Radioactivity

Define Radioactivity :

Radio activity is a spontoneous and self disruptive activity ~~exib~~ exhibited almost entirely by some heavy elements of atomic weights greatere than about 206. occurring in nature. The elements that exihibit this property are called Radio activity elements. The activity emission of powerful rediations called α (alpha), β (Beta) and γ (gamma) rays.

Kinds of Radio activity :

There are two kinds of Radio activity.

1. Natural. 2. Artificial.

Why α, β, γ rays came out spontoneously ?

When molecule atom are loosely bounded, they collison with each other and gather a Lots of energy. Then they came came out from atom.

Characteristic of Radioactive :

1. Distance. (radiation is inversly proportional to the square of the distance)
2. Penetrating power. (There are α, β, γ rays radiation. The Penetrating power depends on type of radiation)
3. Half life.
4. It is spontoneous incident.

State the radioactive decay/ disintegration law and hence show that the radioactive materials disintegrate exponentially.

Decay / Disintegration law :-

The rate of disintegration is governed by an exponential law. This means that the number of atoms that break up at any instant is not affected by environment factors (like temperature, pressure, chemical combination) but is ~~proposed~~ proportional to the ~~no~~ nub. presents at that instant.

or, In other words, equal fraction of radioactive atom disintegrate in equal interval of time.

Rate of disintegration of r.a.m is proportional to the number of atoms remain unchanged at that time

$$\therefore -\frac{dN}{dt} \propto N \quad \text{——— (1)}$$

$$\Rightarrow -\frac{dN}{dt} = \lambda N$$

$$\Rightarrow \frac{dN}{N} = -\lambda dt$$

here,
$\frac{dN}{dt}$ = Rate of . D
also called acti
N = Nub of atoms
(-)ve indicates th
N decreases as t
increases

integrating both sides,

$$\int \frac{dN}{N} = -\lambda \int dt$$

$$\Rightarrow \ln N = -\lambda t + c \quad \text{——— (11)}$$

when, $t = 0$, $N = N_0$

from equ (11)

$$\ln N_0 = 0 + c \quad \therefore c = \ln N_0$$

Put the value in (11)

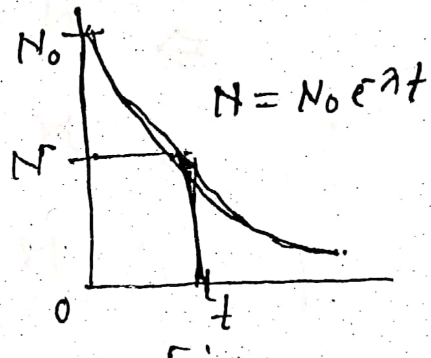
$$\ln N = -\lambda t + \ln N_0$$

$$\Rightarrow \ln N - \ln N_0 = -\lambda t$$

$$\Rightarrow \ln \frac{N}{N_0} = -\lambda t$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \boxed{N = N_0 e^{-\lambda t}}$$



Fig

$\therefore$ Radioactive materials disintegrate exponentially (showed)

Radio active decay Constant :-

we know, $-\frac{dN}{N} = \lambda dt$ ——

$\therefore$

$\therefore \lambda = -\frac{dN}{N}/dt = -\frac{dN}{dt}/N$

Hence, the decay constant may be defined as the ra[illegible] of the ~~atoms~~ number of atoms which disintigrates in unit time to the number of atoms present.

Let, $t = 1/\lambda$

$\therefore N = N_0 e^{-\lambda t} = N_0 e^{-\lambda \cdot 1/\lambda}$

$\Rightarrow N = N_0 e^{-1} = N_0 \cdot \frac{1}{e} = \frac{N_0}{e}$

$\Rightarrow N = \frac{N_0}{2.718}$

$\therefore N = N_0 \, 0.368 \approx \boxed{0.37 N_0}$

Hence the decay constant also be defined as the reciprocal of time during which the no. of radio-active atoms falls to 37 percent of its original value.

$\boxed{\text{Half Life Time}}$:

Half life time defined as the time in which redioactive atoms are reduced to half their initial amount.

we know,

$$N = N_0 e^{-\lambda t}$$

$N = \frac{N_0}{2}$
$t = t_{1/2}$

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\Rightarrow \ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\Rightarrow -0.693 = -\lambda t_{1/2}$$

$$\therefore \boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

Mean or Average Life Time :-

The mean or average life time defined as the ratio of the total life time of all radioactive atoms to the total number of such atoms in it.

$$\tau = \frac{\text{sum of lives all the atoms}}{\text{total number of atoms}}$$

$$\tau = \frac{\text{total life time}}{\text{total no. of atoms}}$$

$$\tau = \frac{\Sigma_0^\infty t \, dN}{N_0}$$

$$\tau = \int_0^\infty \frac{-t \cdot dN}{N_0}$$

$$= \int_0^\infty -t \frac{(-\lambda \cdot N\, dt)}{N_0}$$

$$\frac{dN}{N} = -\lambda dt$$

$$dN = -\lambda N dt$$

$$= \int_0^\infty \frac{t \lambda N\, dt}{N_0}$$

$$N = N_0 e^{-\lambda t}$$

$$= \int_0^\infty \frac{t \lambda \cdot \cancel{N_0} e^{-\lambda t}\, dt}{\cancel{N_0}}$$

$$= \int_0^\infty \lambda t e^{-\lambda t}\, dt$$

$$= \lambda \int_0^\infty t \cdot e^{-\lambda t}\, dt .$$

$$= \lambda \left[t \int e^{-\lambda t} dt - \left\{ \int \frac{d}{dt}(t) \int e^{-\lambda t} dt \right\} dt \right]_0^\infty$$

$$= \lambda \left[t e^{-\lambda t} \cdot \frac{1}{-\lambda} - \int e^{-\lambda t} \cdot \frac{1}{-\lambda}\, dt \right]_0^\infty$$

$$= \lambda \left[-\frac{e^{-\lambda t} \cdot t}{\lambda} - \cancel{\int} \frac{e^{-\lambda t}}{\lambda^2} \right]_0^\infty$$

$$= \lambda \left[0 - \frac{\lambda t e^{-\lambda t} - e^{-\lambda t}}{\lambda^2} \right]_0^\infty$$

$$= \frac{1}{\lambda} \left[-e^{-\lambda t} (\lambda t + 1) \right]_0^\infty .$$

$$= -\frac{1}{\lambda} (0 - 1) = \frac{1}{\lambda} .$$

$$\therefore \boxed{\tau = \frac{1}{\lambda}}$$

Now, $T_{1/2} = \frac{0.693}{\lambda}$

$$\Rightarrow T_{1/2} = \frac{0.693}{1/\tau}$$

$$\therefore \boxed{T_{1/2} = 0.693\,\tau}$$

Activity, $A = \frac{dN}{dt} = -\lambda N$

$$A_0 = \left(\frac{dN}{dt}\right)_{t=0} = -\lambda N_0$$

$$\frac{A}{A_0} = \frac{N}{N_0}$$

$$\therefore A = \frac{A_0 \cancel{N_0} e^{-\lambda t}}{\cancel{N_0}}$$

$$= A_0 e^{-\lambda t}.$$

Example 16.1

The half life of a radioactive substance is 30 days. Calculate (i) the radioactive decay constant, (ii) the time taken for 3/4 of the original number of atoms to disintegrate (iii) the mean life, (iv) the time for 1/8 of the original number of atoms to remain unchanged.

Solution:

(i) Given, $T_{1/2} = 30$ days.

$$T_{1/2} = \frac{0.693}{\lambda}$$

$$\Rightarrow \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{30} = 0.0231 \text{ per day}^{-1}$$

(ii) we know,

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{1}{4} N_0 = N_0 e^{-\lambda t}$$

$$\Rightarrow 1/4 = e^{-\lambda t}$$

$$\Rightarrow \ln(1) - \ln(4) = -\lambda t$$

$$\Rightarrow 1.38 = \lambda t$$

$$\therefore t = \frac{1.38}{0.0231} = 60 \text{ days.}$$

here,

$$N_0 = \frac{3}{4} N_0$$

$$N = N_0 - \frac{3 N_0}{4}$$

$$= \frac{1}{4} N_0 .$$

(iii) Mean life. $\tau = 1/\lambda$

$$\tau = \frac{1}{0.0231} = 43.29 \text{ days.}$$

(iv) we know,

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow \frac{N_0}{8} = N_0 e^{-\lambda t}$$

$$\Rightarrow \ln 1/8 = -\lambda t$$

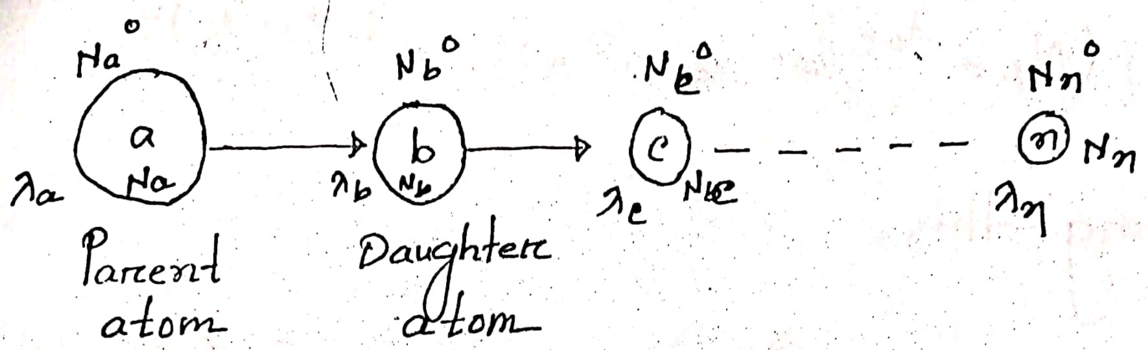
$$\Rightarrow 2.07 = \lambda t$$

$$\therefore t = \frac{2.07}{0.0231} = 90 \text{ days}$$ Ans.

here,

$$N = 1/8 N_0 .$$

Radioactive equlibrium:



$$\frac{dN_b}{dt} = \lambda_a N_a - \lambda_b N_b \quad \text{——— (1)} \longrightarrow \text{(এটা বাদ)}$$

Rate of the Parent Decay,

$$\frac{dN_a}{dt} = -\lambda_a N_a$$

The Rate of net increase the daughter at the same time given by,

$$\frac{dN_b}{dt} = \lambda_a N_a - \lambda_b N_b \quad \text{——— (1)}$$

$$\Rightarrow \frac{dN_b}{dt} = \lambda_a N_a^0 e^{-\lambda_a t} - \lambda_b N_b \qquad [\because N = N_0 e^{-\lambda t}]$$

$$\Rightarrow \frac{dN_b}{dt} + \lambda_b N_b = \lambda_a N_a^0 e^{-\lambda_a t} \quad \text{——— (11)}$$

$$\Rightarrow \frac{dN_b}{dt} \cdot e^{\lambda_b t} + \lambda_b N_b e^{\lambda_b t} = \lambda_a N_a^0 e^{(\lambda_b - \lambda_a)t}$$

[উভপক্ষে $e^{\lambda_b t}$ দ্বারা গুনকরে]

$$\Rightarrow \frac{dN_b}{dt} e^{\lambda_b t} + 0 = \lambda_a N_a^0 e^{(\lambda_b - \lambda_a)t}$$

$$\Rightarrow \frac{d}{dt}\left(N_b e^{\lambda_b t}\right) = \lambda_a N_a^0 e^{(\lambda_b - \lambda_a)t}$$

integrating this,

$$\Rightarrow \int \frac{d}{dt}\left(N_b e^{\lambda_b t}\right) = \int \lambda_a N_a^0 e^{(\lambda_b - \lambda_a)t}$$

$$\Rightarrow N_b e^{\lambda_b t} = \lambda_a \cdot N_a^0 e^{(\lambda_b - \lambda_a)t} \cdot \frac{1}{\lambda_b - \lambda_a} + c$$

$$\Rightarrow N_b e^{\lambda_b t} = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 e^{(\lambda_b - \lambda_a)t} + c \quad \text{--- (iv)}$$

when, $t = 0$, $N_b = 0$

Putting in (iv),

$$0 = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 + c$$

$$\therefore c = -\frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 \quad \text{--- (v)}$$

Putting the value of c in (iv),

$$N_b e^{\lambda_b t} = \frac{\lambda_a}{\lambda_b - \lambda_a} \cdot N_a^0 e^{(\lambda_b - \lambda_a)t} - \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0$$

$$= \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 \left[e^{(\lambda_b - \lambda_a)t} - 1 \right]$$

$$\Rightarrow N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 \left[e^{(\lambda_b - \lambda_a)t} - 1 \right] \cdot e^{-\lambda_b t}$$

$$\Rightarrow N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 \left[e^{-\lambda_a t} - e^{-\lambda_b t} \right].$$

$$\therefore \boxed{N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 \left[e^{-\lambda_a t} - e^{-\lambda_b t} \right]}.$$

Case 1 :- Transient equilibrium :-

Let, $t_a > t_b$, $\lambda_b > \lambda_a$ $[\because \lambda = \frac{1}{t}]$.

Now,
$$N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} \cdot N_a^0 \left[e^{-\lambda_a t} - 0 \right].$$

$$N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a^0 e^{-\lambda_a t}$$

$$\Rightarrow N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a$$

$$\Rightarrow N_b (\lambda_b - \lambda_a) = \lambda_a N_a .$$

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$$\therefore \boxed{N_a \lambda_a = N_b (\lambda_b - \lambda_a)}$$

$$\because \text{nth} \Rightarrow \boxed{N_a \lambda_a = N_b (\lambda_b - \lambda_a) = N_c(\lambda_c - \lambda_b) \ldots \ldots N_n(\lambda_n - \lambda_{n-1})}$$

Case 2: Saccular equilibrium:

Life of time a is $\gg$ Life of time b.

$$t_a \gg t_b$$

$$\lambda_b \gg \lambda_a \qquad [\because t = 1/\lambda].$$

We get,

$$N_b = \frac{\lambda_a}{\lambda_b - 0} N_a^0 \left[e^{-\lambda_a t} - 0\right]$$

$$\Rightarrow N_b = \frac{\lambda_a}{\lambda_b} \cdot N_a^0 e^{-\lambda_a t}$$

$$\Rightarrow N_b = \frac{\lambda_a}{\lambda_b} N_a$$

$$\therefore \boxed{N_a \lambda_a = N_b \lambda_b} = N_c \lambda_c = \ldots \ldots N_n \lambda_n$$

n th,

$$\boxed{N_a\lambda_a = \lambda_b N_b = N_c\lambda_c = \dots\dots N_n\lambda_n}$$

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Charecterestics: (Radioactive)

1. It is a spontoneous incident.
2. It does not affected by temperature.
3. It is a contineous process.
4. It's atomic mass is greater than about 206.

Radioactive Equlibrium:-

In radio active transformation when a daughter element transforms at a same rate at which it is formed from the parent element that state is known as radioactive equilibrium.

$$\underset{\text{Parent}}{A} \longrightarrow \underset{\text{intermediate daughter}}{B} \longrightarrow C$$