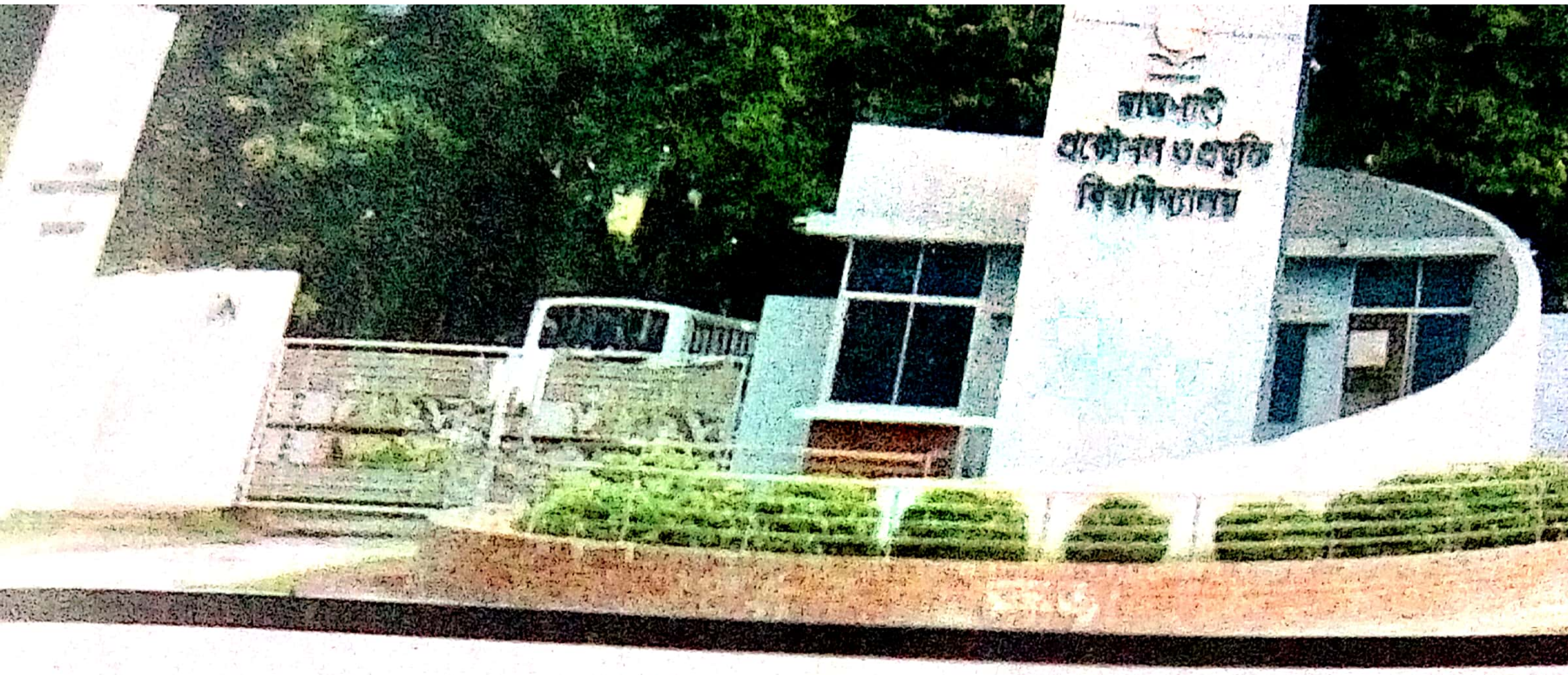

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Mechanics of Materials - I

Stress: (Internal reaction)

Stress is the intensity of internal force developed when an external force is applied on an engineering material.

$$\underbrace{\sigma}_{\text{stress per unit area}} = \frac{P}{\underbrace{A}_{\text{cross sectional area}}}$$

* Stress and strain
* Share force
* Rived and welded connection

* Internal reaction না করলে stress জন্মাবে না।

↳ sustain (বহন) করবে, fail করবে না।

Unit: SI ⟶ N/m^2 (Pascal, Pa) [$1 MPa = 10^6 Pa$]

Strength: It is the property of a material that represents the ability to resist forces or stresses.

Unit strength of a material is usually defined as the stress in the material.

$$\sigma = \frac{dP}{dA}$$ [area ছোট ছোট হলে]

Stress: Tensile ⟶ pull (outward)

Compressive ⟶ push (inward)

Sharing ⟶ subjected to sharing stress.

Strain: (deformation per unit area)

It is a measure of deformation produced by the application of external force.

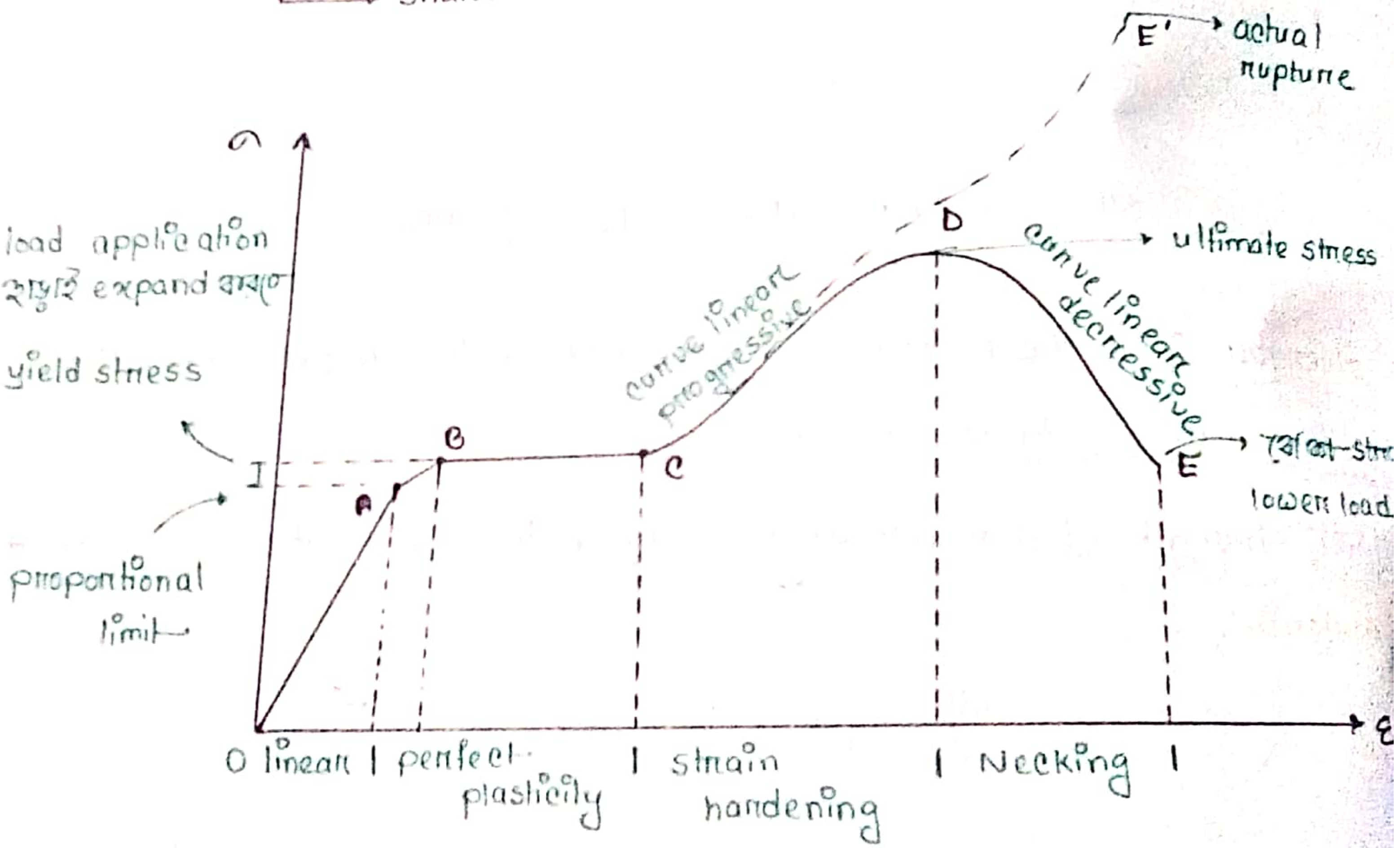
$$\varepsilon = \frac{\Delta}{L}$$

Δ → elongation or shortening of length

L → original length

ε → strain

Strain
- → Tensile
- → compressive
- → share



OA → proportional limit

Keyword:

* yield point, yield stress. (B~C)
* Ultimate stress. (D)
* Necking ⟶ rupture এর আগে, চিকন হয়ে যাওয়া, (D~E)
* Rupture strength (E)
* Ductility ⟶ large permanent strain before fail. (E')
* Brittleness
* Toughness
* Hooke's law
* Poisson's ratio
* Stiffness
* Elastic limit and range
* Plastic " " "
* Actual rupture strength
* Elastic deformation
* Plastic "

$\boxed{\sigma = E \times \varepsilon}$

* Fundamental concept of stress
* Fundamental concept of strain
* Stress - strain diagram

$\boxed{\sigma = P/A}$ $\boxed{\varepsilon = \frac{\Delta}{L}}$

Hooke's law: $\sigma = E \times \varepsilon$

$$\Rightarrow E = \frac{\sigma}{\varepsilon}$$

$$\Rightarrow E = \frac{P/A}{\Delta/L}$$

$$\Rightarrow \boxed{\Delta = \frac{PL}{AE}}$$

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Stress → load apply করে static condition এ থাকে – stress
dynamic " " " – stress develop করবে
→ it should be in equilibrium
$\Sigma F_x = 0$, $\Sigma F_y = 0$

Strain → প্রতি body র একক linear deformation – strain.

Unit: inch/inch কিংবা নেই।

* Stress – strain এর behaviour.

(i) Proportional limit
(ii) Elastic limit
(iii) yield point → without any stress elongate করে।
(iv) Curve linear plastic deformation → আগের অবস্থায় ফেরত আসবেনা।
(v) Rupture strength → lower value তে fail করে (theoritical)
Actually তারও higher value তে গিয়ে fail করে,
ultimate strength বলে.

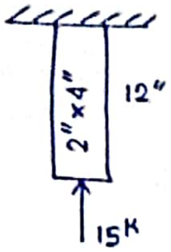
Stiffness → Resistance to bending
(Deformation against load)

$\frac{bh^3}{12}$ → beam এর height

h বেশি → ভার বহন
↓
stiffness বেশি

Practice Problems

Prob: 01



$E = 12\times10^{6}$ psi

① Calculate the stress and deformation.

→

$$\sigma = \frac{P}{A}$$

$$= \frac{15 \times 1000}{2\times4}$$

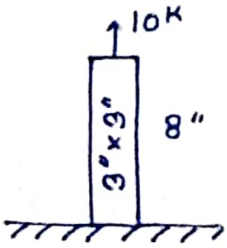
$= 1\cdot875$ psi (c)

$$\Delta = \frac{PL}{AE}$$

$$= \frac{15\times1000\times12}{2\times4\times12\times10^{6}}$$

$= 0\cdot001875$ in (shortening

Ⓘ Calculate the stress and deformation.



→

$$\sigma = \frac{P}{A}$$

$$= \frac{10\times1000}{3\times3}$$

$= 1500$ psi (T)

$$\Delta = \frac{PL}{AE}$$

$= 0\cdot00125$ (elongation)

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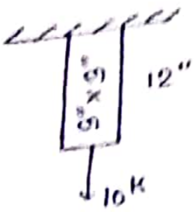
(iii)



$E = 12\times10^6$ psi

$$\sigma = \frac{P}{A}$$

$$= \frac{10\times1000}{3\times3}$$

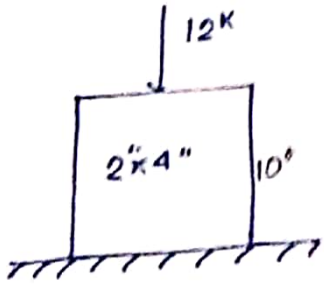
$$=$$

$$\Delta = \frac{PL}{AE}$$

$$= \frac{10\times1000\times12}{3\times3\times12\times10^6}$$

$=$ (elongation)

(iv)



$E = 12\times10^6$ psi

$$\sigma = \frac{P}{A}$$

$$= \frac{12\times1000}{2\times4}$$

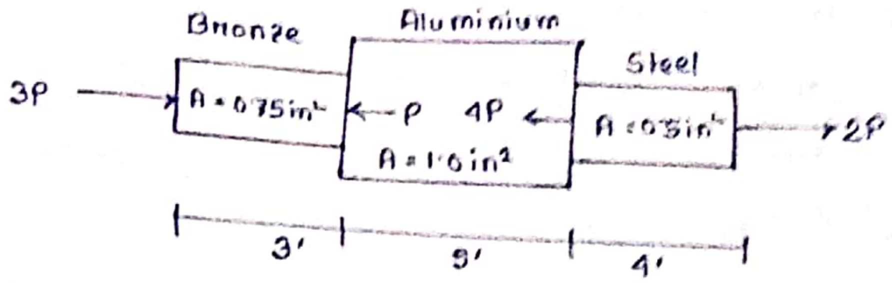
$= 1500$ psi

$$\Delta = \frac{PL}{AE}$$

$$= \frac{12\times1000\times10}{2\times4\times12\times10^6}$$

$= 0.00125$ m (Shortening)

Prob: 02



Determine the stresses in each of following material. P = 3000 lb.

⇒ Part: 1

3P → Bro ← 3P

$\sigma_1 = \frac{P}{A} = \frac{3\times3000}{0.75}$

$= 12000\,psi\ (C)$

Part: 2

2P → Al ← 2P

$\sigma_2 = \frac{P}{A} = \frac{2\times3000}{1.0}$

$= 6000\,psi\ (C)$

Part: 3

2P ← St → 2P

$\sigma_3 = \frac{P}{A} = \frac{2\times3000}{0.5}$

$= 12000\,psi\ (T)$

Prob-02

Calculate the normal stress in the bar.

($E = 10 \times 10^3$ ksi and area, $A = 2.0\ in^2$)

$\rightarrow \quad \Sigma F_x = 0$

$\Rightarrow 400 + P = 300 + 200$

$\Rightarrow P = 100^k \ (\rightarrow)$

Part-01

$\sigma_{AB} = \frac{400}{2} = 200$ ksi (C)

Part-02

$\sigma_{BC} = \frac{100}{2}$

$= 50$ ksi (C)

Part-03

$\sigma_{CD} = \frac{100}{2}$

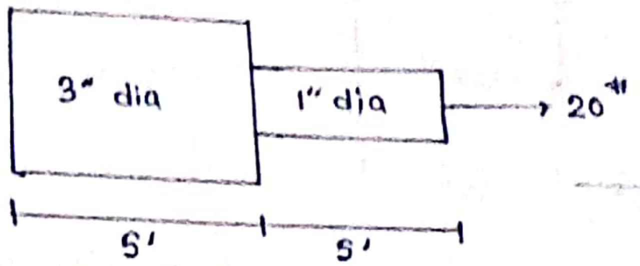
$= 50$ ksi (T)

A ছোট → stress বেশি

→ lb

Prob: 01

Calculate total deformation and max stress. ($E = 12\times10^6$ psi)



$\left\{ \begin{array}{l} A = \frac{\pi}{4} d^2 \\ \text{or, } A = \pi r^2 \end{array} \right.$

→ (P, L, E same)

→ $\Delta_1 = \frac{PL}{AE}$

$= \frac{20\times5\times12}{\pi\times1.5^2\times12\times10^6}$

$= 1.416\times10^{-5}$ in.

$\Delta_2 = \frac{PL}{AE}$

$= \frac{20\times5\times12}{\pi\times0.5^2\times12\times10^6}$

$= 1.27\times10^{-4}$ in.

Total deformation, $\Delta = \Delta_1 + \Delta_2$

$= 1.415\times10^{-4}$ in.

$\therefore$ Max stress $= \frac{20}{(\pi\times0.5)^2} = 25.46$ psi.

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Problem 3

Calculate the normal stress ($E = 15\times10^6$ psi and $A = 5\ in^2$)

(i) Total deformation
(ii) Free body diagram

⇒ As the both ends are fixed, total deformation will be zero.

$\delta_{AB} = +ve$
$\delta_{BC} = -ve$

Total deformation $\delta_{total} = 0$

$$\delta_{AB} + \delta_{BC} = 0 \quad \text{---(i)}$$

$\Sigma F_x = 0$

$$\Rightarrow R_A + R_B = 600\# \quad \text{---(ii)}$$

$$\delta_{AB} = \frac{R_A \times 10}{AE}$$

$$\delta_{BC} = -\frac{R_B \times 12}{AE}$$

From (i) →

$$\frac{10R_A}{AE} - \frac{12R_B}{AE} = 0$$

$$\Rightarrow 5R_A = 6R_B$$

$$\Rightarrow R_A = \frac{6}{5}R_B$$

From (ii) →

$$\frac{6}{5}R_B + R_B = 600$$

$$\Rightarrow R_B = 327.27\#$$

$$\therefore R_A = 272.72\#$$

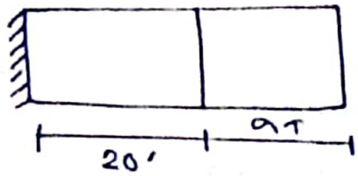
$$\sigma_{AB} = \frac{272.72}{5} = 54.54 \text{ psi (C)}$$

$$\sigma_{BC} = \frac{327.27}{5} = 65.45 \text{ psi (T)}$$

(Ans)

Prob: 06

Calculate the thermal deformation and the required external load to resist thermal deformation.



Thermal deformation

$\boxed{\delta_T = \alpha L T}$

$\Rightarrow$ Given that,

$\alpha = 6.5 \times 10^{-6}$

$A = 3 \text{ in}^2$

$T_1 = 100°F$

$T_2 = 150°F$

$E = 3 \times 10^3$ ksi

$$\delta_T = \alpha L T$$
$$= 6.5 \times 10^{-6} \times 20 \times 12 \times (150 - 100)$$
$$= 0.078 \text{ in.}$$

$$\delta = \frac{PL}{AE}$$

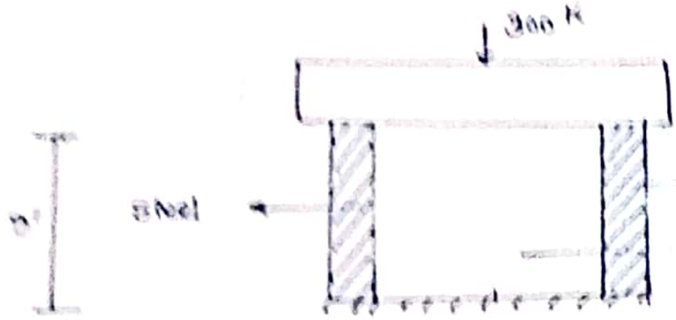
$$\Rightarrow P = \frac{\delta AE}{L}$$
$$= \frac{0.078 \times 3 \times 30 \times 10^3}{20 \times 12}$$
$$= 29.25 \text{ kip.}$$

(Ans)

Stress-strain problems

Problem 08

Compute stress at steel and concrete, $E_s = 30\times10^3$ ksi and $E_c = 3\times10^3$ ksi



- steel, concrete এ same deformation হবে।
- steel, $D_{out} = 7"$
- concrete, $D_{in} = 6"$

but same load নিবেনা, (নি সমান নিবে)
same deformation.

For stress-strain problem,

⇒ ① Geometric relation of geometric deformation

② Conditions of static equilibrium

(Composite structure)

Total load = load taken by steel + load taken by concrete.

From the free body, $P = P_s + P_c$ ——— (i)
$= 300$

From the geometry of deformation, $\Delta_s = \Delta_c$ ——— (ii)

From eqn ② ⟶

$$\frac{P_s\times9\times12}{\pi\times(3.5^2-3^2)\times30\times10^3} = \frac{P_c\times9\times12}{\pi\times3^2\times3\times10^3}$$

$$\Rightarrow P_s = 3.612\ P_c \quad \text{——— (iii)}$$

From eq^n (ii) and (iii) →

$P_c = 65.03^k$ and $P_s = 234.93^k$.

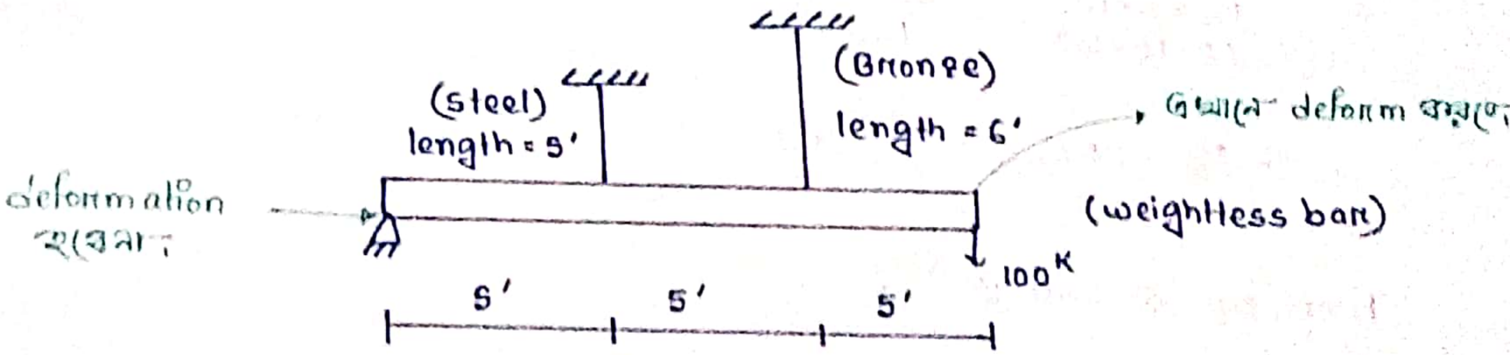
$\sigma_s = \frac{P_s}{A_s} = \frac{234.93}{\pi \times (3.5^2 - 3^2)} = 23.0009$ ksi (c)

$\sigma_c = \frac{P_c}{A_c} = \frac{65.03}{\pi \times 3^2} = 2.3$ ksi (c)

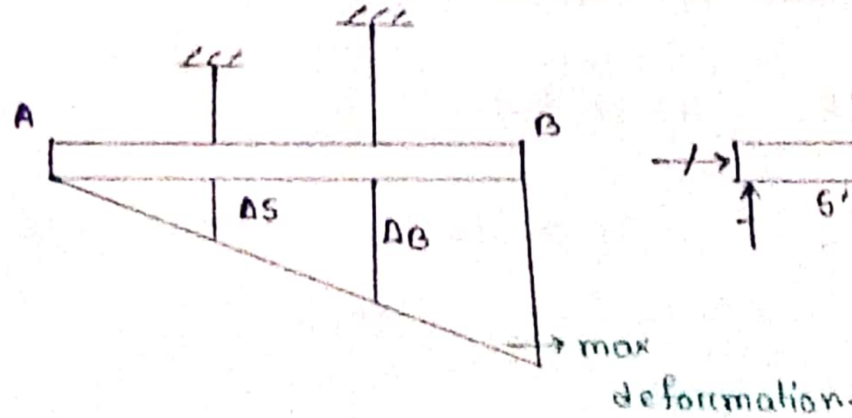
Prob: 9

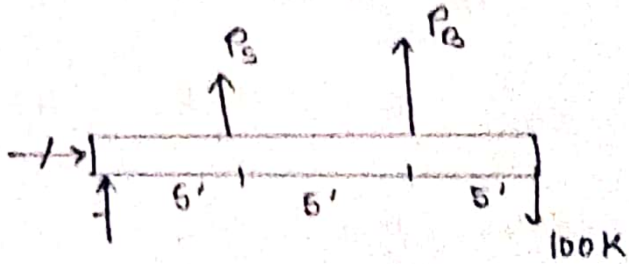
Compute stress in each bar,

(Given $A_S = 1.0\ in^2$, $A_B = 1.2\ in^2$, $E_S = 30 \times 10^3$ ksi, $E_B = 12 \times 10^3$ ksi)



→





Free body diagram.

Geometric relation of elastic deformation.

$\Sigma M_A = 0$

$\Rightarrow 5 \times P_S + 10 \times P_B = 15 \times 100$

$\Rightarrow P_S + 2P_B = 300$ ——— (1)

From the geometric relation of the elastic deformation (Similar triangle) →

$$\frac{\Delta_S}{\Delta_B} = \frac{5}{10}$$

$\Rightarrow \Delta_B = 2\Delta_S$ ——— (2)

From eqn (2) →

$$\frac{P_B \times 6 \times 12}{1.2 \times 12 \times 10^3} = \frac{2 \times P_S \times 5 \times 12}{1 \times 30 \times 10^3}$$

$\Rightarrow P_B = 1.2 P_S$ ——— (3)

From eqn (1) and (3) →

$P_B = 92.307^K$ and $P_S = 115.38^K$.

Stress, $\sigma_S = \frac{115.38}{1} = 115.38$ ksi

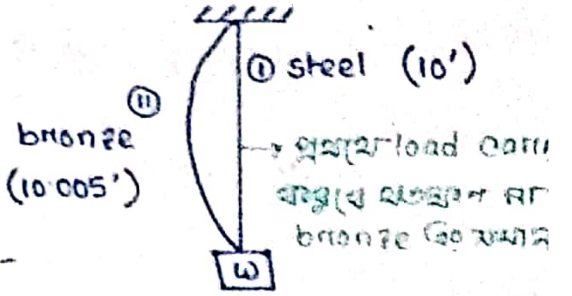
Stress, $\sigma_B = \frac{92.307}{1.2} = 76.92$ ksi

-(Ans)

Prob: 10

Find the stress and the final length of two strings.

(Given $A_S = 0.5\ in^2$, $A_B = 0.4\ in^2$, $E_S = 30 \times 10^3$ ksi, $E_B = 2 \times 10^3$ ksi, $L_S = 10'$, $L_B = 10.005'$, $\omega = 200$ kip)



$\Rightarrow L_2 - l_1 = 10.005' - 10' = 0.005'$

$\Delta_{S_1} = \frac{PL}{AE} \Rightarrow 0.005 \times 12 = \frac{P_{S_1} \times 10 \times 12}{0.5 \times 30 \times 10^3}$

$\Rightarrow P_{S_1} = 7.5^K$

Rest load $= 200 - 7.5 = 192.5$ kip

Hence: $P_B + P_{S_2} = 192.5$ ——— ①

and $\Delta_B = \Delta_{S_2}$ ——— ②

$\Rightarrow \frac{P_B \times 10.005 \times 12}{0.4 \times 20 \times 10^3} = \frac{P_{S_2} \times 10.005 \times 12}{0.5 \times 30 \times 10^3}$

$\Rightarrow P_{S_2} = 1.875\ P_B$ ——— ③

From eqn ① and ③ ⟶

$P_B = 66.96$ kip and $P_{S_2} = 125.54$ kip.

$\sigma_S = \sigma_{S_1} + \sigma_{S_2}$

$= (7.5/0.5) + (125.54/0.5)$

$= 266.087$ ksi.

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$\sigma_B = P_B / A_B = \dfrac{66.86}{0.4} = 167.04$ ksi

$\delta_s = \delta_{s_1} + \delta_{s_2}$

$= [(7.5 \times 10 \times 12)/(0.5 \times 30 \times 10^3)] + [(125.54 \times 10.005 \times 12)/(0.5 \times 30 \times 10^3)]$

$= 1.0648''$

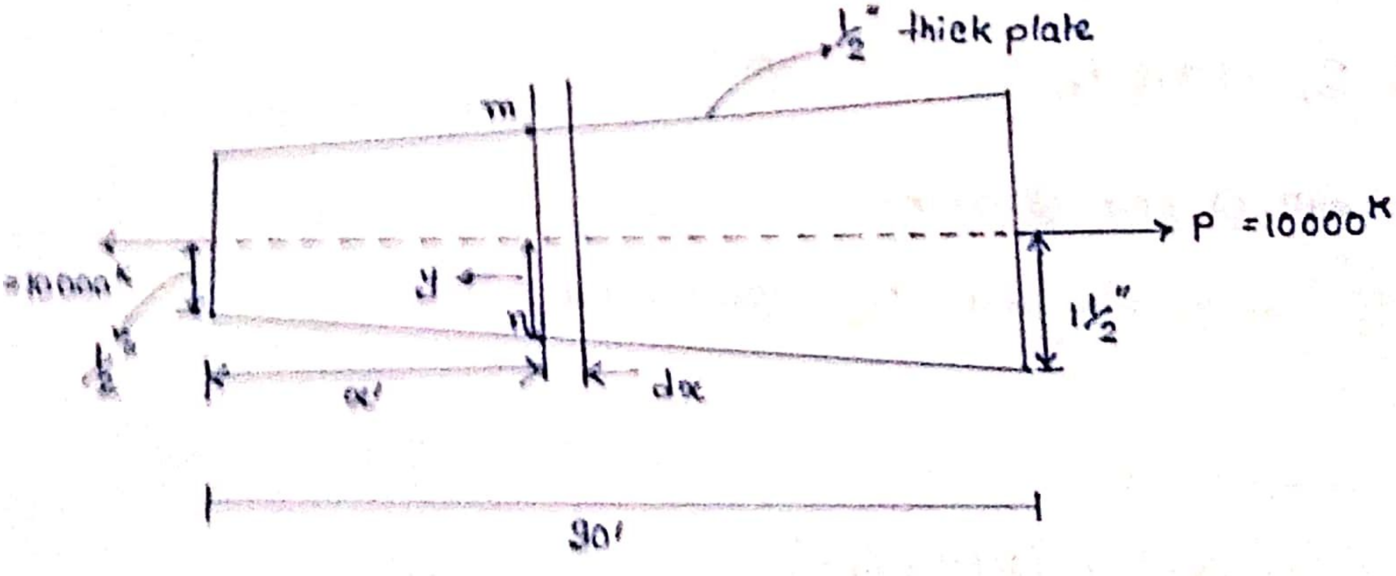
$\delta_B = [(66.86 \times 10.005 \times 12)/(0.4 \times 20 \times 10^3)]$

$= 1.005''$

(Ans)

Prob: 11

Compute the total elongation ($E = 30 \times 10^6$ psi)



→ Since the cross sectional area is not constant, equation $\delta = \frac{PL}{AE}$ can not be applied directly.

At section m-n, distant from x' from the smaller end, the half width 'y' is found from the geometry to be —

$$y = \frac{1}{2} + \frac{x}{30}$$

And the area at that section is,

$$A = \frac{1}{2} \times (2y) = \left(\frac{1}{2} + \frac{x}{30}\right) \text{ sq in.}$$

At section m-n, in a differential length dx, the elongation may be found from —

$$d\delta = \frac{10000 \times dx}{\left(\frac{1}{2} + \frac{x}{30}\right) \times 30 \times 10^{6}}$$

$$= 10^{-2} \times \frac{dx}{(15+x)}$$

From which the total elongation is —

$$\delta = 10^{-2} \times \int_0^{30} \frac{dx}{x+15} = 10^{-2} \times \left[\log_e(15+x)\right]_0^{30}$$

$$= 10^{-2} \times \log_e \frac{45}{15}$$

$$= 10^{-2} \times \log_e 3$$

$$= 0.01098$$

$$= 0.011'$$

(Ans)

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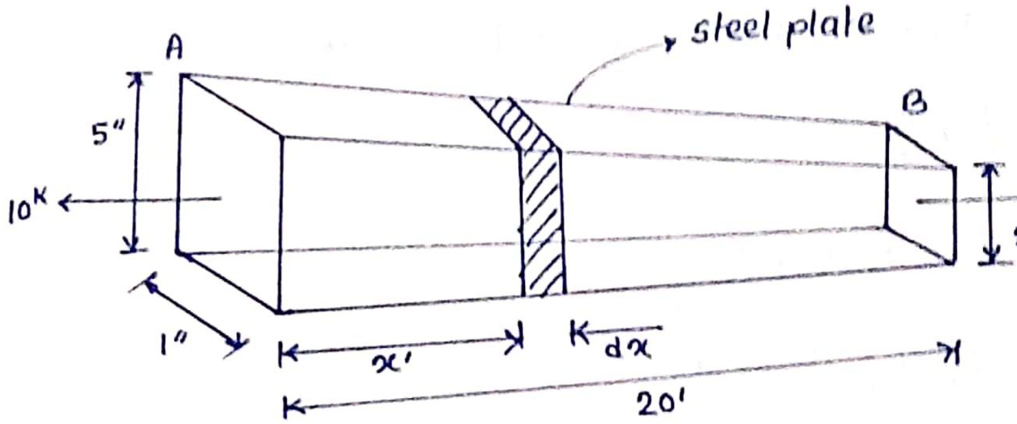
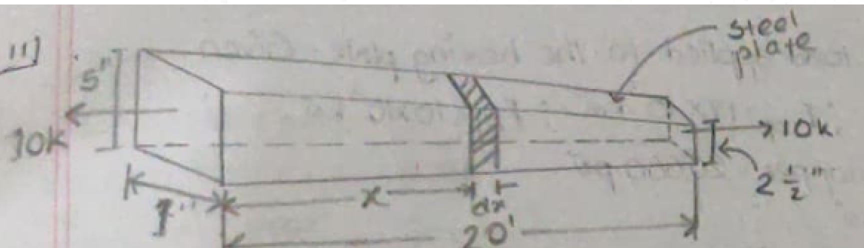
Prob: 12

Compute the elongation of the steel flat plate ($E = 30\times10^6$ psi)



11)



$$\frac{Y-5}{X-0} = \frac{5-2.5}{0-20}$$

$$\Rightarrow \frac{Y-5}{X} = \frac{(5-2.5)\times X}{20}$$

$$\Rightarrow Y = 5 - \frac{(5-2.5)x}{20}$$

Width of a plate at a distance 'x' from $A = 5-(5-2.5)\times x/8$

Cross-sectional area of the bar at this section $= 1\times(5-x/8) = (5-x/8)$ sq in

At this section, in a differential length dx, the elongation may be found from–

$$d\delta = \frac{10{,}000\times dx}{(5-\frac{x}{8})\times30\times10^6} = 10^{-2}\times\frac{dx}{150-3.75x}$$

From which the total elongation is –

$$\delta = 10^{-2}\times\int_0^{20}\frac{dx}{150-3.75x} = \frac{10^{-2}}{-3.75}\times\Big[\log_e\left(150-3.75x\right)\Big]_0^{20}$$

$$= \frac{10^{-2}}{-3.75}\left[\ln 75-\ln 150\right] = \frac{1}{375}\times\ln\frac{150}{75}$$

$$= \frac{1}{375}\log_e 2 = 0.00185 \text{ ft}$$

(Ans)

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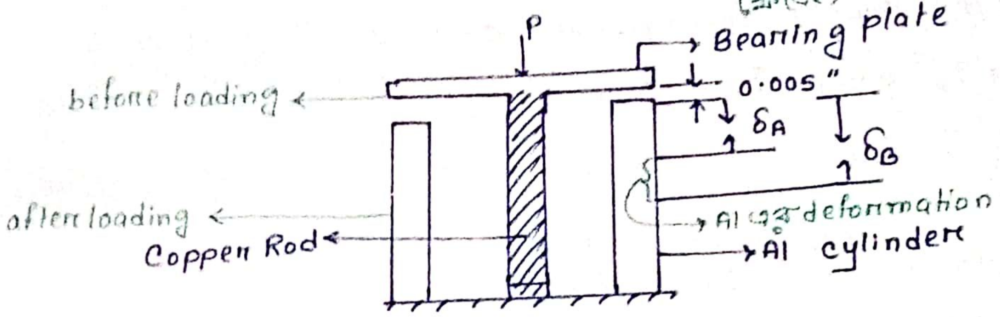
Prob: 13

Compute the maximum load applied to the bearing plate. (Given $A_{cop} = 2\,in^2$, $A_{Al} = 3\,in^2$, $E_{cop} = 17\times10^3$ ksi, $E_{Al} = 10\times10^3$ ksi)

Allowable stress for copper = 20000 psi and aluminium = 10000 psi.

(যেকোনটার limit exit করা যাবে না যেভাবেই)



$\Rightarrow$ Considering the geometric relation of elastic deformation of copper and aluminium cylinder.. we find –

$$\delta_{cop} = \delta_{Al} + 0{\cdot}005 \quad \text{——— (1)}$$

Again $\quad P = P_{cop} + P_{Al} \quad$ ——— (2)

$$\left(\frac{S\times L}{E}\right)_{cop} = \left(\frac{S\times L}{E}\right)_{Al} + 0{\cdot}005$$

$$\Rightarrow \frac{S_{cop}\times 10{\cdot}005}{17\times10^6} = \frac{S_{Al}\times 10}{10\times10^6} + 0{\cdot}005$$

$$\Rightarrow S_{cop} = 1{\cdot}7S_{Al} + 8495{\cdot}75 \quad \text{——— (3)}$$

eq^n ③ is the governing relation between stresses. Using

$S_{Al} = 10000$ psi, we get,

$S_{cop} = 25495.75$ psi which is over stressed.

Therefore Copper governs and corresponding stress in the Al is determined from eq^n ③ —

$$20000 = 1.7\, S_{Al} + 8495.75$$

$$\Rightarrow S_{Al} = 6767.21 \text{ psi}$$

Total safe load is given by —

$$P = P_{cop} + P_{Al}$$

$$\Rightarrow P = S_{cop} A_{cop} + S_{Al} A_{Al}$$

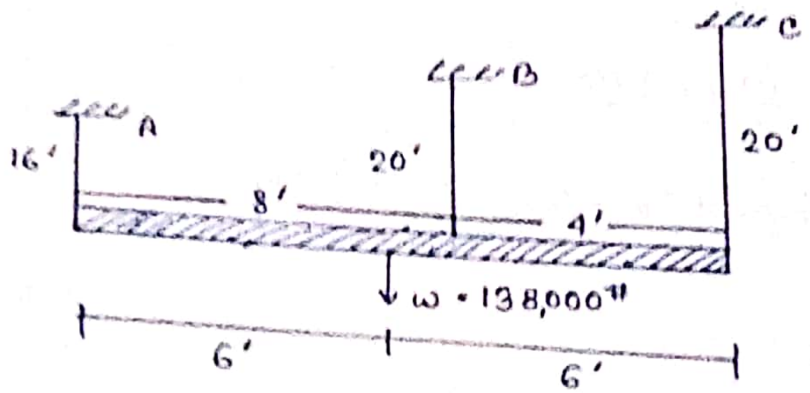
$$\Rightarrow P = 2 \times 20000 + 3 \times 6767.21$$

$$= 60301.62 \text{ lb}$$

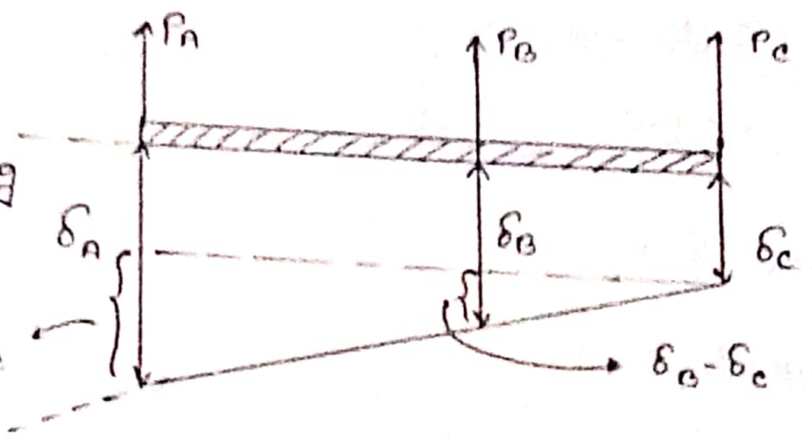
(Ans)

Prob: 14

Compute loads on each bar (Area and materials of the bars are same)



⇒



Geometry of elastic deformation.

From the free body diagram:

$$P_A + P_B + P_C = 138000 \quad \text{------} (1)$$

Again for the geometric relation of elastic deformation:

$$\frac{\delta_B - \delta_C}{\delta_A - \delta_C} = \frac{4}{12} \quad \text{------} (2)$$

$$\Rightarrow \delta_A - 3\delta_B + 2\delta_C = 0$$

$$\Rightarrow \frac{P_A \times 16}{AE} - \frac{3P_B \times 20}{AE} + \frac{2P_C \times 20}{AE} = 0$$

$$\Rightarrow P_A - 2.75 P_B + 2.5 P_C = 0 \quad \text{--- (3)}$$

Equating eq^n (1) and (3) $\longrightarrow$

$$4.75 P_B - 1.5 P_C = 138000 \quad \text{--- (4)}$$

$\Sigma M_A = 0$

$$\Rightarrow 138000 \times 6 = P_B \times 8 + P_C \times 12$$

$$\Rightarrow P_C + 0.66 P_B = 69000 \quad \text{--- (5)}$$

From eq^n (4) and (5) $\longrightarrow$

$P_A = 55000$ lb

$P_B = 42000$ lb

$P_C = 41000$ lb

(Ans)

Concepts of thermal stress-strain: (Thermal + load deformation)

General procedure for computing the loads and stresses caused when thermal deformation is prevented —

Structure is relieved of all applied loads and temp. deformation can occure freely. Represents this deformation on a sketch and effect.

Now imagene loads applied to the structure to restore to the specific conditions of restraint. Represents these loads and corresponding load deformations on the sketch.

The geometric relations between the temp. and load deformation on the sketch give eqns which together with the eqn of static equllibrium may be solved for all unknown quantities.

#

1. Temp. deformation and sketch. (শুধুমাত্র temp)
2. Load deformation and sketch. (শুধুমাত্র load)
3. Geometric relation of temp. and load deformation.
4. Condition of static equillibrium.
5. All unknows parameteres will be solved.

* শুধুমাত্র temp
* শুধুমাত্র load } Geomety draw.

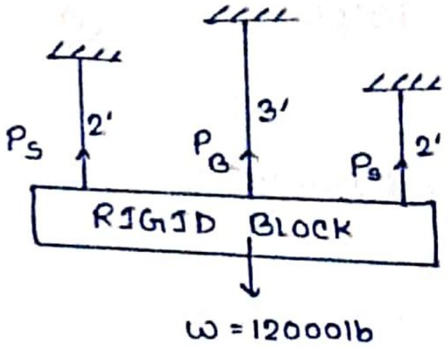
Prob: 15 Compute the stress in each bar.

Given, $A_S = 0.75\ in^2$ $A_B = 1.5\ in^2$

$E_S = 30 \times 10^6$ psi $E_B = 12 \times 10^6$ psi

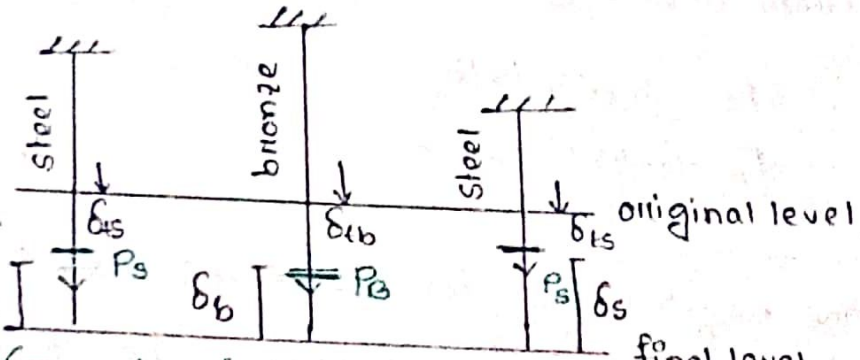
$\alpha_S = 6.5 \times 10^{-6}$ $\alpha_B = 10 \times 10^{-6}$ → বেশি expand করবে।

temp. change = 100°F



→

Steel thermal deformation বড়, load deformation থেকে



Geometry of elastic deformation.

From the geometric relation of elastic deformation in the following figure, we obtain the following relation betn the deformation –

$\delta_{Ts} + \delta_{Ps} = \delta_{Tb} + \delta_{Pb}$

$\Rightarrow (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_s = (\alpha L \Delta T)_b + \left(\frac{PL}{AE}\right)_b$

$\Rightarrow (6.5 \times 10^{-6} \times 2 \times 100) + \left(\frac{P_s \times 2}{0.75 \times 30 \times 10^{6}}\right) = (10 \times 10^{-6} \times 3 \times 100) + \left(\frac{P_B \times 3}{1.5 \times 12 \times 10^{6}}\right)$

$\Rightarrow P_s - 1.875 P_B = 19125$ ——— (1)

• From the free body diagram –

• $\Sigma Y = 0$

$\Rightarrow 2P_s + P_B = 12000$ ——— (2)

• From (1) and (2) →

$P_s = 8763.16\ lb$

$P_B = -5526.32\ lb$

• The negative sign for P_B means that the load P_B acts oppositely to that assumed. So, the stress are –

$S_s = 8763.16 \div 0.75 = 11684.21$ psi (T)

$S_B = 5526.32 \div 1.50 = 3684.21$ psi (C)

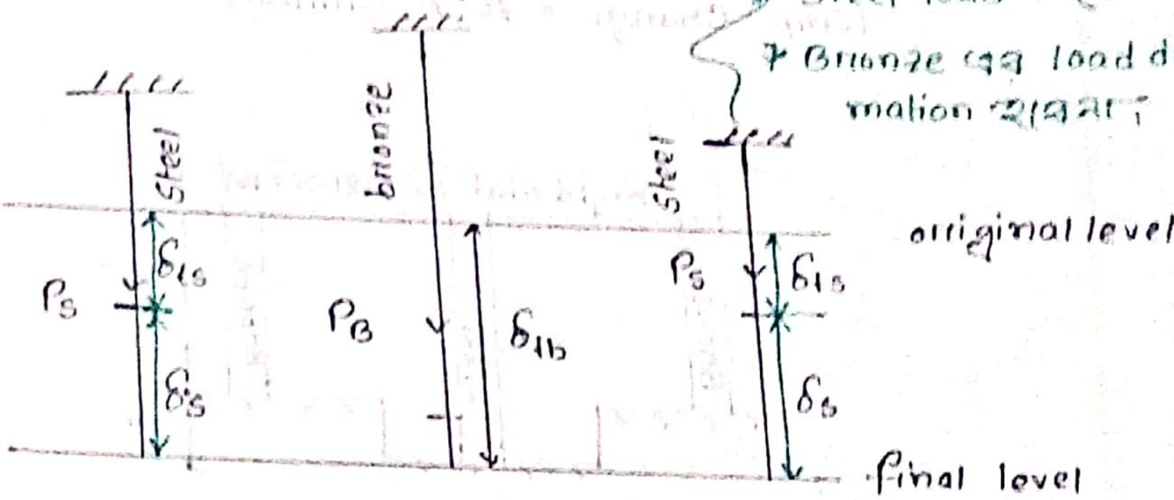
Prob: 16 Determine the temp. rise necessary to cause all the applied loads to be supported by the steel rods.

(Area and materials of the bar are same as prob: 15)

Soln: Geometric relation betn the load and thermal deformation is —

* Steel load নিয়েছে

* Bronze এর load deformation হয়েছে।



$$\delta_{tb} = \delta_{ts} + \delta_s$$

$$\Rightarrow (\alpha L \Delta T)_b = (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_s$$

$$\Rightarrow (10\times10^{-6} \times 3 \times \Delta T) = (6.5\times10^{-6} \times 2 \times \Delta T) + \left(\frac{6000\times 2}{0.75\times 30\times10^{6}}\right)$$

$$\Rightarrow \Delta T = 31.4^\circ F$$

(Ans)

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Prob: 17

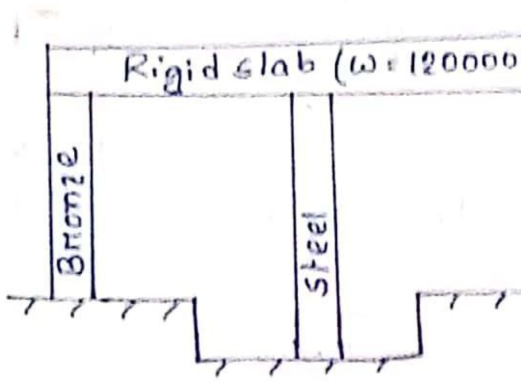
At what temp. will the stress in the

Given: $A_s = 10 \text{ in}^2$

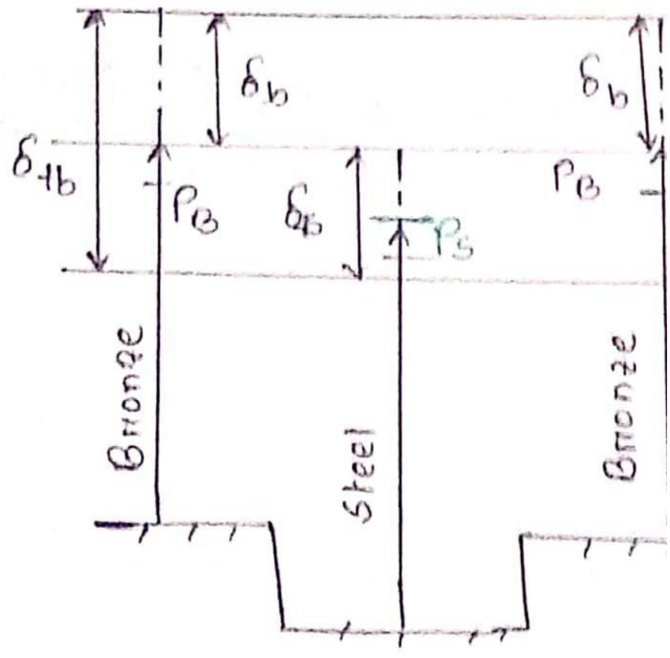
$E_s = 30 \times 10^6$ psi

$\alpha_s = 6.5 \times 10^{-6}$

Temp. Change = 70° F (Initial



⇒ Geometric relation between deformation is —



Geometry of elastic.

$$\delta_{tb} + \delta_b = \delta_{tb}$$

$$\Rightarrow (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_b = (\alpha L \Delta T)_b$$

$$\Rightarrow (6.5 \times 10^{-6} \times 12 \times \Delta T) + \left(\frac{6000 \times 10}{10 \times 12 \times 10^{6}}\right) = (10.5 \times 10^{-6} \times 10 \times \Delta T)$$

$$\Rightarrow 78\,\Delta T + 5000 = 105\,\Delta T$$

$$\Rightarrow \Delta T = 185.15\,°F$$

Hence, temp change $= 185.15 + 70 = 255.18\,°F$

(Ans)