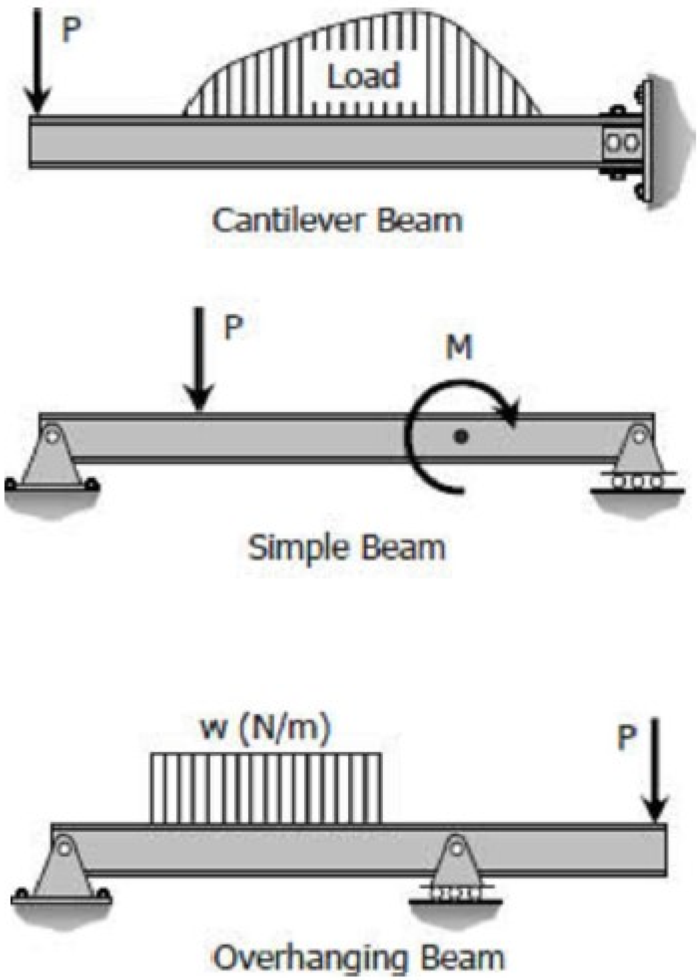
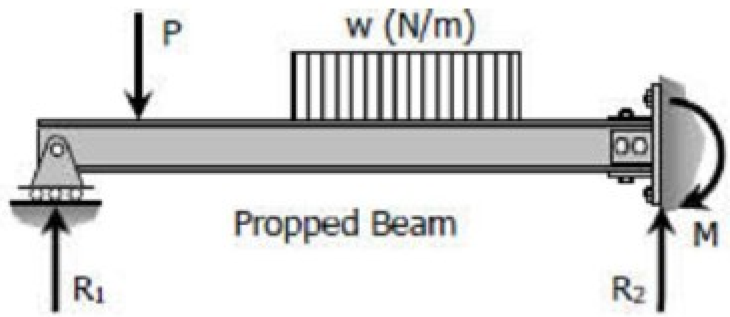
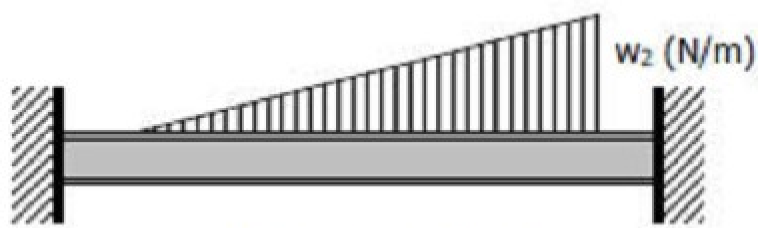
CE 2111
MECHANICS OF MATERIALS - I

Statically Determinate Beam

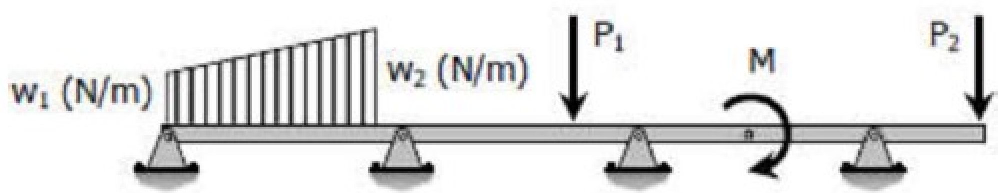




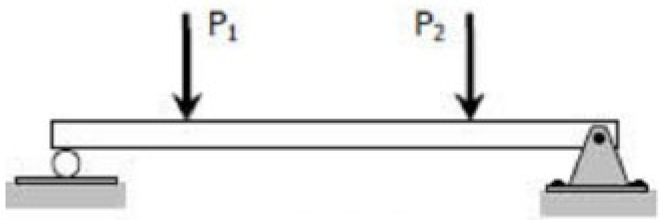
Propped Beam



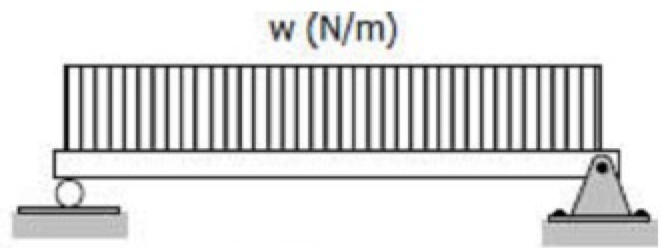
Fixed or Restrained Beam



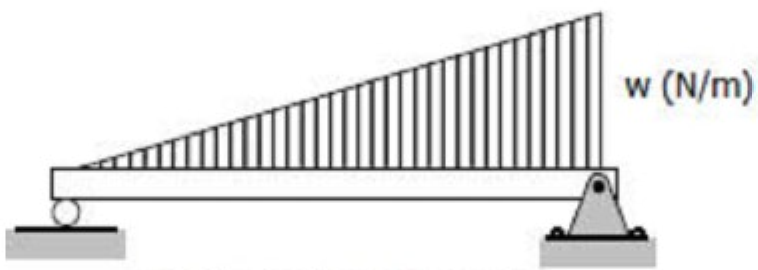
Continuous Beam



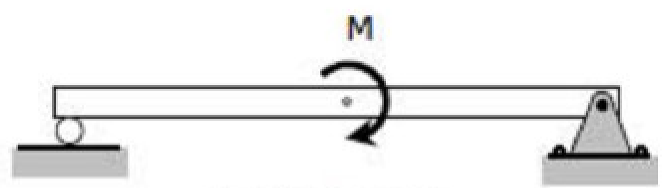
Concentrated Loads



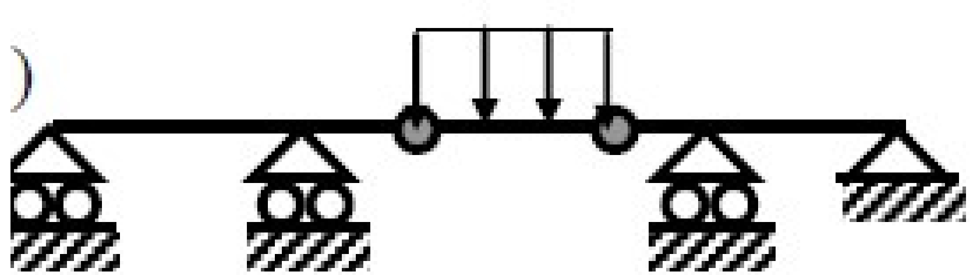
Uniform Load



Uniformly Varying Load



Applied Couple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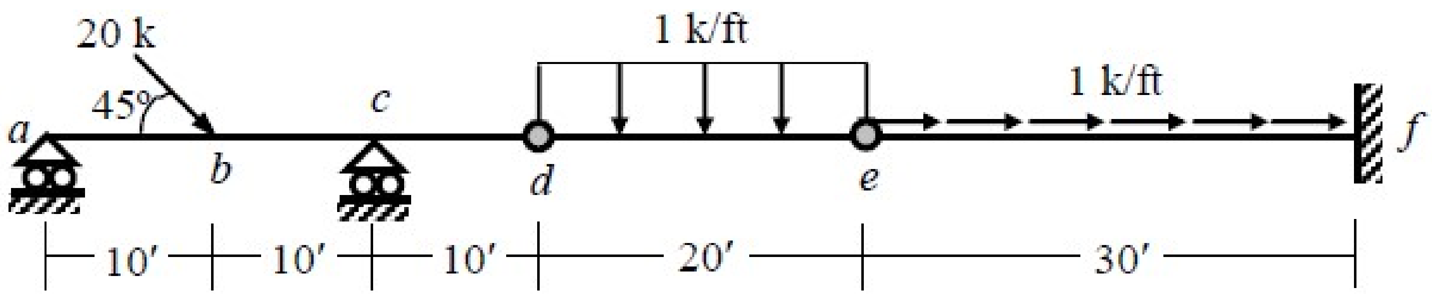
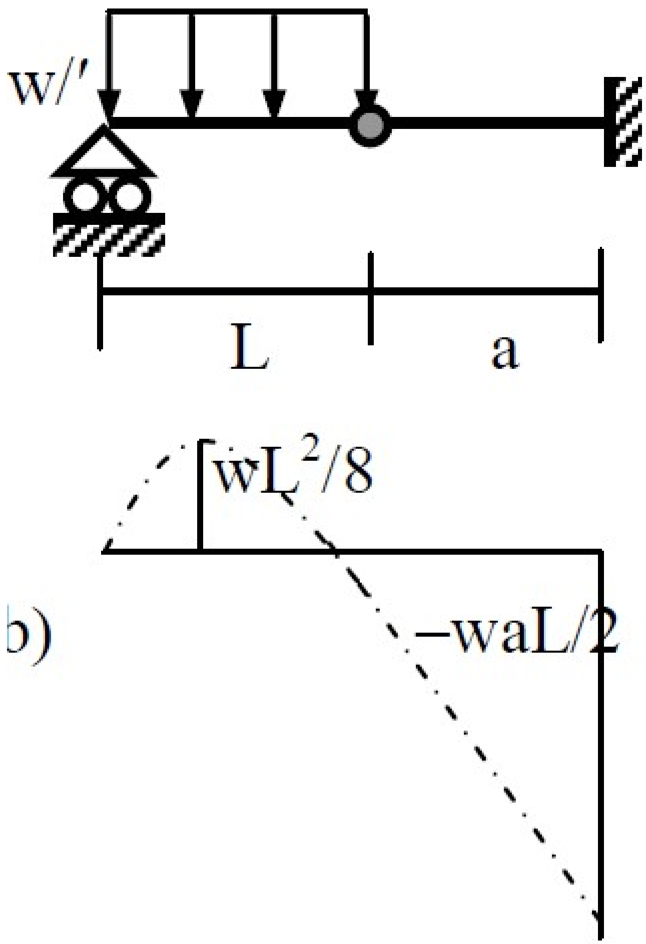
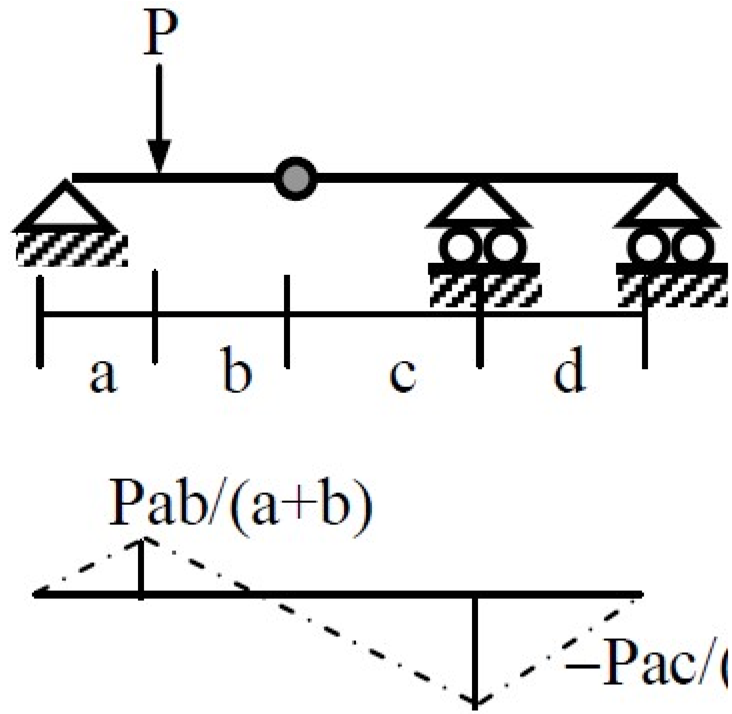
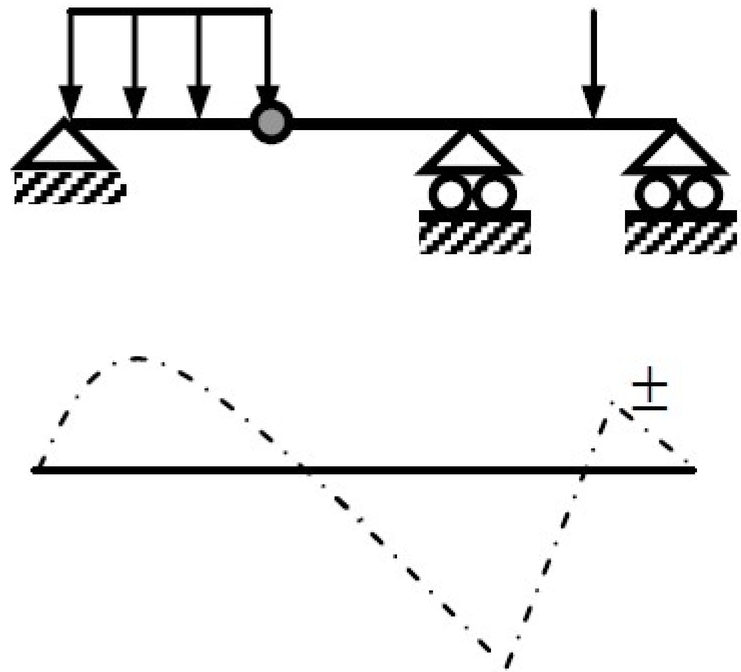
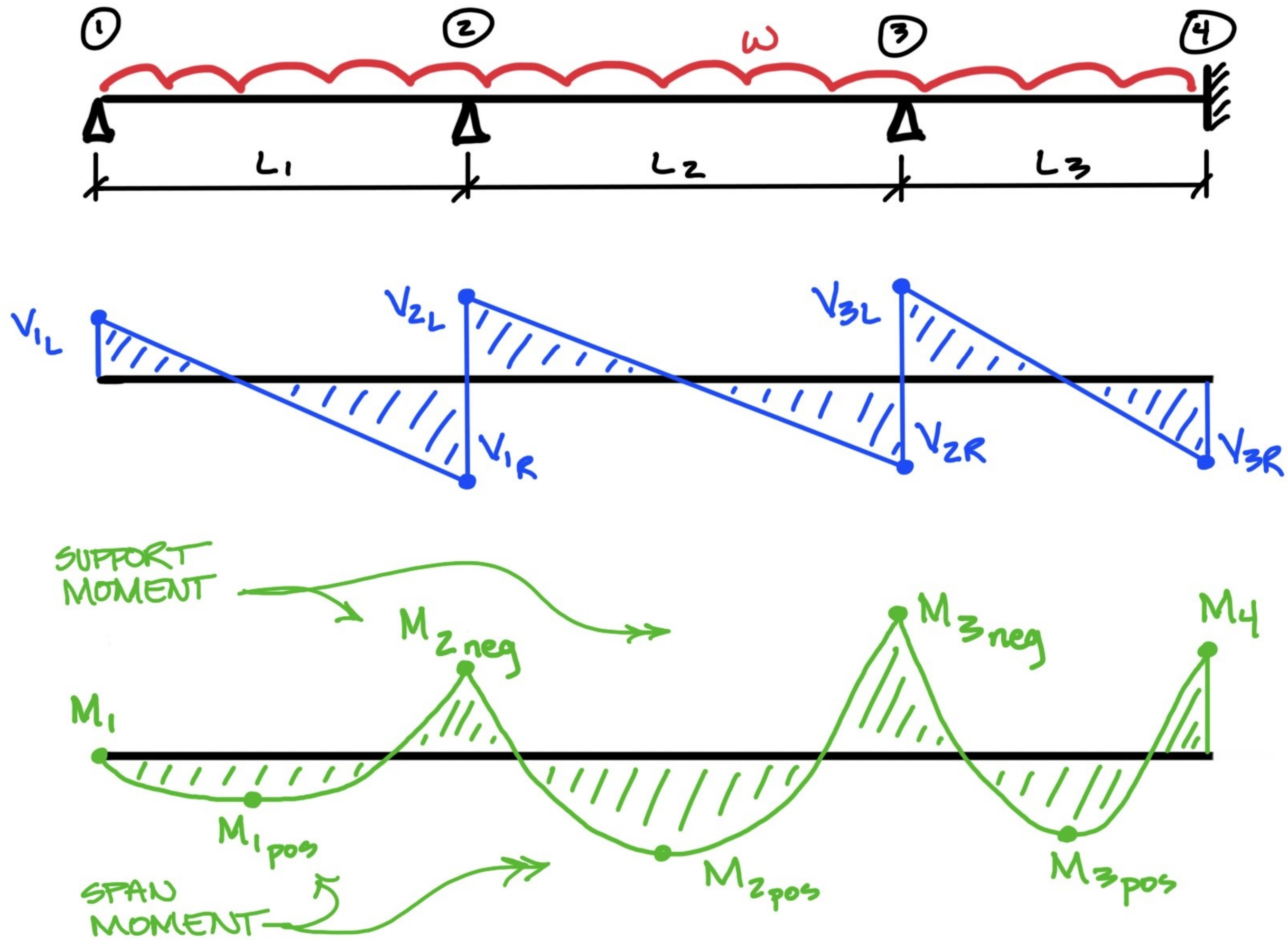
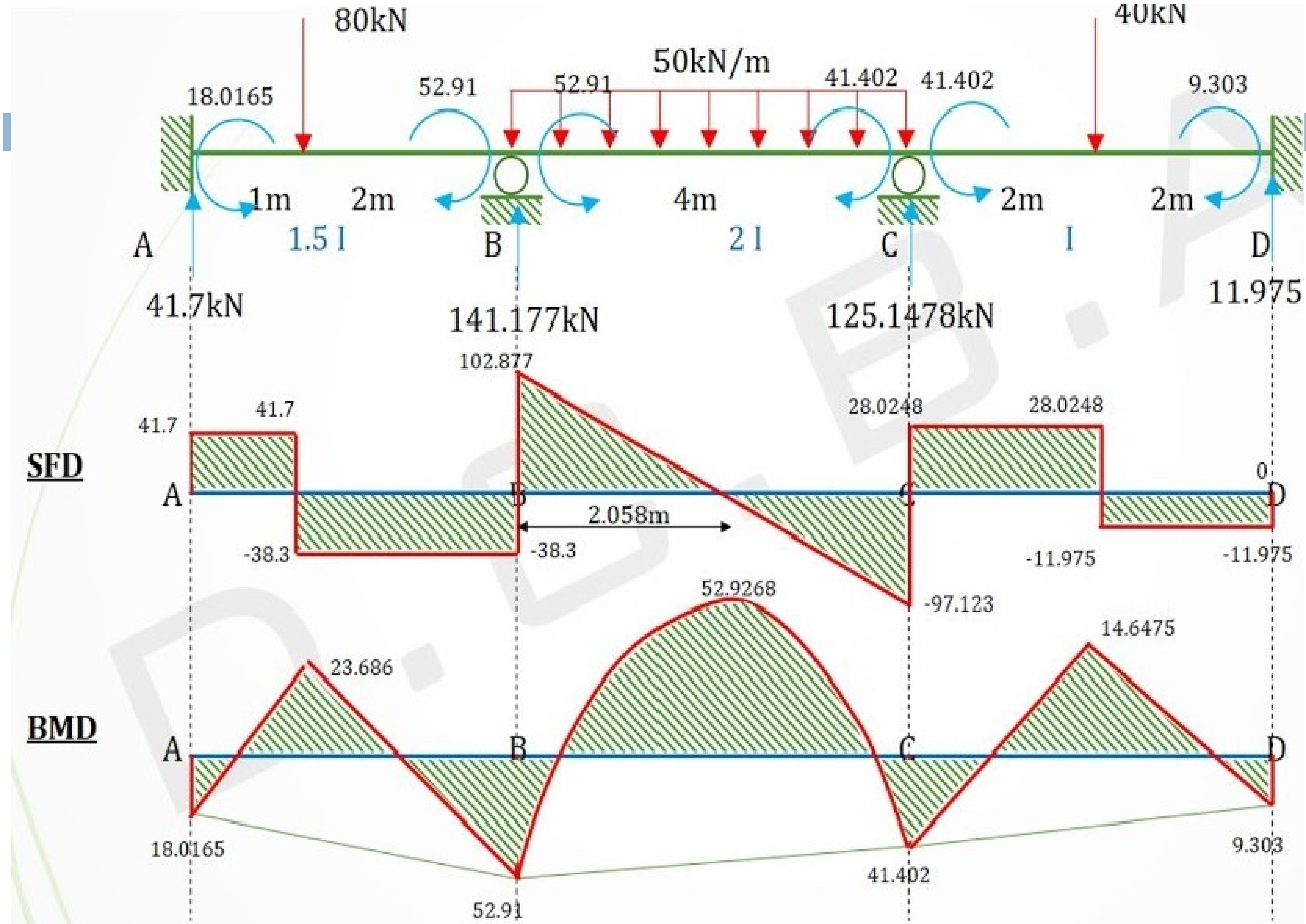

20 k
45°
1 k/ft
1 k/ft
a
b
c
d
e
f
10′
10′
10′
20′
30′


w/'
L
a
wL²/8
b)
−waL/2

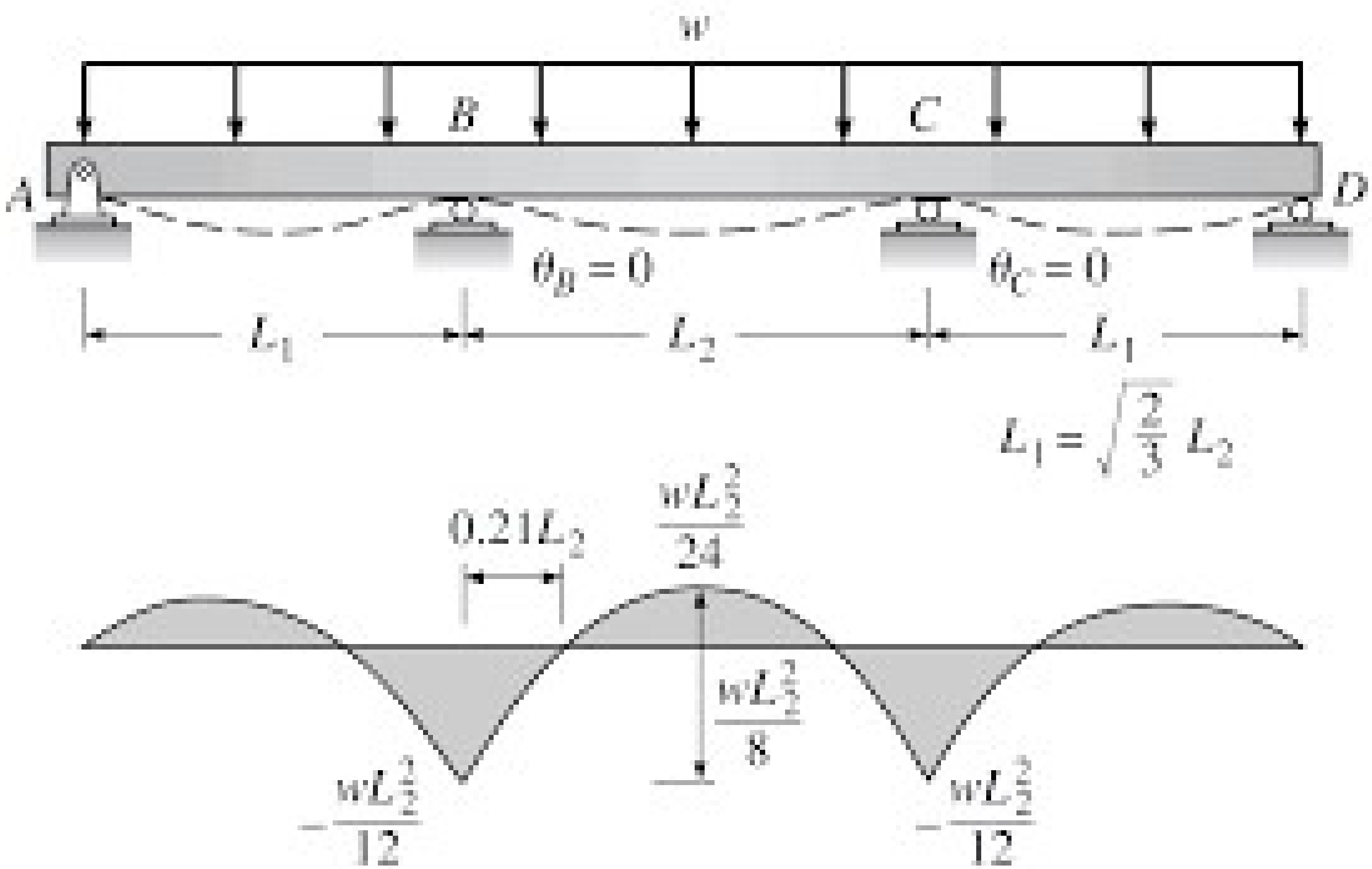

P
a
b
c
d
Pab/(a+b)
−Pac/(



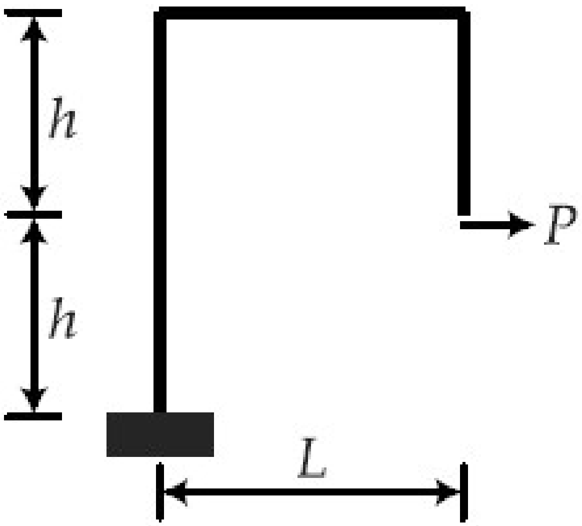

1
2
3
4
w
L1
L2
L3
V1L
V2L
V3L
V1R
V2R
V3R
SUPPORT MOMENT
M2neg
M3neg
M4
M1
M1pos
M2pos
M3pos
SPAN MOMENT

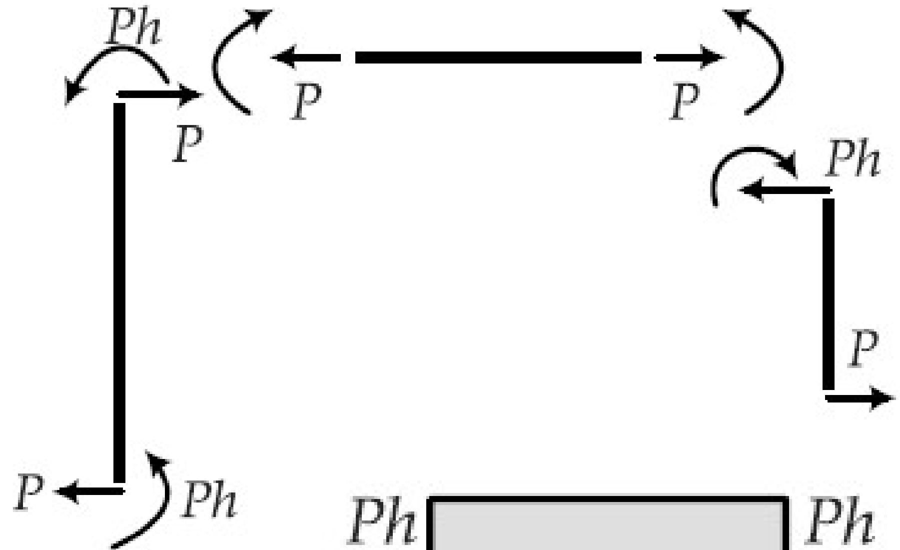


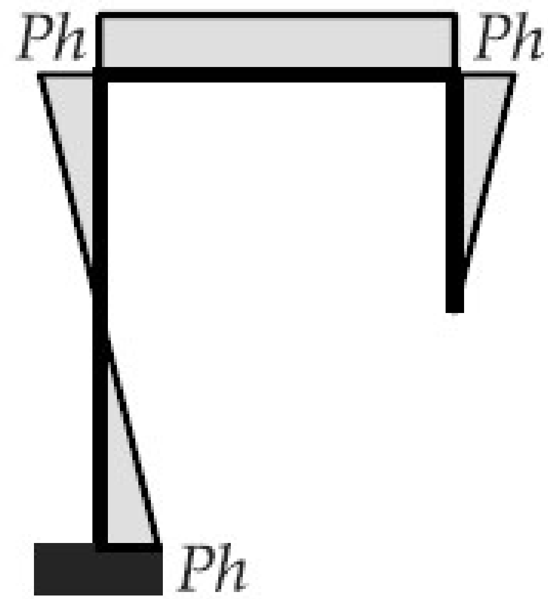
80kN
40kN
50kN/m
18.0165
52.91
52.91
41.402
41.402
9.303
1m
2m
4m
2m
2m
1.5 I
2 I
I
A
B
C
D
41.7kN
141.177kN
125.1478kN
11.975
SFD
102.877
41.7
41.7
28.0248
28.0248
0
-38.3
-38.3
2.058m
-11.975
-11.975
-97.123
BMD
52.9268
23.686
14.6475
18.0165
52.91
41.402
9.303

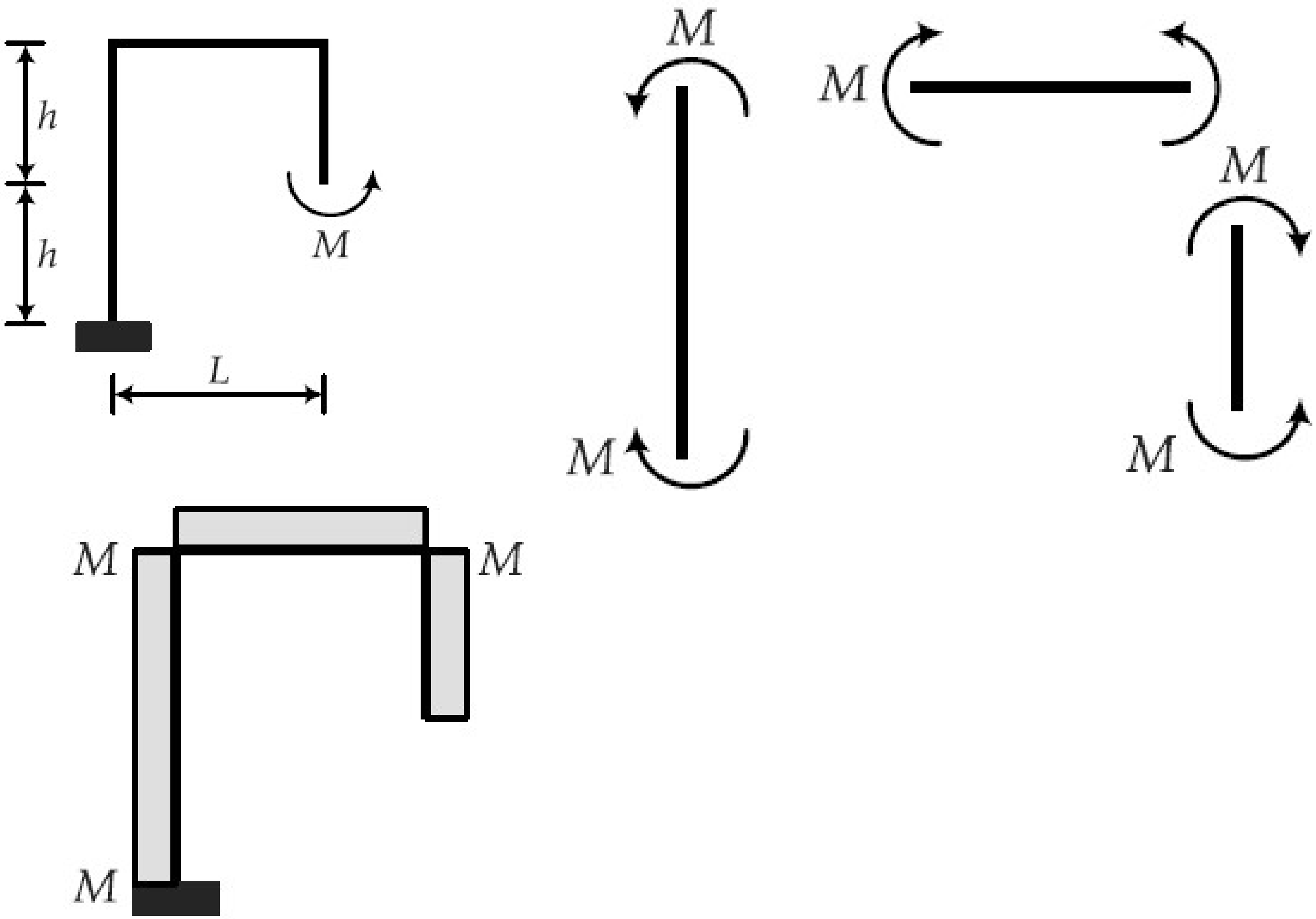


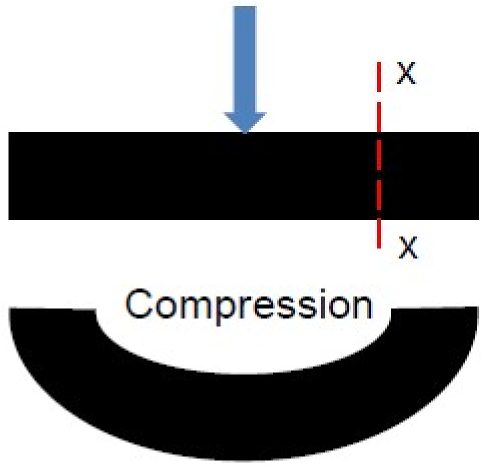
w
B
C
A
D
$\theta_B = 0$
$\theta_C = 0$
L_1
L_2
L_1
$L_1 = \sqrt{\frac{2}{3}}\, L_2$
$0.21L_2$
$\frac{wL_2^2}{24}$
$\frac{wL_2^2}{8}$
$-\frac{wL_2^2}{12}$
$-\frac{wL_2^2}{12}$

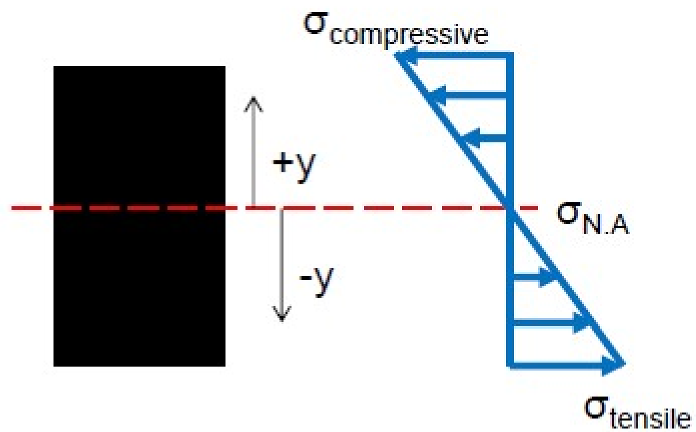

h
h
P
L

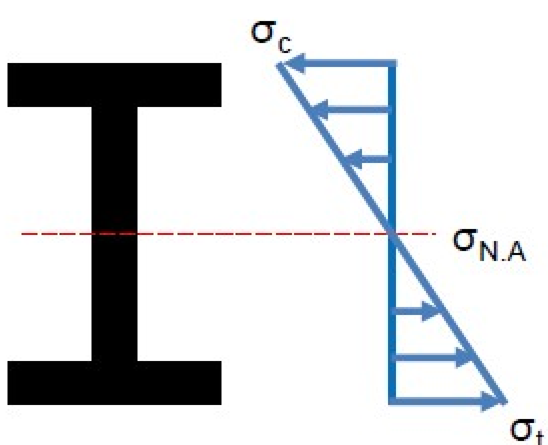

Ph
P
P
P
Ph
P
P
Ph


Ph
Ph
Ph

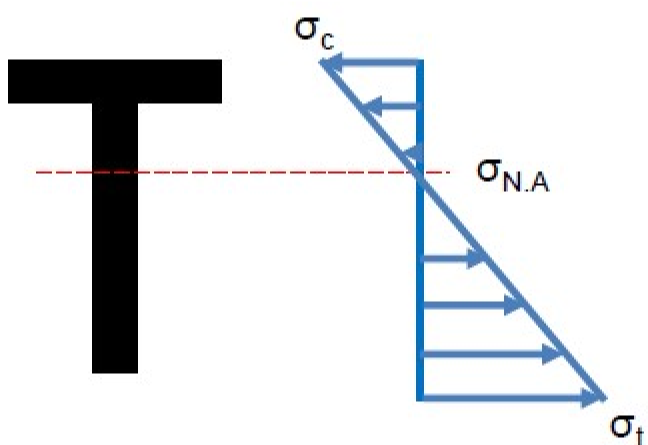

h
h
M
L
M
M
M
M
M
M
M
M
M





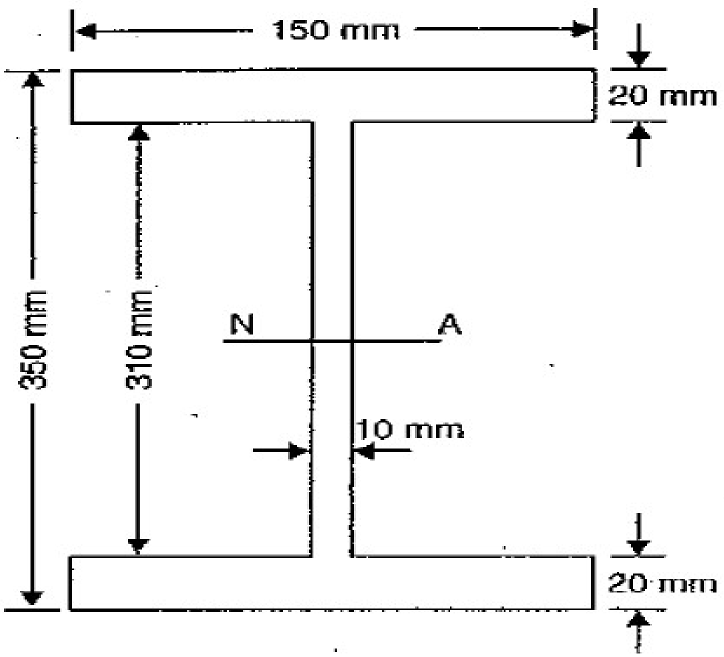


Stress distribution on **symmetrical beam**



Stress distribution on **unsymmetrical beam**

Calculate the maximum shear stress and draw the shear stress distribution across the section. Determine the total shear resisted by the web. **V= 40 kN**

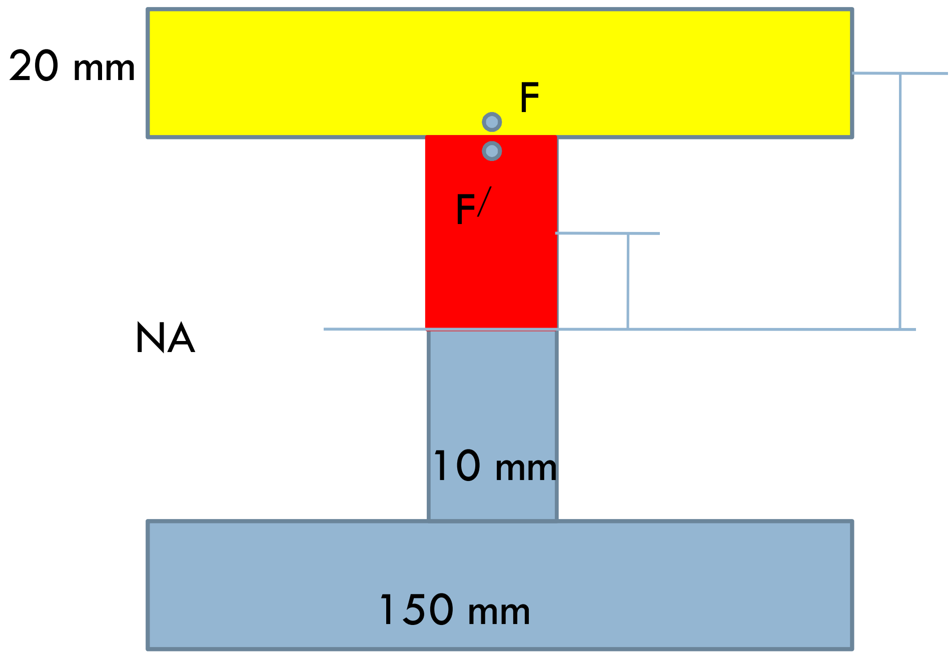


$$I = \frac{150 \times 350^3}{12} - \frac{140 \times 310^3}{12} \text{ mm}^4$$

$$= 535937500 - 347561666.6$$

$$= 188375833.4 \text{ mm}^4.$$

$$\tau_{max} = \frac{F \times A \times \bar{y}}{I \times b}$$



Q = Ay/ = Moment of the area above the neutral axis about the neutral axis

$$= (150 \times 20) \times \left(\frac{310}{2} + \frac{20}{2}\right) + \left(\frac{310}{2} \times 10\right) \times \left(\frac{310}{2} \times \frac{1}{2}\right) = 615125\ mm^3$$

$$\tau_{max} = \frac{40,000 \times 615125}{188375833.4 \times 10} = 13.06\ \text{N/mm}^2.$$

Shear stress at the upper edge of the flange is zero

Shear stress at the bottom edge of the flange

Q_F =

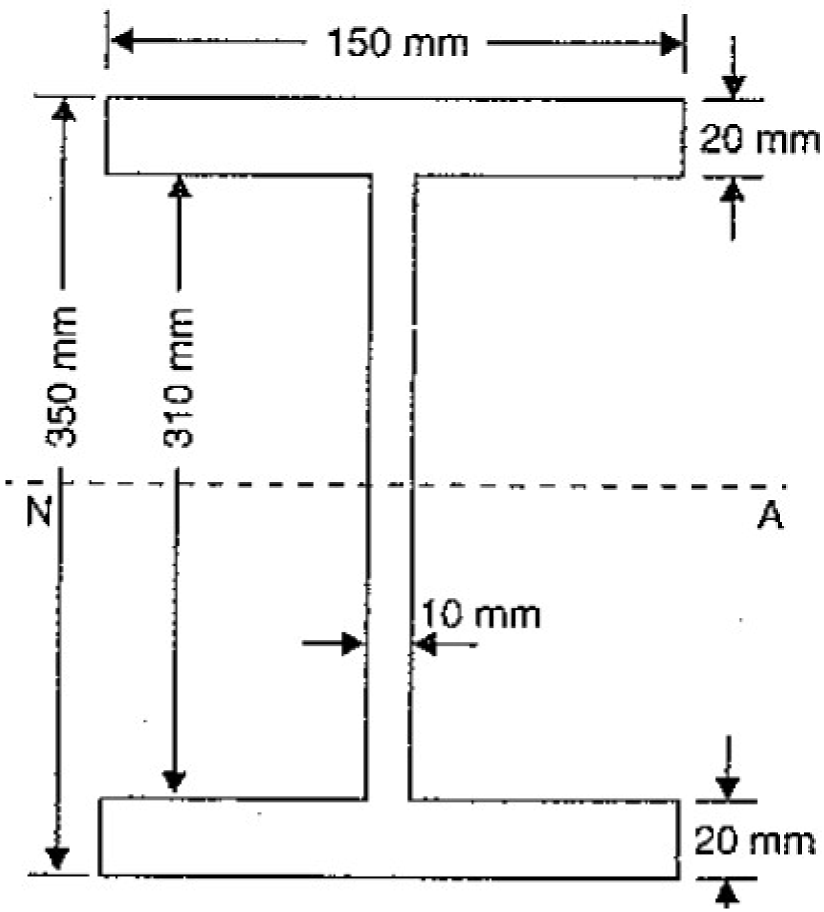
A

$$= (150 \times 20) \times \left(\frac{310}{2} + \frac{20}{2}\right) = 495000\ \text{mm}^3$$

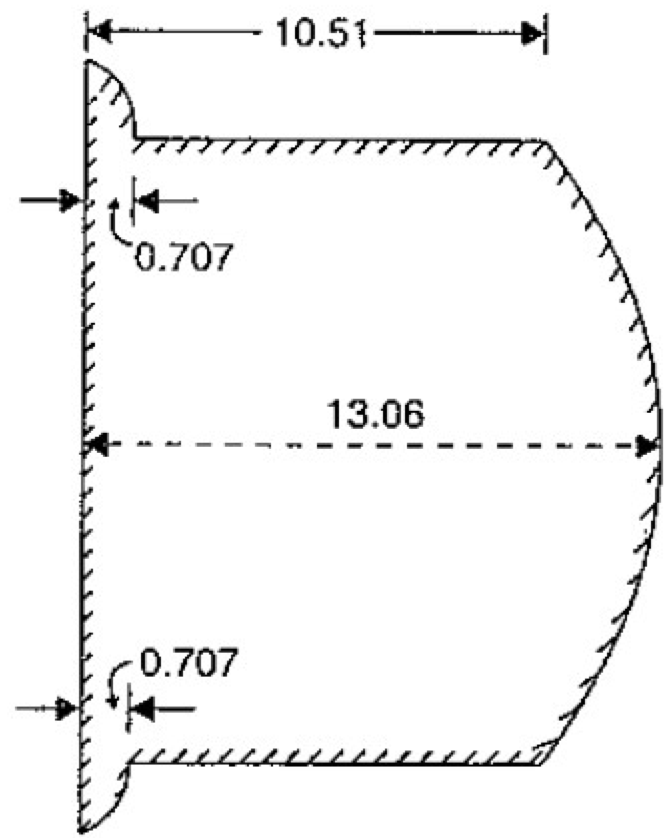
$$\tau_F = \frac{VQ}{Ib} = \frac{40000 * 495000}{188375833 * 150} = 0.707 N / mm^2$$

$Q_{F^/}$ = Q_F = 495000 mm^3

$$\tau_{F^/} = \frac{VQ}{Ib} = \frac{40000 * 495000}{188375833 * 10} = 10.5 N / mm^2$$

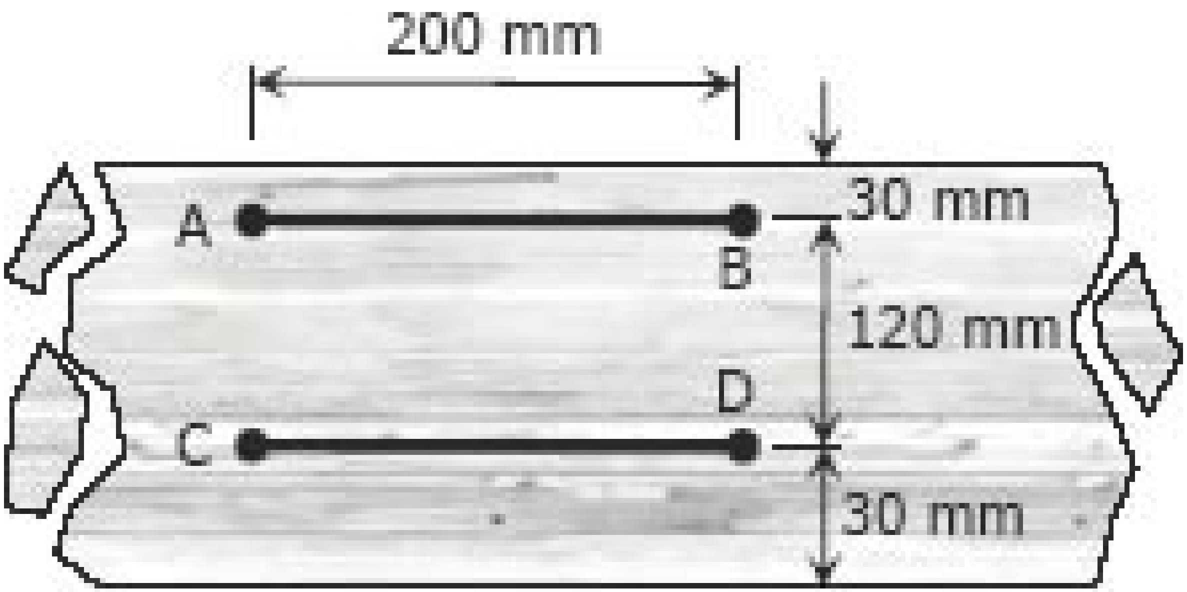


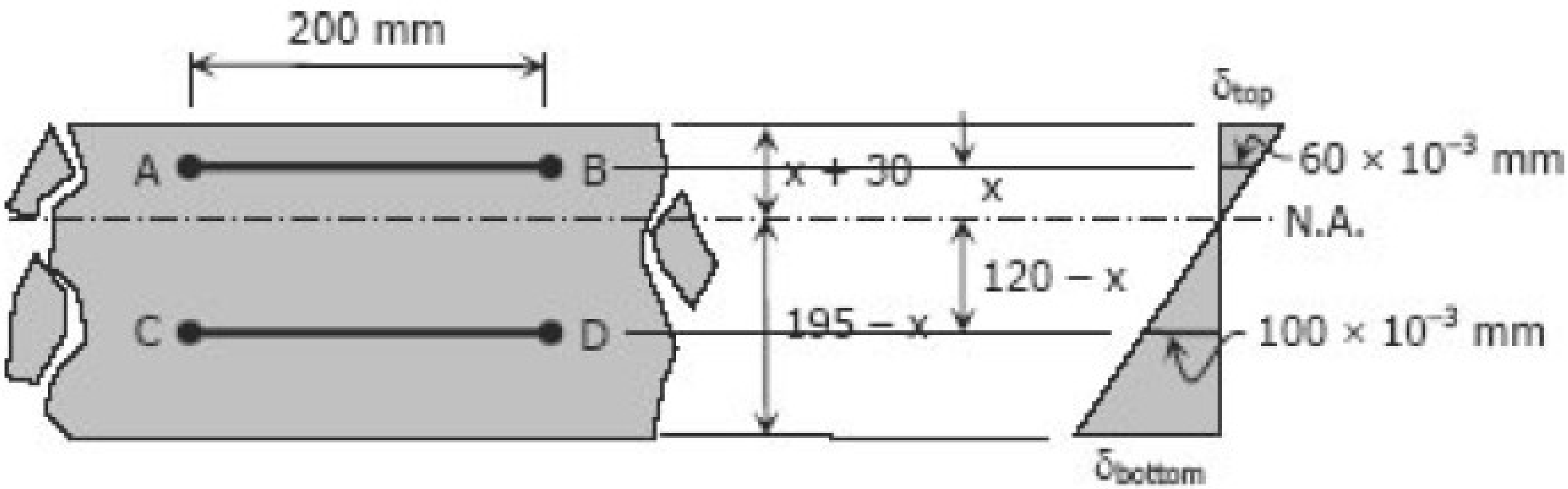
150 mm
20 mm
350 mm
310 mm
N
A
10 mm
20 mm



10.51
0.707
13.06
0.707

In a laboratory test of a beam loaded by end couples, the fibers at layer AB in Fig. P-507 are found to increase 60 × 10–3 mm whereas those at CD decrease 100 × 10–3 mm in the 200-mm-gage length. Using E = 70 GPa, determine the flexural stress in the top and bottom fibers.





$$\frac{x}{60\times10^{-3}} = \frac{120-x}{100\times10^{-3}}$$

$$x = 0.6(120 - x)$$

$$x + 0.6x = 0.6(120)$$

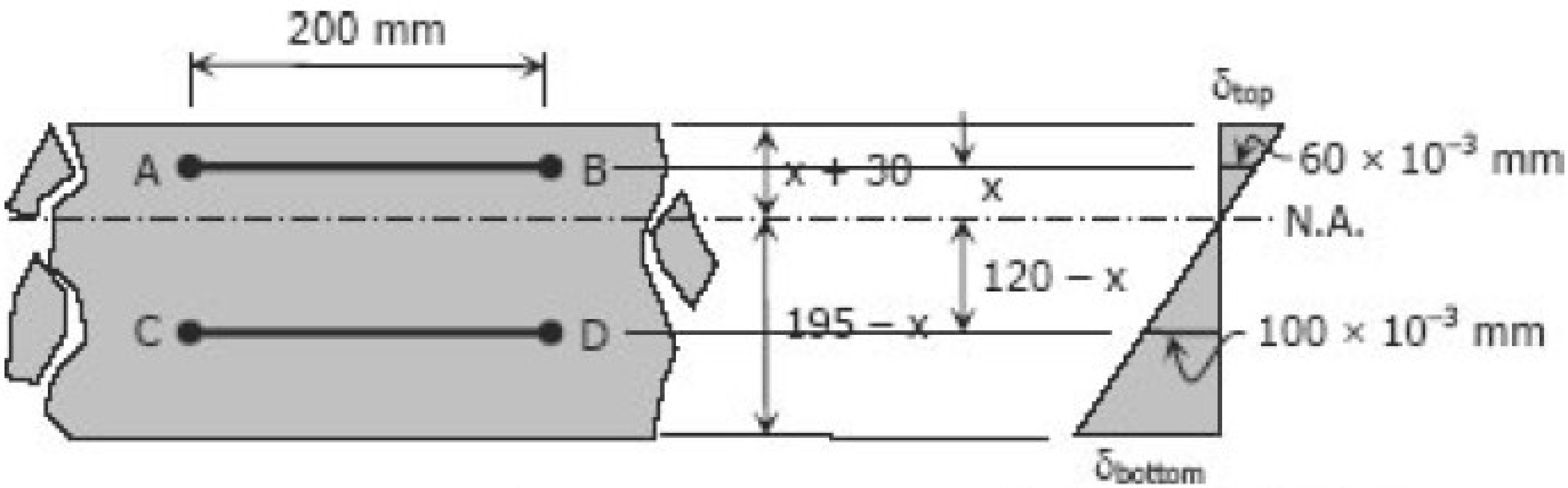
$$1.6x = 72$$

$$x = 45 \text{ mm}$$

$$\frac{\delta_{top}}{x+30} = \frac{60\times10^{-3}}{x}$$

$$\delta_{top} = \frac{60\times10^{-3}}{45}(45+30)$$

$$\delta_{top} = 0.1 \text{ mm lengthening}$$



$$\frac{\delta_{bottom}}{195-x} = \frac{100\times10^{-3}}{120-x}$$

$$\delta_{bottom} = \frac{100\times10^{-3}}{120-45}(195-45)$$

δ_{bottom} = 0.2 mm shortening

From Hooke's Law

$$f_b = E\varepsilon$$

$$f_b = \frac{E\delta}{L}$$

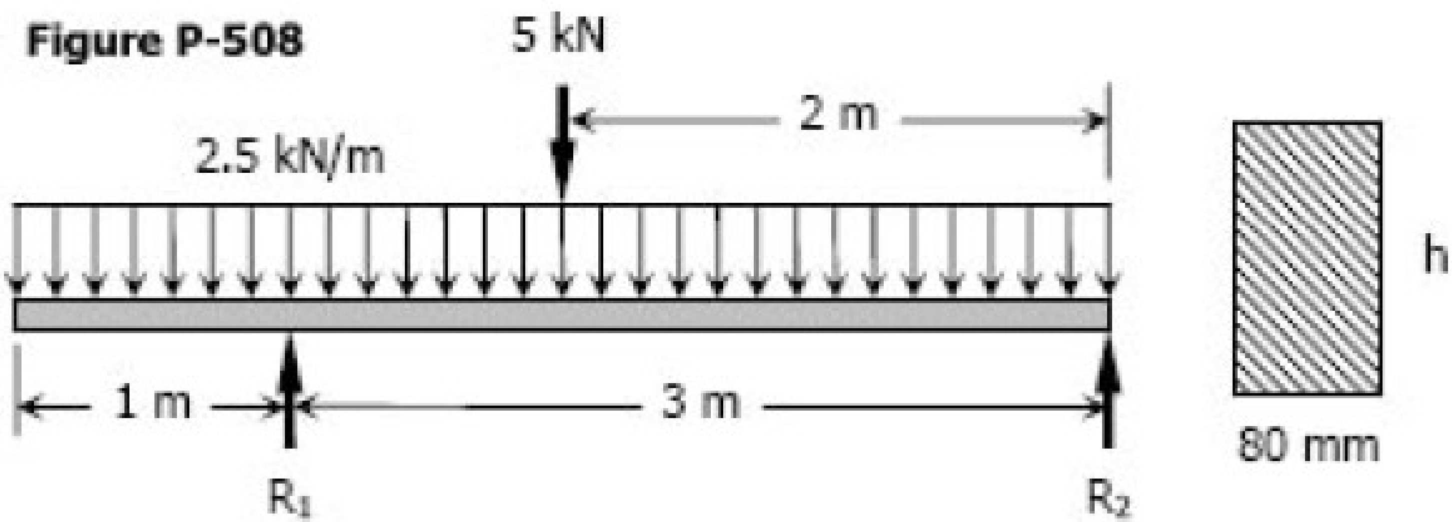
$$(f_b)_{top} = \frac{70000(0.1)}{200}$$

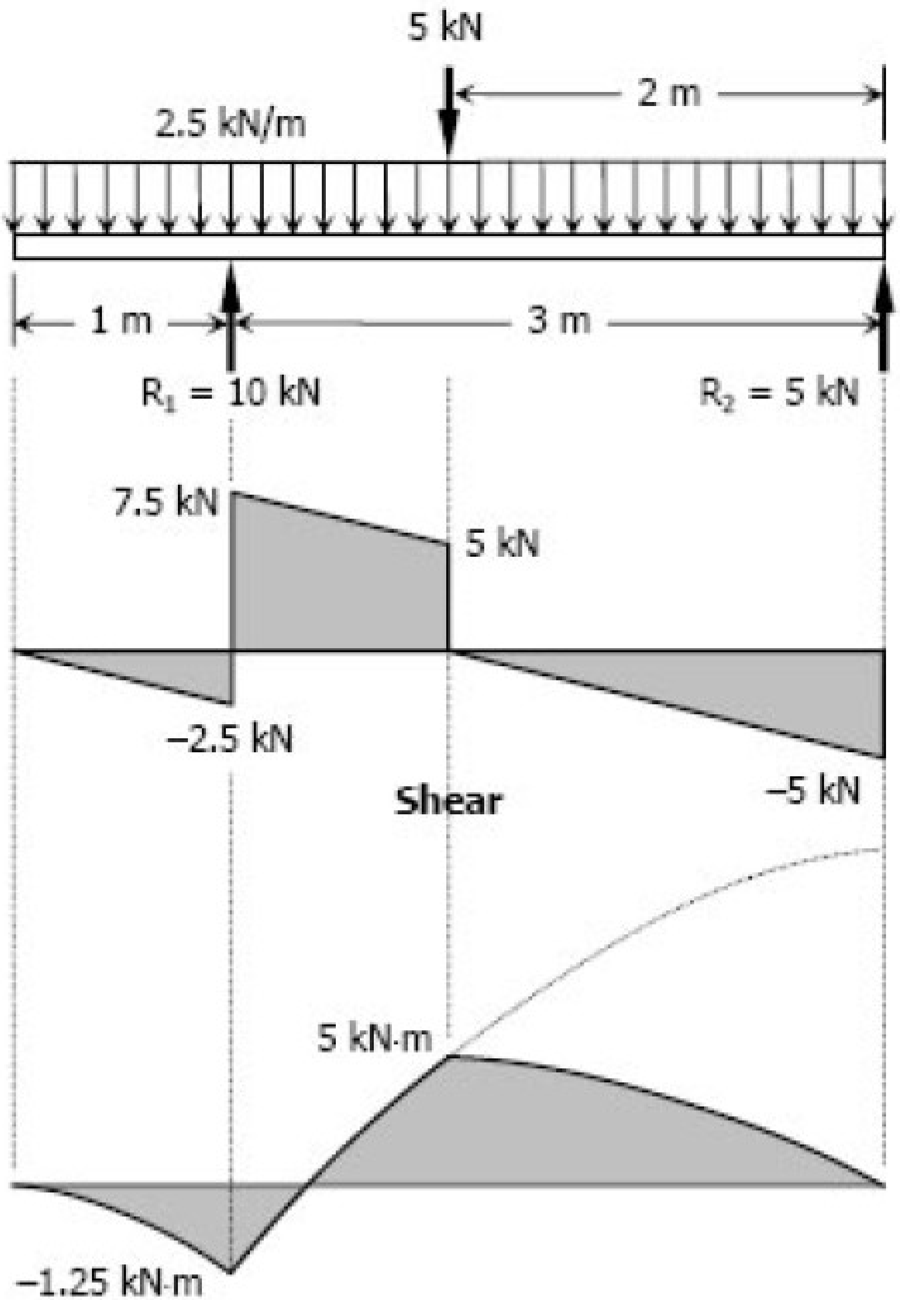
= 35 MPa tension

$$(f_b)_{bottom} = \frac{70000(0.2)}{200}$$

= 70 MPa compression

Determine the minimum height h of the beam shown in Fig. if the flexural stress is not to exceed 20 MPa.





$$\Sigma M_{R2} = 0$$
$$3R_1 = 2(5) + 2(2.5)(4)$$
$$R_1 = 10 \text{ kN}$$

$$\Sigma M_{R1} = 0$$
$$3R_2 = 1(5) + 1(2.5)(4)$$
$$R_2 = 5 \text{ kN}$$

$$I = \frac{bh^3}{12} = \frac{80h^3}{12}$$
$$= \tfrac{20}{3} h^3$$

$$c = \tfrac{1}{2} h$$

$$f_b = \frac{Mc}{I}$$

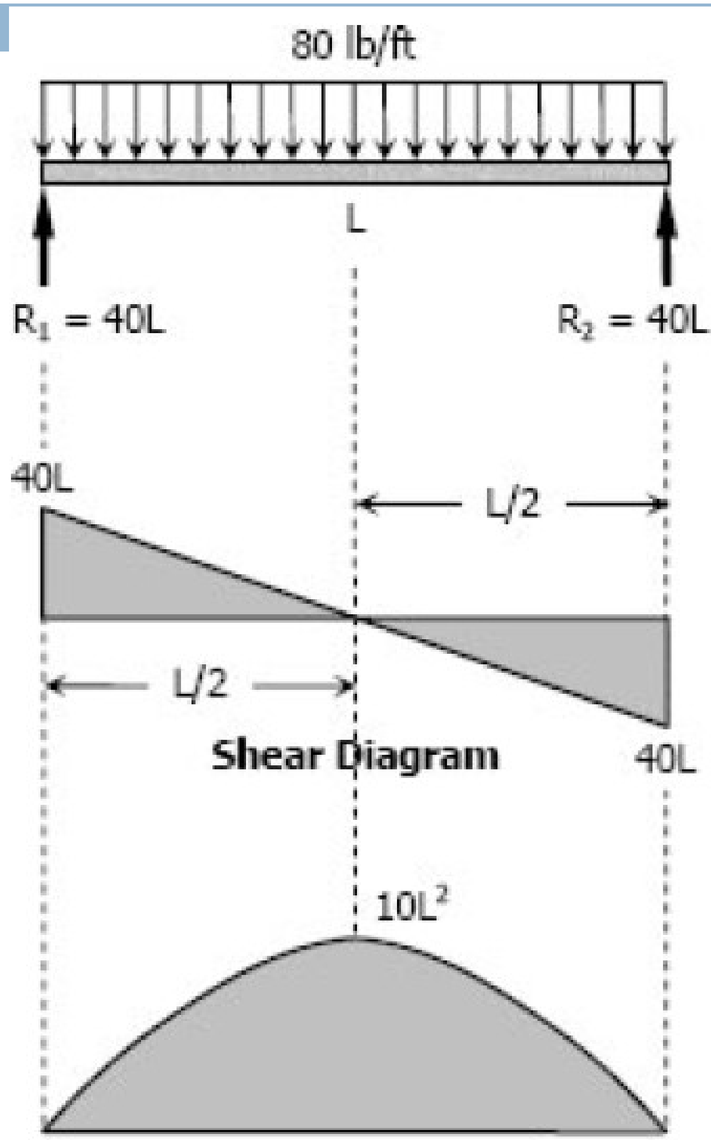
$$M = 5 \text{ kN·m}$$
$$= 5(1000)^2 \text{ N·mm}$$

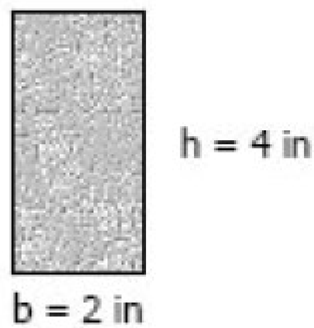
$$f_b = 20 \text{ MPa}$$

$$20 = \frac{5(1000)^2 (\frac{1}{2}h)}{\frac{20}{3}h^3}$$

$$h^2 = 18\,750$$

$$h = 137 \text{ mm}$$





$$I = \frac{bh^3}{12} = \frac{2(4)^3}{12}$$

$$= \tfrac{32}{3}\ \text{in}^4$$

$$c = h / 2 = 2\ \text{in}$$

$$(f_b)_{max} = \frac{Mc}{I}$$

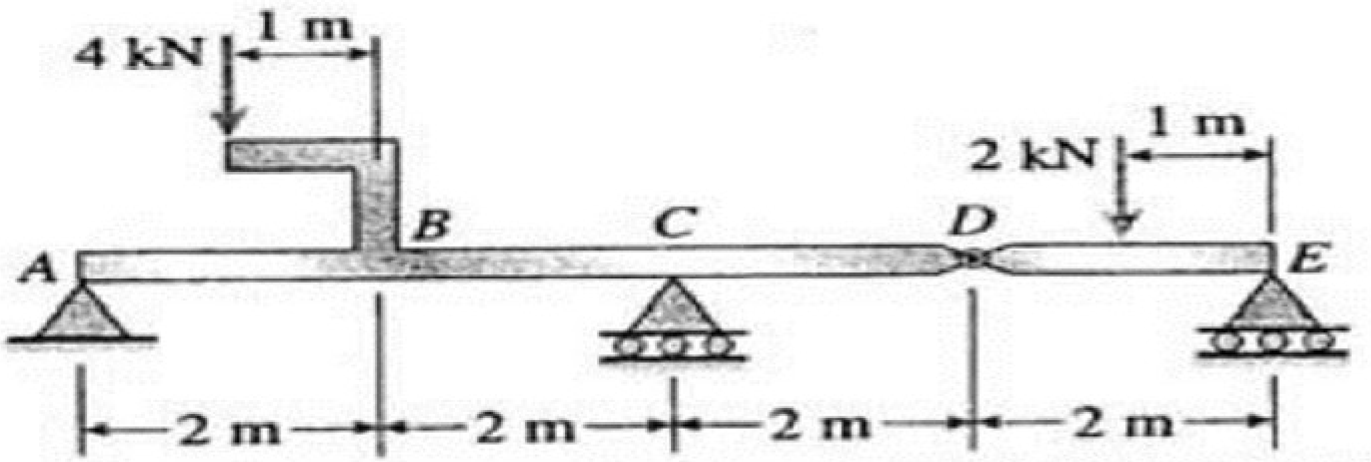
$$(f_b)_{max} = 3000 \text{ psi}$$

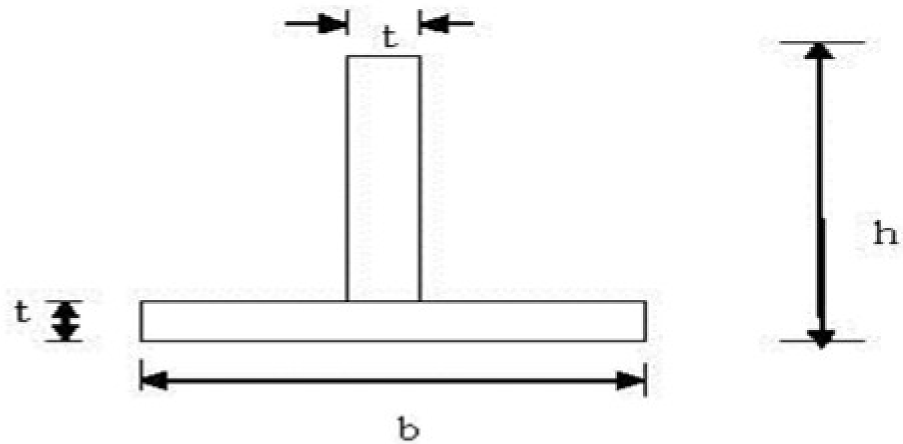
$$M = 10L^2 \text{ lb-ft}$$

$$3000 = \frac{10L^2(12)(2)}{32/3}$$

$$L = 11.55 \text{ ft}$$

Compute the Maximum tensile and Compressive Stresses in the beam

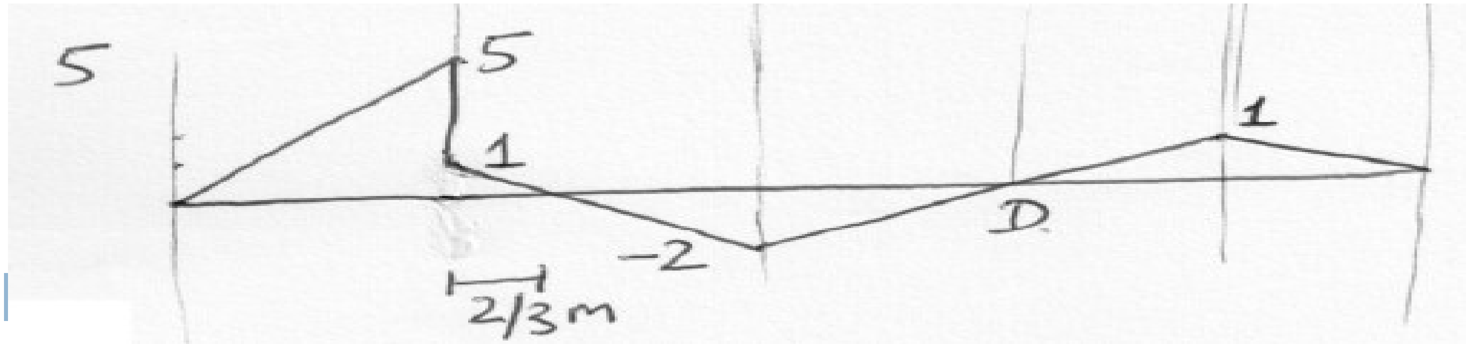


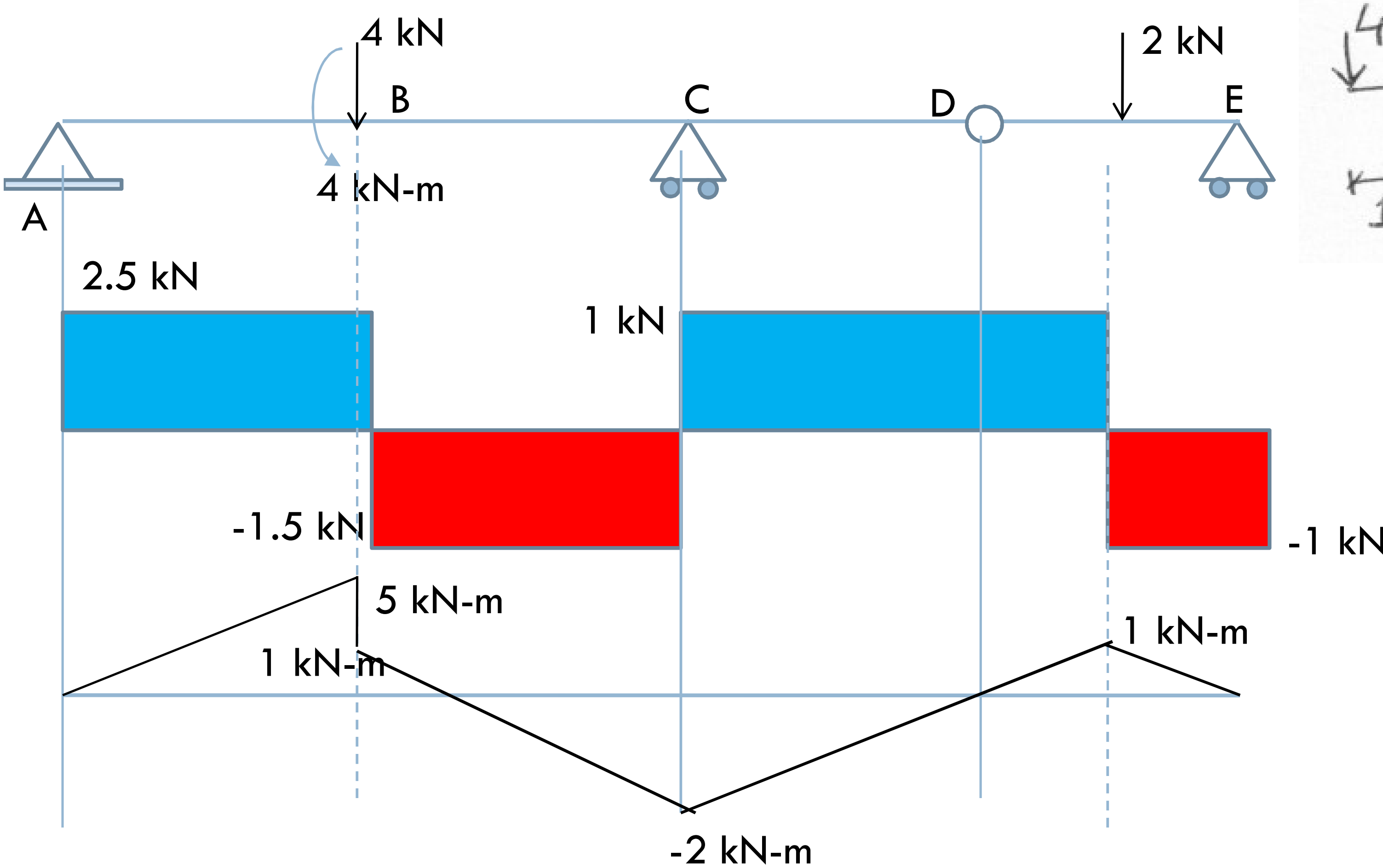


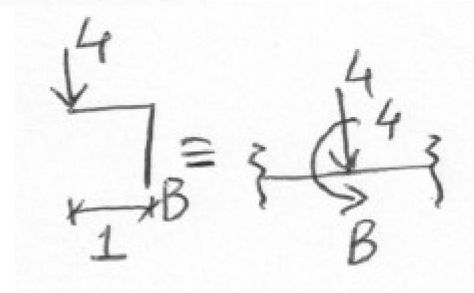
b =60 mm

h = 75 mm

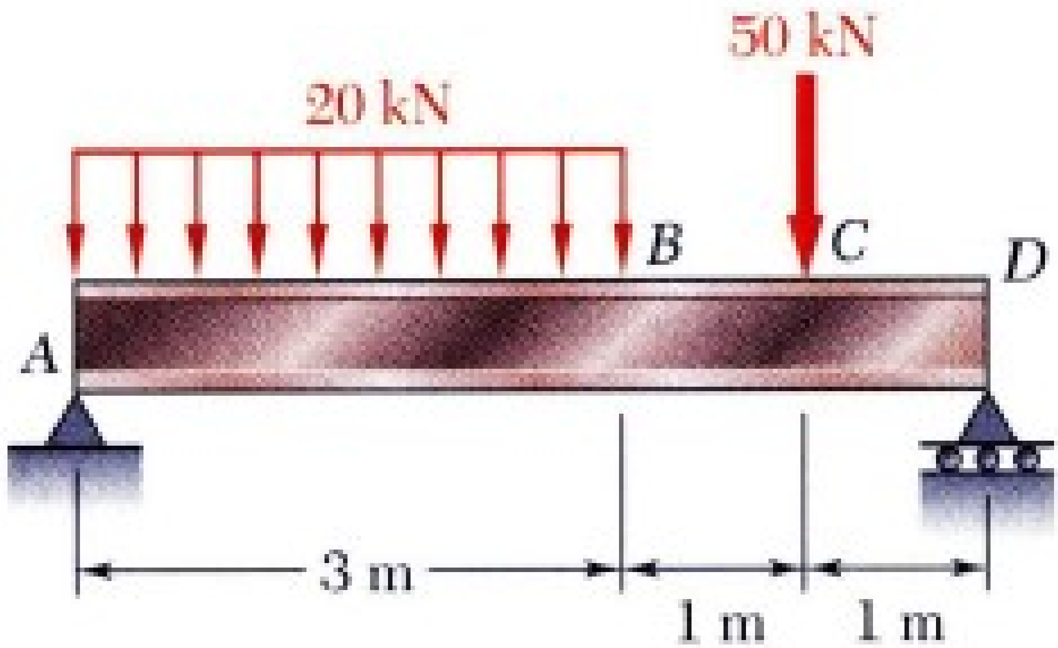
t = 10 mm

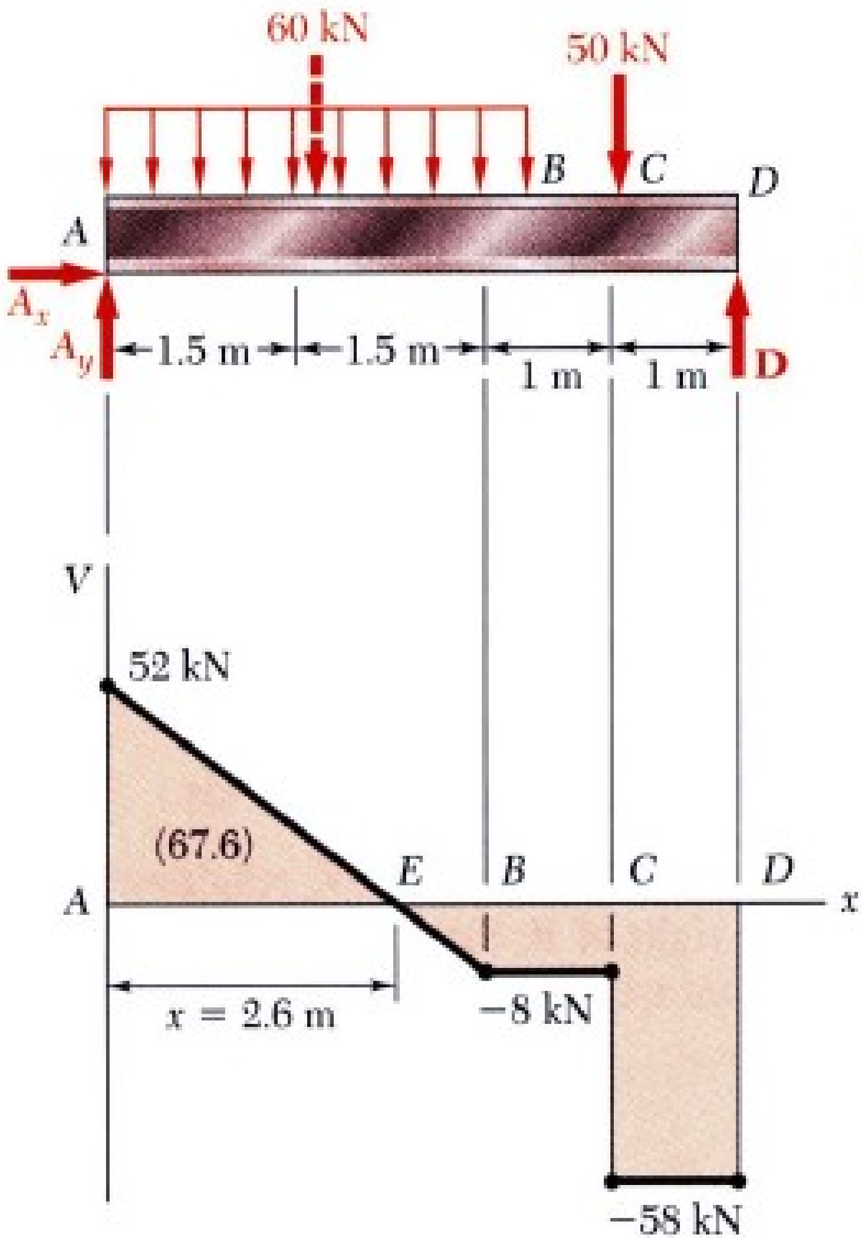

5
5
1
1
D
-2
2/3m


4 kN
2 kN
B
C
D
E
A
4 kN-m
2.5 kN
1 kN
-1.5 kN
-1 kN
5 kN-m
1 kN-m
1 kN-m
-2 kN-m


4
4
4
B
1
B

- Simply supported steel beam is to carry loads as shown. Allowable normal stress for steel used is 160 Mpa. Select the wide-flange shape that should be used.





$$\sum M_A = 0 = D(5\,\mathrm{m}) - (60\,\mathrm{kN})(1.5\,\mathrm{m}) - (50\,\mathrm{kN})(4\,\mathrm{m})$$

$$D = 58.0\,\mathrm{kN}$$

$$\sum F_y = 0 = A_y + 58.0\,\mathrm{kN} - 60\,\mathrm{kN} - 50\,\mathrm{kN}$$

$$A_y = 52.0\,\mathrm{kN}$$

Maximum bending moment occurs at *V = 0 or x = 2.6 m.*

M_{max} = Area under shear curve A to E = 0.5*52*2.6 = 67.6 kN-m.

Determine minimum acceptable section

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{67.6\,\text{kN}\cdot\text{m}}{160\,\text{MPa}}$$

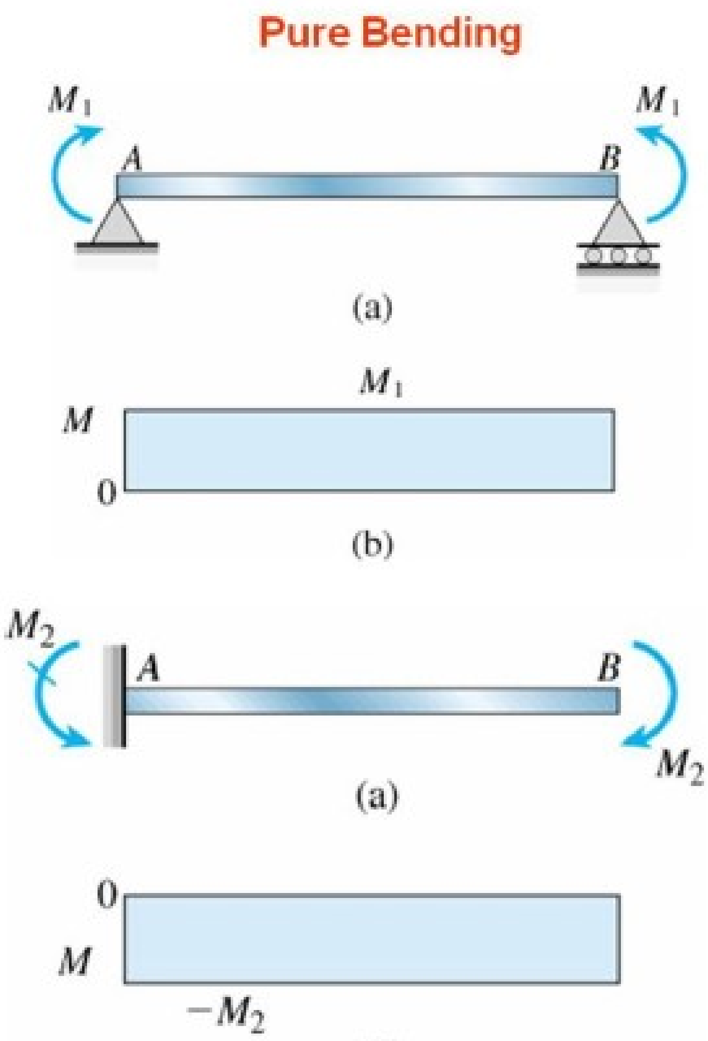
$$= 422.5\times10^{-6}\,\text{m}^3 = 422.5\times10^{3}\,\text{mm}^3$$

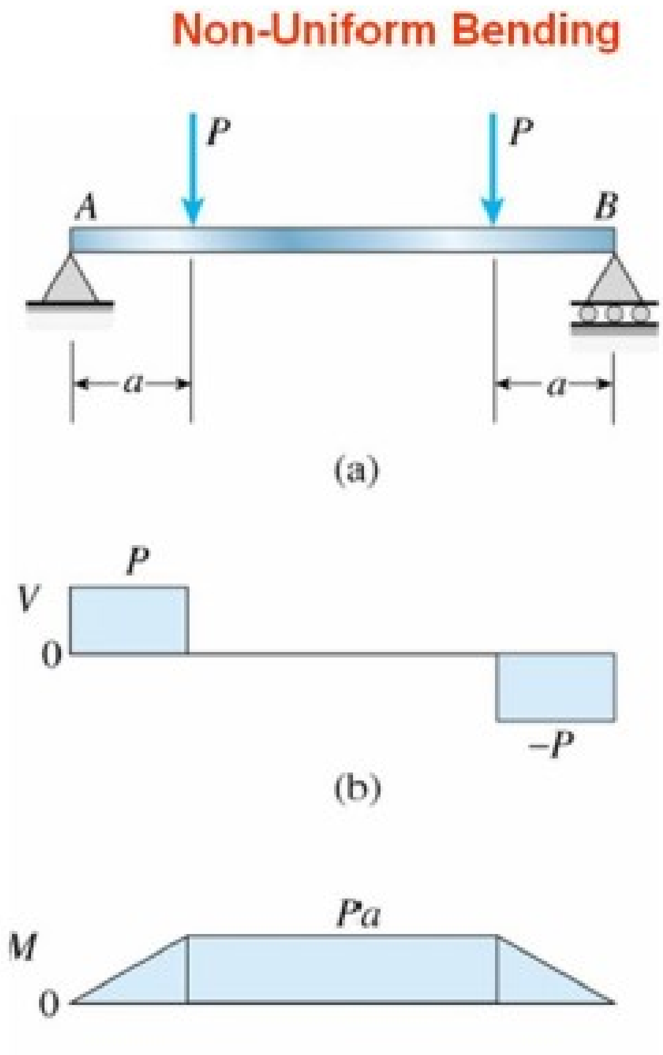
Choose best standard section which meets this criteria.

Shape	$S\times10^3$, A
W410×38.8	629, 4950
W360×32.9	475, 4190
W310×38.7	547, 4940
W250×44.8	531, 5700
W200×46.1	451, 5880

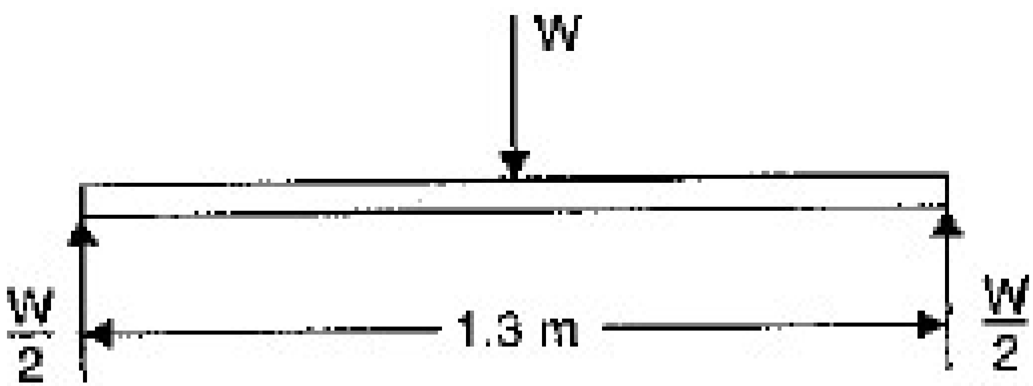
$W360\times32.9$

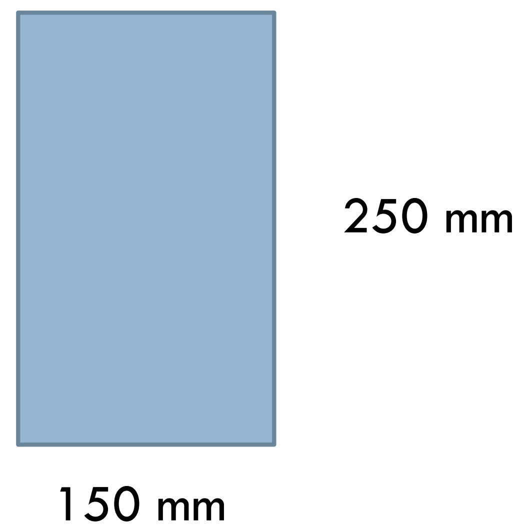
Pure Bending





- A simply supported wooden beam 150 x 250 mm is carrying a concentrated load W at the center. Determine the the safe load W if the permissible stress in bending is 7 Mpa and in shearing 1 Mpa.





$M_{max} = WL/4$ $V_{max} = W/2$

- M_{max} = W*1.3*1000/4 = 325 W N-mm
- V_{max} = W/2 N
- I = bh3/12 = 195312500 mm^4
- Q = $bh^2/4$
- Bending stress = 7 Mpa
- Shearing stress = 1 Mpa

σ = My/I

7 = 325W*125/ 195312500

W = 33653.8 N

$$\tau = \frac{VQ}{Ib} = \quad W = 50{,}000 \text{ N}$$

Safe load is the minimum of two = 33,653.8 N