

# 0 of Thermodynamics

'09, '07, '05, '02

## 0) Zeroth law of Thermodynamics

1st class test

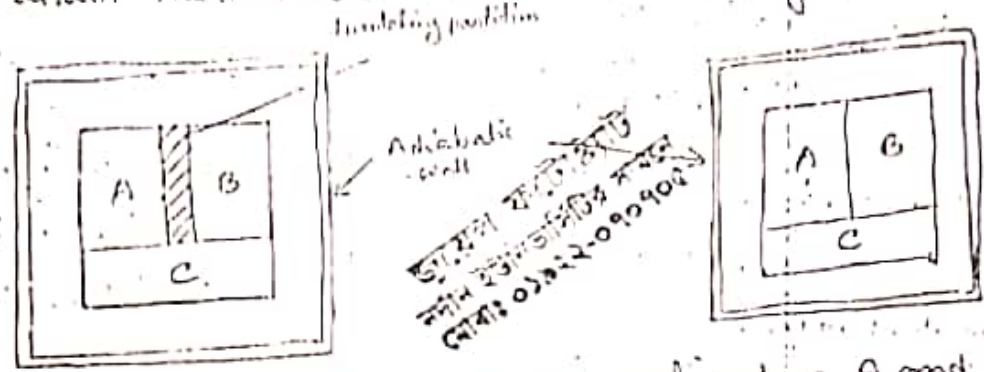
10, 10, 10

separately

If two systems in thermal equilibrium

with a third one are also in thermal equilibrium with each other.

Explanation: We consider two systems A and B insulated from each other but in good thermal contact with a common system C. Systems A and B will attain thermal equilibrium with third system C.



Now if the insulating partition is removed and systems A and B are brought into thermal contact, we find that there is no further change. This means that the systems A and B are also in thermal equilibrium with each other. This is known as zeroth law of thermodynamics.

1) 1st law of thermodynamics - states that the amount of heat given to a system is equal to the sum of the increase in the internal energy of the system and the external work done.

For a very small change in the state of the system, the law is expressed in the differential form as

$$\delta Q = dU + \delta W \quad \text{--- (1)}$$

where,  $\delta Q$  and  $\delta W$  are not perfect differentials but  $dU$  is a perfect differential because  $U$  is a function of the state of the system.

06/09/2016

Applications of First law of Thermodynamics

The internal energy of a system is a single valued function of the state variables viz. pressure, volume, temperature etc. In the case of gas, any two of the variables P, V, T are sufficient to define completely its state. If V and T are chosen as the independent variables,

$$U = f(V, T) \quad \text{--- (i)}$$

Differentiating equation (i)

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad \text{--- (ii)}$$

If an amount of heat  $\delta Q$  is supplied to a thermodynamical system, say an ideal gas and if the volume increases by  $dV$  at constant pressure  $P$  then according to the first law of thermodynamics

$$\delta Q = dU + \delta W \quad \text{--- (iii)}$$

Here:  $\delta W = P \cdot dV$

$$\therefore \delta Q = dU + P dV \quad \text{--- (iv)}$$

Substituting the value of  $dU$  from equation (ii)

$$\delta Q = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + P dV$$

Dividing both sides by  $dT$ .

$$\frac{\delta Q}{dT} = \left(\frac{\partial U}{\partial T}\right)_V + \left(\frac{\partial U}{\partial V}\right)_T \frac{dV}{dT} + P \frac{dV}{dT} \quad \text{--- (v)}$$

$$\text{or, } \left(\frac{\delta Q}{dT}\right) = \left(\frac{\partial U}{\partial T}\right)_V + \left[ P + \left(\frac{\partial U}{\partial V}\right)_T \right] \frac{dV}{dT} \quad \text{--- (vi)}$$

If the gas is heated at constant volume

$$\left(\frac{\delta Q}{dT}\right)_V = C_V \quad \text{and} \quad \frac{dV}{dT} = 0$$

$$\therefore \left(\frac{\delta Q}{dT}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = C_V$$

# Adiabatic Equation of a Perfect gas

2016

Consider 1 gram of the working substance perfectly insulated from the surrounding. If the external work done is  $dW$  and the change in internal energy is  $dU$ , then according to the first law of thermodynamics.

$$dQ = dU + dW$$

$$\therefore dQ = dU + PdV \quad \text{--- (i)}$$

here  $P$  is the pressure of the gas and  $dV$  is the change in Volume.  
For an adiabatic process,  $dQ = 0$

$$dU + PdV = 0 \quad \text{--- (ii)}$$

As the external work is done by the gas at the cost of its internal energy, there is fall in temperature by  $dT$ .

$$dU = C_v dT \quad \text{--- (iii)}$$

For an ideal gas,  $PV = RT$ .

Differentiating the above equation, we have,

$$PdV + VdP = RdT$$

$$\Rightarrow dT = \frac{PdV + VdP}{R} \quad \text{--- (iv)}$$

Substituting the value of  $dT$  in equation (ii)

$$C_v \left( \frac{PdV + VdP}{R} \right) + PdV = 0$$

$$\Rightarrow C_v [PdV + VdP] + RPdV = 0$$

But  $C_p - C_v = R$ , then

$$C_v [PdV + VdP] + (C_p - C_v) PdV = 0$$

$$\therefore C_v VdP + C_p PdV = 0$$

Dividing by  $C_v PV$

$$\frac{dP}{P} + \left( \frac{C_p}{C_v} \right) \frac{dV}{V} = 0$$

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Isothermal process

The system in which the system is perfectly conducting to the surroundings and temperature remains constant throughout the process is called isotherm process. During the process the temperature of the working substance remains constant. It can absorb heat from and gives heat to the surround.

$$PV = RT$$

07 Adiabatic process

The process in which the system is perfectly insulated from the surroundings it can neither give nor take heat from surroundings is called adiabatic process. All along the process there is change in temperature.

When the gas is heated at constant pressure.

$$\left(\frac{\delta Q}{dT}\right) = C_p$$

From equation (iv)

$$C_p = \left(\frac{\partial U}{\partial T}\right)_V + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \left(\frac{\partial V}{\partial T}\right)_P$$

$$\therefore C_p = C_v + \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \left(\frac{\partial V}{\partial T}\right)_P$$

$$\text{or, } C_p - C_v = \left[P + \left(\frac{\partial U}{\partial V}\right)_T\right] \left(\frac{\partial V}{\partial T}\right)_P$$

From Joules experiment, for an ideal gas on opening the stopcock no work was done and no heat transfer took place. So  $\delta Q = 0$ . Therefore  $dU = 0$ . Even though the volume changed, while the temperature is constant, there is no change in the internal energy.

$$\left(\frac{\partial U}{\partial V}\right)_T = 0$$

From the ideal gas equation

$$PV = RT$$

$$\text{or, } P \left(\frac{\partial V}{\partial T}\right)_P = R$$

$$\therefore C_p - C_v = P \left(\frac{\partial V}{\partial T}\right)_P + \left(\frac{\partial U}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

$$\text{But, } \left(\frac{\partial U}{\partial V}\right)_T = 0$$

$$\therefore C_p - C_v = P \left(\frac{\partial V}{\partial T}\right)_P$$

$$\therefore C_p - C_v = R$$

The formula is known as Mayer's relation.

Here  $C_p$ ,  $C_v$  and  $R$  are expressed in the same unit.

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Adiabatic and isothermal

an isothermal process  
 $PV = \text{constant}$

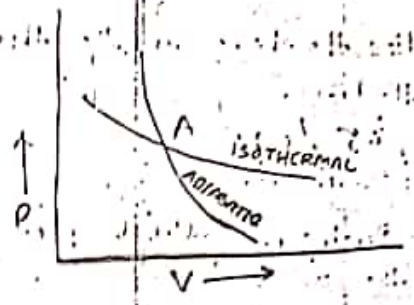
Differentiating  $\therefore P dV + V dP = 0$   
 $\therefore \frac{dP}{dV} = -\frac{P}{V}$

In an adiabatic process  
 $PV^\gamma = \text{constant}$

Differentiating  
 $P\gamma V^{\gamma-1} dV + V^\gamma dP = 0$   
 $\therefore \frac{dP}{dV} = -\gamma \frac{P}{V}$

Therefore, the slope  $(\frac{dP}{dV})$  of an adiabatic is  $\gamma$  times the slope of the isothermal.

Hence, the adiabatic curve is steeper than the isothermal curve at a point A where the two types curves intersect each other.



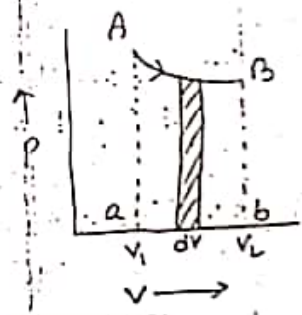
29 Work done during an Isothermal Process:

When a gas is allowed to expand isothermally work is done by it. Let the initial and final volumes be  $V_1$  and  $V_2$  respectively.

Work done,  $W = \int_{V_1}^{V_2} P dV$   
 $= \text{Area ABba}$

Considering one gram molecule of the gas

$PV = RT$   
 or,  $P = \frac{RT}{V}$   
 $\therefore W = RT \int \frac{dV}{V} = RT \ln \frac{V_2}{V_1}$   
 $\therefore W = RT \times 2.303 \log_{10} \frac{V_2}{V_1}$



$$\therefore \frac{dP}{P} + \gamma \frac{dV}{V} = 0 \quad \left[ \because \frac{C_P}{C_V} = \gamma \right]$$

Integrating both sides

$$\log P + \gamma \log V = \text{constant}$$

$$\text{or, } \log PV^\gamma = \text{constant}$$

$$\boxed{PV^\gamma = \text{constant}}$$

This is the relation between pressure and volume during an adiabatic process and is known as Poisson's Law.

Taking  $PV = RT$

$$\text{or, } P = \frac{RT}{V}$$

$$\therefore \left( \frac{RT}{V} \right) V^\gamma = \text{constant}$$

But, R is constant

$$RTV^{\gamma-1} = \text{constant}$$

$$\boxed{\therefore TV^{\gamma-1} = \text{constant}}$$

Also,  $V = \frac{RT}{P}$

$$P \left[ \frac{RT}{P} \right]^\gamma = \text{constant}$$

$$\text{or, } \frac{R^\gamma T^\gamma}{P^{\gamma-1}} = \text{constant}$$

$$\text{or, } \frac{T^\gamma}{P^{\gamma-1}} = \text{constant}$$

$$\therefore TP^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

$$\left( \frac{P_1}{T_1} \right)^{\frac{1-\gamma}{\gamma}} = \left( \frac{P_2}{T_2} \right)^{\frac{1-\gamma}{\gamma}}$$

$$T_1 P_1^{\frac{1-\gamma}{\gamma}} = (P_2 T_2)^{\frac{1-\gamma}{\gamma}}$$

$$\frac{P_1^{\gamma-1}}{T_1} = \frac{P_2^{\gamma-1}}{T_2}$$

Resultant pressure,  $P = P + p = \left( P + \frac{a}{V^2} \right)$

where  $p$  is observed pressure

### Correction for Volume

Due to the finite size of gas molecules, the actual space available for the movement of the molecules is less than the volume of the container.

Let the radius of the molecule is  $r$

then the volume of the molecule,  $x = \frac{4}{3} \pi r^3$

The centre of any two molecules can approach each other only by a minimum distance of  $2r$

The volume of sphere of influence of each molecule

$$s = \frac{4}{3} \pi (2r)^3 = 8 \times \frac{4}{3} \pi r^3 = 8x$$

Let the volume of a container be  $V$

The volume available for the first molecule =  $V$

The volume available for the second molecule =  $V - s$

The volume available for the  $n$ th molecule =  $V - (n-1)s$

The average space available for each molecule

$$= \frac{V + (V - s) + (V - 2s) + \dots + [V - (n-1)s]}{n}$$

$$= \frac{n \cdot V - \frac{s}{n} \{ 1 + 2 + 3 + \dots + (n-1) \}}{n}$$

$\sim \frac{n \cdot n}{2}$

## van der Waals Equation of State

laws of thermodynamics

While deriving the perfect gas equation  $PV = RT$  on the basis of kinetic theory, it was assumed that

- i) The size of the gas molecule is negligible.
- ii) The force of intermolecular attraction is negligible.

But in actual practice, at high pressure the size of the molecule of the gas becomes significant and can not be neglected; in comparison with the volume of the gas.

Also at high pressure, the molecules come closer and the forces of molecular attraction are appreciable.

① Correction for Pressure: A molecule in the interior of gas, experiences force of attraction in all directions, and the resultant cohesive force is zero. A molecule near the walls of the container, experience a resultant force inwards. Due to this reason that observed pressure of a gas is less than the <sup>actual</sup> ~~observed~~ pressure. The correction for pressure  $p$  depends on

- i) The number of molecules striking a unit area per unit time of the walls of the container and
- ii) The number of molecule present in a gas give volume.

$p \propto \frac{1}{V^2}$  and  $p \propto \frac{1}{V}$   
or  $p = \frac{a}{V^2}$   
Here,  $a$  is a constant and  $V$  is the volume of the gas.

Also,  $P_1 V_1 = P_2 V_2$   
 $\frac{V_2}{V_1} = \frac{P_1}{P_2}$

$\therefore W = RT \times 2.303 \log_{10} \frac{P_1}{P_2}$

Here, the change in the thermal energy of the system is zero. So, the heat com absorbed by the system is equal to the work done by it.

'09, '05, '09

7. Work done during an Adiabatic process.

during an adiabatic process, the system is thermally insulated from the surroundings.

$W = \int_{V_1}^{V_2} P dV = \text{Area ABba}$

during an adiabatic process,  $PV^\gamma = \text{constant} = K$   
 $P = \frac{K}{V^\gamma}$

$\therefore W = K \int_{V_1}^{V_2} \frac{dV}{V^\gamma}$   
 $= \frac{K}{1-\gamma} \left[ \frac{1}{V_2^{1-\gamma}} - \frac{1}{V_1^{1-\gamma}} \right]$

Since A and B lie on the same adiabatic,  $P_1 V_1^\gamma = P_2 V_2^\gamma = K$

$W = \frac{1}{1-\gamma} \left[ \frac{K}{V_2^{1-\gamma}} - \frac{K}{V_1^{1-\gamma}} \right]$   
 $= \frac{1}{1-\gamma} \left[ \frac{P_2 V_2^\gamma}{V_2^{1-\gamma}} - \frac{P_1 V_1^\gamma}{V_1^{1-\gamma}} \right]$   
 $= \frac{1}{1-\gamma} [P_2 V_2 - P_1 V_1] \dots \dots \textcircled{1}$

∴ Taking  $T_1$  and  $T_2$  as the temperature at points A and B respectively and considering 1 gm molecule of the gas.  
 $P_1 V_1 = RT_1$  and  $P_2 V_2 = RT_2$

Substituting the values in equation  $\textcircled{1}$ .

$W = \frac{1}{1-\gamma} [RT_2 - RT_1]$

$W = \frac{R}{1-\gamma} [T_2 - T_1]$

Thus, the work done along any adiabatic between two isotherms is independent of the particular adiabatic.

05

## Critical Constants: ( $T_c, P_c, V_c$ )

① Critical temperature: It is defined as the maximum temperature at which a gas can be liquefied by the increase of pressure alone. Above this temperature, the gas can not be liquefied how so ever large the applied pressure may be.

② Critical pressure: The pressure necessary to liquefy a gas at critical temperature is called the critical pressure.

③ Critical volume: It is the volume which unit mass of a gas occupies at the critical temperature and critical pressure.

④ Critical point: It is that point on the isotherm for the critical temperature for the critical temperature at which the change of state from the gaseous state to the liquid state takes place under constant values of  $P_c$  and  $V_c$ .

N.B. These three quantities  $T_c, P_c$  and  $V_c$  are called critical constants of a gas.

The critical temperature and the corresponding value of pressure and volume at the critical point are called ~~as~~ critical constants.

$$= V - \frac{S}{n} \cdot \frac{n(n-1)}{2}$$

$$= V - \frac{nS}{2} - \frac{S}{2}$$

As the number of molecules is very large  $\frac{S}{2}$  can be neglected.

∴ Average space available of each molecule

$$= V - \frac{nS}{2}$$

$$= V - \frac{n(8x)}{2}$$

$$= V - 4nx$$

$$= V - b$$

where,  $b = 4(nx) =$  four times the actual volume of molecules.

Thus the corrected gas equation is.

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where 'a' and 'b' are Vander walls constant.

This equation is called Vander walls equation.



Work done from D to A (Adiabatic process)

$$W_4 = \int_{V_4}^{V_1} P dV = \frac{K R (T_1 - T_2)}{\gamma - 1} = \text{area DFEA}$$

Total work done,  $W = W_1 + W_2 + W_3 + W_4$

$$= R T_1 \log \frac{V_2}{V_1} + \frac{K [T_1 - T_2]}{\gamma - 1} - R T_2 \log \frac{V_3}{V_4} + \frac{K [T_1 - T_2]}{\gamma - 1}$$

$$= R T_1 \log \frac{V_2}{V_1} - R T_2 \log \frac{V_3}{V_4} \quad \text{--- (1)}$$

The point A and D are on the same adiabatic.

$$T_1 V_1^{\gamma - 1} = T_2 V_4^{\gamma - 1}$$

$$\frac{T_1}{T_2} = \left( \frac{V_4}{V_1} \right)^{\gamma - 1} \quad \text{--- (ii)}$$

The point B and C are on the same adiabatic.

$$T_1 V_2^{\gamma - 1} = T_2 V_3^{\gamma - 1}$$

$$\frac{T_1}{T_2} = \left( \frac{V_3}{V_2} \right)^{\gamma - 1} \quad \text{--- (iii)}$$

From eqn (ii) and (iii)

$$\left( \frac{V_4}{V_1} \right)^{\gamma - 1} = \left( \frac{V_3}{V_2} \right)^{\gamma - 1}$$

$$\text{or } \frac{V_4}{V_1} = \frac{V_3}{V_2}$$

$$\therefore \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

Substitution in eqn (1) we get.

$$\text{Total work done } W = R T_1 \log \frac{V_2}{V_1} - R T_2 \log \frac{V_3}{V_4}$$

$$= R (T_1 - T_2) \log \frac{V_2}{V_1}$$

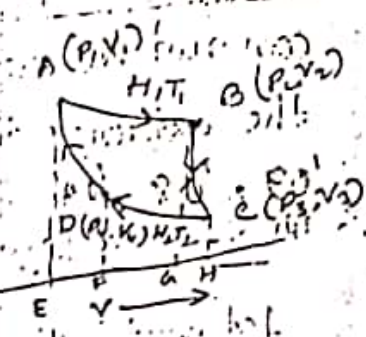
$$\therefore W = H_1 - H_2 = Q_1 - Q_2$$

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07, 06, 09, '03  
Application of Second law of Thermodynamics  
Work done by the engine per cycle.

Carnot's cycle: In order to obtain a continuous supply of work, the working substance is subjected to the following cycle of quasi-static operations known as Carnot's cycle.

(i) Place the engine containing the working substance over the source at a temperature  $T_1$ .



Work done from A to B (isothermal process)

$$W_1 = \int_{V_1}^{V_2} P dV = RT_1 \log \frac{V_2}{V_1} = \text{area } ABCGE \quad [PV = RT]$$

(ii) Work done from B to C (Adiabatic process)

$$W_2 = \int_{V_2}^{V_3} P dV = RK \int_{V_2}^{V_3} \frac{dV}{V^\gamma} = RK \left[ \frac{V^{\gamma-1}}{\gamma-1} \right]_{V_2}^{V_3} \quad [PV^\gamma = K]$$

$$= \frac{KR(T_1 - T_2)}{\gamma - 1} = \text{area } BCFG$$

(iii) Work done from C to D (isothermal process)

$$W_3 = \int_{V_3}^{V_4} P dV = RT_2 \log \frac{V_4}{V_3}$$

$$= -RT_2 \log \frac{V_3}{V_4} = \text{area } CDHF$$

## Reversible process

A process is said to be reversible if it can be retracted back exactly in the opposite direction so that the working substance passes exactly in the same state in all respect as in the case of direct process.

In a reversible process there is no loss of heat due to friction or other

Irreversible process: An irreversible process is one which can not be retracted in opposite direction by changing the external conditions infinitesimally.

## Second law of Thermodynamics:

1st part: It is impossible to get continuous supply of work from a body by cooling it to a temperature lower than that of its surroundings. (Lord Kelvin's statement).

2nd part: It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing any external work on the substance.

Planck's statement: It is impossible to construct an engine which working in a complete cycle, will produce no effect other than the raising of a weight and the cooling of a heat reservoir.

Kelvin-Planck statement: It is impossible to construct engine which operating in a cycle, has the sole effect of extracting heat from a reservoir and performing an equivalent amount of work.

Clausius Statement: It is impossible for a self-acting machine working in a cycle process, unaided by external agency, to transfer heat from a body at a lower temperature to a body at a higher temperature.

From one adiabatic to other adiabatic heat energy is either absorbed or rejected. The quantity of heat absorbed or rejected is not constant but it depends upon the higher temp. and the temp. more in the heat energy absorbed or rejected is and vice versa. The quantity  $\frac{H}{T}$  between two adiabatics is constant and this is called the change in entropy. Let the entropy for the adiabatics L and M be  $S_1$  and  $S_2$  respectively.

$$\therefore S_2 - S_1 = \frac{H}{T} = \text{constant}$$

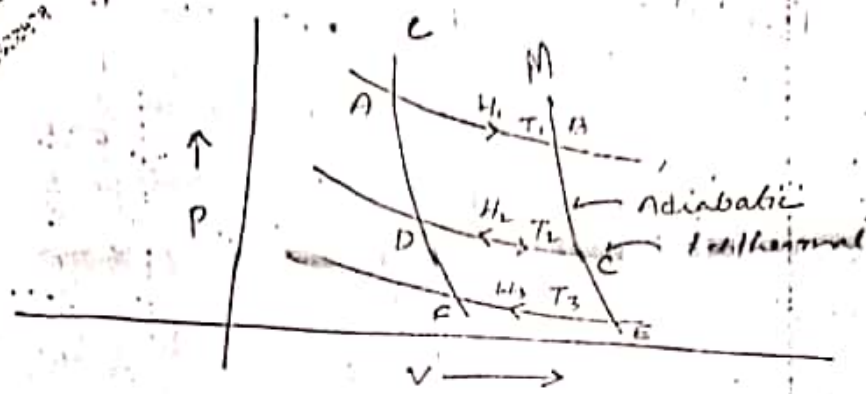
If the adiabatics are very close, and the heat absorbed or rejected is  $\delta H$  at a temp.  $T$ ,

$$\text{change in entropy, } dS = \frac{\delta H}{T} \quad \text{--- (1)}$$

$$\text{In general the change in entropy} = \int_{S_1}^{S_2} dS = S_2 - S_1 = \int_A^B \frac{\delta H}{T} \quad \text{--- (2)}$$

But along the adiabatic,  $\delta H = 0$ , Therefore, the change in entropy along an adiabatic is zero or entropy remains constant.

Entropy is a disorder of Thermodynamics system  
 as the measurement of degree of disorder of state,  
 $\therefore ds = \frac{dq}{T}$  where,  $dq$  = heat rejected by the system



Consider adiabatic L and M on the P-V diagram. All along the adiabatic L with change in pressure here is in volume and temperature. This shows that all along the adiabatics L or M. There is change of Temp ABCD represents the Carnot's reversible cycle. From A to B heat energy  $H_1$  is absorbed at Temp.  $T_1$  from C to D heat energy  $H_2$  is rejected at Temp.  $T_2$ .

$\frac{H_1}{T_1} = \frac{H_2}{T_2}$  similarly considering the cycle

DEF

$\frac{H_2}{T_2} = \frac{H_3}{T_3}$

Efficiency: The efficiency of the heat engine is the rate of quantity of heat converted into work per cycle to the total amount of heat absorbed per cycle.

$$\text{Efficiency: } \eta = \frac{\text{Useful Output}}{\text{Input}} = \frac{W}{H_1}$$

$$= \frac{H_1 - H_2}{H_1} = 1 - \frac{H_2}{H_1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Q. Find the efficiency of a Carnot's engine working between the steam point and the ice point.

Here, ice point,  $T_2 = 0^\circ\text{C} = 273\text{ K}$

steam point,  $T_1 = 100^\circ\text{C} = 373\text{ K}$

$$\eta = 1 - \frac{273}{373} = 0.2652$$

$$\therefore \eta = 26.52\%$$

## KINETIC THEORY OF GASES:

- Postulates of Kinetic theory of gases are given below
- ① The gas is composed of small indivisible particles called molecules.
  - ② The properties of the individual molecules are the same as that of the gas as a whole.
  - ③ The distance between molecules is large as compared to that of a solid or liquid and hence the force of intermolecular attraction are negligible.
  - ④ The molecules are continuously in motion with varying velocities and the molecules move in straight lines between two consecutive collisions.
  - ⑤ The size of the molecules infinitesimally small as compared to the average distance traversed by molecule between two consecutive collisions.
  - ⑥ The molecules are perfectly hard elastic sphere and the whole of their energy is kinetic.

### Expression for pressure exerted by a gas:

Brij Lal - Heat and Thermodynamics. Page - 7.

First law of thermodynamics states that the amount of heat given to a system is equal to the sum of the increase in the internal energy of the system and the external work done.

$$dQ = dU + dW$$

### Thermometry:

It is the measurement of thermometer.

# THERMODYNAMICS

## The zeroth law of thermodynamics:

In every measurement of temperature, the correctness of a law is tacitly assumed. This law, known as the zeroth law of thermodynamics, states that when any two bodies are each ~~separately~~ separately in thermal equilibrium with a third, then they are also in thermal equilibrium with each other.

## Constant volume hydrogen thermometer (Gas thermometer) (Principle)

If  $\gamma$  represents the coefficient of increase of pressure at constant volume, then  $P_{100} = P_0(1 + \gamma \cdot 100)$ .  $P_t = P_0(1 + \gamma t)$

Similarly,  $P_t = P_0(1 + \gamma t)$

$\therefore P_t - P_0 = P_0 \gamma t$  --- (ii)

Dividing (ii) by (i) we get,  $\frac{t}{100} = \frac{P_t - P_0}{P_{100} - P_0}$

$\therefore t = \frac{P_t - P_0}{P_{100} - P_0} \times 100$

It is based on the principle that when volume of gas is kept constant, the pressure increases with rising temperature.

## Resistance thermometer (Principle)

It is based on the principle of change of resistance with temperature.

$R_t = R_0(1 + \alpha t)$  and  $R_{100} = R_0(1 + \alpha \cdot 100)$

we get,  $R_t - R_0 = R_0 \alpha t$

$R_{100} - R_0 = R_0 \alpha \cdot 100$

$\therefore t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$

- ① The principle of operation
- ② The construction
- ③ The uses
- ④ The calibration

- ① The principle of operation
- ② The construction
- ③ The uses
- ④ The calibration

... ..

Advantages and disadvantages of  $H_2$  gas

thermometer:

constant volume thermometer  
can be used over a wide range of

temperature with  $H_2$  gas and  
platinum-iridium bulb. The range is  
 $-200^\circ C$  to  $500^\circ C$

By using  $N_2$  gas we can measure  
temperature up to  $1500^\circ C$ .

By using He gas we can measure  
temp. below  $-200^\circ C$

Q8, b5

Construction of platinum resistance thermometer

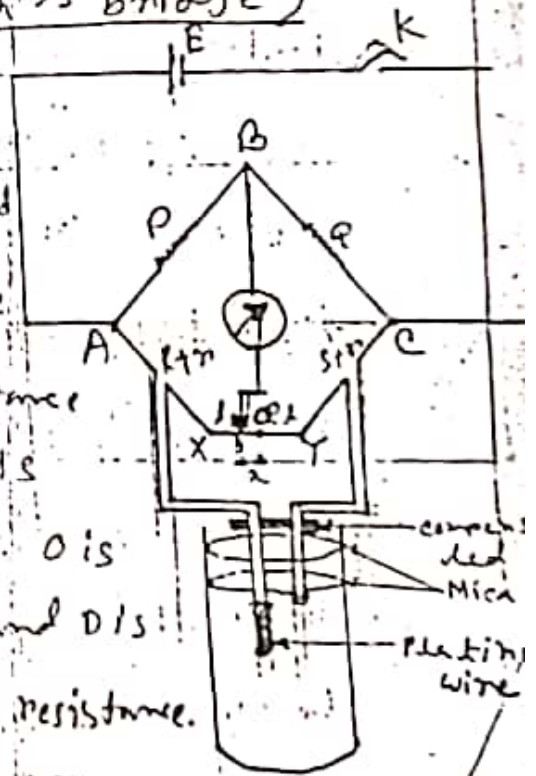
(Callender and Griffith's bridge)

There is a inductive less

coil. P and Q are the standard  
resistance, R is the resistance of the  
platinum wire, r is the resistance  
of the compensating leads, XY is

the bridge wire of length 2l. O is  
the center of the bridge wire and D is  
the null point. S is a standard resistance.

$\therefore XO = l - x$  and  $OY = l + x$



Application of the first law thermodynamics.

Thermometric property of thermometers:

- (i) design or construction;
- (ii) calibration
- (iii) sensibility

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Thermometric property of following thermometers

- (1) Liquid thermometers: Expansion or compression of volume w.r. to temperature (Mercury, Hg)
- (2) Gas thermometer: constant volume, expansion or compression of pressure w.r. to temperature (Hydrogen gas,  $H_2$ )
- (3) Resistance thermometer: change of resistance w.r. to temperature (Platinum, Pt)

Construction of  $H_2$  gas thermometer

It consists of a platinum-

iridium bulb B connected to the

tube A. Reservoir R containing

mercury (Hg) is connected to

the tubes C and A as shown

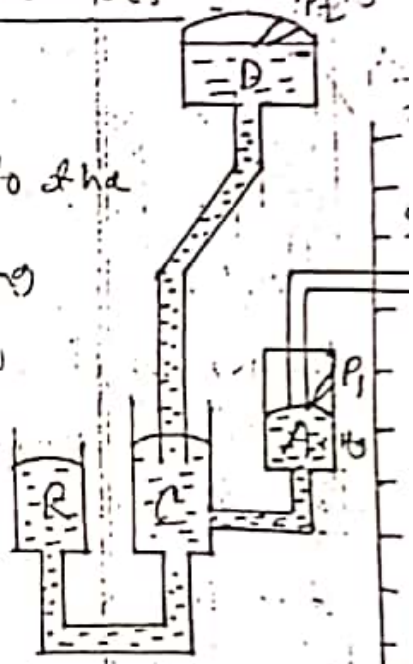
in the figure. A moveable

barometric tube with a

float is placed to a stand. It can

be moved vertically as desired.  $P_1$  and  $P_2$

are in vertical line.





∴ from eqn<sup>n</sup> (2),

$$\frac{1}{2} (mN) c^2 = \frac{3}{2} RT$$

$$\Rightarrow \frac{1}{2} m c^2 = \frac{3}{2} \left( \frac{R}{N} \right) T$$

$$\Rightarrow \frac{1}{2} m c^2 = \frac{3}{2} kT \quad [k \rightarrow \text{Boltzmann constant}]$$

$$\Rightarrow E = \frac{3}{2} kT \quad \text{--- (4)}$$

From eqn<sup>n</sup> (4), the kinetic energy of a molecule directly proportional to the absolute temperature of a gas.

Thus at absolute zero (0) temp, the kinetic energy should be zero (0). It means at absolute zero (0) temp, the molecules are in a perfect state of rest and have no kinetic energy.

### Ex. 1. Degrees of freedom:

The total number of independent variables required to describe completely the state of motion of a body are called its degrees of freedom.

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RMS velocity as a function of absolute temperature: (Energy)  $\propto T$

The pressure of a gas according to kinetic theory,

$$P = \frac{1}{3} \rho c^2$$

$$\Rightarrow P = \frac{1}{3} \frac{m}{V} c^2$$

$$\Rightarrow PV = \frac{1}{3} Mc^2 \quad \text{--- (1)}$$

considering one gm molecule of the gas at temp  $T$  K,

$$PV = RT \quad \text{--- (2)}$$

From eqn<sup>n</sup> (1) and (2),

$$\frac{1}{3} Mc^2 = RT$$

$$\Rightarrow \frac{1}{2} Mc^2 = \frac{3}{2} RT \quad \text{--- (3)}$$

Let the mass of each molecule be  $m$  and Avogadro's no. be  $N$ ,

$$\therefore M = m \times N$$

$$\therefore X_D = d - x \text{ and } dY = d + x$$

$x$  is the distance bet<sup>n</sup> the centre  $O$  and the null point  $D$ .

Let us consider, resistance of per. unit length of bridge wire is  $\rho$ . So, total resistance is  $2l\rho$ .

We know,  $\frac{P}{A} = \frac{R}{S} \quad \text{--- (1)}$

If  $\rho = a$ , then from eqn<sup>n</sup> (1):

$$l = \frac{R}{S}$$

$$\therefore R = S$$

We can write,  $R + 2x(l-x)\rho = S + 2x(l+x)\rho$

$$R = S + 2x\rho \quad \text{--- (2)}$$

08/05  
Advantages and disadvantages of resistance thermometers

This can be used to measure temp. rang

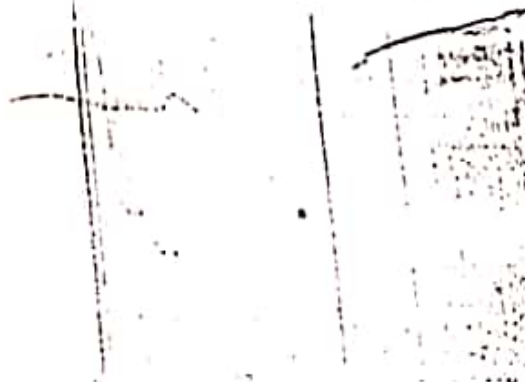
from  $-200^\circ\text{C}$  to  $1200^\circ\text{C}$

08/05  
Correction:

$$R_t = R_0 (1 + \alpha t + \beta t^2)$$

this is the standard eqn<sup>n</sup>, not neglecting  $\beta$  though  $\beta$  is too small.

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the free path is not constant. Therefore the term mean free path is used to indicate the mean distance travelled by a molecule bet<sup>n</sup> two collisions. If the total distance travelled after  $N$  collision is  $S$ , then mean free path,

$$\lambda = \frac{S}{N} \quad \text{--- (1)}$$

of  $\lambda$  Mean free path,  $\lambda \propto \frac{1}{d^2}$

Let us consider a molecule of diameter  $d$  and velocity  $v$  consist

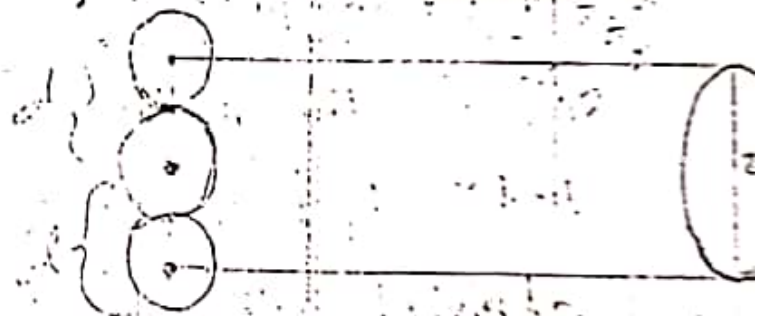
to consist

$$\therefore \text{Area, } A = \pi d^2$$

$$\therefore \text{Volume of cylinder, } V = \pi d^2 v$$

If per cc no. of molecules is  $n$ , the total no. of molecule =  $\pi d^2 v n$

$$\therefore \Delta D = d - r \quad \text{and} \quad \Delta Y = d + r$$



Thus, the energy associated with each degree of freedom is  $\frac{1}{2}kT$ . This represents the theorem or law of equipartition of energy.

It states that for any dynamical system in thermal equilibrium, the total energy is divided equally among all the degrees of freedom and the energy associated per degree of freedom is  $\frac{1}{2}kT$ .

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07  
Q. Mean free path:

The path covered by a molecule between two consecutive collision in a straight line is called free path.

The direction of a molecule is changed after each collision. After a no. of collisions, the total path appears to be zig-zag and

0.3, 0.4  
Q Law of equipartition of energy :

According to kinetic theory of gases, the mean kinetic energy of a gas molecule at a temp.  $T$  is given by,

$$\frac{1}{2} m c^2 = \frac{3}{2} kT$$

$$\text{But, } c^2 = u^2 + v^2 + w^2$$

As  $x$ ,  $y$  and  $z$  are all equivalent, mean square velocities along these axes are equal

$$\therefore u^2 = v^2 = w^2$$

$$\text{Hence, } u^2 = v^2 = w^2 = \frac{1}{2} c^2$$

$$\Rightarrow \frac{1}{2} m u^2 = \frac{1}{2} m v^2 = \frac{1}{2} m w^2$$

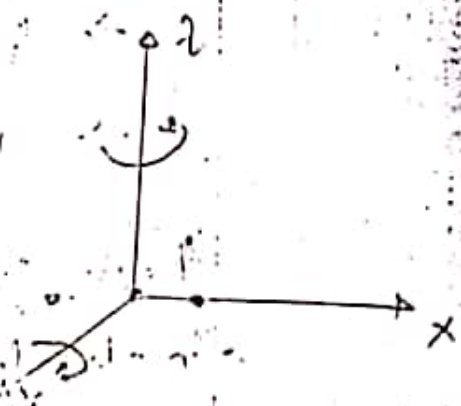
$$\therefore \frac{1}{2} m c^2 = 3 \left[ \frac{1}{2} m u^2 \right] = 3 \left[ \frac{1}{2} m v^2 \right] = 3 \left[ \frac{1}{2} m w^2 \right] = \frac{9}{2} kT$$

$$\therefore \frac{1}{2} m u^2 = \frac{1}{2} kT$$

$$\therefore \frac{1}{2} m v^2 = \frac{1}{2} kT$$

$$\therefore \frac{1}{2} m w^2 = \frac{1}{2} kT$$

$$\therefore \Delta D = d \cdot n \text{ and } \Delta Y = d \cdot x$$





## Derivation of gas eqn

From Kinetic theory,

$$P = \frac{1}{3} \rho c^2$$

$$= \frac{1}{3} \frac{m}{V} c^2$$

$$\therefore PV = \frac{1}{3} mc^2$$

considering one gram of molecule of a gas at an absolute temp.  $T$ . The mean energy of the molecules,

$$= \frac{1}{2} mc^2$$

$$= \frac{1}{2} m N c^2$$

$$\therefore PV = \frac{1}{3} m N c^2$$

$$= \frac{2}{3} N \cdot \frac{1}{2} mc^2 \quad \text{--- (1)}$$

$$\text{mean kinetic energy of a molecule} = \frac{1}{2} mc^2$$
$$= \frac{3}{2} kT$$

$$\text{From eqn (1), } PV = \frac{2}{3} N \cdot \frac{3}{2} kT$$

$$\therefore PV = NkT$$

$$\therefore xD = d - n \quad \text{and} \quad dY = d + x$$

Force per unit area on the wall BCF of ADEH is equal to the pressure  $P_x$

$$\therefore P_x = \frac{m}{\Delta x \Delta y} (u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2)$$

Similarly the pressure,

$$P_y = \frac{m}{\Delta y \Delta z} (v_1^2 + v_2^2 + \dots + v_n^2)$$

$$\text{and } P_z = \frac{m}{\Delta x \Delta z} (w_1^2 + w_2^2 + \dots + w_n^2)$$

As the pressure is same in all directions

$$P = \frac{P_x + P_y + P_z}{3}$$

$$= \frac{m}{3 \Delta x \Delta y \Delta z} [(u_1^2 + v_1^2 + w_1^2) + (u_2^2 + v_2^2 + w_2^2) + \dots + (u_n^2 + v_n^2 + w_n^2)]$$

$$= \frac{m}{3 \Delta x \Delta y \Delta z} [c_1^2 + c_2^2 + c_3^2 + \dots + c_n^2] \quad \text{--- (1)}$$

Let  $c$  be the root-mean-square velocity of the molecules, then  $c^2 = \frac{c_1^2 + c_2^2 + \dots + c_n^2}{n}$

$$\therefore n c^2 = c_1^2 + c_2^2 + \dots + c_n^2$$

$$\therefore \Delta x \Delta y \Delta z = \Delta x \Delta y \Delta z \quad \text{and } \Delta x = \Delta x$$

considering a molecule P moving in a random direction with a velocity  $c_1$ .

The velocity can be resolved into three mutually perpendicular components  $u_1$ ,  $v_1$  and  $w_1$  along the x, y and z axes respectively.

$$c_1^2 = u_1^2 + v_1^2 + w_1^2$$

The component of the velocity with which the molecule P will strike the opposite face BCFG is  $u_1$  and the momentum of the molecule is  $mu_1$ .

This molecule is reflected back with the same momentum,  $mu_1$ , in an opposite direction and after travelling a distance  $l$  will strike the opposite face ADHI.

$$\therefore XO = l - x \text{ and } OY = l + x$$

∴ from eqn (1),

$$P = \frac{m}{3V} n c^2 \quad [\because d^3 = V = \text{volume}]$$

— (2)

But,  $M = mn$ , where,  $M = \text{mass of gas of } V \text{ \& volume}.$

$m = \text{mass of each molecule}$

$n = \text{no. of molecule in volume } V$

no. of molecules in volume  $V$

$$n = \frac{M}{m}$$

∴ from eqn (2),

$$P = \frac{m n c^2}{3V} = \frac{M}{V} \cdot \frac{c^2}{3}$$

$$\therefore P = \frac{1}{3} \rho c^2 \quad \text{--- (3)} \quad [\because \frac{M}{V} = \rho = \text{density}]$$

$$\Rightarrow c^2 = \frac{3P}{\rho}$$

$$\therefore c = \sqrt{\frac{3P}{\rho}}$$

### Kinetic interpretation of Temperature:

considering a system of  $N$  molecules of monoatomic gas. If  $E$  is the total energy of the system, then some of the energy will

be in the form of translational energy, and the remaining may be rotational, vibrational potential energies of the molecules.

Let  $E_t$  be the translational energy then according to the law of equipartition of energies,

$$\frac{E_t}{N} = \frac{3}{2} K.T \quad \text{--- (1)} \quad \left[ K \rightarrow \text{Boltzmann constant} \right]$$

But  $\frac{E_t}{N}$  is nothing but the average translational energy per molecule,

$$\bar{E}_t = \frac{E_t}{N} = \frac{3}{2} K.T$$

0.8, 0.5, 0.2  $\bar{E}_t \propto T$

C.T.  $\Rightarrow$  Change in entropy in a reversible process:

From A to B the process is isothermal,

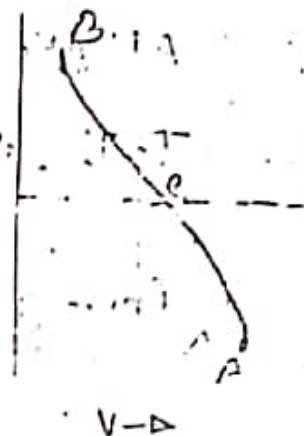
$$\therefore \text{Entropy, } S_1 = \frac{K_1}{T_1}$$

$$\therefore X D = d - x \quad \text{and} \quad -D Y = d + x$$



## Estimation of critical constants:

Considering the critical isothermal AEB as shown in figure. At the critical point C, the curve is horizontal. Therefore at point C, the slope  $\left(\frac{dp}{dv}\right) = 0$ .



At this point, the tangent also crosses the curve. Therefore the tangent at such a point is said to be stationary and the point is called 'the point of inflexion'. At the point of inflexion,  $\frac{d^2p}{dv^2} = 0$ .

Hence at critical point C,  $\frac{dp}{dv} = 0$  and  $\frac{d^2p}{dv^2} = 0$

The van der Waals equation is  $\left(p + \frac{a}{v^2}\right)(v-b) = RT$

$$p = \frac{RT}{v-b} - \frac{a}{v^2} \quad \text{--- (1)}$$

$$\therefore \frac{dp}{dv} = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3} \quad \text{--- (2)}$$

from B to C the process is adiabatic. So the change in entropy is zero (0).

from C to D the process is isothermal. So, the change in entropy,  $S_2 = \frac{H_2}{T_1}$ .

from D to A the process is adiabatic. So, the change of entropy is zero (0).

So, the gain of entropy of the working substance during the cycle ABCDA =  $\frac{H_1}{T_1} - \frac{H_2}{T_2}$ .

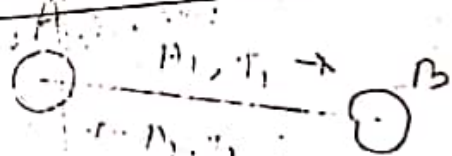
We know for a reversible process the loss of energy is zero (0). That is why,

$$\frac{H_1}{T_1} - \frac{H_2}{T_2} = 0$$

$$dS = 0 \quad (\text{constant})$$

Q8 Change in entropy in a irreversible process:

$$\begin{aligned} \text{loss in entropy} &= \frac{H_1}{T_1} \\ \text{gain in " " " } &= \frac{H_2}{T_2} \end{aligned}$$



$$\therefore \text{Change in entropy} = \frac{W_2}{T_2} - \frac{W_1}{T_1}$$

As  $T_1 > T_2$ , thus the entropy of the system increases in irreversible process.

Q.14 Prove that for a diatomic gas  $\gamma = 1.40$

Proof: Energy associated with each degree of freedom =  $\frac{1}{2} kT$

$\therefore$  Energy associated with 5 degree of

$$\text{freedom} = \frac{5}{2} kT$$

Consider 1 gm molecule of gas,

energy associated with 1 gm molecule of

$$\text{a diatomic gas} = N \times \frac{5}{2} kT = \frac{5}{2} NkT$$

$$\therefore U = \frac{5}{2} RT$$

$$\text{and } c_v = \frac{dU}{dT}$$

$$\text{But, } c_p - c_v = R$$

$$\therefore c_p = c_v + R = \frac{5}{2} R + R = \frac{7}{2} R$$

Therefore, the ratio of specific heats,

$$\therefore \gamma = \frac{c_p}{c_v} = \frac{\frac{7}{2} R}{\frac{5}{2} R} = 1.4$$

The change in momentum due to the impact is,  $mu_1 - (-mu_1) = 2mu_1$

∴ The time interval between two successive impacts on the wall BCFG is

$$\frac{2l}{u_1} \text{ seconds}$$

$$\therefore \text{No. of impacts per second} = \frac{1}{\frac{2l}{u_1}} = \frac{u_1}{2l}$$

$$\begin{aligned} \text{Change in momentum produced in one} \\ \text{second due to this molecule} &= 2mu_1 \times \frac{u_1}{2l} \\ &= \frac{mu_1^2}{l} \end{aligned}$$

The force  $F_x$  due to the impact of all the molecules in one second

$$= \frac{m}{d} [u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2]$$



$$= \frac{m}{d} \times \frac{u_1^2}{2l}$$



$$\text{and } \frac{d^2P}{dV^2} = \frac{2RT}{(V-b)^3} - \frac{6a}{V^4} \quad \text{--- (3)}$$

At the critical point C  
 $T = T_c, P = P_c, V = V_c, \frac{dP}{dV} = 0$  and  $\frac{d^2P}{dV^2} = 0$

$$\text{from (1), } P_c = \frac{RT_c}{(V_c-b)} - \frac{a}{V_c^2} \quad \text{--- (4)}$$

$$\text{from (2), } 0 = -\frac{RT_c}{(V_c-b)^2} + \frac{2a}{V_c^3}$$

$$\Rightarrow \frac{RT_c}{(V_c-b)^2} = \frac{2a}{V_c^3} \quad \text{--- (5)}$$

$$\text{from (3), } 0 = \frac{2RT_c}{(V_c-b)^3} - \frac{6a}{V_c^4}$$

$$\Rightarrow \frac{2RT_c}{(V_c-b)^3} = \frac{6a}{V_c^4} \quad \text{--- (6)}$$

Dividing (5) by (6),

$$\frac{V_c-b}{2} = \frac{V_c}{3}$$

$$\Rightarrow 3V_c - 3b = 2V_c$$

$$\therefore V_c = 3b \quad \text{--- (7)}$$

$\therefore \frac{dP}{dV} = 0$  and  $\frac{d^2P}{dV^2} = 0$

from (5),

$$\frac{RT_c}{(3b-b)^2} = \frac{2a}{(3b)^3}$$

$$\Rightarrow \frac{RT_c}{4b^2} = \frac{2a}{27b^3}$$

$$T_c = \frac{8a}{27bR} \quad \text{--- (6)}$$

from (4),  $P_c = \frac{8aR}{27bR(3b-b)} = \frac{a}{(9b)^2}$

$$= \frac{8a}{27 \times 2b^2} = \frac{a}{9b^2}$$

$$= \frac{a}{9b^2} \left( \frac{8}{9} - 1 \right) = \frac{a}{9b^2} \times \frac{1}{3}$$

$$\therefore P_c = \frac{a}{27b^2} \quad \text{--- (7)}$$

105  
Q) Physical importance of entropy:

entropy,  $dS = \frac{dQ}{T}$

$$\text{or, } S_B - S_A = \int_A^B \frac{dQ}{T}$$

① Just like temp. remains constant in isothermal process, the entropy remains constant in an adiabatic process.

But  $N \times k = R$

$$PV = RT$$

where,  $P = \text{Pressure} = \text{dynes/cm}^2$

$R = 8.31 \times 10^7 \text{ ergs/gm mol-K}$

$T = \text{absolute temp.} = \text{Kelvin (K)}$

$V = \text{Volume of per gm molecule}$

06, 03

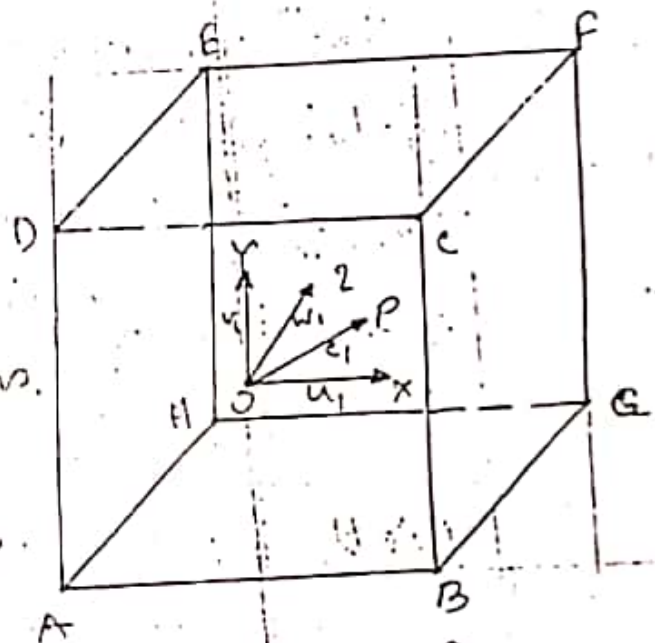
A Expression for the pressure exerted by a gas :

Considering the cubical vessel ABCDEFGH of side  $d$  cm containing gas.

Volume of the vessel  $= d^3 \text{ cm}^3$

Let,  $m = \text{mass of each molecule}$

$n = \text{no. of molecules present in vessel}$



① It is a definite single valued function of the thermodynamic variables describing the state of a working substance.

② In every natural process (i.e. irreversible change) there is always an increase in entropy.

③ The 2<sup>nd</sup> law of thermodynamics can be stated in terms of entropy of a system.

④ Due to increase in entropy, unavailable energy increases.

⑤ According to free man Drossin, the entropy is a measure of disorderliness. This disorderliness can be evaluated by using the relation,

$$S = k \log W$$

where  $k$  is the Boltzmann constant and  $W$  is the disorder parameter.

$$\therefore X D = d - x \quad \text{and} \quad -D Y = d + x$$