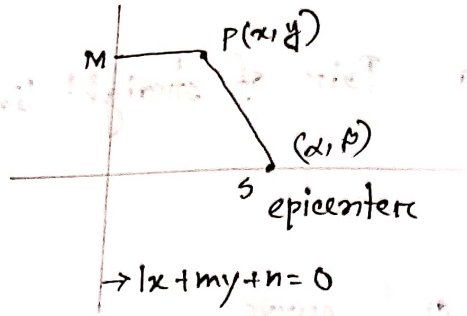
Pairs of straight lines

General discussion



$$\frac{SP}{PM} = \text{constant}$$
$$= \text{eccentricity } (e)$$

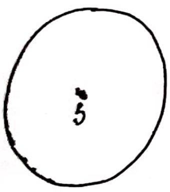
$$SP = e \cdot PM$$

$$\boxed{\sqrt{(x-\alpha)^2+(y-\beta)^2} = e\left|\frac{lx+my+n}{\sqrt{l^2+m^2}}\right|}$$

→ General equation of conics

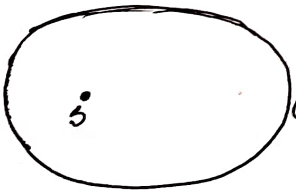
$$ax^2+by^2+2hxy+2gx+2fy+c=0$$

circle



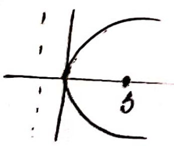
$$e = \frac{\cancel{PM}\,SP}{\alpha} = 0$$

ellipse



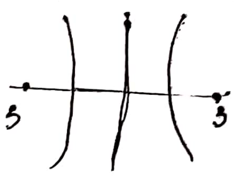
$0 < e < \cancel{0}1$

Parabola



$$e = \frac{\cancel{PM}\,SP}{PM} = 1$$

Hyperbola



$$e = \frac{SP}{PM}$$
$$e > 1$$

straight line

$$e = \infty$$

General equation of conics

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ Pair of straight lines

$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0$ curve

- → circle $a=b \quad h=0$
- → parabola $ab-h^2=0$
- → ellipse $ab-h^2>0$
- → hyperbola $ab-h^2<0$

Theorem-01

Find the condition that the general equation of 2nd degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ may represent a pair of straight

Soln

Given that

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

$$\Rightarrow ax^2+2(hy+g)x+by^2+2fy+c=0.$$

$$\therefore x=\frac{-2(hy+g)\pm\sqrt{4(hy+g)^2-4a(by^2+2fy+c)}}{2a}$$

$$x=\frac{-(hy+g)\pm\sqrt{(h^2-ab)y^2+2(gh-af)y+(g^2-ac)}}{a}$$

It will be straight lines if,

$$4(gh-af)^2-4(h^2-ab)(g^2-ac)=0$$

$$\Rightarrow \boxed{abc+2fgh-af^2-bg^2-ch^2=0}$$ Ans.

Theorem-02

Angle between the lines represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$

Soln

let,

two straight lines are

$$l_1x+m_1y+n_1=0 \quad \text{------(I)}$$

$$l_2x+m_2y+n_2=0 \quad \text{------(II)}$$

From equ (I)

$$y=-\frac{l_1}{m_1}x-\frac{n_1}{m_1} \quad \text{------(III)}$$

From equ (II)

$$y=-\frac{l_2}{m_2}x-\frac{n_2}{m_2} \quad \text{------(IV)}$$

$$\tan\theta = \frac{-\frac{l_1}{m_1} - \left(-\frac{l_2}{m_2}\right)}{1 + \left(-\frac{l_1}{m_1} \cdot -\frac{l_2}{m_2}\right)}$$

$$\tan\theta = \frac{l_1 m_2 - l_2 m_1}{m_1 m_2 + l_1 l_2}$$

again,

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = (l_1x + m_1y + n_1)(l_2x + m_2y + n_2)$$

from the equation, we get

$$\left.\begin{aligned} &* \; n_1 n_2 = c \\ &* \; l_1 l_2 = a \\ &* \; m_1 m_2 = b \\ &* \; 2h = l_1 m_2 + l_2 m_1 \\ &* \; 2g = l_1 n_2 + l_2 n_1 \\ &* \; 2f = n_1 m_2 + m_1 n_2 \end{aligned}\right\}$$

for equation

$l_1x + m_1y + n_1 = 0$

$l_2x + m_2y + n_2 = 0$

$$\tan\theta = \frac{\sqrt{(l_1 m_2 + l_2 m_1)^2 - 4 l_1 l_2 m_1 m_2}}{a+b}$$

$$= \frac{\sqrt{4h^2 - 4ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\therefore \boxed{\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}}$$

Ans.

condition-01: $\theta = 90°$ (the lines will be perpendiculare)

$a+b=0$ *

condition-02: $\theta = 0°$ (the lines will be parallel)

$h^2 - ab = 0$ *

Theorem-03

Prove that the homogeneous equation of 2^{nd} degree $ax^2 + 2hxy + by^2 = 0$ will represent a pair of straight lines passing through the origin.

Proof:

let, two straight lines passing through the origin

$$l_1x + m_1y = 0 \text{ ------ (I)}$$
$$l_2x + m_2y = 0 \text{ ---- (II)}$$

equ (I)×(II)

$$(l_1x + m_1y)(l_2x + m_2y) = 0$$
$$\Rightarrow l_1l_2x^2 + (l_1m_2 + l_2m_1)xy + m_1m_2y^2 \text{ ---- (III)}$$

we know,

$$l_1l_2 = a$$
$$2h = l_1m_2 + l_2m_1$$
$$b = m_1m_2$$

So, from the equ (III)

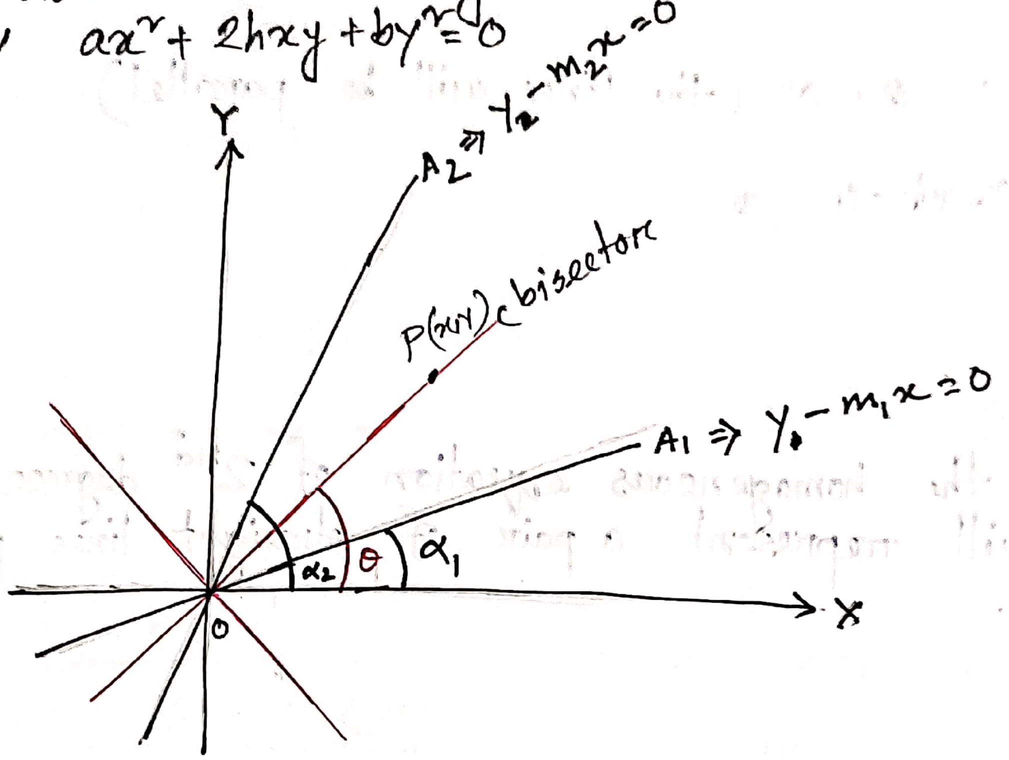
$$\boxed{ax^2 + 2hxy + by^2 = 0}$$

Ans.

Theorem-04

Find the bisectors of the angles between the lines represented by $ax^2 + 2hxy + by^2 = 0$

Soln



$\tan\alpha_1 = m_1$

$\tan\alpha_2 = m_2$

$$\angle XOC = \angle XOA_1 + \angle A_1OC$$

$$= \alpha_1 + \frac{1}{2}\angle A_1OA_2$$

$$= \alpha_1 + \frac{1}{2}\left[\angle XOA_2 - \angle XOA_1\right]$$

$$= \alpha_1 + \frac{1}{2}(\alpha_2 - \alpha_1)$$

$$2\theta = \alpha_1 + \alpha_2$$

$$\tan 2\theta = \tan(\alpha_1 + \alpha_2)$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{\tan\alpha_1 + \tan\alpha_2}{1 - \tan\alpha_1 . \tan\alpha_2} = \frac{m_1 + m_2}{1 - m_1 m_2}$$

// again,

$$ax^2 + 2hxy + by^2 = 0 \quad \text{———— (1)}$$

$$(y - m_1x)(y - m_2x) = 0$$

$$y^2 - (m_1 + m_2)xy + m_1m_2x^2 = 0$$

$$\Rightarrow m_1m_2x^2 - (m_1 + m_2)xy + y^2 = 0 \quad \text{———— (ii)}$$

$$\frac{m_1m_2}{a} = \frac{-(m_1+m_2)}{2h} = \frac{1}{b}$$

$$* \quad m_1m_2 = \frac{a}{b}$$

$$* \quad m_1 + m_2 = -\frac{2h}{b}$$

for equation

$$y - m_1x = 0$$

$$y - m_2x = 0$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{m_1+m_2}{1-m_1m_2} = \frac{-\frac{2h}{b}}{1-\frac{a}{b}}$$

$$\Rightarrow \frac{2\frac{y}{x}}{1-\frac{y^2}{x^2}} = \frac{-\frac{2h}{b}}{1-\frac{a}{b}}$$

$$\Rightarrow \boxed{\frac{x^2-y^2}{a-b} = \frac{xy}{h}}$$

Q-01

Find the straight lines from the equation $2x^2+7xy+6y^2+7x+11y+3=0$

Soln

Given that,

$$2x^2+7xy+6y^2+7x+11y+3=0$$

$$\Rightarrow 2x^2+x(7y+7)+6y^2+11y+3=0$$

$$x=\frac{-7(y+1)\pm\sqrt{49(y+1)^2-4\times2(6y^2+11y+3)}}{4}$$

$$\Rightarrow 4x=-7(y+1)\pm\sqrt{49(y^2+2y+1)-48y^2-88y-24}$$

$$\Rightarrow 4x=-7(y+1)\pm\sqrt{y^2+10y+25}$$

$$\Rightarrow 4x=-7(y+1)\pm\sqrt{(y+5)^2}$$

$$\Rightarrow 4x=-8y-12$$

$$\Rightarrow x+2y+3=0 \quad \text{Ans.}$$

$$4x=-6y-2$$

$$2x+3y+1=0 \quad \text{Ans}$$

Q-02

Prove that when lines are parallel, then $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$

Soln

for straight lines we know,

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$

when $\theta = 0$,

$$h^2 - ab = 0$$

$$h = \sqrt{ab}$$

so,

$$\cancel{ab}c + 2fg\sqrt{ab} - af^2 - bg^2 - \cancel{ab}c = 0$$

$$\Rightarrow (\sqrt{a}f)^2 + (\sqrt{b}g)^2 - 2\sqrt{a}f\sqrt{b}g = 0$$

$$\Rightarrow (f\sqrt{a} - g\sqrt{b})^2 = 0$$

$$\Rightarrow \sqrt{a}f = \sqrt{b}g \quad \text{———} *$$

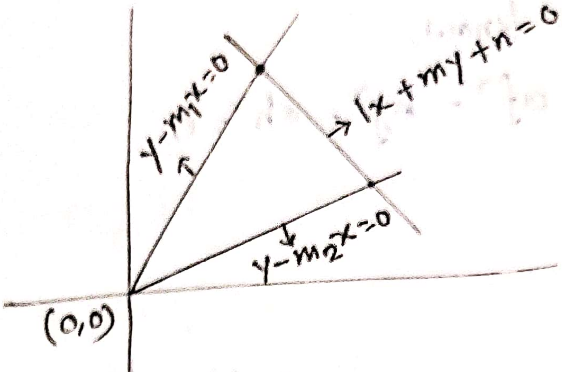
$\sqrt{ab}f = bg$	$af = \sqrt{ab}\,g$
$hf = bg$	$af = hg$
$\frac{h}{b} = \frac{g}{f}$	$\frac{a}{h} = \frac{g}{f}$

$$\therefore \frac{a}{h} = \frac{h}{b} = \frac{g}{f} \quad \boxed{\text{Prove}}$$

Q-03

Find the area of the triangle formed by the lines represented by $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$

Soln



let, the pair straight lines are

$$y - m_1x = 0 \quad \text{——— (I)}$$
$$y - m_2x = 0 \quad \text{——— (II)}$$

and

$$lx + my + n = 0 \quad \text{——— (III)}$$

The connecting point of (I) and (III) equation

$$y = m_1x$$
$$lx + mm_1x + n = 0$$
$$x = -\frac{n}{(l + m_1m)}$$

$$x = \frac{y}{m_1}$$
$$\frac{ly}{m_1} + my + n = 0$$
$$\Rightarrow yl + mm_1y + m_1n = 0$$
$$y = -\frac{m_1n}{l + mm_1}$$

$$(x, y) = \left\{ -\frac{n}{l + m_1m}, -\frac{m_1n}{l + mm_1} \right\}$$

The connecting point of (II) and (III) equation

$$(x, y) = \left\{ -\frac{n}{l + mm_2}, -\frac{m_2n}{l + mm_2} \right\}$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -\frac{n}{l+mm_1} & -\frac{m_1 n}{l+mm_1} & 1 \\ -\frac{n}{l+mm_2} & -\frac{m_2 n}{l+mm_2} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left\{\frac{n^2 m_2}{(l+mm_1)(l+mm_2)} - \frac{n^2 m_1}{(l+mm_1)(l+mm_2)}\right\}$$

$$= \frac{1}{2}\left\{\frac{n^2(m_2-m_1)}{(l+mm_1)(l+mm_2)}\right\}$$

$$= \frac{\frac{n^2}{2}\sqrt{(m_2+m_1)^2-4m_1m_2}}{l^2+lm(m_1+m_2)+m^2m_1m_2}$$

$$= \frac{\frac{n^2}{2}\sqrt{\left(-\frac{2h}{b}\right)^2-4\frac{a}{b}}}{l^2+lm\left(-\frac{2h}{b}\right)+m^2\frac{a}{b}}$$

$$= \frac{n^2\sqrt{\frac{h^2}{b^2}-\frac{ab}{b^2}}}{l^2-\frac{2hlm}{b}+\frac{am^2}{b}}$$

$$= \frac{n^2\sqrt{h^2-ab}}{bl^2-2hlm+am^2} \quad \text{Ans.}$$

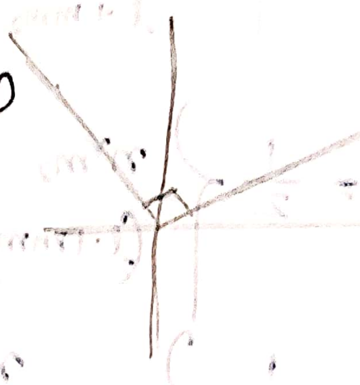
Q-04

If the lines joining the origin and the point of intersection of curves $ax^2+2hxy+by^2+2gx=0$ and $a_1x^2+2h_1xy+b_1y^2+$ are mutually perpendicular, then prove that $g(a_1+b_1)=g_1(a+b)$

Soln

$$ax^2+2hxy+by^2=-2gx \quad \text{---(i)}$$

$$a_1x^2+2h_1xy+b_1y^2=-2g_1x \quad \text{---(ii)}$$



equ (i) ÷ (ii)

$$\frac{ax^2+2hxy+by^2}{a_1x^2+2h_1xy+b_1y^2}=\frac{g}{g_1}$$

$$\Rightarrow ag_1x^2+2hg_1xy+bg_1y^2=a_1gx^2+2h_1gxy+b_1gy^2$$

$$\Rightarrow x^2(ag_1-a_1g)+2xy(hg_1-h_1g)+y^2(bg_1-b_1g)=0$$

If $\theta=90^\circ$

then,

$$a+b=0$$

$$ag_1-a_1g=-bg_1+b_1g$$

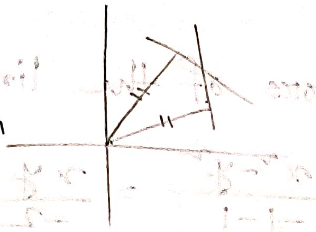
$$\Rightarrow g(a_1+b_1)=g_1(a+b)$$

Ans.

Q-05 *

Prove that the straight lines represented by the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ will be the equidistant from the origin, if $f^4-g^4=c(bf^2-ag^2)$

Soln



let represented by the given equation

$$lx+my+n=0 \quad \text{------(I)}$$

and $$l'x+m'y+n'=0 \quad \text{------(II)}$$

Since,

they are equidistance from the origin

$$\frac{n}{\sqrt{l^2+m^2}} = \frac{n'}{\sqrt{l'^2+m'^2}}$$

$$\Rightarrow n^2(l'^2+m'^2) = n'^2(l^2+m^2)$$

$$\Rightarrow n^2l'^2 - n'^2l^2 = n'^2m^2 - nm'^2$$

$$\Rightarrow (nl'+n'l)(nl'-n'l) = (n'm+m'n)(n'm-m'n)$$

$$\Rightarrow (nl'+n'l)\sqrt{(nl'+n'l)^2-4nn'll'} = (n'm+m'n)\sqrt{(n'm+m'n)^2-4nn'mm'}$$

$$\Rightarrow 2g\sqrt{4g^2-4ac} = \sqrt{4f^2-4bc} \times 2f$$

$$\Rightarrow g\sqrt{g^2-ac} = f\sqrt{f^2-bc}$$

$$\Rightarrow g^4 - g^2ac = f^4 - f^2bc$$

$$\Rightarrow \cancel{f^4 \quad f^4 = c(}$$

$$\Rightarrow f^4 - g^4 = c(bf^2-ag^2) \quad \text{Ans.}$$

Q-06

Prove that the lines $y^2-4xy-x^2=0$ and $y^2+xy-x^2=0$ bisect the angles between one another.

Soln

bisectors of the lines $y^2-4xy-x^2=0$ is

$$\frac{x^2-y^2}{-1-1}=\frac{xy}{-2}$$

$$\Rightarrow x^2-xy-y^2=0 \quad \text{Proved}$$

Again bisectors of the lines $x^2-xy-y^2=0$

$$\frac{x^2-y^2}{2}=\frac{xy}{-\frac{1}{2}}$$

$$\Rightarrow y^2-4xy-x^2=0 \quad \text{Proved}$$

Q-07

Show that the equation of the lines bisecting the angles between the bisectors of the pair of lines $ax^2+2hxy+by^2=0$ be $(a-b)(x^2-y^2)+4hxy=0$

Soln

bisectors of the equ $ax^2+2hxy+by^2=0$ is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

Or $hx^2-hy^2=(a-b)xy$

Or, $hx^2-(a-b)xy-hy^2=0$ ——— (1)

again bisectors of the equ (1) is

$$\frac{x^2-y^2}{2h}=\frac{2xy}{-(a+b)}$$

$$\Rightarrow (a-b)(x^2-y^2)+4xyh=0 \quad \text{Ans.}$$

Extra math

Q-08

The axes being rectangular, find the equation to the pair of straight lines meeting at the origin which are perpendicular to the pair given by the equation, $ax^2+2hxy+by^2=0$

Soln

The line represented by $ax^2+2hxy+by^2=0$ be,

$m_1m_2 = a/b$

$m_1+m_2 = -2h/b$

$y-m_1x=0$ and $y=m_2x$

Now, the equation are perpendicular to above lines and passes through the origin are,

$ym_1+x=0$ and $my_2+x=0$

So, the equation will be

$(x+ym_1)(x+ym_2)=0$

$\Rightarrow x^2+xym_2+xym_1+m_1m_2y^2=0$

$\Rightarrow x^2+xy(m_1+m_2)+m_1m_2y^2=0$

$\Rightarrow x^2+xy\left(-\frac{2h}{b}\right)+a/b\,y^2=0$

$\Rightarrow bx^2-2hxy+ay^2=0$ Ans.

Q-9

If the pair of straight lines $x^2 - 2axy - y^2 = 0$ and $x^2 - 2bxy - y^2 = 0$ be such that each pair bisects the angle between the other pair. prove that $ab = -1$

Soln

Equation to the bisector of first pair,

$$\frac{x^2 - y^2}{1-(-1)} = \frac{xy}{-a}$$

$$x^2 - y^2 = \frac{2xy}{-a} \quad \text{------} (I)$$

again,
$$x^2 - y^2 = \frac{2bxy}{1}$$

Or, $$x^2 - y^2 = \frac{2xy}{1/b} \quad \text{------} (II)$$

Compare (I) and (II) equation

$$-a = 1/b$$

$$\Rightarrow ab = -1 \quad \text{(Proved)}$$

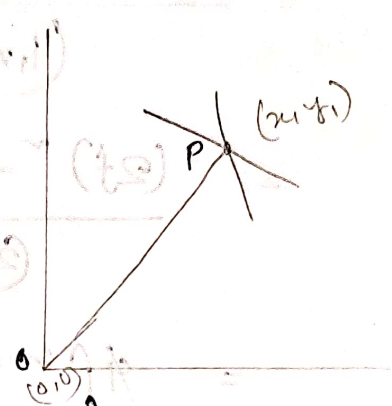
Q-10

If the general equation of second degree represents two straight lines prove that the square of their point of intersection from the origin will be $\dfrac{c(a+b)-f^2-g^2}{ab-h^2}$

Soln:

We know the equation of second degree,

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$



Let the two straight lines;

$$l_1x+m_1y+n_1=0 \text{ -------- (i)}$$
$$l_2x+m_2y+n_2=0 \text{ ---- (ii)}$$

Let, two lines intersects at points $P(x_1,y_1)$

So,

$$l_1x_1+m_1y_1+n_1=0$$
$$l_2x_1+m_2y_1+n_2=0$$

By cross multiplication

$$\frac{x_1}{m_1n_2 \pm m_2n_1} = \frac{y_1}{n_1l_2-n_2l_1} = \frac{1}{l_1m_2-m_1l_2}$$

$$\therefore x_1 = \frac{m_1n_2-m_2n_1}{l_1m_2-m_1l_2} \quad , \quad y_1 = \frac{n_1l_2-n_2l_1}{l_1m_2-m_1l_2}$$

From graph,

$$OP^2 = (x_1-0)^2 + (y_1-0)^2$$
$$= x_1^2 + y_1^2$$
$$= \left(\frac{m_1n_2-m_2n_1}{l_1m_2-m_1l_2}\right)^2 + \left(\frac{n_1l_2-n_2l_1}{l_1m_2-m_1l_2}\right)^2$$

$$= \frac{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - l_1n_2)^2}{(l_1m_2 - m_1l_2)^2}$$

$$= \frac{(m_1n_2 + m_2n_1)^2 - 4m_1m_2n_1n_2 + (n_1l_2 + l_1n_2)^2 - 4l_1l_2n_1n_2}{(l_1m_2 + m_1l_2)^2 - 4l_1l_2m_1m_2}$$

$$= \frac{(2f)^2 - 4bc + (2g)^2 - 4ac}{(2h)^2 - 4ab}$$

$$= \frac{\cancel{4}f^2 - \cancel{4}bc + \cancel{4}g^2 - \cancel{4}ac}{\cancel{4}h^2 - \cancel{4}ab}$$

$$= \frac{c(a+b) - f^2 - g^2}{ab - h^2}$$ Ans.

Q-11

Find the condition that the pair of straight lines $ax^2+2hxy+by^2=0$ and $a'x^2+2h'xy+b'y^2=0$ should have a common line.

Or,

If one of the lines $ax^2+2hxy+by^2=0$ be coincident to one of the lines $a'x^2+2h'xy+b'y^2=0$, then prove that, $(ab'-a'b)^2=4(ah'-a'h)(bh'-b'h)$

Solution

The given equation

$$ax^2+2hxy+by^2=0 \quad \text{------ (i)}$$

and $$a'x^2+2h'xy+b'y^2=0 \quad \text{------ (ii)}$$

let, the common line is

$$y=mx$$

Or, $$\frac{y}{x}=m$$

Now, dividing the equation (i) and (ii) by x^2

$$a+2h\frac{y}{x}+b\left(\frac{y}{x}\right)^2=0$$

$$\Rightarrow a+2hm+bm^2=0 \quad \text{------ (iii)}$$

and,

$$a'+2h'\frac{y}{x}+b'\left(\frac{y}{x}\right)^2=0$$

$$\Rightarrow a'+2h'm+b'm^2=0 \quad \text{------ (iv)}$$

Solving the equation (iii) and (iv)

$$\frac{m^2}{a'2h-a2h'}=\frac{m}{ab'-a'b}=\frac{1}{2bh'-2b'h}$$

$$\therefore m^2=\frac{a'h-h'a}{bh'-b'h} \quad \text{------ (v)}$$

and

$$m = \frac{ab' - a'b}{2h'b - 2hb'}$$

$$m^2 = \frac{(ab' - a'b)^2}{(2h'b - 2hb')^2} \quad \text{———} \quad (vi)$$

from (v) and (vi)

$$\frac{a'h - h'a}{bh' - b'h} = \frac{(ab' - a'b)^2}{(2h'b - 2hb')^2}$$

$$\Rightarrow \frac{a'h - h'a}{bh' - b'h} = \frac{(ab' - a'b)^2}{4(h'b - hb')^2}$$

$$\Rightarrow \frac{(a'h - h'a).4(h'b - hb')^2}{bh' - b'h} = (ab' - a'b)^2$$

$$\Rightarrow (ab' - a'b)^2 = 4(ah' - ha')(h'b - hb') \quad \underline{\text{Ans.}}$$

Q-12

Find the equation of the bisectors of the angles between the lines represented by the equation $2x^2+7xy+6y^2+13x+22y+20=0$

Soln

Given that,

$$2x^2+7xy+6y^2+13x+22y+20=0$$

$$2x^2+x(7y+13)+6y^2+22y+20=0$$

$$x=\frac{-(7y+13)\pm\sqrt{(7y+13)^2-4\times2(6y^2+22y+20)}}{4}$$

$$4x=-(7y+13)\pm\sqrt{49y^2+1282y+169-48y^2-176y-160}$$

$$=-(7x+13)\pm\sqrt{y^2+6y+9}$$

$$=-7x-13\pm(y+3)$$

So, the straight lines will be,

$4x=-7x-13+y+3$	$4x=-7x-13-y-3$
$\Rightarrow 11x-y+10=0$	$11x+y+16=0$

The bisectors will be

$$\frac{11x-y+10}{\sqrt{11^2+1^2}}=\pm\frac{11x+y+16}{\sqrt{11^2+1^2}}$$

$\Rightarrow \cancel{11x}-y+10=\cancel{11x}+y+16$	$11x-\cancel{y}+10=-11x-\cancel{y}-16$
$\Rightarrow 2y+16=0$ Ans.	$22x+26=0$ Ans.

Q-13
Prove that, the pair of the lines joining the origin to the point of intersection of the curve. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ by the straight lines $lx+my+n=0$ are coincident if $a^2l^2+b^2m^2=n^2$

Soln

The given equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{------ (i)}$$

and

$$lx+my+n=0 \quad \text{------ (ii)}$$

From equ (ii)

$$lx+my=-n$$

$$\Rightarrow \frac{lx}{-n}+\frac{my}{-n}=1$$

$$\Rightarrow \frac{(lx+my)^2}{n^2}=1 \quad (\text{squaring})$$

Putting the value in equ (i)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{(lx+my)^2}{n^2}$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{l^2x^2}{n^2}-\frac{2lmxy}{n^2}-\frac{m^2y^2}{n^2}=0$$

$$\Rightarrow \left(\frac{1}{a^2}-\frac{l^2}{n^2}\right)x^2+\left(\frac{1}{b^2}-\frac{m^2}{n^2}\right)y^2-\frac{2xylm}{n^2}=0$$

This equation represents a pair of straight line through the origin

The two lines will be coincide, if.

$$h^2-ab=0$$

$$h^2=ab$$

$$\left(\frac{lm}{n^2}\right)^2=\left(\frac{1}{a^2}-\frac{l^2}{n^2}\right)\left(\frac{1}{b^2}-\frac{m^2}{n^2}\right)$$

$$\Rightarrow \cancel{\frac{l^2m^2}{n^4}} = \frac{1}{a^2b^2} - \frac{m^2}{a^2n^2} - \frac{l^2}{n^2b^2} + \cancel{\frac{l^2m^2}{n^4}}$$

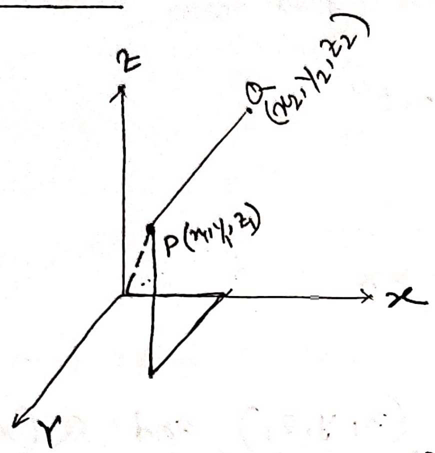
$$\Rightarrow \frac{1}{a^2b^2} - \frac{m^2}{a^2n^2} - \frac{l^2}{n^2b^2} = 0$$

$$\Rightarrow \frac{n^2 - b^2m^2 - a^2l^2}{a^2b^2n^2} = 0$$

$$\Rightarrow a^2l^2 + b^2m^2 = n^2 \quad \text{(Proved)}$$

RECTANGULAR CO-ORDINATES
[3D]

General discussion



distance between two points of three dimension

$$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

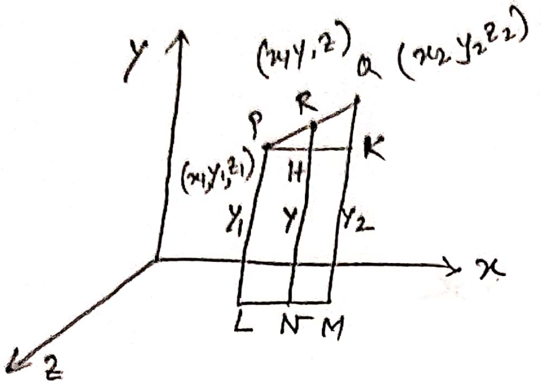
Topic of discussion -

- Theorem -01 - Section ~~ratio~~ formula
- Theorem -02 - Centre of gravity
- Theorem -03 - Direction cosine of a line (D.C.L)
- Theorem -04 - Direction ratio
- Theorem -05 - Projection of line
- Theorem -06 - Angle between two lines.

Theorem - 01

To find the co-ordinates of the point which divides the straight lines joining two given points in a given ratio.

Solution:



Let two given points be $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ and $R(x, y, z)$ be a point of PQ. Such that $PR : RQ = m_1 : m_2$

Now, two triangle $\triangle HPR$ and $\triangle KPQ$ are similar, then,

$$\frac{m_1}{m_2} = \frac{PR}{RQ} \Rightarrow \frac{m_2}{m_1} = \frac{RQ}{PR}$$

$$\Rightarrow \frac{m_1+m_2}{m_1} = \frac{PR+RQ}{PR} = \frac{PQ}{PR} = \frac{KQ}{RH} = \frac{MQ-MK}{NR-NH}$$

$$\Rightarrow \frac{m_1+m_2}{m_1} = \frac{y_2-y_1}{y-y_1}$$

$$\Rightarrow m_1y + m_2y - \cancel{m_1y_1} - m_2y_1 = m_1y_2 - \cancel{m_1y_1}$$

$$\Rightarrow \boxed{y = \frac{m_1y_2 + m_2y_1}{m_1+m_2}}$$

Similarly

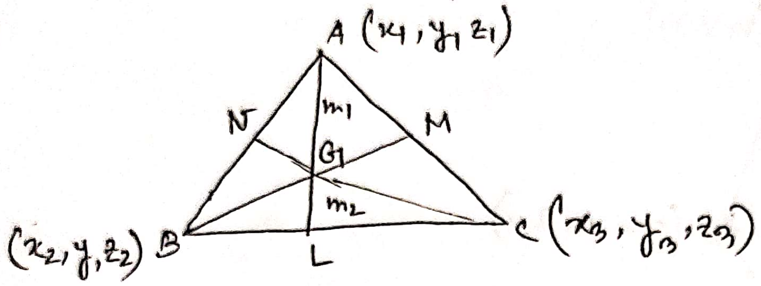
$$\boxed{z = \frac{m_1z_2 + m_2z_1}{m_1+m_2}}$$

and $$\boxed{x = \frac{m_1x_2 + m_2x_1}{m_1+m_2}}$$

Theorem - 02

1) Find the centre of gravity of the coordinates of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3)

Solution



L, the mid point of the BC, so the co-ordinates of L will be,

$$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$$

The cetre of the gravity of the $\triangle ABC$ which divides AL in the ratio 2:1, so the co-ordinate will be

Then,

$$x = \frac{2\left(\frac{x_2+x_3}{2}\right) + 1 \cdot x_1}{2+1}$$

$$= \frac{x_1+x_2+x_3}{3} \quad \text{Ans.}$$

$$y = \frac{\cancel{2}\left(\frac{y_2+y_3}{\cancel{2}}\right) + 1 \cdot y_1}{2+1}$$

$$= \frac{y_1+y_2+y_3}{3}$$

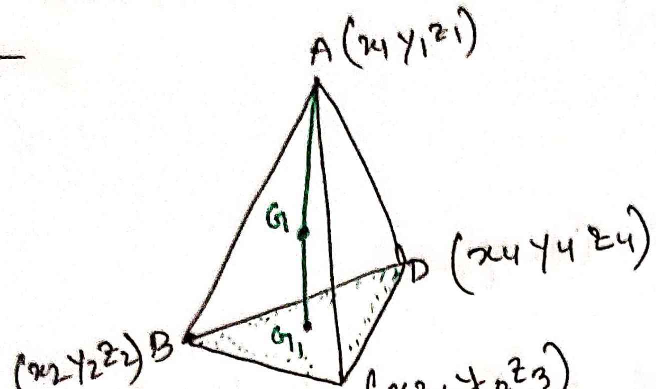
$$z = \frac{z_1+z_2+z_3}{3}$$

$$\therefore (x, y, z) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

Ans.

3) Find the co-ordinates of the centre of gravity of the tetrahedron which vertices are, $(x_1 y_1 z_1)$, $(x_2 y_2 z_2)$ $(x_3 y_3 z_3)$, (x_4, y_4, z_4)

Soln



The center of the gravity for $\triangle BCD$ will be

$$G_1 = \left(\frac{x_2+x_3+x_4}{3}, \frac{y_2+y_3+y_4}{3}, \frac{z_2+z_3+z_4}{3} \right)$$

The center of the gravity of tetrahedron which divides AG_1 at the ratio of 3:1

$$\therefore x = \frac{3\,\frac{x_2+x_3+x_4}{3} + 1x_1}{3+1}$$

$$= \frac{x_1+x_2+x_3+x_4}{4}$$

$$y = \frac{3\,\frac{y_2+y_3+y_4}{3} + 1y_1}{4}$$

$$= \frac{y_1+y_2+y_3+y_4}{4}$$

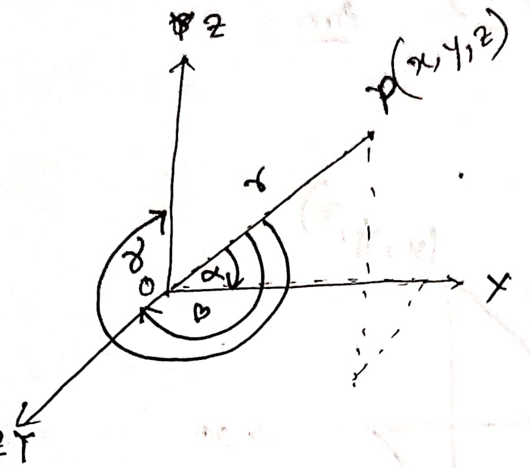
$$z = \frac{z_1+z_2+z_3+z_4}{4}$$

Ans.

Theorem-03 Direction cosine of a line:

Prove that, $l^2+m^2+n^2=1$ or, $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, or $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$ where l, m, n is the direction cosine of a line.

Solution



Soln

Let, OP be the straight line whose direction cosine are $\cos\alpha, \cos\beta, \cos\gamma$

Again, suppose $P(x,y,z)$ be any point on OP, Now the projection of OP axes,

$$x = r\cos\alpha = rl \quad \text{---} (i)$$

$$y = r\cos\beta = rm \quad \text{---} (ii)$$

$$z = r\cos\gamma = rn \quad \text{---} (iii)$$

Now,

$$OP^2 = x^2+y^2+z^2 = r^2$$

again, squaring (i)+(ii)+(iii)

$$x^2+y^2+z^2 = r^2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)$$

$$\Rightarrow r^2 = r^2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)$$

$$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1 \quad \text{Ans.}$$

$$\Rightarrow l^2+m^2+n^2 = 1 \quad \text{Ans.}$$

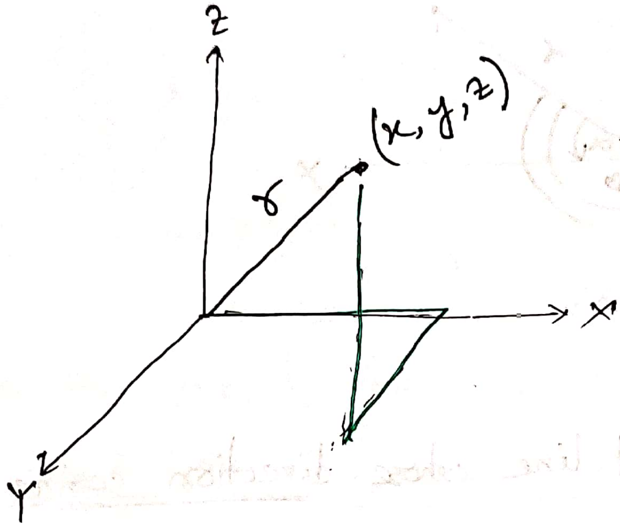
$\cos\alpha = l$
$\cos\beta = m$
$\cos\gamma = n$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$\Rightarrow \underline{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2} \quad \underline{\text{Ans.}}$$

<u>Vectore analysis</u>



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= rl\hat{i} + rm\hat{j} + rn\hat{k}$$

$$\vec{r} = r(l\hat{i} + m\hat{j} + n\hat{k})$$

$$\frac{\vec{r}}{|r|} = (l\hat{i} + m\hat{j} + n\hat{k})$$

$$\underline{\hat{r} = l\hat{i} + m\hat{j} + n\hat{k}} \; *$$

$$|\hat{r}| = \sqrt{l^2 + m^2 + n^2} = 1$$

$$\text{Or, } \underline{l^2 + m^2 + n^2 = 1} \; *$$

<u>Defination</u>

The defiection cosine of a vector are the cosines of the angle between the vector and three co-ordinate axes.

Theorem-04 Direction Ratio

Any three numbers a, b, c which are proportional to the direction cosines l, m, n respectively of the given line are called direction ratios of the given line.

From the defination -

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \lambda$$

$$l = a\lambda \quad \text{(i)}$$

$$m = b\lambda \quad \text{(ii)}$$

$$n = c\lambda \quad \text{(iii)}$$

$(i)^2 + (ii)^2 + (iii)^2$

$$l^2 + m^2 + n^2 = \lambda^2(a^2+b^2+c^2)$$

We know, $l^2+m^2+n^2 = 1$

$$\Rightarrow \lambda = \pm \frac{1}{\sqrt{a^2+b^2+c^2}}$$

So,

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$\cos\alpha = \frac{a}{\sqrt{\Sigma a^2}}$

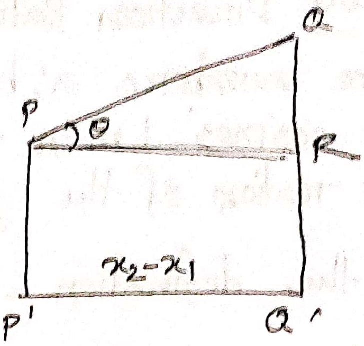
relation between DC and DR

$\cos\beta = \frac{b}{\sqrt{\Sigma a^2}}$

$\cos\gamma = \frac{c}{\sqrt{\Sigma a^2}}$

where l, m, n are direction cosines and a, b, c are direction ratios.

Theorem-05 Projection of Lines



$P'Q'$ is the projection of line.

$$PR = P'Q'$$

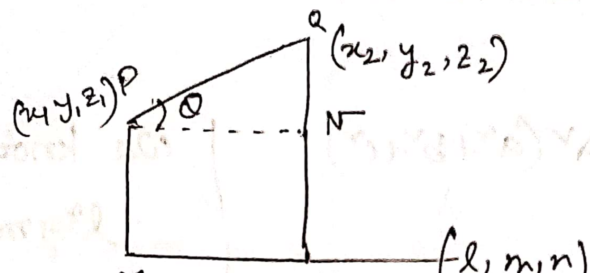
$$\frac{PR}{PQ} = \cos\theta$$

$\therefore PR = PQ\cos\theta$

$\therefore P'Q' = PQ\cos\theta$

$\therefore \underbrace{x_2-x_1 = PQ\cos\theta}$ Projection of line

** Projection of a line joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) on another line whose direction cosine are l, m, n



Projection of line

$\cancel{PQ\cos\theta} = \cancel{(x_2-x_1)l + (y_2-y_1)m + (z_2-z_1)n}$

we know from theorem 06

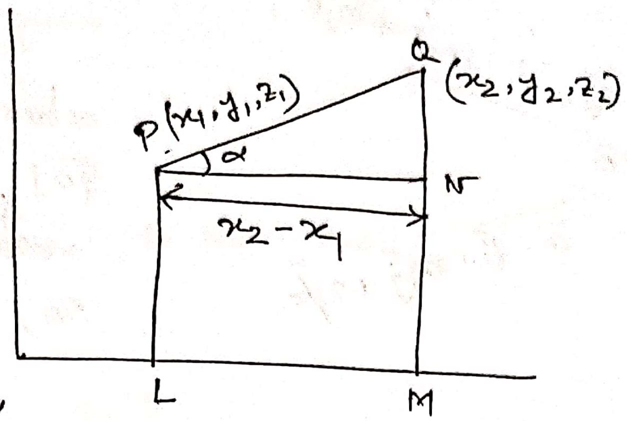
Angle cosine two lines is

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

Projection of line $= \frac{x_2-x_1}{PQ} \times l + \frac{y_2-y_1}{PQ} \times m + \frac{z_2-z_1}{PQ} \times n$

$\therefore \underline{PQ\cos\theta} = (x_2-x_1)l + (y_2-y_1)m + (z_2-z_1)n$

Direction ratio



$LM = PN = PQ \cos\alpha$

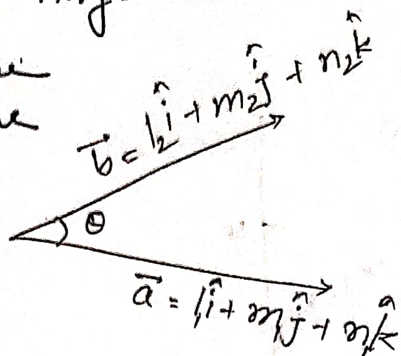
$$\cos\alpha = \frac{x_2 - x_1}{PQ}$$

$$\cos\beta = \frac{y_2 - y_1}{PQ}$$

$$\cos\gamma = \frac{z_2 - z_1}{PQ}$$

Theorem-06 Angle between two lines.

when given the derection cosine



where $\vec{a}$, $\vec{b}$ are unique vector.

so, $|\vec{a}| = 1$

$|\vec{b}| = 1$

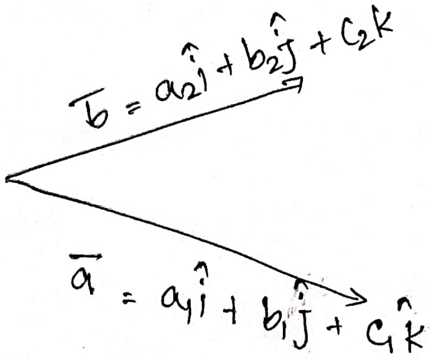
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\sqrt{(l_1^2 + m_1^2 + n_1^2)} \times \sqrt{l_2^2 + m_2^2 + n_2^2}}$$

$$\therefore \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2 \quad \text{Ans.}$$

$$\theta = \cos^{-1}(l_1 l_2 + m_1 m_2 + n_1 n_2)$$

when given the direction ratio



$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|.|\vec{b}|}$$

$$= \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Math-01

Show that when two lines are parallel, the direction cosine of two lines are equal. Or, $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$

Soln

We know that:

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

We know,

$$\sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$$

$$= (l_1^2 + m_1^2 + n_1^2) \times (l_2^2 + m_2^2 + n_2^2) - (l_1 l_2 + m_1 m_2 + n_1 n_2)^2$$

$$\text{Or, } \sin^2\theta = (m_2 n_1 - n_2 m_1)^2 + (n_1 l_2 - l_1 n_2)^2 + (m_2 l_1 - l_2 m_1)^2$$

$$\text{Or } \sin\theta = \sqrt{(m_2 n_1 - n_2 m_1)^2 + (n_1 l_2 - l_1 n_2)^2 + (m_2 l_1 - l_2 m_1)^2}$$

$$\sin\theta = \sqrt{\sum (m_2 n_1 - n_2 m_1)^2} \quad *$$

When two lines are parallel θ will be $0°$

then, $\sin\theta = 0$.

$$\text{So, } (m_2 n_1 - n_2 m_1)^2 + (n_1 l_2 - l_1 n_2)^2 + (m_2 l_1 - l_2 m_1)^2 = 0$$

$(m_2 n_1 - n_2 m_1)^2 = 0$	$(n_1 l_2 - l_1 n_2)^2 = 0$	$(m_2 l_1 - l_2 m_1)^2 = 0$
$\Rightarrow \dfrac{m_2}{m_1} = \dfrac{n_2}{n_1}$	$\dfrac{n_2}{n_1} = \dfrac{l_2}{l_1}$	$\dfrac{m_2}{m_1} = \dfrac{l_2}{l_1}$

$$\text{So, } \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{\sqrt{l_1^2 + m_1^2 + n_1^2}}{\sqrt{l_2^2 + m_2^2 + n_2^2}} = \frac{1}{1}$$

So, $l_1 = l_2$; $m_1 = m_2$ and $n_1 = n_2$ Ans.

Math-02 show that,
when two lines are parallel, the direction ~~cosine~~ ratio of the two lines are propotional. or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
and when two lines are perpendicular, show that
$a_1a_2 + b_1b_2 + c_1c_2 = 0$

Soln we know, angle cosine between two lines

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \times \sqrt{a_2^2+b_2^2+c_2^2}}$$

when they are perpendicular, $\cos\theta = 0$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$ Ans.

if they are parallel, we know,

$$l_1 = l_2$$

$$\Rightarrow \frac{a_1}{\sqrt{\Sigma a_1^2}} = \frac{a_2}{\sqrt{\Sigma a_2^2}}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}} \quad \cdots\cdots (i)$$

$$m_1 = m_2$$

$$\Rightarrow \frac{b_1}{\sqrt{\Sigma a_1^2}} = \frac{b_2}{\sqrt{\Sigma a_2^2}}$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}} \quad \cdots\cdots (ii)$$

$$n_1 = n_2$$

$$\Rightarrow \frac{c_1}{\sqrt{\Sigma a_1^2}} = \frac{c_2}{\sqrt{\Sigma a_2^2}}$$

$$\Rightarrow \frac{c_1}{c_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}} \quad \cdots\cdots (iii)$$

from ~~by~~ the equ (i), (ii) (iii)

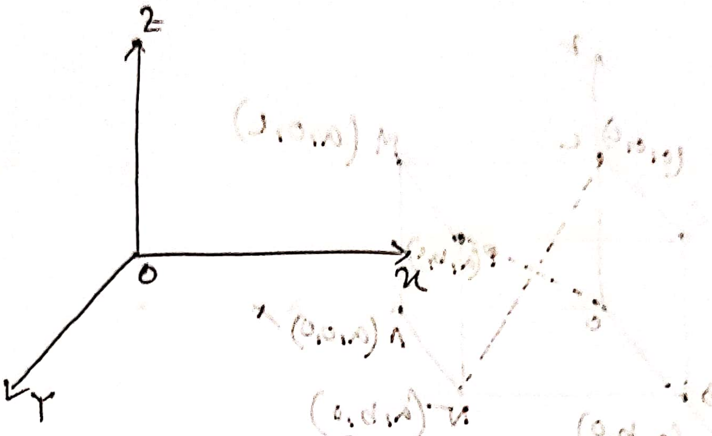
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Math-02

perpendicular

Find out the direction cosine of a line with respect to other two lines which are also perpendicular to one another. Given that the D.C of those two lines are (l_1, m_1, n_1) and (l_2, m_2, n_2)

Soln



Let the Direction cosine of 3rd line is (l, m, n)

And given, Direction cosine, $O_x = (l_1, m_1, n_1)$

$O_y = (l_2, m_2, n_2)$

Because of they are perpendicular

$$l l_1 + m m_1 + n n_1 = 0$$

$$l l_2 + m m_2 + n n_2 = 0$$

$$\therefore \frac{l}{m_1 n_2 - m_2 n_1} = \frac{m}{n_2 l_1 - n_1 l_2} = \frac{n}{l_1 m_2 - m_1 l_2}$$

$$= \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{\Sigma (m_1 n_2 - m_2 n_1)^2}} = \frac{1}{\sin\theta} = \frac{1}{\sin 90^\circ} = 1$$

$$\therefore l = m_1 n_2 - m_2 n_1$$
$$m = n_1 l_2 - n_2 l_1$$
$$n = l_1 m_2 - m_1 l_2$$

Ans.

Math-02 Show that,

when two lines are parallel, the direction cosine ratio of the two lines are proportional. or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and when two lines are perpendicular, show that $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Soln we know, angle cosine between two lines

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2} \times \sqrt{a_2^2+b_2^2+c_2^2}}$$

when they are perpendicular, $\cos\theta = \cancel{8}\ 0$

$$a_1a_2 + b_1b_2 + c_1c_2 = \cancel{0}\ 0 \quad \text{Ans.}$$

if they are parallel, we know,

$$l_1 = l_2$$

$$\Rightarrow \frac{a_1}{\sqrt{\Sigma a_1^2}} = \frac{a_2}{\sqrt{\Sigma a_2^2}}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}} \quad \cdots\cdots (i)$$

$$\cancel{m} \quad m_1 = m_2$$

$$\Rightarrow \frac{b_1}{\sqrt{\Sigma a_1^2}} = \frac{b_2}{\sqrt{\Sigma a_2^2}}$$

$$\Rightarrow \frac{b_1}{b_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}} \quad \cdots\cdots (ii)$$

$$n_1 = n_2$$

$$\Rightarrow \frac{c_1}{\sqrt{\Sigma a_1^2}} = \frac{c_2}{\sqrt{\Sigma a_2^2}}$$

$$\Rightarrow \frac{c_1}{c_2} = \frac{\sqrt{\Sigma a_1^2}}{\sqrt{\Sigma a_2^2}} \quad \cdots\cdots (iii)$$

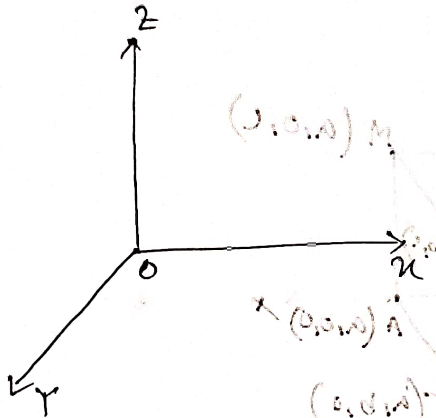
from by the equ (i), (ii) (iii)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Math-02

Find out the direction cosine of a line with respect to other two lines which are also perpendicular to one another. Given that these D.C of those two lines are (l_1, m_1, n_1) and (l_2, m_2, n_2)

Soln



let the Direction cosine of 3rd line is (l, m, n)

And given, Direction cosine, $Ox = (l_1, m_1, n_1)$

$Oy = (l_2, m_2, n_2)$

Because of they are perpendicular

$$ll_1 + mm_1 + nn_1 = 0$$

$$ll_2 + mm_2 + nn_2 = 0$$

$$\therefore \frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_2l_1 - n_1l_2} = \frac{n}{l_1m_2 - m_1l_2}$$

$$= \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{\Sigma(m_1n_2 - m_2n_1)^2}} = \frac{1}{\sin\theta} = \frac{1}{\sin 90^\circ} = 1$$

$$\therefore l = m_1n_2 - m_2n_1$$

$$m = n_1l_2 - n_2l_1$$

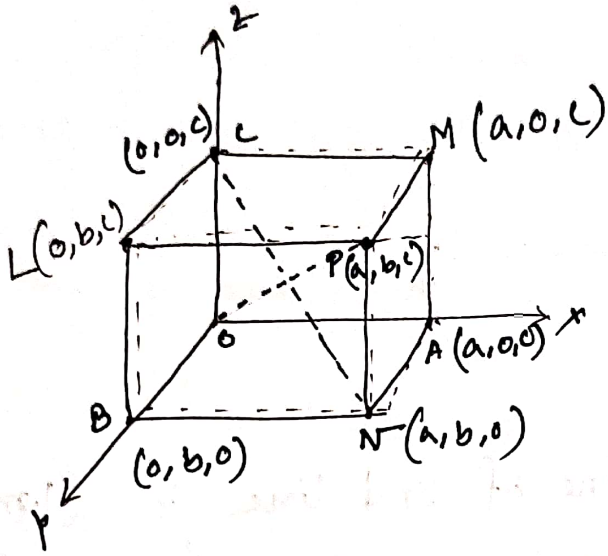
$$n = l_1m_2 - m_1l_2$$

Ans.

Math-03

If the edeges of a reectangulare parallelopiped are a, b, c show that, the angle between four diagonals are given by
$$\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$$

Soln



Direction cosine of $\underline{OP}$ are $\frac{a}{\sqrt{\Sigma a^2}}, \frac{b}{\sqrt{\Sigma a^2}}, \frac{c}{\sqrt{\Sigma a^2}}$

Direction cosine of OM are $\frac{a}{\sqrt{\Sigma a^2}}, \frac{-b}{\sqrt{\Sigma a^2}}, \frac{c}{\sqrt{\Sigma a^2}}$

Direction cosine of $\underline{CN}$ are $\frac{a}{\sqrt{\Sigma a^2}}, \frac{b}{\sqrt{\Sigma a^2}}, \frac{-c}{\sqrt{\Sigma a^2}}$

Direction cosine of AL are $\frac{-a}{\sqrt{\Sigma a^2}}, \frac{b}{\sqrt{\Sigma a^2}}, \frac{c}{\sqrt{\Sigma a^2}}$

The angle between OP and CN, therefore is
$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$\therefore \cos\theta = \frac{a}{\sqrt{\Sigma a^2}} \times \frac{a}{\sqrt{\Sigma a^2}} + \frac{b}{\sqrt{\Sigma a^2}} \times \frac{b}{\sqrt{\Sigma a^2}} + \frac{c}{\sqrt{\Sigma a^2}} \times \frac{-c}{\sqrt{\Sigma a^2}}$$

$$= \frac{a^2}{\Sigma a^2} + \frac{b^2}{\Sigma a^2} - \frac{c^2}{\Sigma a^2}$$

$$= \frac{a^2+b^2-c^2}{a^2+b^2+c^2}$$

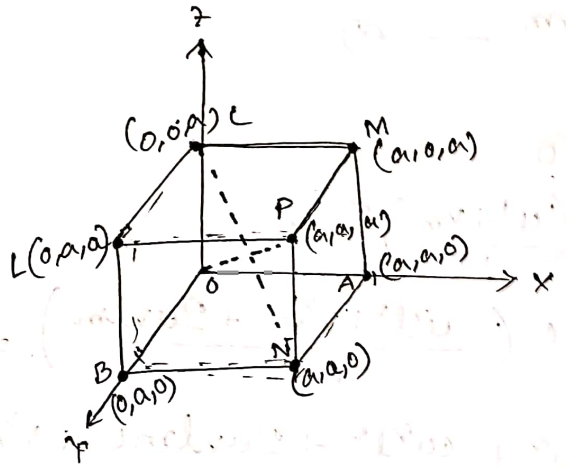
Similarly the angle between the other five of the pairs of diagonal can be found. So, the angle between six diagonals

$$\theta = \cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$$ showed

Math-04

Prove that, the angle between two diagonals of a cubic is $\cos^{-1} 1/3$

Soln



DC. of OP are, $\frac{a}{\sqrt{a^2+a^2+a^2}} = \frac{a}{\sqrt{3a^2}}, \frac{a}{\sqrt{3a^2}}, \frac{a}{\sqrt{3a^2}}$

$$= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

DC of CN, $\frac{a}{\sqrt{3a^2}}, \frac{a}{\sqrt{3a^2}}, -\frac{a}{\sqrt{3a^2}}$

$$= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$$

$$\cos\theta = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{1}{3}$$

$$\theta = \cos^{-1}\frac{1}{3}$$ (Proved)

Math-05

Show that the straight two lines whose direction cosine are connected by the relation $ul+vm+wn=0$ and $al^2+bm^2+cn^2=0$ are perpendicular if $u^2(b+c)+v^2(c+a)+w^2(a+b)$ and parallel if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=0$

Soln

Given equation,

$$ul+vm+wn=0$$

$$n=-\frac{ul+vm}{w} \quad \text{--- (i)}$$

Now,

$$al^2+bm^2+cn^2=0$$

$$\Rightarrow al^2+bm^2+c\left(-\frac{ul+vm}{w}\right)^2=0$$

$$\Rightarrow al^2+bm^2+c\left(\frac{u^2l^2+v^2m^2+2uvlm}{w^2}\right)=0$$

$$\Rightarrow al^2w^2+bm^2w^2+cu^2l^2+2uvlmc+v^2m^2c=0$$

$$\Rightarrow (aw^2+cu^2)l^2+(bw^2+v^2c)m^2+2uvlmc=0$$

$a_1x^2+b_1x+c_1=0$ $$\Rightarrow (aw^2+cu^2)\frac{l^2}{m^2}+(bw^2+v^2c)+2uvc\frac{l}{m}=0$$

Again, if

$$m=-\left(\frac{ul+wn}{v}\right)$$

then, $$al^2+b\left(\frac{ul+wn}{v}\right)^2+cn^2=0$$

$$\Rightarrow (av^2+bu^2)\frac{l^2}{n^2}+2ub\frac{l}{n}w+(bw^2+cv^2)=0$$

$a_2x^2+b_2x+c_2=0$ $$\Rightarrow (av^2+bu^2)\frac{l^2}{n^2}+2ubw\frac{l}{n}+(bw^2+cv^2)=0$$

$=ul\,wn\,b$

let,

$\frac{l_1}{m_1}, \frac{l_2}{m_2}$ are product of first equa

so,

$$\frac{l_1}{m_1} \times \frac{l_2}{m_2} = \frac{b\omega^2 + v^2c}{a\omega^2 + cu^2}$$

[illegible]

$$\Rightarrow \frac{l_1 l_2}{b\omega^2 + cv^2} = \frac{m_1 m_2}{a\omega^2 + cu^2}$$

let,

$\frac{l_1}{n_1}, \frac{l_2}{n_2}$ are product of 2nd equa

so,

$$\frac{l_1}{n_1} \times \frac{l_2}{n_2} = \frac{b\omega^2 + cv^2}{av^2 + bu^2}$$

$$\Rightarrow \frac{l_1 l_2}{b\omega^2 + cv^2} = \frac{n_1 n_2}{av^2 + bu^2}$$

⋮