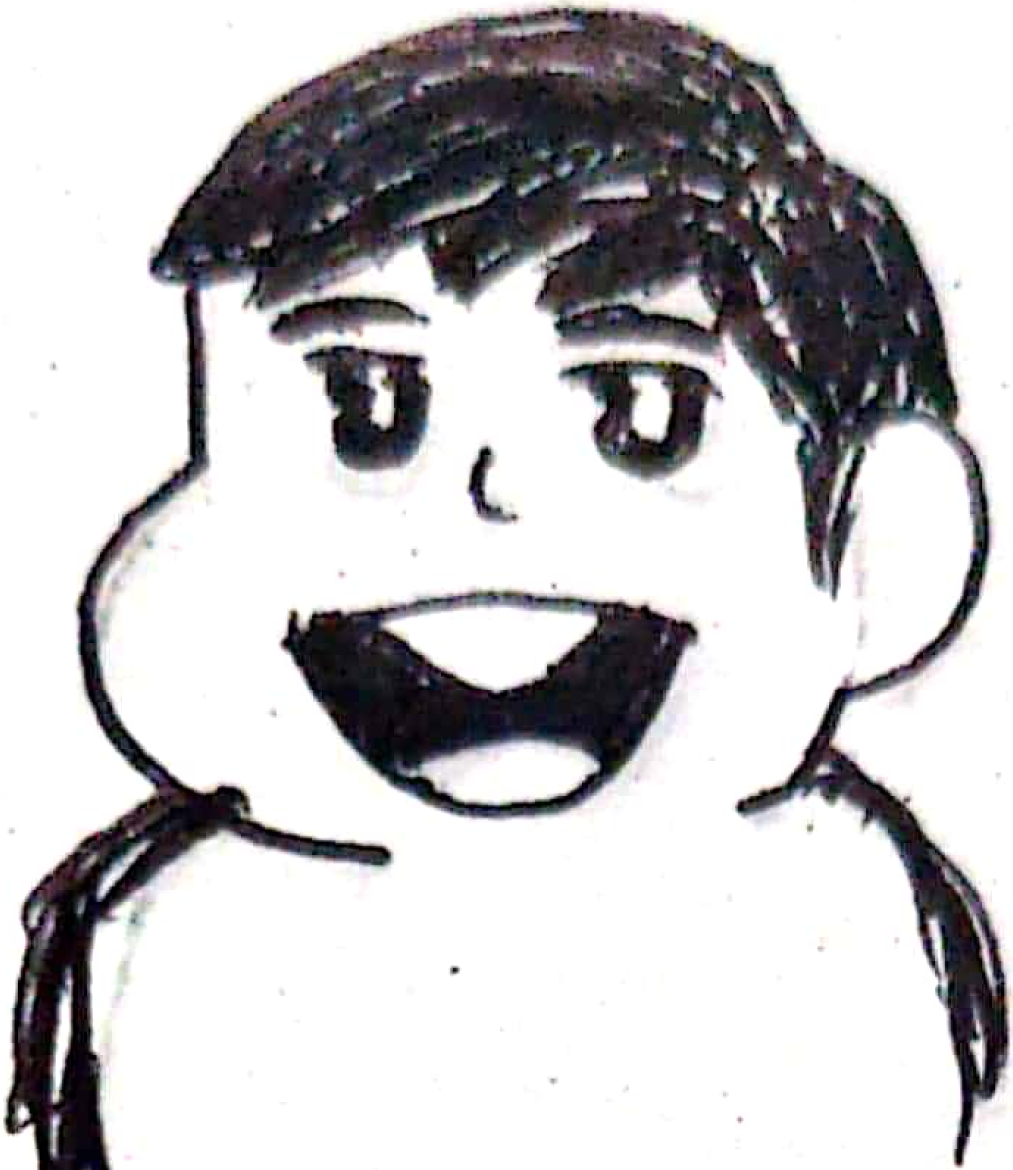

"Abdullah-Al-Mamun" (CE-13)

Relativity

What is special theory of relativity? Write the postulates of special theory of relativity. 13

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The special theory of relativity is length / space, time and mass.

Length/Space, time, mass

Postulates / General postulates of Einstein's special theory of relativity :-

Einstein's
Einstein's
Einstein's

① The laws of physics may be expressed in equations having the same form in all inertial frames of reference i.e., frames of reference moving at constant velocity with respect to one another.

(2)

(11) The velocity of light in free space (vacuum) is a constant (same for all observers), independent ~~non~~ not only of the direction of propagation but also of the relative velocity of the source and the observer.

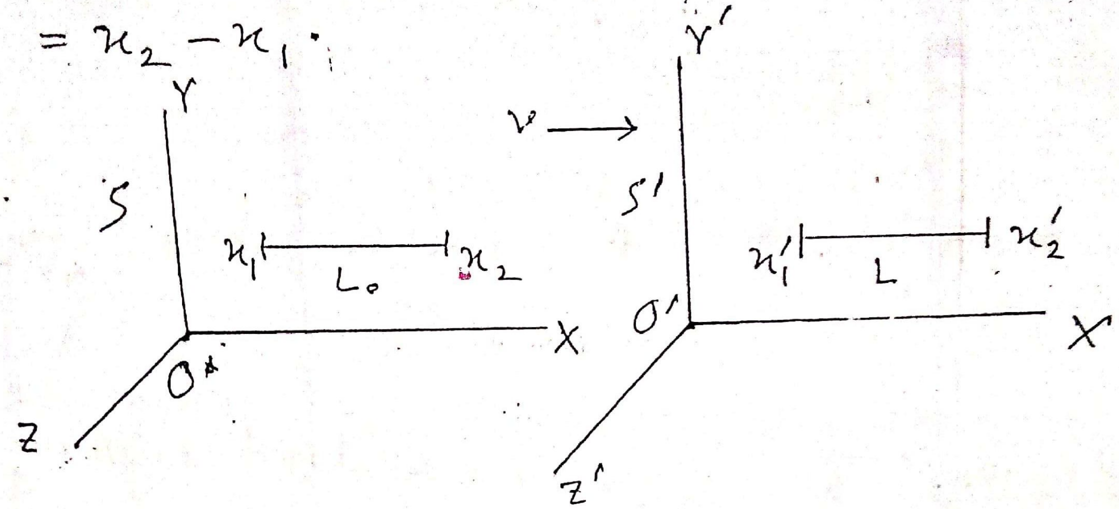
□ Define length contraction and time dilation and also derive their expressions. [13]

<u>Length contraction:</u> ~~Then~~ The length of an object is measured to be shorter when it is moving than when it is at rest.

<u>Derivation:</u> Let a rod of length L_0 parallel to the x axis and having the co-ordinates x_2 and x_1 in the reference frame S.

An observer in the reference frame S measures the length of the rod as

$L_0 = x_2 - x_1$.



Consider another reference frame S' which is with a velocity v with respect to S. The observer measured the length from this frame.

$$L = x_2' - x_1'$$

According to inverse ~~lorength~~ Lorentz transformation we get,

$$x_1 = \frac{x_1' + vt'}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad x_2 = \frac{x_2' + vt'}{\sqrt{1 - v^2/c^2}}.$$

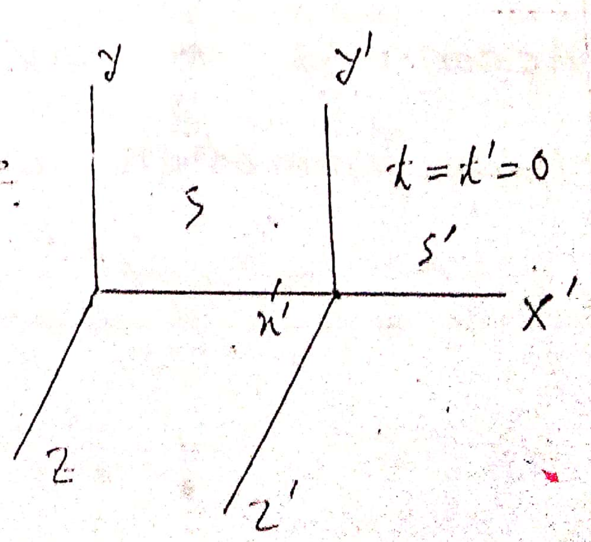
$$\therefore L_0 = \frac{x_2' - x_1'}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow L_0 = \frac{L}{\sqrt{1 - v^2/c^2}}$$

$$\therefore L = L_0 \sqrt{1 - v^2/c^2}$$

Time dilation: The time measured with a reference frame at rest is larger ~~that~~ than the time measured from a reference frame moving with a constant velocity.

Derivation: Suppose at any instant the two reference frame co-incide at $t = t' = 0$. The observer in S' notes the time at any instant in his clock is t_1' and



~~and~~ the observer in S notes the ~~the~~ time as t_1

Let t_2' and t_2 be the times measured by the two observer at the same instant. Consider t_0 be the interval of time as assumed by the observer in S' and t the interval of time measured by the observer in S

$$\therefore t_0 = t_2' - t_1' \quad \text{———} \quad (i)$$

$$t = t_2 - t_1 \quad \text{———} \quad (ii)$$

According to inverse of Lorentz transformation

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \text{and} \quad t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

$$\therefore t = \frac{t_2' - t_1'}{\sqrt{1 - v^2/c^2}} \quad \left[\text{From (i) and (ii)}\right]$$

$$\Rightarrow t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

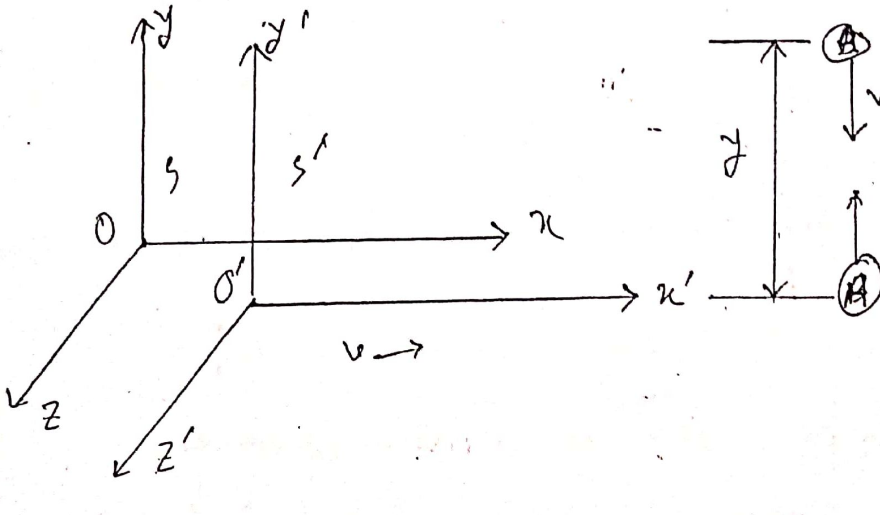
06

Define rest mass and effective mass of a body. Prove that the relativistic formula, $m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$ [12]

Rest mass: The physical mass of a body when it is regarded as being rest. The mass of an object that is at rest relative to an observer is called rest mass. The increase in motion decreases the rest masses.

Effective mass: The physical mass of a body when it is regarded as being motion. The mass of an object that is in motion relative to an observer is called effective mass. The increase in motion, increases the effective mass of a body

<u>Proof</u>: consider the elastic collision between two particles A and B as witnessed by observers in the reference frame S and S' which are in uniform relative motion.



The frame S' is moving in the positive x direction with respect to S at the velocity v. Before the collision the particles A and B are respectively at rest in

(08)

s and s'. At the same instant A was thrown in the $+y$ direction at the speed v_A and B was thrown in the $-y$ direction at the speed v_B'.

$\therefore v_A = v_B'$

After collision A rebounds in the $-y$ direction and B rebounds in the $+y$ direction. Let the collision occurs at a distance $\frac{Y}{2}$.

$\therefore y = y' = \frac{Y}{2}$

The round trip time for A in S frame

$$T_0 = \frac{Y}{v_A} \quad \text{———} \quad (1).$$

and hence for B in S' frame

$$T_0 = \frac{Y}{v_B'}$$

In S the speed v_B is found

$$v_B = \frac{Y}{T} \quad \text{———} \quad (11)$$

where,

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}}$$

$$\therefore \quad v_B = \frac{Y\sqrt{1 - v^2/c^2}}{T_0} \qquad \text{[From (ii)]}$$

And for A the speed,

$$v_A = \frac{Y}{T_0}$$

According to the law of conservation of linear momentum,

$$m_A v_A = m_B v_B$$

$$\Rightarrow m_A \cdot \frac{Y}{T_0} = m_B \cdot \frac{Y\sqrt{1 - v^2/c^2}}{T_0}$$

$$\Rightarrow m_A = m_B \sqrt{1 - v^2/c^2}$$

$$\Rightarrow m_B = \frac{m_A}{\sqrt{1 - v^2/c^2}}$$

If $m_A = m_0$ and $m_B = m$ then from the relation

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Derive Einstein's mass-energy relation. [11]

<u>Derivation:</u> According to Newton's 2nd law of motion, we get,

$$F = \frac{d}{dt}(mv)$$

$$\Rightarrow F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{------} \quad (i)$$

Let force F is the cause of displacement dx. So, work done is F.dx and this work done is equal to kinetic energy.

$$dE_K = F.dx$$

$$= \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)dx \quad [\text{From (i)}]$$

$$= m\frac{dv}{dt}\cdot dx + v\frac{dm}{dt}dx$$

$$= m\frac{dx}{dt}\cdot dv + v\frac{dx}{dt}\cdot dm$$

$$= mv\,dv + v^2 dm \quad \text{------} \quad (ii)$$

Again, we know,

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2}{1 - v^2/c^2}$$

$$\Rightarrow m^2 = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2 c^2$$

$$\Rightarrow m^2c^2 = m_0^2c^2 + m^2v^2 \quad \text{———— (iii)}$$

differentiating this equation,

$$[d(m_0^2c^2) = 0]$$

$$2m\,dm\,c^2 = 0 + 2m\,dm\,v^2 + 2v\,dv\,m^2 \; [\cancel{m_0^2c^2} = 0]$$

$$\Rightarrow dm\,c^2 = v^2\,dm + mv\,dv$$

From eqn (ii)

$$dE_K = dm\,c^2 \quad \text{———— (iv)}$$

when, $E_K = 0$, then $m = m_0$

" $E_K = E_K$ then $m = m$

Integrating eqn (iv),

$$\int_0^{E_K} dE_K = c^2 \int_{m_0}^{m} dm$$

$$\Rightarrow E_K = c^2(m - m_0)$$

$$\Rightarrow E_K = mc^2 - m_0c^2 \quad \text{———— (v)}$$

Total energy = kinetic energy + rest mass energy

$$\Rightarrow E = E_k + m_0c^2$$

$$\Rightarrow E = mc^2 \cancel{- m_0c^2} + \cancel{m_0c^2} \quad [\text{From (v)}]$$

$$\therefore E = mc^2$$

And this is the Einstein's mass energy relationship.

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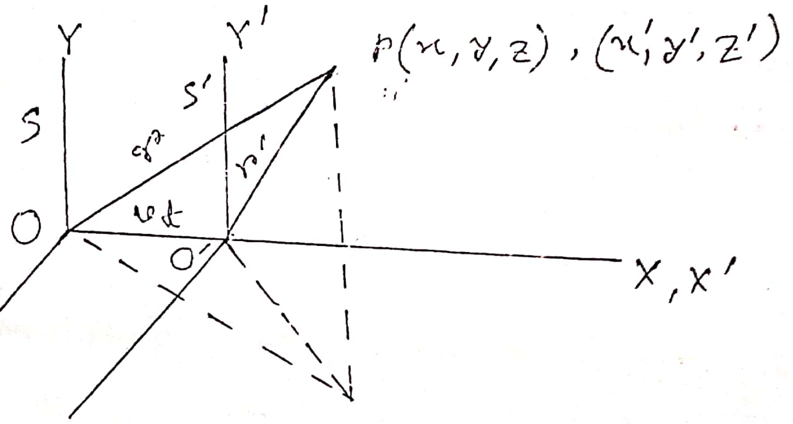
☐ Write a short note on Galileon's transformation.

Ans: The transformation of co-ordinates of a particle from one inertial frame of reference to another inertial frame of reference is called the Galileon's transformation.

Consider two inertial frames of referen ~~s(x,y,z) and~~ S and S'. The observer O' moves with a uniform velocity v relative to O along the x-axis, at time $t = 0$, O and O' are coincident. After time t,

$$OO' = vt$$



Now consider a particle at p.

Here, $r = r' + vt$

~~or, $r' = r +$~~

$\Rightarrow r' = r - vt$

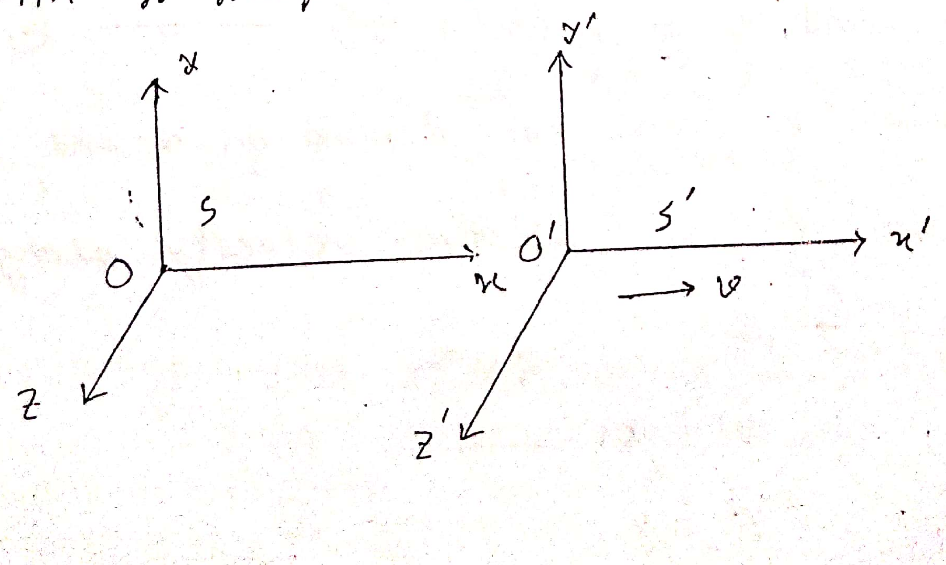
As v is parallel to x-axis separating the vector equation into three components

$$x' = x - vt$$
$$y' = y$$
$$z' = z \quad \text{and} \quad t' = t$$

These set of equations are called ~~Gstolle~~ Galileon's transformations because by using these equations, the results observed in one reference frame can be transformed the other reference frame.

☐ Write a short note on Lorentz transformation.

Let consider two frame S and S'. S' moves with a uniform velocity v with respect to S.



When time $t = t' = 0$, the observer of the frame s measured

$$x = ct \quad \text{——— (i)}$$

and the observer of the frame s' measured

$$x' = ct' \quad \text{——— (ii)}$$

From Galileon's transformation,

$$x' = x - vt$$

$$\text{and } x = x' + vt'$$

We cosidered the ~~eq~~ equation of the next time,

$$x' = k(x - vt) \quad \text{——— (iii)}$$

$$\text{and } x = k(x' + vt') \quad \text{——— (iv)}$$

Here k does not depend on x and t

There is no relative velocity along y and z axis,

$$y' = y \quad \text{——— (v)}$$

$$z' = z \quad \text{——— (vi)}$$

But the magnitude of t and t' is not equal.

From eqn (III) and (IV)

$$x = k\left[k(x - vt) + vt'\right]$$

$$\Rightarrow x = k^2x - k^2vt + kvt'$$

$$\Rightarrow kvt' = x - k^2x + k^2vt$$

$$\Rightarrow t' = \left(\frac{1-k^2}{kv}\right)x + kt \quad \text{——— (VII)}$$

Substituting the value of (III) and (VII) in

eqn (II)

$$k(x - vt) = c\left[\left(\frac{1-k^2}{kv}\right)x + kt\right]$$

$$\Rightarrow kx - kvt = ckt + \left(\frac{1-k^2}{kv}\right)cx$$

$$\Rightarrow x\left[k - \left(\frac{1-k^2}{kv}\right)c\right] = ckt + kvt$$

$$\Rightarrow x = \frac{ct\left(k + \frac{kv}{c}\right)}{\left[k - \left(\frac{1-k^2}{kv}\right)c\right]}$$

Substituting this value in eqn (I) we get,

(18)

$$\frac{ct\left(k+\frac{kv}{c}\right)}{\left[k-\left(\frac{1-k^2}{kv}\right)c\right]} = ct$$

$$\Rightarrow \quad k+\frac{kv}{c} = k-\left(\frac{1-k^2}{kv}\right)c$$

$$\Rightarrow \quad kv = -\left(\frac{1-k^2}{kv}\right)c^2$$

$$\Rightarrow \quad k^2v^2 = -c^2+k^2c^2$$

$$\Rightarrow \quad k^2(c^2-v^2) = c^2$$

$$\Rightarrow \quad k^2 = \frac{c^2}{c^2-v^2}$$

$$\Rightarrow \quad k^2 = \frac{c^2/c^2}{c^2/c^2 - v^2/c^2}$$

$$\Rightarrow \quad k^2 = \frac{1}{1-v^2/c^2}$$

$$\therefore \quad k = \frac{1}{\sqrt{1-v^2/c^2}}$$

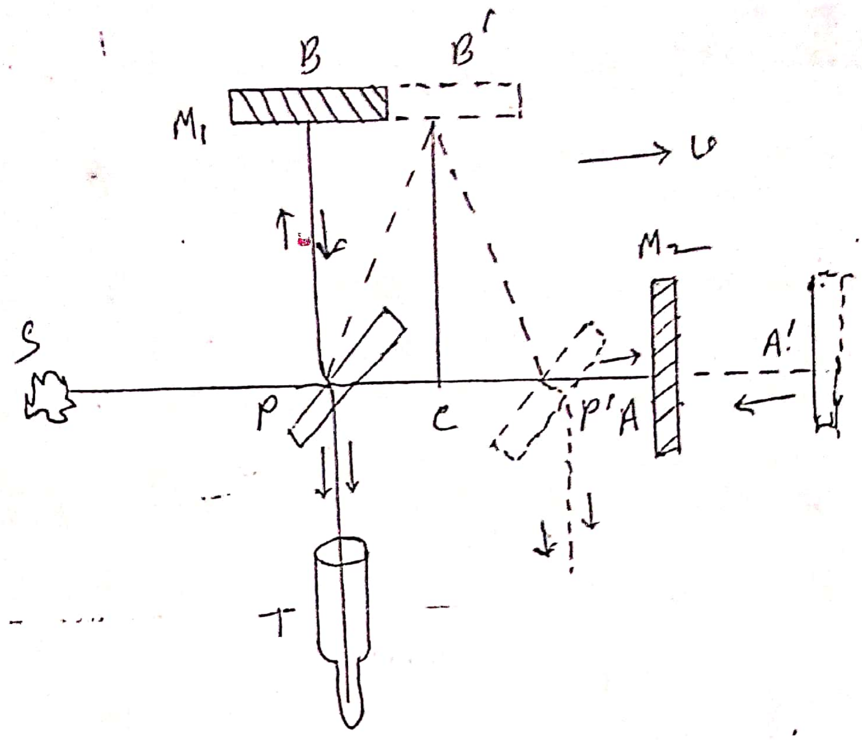
Substituting the value of k in eqn (iii) and (vii)

$$x' = \frac{x-vt}{\sqrt{1-v^2/c^2}} \quad \text{and} \quad x = \frac{x'+vt'}{\sqrt{1-v^2/c^2}}$$

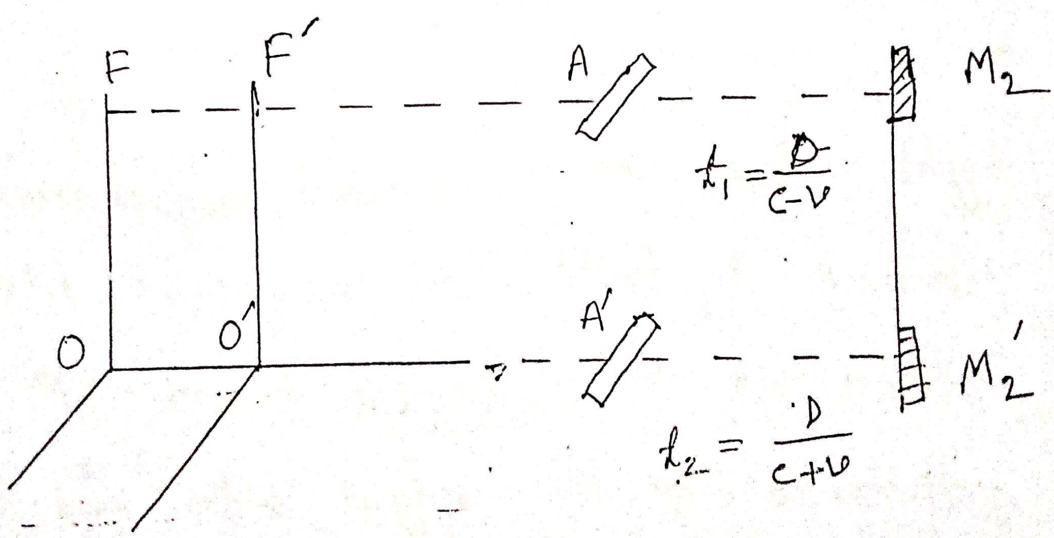
$$t' = \frac{t-vx/c^2}{\sqrt{1-v^2/c^2}} \quad \text{and} \quad t = \frac{t'+vx'/c^2}{\sqrt{1-v^2/c^2}}$$

And these equations are lorentz transformation.

Describe Michelson-Morley experiment. Discuss the results obtained. $\boxed{09}$



Light from an extended monochromatic source S falls on a glass plate 'p' place at 45° to the beam. P is half-silvered on its right side and this surface reflects half of the light up towards M_1 while the other half is transmitted goes towards M_2. The reflected portion falls normally at B on M_1 and transmitted portion falls normal on at A on M_2. Both M_1 and M_2 reflect their beams ~~to~~ back towards P. The beam from M_1 is partly reflected at P and remainder goes on through to the telescope T. A ~~por~~ portion from M_2 is reflected at P and goes to telescope T and the rest goes through the glass plate and is lost.



F is a fixed frame corresponding to the ether medium and F' is the ~~ref~~ reference frame moving with a velocity v.

The velocity of the light in the direction of the movement of the frame $F' = c - v$ and in opposite direction is $c + v$. Let t_1 be the time taken by light to travel from A to M_2 and t_2 be the time to travel from M_2' to A'.

The total time taken by light,

$$t = t_1 + t_2$$

$$= \frac{D}{c-v} + \frac{D}{c+v}$$

$$= \frac{2Dc}{c^2 - v^2}$$

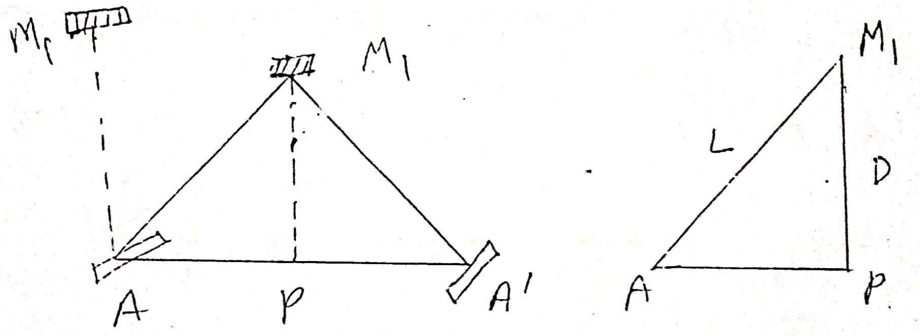
The total distance travelled by the light, $x_1 = t \times c$

$$= \frac{2Dc^2}{c^2 - v^2}$$

$$= 2D \left\{ \frac{c^2+v^2}{c^2} + \frac{v^4}{c^2(c^2-v^2)} \right\}$$

$$\Rightarrow x_1 = 2D\left(1+\frac{v^2}{c^2}\right) \quad \text{——} \quad (1)$$

Let the time taken by the light to travel from A to M_1' be t'. Here M_1 is shifted to M_1' in the time A is shifted to P



The distance $AP = vt'$

But $t' = \frac{L}{c}$

$\therefore AP = \frac{vL}{c}$

From the triangle we get,

$$L^2 = D^2 + \left(\frac{vL}{c}\right)^2$$

$$\Rightarrow L^2\left(1-\frac{v^2}{c^2}\right) = D^2$$

$$\Rightarrow L^2 = \frac{D^2}{\sqrt{\;}}$$

$$\Rightarrow L^2 = \frac{D^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

$$\Rightarrow L = D\left(1 + \frac{v^2}{2c^2}\right) \quad \left[\text{because } \frac{v^2}{c^2} << 1\right]$$

$\therefore$ Total distance travelled by light in time t in going from A to M_1' and back to A' is

$$2L = 2D\left(1 + \frac{v^2}{2c^2}\right)$$

$$\therefore x_2 = 2D\left(1 + \frac{v^2}{2c^2}\right) \quad \text{—— (ii)}$$

From eq^n (i) and (ii), path difference,

$$x_2 - x_1 = 2D\left(1 + \frac{v^2}{2c^2}\right) - 2D\left(1 + \frac{v^2}{c^2}\right)$$

$$= \frac{Dv^2}{c^2}$$

If the apparatus is turned through 90°, the path difference will be $-\frac{Dv^2}{c^2}$.

$\therefore$ The displacement in the inteference fringe is $\frac{Dv^2}{c^2}$.

This negative result suggests that

(24)

the velocity of light is invariable and remains constant in all directions.

~~Thus~~

Define photo electric effect and obtain the Einstein's photoelectric equation. [10]

Photoelectric effect: When light of sufficient high frequency (ultraviolet x-rays γ-rays etc) falls upon a metal surface, electrons are emited from it. This phenomena is known as photo-electric effect. The emitted electrons are called photoelectrons and the current constitutes by the electrons are colle photo-current or photo-electricity.

Einstein's photo-electric equation Considers a photon of energy $h\nu$ incidents upon a metal surface and reject a photo-electron of velocity v then

$$h\nu = \frac{1}{2}mv^2 + W_0 \quad \text{——— (1)}$$

where, W_0 is the work done or energy

spend in just reject the electrons outside the surface of the metal.

If W_0 is the photo-electric work function ... then the frequency of light ν_0 is required for the purpose, given by,

$$W_0 = h\nu_0$$

$$\therefore \nu_0 = \frac{W_0}{h} \quad \text{——— (ii)}$$

Here, ν_0 is known as the threshold frequency. When a radiation of frequency, ν is greater than the threshold frequency, ν_0 then the difference in energy $(h\nu - h\nu_0)$ is used, so that,

$$h\nu = \frac{1}{2} m v_{max}^2 + W_0$$

$$\therefore h\nu = \frac{1}{2} m v_{max}^2 + h\nu_0 \quad \text{——— (iii)}$$

eqn (iii) is known as the Einstein's photo-electric equation.

④ Write down the laws of photoelectric effect [07]

(i) The strength of the photo-electric current (i.e. the number of electrons emitted per second) is directly proportional to the intensity of light or radiation used, provided the frequency of the radiation is kept constant.

(ii) keeping intencity and frequency constan of incident light, the photo electric current will be decreased with the increase of retarding potential v.

(iii) The maximum velocity of an electron emitted varies linearly with the frequency of the incident light but is independent of its intensity.

(iv) The photoelectric emission is an instantaneous process. If there is any time lag between the arrival of light at a metal surface and the emission photo-electrons, it should be less than 3×10^{-9} seconds.

11 State and explain the compton effect. Show that the wavelength of the scattered photon is greater than the wavelength of the incident photon. 12

Compton effect: The scattering of a photon by an electron is called the compton effect. Both energy and momentum are conserved in such an event and as a result the scattered photon has less energy than the incident photon.

A collision between the incident photon and an electron is showed in the figure below. In this process, an x-ray photo strikes on electron and is scattered away from its original direction of motion and begins to move.

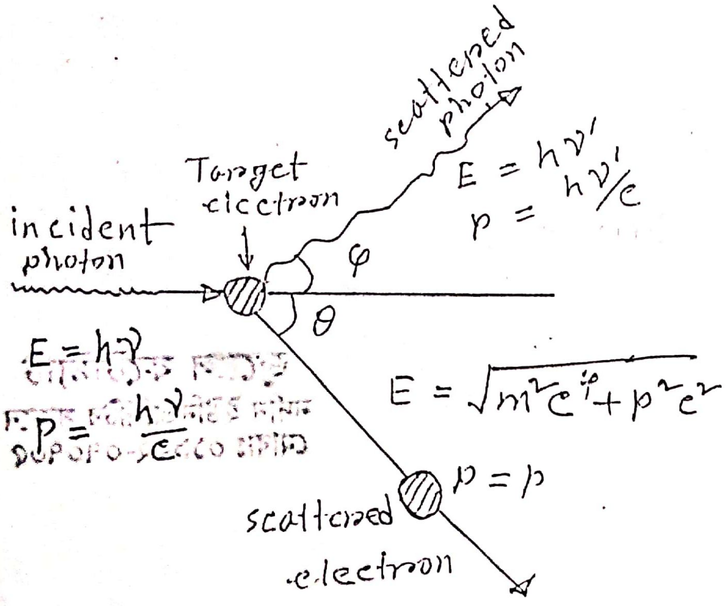


Fig : (a)

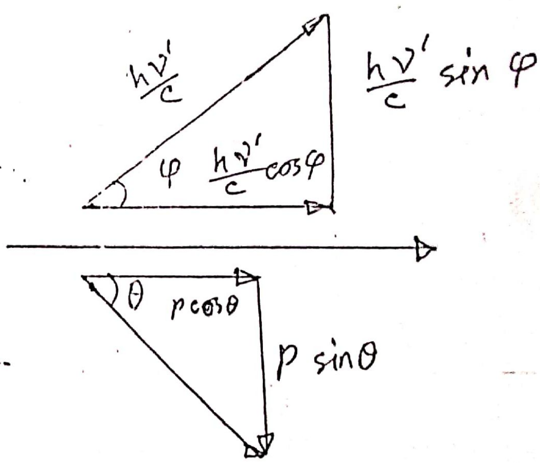


Fig: (b) vector diagram.

Then the photon is losing an amount energy in the collision, that is the same as the kinetic energy (KE) gained by the electron. If the incident photo has the frequency ν associated with it, the scattered photon has the lower frequency ν'

Loss in photon energy = gain in electron energy

$$\Rightarrow \quad h\nu - h\nu' = KE \quad \text{———} \quad (1)$$

The momentum of massless particle is related to its energy by, $E = pc$ ——— (2)

Since the energy of a photon is $h\nu$, its momentum is

$$p = \frac{E}{c} = \frac{h\nu}{c} \quad \text{———} \quad (3)$$

The momentum of the incident and scattered photon are $\frac{h\nu}{c}$ and $\frac{h\nu'}{c}$ respectively and the electron are 0 and p.

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Initial momentum = Final momentum

$$\Rightarrow \quad \frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\varphi + p\cos\theta \quad \text{———} \quad (4)$$

and perpendicular to this directio

$$0 = \frac{h\nu'}{c}\sin\varphi - p\sin\theta \quad \text{———} \quad (5)$$

Here φ is the angle between the directions of the initial and scattered photons and θ is