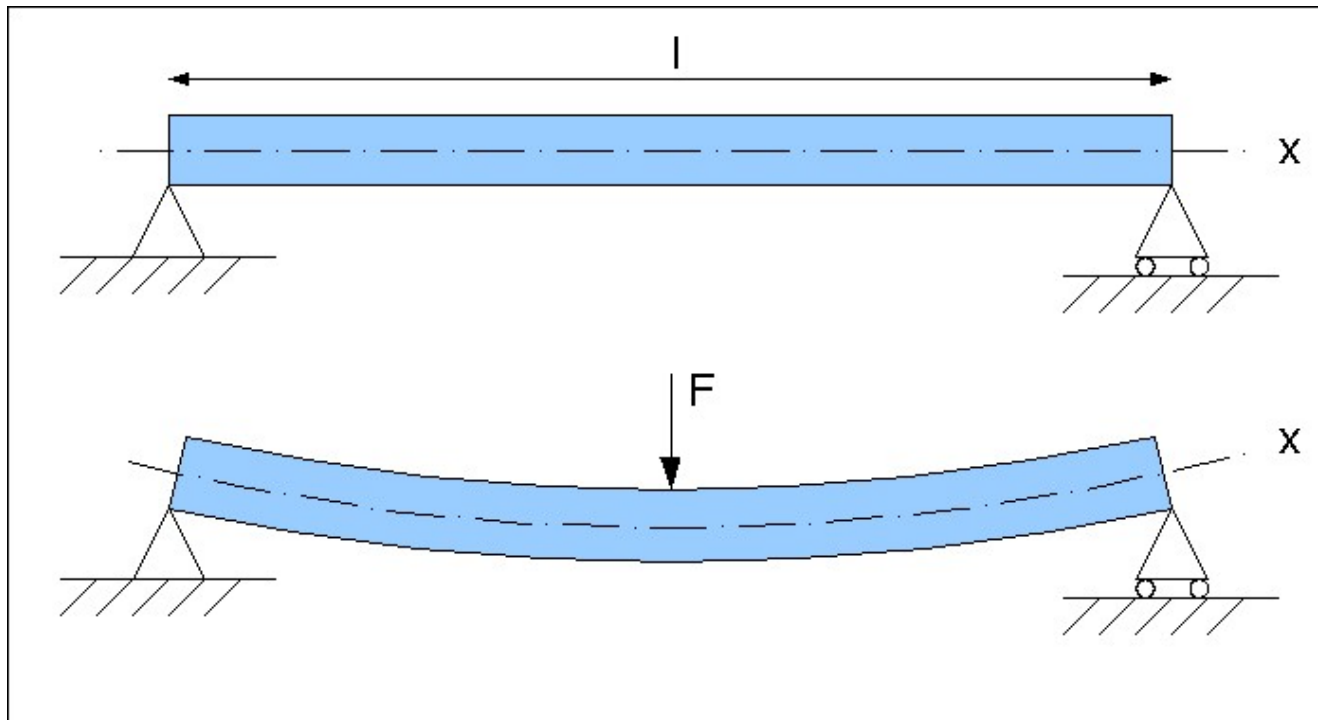


MECHANICS OF MATERIALS-I
CE 2111
BENDING STRESSES IN BEAMS

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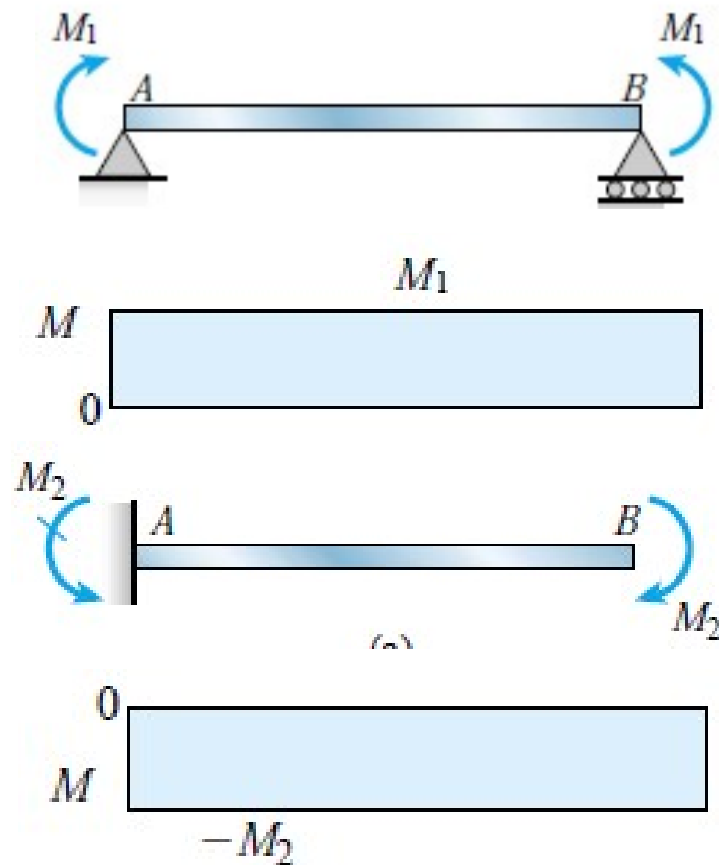
Bending of Beam

- When a 'beam' experiences a bending moment it will change its shape and internal forces (stresses) will be developed.



Pure bending refers to flexure of a beam under a constant bending moment.

Pure bending occurs only in regions of a beam where the shear force is zero



Assumptions



- The material of the beam is homogeneous and isotropic.
- The value of the Young's modulus of elasticity is the same in tension and compression.
- The transverse section which were plane before bending remain plane after bending also.
- The beam is initially straight and all longitudinal filaments bend into circular arcs with a common center of curvature.
- The radius of curvature is so large compared with dimensions of the beam cross section.
- Each layer of the beam is free to expand or contract independently of the layer, above or below it.

Theory of Simple Bending

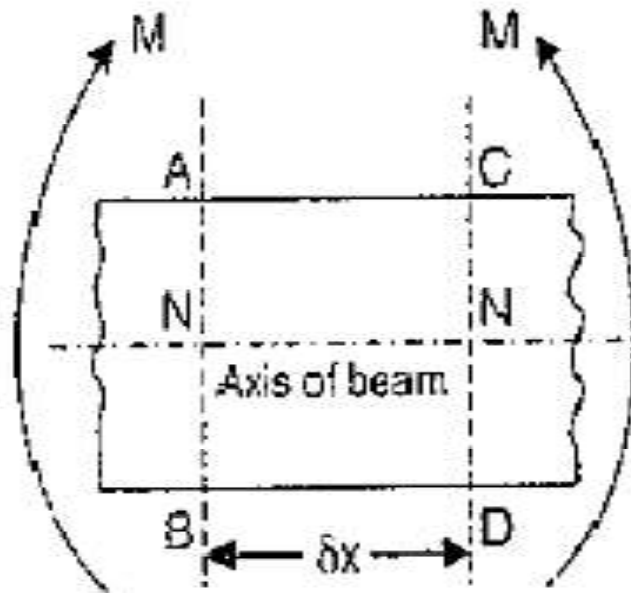


Figure 1

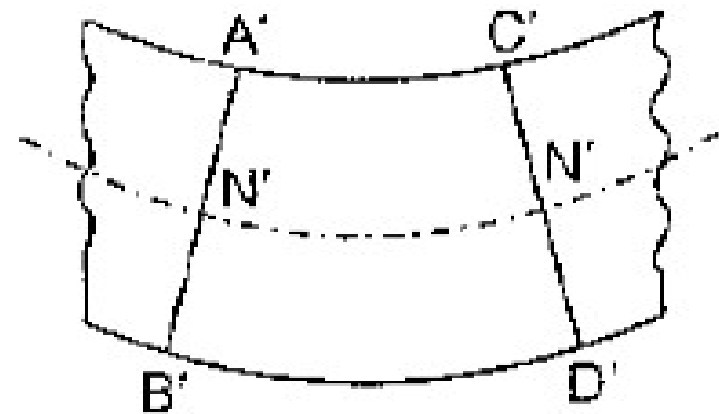


Figure 2

Theory of Simple Bending

- Consider a part of beam under simple bending.
- Consider a small length of the beam. Assume two sections AB and CD which are normal to the axis of the beam N-N.
- Due to the action of the bending moment the part of length δx will be deformed as shown in Figure 2.
- From this figure it is clear that all the layers of the beam, which were originally of the same length, do not remain of the same length any more.
- From Figure 2 it is clear that some layers have been shortened while some of them are elongated.
- There is also a layer which is neither shortened nor elongated. This layer is called neutral layer or neutral surface.

Theory of Simple Bending

- It can also be seen that the top layer has been shortened maximum and the decrease in length of the layer decreases if we proceed towards the neutral surface N-N.
- This means that the compressive stress will be maximum at the top layer.
- Similarly the increase in length will be maximum at the bottom layer.
- Hence the amount of by which a layer increases or decreases in length depends upon the position of the layer with respect to N-N.
- This theory is known as theory of simple bending.

Expression for Bending Stress

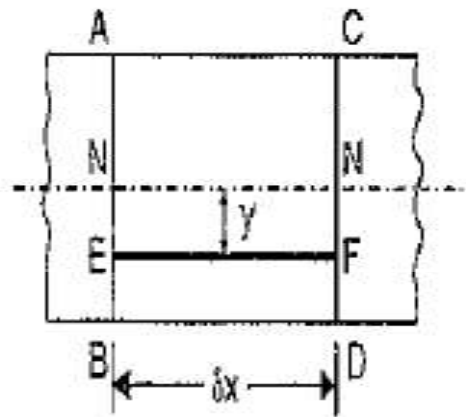


Fig. 3

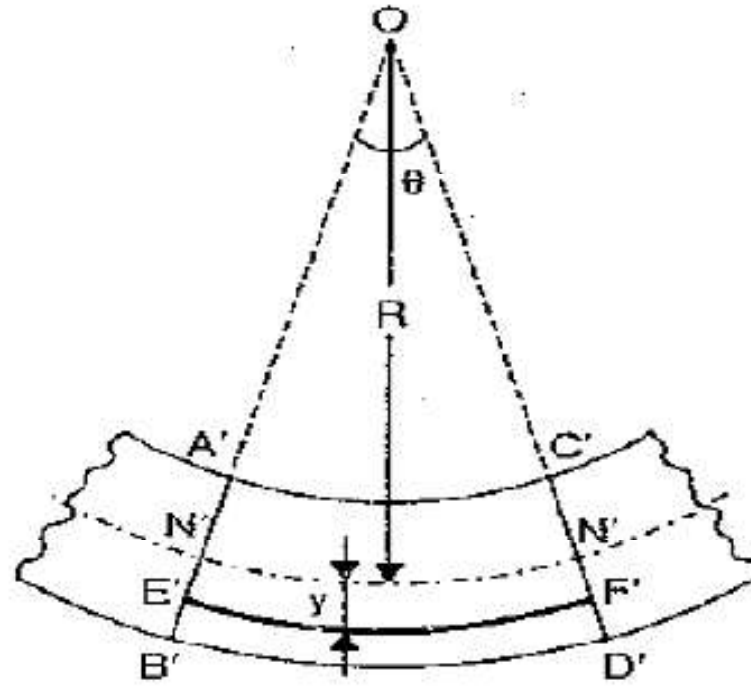


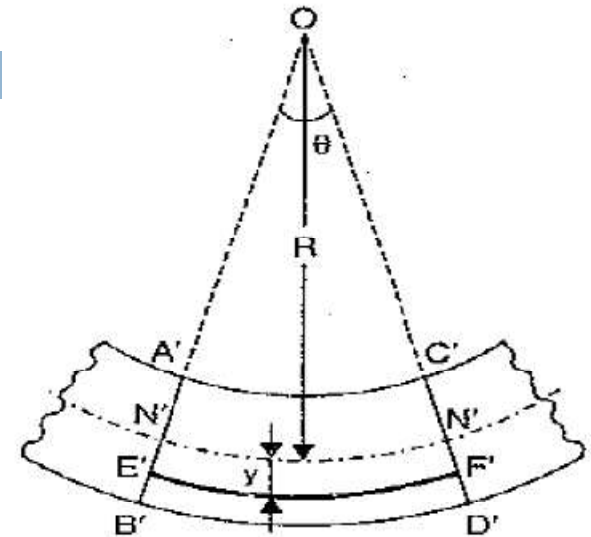
Fig. 4

Expression for Bending Stress

- Consider a small length δx of a beam subjected to simple bending.
- Due to the action of bending, the part of length δx will be deformed as shown in Figure 4.
- Let, A/B and C/D meet at O .
- R = radius of neutral layer N/N
- θ = angle subtended at O by A/B and C/D .
- Consider a layer EF at a distance y below the neutral layer NN . After bending this layer will be elongated to E'/F' .
- Original length of layer $EF = \delta x$
- Length of neutral layer $NN = \delta x$.
- After bending length of neutral layer remains the same.

Expression for Bending Stress

- Therefore, $NN = N'/N' = \delta x$.
- Now from Figure 4, $N'/N' = R * \theta$
- and $E'/F' = (R+y) * \theta$
- but $NN = N'/N' = \delta x$
- Hence, $\delta x = R * \theta$
- Increase in the length of the layer $EF = E'/F' - EF$
- $= (R+y) * \theta - R * \theta = y * \theta$



$$\text{Strain in the layer } EF = \frac{\text{Increase in length}}{\text{Original length}} = \frac{y\theta}{R\theta} = \frac{y}{R}$$

- Since R is constant, strain in a layer is proportional to the distance from the neutral axis.

Expression for Bending Stress

□ Stress Variation

□ σ = Stress in the layer

□ E = Modulus of elasticity

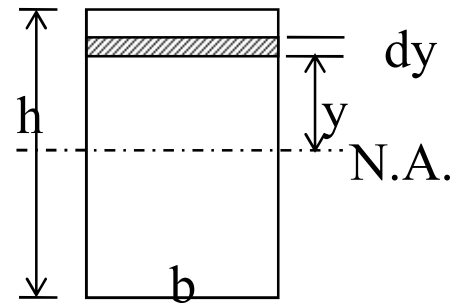
□ Then, stress = Strain x modulus of elasticity

□
$$\sigma = \frac{E}{R} y \quad \therefore \frac{\sigma}{y} = \frac{E}{R}$$

□ Since E and R are constants, therefore stress in any layer is directly proportional to the distance of the layer from the neutral surface.

Expression for Bending Stress

□ Neutral Axis and Moment of Resistance



□ Figure shows the cross section of a beam. Consider a small layer of area dA at a distance y from the neutral axis.

□ Now force on the layer = stress in the layer * area of the layer =

$$\sigma * dA = \frac{E}{R} y * dA$$

□ Total force on the beam section = $\int \frac{E}{R} y * dA = \frac{E}{R} \int y * dA$

Expression for Bending Stress

□ **Moment of Resistance**

- Due to pure bending layers above the N.A. are subjected to compressive stress whereas the layers below the N.A. are subjected to tensile stresses.
- Due to these stresses, the forces will be acting on the layers .
- **These forces will have a moment about the neutral axis. The total moment of these forces about the N.A. for a section is known as the moment of resistance of that section.**
- Moment of the force of the layer about N.A.

$$\square = \frac{E}{R} y * dA * y = \frac{E}{R} y^2 dA$$

Expression for Bending Stress

- Total moment of the forces on the section of the beam =

$$\int \frac{E}{R} y^2 dA$$

- Let M = external moment applied on the beam.
- For equilibrium, external moment = internal moment

$$M = \frac{E}{R} \int y^2 dA$$

- The moment of inertia of the area $I = \int y^2 dA$

- $\therefore M = \frac{EI}{R}$ or $\frac{M}{I} = \frac{E}{R}$

Expression for Bending Stress

$$\therefore \frac{M}{I} = \frac{\sigma}{y} \quad \text{since} \left[\frac{\sigma}{y} = \frac{E}{R} \right]$$

$$\text{Bending stress } \sigma = \frac{My}{I}$$

Important Points of the Flexure Formula

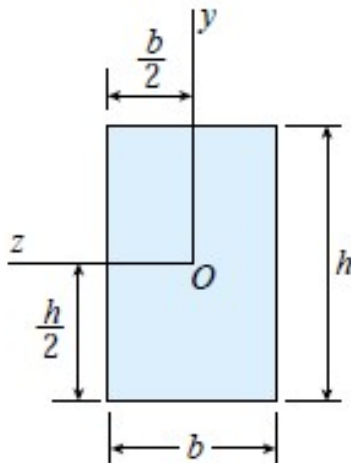
- The cross section of a straight beam *remains plane* when the beam deforms due to bending. This causes tensile stress on one portion of the cross section and compressive stress on the other portion. In between these portions, there exists the *neutral axis* which is subjected to *zero stress*.
- Due to the deformation, the *longitudinal strain varies linearly* from zero at the neutral axis to a maximum at the outer fibers of the beam. Provided the material is homogeneous and linear elastic, then the *stress* also varies in a *linear* fashion over the cross section.
-

Important Points of the Flexure Formula

- The neutral axis passes through the *centroid* of the cross-sectional area. This result is based on the fact that the resultant normal force acting on the cross section must be zero.
- The flexure formula is based on the requirement that the resultant internal moment on the cross section is equal to the moment produced by the normal stress distribution about the neutral axis.
-

Section Modulus

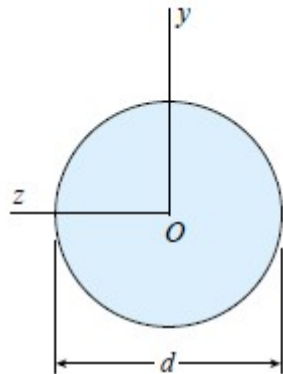
- The term I/c in expression for maximum bending stress is known as section modulus.
- The advantage of expressing the maximum stresses in terms of section moduli arises from the fact that each section modulus combines the beam's relevant cross-sectional properties into a single quantity.



$$I = bh^3/12 \quad c = h/2$$

$$\text{Section modulus, } z = I/c = bh^2/6$$

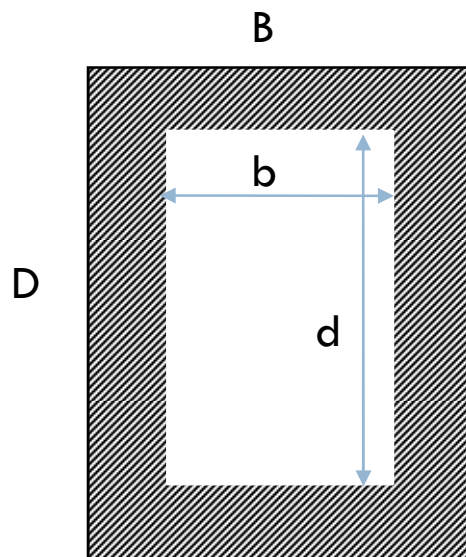
Section Modulus



$$I = \pi d^4 / 64 \quad c = d / 2$$

$$\text{Section modulus, } z = I / c = \pi d^3 / 32$$

Section Modulus



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

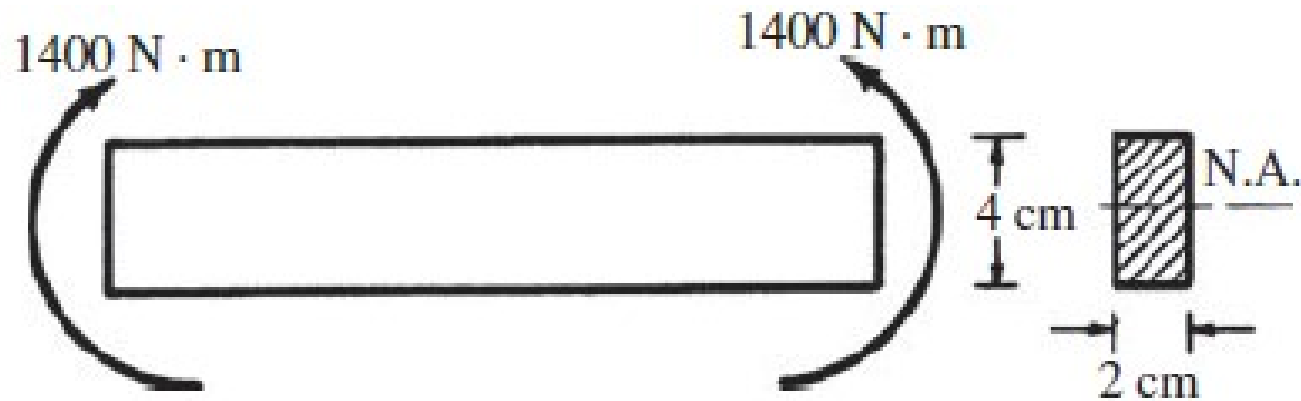
$$c = D/2$$

Section modulus $Z = I/c$

$$Z = \frac{1}{6D} (BD^3 - bd^3)$$

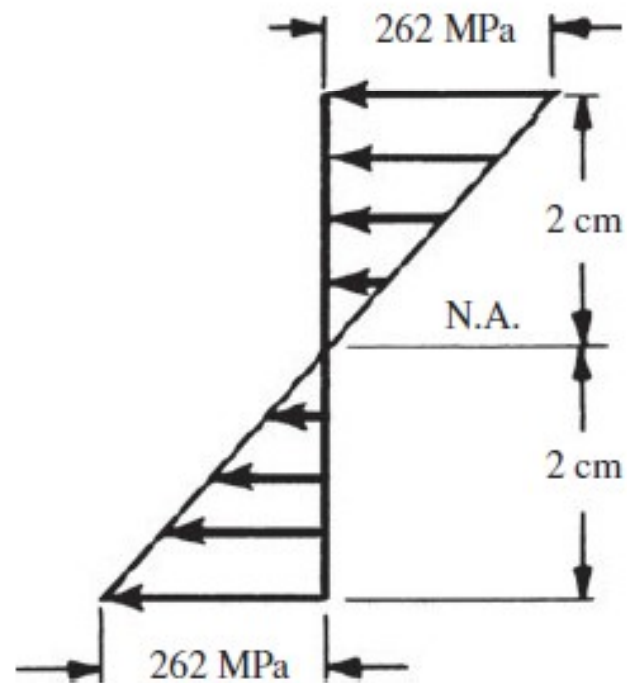
Example #1

- A beam is loaded by a couple of 1400 N-m at each of its ends, as shown in Figure. Determine the maximum bending stress in the beam and indicate the variation of bending stress over the depth of the beam.



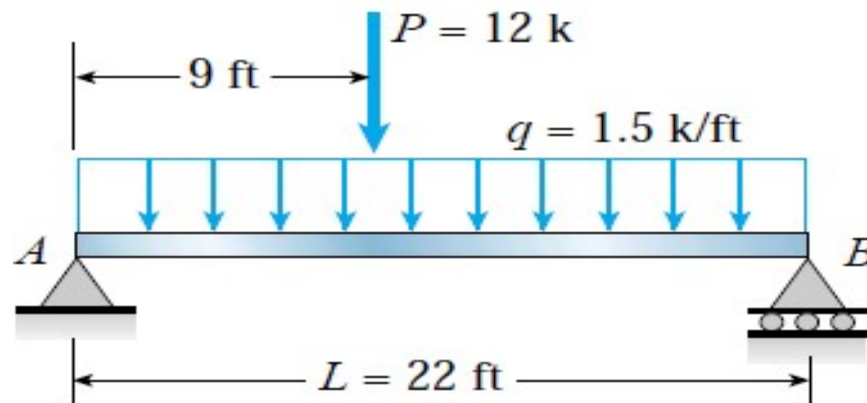
□ :

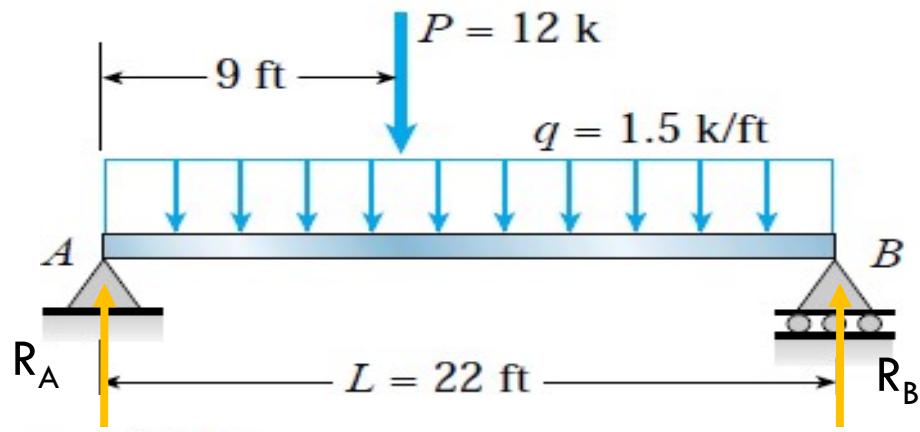
$$\sigma = \frac{Mc}{I} = \frac{1400 \times 0.02}{0.02 \times 0.04^3 / 12} = 262 \times 10^6 \text{ Pa}$$



Example #2

- A simple beam AB of span length 22 ft supports a uniform load of intensity 1.5 k/ft and a concentrated load 12 k . The concentrated load acts at a point 9.0 ft from the left-hand end of the beam. The beam has a cross section of width $b = 9.0\text{ in.}$ and height $h = 27\text{ in.}$

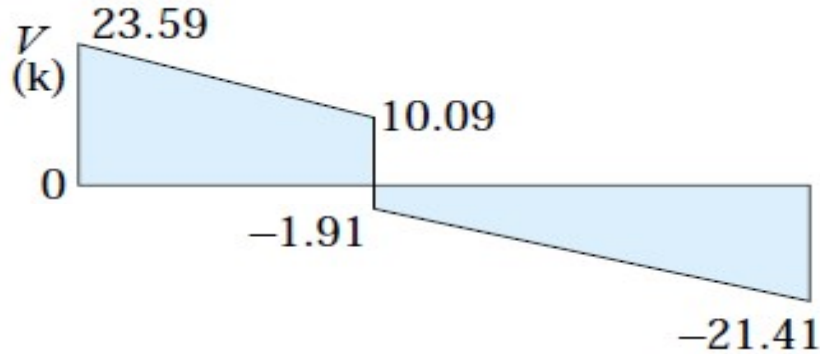




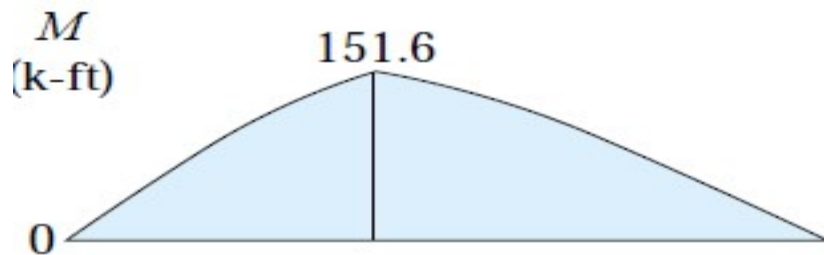
$$\sum M_A = 0$$

$$1.5 * 22 * 22 / 2 + 12 * 9 - R_B * 22 = 0$$

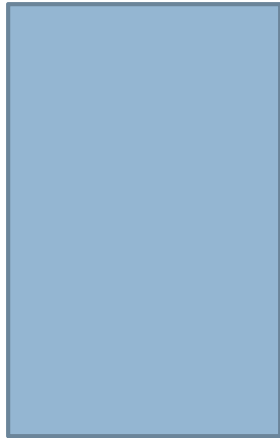
$$R_B = 21.41 \text{ k} \quad R_A = 23.59 \text{ k}$$



$$\begin{aligned} \text{Shear at point load} &= R_A - 1.5 * 9 \\ &= 23.59 - 13.5 \\ &= 10.09 \text{ k} \end{aligned}$$



$$\begin{aligned} \text{Moment at point load} &= R_A * 9 - 1.5 * 9 * 9 / 2 \\ &= 23.59 * 9 - 13.5 * 4.5 \\ &= 151.6 \text{ k-ft} \end{aligned}$$



$h = 27 \text{ in.}$

$b = 9 \text{ in.}$

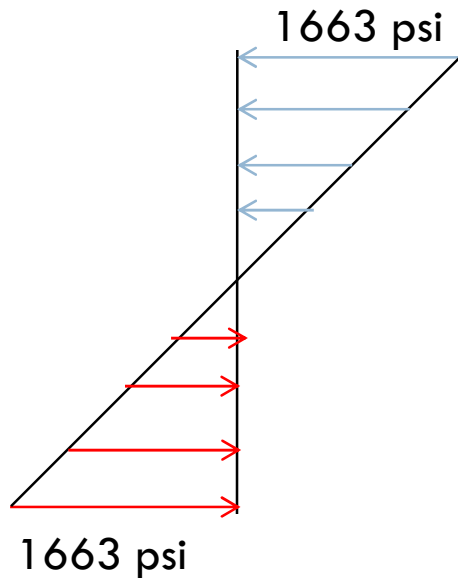
$$\begin{aligned} \text{Moment of inertia } I &= bh^3/12 \\ &= 9 \cdot (27)^3 / 12 \\ &= 14,762.25 \end{aligned}$$

in^4

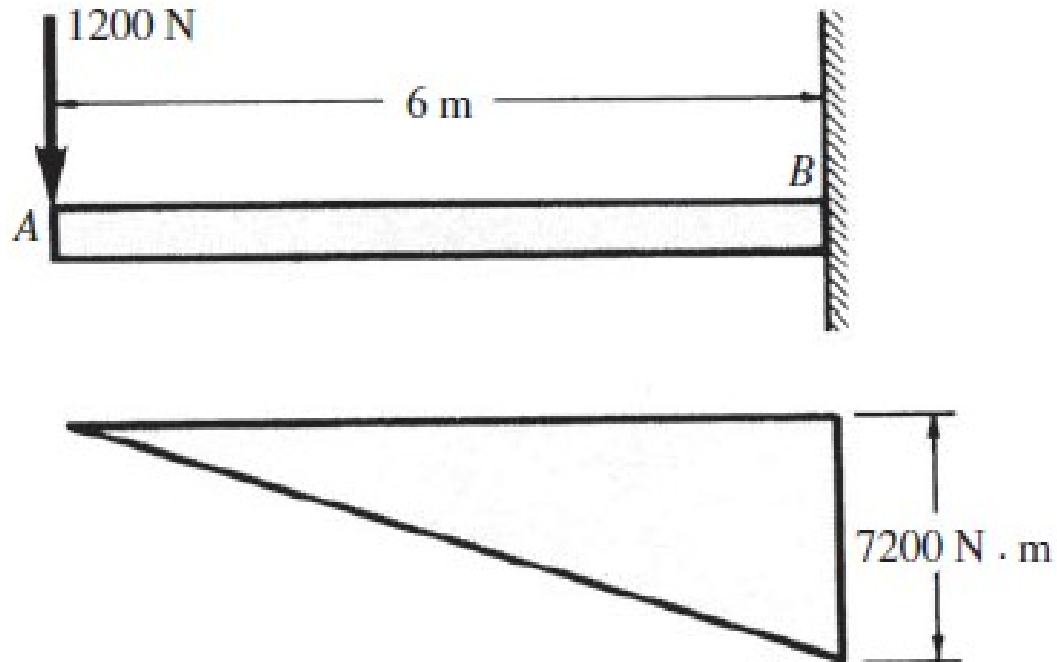
$$c = h/2 = 13.5 \text{ in}$$

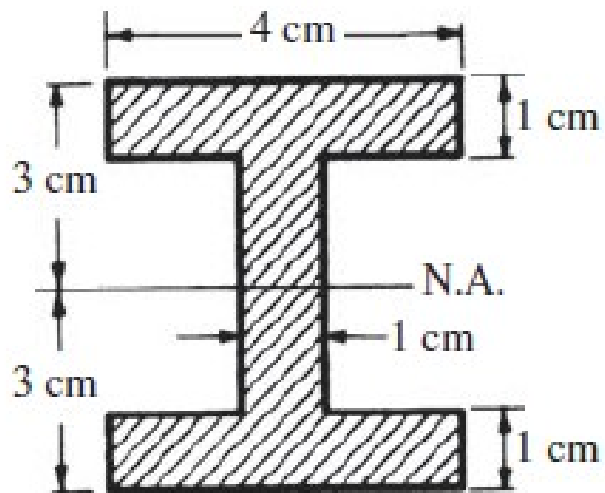
Maximum bending stress

$$\sigma = \frac{Mc}{I} = \frac{151.6 \cdot 12 \cdot 13.5}{14762.25} = 1.663 \text{ ksi}$$



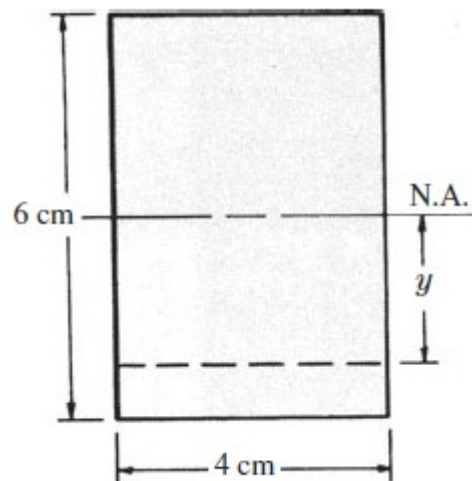
A steel cantilever beam 6 m in length is subjected to a concentrated load of 1200 N acting at the free end of the bar. Determine the magnitude and location of the maximum tensile and compressive bending stresses in the beam.





$$I = \frac{4 \times 6^3}{12} - \frac{3 \times 4^3}{12} = 56 \text{ cm}^4$$

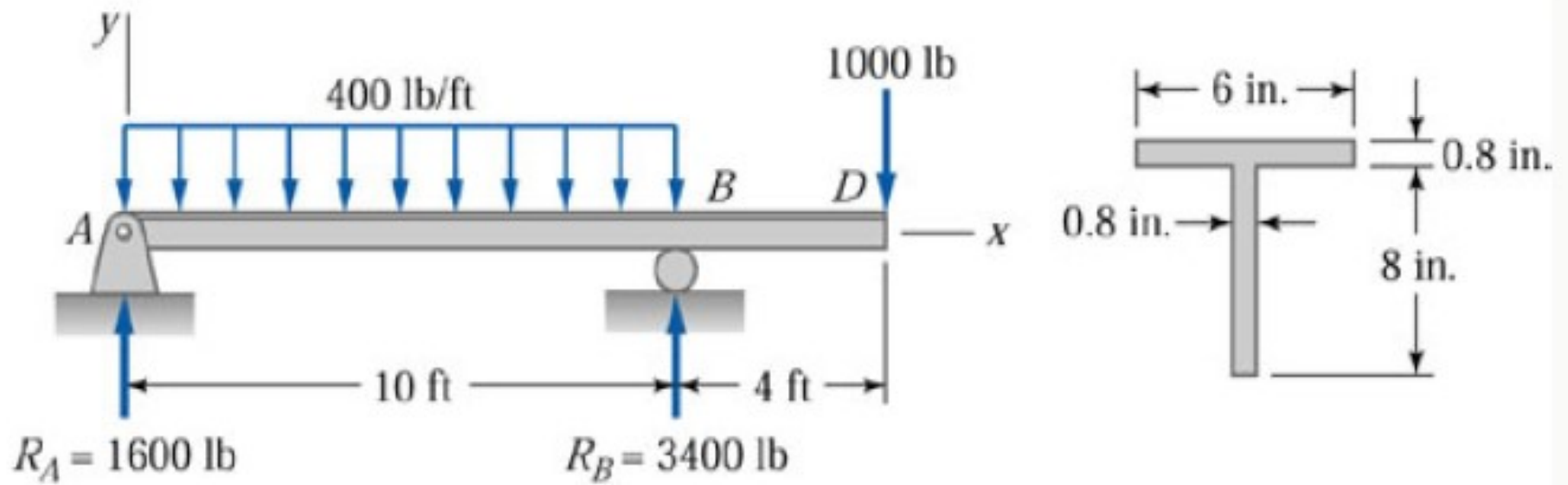
$$\sigma = \frac{Mc}{I} = \frac{7200 \times 0.03}{56 \times 10^{-8}} = 386 \times 10^6 \text{ Pa}$$

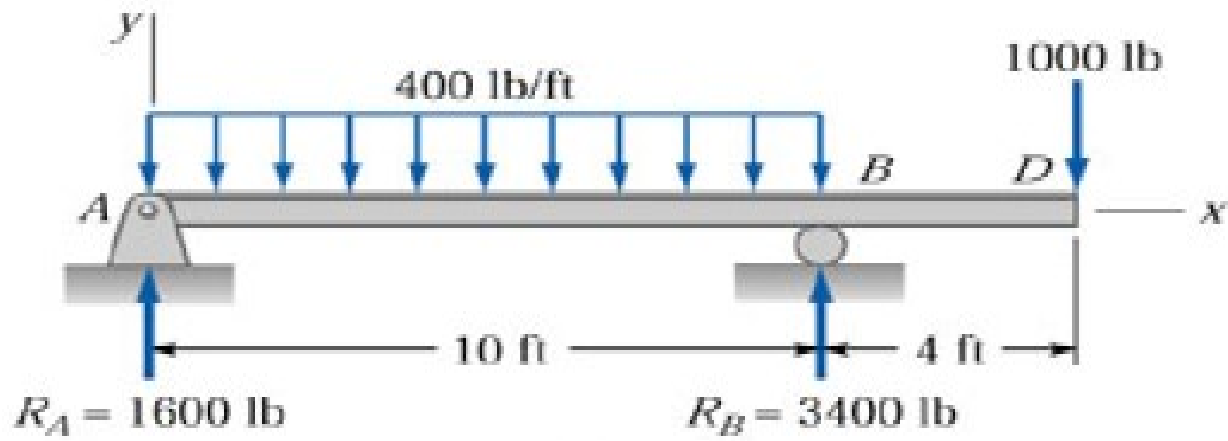


$$\sigma = \frac{My}{I} = \frac{7200 \times 0.03}{0.04 \times 0.06^3 / 12} = 300 \times 10^6 \text{ Pa}$$

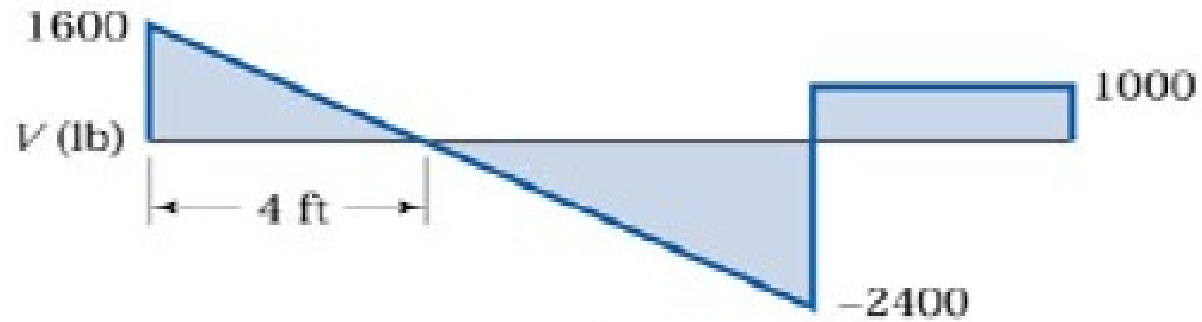
Example #3

The beam shown in Figure below has the T-shaped cross section. Determine the values and locations of the maximum tensile and compressive bending stresses.

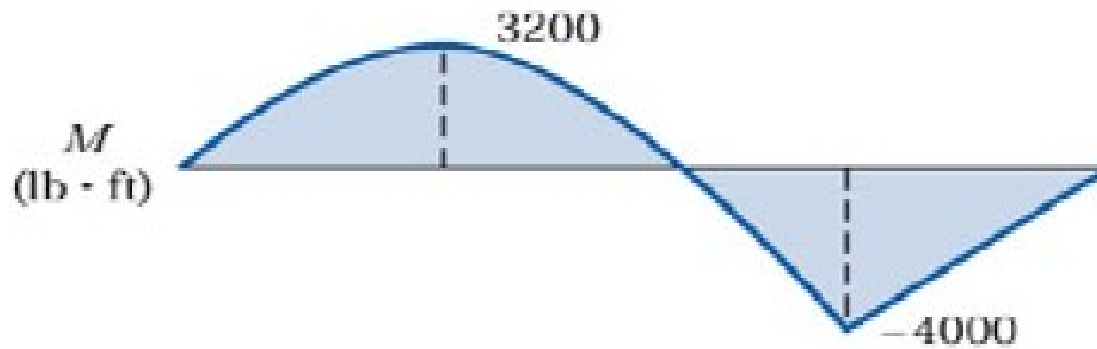




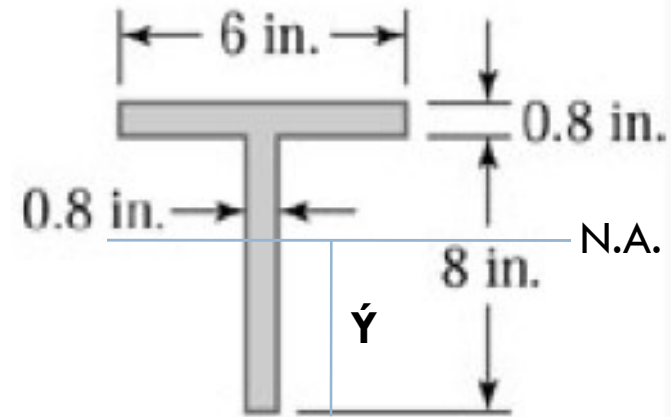
(a)



(b)



$$\bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2} = \frac{6.4(4) + 4.8(8.4)}{6.4 + 4.8} = 5.886 \text{ in.}$$



$$I = \left[\frac{0.8(8)^3}{12} + 6.4(4 - 5.886)^2 \right] + \left[\frac{6(0.8)^3}{12} + 4.8(8.4 - 5.886)^2 \right] = 87.49 \text{ in.}^4$$

$$c_{\text{top}} = 8.8 - 5.886 = 2.914 \text{ in.}$$

$$c_{\text{bottom}} = 5.886$$

Stress at $x = 4\text{ft}$.

$$\sigma_{top} = -\frac{Mc_{top}}{I} = -\frac{(3200 \times 12)(2.914)}{87.49} = -1279 \text{ psi}$$

$$\sigma_{bot} = -\frac{Mc_{bot}}{I} = \frac{(3200 \times 12)(-5.886)}{87.49} = 2580 \text{ psi}$$

Stress at $x = 10$ ft.

$$\sigma_{top} = -\frac{Mc_{top}}{I} = \frac{(-4000 \times 12)(2.914)}{87.49} = 1599 \text{ psi}$$

$$\sigma_{bot} = -\frac{Mc_{bot}}{I} = \frac{(-4000 \times 12) - (5.886)}{87.49} = -3230 \text{ psi}$$

$(\sigma_T)_{\max} = 2580 \text{ psi}$ (bottom of the section at $x = 4 \text{ ft}$)

$(\sigma_c)_{\max} = 3230 \text{ psi}$ (bottom of the section at $x = 10 \text{ ft}$)