Syllabus

- Fundamental Concepts of Stress and Strain;
- Mechanical Properties of Materials;
- Strain Energy, Stresses and Strains in Members Subjected to Tension, Compression, Shear and Temperature Changes;
- Thin Walled Pressure Containers;
- Bending Moment and Shear Force Diagrams of Beams and Frames;
- Flexural and Shearing Stresses in Beams;
- Riveted and Welded Joints;
- Shear Flow and Shear Center.

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FUNDAMENTAL CONCEPTS OF STRESS

- Definition of Stress:
  - Force per unit area.
  - Stress is the intensity of internal force developed when an external force is applied on an engineering material.
- Stress is symbolically expressed as:

$$\sigma = \frac{P}{A}$$

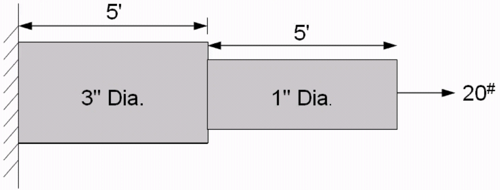
  - Where; σ (Sigma) is the stress per unit area;
  - P is the applied load;
  - A is the cross-sectional area.

FUNDAMENTAL CONCEPTS OF STRESS

- Unit of Stress:
  - SI unit → N/m^2 (Pascal, Pa) ……..[1 MPa = 10^6 Pa]
- US Customary Unit:
  - Psi (lb/in^2) or,
  - Ksi (kip/in^2) ……..[1 Kip = 1,000 lb]
- Examples of Stress:
  - Compressive and Tensile stress of structural steel are nearly equal.
  - Cast Iron take more compressive strength and week in tension.
  - Concrete is strong in compression but week in tension.

STRESS-STRAIN PROBLEMS

- **Problem 4**: Calculate total deformation and maximum stress. ($E = 12 \times 10^6$ psi)



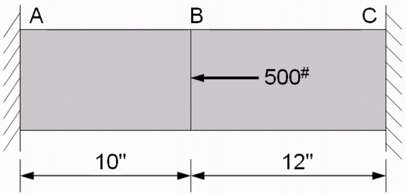
$$\Delta_1 = \frac{PL}{AE} = \frac{20 \times 5 \times 12}{\pi \times 1.5^2 \times 12 \times 10^6} = 1.415 \times 10^{-5}\ in.$$

$$\Delta_2 = \frac{PL}{AE} = \frac{20 \times 5 \times 12}{\pi \times 0.5^2 \times 12 \times 10^6} = 1.27 \times 10^{-4}\ in.$$

- Total Deformation, $\Delta = \Delta_1 + \Delta_2$
  $= 1.415\text{x}10^{-5} + 1.27\text{x}10^{-4}$
  $= 1.415\text{x}10^{-4}$ in. ....*(Ans.)*
- Max. Stress = 20 ÷ ($\pi/0.5^2$) = 25.46 psi ...*(Ans.)*

STRESS-STRAIN PROBLEMS

- **Problem 6**: Calculate the normal stress.
  ($E = 12 \times 10^3$ ksi and Area, $A = 5.0$ in^2)



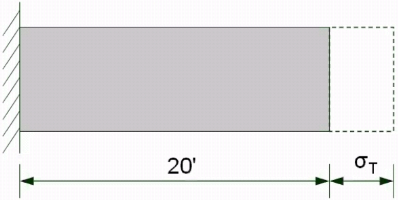
STRESS-STRAIN PROBLEMS

- **Problem 7**: Calculate thermal deformation and the required external load to resist thermal deformation.

Given That:

$\alpha = 6.5 \times 10^{-6}$, $T_1 = 100°F$, $T_2 = 150°F$,

$A = 3\ in^2$ & $E = 30 \times 10^3$ ksi



$$\Delta_T = \alpha L T$$

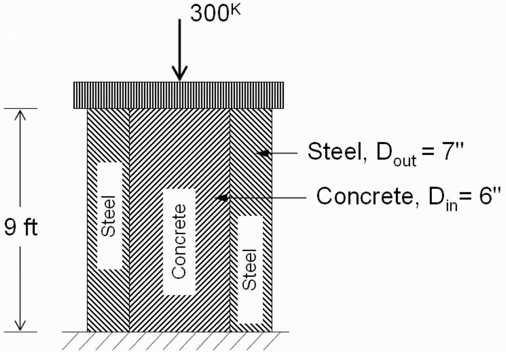
$$\Delta_T = 6.5 \times 10^{-6} \times 20 \times 12 \times (150 - 100) = 0.078 \ (inch) \ ....(Ans)$$

$$\Delta = \frac{PL}{AE} \Rightarrow P = \frac{\Delta A E}{L}$$

$$P = \frac{0.078 \times 3 \times 30 \times 10^3}{20 \times 12} = 29.25\ Kip \ ....(Ans)$$

STRESS-STRAIN PROBLEMS

- **Problem 8:** Compute stress at steel and concrete.
  (Es = 30 x 10³ ksi and Ec = 3 x 10³ ksi)



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STRESS-STRAIN PROBLEMS

Solution:

- From the free body: P = Ps + Pc ....(1)
- From the geometry of deformation, $\Delta s = \Delta c$ ....(2)
- From Eq^n - 2:

$$\frac{P_S \times 9 \times 12}{\pi(3.5^2 - 3^2) \times 30 \times 10^3} = \frac{P_C \times 9 \times 12}{\pi \times 3^2 \times 3 \times 10^3}$$

Ps = 3.612 Pc.....(3)

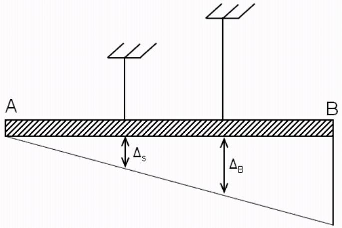
- From Eq^n 1 & 3: Pc = 65.03^K & Ps = 234.93^K

$$\sigma_S = \frac{P_S}{A_S} = \frac{234.93}{\pi \times (3.5^2 - 3^2)} = 23.009\ ksi \quad (comp.)......(Ans)$$

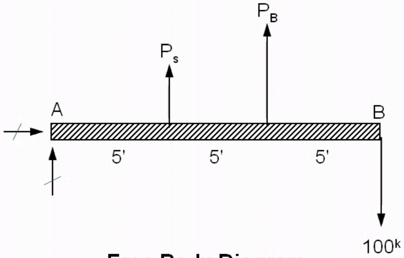
$$\sigma_C = \frac{P_C}{A_C} = \frac{65.03}{\pi \times 3^2} = 2.3\ ksi\ (comp.)......(Ans.)$$

STRESS-STRAIN PROBLEMS

- **Solution**:



Geometric Relation of Elastic Deformation



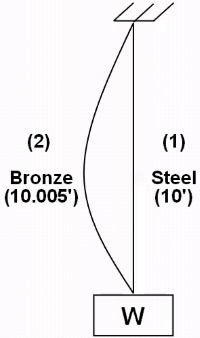
Free Body Diagram

STRESS-STRAIN PROBLEMS

- **<u>Problem</u> 10:** Find the stress and the final length of two strings. *(Given: $A_S = 0.5\ in^2$; $A_B = 0.4\ in^2$;*
  $E_S = 30 \times 10^3\ ksi$; $E_B = 20 \times 10^3\ ksi$;
  $L_S = 10\ ft$; $L_B = 10.005\ ft$ & $W = 200\ kip$)

Solution:

$$L_2 - L_1 = 10.005 - 10 = 0.005\ ft$$



$$\Delta_{S1} = \frac{P \times L}{A \times E}$$

$$\Rightarrow 0.005 \times 12 = \frac{P_{S1} \times 10 \times 12}{0.5 \times 30 \times 10^3} \Rightarrow P_{S1} = 7.5^K$$

STRESS-STRAIN PROBLEMS

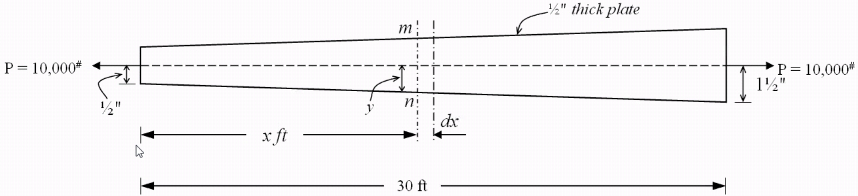
- Rest Load = 200 – 7.5 = 192.5 kip
- Hence; $P_B + P_{S2} = 192.5$ ……. (1)
- And; $\Delta_B = \Delta_{S2}$ …..(2)
- $$\Rightarrow \frac{P_B \times 10.005 \times 12}{0.4 \times 20 \times 10^3} = \frac{P_{S2} \times 10.005 \times 12}{0.5 \times 30 \times 10^3}$$

$$\Rightarrow P_{S2} = 1.875 P_B \text{ ....}(3)$$

- From Eq. (1 & 3) : P_B = 66.96 kip & P_{S2} = 125.54 kip

STRESS-STRAIN PROBLEMS

- **Problem:11**
- Compute the total elongation ($E = 30 \times 10^6$ psi).



- **Solution**:
  Since the cross-sectional area is not constant; Equation $\Delta = \dfrac{PL}{AE}$ can not be applied directly.

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STRESS-STRAIN PROBLEMS

- At section *m-n*, distant from *x ft* from the smaller end, the half width '*y*' is found from the geometry to be –

$$y = \frac{1}{2} + \frac{x}{30}$$

- And the area at that section is;

$$A = \frac{1}{2} \times (2y) = (\frac{1}{2} + \frac{x}{30})\, sq.in$$

STRESS-STRAIN PROBLEMS

- At section *m-n*, in a differential length *dx*, the elongation may be found from –

$$d\delta = \frac{10{,}000 \times dx}{\left(\frac{1}{2} + \frac{x}{30}\right) \times 30 \times 10^{6}} = 10^{-2} \times \frac{dx}{(15 + x)}$$

- From which the total elongation is -

$$\delta = 10^{-2} \times \int_0^{30} \frac{dx}{(x + 15)} = 10^{-2} \times \left[\log_e (15 + x)\right]_0^{30}$$

$$= 10^{-2} \times \log_e \frac{45}{15} = 10^{-2} \times \log_e 3 = 0.01098 = 0.011\ ft. \quad (Ans.)$$

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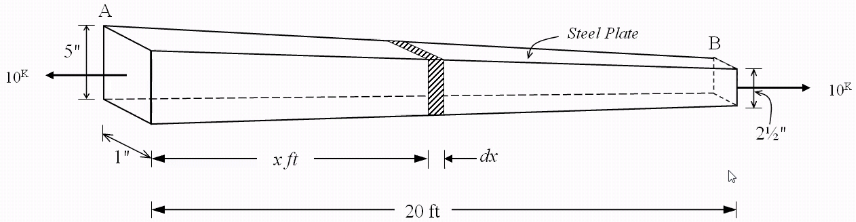
STRESS-STRAIN PROBLEMS

- **Problem:12**
  Compute the elongation of the steel flat plate. ($E = 30 \times 10^6$ psi).



- **Solution**:
  Width of the plate at a distance 'x' from 'A' = $5 - (5 - 2.5) \times x/8$
  Cross-sectional area of the bar at this section = $1 \times (5 - x/8) = (5 - x/8)$ sq.in

STRESS-STRAIN PROBLEMS

- At this section, in a differential length dx, the elongation may be found from –

$$d\delta = \frac{10{,}000 \times dx}{(5 - \frac{x}{8}) \times 30 \times 10^6} = 10^{-2} \times \frac{dx}{(150 - 3.75x)}$$

- From which the total elongation is -

$$\delta = 10^{-2} \times \int_0^{20} \frac{dx}{(150 - 3.75x)} = \frac{10^{-2}}{-3.75} \times \left[\log_e(150 - 3.75x)\right]_0^{20}$$

$$= \frac{10^{-2}}{-3.75} \times \left[\log_e 75 - \log_e 150\right] = \frac{1}{375} \times \log_e(\frac{150}{75})$$

$$= \frac{1}{375} \times \log_e 2 = 0.00185\ ft.\ (Ans.)$$

STRESS-STRAIN PROBLEMS

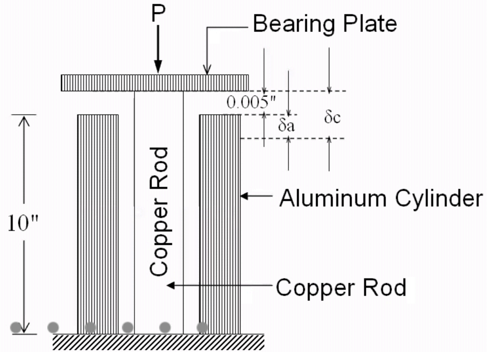
- **Problem 13:**
- Compute the maximum load applied to the bearing plate.

  (Given: $A_{Cop} = 2\ in^2$; $A_{Al} = 3\ in^2$;

  $E_{Cop} = 17 \times 10^3$ ksi; $E_{Al} = 10 \times 10^3$ ksi;

  Allowable Stress for Copper = 20,000 psi &

  Aluminum = 10,000 psi)



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STRESS-STRAIN PROBLEMS

- **Solution**:
- Considering the geometric relation of elastic deformation of copper and aluminum cylinder; we find –

$$\delta_{Cop} = \delta_{Al} + 0.005 \ldots\ldots\ldots\ldots (1)$$

Again; $P = P_{Cop} + P_{Al} \ldots\ldots\ldots\ldots\ldots\ldots (2)$

$$\left(\frac{S \times L}{E}\right)_{Cop} = \left(\frac{S \times L}{E}\right)_{Al} + 0.005$$

$$\Rightarrow \left(\frac{S_{Cop} \times 10.005}{17 \times 10^6}\right) = \left(\frac{S_{Al} \times 10}{10 \times 10^6}\right) + 0.005$$

$$\Rightarrow S_{Cop} = 1.7 S_{Al} + 8495.75 \ldots\ldots\ldots\ldots (3)$$

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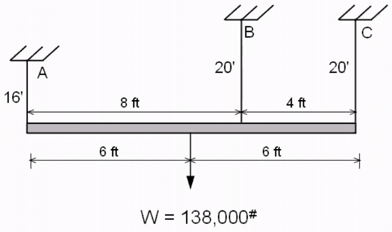
STRESS-STRAIN PROBLEMS

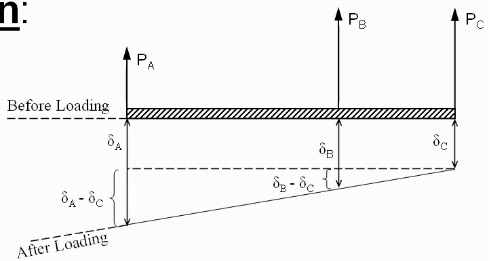
- **Problem 14**: Compute loads on each bar.

(Area and Materials of all bars are same.)



STRESS-STRAIN PROBLEMS

- **Solution**:



Geometry of Elastic Deformation

- From the Free body Diagram:

$$P_A + P_B + P_C = 138,000\ldots..(1)$$

STRESS-STRAIN PROBLEMS

- Again for the geometric relation of elastic deformation:

$$\frac{\delta_B - \delta_C}{\delta_A - \delta_C} = \frac{4}{12} \text{ ....(2)}$$

$$\Rightarrow \delta_A - 3\delta_B + 2\delta_C = 0$$

$$\Rightarrow \frac{P_A \times 16}{A \times E} - \frac{3P_B \times 20}{A \times E} + \frac{2P_C \times 20}{A \times E} = 0$$

$$\Rightarrow P_A - 3.75P_B + 2.5P_C = 0 \text{.........(3)}$$

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STRESS-STRAIN PROBLEMS

- Equating Equation (1) & (3) :

  $4.75\ P_B - 1.5\ P_C = 138{,}000$……(4)
- $\sum M_A = 0$

  or, $138000 \times 6 = P_B \times 8 + P_C \times 12$

  or, $P_C + 0.66\ P_B = 69{,}000$……..(5)
- From Equation (4) & (5):

  $P_A = 55{,}000$ lb

  $P_B = 42{,}000$ lb

  $P_C = 41{,}000$ lb………(Ans)

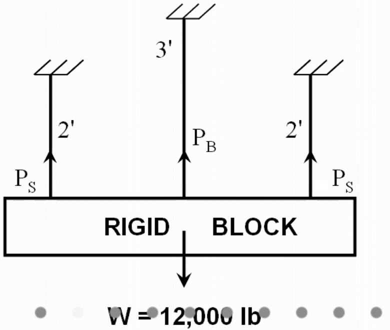
CONCEPTS OF THERMAL STRESS-STRAIN

- General procedure for computing the loads and stresses caused when thermal deformation is prevented –
  1. Structure is relieved of all applied loads and temperature deformation can occur freely. Represent these deformations on a sketch and effect.
  2. Now imagine loads applied to the structure to restore it to the specified conditions of restraint. Represent these loads and corresponding load deformations on the sketch.
  3. The geometric relations between the temperature and load deformations on the sketch give equations which, together with the equation of static equilibrium, may be solved for all unknown quantities.

THERMAL STRESS-STRAIN PROBLEMS

- **Problem 15**: Compute the stress in each bar.

Given: $A_S = 0.75\ in^2$; $A_B = 1.5\ in^2$;
$E_S = 30 \times 10^6$ psi; $E_B = 12 \times 10^6$ psi;
$\alpha_S = 6.5 \times 10^{-6}$; $\alpha_B = 10 \times 10^{-6}$ &
Temp Change = 100°F



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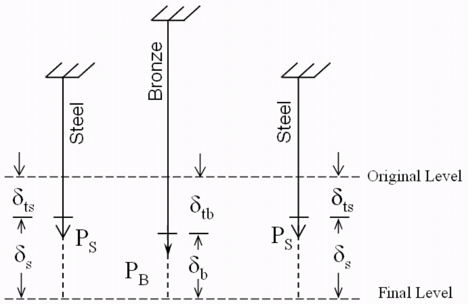
THERMAL STRESS-STRAIN PROBLEMS

- From the geometric relation of elastic deformation in the following figure, we obtain the following relation between the deformation –



Geometry of Elastic Deformation

THERMAL STRESS-STRAIN PROBLEMS

$$\Rightarrow \delta_{ts} + \delta_s = \delta_{tb} + \delta_b$$

$$\Rightarrow (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_S = (\alpha L \Delta T)_b + \left(\frac{PL}{AE}\right)_b$$

$$\Rightarrow (6.5 \times 10^{-6} \times 2 \times 100) + \left(\frac{P_S \times 2}{0.75 \times 30 \times 10^6}\right) =$$

$$(10 \times 10^{-6} \times 3 \times 100) + \left(\frac{P_B \times 3}{1.5 \times 12 \times 10^6}\right)$$

$$\Rightarrow P_S - 1.875 P_B = 19125 \;\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

THERMAL STRESS-STRAIN PROBLEMS

- From the Free-body diagram –
- $\sum Y = 0; \ 2P_S + P_B = 12{,}000 \ldots.(2)$
- From (1) & (2) –

$$P_S = 8{,}763.16 \text{ lb}$$

$$P_B = -5{,}526.32 \text{ lb}$$

- The negative sign for P_B means that load P_B acts oppositely to that assumed. So, the stresses are –

$$S_S = 8{,}763.16 \div 0.75 = 11{,}684.21 \text{ psi (Tension)}$$

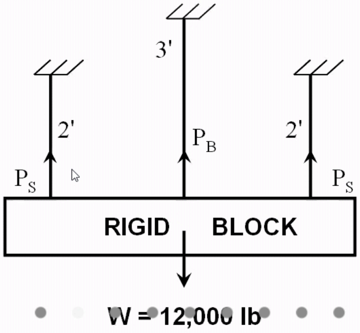
$$S_B = 5{,}526.32 \div 1.50 = 3{,}684.21 \text{ psi (Comp.)}$$

(Ans.)

THERMAL STRESS-STRAIN PROBLEMS

- **Problem 15**: Compute the stress in each bar.

Given: $A_S = 0.75\ in^2$; $A_B = 1.5\ in^2$; $E_S = 30 \times 10^6$ psi; $E_B = 12 \times 10^6$ psi; $\alpha_S = 6.5 \times 10^{-6}$; $\alpha_B = 10 \times 10^{-6}$ & *Temp Change = 100°F*



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THERMAL STRESS-STRAIN PROBLEMS

- From the Free-body diagram –
- $\sum Y = 0; \; 2P_S + P_B = 12{,}000 \ldots.(2)$
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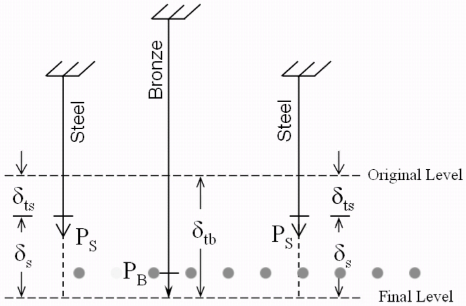
(Ans.)

THERMAL STRESS-STRAIN PROBLEMS

- **<u>Problem 16</u>**: Determine the temperature rise necessary to cause all the applied loads to be supported by the steel rods.
  (Area and Materials of all bars are same as in Problem 15.)

<u>Solution</u>:

Geometric relation between the load and thermal deformation is –



THERMAL STRESS-STRAIN PROBLEMS

$$\Rightarrow \delta_{tb} = \delta_{ts} + \delta_{s}$$

$$\Rightarrow (\alpha.L.\Delta T)_b = (\alpha.L.\Delta T)_S + \left(\frac{P.L}{A.E}\right)_S$$

$$\Rightarrow (10 \times 10^{-6} \times 3 \times \Delta T) = (6.5 \times 10^{-6} \times 2 \times \Delta T) + \left(\frac{6000 \times 2}{0.75 \times 30 \times 10^{6}}\right)$$

$$\Rightarrow \Delta T = 31.4°F \text{ ....} (Ans)$$

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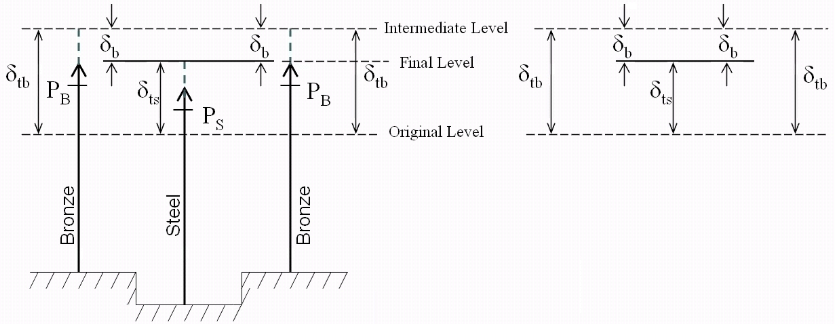
THERMAL STRESS-STRAIN PROBLEMS

Solution:

Geometric relation between the load and thermal deformation is –



Geometry of Elastic Deformation

THERMAL STRESS-STRAIN PROBLEMS

$$\Rightarrow \delta_{ts} + \delta_b = \delta_{tb}$$

$$\Rightarrow (\alpha L \Delta T)_s + \left(\frac{PL}{AE}\right)_b = (\alpha L \Delta T)_b$$

$$\Rightarrow (6.5 \times 10^{-6} \times 12 \times \Delta T) + \left(\frac{60000 \times 10}{10 \times 12 \times 10^6}\right) = (10.5 \times 10^{-6} \times 10 \times \Delta T)$$

$$\Rightarrow 78 \Delta T + 5000 = 105 \Delta T$$

$$\Rightarrow \Delta T = 185.15\,°F$$

$$Hence\ Temp\ Change = 185.15 + 70 = 255.18\,°F \ .....(Ans.)$$

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