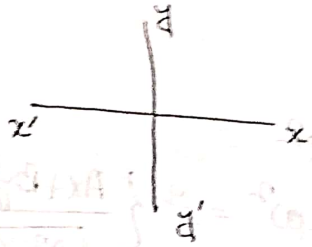
A.E day

Math (MMR)

Course no – 1201

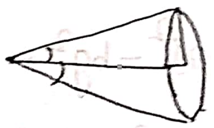
Geometry (2D+3D):



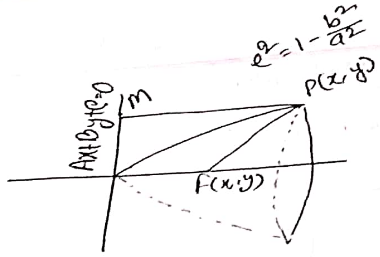
$y = f(x)$
↓
Independent

$x^2 - y^2 = a^2$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$



angle গুলো সমান → semi vertical angle বলে



$\frac{PF}{PM} = e$

$e=1$
$e>1$
$e<1$

$\Rightarrow PF^2 = PM^2 \times e^2$

$\Rightarrow (x-\alpha)^2 + (y-\beta)^2 = e^2 \left\{ \frac{Ax+By+C}{\sqrt{A^2+B^2}} \right\}^2$

$(\quad)^{a} x^2 + (\quad)^{2h} xy + (\quad)^{b} y^2 + (\quad)^{2g} x + (\quad)^{2f}$

$+ \underset{c}{con} = 0$

$\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

যদি 0 → সরলরেখা হয় (২টি)
" 0 না হয় → curve হবে

① $a+b=0$ ⟶ ~~Strate~~ সরলরেখা perpendicular হবে
⑪ $h^2 - ab = 0$ ⟶ " ২টি parallal

$e=1$
$e>1$
$e<1$

① $\Delta \neq 0$ $\quad a=b$, $h=0$ (Circle)

② $\Delta \neq 0$ $\quad ab-h^2=0 \longrightarrow$ Parabola (পরাবৃত্ত)

③ $\Delta \neq 0$ $\quad ab-h^2>0 \longrightarrow$ ellipse (উপবৃত্ত)

$9x^2+24xy+16y^2+22x+46y+9=0$

$$ab-h^2 = 9\times16-(12)^2=0$$

$$(3x+4y)^2+22x$$

$25x^2+2xy+25y^2-130x-130y+160=0$

$$25\times25-1^2=?>0$$

④ $\Delta \neq 0$ $\quad ah-h^2<0 \longrightarrow$ hyperbola

$$6x^2+5xy-6y^2-4x+7y+160$$

$$(6)(-6)-\left(\tfrac{5}{2}\right)^2 = ? <0$$

হতে

Pair of straigth line: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$x+2y+3=0$

$2x+3y+1=0$

$(x+2y+3)(2x+3y+1)=0$

$\Rightarrow 2x^2+7xy+6y^2+7x+11y+3$

$a = 2 \quad b = 6$

$c = 3 \quad h = 7/2$

$g = 7/2 \quad f = 11/2$

~~$abc + 2hxy + ax^2 = by^2 + ch^2$~~

$abc + 2fgh - af^2 - bg^2 - ch^2$

$= 0$

$x+2xy+3=0$

$2x+3y+1=0$

$Ax^2+Bx+C = 0$

$$\Rightarrow x = \frac{-B \pm \sqrt{B^2-4AC}}{\cancel{2b}\ 2A}$$

$ax^2 + 2(hy+g)x + (by^2+2fy+c)=0$

$$x = \frac{-2(hy+g) \pm \sqrt{4(hy+g)^2 - 4\times A(by^2+2fy+c)}}{2a}$$

$Ax^2+Bx+C=0$

$$\Rightarrow x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

x_1, x_2

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$\Rightarrow ax^2+2(hy+g)x+by^2+2fy+c=0$

$$x=\frac{-2(hy+g)\pm\sqrt{4(hy+g)^2-4a.(by^2+2fy+c)}}{2.a}$$

$$=\frac{-2(hy+g)\pm 2\sqrt{(h^2-ab)y^2+2y(hg-fa)+(g^2-ac)}}{2a}$$

$$=\frac{-(hy+g)\pm\sqrt{(h^2-ab)y^2+2y(hg-fa)+(g^2-ac)}}{a}$$

$\boxed{b^2-4ac=0}$

$4(hy+g)^2-4a(by^2+2fy+c)=0$

$\Rightarrow (hy+g)^2=a(by^2+2fy+c)$

$2fy+c=0$

$(3y+1)=($

$6y^2+7$

$b=6$

$ha=3$

$f=$

$2xy+$

$+3y+$

Vaxitet

$+2(hy+g)x+(by^2+2fy+c)=0$

$$x=\frac{-2(hy+g)\pm\sqrt{4(hy+g)^2-4\times A(by^2+2fy+}}{2a}$$

$-fa)^2 - 4\cdot(h^2-ab)(g^2-ac)=0$

$-fa)^2 - (h^2-ab)(g^2-ac)=0$

$g^2 - 2hgfa + f^2a^2 - (h^2g^2 - h^2ac$

$- abg^2 + a^2bc) = 0$

$g^2 - 2hgfa + f^2a^2 - \cancel{h^2g^2} + h^2ac + abg^2 - a^2bc = 0$

$2hgf + f^2a + h^2c + bg^2 - abc = 0$

eh $\boxed{abc + 2hgf - af^2 - bg^2 - ch^2 = 0}$

$\Rightarrow 2x^2 + 3xy + x + 4xy + 6y^2 + 2\ldots$

$+ 6x + 9y + 3 = 0$

Comparison $\Rightarrow 2x^2 + 7xy + 6y^2 + 7x + 11y + 3 = 0$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$a = 2$

$h = \frac{7}{2}$ $\quad f = \frac{11}{2}$

$b = 6$ $\quad c = 3$

$x + 2y + \ldots$

$(hg-fa)^2 - (h^2-ab)(g^2-ac) = 0$

$\Rightarrow h^2g^2 - 2hgfa + f^2a^2 - (h^2g^2 - h^2ac - abg^2 + a^2bc) = 0$

$\Rightarrow h^2g^2 - 2hgfa + f^2a^2 - h^2g^2 + h^2ac + abg^2 - a^2bc = 0$

$\Rightarrow -2hgf + f^2a + h^2c + bg^2 - abc = 0$

$\Rightarrow \boxed{abc + 2hgf - af^2 - bg^2 - ch^2 = 0}$

$\Rightarrow 2x^2 + 3xy + x + 4xy + 6y^2 + 2y + 6x + 9y + 3 = 0$

compare

$\Rightarrow 2x^2 + 7xy + 6y^2 + 7x + 11y + 3 = 0$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$a = 2$, $h = \frac{7}{2}$, $b = 6$, $g = \frac{7}{2}$, $f = \frac{11}{2}$, $c = 3$

$x + 2y + 3 = 0$

$2x + 3y + 1 = 0$

$abc + 2hgf - af^2 - bg^2 - ch^2$

$= 2\times6\times3 + 2\times\frac{7}{2}\times\frac{7}{2}\times\frac{11}{2} - 2\left(\frac{11}{2}\right)^2 - 6\left(\frac{7}{2}\right)^2 - 3\times\left(\frac{7}{2}\right)^2$

$= 0$

Vaxitet

$ax^2 + 2(hy+g)x + (by^2 + 2fy + c) = 0$

$$x = \frac{-2(hy+g) \pm \sqrt{4(hy+g)^2 - 4\times a(by^2+2fy+c)}}{2a}$$

$$x = \frac{-(hy+g) \pm \sqrt{(h^2-ab)y^2 + 2(gh-af)y + (g^2-ac)}}{a}$$

$$4(gh-af)^2 - 4(h^2-ab)(g^2-ac) = 0 \qquad \boxed{B^2 - 4AC = 0}$$

$$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

Pair of straight line

A-Day

25-08-19

Q.4

Q. Determine the angle between the lines represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$\Delta \equiv abc+2fgh-af^2-bg^2-ch^2=0$

$l_1x+m_1y+h_1=0$

$l_2x+m_2y+h_2=0$

$x^2 \quad m \quad y^2 \quad x \quad y$

$(l_1x+m_1y+h_1)(l_2x+m_2y+h_2)=0$

$n_1m_2+n_2m_1=2f,$

$n_1l_2+n_2l_1=2g$

$l_1m_2+l_2m_1=2h$

(i) $\rightarrow$

$y=-\frac{l_1x}{m}-\frac{n_1}{m_1}$

(ii) $\longrightarrow$

$y=-\frac{l_2x}{m_2}-\frac{n_2}{m_2}$

$\tan\theta=\frac{-\frac{l_1}{m_1}-\left(-\frac{l_2}{m_2}\right)}{1+\left(\frac{l_1}{m_1}\right)\left(-\frac{l_2}{m_2}\right)}$

$\left[\tan\theta=\frac{m_1-m_2}{1+m_1m_2}\right]$

$$\Rightarrow \tan\theta = \frac{-\frac{l_1}{m_1} + \frac{l_2}{m_2}}{1 \mp \frac{l_1 l_2}{m_1 m_2}}$$

$(a-b)^2$

$$\Rightarrow \tan\theta = \frac{-l_1 m_2 + l_2 m_1}{m_1 m_2 - l_1 l_2}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{(m_1 l_2 + m_2 l_1)^2 - 4 l_1 l_2 m_1 m_2}}{l_1 l_2 + m_1 m_2}$$

$$\Rightarrow \boxed{\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}}$$

$h^2 - ab = 0$ (parallal)

$a+b=0$ यदि $\theta = 90^\circ$ (perpendicular हैं)

$$\boxed{\begin{array}{l} \theta = 90^\circ \longrightarrow h^2 - ab = 0 \\ \theta = 0^\circ \longrightarrow h^2 - ab = 0 \end{array}}$$

When $\theta = 0^\circ \rightarrow$

$h^2 - ab = 0$ — (i)

$\frac{a}{h} = \frac{\cancel{b}h}{b}$ — (ii)

Q. Show that the eqn $ax^2+2hxy+by^2+2gx+2fy+c=0$ repr... two parrallal straigth lines if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$??

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow abc + 2gf\sqrt{ab} - \sqrt{a}f^2 - \sqrt{b}g^2 - cab = 0$

$\Rightarrow (f\sqrt{a} - g\sqrt{b})^2 = 0$

$\Rightarrow \sqrt{a}f = g\sqrt{b}$

$\Rightarrow \sqrt{ab}f = bg$

$\Rightarrow hf = bg$

$\Rightarrow \frac{h}{b} = \frac{g}{f}$

$\therefore \boxed{\frac{a}{h} = \frac{h}{b} = \frac{g}{f}}$

[Proved]

OR Prove that $\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$

$$\boxed{ax^2+2hxy+by^2=0}$$

$y=m_1x$ $\quad$ $y=m_1x+c_1$

$y=m_2x$ $\quad$ $y=m_2x+c_2$

$$(ax^2+2hxy+by^2)=(y-m_1x)(y-m_2x)$$

$$(y-m_1x)(y-m_2x)=0$$

$$\boxed{m_1m_2x^2+(m_1+m_2)xy+y^2=0}$$

$$\frac{m_1m_2}{a}=\frac{-(m_1+m_2)}{2h}=\frac{1}{b}$$

$$m_1m_2=\frac{a}{b}$$

$$m_1+m_2=-\frac{2h}{b}$$

$$\boxed{\tan\theta=\frac{m_1-m_2}{1+m_1m_2}}=\frac{2\sqrt{h^2-ab}}{a+b}$$

$$=\frac{\sqrt{(m_1+m_2)^2-4m_1m_2}}{1+m_1m_2}$$

$$=\frac{2\sqrt{h^2-ab}}{a+b}$$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

$$\boxed{\begin{array}{l} l_1x + m_1y + h_1 = 0 \\ l_2x + m_2y + h_2 = 0 \end{array}}$$

Parallel equation

$l_1x + m_1y = 0$ [Pass (0,0) point]

$l_2x + m_2y = 0$ [" " "]

$(l_1x + m_2y)(l_2x + m_2y) = 0$

$\Rightarrow l_1l_2x^2 + (l_1m_2 + l_2m_1)xy + m_1m_2y^2 = 0$

$\Rightarrow ax^2 + 2hxy + by^2 = 0$

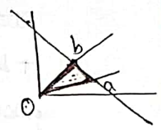
[Proved]

$ax^2+2hxy+by^2=0$

$lx+my+h=0$

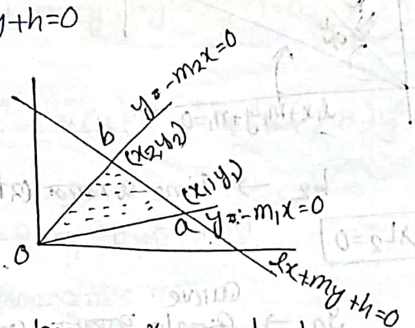
$y=m_1x$

$y=m_2x$



find the area of triangle OAB fromed by the li·

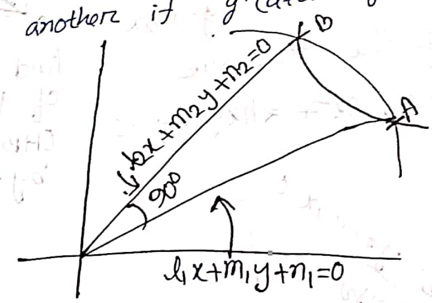
Find the area of the triangle formed by the line $ax^2+2hxy+by^2=0$ and $lx+my+h=0$



$$\Delta Oab = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & 1 \\ y_1 & y_2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}(x_1y_2 - x_2y_1)$$

$$\left.\begin{aligned} lx_1+my_1+h=0 \\ y-m_1x_1=0 \end{aligned}\right\} \begin{aligned} x_1=0 \\ y_1=0 \end{aligned} \quad \left|\quad \left.\begin{aligned} l_2x_2+m_2y_2+h_2=0 \\ y_2-m_2x_2=0 \end{aligned}\right\} \begin{aligned} x_2=0 \\ y_2=0 \end{aligned}\right.$$

☐ Show that the straight lines joining the angle to the point of intersection of the two corners $ax^2+2hxy+by^2+2gx=0$ and $a'x^2+2h'xy+b'y^2+2g'x=0$ will be at right angle to one another if $g'(a+b)=g(a'+b')$



L_1 L_2 → Line হলে (ধরি)

$\boxed{L_1+\lambda L_2=0}$

S_1 S_2 → curve ~~Circle~~ হলে (ধরি)

$\boxed{S_1+\lambda S_2=0}$

$\lambda=?$

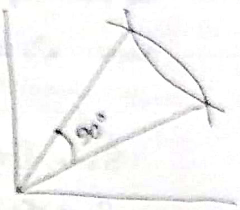
31-08-19

E-Day

Ans:

$ax^2+2hxy+by^2+2gx=0$

$a_1'x^2+2h'xy+b'y^2+2g'x=0$



$ax^2+2hxy+by^2+2gx+$

$+\lambda(a_1x^2+2h'xy+b'y^2+2g'x)=0$

$\Rightarrow (a+\lambda a_1')x^2+2(h+h')xy+(b+\lambda b')y^2+2(g+g')x=0$ —— (iii)

$ax^2+2hxy+by^2$ —— (iv)

(iii) ও (iv) homogenius হলে,

$g+\lambda g'=0$

$\Rightarrow \lambda=-\frac{g}{g'}$

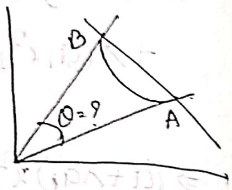
$\therefore ax^2+2hxy+by^2+2gx-\frac{g}{g'}(a'x^2+2h'xy+b'y^2+2g'x)=0$

[illegible] perpendicular [illegible]

$\Rightarrow (g'a-ga')x^2+(bg'-gb')=0$

$\Rightarrow g(a'+b')=g'(a+b)$ [Proved]

Q. Find the angle between the lines joining the angle to the point of intersection of the lines $y = 3x+2$ at the curve $x^2 + 2xy + 5y^2 + 4x + 8y - 11 = 0$.



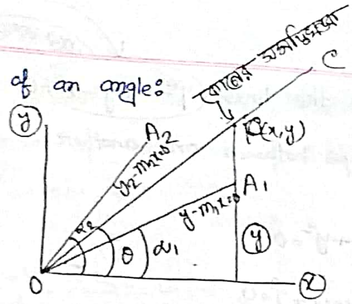
$$\left.\begin{aligned} S + \lambda L &= 0 \\ S_1 + \lambda S_2 &= 0 \end{aligned}\right\} \text{— (1)}$$

$$ax^2 + 2hxy + by^2 = 0 \text{ — (ii)}$$

- - - - - - -

$$\theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a+b}$$

By section of an angle:



$$\boxed{\angle A_1OC = \angle A_2OC}$$

$$m_1 = \tan\alpha_1$$
$$m_2 = \tan\alpha_2$$

$$\angle XOC = \angle XOA_1 + \angle A_1OC$$
$$= \angle XOA_1 + \tfrac{1}{2}\angle A_1OA_2$$
$$= \angle XOA_1 + \tfrac{1}{2}\{(\angle XOA_2 - \angle XOA_1)\}$$
$$= \alpha_1 + \tfrac{1}{2}(\alpha_2 - \alpha_1)$$

$$\Rightarrow 2\angle XOC = 2\alpha_1 + \alpha_2 - \alpha_1$$

$$\Rightarrow 2\angle XOC = \alpha_1 + \alpha_2$$

$$\Rightarrow \tan(2\theta) = \tan(\alpha_1 + \alpha_2)$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{\tan\alpha_1 + \tan\alpha_2}{1-\tan\alpha_1\tan\alpha_2} = \frac{m_1+m_2}{1-m_1m_2}$$

$$\Rightarrow \frac{2\frac{y}{x}}{1-\frac{y^2}{x^2}} = \frac{-\frac{2h}{b}}{1-\frac{a}{b}} \quad \Big| \quad \Rightarrow \frac{x^2-y^2}{a-b} = \frac{xy}{h}$$

Prove that the lines $y^2-4ay-x^2=0$ and y^2+xy ... bisect the angle between one another

একে অপরের দ্বিখণ্ডক

$$x^2-4ay+y^2=0$$

$$\Rightarrow -x^2-4xy+y^2=0$$

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

according

$$\frac{x^2-y^2}{-2}=\frac{xy}{2}$$

$$\Rightarrow y^2+xy-x^2=0$$

$$-x^2+xy+y^2$$

$$\Rightarrow \frac{x^2-y^2}{-1-1}=\frac{xy}{1/2}$$

$$\Rightarrow x^2-y^2=-2(2xy)$$

$$\Rightarrow y^2-4xy+x^2=0$$ [Proved]

Show the equation the lines bisecting the angle between the bisectors of the pair of lines $ax^2+2hxy+by^2=0$ is $(a-b)(x^2-y^2)+4hxy=0$

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

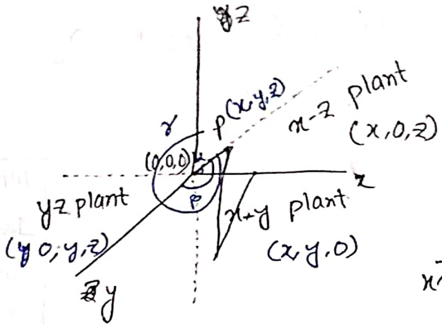
$$\Rightarrow h(\quad)x^2+(\quad)xy+(\quad)y^2=0$$

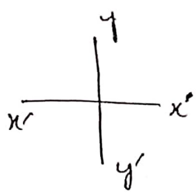
MMR 3.1 → Bisectors

1st & 2nd pair of straight lines.

3D

3D





Direction process:

x, y, z axis এর সাথে কত angle এ আছে

$\cos\alpha = \frac{x}{OP} = \frac{x}{r}$

$\Rightarrow x = r\cos\alpha = pl \quad [l = \cos\alpha]$

$\Rightarrow \cos\beta = \frac{y}{OP} = \frac{y}{r}$

$\Rightarrow y = r\cos\beta = rm \quad [m = \cos\beta)$

$\cos\gamma = \frac{z}{OP} = \frac{z}{r}$

$\Rightarrow z = r\cos\gamma = rn \quad (n = \cos\gamma)$

$\therefore x = rl$ ——— (i)

$y = rm$ ——— (ii)

$z = rn$ ——— (iii)

(i)^2 + (ii)^2 + (iii)^2

$x^2+y^2+z^2 = r^2(l^2+m^2+n^2)$

$\therefore OP^2 = r^2 = (x-0)^2+(y-0)^2+(z-0)^2$

$= x^2+y^2+z^2$

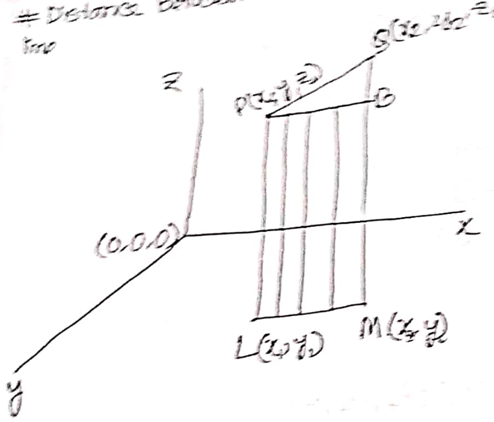
$\Rightarrow r^2 = r^2 \cdot (l^2+m^2+n^2)$

$\Rightarrow l^2+m^2+n^2 = 1$

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Distance between the two points of a straight line

A-Day
02-08-19
02-09-19



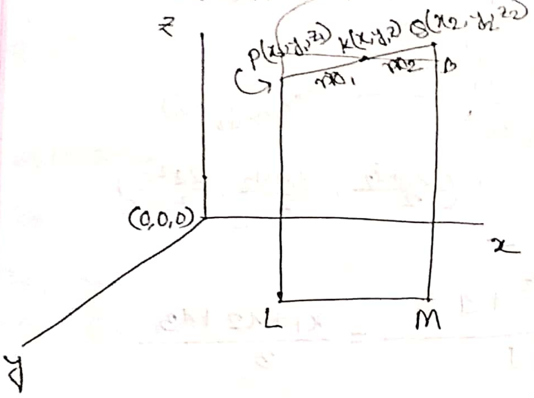
$\boxed{PQ^2 = ?}$

$$LM^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

ΔPBQ
$\angle PBQ$

$$PQ^2 = PB^2 + QB^2$$
$$= LM^2 + QB^2$$
$$= LM^2 + (QM - MB)^2$$
$$= (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_2 - z_1)^2$$
$$\Rightarrow PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

Section formula:



PQ কে AB সমান্তরাল AB করে।

$\triangle ADK$, $\triangle QBK$

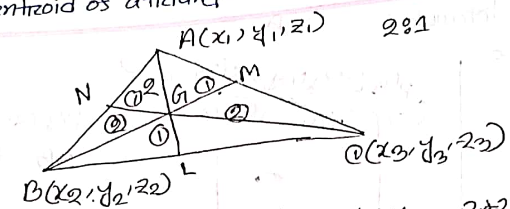
$$\frac{m_1}{m_2} = \frac{PK}{QK} = \frac{AP}{QP} = \frac{AL-PL}{QM-MB} = \frac{z-z_1}{z_1-z_2}$$

$$z = \frac{m_2 z_1 + m_1 z_2}{m_1 + m_2}$$

$$x = \frac{m_2 x_1 + m_1 x_2}{m_1 + m_2}$$

Centroid of a triangle:

2:1



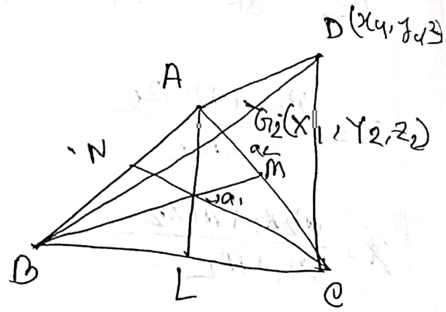
$$L\left(\frac{x_3+x_2}{2}, \frac{y_3+y_2}{2}, \frac{z_3+z_2}{2}\right)$$

$$X = \frac{2 \cdot x \cdot \frac{x_1+x_2}{2} + 1 \cdot x_1}{2+1} = \frac{x_1+x_2+x_3}{3}$$

$$Y = \frac{y_1+y_2+y_3}{3}$$

$$Z = \frac{z_1+z_2+z_3}{3}$$

Center of pyramid:



$$X = \frac{3 \cdot \frac{x_1+x_2+x_3}{3} + 1 \cdot x_1}{3+1}$$

$$= \frac{x_1+x_2+x_3+x_4}{4}$$

$a_1(X_1, Y_1, Z_1)$

$a_2(X_2, Y_2, Z_2)$

$$Y = \frac{y_1+y_2+y_3+y_4}{4}$$

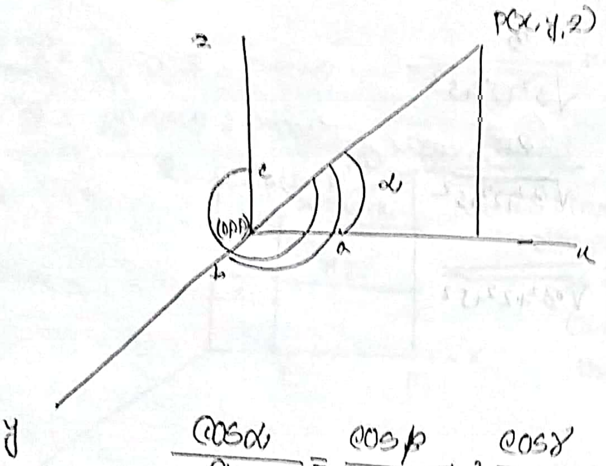
$$Z = \frac{z_1+z_2+z_3+z_4}{4}$$

Direction rotation of a line:

Direction rotation of a line:

Direction cosines:

Direction Ratio:



$$\frac{\cos\alpha}{a} = \frac{\cos\beta}{b} = \frac{\cos\gamma}{c} = \frac{\sqrt{\cos^2\alpha+\cos^2\beta+\cos^2\gamma}}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{1}{\sqrt{\Sigma a^2}}$$

$$\cos\alpha = \frac{a}{\sqrt{\Sigma a^2}}$$

$$\cos\beta = \frac{b}{\sqrt{\Sigma a^2}}$$

$$\cos\gamma = \frac{c}{\sqrt{\Sigma a^2}}$$

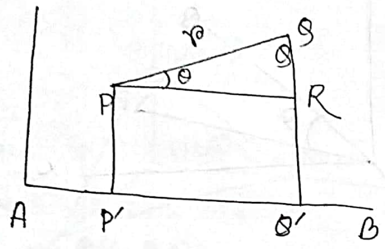
when $a=3$, $b=2$, $c=5$

$$\cos\alpha = \frac{3}{\sqrt{3^2+2^2+5^2}}$$

$$\cos\beta = \frac{2}{\sqrt{3^2+2^2+5^2}}$$

$$\cos\gamma = \frac{5}{\sqrt{3^2+2^2+5^2}}$$

⊞ Projection of a line:



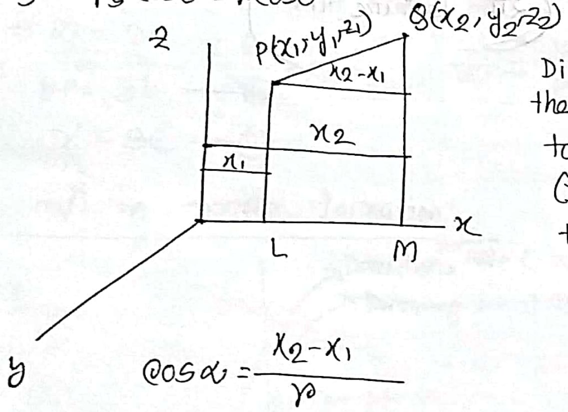
$\triangle PQR$

$$\cos\theta = \frac{PR}{PQ}$$

$\Rightarrow PR = PQ\cos\theta = r\cos\theta$

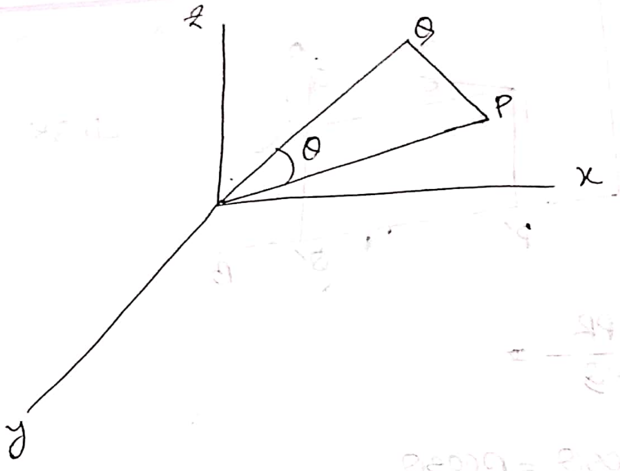
$P'Q' = PQ\cos\theta = r\cos\theta$

⊞



Direction cosine gives the proportional value to the direction ratio (Direction ratio-এর theory টা লিখতে হবে)

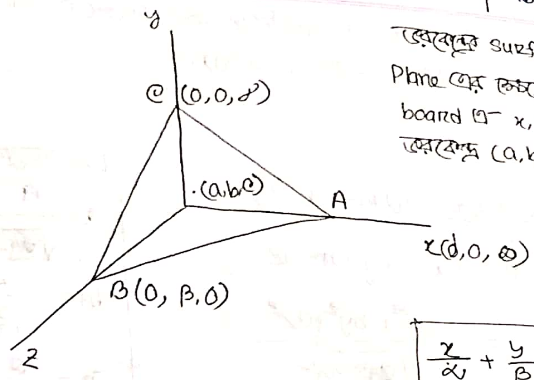
$$\cos\alpha = \frac{x_2 - x_1}{r}$$



(২টি Lecture বাকি)

মারিয়া

E-Day
16-09-19



কেন্দ্রের surface এর eqn:
Plane এর ক্ষেত্রে Drawing board এ x, y, z axis আছে
কেন্দ্র (a,b,c)।

$$\boxed{\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1}$$

$$a = \frac{x_1+x_2+x_3}{3} = \frac{\alpha+0+0}{3}$$

$$\Rightarrow \alpha = 3a \text{ — (i)}$$

$$\beta = 3b \text{ — (ii)}$$

$$\gamma = 3c \text{ — (iii)}$$

From eqn →

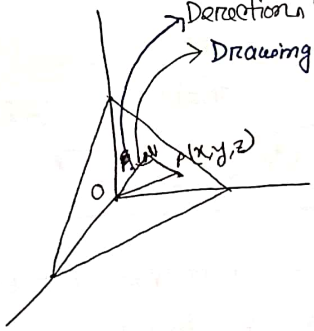
$$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$$

Plane eqn in normal formation:

Direction cosine of PQ = l m n

Drawing board এর উপর লম্ব

P is any point



OP = P = plan of projection of OP OP ON

$= (x-0)l + (y-0)m + (z-0)n$

$p =$

$P_1 \cos\theta = (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$

$\therefore P = lx + my + nz$

$= \frac{ax + by + cz}{\sqrt{\Sigma a^2}}$

D.C অনুযায়ী

$l = \frac{a}{\sqrt{\Sigma a^2}}$

$m = \frac{b}{\sqrt{\Sigma a^2}}$

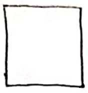
$n = \frac{c}{\sqrt{\Sigma a^2}}$

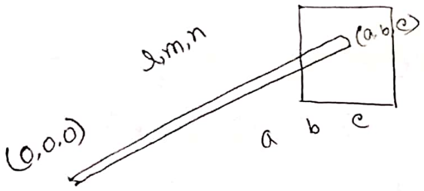
$ax + by + cz = P\sqrt{\Sigma a^2} \Rightarrow ax + by + cz = P\sqrt{\Sigma a^2} = -d$

constant

$\Rightarrow \boxed{ax + by + cz + d = 0}$

This is the general equation of plane

ঢ  → একটি আতার surface উপর একটি বিন্দু আছে x দিয়ে surface এর লম্ব বরাবর লম্ব টানলাম, লম্ব টানার ফলে surface এর eqn কত?



$\lambda = lx + my + nz$

$$l = \frac{a}{\sqrt{\Sigma a^2}} \qquad m = \frac{b}{\sqrt{\Sigma a^2}} \qquad n = \frac{c}{\sqrt{\Sigma a^2}}$$

$$\lambda = lx + my + nz = \frac{ax + by + cz}{\sqrt{\Sigma a^2}} = \frac{a^2 + b^2 + c^2}{(a^2 + b^2 + c^2)^{1/2}} = (a^2 + b^2 + c^2)^{1/2}$$

$$\frac{ax}{\sqrt{\Sigma a^2}} + \frac{by}{\sqrt{\Sigma a^2}} + \frac{cz}{\sqrt{\Sigma a^2}} = (a^2 + b^2 + c^2)^{1/2}$$

$$\Rightarrow \boxed{ax^2 + by^2 + cz^2 = (a^2 + b^2 + c^2)^{1/2}}$$

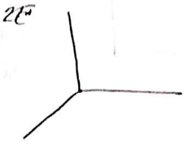
a,b,c → direction ratio of the normal (লম্ব)

[This is the equation of the point of the whole surface]

This is the ~~normal~~ eqn of plane in normal.

$\boxed{ax+by+cz+d=0 \text{ (Let)}}$ Plane shied এ eq^n কত?

এর জন্য perpendicular হবে



Perpendicular ভাবে Plan রাখলে তার eq^n কত?

$x=0$

$\Rightarrow ax + 0.y + 0.z = 0$

হত

$a.1+b.0+c=0=0$

x-axis এর জন্য

$\Rightarrow a=0$

$\boxed{by+cz+d=0}$ → This is the equation.

Similarly y, z axis বরাবর নিতে হবে

↳ $ax+cz+d=0$

$ax+by+d=0$

$2x+3y+ -z+4$

Determine the 3 planes theory in the intersection of the plans $x+y+z-1=0$ and $2x+3y-z=0$ ~~whist~~ which are parallal to the ~~tee~~ three condinates axises.

$L_1+\lambda L_2=0$ [যখন 2ft Straigth line দেয় করে]

$\Rightarrow (x+y+z-1)+\lambda(2x+3y-z+4)=0$

$\Rightarrow (x+y+z-1)+2\lambda x+3\lambda y-\cancel{2\lambda=0}-\lambda z+4\lambda=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z+(4\lambda-1)=0$

(i) if parallal to x axis then $(1+2\lambda)=0$

$\Rightarrow \lambda=-\frac{1}{2}$

Then the equation : $y-3z+6=0$

(ii) If parallal to y axis then $(1+3\lambda)=0$

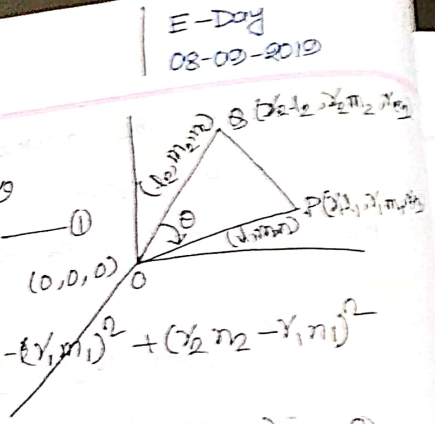
$\Rightarrow \lambda=-\frac{1}{3}$

(iii) If parallal to z axis then : $-1-\lambda=0$

$\Rightarrow \lambda=1$

Then equation : $3x+4y+3=0$

E-Day
08-09-2019



$\triangle OPQ$

$$PQ^2 = OP^2 + OQ^2 - 2OP.OQ\cos\theta$$
$$= r_1^2 + r_2^2 - 2r_1r_2\cos\theta \quad \text{--- (i)}$$

$$PQ^2 = (r_2 l_2 - r_1 l_1)^2 + (r_2 m_2 - r_1 m_1)^2 + (r_2 n_2 - r_1 n_1)^2$$
$$= r_1^2 + r_2^2 - 2r_1r_2(l_1l_2 + m_1m_2 + n_1n_2) \quad \text{--- (ii)}$$

$$l_1^2 + m_1^2 + n_1^2 = 1$$
$$l_2^2 + m_2^2 + n_2^2 = 1$$

(i) = (ii)

$$\Rightarrow \boxed{\cos\theta = l_1l_2 + m_1m_2 + n_1n_2}$$

$$\therefore \theta = \cos^{-1}(l_1l_2 + m_1m_2 + n_1n_2)$$

$$\therefore \sin^2\theta = 1 - \cos^2\theta = 1^2 \cdot 1^2 - \cos^2\theta = (l_1^2 + m_1^2 + n_1^2)(l_2^2 + m_2^2 + n_2^2) - (l_1l_2 + m_1m_2 + n_1n_2)^2$$

$$= (n_1m_2 - n_2m_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2$$

$$\therefore \sin\theta = \sqrt{\Sigma(n_1m_2 - n_2m_1)^2}$$

$\theta = 90^\circ$ হলে $\therefore \boxed{l_1l_2 + m_1m_2 + n_1n_2 = 0}$

$\theta = 0^\circ$ হলে $\quad \sin\theta = 0 \rightarrow \sqrt{\Sigma(n_1 m_2 - n_2 m_1)^2} = 0$

$$\Rightarrow n_1 m_2 - n_2 m_1 = 0$$

$$\Rightarrow \boxed{\frac{m_1}{m_2} = \frac{n_1}{n_2}}$$

$$n_1 l_2 - n_2 l_1 = 0$$

$$\Rightarrow \boxed{\frac{n_1}{n_2} = \frac{l_1}{l_2}}$$

$$l_1 m_2 - l_2 m_1 = 0$$

$$\Rightarrow \boxed{\frac{l_1}{l_2} = \frac{m_1}{m_2}} \qquad (\because l_1 = l_2 / m_1 = m_2 / n_1 = n_2)$$

$$\Rightarrow \boxed{\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{\sqrt{l_1^2 + m_1^2 + n_1^2}}{\sqrt{l_2^2 + m_2^2 + n_2^2}}}$$

both line parallel হবে

$$l = \frac{a}{\sqrt{\Sigma a^2}}$$

$$\therefore l_1 = \frac{a_1}{\sqrt{\Sigma a^2}}, \quad m_1 = \frac{b_1}{\sqrt{\Sigma a^2}} \quad n_1 = \frac{c_1}{\sqrt{\Sigma a^2}}$$

// $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$$= \frac{a_1}{\sqrt{\Sigma a^2}} \cdot \frac{a_2}{\sqrt{\Sigma a^2}} + \frac{b_1}{\sqrt{\Sigma a_1^2}} \cdot \frac{b^2}{\sqrt{\Sigma a^2}} + \frac{c_1}{\sqrt{\Sigma a^2}} \cdot \frac{c_2}{\sqrt{\Sigma a^2}}$$

$$= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\Sigma a_1^2}\sqrt{\Sigma a_2^2}}$$

$\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow \boxed{a_1 a_2 + b_1 b_2 + c_1 c_2 = 0}$

$\theta = 0^\circ$ হলে $\frac{a_1}{a_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}}$; $\frac{b_1}{b_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}}$

$$\frac{c_1}{c_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}}$$

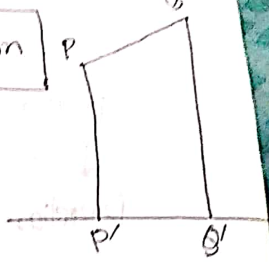
$\therefore \boxed{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$ $\Rightarrow$ দুইটি line parallel হলে তাদের ratio গুলোর অনুপাত সমান

Projection of line: একটি line এর image অপর line এ পড়া

$$\therefore \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\Rightarrow \cos\theta = \left(\frac{x_1 - x_2}{PQ}\right) l + \left(\frac{y_2 - y_1}{PQ}\right) m + \left(\frac{z_2 - z_1}{PQ}\right) n$$

$$\Rightarrow \boxed{PQ \cos\theta = (x_2 - x_1) l + (y_2 - y_1) m + (z_2 - z_1) n}$$



$$\boxed{P'Q' = PQ \cos\theta}$$

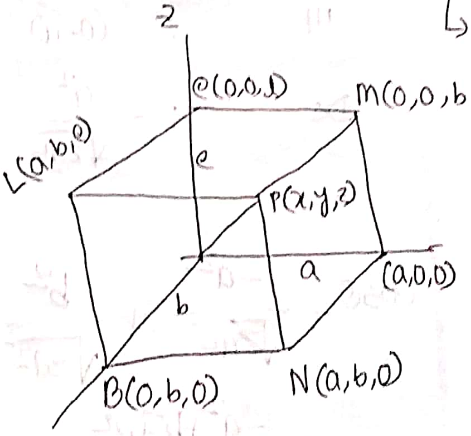
?

E $$\theta = \cos^{-1} \frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}$$

$$\therefore \cos\theta = \frac{-a^2 + b^2 + c^2}{a^2 + b^2 + c^2}$$

$$\cos\theta = \frac{a^2 - b^2 + c^2}{a^2 + b^2 + c^2}$$

$$\cos\theta = \frac{a^2 + b^2 - c^2}{a^2 + b^2 + c^2}$$



OA, OP, AL [illegible] → $x_0 = x_1$

$y_0 = y_1$

$z_0 = z_1$

OP : $(a-0)$ $(b-0)$ $(c-0)$

a b c

D.C : $\dfrac{a}{\sqrt{\sum a^2}}$ $\dfrac{b}{\sqrt{\sum a^2}}$ $\dfrac{c}{\sqrt{\sum a^2}}$, $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

Direction cosine

again $a\, l_1$ $b m_1$ $c\, n_1$

AL : $(0-a)$ $b-0$ $c-0$

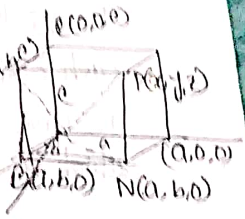
DC : $-\dfrac{a}{\sqrt{\sum a^2}}$ $\dfrac{b}{\sqrt{\sum a^2}}$ $\dfrac{c}{\sqrt{\sum a^2}}$

$$\cos\theta = \frac{-a^2}{\sqrt{\sum a^2}} + \frac{b^2}{\sqrt{\sum a^2}} + \frac{c^2}{\sqrt{\sum a^2}}$$

$$= \frac{-a^2+b^2+c^2}{a^2+b^2+c^2}$$

$\therefore \cos\theta = \frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}$

if square then $a=b=c$



$OP : \quad (a-0) \qquad (a-0) \qquad (a-0)$

$\qquad a \qquad a \qquad a$

$DC : \quad \frac{1}{\sqrt{3}} \qquad \frac{1}{\sqrt{3}} \qquad \frac{1}{\sqrt{3}}$

$\qquad l_1 \qquad m_1 \qquad n_1$

$AL : \quad (0-a) \qquad (b-0) \qquad (c-0)$

$\qquad -a \qquad b \qquad c$

$\theta = \cos^{-1}(1/3)$

$\cos\theta = -\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}$

$D.C : \quad l_1 \quad m_1 \quad n_1 \quad ; \quad l_2 \quad m_2 \quad n_2$

দুইটি perpendicular line এর D.C

দুটি Line perpendicular হয়ে আছে। যাদের direction cosine দেয়া আছে। তাদের উপর perpendicular ভাবে অবস্থিত তৃতীয় lineটির direction cosine

9-9-19
A-Day

Theory: Two lines are perpendicular to each other and their direction cosine is give. Find the direction cosine of third line which is also perpendicular to that two lines.

let the two lines

$$ll_1 + mm_1 + nn_1 = 0$$
$$ll_2 + mm_2 + nn_2 = 0$$

From there two equation,

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

$$= \frac{\sqrt{l^2+m^2+n^2}}{\left(\sqrt{\Sigma(m_1n_2 - n_1m_2)}\right)^2}$$

$$= \frac{1}{\sin\theta} = \frac{1}{1} = 1$$

[$\theta = 90^0$ cause the two lines are perpendicular]

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$$= \frac{a}{\sqrt{3a^2}} \cdot \frac{a}{\sqrt{3a^2}} + \frac{b}{\sqrt{3a^2}} \cdot \frac{b}{\sqrt{3a^2}} + \frac{c}{\sqrt{3a^2}} \cdot \frac{c}{\sqrt{3a^2}}$$

$$= \frac{a}{3a^2} + \frac{b^2}{3a^2} - \frac{c^2}{3a^2} = \frac{a^2+b^2-c^2}{3a^2} = \frac{a^2+b^2-c^2}{a^2+b^2+c^2}$$

Again from the diagonal AL and ON

$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$$= \frac{-a}{\sqrt{3a^2}} \cdot \frac{a}{\sqrt{3a^2}} + \frac{b}{\sqrt{3a^2}} \cdot \frac{b}{\sqrt{3a^2}} + \frac{c}{\sqrt{3a^2}} \cdot \frac{-c}{\sqrt{3a^2}}$$

[7] Show that the straight lines whose direction cosines are given by the equation

$$ul + vm + wn = 0 \;;\; al^2 + bm^2 + cn^2 = 0$$

are perpendicular if $u^2(b+c) + v^2(c+a) + w^2(a+b) = 0$

are parallel $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = 0$

Soln: $ul + vm + wn = 0 \quad n = -\frac{ul+vm}{w}$

putting the value of n in $al^2 + bm^2 + cn^2 = 0$

$$al^2 + bm^2 + cn^2 = 0$$

$$\Rightarrow al^2 + bm^2 + c\left[\frac{-lu-mv}{w}\right]^2 = 0$$

$$\Rightarrow al^2 + bm^2 + c \cdot \frac{l^2u^2 + m^2v^2 + 2lmuv}{w^2} = 0$$

$$\Rightarrow aw^2l^2 + bw^2m^2 + cl^2u^2 + cm^2v^2 + 2lmuvc = $$

$$\Rightarrow l^2(aw^2 + cu^2) + m^2(bw^2 + cv^2) + 2lmuvc =$$

Dividing this eqn by m^2

$$(aw^2+cu^2)\frac{l^2}{m^2}+2cuv\frac{l}{m}+(bw^2+cv^2)=0$$

Let,

$A=aw^2+cu^2$

$B=2cuv$

$C=bw^2+cv^2$

$$A\frac{l^2}{m^2}+B\frac{l}{m}+C=0$$

$$\frac{l_1}{m_1}\cdot\frac{l_2}{m_2}=\frac{C}{A}$$

$$\Rightarrow \frac{l_1 l_2}{m_1 m_2}=\frac{bw^2+cv^2}{aw^2+cu^2}$$

$$\Rightarrow \frac{l_1 l_2}{bw^2+cv^2}=\frac{m_1 m_2}{aw^2+cu^2}$$

Again, $ul+vm+wn=0$; $\Rightarrow m=-\frac{ul+wn}{v}$

$$al^2+bm^2+cn^2=0$$

$$\Rightarrow al^2+b\cdot\left[-\frac{ul+wn}{v}\right]^2+cn^2=0$$

$$\Rightarrow al^2v^2+bu^2l^2+2uwnlb+w^2n^2b+cn^2v^2=0$$

$$\Rightarrow l^2(av^2+bu^2)+n^2(bw^2+cv^2)+2uwnlb=0$$

$A'=av^2+bu^2$

$B'=bw^2+cv^2$

$C'=2uwb$

$$A'\frac{l^2}{n^2}+B'\frac{l}{n}+C'=0$$

$$\frac{l_1}{m_1 n_1} \cdot \frac{l_2}{n_2} = \frac{c}{a}$$

$$\Rightarrow \frac{l_1 l_2}{n_1 n_2} = \frac{cv^2+bw^2}{av^2+bu^2}$$

$$\Rightarrow \frac{l_1 l_2}{cv^2+bw^2} = \frac{n_1 n_2}{av^2+bu^2}$$

$$\frac{l_1 l_2}{bw^2+cv^2} = \frac{m_1 m_2}{aw^2+cu^2} = \frac{n_1 n_2}{av^2+bu^2} = \frac{1}{\sin\theta} = \frac{1}{1} = 1$$

For perpendicular,

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

$$\Rightarrow (bw^2+cv^2) + (aw^2+cu^2) + (av^2+bu^2) = 0$$

$$\Rightarrow u^2(b+c) + v^2(c+a) + w^2(a+b) = 0$$

For parallel:

$$b^2 = 4ac$$

$$\Rightarrow (2cuv)^2 = 4(aw^2+cu^2)(bw^2+cv^2)$$

$$\Rightarrow 4c^2u^2v^2 = 4(abw^4 + acw^2v^2 + bcu^2w^2 + c^2u^2v^2)$$

$$\Rightarrow abw^2 + acv^2 + bcu^2 = 0$$

$$\Rightarrow \frac{ab}{c^2}w^2 + \frac{a}{c}v^2 + \frac{b}{c}u^2 = 0$$

$$\Rightarrow a\frac{w^2}{c} + \frac{a}{ab}v^2 + \frac{b}{ab}u^2 = 0$$

$$\Rightarrow \frac{w^2}{c} + \frac{v^2}{b} + \frac{u^2}{a} = 0$$

$$\Rightarrow \frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = 0 \text{ [Showed]}$$

A-Day
17-09-19

দুটি plane এর General plane from

$$ax_1 + by_1 + cz_1 + d = 0 \longrightarrow a_1, b_1, c_1 \text{ (দিককোসাইন)}$$

$$ax_2 + by_2 + cz_2 + d = 0 \longrightarrow a_2, b_2, c_2 \text{ (দিককোসাইন)}$$

$$\boxed{\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2}$$

→ যেকোনো line এর দুইটা যারা plane এ perpendicular.

$$l_1 = \frac{a_1}{\sqrt{\Sigma a^2}} \qquad m_1 = \frac{b_1}{\sqrt{\Sigma a^2}} \qquad n_1 = \frac{c_1}{\sqrt{\Sigma a^2}}$$

$$l_2 = \frac{a_2}{\sqrt{\Sigma a^2}} \qquad m_2 = \frac{b_2}{\sqrt{\Sigma a^2}} \qquad n_2 = \frac{c_2}{\sqrt{\Sigma a^2}}$$

$$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\Sigma a^2} \cdot \sqrt{\Sigma a^2}}$$

$$\therefore \sin\theta = \frac{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (a_1 c_1 - a_2 c_2)^2 + (a_1 b_2 - a_2 b_1)^2}}{\sqrt{\Sigma a^2} \cdot \sqrt{\Sigma a^2}}$$

If $\theta = 90°$ then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

$$b_1 a_2 - b_2 a_1 = 0$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{a_1}{a_2}$$

Similarly $\frac{a_1}{a_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\boxed{\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$$

Q. $a_1x + b_1y + c_1z + d_1 = 0$ (α, β, γ)

0 নির্ণেয় সমতল

যদি Plane দেওয়া থাকে, ঐ Plane এর সাপেক্ষে অপর plane এর equation বের করতে বলে যা Parallel হবে এবং (α, β, γ) দিয়ে যায়

$$a_1x + b_1y + c_1z + d_1 + \lambda(a_2x + b_2y + c_2z + d_2) = 0$$

$$\Rightarrow (a_1 + \lambda a_2)x + (b_1 + \lambda b_2)y + (c_1 + c_2\lambda)z + \underbrace{\cancel{d}(1+\lambda)}_{(d_1+\lambda d_2)} = 0$$

এভাবে লেখা যায় নির্ণেয় হবে

$\boxed{a_3x + b_3y + c_3z + d = 0}$ → এটি নির্ণেয় হবে

<u>Sample question:</u> $x + 3x + 5y + k = 0$ → যা $(0,3,5)$ দিয়ে যায়

$(0,3,5)$

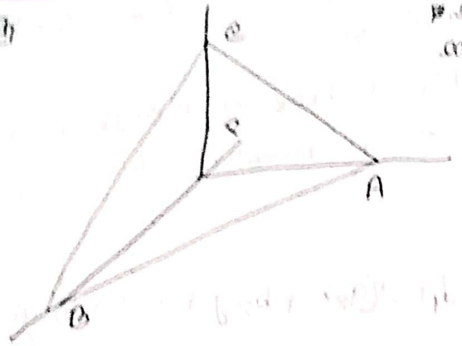
$x + 3x + 5y + k = 0$

$x + 3y + \cancel{5z}k = 0$

→ Parallel line পাওয়া যাবে

এটি ওয় insert করলে যাবে line বের করতে হবে।

(ii)



কেন্দ্র নির্ণয়ের [illegible]
তাহলে নির্ণয়ের [illegible]
equation

$$x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$$

সমতলের equation:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$p = \frac{1}{\sqrt{1/a^2 + 1/b^2 + 1/c^2}}$$

এখানে P থেকে surface এর উপর লম্বদূরত্বের eqn. বের করতে হয়,

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$$

ধরি,

$$\frac{a+0+0+0}{4} = x'$$

$$\Rightarrow a = 4x'$$

$$b = 4y'$$

$$c = 4z'$$

$$\frac{1}{16x^2} + \frac{1}{16y^2} + \frac{1}{16z^2} = \frac{1}{p^2}$$

$$\Rightarrow x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$$

The pair of straight line এর শর্ত $ax^2+2hxy+by^2+2gx+2fy+c=0$

$\Rightarrow abc+2fgh-af^2-bg^2-ch^2=0$

একাধিক Plane এর ক্ষেত্রে $\boxed{ax^2+by^2+cz^2+2fyz+2gzx+2hxy}$

$\Downarrow$

general equation

যেখানে (i) $abc+2fgh-af^2-bg^2-ch^2=0$ হলে the pair of plane হবে।

(ii) if $abc+2fgh-af^2-bg^2-ch^2\neq 0$ হলে curve হবে।

curve section

Straight line,

$ax^2+bx+c=0$

$l_1x+m_1y+n_1=0$

$l_2x+m_2y+n_2=0$

Plane এর ক্ষেত্রে

$l_1x+m_1y+n_1z=0$

$l_2x+m_2y+n_2z=0$

$Ax^2+Bx+C=0$

$x = \dfrac{-B \pm \sqrt{B^2-4AC}}{2A}$

Now, $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$

$+(by^2+cz^2+2fyz)=0$

Linear [illegible] $B^2-4AC=0$ [illegible]

$$x = \frac{-(hy+gz) \pm \sqrt{(hy+gz)^2 - a(by^2+cz^2+2fyz)}}{a}$$

$$x = \frac{-(hy+gz) \pm \sqrt{(h^2-ab)y^2 + 2yz(gh-af) + (g^2-ac)z^2}}{a}$$

17-Day
23-09-19

(1) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

$a = 2$
$b = -6$
$c = -12$
$f = 9$
$h = 1/2$
$g = 1/2$

$$2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$$

$$\Rightarrow 2x^2 + (2z + y)x + (6y^2 + 12z^2 - 18yz) = 0$$

$$\boxed{Ax^2 + Bx + C = 0}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x + 2y - 2z = 0$$
$$2x - 3y + 6z = 0$$

[illegible]

[illegible]

$$\Rightarrow \theta = \tan^{-1} \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{\Sigma a_1^2} \cdot \times \sqrt{\Sigma a_2^2}}$$

from (1) $\quad l_1x + m_1y + n_1z = 0$
$\quad l_2x + m_2y + n_2z = 0$

$l_1l_2 = a$
$m_1m_2 = b$
$n_1n_2 = c$

$$m_1n_2 + m_2n_1 = 2f$$
$$n_1l_2 + n_2l_1 = 2g$$
$$l_1m_2 + l_2m_1 = 2h$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (m_2l_1 - m_1 …}{l_1l_2 + m_1m_2 + n_1n_2}$$

$$= \frac{(4f^2 - 4bc) + (4g^2 - 4ac) + (4h^2 - 4ab)}{a+b+c}$$

$$\therefore \theta = \tan^{-1}\left[\frac{2\sqrt{f^2+g^2+h^2-bc-ca-ab}}{a+b+c}\right]$$

$\boxed{}$ $~~ax^2 + by^2 + cz^2 + 4by^2 +~~$

$\boxed{}$ $\phi(x, y, z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ —— ①

(α, β, γ) বিন্দু থেকে ① এর যে দুইটি plane যেখানে (তাদের দূরত্ব)

$\rightarrow$ $l_1x + m_1y + n_1z = 0$

$l_2x + m_2y + n_2z = 0$

find the condition for the plane

(α, β, γ) থেকে plane দুইটির দূরত্ব product করতে (গেল)

পার $= \dfrac{\phi(\alpha, \beta, \gamma)}{\sqrt{\Sigma a^2 + 4\Sigma f^2 - 2\Sigma bc}}$

$$\frac{l_1\alpha + m_1\beta + n_1\gamma}{\sqrt{l_1^2+m_1^2+n_1^2}} \times \frac{l_2\alpha + m_2\beta + n_2\gamma}{\sqrt{l_2^2+m_2^2+n_2^2}} = \frac{\phi(\alpha,\beta,\gamma)}{\sqrt{l_1^2l_2^2+m_1^2m_2^2+n_1^2n_2^2 + \Sigma(m_2^2n_1^2 + m_1^2n_2^2)}}$$

$$\cancel{\phi(\alpha,\beta,\gamma)} = \frac{a\alpha^2 + b\beta^2 + c\gamma^2 + 2f\beta\gamma + 2g\alpha\gamma + 2h\alpha\beta}{\sqrt{l_1^2l_2^2 + m_1^2m_2^2 + n_1^2n_2^2 + \Sigma(m_2^2n_1^2 + m_1^2n_2^2)}}$$

$a^2+b^2=(a+b)^2-2ab$

$$= \frac{\phi(\alpha,\beta,\gamma)}{\sqrt{\Sigma a^2 + 4f^2 - \Sigma bc}}$$

Straight line

$$\boxed{\begin{array}{l} a_1x + b_1y + c_1z + d^2 = 0 \\ a_2x + b_2y + c_2z + d^2 = 0 \end{array}}$$

(x_1, y_1, z_1)

$\Downarrow$

This is the equtn of straugth line in general from.

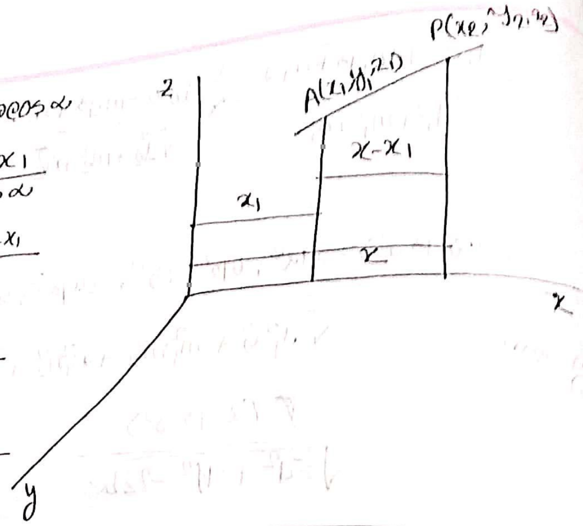
$x - x_1 = r\cos\alpha$

$\Rightarrow r = \dfrac{x - x_1}{\cos\alpha}$

$= \dfrac{x - x_1}{l}$

$\Rightarrow r = \dfrac{y - y_1}{m}$

$\Rightarrow r = \dfrac{z - z_1}{n}$



$$\boxed{\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r}$$

$\hookrightarrow x = rl + x_1$

$y = rl + y_1$

$z = rl + z_1$

$\therefore ax + by + cz + d = 0$

$\Rightarrow a(rl + x_1) + b(rl + y_1) + c(rl + z_1) + d = 0$

$\hookrightarrow r = ?$

then $x = ?$

$y = ?$ হের ধরব

$z = ?$

$$\frac{x+1}{3} = \frac{y-3}{1} = \frac{z-7}{-2} = r \text{ (say)}$$

$x = 3r - 1$

$y = r + 3$

$z = -2r + 7$

$$x + 2y + z - 1 = 0$$

$\Rightarrow 3r - 1 - 2r - 6 + (2r + 7) - 1 = 0$

$\Rightarrow -r - 1 = 0$

$\Rightarrow r = -1$

$\therefore x = -4,\ y = 2,\ z = 9$

$\Rightarrow$ $(1, 0, 3)$ বিন্দু আলোকে, $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} \Rightarrow \frac{x}{-3} = \frac{y}{7} = \frac{z}{2}$

Find the equation pass through the point 2টি line এর উপর perpendicular

$\Rightarrow$ any equ of straight line will passes through the point $(1, 0, 3)$, Find the d.c of the eqn.

$$\frac{x-1}{l} = \frac{y-0}{m} = \frac{z-3}{n} \quad \text{— (4)}$$

(i) + (ii),

$$2l + 5m + n = 0$$

$$-2l + 2m + 9n = 0$$

$$\frac{l}{[illegible]} = \frac{m}{[illegible]} = \frac{n}{[illegible]}$$

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

[illegible]

[illegible]

$\to$ Direction ratio (a_1, b_1, c_1)

$$a_1x + b_1y + c_1z + d_1 = 0$$
$$a_2x + b_2y + c_2z + d_2 = 0$$

দুটি straight line এর সমীকরণ Symmetric form এ নিলে

$\to$ Direction ratio (a_2, b_2, c_2)

Straight line টি [illegible] form এ প্রকাশ হবে

$$\boxed{\frac{x_1 - x_2}{l} = \frac{y_1 - y_2}{m} = \frac{z_1 - z_2}{n}}$$

(a_1, b_1, c_1)

(a_2, b_2, c_2)

l, m, n এর মান কে বের করতে হবে

$\Rightarrow$ Solⁿ:

$$l \cdot \frac{a_1}{\sqrt{\Sigma a^2}} + m \frac{b_1}{\sqrt{\Sigma b^2}} + n \frac{c_1}{\sqrt{\Sigma c^2}} = 0$$

$$a_1 l + b_1 m + c_1 n = 0 \qquad a_2 l + b_2 m + c_2 n = 0$$

$$\frac{l}{b_1c_2 - b_2c_1} = \frac{m}{a_2c_1 - a_1c_2} = \frac{n}{a_1b_2 - a_2b_1}$$

(x_1, y_1, z_1)

$$a_1x_1 + b_1y_1 + c_1z_1 + d_1 = 0$$

$$a_2x_2 + b_2y_2 + c_2z_2 + d_2 = 0$$

Let, $z_1 = 0$

$$a_1 x_1 + b_1 y_1 + 0 + d_1 = 0$$

$$a_2 x_2 + b_2 y_2 + 0 + d_2 = 0$$

$$\frac{x_1}{b_1 d_2 - b_2 d_1} = \frac{y_1}{d_1 a_2 - a_1 d_2} = \frac{1}{a_1 b_2 - a_2 b_1}$$

$$x_1 = \frac{b_1 d_2 - b_2 d_1}{a_1 b_2 - a_2 b_1}$$

$$y_1 = \frac{d_1 a_2 - a_1 d_2}{a_1 b_2 - a_2 b_1}$$

$$\boxed{\begin{aligned} x + y + z - 1 &= 0 \\ 7x - 3y + 5z + 2 &= 0 \end{aligned}}$$

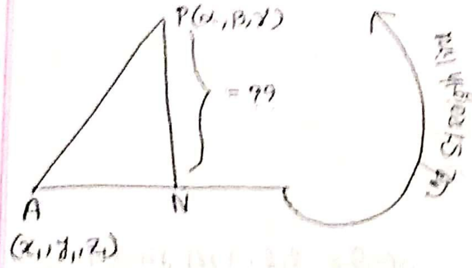
$a_1 = 1$
$b_1 = 1$
$c_1 = 1$
$d_1 = -1$

$a_2 = 7$
$b_2 = -3$
$c_2 = 5$
$d_2 = 2$

[P] (α, β, γ)

Determined the perpendicular distance of a point on the straight line?

$$\frac{x_1 - x_2}{l} = \frac{y_1 - y_2}{m} = \frac{z_1 - z_2}{n}$$



$PA^2 = PN^2 + AN^2$

$\Rightarrow PN^2 = PA^2 - AN^2$

$AP^2 = (x_1 - \alpha)^2 + (y_1 - \beta)^2 + (z_1 - \gamma)^2$

AN = Projection of AP on AN

$= (x_1 - \alpha)l + (y_1 - \beta)m + (z_1 - \gamma)n$

Projection of AP on AN

$$\frac{x+3}{1} = \frac{y-7}{-5} = \frac{z+1}{7}$$

$(\alpha, \beta, \gamma) = (0, 2, 7)$

$x_1 = -3$, $y_1 = 7$, $z_1 = 1$

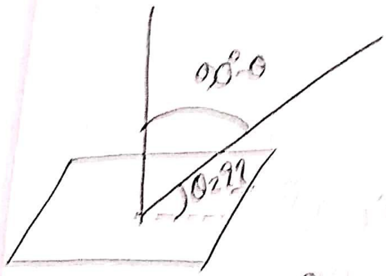
$l = 1$, $m = -5$, $n = 7$

(α, β, γ)

[9] Determine the angle

$$ax+by+cz+d=0$$

$$\frac{x_1-x_2}{l} = \frac{y_1-y_2}{m} = \frac{z_1-z_2}{n}$$



$\cos\theta = l_1l_2+m_1m_2+n_1n_2$

calculation হবে normal এর সাথে

$$\cos(90°-\theta) = l\cdot\frac{a}{\sqrt{\Sigma a^2}} + m\cdot\frac{a}{\sqrt{\Sigma a^2}} + n\cdot\frac{a}{\sqrt{\Sigma a^2}} = \frac{al+bm+cn}{\sqrt{\Sigma a^2}}$$

$$al+bm+cn=0$$

এটি plane থেকে এটি line কে contain করে

▫ Find the condition between the line and the plane meet (meeting point)

$$ax+by+cz+d=0$$

$$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n} = r \text{ (say)}$$

$$x = rl + x_1$$

$$y = rm + y_1$$

$$z = rn + z_1$$

$$a(rl+x_1) + b(rm+y_1) + c(rn+z_1) + d = 0$$

$$\Rightarrow (al+mb+cn)r + (ax_1+by_1+cz_1+d) = 0$$

$$\Rightarrow \boxed{\begin{aligned} &al+bm+cn = 0 \\ &ax_1+by_1+cz_1+d=0 \end{aligned}}$$

condition:

▫ Plane কে line contain করলে

(i) যেকোনো point এ satisfied করবে

(ii) normal আর তা line এর সাথে perpendicular হবে।

▫ line কে plane contain করলে

৪। $\boxed{\dfrac{x-x_1}{l} = \dfrac{y-y_1}{m} = \dfrac{z-z_1}{n}}$ line কে plane contain করলে শর্ত

$ax+by+cz+d=0$

$ax_1+by_1+cz_1+d=0$

(–) করে,

$$\boxed{\begin{array}{c} a(x-x_1)+b(y-y_1)+c(z-z_1)=0 \\ \hline al+bm+cn=0 \end{array}}$$

equation of required line

৫। Find the equation of plane যেটা

$\dfrac{x-x_1}{l_1} = \dfrac{y-y_1}{m_1} = \dfrac{z-z_1}{n_1}$ রেখা দিয়ে যায় আর

$\dfrac{x_1-x_2}{l_2} = \dfrac{y-y_2}{m_2} = \dfrac{z-z_2}{n_2}$ এর সাথে perpendicular

হবে তার equation বের কর?

We assume that any line

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$$

→ Plane এর equation

$al_1+bm_1+cn_1=0$ ↳ normal to the line.

perpendicular

$al_2+bm_2+cn_2=0$

Q. [illegible]

$$x^2 + 4xy + 3y^2 + (4x + 8y) - 11 = 0$$

$$y = 2x + 2$$

[illegible] $\frac{2\sqrt{h^2-ab}}{a+b}$

$$\frac{y-2x}{2} = 1$$

$$x^2 + 4xy + 3y^2 + (4x+8y)\left(\frac{y-2x}{2}\right) - 11\left(\frac{y-2x}{2}\right)^2 = 0$$

$$\Rightarrow 7x^2 - 14xy + y^2 = 0$$

$a = 7$

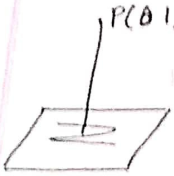
$b = 1$

$c = 7$

Find the distance of the point $(1, -2, 3)$ from the plane $x - y - z = 0$ measured parallal is the line $\frac{x}{2} = \frac{y}{3} = \frac{z-1}{-6}$

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = p \text{ (say)}$$

$P(1, -2, 3)$



$x = 2p + 1$

$y = 3p - 2$

$z = -6p + 9$

একটি plane এর eq^n বের করতে হবে যা $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ এর সাথে contain করে এবং যা perpendicular হবে $\boxed{\frac{x}{m} = \frac{y}{n} = \frac{z}{l} \text{ ও } \frac{x}{n} = \frac{y}{l} = \frac{z}{m}}$ এর সাথে এ দুইটি line এর সাথে

$$\begin{vmatrix} x & y & z \\ m & n & l \\ n & l & m \end{vmatrix} = 0$$

$$x(nm - l^2) - y(m^2 - nl) + z(ml - n^2) = 0$$

$$\boxed{\begin{array}{l} ax + by + cz = 0 \\ al + bm + cn = 0 \end{array}}$$

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

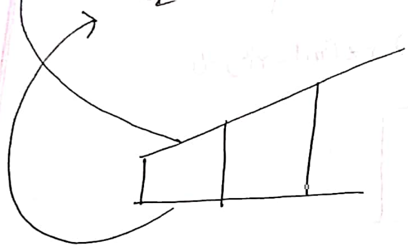
$$al + bm + cn = 0$$

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$$

$$\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$$

২টি true line এদের মধ্যে minimum distance বের করতে হবে।

Find shortest distance between two lines and the equation of the shortest distance?



Distance of LM $= $ proj of AB on LM

$= (x_2 - x_1)l + (y_2 - y_1)m + (z_2 - z_1)n$

LM is perpendicular on the plane

so $ll_1 + mm_1 + nn_1 = 0$

$ll_2 + mm_2 + nn_2 = 0$

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - l_1n_2} = \frac{n}{l_1m_2 - m_1l_2}$$

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$$

$$\begin{vmatrix} x & y & z \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0$$

$$\Rightarrow x(m_1 n - m n_1) - y(n l_1 - l n_1) + z(l_1 m - l m_1) = 0$$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0$$

$$\Rightarrow (x - x_1)(m_1 n - m n_1) - y(y - y_1)(n l_1 - n_1 l) + (z - z_1)(l_1 m - l m_1) = 0$$

$$ax + by + cz + d = 0$$

$$\begin{vmatrix} x - x_2 & y - y_2 & z - z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix} = 0$$

$$\Rightarrow (x - x_2)(m_2 n - n m_2) - (y - y_2)(l_2 n - l n_2) + (z - z_2)(l_2 m - l m_2) = 0$$

$$a_2 x + b_2 y + c_2 z + d_2 = 0$$

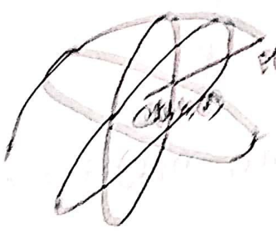
$$\boxed{\begin{aligned} a_1x+b_1y+c_1z+d_1=0 \\ a_2x+b_2y+c_2z+d_2=0 \end{aligned}}$$

CT
Coordinate geometry

8th - E Day
21-10-19

Sphere

Sphere



নির্দিষ্ট দূরত্ব হলো radius

$CP^2 = r^2$

$(x-a)^2+(y-b)^2+(z-c)^2 = r^2$

$$\boxed{a=b=c=0} \quad \boxed{x^2+y^2+z^2=r^2}$$

$\Rightarrow x^2+a^2-2xa+y^2+b^2-2yb+z^2+c^2-2zc-r^2=0$

$\Rightarrow x^2+y^2+z^2+2(-a)x+2(-b)y+2(-c)z+\underbrace{\boxed{-r^2+a^2+b^2+c^2}}_{\text{constant}}=0$

$\Rightarrow x^2+y^2+z^2+2ux+2vy+2wz+d=0$

$\Rightarrow (x+u)^2+(y+v)^2+(z+w)^2+d-u^2-v^2-w^2=0$

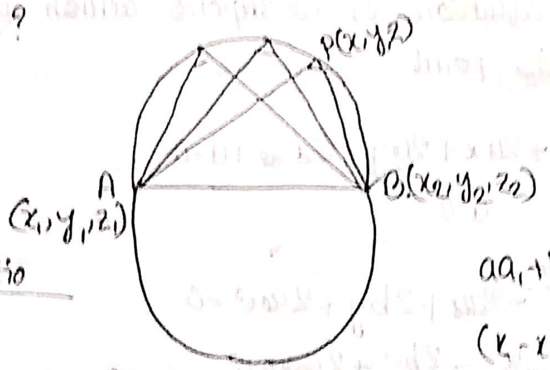
$\Rightarrow \{x-(-u)\}^2+\{y-(-v)\}^2+\{z-(-w)\}^2=\{\sqrt{u^2+v^2+w^2-d}\}^2$

where : $(-u, -v, -w)$
radius : $\sqrt{u^2+v^2+w^2-d}$

$$x^2+y^2+z^2+2x-y+6z+3=0$$

where, $(1, -\frac{1}{2}, 3) \rightarrow$ center
Radius $\sqrt{1^2+(-\frac{1}{2})^2+3^2-3} =$

দুটি A (x_1, y_1, z_1) B (x_2, y_2, z_2) এর equation বের কর
যদি point যোগ করলে যে straight line হয় তা হবে ঐ sphere এর diameter?



APDR এর Ratio
$x-x_1$
$y-y_1$
$z-z_1$

BPDR
$x-x_2$
$y-y_2$
$z-z_2$

$$aa_1+bb_1+cc_1=0$$

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+(z-z_1)(z-z_2)=0$$

$+15+19$
29

$-7z-29$

$A(1, 3, 7)$
$B(0, 5, 2)$

$(x-1)(x-0) + (y-3)(y-5) + (z-7)(z-2) = 0$

$\Rightarrow \boxed{x^2+y^2+z^2-x-8y-9z+29=0}$

This is the equation of sphere

Q] $(0,0,0)$, $(-a,b,c)$, $(a,-b,c)$, $(a,b,-c)$

Find the equation of the sphere which passes through this point.

$$x^2+y^2+z^2+2ux+2vy+2zw+d=0$$

$$d=0$$

$$a^2+b^2+c^2-2au+2by+2wc=0$$

$$\Rightarrow a^2+b^2+c^2+2au-2bv+2wc=0$$

$$\Rightarrow a^2+b^2+c^2+2au+2bv-2wc=0$$

$$\Rightarrow 2(a^2+b^2+c^2)+2au=0$$

$u = -\dfrac{a^2+b^2+c^2}{2a}$ $\quad v = -\dfrac{a^2+b^2+c^2}{2b}$ $\quad w = -\dfrac{a^2+b^2+c^2}{2c}$

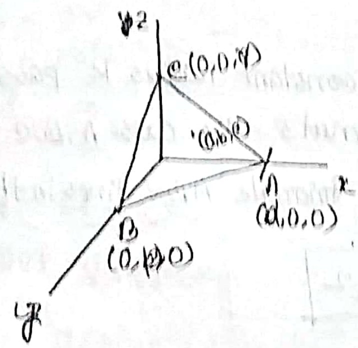
$$x^2+y^2+z^2 - \frac{a^2+b^2+c^2}{a}x - \frac{a^2+b^2+c^2}{b}y - \frac{a^2+b^2+c^2}{c}z = 0$$

Center : $\left[\frac{a^2+b^2+c^2}{2a}, \frac{a^2+b^2+c^2}{2b}, \frac{a^2+b^2+c^2}{2c}\right]$

Radius: $\sqrt{u^2+v^2+w^2-d}$

$$= \frac{a^2+b^2+c^2}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

Ex



Plane ধরা হলো ABC যা x-অক্ষ, y অক্ষ, z অক্ষকে A, B, C তে ছেদ করে।
Center এর
A Sphere OABC এর

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$

$$\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=0$$

$$x^2+y^2+z^2-\alpha x-\beta y-\gamma z=0$$

$$\Rightarrow x^2+y^2+z^2+2ux+2vy+2wz+ \quad =0$$

$$\alpha+2x=0$$

$$\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=1$$

$\alpha = 2x'$ $\qquad$ x', y', z'

$\beta = 2y'$

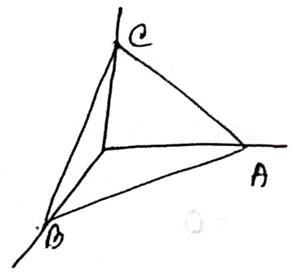
$c = 2z'$

$$\frac{a}{x'} + \frac{b}{by'} + \frac{c}{z'} = 0$$

the locus of the centroyed of the sphere is

A ~~Spa~~ sphere of constant radius k pass through in the origin of meets the axes A, B, C. Prove that the centroid of the triangle ABC lines in the sphere.

$$\boxed{9(x^2+y^2+z^2) = 4k^2}$$



$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$x^2+y^2+z^2-ax-by-cz=0$

center (a, b, c)

radius $\sqrt{a^2+b^2+c^2}$

$a^2+b^2+c^2=k^2$

$\hookrightarrow$ constant distance

x', y', z'

$x' = \frac{a}{3}$

$a = 3x'$

$b = 3y'$

$c = 3z'$

$9(x'^2+y'^2+z'^2) = 4k^2$

center (a, b, c)

radius $\sqrt{a^2+b^2+c^2}$

$9(x^2+y^2+z^2) = 4k^2$

$\boxed{\text{10th-A day} \quad \text{CT হবে}}$

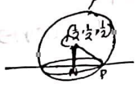
rough

ve

12th—E Day
24-11-19

Q. Find the radius and ~~circle~~ center of the circle

$x^2+y^2+z^2-x-y-z-100;\ 1x+1y+1z=0$



$\text{Radius} = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+1}$

$PN^2 = CP^2 - CN^2$

$\Rightarrow PN^2 = (\quad) - \frac{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}{\sqrt{1^2+1^2+1^2+1}} = ?$

$\frac{x-\frac{1}{2}}{1} = \frac{y-\frac{1}{2}}{1} = \frac{z-\frac{1}{2}}{1} = r$ (say)

$x = r+\frac{1}{2}$

$y = r+\frac{1}{2}$

$z = r+\frac{1}{2}$

$\left(r+\frac{1}{2}\right)+\left(r+\frac{1}{2}\right)+\left(r+\frac{1}{2}\right)=0$

$\Rightarrow r = 9$

8. Find the eqn of the sphere whose center is the point (1,2,3) and which touch the plane $(3x+2y+z+4=0$

Find also the radius of the circle in which the plane sphere cuts by the plane $x+y+z=0$

$$\text{Radius} = \frac{3\cdot1+2\cdot2+1\cdot3+4}{\sqrt{3^2+2^2+1^2}}$$