines on the grating is 2620 in one inch?

Solution

The maximum possible values of

$\sin\theta_n = 1$

$(a+b) = n\lambda$

$\lambda = 5000 \times 10^{-8}$ cm

$a+b = \frac{1}{\frac{2620}{2.54}} = \frac{2.54}{2620}$ cm

$n = \frac{a+b}{\lambda}$

$= \frac{2.54 \times 10^{8}}{2620 \times 5000}$

$n = 19$

The highest order of the spectrum that can be seen is 19.

Q30 A parallel beam of monochromatic light is allowed to be incident normally on a plane transmission grating having 5000 lines/cm and the second order spectral line is found to be diffracted through 30°. Calculate the wavelength of light.

Solution

$(a+b)\sin\theta = n\lambda$

$(a+b) = \frac{1}{5000}$ cm

$\theta = 30^\circ, n = 2$

$\lambda = \frac{(a+b)\sin\theta}{n}$

$= \frac{1 \times \sin 30^\circ}{2 \times 5000} = 5 \times 10^{-5}$ cm $= 5000$ Å