Design for Flexure and Shear

To determine the load capacity or the size of beam section, it must satisfy the allowable stresses in both flexure (bending) and shear. Shearing stress usually governs in the design of short beams that are heavily loaded, while flexure is usually the governing stress for long beams. In material comparison, timber is low in shear strength than that of steel.

For any cross-sectional shape, flexure and shear are given in the following formulas:

Flexure Formula

$$f_b = \frac{Mc}{I}$$

Horizontal Shear Stress

$$f_v = \frac{VQ}{Ib}$$

For rectangular beam, the following defines for flexure and shear:

Flexure formula for rectangular beam

$$f_b = \frac{6M}{bd^2}$$

Horizontal shear stress for rectangular beam

$$f_v = \frac{3V}{2bd}$$

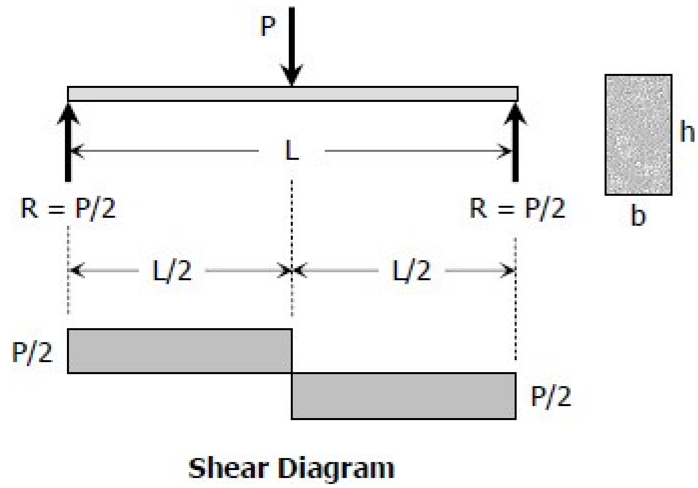
Where
f_b = flexure stress
f_v = bending stress
M = maximum moment applied to the beam
V = maximum shear applied to the beam
I = moment of inertia about the neutral axis
Q = moment of area
b = breadth
d = depth

Solution to Problem 580 | Design for Flexure and Shear

Problem 580
A rectangular beam of width b and height h carries a central concentrated load P on a simply

supported span of length L. Express the maximum f_v in terms of maximum f_b.

Solution 580



From the figure:

$V_{max} = \frac{1}{2}P$

$M_{max} = \frac{1}{2}L\left(\frac{1}{2}P\right) = \frac{1}{4}PL$

From flexure formula:

$$f_b = \frac{6M}{bh^2} = \frac{6(\frac{1}{4}PL)}{bh^2}$$

$$f_b = \frac{3PL}{2bh^2}$$

$$bh = \frac{3PL}{2f_b\,h}$$

From shear stress formula:

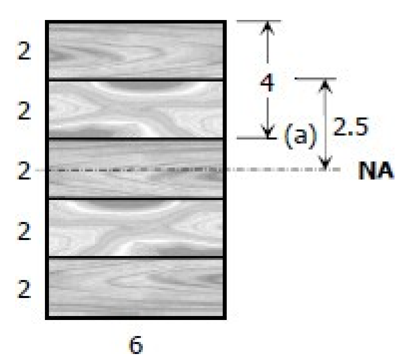
$$f_b = \frac{3V}{2bh} = \frac{3(\frac{1}{2}P)}{2\left(\dfrac{3PL}{2f_b\,h}\right)}$$

$$f_v = \frac{f_b\,h}{2L} \qquad \textit{answer}$$

Problem 581

A laminated beam is composed of five planks, each 6 in. by 2 in., glued together to form a section 6 in. wide by 10 in. high. The allowable shear stress in the glue is 90 psi, the allowable shear stress in the wood is 120 psi, and the allowable flexural stress in the wood is 1200 psi. Determine the maximum uniformly distributed load that can be carried by the beam on a 6-ft simple span.

Solution 581



Maximum moment for simple beam

$M_{max} = \frac{1}{8}w_o\,L^2$

$M_{max} = \frac{1}{8}w_o\,(6^2)$

$M_{max} = 4.5w_o$ lbmiddot;ft

Maximum shear for simple beam

$V_{max} = \frac{1}{2}w_o\,L$

$V_{max} = \frac{1}{2}w_o\,(6)$

$V_{max} = 3w_o$ lb

For bending stress of wood

$$f_b = \frac{6M}{bd^2}$$

$$120 = \frac{6(4.5w_o)(12)}{6(10^3)}$$

$$w_o = 2\,222.22 \text{ lb/ft}$$

For shear stress of wood

$$(f_v)_{wood} = \frac{3V}{2bd}$$

$$120 = \frac{3(3w_o)}{2(6)(10)}$$

$$w_o = 1600 \text{ lb/ft}$$

For shear stress in the glued joint

$$(f_v)_{glue} = \frac{VQ}{Ib}$$

Where:

$$Q = 6(4)(2.5) = 60 \text{ in}^3$$

$$I = \frac{bd^3}{12} = \frac{6(10^3)}{12} = 500 \text{ in}^4$$

$$b = 6 \text{ in}$$

Thus,

$$90 = \frac{3w_o(60)}{500(6)}$$

$$w_o = 1250 \text{ lb/ft}$$

Use $\mathbf{w_o = 1250}$ **lb/ft** for safe value of uniformly distributed load. *answer*

Problem 582

Find the cross-sectional dimensions of the smallest square beam that can be loaded as shown in Fig. P-582 if $f_v \leq 1.0$ MPa and $f_b \leq 8$ MPa.

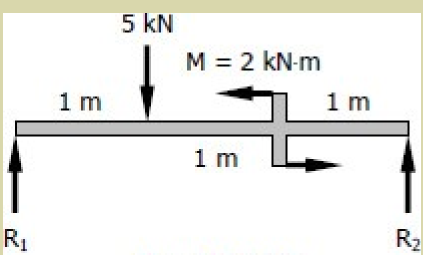


Figure P-582

Solution 582

$\Sigma M_{R2} = 0$

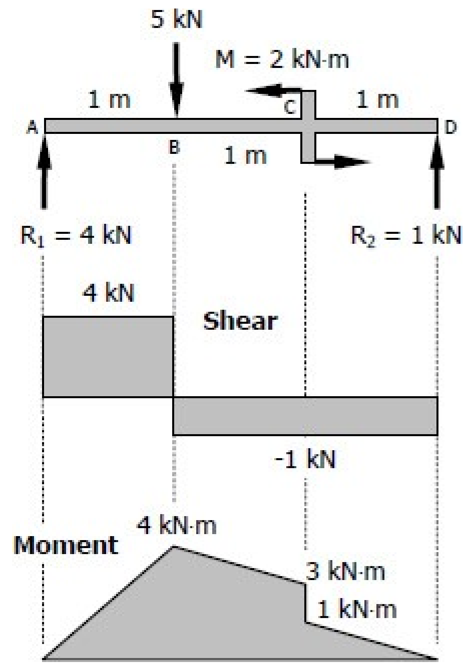
$3R_1 = 5(2) + 2$

$R_1 = 4 \text{ kN}$

$\Sigma M_{R2} = 0$

$3R_2 + 2 = 5(1)$

$R_2 = 1 \text{ kN}$



Based on bending stress (square b = d):

$f_b = \dfrac{6M}{bd^2}$

$8 = \dfrac{6(4)(1000^2)}{d^3}$

$d = 144.22 \text{ mm}$

Based on shear stress (square b = d):

$f_v = \dfrac{3V}{2bd}$

$1 = \dfrac{3(4)(1000)}{2d^2}$

$d = 77.46 \text{ mm}$

Use **145 mm** × **145 mm** square beam *answer*

Problem 584

A wide-flange section having the dimensions shown in Fig. P-584 supports a distributed load of w_o lb/ft on a simple span of length L ft. Determine the ratio of the maximum flexural stress to the maximum shear stress.

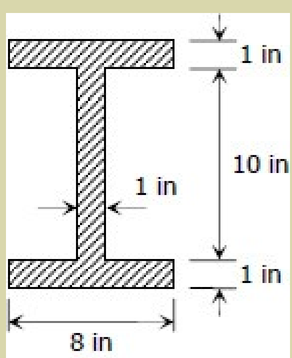


Figure P-584

Solution 584

Bending stress:

$$f_b = \frac{Mc}{I}$$

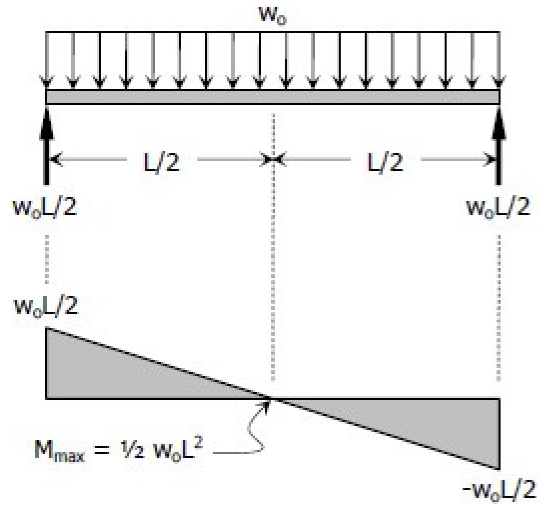
where:

$M = \frac{1}{8}w_oL^2 \ \text{lb}\cdot\text{ft}$

$c = 12/2 = 6 \ \text{in}$

$$I = \frac{8(12^3)}{12} - \frac{7(10^3)}{12}$$

$$I = \frac{1706}{3} \ \text{in}^4$$



Thus,

$$f_b = \frac{\frac{1}{8}w_oL^2(12)(6)}{\frac{1706}{3}}$$

$$f_b = \frac{27}{1706}w_oL^2$$

Shear stress:

$$f_v = \frac{VQ}{Ib}$$

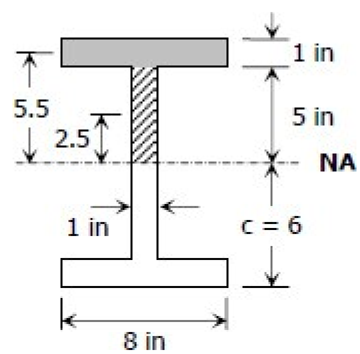
where:

$V = \frac{1}{2}w_oL$

$Q = 8(1)(5.5) + 5(1)(2.5) = 56.5 \ \text{in}^3$

$I = \frac{1706}{3} \ \text{in}^4$ (see computation above)

$b = 1 \ \text{in}$



Thus,

$$f_v = \frac{(\frac{1}{2}w_oL)(56.5)}{\frac{1706}{3}(1)}$$

$$f_v = \frac{339}{6824}w_oL$$

Ratio (flexural stress : shear stress)

$$\text{Ratio} = \frac{\frac{27}{1706}w_oL^2}{\frac{339}{6824}w_oL}$$

$$\text{Ratio} = \frac{36}{113}L$$

$$\text{Ratio} = 0.3186L$$ *answer*

Problem 585

A simply supported beam of length L carries a uniformly distributed load of 6000 N/m and has the cross section shown in Fig. P-585. Find L to cause a maximum flexural stress of 16 MPa. What maximum shearing stress is then developed?

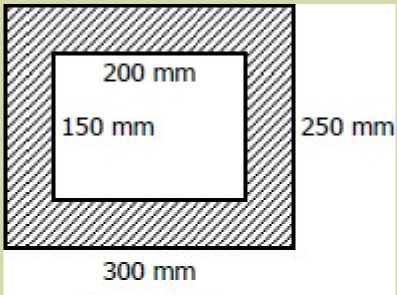


Figure P-585

Solution 585

Flexural Stress

$$f_b = \frac{Mc}{I}$$

Where
f_b = 16 MPa
$M = 1/8\ w_oL^2 = 1/8\ (6000)L^2 = 750L^2$ N·m
c = ½(250) = 125 mm
$I = 300(250^3)/12 - 200(150^3)/12 = 334\ 375\ 000\ mm^4$

Thus,

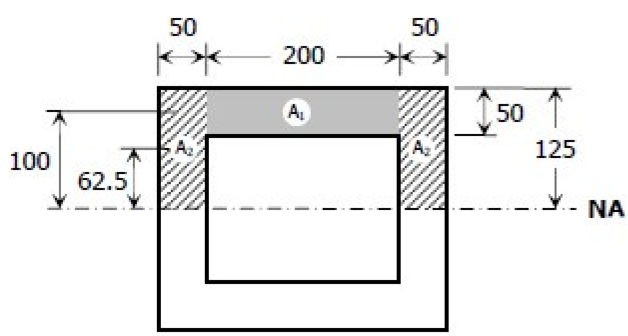
$$16 = \frac{750L^2(1000)(125)}{334\,375\,000}$$

$L = 7.55\ \text{m}$ *answer*

Shearing Stress

$$f_v = \frac{VQ}{Ib}$$

Where
$V = ½\ w_oL = ½(6000)(7.55) = 22\ 650$ N
Q = 10 000(100) + 2(6 250)(62.5)
$Q = 1\ 781\ 250\ mm^3$
b = 2(50) = 100 mm



$A_1 = 200(50) = 10\ 000\ mm^2$
$A_2 = 125(50) = 6\ 250\ mm^2$

Thus,

$$f_v = \frac{22\,650(1\,781\,250)}{334\,375\,000(100)}$$

$f_v = 1.21\ \text{MPa}$ *answer*

Problem 586

The distributed load shown in Fig. P-586 is supported by a box beam having the same cross-section as that in Prob. 585. Determine the maximum value of w_o that will not exceed a flexural stress of 10 MPa or a shearing stress of 1.0 MPa.

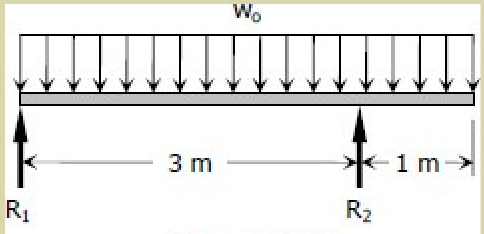
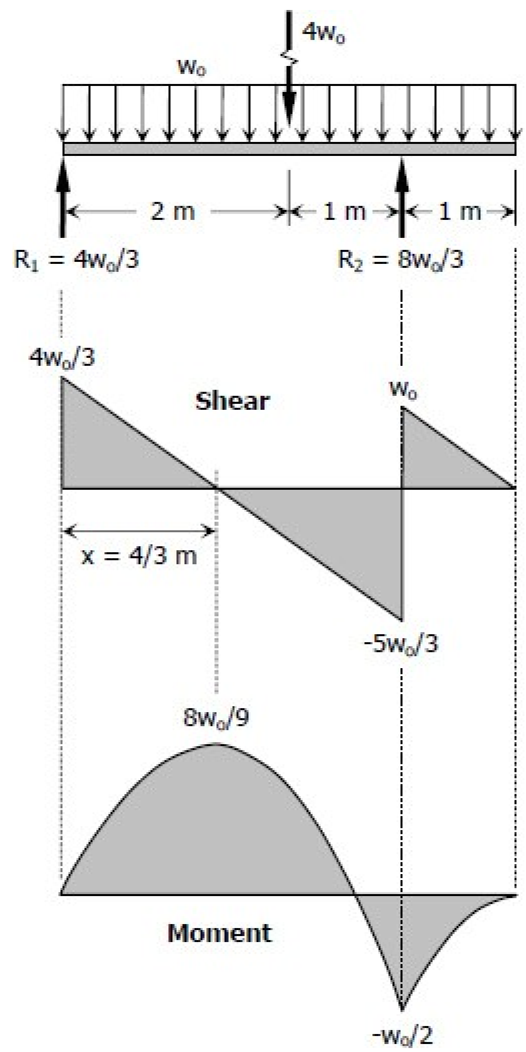


Figure P-586

Solution 586



$\Sigma M_{R2} = 0$

$3R_1 = 4w_o(1)$

$R_1 = \frac{4}{3}w_o$

$\Sigma M_{R1} = 0$

$3R_2 = 4w_o(2)$

$R_2 = \frac{8}{3}w_o$

From shear diagram

$\dfrac{x}{\frac{4}{3}w_o} = \dfrac{3-x}{\frac{5}{3}w_o}$

$\frac{5}{3}x = 4 - \frac{4}{3}x$

$x = \frac{4}{3}\ \text{m}$

Based on allowable bending stress

$f_b = \dfrac{Mc}{I}$

Where (From Solution 585):
c = 125 mm
I = 334 375 000 mm^4

Thus,

$10 = \dfrac{\frac{8}{9}w_o(1000^2)(125)}{334\,375\,000}$

$w_o = 30.09 \text{ kN/m}$

Based on allowable shear stress

$f_v = \dfrac{VQ}{Ib}$

Where (From Solution 585):
Q = 1 781 250 mm^3
I = 334 375 000 mm^4
b = 100 mm

Thus,

$1 = \dfrac{\frac{5}{3} w_o (1000)(1\,781\,250)}{334\,375\,000(100)}$

$w_o = 11.26 \text{ kN/m}$

For safe value of w_o, use $\mathbf{w_o = 11.26}$ **kN/m** *answer*

Problem 587

A beam carries two concentrated loads P and triangular load of 3P as shown in Fig. P-587. The beam section is the same as that in Fig. P-577 on this page. Determine the safe value of P if $f_b \leq 1200$ psi and $f_v \leq 200$ psi.

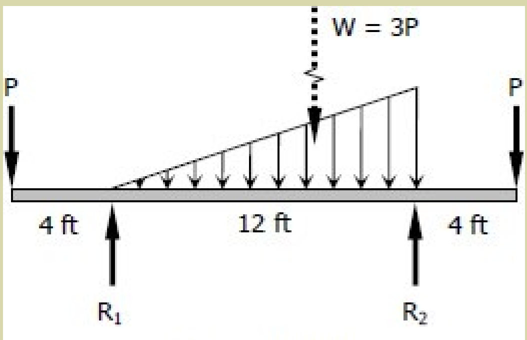


Figure P-587

Solution 587

$\Sigma M_{R2} = 0$

$12R_1 + 4P = 16P + 4(3P)$

$R_1 = 2P$

$\Sigma M_{R1} = 0$

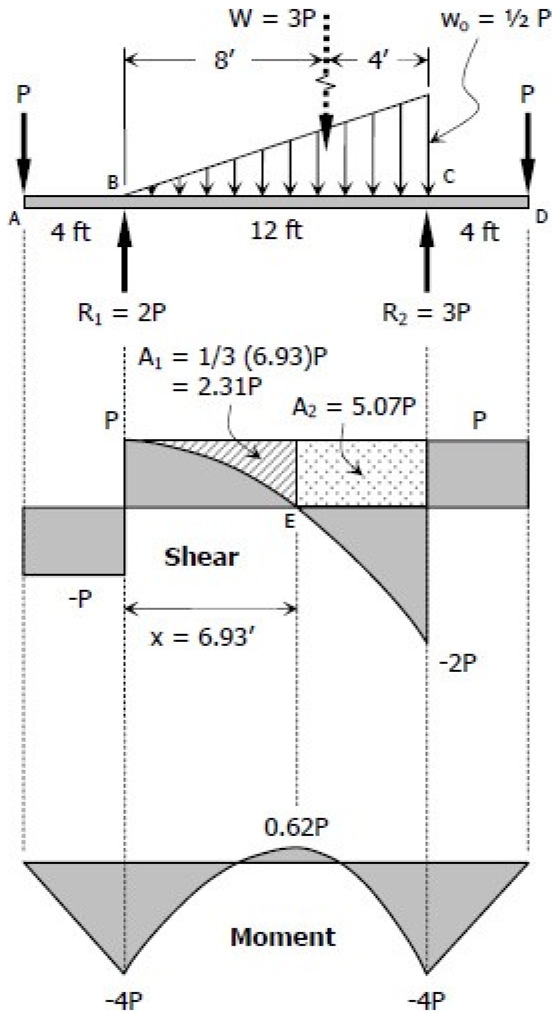
$12R_2 + 4P = 16P + 8(3P)$

$R_2 = 3P$

$W = \frac{1}{2}(12)w_o = 3P$

$w_o = \frac{1}{2}P$

To draw the Shear Diagram



1. $V_A = -P$ lb
2. $V_B = V_A$ + Area in load diagram
   $V_B = -P + 0 = -P$ lb
   $V_{B2} = V_B + R_1 = -P + 2P = P$ lb
3. $V_C = V_{B2}$ + Area in load diagram
   $V_C = P - ½(12)(½P) = -2P$ lb
   $V_{C2} = V_C + R_2 = -2P + 3P = P$ lb
4. $V_D = V_{C2}$ + Area in load diagram
   $V_D = P + 0 = P$
   $V_{D2} = V_D - P = P - P = 0$
5. Shear at AB and CD are rectangular.
6. Shear at BC is parabolic (2nd degree curve).
7. Location of zero shear:
   By squared property of parabola
   $x^2 / P = 12^2 / 3P$
   $x = 6.93$ ft
   $12 - x = 5.07$ ft

To draw the Moment Diagram

1. $M_A = 0$
2. $M_B = M_A$ + Area in shear diagram
   $M_B = 0 - 4P = -4P$ lb·ft
3. $M_E = M_B$ + Area in shear diagram
   $M_E = -4P + 2/3\ (6.93)(P) = 0.62P$ lb·ft
4. $M_C = M_E$ + Area in shear diagram
   $M_C = 0.62P - [\ 1/3\ (12)3P - 2.31P - 5.07P\ ]$
   $M_C = -4P$ lb·ft

5. $M_D = M_C$ + Area in shear diagram
$M_D = -4P + 4P = 0$

6. The moment diagram at AB and CD are straight lines (1st degree curves) while at BC is 3rd degree curve.

Based on allowable bending stress

$$f_b = \frac{Mc}{I}$$

Where (From Solution 577)
c = 6 in
$I = 350.67 \text{ in}^4$

Thus,

$$1200 = \frac{4P(12)(6)}{350.67}$$

$$P = 1461.125 \text{ lb}$$

Based on allowable shear stress

$$f_v = \frac{VQ}{Ib}$$

Where (From Solution 577)
$Q = 35.5 \text{ in}^3$
$I = 350.67 \text{ in}^4$
b = 0.75 in

Thus,

$$200 = \frac{2P(35.5)}{350.67(0.75)}$$

$$P = 740.85 \text{ lb}$$

For safe value of P, use **P = 740.85 lb**. *answer*

Problem 588

The distributed load shown in Fig. P-588 is supported by a wide-flange section of the given dimensions. Determine the maximum value of w_o that will not exceed a flexural stress of 10 MPa or a shearing stress of 1.0 MPa.

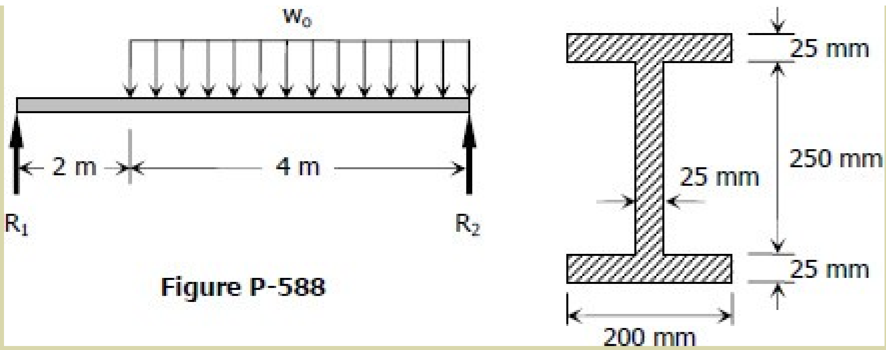
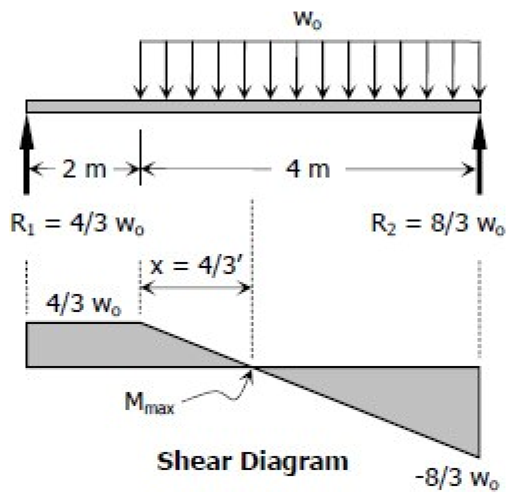


Figure P-588

Solution 588



$\Sigma M_{R2} = 0$

$6R_1 = 4w_o(2)$

$R_1 = \frac{4}{3}w_o \text{ N}$

$\Sigma M_{R1} = 0$

$6R_2 = 4w_o(4)$

$R_2 = \frac{8}{3}w_o \text{ N}$

From the shear diagram

$\dfrac{x}{\frac{4}{3}w_o} = \dfrac{4-x}{\frac{8}{3}w_o}$

$2x = 4 - x$

$x = \frac{4}{3} \text{ m}$

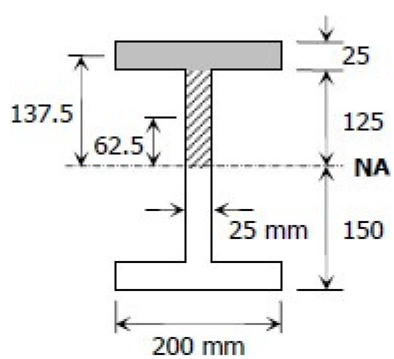
Maximum moment = sum of area in Shear Diagram at the left of point of zero shear

$M_{max} = 2(\frac{4}{3}w_o) + \frac{1}{2}(\frac{4}{3})(\frac{4}{3}w_o)$

$M_{max} = \frac{32}{9}w_o \text{ N}\cdot\text{m}$

Based on allowable flexural stress

$f_b = \dfrac{Mc}{I}$



Where

c = 150 mm

I = $200(300^3)/12 - 175(250^3)/12$

I = 222 135 416.67 mm^4

Thus,

$10 = \dfrac{\frac{32}{9}w_o(1000)(150)}{222\,135\,416.67}$

$w_o = 4165.04 \text{ N/m}$

Based on allowable shear stress

$f_v = \dfrac{VQ}{Ib}$

Where

Q = 200(25)(137.5) + 125(25)(62.5)

Q = 882 812.5 mm^3

I = 222 135 416.67 mm^4

b = 25 mm

Thus,

$1.0 = \dfrac{\frac{8}{3}w_o(882812.5)}{222\,135\,416.67(25)}$

$w_o = 94.36 \text{ N/m}$

For safe value of w_o, use $\mathbf{w_o = 94.36}$ **N/m**. *answer*

Problem 589

A channel section carries a concentrated loads W and a total distributed load of 4W as shown in Fig. P-589. Verify that the NA is 2.17 in. above the bottom and that I_{NA} = 62 in^4. Use these values to determine the maximum value of W that will not exceed allowable stresses in tension of 6,000 psi, in compression of 10,000 psi, or in shear of 8,000 psi.

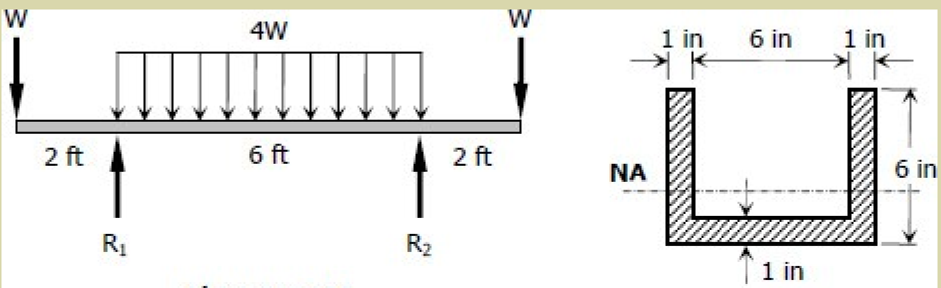


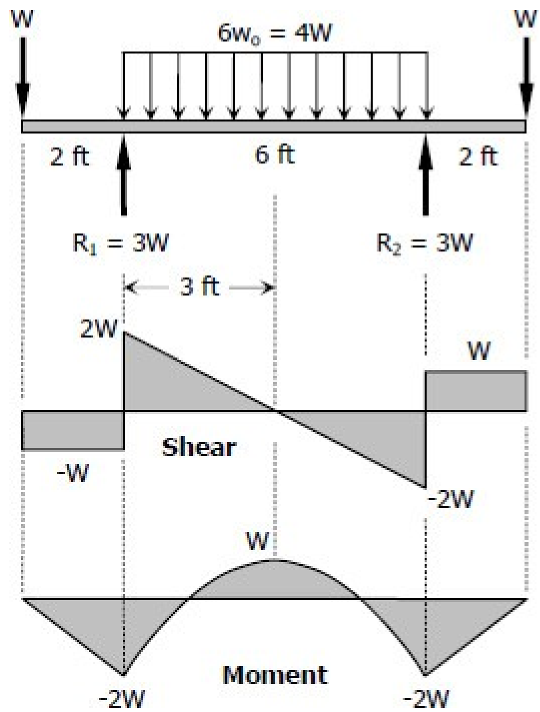
Figure P-589

Solution 589

HideBased on allowable bending stress:

By symmetry

$R_1 = R_2 = \frac{1}{2}(W + 4W + W)$



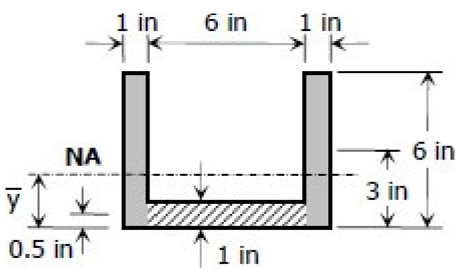
$R_1 = R_2 = 3W$

$6w_o = 4W$

$w_o = \frac{2}{3}W$

$A = 3(6)(1)$

$A = 18\ \text{in}^2$

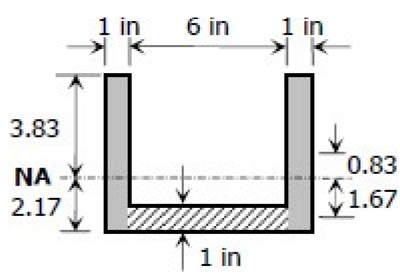


$A\bar{y} = \Sigma ay$

$18\bar{y} = 2[\,6(1)(3)\,] + 6(1)(0.5)$

$\bar{y} = 2.17\ \text{in}$ (*okay!*)

By transfer formula for moment of inertia



$$I_{NA} = 2\left[\frac{1(6^3)}{12} + 6(0.83^2)\right] + \left[\frac{6(1^3)}{12} + 6(1.67^2)\right]$$

$I_{NA} = 61.5002 \text{ in}^4$ (*okay!*)

$$f_b = \frac{Mc}{I}$$

For M = -2W lb·ft

Top fiber in tension

$$6000 = \frac{2W(12)(3.83)}{62}$$

$$W = 4045 \text{ lb}$$

Bottom fiber in compression

$$10\,000 = \frac{2W(12)(2.17)}{62}$$

$$W = 11\,905 \text{ lb}$$

For M = W lb·ft

Top fiber in compression

$$10\,000 = \frac{W(12)(3.83)}{62}$$

$$W = 13\,490 \text{ lb}$$

Bottom fiber in tension

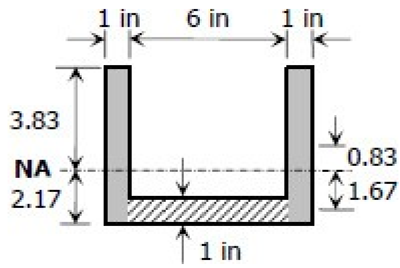
$$6000 = \frac{W(12)(2.17)}{62}$$

$$W = 14\,286 \text{ lb}$$

Based on allowable shear stress:

$$f_v = \frac{VQ}{Ib}$$

Where



$V = 2W$

$Q_{NA} = 2[3.83(1)(3.83/2)] = 14.6689\ \text{in}^3$

$I = 62\ \text{in}^4$

$b = 2\ \text{in}$

Thus,

$$8000 = \frac{2W(14.6689)}{62(2)}$$

$W = 33\,813\ \text{lb}$

For safe value of W, use **W = 4045 lb** *answer*

Problem 590

A box beam carries a distributed load of 200 lb/ft and a concentrated load P as shown in Fig. P-590. Determine the maximum value of P if $f_b \leq 1200$ psi and $f_v \leq 150$ psi.

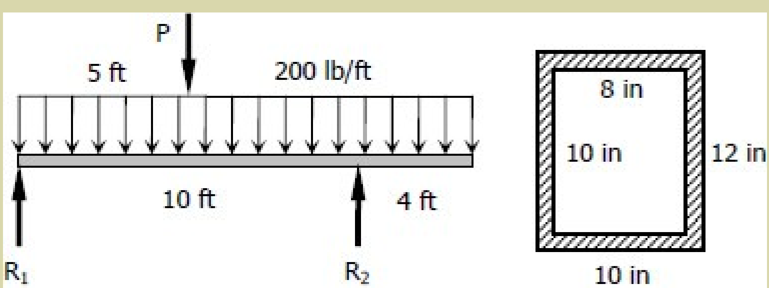


Figure P-590

Solution 590

$\Sigma M_{R2} = 0$

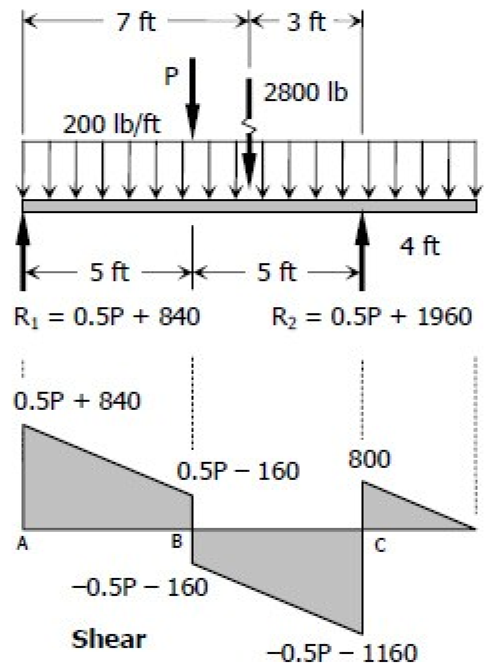
$10R_1 = 5P + 3(2800)$

$R_1 = 0.5P + 840$

$\Sigma M_{R1} = 0$

$10R_2 = 5P + 7(2800)$

$R_2 = 0.5P + 1960$



$M_B = \frac{1}{2}[\,(0.5P + 840) + (0.5P - 160)\,](5)$

$M_B = 2.5P + 1700 \text{ lb}\cdot\text{ft}$

$M_C = M_B - \frac{1}{2}[\,(0.5P + 160) + (0.5P + 1160)\,](5)$

$M_C = M_B - (2.5P + 1320)$

$M_C = (2.5P + 1700) - (2.5P + 3300)$

$M_C = -1600 \text{ lb}\cdot\text{ft}$

Check M_C from the overhang segment

$M_C = -\frac{1}{2}(4)(800)$

$M_C = -1600 \text{ lb}\cdot\text{ft}$ (*okay!*)

Based on allowable bending stress

$f_b = \dfrac{Mc}{I}$

Where

M = 2.5P + 1700 lb·ft

c = 12/2 = 6 in

I = $10(12^3)/12 - 8(10^3)/12 = 773.33 \text{ in}^4$

Thus,

$1200 = \dfrac{(2.5P + 1700)(12)(6)}{773.33}$

$P = 4475.56 \text{ lb}$

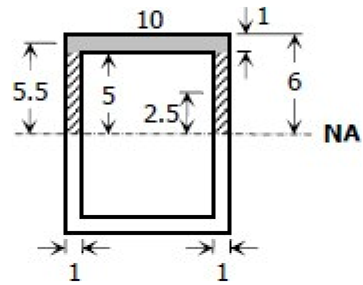
Based on allowable shear stress

$f_v = \dfrac{VQ}{Ib}$

Where

V = 0.5P + 1160 lb

Q = 10(1)(5.5) + 2 [5(1)(2.5)] = 80 in^3
b = 2 in



Thus,

$$150 = \frac{(0.5P + 1160)(80)}{773.33(2)}$$

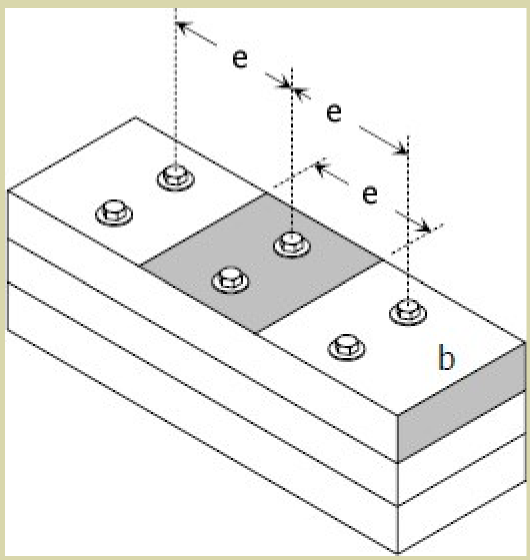
$$P = 3480 \text{ lb}$$

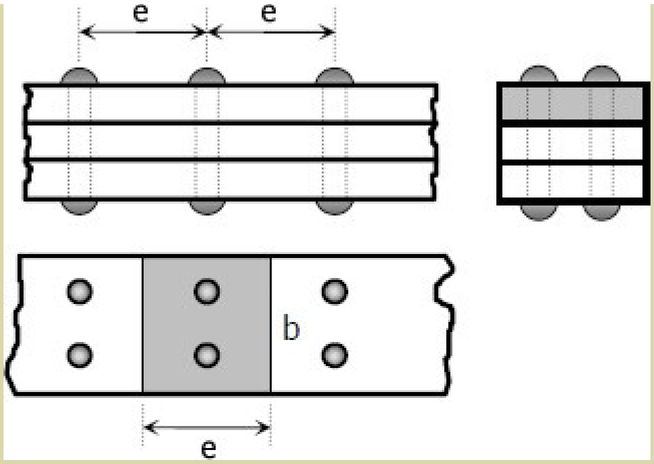
For safe value of P, use **P = 3480 lb** *answer*

Spacing of Rivets or Bolts in Built-Up Beams

When two or more thin layers of beams are fastened together with a bolt or a rivet so that they act as a unit to gain more strength, it is necessary to design the to size or spacing of these bolts or rivets so that it can carry the shearing force acting between each adjacent layers.

Consider the beam shown in the figure.





The shearing stress at the contact surface between the two planks is

$$f_v = \frac{VQ}{Ib}$$

The effective area covered by each bolt group has a length equal to the spacing of the bolts. The total shearing force F acting between the two surfaces must be equal to the total shearing force R produced by the bolts.

$$F = f_v(be) = \frac{VQ}{Ib}be = \frac{VQ}{I}e$$

then,

$$R = \frac{VQ}{I}e$$

where R is the total shearing force to be resisted by the bolts and is equal to the allowable shearing stress × area × number of bolts in the group. R should be taken at the contact surface nearest the neutral axis where the shearing stress is greatest. The spacing of bolts, e, is also called pitch.

Problem 592

A wide flange section is formed by bolting together three planks, each 80 mm by 200 mm, arranged as shown in Fig. P-592. If each bolt can withstand a shearing force of 8 kN, determine the pitch if the beam is loaded so as to cause a maximum shearing stress of 1.4 MPa.

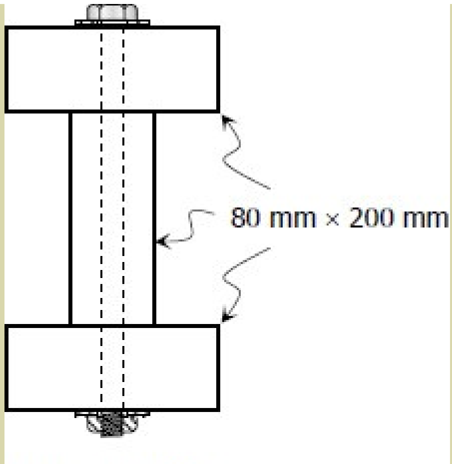


Figure P-592

Solution 592

$$(f_v)_{max} = \frac{VQ_{NA}}{Ib}$$

Where

$(f_v)_{max} = 1.4 \text{ MPa}$

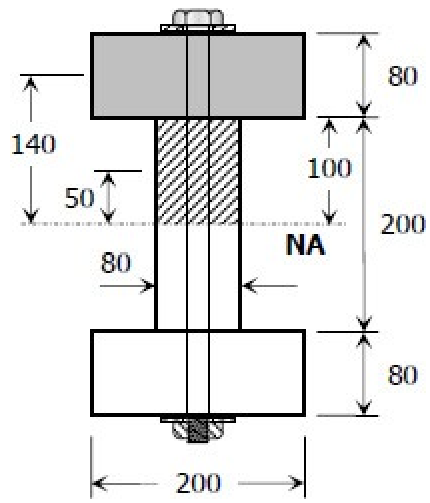
$Q_{NA} = 200(80)(140) + 100(80)(50)$

$Q_{NA} = 2\,640\,000 \text{ mm}^3$

$I = \dfrac{200(360^3)}{12} - \dfrac{120(200^3)}{12}$

$I = 697\,600\,000 \text{ mm}^4$

$b = 80 \text{ mm}$



Thus,

$1.4 = \dfrac{V(2\,640\,000)}{697\,600\,000(80)}$

$V = 29\,595.15 \text{ N}$

Spacing of bolts

$R = \dfrac{VQ_{flange}}{I}e$

$8(1000) = \dfrac{29\,595.15\,[\,200(80)(140)\,]}{697\,600\,000}e$

$e = 84.18 \text{ mm}$ *answer*

Problem 593

A box beam, built up as shown in Fig. P-593, is secured by screws spaced 5 in. apart. The beam supports a concentrated load P at the third point of a simply supported span 12 ft long. Determine the maximum value of P that will not exceed f_v = 120 psi in the beam or a shearing force of 300 lb in the screws. What is the maximum flexural stress in the beam?

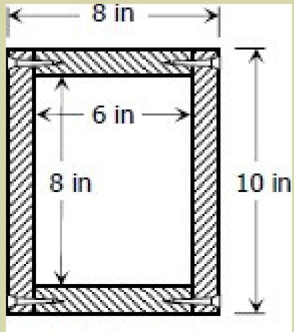


Figure P-539

Solution 593

$\Sigma M_{R2} = 0$

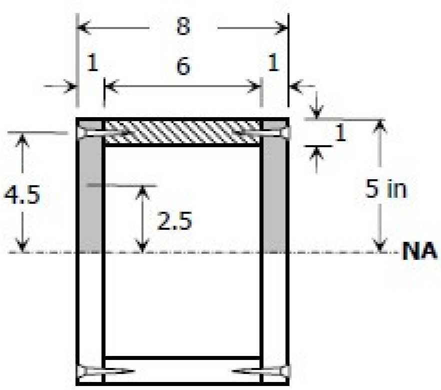
$12R_1 = 8P$

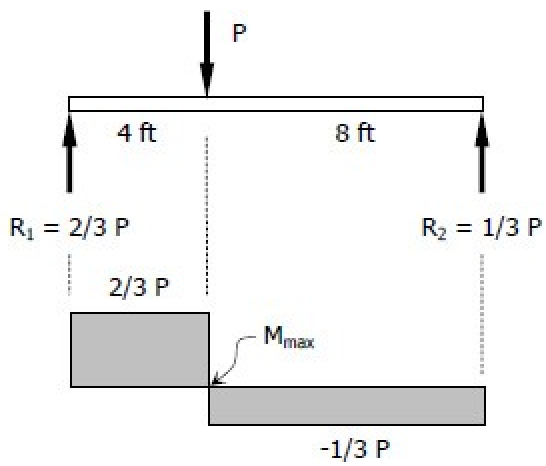
$R_1 = \frac{2}{3}P$

$\Sigma M_{R1} = 0$

$12R_2 = 4P$

$R_2 = \frac{1}{3}P$





$M_{max} = 4(\frac{2}{3}P) = \frac{8}{3}P \text{ lb}\cdot\text{ft}$

Based on allowable shearing force of beam

$f_v = \dfrac{VQ_{NA}}{Ib}$

Where:
V = 2/3 P
Q_{NA} = 6(1)(4.5) + 2 [5(1)(2.5)] = 52 in^3
I = 8(10^3)/12 - 6(8^3)/12 = 410.67 in^4
b = 2 in
f_v = 120 psi

Thus,

$$120 = \frac{\frac{2}{3}P(52)}{410.67(2)}$$

$$P = 3843.1 \text{ lb}$$

Based on allowable shearing force of the screws

$$R = \frac{VQ_{screw}}{I}e$$

Where:
R = 2(300) = 600 lb
V = 2/3 P
Q_{screw} = 6(1)(4.5) = 27 in^3
I = 410.67 in^4
e = 5 in

Thus,

$$600 = \frac{\frac{2}{3}P(27)}{410.67}(5)$$

$$P = 2737.8 \text{ lb}$$

For safe value of P, use **P = 2737.8 lb.** *answer*

Bending stress:

$$f_b = \frac{Mc}{I} = \frac{\frac{3}{8}(2737.8)(12)(5)}{410.67}$$

$$f_b = 150 \text{ psi}$$ *answer*

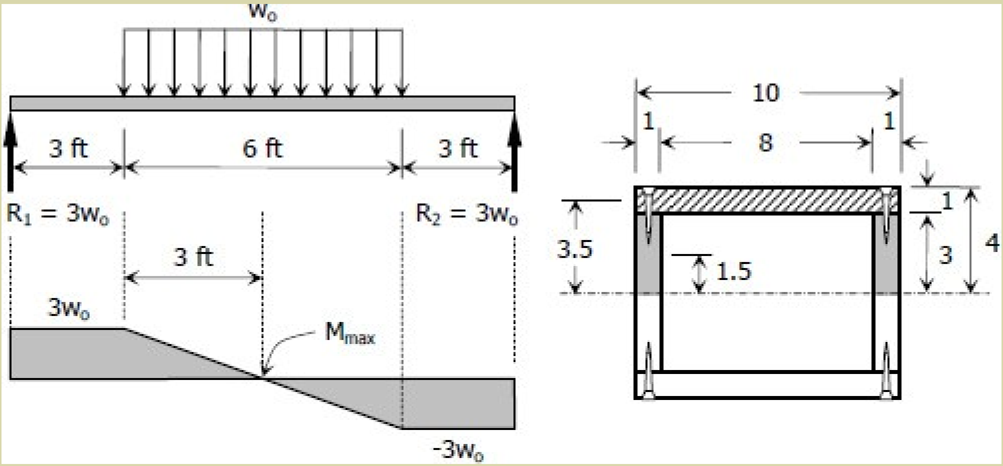
Problem 594

A distributed load of w_o lb/ft is applied over a middle 6 ft of a simply supported span 12 ft long. The beam section is that in Prob. 593, but used here so that the 8-in dimension is vertical. Determine the maximum value of w_o if $f_b \leq 1200$ psi, $f_v \leq 120$ psi, and the screws have a shear strength of 200 lb and a pitch of 2 in.

Solution 594

$$M_{max} = 3(3w_o) + \frac{1}{2}(3)(3w_o)$$

$$M_{max} = 13.5w_o \;\; \text{lb}\cdot\text{ft}$$



$$I = \frac{10(8^3)}{12} - \frac{8(6^3)}{12}$$

$$I = 848/3 \;\; \text{in}^3$$

Based on allowable flexure stress:

$$f_b = \frac{Mc}{I}$$

$$1200 = \frac{13.5w_o(12)(4)}{848/3}$$

$$w_o = 523.46 \;\; \text{lb/ft}$$

Based on shear stress of wood:

$$f_v = \frac{VQ_{NA}}{Ib}$$

$$120 = \frac{3w_o\,[\,10(1)(3.5) + 3(1)(1.5)(2)\,]}{(848/3)(2)}$$

$$w_o = 513.94 \;\; \text{lb/ft}$$

Based on shear strength of screws:

$$R = \frac{VQ_{screws}}{I}e$$

$$2(200) = \frac{3w_o\,[10(1)(3.5)]}{848/3}(2)$$

$$w_o = 538.41 \text{ lb/ft}$$

For safe value of w_o, use w_o = **513.94 lb/ft**. *answer*

Problem 595

A concentrated load P is carried at midspan of a simply supported 12-ft span. The beam is made of 2-in. by 6-in. pieces screwed together, as shown in Fig. P-595. If the maximum flexural stress developed is 1400 psi, find the maximum shearing stress and the pitch of the screws if each screw can resist 200 lb.

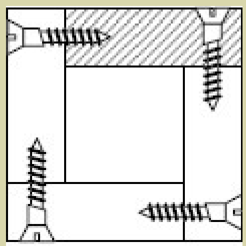


Figure P-595

Solution 595

For concentrated load P at midspan of a simply supported beam of span L = 12 ft.

$V_{max} = \frac{1}{2}P$

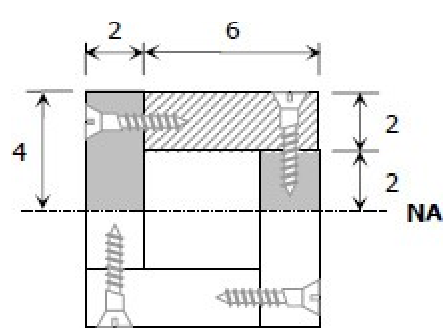
$M_{max} = \frac{1}{4}PL = \frac{1}{4}P(12) = 3P$

From the cross section shown:

$I = \dfrac{8(8^3)}{12} - \dfrac{4(4^3)}{12} = 320 \text{ in}^4$

$Q_{NA} = 4(2)(2) + 6(2)(3) + 2(2)(1) = 56 \text{ in}^3$

$Q_{screw} = 6(2)(3) = 36 \text{ in}^3$



From bending stress

$$f_b = \frac{Mc}{I}$$

$$1400 = \frac{3P(12)(4)}{320}$$

$$P = 3111.11 \text{ lb}$$

Maximum shear stress

$$f_v = \frac{VQ_{NA}}{Ib} = \frac{\frac{1}{2}(3111.11)(56)}{320(4)}$$

$$f_v = 68.06 \text{ psi}$$ *answer*

From strength of screws

$$R = \frac{VQ_{screws}}{I}e$$

$$2(200) = \frac{\frac{1}{2}(3111.11)(36)}{320}s$$

$$s = 2.28 \text{ in}$$ *answer*

Problem 596

Three planks 4 in by 6 in., arranged as shown in Fig. P-596 and secured by bolts spaced 1 ft apart, are used to support a concentrated load P at the center of a simply supported span 12 ft long. If P causes a maximum flexural stress of 1200 psi, determine the bolt diameters, assuming that the shear between the planks is transmitted by friction only. The bolts are tightened to a tension of 20 ksi and the coefficient of friction between the planks is 0.40.

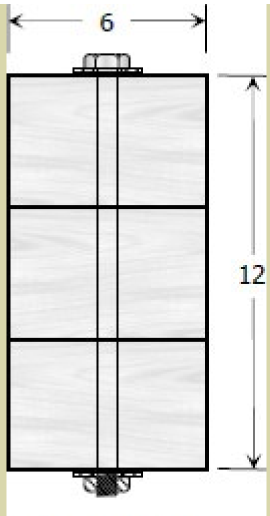


Figure P-596

Solution 596

$$M_{max} = \frac{1}{4}PL = \frac{1}{4}P(12) = 3P \text{ lb} \cdot \text{ft}$$

$$V_{max} = \frac{1}{2}P$$

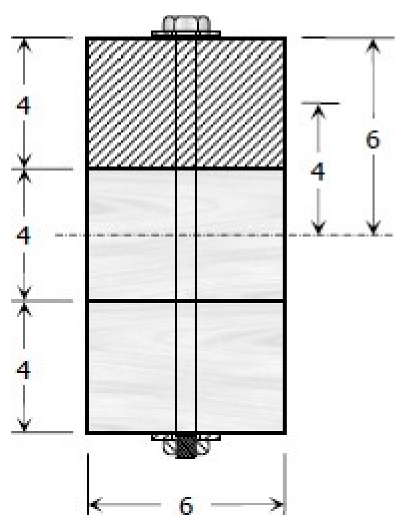
$$I = \frac{1}{12}(6)(123) = 864 \text{ in}^4$$

From allowable flexural stress

$$f_b = \frac{Mc}{I}$$

$$1200 = \frac{3P(12)(16)}{864}$$

$$P = 4800 \text{ lb}$$



Strength of bolt

$$R = \frac{VQ_{1st\ plank}}{I}e = \frac{\frac{1}{2}(4800)\,[\,4(6)(4)\,]}{864}(12)$$

$$R = 3200 \text{ lb}$$

Normal force

$$R = \mu N$$

$$3200 = 0.40N$$

$$N = 8000 \text{ lb}$$

From tensile stress of bolt:

$$\sigma = \frac{Force}{Area}$$

$$20\,000 = \frac{8000}{\frac{1}{4}\pi d^2}$$

$$d = 0.7136 \text{ in}$$ *answer*

Problem 597

A plate and angle girder similar to that shown in Fig. 5-32 is fabricated by riveting the short legs of four 125 × 75 × 13 mm angles to a web plate 1000 mm by 10 mm to form a section 1020 mm deep. Cover plates, each 300 mm × 10 mm, are then riveted to the flange angles making the overall height 1040 mm. The moment of inertia of the entire section about the NA is $I = 4770 \times 10^6 \text{ mm}^4$. Using the allowable stresses specified in Illustrative Problem 591, determine the rivet pitch for 22-mm rivets, attaching the angles to the web plate at a section where V = 450 kN.

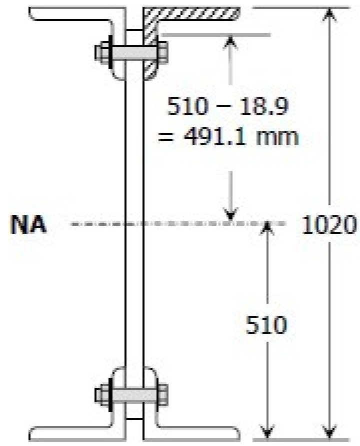
Solution 597

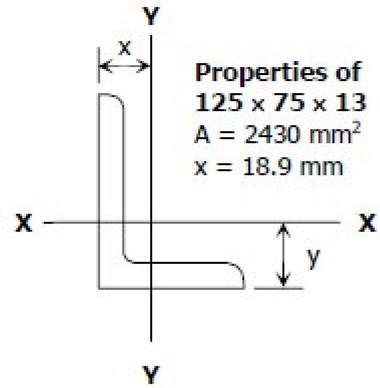
From Illustrative Problem 591

τ = 100 MPa shear stress

σ_b = 220 MPa bearing stress for single shear rivet

σ_b = 280 MPa bearing stress for double shear rivet





Rivet capacity in terms of shear (double shear)

$R_s = 2(A_s\,\tau) = 2\left[\,\frac{1}{4}\pi(22^2)(100)\,\right]$

$R_s = 24\,200\pi\ \text{ N }\ = 24.2\pi\ \text{ kN }\ = 76.03\text{ kN}$

Rivet capacity in terms of bearing (use σ_b = 280 MPa)

$R_b = A_b\,\sigma_b = [\,22(10)\,](280)$

$R_b = 61\,600\ \text{ N }\ = 61.6\ \text{ kN}$

Use R = 61.6 kN for safe value of R

From the strength of rivet

$R = \dfrac{VQ_{angle}}{I}e$

$61.6 = \dfrac{450\,[\,2(2430)(491.1)\,]}{4770\times10^6}e$

$e = 273.58\ \text{ mm}$ *answer*

Problem 598

As shown in Fig. P-598, two C380 × 60 channels are riveted together by pairs of 19-mm rivets spaced 200 mm apart along the length of the beam. What maximum vertical shear V can be applied to the section without exceeding the stresses given in Illustrative Problem 591?

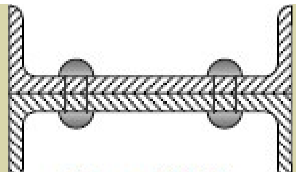


Figure P-598

Solution 598

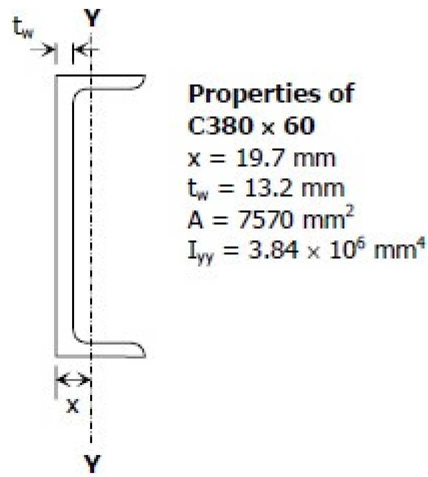
From Illustrative Problem 591
τ = 100 MPa shear stress
σ_b = 220 MPa bearing stress for single shear rivet
σ_b = 280 MPa bearing stress for double shear rivet

By transfer formula for moment of inertia

$$I = \bar{I} + Ad^2$$

$$I_{NA} = 2\left[\,(3.84\times10^6) + 7570(19.7^2)\,\right]$$

$$I_{NA} = 13\,555\,682.6\ \text{mm}^4$$



Rivet capacity in shear (single shear)

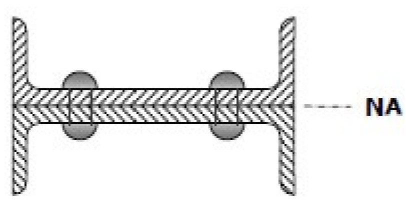
$$R_s = 2(A_s\,\tau) = 2\left[\,\tfrac{1}{4}\pi(19^2)(100)\,\right]$$

$$R_s = 18\,050\pi\ \text{N} = 56.705\ \text{kN}$$

Rivet capacity in bearing (use σ_b = 220 MPa)

$$R_b = 2(A_b\,\sigma_b) = 2\left[\,19(13.2)(220)\,\right]$$

$$R_b = 110\,352\ \text{N} = 110.352\ \text{kN}$$



Use R = 56.705 kN for safe value of R

From strength of rivets

$$R = \frac{VQ_{NA}}{I}e$$

$$56.705 = \frac{V\,[\,7570(19.7)\,]}{13555682.6}(200)$$

$V = 25.77 \text{ kN}$ *answer*

Problem 599

A beam is formed by bolting together two W200 × 100 sections as shown in Fig. P-599. It is used to support a uniformly distributed load of 30 kN/m (including the weight of the beam) on a simply supported span of 10 m. Compute the maximum flexural stress and the pitch between bolts that have a shearing strength of 30 kN.

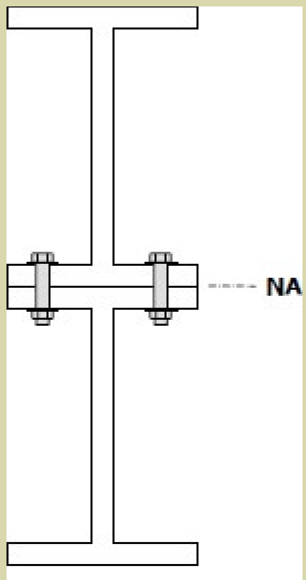


Figure P-599

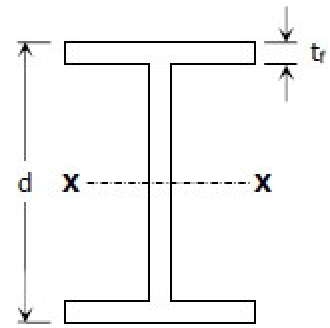
Solution 599

Properties of W200 × 100
A = 12 700 mm^4
t_f = 23.7 mm
d = 229 mm
I_{xx} = 113 × 10^6 mm^4

Maximum moment

$$M_{max} = \frac{w_o L^2}{8} = \frac{30(10^2)}{8}$$

$$M_{max} = 375 \ \text{kN} \cdot \text{m}$$



Maximum shear

$$V_{max} = \frac{w_o L}{2} = \frac{30(10)}{2}$$

$$V_{max} = 150 \ \text{kN}$$

By transfer formula for moment of inertia

$$I_{NA} = 2\left[\,(113 \times 10^6) + (12700)(229/2)^2\,\right]$$

$$I_{NA} = 559\,000\,350 \ \text{mm}^4$$

Maximum flexural stress

$$(\,f_b\,)_{max} = \frac{Mc}{I} = \frac{375(1000^2)(229)}{559\,000\,350}$$

$$(\,f_b\,)_{max} = 153.62 \ \text{MPa}$$ *answer*

Bolt pitch

$$R = \frac{VQ_{NA}}{I}e$$

$$2(30) = \frac{150\left[\,12700(229/2)\,\right]}{559\,000\,350}e$$

$$e = 153.77 \ \text{mm}$$ *answer*

Chapter 06 - Beam Deflections

Deflection of Beams

The deformation of a beam is usually expressed in terms of its deflection from its original unloaded position. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the elastic curve of the beam.

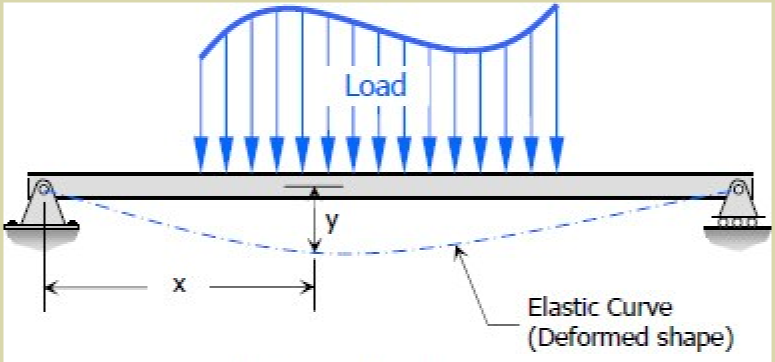


Figure: Elastic curve

Methods of Determining Beam Deflections

Numerous methods are available for the determination of beam deflections. These methods include:

1. Double-integration method
2. Area-moment method
3. Strain-energy method (Castigliano's Theorem)
4. Conjugate-beam method
5. Method of superposition

Of these methods, the first two are the ones that are commonly used

Double Integration Method | Beam Deflections

The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.

In calculus, the radius of curvature of a curve y = f(x) is given by

$$\rho = \frac{[\,1 + (dy/dx)^2\,]^{3/2}}{|\,d^2y/dx^2\,|}$$

In the derivation of flexure formula, the radius of curvature of a beam is given as

$$\rho = \frac{EI}{M}$$

Deflection of beams is so small, such that the slope of the elastic curve dy/dx is very small, and squaring this expression the value becomes practically negligible, hence

$$\rho = \frac{1}{d^2y/dx^2} = \frac{1}{y''}$$

Thus, EI / M = 1 / y''

$$y'' = \frac{M}{EI} = \frac{1}{EI}M$$

If EI is constant, the equation may be written as:

$$EI\,y'' = M$$

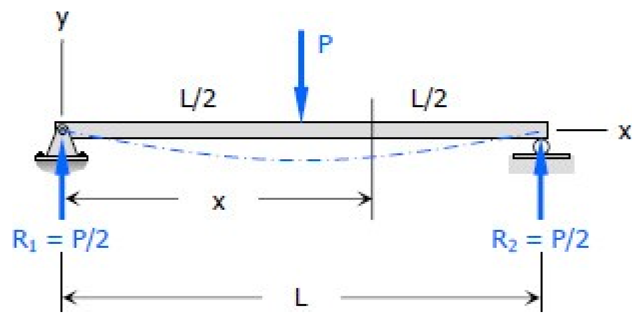
where x and y are the coordinates shown in the figure of the elastic curve of the beam under load, y is the deflection of the beam at any distance x. E is the modulus of elasticity of the beam, I represent the moment of inertia about the neutral axis, and M represents the bending moment at a distance x from the end of the beam. The product EI is called the **flexural rigidity** of the beam.

The first integration y' yields the slope of the elastic curve and the second integration y gives the deflection of the beam at any distance x. The resulting solution must contain two constants of integration since EI y'' = M is of second order. These two constants must be evaluated from known conditions concerning the slope deflection at certain points of the beam. For instance, in the case of a simply supported beam with rigid supports, at x = 0 and x = L, the deflection y = 0, and in locating the point of maximum deflection, we simply set the slope of the elastic curve y' to zero.

Problem 605

Determine the maximum deflection δ in a simply supported beam of length L carrying a concentrated load P at midspan.

Solution 605



$EI\,y'' = \frac{1}{2}Px - P\langle x - \frac{1}{2}L \rangle$

$EI\,y' = \frac{1}{4}Px^2 - \frac{1}{2}P\langle x - \frac{1}{2}L \rangle^2 + C_1$

$EI\,y = \frac{1}{12}Px^3 - \frac{1}{6}P\langle x - \frac{1}{2}L \rangle^3 + C_1x + C_2$

At x = 0, y = 0, therefore, $C_2 = 0$

At x = L, y = 0
$0 = \frac{1}{12}PL^3 - \frac{1}{6}P\langle L - \frac{1}{2}L \rangle^3 + C_1L$

$0 = \frac{1}{12}PL^3 - \frac{1}{48}PL^3 + C_1L$

$C_1 = -\frac{1}{16}PL^2$

Thus,
$EI\,y = \frac{1}{12}Px^3 - \frac{1}{6}P\langle x - \frac{1}{2}L \rangle^3 - \frac{1}{16}PL^2x$

Maximum deflection will occur at x = ½ L (midspan)
$EI\,y_{max} = \frac{1}{12}P(\frac{1}{2}L)^3 - \frac{1}{6}P(\frac{1}{2}L - \frac{1}{2}L)^3 - \frac{1}{16}PL^2(\frac{1}{2}L)$

$EI\,y_{max} = \frac{1}{96}PL^3 - 0 - \frac{1}{32}PL^3$

$$y_{max} = -\frac{PL^3}{48EI}$$

The negative sign indicates that the deflection is below the undeformed neutral axis.

Therefore,

$$\delta_{max} = \frac{PL^3}{48EI} \qquad \textit{answer}$$

Problem 606

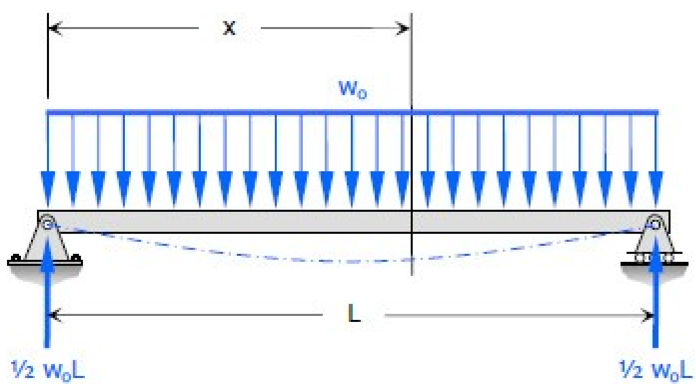
Determine the maximum deflection δ in a simply supported beam of length L carrying a uniformly distributed load of intensity w_o applied over its entire length.

Solution 606

From the figure below

$$EI\,y'' = \tfrac{1}{2}w_oLx - w_ox(\tfrac{1}{2}x)$$

$$EI\,y'' = \tfrac{1}{2}w_oLx - \tfrac{1}{2}w_ox^2$$



$$EI\,y' = \tfrac{1}{4}w_oLx^2 - \tfrac{1}{6}w_ox^3 + C_1$$

$$EI\,y = \tfrac{1}{12}w_oLx^3 - \tfrac{1}{24}w_ox^4 + C_1x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = L, y = 0

$$0 = \tfrac{1}{12}w_oL^4 - \tfrac{1}{24}w_oL^4 + C_1L$$

$C_1 = -\frac{1}{24} w_o L^3$

Therefore,
$EI\,y = \frac{1}{12} w_o L x^3 - \frac{1}{24} w_o x^4 - \frac{1}{24} w_o L^3 x$

Maximum deflection will occur at x = ½ L (midspan)

$EI\,y_{max} = \frac{1}{12} w_o L (\frac{1}{2}L)^3 - \frac{1}{24} w_o (\frac{1}{2}L)^4 - \frac{1}{24} w_o L^3 (\frac{1}{2}L)$

$EI\,y_{max} = \frac{1}{96} w_o L^4 - \frac{1}{384} w_o L^4 - \frac{1}{48} w_o L^4$

$EI\,y_{max} = -\frac{5}{384} w_o L^4$

$\delta_{max} = \dfrac{5 w_o L^4}{384 EI}$ *answer*

Taking W = w_oL:
$\delta_{max} = \dfrac{5 (w_o L)(L^3)}{384 EI}$

$\delta_{max} = \dfrac{5 W L^3}{384 EI}$ *answer*

Problem 607

Determine the maximum value of EIy for the cantilever beam loaded as shown in Fig. P-607. Take the origin at the wall.

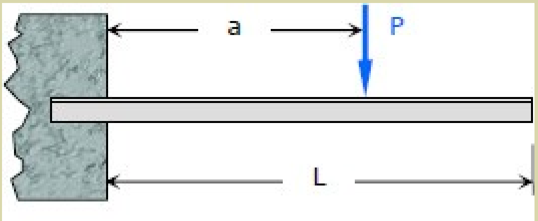


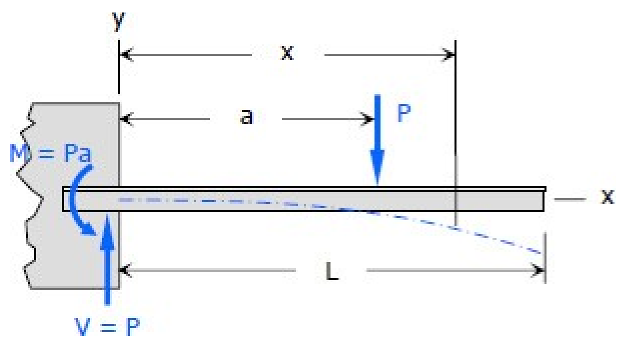
Figure P-607

Solution 607

$EI\,y'' = -Pa + Px - P\langle x - a \rangle$

$EI\,y' = -Pax + \frac{1}{2}Px^2 - \frac{1}{2}P\langle x - a \rangle^2 + C_1$

$EI\,y = -\frac{1}{2}Pax^2 + \frac{1}{6}Px^3 - \frac{1}{6}P\langle x - a \rangle^3 + C_1x + C_2$



At x = 0, y' = 0, therefore $C_1 = 0$
At x = 0, y = 0, therefore $C_2 = 0$

Therefore,
$EI\,y = -\frac{1}{2}Pax^2 + \frac{1}{6}Px^3 - \frac{1}{6}P\langle x - a \rangle^3$

The maximum value of EI y is at x = L (free end)

$EI\,y_{max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}P(L - a)^3$

$EI\,y_{max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}P(L^3 - 3L^2a + 3La^2 - a^3)$

$EI\,y_{max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}PL^3 + \frac{1}{2}PL^2a - \frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$

$EI\,y_{max} = -\frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$

$EI\,y_{max} = -\frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$

$EI\,y_{max} = -\frac{1}{6}Pa^2(3L - a)$ *answer*

Problem 608

Find the equation of the elastic curve for the cantilever beam shown in Fig. P-608; it carries a load that varies from zero at the wall to w_o at the free end. Take the origin at the wall.

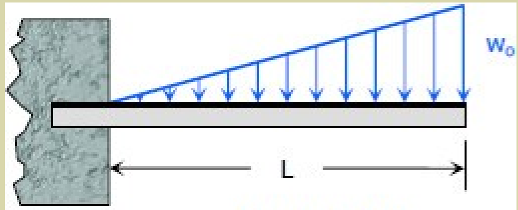
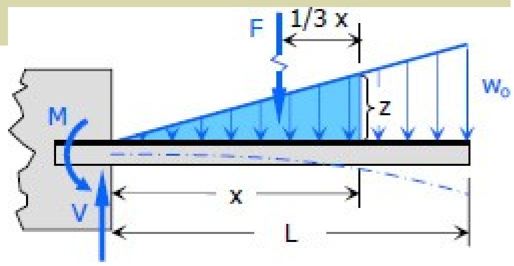


Figure P-608

Solution 608

$$V = \tfrac{1}{2} w_o L$$



$$M = \tfrac{1}{2} w_o L(\tfrac{2}{3} L)$$

$$M = \tfrac{1}{3} w_o L^2$$

By ratio and proportion

$$\frac{z}{x} = \frac{w_o}{L}$$

$$z = \frac{w_o}{L} x$$

$$F = \tfrac{1}{2} x z$$

$$F = \tfrac{1}{2} x \left(\frac{w_o}{L} x \right)$$

$$F = \frac{w_o}{2L} x^2$$

$$EI\, y'' = -M + Vx - F(\tfrac{1}{3} x)$$

$$EI\, y'' = -\tfrac{1}{3} w_o L^2 + \tfrac{1}{2} w_o L x - \tfrac{1}{3} x \left(\frac{w_o}{2L} x^2 \right)$$

$$EI\, y'' = -\frac{w_o L^2}{3} + \frac{w_o L}{2} x - \frac{w_o}{6L} x^3$$

$$EI\,y' = -\frac{w_o L^2}{3}x + \frac{w_o L}{4}x^2 - \frac{w_o}{24L}x^4 + C_1$$

$$EI\,y = -\frac{w_o L^2}{6}x^2 + \frac{w_o L}{12}x^3 - \frac{w_o}{120L}x^5 + C_1 x + C_2$$

At x = 0, y' = 0, therefore $C_1 = 0$
At x = 0, y = 0, therefore $C_2 = 0$

Therefore, the equation of the elastic curve is

$$EI\,y = -\frac{w_o L^2}{6}x^2 + \frac{w_o L}{12}x^3 - \frac{w_o}{120L}x^5 \qquad \textit{answer}$$

Problem 609

As shown in Fig. P-609, a simply supported beam carries two symmetrically placed concentrated loads. Compute the maximum deflection δ.

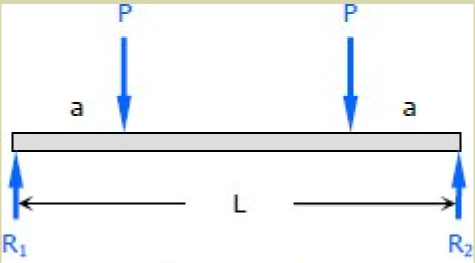


Figure P-609

Solution 609

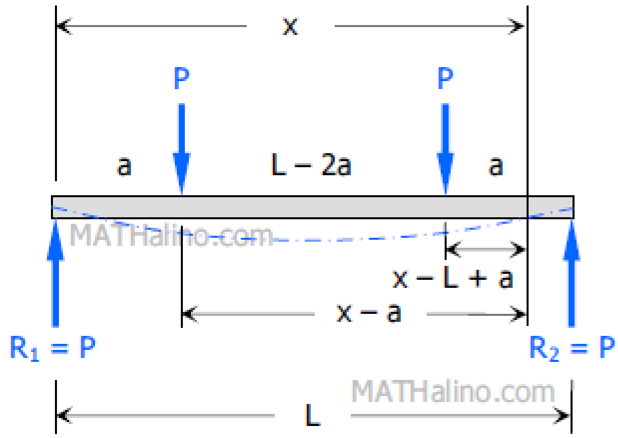
By symmetry

$R_1 = R_2 = P$

$$EI\,y'' = Px - P\langle x - a\rangle - P\langle x - L + a\rangle$$

$$EI\,y' = \tfrac{1}{2}Px^2 - \tfrac{1}{2}P\langle x - a\rangle^2 - \tfrac{1}{2}P\langle x - L + a\rangle^2 + C_1$$

$$EI\,y = \tfrac{1}{6}Px^3 - \tfrac{1}{6}P\langle x - a\rangle^3 - \tfrac{1}{6}P\langle x - L + a\rangle^3 + C_1 x + C_2$$



At x = 0, y = 0, therefore $C_2 = 0$

At x = L, y = 0
$$0 = \frac{1}{6}PL^3 - \frac{1}{6}P\langle L-a\rangle^3 + C_1 L$$

$$0 = PL^3 - P(L^3 - 3L^2 a + 3La^2 - a^3) - Pa^3 + 6C_1 L$$

$$0 = PL^3 - PL^3 + 3PL^2 a - 3PLa^2 + Pa^3 - Pa^3 + 6C_1 L$$

$$0 = 3PL^2 a - 3PLa^2 + 6C_1 L$$

$$0 = 3PLa(L-a) + 6C_1 L$$

$$C_1 = -\frac{1}{2}Pa(L-a)$$

Therefore,
$$EI\,y = \frac{1}{6}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-L+a\rangle^3 - \frac{1}{2}Pa(L-a)x$$

Maximum deflection will occur at x = ½ L (midspan)
$$EI\,y_{max} = \frac{1}{6}P(\tfrac{1}{2}L)^3 - \frac{1}{6}P(\tfrac{1}{2}L-a)^3 - \frac{1}{2}Pa(L-a)(\tfrac{1}{2}L)$$

$$EI\,y_{max} = \frac{1}{48}PL^3 - \frac{1}{6}P[\,\tfrac{1}{2}(L-2a)\,]^3 - \frac{1}{4}PL^2 a + \frac{1}{4}PLa^2$$

$$EI\,y_{max} = \frac{1}{48}PL^3 - \frac{1}{48}P[\,L^3 - 3L^2(2a) + 3L(2a)^2 - (2a)^3\,] - \frac{1}{4}PL^2 a + \frac{1}{4}PLa^2$$

$$EI\,y_{max} = \frac{1}{48}PL^3 - \frac{1}{48}PL^3 + \frac{1}{8}PL^2 a - \frac{1}{4}PLa^2 + \frac{1}{6}Pa^3 - \frac{1}{4}PL^2 a + \frac{1}{4}PLa^2$$

$EI\,y_{max} = -\frac{1}{8}PL^2a + \frac{1}{6}Pa^3$

$EI\,y_{max} = -\frac{1}{24}Pa\,(3L^2 - 4a^2)$

$$y_{max} = -\frac{Pa}{24EI}(3L^2 - 4a^2)$$

$$\delta_{max} = \frac{Pa}{24EI}(3L^2 - 4a^2) \qquad \textit{answer}$$

Problem 610

The simply supported beam shown in Fig. P-610 carries a uniform load of intensity w_o symmetrically distributed over part of its length. Determine the maximum deflection δ and check your result by letting a = 0 and comparing with the answer to Problem 606.

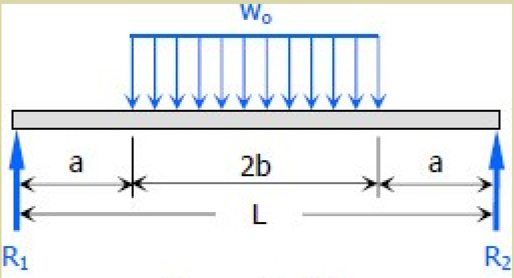


Figure P-610

Solution 610

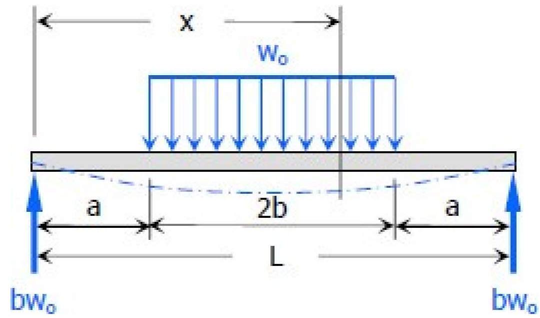
By symmetry

$R_1 = R_2 = bw_o$

$EI\,y'' = bw_ox - \frac{1}{2}w_o\langle x - a\rangle^2$

$EI\,y' = \frac{1}{2}bw_ox^2 - \frac{1}{6}w_o\langle x - a\rangle^3 + C_1$

$EI\,y = \frac{1}{6}bw_ox^3 - \frac{1}{24}w_o\langle x - a\rangle^4 + C_1x + C_2$



At x = 0, y = 0, therefore $C_2 = 0$

At x = a + b, y' = 0

$$0 = \frac{1}{2}bw_o(a+b)^2 - \frac{1}{6}w_o b^3 + C_1$$

$$C_1 = \frac{1}{6}w_o b^3 - \frac{1}{2}bw_o(a+b)^2$$

Therefore,

$$EI\,y = \frac{1}{6}bw_o x^3 - \frac{1}{24}w_o\langle x-a\rangle^4 + \frac{1}{6}w_o b^3 x - \frac{1}{2}bw_o(a+b)^2 x$$

Maximum deflection will occur at x = a + b (midspan)

$$EI\,y_{max} = \frac{1}{6}bw_o(a+b)^3 - \frac{1}{24}w_o b^4 + \frac{1}{6}w_o b^3(a+b) - \frac{1}{2}bw_o(a+b)^3$$

$$EI\,y_{max} = -\frac{1}{3}bw_o(a+b)^3 - \frac{1}{24}w_o b^4 + \frac{1}{6}w_o b^3(a+b)$$

$$EI\,y_{max} = -\frac{1}{24}w_o b\,[\,8(a+b)^3 + b^3 - 4b^2(a+b)\,]$$

Therefore,

$$\delta_{max} = \frac{w_o b}{24EI}[\,8(a+b)^3 + b^3 - 4b^2(a+b)\,]$$ *answer*

Checking:

When a = 0, 2b = L, thus b = ½ L

$$\delta_{max} = \frac{w_o(\frac{1}{2}L)}{24EI}[\,8(0+\frac{1}{2}L)^3 + (\frac{1}{2}L)^3 - 4(\frac{1}{2}L)^2(0+\frac{1}{2}L)\,]$$

$$\delta_{max} = \frac{w_o L}{48EI}[\,L^3 + \frac{1}{8}L^3 - \frac{1}{2}L^3\,]$$

$$\delta_{max} = \frac{w_o L}{48EI}[\,\tfrac{5}{8}L^3\,]$$

$$\delta_{max} = \frac{5w_o L^4}{384EI} \qquad (okay!)$$

Problem 611

Compute the value of EI δ at midspan for the beam loaded as shown in Fig. P-611. If E = 10 GPa, what value of I is required to limit the midspan deflection to 1/360 of the span?

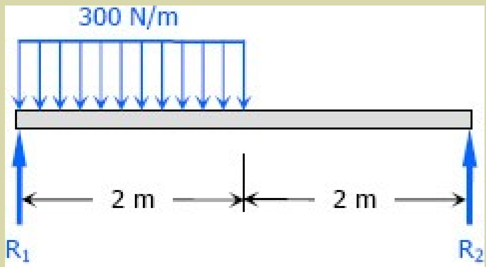


Figure P-611

Solution 611

$$\Sigma M_{R2} = 0$$

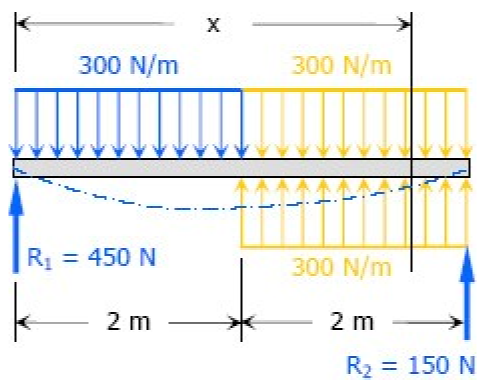
$$4R_1 = 300(2)(3)$$

$$R_1 = 450\text{ N}$$

$$\Sigma M_{R1} = 0$$

$$4R_2 = 300(2)(1)$$

$$R_2 = 150\text{ N}$$



$$EI\,y'' = 450x - \tfrac{1}{2}(300)x^2 + \tfrac{1}{2}(300)\langle\, x - 2\,\rangle^2$$

$$EI\,y'' = 450x - 150x^2 + 150\langle\, x - 2\,\rangle^2$$

$$EI\,y' = 225x^2 - 50x^3 + 50\langle\, x - 2\,\rangle^3 + C_1$$

$$EI\, y = 75x^3 - 12.5x^4 + 12.5\langle x - 2\,\rangle^4 + C_1 x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 4 m, y = 0

$$0 = 75(4^3) - 12.5(4^4) + 12.5(4 - 2)^4 + 4C_1$$

$$C_1 = -450\,\text{N}\cdot\text{m}^2$$

Therefore,

$$EI\, y = 75x^3 - 12.5x^4 + 12.5\langle x - 2\,\rangle^4 - 450x$$

At x = 2 m (midspan)

$$EI\, y_{midspan} = 75(2^3) - 12.5(2^4) + 12.5(2 - 2)^4 - 450(2)$$

$$EI\, y_{midspan} = -500\,\text{N}\cdot\text{m}^3$$

$$EI\, \delta_{midspan} = 500\,\text{N}\cdot\text{m}^3$$

Maximum midspan deflection

$$\delta_{midspan} = \tfrac{1}{360}L = \tfrac{1}{360}(4) = \tfrac{1}{90}\,\text{m}$$

$$\delta_{midspan} = \tfrac{100}{9}\,\text{mm}$$

Thus,

$$10\,000I\,(\tfrac{100}{9}) = 500(1000^3)$$

$$I = 4\,500\,000\,\text{mm}^4$$

$$I = 4.5\times 10^6\,\text{mm}^4$$ *answer*

Problem 612

Compute the midspan value of EI δ for the beam loaded as shown in Fig. P-612.

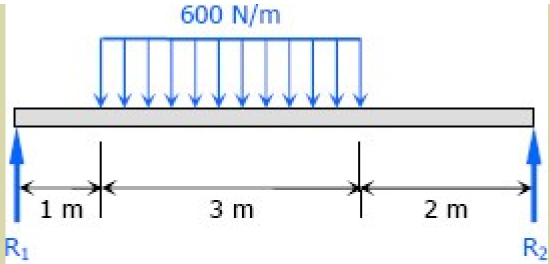
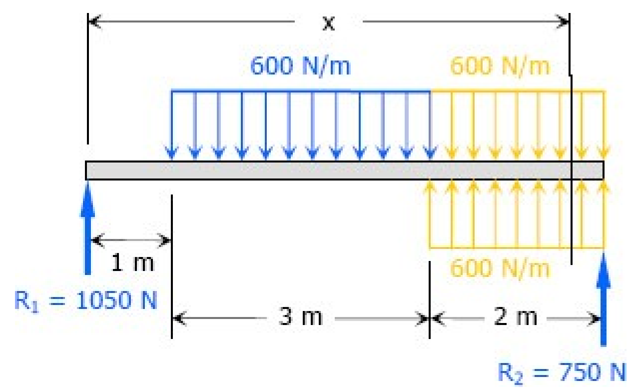


Figure P-612

Solution 612



$\Sigma M_{R2} = 0$

$6R_1 = 600(3)(3.5)$

$R_1 = 1050\,\text{N}$

$\Sigma M_{R1} = 0$

$6R_2 = 600(3)(2.5)$

$R_2 = 750\,\text{N}$

$EI\,y'' = 1050x - \frac{1}{2}(600)\langle\, x - 1\,\rangle^2 + \frac{1}{2}(600)\langle\, x - 4\,\rangle^2$

$EI\,y'' = 1050x - 300\langle\, x - 1\,\rangle^2 + 300\langle\, x - 4\,\rangle^2$

$EI\,y' = 525x^2 - 100\langle\, x - 1\,\rangle^3 + 100\langle\, x - 4\,\rangle^3 + C_1$

$EI\,y = 175x^3 - 25\langle\, x - 1\,\rangle^4 + 25\langle\, x - 4\,\rangle^4 + C_1x + C_2$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 6 m, y = 0
$0 = 175(6^3) - 25(6 - 1)^4 + 25(6 - 4)^4 + 6C_1$

$C_1 = -3762.5\,\text{N}\cdot\text{m}^2$

Therefore,
$$EI\,y = 175x^3 - 25\langle\, x - 1\,\rangle^4 + 25\langle\, x - 4\,\rangle^4 - 3762.5x$$

At midspan, x = 3 m
$$EI\,y_{midspan} = 175(3^3) - 25(3-1)^4 - 3762.5(3)$$

$$EI\,y_{midspan} = -6962.5\,\mathrm{N \cdot m^3}$$

Thus,
$$EI\,\delta_{midspan} = 6962.5\,\mathrm{N \cdot m^3}$$ *answer*

Problem 613

If E = 29 × 10^6 psi, what value of I is required to limit the midspan deflection to 1/360 of the span for the beam in Fig. P-613?

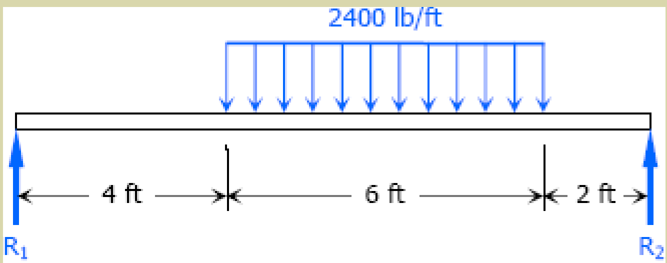


Figure P-613

Solution 613

$$\Sigma M_{R2} = 0$$

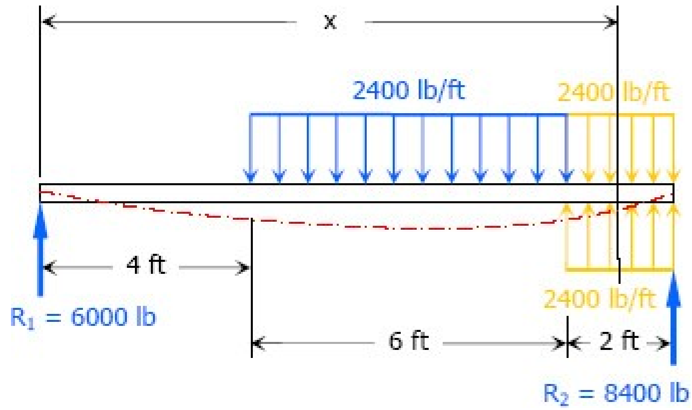
$$12R_1 = 2400(6)(5)$$

$$R_1 = 6000\,\mathrm{lb}$$

$$\Sigma M_{R1} = 0$$

$$12R_2 = 2400(6)(7)$$

$$R_2 = 8400\,\mathrm{lb}$$



$$EI\,y'' = 6000x - \tfrac{1}{2}(2400)\,\langle\, x - 4\,\rangle^2 + \tfrac{1}{2}(2400)\,\langle\, x - 10\,\rangle^2$$

$$EI\,y'' = 6000x - 1200\,\langle\, x - 4\,\rangle^2 + 1200\,\langle\, x - 10\,\rangle^2$$

$$EIy' = 3000x^2 - 400\,\langle\, x - 4\,\rangle^3 + 400\,\langle\, x - 10\,\rangle^3 + C_1$$

$$EI\,y = 1000x^3 - 100\,\langle\, x - 4\,\rangle^4 + 100\,\langle\, x - 10\,\rangle^4 + C_1x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 12 ft, y = 0

$$0 = 1000(12^3) - 100(12 - 4)^4 + 100(12 - 10)^4 + 12C_1$$

$$C_1 = -110\,000\,\text{lb}\cdot\text{ft}$$

Therefore

$$EI\,y = 1000x^3 - 100\,\langle\, x - 4\,\rangle^4 + 100\,\langle\, x - 10\,\rangle^4 - 110\,000x$$

E = 29 × 10^6 psi
L = 12 ft
At midspan, x = 6 ft
y = -1/360 (12) = -1/30 ft = -2/5 in

Thus,
$$EI\,y = 1000x^3 - 100\,\langle\,x-4\,\rangle^4 + 100\,\langle\,x-10\,\rangle^4 - 110\,000x$$

$$(29\times10^6)I(-\tfrac{2}{5}) = [\,1000(6^3) + 100(2^4) - 110\,000(6)\,](12^3)$$

$I = 65.9\ \text{in}^4$ *answer*

Problem 614

For the beam loaded as shown in Fig. P-614, calculate the slope of the elastic curve over the right support.

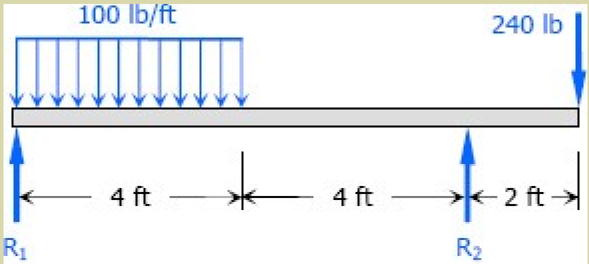


Figure P-614

Solution 614

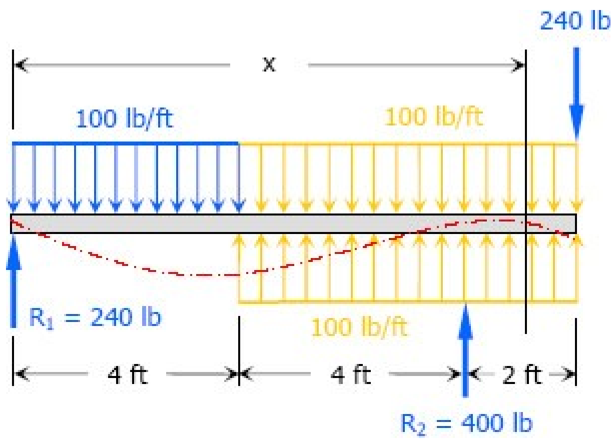
$$\Sigma M_{R2} = 0$$

$$8R_1 + 240(2) = 100(4)(6)$$

$$R_1 = 240\,\text{lb}$$

$$\Sigma M_{R1} = 0$$
$$8R_2 = 240(10) + 100(4)(2)$$

$$R_2 = 400\,\text{lb}$$



$$EI\,y'' = 240x - \tfrac{1}{2}(100)x^2 + \tfrac{1}{2}(100)\,\langle\, x-4\,\rangle^2 + 400\,\langle\, x-8\,\rangle$$

$$EI\,y'' = 240x - 50x^2 + 50\,\langle\, x-4\,\rangle^2 + 400\,\langle\, x-8\,\rangle$$

$$EI\,y' = 120x^2 - \tfrac{50}{3}x^3 + \tfrac{50}{3}\,\langle\, x-4\,\rangle^3 + 200\,\langle\, x-8\,\rangle^2 + C_1$$

$$EI\,y = 40x^3 - \tfrac{25}{6}x^4 + \tfrac{25}{6}\,\langle\, x-4\,\rangle^4 + \tfrac{200}{3}\,\langle\, x-8\,\rangle^3 + C_1x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 8 ft, y = 0
$0 = 40(8^3) - (25/6)(8^4) + (25/6)(4^4) + 8C_1$
$C_1 = -560\ \text{lb}\cdot\text{ft}^2$

Thus,
$$EI\,y' = 120x^2 - \tfrac{50}{3}x^3 + \tfrac{50}{3}\,\langle\, x-4\,\rangle^3 + 200\,\langle\, x-8\,\rangle^2 - 560$$

At the right support, x = 8 ft

$$EI\,y' = 120(8^2) - \tfrac{50}{3}(8^3) + \tfrac{50}{3}(4^3) - 560$$

$$EI\,y' = -\tfrac{1040}{3}\ \text{lb}\cdot\text{ft}^2$$

$$y' = -\frac{1040}{3EI}\ \text{lb}\cdot\text{ft}^2$$ *answer*

Problem 615

Compute the value of EI y at the right end of the overhanging beam shown in Fig. P-615.

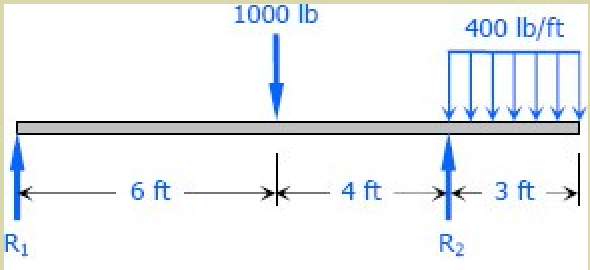


Figure P-615

Solution 615

$\Sigma M_{R2} = 0$

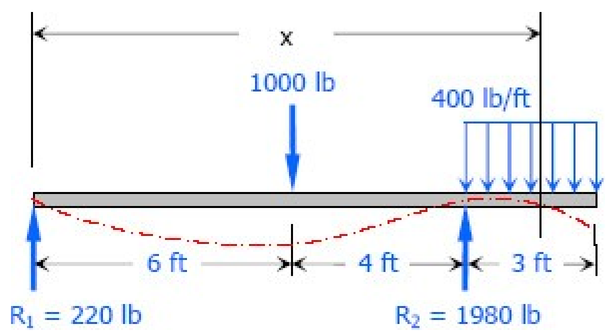
$10R_1 + 400(3)(1.5) = 1000(4)$

$R_1 = 220\,\text{lb}$

$\Sigma M_{R2} = 0$

$10R_2 = 400(3)(11.5) + 1000(6)$

$R_2 = 1980\,\text{lb}$



$EI\,y'' = 220x - 1000\,\langle\,x - 6\,\rangle + 1980\,\langle\,x - 10\,\rangle - \frac{1}{2}(400)\,\langle x - 10\,\rangle^2$

$$EI\,y'' = 220x - 1000\,\langle x-6\rangle + 1980\,\langle x-10\rangle - 200\,\langle x-10\rangle^2$$

$$EI\,y' = 110x^2 - 500\,\langle x-6\rangle^2 + 990\,\langle x-10\rangle^2 - \frac{200}{3}\,\langle x-10\rangle^3 + C_1$$

$$EI\,y = \frac{110}{3}x^3 - \frac{500}{3}\,\langle x-6\rangle^3 + 330\,\langle x-10\rangle^3 - \frac{50}{3}\,\langle x-10\rangle^4 + C_1x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 10 ft, y = 0
$0 = (110/3)(10^3) - (500/3)(4^3) + 10C_1$
$C_1 = -2600$ lb·ft^2

Therefore,
$$EI\,y = \frac{110}{3}x^3 - \frac{500}{3}\,\langle x-6\rangle^3 + 330\,\langle x-10\rangle^3 - \frac{50}{3}\,\langle x-10\rangle^4 - 2600x$$

At the right end of the beam, x = 13 ft

$$EI\,y = \frac{110}{3}(13^3) - \frac{500}{3}(7^3) + 330(3^3) - \frac{50}{3}(3^4) - 2600(13)$$

$$EI\,y = -2850\,\text{lb}\cdot\text{ft}^3$$ *answer*

Problem 616

For the beam loaded as shown in Fig. P-616, determine (a) the deflection and slope under the load P and (b) the maximum deflection between the supports.

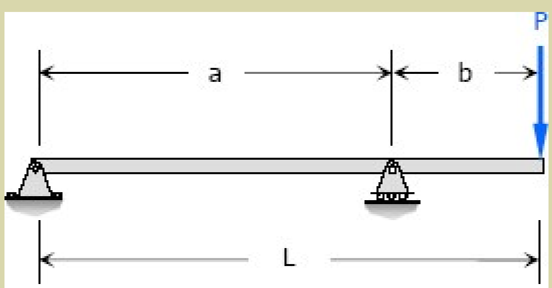


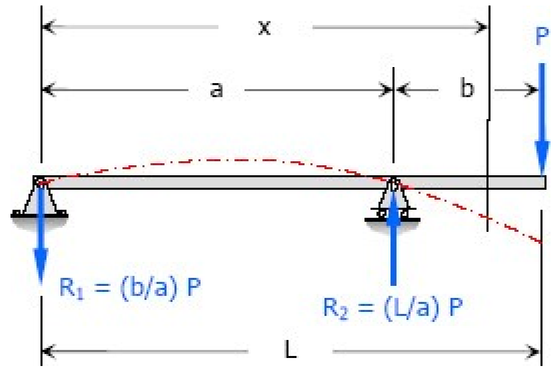
Figure P-616 and P-617

Solution 616

$$\Sigma M_{R2} = 0$$

$$aR_1 = bP$$

$$R_1 = \frac{b}{a}P$$



$$\Sigma M_{R1} = 0$$

$$aR_2 = PL$$

$$R_2 = \frac{L}{a}P$$

$$EI\,y'' = -\frac{b}{a}Px + \frac{L}{a}P\,\langle\, x - a\,\rangle$$

$$EI\,y' = -\frac{b}{2a}Px^2 + \frac{L}{2a}P\,\langle\, x - a\,\rangle^2 + C_1$$

$$EI\,y = -\frac{b}{6a}Px^3 + \frac{L}{6a}P\,\langle\, x - a\,\rangle^3 + C_1x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = a, y = 0
0 = -[b / (6a)] Pa^3 + aC_1
C_1 = (ab/6)P

Therefore,

$$EI\,y' = -\frac{b}{2a}Px^2 + \frac{L}{2a}P\,\langle\, x - a\,\rangle^2 + \frac{ab}{6}P$$

$$EI\,y = -\frac{b}{6a}Px^3 + \frac{L}{6a}P\,\langle\, x - a\,\rangle^3 + \frac{ab}{6}Px$$

Part (a): Slope and deflection under the load P

Slope under the load P: (note x = a + b = L)

$$EI\,y' = -\frac{b}{2a}P(a+b)^2 + \frac{a+b}{2a}Pb^2 + \frac{ab}{6}P$$

$$EI\,y' = -\frac{b}{2a}P(a^2+2ab+b^2) + \frac{ab^2+b^3}{2a}P + \frac{ab}{6}P$$

$$EI\,y' = -\frac{ab}{2}P - b^2P - \frac{b^3}{2a}P + \frac{b^2}{2}P\frac{b^3}{2a}P + \frac{ab}{6}P$$

$$EI\,y' = -\tfrac{1}{2}b^2P - \tfrac{1}{3}abP$$

$$EIy' = -\tfrac{1}{6}b\,(3b+2a)P$$

$$EIy' = -\tfrac{1}{6}b\,[\,2(a+b)+b\,]P$$

$$EIy' = -\tfrac{1}{6}b\,(2L+b)P$$ *answer*

Deflection under the load P: (note x = a + b = L)

$$EI\,y = -\frac{b}{6a}P(a+b)^3 + \frac{a+b}{6a}P(b^3) + \frac{ab}{6}P(a+b)$$

$$EI\,y = -\frac{b}{6a}P(a^3+3a^2b+3ab^2+b^3) + \frac{ab^3+b^4}{6a}P + \frac{ab}{6}P(a+b)$$

$$EI\,y = -\frac{a^2b}{6}P - \frac{ab^2}{2}P - \frac{b^3}{2}P - \frac{b^4}{6a}P + \frac{b^3}{6}P + \frac{b^4}{6a}P + \frac{a^2b}{6}P + \frac{ab^2}{6}P$$

$$EI\,y = -\tfrac{1}{3}ab^2P - \tfrac{1}{3}b^3P$$

$$EI\,y = -\tfrac{1}{3}(a+b)\,b^2P$$

$$EI\,y = -\tfrac{1}{3}Lb^2P$$ *answer*

Part (b): Maximum deflection between the supports

The maximum deflection between the supports will occur at the point where y' = 0.

$$EI\,y' = -\frac{b}{2a}Px^2 + \frac{L}{2a}P\,\langle\, x-a\,\rangle^2 + \frac{ab}{6}P$$

At y' = 0, 〈 x - a 〉 do not exist thus,

$$0 = -\frac{b}{2a}Px^2 + \frac{ab}{6}P$$

$$x^2 = \frac{1}{3}a^2$$

$$x = \frac{1}{\sqrt{3}}a$$

At $x = \frac{1}{\sqrt{3}}a$,

$$EI\,y_{max} = -\frac{b}{6a}P(\tfrac{1}{\sqrt{3}}a)^3 + \frac{ab}{6}P(\tfrac{1}{\sqrt{3}}a)$$

$$EI\,y_{max} = -\frac{a^2b}{6(3\sqrt{3})}P + \frac{a^2b}{6\sqrt{3}}P$$

$$EI\,y_{max} = \frac{a^2b}{6\sqrt{3}}P\left(-\frac{1}{3}+1\right)$$

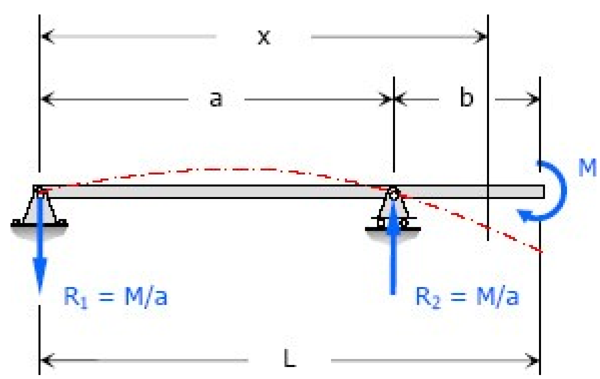
$$EI\,y_{max} = \frac{a^2b}{6\sqrt{3}}P\left(\frac{2}{3}\right)$$

$$EI\,y_{max} = \frac{a^2b}{9\sqrt{3}}P$$ *answer*

Problem 617

Replace the load P in Prob. 616 by a clockwise couple M applied at the right end and determine the slope and deflection at the right end.

Solution 617



$$EI\,y'' = -\frac{M}{a}x + \frac{M}{a}\langle x-a\rangle$$

$$EI\,y' = -\frac{M}{2a}x^2 + \frac{M}{2a}\langle x-a\rangle^2 + C_1$$

$$EI\,y = -\frac{M}{6a}x^3 + \frac{M}{6a}\langle x-a\rangle^3 + C_1 x + C_2$$

At x = 0, y = 0, therefore C2 = 0

At x = a, y = 0
$0 = -(M / 6a)(a^3) + aC_1$
$C_1 = Ma / 6$

Therefore,

$$EI\,y' = -\frac{M}{2a}x^2 + \frac{M}{2a}\langle x-a\rangle^2 + \frac{Ma}{6}$$

$$EI\,y = -\frac{M}{6a}x^3 + \frac{M}{6a}\langle x-a\rangle^3 + \frac{Ma}{6}x$$

Slope at x = a + b

$$EI\,y' = -\frac{M}{2a}(a+b)^2 + \frac{M}{2a}(b^2) + \frac{Ma}{6}$$

$$EI\,y' = -\frac{M}{2a}(a^2+2ab+b^2) + \frac{M}{2a}(b^2) + \frac{Ma}{6}$$

$$EI\,y' = -\tfrac{1}{2}Ma - Mb - \frac{Mb^2}{2a} + \frac{Mb^2}{2a} + \tfrac{1}{6}Ma$$

$$EI\,y' = -\tfrac{1}{3}Ma - Mb$$

$$EI\,y' = -\tfrac{1}{3}M\,(a+3b)$$

$$EI\,y' = -\tfrac{1}{3}M\,[\,(a+b)+2b\,]$$

$$EI\,y' = -\tfrac{1}{3}M\,(L+2b)$$ *answer*

Deflection at x = a + b

$$EI\,y = -\frac{M}{6a}(a+b)^3 + \frac{M}{6a}(b^3) + \frac{Ma}{6}(a+b)$$

$$EI\,y = -\frac{M}{6a}(a^3 + 3a^2b + 3ab^2 + b^3) + \frac{Mb^3}{6a} + \frac{Ma}{6}(a+b)$$

$$EI\,y = -\frac{Ma^2}{6} - \frac{Mab}{2} - \frac{Mb^2}{2} - \frac{Mb^3}{6a} + \frac{Mb^3}{6a} + \frac{Ma^2}{6} + \frac{Mab}{6}$$

$$EI\,y = -\tfrac{1}{3}Mab - \tfrac{1}{2}Mb^2$$

$$EI\,y = -\tfrac{1}{6}Mb\,(2a + 3b)$$

$$EI\,y = -\tfrac{1}{6}Mb\,[\,2(a+b) + b\,]$$

$$EI\,y = -\tfrac{1}{6}Mb\,(2L + b)$$

$$EI\,\delta = \tfrac{1}{6}Mb\,(2L + b)$$ *answer*

Problem 618

A simply supported beam carries a couple M applied as shown in Fig. P-618. Determine the equation of the elastic curve and the deflection at the point of application of the couple. Then letting a = L and a = 0, compare your solution of the elastic curve with cases 11 and 12 in the Summary of Beam Loadings.

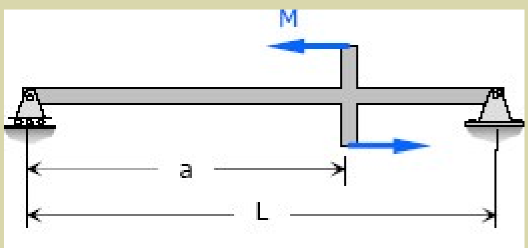


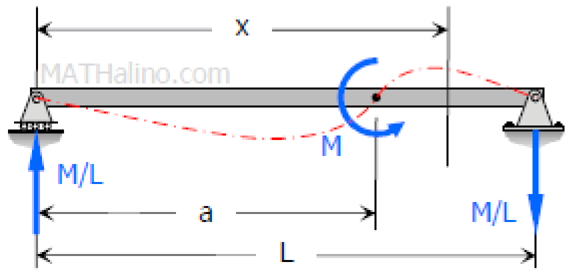
Figure P-618

Solution 618

$$EI\,y'' = \frac{M}{L}x - M\,\langle\,x - a\,\rangle^0$$

$$EI\,y' = \frac{M}{2L}x^2 - M\,\langle\,x - a\,\rangle + C_1$$

$$EI\,y = \frac{M}{6L}x^3 - \tfrac{1}{2}M\,\langle\,x - a\,\rangle^2 + C_1x + C_2$$



At x = 0, y = 0, therefore $C_2 = 0$

At x = L, y = 0

$$0 = \tfrac{1}{6}ML^2 - \tfrac{1}{2}M(L-a)^2 + C_1L$$

$$0 = \tfrac{1}{6}ML^2 - \tfrac{1}{2}M(L^2 - 2La + a^2) + C_1L$$

$$0 = \tfrac{1}{6}ML^2 - \tfrac{1}{2}ML^2 + MLa - \tfrac{1}{2}Ma^2 + C_1L$$

$$0 = -\tfrac{1}{3}ML^2 + MLa - \tfrac{1}{2}Ma^2 + C_1L$$

$$C_1L = \tfrac{1}{3}ML^2 - MLa + \tfrac{1}{2}Ma^2$$

$$C_1 = \tfrac{1}{3}ML - Ma + \frac{Ma^2}{2L}$$

Therefore,

$$EI\,y = \frac{M}{6L}x^3 - \tfrac{1}{2}M\,\langle\,x - a\,\rangle^2 + \left(\tfrac{1}{3}ML - Ma + \frac{Ma^2}{2L}\right)x$$

answer

At x = a

$$EI\,y = \frac{Ma^3}{6L} + \left(\tfrac{1}{3}ML - Ma + \frac{Ma^2}{2L}\right)a$$

$$EI\,y = \frac{2Ma^3}{3L} + \tfrac{1}{3}MLa - Ma^2$$

$$EI\,y = \frac{Ma}{3L}(2a^2 + L^2 - 3La)$$

$$EI\,y = \frac{Ma}{3L}(L^2 - 3La + 2a^2)$$ *answer*

When a = 0 (moment load is at the left support):

$$EI\,y = \frac{M}{6L}x^3 - \tfrac{1}{2}M\,\langle\, x - a\,\rangle^2 + \left(\tfrac{1}{3}ML - Ma + \frac{Ma^2}{2L}\right)x$$

$$EI\,y = \frac{M}{6L}x^3 - \tfrac{1}{2}Mx^2 + \tfrac{1}{3}MLx$$

$$EI\,y = \frac{Mx}{6L}(x^2 - 3Lx + 2L^2)$$

$$EI\,y = \frac{Mx}{6L}(2L^2 - 3Lx + x^2)$$

$$EI\,y = \frac{Mx}{6L}(L - x)(2L - x)$$ *answer*

When a = L (moment load is at the right support):

$$EI\,y = \frac{M}{6L}x^3 - \tfrac{1}{2}M\,\langle\, x - a\,\rangle^2 + \left(\tfrac{1}{3}ML - Ma + \frac{Ma^2}{2L}\right)x$$

$$EI\,y = \frac{M}{6L}x^3 + \left(\tfrac{1}{3}ML - ML + \tfrac{1}{2}ML\right)x$$

$$EI\,y = \frac{M}{6L}x^3 - \tfrac{1}{6}MLx$$

$$EI\,y = \frac{Mx^3 - ML^2x}{6L}$$

$$EI\,y = \frac{-Mx(-x^2 + L^2)}{6L}$$

$$EI\,y = \frac{-MLx(L^2 - x^2)}{6L^2}$$

$$EI\,y = -\frac{MLx}{6L^2}(L^2 - x^2)$$

$$EI\,y = -\frac{MLx}{6}\left(1 - \frac{x^2}{L^2}\right)$$ *answer*

Problem 619

Determine the value of EIy midway between the supports for the beam loaded as shown in Fig. P-619.

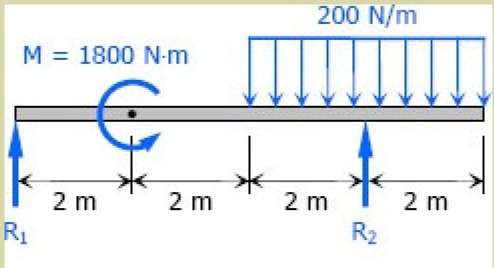
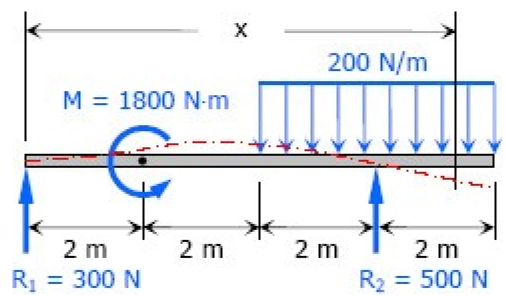


Figure P-619

Solution 619



$$\Sigma M_{R2} = 0$$

$$6R_1 + 200(4)(0) = 1800$$

$$R_1 = 300\,\text{N}$$

$$\Sigma M_{R1} = 0$$

$$6R_2 + 1800 = 200(4)(6)$$

$$R_2 = 500\,\text{N}$$

$$EI\,y'' = 300x - 1800\,\langle\, x - 2\,\rangle^0 + 500\,\langle\, x - 6\,\rangle - \tfrac{1}{2}(200)\,\langle\, x - 4\rangle^2$$

$$EI\,y'' = 300x - 1800\,\langle\, x - 2\,\rangle^0 + 500\,\langle\, x - 6\,\rangle - 100\,\langle\, x - 4\rangle^2$$

$$EI\,y' = 150x^2 - 1800\,\langle\, x - 2\,\rangle + 250\,\langle\, x - 6\,\rangle^2 - \tfrac{100}{3}\,\langle\, x - 4\rangle^3 + C_1$$

$$EI\,y = 50x^3 - 900\,\langle\, x - 2\,\rangle^2 + \tfrac{250}{3}\,\langle\, x - 6\,\rangle^3 - \tfrac{25}{3}\,\langle\, x - 4\rangle^4 + C_1x + C_2$$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 6 m, y = 0
$0 = 50(6^3) - 900(4^2) - (25/3)(2^4) + 6C_1$
$C_1 = 5600/9\ \text{N·m}^3$

Therefore,
$EI\,y = 50x^3 - 900\,\langle x-2\rangle^2 + \frac{250}{3}\,\langle x-6\rangle^3 - \frac{25}{3}\,\langle x-4\rangle^4 + \frac{5600}{9}x$

At x = 3 m
$EI\,y = 50(3^3) - 900(1^2) + \frac{5600}{9}(3)$

$EI\,y = \frac{6950}{3}\,\text{N}\cdot\text{m}^3$ *answer*

Problem 619

Determine the value of EIy midway between the supports for the beam loaded as shown in Fig. P-619.

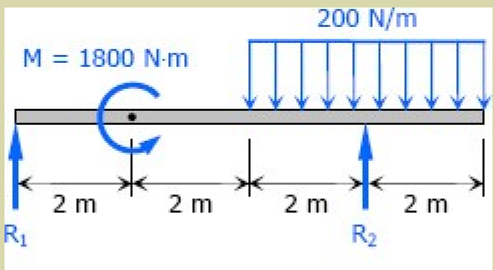


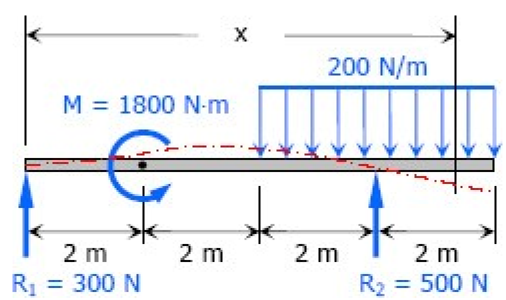
Figure P-619

Solution 619

$\Sigma M_{R2} = 0$

$6R_1 + 200(4)(0) = 1800$

$R_1 = 300\,\text{N}$



$\Sigma M_{R1} = 0$

$6R_2 + 1800 = 200(4)(6)$

$R_2 = 500\,\text{N}$

$EI\,y'' = 300x - 1800\,\langle\, x - 2\,\rangle^0 + 500\,\langle\, x - 6\,\rangle - \frac{1}{2}(200)\,\langle\, x - 4\rangle^2$

$EI\,y'' = 300x - 1800\,\langle\, x - 2\,\rangle^0 + 500\,\langle\, x - 6\,\rangle - 100\,\langle\, x - 4\rangle^2$

$EI\,y' = 150x^2 - 1800\,\langle\, x - 2\,\rangle + 250\,\langle\, x - 6\,\rangle^2 - \frac{100}{3}\,\langle\, x - 4\rangle^3 + C_1$

$EI\,y = 50x^3 - 900\,\langle\, x - 2\,\rangle^2 + \frac{250}{3}\,\langle\, x - 6\,\rangle^3 - \frac{25}{3}\,\langle\, x - 4\rangle^4 + C_1x + C_2$

At x = 0, y = 0, therefore $C_2 = 0$

At x = 6 m, y = 0
$0 = 50(6^3) - 900(4^2) - (25/3)(2^4) + 6C_1$
$C_1 = 5600/9\ \text{N·m}^3$

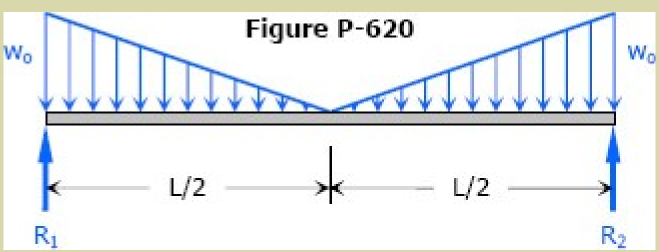
Therefore,
$EI\,y = 50x^3 - 900\,\langle\, x - 2\,\rangle^2 + \frac{250}{3}\,\langle\, x - 6\,\rangle^3 - \frac{25}{3}\,\langle\, x - 4\rangle^4 + \frac{5600}{9}x$

At x = 3 m
$EI\,y = 50(3^3) - 900(1^2) + \frac{5600}{9}(3)$

$EI\,y = \frac{6950}{3}\,\text{N}\cdot\text{m}^3$ *answer*

Problem 620

Find the midspan deflection δ for the beam shown in Fig. P-620, carrying two triangularly distributed loads. (*Hint:* For convenience, select the origin of the axes at the midspan position of the elastic curve.)

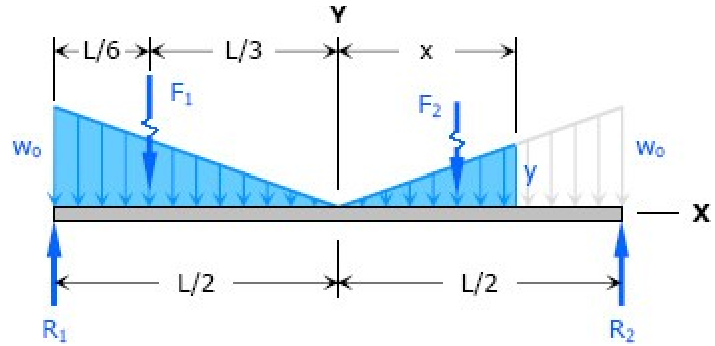


Solution 620

By ratio and proportion:

$$\frac{y}{x} = \frac{w_o}{L/2}$$

$$y = \frac{2w_o}{L}x$$



By symmetry:

$$R_1 = R_2 = \tfrac{1}{2}(L/2)(w_o) = \tfrac{1}{4}w_oL$$

$$F_1 = \tfrac{1}{2}(L/2)(w_o) = \tfrac{1}{4}w_oL$$

$$F_2 = \tfrac{1}{2}xy = \tfrac{1}{2}x\left(\frac{2w_o}{L}x\right) = \frac{w_o}{L}x^2$$

$$EI\,y'' = R_1(x + \tfrac{1}{2}L) - F_1(x + \tfrac{1}{3}L) - F_2(\tfrac{1}{3}x)$$

$$EI\,y'' = \tfrac{1}{4}w_oL(x + \tfrac{1}{2}L) - \tfrac{1}{4}w_oL(x + \tfrac{1}{3}L) - \frac{w_o}{L}x^2(\tfrac{1}{3}x)$$

$$EI\,y'' = \tfrac{1}{4}w_oLx + \tfrac{1}{8}w_oL^2 - \tfrac{1}{4}w_oLx - \tfrac{1}{12}w_oL^2 - \frac{w_o}{3L}x^3$$

$$EI\,y'' = \tfrac{1}{24}w_oL^2 - \frac{w_o}{3L}x^3$$

$$EI\,y' = \tfrac{1}{24}w_oL^2x - \frac{w_o}{12L}x^4 + C_1$$

$$EI\,y = \tfrac{1}{48}w_oL^2x^2 - \frac{w_o}{60L}x^5 + C_1x + C_2$$

At x = 0, y' = 0, therefore $C_1 = 0$

At x = ½L, y = 0

$0 = (1/48)w_oL^2\,(½L)^2 - (w_o60L)(½L)^5 + C_2$

$0 = (1/192)w_o\,L^4 - (1/1920)w_o\,L^4 + C_2$

$C_2 = -(3/640)w_o\,L^4$

Therefore,

$$EI\,y = \frac{1}{48}w_oL^2x^2 - \frac{w_o}{60L}x^5 - \frac{3}{640}w_oL^4$$

At x = 0 (midspan)

$$EI\,y_{max} = -\frac{3}{640}w_oL^4 = -\frac{3}{640}w_oL^4 \times \frac{3}{3}$$

$$EI\,y_{max} = -\frac{9}{1920}w_oL^4$$

Thus,

$$\delta_{midspan} = \frac{9w_oL^4}{1920EI} \qquad \textit{answer}$$

Problem 621

Determine the value of EIδ midway between the supports for the beam shown in Fig. P-621. Check your result by letting a = 0 and comparing with Prob. 606. (Apply the hint given in Prob. 620.)

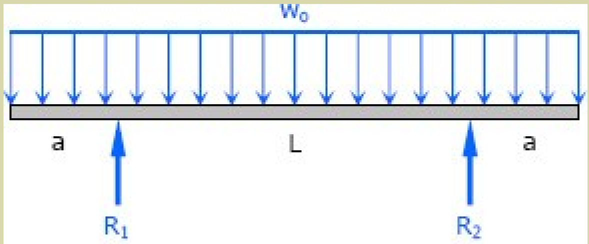
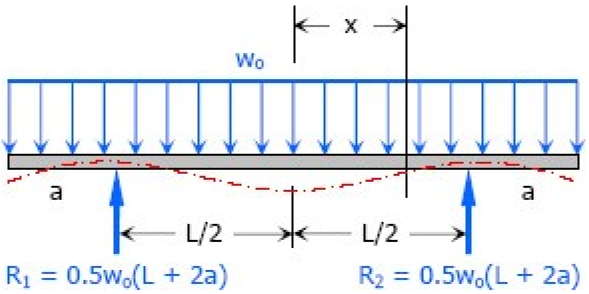


Figure P-621

Solution 621

By symmetry

$$R_1 = R_2 = \frac{1}{2}w_o(L + 2a)$$



$$EI\,y'' = [\,\tfrac{1}{2}w_o(L+2a)\,](x+\tfrac{1}{2}L) - \tfrac{1}{2}w_o(x+\tfrac{1}{2}L+a)^2$$

$$EI\,y'' = \tfrac{1}{2}w_o(L+2a)x + \tfrac{1}{4}w_o(L+2a)L - \tfrac{1}{2}w_o\,[\,x^2 + 2x(\tfrac{1}{2}L+a) + (\tfrac{1}{2}L+a)^2\,]$$

$$EI\,y'' = \tfrac{1}{2}w_o(L+2a)x + \tfrac{1}{4}w_o(L+2a)L - \tfrac{1}{2}w_ox^2 - w_o(\tfrac{1}{2}L+a)x - \tfrac{1}{2}w_o(\tfrac{1}{2}L+a)^2$$

$$EI\,y'' = \tfrac{1}{2}w_o(L+2a)x + \tfrac{1}{4}w_o(L+2a)L - \tfrac{1}{2}w_ox^2 - \tfrac{1}{2}w_o(L+2a)x - \tfrac{1}{8}w_o(L+2a)^2$$

$$EI\,y'' = \tfrac{1}{4}w_o(L+2a)L - \tfrac{1}{2}w_ox^2 - \tfrac{1}{8}w_o(L+2a)^2$$

$$EI\,y' = \tfrac{1}{4}w_o(L+2a)Lx - \tfrac{1}{6}w_ox^3 - \tfrac{1}{8}w_o(L+2a)^2x + C_1$$

$$EI\,y = \tfrac{1}{8}w_o(L+2a)Lx^2 - \tfrac{1}{24}w_ox^4 - \tfrac{1}{16}w_o(L+2a)^2x^2 + C_1x + C_2$$

At x = 0, y' = 0, therefore $C_1 = 0$

At x = ½L, y = 0

$$0 = \tfrac{1}{8}w_o(L+2a)L(\tfrac{1}{2}L)^2 - \tfrac{1}{24}w_o(\tfrac{1}{2}L)^4 - \tfrac{1}{16}w_o(L+2a)^2(\tfrac{1}{2}L)^2 + C_2$$

$$0 = \tfrac{1}{32}w_o(L+2a)L^3 - \tfrac{1}{384}w_oL^4 - \tfrac{1}{64}w_o(L+2a)^2L^2 + C_2$$

$$0 = \tfrac{1}{32}w_oL^4 - \tfrac{1}{16}w_oL^3a - \tfrac{1}{384}w_oL^4 - \tfrac{1}{64}w_o(L^2+4La+4a^2)L^2 + C_2$$

$$0 = \tfrac{1}{32}w_oL^4 - \tfrac{1}{16}w_oL^3a - \tfrac{1}{384}w_oL^4 - \tfrac{1}{64}w_oL^4 - \tfrac{1}{16}w_oL^3a - \tfrac{1}{16}w_oL^2a^2 + C_2$$

$$0 = \tfrac{5}{384}w_oL^4 - \tfrac{1}{16}w_oL^2a^2 + C_2$$

$$C_2 = \tfrac{1}{16}w_oL^2a^2 - \tfrac{5}{384}w_oL^4$$

$$C_2 = \tfrac{1}{384}w_oL^2(24a^2 - 5L^2)$$

Therefore,

$$EI\,y = \tfrac{1}{8}w_o(L+2a)Lx^2 - \tfrac{1}{24}w_ox^4 - \tfrac{1}{16}w_o(L+2a)^2x^2 + \tfrac{1}{384}w_oL^2(24a^2 - 5L^2)$$

At x = 0 (midspan)

$$EI\,y = \tfrac{1}{384}w_oL^2(24a^2 - 5L^2)$$ *answer*

At x = 0 when a = 0

$$EI\,y_{max} = \frac{1}{384} w_o L^2 (0 - 5L^2)$$

$$EI\,y_{max} = -\frac{5w_o L^4}{384}$$

Thus,

$$\delta_{max} = \frac{5w_o L^4}{384EI} \qquad \textit{answer}$$

Moment Diagrams by Parts

The moment-area method of finding the deflection of a beam will demand the accurate computation of the area of a moment diagram, as well as the moment of such area about any axis. To pave its way, this section will deal on how to draw moment diagrams by parts and to calculate the moment of such diagrams about a specified axis.

Basic Principles

1. The bending moment caused by all forces to the left or to the right of any section is equal to the respective algebraic sum of the bending moments at that section caused by each load acting separately.

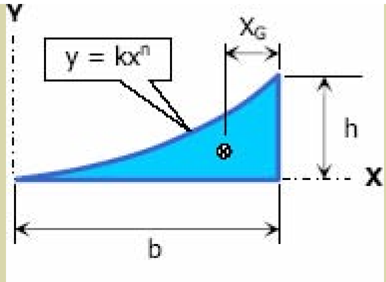
$$M = (\Sigma M)_L = (\Sigma M)_R$$

2. The moment of a load about a specified axis is always defined by the equation of a spandrel

$$y = kx^n$$

where n is the degree of power of x.

The graph of the above equation is as shown below



Area and centroid of moment diagram (spandrel)

and the area and location of centroid are defined as follows.

$$A = \frac{1}{n+1} bh$$

$$X_G = \frac{1}{n+2} b$$

Cantilever Loadings

A = area of moment diagram

M_x = moment about a section of distance x

barred x = location of centoid

Degree = degree power of the moment diagram

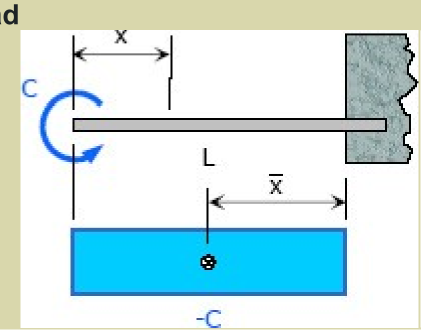
Couple or Moment Load

$A = -CL$

$M_x = -C$

$\bar{x} = \frac{1}{2}L$

Degree: zero



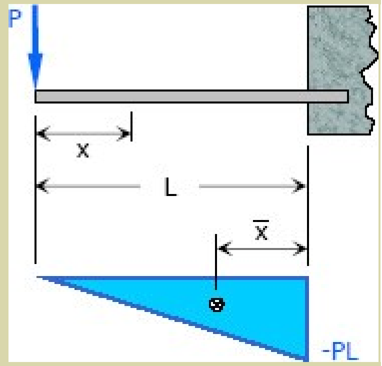
Concentrated Load

$$A = -\frac{1}{2}PL^2$$

$$M_x = -Px$$

$$\bar{x} = \frac{1}{3}L$$

Degree: first



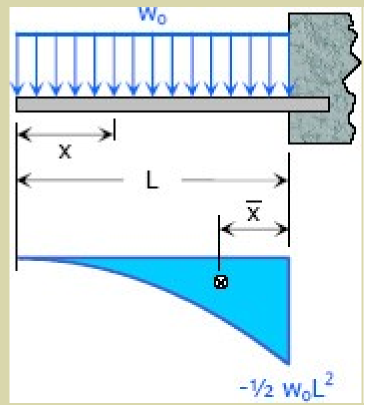
Uniformly Distributed Load

$$A = -\frac{1}{6}w_oL^3$$

$$M_x = -\frac{1}{2}w_ox^2$$

$$\bar{x} = \frac{1}{4}L$$

Degree: second



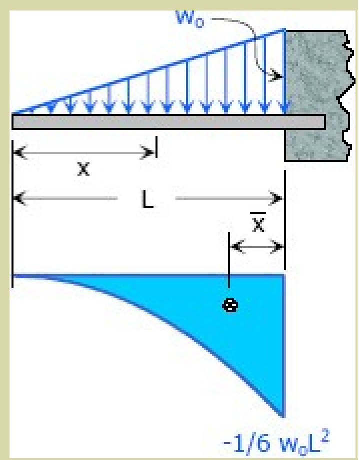
Uniformly Varying Load

$$A = -\frac{1}{24}w_oL^3$$

$$M_x = -\frac{w_o}{6L}x^2$$

$$x = \frac{1}{5}L$$

Degree: third



Problem 624

For the beam loaded as shown in Fig. P-624, compute the moment of area of the M diagrams between the reactions about both the left and the right reaction.

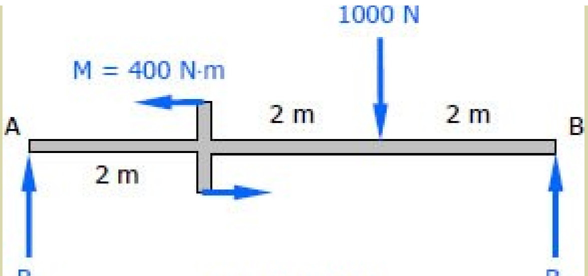


Figure P-624

Solution 624

$$\Sigma M_{R2} = 0$$

$$6R_1 = 400 + 1000(2)$$

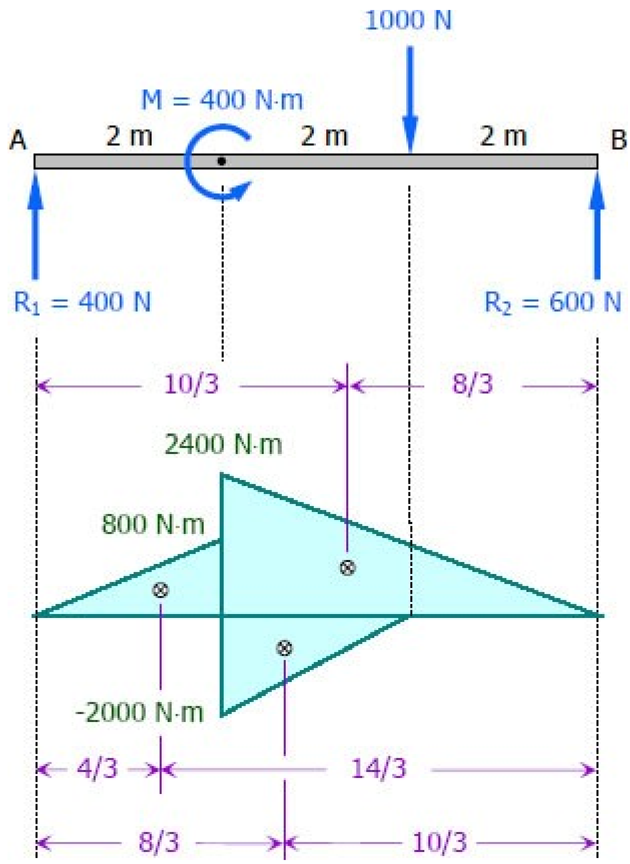
$$R_1 = 400\,\text{N}$$

$$\Sigma M_{R1} = 0$$

$$6R_2 + 400 = 1000(2)$$

$$R_2 = 600\,\text{N}$$

Moment diagram by parts can be drawn in different ways; three are shown below.

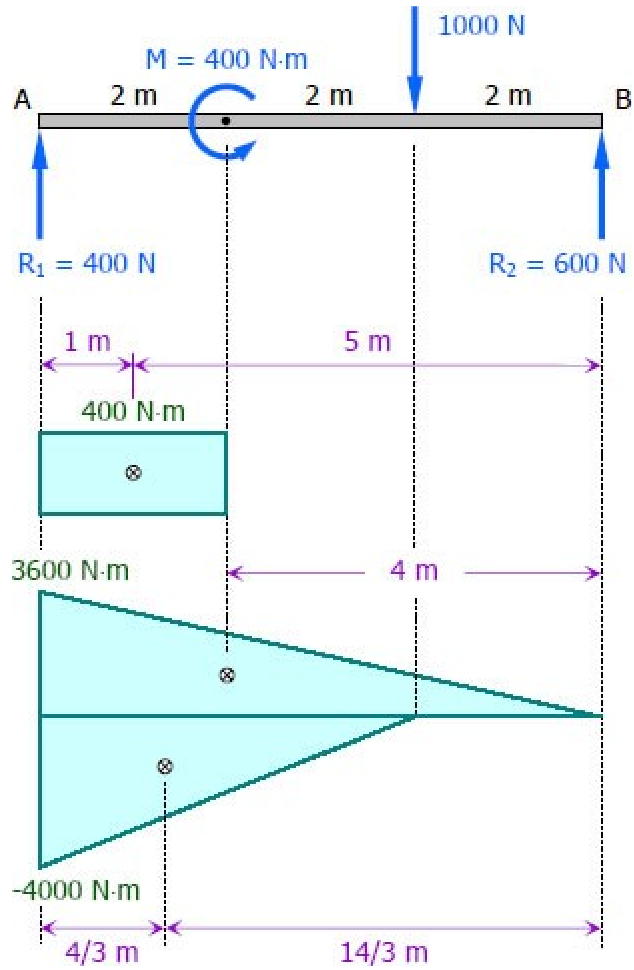


$$(Area_{AB})\bar{X}_A = \frac{1}{2}(2)(800)(\frac{4}{3}) + \frac{1}{2}(4)(2400)(\frac{10}{3}) - \frac{1}{2}(2)(2000)(\frac{8}{3})$$

$$(Area_{AB})\bar{X}_A = 11\,733.33\,\mathrm{N\cdot m^3}$$

$$(Area_{AB})\bar{X}_B = \frac{1}{2}(2)(800)(\frac{14}{3}) + \frac{1}{2}(4)(2400)(\frac{8}{3}) - \frac{1}{2}(2)(2000)(\frac{10}{3})$$

$$(Area_{AB})\bar{X}_B = 9\,866.67\,\mathrm{N\cdot m^3}$$

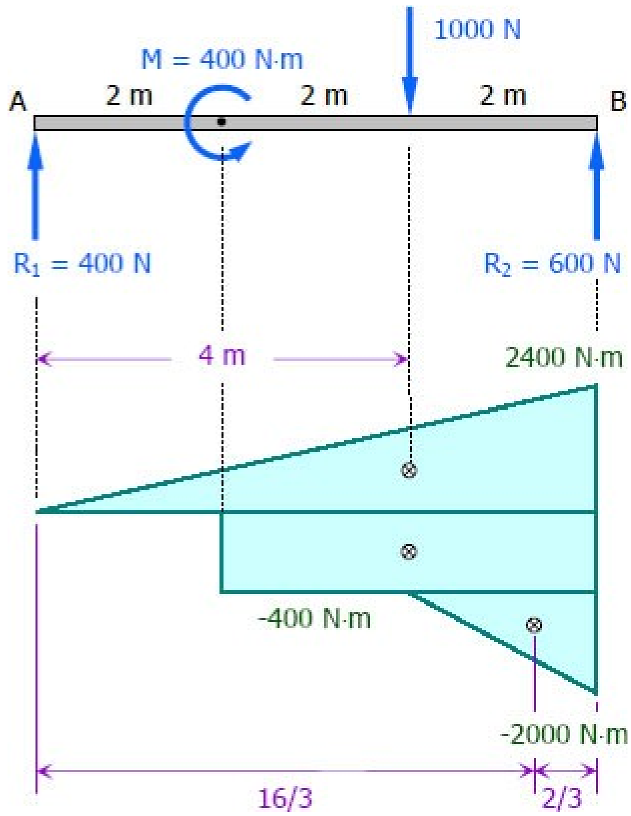


$$(Area_{AB})\bar{X}_A = 400(2)(1) + \frac{1}{2}(6)(3600)(2) - \frac{1}{2}(4)(4000)(\frac{4}{3})$$

$$(Area_{AB})\bar{X}_A = 11\,733.33\,\text{N}\cdot\text{m}^3$$

$$(Area_{AB})\bar{X}_B = 400(2)(5) + \frac{1}{2}(6)(3600)(4) - \frac{1}{2}(4)(4000)(\frac{14}{3})$$

$$(Area_{AB})\bar{X}_B = 9\,866.67\,\text{N}\cdot\text{m}^3$$



$$(Area_{AB})\,\bar{X}_A = \tfrac{1}{2}(6)(2400)(4) - 400(4)(4) - \tfrac{1}{2}(2)(2000)(\tfrac{16}{3})$$

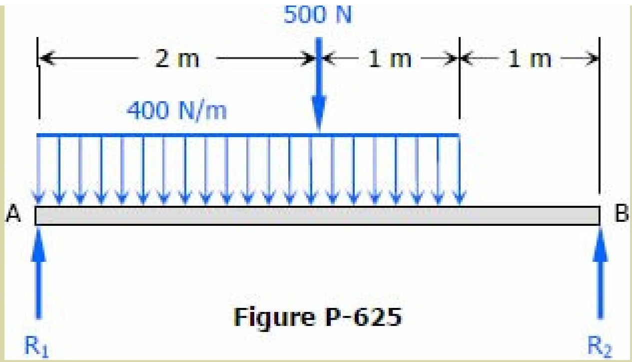
$$(Area_{AB})\bar{X}_A = 11\,733.33\ \text{N}\cdot\text{m}^3$$

$$(Area_{AB})\bar{X}_B = \tfrac{1}{2}(6)(2400)(2) - 400(4)(2) - \tfrac{1}{2}(2)(2000)(\tfrac{2}{3})$$

$$(Area_{AB})\bar{X}_B = 9\,866.67\ \text{N}\cdot\text{m}^3$$

Problem 625

For the beam loaded as shown in Fig. P-625, compute the moment of area of the M diagrams between the reactions about both the left and the right reaction. (Hint: Draw the moment diagram by parts from right to left.)



Solution 625

$\Sigma M_{R2} = 0$

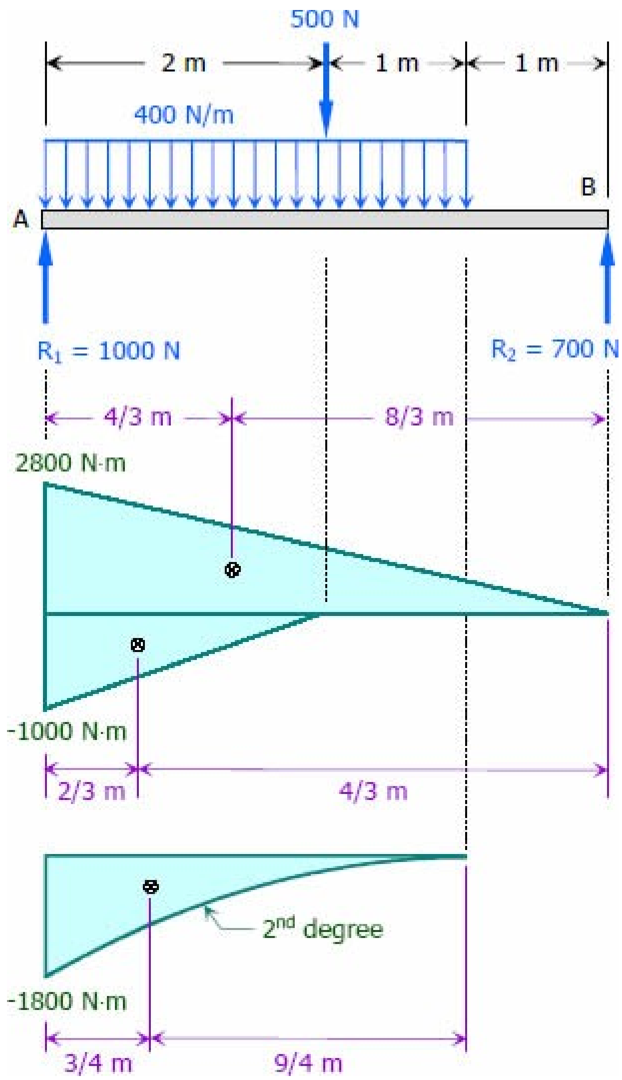
$4R_1 = 400(3)(2.5) + 500(2)$

$R_1 = 1000 \text{ N}$

$\Sigma M_{R1} = 0$

$4R_2 = 400(3)(1.5) + 500(2)$

$R_2 = 700 \text{ N}$



$$(Area_{AB})\,\bar{X}_A = \frac{1}{2}(4)(2800)(\frac{4}{3}) - \frac{1}{2}(2)(1000)(\frac{2}{3}) - \frac{1}{3}(3)(1800)(\frac{3}{4})$$

$$(Area_{AB})\,\bar{X}_A = 5\,450\,\text{N}\cdot\text{m}^3$$ *answer*

$$(Area_{AB})\,\bar{X}_B = \frac{1}{2}(4)(2800)(\frac{8}{3}) - \frac{1}{2}(2)(1000)(\frac{4}{3}) - \frac{1}{3}(3)(1800)(\frac{9}{4}+1)$$

$$(Area_{AB})\,\bar{X}_B = 7\,750\,\text{N}\cdot\text{m}^3$$ *answer*

Problem 626

For the eam loaded as shown in Fig. P-626, compute the moment of area of the M diagrams between the reactions about both the left and the right reaction.

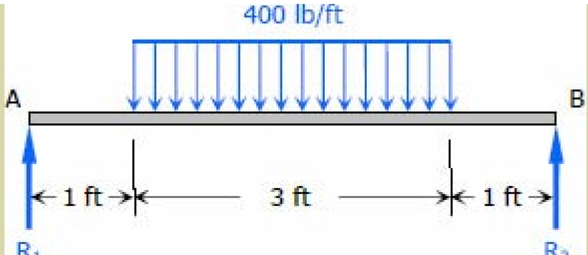


Figure P-626

Solution 626

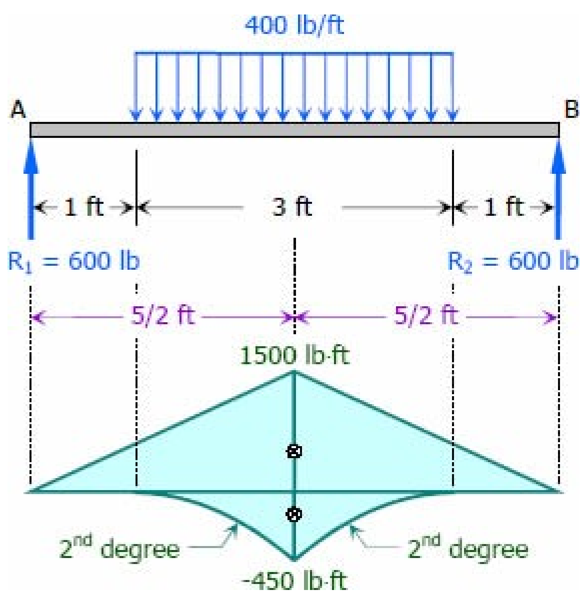
By symmetry

$R_1 = R_2 = \frac{1}{2}(400)(3)$

$R_1 = R_2 = 600 \, \text{lb}$

and

$(Area_{AB}) \, \bar{X}_A = (Area_{AB}) \, \bar{X}_B$



$(Area_{AB}) \, \bar{X}_A = \frac{1}{2}(5)(1500)(\frac{5}{2}) - \frac{1}{3}(3)(450)(\frac{5}{2})$ *answer*

$(Area_{AB})\,\bar{X}_A = 8\,250 \text{lb} \cdot \text{ft}^3$

Thus,
$(Area_{AB})\,\bar{X}_B = 8\,250 \text{lb} \cdot \text{ft}^3$ *answer*

Problem 627

For the beam loaded as shown in Fig. P-627compute the moment of area of the M diagrams between the reactions about both the left and the right reaction. (Hint: Resolve the trapezoidal loading into a uniformly distributed load and a uniformly varying load.)

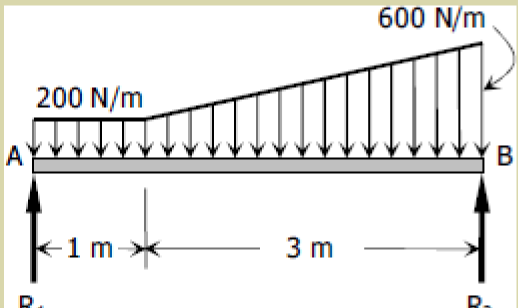


Figure P-627

Solution 627

$\Sigma M_{R2} = 0$

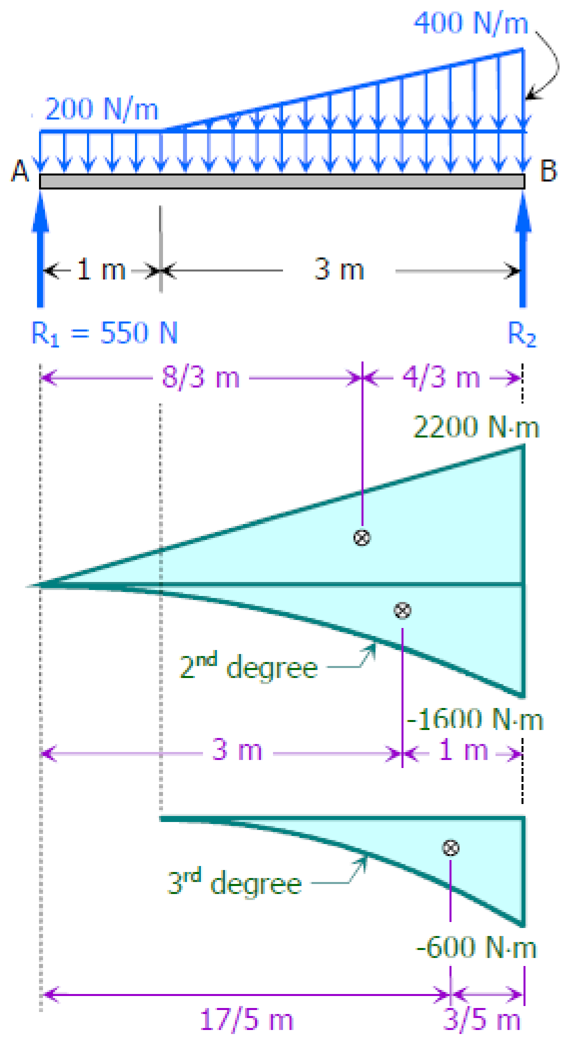
$4R_1 = 200(4)(2) + \frac{1}{2}(3)(400)(1)$

$R_1 = 550\,\text{N}$

$\Sigma M_{R1} = 0$

$4R_2 = 200(4)(2) + \frac{1}{2}(3)(400)(3)$

$R_2 = 850\,\text{N}$



$$(Area_{AB})\,\bar{X}_A = \tfrac{1}{2}(4)(2200)(\tfrac{8}{3}) - \tfrac{1}{3}(4)(1600)(3) - \tfrac{1}{4}(3)(600)(\tfrac{17}{5})$$

$$(Area_{AB})\,\bar{X}_A = 3\,803.33\ \text{N}\cdot\text{m}^3 \qquad \textit{answer}$$

$$(Area_{AB})\,\bar{X}_B = \tfrac{1}{2}(4)(2200)(\tfrac{4}{3}) - \tfrac{1}{3}(4)(1600)(1) - \tfrac{1}{4}(3)(600)(\tfrac{3}{5})$$

$$(Area_{AB})\,\bar{X}_B = 3\,463.33\ \text{N}\cdot\text{m}^3 \qquad \textit{answer}$$

Problem 628

For the beam loaded with uniformly varying load and a couple as shown in Fig. P-628 compute the moment of area of the M diagrams between the reactions about both the left and the right reaction.

Solution 628

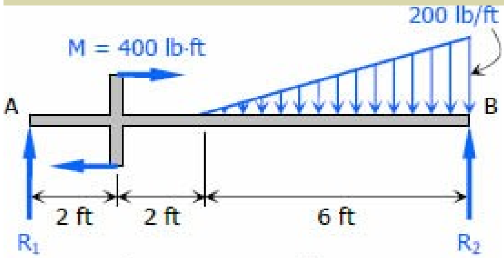


Figure P-628 and P-629

$\Sigma M_{R2} = 0$

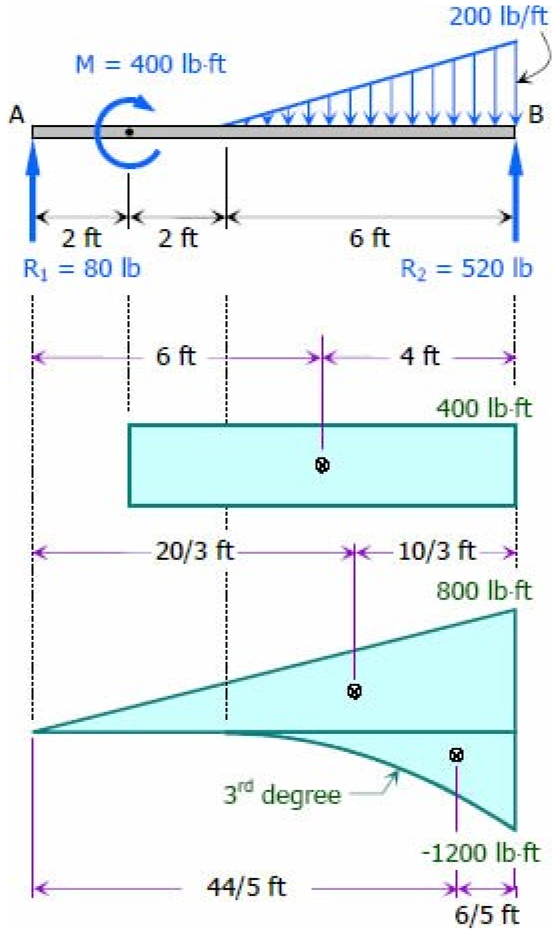
$10R_1 + 400 = \frac{1}{2}(6)(200)(2)$

$R_1 = 80\,\text{lb}$

$\Sigma M_{R1} = 0$

$10R_2 = 400 + \frac{1}{2}(6)(200)(8)$

$R_2 = 520\,\text{lb}$



$$(Area_{AB})\,\bar{X}_A = 400(8)(6) + \tfrac{1}{2}(10)(800)(\tfrac{20}{3}) - \tfrac{1}{4}(6)(1200)(\tfrac{44}{5})$$

$$(Area_{AB})\,\bar{X}_A = 30\,026.67\,\text{lb}\cdot\text{ft}^3$$ *answer*

$$(Area_{AB})\,\bar{X}_B = 400(8)(4) + \tfrac{1}{2}(10)(800)(\tfrac{10}{3}) - \tfrac{1}{4}(6)(1200)(\tfrac{6}{5})$$

$$(Area_{AB})\,\bar{X}_B = 23\,973.33\,\text{lb}\cdot\text{ft}^3$$ *answer*

Problem 629

Solve Prob. 628 if the sense of **the couple is counterclockwise instead of clockwise** as shown in Fig. P-628.

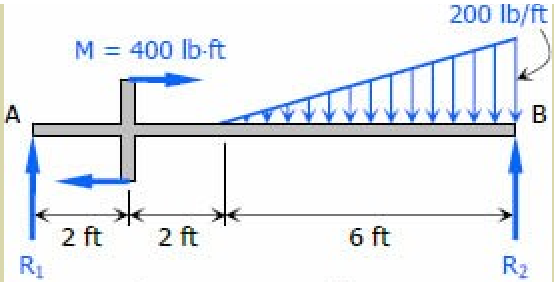


Figure P-628 and P-629

Solution 629

$\Sigma M_{R2} = 0$

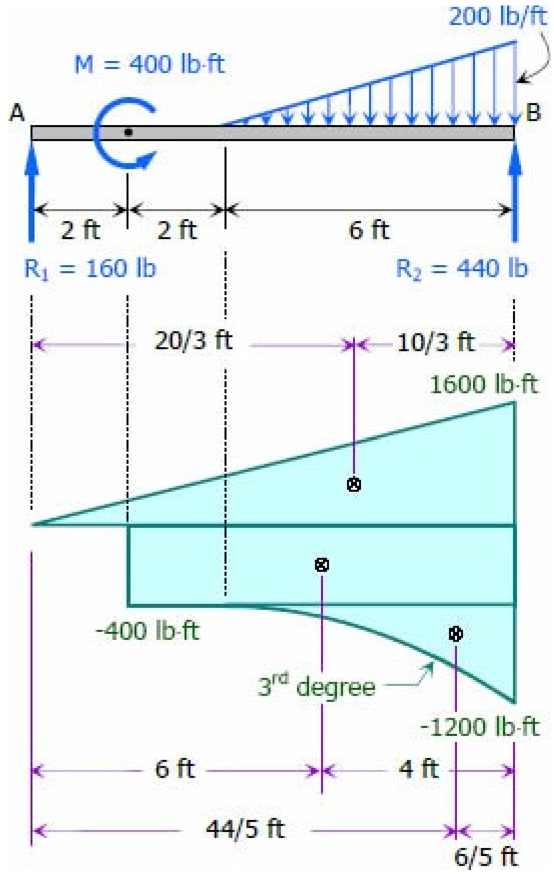
$10R_1 = 400 + \frac{1}{2}(6)(200)(2)$

$R_1 = 160 \, \text{lb}$

$\Sigma M_{R1} = 0$

$10R_2 + 400 = \frac{1}{2}(6)(200)(8)$

$R_2 = 440 \, \text{lb}$



$$(Area_{AB})\,\bar{X}_A = \frac{1}{2}(10)(1600)(\frac{20}{3}) - 400(8)(6) - \frac{1}{4}(6)(1200)(\frac{44}{5})$$

$(Area_{AB})\,\bar{X}_A = 18\,293.33\ \text{lb}\cdot\text{ft}^3$ *answer*

$$(Area_{AB})\,\bar{X}_B = \frac{1}{2}(10)(1600)(\frac{10}{3}) - 400(8)(4) - \frac{1}{4}(6)(1200)(\frac{6}{5})$$

$(Area_{AB})\,\bar{X}_B = 11\,706.67\ \text{lb}\cdot\text{ft}^3$ *answer*

Problem 630

For the beam loaded as shown in Fig. P-630, compute the value of (Area$_{AB}$)barred(X)$_A$. From the result determine whether the tangent drawn to the elastic curve at B slopes up or down to the right. (Hint: Refer to the deviation equations and rules of sign.)

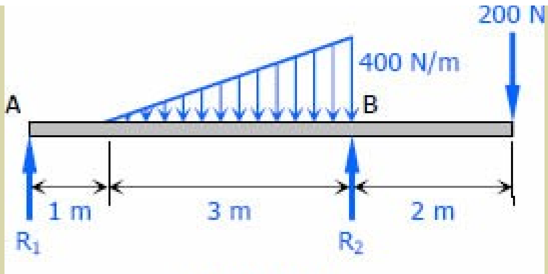
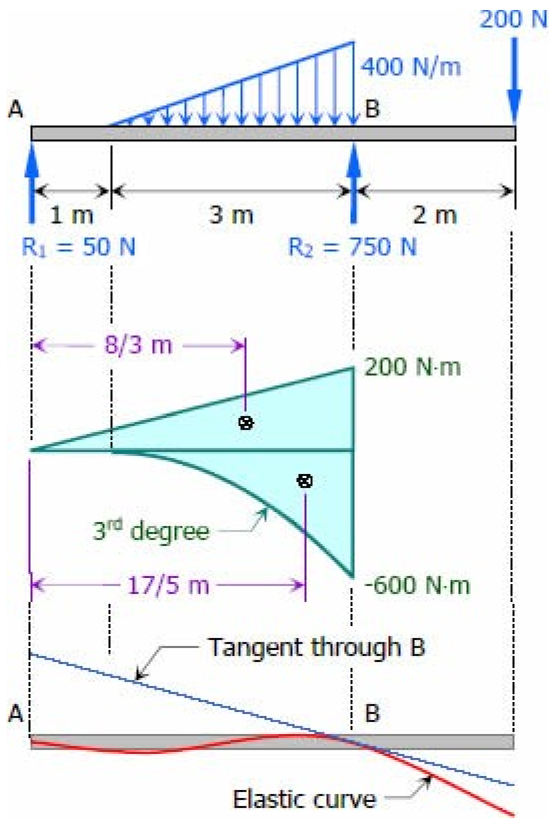


Figure P-630

Solution 630



$\Sigma M_{R2} = 0$

$4R_1 + 200(2) = \frac{1}{2}(3)(400)(1)$

$R_1 = 50 \text{ N}$

$\Sigma M_{R1} = 0$

$4R_2 = 200(6) + \frac{1}{2}(3)(400)(3)$

$R_2 = 750 \text{ N}$

$(Area_{AB})\,\bar{X}_A = \frac{1}{2}(4)(200)(\frac{8}{3}) - \frac{1}{4}(3)(600)(\frac{17}{5})$

$(Area_{AB})\,\bar{X}_A = -463.33 \text{ N}\cdot\text{m}^3$ *answer*

The value of ($Area_{AB}$) barred$(X)_A$ is negative; therefore point A is below the tangent through B, thus **the tangent through B slopes downward to the right**. See the approximate elastic curve shown to the right and refer to the rules of sign for more information.

Problem 631

Determine the value of the couple M for the beam loaded as shown in Fig. P-631 so that the moment of area about A of the M diagram between A and B will be zero. What is the physical significance of this result?

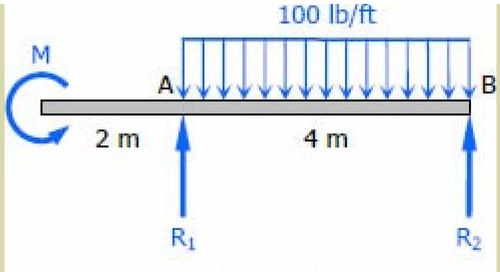
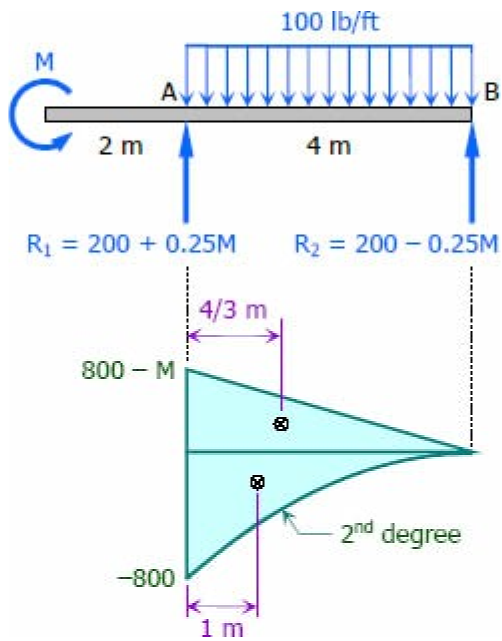


Figure P-631

Solution 631

HideClick here to show or hide the solution



$\Sigma M_A = 0$

$4R_2 + M = 100(4)(2)$

$R_2 = 200 - \frac{1}{4}M$

$\Sigma M_B = 0$

$4R_1 = 100(4)(2) + M$

$R_1 = 200 + \frac{1}{4}M$

$(Area_{AB})\,\bar{X}_A = 0$

$\frac{1}{2}(4)(800 - M)(\frac{4}{3}) - \frac{1}{3}(4)(800)(1) = 0$

$\frac{8}{3}(800 - M) = \frac{3200}{3}$

$M = 400\,\text{lb}\cdot\text{ft}$ *answer*

The uniform load over span AB will cause segment AB to deflect downward. The moment load equal to 400 lb·ft applied at the free end will cause the slope through B to be horizontal making the deviation of A from the tangent through B equal to zero. The downward deflection therefore due to uniform load will be countered by the moment load.

Problem 632

For the beam loaded as shown in Fig. P-632, compute the value of $(Area_{AB})\ \bar{X}_A$. From this result, is the tangent drawn to the elastic curve at B directed up or down to the right? (Hint: Refer to the deviation equations and rules of sign.)

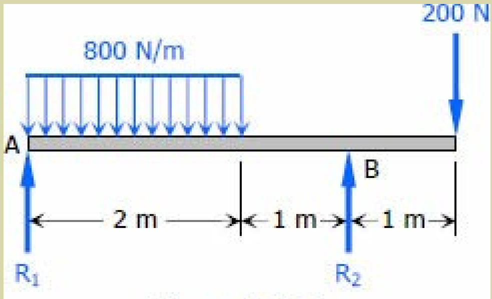


Figure P-632

Solution 632

$\Sigma M_B = 0$

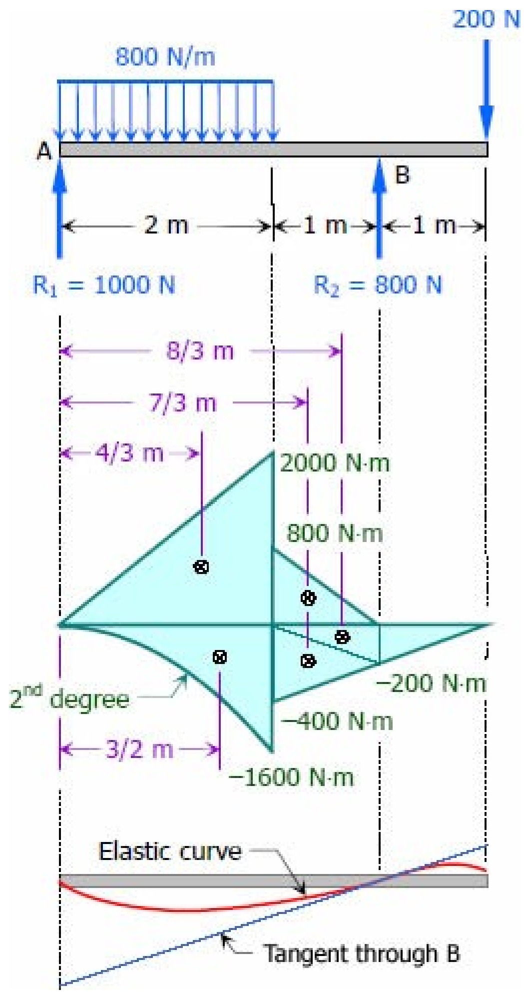
$3R_1 + 200(1) = 800(2)(2)$

$R_1 = 1000\,\text{N}$

$\Sigma M_A = 0$

$3R_2 = 200(4) + 800(2)(1)$

$R_1 = 800\,\text{N}$



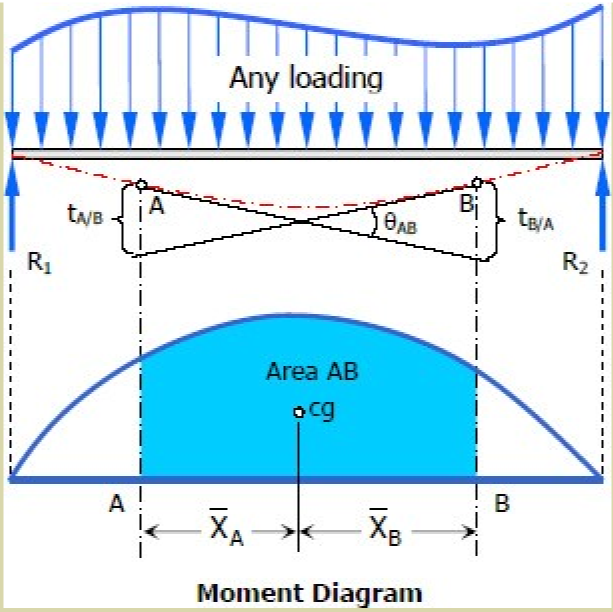
$$(Area_{AB})\,\bar{X}_A = \tfrac{1}{2}(2)(2000)(\tfrac{4}{3})+\tfrac{1}{2}(1)(800)(\tfrac{7}{3})-\tfrac{1}{3}(2)(1600)(\tfrac{3}{2})-\tfrac{1}{2}(1)(400)(\tfrac{7}{3})-\tfrac{1}{2}(1)(200)(\tfrac{8}{3})$$

$$(Area_{AB})\,\bar{X}_A = 1\,266.67\ \text{N}\cdot\text{m}^3$$ *answer*

The value of ($Area_{AB}$) barred$(X)_A$ is positive, therefore point A is above the tangent through B, thus **the tangent through B is upward to the right**. See the approximate elastic curve shown above and refer to the rules of sign for more information.

Area-Moment Method | Beam Deflections

Another method of determining the slopes and deflections in beams is the area-moment method, which involves the area of the moment diagram.



Theorems of Area-Moment Method

Theorem I

The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1/EI multiplied by the area of the moment diagram between these two points.

$$\theta_{AB} = \frac{1}{EI}(Area_{AB})$$

Theorem II

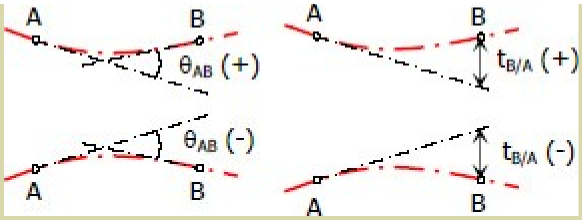
The deviation of any point B relative to the tangent drawn to the elastic curve at any other point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

$$t_{B/A} = \frac{1}{EI}(Area_{AB}) \cdot \bar{X}_B$$

and

$$t_{A/B} = \frac{1}{EI}(Area_{AB}) \cdot \bar{X}_A$$

Rules of Sign

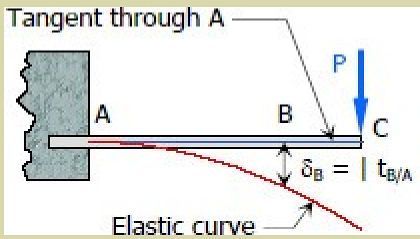


1. The deviation at any point is positive if the point lies above the tangent, negative if the point is below the tangent.

2. Measured from left tangent, if θ is counterclockwise, the change of slope is positive, negative if θ is clockwise.

Deflection of Cantilever Beams | Area-Moment Method

Generally, the tangential deviation t is not equal to the beam deflection. In cantilever beams, however, the tangent drawn to the elastic curve at the wall is horizontal and coincidence therefore with the neutral axis of the beam. The tangential deviation in this case is equal to the deflection of the beam as shown below.



From the figure above, the deflection at B denoted as δ_B is equal to the deviation of B from a tangent line through A denoted as $t_{B/A}$. This is because the tangent line through A lies with the neutral axis of the beam.

Problem 636

The cantilever beam shown in Fig. P-636 has a rectangular cross-section 50 mm wide by h mm high. Find the height h if the maximum deflection is not to exceed 10 mm. Use E = 10 GPa.

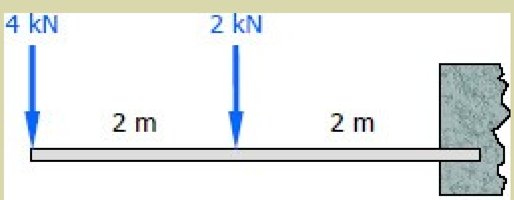
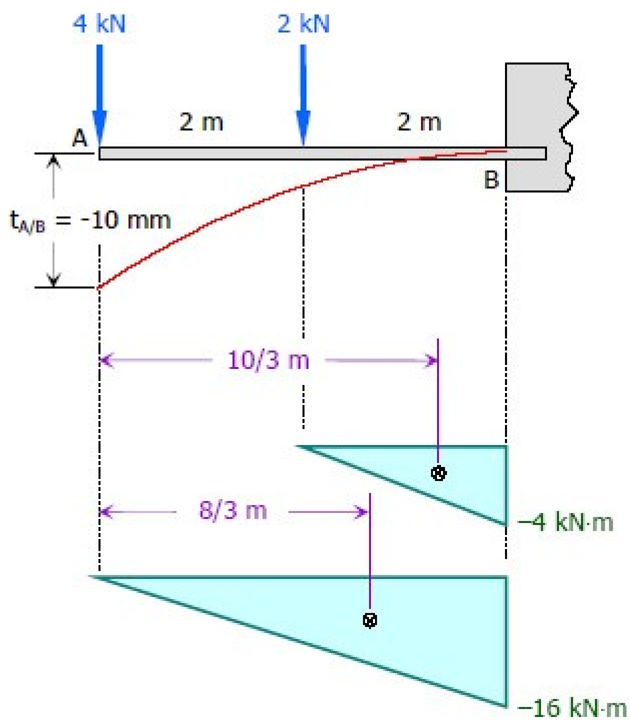


Figure P-636

Solution 636



$$t_{A/B} = \frac{1}{EI}(Area_{AB})\,\bar{X}_A$$

$$-10 = \frac{1}{10\,000\left(\dfrac{50h^3}{12}\right)}\left[-\tfrac{1}{2}(2)(4)(\tfrac{10}{3}) - \tfrac{1}{2}(4)(16)(\tfrac{8}{3})\right](1000^4)$$

$$-10 = \frac{3}{125\,000h^3}\left[-\frac{296}{3}\right](1000^4)$$

$$h^3 = \frac{-296(1000^4)}{125\,000(-10)}$$

$h = 618.67\,\text{mm}$ *answer*

Problem 637

For the beam loaded as shown in Fig. P-637, determine the deflection 6 ft from the wall. Use E = 1.5 × 10^6 psi and I = 40 in^4.

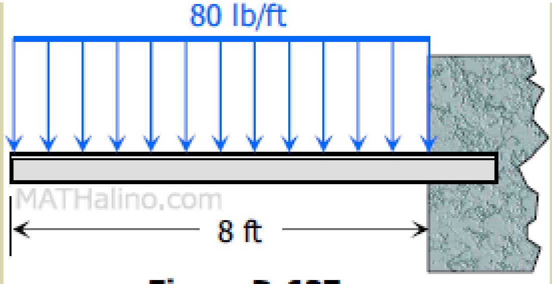


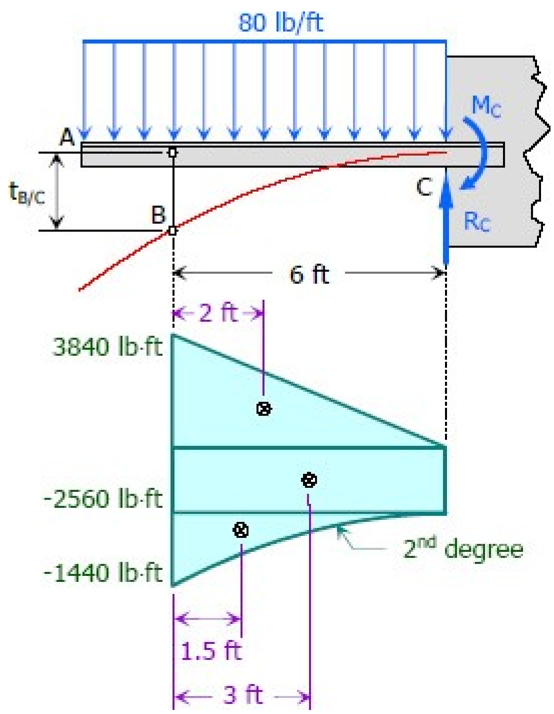
Figure P-637

Solution 637

HideClick here to show or hide the solution

$R_C = 80(8) = 640\ \text{lb}$

$M_C = 80(8)(4) = 2560\,\text{lb}\cdot\text{ft}$



$$t_{B/C} = \frac{1}{EI}(Area_{BC})\bar{X}_B$$

$$t_{B/C} = \frac{1}{EI}[\,\frac{1}{2}(6)(3840)(2) - 6(2560)(3) - \frac{1}{3}(6)(1440)(1.5)\,]\,(12^3)$$

$$t_{B/C} = \frac{1}{EI}[\,27\,360\,]\,(12^3)$$

$$t_{B/C} = \frac{1}{(1.5 \times 10^6)(40)}[\,27\,360\,]\,(12^3)$$

$$t_{B/C} = -0.787968 \text{ in}$$

Thus, $\delta_B = |\,t_{B/C}\,| = 0.787968$ in *answer*

Problem 638

For the cantilever beam shown in Fig. P-638, determine the value of EIδ at the left end. Is this deflection upward or downward?

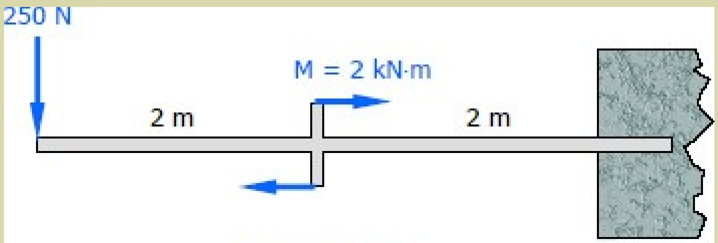
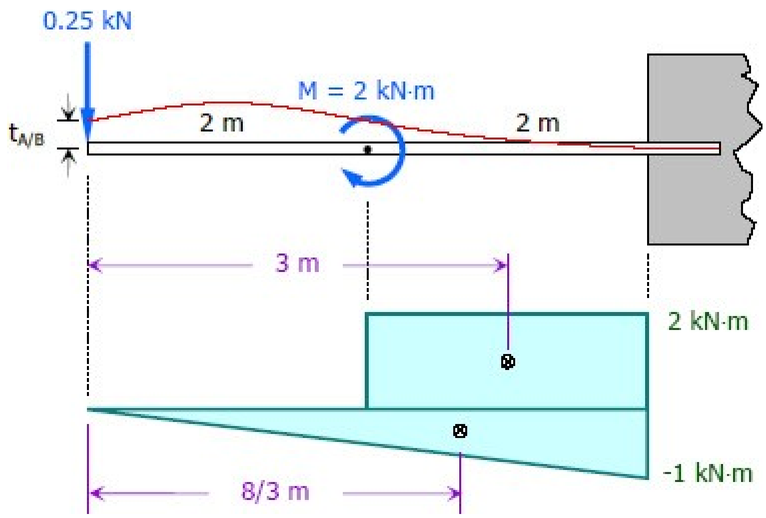


Figure P-638

Solution 638

HideClick here to show or hide the solution



$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = 2(2)(3) - \tfrac{1}{2}(4)(1)(\tfrac{8}{3})$$

$$EI\,t_{A/B} = \tfrac{20}{3} = 6.67\ \text{ kN}\cdot\text{m}^3$$

∴ $EI\delta = 6.67\ \text{kN}\cdot\text{m}^3$ upward *answer*

Problem 639

The downward distributed load and an upward concentrated force act on the cantilever beam in Fig. P-639. Find the amount the free end deflects upward or downward if $E = 1.5 \times 10^6$ psi and $I = 60\ \text{in}^4$.

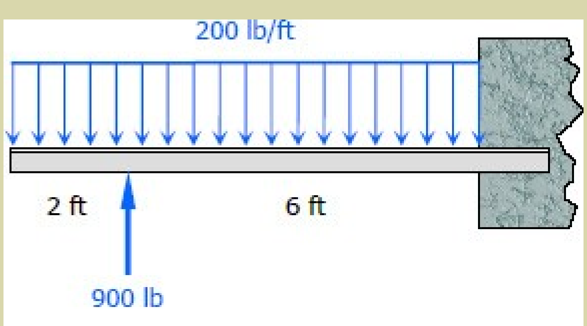
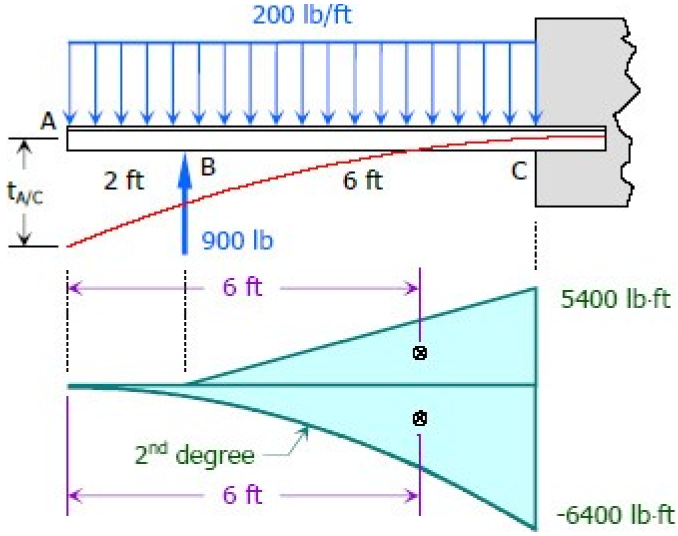


Figure P-639 and P-640

Solution 639

HideClick here to show or hide the solution



$$t_{A/C} = \frac{1}{EI}(Area_{AB})\,\bar{X}_A$$

$$t_{A/C} = \frac{1}{(1.5\times10^6)(60)}\left[\,\tfrac{1}{2}(6)(5400)(6) - \tfrac{1}{3}(8)(6400)(6)\,\right](12^3)$$

$$t_{A/C} = -0.09984 \text{ in}$$

$\therefore$ The free end will move by 0.09984 inch downward. *answer*

Problem 640

Compute the value of δ at the concentrated load in Prob. 639. Is the deflection upward downward?

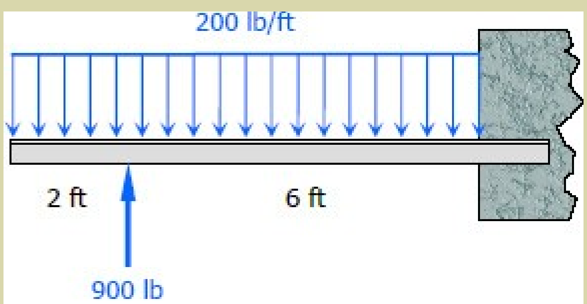


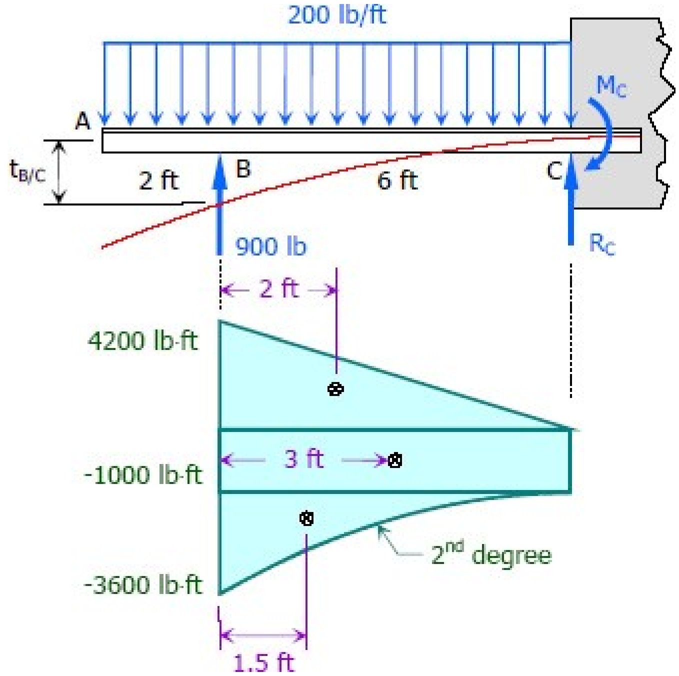
Figure P-639 and P-640

Solution 640

HideClick here to show or hide the solution

$R_C = 200(8) - 900 = 700 \text{ lb}$

$M_C = 200(8)(4) - 900(6) = 1000 \text{ lb} \cdot \text{ft}$



$$t_{B/C} = \frac{1}{EI}(Area_{BC})\,\bar{X}_B$$

$$t_{B/C} = \frac{1}{(1.5 \times 10^6)(60)}[\,\tfrac{1}{2}(6)(4200)(2) - 1000(6)(3) - \tfrac{1}{3}(6)(3600)(1.5)\,](12^3)$$

$$t_{B/C} = -0.06912 \text{ in}$$

∴ $\delta = 0.06912$ inch downward *answer*

Problem 641

For the cantilever beam shown in Fig. P-641, what will cause zero deflection at A?

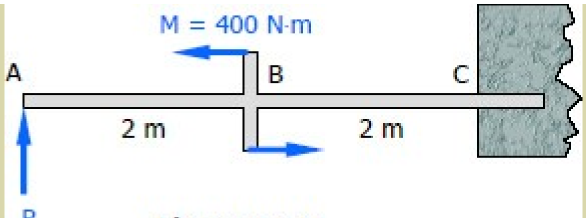
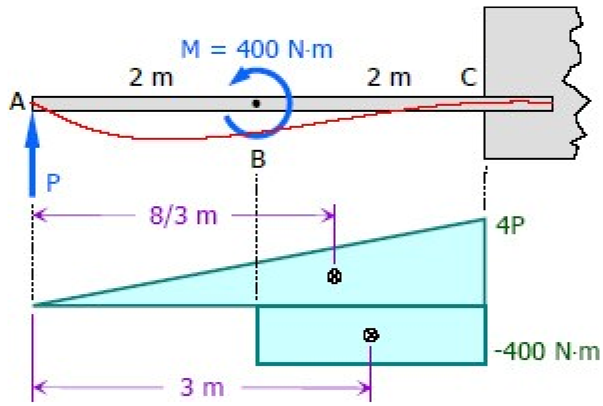


Figure P-641

Solution 641



$$\frac{1}{EI}(Area_{AC})\,\bar{X}_A = 0$$

$$\frac{1}{EI}[\,\frac{1}{2}(4)(4P)(\frac{8}{3}) - 2(400)(3)\,] = 0$$

$P = 112.5\ \text{N}$ *answer*

Problem 642

Find the maximum deflection for the cantilever beam loaded as shown in Figure P-642 if the cross section is 50 mm wide by 150 mm high. Use E = 69 GPa.

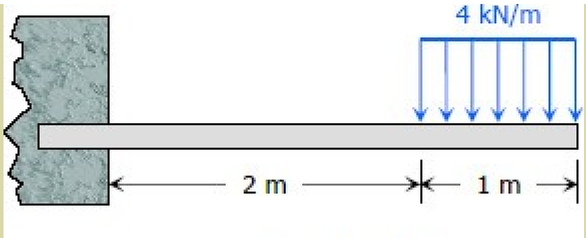


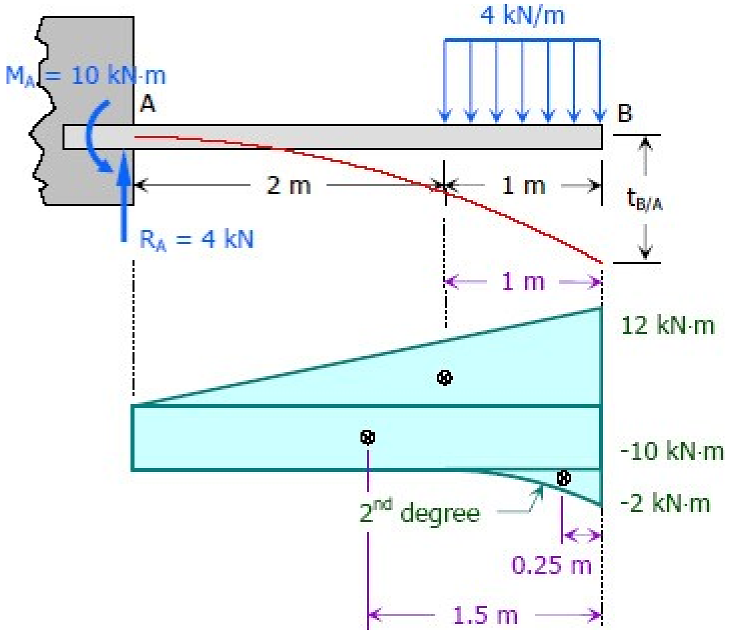
Figure P-642

Solution 642

HideClick here to show or hide the solution

$R_A = 4(1) = 4\,\text{kN}$

$M_A = 4(1)(2.5) = 10\,\text{kN}\cdot\text{m}$



$$t_{B/A} = \frac{1}{EI}(Area_{AB})\,\bar{X}_B$$

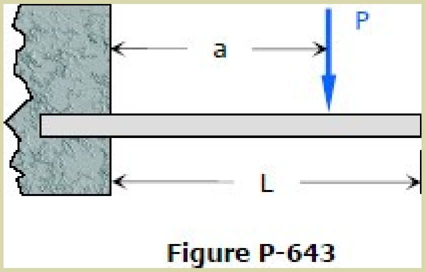
$$t_{B/A} = \frac{1}{69\,000\left[\dfrac{50(150^3)}{12}\right]}\left[\,\tfrac{1}{2}(3)(12)(1) - 3(10)(1.5) - \tfrac{1}{3}(1)(2)(0.25)\,\right](1000^4)$$

$t_{B/A} = -28\,\text{mm}$

∴ $\delta_{max} = 28$ mm *answer*

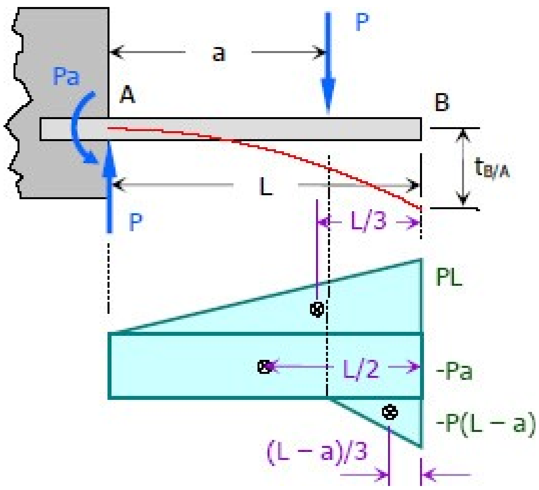
Problem 643

Find the maximum value of EIδ for the cantilever beam shown in Fig. P-643.



Solution 643

HideClick here to show or hide the solution



$EI\,t_{B/A} = (Area_{AB})\,\bar{X}_B$

$EI\,t_{B/A} = \frac{1}{2}L(PL)(\frac{1}{3}L) - PaL(\frac{1}{2}L) - \frac{1}{2}(L-a)P(L-a)[\,\frac{1}{3}(L-a)\,]$

$$EI\,t_{B/A} = \frac{1}{6}PL^3 - \frac{1}{2}PL^2a - \frac{1}{6}P(L-a)^3$$

$$EI\,t_{B/A} = \frac{1}{6}PL^3 - \frac{1}{2}PL^2a - \frac{1}{6}P(L^3 - 3L^2a + 3La^2 - a^3)$$

$$EI\,t_{B/A} = \frac{1}{6}PL^3 - \frac{1}{2}PL^2a - \frac{1}{6}PL^3 + \frac{1}{2}PL^2a - \frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$$

$$EI\,t_{B/A} = -\frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$$

$$EI\,t_{B/A} = -\frac{1}{6}Pa^2(3L-a)$$

Therefore

$EI\,\delta_{max} = \frac{1}{6}Pa^2(3L-a)$ *answer*

Problem 644

Determine the maximum deflection for the beam loaded as shown in Fig. P-644.

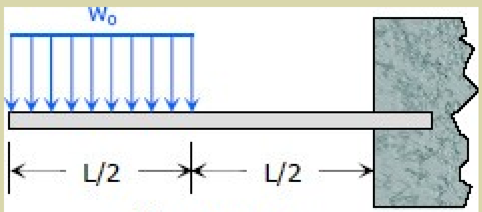


Figure P-644

Solution 644

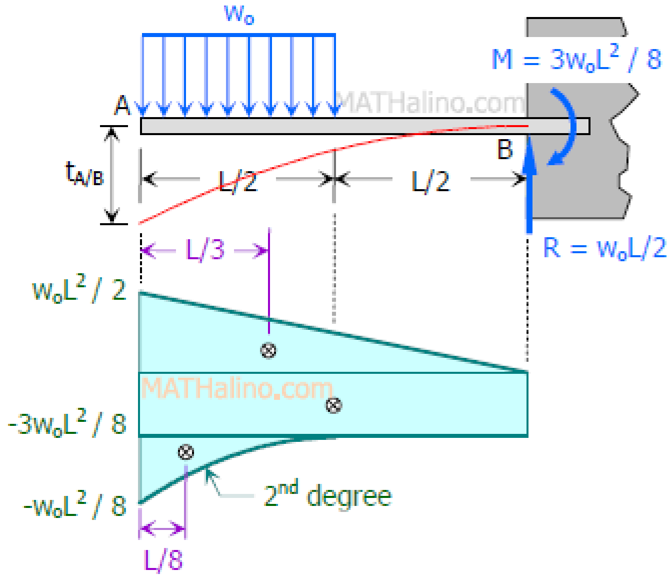
HideClick here to show or hide the solution

$$R = w_o(\tfrac{1}{2}L)$$

$$R = \frac{1}{2}w_oL$$

$$M = w_o(\tfrac{1}{2}L)(\tfrac{3}{4}L)$$

$$M = \frac{3}{8}w_oL^2$$



$$t_{A/B} = \frac{1}{EI}(Area_{AB})\,\bar{X}_A$$

$$t_{A/B} = \frac{1}{EI}[\,\tfrac{1}{2}(L)(\tfrac{1}{2}w_oL^2)(\tfrac{1}{3}L) - \tfrac{3}{8}w_oL^2(L)(\tfrac{1}{2}L) - \tfrac{1}{3}(\tfrac{1}{8}w_oL^2)(\tfrac{1}{2}L)(\tfrac{1}{8}L)\,]$$

$$t_{A/B} = \frac{1}{EI}[\,\tfrac{1}{12}w_oL^4 - \tfrac{3}{16}w_oL^4 - \tfrac{1}{384}w_oL^4\,]$$

$$t_{A/B} = \frac{1}{EI}[-\tfrac{41}{384}w_oL^4]$$

$$t_{A/B} = -\frac{41w_oL^4}{384EI}$$

Therefore

$$\delta_{max} = \frac{41w_oL^4}{384EI} \qquad \textit{answer}$$

Problem 645

Compute the deflection and slope at a section 3 m from the wall for the beam shown in Fig. P-645. Assume that E = 10 GPa and I = 30 × 10^6 mm^4.

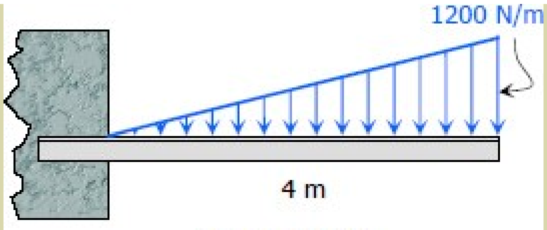


Figure P-645

Solution 645

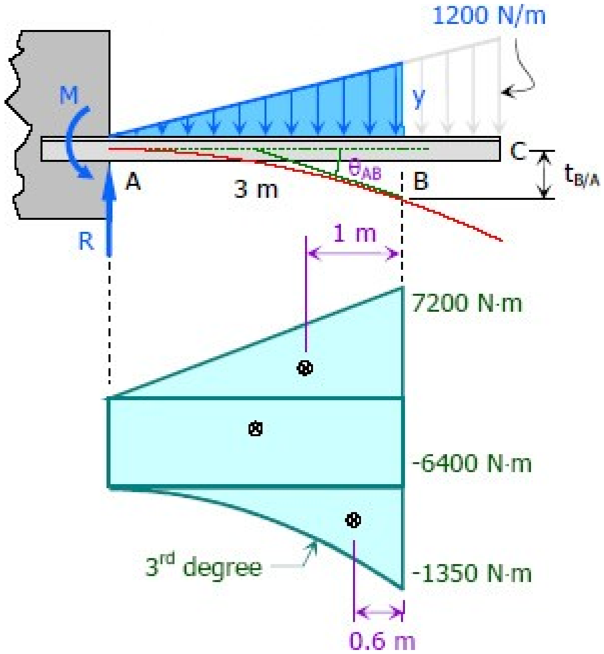
HideClick here to show or hide the solution

$R = \frac{1}{2}(4)(1200)$

$R = 2400\,\text{N}$

$M = \frac{1}{2}(4)(1200)(\frac{8}{3})$

$M = 6400\,\text{N}\cdot\text{m}$



$$\frac{y}{3} = \frac{1200}{4}$$

$$y = 900\ \text{N/m}$$

$$t_{B/A} = \frac{1}{EI}(Area_{AB})\,\bar{X}_B$$

$$t_{B/A} = \frac{1}{EI}[\,\tfrac{1}{2}(3)(7200)(1) - 3(6400)(1.5) - \tfrac{1}{4}(3)(1350)(0.6)\,](1000^3)$$

$$t_{B/A} = \frac{1}{10000(30 \times 10^6)}[\,-18607.5\,](1000^3)$$

$$t_{B/A} = -62.025\ \text{mm}$$

Therefore:

$\delta_B = 62.025\ \text{mm}$ *answer*

$$\theta_{AB} = \frac{1}{EI}(Area_{AB})$$

$$\theta_{AB} = \frac{1}{EI}[\,\tfrac{1}{2}(3)(7200) - 3(6400) - \tfrac{1}{4}(3)(1350)\,](1000^2)$$

$$\theta_{AB} = \frac{1}{10\,000(30 \times 10^6)}[\,-9412.5\,](1000^2)$$

$$\theta_{AB} = -0.031375\ \text{radian}$$

$\theta_{AB} = 1.798\ \text{degree}$ *answer*

Problem 646

For the beam shown in Fig. P-646, determine the value of I that will limit the maximum deflection to 0.50 in. Assume that E = 1.5 × 10^6 psi.

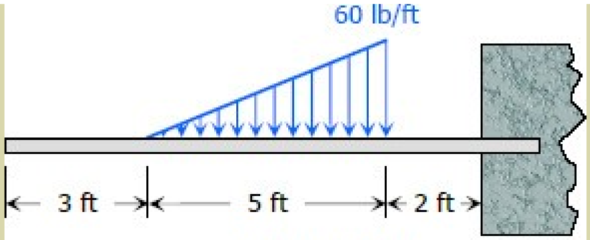


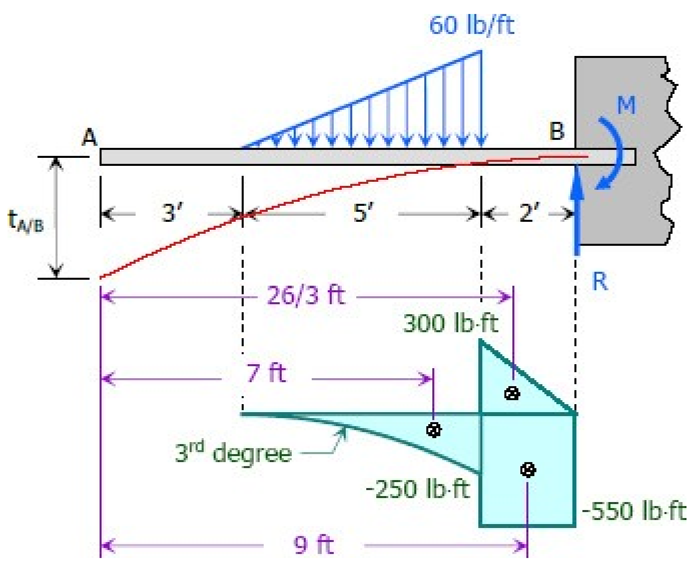
Figure P-646

Solution 646

HideClick here to show or hide the solution

$M = \frac{1}{2}(5)(60)(2 + \frac{5}{3}) = 550\,\text{lb}\cdot\text{ft}$

$R = \frac{1}{2}(5)(60) = 150\,\text{lb}$



$$t_{A/B} = \frac{1}{EI}(Area_{AB})\,\bar{X}_A$$

$$-5 = \frac{1}{EI}[\,\frac{1}{2}(300)(2)(\frac{26}{3}) - 550(2)(9) - \frac{1}{4}(5)(250)(7)\,](12^3)$$

$$-5 = \frac{1}{(1.5\times10^6)I}(-16394400)$$

$I = 2.18592\,\text{in}^4$

Problem 647

Find the maximum value of EIδ for the beam shown in Fig. P-647.

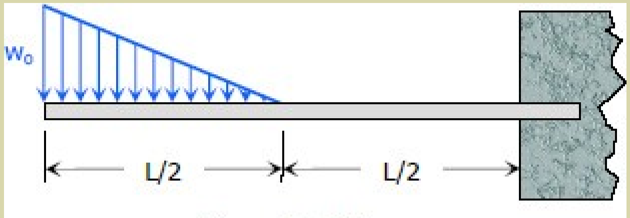


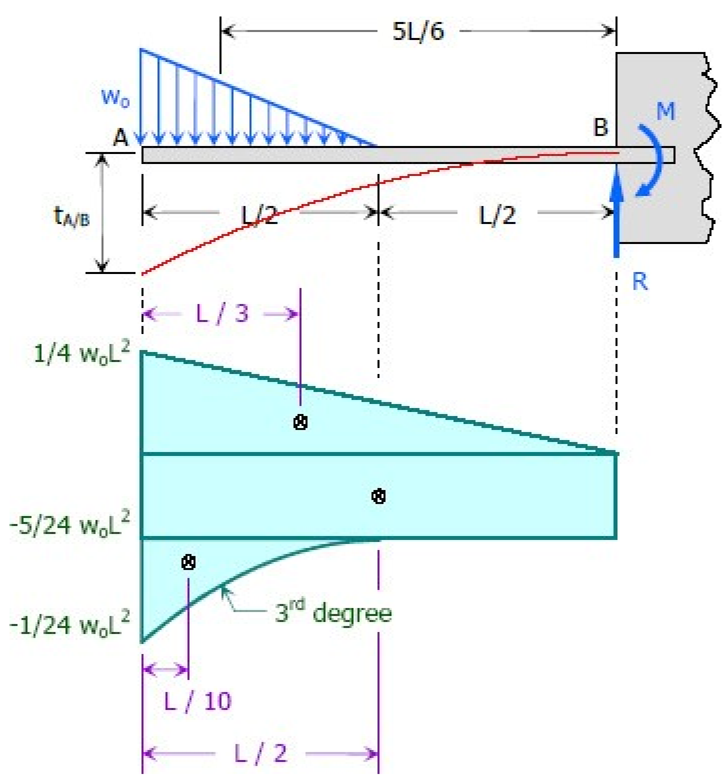
Figure P-647

Solution 647

HideClick here to show or hide the solution

$R = \frac{1}{2}(\frac{1}{2}L)(w_o) = \frac{1}{4}w_oL$

$M = \frac{1}{2}(\frac{1}{2}L)(w_o)(\frac{5}{6}L) = \frac{5}{24}w_oL^2$



$EI\, t_{A/B} = (Area_{AB})\, \bar{X}_A$

$EI\, t_{A/B} = \frac{1}{2}L(\frac{1}{4}w_oL^2)(\frac{1}{3}L) - L(\frac{5}{24}w_oL^2)(\frac{1}{2}L) - \frac{1}{4}(\frac{1}{2}L)(\frac{1}{24}w_oL^2)(\frac{1}{10}L)$

$EI\, t_{A/B} = \frac{1}{24}w_oL^4 - \frac{5}{48}w_oL^4 - \frac{1}{1920}w_oL^4$

$EI\, t_{A/B} = -\frac{121}{1920}w_oL^4$

Therefore
$EI\, \delta_{max} = \frac{121}{1920}w_oL^4$ *answer*

Problem 648
For the cantilever beam loaded as shown in Fig. P-648, determine the deflection at a distance x from the support.

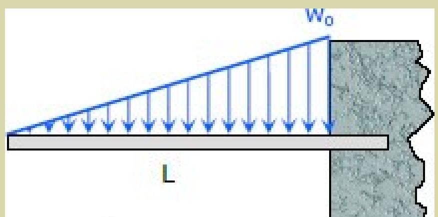


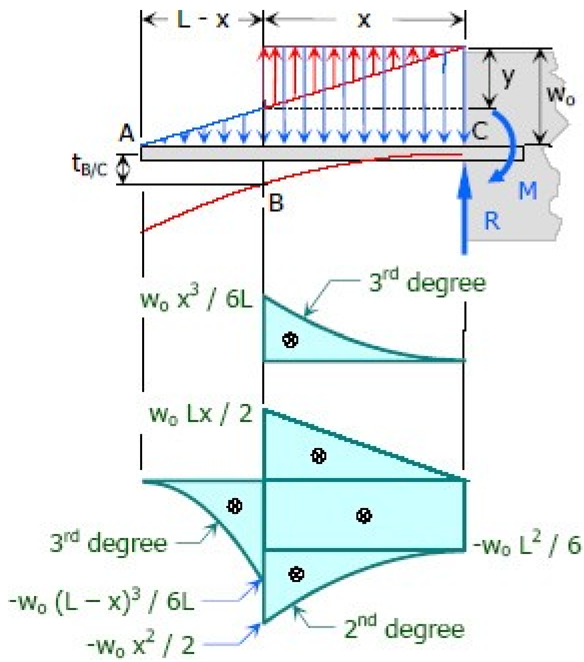
Figure P-648

Solution 648

HideClick here to show or hide the solution

$\frac{y}{x} = \frac{w_o}{L}$

$$y = \frac{w_o}{L}x$$



$$M = \tfrac{1}{2}L(w_o)(\tfrac{1}{3}L) = \tfrac{1}{6}w_oL^2$$

$$R = \tfrac{1}{2}w_oL$$

Moments about B:

Triangular force to the left of B:

$$M_1 = -\tfrac{1}{2}(L-x)(w_o-y)(\tfrac{1}{3})(L-x)$$

$$M_1 = -\tfrac{1}{6}(L-x)^2\left(w_o - \frac{w_ox}{L}\right)$$

$$M_1 = -\frac{w_o(L-x)^3}{6L}$$

Triangular upward force:

$$M_2 = \tfrac{1}{2}(xy)(\tfrac{1}{3}x) = \tfrac{1}{6}x^2\frac{w_ox}{L}$$

$$M_2 = \frac{w_ox^3}{6L}$$

Rectangle (w_o by x):

$$M_3 = -w_ox(\tfrac{1}{2}x) = -\tfrac{1}{2}w_ox^2$$

Reactions R and M:

$$M_4 = Rx = \tfrac{1}{2}w_oLx$$

$$M_5 = -M = -\tfrac{1}{6}w_oL^2$$

Deviation at B with the tangent line through C

$$EI\,t_{B/C} = (Area_{BC})\,\bar{X}_B$$

$$EI\,t_{B/C} = \tfrac{1}{4}x\left(\frac{w_ox^3}{6L}\right)(\tfrac{1}{5}x)+\tfrac{1}{2}x(\tfrac{1}{2}w_oLx)(\tfrac{1}{3}x)-(\tfrac{1}{6}w_oL^2)\,x(\tfrac{1}{2}x)-\tfrac{1}{3}x(\tfrac{1}{2}w_ox^2)(\tfrac{1}{4}x)$$

$$EI\,t_{B/C} = \frac{w_o}{120L}x^5 + \frac{w_o L}{12}x^3 - \frac{w_o L^2}{12}x^2 - \frac{w_o}{24}x^4$$

$$EI\,t_{B/C} = \frac{w_o x^2}{120L}(x^3 + 10L^2 x - 10L^3 - 5Lx^2)$$

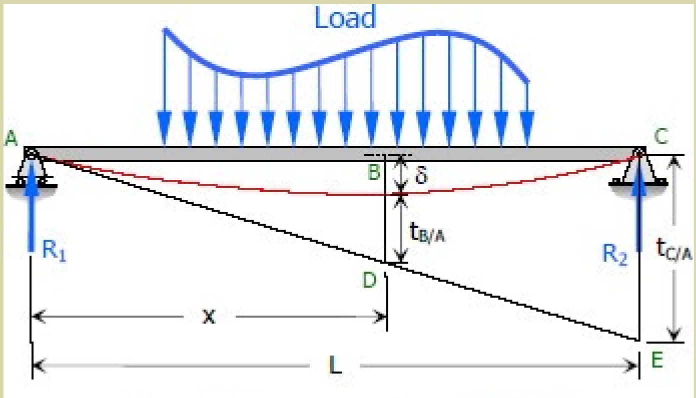
Therefore,

$$EI\,\delta = -\frac{w_o x^2}{120L}(x^3 + 10L^2 x - 10L^3 - 5Lx^2)$$

$$EI\,\delta = \frac{w_o x^2}{120L}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$ *answer*

-Deflections in Simply Supported Beams | Area-Moment Method

The deflection δ at some point B of a simply supported beam can be obtained by the following steps:



Geometry of area-moment method for finding deformation δ in simply supported beam

1. Compute $t_{C/A} = \frac{1}{EI}(Area_{AC})\,\bar{X}_C$
2. Compute $t_{B/A} = \frac{1}{EI}(Area_{AB})\,\bar{X}_B$
3. Solve δ by ratio and proportion (see figure above).

$$\frac{\delta + t_{B/A}}{x} = \frac{t_{C/A}}{L}$$

Problem 653

Compute the midspan value of EIδ for the beam shown in Fig. P-653. (Hint: Draw the M diagram by parts, starting from midspan toward the ends. Also take advantage of symmetry to note that the tangent drawn to the elastic curve at midspan is horizontal.)

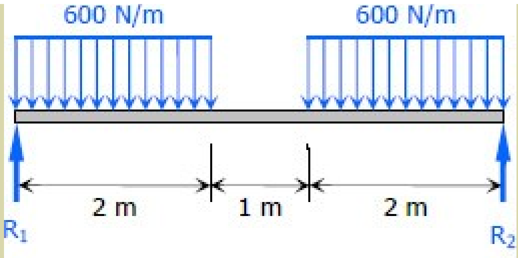
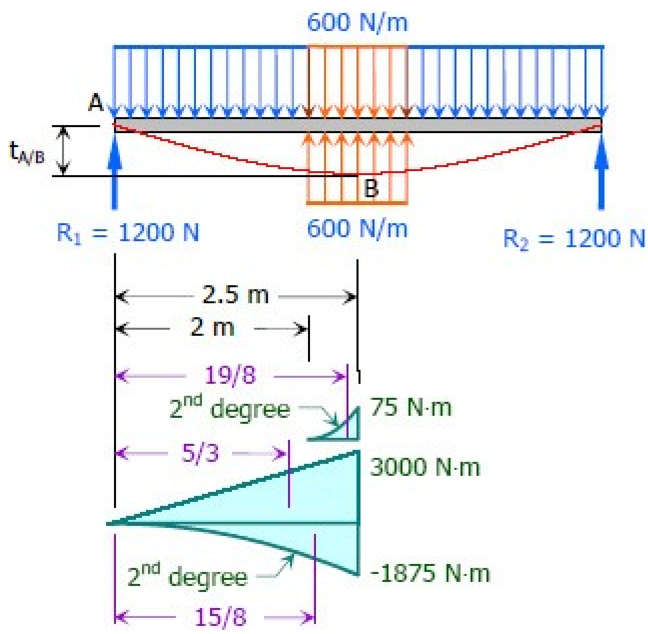


Figure P-653

Solution 653

HideClick here to show or hide the solution

By symmetry:
$R_1 = R_2 = 600(2) = 1200\,\text{N}$



$$t_{A/B} = \frac{1}{EI}(Area_{AB})\,\bar{X}_A$$

$$t_{A/B} = \frac{1}{EI}[\,\tfrac{1}{2}(2.5)(3000)(\tfrac{5}{3}) + \tfrac{1}{3}(0.5)(75)(\tfrac{19}{8}) - \tfrac{1}{3}(2.5)(1875)(\tfrac{15}{8})\,]$$

$$t_{A/B} = \frac{3350}{EI}$$

From the figure
$\delta_{midspan} = t_{A/B}$

Thus
$EI\,\delta_{midspan} = 3350\ \text{N}\cdot\text{m}^3$ *answer*

Problem 654

For the beam in Fig. P-654, find the value of EIδ at 2 ft from R_2. (Hint: Draw the reference tangent to the elastic curve at R_2.)

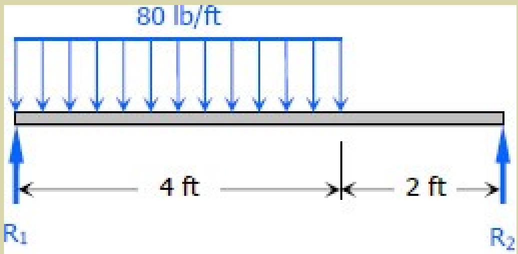


Figure P-654

Solution 654

HideClick here to show or hide the solution

$\Sigma M_{R2} = 0$

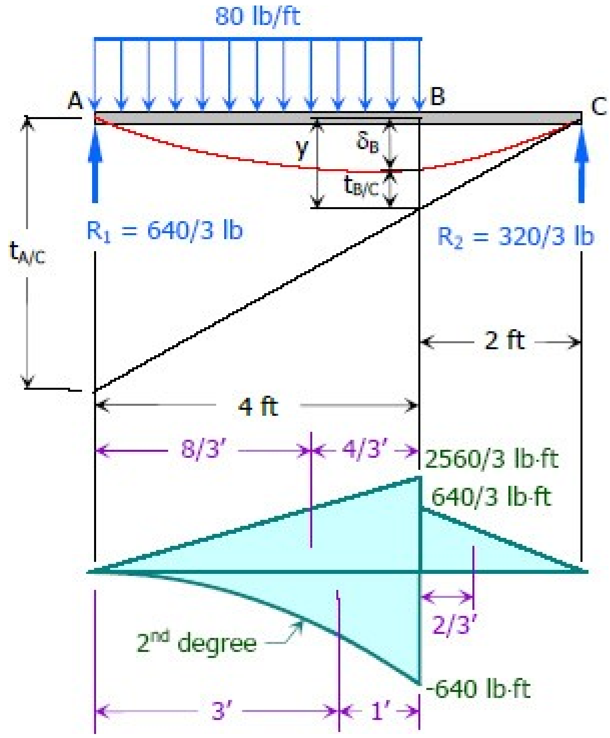
$6R_1 = 80(4)(4)$

$R_1 = \frac{640}{3}\ \text{lb}$

$\Sigma M_{R1} = 0$

$6R_2 = 80(4)(2)$

$R_2 = \frac{320}{3}\ \text{lb}$



$$t_{A/C} = \frac{1}{EI}(Area_{AC})\,\bar{X}_A$$

$$t_{A/C} = \frac{1}{EI}[\,\frac{1}{2}(4)(2560/3)(\frac{8}{3}) + \frac{1}{2}(2)(\frac{640}{3})(4+\frac{2}{3}) - \frac{1}{3}(4)(640)(3)\,]$$

$$t_{A/C} = \frac{8960}{3EI}$$

$$t_{B/C} = \frac{1}{EI}(Area_{BC})\,\bar{X}_B$$

$$t_{B/C} = \frac{1}{EI}[\,\frac{1}{2}(2)(\frac{640}{3})(\frac{2}{3})\,]$$

$$t_{B/C} = \frac{1280}{9EI}$$

By ratio and proportion:

$$\frac{y}{2} = \frac{t_{A/C}}{6}$$

$$y = \frac{2}{6}\left(\frac{8960}{3EI}\right)$$

$$y = \frac{8960}{9EI}$$

$$\delta_B = y - t_{B/C}$$

$$\delta_B = \frac{8960}{9EI} - \frac{1280}{9EI}$$

$$\delta_B = \frac{2560}{3EI}$$

$EI\,\delta_B = \frac{2560}{3}\ \text{lb}\cdot\text{ft}^3$ *answer*

Problem 655

Find the value of EIδ under each concentrated load of the beam shown in Fig. P-655.

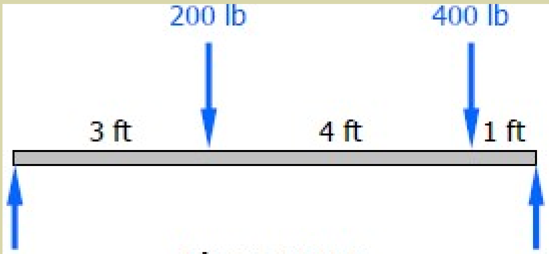


Figure P-655

Solution 655

HideClick here to show or hide the solution

$$\Sigma M_{R2} = 0$$

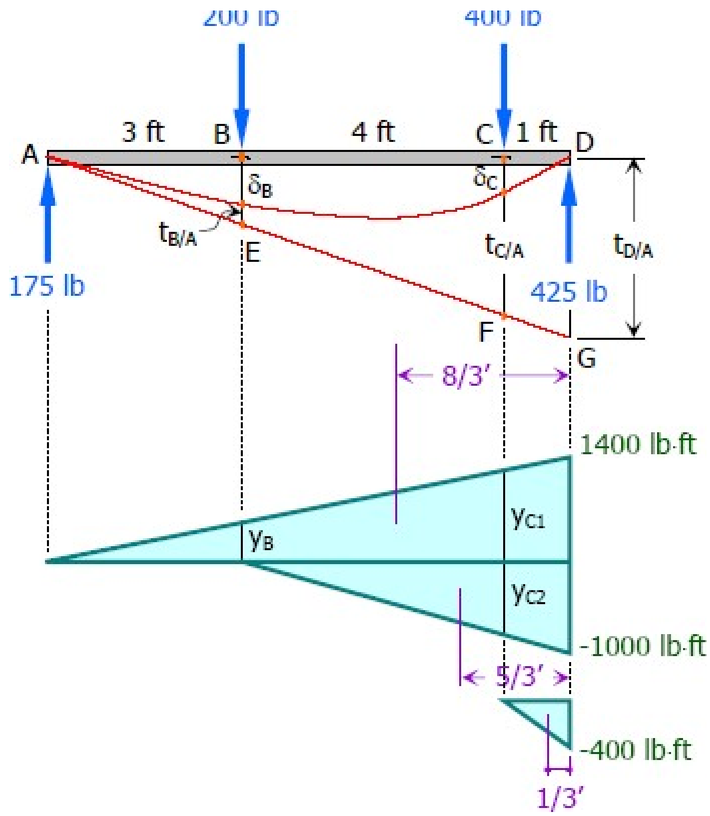
$$8R_1 = 200(5) + 400(1)$$

$$R_1 = 175\ \text{lb}$$

$$\Sigma M_{R1} = 0$$

$$8R_2 = 200(3) + 400(7)$$

$$R_2 = 425 \text{ lb}$$



$$\frac{y_{C1}}{7} = \frac{1400}{8}$$

$$y_{C1} = 1225 \text{ lb}$$

$$\frac{y_{C2}}{4} = \frac{-1000}{5}$$

$$y_{C2} = -800 \text{ lb}$$

$$\frac{y_B}{3} = \frac{1400}{8}$$

$y_B = 525\ \text{lb}$

$EI\,t_{D/A} = (Area_{AD})\,\bar{X}_D$

$EI\,t_{D/A} = \frac{1}{2}(8)(1400)(\frac{8}{3}) - \frac{1}{2}(5)(1000)(\frac{5}{3}) - \frac{1}{2}(1)(400)(\frac{1}{3})$

$EI\,t_{D/A} = 10\,700\ \text{lb}\cdot\text{ft}^3$

$EI\,t_{C/A} = (Area_{AC})\,\bar{X}_C$

$EI\,t_{C/A} = \frac{1}{2}(7)(y_{C1})(\frac{7}{3}) - \frac{1}{2}(4)(y_{C2})(\frac{4}{3})$

$EI\,t_{C/A} = \frac{1}{2}(7)(1225)(\frac{7}{3}) - \frac{1}{2}(4)(800)(\frac{4}{3})$

$EI\,t_{C/A} = \frac{47\,225}{6}\ \text{lb}\cdot\text{ft}^3$

$EI\,t_{B/A} = (Area_{AB})\,\bar{X}_B$

$EI\,t_{C/A} = \frac{1}{2}(3)(y_B)(1)$

$EI\,t_{C/A} = \frac{1}{2}(3)(525)(1)$

$EI\,t_{C/A} = \frac{1575}{2}\ \text{lb}\cdot\text{ft}^3$

By ratio and proportion:

$$\frac{\bar{BE}}{3} = \frac{\bar{CF}}{7} = \frac{t_{D/A}}{8}$$

$\bar{BE} = \frac{3}{8}t_{D/A} = \frac{3}{8}(10\,700) = \frac{8025}{2}$

$\bar{CF} = \frac{7}{8}t_{D/A} = \frac{7}{8}(10\,700) = \frac{18\,725}{2}$

Deflections:

$\delta_B = \bar{BE} - t_{B/A}$

$EI\,\delta_B = EI\,\bar{BE} - EI\,t_{B/A} = \frac{8025}{2} - \frac{1575}{2}$

$EI\,\delta_B = 3225\ \text{lb}\cdot\text{ft}^3$ $\rightarrow$ *answer*

$\delta_C = C\bar{F} - t_{C/A}$

$EI\,\delta_C = EI\,C\bar{F} - EI\,t_{C/A} = \frac{18\,725}{2} - \frac{47\,225}{6}$

$EI\,\delta_C = \frac{4475}{3} = 1491.67\ \text{lb}\cdot\text{ft}^3$ *answer*

Problem 656

Find the value of EIδ at the point of application of the 200 N·m couple in Fig. P-656.

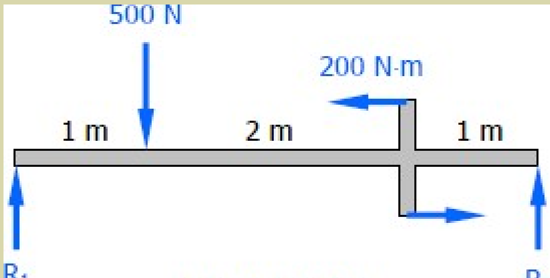


Figure P-656

Solution 656

HideClick here to show or hide the solution

$\Sigma M_{R2} = 0$

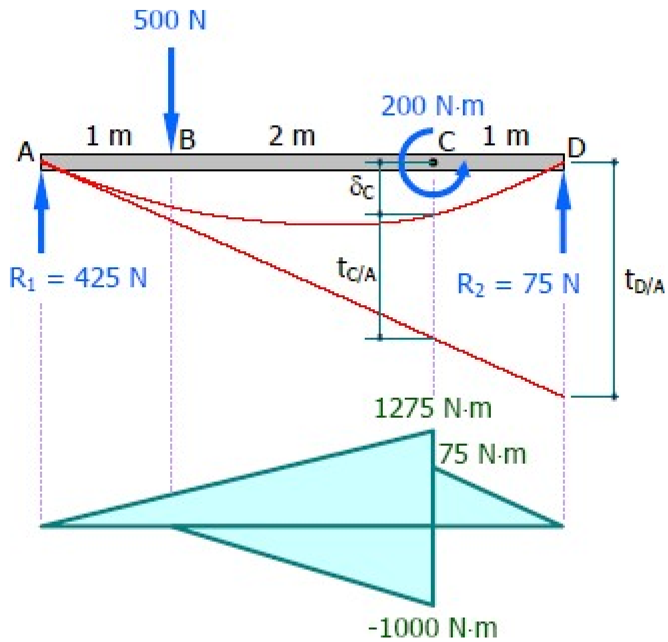
$4R_1 = 500(3) + 200$

$R_1 = 425\ \text{N}$

$\Sigma M_{R1} = 0$

$4R_2 + 200 = 500(1)$

$R_2 = 75\ \text{N}$



$$EI\,t_{D/A} = (Area_{AD})\,\bar{X}_D$$

$$EI\,t_{D/A} = \tfrac{1}{2}(1)(75)(\tfrac{2}{3}) + \tfrac{1}{2}(3)(1275)(2) - \tfrac{1}{2}(2)(1000)(\tfrac{5}{3})$$

$$EI\,t_{D/A} = \tfrac{6550}{3}\ \text{N}\cdot\text{m}^3$$

$$EI\,t_{C/A} = (Area_{AC})\,\bar{X}_C$$

$$EI\,t_{C/A} = \tfrac{1}{2}(3)(1275)(1) - \tfrac{1}{2}(2)(1000)(\tfrac{2}{3})$$

$$EI\,t_{C/A} = \tfrac{7475}{6}\ \text{N}\cdot\text{m}^3$$

$$\frac{\bar{CE}}{3} = \frac{t_{D/A}}{4}$$

$$\bar{CE} = \frac{3}{4}\left(\frac{6550}{3EI}\right) = \frac{3275}{2EI}$$

$$EI\,\bar{CE} = \tfrac{3275}{2}\ \text{N}\cdot\text{m}^3$$

$\delta_C = \bar{CE} - t_{C/A}$

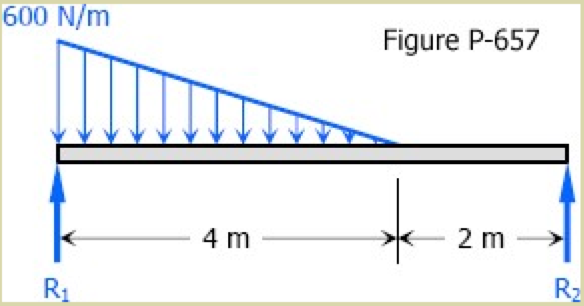
$EI\,\delta_C = EI\,\bar{CE} - EI\,t_{C/A}$

$EI\,\delta_C = \frac{3275}{2} - \frac{7475}{6} = \frac{1175}{3}$

$EI\,\delta_C = 391.67\ \text{N}\cdot\text{m}^3$ *answer*

Problem 657

Determine the midspan value of EIδ for the beam shown in Fig. P-657.



Solution 657

HideClick here to show or hide the solution

$\Sigma M_{R1} = 0$

$$6R_2 = \frac{1}{2}(4)(600)(\frac{4}{3})$$

$$R_2 = \frac{800}{3} \text{ N}$$

$$EI\, t_{A/B} = (Area_{AB})\,\bar{X}_A$$

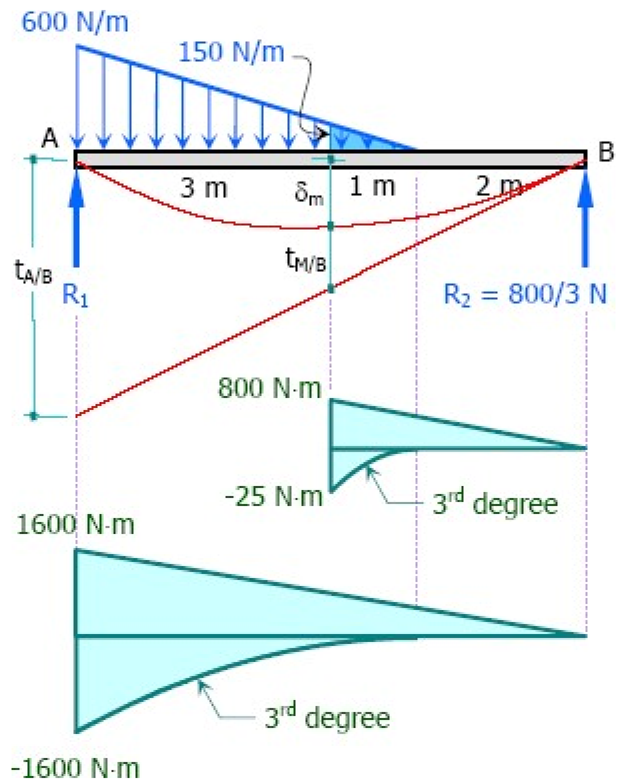
$$EI\, t_{A/B} = \frac{1}{2}(6)(1600)(2) - \frac{1}{4}(4)(1600)(\frac{4}{5})$$

$$EI\, t_{A/B} = 8320\,\text{N}\cdot\text{m}^3$$

$$EI\, t_{M/B} = (Area_{MB})\,\bar{X}_M$$

$$EI\, t_{M/B} = \frac{1}{2}(3)(800)(1) - \frac{1}{4}(1)(25)(\frac{1}{5})$$

$$EI\, t_{M/B} = 1198.75\,\text{N}\cdot\text{m}^3$$



By ratio and proportion:

$$\frac{\delta_m + t_{M/B}}{3} = \frac{t_{A/B}}{6}$$

$$\delta_m + t_{M/B} = \frac{1}{2}t_{A/B}$$

$$EI\,\delta_m + EI\,t_{M/B} = EI\,\frac{1}{2}t_{A/B}$$

$$EI\,\delta_m + 1198.75 = EI\,\frac{1}{2}(8320)$$

$$EI\,\delta_m = 2961.25\,\text{N}\cdot\text{m}^3$$ *answer*

Problem 658

For the beam shown in Fig. P-658, find the value of EIδ at the point of application of the couple.

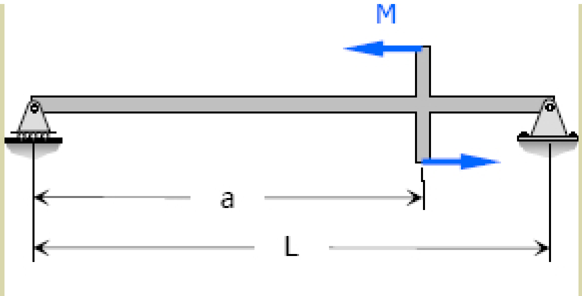
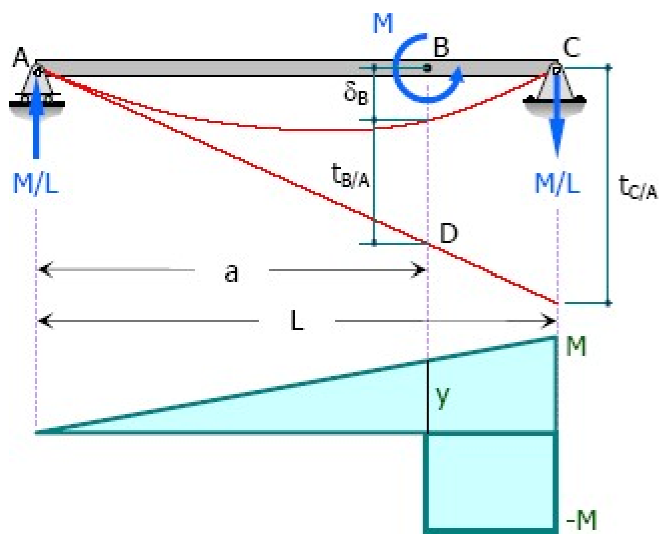


Figure P-658

Solution 658

HideClick here to show or hide the solution



$$\frac{y}{a} = \frac{M}{L}$$

$$y = Ma/L$$

$$EI\,t_{B/A} = (Area_{AB})\,\bar{X}_B$$

$$EI\,t_{B/A} = \tfrac{1}{2}(ay)(\tfrac{1}{3}a)$$

$$EI\,t_{B/A} = \tfrac{1}{6}a^2(Ma/L)$$

$$EI\,t_{B/A} = \frac{Ma^3}{6L}$$

$$EI\,t_{C/A} = (Area_{AC})\,\bar{X}_C$$

$$EI\,t_{C/A} = \tfrac{1}{2}(LM)(\tfrac{1}{3}L) - M(L-a)[\,\tfrac{1}{2}(L-a)\,]$$

$$EI\,t_{C/A} = \tfrac{1}{6}ML^2 - \tfrac{1}{2}M(L-a)^2$$

By ratio and proportion:

$$\frac{\delta_B + t_{B/A}}{a} = \frac{t_{C/A}}{L}$$

$$\delta_B = \frac{a}{L}t_{C/A} - t_{B/A}$$

$$EI\,\delta_B = \frac{a}{L}EI\,t_{C/A} - EI\,t_{B/A}$$

$$EI\,\delta_B = \frac{a}{L}[\,\tfrac{1}{6}ML^2 - \tfrac{1}{2}M(L-a)^2\,] - \frac{Ma^3}{6L}$$

$$EI\,\delta_B = \frac{a}{L}[\,\tfrac{1}{6}ML^2 - \tfrac{1}{2}M(L-a)^2 - \tfrac{1}{6}Ma^2\,]$$

$$EI\,\delta_B = \frac{Ma}{6L}[\,L^2 - 3(L-a)^2 - a^2\,]$$

$$EI\,\delta_B = \frac{Ma}{6L}[\,L^2 - 3(L^2 - 2La + a^2) - a^2\,]$$

$$EI\,\delta_B = \frac{Ma}{6L}[\,L^2 - 3L^2 + 6La - 3a^2 - a^2\,]$$

$$EI\,\delta_B = \frac{Ma}{6L}[\,-3L^2 + 6La - 4a^2\,]$$

$$EI\,\delta_B = -\frac{Ma}{6L}(3L^2 - 6La + 4a^2)$$

The negative sign indicates that the deflection is opposite to the direction sketched in the figure. Thus,

$$EI\,\delta_B = \frac{Ma}{6L}(3L^2 - 6La + 4a^2)$$ upward *answer*

Problem 659

A simple beam supports a concentrated load placed anywhere on the span, as shown in Fig. P-659. Measuring x from A, show that the maximum deflection occurs at x = $\sqrt{[(L^2 - b^2)/3]}$.

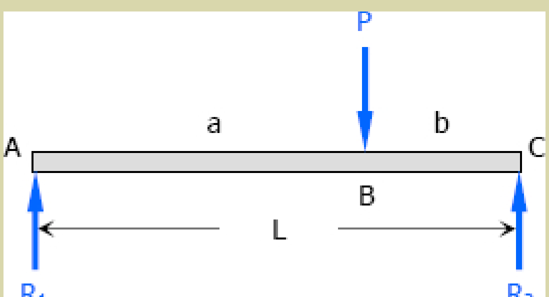
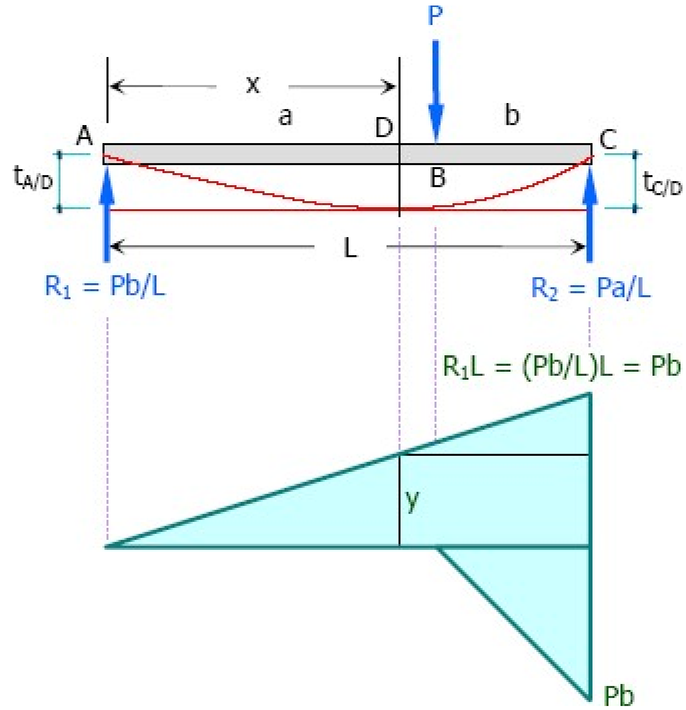


Figure P-659

Solution 659

HideClick here to show or hide the solution



$\Sigma M_{R2} = 0$

$LR_1 = Pb$

$R_1 = Pb/L$

$\Sigma M_{R1} = 0$

$LR_2 = Pa$

$R_2 = Pa/L$

$\dfrac{y}{x} = \dfrac{Pb}{L}$

$y = \dfrac{Pb}{L}x$

$t_{A/D} = \dfrac{1}{EI}(Area_{AD})\,\bar{X}_A$

$t_{A/D} = \dfrac{1}{EI}[\,\frac{1}{2}xy(\frac{2}{3}x\,]$

$t_{A/D} = \dfrac{1}{EI}[\,\frac{1}{3}x^2y\,]$

$t_{A/D} = \dfrac{1}{EI}\left[\,\frac{1}{3}x^2\left(\dfrac{Pb}{L}x\right)\right]$

$t_{A/D} = \dfrac{1}{EI}\dfrac{Pb}{3L}x^3$

$t_{C/D} = \dfrac{1}{EI}(Area_{CD})\,\bar{X}_C$

$t_{C/D} = \dfrac{1}{EI}[\,\frac{1}{6}(L-x)^2(Pb-y) + \frac{1}{2}(L-x)^2y - \frac{1}{6}Pb^3\,]$

$$t_{C/D} = \frac{1}{EI}\left[\tfrac{1}{6}(L-x)^2\left(Pb - \frac{Pb}{L}x\right) + \tfrac{1}{2}(L-x)^2\left(\frac{Pb}{L}x\right) - \tfrac{1}{6}Pb^3\right]$$

$$t_{C/D} = \frac{1}{EI}\left[\tfrac{1}{6}Pb(L-x)^2\left(1 - \frac{x}{L}\right) + \tfrac{1}{2}Pb(L-x)^2\left(\frac{x}{L}\right) - \tfrac{1}{6}Pb^3\right]$$

$$t_{C/D} = \frac{1}{EI}\left[\frac{Pb}{6L}(L-x)^3 + \frac{Pb}{2L}(L-x)^2 x - \frac{Pb^3}{6}\right]$$

From the figure:
$t_{A/D} = t_{C/D}$

$$\frac{1}{EI}\frac{Pb}{3L}x^3 = \frac{1}{EI}\left[\frac{Pb}{6L}(L-x)^3 + \frac{Pb}{2L}(L-x)^2 x - \frac{Pb^3}{6}\right]$$

$$\frac{Pb}{3L}x^3 = \frac{Pb}{6L}(L-x)^3 + \frac{Pb}{2L}(L-x)^2 x - \frac{Pb^3}{6}$$

$$\frac{2x^3}{L} = \frac{(L-x)^3}{L} + \frac{3(L-x)^2 x}{L} - b^2$$

$$2x^3 = (L-x)^3 + 3(L-x)^2 x - Lb^2$$

$$2x^3 = (L^3 - 3L^2x + 3Lx^2 - x^3) + 3(L^2 - 2Lx + x^2)x - Lb^2$$

$$2x^3 = L^3 - 3L^2x + 3Lx^2 - x^3 + 3L^2x - 6Lx^2 + 3x^3 - Lb^2$$

$$0 = L^3 - 3Lx^2 - Lb^2$$

$$0 = L^2 - 3x^2 - b^2$$

$$3x^2 = L^2 - b^2$$

$$x = \sqrt{\frac{L^2 - b^2}{3}} \quad (okay!)$$

Problem 660

A simply supported beam is loaded by a couple M at its right end, as shown in Fig. P-660. Show that the maximum deflection occurs at x = 0.577L.

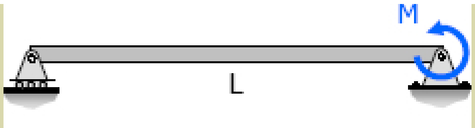


Figure P-660

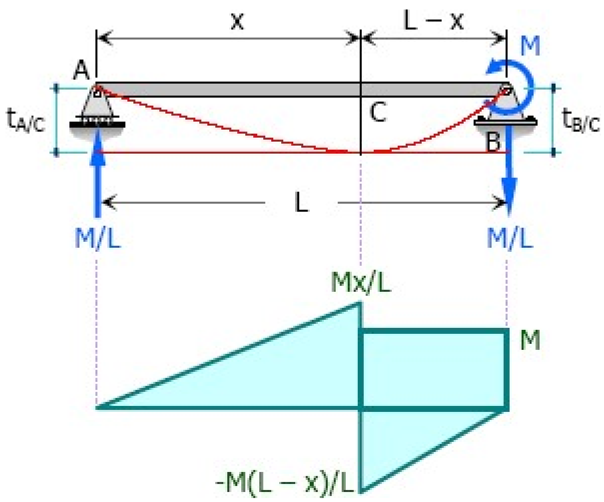
Solution 660

HideClick here to show or hide the solution

$$EI\,t_{A/C} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/C} = \tfrac{1}{2}x\left(\dfrac{Mx}{L}\right)(\tfrac{2}{3}x)$$

$$EI\,t_{A/C} = \dfrac{Mx^3}{3L}$$



$$EI\,t_{B/C} = (Area_{BC})\,\bar{X}_B$$

$$EI\,t_{B/C} = M(L-x)\tfrac{1}{2}(L-x) - \tfrac{1}{2}(L-x)\dfrac{M}{L}(L-x)\tfrac{2}{3}(L-x)$$

$$EI\,t_{B/C} = \tfrac{1}{2}M(L-x)^2 - \dfrac{M}{3L}(L-x)^3$$

$$EI\, t_{B/C} = \frac{M}{6L}(L - x)^2 [\, 3L - 2(L - x)\,]$$

$$EI\, t_{B/C} = \frac{M}{6L}(L - x)^2 (L + 2x)$$

$$EI\, t_{B/C} = \frac{M}{6}(L - x)^2 + \frac{Mx}{3L}(L - x)^2$$

$$EI\, t_{B/C} = \frac{M}{6}(L^2 - 2Lx + x^2) + \frac{Mx}{3L}(L^2 - 2Lx + x^2)$$

$$EI\, t_{B/C} = \frac{ML^2}{6} - \frac{MLx}{3} + \frac{Mx^2}{6} + \frac{MLx}{3} - \frac{2Mx^2}{3} + \frac{Mx^3}{3L}$$

$$EI\, t_{B/C} = \frac{ML^2}{6} - \frac{Mx^2}{2} + \frac{Mx^3}{3L}$$

From the figure
$EI\, t_{A/C} = EI\, t_{B/C}$

$$\frac{Mx^3}{3L} = \frac{ML^2}{6} - \frac{Mx^2}{2} + \frac{Mx^3}{3L}$$

$$\frac{Mx^2}{2} = \frac{ML^2}{6}$$

$x^2 = \frac{1}{3}L^2$

$x = 0.577L$ (*okay!*)

Problem 661

Compute the midspan deflection of the symmetrically loaded beam shown in Fig. P-661. Check your answer by letting a = L/2 and comparing with the answer to Problem 609.

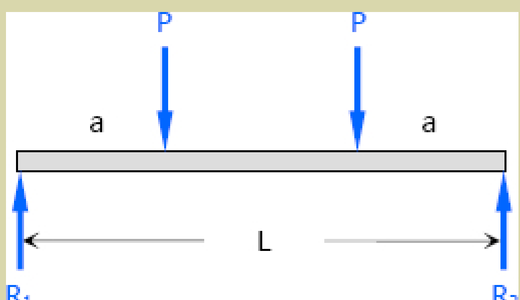
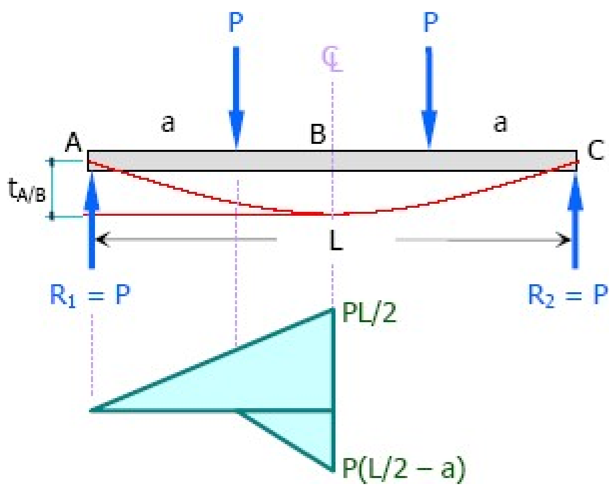


Figure P-661

Solution 661

HideClick here to show or hide the solution

$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$



$EI\,t_{A/B} = \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}PL)[\,\frac{2}{3}(\frac{1}{2})] - \frac{1}{2}(\frac{1}{2}L - a)P(\frac{1}{2}L - a)[\,a + \frac{2}{3}(\frac{1}{2}L - a)\,]$

$EI\,t_{A/B} = \frac{1}{24}PL^3 - \frac{1}{2}Pa(\frac{1}{2}L - a)^2 - \frac{1}{3}P(\frac{1}{2}L - a)^3$

$EI\,t_{A/B} = \frac{1}{24}PL^3 - \frac{1}{2}Pa(\frac{1}{4}L^2 - La + a^2) - \frac{1}{3}P(\frac{1}{8}L^3 - \frac{3}{4}L^2a + \frac{3}{2}La^2 - a^3)$

$EI\,t_{A/B} = \frac{1}{24}PL^3 - \frac{1}{8}PL^2a + \frac{1}{2}PLa^2 - \frac{1}{8}Pa^3 - \frac{1}{24}PL^3 + \frac{1}{4}PL^2a - \frac{1}{2}PLa^2 + \frac{1}{3}Pa^3$

$EI\,t_{A/B} = \frac{1}{8}PL^2a - \frac{1}{6}Pa^3$

$EI\,t_{A/B} = \frac{1}{24}Pa(3L^2 - 4a^2)$ *answer*

When a = ½L

$EI\,t_{A/B} = \frac{1}{24}P(\frac{1}{2}L)[\,3L^2 - 4(\frac{1}{2}L)^2\,]$

$EI\,t_{A/B} = \frac{1}{48}PL[\,3L^2 - L^2\,]$

$$EI\,t_{A/B} = \frac{1}{48}PL(2L^2)$$

$$EI\,t_{A/B} = \frac{1}{24}PL^3 \qquad \rightarrow answer$$

$$EI\,t_{A/B} = 2(\frac{1}{48}PL^3) \quad (okay!)$$

Problem 662

Determine the maximum deflection of the beam shown in Fig. P-662. Check your result by letting a = L/2 and comparing with case 8 in Table 6-2. Also, use your result to check the answer to Prob. 653.

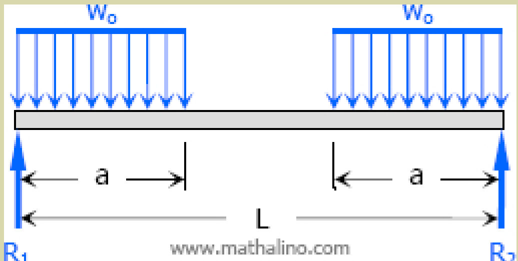
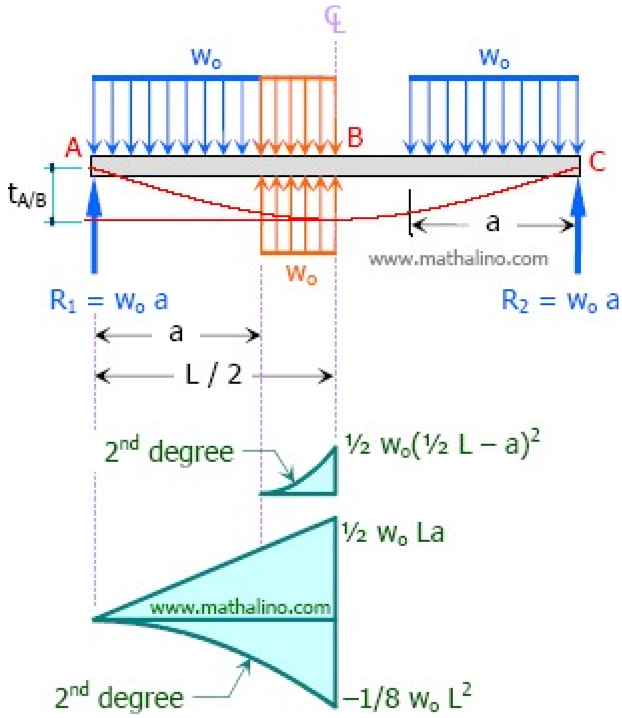


Figure P-662

Solution 662

HideClick here to show or hide the solution



$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$

$EI\,t_{A/B} = \frac{1}{3}(\frac{1}{2}L - a)[\,\frac{1}{2}w_o(\frac{1}{2}L - a)^2\,][\,a + \frac{3}{4}(\frac{1}{2}L - a)\,] + \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}w_oLa)(\frac{1}{3}L) - \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_oL^2)(\frac{3}{8}L)$

$EI\,t_{A/B} = \frac{1}{6}w_oa(\frac{1}{2}L - a)^3 + \frac{1}{8}w_o(\frac{1}{2}L - a)^4 + \frac{1}{24}w_oL^3a - \frac{1}{128}w_oL^4$

$EI\,t_{A/B} = \frac{1}{6}w_oa[\,\frac{1}{2}(L - 2a)\,]^3 + \frac{1}{8}w_o[\,\frac{1}{2}(L - 2a)\,]^4 + \frac{1}{24}w_oL^3a - \frac{1}{128}w_oL^4$

$EI\,t_{A/B} = \frac{1}{48}w_oa(L - 2a)^3 + \frac{1}{128}w_o(L - 2a)^4 + \frac{1}{24}w_oL^3a - \frac{1}{128}w_oL^4$

$EI\,t_{A/B} = \frac{1}{48}w_oa[\,L^3 - 3L^2(2a) + 3L(2a)^2 - (2a)^3\,] + \frac{1}{128}w_o[\,L^4 - 4L^3(2a) + 6L^2(2a)^2 - 4L(2a)^3 + (2a)^4\,] + \frac{1}{24}w_oL^3a - \frac{1}{128}w_oL^4$

$EI\,t_{A/B} = \frac{1}{48}w_oL^3a - \frac{1}{8}w_oL^2a^2 + \frac{1}{4}w_oLa^3 - \frac{1}{6}w_oa^4 + \frac{1}{128}w_oL^4 - \frac{1}{16}w_oL^3a + \frac{3}{16}w_oL^2a^2 - \frac{1}{4}w_oLa^3 + \frac{1}{8}w_oa^4 + \frac{1}{24}w_oL^3a - \frac{1}{128}w_oL^4$

$EI\,t_{A/B} = \frac{1}{16}w_oL^2a^2 - \frac{1}{24}w_oa^4$

$EI\,t_{A/B} = \frac{1}{48}w_oa^2(3L^2 - 2a^2)$ *answer*

Check Problem 653:
w_o = 600 N/m; L = 5 m; a = 2 m

$$EI\,t_{A/B} = \frac{1}{48}(600)(2^2)[\,3(5^2) - 2(2^2)\,]$$

$$EI\,t_{A/B} = 3350\ \text{ N}\cdot\text{m}^3$$ (*okay!*)

When a = L/2 (the load is over the entire span)

$$EI\,t_{A/B} = \frac{1}{48}w_o(\frac{1}{2}L)^2[\,3L^2 - 2(\frac{1}{2}L)^2\,]$$

$$EI\,t_{A/B} = \frac{1}{192}w_oL^2[\,3L^2 - \frac{1}{2}L^2\,]$$

$$EI\,t_{A/B} = \frac{1}{192}w_oL^2[\,\frac{5}{2}L^2\,]$$

$$EI\,t_{A/B} = \frac{5}{384}w_oL^4$$

Therefore

$$\delta_{max} = \frac{5w_oL^4}{384EI}$$

Problem 663

Determine the maximum deflection of the beam carrying a uniformly distributed load over the middle portion, as shown in Fig. P-663. Check your answer by letting 2b = L.

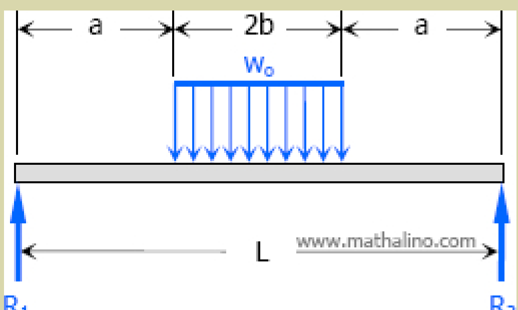
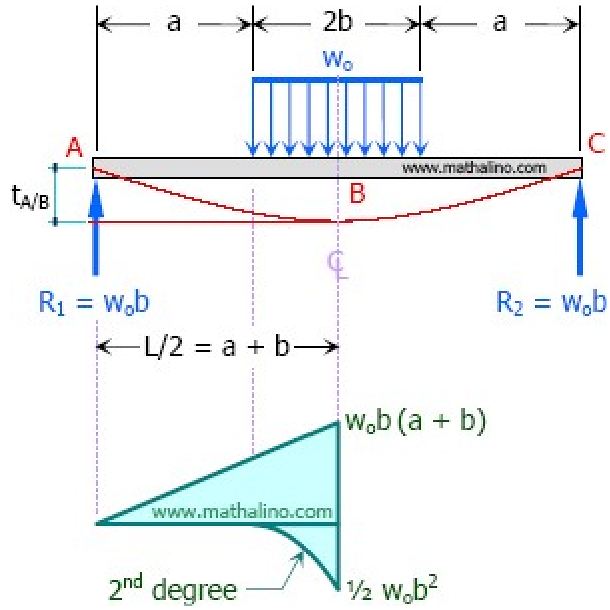


Figure P-663

Solution 663

HideClick here to show or hide the solution



$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = \frac{1}{2}(a+b)[\,w_o b\,(a+b)\,][\,\frac{2}{3}(a+b)\,] - \frac{1}{3}b(\frac{1}{2}w_o b^2)(a+\frac{3}{4}b)$$

$$EI\,t_{A/B} = \frac{1}{3}w_o b(a+b)^3 - \frac{1}{6}w_o b^3(a+\frac{3}{4}b)$$

$$EI\,t_{A/B} = \frac{1}{3}w_o b(a+b)^3 - \frac{1}{24}w_o b^3(4a+3b)$$

$$EI\,t_{A/B} = \frac{1}{3}w_o b(a+b)^3 - \frac{1}{24}w_o b^3[\,(2a+2b)+(a+b)+a\,]$$

$$EI\,t_{A/B} = \frac{1}{3}w_o b(\frac{1}{2}L)^3 - \frac{1}{24}w_o b^3[\,L+\frac{1}{2}L+a\,]$$

$$EI\,t_{A/B} = \frac{1}{24}w_o L^3 b - \frac{1}{24}w_o b^3(\frac{3}{2}L+a)$$

$$EI\,t_{A/B} = \frac{1}{24}w_o L^3 b - \frac{1}{48}w_o b^3(3L+2a)$$

$$EI\,t_{A/B} = \frac{1}{24}w_o L^3 b - \frac{1}{48}w_o b^3[\,3L+(L-2b)\,]$$

$$EI\,t_{A/B} = \frac{1}{24}w_o L^3 b - \frac{1}{48}w_o b^3(4L-2b)$$

$$EI\,t_{A/B} = \frac{1}{24}w_o L^3 b - \frac{1}{24}w_o b^3(2L-b)$$

$$EI\,t_{A/B} = \frac{1}{24}w_o b[\,L^3 - b^2(2L-b)\,]$$

$EI\,t_{A/B} = \frac{1}{24}w_o b(L^3 - 2Lb^2 + b^3)$ *answer*

When 2b = L; b = ½L

$EI\,t_{A/B} = \frac{1}{24}w_o(\frac{1}{2}L)[\,L^3 - 2L(\frac{1}{2}L)^2 + (\frac{1}{2}L)^3\,]$

$EI\,t_{A/B} = \frac{1}{48}w_o L\,[\,L^3 - \frac{1}{2}L^3 + \frac{1}{8}L^3\,]$

$EI\,t_{A/B} = \frac{1}{48}w_o L\,[\,\frac{5}{8}L^3\,]$

$EI\,t_{A/B} = \frac{5}{384}w_o L^4$ (*okay!*)

Problem 664

The middle half of the beam shown in Fig. P-664 has a moment of inertia 1.5 times that of the rest of the beam. Find the midspan deflection. (Hint: Convert the M diagram into an M/EI diagram.)

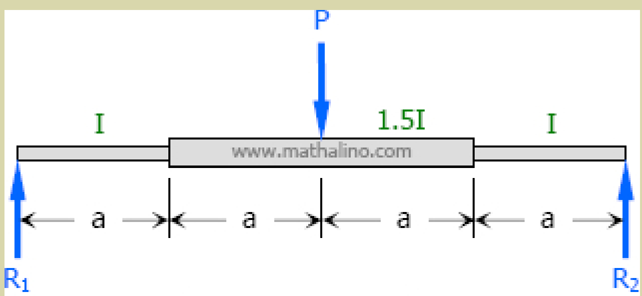
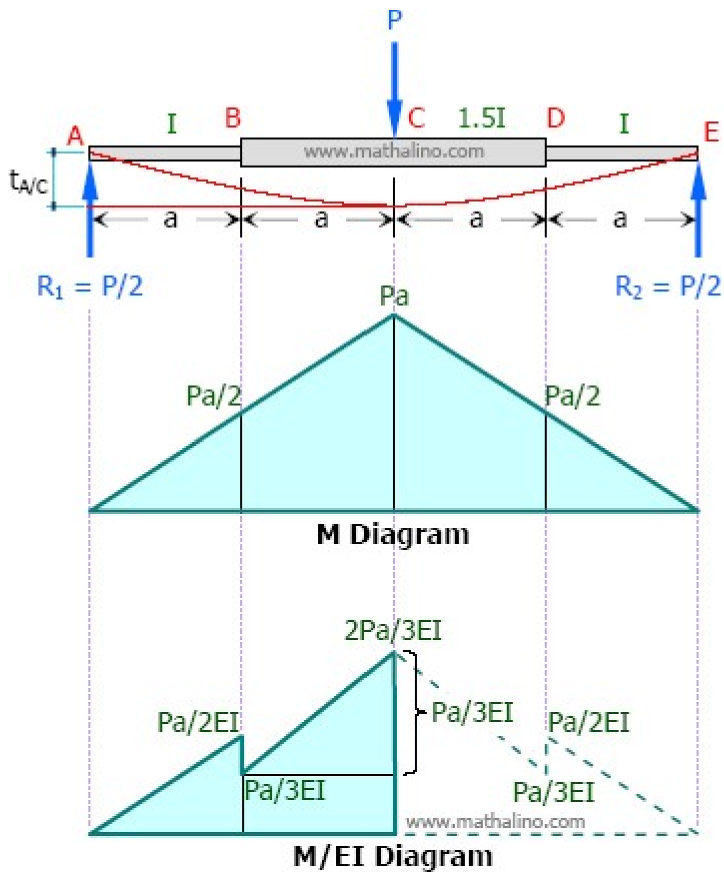


Figure P-664 and P-665

Solution 664

HideClick here to show or hide the solution



$$t_{A/C} = \frac{1}{EI}(Area_{AC})\,\bar{X}_A$$

$$t_{A/C} = \tfrac{1}{2}a\left(\frac{Pa}{2EI}\right)(\tfrac{2}{3}a) + a\left(\frac{Pa}{3EI}\right)(\tfrac{3}{2}a) + \tfrac{1}{2}a\left(\frac{2Pa}{3EI} - \frac{Pa}{3EI}\right)(\tfrac{5}{3}a)$$

$$t_{A/C} = \frac{Pa^3}{6EI} + \frac{Pa^3}{2EI} + \frac{5Pa^3}{18EI}$$

$$t_{A/C} = \frac{17Pa^3}{18EI}$$

Therefore,

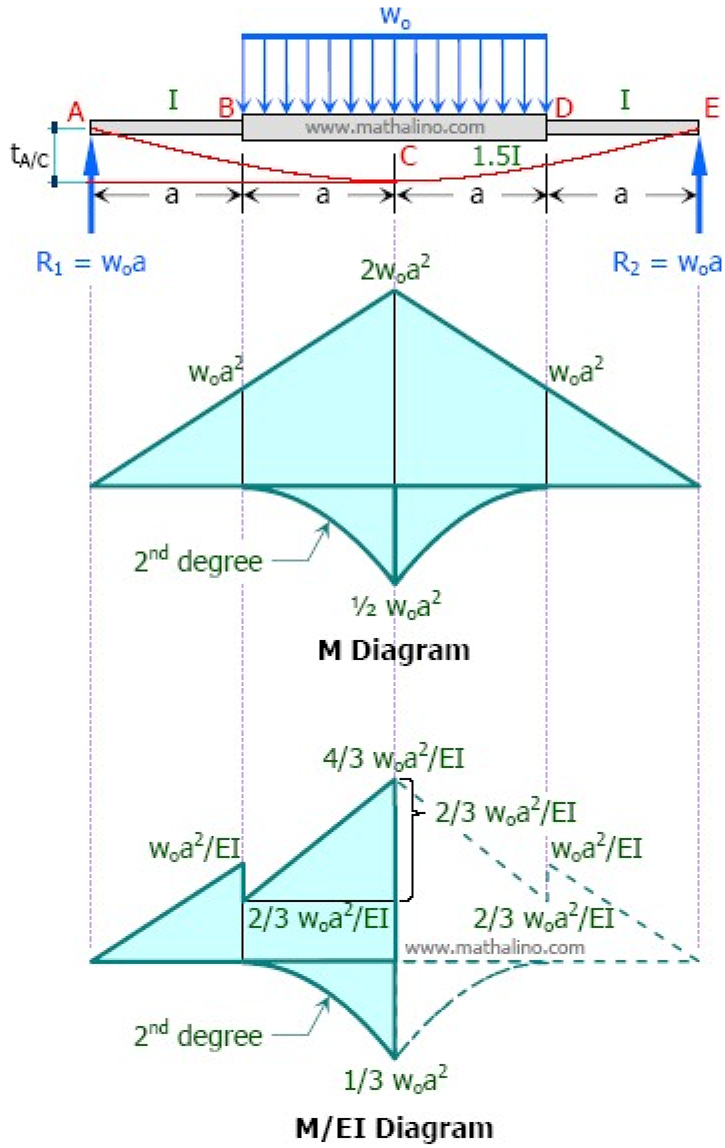
$$\delta_{midspan} = \frac{17Pa^3}{18EI} \qquad \textit{answer}$$

Problem 665

Replace the concentrated load in Prob. 664 by a uniformly distributed load of intensity w_o acting over the middle half of the beam. Find the maximum deflection.

Solution 665

HideClick here to show or hide the solution



$$t_{A/C} = \frac{1}{EI}(Area_{AC})\,\bar{X}_A$$

$$t_{A/C} = \frac{1}{2}a\left(\frac{w_oa^2}{EI}\right)(\frac{2}{3}a) + a\left(\frac{2w_oa^2}{3EI}\right)(\frac{3}{2}a) + \frac{1}{2}a\left(\frac{2w_oa^2}{3EI}\right)(\frac{5}{3}a) - \frac{1}{3}a\left(\frac{w_oa^2}{3EI}\right)(\frac{7}{4}a)$$

$$t_{A/C} = \frac{w_oa^4}{3EI} + \frac{w_oa^4}{EI} + \frac{5w_oa^4}{9EI} - \frac{7w_oa^4}{36EI}$$

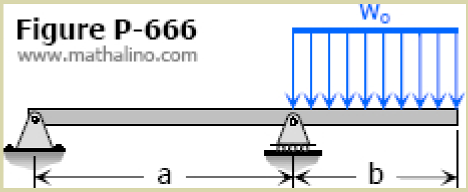
$$t_{A/C} = \frac{61w_oa^4}{36EI}$$

Therefore,

$\delta_{midspan} = \dfrac{61w_oa^4}{36EI}$ *answer*

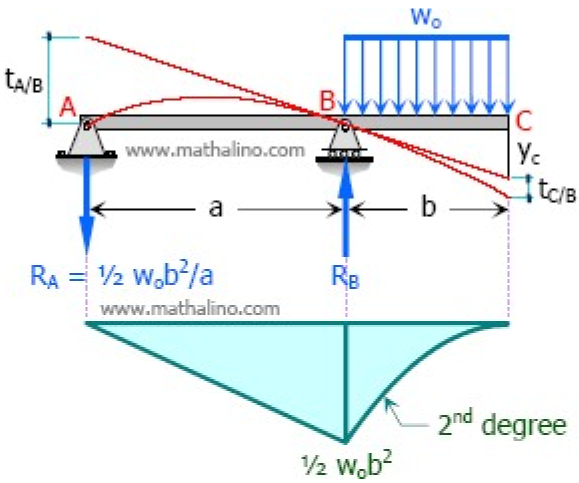
Problem 666

Determine the value of EIδ at the right end of the overhanging beam shown in Fig. P-666.



Solution 666

HideClick here to show or hide the solution



$$\Sigma M_B = 0$$

$$aR_A = w_ob(\frac{1}{2}b)$$

$$R_A = \frac{w_ob^2}{2a}$$

$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = \frac{1}{2}a(\frac{1}{2}w_ob^2)(\frac{2}{3}a)$$

$$EI\,t_{A/B} = \frac{1}{6}w_oa^2b^2$$

$$EI\,t_{C/B} = (Area_{BC})\,\bar{X}_C$$

$$EI\,t_{C/B} = \frac{1}{3}b(\frac{1}{2}w_o b^2)(\frac{3}{4}b)$$

$$EI\,t_{C/B} = \frac{1}{8}w_o b^4$$

$$\frac{y_C}{b} = \frac{t_{A/B}}{a}$$

$$y_C = \frac{b}{a}t_{A/B}$$

$$EI\,y_C = \frac{b}{a}EI\,t_{A/B}$$

$$EI\,y_C = \frac{b}{a}(\frac{1}{6}w_o a^2 b^2)$$

$$EI\,y_C = \frac{1}{6}w_o a b^3$$

$$\delta_C = y_C + t_{C/B}$$

$$EI\,\delta_C = EI\,y_C + EI\,t_{C/B}$$

$$EI\,\delta_C = \frac{1}{6}w_o a b^3 + \frac{1}{8}w_o b^4$$

$$EI\,\delta_C = \frac{1}{24}w_o b^3(4a + 3b) \qquad \textit{answer}$$

Problem 667

Determine the value of EIδ at the right end of the overhanging beam shown in Fig. P-667. Is the deflection up or down?

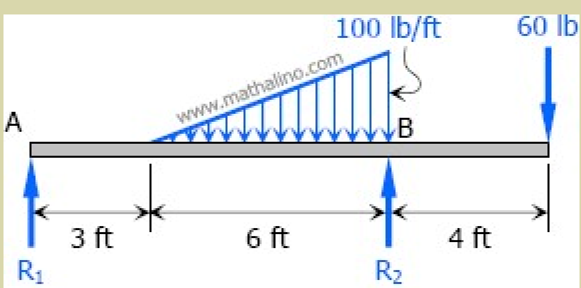


Figure P-667

Solution 667

HideClick here to show or hide the solution

$\Sigma M_{R2} = 0$]

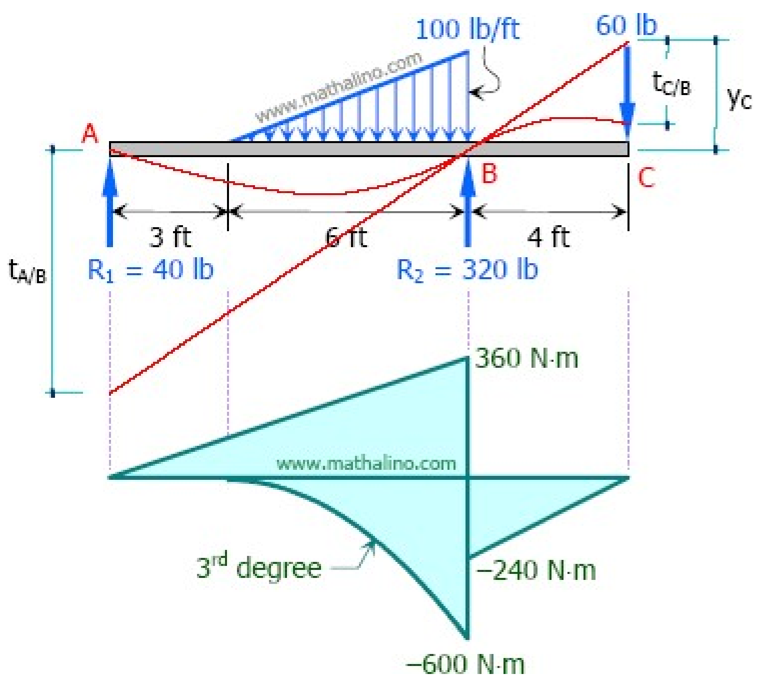
$9R_1 + 4(60) = \frac{1}{2}(6)(100)(2)$

$R_1 = 40 \text{ lb}$

$\Sigma M_{R1} = 0$

$9R_2 = \frac{1}{2}(6)(100)(7) + 13(60)$

$R_2 = 320 \text{ lb}$



$EI\, t_{A/B} = (Area_{AB})\,\bar{X}_A$

$EI\, t_{A/B} = \frac{1}{2}(9)(360)(6) - \frac{1}{4}(6)(600)(\frac{39}{5})$

$$EI\,t_{A/B} = 2700\ \text{ N}\cdot\text{m}^3$$

$$EI\,t_{C/B} = (Area_{BC})\,\bar{X}_C$$

$$EI\,t_{C/B} = -\tfrac{1}{2}(4)(240)(\tfrac{8}{3})$$

$$EI\,t_{C/B} = -1280\ \text{ N}\cdot\text{m}^3$$

The negative sign indicates that the elastic curve is below the tangent line. It is shown in the figure indicated as $t_{C/B}$. See Rules of Sign for Area-Moment Method.

$$\dfrac{y_C}{4} = \dfrac{t_{A/B}}{9}$$

$$y_C = \tfrac{4}{9}t_{A/B}$$

$$EI\,y_C = \tfrac{4}{9}EI\,t_{A/B}$$

$$EI\,y_C = \tfrac{4}{9}(2700)$$

$$EI\,y_C = 1200\ \text{ N}\cdot\text{m}^3$$

Since the absolute value of EI $t_{C/B}$ is greater than the absolute value of EI y_C, the elastic curve is below the undeformed neutral axis (NA) of the beam.

Therefore,

$$EI\,\delta_D = 1280 - 1200$$

$EI\,\delta_D = 80\ \text{ N}\cdot\text{m}^3$ below C (deflection is down) *answer*

Problem 668

For the beam shown in Fig. P-668, compute the value of P that will cause the tangent to the elastic curve over support R_2 to be horizontal. What will then be the value of $EI\delta$ under the 100-lb load?

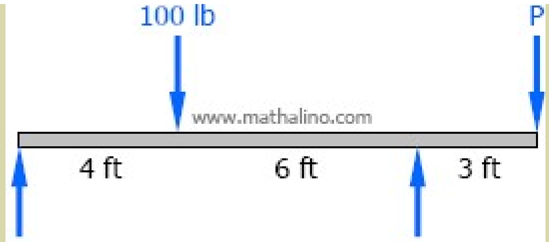


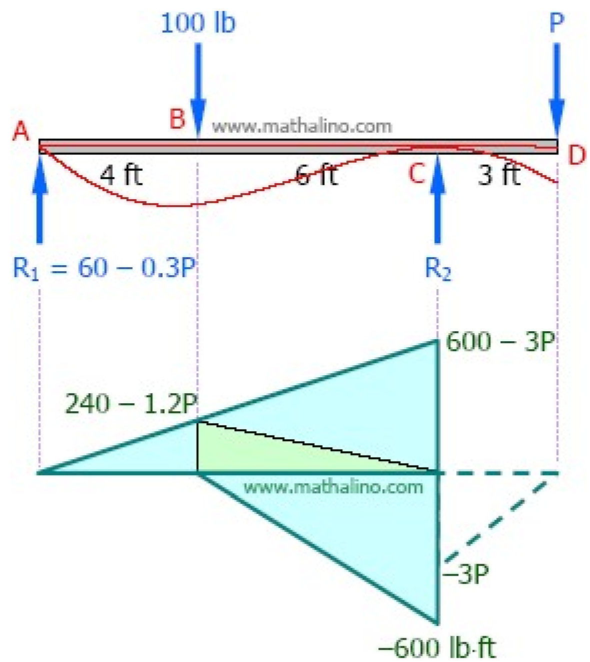
Figure P-668

Solution 668

HideClick here to show or hide the solution

$10R_1 + 3P = 6(100)$

$R_1 = 60 - 0.30P$



$EI\,t_{A/C} = 0$

$(Area_{AC})\,\bar{X}_A = 0$

$\frac{1}{2}(10)(600 - 3P)(\frac{20}{3}) - \frac{1}{2}(6)(600)(8) = 0$

$P = 56\ \text{lb}$ *answer*

Thus,
$240 - 1.2P = 172.8\ \text{lb}$

$600 - 3P = 432\ \text{lb}$

Under the 100-lb load:
$EI\, t_{B/C} = (Area_{BC})\, \bar{X}_B$

$EI\, t_{B/C} = \frac{1}{2}(6)(172.8)(2) + \frac{1}{2}(6)(432)(4) - \frac{1}{2}(6)(600)(4)$

$EI\, t_{B/C} = -979.2\ \text{lb}\cdot\text{ft}^3$

The negative sign indicates that the elastic curve is below the reference tangent.

Therefore,
$EI\, \delta_B = 979.2\ \text{lb}\cdot\text{ft}^3$ downward *answer*

Problem 669

Compute the value of EIδ midway between the supports of the beam shown in Fig. P-669.

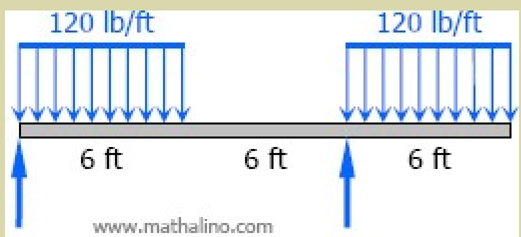
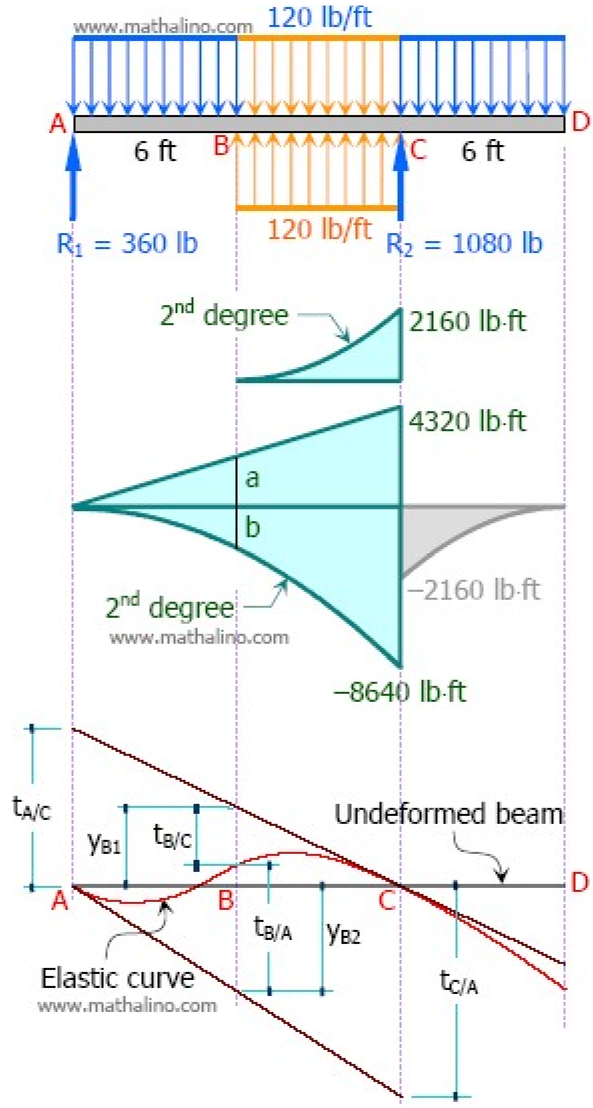


Figure P-669

Solution 669

HideClick here to show or hide the solution

$\Sigma M_{R2} = 0$



$12R_1 + 3(6)(120) = 9(6)(120)$

$R_1 = 360 \text{ lb}$

$\Sigma M_{R1} = 0$

$12R_2 = 3(6)(120) + 15(6)(120)$

$R_2 = 1080 \text{ lb}$

By ratio and proportion:

$\dfrac{a}{6} = \dfrac{4320}{12}$

$a = 2160 \text{ lb} \cdot \text{ft}$

By squared property of parabola:

$\dfrac{b}{6^2} = \dfrac{-8640}{12^2}$

$b = -2160 \text{ lb} \cdot \text{ft}$

$EI\,t_{C/A} = (Area_{AC})\,\bar{X}_C$

$EI\,t_{C/A} = \frac{1}{3}(6)(2160)(\frac{3}{2}) + \frac{1}{2}(12)(4320)(4) - \frac{1}{3}(12)(8640)(3)$

$EI\,t_{C/A} = 6480 \text{ lb} \cdot \text{ft}^3$

$EI\,t_{B/A} = (Area_{AB})\,\bar{X}_B$

$EI\,t_{B/A} = \frac{1}{2}(6a)(2) - \frac{1}{3}(6b)(\frac{3}{2})$

$EI\,t_{B/A} = 6a - 3b$

$EI\,t_{B/A} = 6(2160) - 3(2160)$

$EI\, t_{B/A} = 6480\ \text{lb} \cdot \text{ft}^3$

With the values of EI $t_{C/A}$ and EI $t_{B/A}$, it is obvious that the elastic curve is above point B. The deflection at B (up or down) can also be determined by comparing the values of $t_{B/A}$ and y_{B2}.

By ratio and proportion:

$\dfrac{y_{B2}}{6} = \dfrac{t_{C/A}}{12}$

$y_{B2} = \frac{1}{2} t_{C/A}$

$EI\, y_{B2} = \frac{1}{2} EI\, t_{C/A}$

$EI\, y_{B2} = \frac{1}{2}(6480)$

$EI\, y_{B2} = 3240\ \text{lb} \cdot \text{ft}^3$

Since $t_{B/A}$ is greater than y_{B2}, the elastic curve is above point B as concluded previously.

Therefore,

$EI\, \delta_B = EI\, t_{B/A} - EI\, y_{B2}$

$EI\, \delta_B = 6480 - 3240$

$EI\, \delta_B = 3240\ \text{lb} \cdot \text{ft}^3$ *answer*

You can also find the value EI δ_B by finding $t_{A/C}$, $t_{B/C}$, and y_{B1}. I encourage you to do it yourself.

Problem 670

Determine the value of EIδ at the left end of the overhanging beam shown in Fig. P-670.

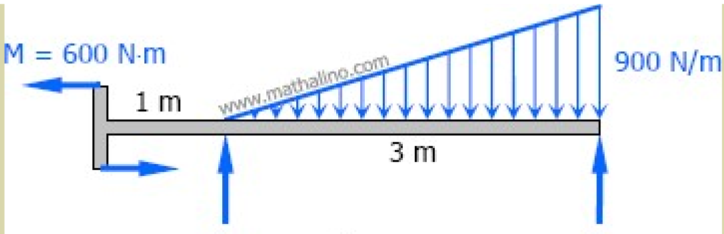


Figure P-670

Solution 670

HideClick here to show or hide the solution

$\Sigma M_{R2} = 0$

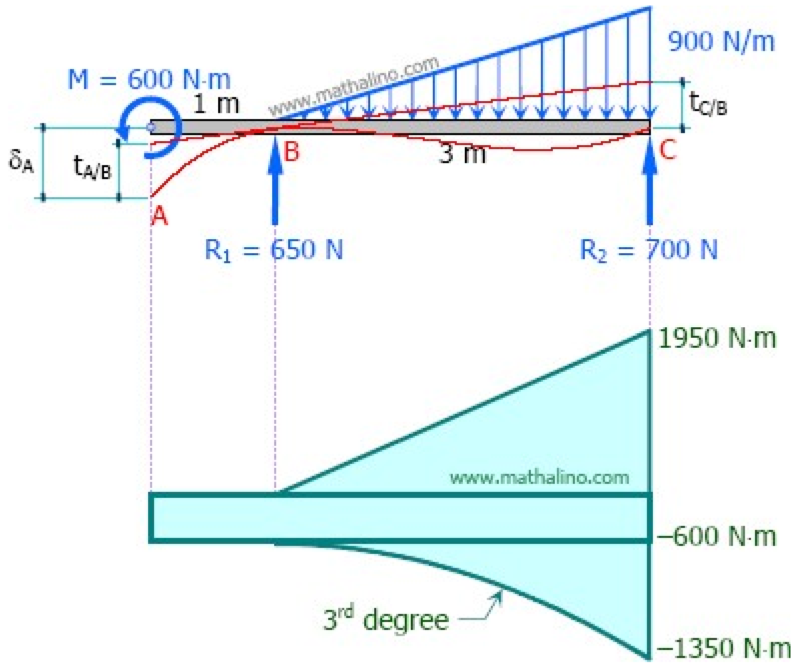
$3R_1 = 600 + \frac{1}{2}(3)(900)(1)$

$R_1 = 650 \text{ N}$

$\Sigma M_{R2} = 0$

$3R_2 + 600 = \frac{1}{2}(3)(900)(2)$

$R_2 = 700 \text{ N}$



$$EI\,t_{C/B} = (Area_{BC})\,\bar{X}_C$$

$$EI\,t_{C/B} = \frac{1}{2}(3)(1950)(1) - 3(600)(\frac{3}{2}) - \frac{1}{4}(3)(1350)(\frac{3}{5})$$

$$EI\,t_{C/B} = -382.5 \;\; \text{N} \cdot \text{m}^3$$

$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = -1(600)(\frac{1}{2})$$

$$EI\,t_{A/B} = -300 \;\; \text{N} \cdot \text{m}^3$$

The negative signs above indicates only the location of elastic curve relative to the reference tangent. It does not indicate magnitude. It shows that the elastic curve is below the reference tangent at points A and C.

By ratio and proportion

$$\frac{\delta_A - t_{A/B}}{1} = \frac{t_{C/B}}{3}$$

$\delta_A = \frac{1}{3} t_{C/B} + t_{A/B}$

$EI\,\delta_A = \frac{1}{3} EI\,t_{C/B} + EI\,t_{A/B}$

$EI\,\delta_A = \frac{1}{3}(382.5) + 300$

$EI\,\delta_A = 427.5\ \text{N}\cdot\text{m}^3$ *answer*

Midspan Deflection | Deflections in Simply Supported Beams

In simply supported beams, the tangent drawn to the elastic curve at the point of maximum deflection is horizontal and parallel to the unloaded beam. It simply means that the deviation from unsettling supports to the horizontal tangent is equal to the maximum deflection. If the simple beam is symmetrically loaded, the maximum deflection will occur at the midspan.

Finding the midspan deflection of a symmetrically loaded simple beam is straightforward because its value is equal to the maximum deflection. In unsymmetrically loaded simple beam however, the midspan deflection is not equal to the maximum deflection. To deal with unsymmetrically loaded simple beam, we will add a symmetrically placed load for each load actually acting on the beam, making the beam symmetrically loaded. The effect of this transformation to symmetry will double the actual midspan deflection, making the actual midspan deflection equal to one-half of the midspan deflection of the transformed symmetrically loaded beam.

Problem 673

For the beam shown in Fig. P-673, show that the midspan deflection is $\delta = (Pb/48EI)(3L^2 - 4b^2)$.

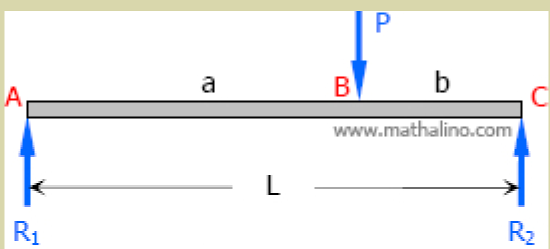
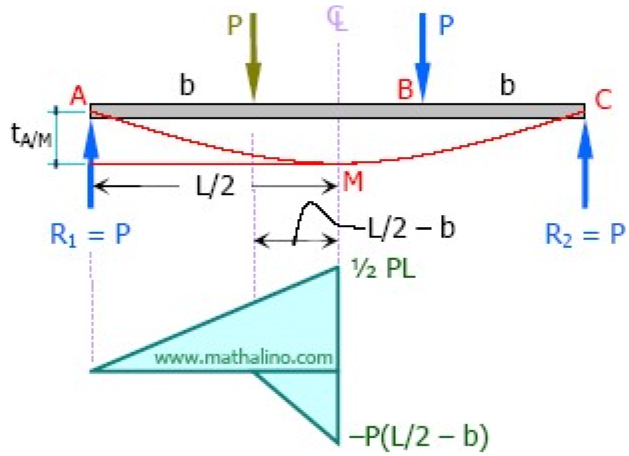


Figure P-673

Solution 673

HideClick here to show or hide the solution



$$EI\,t_{A/M} = (Area_{AM})\,\bar{X}_A$$

$$EI\,t_{A/M} = \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}PL)[\,\frac{2}{3}(\frac{1}{2}L)\,] - \frac{1}{2}(\frac{1}{2}L - b)P(\frac{1}{2}L - b)[\,b + \frac{2}{3}(\frac{1}{2}L - b)\,]$$

$$EI\,t_{A/M} = \frac{1}{24}PL^3 - \frac{1}{2}Pb(\frac{1}{2}L - b)^2 - \frac{1}{3}P(\frac{1}{2}L - b)^3$$

$$EI\,t_{A/M} = \frac{1}{24}PL^3 - \frac{1}{2}Pb\left(\frac{L - 2b}{2}\right)^2 - \frac{1}{3}P\left(\frac{L - 2b}{2}\right)^3$$

$$EI\,t_{A/M} = \frac{1}{24}PL^3 - \frac{1}{8}Pb(L - 2b)^2 - \frac{1}{24}P(L - 2b)^3$$

$$EI\,t_{A/M} = \frac{1}{24}PL^3 - \frac{1}{8}Pb(L^2 - 4Lb + 4b^2) - \frac{1}{24}P(L^3 - 6L^2b + 12Lb^2 - 8b^3)$$

$$EI\,t_{A/M} = \frac{1}{24}PL^3 - \frac{1}{8}PL^2b + \frac{1}{2}PLb^2 - \frac{1}{2}Pb^3 - \frac{1}{24}PL^3 + \frac{1}{4}PL^2b - \frac{1}{2}PLb^2 + \frac{1}{3}Pb^3$$

$$EI\,t_{A/M} = \frac{1}{8}PL^2b - \frac{1}{6}Pb^3$$

$$EI\,t_{A/M} = \frac{1}{24}Pb(3L^2 - 4b^2)$$

$$t_{A/M} = \frac{Pb}{24EI}(3L^2 - 4b^2)$$

$$\delta_{midspan} = \frac{1}{2}t_{A/M}$$

$$\delta_{midspan} = \frac{Pb}{48EI}(3L^2 - 4b^2)$$ (*okay!*)

Problem 674

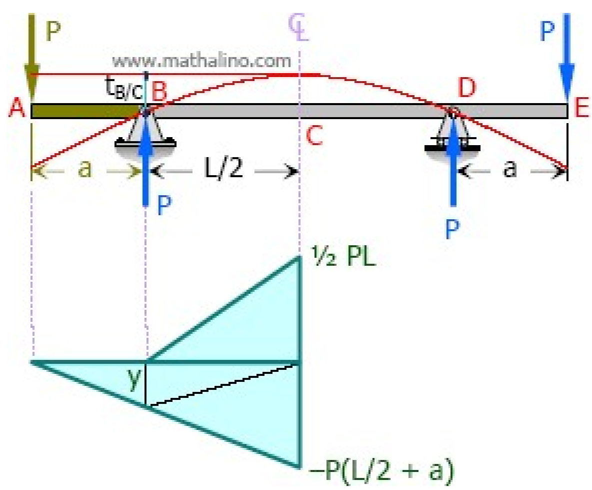
Find the deflection midway between the supports for the overhanging beam shown in Fig. P-674.

Solution 674

HideClick here to show or hide the solution

$$\frac{y}{a} = \frac{-P(\frac{1}{2}L + a)}{\frac{1}{2}L + a}$$

$$y = -Pa$$



$$EI\,t_{B/C} = (Area_{BC})\,\bar{X}_B$$

$$EI\,t_{B/C} = \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}PL)[\,\frac{2}{3}(\frac{1}{2}L)\,] - \frac{1}{2}(\frac{1}{2}L)(y)[\,\frac{1}{3}(\frac{1}{2}L)\,] - \frac{1}{2}(\frac{1}{2}L)[\,P(\frac{1}{2}L + a)\,]\,[\,\frac{2}{3}(\frac{1}{2}L)\,]$$

$$EI\,t_{B/C} = \frac{1}{24}PL^3 - \frac{1}{24}PL^2a - \frac{1}{12}PL^2(\frac{1}{2}L + a)$$

$$EI\,t_{B/C} = \frac{1}{24}PL^3 - \frac{1}{24}PL^2a - \frac{1}{24}PL^3 - \frac{1}{12}PL^2a$$

$EI\,t_{B/C} = -\frac{1}{8}PL^2a$

$2\delta = t_{B/C}$

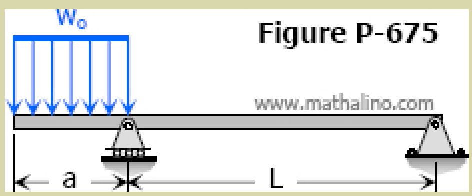
$\delta = \dfrac{1}{2}\left[-\dfrac{1}{8EI}PL^2a\right]$

$\delta = -\dfrac{1}{16EI}PL^2a$

$\delta = \dfrac{1}{16EI}PL^2a \;\; \text{upward}$ *answer*

Problem 675

Repeat Prob. 674 for the overhanging beam shown in Fig. P-675.



Solution 675

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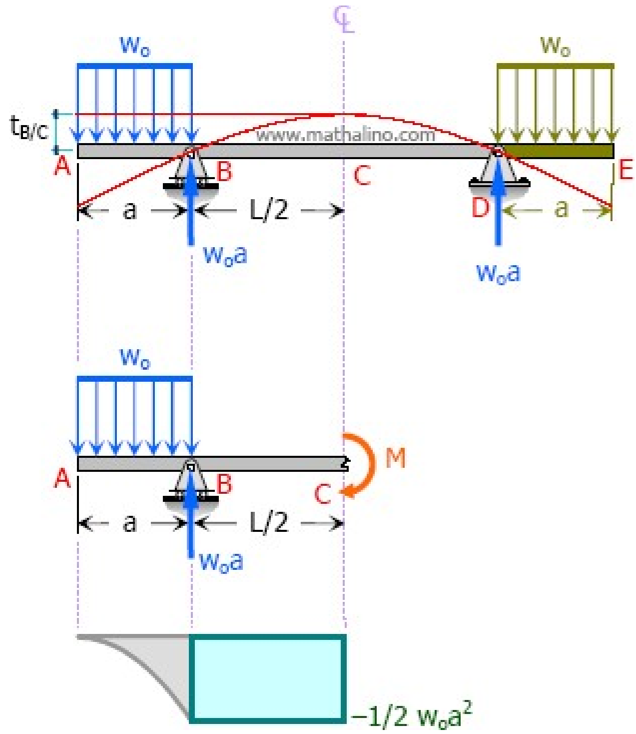
$M = w_o a(\frac{1}{2}L) - w_o a(\frac{1}{2}L + \frac{1}{2}a)$

$$M = \frac{1}{2}w_oLa - \frac{1}{2}w_oLa - \frac{1}{2}w_oa^2$$

$$M = -\frac{1}{2}w_oa^2$$



$$EI\,t_{B/C} = (Area_{BC})\,\bar{X}_B$$

$$EI\,t_{B/C} = \frac{1}{2}L(-\frac{1}{2}w_oa^2)(\frac{1}{4}L)$$

$$EI\,t_{B/C} = -\frac{1}{16}w_oL^2a^2$$

$$t_{B/C} = -\frac{1}{16EI}w_oL^2a^2$$

$$2\delta = t_{B/C}$$

$$2\delta = -\frac{1}{16EI}w_oL^2a^2$$

$$\delta = -\frac{1}{32EI}w_oL^2a^2$$

$$\delta = \frac{1}{32EI}w_oL^2a^2 \text{ upward}$$ *answer*

Problem 676

Determine the midspan deflection of the simply supported beam loaded by the couple shown in Fig. P-676.

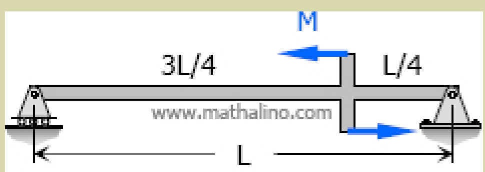


Figure P-676

Solution 676

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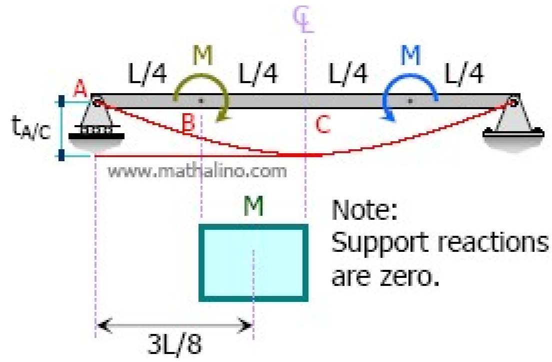
$$EI\,t_{A/C} = (Area_{AC})\,\bar{X}_A$$

$$EI\, t_{A/C} = M\left(\frac{L}{4}\right)\left(\frac{3L}{8}\right)$$

$$t_{A/C} = \frac{3ML^2}{32EI}$$



$$\delta_{midspan} = \tfrac{1}{2} t_{A/C}$$

$$\delta_{midspan} = \frac{1}{2}\left(\frac{3ML^2}{32EI}\right)$$

$$\delta_{midspan} = \frac{3ML^2}{64EI} \qquad answer$$

Problem 677

Determine the midspan deflection of the beam loaded as shown in Fig. P-677.

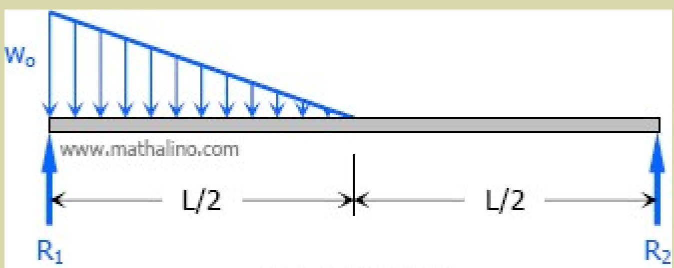
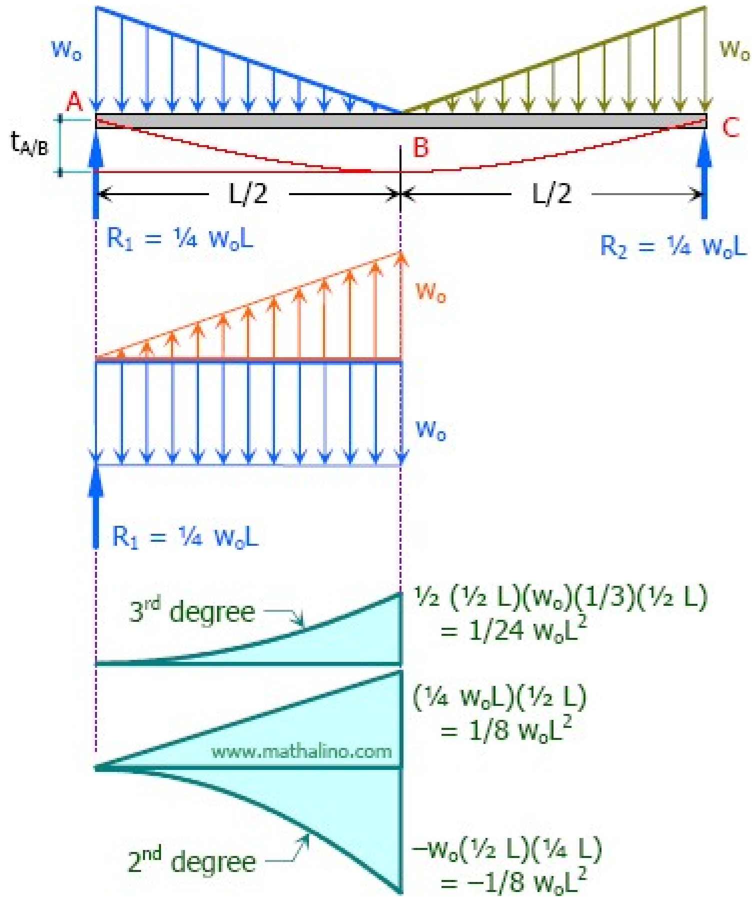


Figure P-677

Solution 677

HideClick here to show or hide the solution



$$EI\, t_{A/B} = (Area_{AB})\, \bar{X}_A$$

$$EI\, t_{A/B} = \frac{1}{4}(\frac{1}{2}L)(\frac{1}{24}w_oL^2)\frac{4}{5}(\frac{1}{2}L) + \frac{1}{2}(\frac{1}{2}L)(\frac{1}{8}w_oL^2)\frac{2}{3}(\frac{1}{2}L) - \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_oL^2)\frac{3}{4}(\frac{1}{2}L)$$

$$EI\, t_{A/B} = \frac{1}{480}w_oL^4 + \frac{1}{96}w_oL^4 - \frac{1}{128}w_oL^4$$

$$EI\, t_{A/B} = \frac{3}{640}w_oL^4$$

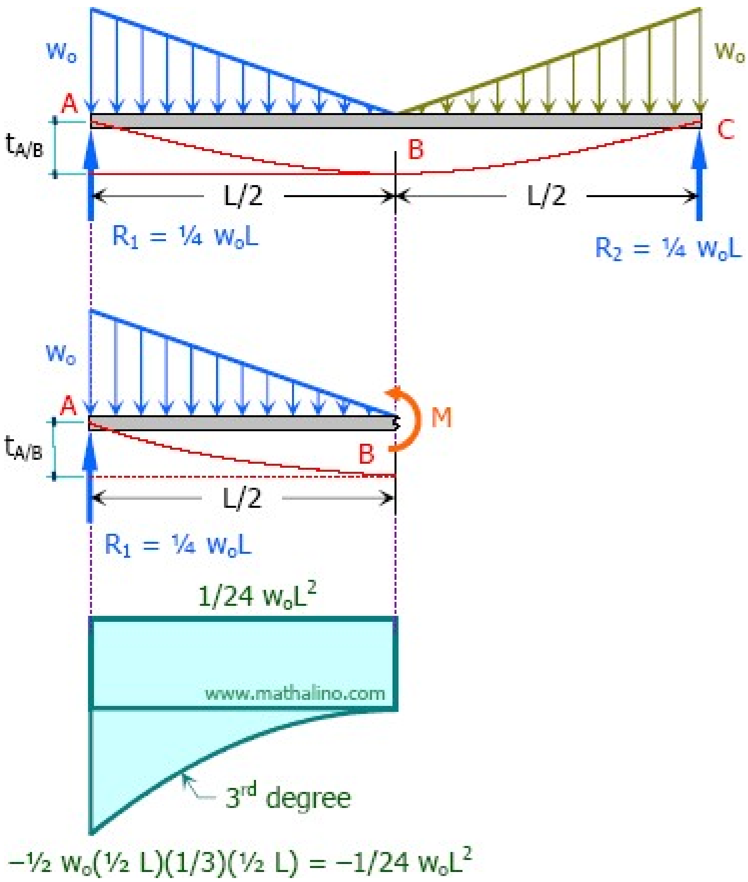
$$t_{A/B} = \frac{3w_oL^4}{640EI}$$

$$\delta_{midspan} = \frac{1}{2}t_{A/B}$$

$$\delta_{midspan} = \frac{1}{2}\left(\frac{3w_oL^4}{640EI}\right)$$

$$\delta_{midspan} = \frac{3w_o L^4}{1280EI} \qquad \textit{answer}$$

HideAnother Solution



$$M = \frac{1}{4}w_o L(\frac{1}{2}L) - \frac{1}{2}(\frac{1}{2}w_o L)\frac{2}{3}(\frac{1}{2}L)$$

$$M = \frac{1}{24}w_o L^2$$

$$EI\, t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\, t_{A/B} = \frac{1}{2}L(\frac{1}{24}w_o L^2)\frac{1}{2}(\frac{1}{2}L) - \frac{1}{4}(\frac{1}{2}L)(\frac{1}{24}w_o L^2)\frac{1}{5}(\frac{1}{2}L)$$

$$EI\, t_{A/B} = \frac{1}{192}w_o L^4 - \frac{1}{1920}w_o L^4$$

$EI\,t_{A/B} = \frac{3}{640} w_o L^4$

$t_{A/B} = \dfrac{3w_o L^4}{640EI}$ $\quad (okay!)$

Problem 678

Determine the midspan value of EIδ for the beam shown in Fig. P-678.

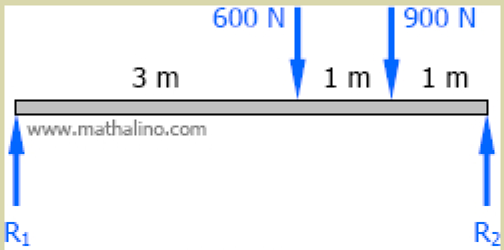
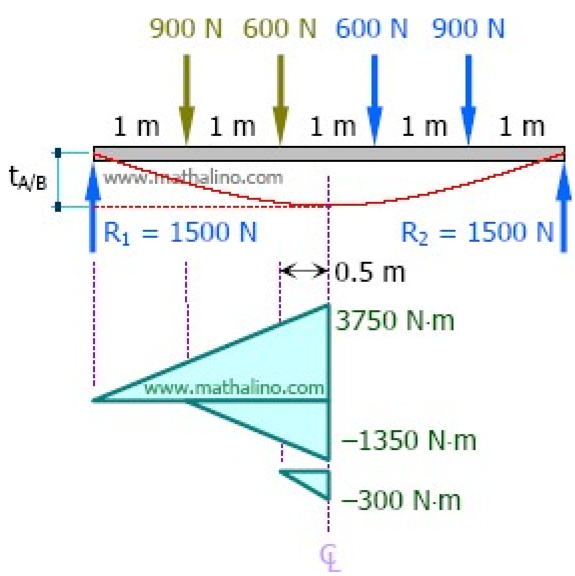


Figure P-678

Solution 678

HideClick here to show or hide the solution



$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$

$$EI\,t_{A/B} = \frac{1}{2}(2.5)(3750)(\frac{5}{3}) - \frac{1}{2}(1.5)(1350)(2) - \frac{1}{2}(0.5)(300)(\frac{7}{3})$$

$$EI\,t_{A/B} = 5612.5 \ \text{N} \cdot \text{m}^3$$

$$EI\,\delta_{midspan} = \frac{1}{2}(EI\,t_{A/B})$$

$$EI\,\delta_{midspan} = \frac{1}{2}(5612.5)$$

$$EI\,\delta_{midspan} = 2806.25 \ \text{N} \cdot \text{m}^3$$ *answer*

Problem 679
Determine the midspan value of EIδ for the beam shown in Fig. P-679 that carries a uniformly varying load over part of the span.

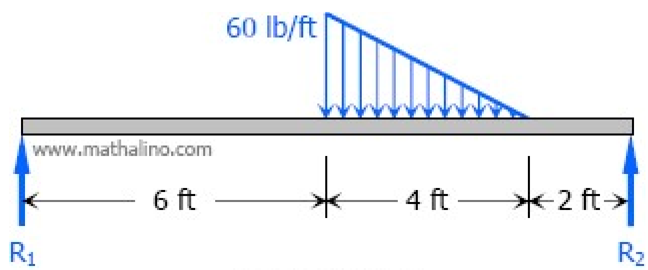


Figure P-679

Solution 679

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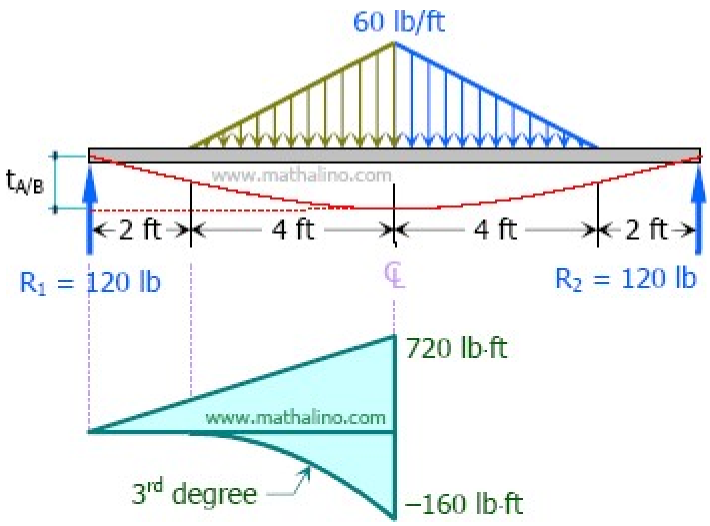

$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = \frac{1}{2}(6)(720)(4) - \frac{1}{4}(4)(160)(\frac{26}{5})$$

$$EI\,t_{A/B} = 7808 \ \text{lb}\cdot\text{ft}^3$$

$$EI\,\delta = \frac{1}{2}(EI\,t_{A/B})$$

$$EI\,\delta = \frac{1}{2}(7808)$$

$$EI\,\delta = 3904 \ \text{lb}\cdot\text{ft}^3 \qquad \textit{answer}$$

Problem 680

Determine the midspan value of EIδ for the beam loaded as shown in Fig. P-680.

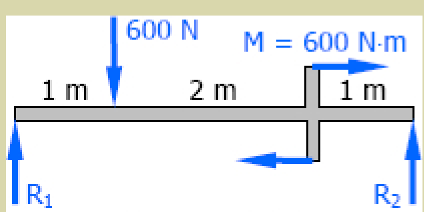

Figure P-680

Solution 680

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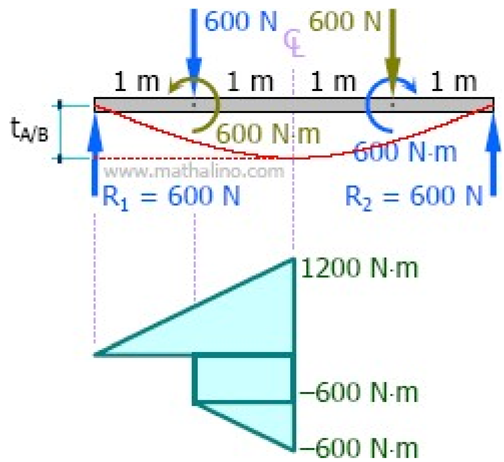


$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = \tfrac{1}{2}(2)(1200)(\tfrac{4}{3}) - 600(1)(\tfrac{3}{2}) - \tfrac{1}{2}(1)(600)(\tfrac{5}{3})$$

$$EI\,t_{A/B} = 200 \ \ \text{N} \cdot \text{m}^3$$

$$EI\,\delta = \tfrac{1}{2}(EI\,t_{A/B})$$

$$EI\,\delta = \tfrac{1}{2}(200)$$

$$EI\,\delta = 100 \ \ \text{N} \cdot \text{m}^3 \qquad \textit{answer}$$

Problem 681

Show that the midspan value of EIδ is $(w_o b/48)(L^3 - 2Lb^2 + b^3)$ for the beam in part (a) of Fig. P-681. Then use this result to find the midspan EIδ of the loading in part (b) by assuming the loading to exceed over two separate intervals that start from midspan and adding the results.

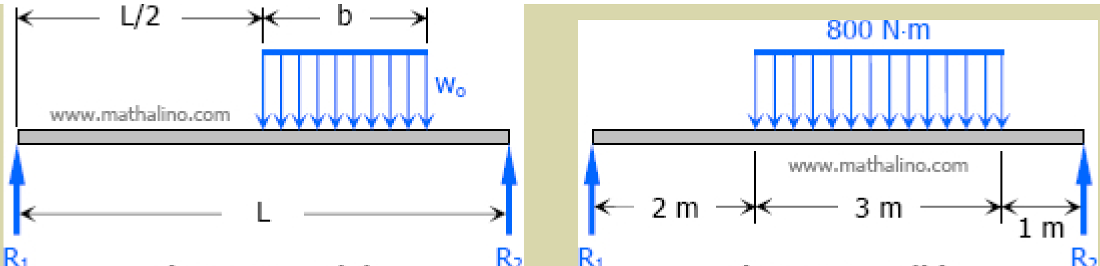


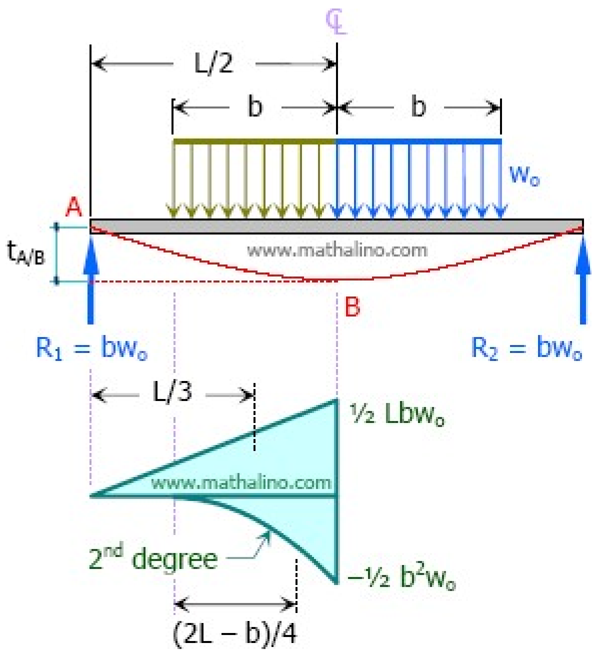
Figure P-681(a)

Figure P-681(b)

Solution 681

HideClick here to show or hide the solution

Part (a)



$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}Lbw_o)(\frac{1}{3}L) - \frac{1}{3}(b)(\frac{1}{2}b^2w_o)\frac{1}{4}(2L - b)$$

$$EI\,t_{A/B} = \frac{1}{24}L^3bw_o - \frac{1}{12}Lb^3w_o + \frac{1}{24}b^4w_o$$

$EI\,t_{A/B} = \frac{1}{24}w_o b(L^3 - 2Lb^2 + b^3)$

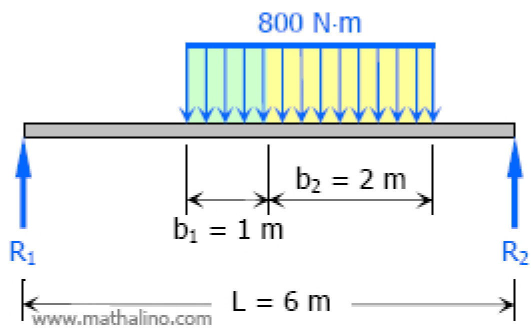
$EI\,\delta = \frac{1}{2}(EI\,t_{A/B})$

$EI\,\delta = \frac{1}{2}[\,\frac{1}{24}w_o b(L^3 - 2Lb^2 + b^3)\,]$

$EI\,\delta = \dfrac{w_o b}{48}(L^3 - 2Lb^2 + b^3)$ *answer*

Part (b)

$EI\,\delta = EI\,\delta_1 + EI\,\delta_2$



$EI\,\delta = \frac{1}{48}w_o b_1(L^3 - 2Lb_1{}^2 + b_1{}^3) + \frac{1}{48}w_o b_2(L^3 - 2Lb_2{}^2 + b_2{}^3)$

$EI\,\delta = \frac{1}{48}(800)(1)[\,6^3 - 2(6)(1^2) + 1^3\,] + \frac{1}{48}(800)(2)[\,6^3 - 2(6)(2^2) + 2^3\,]$

$EI\,\delta = 3416.67 + 5866.67$

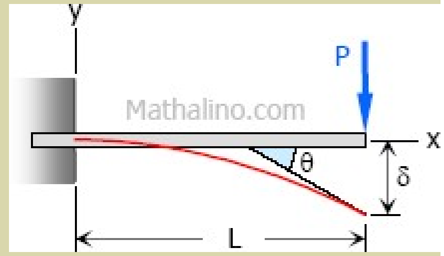
$EI\,\delta = 9283.34\ \text{ N}\cdot\text{m}^3$ *answer*

Method of Superposition | Beam Deflection

The slope or deflection at any point on the beam is equal to the resultant of the slopes or deflections at that point caused by each of the load acting separately.

Rotation and Deflection for Common Loadings

Case 1: Concentrated load at the free end of cantilever beam



Maximum Moment

$$M = -PL$$

Slope at end

$$\theta = \frac{PL^2}{2EI}$$

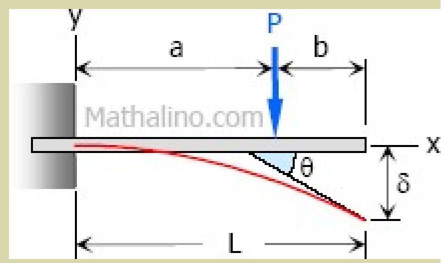
Maximum deflection

$$\delta = \frac{PL^3}{3EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Px^2}{6}(3L - x)$$

Case 2: Concentrated load at any point on the span of cantilever beam



Maximum Moment

$$M = -Pa$$

Slope at end

$$\theta = \frac{Pa^2}{2EI}$$

Maximum deflection

$$\delta = \frac{Pa^3}{6EI}(3L - a)$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Px^2}{6}(3a - x) \text{ for } 0 < x < a$$

$$EI\,y = \frac{Pa^2}{6}(3x - a) \text{ for } a < x < L$$

Case 3: Uniformly distributed load over the entire length of cantilever beam

Maximum Moment

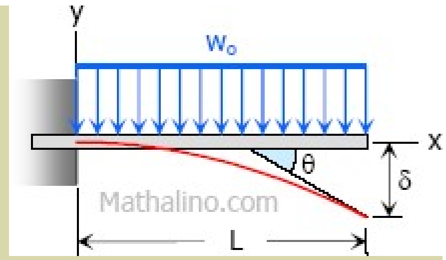
$$M = -\frac{w_oL^2}{2}$$

Slope at end

$$\theta = \frac{w_o L^3}{6EI}$$

Maximum deflection

$$\delta = \frac{w_o L^4}{8EI}$$



Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x^2}{120L}(6L^2 - 4Lx + x^2)$$

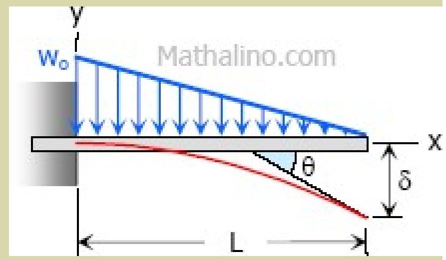
Case 4: Triangular load, full at the fixed end and zero at the free end, of cantilever beam

Maximum Moment

$$M = -\frac{w_o L^2}{6}$$

Slope at end

$$\theta = \frac{w_o L^3}{24EI}$$



Maximum deflection

$$\delta = \frac{w_o L^4}{30EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x^2}{120L}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

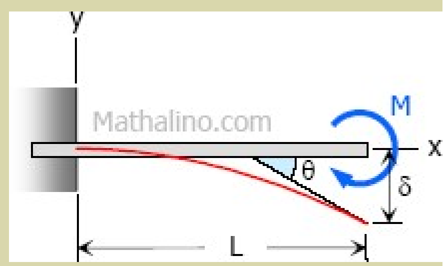
Case 5: Moment load at the free end of cantilever beam

Maximum Moment

$$M = -M$$

Slope at end

$$\theta = \frac{ML}{EI}$$



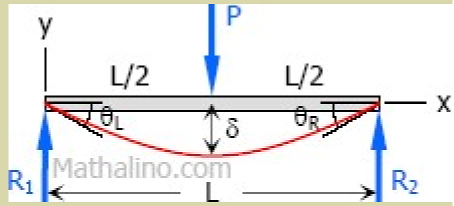
Maximum deflection

$$\delta = \frac{ML^2}{2EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Mx^2}{2}$$

Case 6: Concentrated load at the midspan of simple beam



Maximum Moment

$$M = \frac{PL}{4}$$

Slope at end

$$\theta_L = \theta_R = \frac{PL^2}{16EI}$$

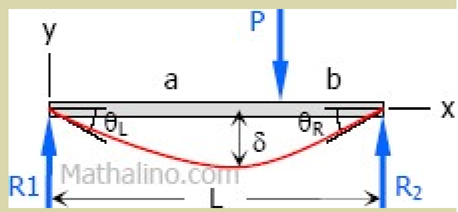
Maximum deflection

$$\delta = \frac{PL^3}{48EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Px}{12}(\tfrac{3}{4}L^2 - x^2) \text{ for } 0 < x < \tfrac{1}{2}L$$

Case 7: Concentrated load at any point on simple beam



Maximum Moment

$$M = \frac{Pab}{L} \text{ at } x = a$$

Slope at end

$$\theta_L = \frac{Pb(L^2 - b^2)}{6EIL}$$

$$\theta_R = \frac{Pa(L^2 - a^2)}{6EIL}$$

Maximum deflection

$$\delta = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\,EIL} \text{ at } x = \sqrt{\frac{L^2 - b^2}{3}}$$

Deflection at the center (not maximum)

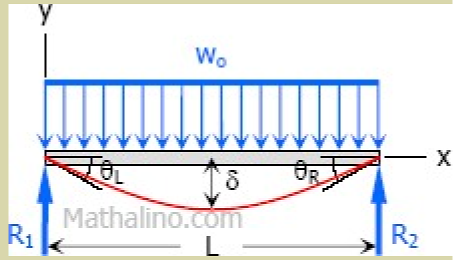
$$\delta = \frac{Pb}{48EI}(3L^2 - 4b^2) \text{ when } a > b$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Pbx}{6L}(L^2 - x^2 - b^2) \text{ for } 0 < x < a$$

$$EI\,y = \frac{Pb}{6L}\left[\frac{L}{b}(x - a)^3 + (L^2 - b^2)x - x^3\right] \text{ for } a < x < L$$

Case 8: Uniformly distributed load over the entire span of simple beam



Maximum Moment

$$M = \frac{w_o L^2}{8}$$

Slope at end

$$\theta_L = \theta_R = \frac{w_o L^3}{24EI}$$

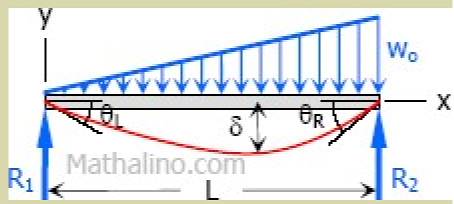
Maximum deflection

$$\delta = \frac{5 w_o L^4}{384EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x}{24}(L^3 - 2Lx^2 + x^3)$$

Case 9: Triangle load with zero at one support and full at the other support of simple beam



Maximum Moment

$$M = \frac{w_o L^2}{9\sqrt{3}}$$

Slope at end

$$\theta_L = \frac{7 w_o L^3}{360EI}$$

$$\theta_R = \frac{8 w_o L^3}{360EI}$$

Maximum deflection

$$\delta = \frac{2.5 w_o L^4}{384EI} \text{ at } x = 0.519L$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x}{360L}(7L^4 - 10L^2x^2 + 3x^4)$$

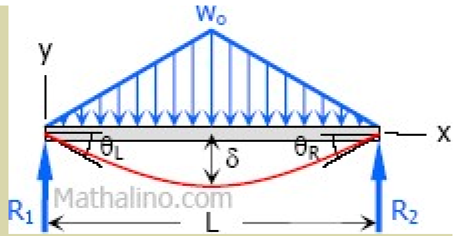
Case 10: Triangular load with zero at each support and full at the midspan of simple beam

Maximum Moment

$$M = \frac{w_o L^2}{12}$$

Slope at end

$$\theta_L = \theta_R = \frac{5w_o L^3}{192EI}$$



Maximum deflection

$$\delta = \frac{w_o L^4}{120EI}$$

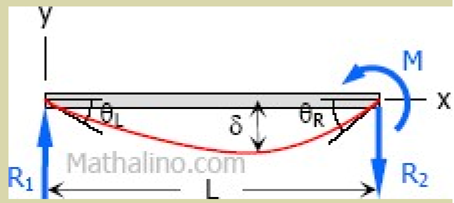
Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x}{960L}(25L^4 - 40L^2x^2 + 16x^4) \text{ for } 0 < x < \frac{L}{2}$$

Case 11: Moment load at the right support of simple beam

Maximum Moment

$$M = M$$



Slope at end

$$\theta_L = \frac{ML}{6EI}$$

$$\theta_R = \frac{ML}{3EI}$$

Maximum deflection

$$\delta = \frac{ML^2}{9\sqrt{3}\,EI} \text{ at } x = \frac{L}{\sqrt{3}}$$

Deflection at the center (not maximum)

$$\delta = \frac{ML^2}{16EI}$$

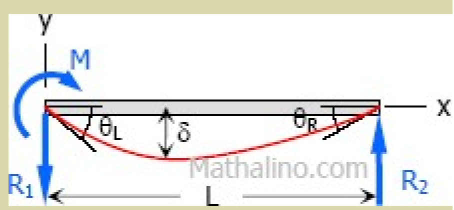
Deflection Equation (y is positive downward)

$$EI\,y = \frac{MLx}{6}\left(1 - \frac{x^2}{L^2}\right)$$

Case 12: Moment load at the left support of simple beam

Maximum Moment

$$M = M$$



Slope at end

$$\theta_L = \frac{ML}{3EI}$$

$$\theta_R = \frac{ML}{6EI}$$

Maximum deflection

$$\delta = \frac{ML^2}{9\sqrt{3}\,EI} \text{ at } x = \left(L - \frac{L}{\sqrt{3}}\right)$$

Deflection at the center (not maximum)

$$\delta = \frac{ML^2}{16EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Mx}{6L}(L - x)(2L - x)$$

Problem 685

Determine the midspan value of EIδ for the beam loaded as shown in Fig. P-685. Use the method of superposition.

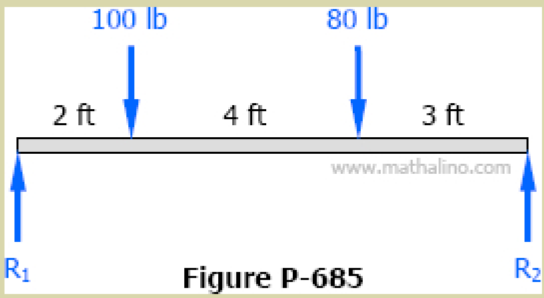


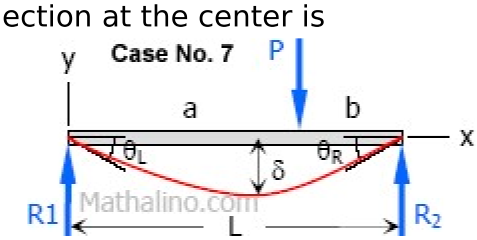
Figure P-685

Solution 685

HideClick here to show or hide the solution

From Case No. 7 of Summary of Beam Loadings, deflection at the center is

$$\delta = \frac{Pb}{48EI}(3L^2 - 4b^2) \text{ when } a > b$$



Thus, for Fig. P-685

EI $\delta_{midspan}$ = EI $\delta_{midspan}$ due to 100 lb force + EI $\delta_{midspan}$ due to 80 lb force

$$EI\,\delta_{midspan} = \sum \frac{Pb}{48}(3L^2 - 4b^2)$$

$$EI\,\delta_{midspan} = \frac{100(2)}{48}[\,3(9^2) - 4(2^2)\,] + \frac{80(3)}{48}[\,3(9^2) - 4(3^2)\,]$$

$$EI\,\delta_{midspan} = 945.83 + 1035$$

$$EI\,\delta_{midspan} = 1980.83\ \text{lb}\cdot\text{ft}^3$$ *answer*

Problem 686

Determine the value of EIδ under each concentrated load in Fig. P-686.

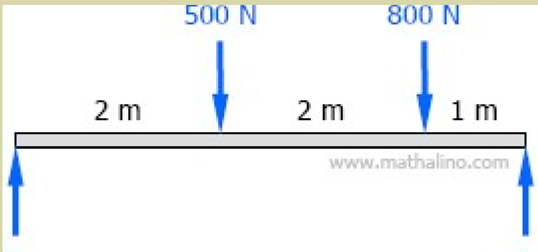


Figure P-686

Solution 686

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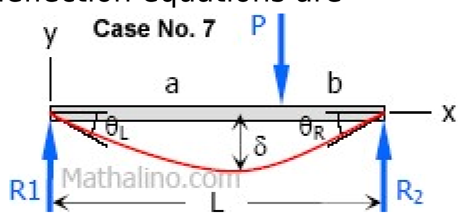
From Case No. 7 of Summary of Beam Loadings, the deflection equations are



$$EI\,y = \frac{Pbx}{6L}(L^2 - x^2 - b^2)\ \text{for}\ 0 < x < a$$

$$EI\,y = \frac{Pb}{6L}\left[\frac{L}{b}(x-a)^3 + (L^2 - b^2)x - x^3\right]\ \text{for}\ a < x < L$$

The point under the load P is generally located at $x = a$ and at this point, both equations above will become

$$EI\,y = \frac{Pab}{6L}(L^2 - a^2 - b^2)$$

Deflection under the 500 N load

EIδ = EIδ due to 500 N load + EIδ due to 800 N load

$$EI\,\delta = \frac{500(2)(3)}{6(5)}(5^2 - 2^2 - 3^2) + \frac{800(1)(2)}{6(5)}(5^2 - 2^2 - 1^2)$$

$$EI\,\delta = 1200 + 1066.67$$

$$EI\,\delta = 2266.67 \text{ N}\cdot\text{m}^3 \qquad \textit{answer}$$

Deflection under the 800 N load

EIδ = EIδ due to 500 N load + EIδ due to 800 N load

$$EI\,\delta = \frac{500(3)}{6(5)}\left[\frac{5}{3}(4-2)^3 + (5^2 - 3^2)(4) - 4^3\right] + \frac{800(1)(4)}{6(5)}(5^2 - 4^2 - 1^2)$$

$$EI\,\delta = 666.67 + 853.33$$

$$EI\,\delta = 1520 \text{ N}\cdot\text{m}^3 \qquad \textit{answer}$$

Problem 687

Determine the midspan deflection of the beam shown in Fig. P-687 if E = 10 GPa and I = 20 × 10^6 mm^4.

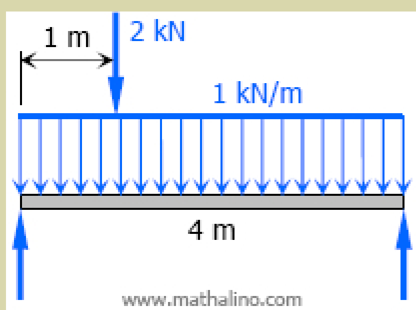
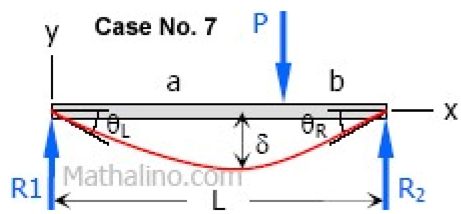


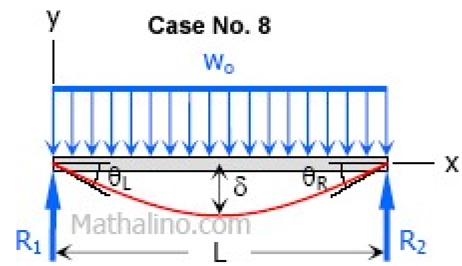
Figure P-687

Solution 687

HideClick here to show or hide the solution

From Case No. 7, midspan deflection is $\delta = \dfrac{Pb}{48EI}(3L^2 - 4b^2) \text{ when } a > b$





From Case No. 8, midspan deflection is $\delta = \dfrac{5w_oL^4}{384EI}$

Midspan deflection of the given beam

EIδ = EIδ due to 2 kN concentrated load + EIδ due to 1 kN/m uniform loading

$$EI\,\delta = \frac{Pb}{48}(3L^2 - 4b^2) + \frac{5w_oL^4}{384}$$

$$EI\,\delta = \frac{2(1)}{48}[\,3(4^2) - 4(1^2)\,] + \frac{5(1)(4^4)}{384}$$

$$EI\,\delta = \tfrac{11}{6} + \tfrac{10}{3}$$

$$EI\,\delta = \tfrac{31}{6}\text{ kN}\cdot\text{m}^3$$

$$\delta = \frac{\frac{31}{6}}{EI}$$

$$\delta = \frac{\frac{31}{6}(1000^4)}{10000(20 \times 10^6}$$

$\delta = 25.83\text{ mm}$ *answer*

Problem 688
Determine the midspan value of EIδ at the left end of the beam shown in Fig. P-688.

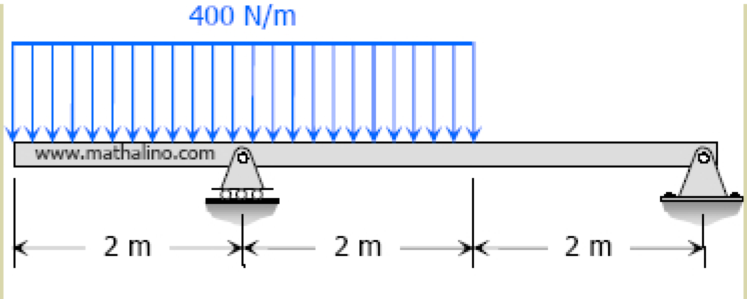


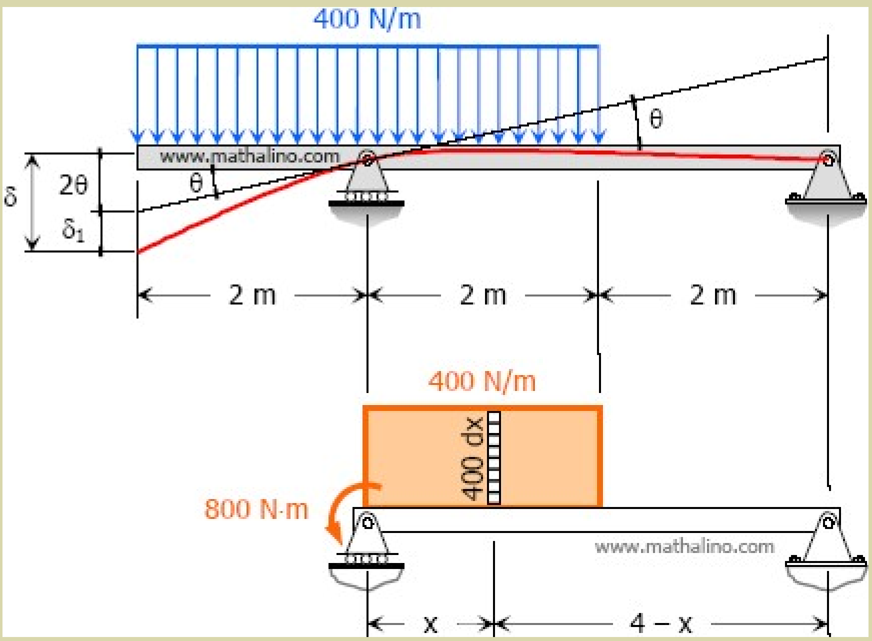
Figure P-688

Solution 688

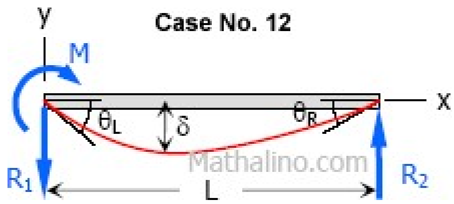
HideClick here to show or hide the solution

From the figure below, the total deformation at the end of overhang is

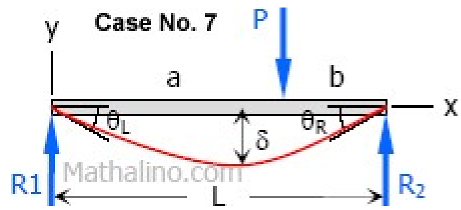
$$\delta = 2\theta + \delta_1$$



The rotation θ at the left support is combination of Case No. 12 and by integration of Case No. 7.



$$\theta_L = \frac{ML}{3EI}$$



$$\theta_L = \frac{Pb(L^2 - b^2)}{6EIL}$$

Solving for θ

EIθ = EIθ due to 800 N·m moment at left support - EIθ due to 400 N/m uniform load

$$EI\,\theta = \frac{800(4)}{3} - \int_0^2 \frac{400dx(4-x)[\,4^2 - (4-x)^2\,]}{6(4)}$$

$$EI\,\theta = \frac{3200}{3} - \frac{50}{3}\int_0^2 [\,16(4-x) - (4-x)^3\,]dx$$

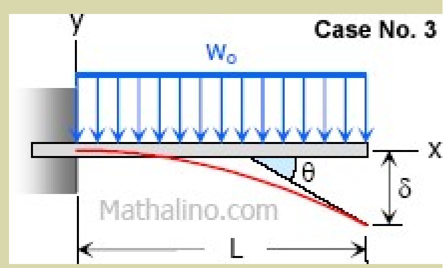
$$EI\,\theta = \frac{3200}{3} - \frac{50}{3}\left[-8(4-x)^2 + \frac{(4-x)^4}{4}\right]_0^2$$

$$EI\,\theta = \frac{3200}{3} - \frac{50}{3}\left[-8(2^2) + \frac{2^4}{4}\right] + \frac{50}{3}\left[-8(4^2) + \frac{4^4}{4}\right]$$

$$EI\,\theta = \frac{3200}{3} + \frac{1400}{3} - \frac{3200}{3}$$

$$EI\,\theta = \frac{1400}{3}\ \text{ N}\cdot\text{m}^2$$

Apply Case No. 3 for solving δ_1. From Case No. 3:



$$\delta = \frac{w_o L^4}{8EI}$$

Solving for δ_1:

$$EI\,\delta_1 = \frac{400(2^4)}{8}$$

$$EI\,\delta_1 = 800 \;\; \text{N}\cdot\text{m}^3$$

Total deflection at the free end

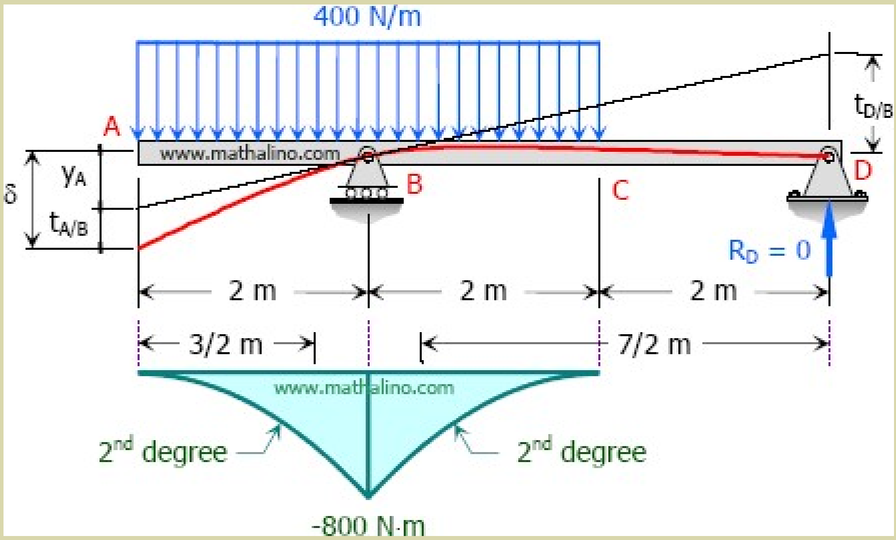
$$EI\,\delta = 2EI\,\theta + EI\,\delta_1$$

$$EI\,\delta = 2\left(\frac{1400}{3}\right) + 800$$

$$EI\,\delta = \frac{5200}{3} \;\; \text{N}\cdot\text{m}^3$$ *answer*

HideAnother Solution (Area-moment method)

This problem can be done with less effort by area-moment method.



$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{A/B} = -\tfrac{1}{3}(2)(800)(\tfrac{3}{2})$$

$$EI\,t_{A/B} = -800 \;\; \text{N}\cdot\text{m}^3$$

$$EI\,t_{D/B} = (Area_{AB})\,\bar{X}_A$$

$$EI\,t_{D/B} = -\tfrac{1}{3}(2)(800)(\tfrac{7}{2})$$

$$EI\,t_{D/B} = -\frac{5600}{3}\ \text{N}\cdot\text{m}^3$$

The negative sign above indicates that the elastic curve is below the tangent line.

$$\frac{y_A}{2} = \frac{t_{D/B}}{4}$$

$$y_A = \frac{1}{2} t_{D/B}$$

$$EI\,y_A = \frac{1}{2}(EI\,t_{D/B})$$

$$EI\,y_A = \frac{1}{2}(\frac{5600}{3})$$

$$EI\,y_A = \frac{2800}{3}\ \text{N}\cdot\text{m}^3$$

$$EI\,\delta = EI\,y_A + EI\,t_{A/B}$$

$$EI\,\delta = \frac{2800}{3} + 800$$

$$EI\,\delta = \frac{5200}{3}\ \text{N}\cdot\text{m}^3$$

$$EI\,\delta = \frac{5200}{3}\ \text{N}\cdot\text{m}^3$$ *(okay!)*

Problem 689

The beam shown in Fig. P-689 has a rectangular cross section 4 inches wide by 8 inches deep. Compute the value of P that will limit the midspan deflection to 0.5 inch. Use E = 1.5 × 10^6 psi.

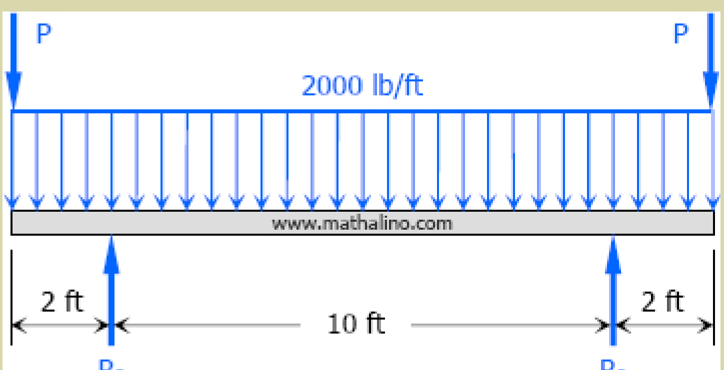


Figure P-689

Solution 689

HideClick here to show or hide the solution

The overhang is resolved into simple beam with end moments. The magnitude of end moment is,

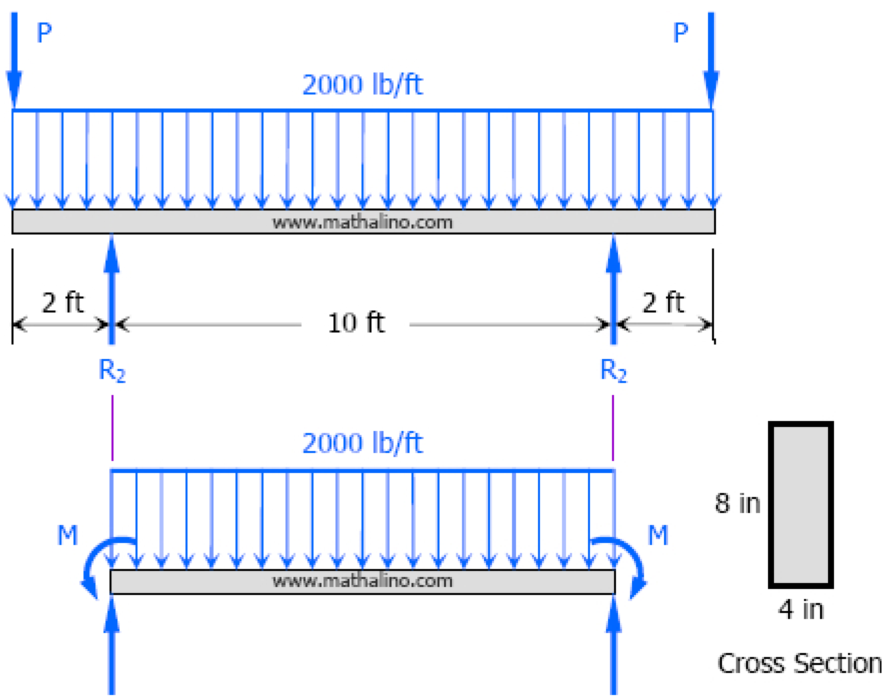
$M = 2000(2)(1) + 2P$

$M = 4000 + 2P$

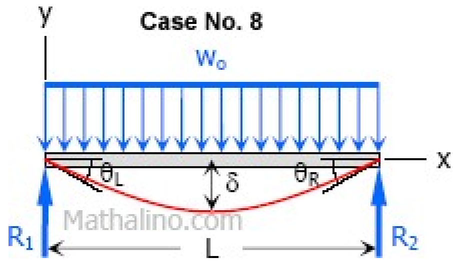
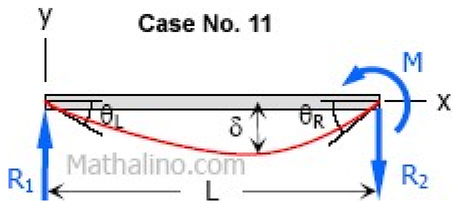
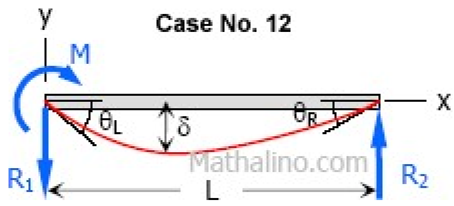
Moment of inertia of beam section

$I = \dfrac{bd^3}{12} = \dfrac{4(8^3)}{12}$

$I = \dfrac{512}{3} \ \text{in}^4$



The midspan deflection is a combination of deflection due to uniform load and two end moments. Use Case No. 8 and Cases No. 8, 11, and 12 to solve for the midspan deflection.

Type of Loading	Midspan Deflection
	$\delta = \dfrac{5w_oL^4}{384EI}$
	$\delta = \dfrac{ML^2}{16EI}$
	$\delta = \dfrac{ML^2}{16EI}$

$$\delta_{midspan} = \frac{5w_oL^4}{384EI} - 2\left[\frac{ML^2}{16EI}\right]$$

$$0.5 = \frac{5(2000)(10^4)(12^3)}{384(1.5\times10^6)(\frac{512}{3})} - 2\left[\frac{(4000+2P)(10^2)(12^3)}{16(1.5\times10^6)(\frac{512}{3})}\right]$$

$$0.5 = \frac{225}{128} - \frac{27(4000+2P)}{320\,000}$$

$$160\,000 = 562\,500 - 27(4000+2P)$$

$$27(4000+2P) = 402\,500$$

$2P = 10\,907.40$

$P = 5453.7 \text{ lb}$ *answer*

Problem 690

The beam shown in Fig. P-690 has a rectangular cross section 50 mm wide. Determine the proper depth d of the beam if the midspan deflection of the beam is not to exceed 20 mm and the flexural stress is limited to 10 MPa. Use E = 10 GPa.

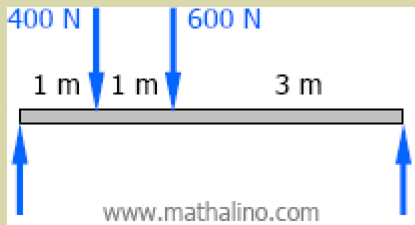
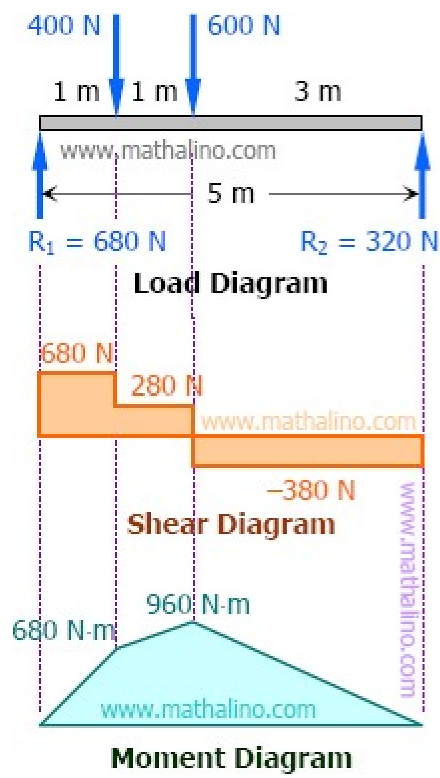


Figure P-690

Solution 690

HideClick here to show or hide the solution



$\Sigma M_{R2} = 0$

$5R_1 = 4(400) + 3(600)$

$R_1 = 680 \text{ N}$

$\Sigma M_{R1} = 0$

$5R_2 = 1(400) + 2(600)$

$R_2 = 320 \text{ N}$

Based on allowable flexural stress

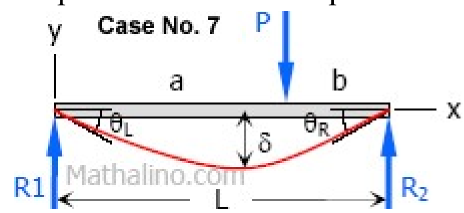
$$(f_b)_{max} = \frac{6M_{max}}{bd^2}$$

$$10 = \frac{6(960)(1000)}{50d^2}$$

$d^2 = 11\,520$

$d = 107.33\ \text{mm}$

Based on allowable midspan deflection. Use Case No. 7, the midspan deflection of simple beam under concentrated load is given by



$$\delta = \frac{Pb}{48EI}(3L^2 - 4b^2) \text{ when } a > b$$

For the given beam, the midspan deflection is the sum of the midspan deflection of each load acting separately.

$$\delta = \sum \frac{Pb}{48EI}(3L^2 - 4b^2)$$

$$20 = \frac{400(1)(1000^3)}{48(10\,000I)}[\,3(5^2) - 4(1^2)\,] + \frac{600(2)(1000^3)}{48(10\,000I)}[\,3(5^2) - 4(2^2)\,]$$

$$\frac{20(10\,000I)}{1000^3} = \frac{400(1)}{48}[\,3(5^2) - 4(1^2)\,] + \frac{600(2)}{48}[\,3(5^2) - 4(2^2)\,]$$

$$\frac{I}{5000} = \frac{1775}{3} + 1475$$

$$\frac{I}{5000} = \frac{6200}{3}$$

$I = 10\,333\,333.33$

$$\frac{bd^3}{12} = 10\,333\,333.33$$

$$\frac{50d^3}{12} = 10\,333\,333.33$$

$d^3 = 2\,480\,000$

$d = 135.36\ \text{mm}$

Use **d = 135.36 mm** *answer*

Problem 691

Determine the midspan deflection for the beam shown in Fig. P-691. (Hint: Apply Case No. 7 and integrate.)

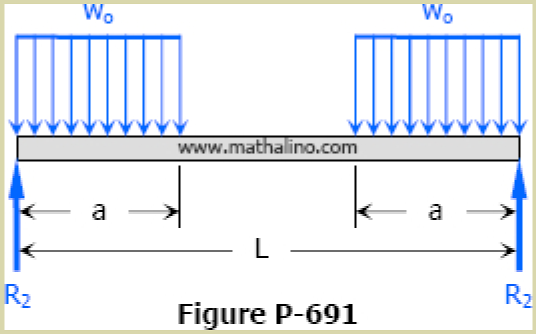
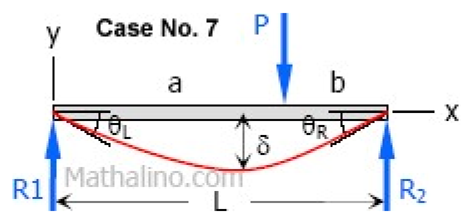


Figure P-691

Solution 691

HideClick here to show or hide the solution

From Case No. 7, the midspan deflection is

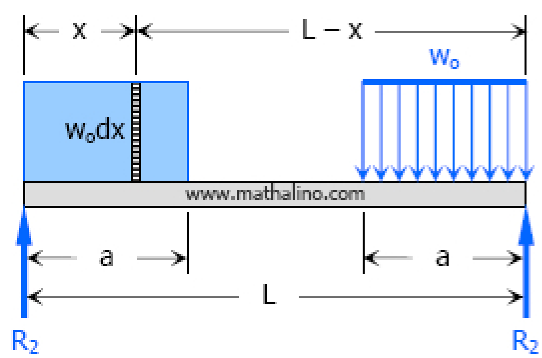


$$\delta = \frac{Pb}{48EI}(3L^2 - 4b^2) \text{ when } a > b$$

For the given beam

$$P = w_o dx$$

$$b = x$$



$$\delta = 2\int_0^a \frac{(w_o dx)x}{48EI}(3L^2 - 4x^2)$$

$$\delta = \frac{w_o}{24EI}\int_0^a (3L^2x - 4x^3)dx$$

$$\delta = \frac{w_o}{24EI}\left[\frac{3L^2x^2}{2} - x^4\right]_0^a$$

$$\delta = \frac{w_o}{24EI}\left[\frac{3L^2x^2 - 2x^4}{2}\right]_0^a$$

$$\delta = \frac{w_o}{48EI}\left[3L^2a^2 - 2a^4\right]$$

$$\delta = \frac{w_o a^2}{48EI}(3L^2 - 2a^2)$$ *answer*

Problem 692

Find the value of EIδ midway between the supports for the beam shown in Fig. P-692. (Hint: Combine Case No. 11 and one half of Case No. 8.)

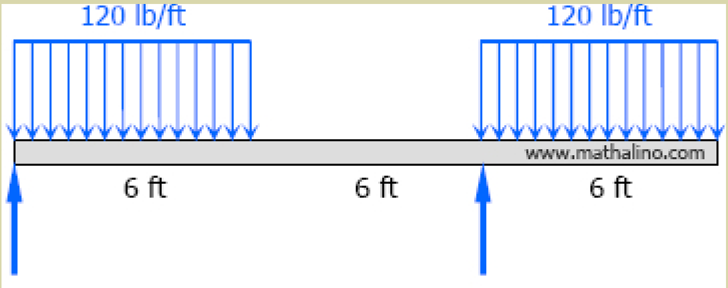


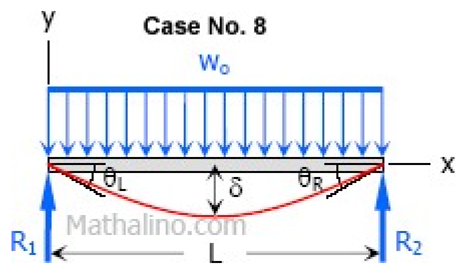
Figure P-692

Solution 692

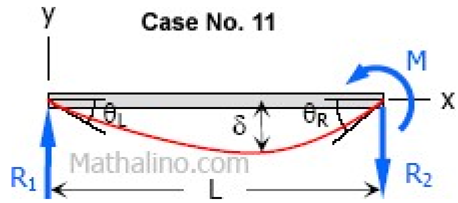
HideClick here to show or hide the solution

The midspan deflection from Case No. 8 and Case No. 11 are respectively,

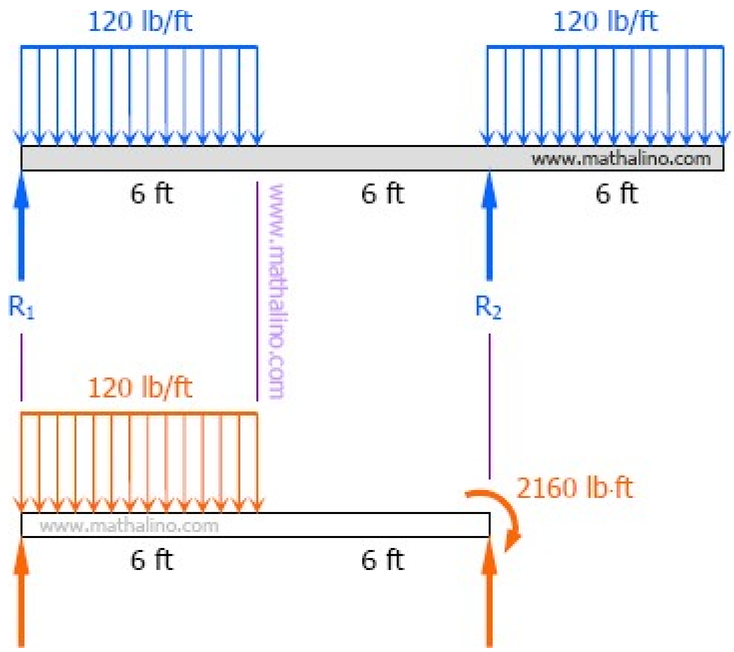
$$\delta = \frac{5w_oL^4}{384EI}$$



$$\delta = \frac{ML^2}{16EI}$$



The given beam is transformed into a simple beam with end moment at the right support due to the load at the overhang as shown in the figure below.



EIδ = ½ of EIδ due to uniform load over the entire span - EIδ due to end moment

$$EI\,\delta_{midspan} = \frac{1}{2}\left[\frac{5(120)(12^4)}{384}\right] - \frac{2160(12^2)}{16}$$

$$EI\,\delta_{midspan} = 16\,200 - 19\,440$$

$$EI\,\delta_{midspan} = -3\,240$$

$EI\,\delta_{midspan} = 3\,240 \text{ lb}\cdot\text{ft}^3 \text{ upward}$ *answer*

Problem 693

Determine the value of EIδ at the left end of the overhanging beam in Fig. P-693.

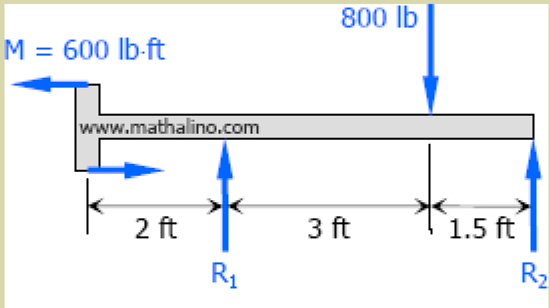


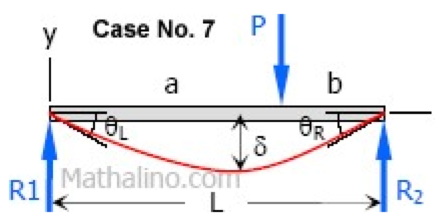
Figure P-693

Solution 693

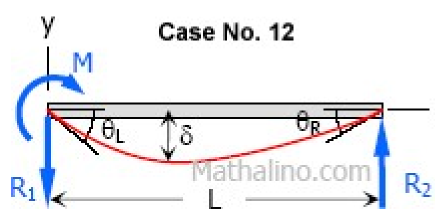
HideClick here to show or hide the solution

The rotation at the left support is the combination of Case No. 7 and Case No. 12.

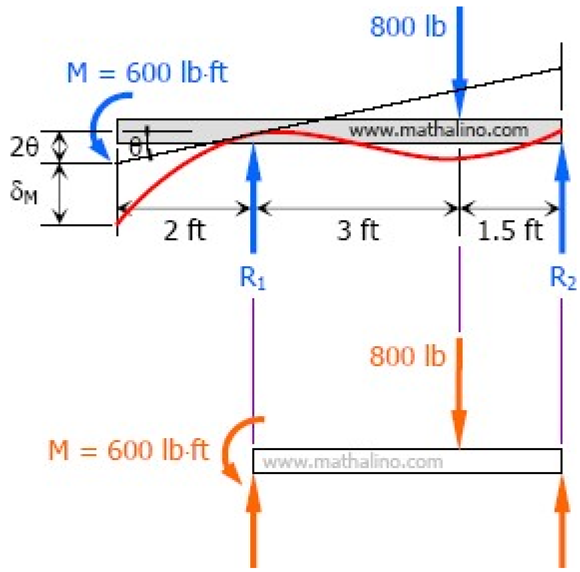
$$\theta_L = \frac{Pb(L^2 - b^2)}{6EIL}$$



$$\theta_L = \frac{ML}{3EI}$$



The overhang beam is transformed into a simple beam and the end moment at the free end of the overhang is carried to the left support of the transformed beam.



$$\theta = \frac{Pb(L^2 - b^2)}{6EIL} - \frac{ML}{3EI}$$

$$\theta = \frac{800(1.5)(4.5^2 - 1.5^2)}{6EI(4.5)} - \frac{600(4.5)}{3EI}$$

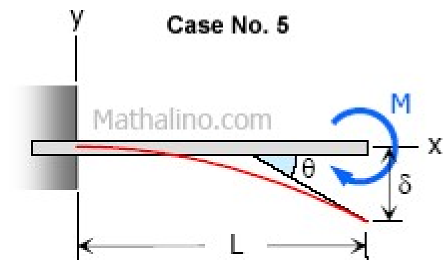
$$\theta = \frac{800}{EI} - \frac{900}{EI}$$

$$\theta = -\frac{100}{EI}$$

The negative sign indicates that the rotation at the left end contributed by the end moment (taken as negative) is greater than the rotation at the left end contributed by the concentrated load (taken as positive).

From Case No. 5, the end deflection is

$$\delta = \frac{ML^2}{2EI}$$



The deflection at the overhang due to moment load alone is

$$\delta_M = \frac{600(2^2)}{2EI}$$

$$\delta_M = \frac{1200}{EI}$$

Total deflection at the left end of the given beam is

$$\delta = 2\theta + \delta_M$$

$$\delta = 2\left(\frac{100}{EI}\right) + \frac{1200}{EI}$$

$$\delta = \frac{1400}{EI}$$ *answer*

Problem 694

The frame shown in Fig. P-694 is of constant cross section and is perfectly restrained at its lower end. Compute the vertical deflection caused by the couple M.

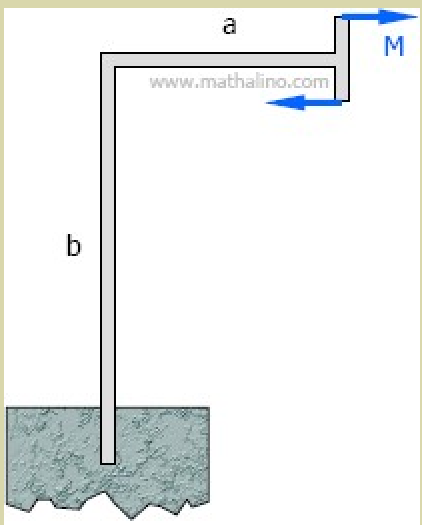
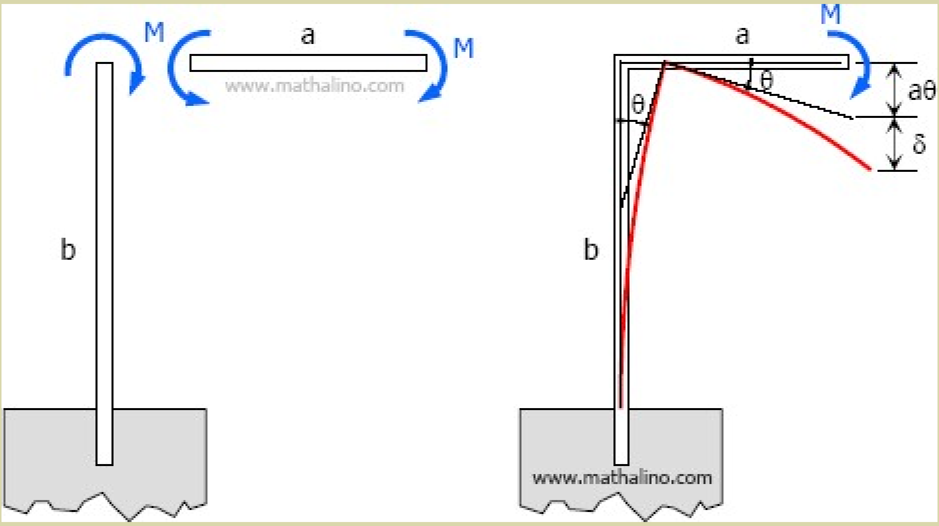


Figure P-694 and P-695

HideClick here to read or hide Solution 694



$$\delta_v = a\theta + \delta$$

$$\delta_v = a\left(\frac{Mb}{EI}\right) + \frac{Ma^2}{2EI}$$

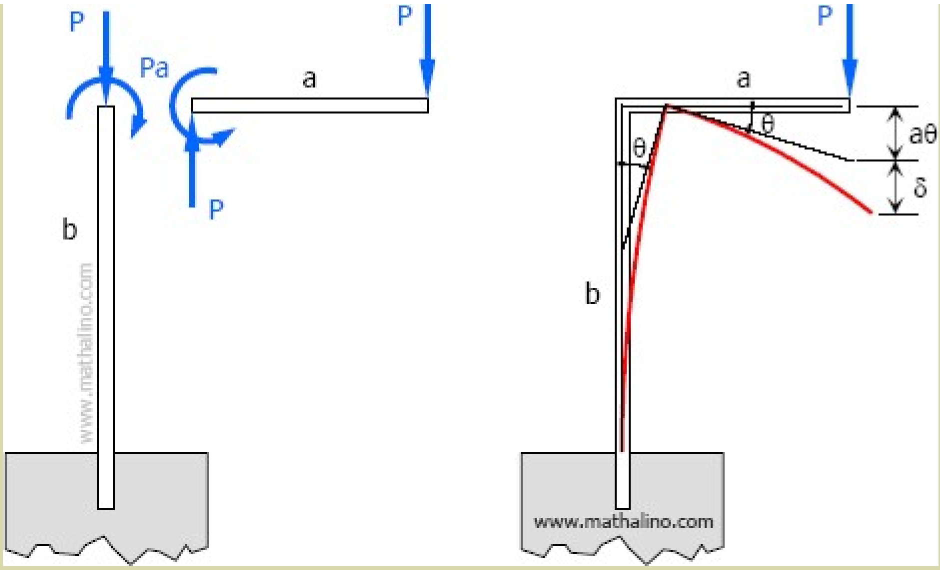
$$\delta_v = \frac{Mba}{EI} + \frac{Ma^2}{2EI}$$

$$\delta_v = \frac{Ma}{2EI}(2b + a)$$ *answer*

Problem 695

Solve Problem 694 if the couple is replaced by a downward load P.

HideClick here to read or hide Solution 695



$$\delta_v = a\theta + \delta$$

$$\delta_v = a\left(\frac{Pab}{EI}\right) + \frac{Pa^3}{3EI}$$

$$\delta_v = \frac{Pa^2b}{EI} + \frac{Pa^3}{3EI}$$

$$\delta_v = \frac{Pa^2}{3EI}(3b + a)$$ *answer*

Problem 696

In Fig. P-696, determine the value of P for which the deflection under P will be zero.

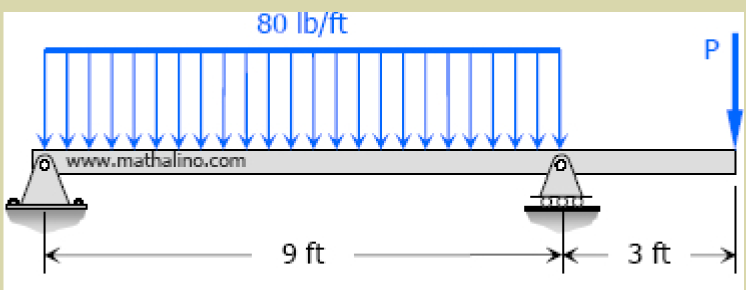
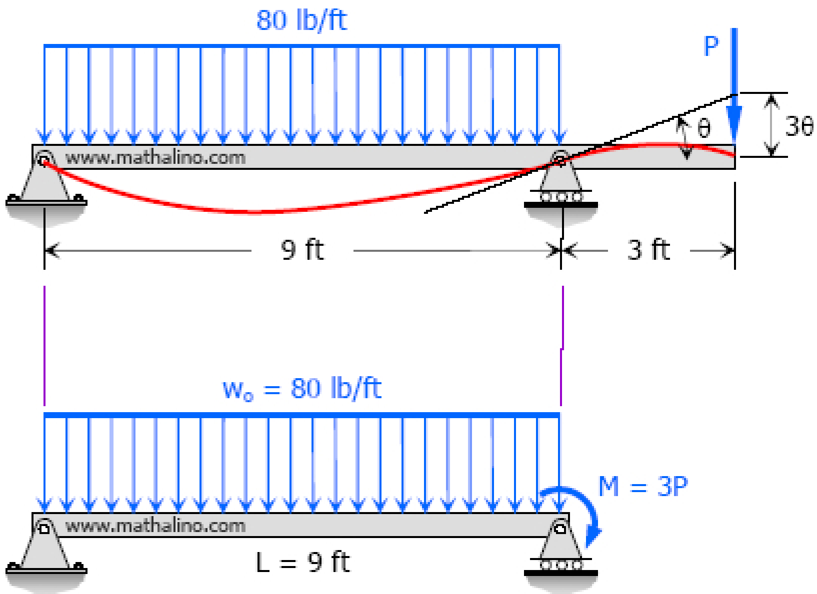


Figure P-696

HideClick here to read or hide Solution 696

Apply Case No. 8 and Case No. 11 to find the slope at the right support.



$$\theta = \frac{w_o L^3}{24EI} - \frac{ML}{3EI}$$

$$\theta = \frac{80(9^3)}{24EI} - \frac{3P(9)}{3EI}$$

$$\theta = \frac{2430}{EI} - \frac{9P}{EI}$$

$$3\theta = \frac{7290}{EI} - \frac{27P}{EI}$$

Use Case No. 1 for the deflection at the free end due to concentrated load P.

$$\delta = \frac{PL^3}{3EI}$$

$$\delta = \frac{P(3^3)}{3EI}$$

$$\delta = \frac{9P}{EI}$$

$$\delta = 3\theta$$

$$\frac{36P}{EI} = \frac{7290}{EI}$$

$P = 202.5 \text{ lb}$ *answer*

Problem 697

For the beam in Prob. 696, find the value of P for which the slope over the right support will be zero.

From Solution 696,

$$\theta = \frac{2430}{EI} - \frac{9P}{EI}$$

$$0 = \frac{2430}{EI} - \frac{9P}{EI}$$

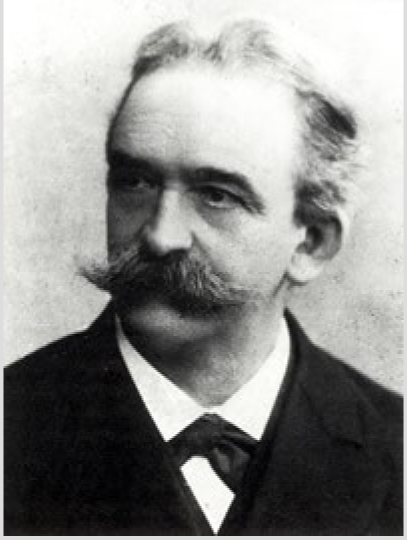
$$\frac{9P}{EI} = \frac{2430}{EI}$$

$P = 270 \text{ lb}$ *answer*

Conjugate Beam Method | Beam Deflection

Slope on real beam = Shear on conjugate beam
Deflection on real beam = Moment on conjugate beam

Properties of Conjugate Beam

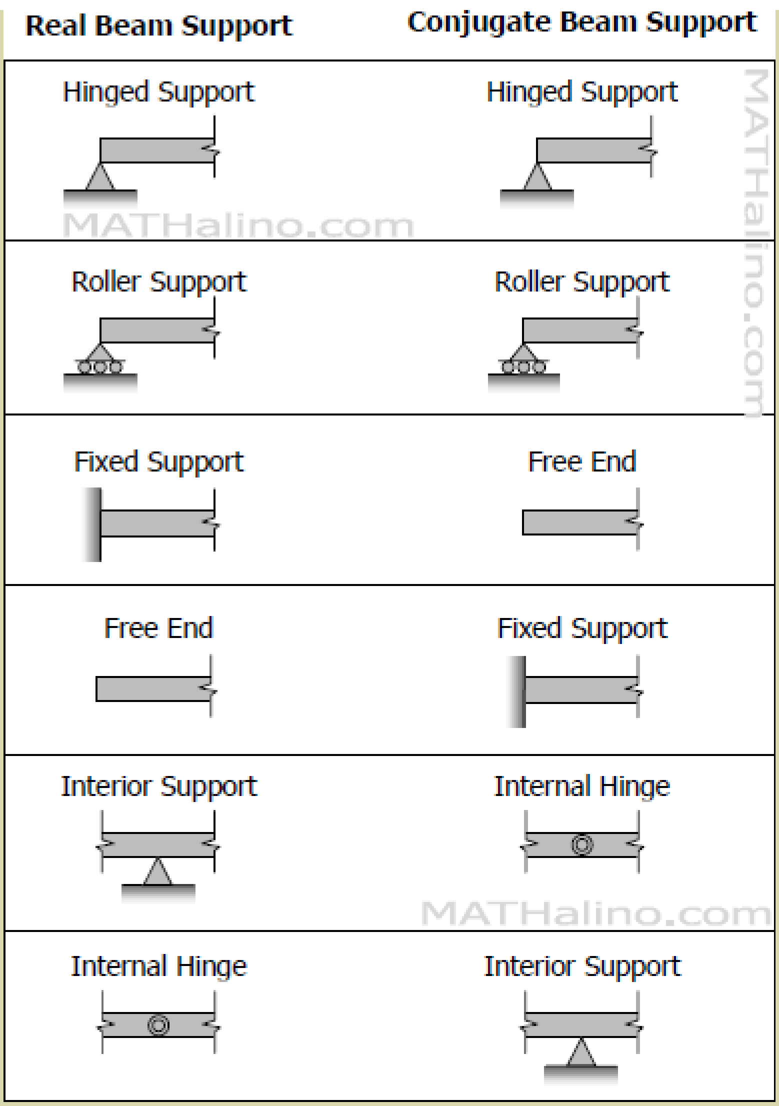


Engr. Christian Otto Mohr

1. The length of a conjugate beam is always equal to the length of the actual beam.
2. The load on the conjugate beam is the M/EI diagram of the loads on the actual beam.
3. A simple support for the real beam remains simple support for the conjugate beam.
4. A fixed end for the real beam becomes free end for the conjugate beam.
5. The point of zero shear for the conjugate beam corresponds to a point of zero slope for the real beam.
6. The point of maximum moment for the conjugate beam corresponds to a point of maximum deflection for the real beam.

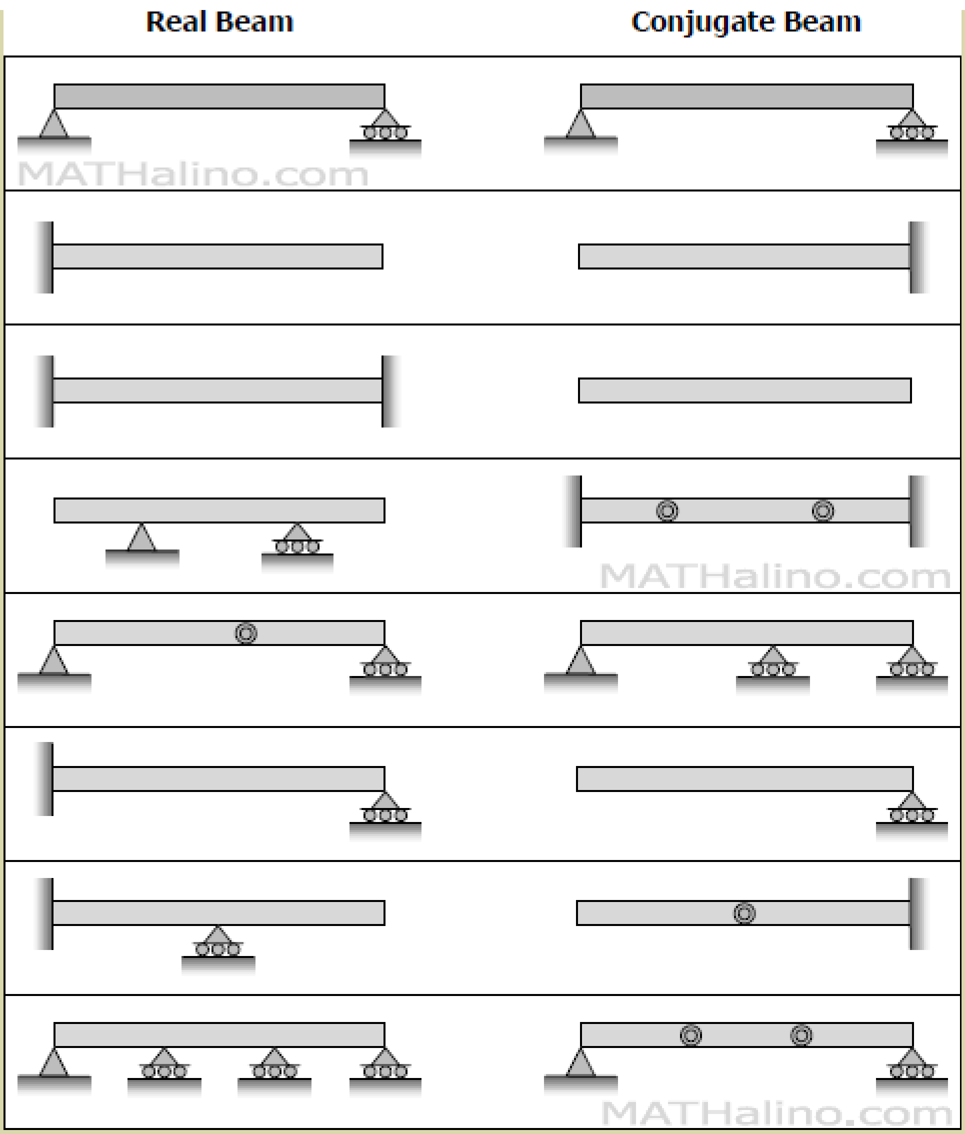
Supports of Conjugate Beam

Knowing that the slope on the real beam is equal to the shear on conjugate beam and the deflection on real beam is equal to the moment on conjugate beam, the shear and bending moment at any point on the conjugate beam must be consistent with the slope and deflection at that point of the real beam. Take for example a real beam with fixed support; at the point of fixed support there is neither slope nor deflection, thus, the shear and moment of the corresponding conjugate beam at that point must be zero. Therefore, the conjugate of fixed support is free end.



Examples of Beam and its Conjugate

The following are some examples of beams and its conjugate. Loadings are omitted.



Problem 653

Compute the midspan value of EIδ for the beam shown in Fig. P-653. (Hint: Draw the M diagram by parts, starting from midspan toward the ends. Also take advantage of symmetry.

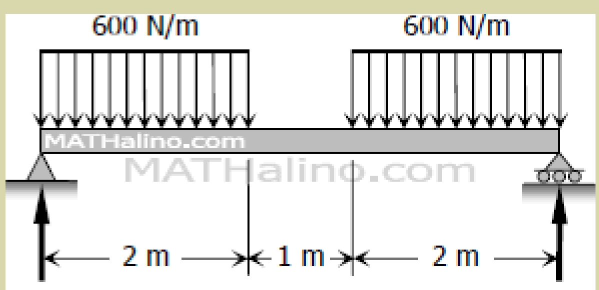


Figure P-653

Solution 653 (Using Moment Diagram by Parts)

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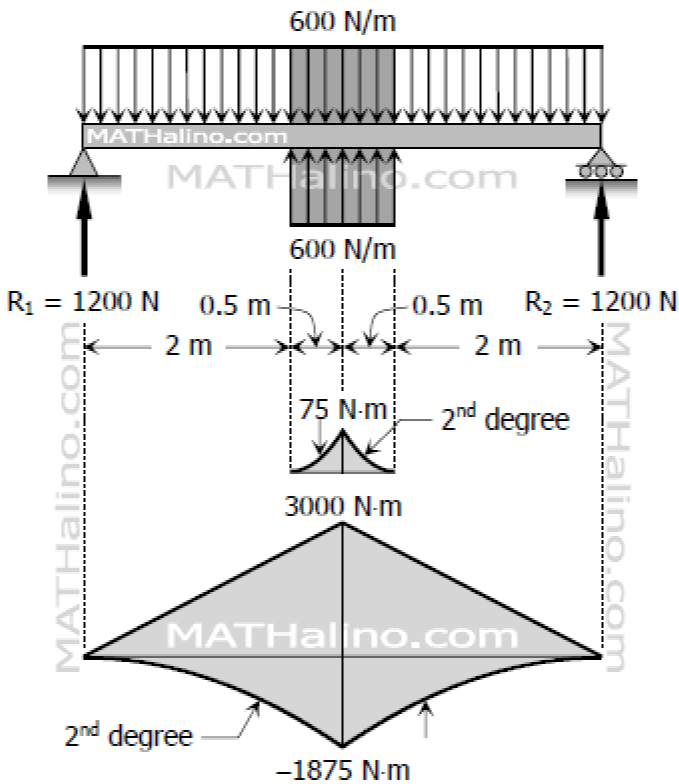
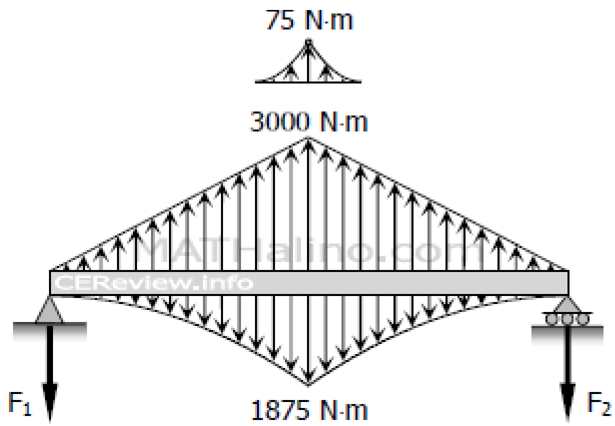
By symmetry,

$R_1 = R_2 = 2(600)$

$R_1 = R_2 = 1200 \text{ N}$



Moment Diagram by Parts



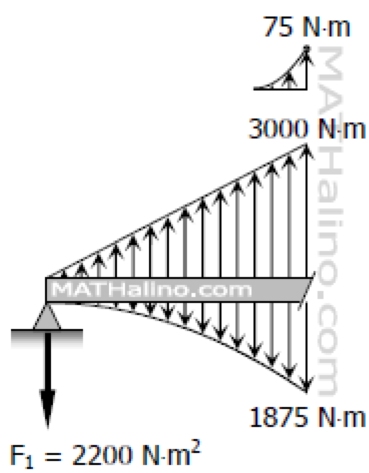
Conjugate Beam Loaded with M/EI Diagram

The loads of conjugate beam are symmetrical, thus,

$F_1 = F_2 = \frac{1}{2}[\,\frac{1}{2}(5)(3000) + \frac{1}{3}(1)(75) - \frac{1}{3}(5)(1875)\,]$

$F_1 = F_2 = 2200 \ \text{N} \cdot \text{m}^2$

For this beam, the maximum deflection will occur at the midspan.



$M_{midspan} = \frac{1}{2}(2.5)(3000)[\frac{1}{3}(2.5)] + \frac{1}{3}(0.5)(75)[\frac{1}{4}(0.5)] - \frac{1}{3}(2.5)(1875)[\frac{1}{4}(2.5)] - 2200(2.5)$

$M_{midspan} = -3350 \ \text{N} \cdot \text{m}^3$

Therefore, the maximum deflection is

$EI\ \delta_{max} = M_{midspan}$

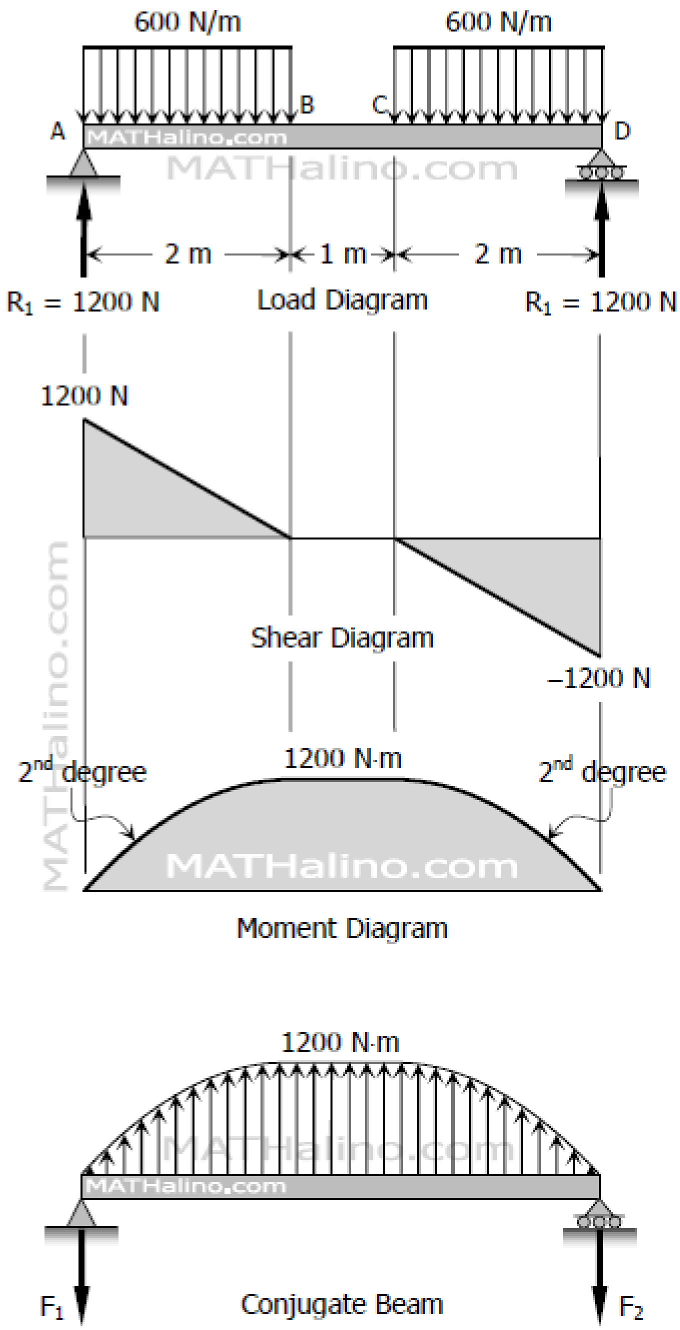
$EI\ \delta_{max} = -3350\ \text{ N}\cdot\text{m}^3$

$EI\ \delta_{max} = 3350\ \text{ N}\cdot\text{m}^3$ below the neutral axis *answer*

Another Solution (Using the Actual Moment Diagram)

HideClick here to show or hide the solution

(*Conjugate beam method using the actual moment diagram*)

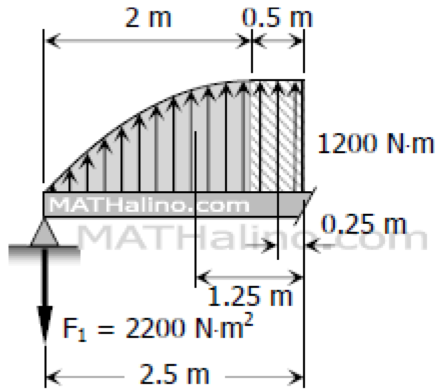


By symmetry of conjugate beam

$F_1 = F_2 = \frac{1}{2}[\,\frac{2}{3}(2)(1200) + 1200(1) + \frac{2}{3}(2)(1200)\,]$

$F_1 = F_2 = 2200\ \text{N} \cdot \text{m}^2$

The maximum deflection will occur at the midspan of this beam



$$M_{midspan} = \frac{2}{3}(2)(1200)(1.25) + 0.5(1200)(0.25) - 2.5(2200)$$

$$M_{midspan} = -3350 \ \text{N} \cdot \text{m}^3$$

Therefore, the maximum deflection is

$$EI\ \delta_{max} = M_{midspan}$$

$$EI\ \delta_{max} = -3350 \ \text{N} \cdot \text{m}^3$$

$EI\ \delta_{max} = 3350 \ \text{N} \cdot \text{m}^3$ below the neutral axis *okay!*

Problem 654

For the beam in Fig. P-654, find the value of EIδ at 2 ft from R_2.

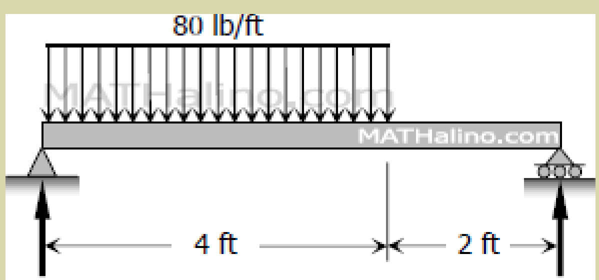


Figure P-654

Solution 654

HideClick here to show or hide the solution

Solving for reactions

$\Sigma M_{R2} = 0$

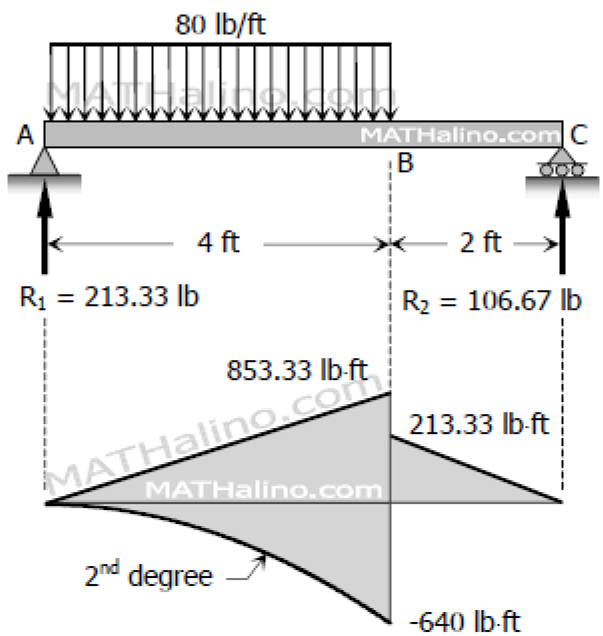
$6R_1 = 80(4)(4)$

$R_1 = 213.33 \text{ lb}$

$\Sigma M_{R1} = 0$

$6R_2 = 80(4)(2)$

$R_2 = 106.67 \text{ lb}$



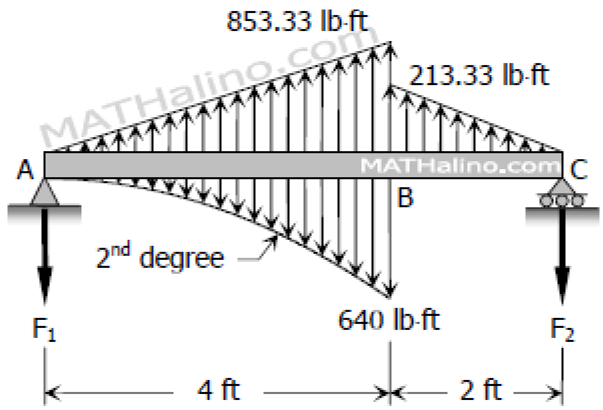
Moment Diagram by Parts

From the conjugate beam

$\Sigma M_A = 0$

$6F_2 + \frac{1}{3}(4)(640)[\frac{3}{4}(4)] = \frac{1}{2}(4)(853.33)[\frac{2}{3}(4)] + \frac{1}{2}(2)(213.33)[4 + \frac{1}{3}(2)]$

$F_2 = 497.77 \text{ lb} \cdot \text{ft}^2$



Conjugate Beam Loaded with M/EI Diagram

$M_B = \frac{1}{2}(2)(213.33[\frac{1}{3}(2)] - 2F_2$

$M_B = \frac{1}{2}(2)(213.33[\frac{1}{3}(2)] - 2(497.77)$

$M_B = -853.32 \text{ lb} \cdot \text{ft}^3$

Thus, the deflection at B is
$EI\,\delta_B = M_B$

$EI\,\delta_B = -853.32 \text{ lb} \cdot \text{ft}^3$

$EI\,\delta_B = 853.32 \text{ lb} \cdot \text{ft}^3$ downward *answer*

Problem 655

Find the value of EIδ under each concentrated load of the beam shown in Fig. P-655.

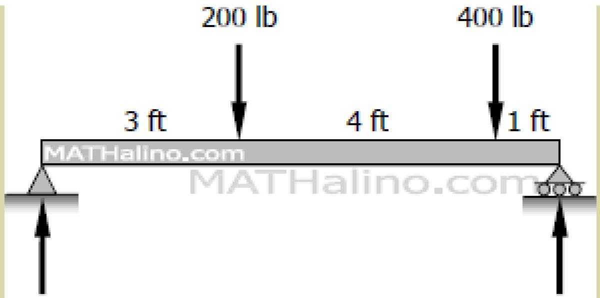


Figure P-655

Solution 655

HideClick here to show or hide the solution

$\Sigma M_D = 0$

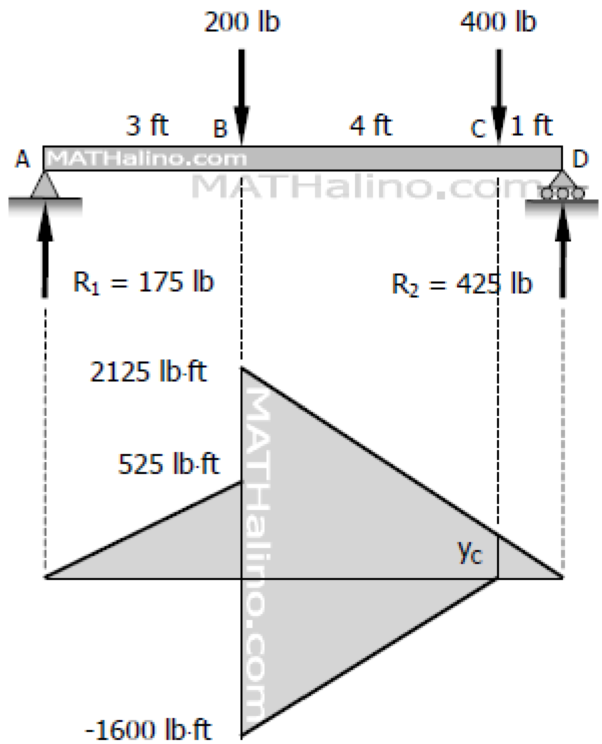
$8R_1 = 200(5) + 400(1)$

$R_1 = 175 \text{ lb}$

$\Sigma M_A = 0$

$8R_2 = 200(3) + 400(7)$

$R_2 = 425 \text{ lb}$



Moment Diagram by Parts

By ratio and proportion

$$\frac{y_C}{1} = \frac{2125}{5}$$

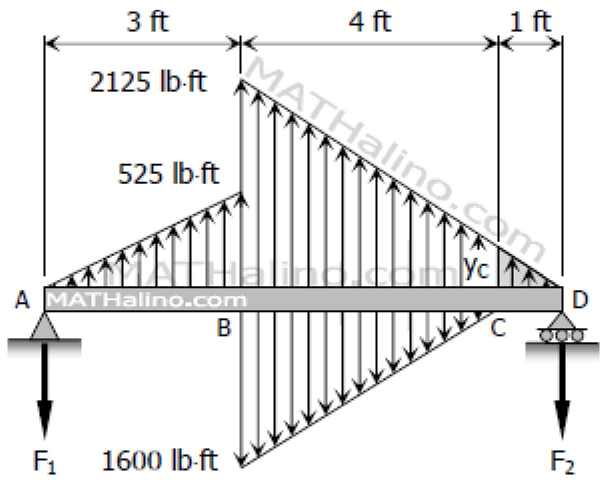
$$y_C = 425 \text{ lb} \cdot \text{ft}$$

From the conjugate beam

$$\Sigma M_D = 0$$

$$8F_1 + \frac{1}{2}(4)(1600)[1 + \frac{2}{3}(4)] = \frac{1}{2}(3)(525)[5 + \frac{1}{3}(3)] + \frac{1}{2}(5)(2125)[\frac{2}{3}(5)]$$

$$F_1 = 1337.5 \text{ lb} \cdot \text{ft}^2$$



Conjugate Beam under M/EI Load

$\Sigma M_A = 0$

$8F_2 + \frac{1}{2}(4)(1600)[\frac{1}{3}(4)] = \frac{1}{2}(3)(525)[\frac{2}{3}(3)] + \frac{1}{2}(5)(2125)[3 + \frac{1}{3}(5)]$

$F_2 = 1562.5 \text{ lb} \cdot \text{ft}^2$

Consider the section to the left of B in conjugate beam
$M_B = \frac{1}{2}(3)(525)[\frac{1}{3}(3)] - 3F_1$

$MB = 787.5 - 3(1337.5)$

$M_B = -3225 \text{ lb} \cdot \text{ft}^3$

Thus, the deflection at B is
$EI\ \delta_B = M_B$

$EI\ \delta_B = 3225 \text{ lb} \cdot \text{ft}^3$ *answer*

Consider the section to the right of C in conjugate beam
$M_C = \frac{1}{2}(1)(y_C)[\frac{1}{3}(1)] - 1F_2$

$M_C = \frac{1}{2}(1)(425)[\frac{1}{3}(1)] - 1(1562.5)$

$M_C = -1491.67 \ \text{lb} \cdot \text{ft}^3$

Thus, the deflection at C is
$EI \ \delta_C = M_C$

$EI \ \delta_C = -1491.67 \ \text{lb} \cdot \text{ft}^3$

$EI \ \delta_C = 1491.67 \ \text{lb} \cdot \text{ft}^3$ downward *answer*

Problem 656

Find the value of EIδ at the point of application of the 200 N·m couple in Fig. P-656.

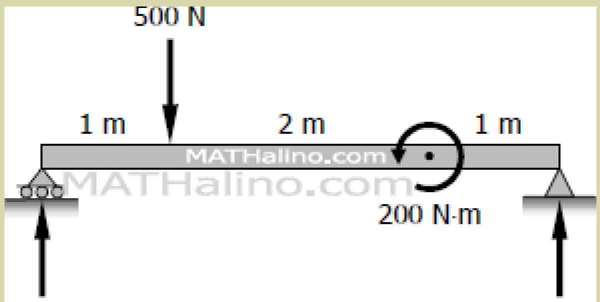


Figure P-656

Solution 656

HideClick here to show or hide the solution

From the real beam
$\Sigma M_D = 0$

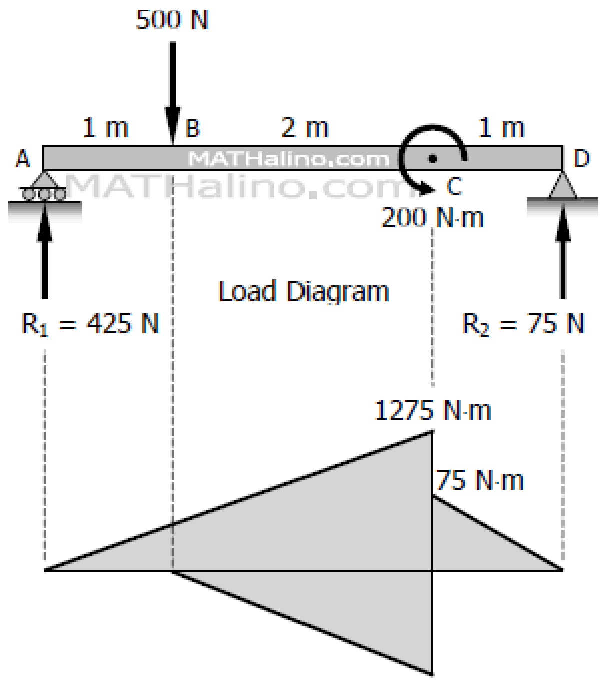
$4R_1 = 3(500) + 200$

$R_1 = 425 \ \text{N}$

$\Sigma M_A = 0$

$4R_2 + 200 = 1(500)$

$R_2 = 75 \ \text{N}$



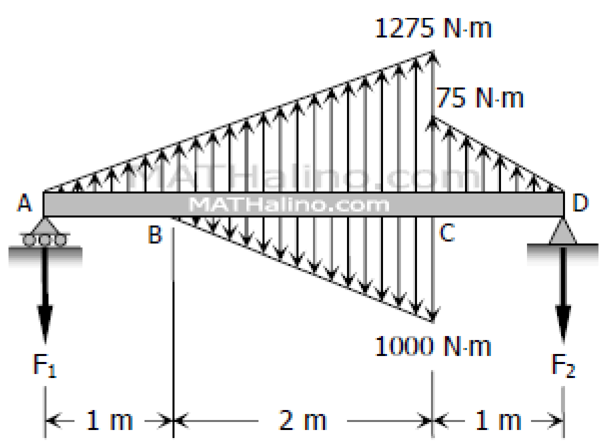
Moment Diagram by Parts

From the conjugate beam

$\Sigma M_A = 0$

$$4F_2 + \frac{1}{2}(2)(1000)[1 + \frac{2}{3}(2)] = \frac{1}{2}(3)(1275)[\frac{2}{3}(3)] + \frac{1}{2}(1)(75)[3 + \frac{1}{3}(1)]$$

$$F_2 = 404.17 \text{ N} \cdot \text{m}^3$$



Conjugate Beam under M/EI Load

$M_C = \frac{1}{2}(1)(75)[\frac{1}{3}(1)] - 1(F_2)$

$M_C = 12.5 - 1(404.17)$

$M_C = -391.67 \text{ N} \cdot \text{m}^3$

Therefore, the deflection at C is
$EI\,\delta_C = M_C$

$EI\,\delta_C = -391.67 \text{ N} \cdot \text{m}^3$

$EI\,\delta_C = 391.67 \text{ N} \cdot \text{m}^3$ downward *answer*

Problem 657

Determine the midspan value of EIδ for the beam shown in Fig. P-657.

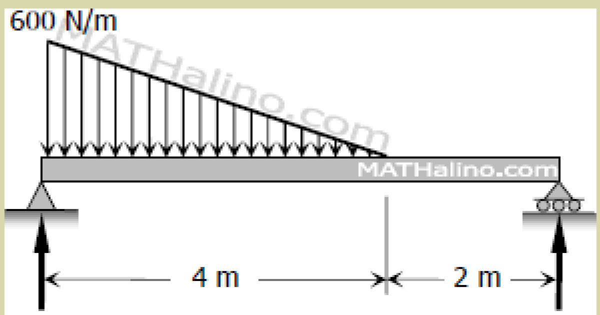


Figure P-657

Solution 657

HideClick here to show or hide the solution

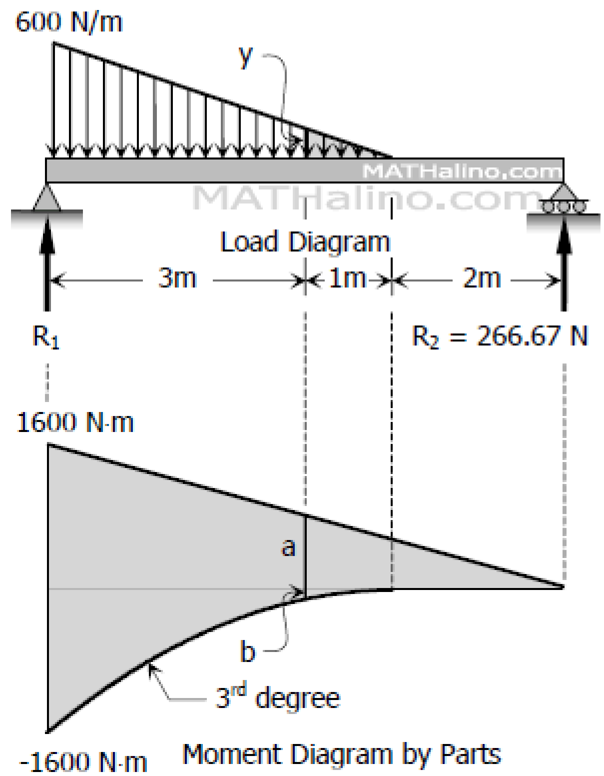
From the load diagram
$\Sigma M_{R1} = 0$

$6R_2 = \frac{1}{2}(4)(600)[\frac{1}{3}(4)]$

$R_2 = 266.67 \text{ N}$

$\frac{y}{1} = \frac{600}{4}$

$y = 150 \;\; \text{N/m}$



From the moment diagram
$a = 3R_2 = 3(266.67)$

$a = 800 \;\; \text{N} \cdot \text{m}$

$b = -\frac{1}{2}(1)(y)[\frac{1}{3}(1)]$

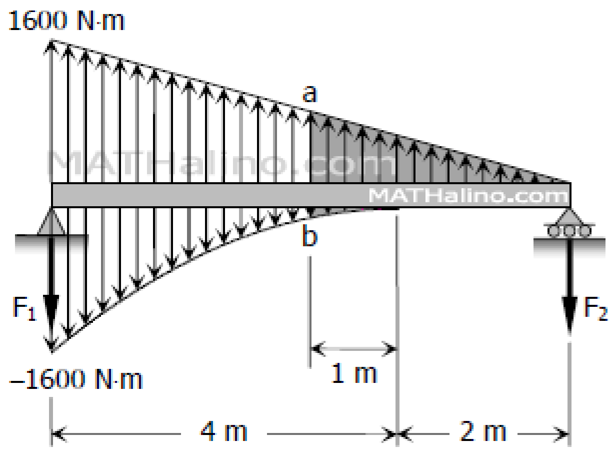
$b = -\frac{1}{6}y = -\frac{1}{6}(150)$

$b = -25 \;\; \text{N} \cdot \text{m}$

From the conjugate beam
$\Sigma M_{F1} = 0$

$6F_2 + \frac{1}{4}(4)(1600)[\frac{1}{5}(4)] = 12(6)(1600)[\frac{1}{3}(6)]$

$F_2 = 1386.67 \;\; \text{N} \cdot \text{m}^2$



Conjugate Beam under M/EI Load

$M_{midspan} = \frac{1}{2}(3a)[\frac{1}{3}(3)] - \frac{1}{4}(1b)[\frac{1}{5}(1)] - 3F_2$

$M_{midspan} = \frac{1}{2}(3)(800)[\frac{1}{3}(3)] - \frac{1}{4}(1)(25)[\frac{1}{5}(1)] - 3(1386.67)$

$M_{midspan} = -2961.25 \ \text{N} \cdot \text{m}^3$

Thus, the deflection at the midspan is

$EI\,\delta_m = M_{midspan}$

$EI\,\delta_m = -2961.25 \ \text{N} \cdot \text{m}^3$

$EI\,\delta_m = 2961.25 \ \text{N} \cdot \text{m}^3$ below the neutral axis *answer*

Problem 658

For the beam shown in Fig. P-658, find the value of EIδ at the point of application of the couple.

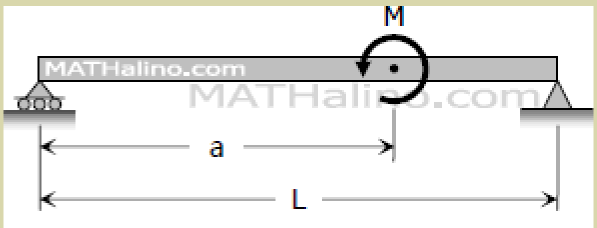
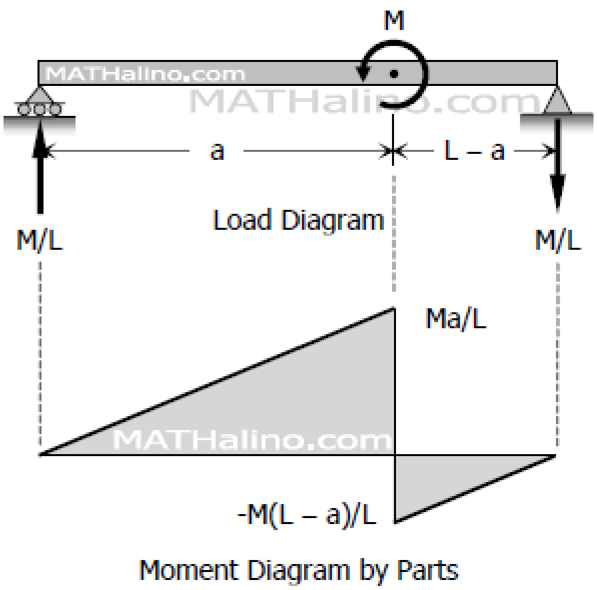


Figure P-658

Solution 658

HideClick here to show or hide the solution



From the conjugate beam
$\Sigma M_A = 0$

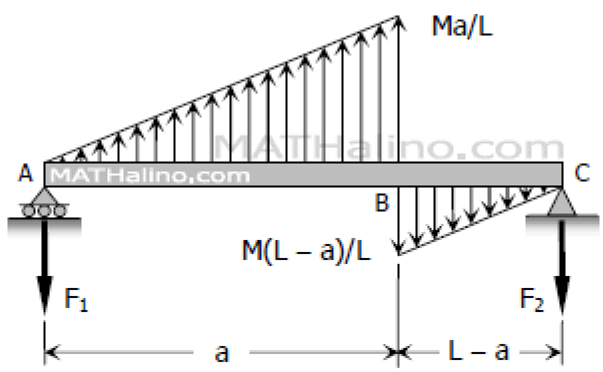
$$LF_2 + \frac{1}{2}(L-a)[M(L-a)/L][a + \frac{1}{3}(L-a)] = \frac{1}{2}a(Ma/L)[\frac{2}{3}a]$$

$$LF_2 + \frac{M(L-a)^2}{2L}[\frac{1}{3}L + \frac{2}{3}a] = \frac{Ma^3}{3L}$$

$$LF_2 + \frac{M(L-a)^2(L+2a)}{6L} = \frac{Ma^3}{3L}$$

$$LF_2 = \frac{Ma^3}{3L} - \frac{M(L-a)^2(L+2a)}{6L}$$

$$F_2 = \frac{Ma^3}{3L^2} - \frac{M(L-a)^2(L+2a)}{6L^2}$$



Conjugate Beam under M/EI Load

$$F_2 = \frac{M}{6L^2}[2a^3 - (L-a)^2(L+2a)]$$

$$F_2 = \frac{M}{6L^2}[2a^3 - (L^2 - 2aL + a^2)(L+2a)]$$

$$F_2 = \frac{M}{6L^2}[2a^3 - (L^3 - 2aL^2 + a^2L + 2aL^2 - 4a^2L + 2a^3)]$$

$$F_2 = \frac{M}{6L^2}[-L^3 + 3a^2L]$$

$$F_2 = \frac{M}{6L}(3a^2 - L^2)$$

$$M_B = -(L-a)F_2 - \tfrac{1}{2}(L-a)[M(L-a)/L][\tfrac{1}{3}(L-a)]$$

$$M_B = -(L-a)\frac{M}{6L}(3a^2 - L^2) - \tfrac{1}{2}(L-a)[M(L-a)/L][\tfrac{1}{3}(L-a)]$$

$$M_B = -\frac{M(L-a)}{6L}[(3a^2 - L^2) + (L-a)^2]$$

$$M_B = -\frac{M(L-a)}{6L}[3a^2 - L^2 + L^2 - 2aL + a^2]$$

$$M_B = -\frac{M(L-a)}{6L}(4a^2 - 2aL)$$

$$M_B = -\frac{M(L-a)}{6L}[-2a(L-2a)]$$

$$M_B = \frac{Ma}{3L}(L-a)(L-2a)$$

$$M_B = \frac{Ma}{3L}(L^2 - 3aL + 2a^2)$$

Thus,

$$EI\,\delta_B = \frac{Ma}{3L}(L^2 - 3aL + 2a^2)$$ *answer*

Strain Energy Method (Castigliano's Theorem) | Beam Deflection

Italian engineer Alberto Castigliano (1847 – 1884) developed a method of determining deflection of structures by strain energy method. His *Theorem of the Derivatives of Internal Work of Deformation* extended its application to the calculation of relative rotations and displacements between points in the structure and to the study of beams in flexure.

Energy of structure is its capacity of doing work and *strain energy* is the internal energy in the structure because of its deformation. By the principle of conservation of energy,

$$U = W_i$$

where U denotes the strain energy and W_i represents the work done by internal forces. The expression of strain energy depends therefore on the internal forces that can develop in the member due to applied external forces.

Castigliano's Theorem for Beam Deflection

For linearly elastic structures, the partial derivative of the strain energy with respect to an applied force (or couple) is equal to the displacement (or rotation) of the force (or couple) along its line of action.

$$\delta = \frac{\partial U}{\partial P} \quad \text{or} \quad \theta = \frac{\partial U}{\partial \bar{M}}$$

Where δ is the deflection at the point of application of force P in the direction of P, θ is the rotation at the point of application of the couple $\bar{M}$ in the direction of $\bar{M}$, and U is the strain energy.

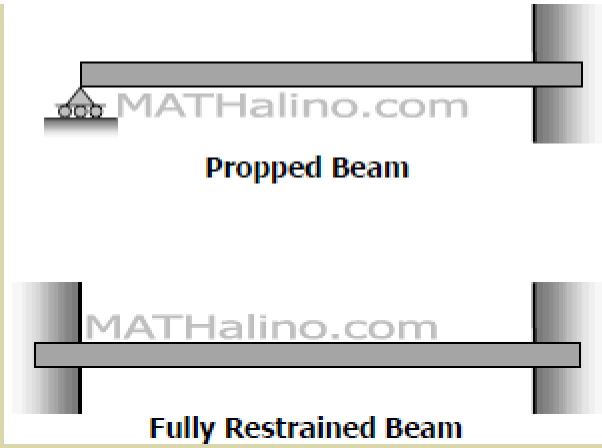
The strain energy of a beam was known to be $U = \int_0^L \frac{M^2}{2EI} dx$. Finding the partial derivative of this expression will give us the equations of Castigliano's deflection and rotation of beams. The equations are written below for convenience.

$$\delta = \int_0^L \left(\frac{\partial M}{\partial P} \right) \frac{M}{EI} dx \quad \text{and} \quad \theta = \int_0^L \left(\frac{\partial M}{\partial \bar{M}} \right) \frac{M}{EI} dx$$

Chapter 07 - Restrained Beams

Restrained Beams

In addition to the equations of static equilibrium, relations from the geometry of elastic curve are essential to the study of indeterminate beams. Such relations can be obtained from the study of deflection and rotation of beam. This section will focus on two types of indeterminate beams; the propped beams and the fully restrained beams.



A propped beam is fixed at one end and propped either at the other end or at any other point along its span. If the simple support is removed, propped beam will become cantilever beam. Fully restrained beam is fixed at both ends as shown in the figure above.

Deflection and Rotation of Propped Beam

Unless otherwise specified, the boundary conditions of propped beams are as follows.

- Deflection at both ends is zero.
- Rotation at fixed support is zero.

Deflection and Rotation of Fully Restrained Beam

Unless otherwise specified, the boundary conditions of propped beams are as follows.

- Deflection at both ends is zero.
- Rotation at both ends is zero.

Application of Double Integration and Superposition Methods to Restrained Beams

Superposition Method

There are 12 cases listed in the method of superposition for beam deflection.

- Cantilever beam with...

1. concentrated load at the free end.
2. concentrated load anywhere on the beam.
3. uniform load over the entire span.

4. triangular load with zero at the free end

5. moment load at the free end.

- Simply supported beam with...

1. concentrated load at the midspan.

2. concentrated load anywhere on the beam span.

3. uniform load over the entire span.

4. triangular load which is zero at one end and full at the other end.

5. triangular load with zero at both ends and full at the midspan.

6. moment load at the right support.

7. moment load at the left support.

See beam deflection by superposition method

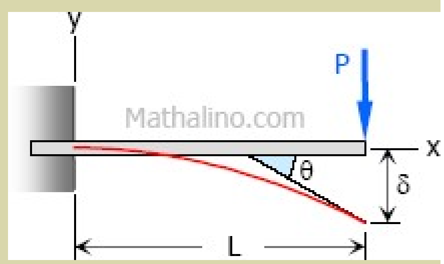
(The slope or deflection at any point on the beam is equal to the resultant of the slopes or deflections at that point caused by each of the load acting separately.

Rotation and Deflection for Common Loadings

Case 1: Concentrated load at the free end of cantilever beam

Maximum Moment

$$M = -PL$$



Slope at end

$$\theta = \frac{PL^2}{2EI}$$

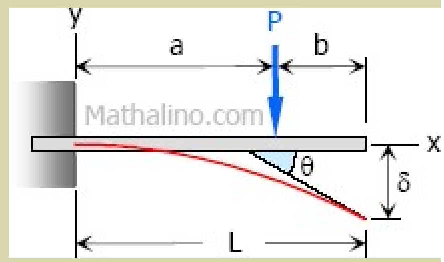
Maximum deflection

$$\delta = \frac{PL^3}{3EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Px^2}{6}(3L - x)$$

Case 2: Concentrated load at any point on the span of cantilever beam



Maximum Moment

$$M = -Pa$$

Slope at end

$$\theta = \frac{Pa^2}{2EI}$$

Maximum deflection

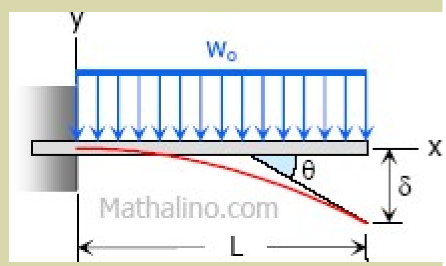
$$\delta = \frac{Pa^3}{6EI}(3L - a)$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Px^2}{6}(3a - x) \text{ for } 0 < x < a$$

$$EI\,y = \frac{Pa^2}{6}(3x - a) \text{ for } a < x < L$$

Case 3: Uniformly distributed load over the entire length of cantilever beam



Maximum Moment

$$M = -\frac{w_o L^2}{2}$$

Slope at end

$$\theta = \frac{w_o L^3}{6EI}$$

Maximum deflection

$$\delta = \frac{w_o L^4}{8EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x^2}{120L}(6L^2 - 4Lx + x^2)$$

Case 4: Triangular load, full at the fixed end and zero at the free end, of cantilever beam

Maximum Moment

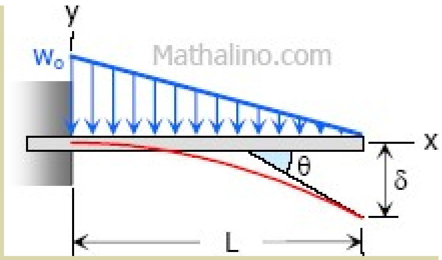
$$M = -\frac{w_o L^2}{6}$$

Slope at end

$$\theta = \frac{w_o L^3}{24EI}$$

Maximum deflection

$$\delta = \frac{w_o L^4}{30EI}$$



Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x^2}{120L}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

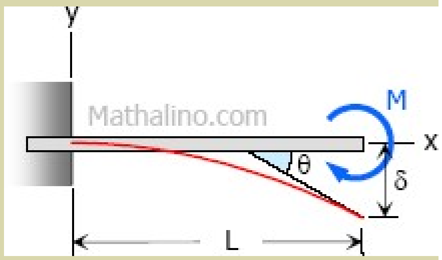
Case 5: Moment load at the free end of cantilever beam

Maximum Moment

$$M = -M$$

Slope at end

$$\theta = \frac{ML}{EI}$$



Maximum deflection

$$\delta = \frac{ML^2}{2EI}$$

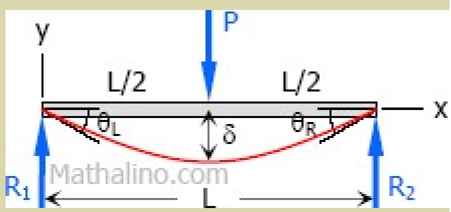
Deflection Equation (y is positive downward)

$$EI\,y = \frac{Mx^2}{2}$$

Case 6: Concentrated load at the midspan of simple beam

Maximum Moment

$$M = \frac{PL}{4}$$



Slope at end

$$\theta_L = \theta_R = \frac{PL^2}{16EI}$$

Maximum deflection

$$\delta = \frac{PL^3}{48EI}$$

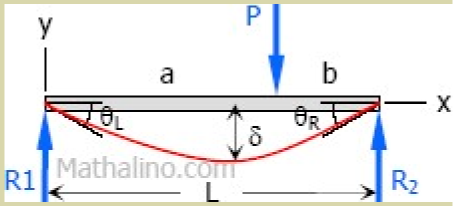
Deflection Equation (y is positive downward)

$$EI\,y = \frac{Px}{12}(\tfrac{3}{4}L^2 - x^2) \text{ for } 0 < x < \tfrac{1}{2}L$$

Case 7: Concentrated load at any point on simple beam

Maximum Moment

$$M = \frac{Pab}{L} \text{ at } x = a$$



Slope at end

$$\theta_L = \frac{Pb(L^2 - b^2)}{6EIL}$$

$$\theta_R = \frac{Pa(L^2 - a^2)}{6EIL}$$

Maximum deflection

$$\delta = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}\,EIL} \text{ at } x = \sqrt{\frac{L^2 - b^2}{3}}$$

Deflection at the center (not maximum)

$$\delta = \frac{Pb}{48EI}(3L^2 - 4b^2) \text{ when } a > b$$

Deflection Equation (y is positive downward)

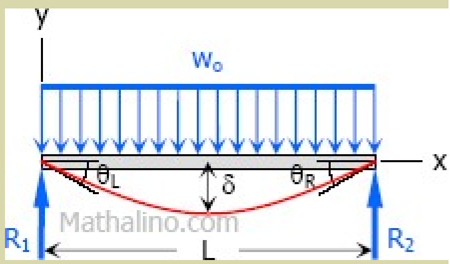
$$EI\,y = \frac{Pbx}{6L}(L^2 - x^2 - b^2) \text{ for } 0 < x < a$$

$$EI\,y = \frac{Pb}{6L}\left[\frac{L}{b}(x-a)^3 + (L^2 - b^2)x - x^3\right] \text{ for } a < x < L$$

Case 8: Uniformly distributed load over the entire span of simple beam

Maximum Moment

$$M = \frac{w_o L^2}{8}$$



Slope at end

$$\theta_L = \theta_R = \frac{w_o L^3}{24EI}$$

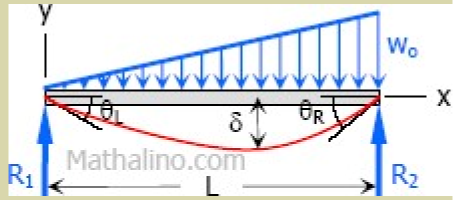
Maximum deflection

$$\delta = \frac{5 w_o L^4}{384EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x}{24}(L^3 - 2Lx^2 + x^3)$$

Case 9: Triangle load with zero at one support and full at the other support of simple beam



Maximum Moment

$$M = \frac{w_o L^2}{9\sqrt{3}}$$

Slope at end

$$\theta_L = \frac{7w_o L^3}{360EI}$$

$$\theta_R = \frac{8w_o L^3}{360EI}$$

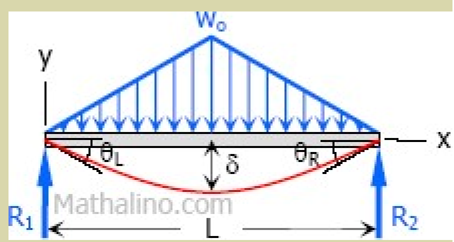
Maximum deflection

$$\delta = \frac{2.5w_o L^4}{384EI} \text{ at } x = 0.519L$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x}{360L}(7L^4 - 10L^2x^2 + 3x^4)$$

Case 10: Triangular load with zero at each support and full at the midspan of simple beam



Maximum Moment

$$M = \frac{w_o L^2}{12}$$

Slope at end

$$\theta_L = \theta_R = \frac{5w_o L^3}{192EI}$$

Maximum deflection

$$\delta = \frac{w_o L^4}{120EI}$$

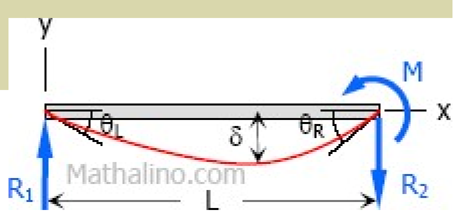
Deflection Equation (y is positive downward)

$$EI\,y = \frac{w_o x}{960L}(25L^4 - 40L^2x^2 + 16x^4) \text{ for } 0 < x < \frac{L}{2}$$

Case 11: Moment load at the right support of simple beam

Maximum Moment

$$M = M$$



Slope at end

$$\theta_L = \frac{ML}{6EI}$$

$$\theta_R = \frac{ML}{3EI}$$

Maximum deflection

$$\delta = \frac{ML^2}{9\sqrt{3}\,EI} \text{ at } x = \frac{L}{\sqrt{3}}$$

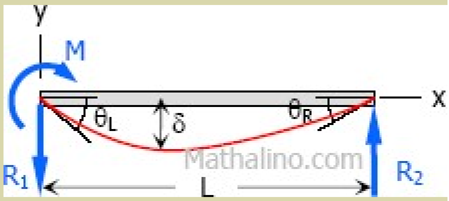
Deflection at the center (not maximum)

$$\delta = \frac{ML^2}{16EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{MLx}{6}\left(1 - \frac{x^2}{L^2}\right)$$

Case 12: Moment load at the left support of simple beam



Maximum Moment

$$M = M$$

Slope at end

$$\theta_L = \frac{ML}{3EI}$$

$$\theta_R = \frac{ML}{6EI}$$

Maximum deflection

$$\delta = \frac{ML^2}{9\sqrt{3}\,EI} \text{ at } x = \left(L - \frac{L}{\sqrt{3}}\right)$$

Deflection at the center (not maximum)

$$\delta = \frac{ML^2}{16EI}$$

Deflection Equation (y is positive downward)

$$EI\,y = \frac{Mx}{6L}(L - x)(2L - x)$$

)for details.

Double Integration Method

Moment at any exploratory section

$$EIy'' = M$$

Slope of the beam at any point

$$EIy' = \int M + C_1$$

Deflection of beam at any point

$$EIy = \int \int M + C_1 x + C_2$$

Problem 704

Find the reactions at the supports and draw the shear and moment diagrams of the propped beam shown in Fig. P-704.

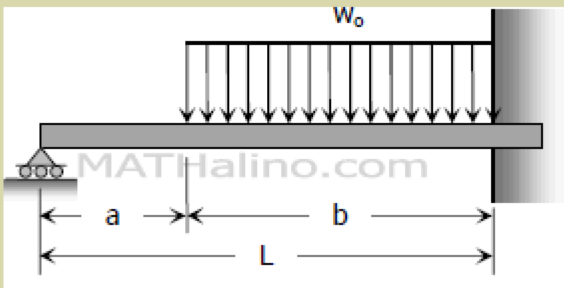


Figure P-704

Solution by Double Integration Method

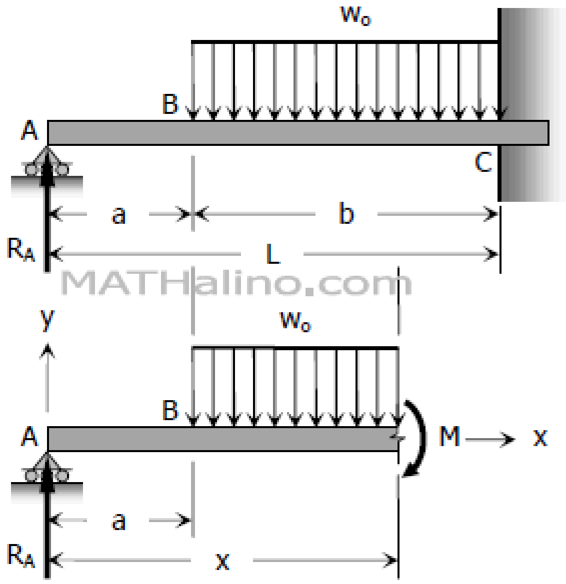
HideClick here to show or hide the solution

$EI\,y'' = M$

$EI\,y'' = R_A x - \frac{1}{2} w_o \langle x - a \rangle^2$

$EI\,y' = \frac{1}{2} R_A x^2 - \frac{1}{6} w_o \langle x - a \rangle^3 + C_1$

$EI\,y = \frac{1}{6}R_A x^3 - \frac{1}{24}w_o\langle x - a\rangle^4 + C_1 x + C_2$



Apply boundary conditions to solve for integration constants C_1 and C_2:

At $x = 0$, $y = 0$, hence $C_2 = 0$

At $x = L$, $y = 0$, hence

$$0 = \frac{1}{6}R_A L^3 - \frac{1}{24}w_o(L - a)^4 + C_1 L + 0$$

$$C_1 L = \frac{1}{24}w_o(L - a)^4 - \frac{1}{6}R_A L^3$$

$$C_1 = \frac{w_o}{24L}(L - a)^4 - \frac{R_A L^2}{6}$$

At $x = L$, $y' = 0$, hence

$$0 = \frac{1}{2}R_A L^2 - \frac{1}{6}w_o(L - a)^3 + \frac{w_o}{24L}(L - a)^4 - \frac{R_A L^2}{6}$$

$$\frac{R_A L^2}{2} - \frac{w_o}{6}(L - a)^3 + \frac{w_o}{24L}(L - a)^4 - \frac{R_A L^2}{6} = 0$$

$$\frac{R_A L^2}{3} = \frac{w_o}{6}(L{-}a)^3 - \frac{w_o}{24L}(L{-}a)^4$$

$$R_A = \frac{w_o}{2L^2}(L{-}a)^3 - \frac{w_o}{8L^3}(L{-}a)^4$$

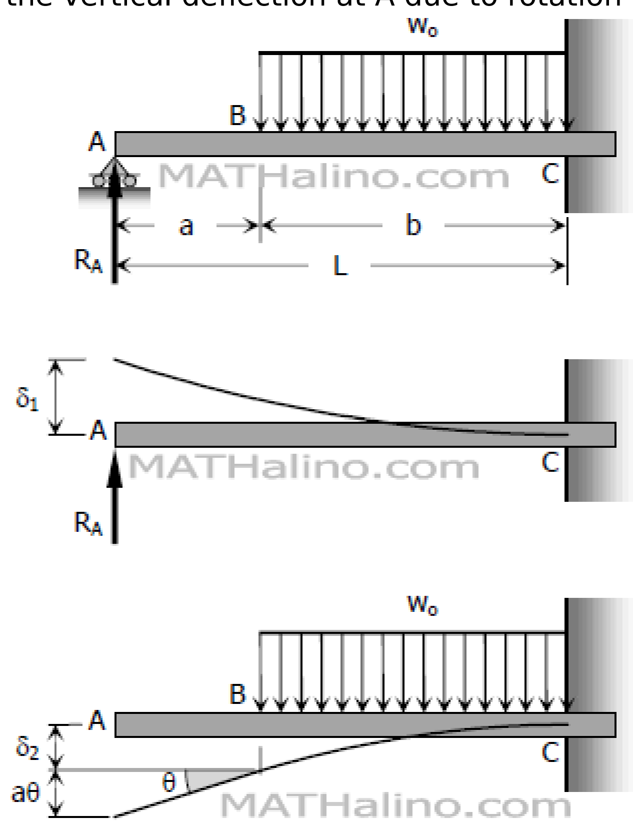
$$R_A = \frac{w_o b^3}{2L^2} - \frac{w_o b^4}{8L^3}$$

$$R_A = \frac{w_o b^3}{8L^3}(4L - b) \qquad \textit{answer}$$

Solution by Superposition Method

HideClick here to show or hide the solution

Deflection at A is zero. Thus, the deflection due to R_A denoted by δ_1 is equal to the sum of deflection at B denoted as δ_2 and the vertical deflection at A due to rotation of B which is denoted by θ.



$$\delta_1 = \delta_2 + a\theta$$

$$\frac{R_A L^3}{3EI} = \frac{w_o b^4}{8EI} + a\left(\frac{w_o b^3}{6EI}\right)$$

$$\frac{R_A L^3}{3} = \frac{w_o b^4}{8} + \frac{w_o a b^3}{6}$$

$$R_A = \frac{3w_o b^4}{8L^3} + \frac{w_o a b^3}{2L^3}$$

$$R_A = \frac{w_o b^3}{L^3}\left(\frac{3b}{8} + \frac{a}{2}\right)$$

$$R_A = \frac{w_o b^3}{L^3}\left(\frac{3b}{8} + \frac{L-b}{2}\right)$$

$$R_A = \frac{w_o b^3}{L^3}\left(\frac{3b + 4L - 4b}{8}\right)$$

$$R_A = \frac{w_o b^3}{8L^3}(4L - b) \qquad \textit{answer}$$

Shear and Moment Diagrams

HideClick here to show or hide the solution

The reaction at the simple support R_A was solved by two different methods above.

$$R_A = \frac{w_o b^3}{8L^3}(4L – b)$$ *answer*

With value of R_A known, it is now easy to solve for M_C and R_C.

Solving for fixed-end moment

$$M_C = R_A L – 0.5 w_o b^2$$

$$M_C = \frac{w_o b^3}{8L^2}(4L – b) – \frac{w_o b^2}{2}$$

$$M_C = \frac{w_o b^2}{8L^2}[\,b(4L – b) - 4L^2\,]$$

$$M_C = \frac{w_o b^2}{8L^2}[\,4Lb – b^2 - 4L^2\,]$$

$$M_C = -\frac{w_o b^2}{8L^2}(4L^2 - 4Lb + b^2)$$ *answer*

Solving for fixed-end shear

$$\Sigma F_V = 0$$

$$R_C + R_A = w_o b$$

$$R_C + \frac{w_o b^3}{8L^3}(4L – b) = w_o b$$

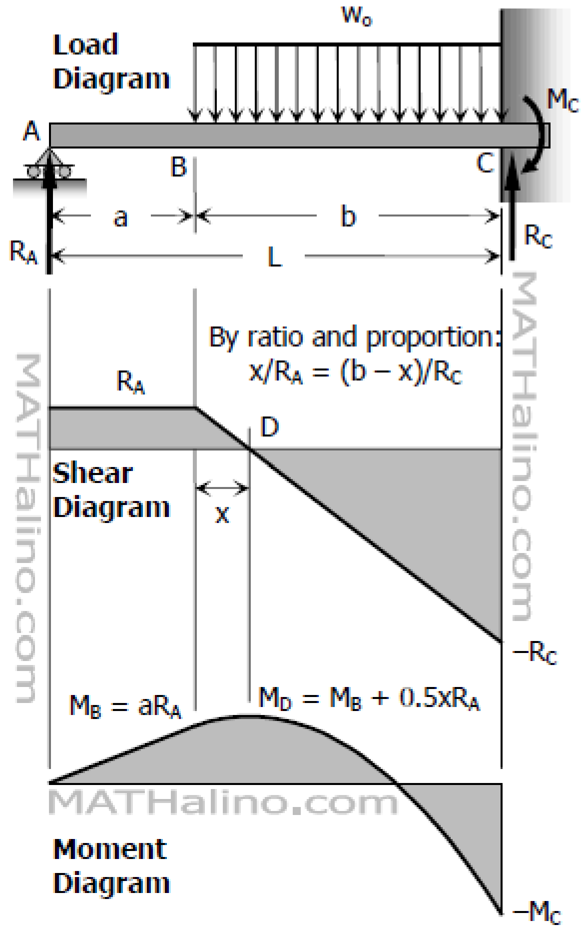
$$R_C = -\frac{w_o b^3}{8L^3}(4L – b) + w_o b$$

$$R_C = \frac{w_o b}{8L^3}[\,-b^2(4L – b) + 8L^3\,]$$

$$R_C = \frac{w_o b}{8L^3}[-4Lb^2 + b^3 + 8L^3]$$

$$R_C = \frac{w_o b}{8L^3}(8L^3 - 4Lb^2 + b^3)$$ *answer*

Shape of Shear Diagram



1. The shear at A is R_A
2. There is no load between segment BC, thus, the shear over BC is uniform and equal to R_A.
3. The load over segment CD is uniform and downward, thus, the shear on this segment is linear and decreasing from R_A to $-R_C$.
4. The shear diagram between BC is zero at D. The location of D can be found by ratio and proportion of the shear triangles of segments BD and DC.

Shape of Moment Diagram

1. The moment at A is zero.
2. The shear between AB is uniform and positive, thus the moment between AB is linearly increasing (straight line) from zero to aR_A.
3. The moment at B is aR_A which is equal to the area of the shear diagram of segment BC.
4. The shear between BC is linear with zero at point D, thus the moment diagram of segment BC is a second degree curve (parabola) with vertex at D. The parabola is open downward because the shear from B to C is decreasing.
5. The moment at D is equal to the sum of the moment at B and the area of the shear diagram of segment BD.

6. Finally, the moment at C is equal to the moment at D minus the area of the shear diagram of segment DC. You can check the accuracy of the moment diagram by finding the moment at C in the load diagram.

In the event that you need to determine the location of zero moment (for construction joint most probably), compute for the area of the moment diagram from D to C and use the squared property of parabola to locate the zero moment. An easier way to find the point of zero moment is to write the moment equation (similar to what we did in double integration method above), equate the equation to zero and solve for x.

Problem 705

Find the reaction at the simple support of the propped beam shown in Fig. P-705 and sketch the shear and moment diagrams.

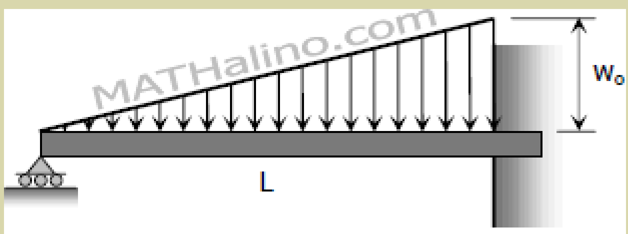
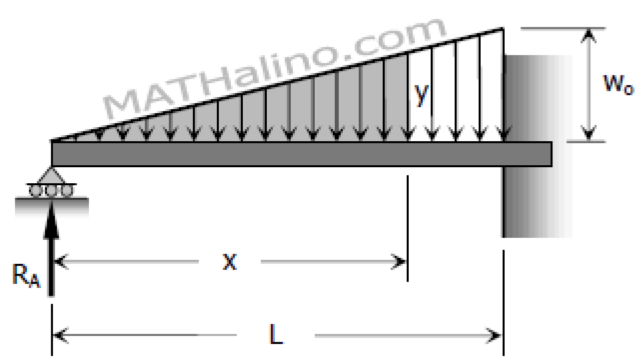


Figure P-705

Solution of Propped Reaction by Double Integration Method

HideClick here to show or hide the solution



$$\frac{y}{x} = \frac{w_o}{L}$$

$$y = \frac{w_o x}{L}$$

Moment at x:

$$M = R_A x - \frac{1}{2}xy(\frac{1}{3}x)$$

$$M = R_A x - \frac{1}{6}x^2 y$$

$$M = R_A x - \frac{x^2}{6}\left(\frac{w_o x}{L}\right)$$

$$M = R_A x - \frac{w_o x^3}{6L}$$

Thus,

$$EI\, y'' = R_A x - \frac{w_o x^3}{6L}$$

$$EI\, y' = \frac{R_A x^2}{2} - \frac{w_o x^4}{24L} + C_1$$

$$EI\, y = \frac{R_A x^3}{6} - \frac{w_o x^5}{120L} + C_1 x + C_2$$

At x = 0, y = 0, thus $C_2 = 0$

At x = L, y’ = 0

$$0 = \frac{R_A L^2}{2} - \frac{w_o L^4}{24L} + C_1$$

$$C_1 = \frac{w_o L^3}{24} - \frac{R_A L^2}{2}$$

Thus, the deflection equation is

$$EI\, y = \frac{R_A x^3}{6} - \frac{w_o x^5}{120L} + \left(\frac{w_o L^3}{24} - \frac{R_A L^2}{2} \right) x$$

At x = L, y = 0

$$0 = \frac{R_A L^3}{6} - \frac{w_o L^5}{120L} + \left(\frac{w_o L^3}{24} - \frac{R_A L^2}{2} \right) L$$

$$0 = \frac{R_A L^3}{6} - \frac{w_o L^4}{120} + \frac{w_o L^4}{24} - \frac{R_A L^3}{2}$$

$$0 = -\frac{R_A L^3}{3} + \frac{w_o L^4}{30}$$

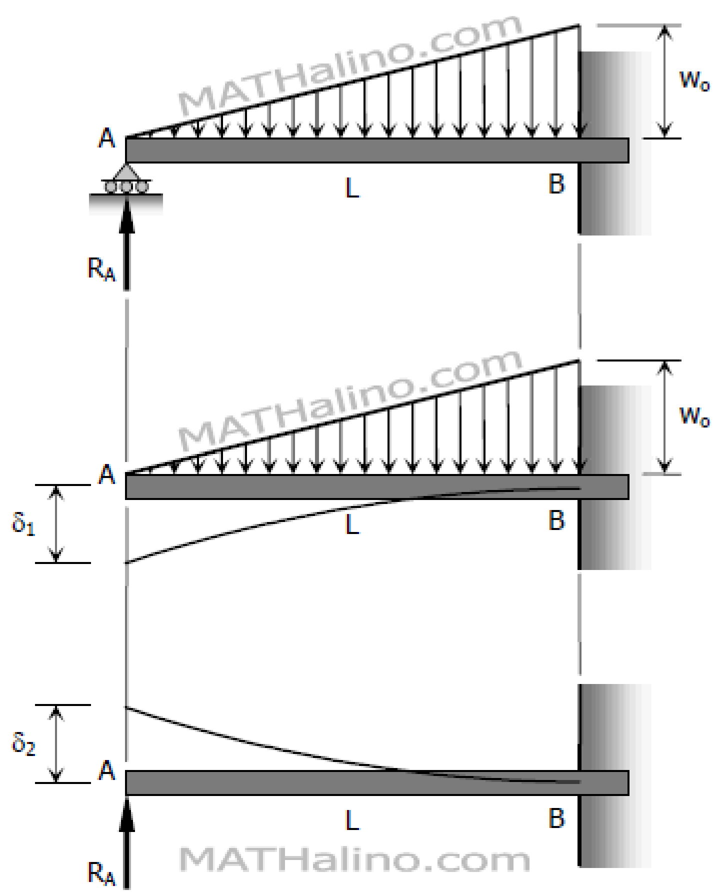
$$\frac{R_A L^3}{3} = \frac{w_o L^4}{30}$$

$$R_A = \frac{w_o L}{10}$$ *answer*

Solution of Propped Reaction by the Method of Superposition

HideClick here to show or hide the solution

Resolve the propped beam into two cantilever beams, one with uniformly varying load and the other with concentrated load as shown below. The concentrated load is the reaction at A.



The deflection at A is zero. Thus, by superposition method, the deflection due to triangular load is equal to the deflection due to concentrated load.

$\delta_1 = \dfrac{w_o L^4}{30EI}$ → deflection due to triangular load

$\delta_2 = \dfrac{R_A L^3}{3EI}$ → deflection due to concentrated load

$\delta_1 = \delta_1$

$$\frac{w_o L^4}{30EI} = \frac{R_A L^3}{3EI}$$

$R_A = \dfrac{w_o L}{10}$ *answer*

Sketching the Shear and Moments Diagrams

HideClick here to show or hide the solution

From the solutions above,

$$R_A = \frac{w_o L}{10}$$

Solving for reaction at B, R_B

$\Sigma F_V = 0$

$$R_A + R_B = \tfrac{1}{2} L w_o$$

$$\frac{w_o L}{10} + R_B = \frac{w_o L}{2}$$

$R_B = \dfrac{2 w_o L}{5}$ *answer*

Solving for moment at B, M_B

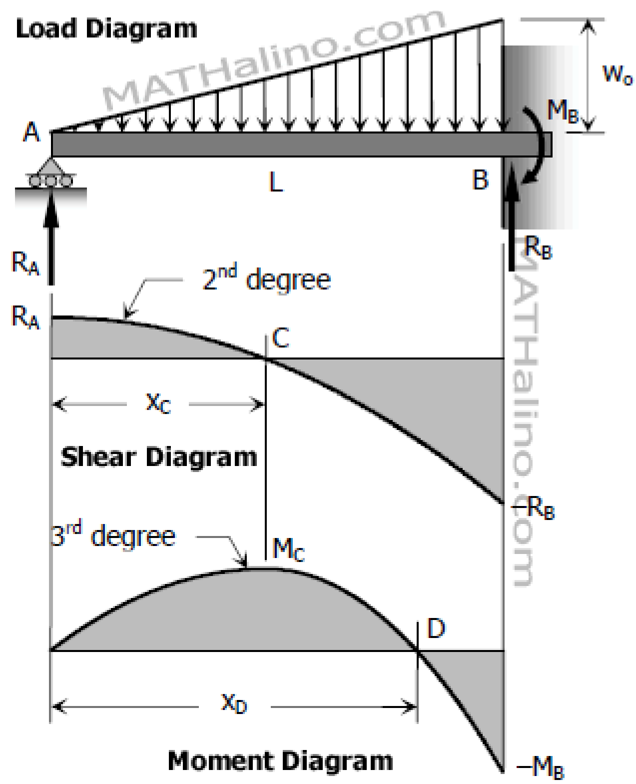
$M_B = R_A L – \frac{1}{2} L w_o (\frac{1}{3} L)$

$$M_B = \left(\frac{w_o L}{10}\right) - \frac{w_o L^2}{6}$$

$$M_B = \frac{w_o L^2}{10} - \frac{w_o L^2}{6}$$

$$M_B = -\frac{w_o L^2}{15}$$ *answer*

To Draw the Shear Diagram



1. The shear at A is equal to R_A
2. The load between AB is negatively increasing from zero at A to w_o at B, thus, the slope of the shear diagram between A and B is decreasing from zero at A to $-w_o$ at B.
3. The load between AB is linear, thus, the shear diagram between AB is a parabola (2nd degree curve) with vertex at A and open downward as stated with the decreasing slope in number 2.
4. The shear at B is equal to $-R_B$. See the magnitude of R_B in the solution above. To compute; shear at B = shear at A - load between AB.
5. The shear diagram between AB is zero at C as shown. Location of C, denoted by x_C can be found by squared property of parabola as follows.

$$\frac{{x_C}^2}{R_A} = \frac{L^2}{R_A + R_B}$$

$${x_C}^2 = \frac{R_A L^2}{R_A + R_B}$$

$${x_C}^2 = \frac{(\frac{1}{10} w_o L) L^2}{\frac{1}{10} w_o L + \frac{2}{5} w_o L}$$

$${x_C}^2 = \frac{\frac{1}{10} w_o L^3}{\frac{1}{2} w_o L}$$

${x_C}^2 = \frac{1}{5}L^2$

$x_C = \frac{1}{\sqrt{5}}L$ from left support

To Draw the Moment Diagram

1. The shear diagram from A to B is a 2nd degree curve, thus, the moment diagram between A and B is a third degree curve.

2. The moment at A is zero.

3. Moment at C is equal to the moment A plus the area of shear diagram between A and C.

   $M_C = \frac{2}{3}x_C R_A$

   $M_C = \frac{2}{3}(\frac{1}{\sqrt{5}}L)(\frac{1}{10}w_o L)$

   $M_C = \dfrac{w_o L^2}{15\sqrt{5}}$

4. The moment at B is equal to $-M_B$, see the magnitude of M_B from the solution above. It is more easy to compute the moment at B by using the load diagram instead of shear diagram. In case, you need to solve the moment at B by the use of shear diagram; $M_B = M_C$ - Area of shear between CB. You can follow the link for an example of finding the area of shear diagram of similar shape.

5. The moment is zero at point D. To locate this point, equate the moment equation developed in double integration method to zero.

   $M = R_A x - \dfrac{w_o x^3}{6L}$ See double integration method above for finding M.

   $M = \dfrac{w_o L x}{10} - \dfrac{w_o x^3}{6L}$

   At D, $x = x_D$ and $M = 0$

   $0 = \dfrac{w_o L x_D}{10} - \dfrac{w_o {x_D}^3}{6L}$

$$\frac{w_o {x_D}^3}{6L} = \frac{w_o L x_D}{10}$$

$$\frac{{x_D}^2}{6L} = \frac{L}{10}$$

$${x_D}^2 = \frac{3L^2}{5}$$

$$x_D = \frac{\sqrt{3}L}{\sqrt{5}} = \frac{\sqrt{3}L}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$x_D = \frac{\sqrt{15}}{5}L$ from left support

Example 03

The propped beam shown in Fig. P -706 is loaded by decreasing triangular load varying from w_o from the simple end to zero at the fixed end. Find the support reactions and sketch the shear and moment diagrams

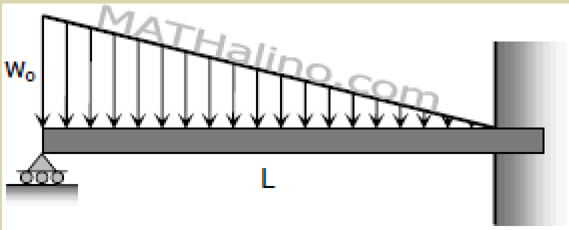


Figure P-706

Solution by Double Integration Method

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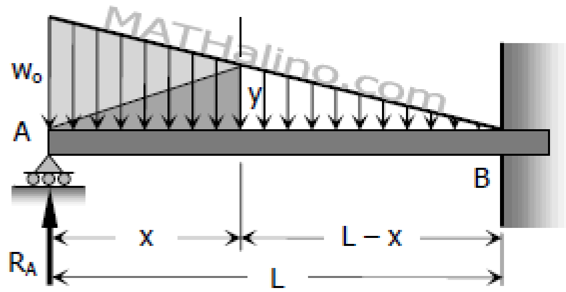
By ratio and proportion:

$$\frac{y}{L - x} = \frac{w_o}{L}$$

$$y = \frac{w_o}{L}(L - x)$$



Solving for moment equation

$M = R_A x - \frac{1}{2}(x)(w_o)(\frac{2}{3}x) - \frac{1}{2}(x)(y)(\frac{1}{3}x)$

$M = R_A x - \frac{1}{3}w_o x^2 - \frac{1}{6}x^2 y$

$M = R_A x - \frac{1}{3}w_o x^2 - \frac{1}{6}x^2 \left[\frac{w_o}{L}(L-x)\right]$

$M = R_A x - \dfrac{w_o}{3}x^2 - \dfrac{w_o}{6L}(Lx^2 - x^3)$

Doing the double integration

$EI\,y'' = R_A x - \dfrac{w_o}{3}x^2 - \dfrac{w_o}{6L}(Lx^2 - x^3)$

$EI\,y' = \dfrac{R_A}{2}x^2 - \dfrac{w_o}{9}x^3 - \dfrac{w_o}{6L}\left(\dfrac{Lx^3}{3} - \dfrac{x^4}{4}\right) + C_1$

$EI\,y = \dfrac{R_A}{6}x^3 - \dfrac{w_o}{36}x^4 - \dfrac{w_o}{6L}\left(\dfrac{Lx^4}{12} - \dfrac{x^5}{20}\right) + C_1 x + C_2$

Boundary conditions

At x = 0, y = 0, $C_2 = 0$

At x = L, y = 0

$0 = \dfrac{R_A L^3}{6} - \dfrac{w_o L^4}{36} - \dfrac{w_o}{6L}\left(\dfrac{L^4}{12} - \dfrac{L^4}{20}\right) + C_1 L + 0$

$0 = \dfrac{R_A L^3}{6} - \dfrac{w_o L^4}{36} - \dfrac{w_o L^4}{180} + C_1 L$

$$C_1 = \frac{w_o L^3}{30} - \frac{R_A L^2}{6}$$

At x = L, y' = 0

$$0 = \frac{R_A L^2}{2} - \frac{w_o L^3}{9} - \frac{w_o}{6L}\left(\frac{L^4}{3} - \frac{L^4}{4}\right) + \left(\frac{w_o L^3}{30} - \frac{R_A L^2}{6}\right)$$

$$0 = \frac{R_A L^2}{2} - \frac{w_o L^3}{9} - \frac{w_o L^3}{72} + \frac{w_o L^3}{30} - \frac{R_A L^2}{6}$$

$$\frac{R_A L^2}{3} = \frac{11 w_o L^3}{120}$$

$$R_A = \frac{11 w_o L}{40} \qquad \textit{answer}$$

Solution by Superposition Method

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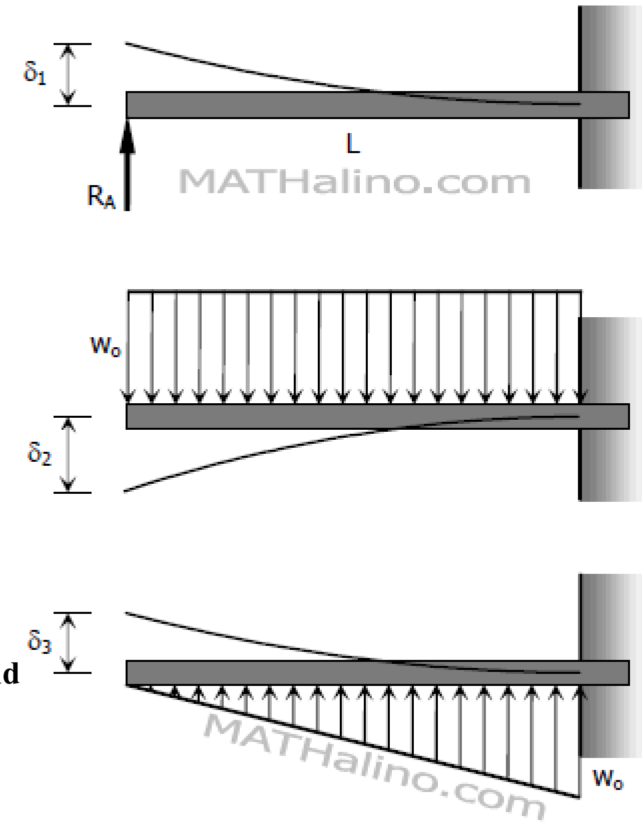
Decreasing triangular load is not listed in the summary of beam loadings. It is therefore necessary to resolve this load into loads that are in the list. In this case, the load was resolved into uniformly distributed load and upward triangular load as shown. The sum of such loads is equal to the one that is given in the problem.

From the figure, it is clear that



$\delta_1 + \delta_3 = \delta_2$

$$\frac{R_A L^3}{3EI} + \frac{w_o L^4}{30EI} = \frac{w_o L^4}{8EI}$$

$$\frac{R_A}{3} + \frac{w_o L}{30} = \frac{w_o L}{8}$$

$$\frac{R_A}{3} = \frac{11 w_o L}{120}$$

$$R_A = \frac{11 w_o L}{40}$$ *answer*

Superposition Method Using Point Load and Integration

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Another way of solving the reaction at A is by the use of derivative. From Case No. 2, the deflection at the free end of cantilever beam is

$$\delta = \frac{Pa^2}{6EI}(3L - a)$$

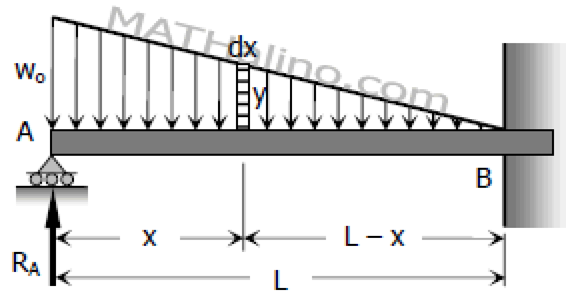
For this problem $P = y\,dx$, $a = L - x$, and $\delta = d\delta_2$.

$$d\delta_2 = \frac{y\,dx(L-x)^2}{6EI}[3L - (L - x)]$$

From the figure,

$$\frac{y}{L-x} = \frac{w_o}{L}$$

$$y = \frac{w_o}{L}(L - x)$$



Thus,

$$d\delta_2 = \frac{\left[\dfrac{w_o}{L}(L - x)\right](L - x)^2}{6EI}[\,3L - (L - x)\,]\,dx$$

$$d\delta_2 = \frac{w_o(L - x)^3}{6L\,EI}[\,3L - (L - x)\,]\,dx$$

$$d\delta_2 = \frac{w_o}{2EI}(L - x)^3\,dx - \frac{w_o}{6L\,EI}(L - x)^4\,dx$$

$$\delta_2 = \frac{w_o}{2EI}\int_0^L (L - x)^3\,dx - \frac{w_o}{6L\,EI}\int_0^L (L - x)^4\,dx$$

δ_1 = end deflection due to R_A

$$\delta_1 = \frac{R_A L^3}{3EI}$$

$$\delta_1 = \delta_2$$

$$\frac{R_A L^3}{3EI} = \frac{w_o}{2EI}\int_0^L (L - x)^3\,dx - \frac{w_o}{6L\,EI}\int_0^L (L - x)^4\,dx$$

$$R_A = \frac{3w_o}{2L^3}\int_0^L (L - x)^3\,dx - \frac{w_o}{2L^4}\int_0^L (L - x)^4\,dx$$

$$R_A = \frac{3w_o}{2L^3}\left[-\frac{(L - x)^4}{4}\right] - \frac{w_o}{2L^4}\left[-\frac{(L - x)^5}{5}\right]$$

$$R_A = -\frac{3w_o}{8L^3}\left[0^4 - L^4\right] + \frac{w_o}{10L^4}\left[0^5 - L^5\right]$$

$$R_A = \frac{3w_oL}{8} - \frac{w_oL}{10}$$

$$R_A = \frac{11w_oL}{40}$$ *answer*

Shear and Moment Diagrams

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The reaction at the simple support R_A was solved using two different methods above.

$R_A = \dfrac{11w_oL}{40}$ *answer*

Solving for vertical reaction at B

$\Sigma F_V = 0$

$R_A + R_B = \frac{1}{2}w_oL$

$\dfrac{11w_oL}{40} + R_B = \dfrac{w_oL}{2}$

$R_B = \dfrac{9w_oL}{40}$ *answer*

Solving for moment reaction at B

$M_B = R_AL - \frac{1}{2}w_oL(\frac{2}{3}L)$

$M_B = \left(\dfrac{9w_oL}{40}\right)L - \dfrac{w_oL^2}{3}$

$M_B = \dfrac{9w_oL^2}{40} - \dfrac{w_oL^2}{3}$

$M_B = -\dfrac{7w_oL^2}{120}$ *answer*

To Draw the Shear Diagram

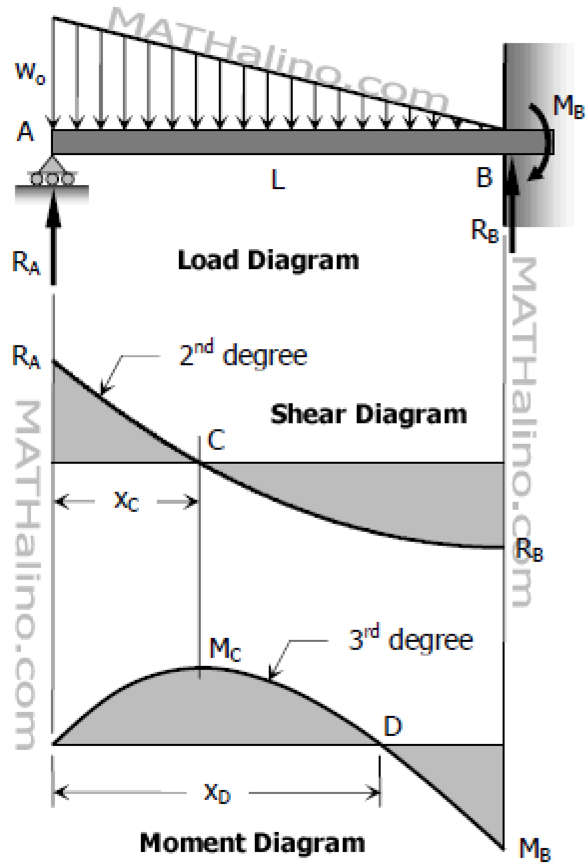
1. The shear at A is equal to R_A

2. The load at AB is increasing from $-w_o$ at A to zero at B, thus, the slope of shear diagram from A to B is also increasing from $-w_o$ at A to zero at B.

3. The load between AB is 1st degree (linear), thus, the shear diagram between AB is 2nd degree (parabolic) with vertex at B and open upward.

4. The magnitude of shear at B is equal to $-R_B$. It is equal to the shear at A minus the triangular load between AB. See the magnitude of R_B above.

5. The shear diagram from A to B will become zero somewhere along AB, the point is denoted by C in the figure. To locate point C, two solutions are presented below.



Location of C, the point of zero shear

Point C is the location of zero shear which may also be the location of maximum moment.

By squared property of parabola

$$\frac{(L - x_C)^2}{R_B} = \frac{L^2}{R_A + R_B}$$

$$(L - x_C)^2 = \frac{R_B L^2}{R_A + R_B}$$

$$(L - x_C)^2 = \frac{(\frac{9}{40} w_o L) L^2}{\frac{11}{40} w_o L + \frac{9}{40} w_o L}$$

$$(L - x_C)^2 = \frac{\frac{9}{40} w_o L^3}{\frac{1}{2} w_o L}$$

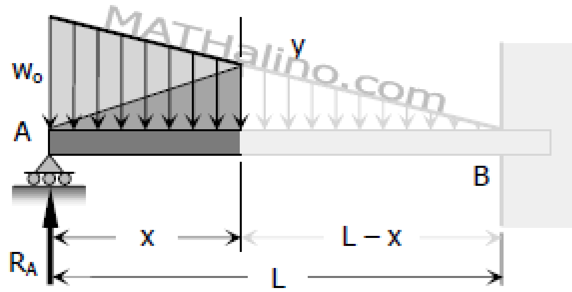
$$(L - x_C)^2 = \frac{9}{20} L^2$$

$$L - x_C = \pm \frac{3}{\sqrt{20}} L$$

$$x_C = \left(1 \pm \frac{3}{\sqrt{20}}\right) L$$

$x_C = 1.6708L$ (*absurd*)
$x_C = 0.3292L$ *answer*

By shear equation
Another method of solving for x_C is to pass an exploratory section anywhere on AB and sum up all the vertical forces to the left of the exploratory section. The location of x_C is where the sum of all vertical forces equate to zero. Consider the figure shown to the right. Note that this figure is the same figure we used to find the reaction R_A by double integration method shown above. The double integration method shows the relationship of x and y which is $y = \dfrac{w_o}{L}(L - x)$.



Sum of all vertical forces
$\Sigma F_V = R_A - \frac{1}{2} w_o x - \frac{1}{2} xy$

$$\Sigma F_V = \frac{11 w_o L}{40} - \frac{w_o x}{2} - \frac{w_o x}{2L}(L - x)$$

$$\Sigma F_V = \frac{11 w_o L}{40} - \frac{w_o x}{2} - \frac{w_o x}{2} + \frac{w_o x^2}{2L}$$

$$\Sigma F_V = \frac{11 w_o L}{40} - w_o x + \frac{w_o x^2}{2L}$$

At point C, $\Sigma F_V = 0$ and $x = x_C$

$$0 = \frac{11 w_o L}{40} - w_o x_C + \frac{w_o {x_C}^2}{2L}$$

$$0 = \frac{11L}{40} - x_C + \frac{{x_C}^2}{2L}$$

$$0 = \tfrac{11}{20} L^2 - 2L x_C + {x_C}^2$$

$${x_C}^2 - 2L x_C = -\tfrac{11}{20} L^2$$

$${x_C}^2 - 2L x_C + L^2 = -\tfrac{11}{20} L^2 + L^2$$

$$(x_C - L)^2 = \frac{9}{20}L^2$$

$$x_C - L = \pm\frac{3}{\sqrt{20}}L$$

$$x_C = \left(1 \pm \frac{3}{\sqrt{20}}\right)L$$

$x_C = 1.6708L$ *(absurd)*
$x_C = 0.3292L$ *answer*

To Draw the Moment Diagram

1. The moment at simple support A is zero.

2. The shear diagram of AB is 2nd degree curve, thus, the moment diagram between AB is 3rd degree curve.

3. The moment at C can be computed in two ways; (a) by solving the area of shear diagram between A and C, and (b) by using the moment equation. For method (a), see the following links for similar situation of solving a partial area of parabolic spandrel.

    - Simple beam with triangular load
    - Simple beam with rectangular and trapezoidal loads

    The solution below is using the approach mentioned in (b). From double integration method of solving R_A, the moment equation is given by

$$M = R_A x - \frac{w_o}{3}x^2 - \frac{w_o}{6L}(Lx^2 - x^3)$$

$$M = \frac{11w_oL}{40}x - \frac{w_o}{3}x^2 - \frac{w_o}{6L}(Lx^2 - x^3)$$

For $x = x_C = 0.3292L$, $M = M_C$

$$M_C = \frac{11w_oL}{40}(0.3292L) - \frac{w_o}{3}(0.3292L)^2 - \frac{w_o}{6L}[\,L(0.3292L)^2 - (0.3292L)^3\,]$$

$$M_C = 0.0423w_oL^2$$

4. In the same manner of solving for M_C, M_B can be found by using x = L. Thus,

$$M_B = \frac{11w_oL^2}{40} - \frac{w_oL^2}{3} - \frac{w_o}{6L}(L^3 - L^3)$$

$$M_B = -\frac{7w_oL^2}{120}$$ which confirms the solution above for M_B.

5. To locate the point of zero moment denoted by D in the figure, we will again use the moment equation; now with M = 0 and x = x_D.

$$0 = \frac{11w_oL}{40}x_D - \frac{w_o}{3}{x_D}^2 - \frac{w_o}{6L}(L{x_D}^2 - {x_D}^3)$$

$$0 = \tfrac{11}{40}L - \frac{1}{3}x_D - \frac{x_D}{6L}(L - x_D)$$

$$0 = \tfrac{33}{20}L^2 - 2Lx_D - x_D(L - x_D)$$

$$0 = \tfrac{33}{20}L^2 - 3Lx_D + {x_D}^2$$

$${x_D}^2 - 3Lx_D = -\tfrac{33}{20}L^2$$

$${x_D}^2 - 3Lx_D + \tfrac{9}{4}L^2 = -\tfrac{33}{20}L^2 + \tfrac{9}{4}L^2$$

$$(x_D - \tfrac{3}{2}L)^2 = \tfrac{3}{5}L^2$$

$$x_D - \tfrac{3}{2}L = \pm\tfrac{3}{5}L$$

$$x_D = \left(\tfrac{3}{2} \pm \tfrac{3}{5}\right) L$$

$x_D = 2.2746L$ (*absurd*)
$x_D = 0.7254L$ *answer*

Problem 707

A couple M is applied at the propped end of the beam shown in Fig. P-707. Compute R at the propped end and also the wall restraining moment.

Figure P-707

Solution 04

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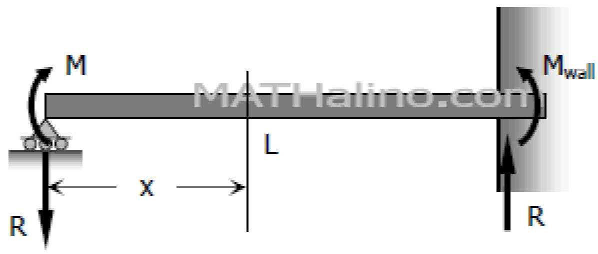
The moment at any point point on the beam which is at distance x from the left support is
$M_x = M - Rx$



By double integration method
$EI\,y'' = M_x$

$EI\,y'' = M - Rx$

$EI\,y' = Mx - \frac{1}{2}Rx^2 + C_1$

$EI\,y = \frac{1}{2}Mx^2 - \frac{1}{6}Rx^3 + C_1x + C_2$

Boundary conditions
At x = 0, y = 0; $C_2 = 0$

At x = L, y = 0;
$0 = \frac{1}{2}ML^2 - \frac{1}{6}RL^3 + C_1L$

$C_1 = \frac{1}{6}RL^2 - \frac{1}{2}ML$

At x = L, y' = 0;
$0 = ML - \frac{1}{2}RL^2 + (\frac{1}{6}RL^2 - \frac{1}{2}ML)$

$0 = M - \frac{1}{2}RL + \frac{1}{6}RL - \frac{1}{2}M$

$\frac{1}{3}RL = \frac{1}{2}M$

$R = \dfrac{3M}{2L}$ *answer*

Problem 708

Two identical cantilever beams in contact at their ends support a distributed load over one of them as shown in Fig. P-708. Determine the restraining moment at each wall.

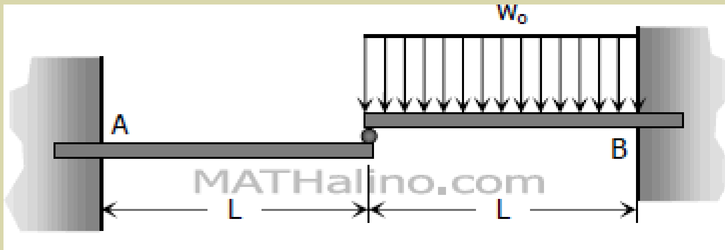


Figure P-708

Solution

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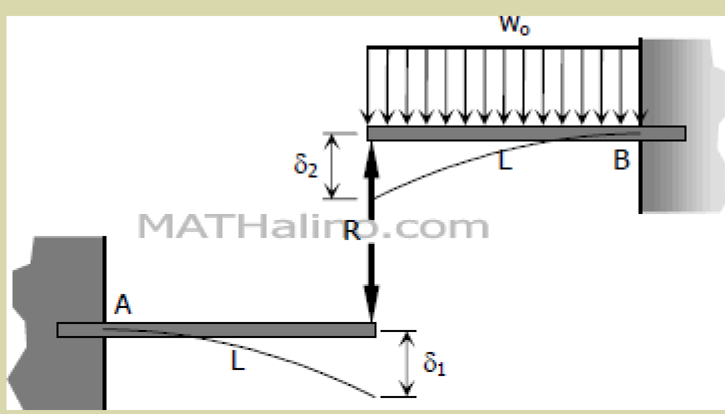

$$\delta_1 = \frac{RL^3}{3EI}$$

$$\delta_2 = \frac{w_oL^4}{8EI} - \frac{RL^3}{3EI}$$

$$\delta_1 = \delta_2$$

$$\frac{RL^3}{3EI} = \frac{w_oL^4}{8EI} - \frac{RL^3}{3EI}$$

$$\frac{2RL^3}{3EI} = \frac{w_oL^4}{8EI}$$

$$R = \frac{3w_oL}{16}$$

$$M_A = -RL$$

$$M_A = -\frac{3w_oL^2}{16} \qquad \textit{answer}$$

$$M_B = RL - w_oL(\tfrac{1}{2}L)$$

$$M_B = \frac{3w_oL^2}{16} - \frac{w_oL^2}{2}$$

$$M_B = -\frac{5w_oL^2}{16} \qquad \textit{answer}$$

Example 06

The beam in Figure PB-006 is supported at the left by a spring that deflects 1 inch for each 300 lb. For the beam E = 1.5 × 10^6 psi and I = 144 in^4. Compute the deflection of the spring.

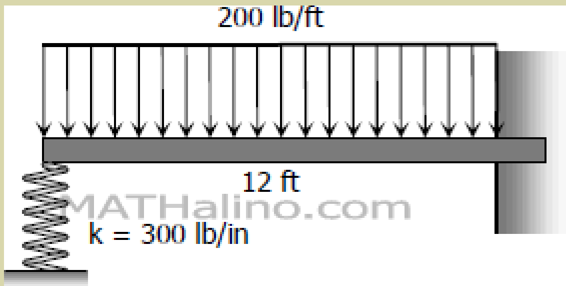

Figure P-709

Solution 06

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Assume there is no spring support at the left end

$$\delta = \frac{w_oL^4}{8EI} = \frac{200(12^4)(12^3)}{8(1.5\times10^6)(144)}$$

$$\delta = 4.1472 \text{ in}$$

Considering the spring reaction

$$\delta - \delta_{spring} = \delta_R$$

$$4.1472 - \frac{R}{k} = \frac{RL^3}{3EI}$$

$$4.1472 - \frac{R}{300} = \frac{R(12^3)(12^3)}{3(1.5\times10^6)(144)}$$

$$4.1472 = 0.0079413R$$

$$R = 522.23 \text{ lb}$$

$$\delta_{spring} = \frac{R}{k} = \frac{522.23}{300}$$

$$\delta_{spring} = 1.74 \text{ in} \qquad \textit{answer}$$

Problem 710

Two timber beams are mounted at right angles and in contact with each other at their midpoints. The upper beam A is 2 in wide by 4 in deep and simply supported on an 8-ft span; the lower beam B is 3 in wide by 8 in deep and simply supported on a 10-ft span. At their cross-over point, they jointly support a load P = 2000 lb. Determine the contact force between the beams.

Solution

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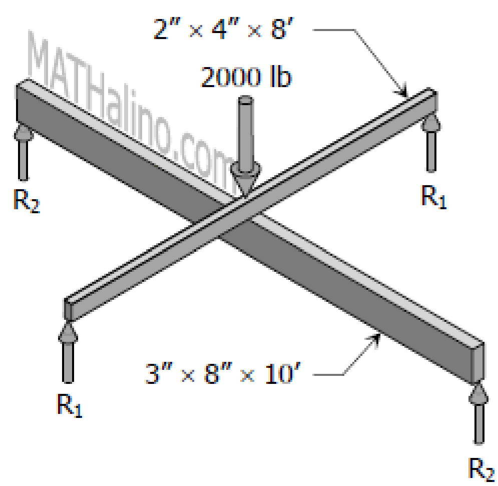
Let
R = contact force between beams A and B
Subscript ($_A$) = for upper beam
Subscript ($_B$) = for lower beam



Moment of inertia

$$I_A = \frac{2(4^3)}{12} = \frac{32}{3} \text{ in}^4$$

$$I_B = \frac{3(8^3)}{12} = 128 \text{ in}^4$$

Note:
The midspan deflection of a simple beam loaded with

concentrated force at the midpoint is given by

$$\delta = \frac{PL^3}{48EI}$$

See Case No. 6 in the Summary of Beam Loadings.

The midspan of upper beam A is under 2000 lb applied load and contact force R. R will act upward at beam A.

$$\delta_A = \frac{2000(8^3)(12^3)}{48E(\frac{32}{3})} - \frac{R(8^3)(12^3)}{48E(\frac{32}{3})}$$

$$\delta_A = \frac{3\,456\,000}{E} - \frac{1728R}{E}$$

The lower beam B is subjected by the contact force R at midspan.

$$\delta_B = \frac{R(10^3)(12^3)}{48E(128)}$$

$$\delta_B = \frac{1125R}{4E}$$

The deflections of upper beam A and lower beam B are obviously equal. R will act downward at beam B.

$$\delta_A = \delta_B$$

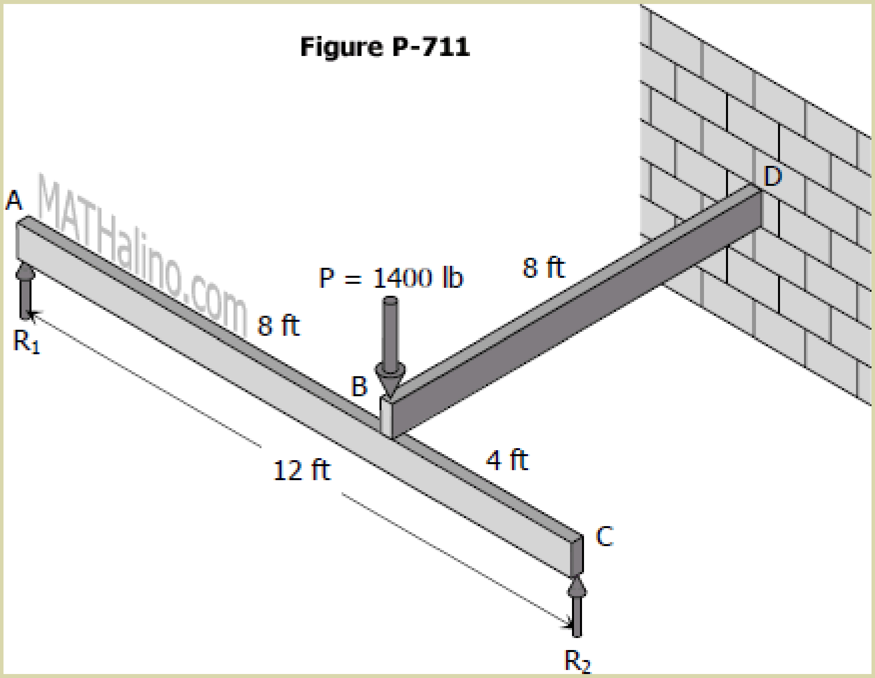
$$\frac{3\,456\,000}{E} - \frac{1728R}{E} = \frac{1125R}{4E}$$

$$3\,456\,000 = \frac{8037R}{4}$$

$R = 1720.04\ \text{lb}$ *answer*

Problem 711

A cantilever beam BD rests on a simple beam AC as shown in Fig. P-711. Both beams are of the same material and are 3 in wide by 8 in deep. If they jointly carry a load P = 1400 lb, compute the maximum flexural stress developed in the beams.



Solution

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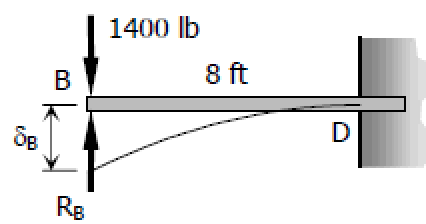
For cantilever beam BD
From Case No. 1 of beam loading cases, the maximum deflection at the end of cantilever beam due to concentrated force at the free end is given by

$$\delta = \frac{PL^3}{3EI}$$

Thus,

$$\delta_B = \frac{1400(8^3)}{3EI} - \frac{R_B(8^3)}{3EI}$$

$$\delta_B = \frac{716\,800}{3EI} - \frac{512R_B}{3EI}$$



For the simple beam AC

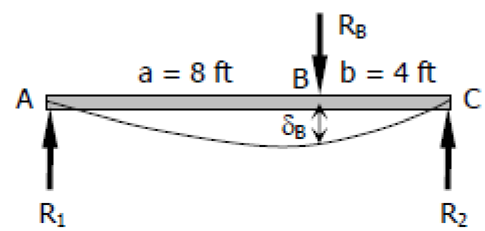
The deflection at distance x from Case No. 7 of different beam loadings is

$$EI\,y = \frac{Pbx}{6L}(L^2 – x^2 – b^2) \quad \text{for} \quad 0xa$$

For $x = a$, the deflection equation will become

$$EI\,y = \frac{Pab}{6L}(L^2 – a^2 – b^2)$$

For beam AC; P = R_B, a = 8 ft, b = 4 ft, and L = 12 ft.



$$\delta_B = \frac{R_B(8)(4)}{6(12)EI}(12^2 – 8^2 – 4^2)$$

$$\delta_B = \frac{4R_B}{9EI}(64)$$

$$\delta_B = \frac{256R_B}{9EI}$$

Solving for the contact force, R_B

$\delta_B = \delta_B$

$$\frac{716\,800}{3EI} - \frac{512R_B}{3EI} = \frac{256R_B}{9EI}$$

$$\frac{716\,800}{3EI} = \frac{1972R_B}{9EI}$$

$R = 1200 \text{ lb}$

Determining the maximum moment

The maximum moment on cantilever beam will occur at D

$M_D = 1200(8) – 1400(8)$

$M_D = -1600 \text{ lb}\cdot\text{ft}$

The maximum moment on simple beam will occur at point B.

$$M_B = \frac{Pab}{L} = \frac{1200(8)(4)}{12}$$

$$M_B = 3200 \text{ lb}\cdot\text{ft}$$

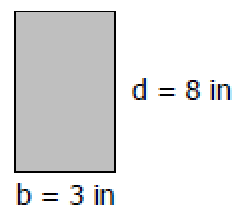
Maximum moment is at point B

$M_{max} = 3200 \text{ lb}\cdot\text{ft}$

Solving for maximum flexural stress

The bending stress of rectangular beam is given by

$$f_b = \frac{6M}{bd^2}$$



Thus,

$$(f_b)_{max} = \frac{6(3200)(12)}{3(8^2)}$$

$(f_b)_{max} = 1200 \text{ psi}$ *answer*

Problem 712

There is a small initial clearance D between the left end of the beam shown in Fig. P-712 and the roller support. Determine the reaction at the roller support after the uniformly distributed load is applied.

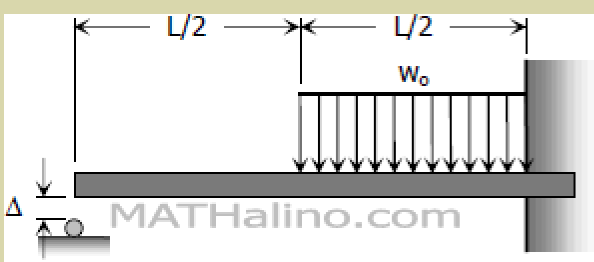


Figure P-712

Solution

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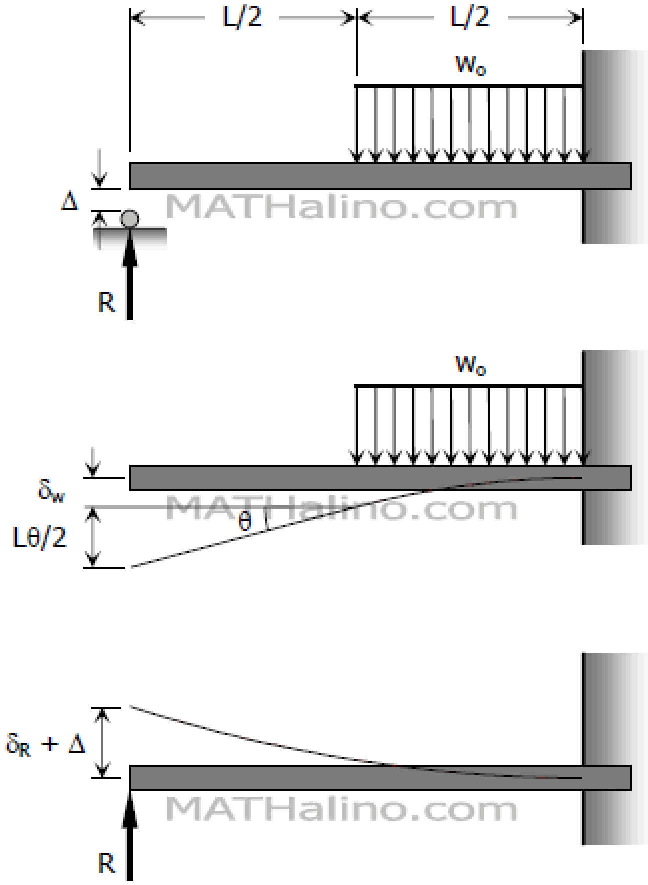
$\delta_R + \Delta = \delta_w + \frac{1}{2}L\theta$



See Case 1 and Case 3 of Superposition Method for formulas:

$$\dfrac{RL^3}{3EI} + \Delta = \dfrac{w_o(\frac{1}{2}L)^4}{8EI} + \dfrac{L}{2}\left[\dfrac{w_o(\frac{1}{2}L)^3}{6EI} \right]$$

$$\dfrac{RL^3}{3EI} + \Delta = \dfrac{w_oL^4}{128EI} + \dfrac{w_oL^4}{96EI}$$

$$\dfrac{RL^3}{3EI} + \Delta = \dfrac{7w_oL^4}{384EI}$$

$$\dfrac{RL^3}{3EI} = \dfrac{7w_oL^4}{384EI} - \Delta$$

$$R = \frac{7w_oL}{128} - \frac{3EI\Delta}{L^3}$$

$$R = \frac{1}{128L^3}(7w_oL^4 - 384EI\Delta)$$ *answer*

Problem 713

Determine the end moment and midspan value of EIδ for the restrained beam shown in Fig. PB-010. (Hint: Because of symmetry, the end shears are equal and the slope is zero at midspan. Let the redundant be the moment at midspan.)

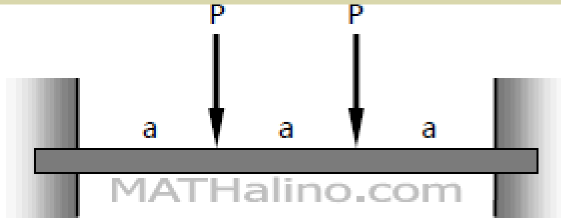


Figure P-713

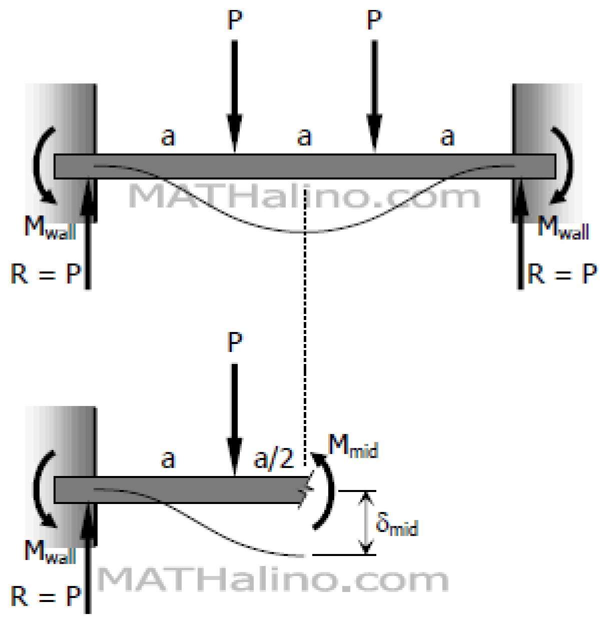
Solution 10

HideClick here to show or hide the solution

$$\Sigma\theta_{mid} = 0$$

See Case 2 and Case 5 of superposition method:

$$\frac{M_{mid}(\frac{3}{2}a)}{EI} - \frac{Pa^2}{2EI} = 0$$



$$\frac{3aM_{mid}}{2EI} = \frac{Pa^2}{2EI}$$

$$M_{mid} = \frac{1}{3}Pa$$

Thus,

$$M_{wall} = M_{mid} - Pa$$

$$M_{wall} = \frac{1}{3}Pa - Pa$$

$$M_{wall} = -\frac{2}{3}Pa$$ *answer*

Again, see Case 2 and Case 5 of superposition method:

$$\delta_{mid} = \frac{Pa^2}{6EI}[\,3(\tfrac{3}{2}a) - a\,] - \frac{M_{mid}(\frac{3}{2}a)^2}{2EI}$$

$$\delta_{mid} = \frac{Pa^2}{6EI}(\tfrac{7}{2}a) - \frac{\frac{1}{3}Pa(\frac{9}{4}a^2)}{2EI}$$

$$\delta_{mid} = \frac{7Pa^3}{12EI} - \frac{3Pa^3}{8EI}$$

$$\delta_{mid} = \frac{5Pa^3}{24EI}$$

$$EI\delta_{mid} = \frac{5}{24}Pa^3$$ *answer*

Problem 714

Determine the end moments of the restrained beam shown in Fig. P-714.

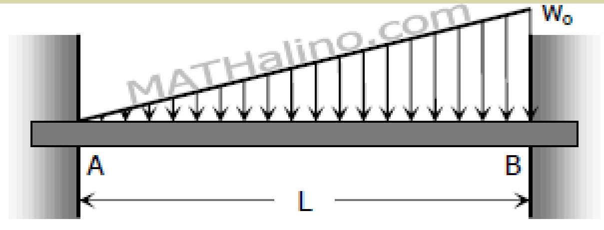


Figure P-714

Solution

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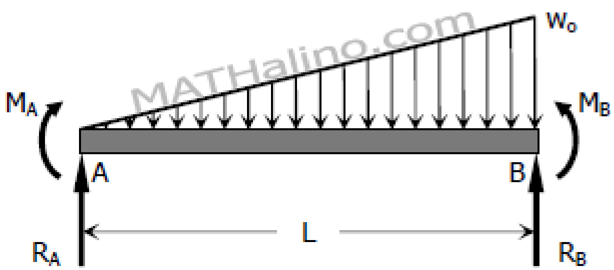
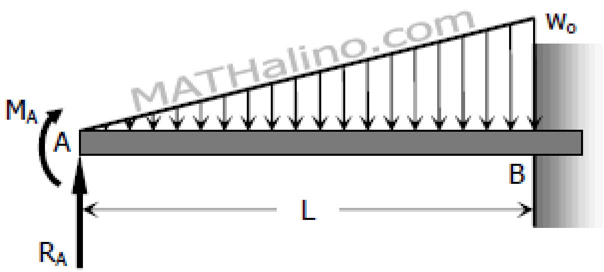
$$\delta_A = 0$$

$$\delta_{triangular\ load} - \delta_{fixed\ end\ moment} - \delta_{reaction\ at\ A} = 0$$





For formulas, see Case 1, Case 4, and Case 5 in superposition method

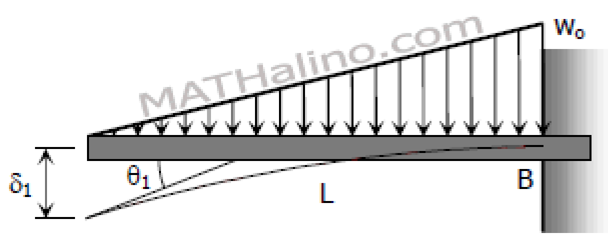
$\delta_1 - \delta_2 - \delta_3 = 0$

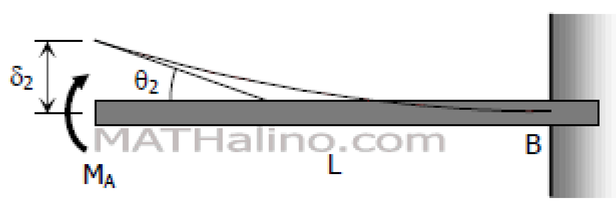
$$\frac{w_o L^4}{30EI} - \frac{M_A L^2}{2EI} - \frac{R_A L^3}{3EI} = 0$$

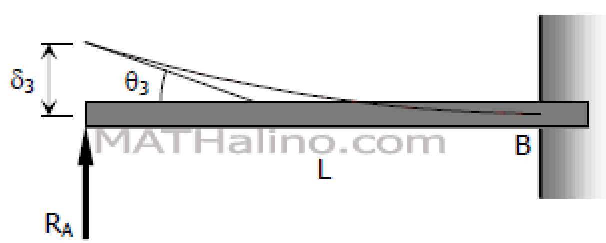
$$\frac{w_o L^2}{30} - \frac{M_A}{2} - \frac{R_A L}{3} = 0$$

$$\frac{w_o L^2}{30} - \frac{M_A}{2} = \frac{R_A L}{3}$$

$$R_A = \frac{w_o L}{10} - \frac{3M_A}{2L}$$







$\theta_A = 0$

$$\theta_{triangular\ load} - \theta_{fixed\ end\ moment} - \theta_{reaction\ at\ A} = 0$$

$$\theta_1 - \theta_2 - \theta_3 = 0$$

$$\frac{w_o L^3}{24EI} - \frac{M_A L}{EI} - \frac{R_A L^2}{2EI} = 0$$

$$\frac{w_o L^2}{24} - M_A - \frac{R_A L}{2} = 0$$

$$w_o L^2 - 24M_A - 12R_A L = 0$$

$$w_o L^2 - 24M_A - 12\left(\frac{w_o L}{10} - \frac{3M_A}{2L}\right)L = 0$$

$$w_o L^2 - 24M_A - \tfrac{6}{5}w_o L^2 + 18M_A = 0$$

$$-6M_A - \tfrac{1}{5}w_o L^2 = 0$$

$$6M_A = -\tfrac{1}{5}w_o L^2$$

$M_A = -\tfrac{1}{30}w_o L^2$ *answer*

$$R_A = \frac{w_o L}{10} - \frac{3(-\frac{1}{30}w_o L^2)}{2L}$$

$$R_A = \frac{w_o L}{10} + \frac{w_o L}{20}$$

$$R_A = \tfrac{3}{20}w_o L$$

$$M_B = M_A + R_A L - \tfrac{1}{2}w_o L(\tfrac{1}{3}L)$$

$$M_B = -\tfrac{1}{30}w_o L^2 + (\tfrac{3}{20}w_o L)L - \tfrac{1}{2}w_o L(\tfrac{1}{3}L)$$

$$M_B = -\tfrac{1}{30}w_o L^2 + \tfrac{3}{20}w_o L^2 - \tfrac{1}{6}w_o L^2$$

$M_B = -\tfrac{1}{20}w_o L^2$ *answer*

Problem 12

Determine the moment and maximum EIδ for the restrained beam shown in Fig. RB-012. (Hint: Let the redundants be the shear and moment at the midspan. Also note that the midspan shear is zero.)

Figure P-715

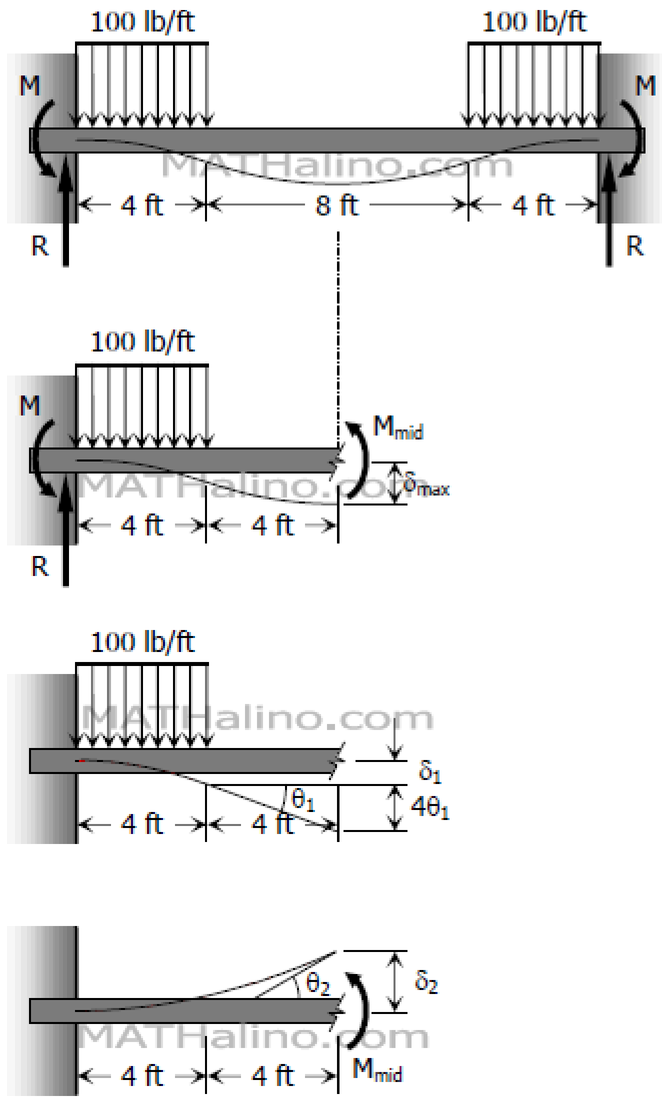
Solution 12

HideClick here to show or hide the solution

$\theta_{mid} = 0$

$\theta_{uniform\ load} - \theta_{moment\ at\ midspan} = 0$

$\theta_1 - \theta_2 = 0$



See Case 3 and Case 5 of superposition method

$$\frac{100(4^3)}{6EI} - \frac{M_{mid}(8)}{EI} = 0$$

$$8M_{mid} = 1066.67$$

$$M_{mid} = 133.33 \text{ ft} \cdot \text{lb}$$

Moment at the fixed support (end moment):

$$M = M_{mid} - 100(4)(2)$$

$M = 133.33 - 800$

$M = -666.67 \text{ ft} \cdot \text{lb}$ *answer*

By symmetry, maximum deflection will occur at the midspan

$\delta_{max} = \delta_1 + 4\theta_1 - \delta_2$

$$\delta_{max} = \frac{100(4^4)}{8EI} + 4\left[\frac{100(4^3)}{6EI}\right] - \frac{M_{mid}(8^2)}{2EI}$$

$$EI\delta_{max} = \frac{100(256)}{8} + \frac{4(100)(64)}{6} - \frac{133.33(64)}{2}$$

$EI\delta_{max} = 3200 + 4266.67 - 4266.67$

$EI\delta_{max} = 3200 \text{ lb} \cdot \text{ft}^3$ *answer*

Note:

The fixed reactions are equal to $R = 100(4) = 400 \text{ lb}$. In the FBD of half left of the beam, R equates to the uniform load; this made the vertical shear at the midspan equal to zero.

Application of Area-Moment Method to Restrained Beams

See deflection of beam by moment-area method for details.

Rotation of beam from A to B

$$\theta_{AB} = \frac{1}{EI}(\text{Area}_{AB})$$

Deviation of B from a tangent line through A

$$t_{B/A} = \frac{1}{EI}(Area_{AB})\,\bar{X}_B$$

Problem 704

Find the reaction at the simple support of the propped beam shown in Figure PB-001 by using moment-area method.

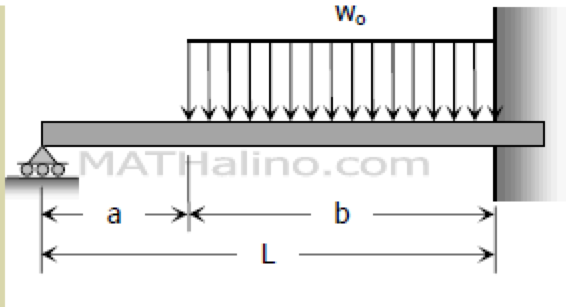


Figure P-704

Solution

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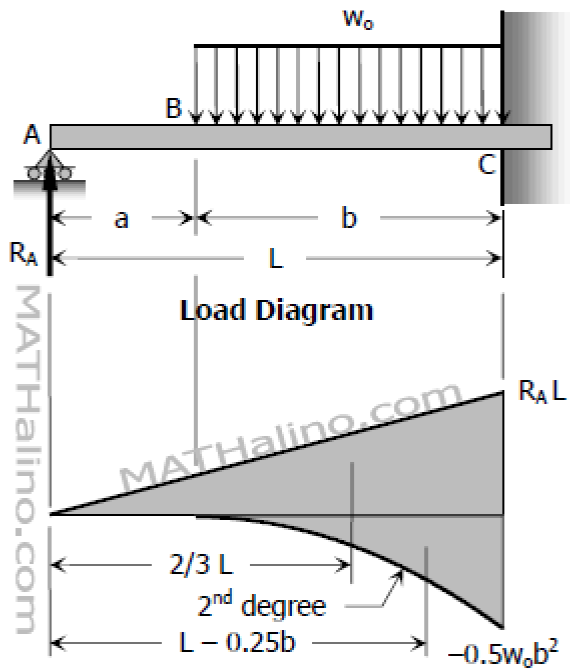
The moment at C due to reaction R_A is R_AL and the moment at C due to uniform load w_o is $-w_ob(0.5b) = -½w_ob^2$.



The deviation at A from tangent line through C is zero. Thus,

$$EI\,t_{A/C} = (Area_{AC})\bar{X}_A = 0$$

$\frac{1}{2}L(R_AL)(\frac{2}{3}L) - \frac{1}{3}b(\frac{1}{2}w_ob^2)(L - \frac{1}{4}b) = 0$

$\frac{1}{3}R_AL^3 - \frac{1}{6}w_ob^3(L - \frac{1}{4}b) = 0$

$\frac{1}{3}R_AL^3 - \frac{1}{24}w_ob^3(4L - b) = 0$

$\frac{1}{3}R_AL^3 = \frac{1}{24}w_ob^3(4L - b)$

$R_A = \dfrac{w_ob^3}{8L^3}(4L - b)$ *answer*

Problem 707

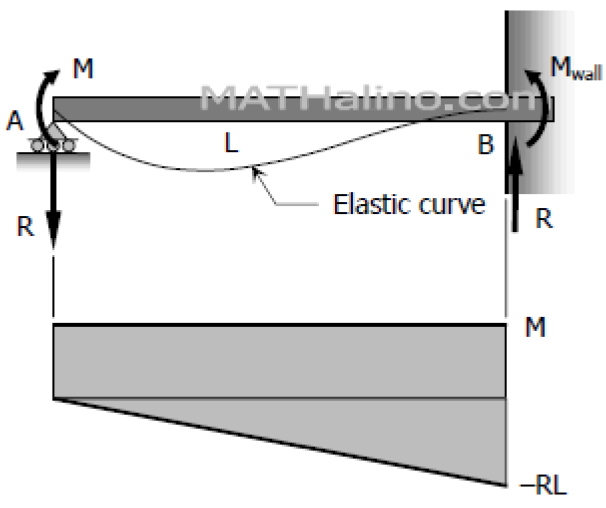
For the propped beam shown in Fig. P-707, solved for vertical reaction R at the simple support.

Figure P-707

Solution

HideClick here to show or hide the solution

Taking the fixed support to be the moment center, the moment diagram by parts is shown to the right.



$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A = 0$

$ML(\frac{1}{2}L) - \frac{1}{2}L(RL)(\frac{2}{3}L) = 0$

$\frac{1}{2}ML^2 - \frac{1}{3}RL^3 = 0$

$R = \dfrac{3M}{2L}$ *answ*

Problem 719

For the propped beam shown in Fig. P-719, determine the propped reaction R and the midspan value of EIδ.

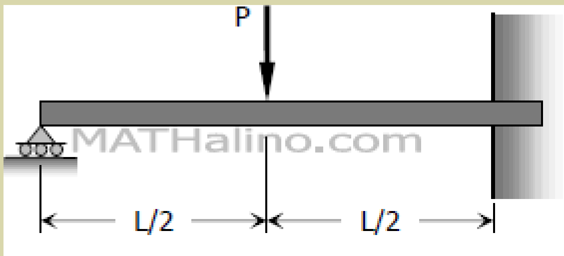


Figure P-719

Solution

HideClick here to show or hide the solution

Solving for the propped reaction R

$EI\,t_{A/B} = 0$

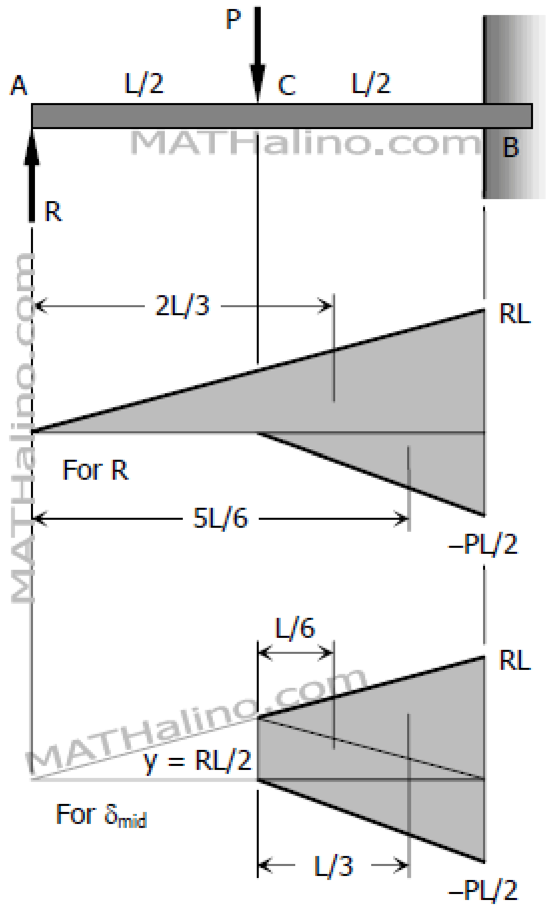
$(Area_{AB}) \cdot \bar{X}_A = 0$

$\frac{1}{2}L(RL)(\frac{2}{3}L) - \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}PL)(\frac{5}{6}L) = 0$

$\frac{1}{3}RL^3 - \frac{5}{48}PL^3 = 0$

$\frac{1}{3}R - \frac{5}{48}P = 0$

$R = \frac{5}{16}P$ *answer*



Solving for the midspan deflection δ_{mid}

$$\frac{y}{\frac{1}{2}L} = \frac{RL}{L}$$

$$y = \tfrac{1}{2}RL$$

$$EI\,t_{C/B} = (Area_{BC}) \cdot \bar{X}_A$$

$$EI\,t_{C/B} = \tfrac{1}{2}(\tfrac{1}{2}L)(RL)(\tfrac{1}{3}L) + \tfrac{1}{2}(\tfrac{1}{2}L)(\tfrac{1}{2}RL)(\tfrac{1}{6}L) - \tfrac{1}{2}(\tfrac{1}{2}L)(\tfrac{1}{2}PL)(\tfrac{1}{3}L)$$

$$EI\,t_{C/B} = \tfrac{1}{12}RL^3 + \tfrac{1}{48}RL^3 - \tfrac{1}{24}PL^3$$

$$EI\,t_{C/B} = \tfrac{5}{48}RL^3 - \tfrac{1}{24}PL^3$$

$$EI\,t_{C/B} = \tfrac{5}{48}(\tfrac{5}{16}P)L^3 - \tfrac{1}{24}PL^3$$

$EI\,t_{C/B} = \frac{25}{768}PL^3 - \frac{1}{24}PL^3$

$EI\,t_{C/B} = -\frac{7}{768}PL^3$

Thus,
$EI\,\delta_{mid} = \frac{7}{768}PL^3$ *answer*

Problem 720

Find the reaction at the simple support of the propped beam shown in Fig. P-705 by using moment-area method.

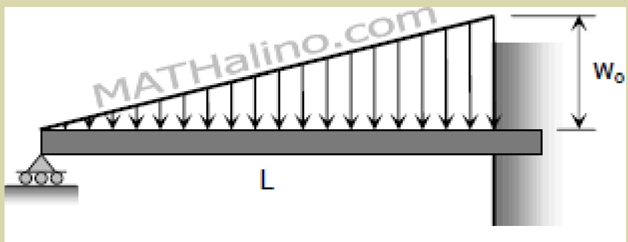


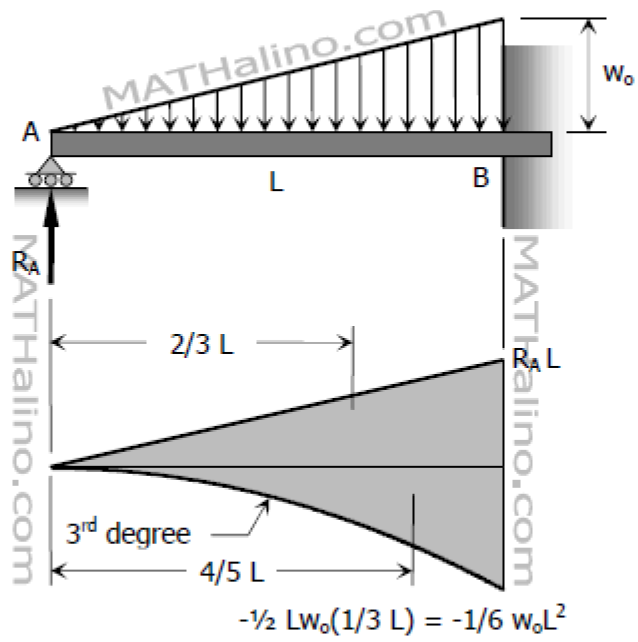
Figure P-705

Solution

HideClick here to show or hide the solution

The moment at B due to R_A is R_AL and the moment at B due to triangular load is $-1/6\ w_oL^2$

Solution of R_A by Moment-Area Method



$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A = 0$$

$$\frac{1}{2}L(R_AL)(\frac{2}{3}L) - \frac{1}{4}L(\frac{1}{6}w_oL^2)(\frac{4}{5}L) = 0$$

$$\frac{1}{3}R_AL^3 - \frac{1}{30}w_oL^4 = 0$$

$$\frac{1}{3}R_AL^3 = \frac{1}{30}w_oL^4$$

$$R_A = \frac{w_oL}{10} \qquad \textit{answer}$$

Problem 721

By the use of moment-are method, determine the magnitude of the reaction force at the left support of the propped beam in Fig. P-706.

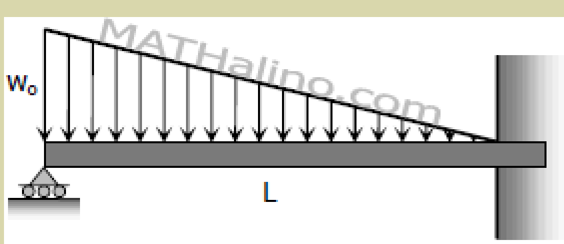


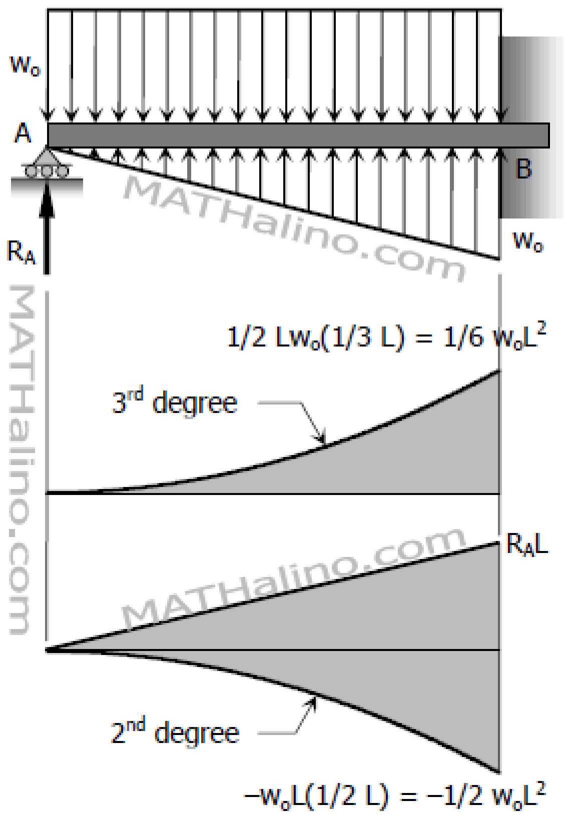
Figure P-706

Solution

HideClick here to show or hide the solution

Transform the triangular load into a downward uniformly distributed load and upward increasing load. We do this so that we can easily draw the moment diagram by parts with moment center at the fixed support.

Solution of R_A by Moment-Area Method



$$EI\,t_{A/B} = (Area_{AB})\,\bar{X}_A = 0$$

$$\frac{1}{4}(L)(\frac{1}{6}w_o L^2)(\frac{4}{5}L) + \frac{1}{2}(L)(R_A L)(\frac{2}{3}L) - \frac{1}{3}(L)(\frac{1}{2}w_o L^2)(\frac{3}{4}L) = 0$$

$$\frac{w_o L^4}{30} + \frac{R_A L^3}{3} - \frac{w_o L^4}{8} = 0$$

$$\frac{R_A L^3}{3} = \frac{11 w_o L^4}{120}$$

$$R_A = \frac{11w_oL}{40} \qquad \textit{answer}$$

Problem 722

For the beam shown in Fig. P-722, compute the reaction R at the propped end and the moment at the wall. Check your results by letting b = L and comparing with the results in Problem 707.

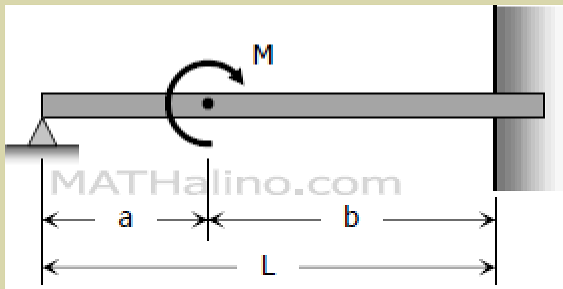
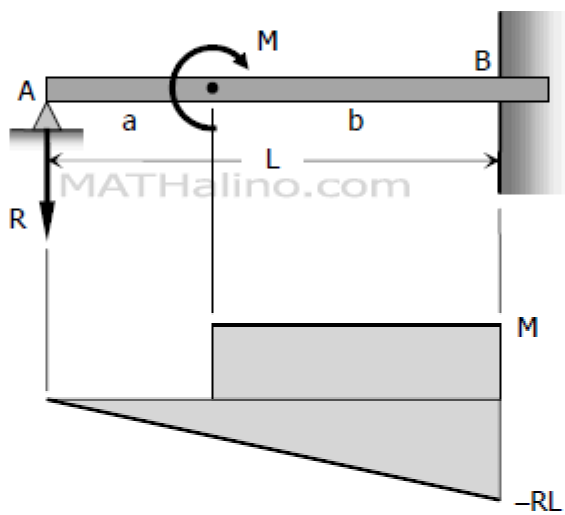


Figure P-722

Solution

HideClick here to show or hide the solution



$$EI\ t_{A/B} = 0$$

$$(Area_{AB}) \cdot \bar{X}_A = 0$$

$$Mb(L - \tfrac{1}{2}b) - \tfrac{1}{2}L(RL)(\tfrac{2}{3}L) = 0$$

$$MbL - \tfrac{1}{2}Mb^2 - \tfrac{1}{3}RL^3 = 0$$

$$\tfrac{1}{3}RL^3 = MbL - \tfrac{1}{2}Mb^2$$

$$R = \frac{3Mb}{L^2} - \frac{3Mb^2}{2L^3}$$

$$R = \frac{3Mb}{2L^3}(2L - b) \qquad \textit{answer}$$

$$M_{wall} = M - RL$$

$$M_{wall} = M - \frac{3Mb}{2L^3}(2L - b)L$$

$$M_{wall} = M - \frac{3Mb}{2L^2}(2L - b)$$ *answer*

When $b = L$

$$R = \frac{3ML}{2L^3}(2L - L) = \frac{3M}{2L}$$

$$M_{wall} = M - \frac{3ML}{2L^2}(2L - L) = M - \frac{3}{2}M = -\frac{3}{2}M$$

See Problem 707 for propped beam with moment load at the simple support for comparison.

Problem 723

Find the reaction R and the moment at the wall for the propped beam shown in Fig. P-723.

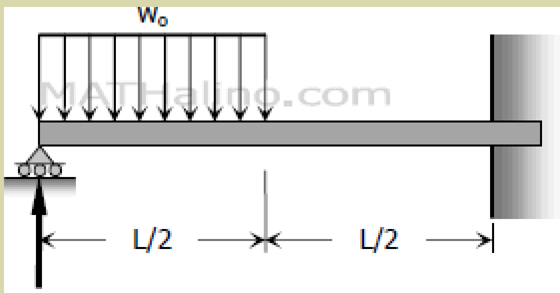


Figure P-723

Solution

$$EI\ t_{A/B} = 0$$

$$\frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_oL^2)(\frac{7}{8}L) + \frac{1}{2}(L)(RL)(\frac{2}{3}L) - \frac{1}{3}(L)(\frac{1}{2}w_oL^2)(\frac{3}{4}L) = 0$$

$$\frac{7}{384}w_oL^4 + \frac{1}{3}RL^3 - \frac{1}{8}w_oL^4 = 0$$

$$\frac{7}{384}w_oL + \frac{1}{3}R - \frac{1}{8}w_oL = 0$$

$$\frac{1}{3}R = \frac{1}{8}w_oL - \frac{7}{384}w_oL$$

$$\frac{1}{3}R = \frac{41}{384}w_oL$$

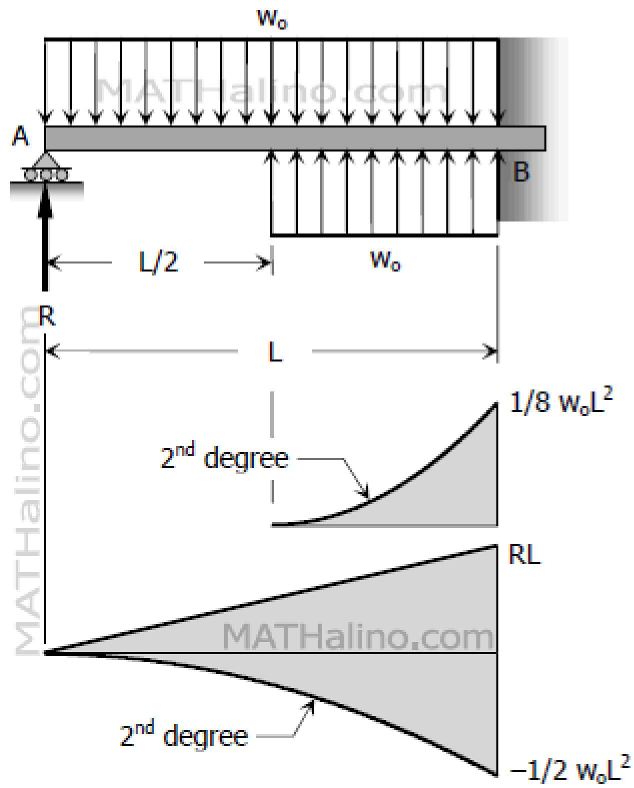
$$R = \frac{41}{128}w_oL \qquad \textit{answer}$$

$$M_{wall} = RL + \frac{1}{8}w_oL^2 - \frac{1}{2}w_oL^2$$

$$M_{wall} = (\frac{41}{128}w_oL)L + \frac{1}{8}w_oL^2 - \frac{1}{2}w_oL^2$$

$$M_{wall} = \frac{41}{128}w_oL^2 + \frac{1}{8}w_oL^2 - \frac{1}{2}w_oL^2$$

$$M_{wall} = -\frac{7}{128}w_oL^2 \qquad \textit{answer}$$



Problem 724

The beam shown in Fig. P-724 is only partially restrained at the wall so that, after the uniformly distributed load is applied, the slope at the wall is $w_oL^3/48EI$ upward to the right. If the supports remain at the same level, determine R.

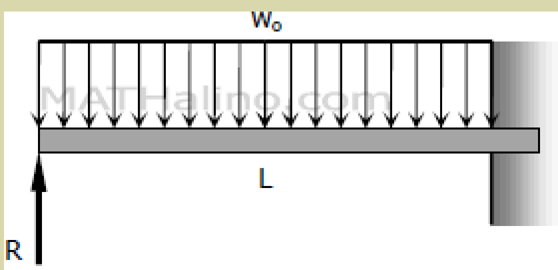
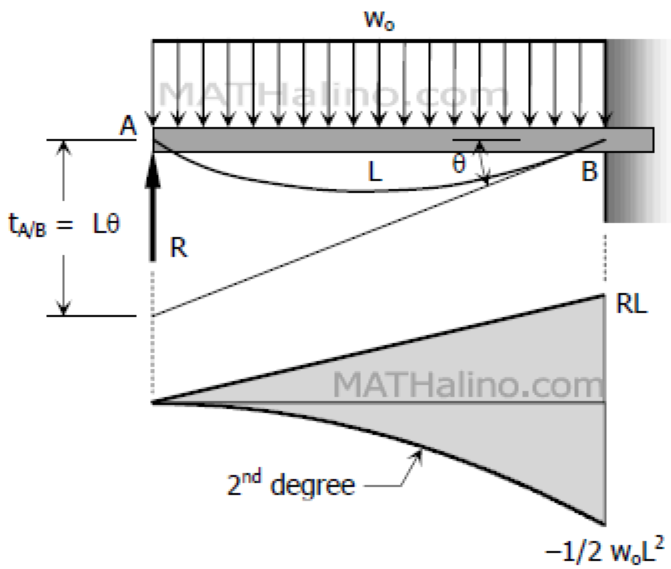


Figure P-724

Solution



$$\theta = \frac{w_o L^3}{48EI}$$

$$L\theta = \frac{w_o L^4}{48EI}$$

$$t_{A/B} = L\theta$$

$$t_{A/B} = \frac{w_o L^4}{48EI}$$

$$EI\ t_{A/B} = \tfrac{1}{48} w_o L^4$$

$$\tfrac{1}{2}(L)(RL)(\tfrac{2}{3}L) - \tfrac{1}{3}(L)(\tfrac{1}{2}w_o L^2)(\tfrac{3}{4}L) = \tfrac{1}{48} w_o L^4$$

$$\tfrac{1}{3}RL^3 - \tfrac{1}{8}w_o L^4 = \tfrac{1}{48} w_o L^4$$

$$\tfrac{1}{3}RL^3 = \tfrac{7}{48} w_o L^4$$

$$R = \tfrac{7}{16} w_o L^4 \qquad \textit{answer}$$

Problem 725

If the support under the propped beam in Problem 724 settles an amount δ, show that the propped reaction decreases by $3EI\delta/L^3$.

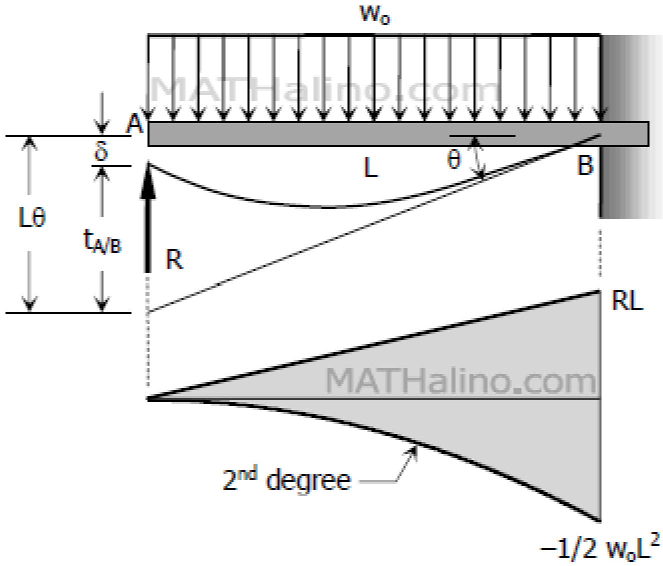
Solution

HideClick here to show or hide the solution

$$\theta = \frac{w_o L^3}{48EI}$$

$$EI\ \theta = \tfrac{1}{48} w_o L^3$$

$$L(EI\ \theta) = \tfrac{1}{48} w_o L^4$$



$t_{A/B} = L\ \theta - \delta$

$EI\ t_{A/B} = L(EI\ \theta) - EI\ \delta$

$\frac{1}{3}RL^3 = \frac{7}{48}w_oL^4 - EI\ \delta$

$$R = \frac{7}{16}w_oL - \frac{3EI\ \delta}{L^3}$$

The quantity $\frac{7}{16}w_oL$ is the simple reaction when there is no settlement δ at the propped support, thus the reaction R decreased by $3EI\delta/L^3$.

Problem 726

A beam of length L, perfectly restrained at both ends, supports a concentrated load P at midspan. Determine the end moment and maximum deflection.

Solution

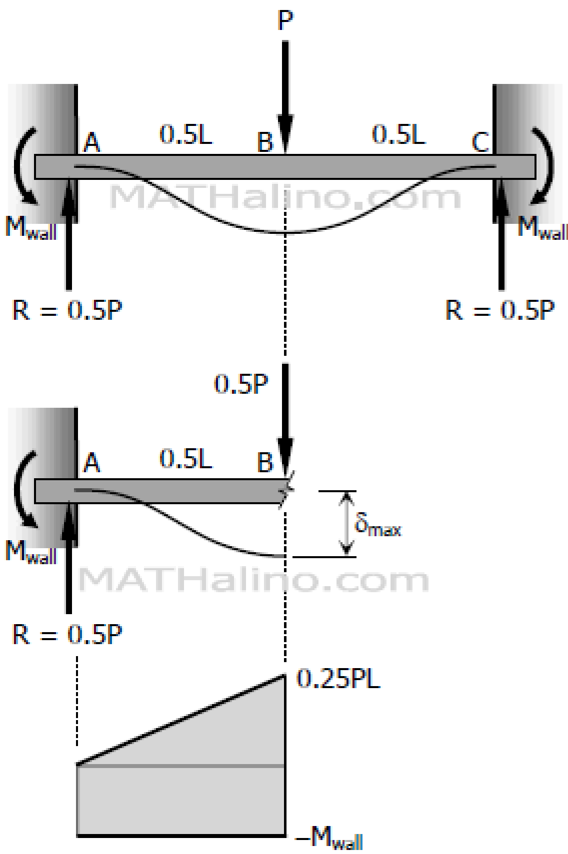
HideClick here to show or hide the solution

$EI\ \theta_{AB} = 0$

$\frac{1}{2}(0.5L)(0.25PL) - 0.5L(M_{wall}) = 0$

$\frac{1}{2}M_{wall}L = \frac{1}{16}PL^2$

$M_{wall} = \frac{1}{8}PL$ *answer*



$\delta_{max} = t_{A/B}$

$EI\ \delta_{max} = EI\ t_{A/B}$

$EI\ \delta_{max} = (Area_{AB}) \cdot \bar{X}_A$

$EI\ \delta_{max} = \frac{1}{2}(0.5L)(0.25PL)[\,\frac{2}{3}(0.5L)\,] - 0.5L(M_{wall})[\,\frac{1}{2}(0.5L)\,]$

$EI\ \delta_{max} = \frac{1}{48}PL^3 - \frac{1}{8}M_{wall}L^2$

$EI\ \delta_{max} = \frac{1}{48}PL^3 - \frac{1}{8}(\frac{1}{8}PL)L^2$

$EI\ \delta_{max} = \frac{1}{48}PL^3 - \frac{1}{64}PL^3$

$EI\ \delta_{max} = \frac{1}{192}PL^3$ *answer*

Problem 727

Repeat Problem 726 assuming that the concentrated load is replaced by a uniformly distributed load of intensity w_o over the entire length.

Solution

HideClick here to show or hide the solution

$EI\ \theta_{AB} = 0$

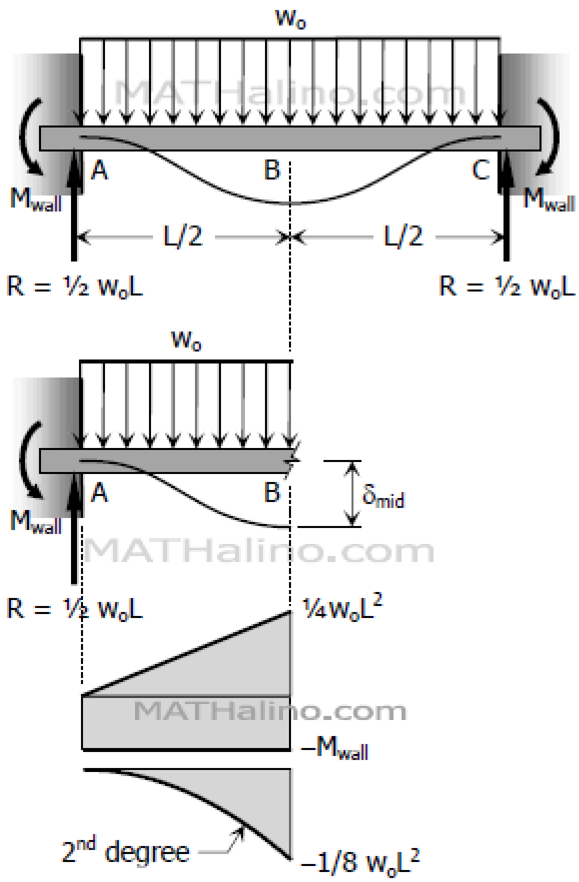
$\frac{1}{2}(\frac{1}{2}L)(\frac{1}{4}w_oL^2) - \frac{1}{2}L(M_{wall}) - \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_oL^2) = 0$

$\frac{1}{16}w_oL^3 - \frac{1}{2}M_{wall}L - \frac{1}{48}w_oL^3 = 0$

$\frac{1}{2}M_{wall}L = \frac{1}{24}w_oL^3$

$M_{wall} = \frac{1}{12}w_oL^2$ *answer*

Note that the actual bending moment is a negative moment (bending the beam downward) as shown in the figure. The answer above is positive which indicates that our assumption of downward arrows is correct. Had we assumed positive M_{wall} (upward arrows), the answer would be negative pointing out that the actual moment is negative (or downward arrows).



$$\delta_{max} = t_{A/B}$$

$$EI\ \delta_{max} = EI\ t_{A/B}$$

$$EI\ \delta_{max} = (Area_{AB}) \cdot \bar{X}_A$$

$$EI\ \delta_{max} = \frac{1}{2}(\frac{1}{2}L)(\frac{1}{4}w_oL^2)[\,\frac{2}{3}(\frac{1}{2}L)\,] - \frac{1}{2}L(M_{wall})[\,\frac{1}{2}(\frac{1}{2}L)\,]$$
$$- \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_oL^2)[\,\frac{3}{4}(\frac{1}{2}L)\,]$$

$$EI\ \delta_{max} = \frac{1}{16}w_oL^3(\frac{1}{3}L) - \frac{1}{2}M_{wall}L(\frac{1}{4}L) - \frac{1}{48}w_oL^3(\frac{3}{8}L)$$

$$EI\ \delta_{max} = \frac{1}{48}w_oL^4 - \frac{1}{8}M_{wall}L^2 - \frac{1}{128}w_oL^4$$

$$EI\ \delta_{max} = \frac{1}{48}w_oL^4 - \frac{1}{8}(\frac{1}{12}w_oL^2)L^2 - \frac{1}{128}w_oL^4$$

$$EI\ \delta_{max} = \frac{1}{48}w_oL^4 - \frac{1}{96}w_oL^4 - \frac{1}{128}w_oL^4$$

$$EI\ \delta_{max} = \frac{1}{384}w_oL^4 \qquad \textit{answer}$$

Problem 728

Determine the end moment and maximum deflection of a perfectly restrained beam loaded as shown in Fig. P-728.

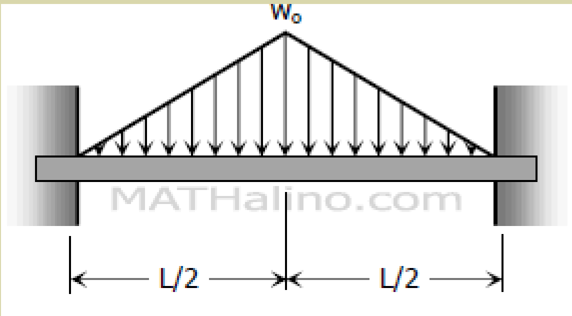


Figure P-728

Solution

HideClick here to show or hide the solution

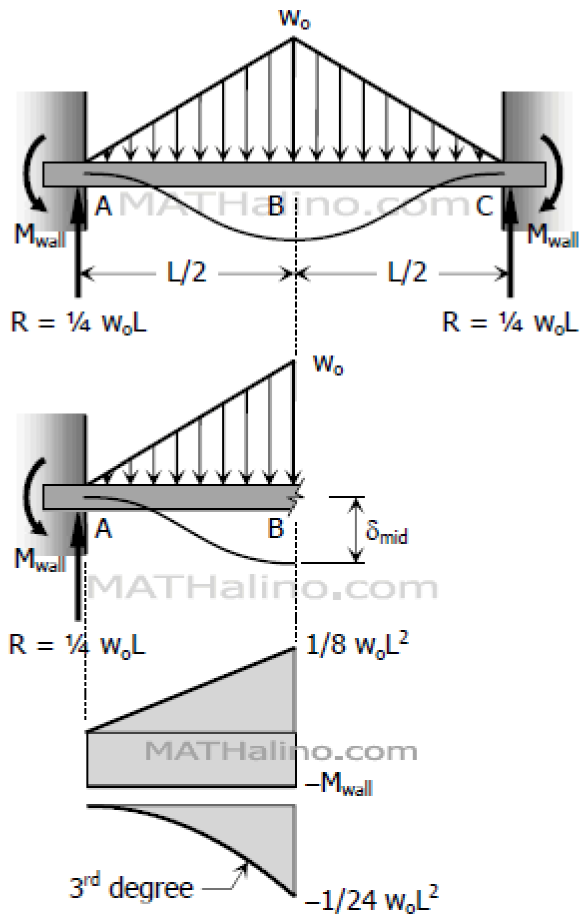
$EI\ \theta_{AB} = 0$

$\frac{1}{2}(\frac{1}{2}L)(\frac{1}{8}w_oL^2) - \frac{1}{2}L(M_{wall}) - \frac{1}{4}(\frac{1}{2}L)(\frac{1}{24}w_oL^2) = 0$

$\frac{1}{32}w_oL^3 - \frac{1}{2}M_{wall}L - \frac{1}{192}w_oL^3 = 0$

$\frac{1}{2}M_{wall}L = \frac{5}{192}w_oL^3$

$M_{wall} = \frac{5}{96}w_oL^2$



$\delta_{max} = t_{A/B}$

$EI\ \delta_{max} = EI\ t_{A/B}$

$EI\ \delta_{max} = (Area_{AB}) \cdot \bar{X}_A$

$EI\ \delta_{max} = \frac{1}{2}(\frac{1}{2}L)(\frac{1}{8}w_oL^2)[\,\frac{2}{3}(\frac{1}{2}L)\,] - \frac{1}{2}L(M_{wall})[\,\frac{1}{2}(\frac{1}{2}L)\,]$
$- \frac{1}{4}(\frac{1}{2}L)(\frac{1}{24}w_oL^2)[\,\frac{4}{5}(\frac{1}{2}L)\,]$

$EI\ \delta_{max} = \frac{1}{32}w_oL^3(\frac{1}{3}L) - \frac{1}{2}M_{wall}L(\frac{1}{4}L) - \frac{1}{192}w_oL^3(\frac{2}{5}L)$

$EI\ \delta_{max} = \frac{1}{96}w_oL^4 - \frac{1}{8}M_{wall}L^2 - \frac{1}{480}w_oL^4$

$EI\ \delta_{max} = \frac{1}{96}w_oL^4 - \frac{1}{8}(\frac{5}{96}w_oL^2)L^2 - \frac{1}{480}w_oL^4$

$EI\ \delta_{max} = \frac{1}{96}w_oL^4 - \frac{5}{768}w_oL^4 - \frac{1}{480}w_oL^4$

$EI\ \delta_{max} = \frac{7}{3840} w_o L^4$ *answer*

Problem 729

For the restrained beam shown in Fig. P-729, compute the end moment and maximum EIδ.

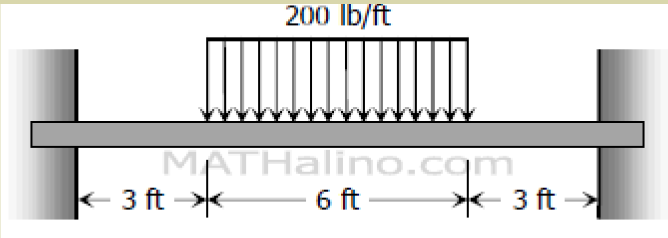


Figure P-729

Solution

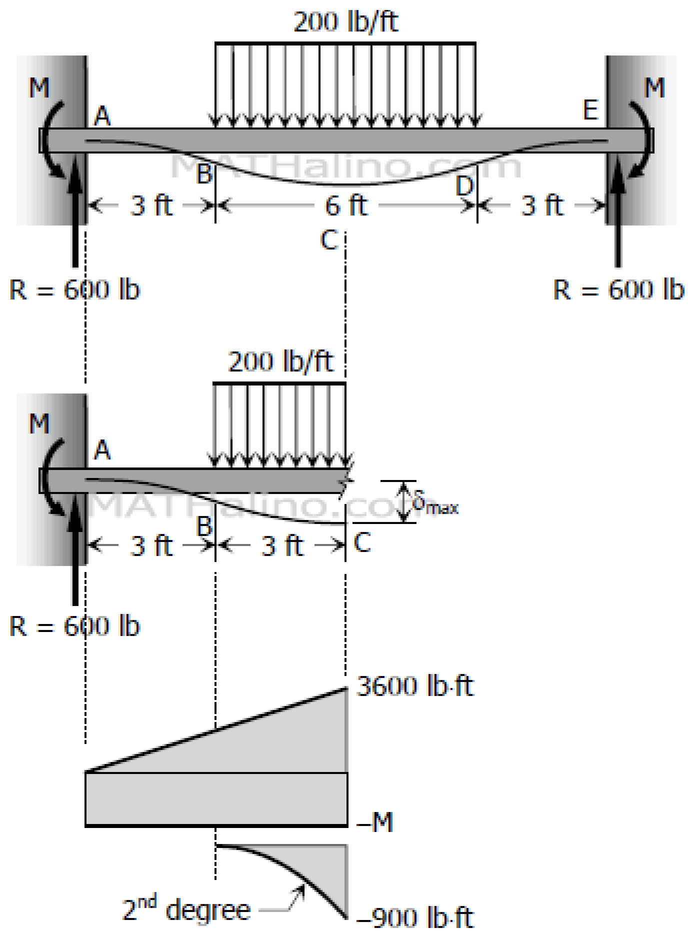
HideClick here to show or hide the solution

$EI\ \theta_{AC} = 0$

$\frac{1}{2}(6)(3600) - 6M - \frac{1}{3}(3)(900) = 0$

$10\,800 - 6M - 900 = 0$

$M = 1650\ \text{lb}\cdot\text{ft}$ *answer*



$$\delta_{max} = t_{A/C}$$

$$EI\ \delta_{max} = EI\ t_{A/C}$$

$$EI\ \delta_{max} = (Area_{AC}) \cdot \bar{X}_A$$

$$EI\ \delta_{max} = \frac{1}{2}(6)(3600)[\,\frac{2}{3}(6)\,] - 6M(3) - \frac{1}{3}(3)(900)[\,3 + \frac{3}{4}(3)\,]$$

$$EI\ \delta_{max} = 10\,800(4) - 18M - 900(5.25)$$

$$EI\ \delta_{max} = 10\,800(4) - 18(1650) - 900(5.25)$$

$$EI\ \delta_{max} = 8775\ \text{lb}\cdot\text{ft}^3$$ *answer*

Problem 703

Determine the end moment and maximum deflection for a perfectly restrained beam loaded as shown in Fig. P-730.

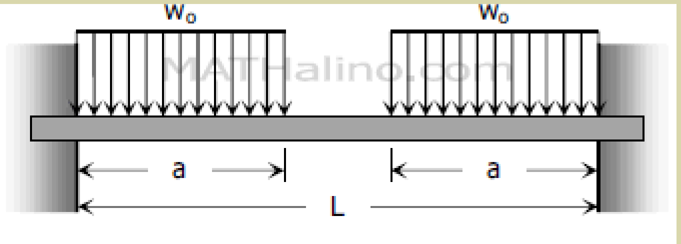


Figure P-730

Solution

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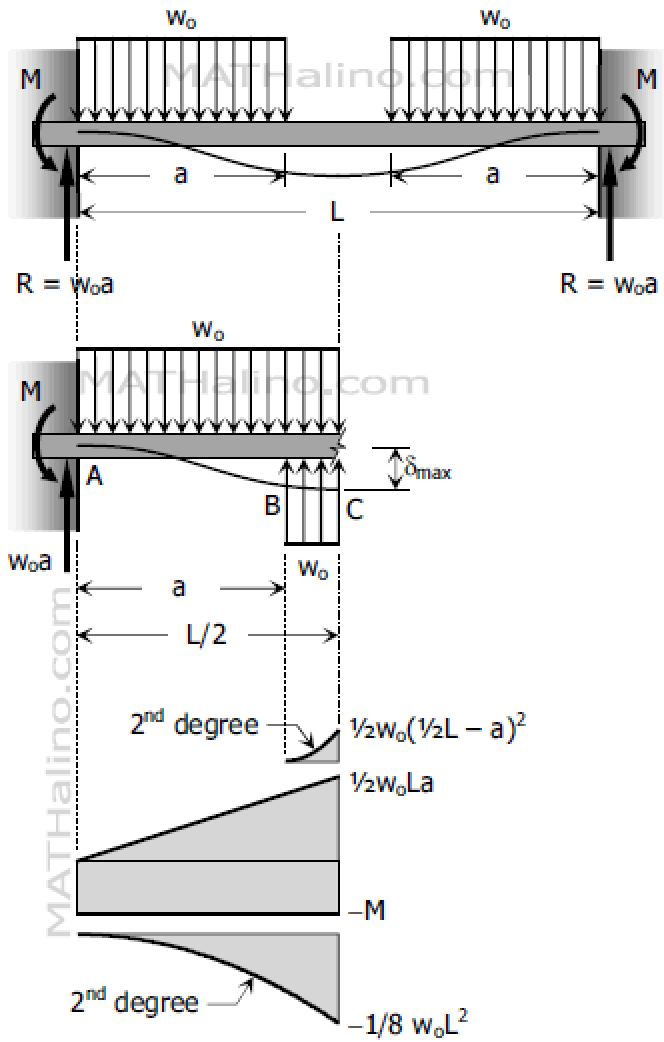


$EI\ \theta_{AC} = 0$

$\frac{1}{3}(\frac{1}{2}L - a)[\,\frac{1}{2}w_o(\frac{1}{2}L - a)^2\,] + \frac{1}{2}(\frac{1}{2}L)(\frac{1}{2}w_oLa) - \frac{1}{2}LM - \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_oL^2) = 0$

$\frac{1}{6}w_o(\frac{1}{2}L - a)^3 + \frac{1}{8}w_oL^2a - \frac{1}{2}LM - \frac{1}{48}w_oL^3 = 0$

$\frac{1}{2}LM = \frac{1}{6}w_o(\frac{1}{2}L - a)^3 - \frac{1}{48}w_oL^3 + \frac{1}{8}w_oL^2a$

$\frac{1}{2}LM = \frac{1}{48}w_o(L - 2a)^3 - \frac{1}{48}w_oL^2(L - 6a)$

$\frac{1}{2}LM = \frac{1}{48}w_o[\,(L - 2a)^3 - L^2(L - 6a)\,]$

$M = \dfrac{w_o}{24L}[\,L^3 - 3L^2(2a) + 3L(2a)^2 - (2a)^3 - L^3 + 6L^2a\,]$

$$M = \frac{w_o}{24L}[\,L^3 - 6L^2a + 12La^2 - 8a^3 - L^3 + 6L^2a\,]$$

$$M = \frac{w_o}{24L}[\,12La^2 - 8a^3\,]$$

$$M = \frac{w_o}{24L}[\,4a^2(3L - 2a)\,]$$

$$M = \frac{w_o a^2}{6L}(3L - 2a)$$ *answer*

$$\delta_{max} = t_{A/C}$$

$$EI\ \delta_{max} = EI\ t_{A/C}$$

$$EI\ \delta_{max} = (Area_{AC}) \cdot \bar{X}_A$$

$$EI\ \delta_{max} = \tfrac{1}{3}(\tfrac{1}{2}L - a)[\,\tfrac{1}{2}w_o(\tfrac{1}{2}L - a)^2\,][\,a + \tfrac{3}{4}(\tfrac{1}{2}L - a)\,]$$
$$+ \tfrac{1}{2}(\tfrac{1}{2}L)(\tfrac{1}{2}w_o La)[\,\tfrac{2}{3}(\tfrac{1}{2}L)\,] - \tfrac{1}{2}LM[\,\tfrac{1}{2}(\tfrac{1}{2}L)\,]$$
$$- \tfrac{1}{3}(\tfrac{1}{2}L)(\tfrac{1}{8}w_o L^2)[\,\tfrac{3}{4}(\tfrac{1}{2}L)\,]$$

$$EI\ \delta_{max} = \tfrac{1}{6}w_o(\tfrac{1}{2}L - a)^3(\tfrac{1}{4}a + \tfrac{3}{8}L) + \tfrac{1}{8}w_o L^2 a(\tfrac{1}{3}L)$$
$$- \tfrac{1}{2}LM(\tfrac{1}{4}L) - \tfrac{1}{48}w_o L^3(\tfrac{3}{8}L)$$

$$EI\ \delta_{max} = \tfrac{1}{384}w_o(L - 2a)^3(2a + 3L) + \tfrac{1}{24}w_o L^3 a - \tfrac{1}{8}L^2 M - \tfrac{1}{128}w_o L^4$$

$$EI\ \delta_{max} = \tfrac{1}{384}w_o[\,L^3 - 3L^2(2a) + 3L(2a)^2 - (2a)^3\,](2a + 3L)$$
$$+ \tfrac{1}{24}w_o L^3 a - \tfrac{1}{8}L^2 M - \tfrac{1}{128}w_o L^4$$

$$EI\ \delta_{max} = \tfrac{1}{384}w_o(L^3 - 6L^2a + 12La^2 - 8a^3)(2a + 3L)$$
$$+ \tfrac{1}{24}w_o L^3 a - \tfrac{1}{8}L^2 M - \tfrac{1}{128}w_o L^4$$

$$EI\ \delta_{max} = \tfrac{1}{384}w_o(2L^3a - 12L^2a^2 + 24La^3 - 16a^4 + 3L^4 - 18L^3a + 36L^2a^2$$
$$- 24La^3) + \tfrac{1}{24}w_o L^3 a - \tfrac{1}{8}L^2 M - \tfrac{1}{128}w_o L^4$$

$$EI\ \delta_{max} = \tfrac{1}{384}w_o(24L^2a^2 - 16a^4 + 3L^4 - 16L^3a) + \tfrac{1}{24}w_o L^3 a - \tfrac{1}{8}L^2 M - \tfrac{1}{128}w_o L^4$$

$$EI\ \delta_{max} = (\tfrac{1}{16}w_o L^2 a^2 - \tfrac{1}{24}w_o a^4 + \tfrac{1}{128}w_o L^4 - \tfrac{1}{24}w_o L^3 a) + \tfrac{1}{24}w_o L^3 a$$
$$- \tfrac{1}{8}L^2 M - \tfrac{1}{128}w_o L^4$$

$$EI\ \delta_{max} = \tfrac{1}{16}w_o L^2 a^2 - \tfrac{1}{24}w_o a^4 - \tfrac{1}{8}L^2 M$$

$$EI\ \delta_{max} = \frac{1}{16}w_oL^2a^2 - \frac{1}{24}w_oa^4 - \frac{1}{8}L^2\left[\frac{w_oa^2}{6L}(3L-2a)\right]$$

$$EI\ \delta_{max} = \frac{1}{16}w_oL^2a^2 - \frac{1}{24}w_oa^4 - \frac{1}{48}w_oLa^2(3L-2a)$$

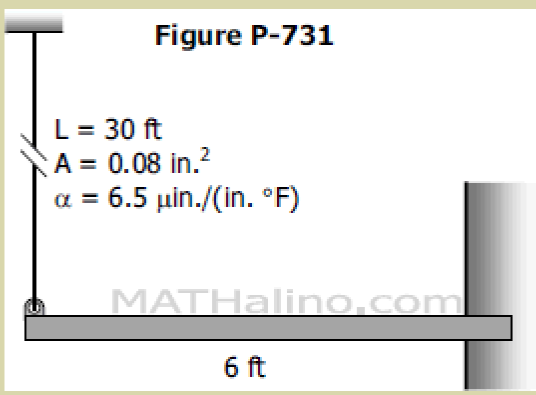
$$EI\ \delta_{max} = \frac{1}{16}w_oL^2a^2 - \frac{1}{24}w_oa^4 - \frac{1}{16}w_oL^2a^2 + \frac{1}{24}w_oa^3$$

$$EI\ \delta_{max} = -\frac{1}{24}w_oa^4 + \frac{1}{24}w_oLa^3$$

$$EI\ \delta_{max} = \frac{1}{24}w_oa^3(L-a) \qquad \textit{answer}$$

Problem 731

The beam shown in Fig. P-731 is connected to a vertical rod. If the beam is horizontal at a certain temperature, determine the increase in stress in the rod if the temperature of the rod drops 90°F. Both the beam and the rod are made of steel with E = 29 × 10^6 psi. For the beam, use I = 154 in.^{4}



Solution

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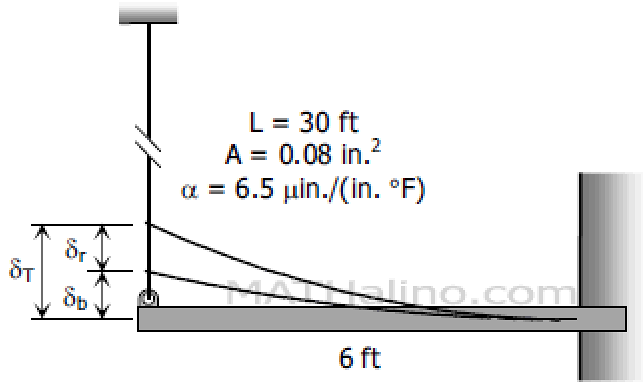
Assuming complete freedom for the rod, the deformation due to drop of temperature is...

$\delta_T = \alpha L\,\Delta T$

$$\delta_T = 0.0000065(30)(90)$$

$$\delta_T = 0.01755 \text{ ft} = 0.2106 \text{ in.}$$



subscript b ($_b$) = refers to the beam
subscript r ($_r$) = refers to the rod

$$\delta_b + \delta_r = \delta_T$$

$$\left(\frac{PL^3}{3EI}\right)_b + \left(\frac{PL}{AE}\right)_r = 0.2106$$

$$\left(\frac{PL^3}{3I}\right)_b + \left(\frac{PL}{A}\right)_r = 0.2106E$$

$$\frac{P(6^3)(12^3)}{3(154)} + \frac{P(30)(12)}{0.08} = 0.2106(29 \times 10^6)$$

$$807.90P + 4500P = 6\,107\,400$$

$$5307.9P = 6\,107\,400$$

$$P = 1150.62 \text{ lb} = 1.151 \text{ kips}$$

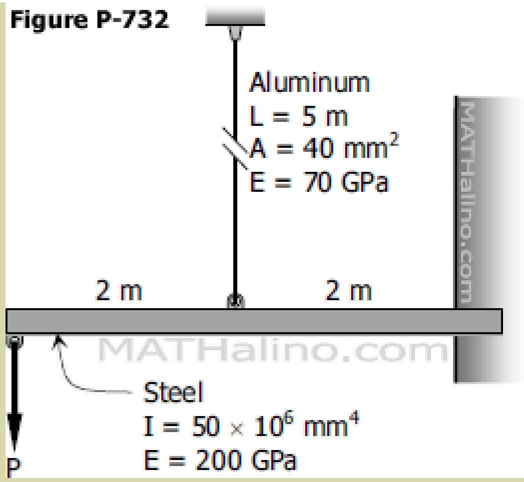
Stress increase on the rod

$$\sigma = \frac{P}{A} = \frac{1.151}{0.08}$$

$$\sigma = 14.39 \text{ ksi}$$ *answer*

Problem 732

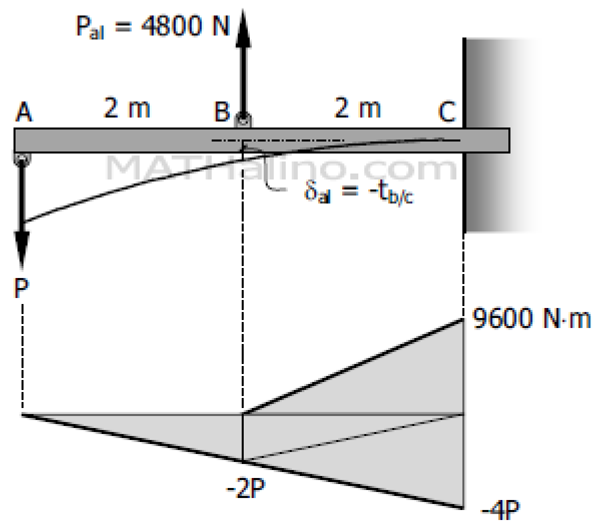
The midpoint of the steel in Fig. P-732 is connected to the vertical aluminum rod. Determine the maximum value of P if the stress in the rod is not to exceed 120 MPa.



Solution 732

HideClick here to show or hide the solution

For the aluminum rod



$P_{al} = (\sigma A)_{al}$

$P_{al} = 120(40)$

$P_{al} = 4800 \text{ N}$

$\delta_{al} = \left(\dfrac{\sigma L}{E}\right)_{al}$

$\delta_{al} = \dfrac{120(5)(1000)}{70000}$

$\delta_{al} = \frac{60}{7} \text{ mm}$

For the steel beam

$t_{B/C} = \dfrac{1}{EI}(Area_{BC}) \cdot \bar{X}_B$

$t_{B/C} = \dfrac{1}{200\,000(50 \times 10^6)}\left[\frac{1}{2}(2)(9600)(\frac{4}{3}) - \frac{1}{2}(2)(2P)(\frac{2}{3}) - \frac{1}{2}(2)(4P)(\frac{4}{3})\right](1000^3)$

$$t_{B/C} = \frac{1}{10\,000}(25600 - \frac{2}{3}P)$$

$$t_{B/C} = \frac{64}{25} - \frac{1}{1500}P$$

From the figure

$$\delta_{al} = -t_{B/C}$$

$$\frac{60}{7} = -(\frac{64}{25} - \frac{1}{1500}P)$$

$$\frac{1}{1500}P = \frac{1948}{175}$$

$$P = 16\,697.14 \text{ N}$$

$$P = 16.7 \text{ kN} \qquad \textit{answer}$$

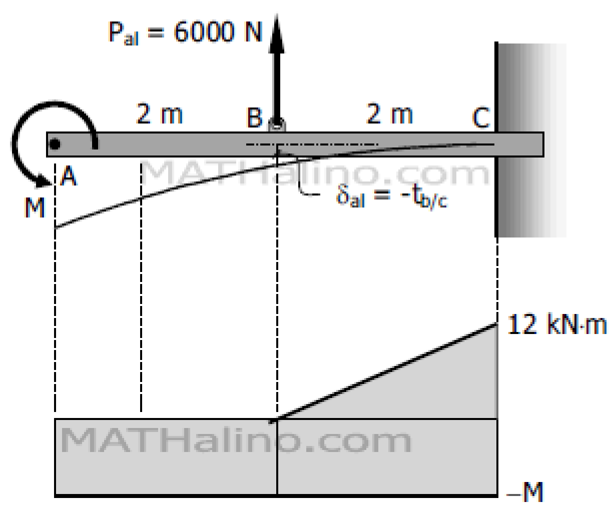
Problem 733

The load P in Prob. 732 is replaced by a counterclockwise couple M. Determine the maximum value of M if the stress in the vertical rod is not to exceed 150 MPa.

Solution 733

HideClick here to show or hide the solution

For the aluminum rod



$$P_{al} = (\sigma A)_{al}$$

$$P_{al} = 150(40)$$

$$P_{al} = 6000 \text{ N}$$

$$\delta_{al} = \left(\frac{\sigma L}{E}\right)_{al}$$

$$\delta_{al} = \frac{150(5)(1000)}{70\,000}$$

$$\delta_{al} = \frac{75}{7} \text{ mm}$$

For the steel beam

$$t_{B/C} = \frac{1}{(EI)_{st}} (\text{Area}_{BC}) \cdot \bar{X}_B$$

$$t_{B/C} = \frac{1}{200\,000(50 \times 10^6)} \left[\frac{1}{2}(2)(12\,000)(\frac{4}{3} - 2M) \right] (1000^3)$$

$$t_{B/C} = \frac{1}{10\,000}(16\,000 - 2M)$$

$$t_{B/C} = \frac{8}{5} - \frac{1}{5000} M$$

From the figure

$$\delta_{al} = -t_{B/C}$$

$$\frac{75}{7} = -(\frac{8}{5} - \frac{1}{5000} M)$$

$$\frac{1}{5000} M = \frac{431}{35}$$

$$P = 61\,571.43 \text{ N}$$

$P = 61.57$ kN *answer*

Problem 734

Determine the end moments for the restrained beams shown in Fig. P-734.

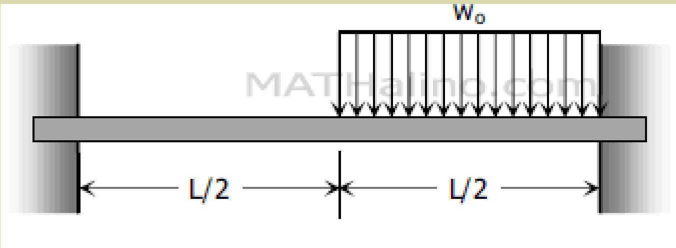


Figure P-734

Solution 734

HideClick here to show or hide the solution

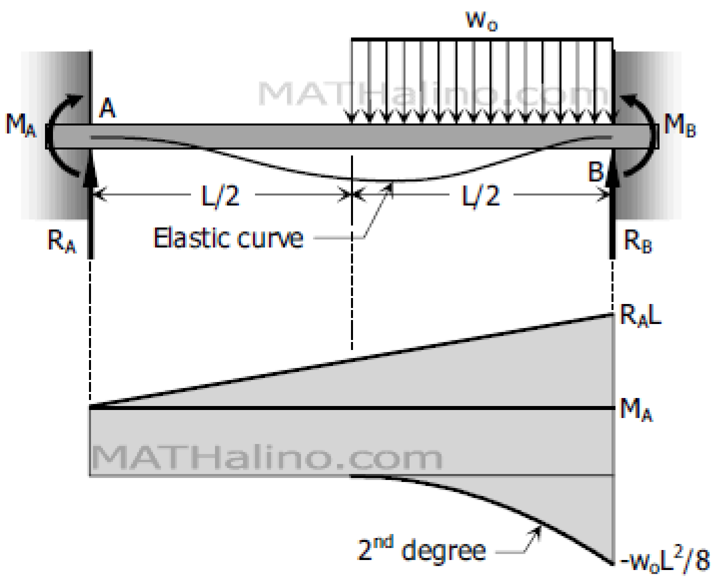
The angle between tangents through A and B is zero

$$EI\,\theta_{AB} = (\text{Area}_{AB}) = 0$$

$$\frac{1}{2}L(R_A L) + MAL - \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_o L^2) = 0$$

$\frac{1}{2}R_A L^2 + M_A L - \frac{1}{48}w_o L^3 = 0$

$M_A = \frac{1}{48}w_o L^2 - \frac{1}{2}R_A L$ → equation (1)



The deviation of B from a tangent line through A is zero

$EI\, t_{B/A} = (\text{Area}_{AB}) \cdot \bar{X}_B = 0$

$\frac{1}{2}L(R_A L)(\frac{1}{3}L) + M_A L(\frac{1}{2}L) - \frac{1}{3}(\frac{1}{2}L)(\frac{1}{8}w_o L^2)[\,\frac{1}{4}(\frac{1}{2}L)\,] = 0$

$\frac{1}{6}R_A L^3 + \frac{1}{2}M_A L^2 - \frac{1}{384}w_o L^4 = 0$

$64R_A L + 192M_A = w_o L^2$ → equation (2)

Substitute M_A of equation (1) to equation (2) above

$64R_A L + 192(\frac{1}{48}w_o L^2 - \frac{1}{2}R_A L) = w_o L^2$

$64R_A L + 4w_o L^2 - 96R_A L = w_o L^2$

$32R_A L = 3w_o L^2$

$R_A = \frac{3}{32}w_o L$

Substitute R_A to equation (1)

$$M_A = \frac{1}{48}w_oL^2 - \frac{1}{2}(\frac{3}{32}w_oL)L$$

$$M_A = \frac{1}{48}w_oL^2 - \frac{3}{64}w_oL^2$$

$$M_A = -\frac{5}{192}w_oL^2 \qquad \textit{answer}$$

Sum up moments at right support B

$M_B = M_A + R_AL - \frac{1}{8}w_oL^2$ $\rightarrow$ see the right end of moment diagram by parts

$$M_B = -\frac{5}{192}w_oL^2 + (\frac{3}{32}w_oL)L - \frac{1}{8}w_oL^2$$

$$M_B = -\frac{5}{192}w_oL^2 + \frac{3}{32}w_oL^2 - \frac{1}{8}w_oL^2$$

$$M_B = -\frac{11}{192}w_oL^2 \qquad \textit{answer}$$

Problem 735

The beam shown in Fig. P-735 is perfectly restrained at A but only partially restrained at B, where the slope is $w_oL^3/48EI$ directed up to the right. Solve for the end moments.

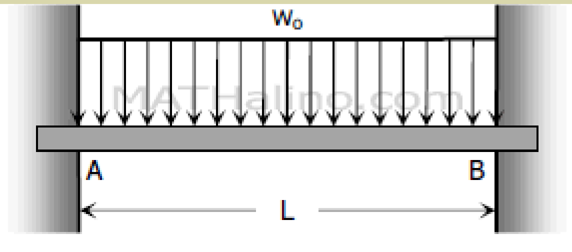


Figure P-735

Solution 735

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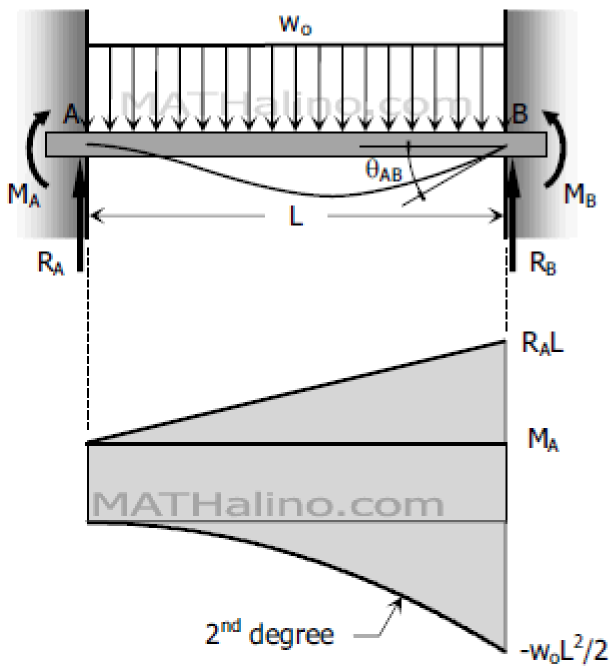
$$EI\,\theta_{AB} = (Area_{AB})$$

$$EI\left(\frac{w_oL^3}{48EI}\right) = \frac{1}{2}(R_AL)L + M_AL - \frac{1}{3}(\frac{1}{2}w_oL^2)L$$

$$\frac{1}{48}w_oL^3 = \frac{1}{2}R_AL^2 + M_AL - \frac{1}{6}w_oL^3$$

$M_A L = \frac{3}{16} w_o L^3 - \frac{1}{2} R_A L^2$

$M_A = \frac{3}{16} w_o L^2 - \frac{1}{2} R_A L$ → Equation (1)



$EI\, t_{B/A} = (Area_{AB}) \cdot \bar{X}_B = 0$

$\frac{1}{2}(R_A L)L(\frac{1}{3}L) + M_A L(\frac{1}{2}L) - \frac{1}{3}(\frac{1}{2}w_o L^2)L(\frac{1}{4}L) = 0$

$\frac{1}{6}R_A L^3 + \frac{1}{2}M_A L^2 - \frac{1}{24}w_o L^4 = 0$

$\frac{1}{6}R_A L + \frac{1}{2}M_A - \frac{1}{24}w_o L^2 = 0$

Substitute M_A defined in equation (1)

$\frac{1}{6}R_A L + \frac{1}{2}(\frac{3}{16}w_o L^2 - \frac{1}{2}R_A L) - \frac{1}{24}w_o L^2 = 0$

$\frac{1}{6}R_A L + \frac{3}{32}w_o L^2 - \frac{1}{4}R_A L - \frac{1}{24}w_o L^2 = 0$

$\frac{1}{12}R_A L = \frac{5}{96}w_o L^2$

$R_A = \frac{5}{8}w_o L$

From equation (1)

$M_A = \frac{3}{16}w_oL^2 - \frac{1}{2}(\frac{5}{8}w_oL)L$

$M_A = \frac{3}{16}w_oL^2 - \frac{5}{16}w_oL^2$

$M_A = -\frac{1}{8}w_oL^2$ *answer*

From moment diagram by parts

$M_B = R_AL + M_A - \frac{1}{2}w_oL^2$

$M_B = (\frac{5}{8}w_oL)L - \frac{1}{8}w_oL^2 - \frac{1}{2}w_oL^2$

$M_B = \frac{5}{8}w_oL^2 - \frac{1}{8}w_oL^2 - \frac{1}{2}w_oL^2$

$M_B = 0$ *answer*

Problem 736

Determine the end shears and end moments for the restrained beam shown in Fig. P-736 and sketch the shear and moment diagrams.

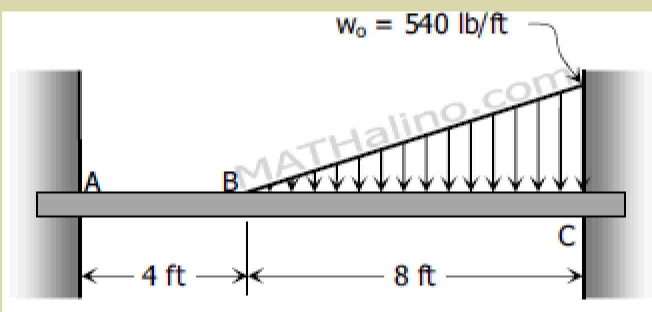


Figure P-736

Solution 736

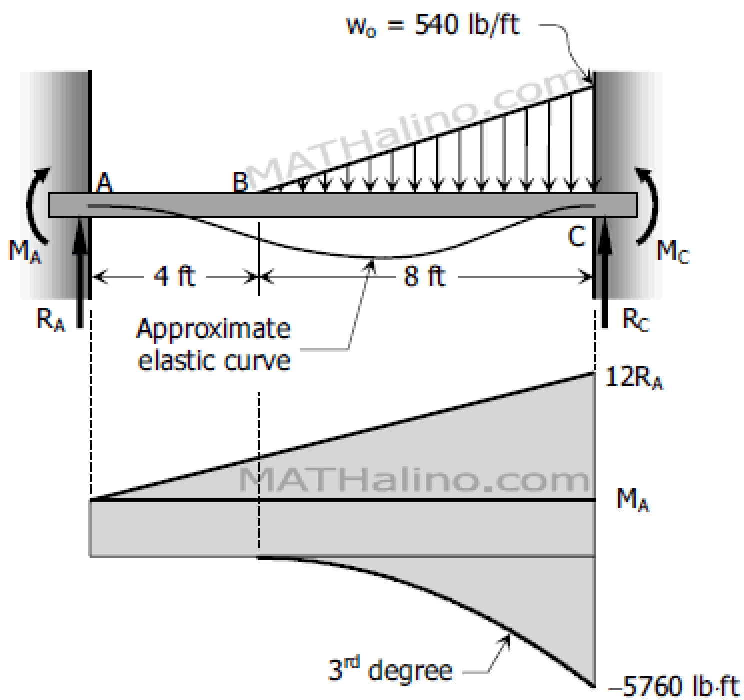
HideClick here to show or hide the solution

$EI\,\theta_{AC} = (Area_{AC}) = 0$

$\frac{1}{2}(12)(12R_A) + 12M_A - \frac{1}{4}(8)(5760) = 0$

$72R_A + 12M_A - 11\,520 = 0$

$6R_A + M_A = 960 \;\rightarrow\;$ Equation (1)



$$EI\,t_{C/A} = (Area_{AC}) \cdot \bar{X}_C = 0$$

$$\frac{1}{2}(12)(12R_A)[\,\frac{1}{3}(12)\,] + 12M_A[\,\frac{1}{2}(12)\,] - \frac{1}{4}(8)(5760)[\,\frac{1}{5}(8)\,] = 0$$

$$288R_A + 72M_A - 18\,432 = 0$$

$4R_A + M_A = 256 \;\rightarrow\;$ Equation (2)

Solving Equations (1) and (2)

$R_A = 352\ \text{lb}$ *answer*

$M_A = -1152\ \text{lb} \cdot \text{ft}$ *answer*

Checking

$$EI\,t_{A/C} = (Area_{AC}) \cdot \bar{X}_A = 0$$

$$\frac{1}{2}(12)(12R_A)[\,\frac{2}{3}(12)\,] + 12M_A[\,\frac{1}{2}(12)\,] - \frac{1}{4}(8)(5760)[\,4 + \frac{4}{5}(8)\,] = 0$$

$576R_A + 72M_A - 119\,808 = 0$

$576(352) + 72(-1152) - 119808 = 0$

$0 = 0$ *okay!*

$M_C = 12R_A + M_A - 5760$

$M_C = 12(352) - 1152 - 5760$

$M_C = -2688 \text{ lb} \cdot \text{ft}$ *answer*

$\Sigma F_V = 0$

$R_A + R_C = \frac{1}{2}(8)(540)$

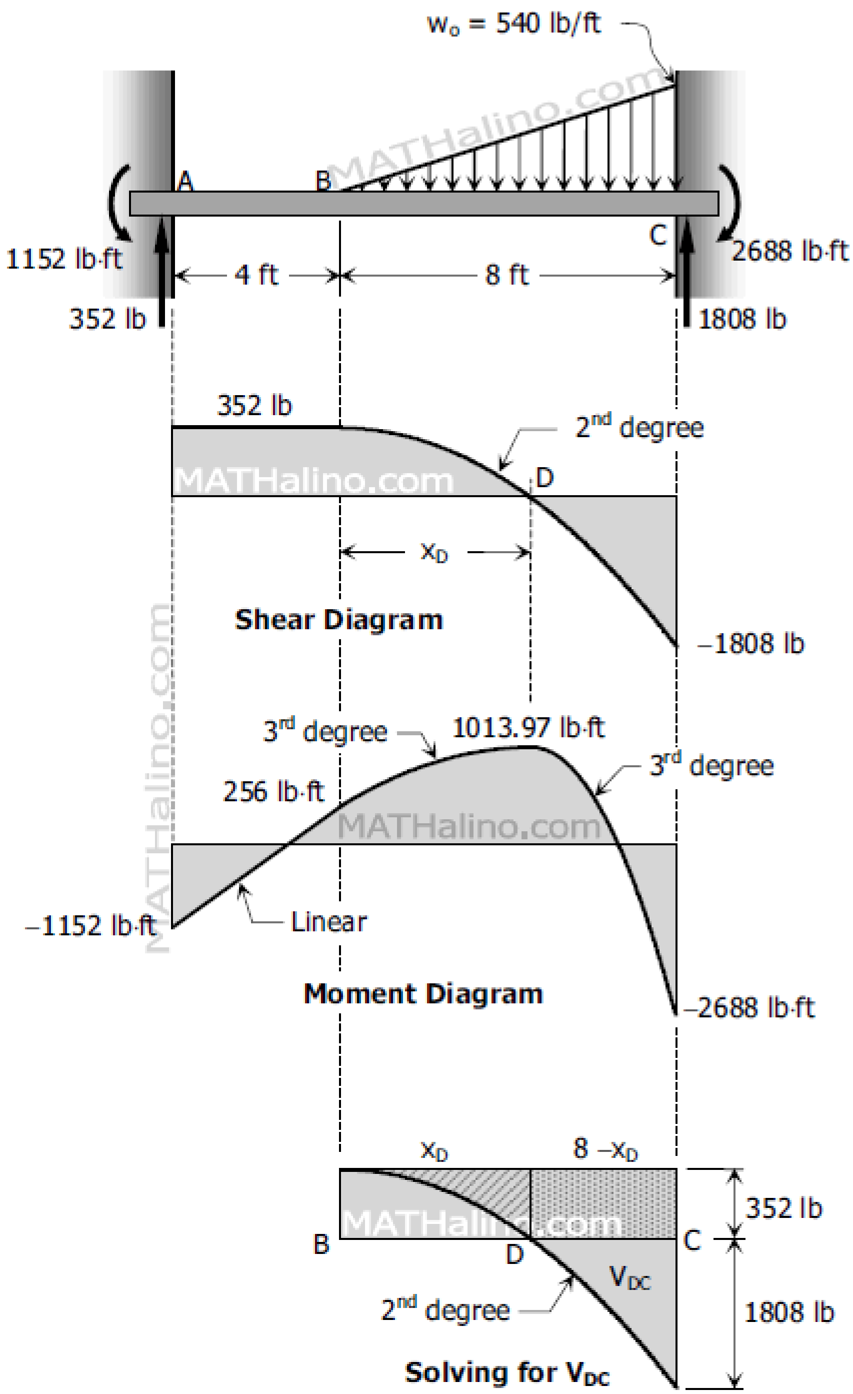
$352 + R_C = 2160$

$R_C = 1\,808 \text{ lb}$ *answer*

To draw the shear diagram

1. V_A = 352 lb
2. $V_B = V_A + Load_{AB}$
   V_B = 352 + 0
   V_B = 352 lb
3. There is no load between AB, thus, shear in AB is constant.
4. $V_C = V_B + Load_{BC}$
   V_C = 352 - (1/2)(8)(540)
   V_C = -1808 lb
5. Load between B and C is linearly decreasing from zero to -540 lb/ft, thus, shear in segment BC is a concave downward second degree curve (parabola) with vertex at B.
6. Location of point D by squared property of parabola:
   $\frac{{x_D}^2}{352} = \frac{8^2}{3521808}$

$x_D = 3.23 \text{ ft}$ to the right of B



To draw the moment diagram

1. M_A = -1152 lb·ft

2. $M_B = M_A + V_{AB}$
   M_B = -1152 + 352(4)
   M_B = 256 lblb·ftft

3. The shear between A and B is uniform and positive, thus, the moment in AB is linear and increasing.

4. $M_D = M_B + V_{BD}$
   $M_D = 256 + (2/3)(x_D)(352)$
   M_D = 256 + (2/3)(3.23)(352)
   M_D = 1013.97 lb·ft

5. $M_C = M_D + V_{DC}$

   Solving for VDC
   $V_{DC} = (-1/3)(8)(352 + 1808) + (1/3)(x_D)(352) + 352(8 - x_D)$
   V_{DC} = -5760 + (1/3)(3.23)(352) + 352(8 - 3.23)
   V_{DC} = -3701.97 lb

   Thus,
   M_C = 1013.97 - 3701.97
   M_C = -2688 lb·ft

6. The shear diagram from B to C is a parabola, thus, the moment diagram of segment BC is a third degree curve. The value of shear from B to C decreases, thus, the slope of moment diagram between B and C also decreases making the cubic curve concave downward.

7. **Problem 737**

In the perfectly restrained beam shown in Fig. P-737, support B has settled a distance Δ below support A. Show that $M_B = -M_A = 6EI\Delta/L^2$.

8.

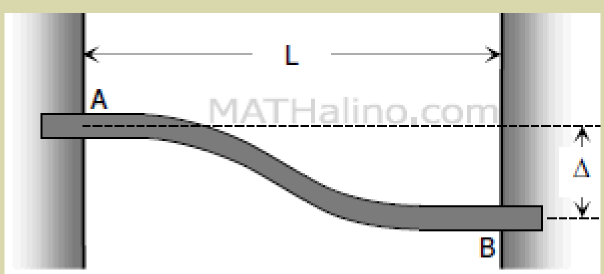


Figure P-737

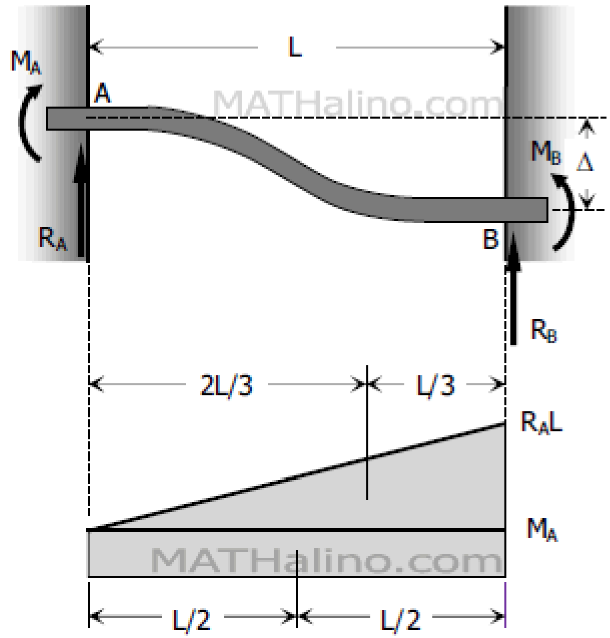
9.

10. **Solution 737**

11. Hide Click here to show or hide the solution

12. Rotation between A and B

$$\theta_{AB} = \frac{1}{EI}(Area_{AB}) = 0$$



13. $\frac{1}{2}L(R_A L) + M_A L = 0$

14. $\frac{1}{2}R_A L^2 + M_A L = 0$

15. $R_A L + 2M_A = 0$

16. $R_A = -\frac{2M_A}{L}$ → equation (1)

17. Deviation of B from tangent through A

$$t_{B/A} = \frac{1}{EI}(Area_A B) \cdot \bar{X}_B = -\Delta$$

18. $\frac{1}{2}L(R_A L)(\frac{1}{3}L) + M_A L(\frac{1}{2}L) = -EI\Delta$

19. $\frac{1}{6}R_A L^3 + \frac{1}{2}M_A L^2 = -EI\Delta$

20. $R_A L^3 + 3M_A L^2 = -6EI\Delta$

21. Substitute R_A defined in equation (1)

$$\left(-\frac{2M_A}{L}\right)L^3 + 3M_A L^2 = -6EI\Delta$$

22. $-2M_A L^2 + 3M_A L^2 = -6EI\Delta$

23. $M_A L^2 = -6EI\Delta$

24. $M_A = -6EI\Delta / L^2$

25. Substitute M_A to equation (1)

$$R_A = -\frac{2(-6EI\Delta/L^2)}{L}$$

26. $$R_A = \frac{12EI\Delta}{L^3}$$

27. Moment at B (See moment diagram by parts)

$$M_B = M_A + R_A L$$

28. $$M_B = -6EI\Delta/L^2 + (12EI\Delta/L^3)L$$

29. $$M_B = -6EI\Delta/L^2 + 12EI\Delta/L^2$$

30. $$M_B = 6EI\Delta/L^2$$

31. Thus,

$$M_B = -M_A = 6EI\Delta/L^2$$ *okay!*

32. **Problem 738**

A perfectly restrained beam is loaded by a couple M applied where shown in Fig. P-738. Determine the end moments.

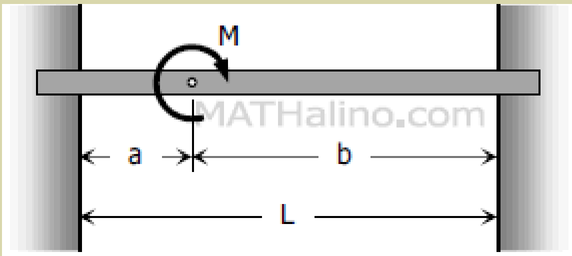
33. 

Figure P-738

34.

35. **Solution 738**

36. [HideClick here to show or hide the solution]

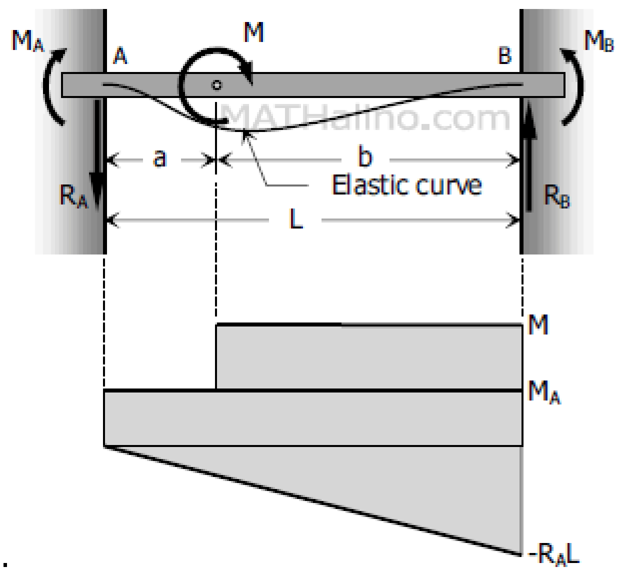
37. Rotation of AB is zero

$$EI\theta = (Area_{AB}) = 0$$

38. $$Mb + M_A L - \tfrac{1}{2}L(R_A L) = 0$$

39. $Mb + M_A L = \frac{1}{2} R_A L^2$

40. $R_A = \dfrac{2(Mb + M_A L)}{L^2}$ → equation (1)



41.

42.

43. Deviation of B from a tangent line through A is zero

$EI\, t_{B/A} = (Area_{AB})\bar{X}_B = 0$

44. $Mb(\frac{1}{2}b) + M_A L(\frac{1}{2}L) - \frac{1}{2}L(R_A L)(\frac{1}{3}L) = 0$

45. $\frac{1}{2}Mb^2 + \frac{1}{2}M_A L^2 - \frac{1}{6}R_A L^3 = 0$

46. $3Mb^2 + 3M_A L^2 - R_A L^3 = 0$

47. Substitute R_A of equation (1) to the equation above

$3Mb^2 + 3M_A L^2 - \left[\dfrac{2(Mb + M_A L)}{L^2}\right] L^3 = 0$

48. $3Mb^2 + 3M_A L^2 - 2MbL - 2M_A L^2 = 0$

49. $M_A L^2 = 2MbL - 3Mb^2$

50. $M_A L^2 = Mb(2L - 3b)$

51. $M_A L^2 = Mb\,[\,2L - 3(L - a)\,]$

52. $M_A L^2 = Mb(3a - L)$

53. $M_A = \dfrac{Mb}{L^2}(3a - L)$

54. $M_A = \dfrac{Mb}{L}\left(\dfrac{3a}{L} - 1\right)$ *answer*

55. From equation (1)
$R_A = \dfrac{2Mb}{L^2} + \dfrac{2M_A}{L}$

56. $R_A = \dfrac{2Mb}{L^2} + \dfrac{2}{L}\left[\dfrac{Mb}{L}\left(\dfrac{3a}{L} - 1\right)\right]$

57. $R_A = \dfrac{2Mb}{L^2} + \dfrac{6Mab}{L^3} - \dfrac{2Mb}{L^2}$

58. $R_A = \dfrac{6Mab}{L^3}$

59. From moment diagram by parts
$M_B = M + M_A - R_A L$

60. $M_B = M + \dfrac{Mb}{L}\left(\dfrac{3a}{L} - 1\right) - \dfrac{6Mab}{L^3}(L)$

61. $M_B = M + \dfrac{3Mab}{L^2} - \dfrac{Mb}{L} - \dfrac{6Mab}{L^2}$

62. $M_B = M - \dfrac{Mb}{L} - \dfrac{3Mab}{L^2}$

63. $M_B = M - \dfrac{M(L - a)}{L} - \dfrac{3Mab}{L^2}$

64. $M_B = M - M + \dfrac{Ma}{L} - \dfrac{3Mab}{L^2}$

65. $M_B = \dfrac{Ma}{L} - \dfrac{3Mab}{L^2}$

66. $M_B = -\dfrac{Ma}{L}\left(\dfrac{3b}{L} - 1\right)$ *answer*

Fixed-end moments of fully restrained beam

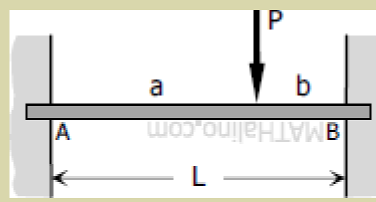
Summary for the value of end moments and deflection of perfectly restrained beam carrying various loadings. Note that for values of EIy, y is positive downward.

Case 1: Concentrated load anywhere on the span of fully restrained beam

End moments

$$M_A = -\frac{Pab^2}{L^2}$$

$$M_B = -\frac{Pa^2b}{L^2}$$



Value of EIy

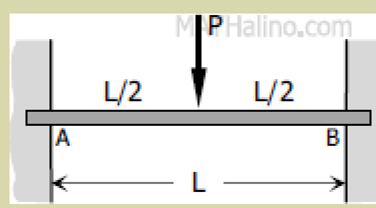
$$\text{Midspan } EI\,y = \frac{Pb^2}{48}(3L - 4b)$$

Note: only for b > a

Case 2: Concentrated load on the midspan of fully restrained beam

End moments

$$M_A = M_B = -\frac{PL}{8}$$



Value of EIy

$$\text{Maximum } EI\,y = \frac{PL^3}{192}$$

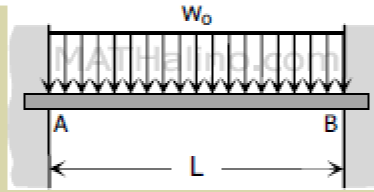
Case 3: Uniformly distributed load over the entire span of fully restrained beam

End moments

$$M_A = M_B = -\frac{w_oL^2}{12} = -\frac{WL}{12}$$

Value of EIy

$$\text{Maximum } EI\,y = \frac{w_o L^4}{384} = \frac{WL^3}{384}$$

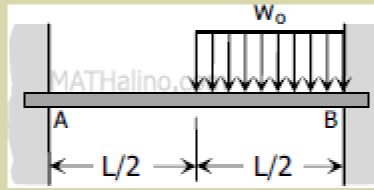


Case 4: Uniformly distributed load over half the span of fully restrained beam

End moments

$$M_A = -\frac{5w_o L^2}{192} = -\frac{5WL}{96}$$

$$M_B = -\frac{11w_o L^2}{192} = -\frac{11WL}{96}$$



Value of EIy

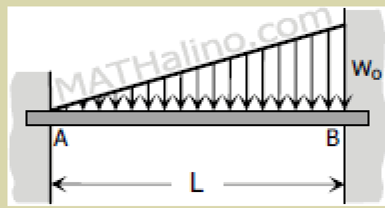
$$\text{Midspan } EI\,y = \frac{w_o L^4}{384} = \frac{WL^3}{384}$$

Case 5: Triangular load over the entire span of fully restrained beam

End moments

$$M_A = -\frac{w_o L^2}{30} = -\frac{WL}{15}$$

$$M_B = -\frac{w_o L^2}{20} = -\frac{WL}{10}$$



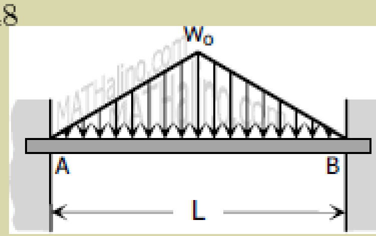
Value of EIy

$$\text{Midspan } EI\,y = \frac{w_o L^4}{768} = \frac{WL^3}{384}$$

Case 6: Isosceles triangle loadings over the entire span of fully restrained beam

End moments

$$M_A = M_B = -\frac{5w_o L^2}{96} = -\frac{5WL}{48}$$



Value of EIy

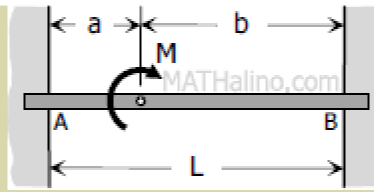
$$\text{Maximum } EI\,y = \frac{7w_o L^4}{3840} = \frac{7WL^3}{1920}$$

Case 7: Moment load anywhere on the span of fully restrained beam

End moments

$$M_A = \frac{Mb}{L}\left(\frac{3a}{L} - 1\right)$$

$$M_B = -\frac{Ma}{L}\left(\frac{3b}{L} - 1\right)$$

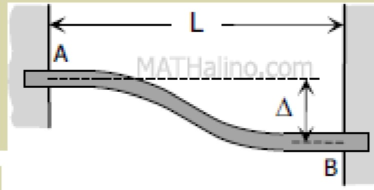


Case 8: Fully restrained beam with one support settling

End moments

$$M_A = -\frac{6EI\Delta}{L^2}$$

$$M_B = \frac{6EI\Delta}{L^2}$$



Chapter 08 - Continuous Beams

Continuous beams are those that rest over three or more supports, thereby having one or more redundant support reactions.

These section includes

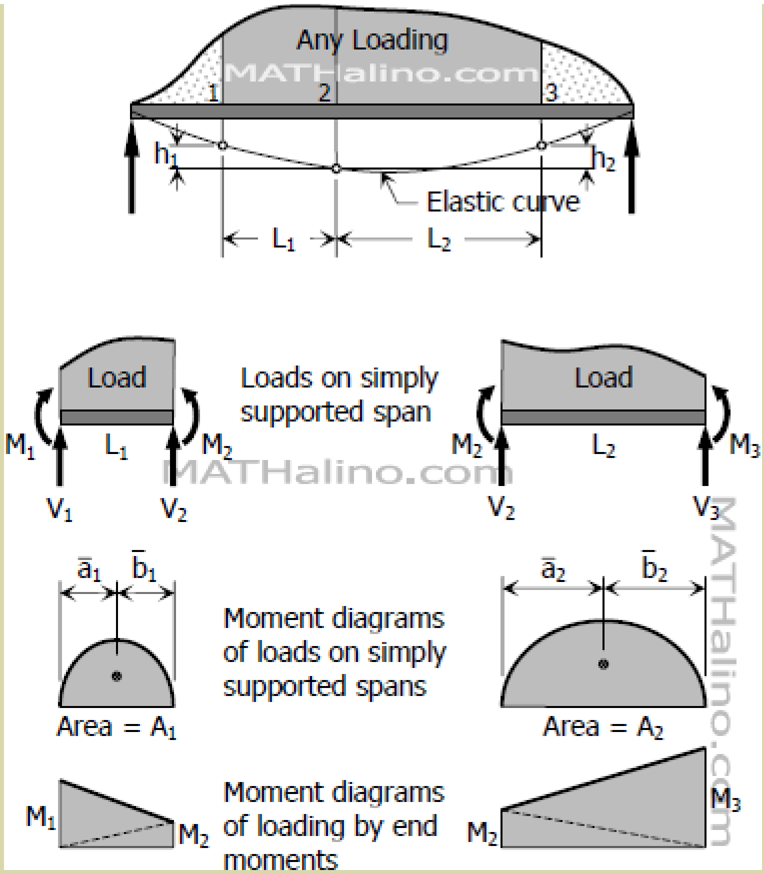
1. Generalized form of three-moment equation
2. Factors for three-moment equation
3. Application of the three-moment equation
4. Reactions of continuous beams
5. Shear and moment diagrams of continuous beams
6. Continuous beams with fixed ends
7. Deflection determined by three-moment equation
8. Moment distribution method

Explore the links under this page for available topics. Any topic not on the link only means that it is not yet available in this site.

The Three-Moment Equation

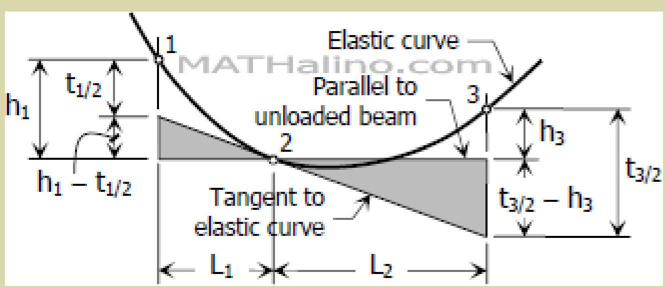
The three-moment equation gives us the relation between the moments between any three points in a beam and their relative vertical distances or deviations. This method is widely used in finding the reactions in a continuous beam.

Consider three points on the beam loaded as shown.



From proportions between similar triangles:

$$\frac{h_1 - t_{1/2}}{L_1} = \frac{t_{3/2} - h_3}{L_2}$$



$$\frac{h_1}{L_1} - \frac{t_{1/2}}{L_1} = \frac{t_{3/2}}{L_2} - \frac{h_3}{L_2}$$

$$\frac{t_{1/2}}{L_1} + \frac{t_{3/2}}{L_2} = \frac{h_1}{L_1} + \frac{h_3}{L_2} \quad \rightarrow \quad \text{equation (1)}$$

Values of $t_{1/2}$ and $t_{3/2}$

$$t_{1/2} = \frac{1}{E_1 I_1}(\text{Area}_{1-2}) \cdot \bar{X}_1$$

$$t_{1/2} = \frac{1}{E_1 I_1}\left[A_1\bar{a}_1 + (\tfrac{1}{2}M_1L_1)(\tfrac{1}{3}L_1) + (\tfrac{1}{2}M_2L_1)(\tfrac{2}{3}L_1)\right]$$

$$t_{1/2} = \frac{1}{6E_1 I_1}(6A_1\bar{a}_1 + M_1{L_1}^2 + 2M_2{L_1}^2)$$

$$t_{3/2} = \frac{1}{E_2 I_2}(\text{Area}_{2-3}) \cdot \bar{X}_3$$

$$t_{3/2} = \frac{1}{E_2 I_2}\left[A_2\bar{b}_2 + (\tfrac{1}{2}M_2L_2)(\tfrac{2}{3}L_2) + (\tfrac{1}{2}M_3L_2)(\tfrac{1}{3}L_2)\right]$$

$$t_{3/2} = \frac{1}{6E_2 I_2}(6A_2\bar{b}_2 + 2M_2{L_2}^2 + M_3{L_2}^2)$$

Substitute $t_{1/2}$ and $t_{3/2}$ to equation (1)

$$\frac{1}{6E_1 I_1}\left(\frac{6A_1\bar{a}_1}{L_1} + M_1L_1 + 2M_2L_1\right) + \frac{1}{6E_2 I_2}\left(\frac{6A_2\bar{b}_2}{L_2} + 2M_2L_2 + M_3L_2\right) = \frac{h_1}{L_1} + \frac{h_3}{L_2}$$

Multiply both sides by 6

$$\frac{1}{E_1 I_1}\left(\frac{6A_1\bar{a}_1}{L_1} + M_1L_1 + 2M_2L_1\right) + \frac{1}{E_2 I_2}\left(\frac{6A_2\bar{b}_2}{L_2} + 2M_2L_2 + M_3L_2\right) = 6\left(\frac{h_1}{L_1} + \frac{h_3}{L_2}\right)$$

Distribute 1/EI

$$\frac{6A_1\bar{a}_1}{E_1I_1L_1} + \frac{M_1L_1}{E_1I_1} + \frac{2M_2L_1}{E_1I_1} + \frac{6A_2\bar{b}_2}{E_2I_2L_2} + \frac{2M_2L_2}{E_2I_2} + \frac{M_3L_2}{E_2I_2} = 6\left(\frac{h_1}{L_1} + \frac{h_3}{L_2}\right)$$

Combine similar terms and rearrange

$$\frac{M_1L_1}{E_1I_1} + 2M_2\left(\frac{L_1}{E_1I_1} + \frac{L_2}{E_2I_2}\right) + \frac{M_3L_2}{E_2I_2} + \frac{6A_1\bar{a}_1}{E_1I_1L_1} + \frac{6A_2\bar{b}_2}{E_2I_2L_2} = 6\left(\frac{h_1}{L_1} + \frac{h_3}{L_2}\right)$$

If E is constant this equation becomes,

$$\frac{M_1L_1}{I_1} + 2M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + \frac{M_3L_2}{I_2} + \frac{6A_1\bar{a}_1}{I_1L_1} + \frac{6A_2\bar{b}_2}{I_2L_2} = 6E\left(\frac{h_1}{L_1} + \frac{h_3}{L_2}\right)$$

If E and I are constant then,

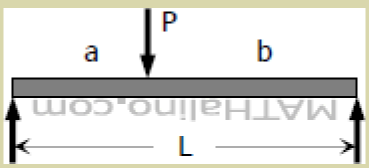
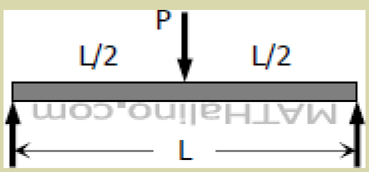
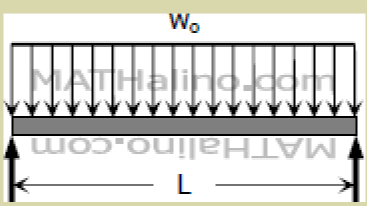
$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 6EI\left(\frac{h_1}{L_1} + \frac{h_3}{L_2}\right)$$

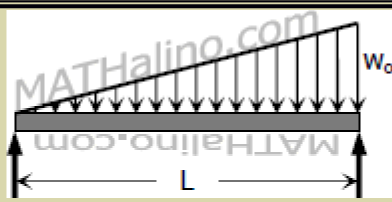
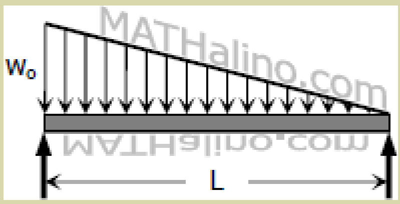
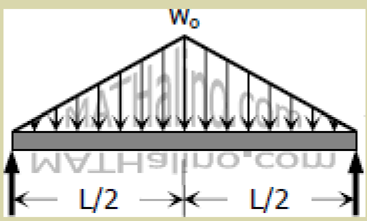
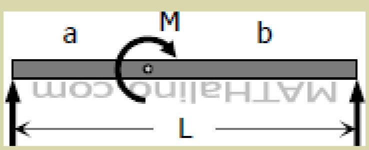
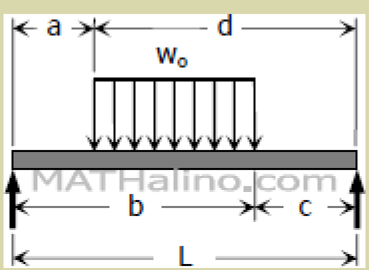
For the application of three-moment equation to continuous beam, points 1, 2, and 3 are usually unsettling supports, thus h_1 and h_3 are zero. With E and I constants, the equation will reduce to

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 0$$

Factors for the three-moment equation

The table below list the value of $6A\bar{a}/L$ and $6A\bar{b}/L$ for different types of loading.

Type of Loading	$\frac{6A\bar{a}}{L}$	$\frac{6A\bar{b}}{L}$
Concentrated load anywhere on the span. 	$\frac{Pa}{L}(L^2 - a^2)$	$\frac{Pb}{L}(L^2 - b^2)$
Concentrated load at the midspan. 	$\frac{3PL^2}{8}$	$\frac{3PL^2}{8}$
Uniform load over the entire span. 	$\frac{w_oL^3}{4}$	$\frac{w_oL^3}{4}$
Increasing triangular load on the whole span.	$\frac{8w_oL^3}{60}$	$\frac{7w_oL^3}{60}$

<table>
<tr><td>
</td><td></td><td></td></tr>
<tr><td>Decreasing triangular load on the whole span.

</td><td>$\frac{7w_oL^3}{60}$</td><td>$\frac{8w_oL^3}{60}$</td></tr>
<tr><td>Isosceles triangular load over the entire span.

</td><td>$\frac{5w_oL^3}{32}$</td><td>$\frac{5w_oL^3}{32}$</td></tr>
<tr><td>Moment load at any point on the span.

</td><td>$-\frac{M}{L}(3a^2 - L^2)$</td><td>$+\frac{M}{L}(3b^2 - L^2)$</td></tr>
<tr><td rowspan="2">General uniform loading.

</td><td colspan="2">$\frac{6A\bar{a}}{L} = \frac{w_o}{4L}[\,b^2(2L^2 - b^2) - a^2(2L^2 - a^2)\,]$</td></tr>
<tr><td colspan="2">$\frac{6A\bar{b}}{L} = \frac{w_o}{4L}[\,d^2(2L^2 - d^2) - c^2(2L^2 - c^2)\,]$</td></tr>
</table>

Problem 813

Determine the moment over the support R_2 of the beam shown in Fig. P-813.

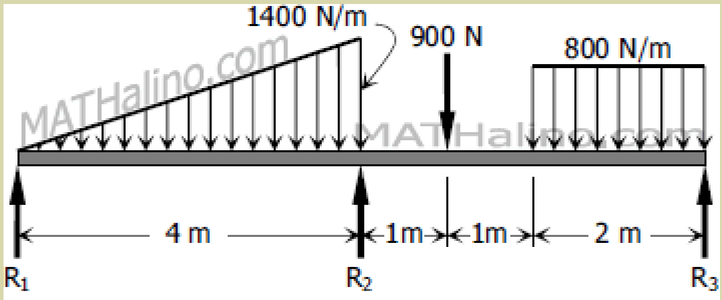


Figure P-813

Solution 813

HideClick here to show or hide the solution

$$M_1L_1 + 2M_2(L_1+L_2) + M_3L_2 + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 0$$

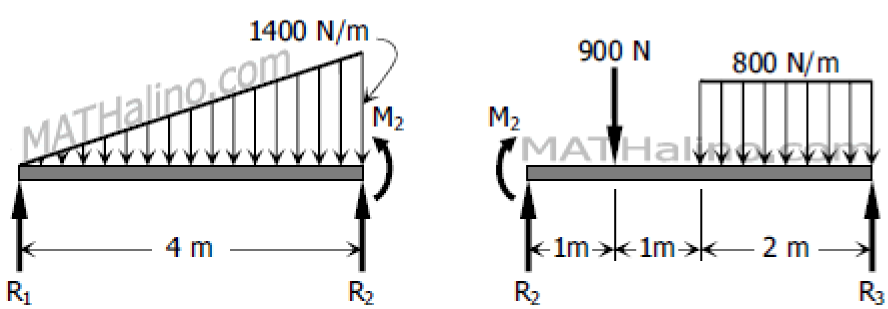
Where

$M_1 = M_3 = 0$

$L_1 = L_2 = 4 \text{ m}$

$$\frac{6A_1\bar{a}_1}{L_1} = \frac{8w_oL^3}{60} = \frac{8(1400)(4^3)}{60}$$

$$\frac{6A_1\bar{a}_1}{L_1} = 11\,946.67 \text{ N}\cdot\text{m}^2$$



$$\frac{6A_2\bar{b}_2}{L_2} = \frac{Pb}{L}(L^2 - b^2) + \frac{w_o d^2}{4L}(2L^2 - d^2)$$

$$\frac{6A_2\bar{b}_2}{L_2} = \frac{900(3)}{4}(4^2 - 3^2) + \frac{800(2^2)}{4(4)}[\,2(4^2) - 2^2\,]$$

$$\frac{6A_2\bar{b}_2}{L_2} = 10\,325 \text{ N}\cdot\text{m}^2$$

Thus,

$0 + 2M_2(4+4) + 0 + 11\,946.67 + 10\,325$

$M_2 = -1391.98 \text{ N}\cdot\text{m}$ *answer*

Problem 814

Find the moment at R_2 of the continuous beam shown in Fig. P-814.

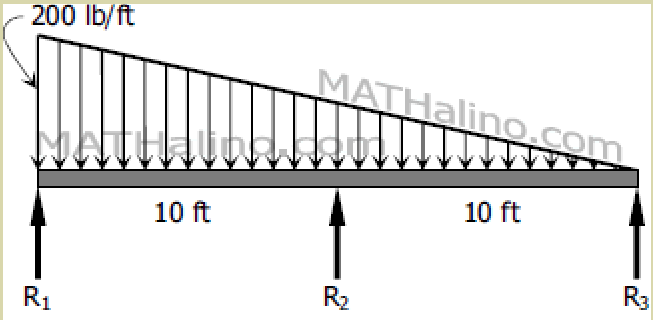


Figure P-814

Solution 814

HideClick here to show or hide the solution

$$M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 + \frac{6A_1\bar{a}_1}{L_1} + \frac{6A_2\bar{b}_2}{L_2} = 0$$

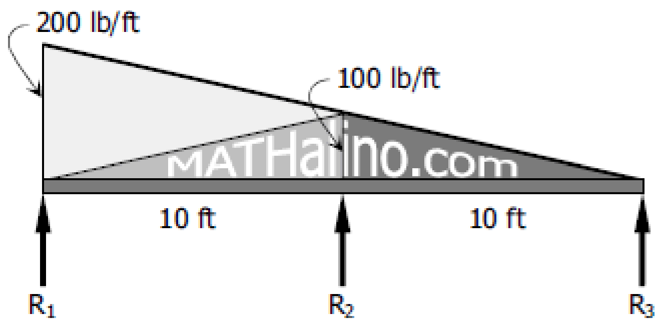
Where

$M_1 = M_3 = 0$

$$L_1 = L_2 = 10 \text{ ft}$$

$$\frac{6A_1\bar{a}_1}{L_1} = \frac{7w_oL^3}{60} + \frac{8w_oL^3}{60} = \frac{7(200)(10^3)}{60} + \frac{8(100)(10^3)}{60}$$

$$\frac{6A_1\bar{a}_1}{L_1} = 36\,666.67 \text{ lb}\cdot\text{ft}^2$$



$$\frac{6A_2\bar{b}_2}{L_2} = \frac{8w_oL^3}{60} = \frac{8(100)(10^3)}{60}$$

$$\frac{6A_2\bar{b}_2}{L_2} = 13\,333.33 \text{ lb}\cdot\text{ft}^2$$

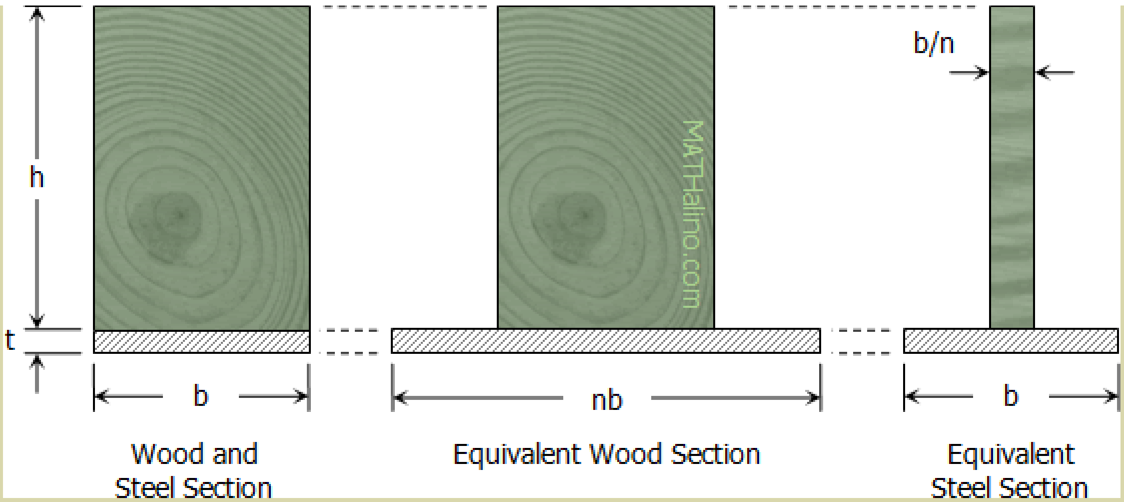
Thus,

$$0 + 2M_2(10 + 10) + 0 + 36\,666.67 + 13\,333.33 = 0$$

$M_2 = -1250 \text{ lb}\cdot\text{ft}$ *answer*

Chapter 10 - Reinforced Beams

Flexure formula do not apply directly to composite beams because it was based on the assumption that the beam was homogeneous. It is therefore necessary to transform the composite material into equivalent homogeneous section. To do this, consider a steel and wood section to be firmly bolted together so that they can act as one unit. Shown below are the composite wood and steel section and the corresponding equivalent in wood and steel sections.



The quantity n is the ratio of the moduli of elasticity of stronger material to the weaker material. In the above case, $n = E_s / E_w$.

The use of the above concept is governed by the following assumptions:

1. The composite materials are firmly bonded together and act as a unit.
2. The strain and load capacities of each material remain unchanged.
3. The strains of any two adjacent materials at their junction point are equal.
4. The loads carried by equivalent fibers are equal.
5. The equivalent fibers must be at the same distance from the neutral axis as the original fibers.
6. The flexure formula can be applied only to the equivalent section.

Beams with Different Materials

From assumption no. (3) in the previous page: The strains of any two adjacent materials at their junction point are equal.

$$\epsilon_s = \epsilon_w$$

$$\frac{f_{bs}}{E_s} = \frac{f_{bw}}{E_w}$$

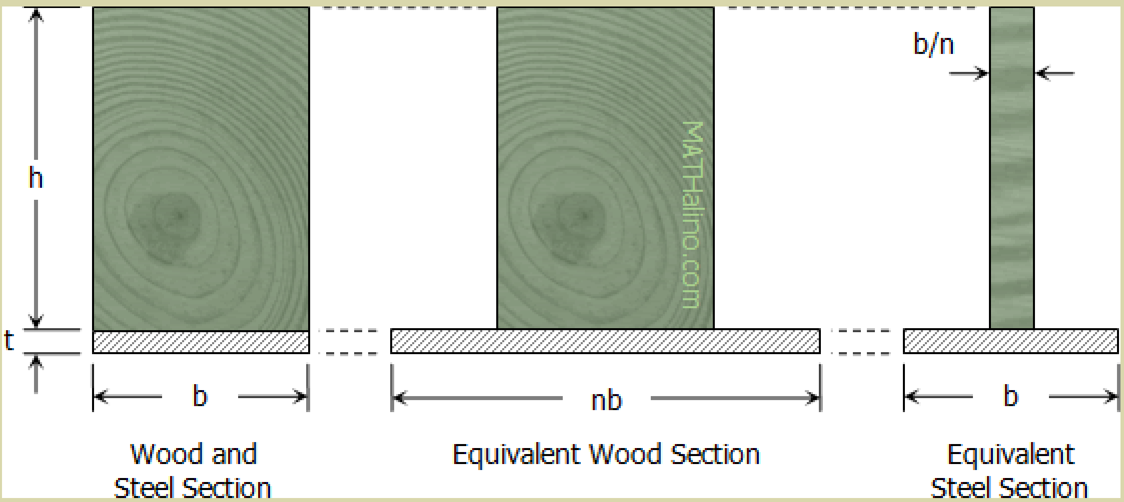
$$\frac{f_{bs}}{f_{bw}} = \frac{E_s}{E_w}$$

We let the moduli ratio be equal to n

$$n = \frac{E_s}{E_w}$$

It will follow also that

$$f_{bw} = \frac{f_{bs}}{n}$$



From assumption no. (4): The loads carried by equivalent fibers are equal.

$$P_w = P_s$$

$$f_{bw}\, A_w = f_{bs}\, A_s$$

$$A_w = \frac{f_{bs}}{f_{bw}} \cdot A_s$$

$$A_w = \frac{E_s}{E_w} \cdot A_s$$

The equivalent area of steel in wood is

$$A_w = n\, A_s$$

Problem 1002

A timber beam is reinforced with steel plates rigidly attached at the top and bottom as shown in Fig. P-1002. By what amount is moment increased by the reinforcement if n = 15 and the allowable stresses in the wood and steel are 8 MPa and 120 MPa, respectively?

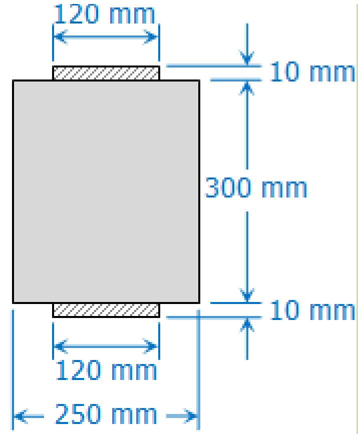


Figure P-1002, P-1003, & P-1004

Solution 1002

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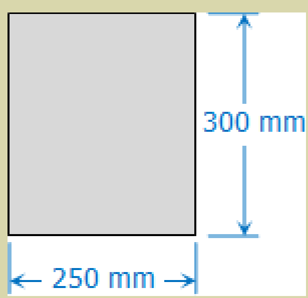
From the flexure formula

$$f_b = \frac{Mc}{I}$$

$$M = \frac{f_b I}{c}$$

Without the steel plate reinforcement



$$I = \frac{250(300^3)}{12} = 562\,500\,000 \text{ mm}^4$$

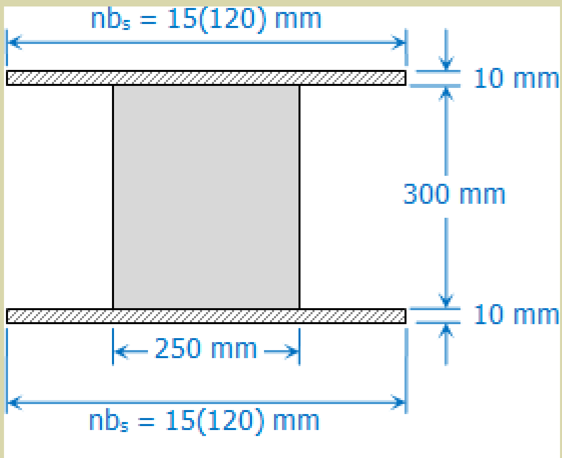
$$c = \frac{300}{2} = 150 \text{ mm}$$

$$M_1 = \frac{8(562\,500\,000)}{150}$$

$$M_1 = 30\,000\,000 \text{ N} \cdot \text{mm}$$

$$M_1 = 30 \text{ kN} \cdot \text{m}$$

With the steel plate reinforcement



Equivalent Wood Section

$$I = \frac{15(120)(300+10+10)^3}{12} - \frac{[15(120)-250](300^3)}{12} = 1\,427\,700\,000 \text{ mm}^4$$

$$c = \frac{300+10+10}{2} = 160 \text{ mm}$$

Convert steel to wood, use $f_{bw} = f_{bs} / n$

$$M_2 = \frac{\frac{120}{15}(1\,427\,700\,000)}{160}$$

$$M_2 = 71\,385\,000 \text{ N} \cdot \text{mm}$$

$$M_2 = 71.4 \text{ kN} \cdot \text{m}$$

Increase in moment capacity
$\Delta M = M_2 - M_1 = 71.4 - 30$

$\Delta M = 41.4 \text{ kN}\cdot\text{m}$ *answer*

Problem 1003

A simply supported beam 4 m long has the cross section shown in Fig. P-1002. It carries a uniformly distributed load of 20 kN/m over the middle half of the span. If n = 15, compute the maximum stresses in the wood and the steel.

Solution 1003

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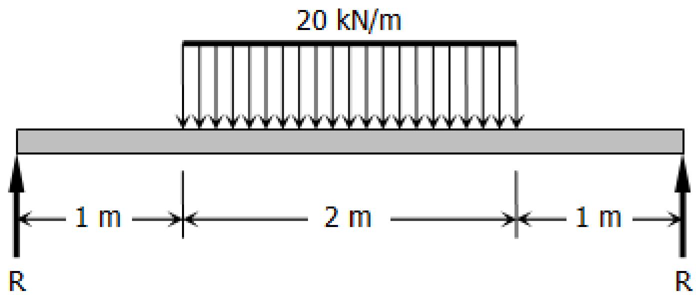


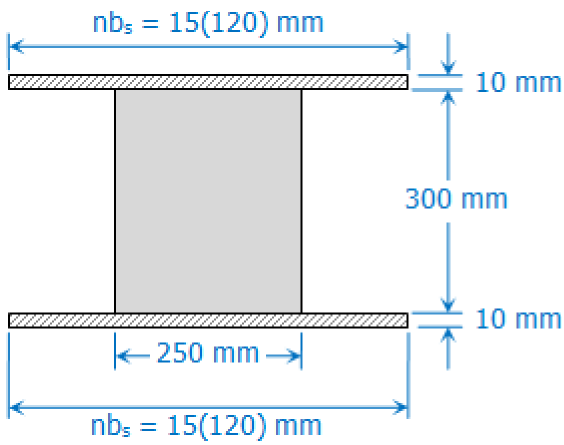
$R = \frac{1}{2}(20)(2) = 20 \text{ kN}$

Maximum moment will occur at the midspan:
$M_{max} = 2R - 20(1)(0.5) = 2(20) - 10$

$M_{max} = 30 \text{ kN}\cdot\text{m}$

Flexural stress:
$f_b = \dfrac{My}{I}$

Where:
$M = M_{max} = 30 \text{ kN} \cdot \text{m}$



Equivalent Wood Section

$$I = \frac{15(120)(300+10+10)^3}{12} - \frac{[15(120)-250](300^3)}{12}$$

$$I = 1\,427\,700\,000 \text{ mm}^4$$

Maximum stress in the wood (y = 150 mm)

$$f_{bw} = \frac{30(150)(1000^2)}{1\,427\,700\,000}$$

$$f_{bw} = 3.152 \text{ MPa}$$

Maximum stress in the steel (y = 160 mm)

$$\frac{f_{bs}}{15} = \frac{30(160)(1000^2)}{1\,427\,700\,000}$$

$$f_{bs} = 47.279 \text{ MPa}$$

Problem 1004

Repeat Problem 1002 assuming that the reinforcement consists of aluminum plates for which the allowable stress is 80 MPa. Use n = 5.

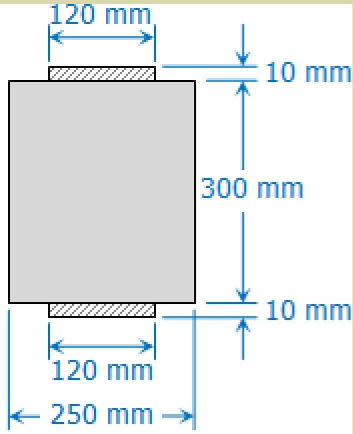


Figure P-1002, P-1003, & P-1004

Solution 1004

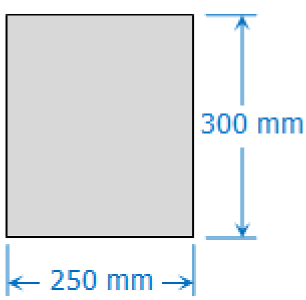
HideClick here to show or hide the solution

From the flexure formula

$$f_b = \frac{Mc}{I}$$

$$M = \frac{f_b I}{c}$$

Without the aluminum plate reinforcement



$$I = \frac{250(300^3)}{12} = 562\,500\,000 \text{ mm}^4$$

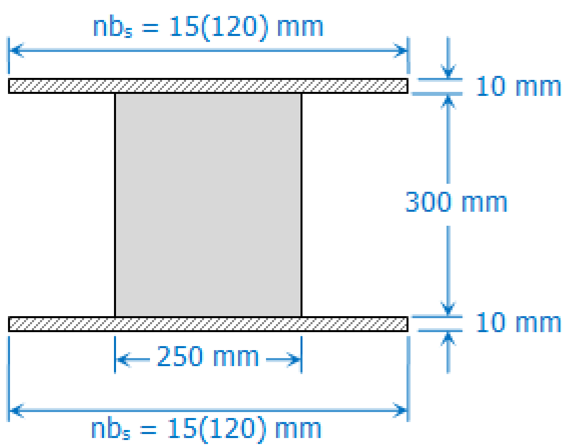
$$c = \frac{300}{2} = 150 \text{ mm}$$

$$M_1 = \frac{8(562\,500\,000)}{150}$$

$$M_1 = 30\,000\,000 \text{ N} \cdot \text{mm}$$

$$M_1 = 30 \text{ kN} \cdot \text{m}$$

With the aluminum plate reinforcement



Equivalent Wood Section

$$I = \frac{5(120)(300+10+10)^3}{12} - \frac{[\,5(120) - 250\,](300^3)}{12} = 850\,900\,000 \text{ mm}^4$$

$$c = \frac{300+10+10}{2} = 160 \text{ mm}$$

Convert aluminum to wood, use $f_{bw} = f_{ba} / n$

$$M_2 = \frac{\frac{80}{5}(850\,900\,000)}{160}$$

$$M_2 = 85\,090\,000 \text{ N}\cdot\text{mm}$$

$$M_2 = 85.09 \text{ kN}\cdot\text{m}$$

Increase in moment capacity

$$\Delta M = M_2 - M_1 = 85.09 - 30$$

$$\Delta M = 55.09 \text{ kN}\cdot\text{m} \qquad \textit{answer}$$

Problem 1005

A timber beam 6 in. by 10 in. is reinforced only at the bottom by a steel plate as shown in Fig. P-1005. Determine the concentrated load that can be applied at the center of a simply supported span 18 ft long if n = 20, $f_s \le 18$ ksi and $f_w \le 1200$ psi. Show that the neutral axis is 7.1 in. below the top and that $I_{NA} = 1160 \text{ in.}^4$.

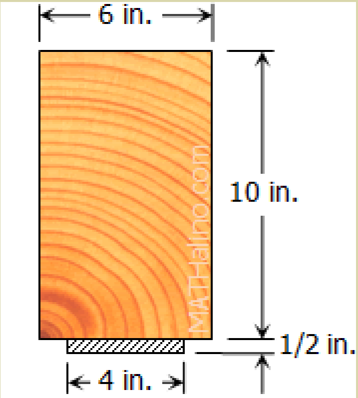


Figure P-1005, P-1006, & P-1007

Solution 1005

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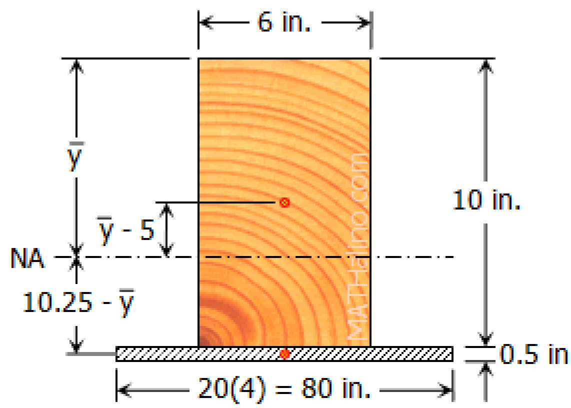
From the equivalent wood section shown

$$A_w = 6(10) = 60 \text{ in}^2$$

$A_s = 80(0.5) = 40\ \text{in}^2$

$A = A_w + A_s = 100\ \text{in}^2$



Equivalent wood section

Location of NA from the top of the beam

$A\bar{y} = A_w y_w + A_s y_s$

$100\bar{y} = 60(5) + 40(10.25)$

$\bar{y} = 7.1\ \text{in}$

Moment of inertia of each section

$$I_g = \frac{bd^3}{12}$$

$$I_{gw} = \frac{6(10^3)}{12} = 500\ \text{in}^4$$

$$I_{gs} = \frac{80(0.5^3)}{12} = \frac{5}{6}\ \text{in}^4$$

Moment of inertia about NA

$I = \Sigma(I_g + Ad^2)$

$$I_{NA} = [\,I_{gw} + A_w(\bar{y} - 5)^2\,] + [\,I_{gs} + A_s(10.25 - \bar{y})^2\,]$$

$$I_{NA} = [\,500 + 60(7.1 - 5)^2\,] + [\,\tfrac{5}{6} + 40(10.25 - 7.1)^2\,]$$

$$I_{NA} = \frac{3487}{3}\ \text{in}^4 = 1162.33\ \text{in}^4$$

Maximum moment that the beam can carry

$$f_b = \frac{My}{I}$$

$$M = \frac{f_b I}{y}$$

Based on strength of wood ($y = \bar{y} = 7.1 \text{ in}$)

$$M_w = \frac{1200(\frac{3487}{3})}{7.1}$$

$$M_w = 196\,450.70 \text{ lb}\cdot\text{in}$$

$$M_w = 16\,370.89 \text{ lb}\cdot\text{ft}$$

Based on strength of wood ($y = 10.5 - \bar{y} = 3.4 \text{ in}$)

$$M_s = \frac{\frac{18\,000}{20} \times \frac{3487}{3}}{3.4}$$

$$M_s = 307\,676.47 \text{ lb}\cdot\text{in}$$

$$M_s = 25\,639.70 \text{ lb}\cdot\text{ft}$$

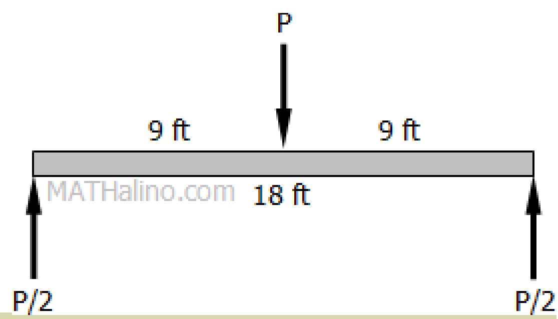
For safe value of M, use the smaller of the two
$M = 16\,370.89 \text{ lb}\cdot\text{ft}$

Safe value of P at the midspan that beam can carry

$$M = \frac{PL}{4}$$

$$16\,370.89 = \frac{P(18)}{4}$$

$P = 3637.98 \text{ lb}$ *answer*



Problem 1006

Determine the width b of the 1/2-in. steel plate fastened to the bottom of the beam in Problem 1005 that will simultaneously stress the wood and the steel to their permissible limits of 1200 psi and 18 ksi, respectively.

Solution 1006

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$$f_b = \frac{My}{I}$$

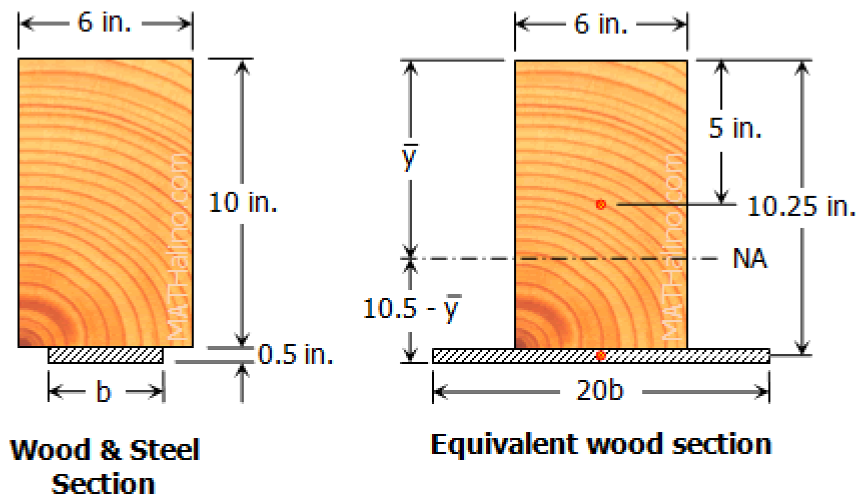
$$\frac{M}{I} = \frac{f_b}{y}$$

At the extreme wood fiber

$$\left(\frac{M}{I}\right)_{\rm wood} = \frac{1200}{\bar{y}}$$

At the extreme steel fiber

$$\left(\frac{M}{I}\right)_{\rm steel} = \frac{18\,000/20}{10.5 - \bar{y}} = \frac{900}{10.5 - \bar{y}}$$



$$\left(\frac{M}{I}\right)_{\rm wood} = \left(\frac{M}{I}\right)_{\rm steel}$$

$$\frac{1200}{\bar{y}} = \frac{900}{10.5 - \bar{y}}$$

$$1200(10.5 - \bar{y}) = 900\bar{y}$$

$$12\,600 - 1200\bar{y} = 900\bar{y}$$

$2100\bar{y} = 12\,600$

$\bar{y} = 6 \text{ in}$

You can also find $\bar{y}$ with the aid of stress diagram as follows:

ShowClick here to show or hide the solution

$A_w = 6(10) = 60 \text{ in}^2$

$A_s = 0.5(20b) = 10b$

$A = A_w + A_s = 60 + 10b$

$A\bar{y} = A_w y_w + A_s y_s$

$(60 + 10b)(6) = 60(5) + 10b(10.25)$

$360 + 60b = 300 + 102.5b$

$42.5b = 60$

$b = \frac{24}{17} \text{ in} = 1.41 \text{ in}$ *answer*

Problem 1007

A uniformly distributed load of 300 lb/ft (including the weight of the beam) is simply supported on a 20-ft span. The cross section of the beam is described in Problem 1005. If n = 20, determine the maximum stresses produced in the wood and the steel.

Solution 1007

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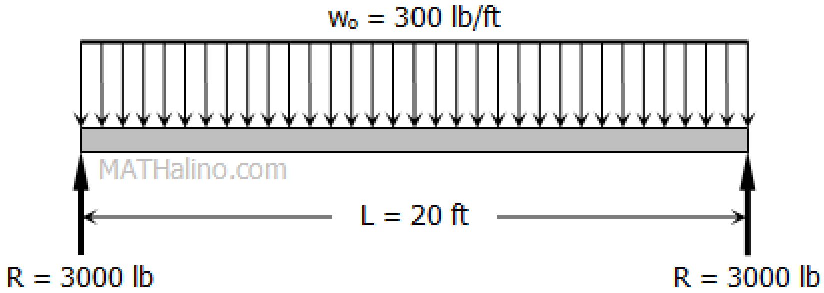


Maximum moment will occur at the midspan

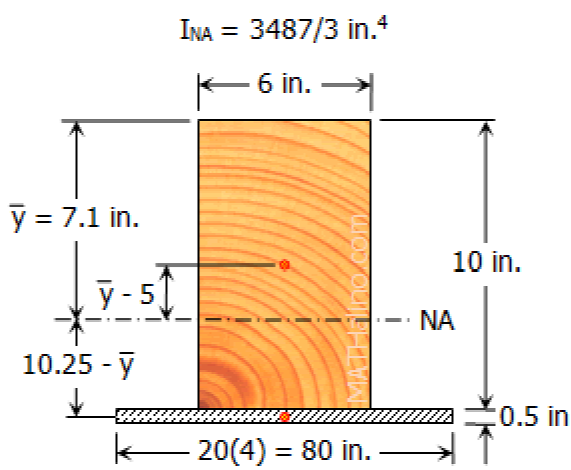
$$M_{max} = \frac{w_o L^2}{8} = \frac{300(20^2)}{8}$$

$$M_{max} = 15\,000 \text{ lb} \cdot \text{ft}$$

From Solution 105:

$$\bar{y} = 7.1 \text{ in}$$

$$I_{NA} = \tfrac{3487}{3} \text{ in}^4$$



Equivalent wood section

$$f_b = \frac{Mc}{I}$$

At the extreme wood fiber ($c = \bar{y}$)

$$f_{bw} = \frac{M_{max}\,\bar{y}}{I_{NA}}$$

$$f_{bw} = \frac{15\,000(7.1)(12)}{\tfrac{3487}{3}}$$

$f_{bw} = 1099.51 \text{ psi}$ *answer*

At the extreme steel fiber ($c = 10.5 - \bar{y}$)

$$\frac{f_{bs}}{n} = \frac{M_{max}(10.5 - \bar{y})}{I_{NA}}$$

$$\frac{f_{bs}}{20} = \frac{15\,000(10.5 - 7.1)(12)}{\frac{3487}{3}}$$

$f_{bs} = 10\,530.54$ psi *answer*

Problem 1008

A timber beam 150 mm wide by 250 mm deep is to be reinforced at the top and bottom by steel plates 10 mm thick. How wide should the steel plates be if the beam is to resist a moment of 40 kN·m? Assume that n = 15 and the allowable stresses in the wood and steel are 10 MPa and 120 MPa, respectively.

Solution 1008

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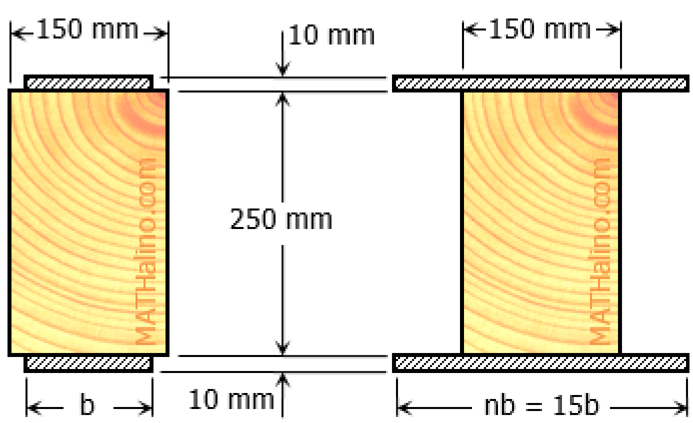
$M = 40 \text{ kN} \cdot \text{m}$

$$I = \frac{15b(270^3)}{12} - \frac{(15b - 150)(250^3)}{12}$$

$$I = 24\,603\,750b - 19\,531\,250b + 195\,312\,500$$

$$I = 5\,072\,500b + 195\,312\,500$$



$$f_b = \frac{Mc}{I}$$

Based on allowable flexural stress of steel:

$f_b = 120/n$

$c = \frac{1}{2}(250) + 10 = 135 \text{ mm}$

Thus,

$$\frac{120}{15} = \frac{40(1000^2)(135)}{5\,072\,500b + 195\,312\,500}$$

$b = 94.57 \text{ mm}$

Based on allowable flexural stress of wood:

$f_b = 10 \text{ MPa}$

$c = \frac{1}{2}(250) = 125 \text{ mm}$

Thus,

$$10 = \frac{40(1000^2)(125)}{5\,072\,500b + 195\,312\,500}$$

$b = 60.01 \text{ mm}$

For stronger section, use $b = 94.6 \text{ mm}$ *answer*

Problem 1009

A timber beam 150 mm wide by 200 mm deep is to be reinforced at the top and bottom by aluminum plates 6 mm thick. Determine the width of the aluminum plates if the beam is to resist a moment of 14 kN·m. Assume n = 5 and take the allowable stresses as 10 MPa and 80 MPa in the wood and aluminum, respectively.

Solution 1009

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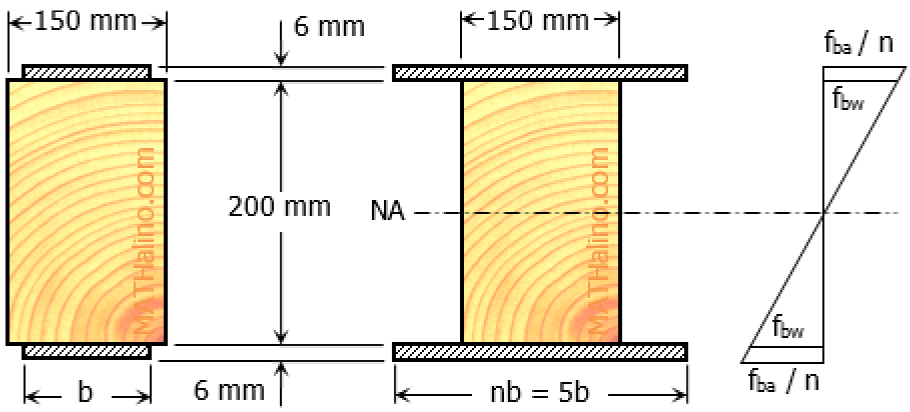
$M = 14 \text{ kN} \cdot \text{m}$

$$I = \frac{5b(212^3)}{12} - \frac{(5b - 150)(200^3)}{12}$$

$$I = 3\,970\,053.33b - 3\,333\,333.33b + 100\,000\,000$$

$$I = 636\,720b + 100\,000\,000$$



$$f_b = \frac{Mc}{I}$$

Based on allowable flexural stress of aluminum:

$$f_b = 80/n$$

$$c = \tfrac{1}{2}(200) + 6 = 106 \text{ mm}$$

Thus,

$$\frac{80}{n} = \frac{14(1000^2)(106)}{636\,720b + 100\,000\,000}$$

$$I = -11.38 \text{ mm}$$

Based on allowable flexural stress of wood:

$f_b = 10 \text{ MPa}$

$c = \frac{1}{2}(100) = 100 \text{ mm}$

Thus,

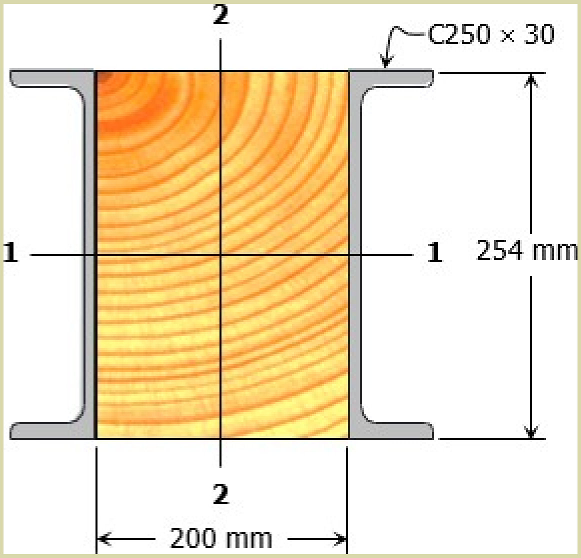
$$10 = \frac{14(1000^2)(100)}{636\,720b + 100\,000\,000}$$

$b = 62.82 \text{ mm}$

For stronger section, use $b = 62.82 \text{ mm}$ *answer*

Problem 1010

A pair of C250 × 30 steel channels are securely bolted to wood beam 200 mm by 254 mm, as shown in Fig. P-1010. From Table B-2 in Appendix B, the depth of the channel is also 254 mm.) If bending occurs about the axis 1-1, determine the safe resisting moment if the allowable stresses σ_s = 120 MPa and σ_w = 8 MPa. Assume n = 20.



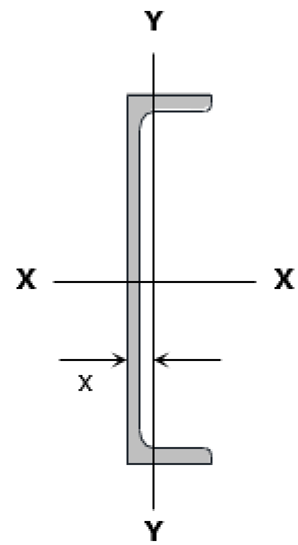
Problem 1011

In Problem 1010, determine the safe resisting moment if bending occurs about axis 2-2.

Solution

HideClick here to show or hide the solution

Relevant data for C250 × 30 steel channel



$I_x = 32.7 \times 10^6 \text{ mm}^4$

$I_y = 1.16 \times 10^6 \text{ mm}^4$

$x = 15.3 \text{ mm}$

$\text{Area} = 3780 \text{ mm}^2$

$\text{Depth} = 254 \text{ mm}^2$

$\text{Flange width} = 69 \text{ mm}^2$

Equivalent wood area

$A_w = 200(254) = 50\,800 \text{ mm}^2$

$A_s = 2(n \times \text{Area}) = 2(20 \times 3780) = 151\,200 \text{ mm}^2$

$A = A_w + A_s = 50\,800 + 151\,200 = 202\,000 \text{ mm}^2$

Solution 1010: Bending occurs at section 1-1

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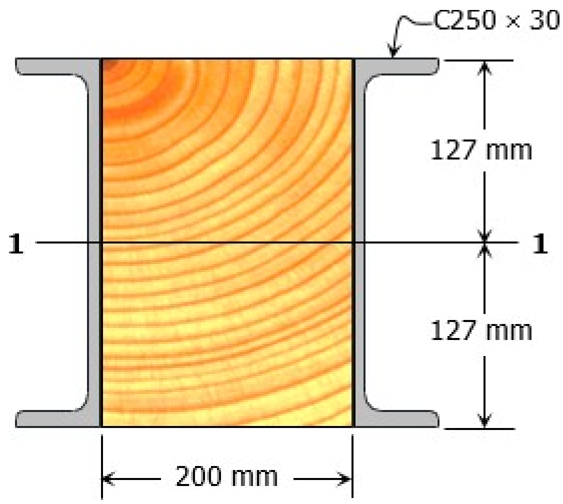
$$I_{1-1} = I_w + 2nI_x = \frac{200(254^3)}{12} + 2(20)(32.7 \times 10^6)$$

$$I_{1-1} = 1\,581\,117\,733 \text{ mm}^4$$



Measured from Section 1-1, the distance of extreme wood fiber and extreme steel fiber are equal. In this case, no need to investigate both materials.

$$\frac{f_{bs}}{n} = \frac{120}{20} = 6 \text{ MPa}$$

$$f_{bw} = 8 \text{ MPa}$$

Since $f_{bs} / n < f_{bw}$, steel is critical.

$$\frac{f_{bs}}{n} = \frac{Mc}{I_{1-1}}$$

$$6 = \frac{M(127)}{1\,581\,117\,733}$$

$$M = 74\,698\,475.57 \text{ N} \cdot \text{mm}$$

$M = 74.7 \text{ kN} \cdot \text{m}$ *answer*

Solution 1011: Bending occurs at section 2-2

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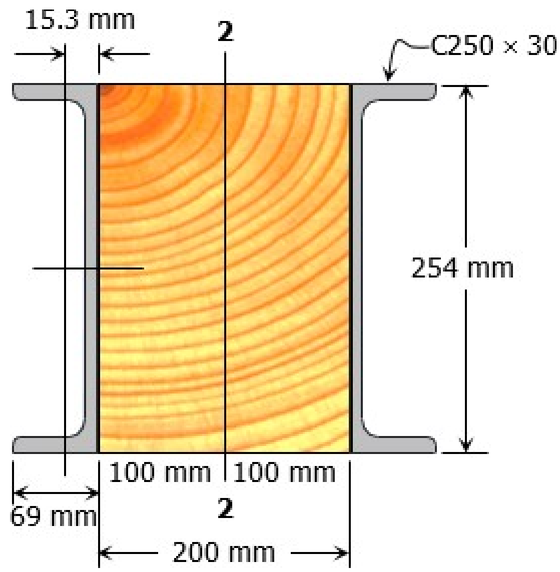
$$I_{2-2} = I_w + 2n[\, I_y + \text{Area}(100 + x)^2\,]$$

$$I_{2-2} = \frac{254(200^3)}{12} + 2(20)[\,(1.16 \times 10^6) + 3780(100 + 15.3)^2\,]$$

$$I_{2-2} = 2\,225\,799\,741 \text{ mm}^4$$



Since $\left(\dfrac{f_{bs}}{n} = 6 \text{ MPa}\right) < (f_{bw} = 8 \text{ MPa})$ and the extreme steel fiber is farther than extreme wood fiber from Section 2-2, steel is critical. In this case, no need to investigate the wood.

$$\frac{f_{bs}}{n} = \frac{Mc}{I_{2-2}}$$

$$6 = \frac{M(100 + 69)}{2\,225\,799\,741}$$

$$M = 79\,022\,476.01 \text{ N} \cdot \text{mm}$$

$M = 79.02 \text{ kN} \cdot \text{m}$ *answer*

From the above results, this section is stronger in bending at Section 2-2.