Relativity (Special Theory)

What is Relativity?

The process of relative measurement of a body with one respect to another is called relativity.

Theory of Einstein :-

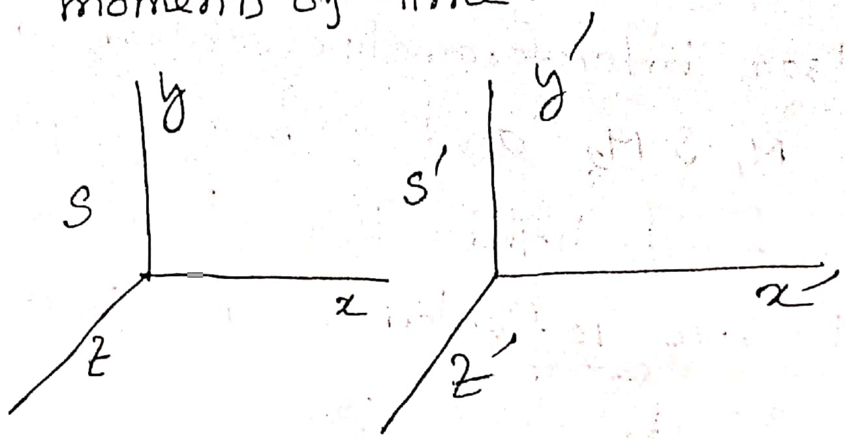
There are two Theory of einstein.

1. Special theory of relativity. (Space or Length, Mass & time)
2. General theory of relativity. (theory of Gravitation, it describes the forces of gravitation)

Reference Frame :-

A framework that is used for the observation and mathmetical description of physics phenomena and the formulation of physics Laws, usaullay consisting of an obserever, a co-ordinate system and a clock or clock assingning times at positions with respect to co-ordinate system.

or, A reference frame may refere to a co-ordinate system to represent and measurement properties of object, such as their position and orientation at different moments of time.



Reference frame are two kinds :-

1. Inertia Reference Frame :- The frame which moves with constant velocity is called Inertia Reference frame.

2. Non Inertia reference Frame :- A non-inertia reference frame is a frame of reference that is undergoing acceleration with respect to inertia frame.

Postulates of Special Theory :-

1. The Principle of Relativity :- The laws of physics are same in all intertia frames of reference.

2. The principle of Invariant Light Speed :- The speed of Light c is constant, independent of the relative motion of the source and observer.

Michelson-Mosly Experiment in Interpherometer :-
A michelson interferometer consists minimally of mirrors M_1 & M_2 and a beam spilter M. A Source S emits light that hits the beam spilter. All The light are reflected and transmitted in detector E.

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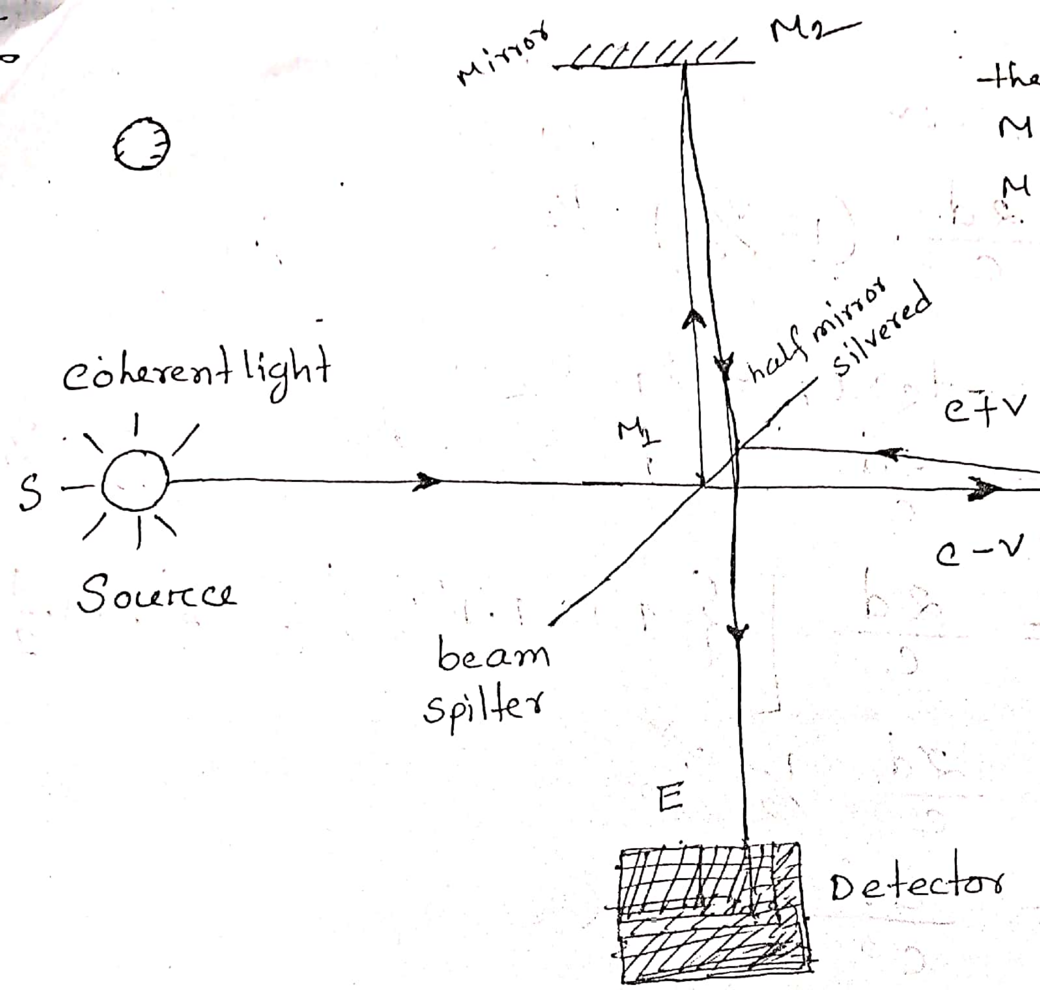

the distance between $MM_1 = d$ and $M_1M_2 = d$.
$MM_2 = 2d$.

Travelling time of Light in MM_1M_2

$$t_1 = \frac{d}{c+v} + \frac{d}{c-v} = \frac{2dc}{c^2 - v^2}$$

$$= \frac{2dc}{c^2\left(1-v^2/c^2\right)}$$

$$= \frac{2d}{c\left(1-v^2/c^2\right)}$$

$$= \frac{2d}{c} \cdot \left(1-v^2/c^2\right)^{-1} \quad \text{——— (1)}$$

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Magnitude

$\Delta N =$

$\therefore$

$$t_2 = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{(1 - v^2/c^2)}}$$

$$= \frac{2d}{c} \cdot \left(1 - v^2/c^2\right)^{-1/2} \quad \text{———} \quad (ii)$$

w,

$$\Delta t = -t_2 + t_1 = t_1 - t_2$$

$$= \frac{2d}{c} \left\{ \left(1 - v^2/c^2\right)^{-1} - \left(1 - v^2/c^2\right)^{-1/2} \right\}$$

$$= \frac{2d}{c} \left[\left\{ 1 + 1 \cdot 1 \left(\frac{v^2}{c^2}\right)^1 \right\} - \left\{ 1 + 1 \cdot \frac{1}{2} \cdot \left(v^2/c^2\right) \right.\right.$$

$$= \frac{\cancel{2}d}{c} \, \frac{1}{\cancel{2}} \, v^2/c^2$$

$$= \frac{dv^2}{c^3} \quad \text{———} \quad (iii)$$

nge shift,

$$\Delta N = \frac{2\Delta t}{T}$$

$$= \frac{2dv^2}{c^3 T}$$

$$= \frac{2dv^2 \cdot c}{c^3 \cdot \lambda} \qquad \left[\because T = \lambda/c\right].$$

$$= \frac{2d}{\lambda} \cdot \left(v^2/c^2\right)$$

$$\therefore \Delta N = \frac{2d}{\lambda} \left(v/c\right)^2$$

$\therefore$ Therefore, the fringe is not shift, so the velocity of light would be same.

Magnitude of ΔN,

$$\therefore \Delta N = \frac{2d}{\lambda}\left(\frac{v}{c}\right)^2$$

$$= \frac{2\times 11}{5.9\times 10^{-7}} \times \left(\frac{\cancel{3}\times 10^{4}}{\cancel{3}\times 10^{8}}\right)^2$$

$$= 0.37 \approx 0.4 \text{ fringes.}$$

$d = 11\ m$
$\lambda = 5.9\times 10^{-7}\ m^{-1}$
$v = 3\times 10^{4}\ ms^{-1}$
$c = 3\times 10^{8}\ ms^{-1}$

Problem :- In the Michelson-Morley Experiment, the wavelenght of monochomatic light is used 5000 Å. what will be the expected fringe-shift on the basic of stationary ethere hypothesis if the effective length of each path be 5 meteres? (velocity of the earth $= 3\times 10^4\ m/s$, $c = 3\times 10^8\ m/s$ and $1\ A.U = 10^{-10} m$)

Soln:

We know,

$$\Delta N = \frac{2D}{\lambda}\cdot\frac{v^2}{c^2}$$

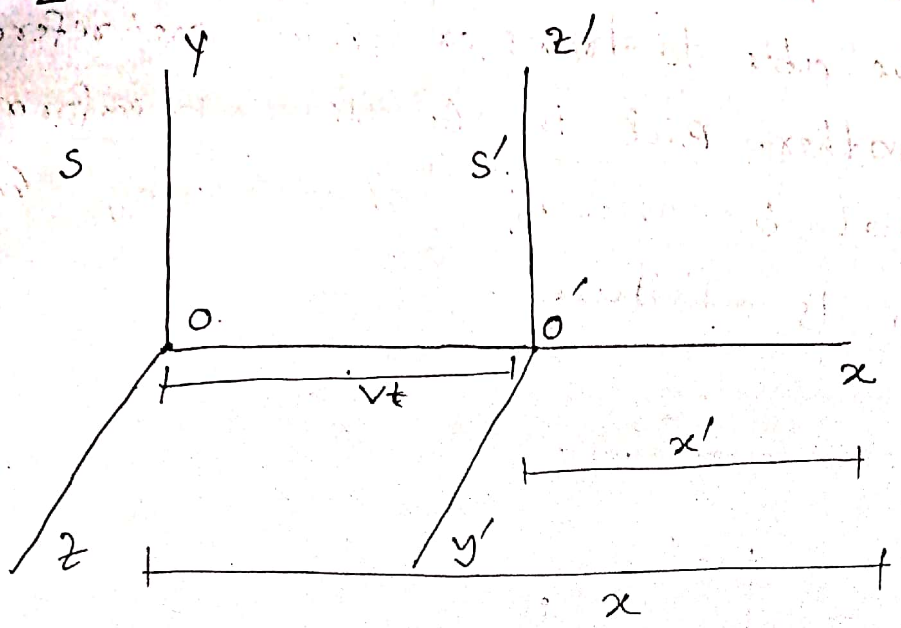
$$= \frac{2\times\cancel{5}}{\cancel{5}\times 10^{-7}} \times \left(\frac{\cancel{3}\times 10^{4}}{\cancel{3}\times 10^{8}}\right)^2$$

$$= 0.2 \text{ or } 1/5 \text{ fringes.}$$

here,
$v = 3\times 10^4\ m/s$
$c = 3\times 10^8\ m/s$
$d = 5\ m$
$\lambda = 5000\ Å$
$= 5000\times 10^{-10} m$
$= 5\times 10^{-7}\ m$

The expected fringe shift is one-fifth of a fringe-width.

Galilien Transformation :-



x direction,

$$x' = x - vt \quad \text{——— (1)}$$

$$x = x' + vt \quad \text{——— (11)}$$

Since there is no relative motion is in y and z direction

$$y = y'$$

$$z = z'$$

Galilean transformation are violet both of the two special theory of relativity. Acceleration are invarint under galilean transformation. But In special theory we know that velocity is constant with respect one reference frame to another. In another time is not transfered ($t' = t$) in one frame to another. But, time should be able to transfer from another.

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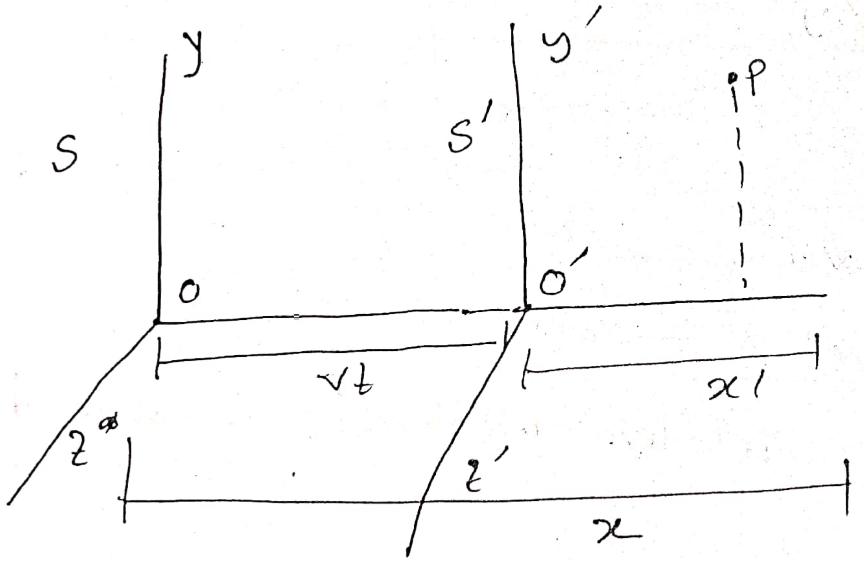
one reference frame to another. And the e... should also be able to transfer from one referen... frame to another. But in Galilean transformation, those are not be shown/possible. That's why, Galilean transformation is failure.

Lorenz Transformation:



At o observer the reference frame S is,

$$x^2 + y^2 + z^2 = c^2 t^2 \quad \text{——— (I)}$$

in s' frame,

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \quad \text{——— (II)}$$

Let, $t = t' = 0$ $\qquad y' = y$

$\qquad\qquad\qquad\qquad\qquad z' = z$

$$x = ct \quad \text{——— (III)}$$

$$x' = ct' \quad \text{——— (IV)}$$

Let a reasonable guess as to kind of relationship between x and x' be,

$$x' = K(x - vt) \quad \text{—— (V)}$$

$$x = K(x' + vt') \quad \text{—— (VI)}$$

where, K is the factor of proportionality that does not depend upon either x or t.

Now, Put the value x' in equ (VI)

$$x = K\{K(x-vt) + vt'\}$$

$$\Rightarrow x = K^2(x-vt) + Kvt'$$

$$\Rightarrow Kvt' = x - K^2x + K^2vt$$

$$\Rightarrow Kvt' = x(1-K^2) + K^2vt$$

$$\Rightarrow t' = \frac{K^2vt}{Kv} + \frac{x(1-K^2)}{Kv}$$

$$\Rightarrow t' = Kt + \frac{x}{v}\frac{(1-K^2)}{K}$$

$$\therefore t' = K\left\{t + \frac{x}{v}\left(\frac{1}{K^2} - 1\right)\right\} \quad \text{—— (VII)}$$

Put the t' value in equ (II)

$$x'^2 + y'^2 + z'^2 = c^2K^2\left\{\frac{x}{v}\left(\frac{1}{K^2}-1\right) + t\right\}^2$$

$$\Rightarrow K^2(x-vt)^2 + y^2 + z^2 = c^2K^2\left\{\frac{x^2}{v^2}\left(\frac{1}{K^2}-1\right)^2 + \frac{2x}{v}\left(\frac{1}{K^2}-1\right)t + t^2\right\}$$

$$\Rightarrow K^2(x^2 - 2xvt + v^2t^2) + y^2 + z^2 = c^2K^2 \frac{x^2}{v^2}\left(\frac{1}{K^2}-1\right) + c^2K^2 \cdot \frac{2xt}{v} \cdot \left(\frac{1}{K^2}-1\right) + c^2K^2t^2$$

$$\Rightarrow K^2x^2 - c^2K^2\frac{x^2}{v^2}\left(\frac{1}{K^2}-1\right)^2 - K^2 2xvt - c^2K^2\frac{2xt}{v}\left(\frac{1}{K^2}-1\right) + y^2 + \cancel{K}z^2 = c^2K^2t^2 - K^2v^2t^2$$

$$\Rightarrow x^2\left[K^2 - \frac{c^2K^2}{v^2}\left(\frac{1}{K^2}-1\right)^2\right] - xt\left[2vK^2 + \frac{2c^2K^2}{v}\cdot\left(\frac{1}{K^2}-1\right)\right] + y^2 + z^2 = t^2(c^2K^2 - K^2v^2) \quad \text{—— (VIII)}$$

NOW, from equ. (II),

$$x'^2 + y'^2 + z'^2 = c^2t'^2$$

$$\Rightarrow (x - vt)^2 + y^2 + z^2 = ct^2$$

$$\Rightarrow x^2 - 2xvt + v^2t^2 + y^2 + z^2 = c^2t^2$$

$$\Rightarrow x^2 - 2vxt + y^2 + z^2 = c^2t^2 - v^2t^2 \quad \text{—— (IX)}$$

$t' = t$
$y' = y$
$z' = z$
$x' = x - vt$
$x = x' - vt'$

equ. (VIII) & (IX) are similiar, so, the quantities in equ (VIII) must be equal to 1, 0 and c^2.

So,
$$2vK^2 + \frac{2c^2K^2}{v}\left(\frac{1}{K^2}-1\right) = 0$$

$$\Rightarrow v^2K^2 + c^2K^2\left(\frac{1}{K^2}-1\right) = 0$$

$$\Rightarrow \left(\frac{1}{K^2}-1\right) = -\frac{v^2}{c^2}$$

$$\Rightarrow \frac{1}{K^2} = 1-\frac{v^2}{c^2}$$

$$\therefore \boxed{K = \frac{1}{\sqrt{1-v^2/c^2}}}$$

Now, put the value in equ. (v),

$$x' = \frac{1}{\sqrt{1-v^2/c^2}}(x-vt)$$

$$\boxed{x' = \frac{x-vt}{\sqrt{1-v^2/c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = K\left(t-\frac{vx}{c^2}\right)$$

$$\boxed{\therefore t' = \frac{t-vx/c^2}{\sqrt{1-v^2/c^2}}}$$

In obtaining these transformation the positive root has been choosen, so that $v=0$ implies $x'=x$ and $t'=t$.

So, Now the equation should be able to transfer one frame to another. this is Loren'z transformation.

Inverse Lorenz's Transformation :-

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Length or Space Contraction :-

Length contraction is the phenomenon of a decrease in Length of an object as measured by an observ which is traveling at any non-zero velocity relati to the Object.

What is the length of the rod measured by an observ in the frame of reference S relative to which the rod (or frame) is moving with a velocity v?

Let, the Observer is in S frame, so the length L be the measured by the observer S is,

$$L = x_2 - x_1$$

also, observer S',

$$L_0 = x'_2 - x'_1$$

Now, applying Lorenz transformation we have,

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

So, $L_0 = x_2' - x_1'$

$$= \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}} - \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{L}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \boxed{L = L_0 \sqrt{1 - v^2/c^2}}$$

Time Dilation :-

Time dilation is a difference of elapsed time between two events as measured by observer eithere moving relative to each othere. ore differentlly situated from a gravitational mass or masses.

Let, the frame of reference S' is moving with a velocity v, along the positive x-direction.

So, the observer in S' is,

$$t_0 = t_2' - t_1' \quad \text{———} \quad (1)$$

and, the observer in S is,

$$t = t_2 - t_1 \quad \text{———} \quad (11)$$

Now, according to Lorenz transformation, we have

$$t = \frac{t' + \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

∴ The time intervel between two events as stationary frame S, is given by,

$$\Delta t = t_2 - t_1$$

$$= \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}} - \frac{t_1' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{t_2' - t_1'}{\sqrt{1 - v^2/c^2}}$$

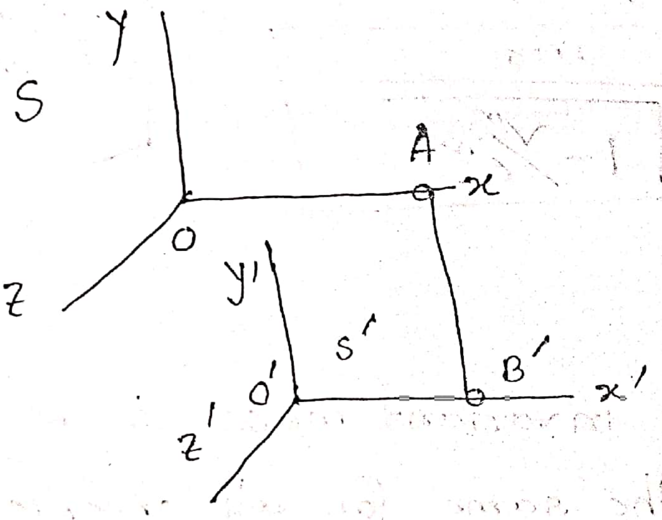
$$= \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \boxed{\Delta t = \frac{t_0}{\sqrt{1 - v^2/c^2}}}$$

Mass of Relativity :-

The word mass is given two meaning in special theory of relativity. One (rest mass or invarint mass) is an invariant quality which is the same for all observers in all reference frame. the other (relativistic mass) is dependent on the velocity of the observer.

Derivation :-



$$V_A = V_{B'}$$

For Consider two frames. One of reference is S and other S'. S' reference is moving with velocity v, but S frame is not moving. So, the mass of S frame is m. Now,

and S' frame is

For S frames, $T_0 = \frac{Y}{V_A}$ —— (1)

S' ,, , $T = \frac{Y}{V_{B'}}$ —— (11)

Now,

$$m_A v_A = m_B' v_B'$$

$$\Rightarrow m_A \frac{Y}{T_0} = m_B' \frac{Y}{T}$$

$$\Rightarrow m_A \cdot \frac{1}{T_0} = m_B' \cdot \frac{\sqrt{1-v^2/c^2}}{T_0} \qquad \left[\because T = \frac{T_0}{\sqrt{1-v^2/c^2}}\right.$$

$$\Rightarrow m_A = m_B' \sqrt{1-v^2/c^2}$$

$$\Rightarrow m_B' = \frac{m_A}{\sqrt{1-v^2/c^2}} \qquad \left[\because m_A = m_0,\ m_B' = m\right.$$

$$\therefore \boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}}}$$

Rest mass :-

Rest mass or invariant mass is an invariant quality which is the same fore all observers in all reference frame. In other word, the rest mass of an object is the mass it has when its velocity is zero. It is ~~define~~ denoted by m_0.

Relativistic mass :

Relativistic mass defined as the mass which is the dependent on the velocity of the observer of reference frame. It is denoted by m.

Relationship between Total energy, the rest energy and the momentum :-

we know,

$$E = mc^2 = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} \quad \text{——— (I)}$$

the momentum,

$$P = mv = \frac{m_0}{\sqrt{1-v^2/c^2}} v \quad \text{——— (II)}$$

From equ (I),

$$E = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \frac{E}{c^2} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \frac{E^2}{c^4} = \frac{m_0^2}{\cancel{\sqrt{1-v^2/c^2}}}$$

$$\Rightarrow E^2/c^2 = \frac{m_0^2 c^2}{\cancel{\sqrt{1-v^2/c^2}}}$$

$$\Rightarrow \left(E/c\right)^2 = \frac{m_0^2 c^2}{(1-v^2/c^2)}$$

$$\Rightarrow m_0^2 c^2 = (E/c)^2 - \frac{E^2}{c^2} \cdot \frac{v^2}{c^2}$$

$$\Rightarrow m_0^2 c^2 = \left(\frac{E}{c}\right)^2 - \frac{m^2 \cancel{c^4}}{\cancel{c^2}} \cdot \frac{v^2}{c^2}$$

$$\Rightarrow m_0^2 c^2 = \left(\frac{E}{c}\right)^2 - m^2 v^2$$

$$\Rightarrow m_0^2 c^2 = \frac{E^2}{c^2} - p^2$$

$$\Rightarrow E^2 = m_0^2 c^4 + c^2 p^2$$

$$\therefore \boxed{E^2 = m_0^2 c^4 + p^2 c^2}$$

It is the relationship between total enery, rest enery and momentum.

Now, with no rest mass ($m_0 = 0$) can still have a momentum, $p = \frac{E}{c} = mc$ as a in the case of a photon compton effect.

Now, $E = m_0 c^2 + E_K$

$$\Rightarrow E^2 = m_0^2 c^4 + E_K^2 \quad \text{[squring]}$$

$$\Rightarrow m_0^2 c^4 + p^2 c^2 = m_0^2 c^4 + E_K^2$$

$$\Rightarrow p^2 c^2 = 2 m_0^2 c^4 + E_K^2$$

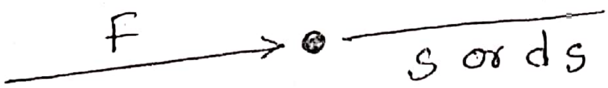
$$\Rightarrow p = \sqrt{2 m_0^2 c^2 + \frac{E_K^2}{c^2}}$$

$$\Rightarrow p = \sqrt{2 \cdot m_0 \cdot m_0 c^2 + \frac{E_K^2}{c^2}}$$

$$\therefore p = \sqrt{2 m_0 E_K + \frac{E_K^2}{c^2}}$$

This equ. is classical formula of momentum and relativistic correction term.

Einstein Mass-Energy Relation:-



NOW,

$$K.E = \int_0^s F.ds$$

$$= \int_0^s ma.ds = \int_0^s m.\frac{v}{t}\,ds$$

$$= \int_0^{mv} \frac{d}{dt}(mv)\,ds \qquad \left|\; \because v = \frac{ds}{dt},\; ds = v.dt \right.$$

$$= \int_0^{mv} \frac{d}{dt}(mv).\,v.dt$$

$$= \int_0^{mv} v\,d(mv)$$

$$= \int_0^{mv} v.(m.dv + v.dm)$$

$$= \int_0^{mv} (mv\,dv + v^2 dm.) \quad \text{——— (1)}$$

We know,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow 1 - v^2/c^2 = \left(\frac{m_0}{m}\right)^2$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2$$

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differentiate with respect to mv,

$$2m\,dm\,c^2 - 2m\,dm\,v^2 - 2v\,dv \cdot m^2 = 0$$

$$\Rightarrow c^2 m\,dm = m^2 v\,dv + v^2 m\,dm$$

$$\Rightarrow c^2 dm = mv\,dv + v^2 dm \quad \text{——— (11)}$$

From equ (1) Put the value,

$$K.E = \int_0^{mv} c^2 dm$$

$$= \int_{m_0}^{m} c^2 dm$$

$$= c^2 [m]_{m_0}^{M}$$

$$= c^2 [m - m_0]$$

$$= mc^2 - m_0 c^2$$

$$\Rightarrow K.E + m_0 c^2 = mc^2$$

$$\therefore \boxed{E = mc^2}$$

$\because$ Total Energy
$E = K.E + P.E$
$= K.E + m_0 c^2$

☐ What is the special theory of Relativity ?

In Physics, Special theory of relativity defines as it is the generally ~~and~~ accepted and experimentally well-confirmed physical theory regarding the relation between space and time.