## Stress:

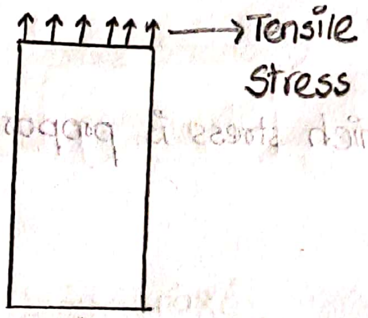
Intensity of internal force developed when an external force is applied on an engineering material.

→ Symbol :- $\sigma = \frac{P}{A}$ ; $\boxed{\sigma = \frac{dP}{dA}}$ → More precise

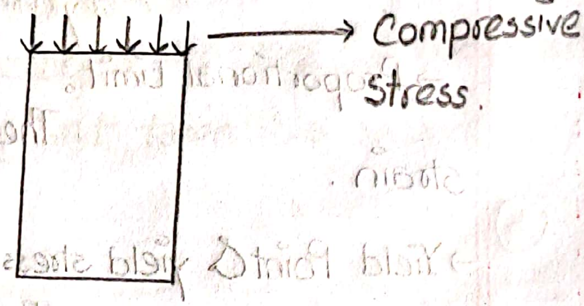
## Strength :

Property of a material that represents its ability to resist internal forces/stresses.

→ Tensile Stress :




→ Compressive Stress :

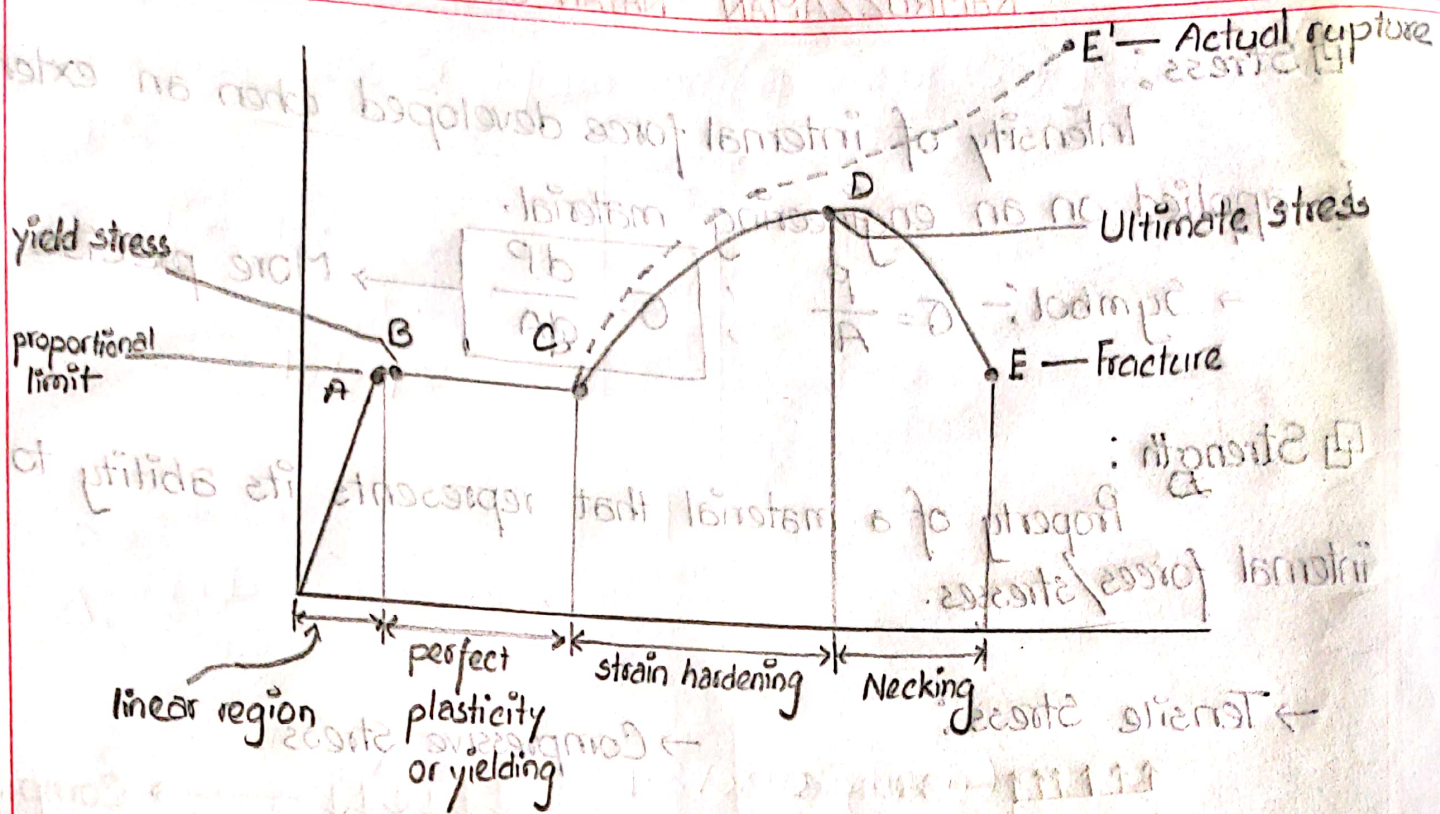



## Strain:

Measure of deformation produced by the application of external force.

→ No dimension

→ Types
- Tensile
- Compressive
- Shearing.




→ Proportional Limit:

The limit till which stress is proportional to strain.

→ Yield Point & yield stress:

The stress beyond which materials become plastic is yield point.

The value of stress at a yield point is yield stress.

→ Ultimate stress:

The maximum value of stress a material can resist.

→ Necking:

A mode of tensile deformation where relatively large amounts of strain localize disproportionately in a small region of material.

→Rupture Strength: The stress at specified environmental conditions to produce rupture in a fixed amount of time

→Elastic Limit & Range: The point beyond which ~~elastic limit~~ upon the removal of load, deformation is permanent.

The Hook's law is satisfied in this range.

→Plastic Limit & Range: The stress range in which a material will not fail when subjected to a force but will not recover completely so that a permanent deformation results when the force is removed.

→Actual Rupture Strength: Nominal stress developed at rupture. not necessarily equal to the ultimate strength and necking isn't taken into account

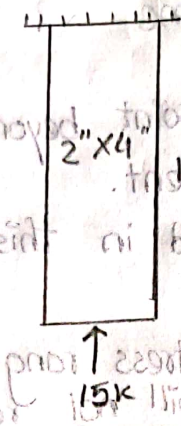
→Elastic Deformation: A temporary change of shape that is self reversing after the force is removed, so that object returns to its original shape.

→Plastic Deformation: The permanent distortion that occurs when a material is subjected to ~~tensile~~ stresses exceeding its yield stress and cause it to alongate, compress, buckle, bend & twist.

# Problems

1. (a) Calculate stress & deformation. $(E = 12\times10^{6}\ Psi)$

Sol$^n$:



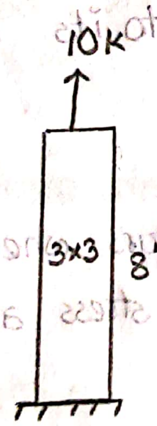

$$\sigma_1 = \frac{P}{A} = \frac{15\times1000}{2\times4} = 1875\ psi$$

$$\Delta_1 = \frac{PL}{AE} = \frac{15\times1000\times12}{2\times4\times12\times10^{6}}$$

$$\cancel{7.5\times10^{-4}} = 1.875\times10^{-3}\ \{\text{inch}\}$$

[Shortening]

(b)



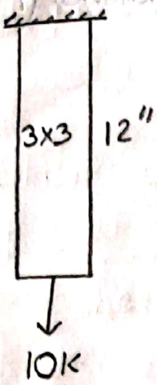

Sol$^n$: $\sigma_2 = \frac{P}{A} = \frac{10\times10^{3}}{3\times3} = 1111.11\ Psi$

$$\Delta_2 = \frac{PL}{AE} = \frac{10\times10^{3}\times8}{3\times3\times12\times10^{6}}$$

$$= 7.41\times10^{-4}\ \text{inch}$$

(Elongation).

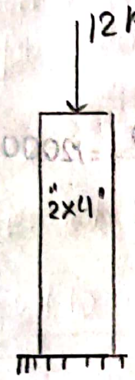
(c)




$$\sigma_1 = \frac{P_1}{A_1} = \frac{10\times1000}{3\times3} = 1111.11 \text{ psi}$$

$$\Delta_1 = \frac{P_1 L_1}{A_1 E_1} = \frac{10000\times12}{9\times12\times10^6} = 1.11\times10^{-3} \text{ in (Elongation).}$$

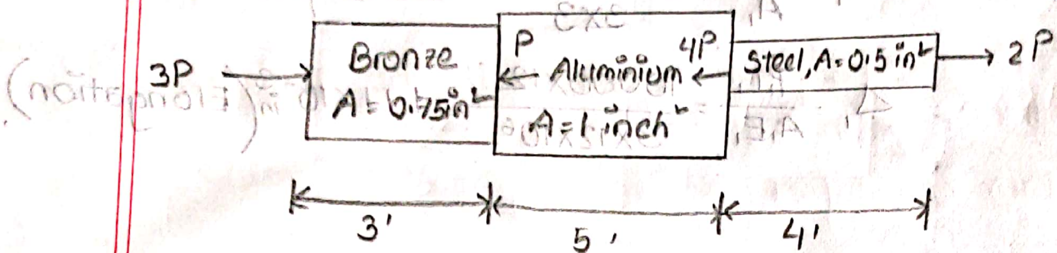
(d)




$$\sigma_1 = \frac{P_2}{A_2} = \frac{12000}{8} = 1500 \text{ psi}$$

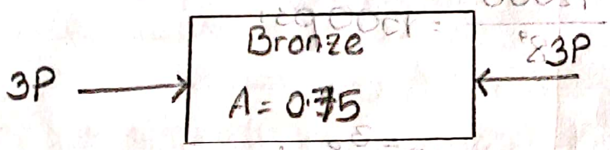
$$\Delta_2 = \frac{P_2 L_2}{A_2 E_2} = 1.25\times10^{-3} \text{ inch (shortening).}$$

2) Determine the stresses in each of the following material (P = 3000 lb)



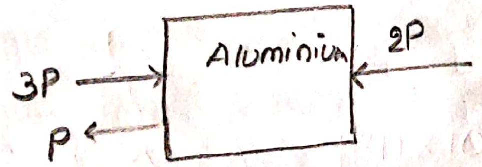

Solution:

Part 1:




$$\sigma_1 = \frac{3P}{A_1} = \frac{3 \times 3000}{0.75} = 12000 \text{ psi}$$

Part 2:

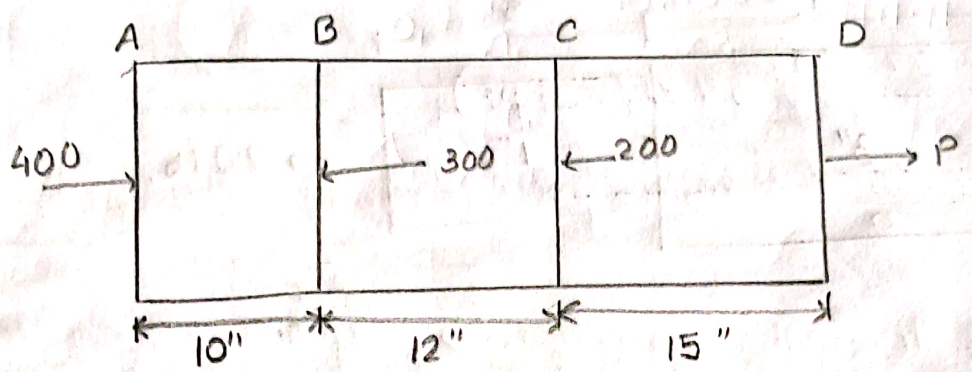



$$\sigma_2 = \frac{2P}{A_2} = \frac{2 \times 3000}{1} = 6000 \text{ psi}$$

Part 3:


$$\sigma_3 = \frac{2P}{A_3} = \frac{2 \times 3000}{0.5} = 12000 \text{ psi}$$

3) Calculate the normal stress in the bar. ($E = 12 \times 10^3$ ksi & Area, $A = 2$ inch$^2$)




$\sum F_x = 0$.

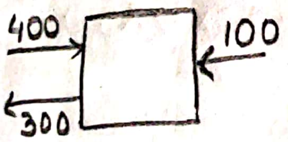
$400 + P = 300 + 200$

$\Rightarrow P = 100$

Part AB :

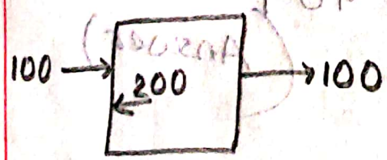

$\sigma_{AB} = \frac{400}{2} = 200$ psi

Part BC :



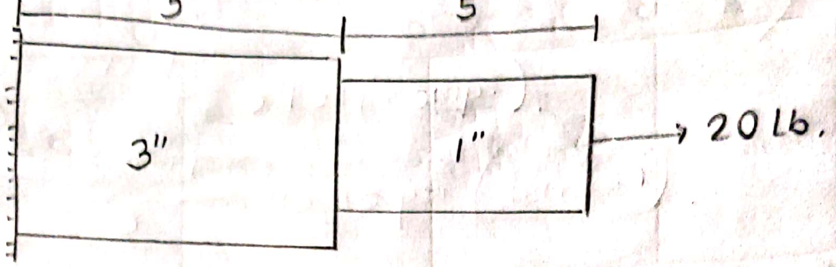

$\sigma_{BC} = \frac{100}{2} = 50$ psi

Part CD :

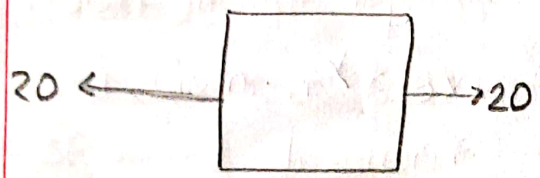



$\sigma_{CD} = \frac{100}{2} = 50$ psi

4) Calculate total deformation and maximum stress. $E = 12 \times 10^6$ psi




Part 1.:




$$\Delta_1 = \frac{20 \times 5 \times 12}{\frac{\pi \times 9}{4} \times 12 \times 10^6} = 1.41 \times 10^{-5} \text{ inch}$$

Part 2:

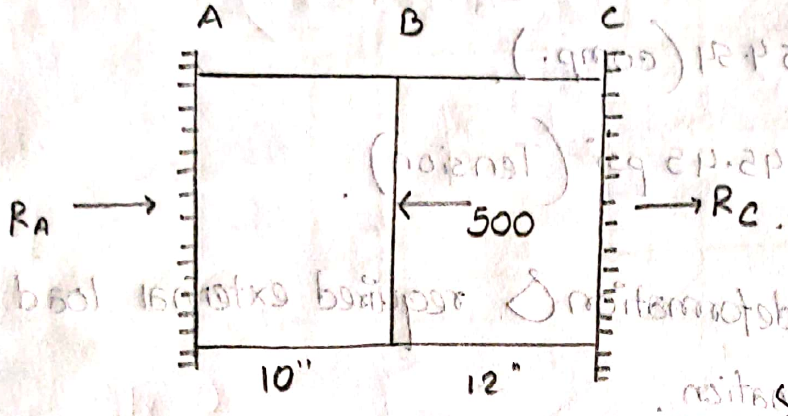

$$\Delta_2 = \frac{20 \times 5 \times 12}{\frac{\pi \times 1}{4} \times 12 \times 10^6} = 1.27 \times 10^{-4} \text{ inch}$$

$$\therefore \Delta = \Delta_1 + \Delta_2 = 1.41 \times 10^{-4} \text{ inch.}$$

$$\text{Maximum Stress} = \sigma = \frac{P}{A} = \frac{20}{\pi \times 0.5^2} = 25.46 \text{ psi}$$

(Answer)

৫।




এসব ক্ষেত্রে individual deformation অবশ্যই হবে। Total deformation 0 হতে পারে।

$$\delta_{AB} = -\frac{R_A \times 10}{AE} \quad , \quad \delta_{BC} = \frac{R_C \times 12}{AE}$$

As the both ends are fixed, deformation will be zero.

$\Delta_{total} = 0 \qquad \therefore \Delta_{AB} + \Delta_{BC} = 0$ ——— (i)

$\sum F_x = 0 \quad R_A + R_C = 500$ ——— (ii)

$$\Delta_{AB} = -\frac{R_A \times 10}{AE} \quad ; \quad \Delta_{BC} = \frac{R_C \times 12}{AE}$$

From (i),

$$\frac{R_C \times 12}{AE} - \frac{10 R_A}{AE} = 0$$

$\Rightarrow 5R_A = 6R_C$

$\Rightarrow R_A = \frac{6}{5} R_C$ ——— (iii)

Now, $\frac{6}{5} R_C + R_C = 500$

$\Rightarrow R_C = 227.27$

6) $\therefore R_A = 272.72$.

$$\Delta_{AB} = \frac{272.72}{5} = 54.54 \text{ (comp.)}.$$

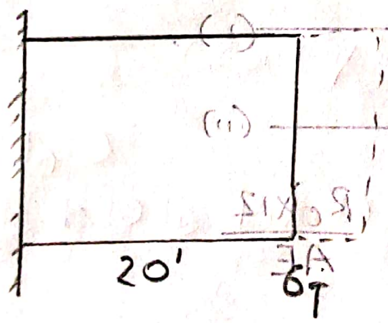
$$\Delta_{BC} = \frac{227.27}{5} = 45.45 \text{ psi (Tension)}.$$

6) Calculate Thermal deformation & required external load to resist thermal deformation.

Given, $\alpha = 6.5 \times 10^{-6}$ | $A = 3$ inch$^2$

$T_1 = 100°F$ | $E = 30 \times 10^3$ ksi

$T_2 = 150°F$

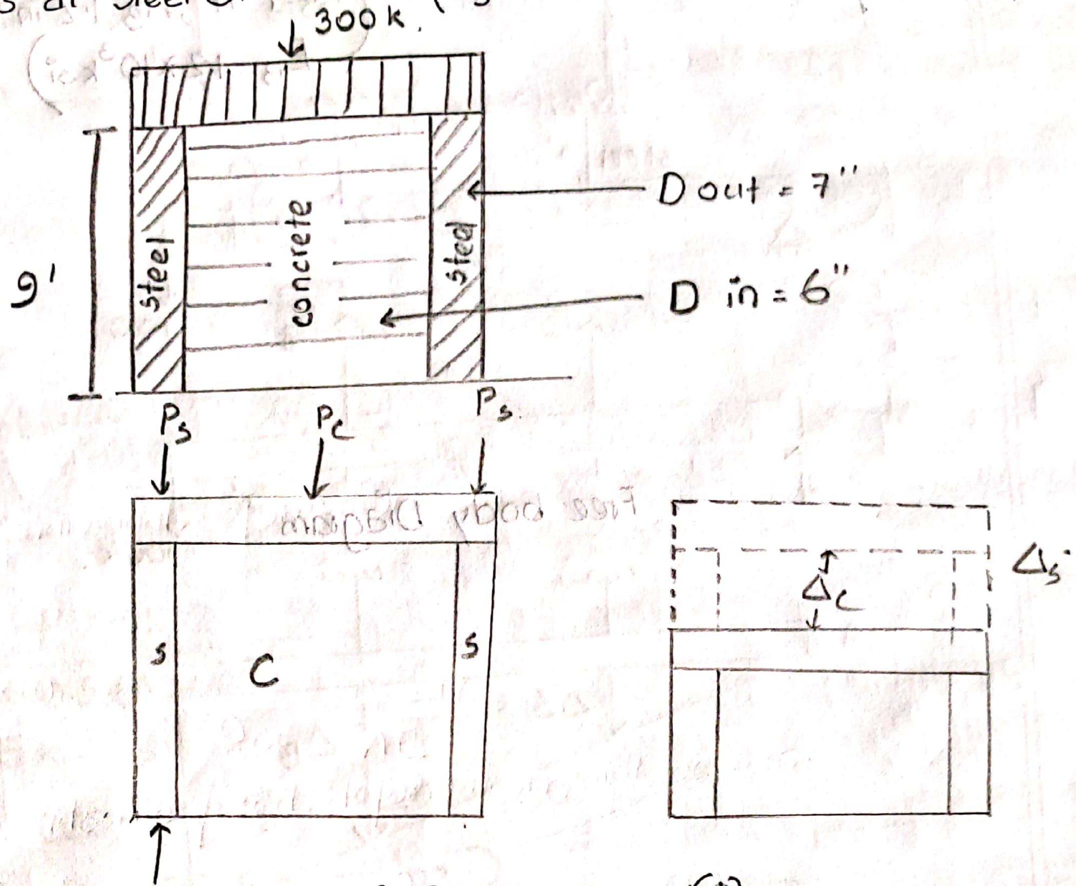



$$\delta_T = \alpha L \Delta T$$
$$= 6.5 \times 10^{-6} \times 20 \times 12 \times 50$$
$$= 0.078$$

$$\delta = \frac{PL}{AE}$$

$$P = \frac{\delta AE}{L}$$
$$= \frac{0.078 \times 3 \times 30 \times 10^6}{20 \times 12}$$
$$= 29.25 \times 10^3 \text{ lb (Ans)}$$

7) Compute stress at steel & concrete ($E_s = 30\times10^3$ ksi & $E_c = 3\times10^3$ ksi).




<u>Solution</u>:

From the free body, $P = P_s + P_c$ ——— (i)

From the geometry of deformation, $\Delta_s = \Delta_c$ ——— (ii)

From (ii),

$$\frac{P_s \times 9 \times 12}{\pi(3.5^2-3^2)\times 30\times10^3} = \frac{P_c \times 9 \times 12}{\pi \times 3^2 \times 3\times10^3}$$

$\Rightarrow P_s = 3.612\ P_c$ ——— (iii)

From eq$^n$ (i) & (iii),

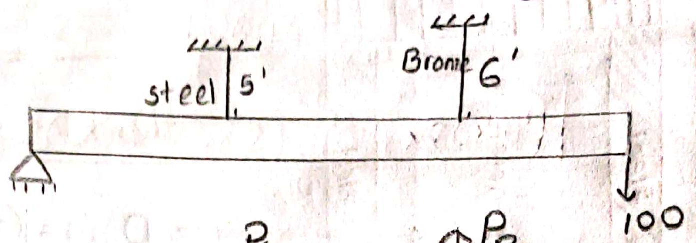
$P_c = 65.03$ k & $P_s = 234.93$ k

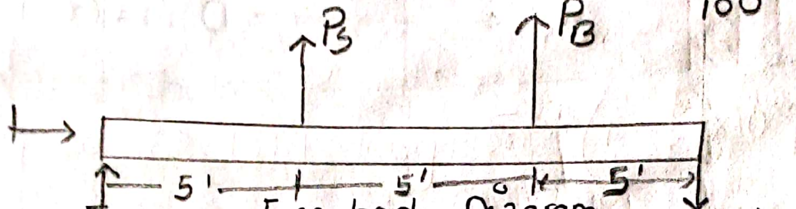
$$\sigma_s = \frac{P_s}{A_s} = \frac{234.93}{\pi(3.5^2-3^2)} = 23.009 \text{ ksi (C)}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{65.03}{\pi \times 3^2} = 2.3 \text{ ksi (C)}$$

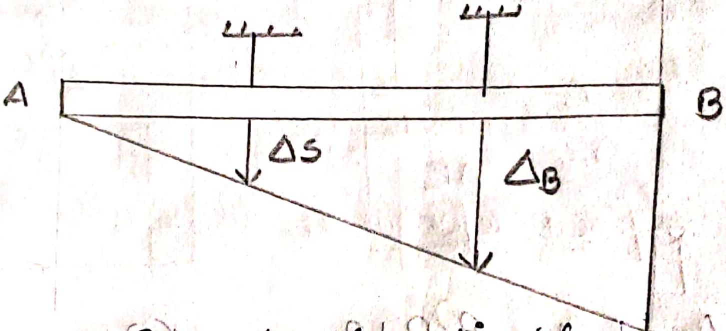
(Answer)

8) Compute stress in each bar. ($A_S = 1\ in^2$; $A_B = 1.2\ in^2$; $E_S = 30\times10^3\,ksi$, $E_B = 12\times10^3\,ksi$)



Free body Diagram



Geometry of elastic deformation.

$\sum M_A = 0$

$5P_S + 10P_B = 15\times100$

$\Rightarrow P_S + 2P_B = 300$ —— (i)

From geometric relation to the elastic deformation (similar triangle) -

$\frac{\Delta_S}{\Delta_B} = \frac{5}{10} \Rightarrow \Delta_B = 2\Delta_S$ —— (ii).

From (ii),

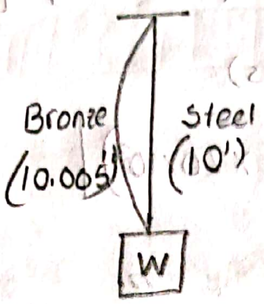
$$\frac{P_B\times6\times12}{1.2\times12\times10^3} = \frac{P_S\times5\times12\times2}{1\times30\times10^3} \Rightarrow P_B = 1.2P_S$$

From (i) & (iii), $P_B = 92.307k$ & $P_S = 115.3\ k$.

Stress, $\sigma_S = 115.3/1 = 115.3\ ksi$ | $P_B = \frac{92.307}{1.2} = 76.92\,ksi$

Ans

9) Find the stress & final length of two strings. (Given, $A_S = 0.5\ in^2$, $A_B = 0.4\ in^2$, $E_S = 30\times10^3$ ksi, $E_B = 20\times10^3$ ksi, $L_S = 10$ ft, $L_B = 10.005$ ft, $W = 200$ k)




Solution: $L_2 - L_1 = (10.005 - 10) = 0.005$ ft

$$\Delta_{s_1} = \frac{PL}{AE}$$

$$\Rightarrow 0.005\times12 = \frac{P_{s_1}\times10\times12}{0.5\times30\times10^3}$$

$$\Rightarrow P_{s_1} = 7.5\text{ k}.$$

Rest load $= 200 - 7.5 = 192.5$ k.

Hence, $P_B + P_{s_2} = 192.5$ —— (i)

& $\Delta_B = \Delta_{s_2}$ —— (ii)

$$\frac{P_B\times10.005\times12}{0.4\times20\times10^3} = \frac{P_{s_2}\times10.005\times12}{0.5\times30\times10^3}$$

$\Rightarrow P_{s_2} = 1.875\,P_B$ —— (iii)

From (i) & (iii) $P_B = 66.96$ k ; $P_{s_2} = 125.54$ k.

~~For~~ $\sigma_s = \sigma_{s_1} + \sigma_{s_2}$

$= (7.5 \div 0.5) + (125.54 \div 0.5) = 266.087$ ksi

$\sigma_B = \frac{P_B}{A_B} = (66.96 \div 0.4) = 167.04$ ksi

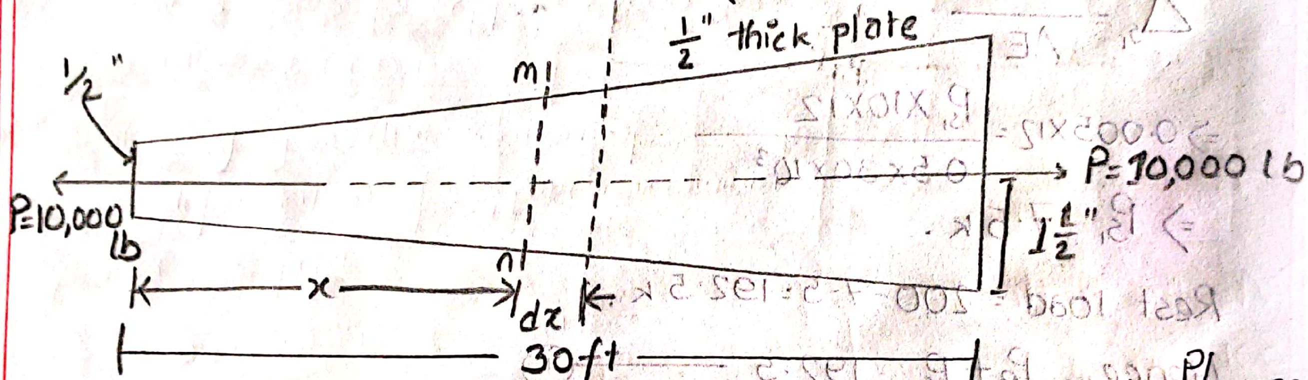
$$\Delta_S = \Delta_{S_1} + \Delta_{S_2} = \left[\left(7.5\times10\times12\right)\div\left(0.5\times30\times10^3\right)\right] + \left[\left(125.54\times10.005\times12\right)\div\left(0.5\times30\times10^3\right)\right]$$

$= 1.0648''$ (Ans)

$$\Delta_B = \left[\left(66.96\times10.005\times12\right)\div\left(0.4\times20\times10^3\right)\right]$$

$= 1.005''$ (Answer)

19 Compute the total elongation. $\left(E = 30\times10^6 \text{ psi}\right)$




Solution: Since cross sectional area is not constant, equation $\Delta = \frac{PL}{AE}$ cannot be applied directly.

At section m-n, distance is $x$ ft from smaller end, the half width 'y' found from geometry is,

$$y = \frac{1}{2} + x/30$$

The area at the section is.

$$A = \frac{1}{2}\times2y = \left(\frac{1}{2}+\frac{x}{30}\right) \text{sq. inch.}$$

At section m-n in a differential length, dx, the elongation may be found from-

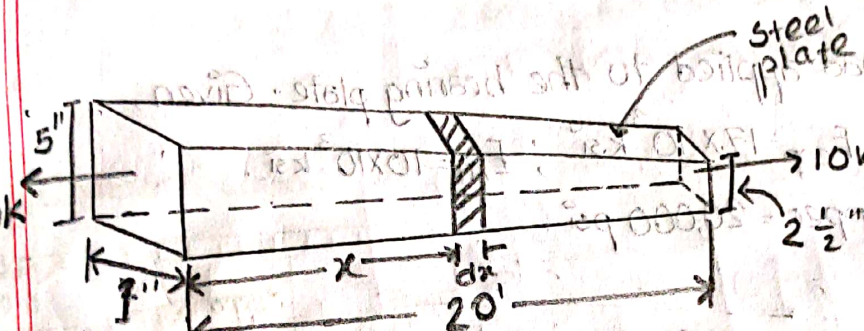
$$d\delta = \frac{10000\times dx}{\left(\frac{1}{2}+\frac{x}{30}\right)\times30\times10^6} = 10^{-2}\times\frac{dx}{15+x}.$$

From which the total elongation is -

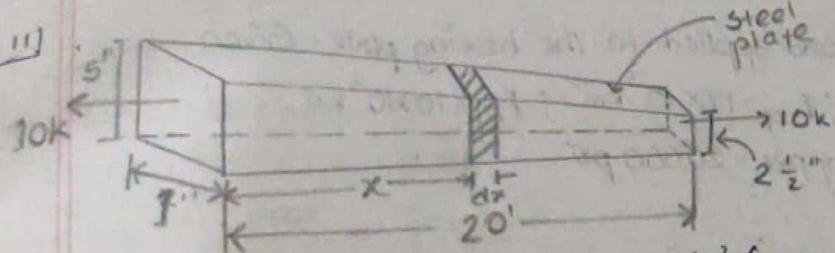
$$\delta = 10^{-2}\int_0^{30}\frac{dx}{x+15} = 10^{-2}\times\Big[\ln(x+15)\Big]_0^{30}.$$

$$= 10^{-2}\ln\frac{45}{15} = 0.011\ ft. \quad \text{(Ans)}$$

11)




11)




$$\frac{Y-5}{X-0} = \frac{5-2.5}{0-20}$$

$$\Rightarrow \frac{Y-5}{X} = \frac{(5-2.5)\times x}{20}$$

$$\Rightarrow Y = 5 - \frac{(5-2.5)x}{20}$$

Width of a plate at a distance 'x' from A $= 5-(5-2.5)\times x/8$

Cross-sectional area of the bar at this section $= 1\times(5-x/8) = (5-x/8)$ sq in

At this section, in a differential length dx, the elongation may be found from-

$$d\delta = \frac{10{,}000\times dx}{(5-\frac{x}{8})\times 30\times 10^6} = 10^{-2}\times\frac{dx}{150-3.75x}$$

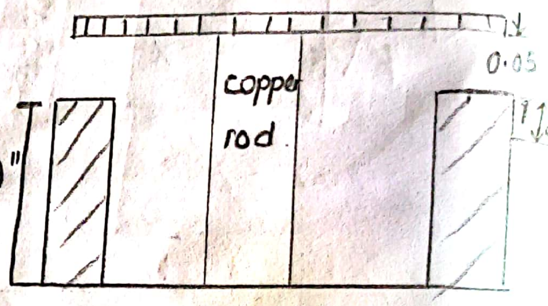
From which the total elongation is -

$$\delta = 10^{-2}\times\int_0^{20}\frac{dx}{150-3.75x} = \frac{10^{-2}}{-3.75}\times\Big[\log_e(150-3.75x)\Big]_0^{20}$$

$$= \frac{10^{-2}}{-3.75}\Big[\ln 75 - \ln 150\Big] = \frac{1}{375}\times\ln\frac{150}{75}.$$

$$= \frac{1}{375}\log_e 2 = 0.00185\ ft \quad \text{(Ans)}$$

121 Compute maximum load applied to the bearing plate. Given,
$A_{cop} = 2\ in^2$ ; $A_{AL} = 3 in^2$ ; $E_{cop} = 17 \times 10^3\ ksi$ ; $E_{Al} = 10 \times 10^3\ ksi$.
Allowable stress for copper = 20,000 psi.
Aliminium = 10.000 psi




solution: Considering geometric relation of elastic deformation of copper & aluminium cylinder we find,

$$\delta_{cop} = \delta_{AL} + 0.005 \quad \text{——— (i)}$$

$$P = P_{cop} + P_{AL} \quad \text{——— (ii)}$$

$$\left(\frac{S \times L}{E}\right)_{cop} = \left(\frac{S \times L}{E}\right)_{AL} + 0.005$$

$$\Rightarrow \frac{S_{cop} \times 10.005}{17 \times 10^6} = \frac{S_{Al} \times 10}{10 \times 10^6} + 0.005.$$

$$\Rightarrow S_{cop} = 1.7 S_{AL} + 8495.75 \quad \text{——— (iii)}$$

Equation (iii) is the governing relation between stresses. Using $S_{AL} = 10000$ psi, we get, $S_{cop} = 25495.75$ psi - which is over stress

Therefore, copper governs and the corresponding stresses in the aluminium is determined from equation (3).

$2000 = 1.7 S_{AL} + 8495.75$

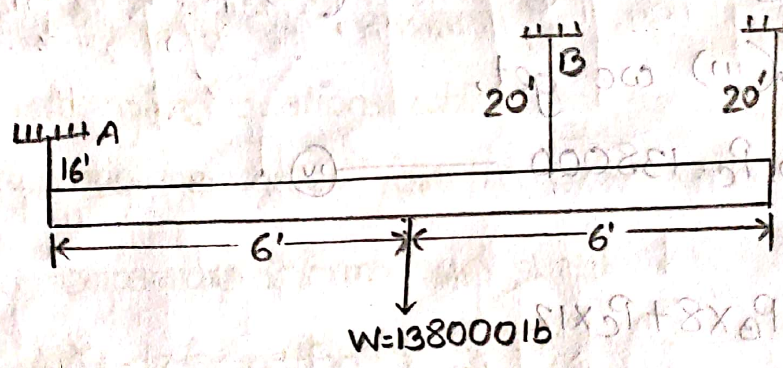
$\Rightarrow S_{AL} = 6.76721$ psi

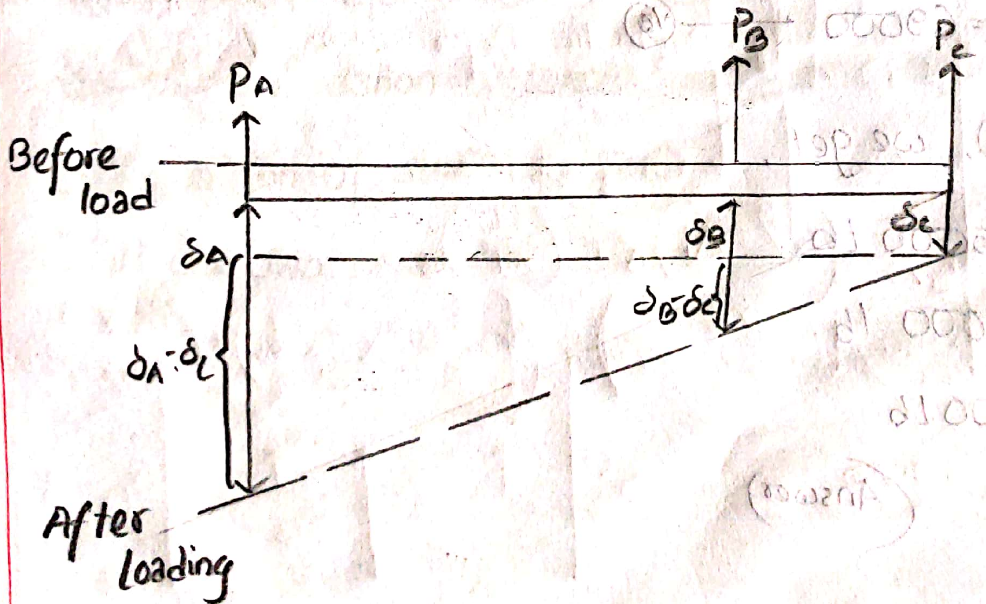
The total safe load is given by,

$P = P_{cop} + P_{AL} = S_{cop} A_{cop} + S_{AL} A_{AL}$.

$= 2 \times 20{,}000 + 2 \times 6767.21 = 60301.62$ lb (Ans)

13) Compute Loads on each bar (Area & materials of all bars are same).



Solution: From the free body diagram,

$$P_A + P_B + P_C = 138000 \quad \text{--- (i)}$$

Again for geometric relation of elastic deformation:

$$\frac{\delta_B - \delta_C}{\delta_A - \delta_C} = \frac{4}{12} \quad \text{--- (ii)}$$

$$\Rightarrow \delta_A - 3\delta_B + 2\delta_C = 0$$

$$\Rightarrow \frac{P_A \times 16}{AE} - \frac{3 P_B \times 20}{AE} + \frac{2 P_C \times 20}{AE} = 0.$$

$$\Rightarrow P_A - 3.75 P_B + 2.5 P_C = 0 \quad \text{--- (iii)}$$

Equating (i) & (iii) we get,

$$4.75 P_B - 1.5 P_C = 138000 \quad \text{--- (iv)}$$

$$\Sigma M_A = 0$$

$$138000 \times 6 = P_B \times 8 + P_C \times 12$$

$$\Rightarrow P_C + 0.66 P_B = 69000 \quad \text{--- (v)}$$

From (iv) & (v), we get,

$P_A = 55000$ lb.

$P_B = 42000$ lb.

$P_C = 41000$ lb

(Answer)

# Concept of Thermal Stress-Strain

General procedure for computing the loads and stresses caused when thermal deformation is prevented →

→ Structure is relived of all applied loads & temparature deformation can occur freely. Represent these deformation on a sketch & effect.

→ Now imagine loads being applied to the structure to restore it to the specific conditions of restraint. Represent these loads and corresponding load deformations on the sketch.

→ The geometric relations between the temparature & load deformations on the sketch give equations which, together with the equation of static equilibrium, may be solved for all unknown quantities.

(i) Only temparature deformation & sketch.

(ii) Only load deformation & sketch.

(iii) Geometric relation of temparature & load deformation.

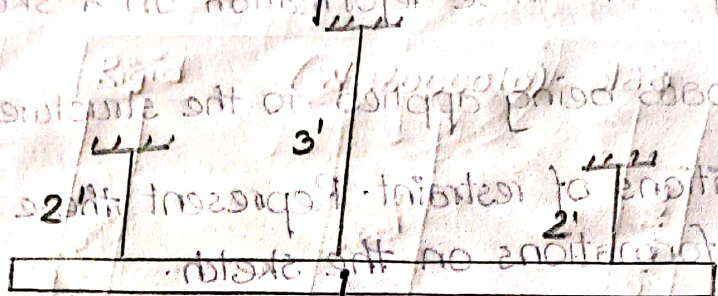
(iv) Condition of static equilibrium.

(v) All unknown parameters will be solved.

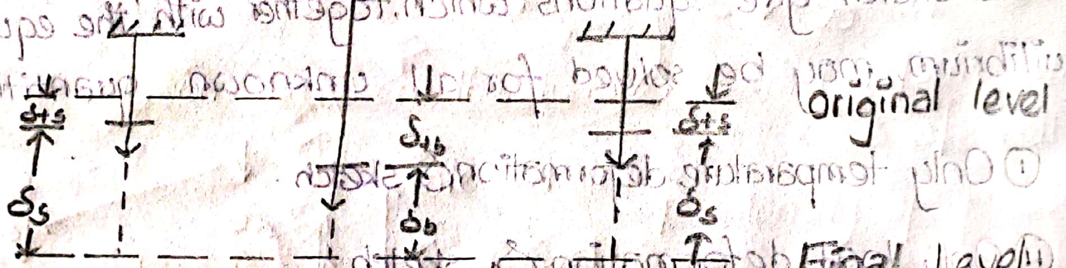
141 Compute the stress in each bar.

Given.

$A_s = 0.75\ in^2$ | $A_B = 1.5\ in^2$
$E_s = 30\times10^6\ psi$ | $E_B = 12\times10^6\ psi$
$\alpha_s = 6.5\times10^{-6}$ | $\alpha_B = 10\times10^{-6}$




From geometric relation of elastic deformation in the following figure, we obtain following relation between the deformation.




Geometry of Elastic Deformation.

$$\delta_{ts} + \delta_s = \delta_{tB} + \delta_B$$

$$\Rightarrow (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_s = (\alpha L \Delta T)_B + \left(\frac{PL}{AE}\right)_B$$

$$\Rightarrow \left(6.5\times10^{-6}\times2\times100\right) + \left(\frac{P_s\times2}{.75\times30\times10^6}\right) = \left(10\times10^{-6}\times3\times100\right) + \left(\frac{P_B\times3}{1.5\times12\times10^6}\right)$$

$$\Rightarrow P_s - 1.875P_B = 19125 \quad \text{——} (1)$$

From the free body diagram.

$\Sigma Y = 0$ ; $2P_s + P_B = 12000$ ——— (2)

From (1) & (2),

$P_s = 8763.16$ lb

$P_B = -5526.32$ lb

(−) for $P_B$ means load $P_B$ acts oppositely to that assumed.

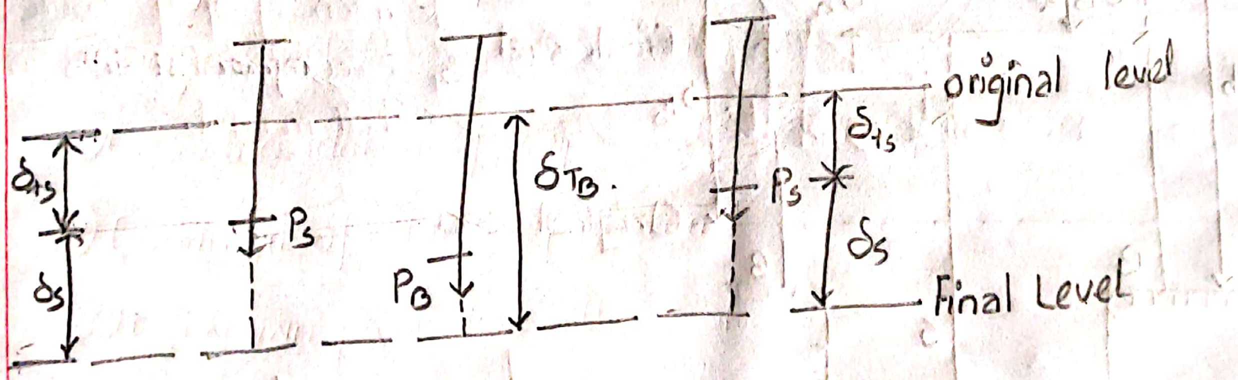
So, stresses are,

$\sigma_s = 8763.16 \div 0.75 = 11684.21$ (T)

$\sigma_B = 5526.32 \div 1.50 = 3684.21$ (C)

(Answer)

(15) Determine the temparature rise necessary to cause all the applied loads to be supported by steel rods.




$\delta_{TB} = \delta_{Ts} + \delta_s$.

$\Rightarrow (\alpha L \Delta T)_B = (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_s$.

$= \left(10 \times 10^{-6} \times 3 \times \Delta T\right) = \left(6.5 \times 10^{-6} \times 2 \times \Delta T\right) + \left(\frac{6000 \times 2}{0.75 \times 30 \times 10^{-6}}\right)$

$\Rightarrow \Delta T =$

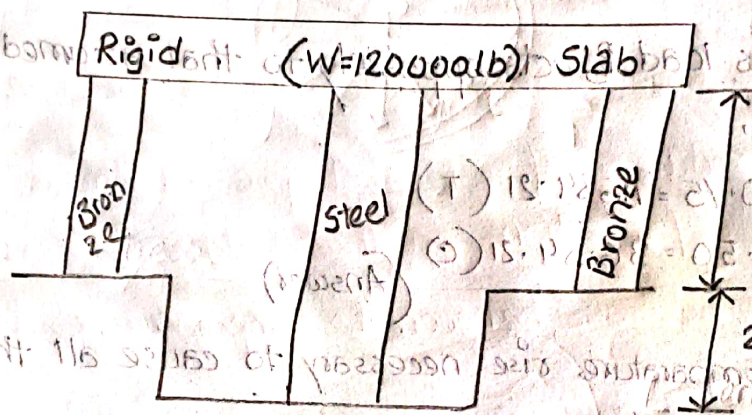
161| At what temparature will the stress in the steel rod be zero?

$A_S = 10 \text{ in}^2$ | $A_B = 10 \text{ in}^2$

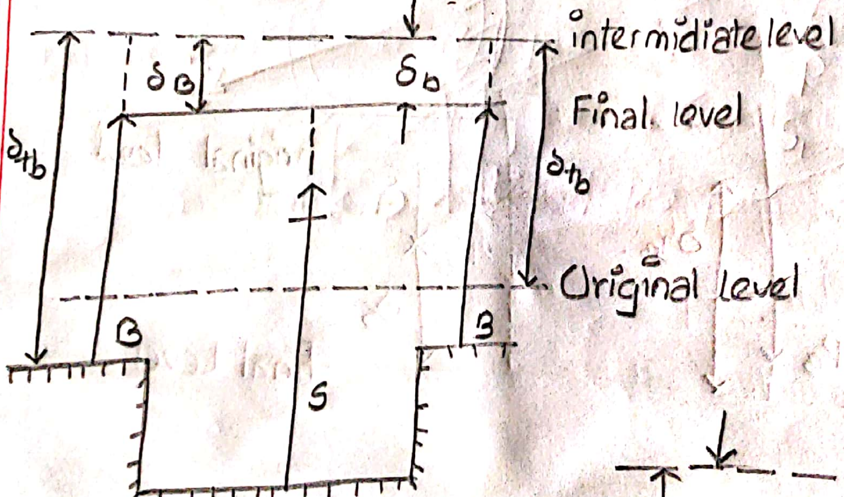
$E_S = 30 \times 10^6$ psi | $E_B = 12 \times 10^6$

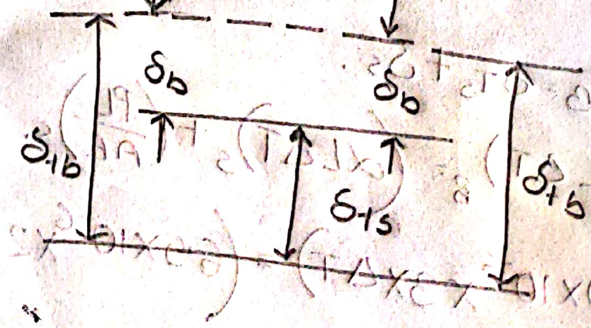
$\alpha_S = 6.5 \times 10^{-6}$ | $\alpha_B = 10.5 \times 10^{-6}$

$\Delta T = 70°F$ (initial).




Geometric relation between the load & thermal deformation?



$\delta_{ts} + \delta_b = \delta_{tb}$.

$\Rightarrow \left(\alpha L \Delta T\right)_s + \left(\frac{PL}{AE}\right)_b = \left(\alpha L \Delta T\right)_b$

$\Rightarrow \left(6.5\times10^{-6}\times12\Delta T\right) + \left(\frac{60000\times10}{10\times12\times10^{6}}\right) = \left(10.5\times10^{-6}\times10\Delta T\right)$.

$\Rightarrow 78\Delta T + 5000 = 105\,\Delta T$.

$\Rightarrow \Delta T = 185.15\,^\circ F$.

Hence, Temp change $= 185.15 + 70 = 255.18\ F$

## THIN WALL CYLINDER

→ **Definition:**

If the thickness of vessels in cylindrical and spherical forms (such as tanks, boilers, compressed air recievers) is less than 1/10th to 1/15th of its diameter, it is known as thin shelled / thin walled cylinder.

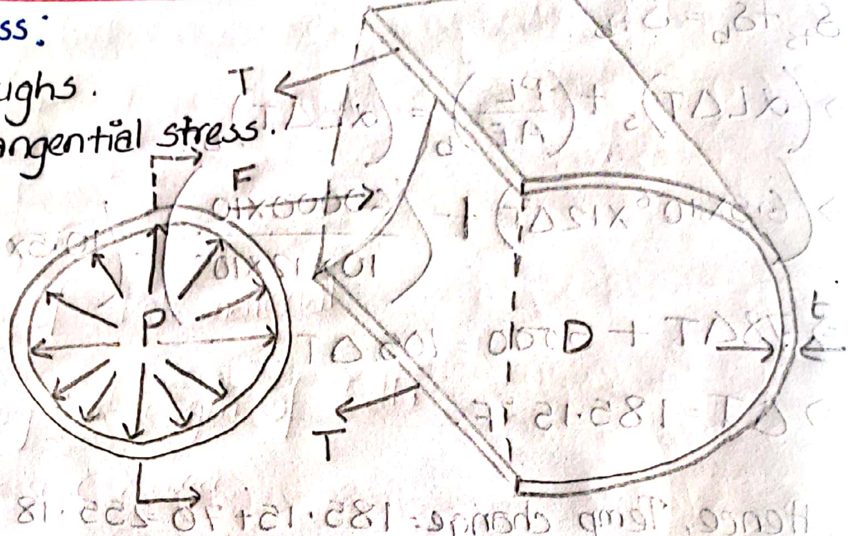
→ **Stresses in thin cylindrical wall:**

Whenever a cylindrical shell is subjected to an internal pressure, its walls are subjected to tensile stresses.

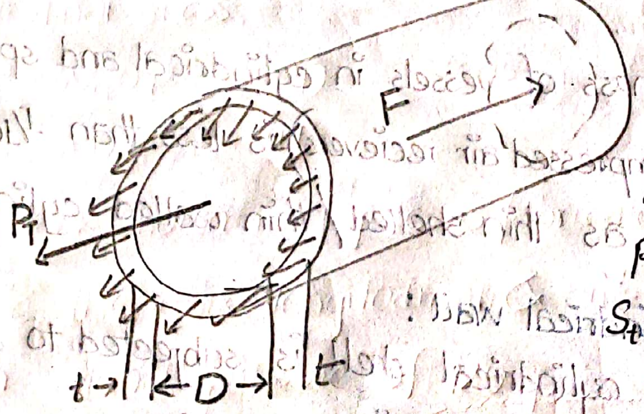
→ 2 types of tensile stresses act on the walls of cylindrical shells.

→ Circumferencial stress:

* Split into two troughs.
* Known as hoop/tangential stress.




→ Longitudinal Stress:




L = Length of shell
D = inner diameter of shell
t = thickness of shell
P = intensity of pressure.
$S_H$ = circumferential stress in shell material.

→ Analytical determination of circumferential stress:

* Total pressure or bursting force along the diameter of the shell.

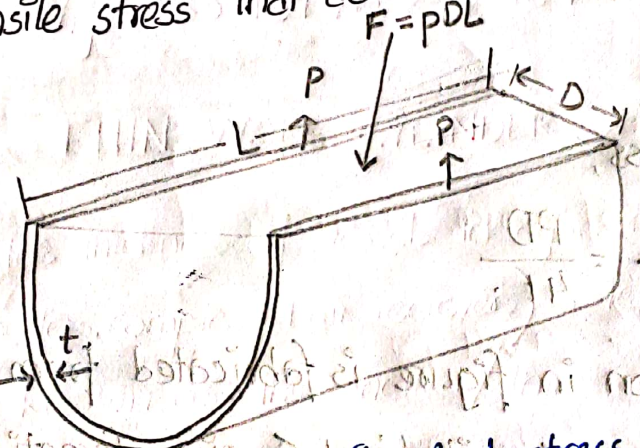
$P$ = intensity of pressure X area.

$P = p \times D \times L$.

Circumferential stress in the shell:

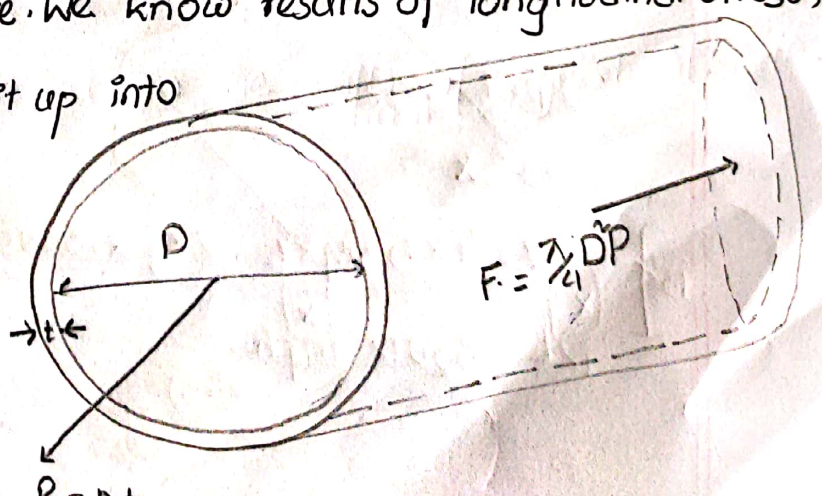
$$S_t = \frac{\text{total pressure}}{\text{Resisting section}} = \frac{p \times D \times L}{2tL} = \frac{pD}{2t}$$

This is a tensile stress that acts across its diameter (A-A).




→ Analytical determination of longitudinal stress

① Consider the same cylindrical shell subjected to the same internal pressure as shown in figure. We know results of longitudinal stress, cylinder has a tendency to split up into two cylinders.



L = length.

D = inner diameter

t = thickness.

P = intensity of pressure.

$S_L$ = longitudinal stress of shell material

Total pressure along the length of shell,

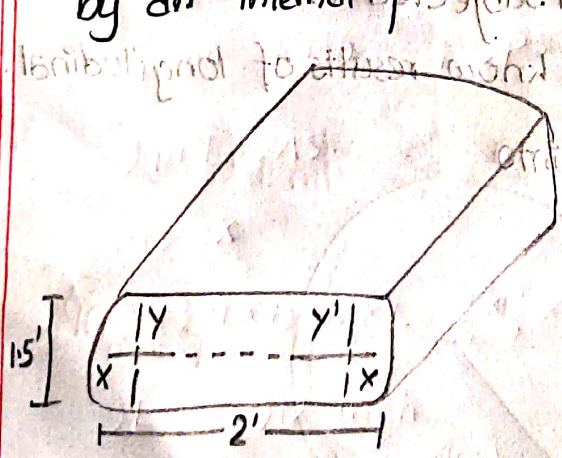
P = intensity of pressure X area.

$= p \times \cancel{\phi}\,\pi D^2/4$

longitudinal stress,

$$S_L = \frac{\pi/4\, D^2 P}{\pi D t} = \frac{PD}{4t}.$$

17). The tank shown in figure is fabricated from $\frac{1}{8}$" steel plate. Calculate the max longitudinal & circumferential stress caused by an internal pressure of 125 psi.



Solution:

$$S_t = \frac{\text{Total Pressure}}{\text{Resisting section}}$$

$$= \frac{PD}{2t} = \frac{125 \times (24+9+9)}{2 \times .125} = 21000 \text{ psi}$$

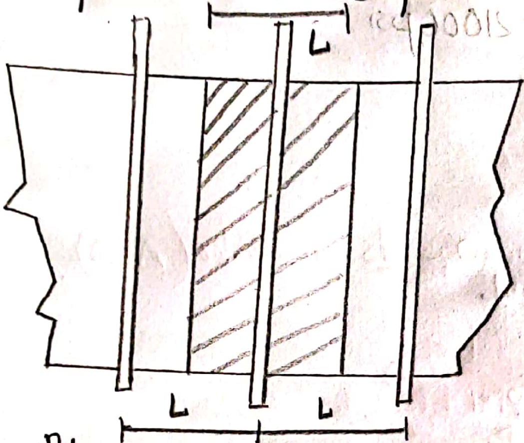
$$S_L = \frac{P \times \left(LD + \frac{\pi D^2}{4}\right)}{2Lt + \pi Dt}$$

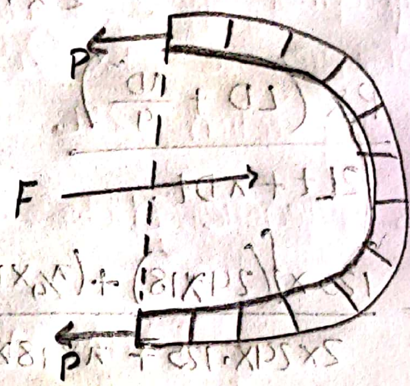
$$= \frac{125 \times \{(24 \times 18) + (\pi/4 \times 18^2)\}}{2 \times 24 \times .125 + \pi \times 18 \times .125}$$

$$= 6566 \text{ psi}$$

(Ans)

181 Find the max spacing between hoopes under a head of water of 100 ft of a large pipe of wooden staves bound together by steel hoops of $\frac{1}{2}$ sq. inch in area. Given, diameter of pipe 5' & max hoop stress 18000 psi.




Sol$^n$:

For 100' head of water, intensity of pressure,

$$P = \frac{62.5 \times 100 \times 12}{12 \times 12 \times 12} = 43.4 \text{ psi}.$$

Let, max spacing between hoops is L

Hence bursting force, $F = pDL$.

from FBD, $F = 2P$.

$\Rightarrow pDL = 2AS$.

$= \cancel{43.5\times}$

$\Rightarrow 43.4 \times 5 \times 12 \times L = 2 \times \frac{1}{2} \times 18000$

$\Rightarrow L = 6.912$ inch

(Ans)