CE 2121 (Fluid Mechanics-I)

Text Book	Writer	Link
A Textbook of Fluid Mechanics and Hydraulic Machines	Dr. R. K. Bansal	http://www.mediafire.com/file/d61ib5kkt62a1vy/Dr._R._K._Bansal.pdf/file
A Textbook of Fluid Mechanics	R. K. Rajput	Soft Copy not available
Hydraulics and Fluid Mechanics including Hydraulic Machines	Dr. P. N. Modi	http://www.mediafire.com/file/7lc8041nqobco7s/Dr._P._N._Modi.pdf/file

Syllabus related to Fluid

Semester	Courses
3rd	CE 2121 Fluid Mechanics
3rd	CE 2122 Fluid Mechanics Sessional
5th	CE 3121 Engineering Hydraulics
5th	CE 3122 Engineering Hydraulics Sessional
7th	CE 4121 Irrigation and Flood Engineering
8th	CE 4227 Hydraulic Structures
8th	CE 4220 Water Resources Engineering Sessional-I

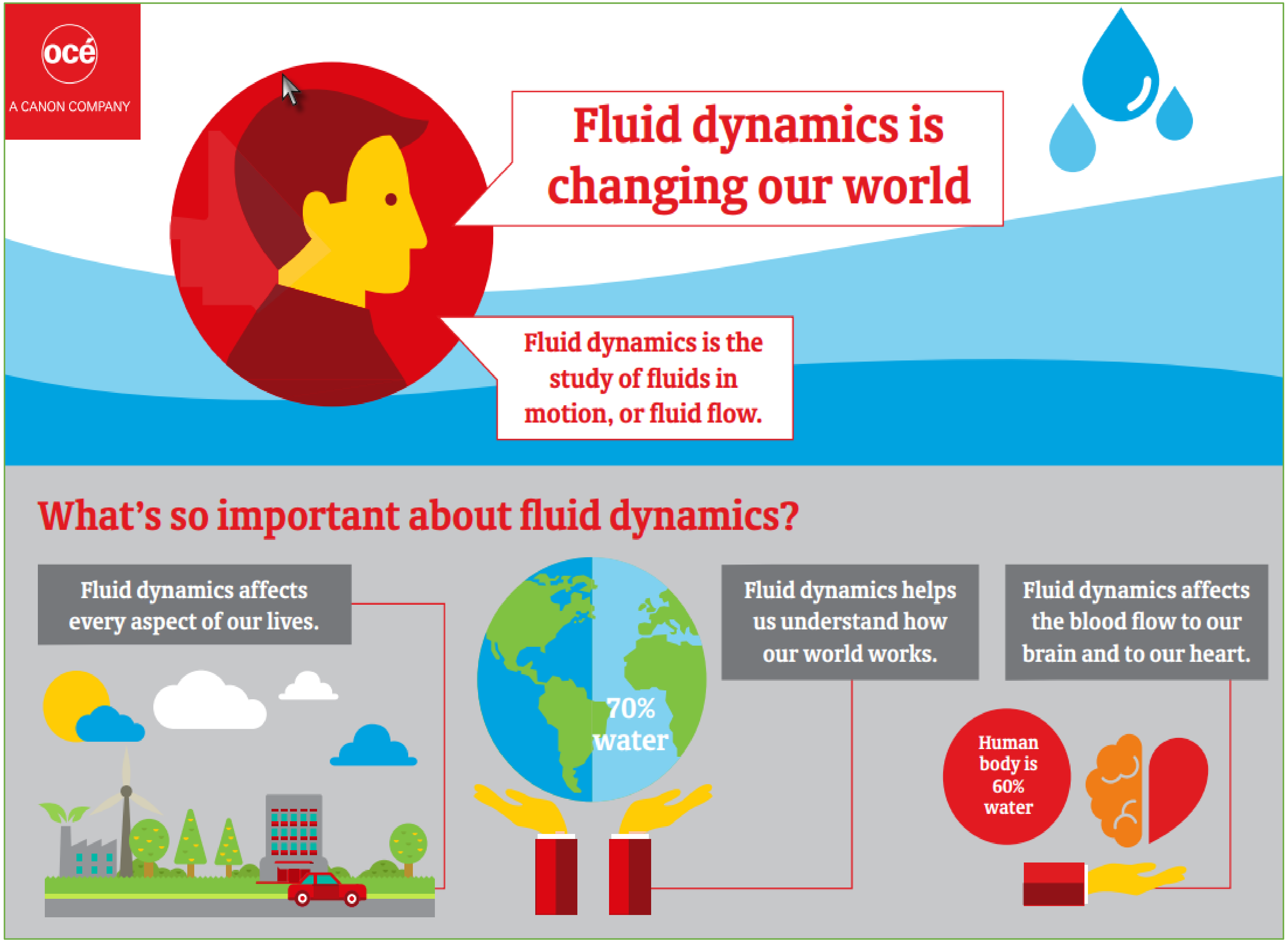
Why study of fluid mechanics is essential for civil engineering?

Organization	Full Form
BWDB	Bangladesh Water Development Board
WASA	Water Supply and Sewerage Authorities
BADC	Bangladesh Agricultural Development Corporation
BMDA	Barind Multipurpose Development Authority
IWM	Institute of Water Modeling

Course Content

1. Fluid Dynamics.
2. Fluid Measurement.
3. Flow Through Pipes.
4. Laminar and Turbulent Flow.
5. Study in Compressible flow in Pressure Conduits.
6. Similitude and Dimensional analysis.

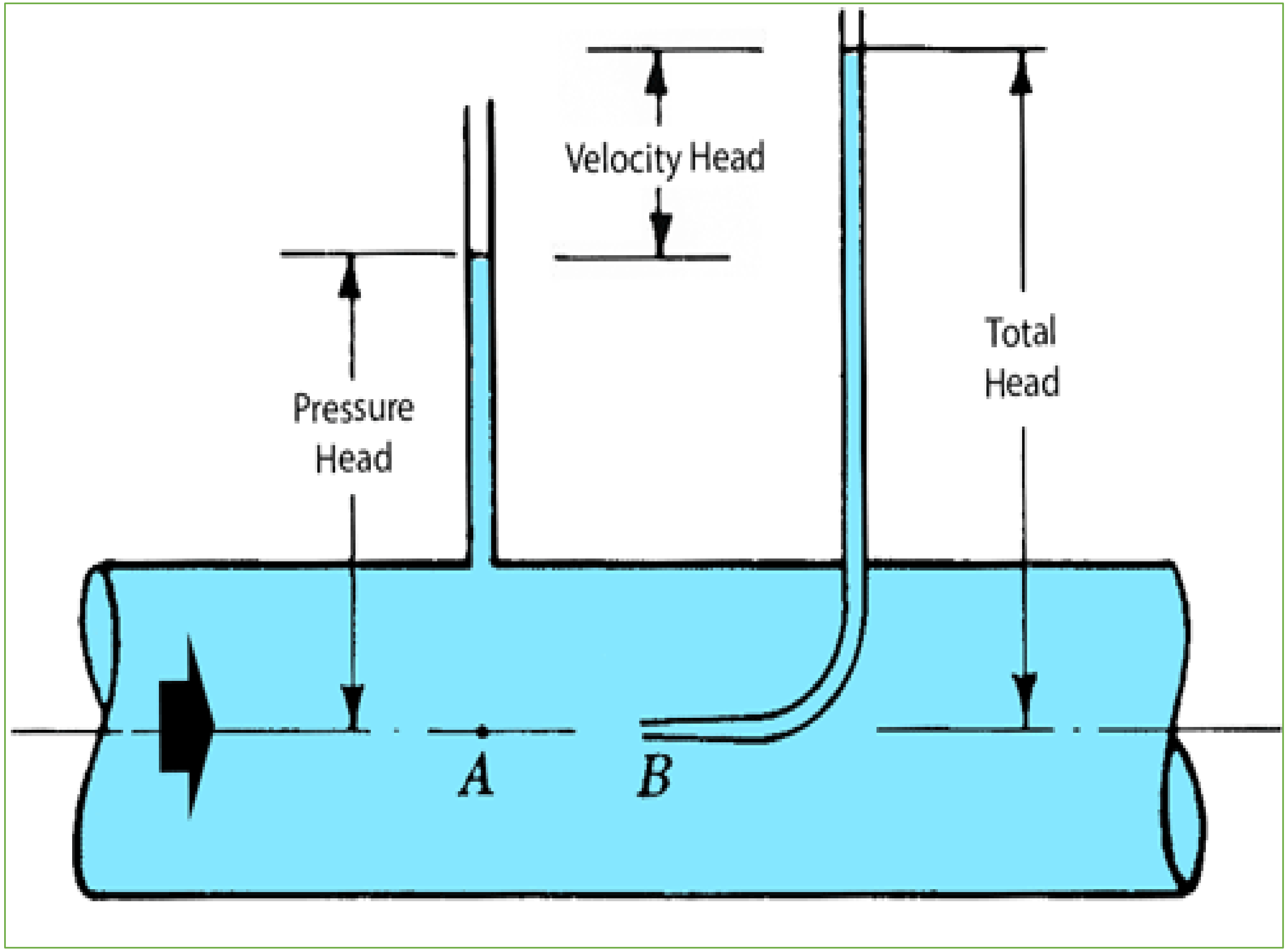
Fluid Dynamics



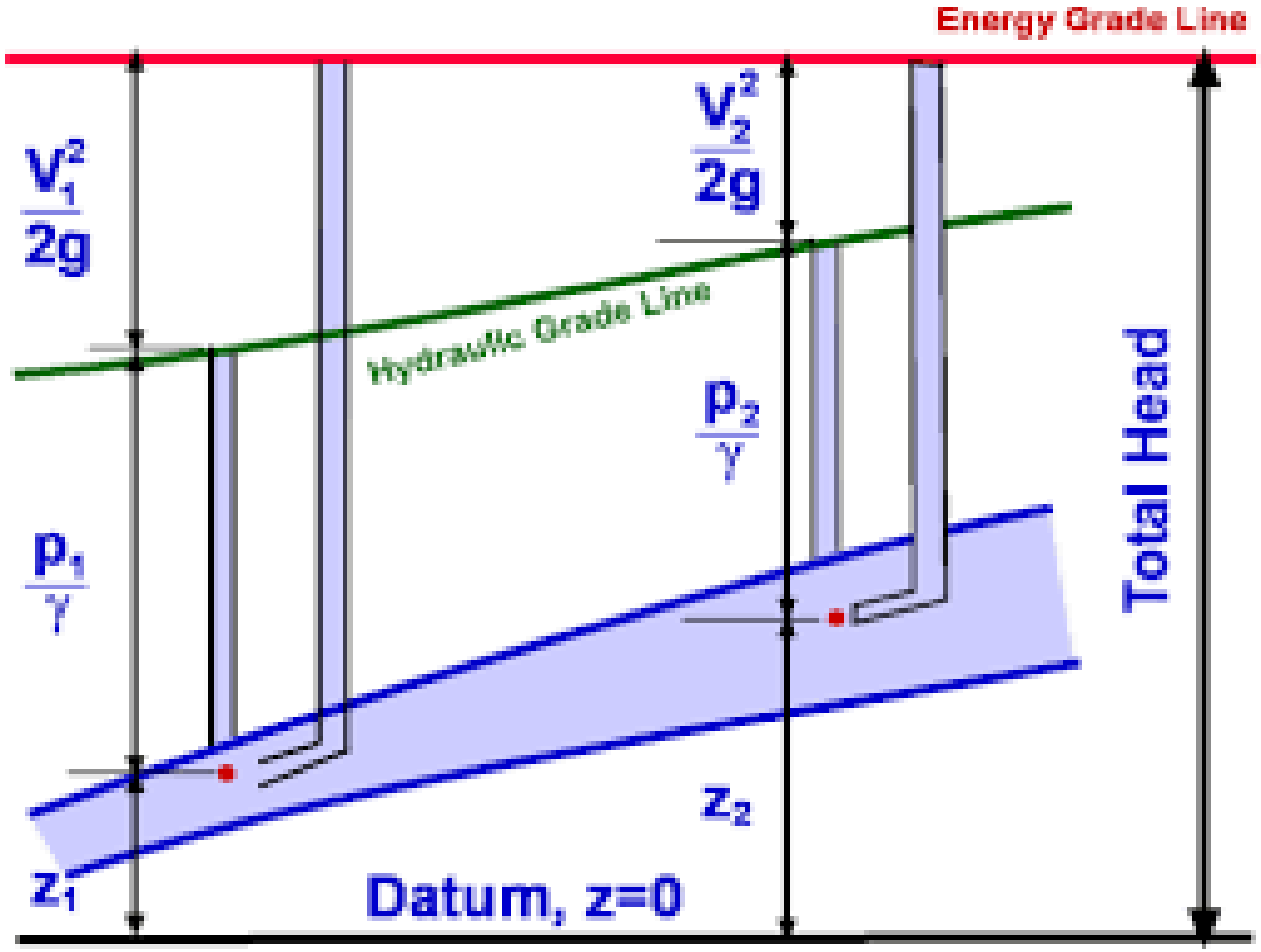
Types of Head of Fluid in Motion

1. **Potential head or Potential energy**: This is due to configuration or position above some suitable datum line. It is denoted by z.
2. **Velocity head or kinetic energy**: This is due to velocity of flowing liquid and is measured as $V^2/2g$.
3. **Pressure head or Pressure energy**: This is due to the pressure of liquid and reckoned as p/γ or p/w

Total Head



Total Head



Total Head = Potential head+ Pressure Head + Velocity Head

H= Z + $V^2/2g$ + p/γ meter of Liquid

Example Problem 1

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross section, which is 5m above the datum line.

Solution. Given :

Diameter of pipe $= 5 \text{ cm} = 0.5 \text{ m}$

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0 \text{ m/s}$

Datum head, $z = 5 \text{ m}$

Total head = pressure head + kinetic head + datum head

Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{\rho \text{ for water} = 1000 \dfrac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \dfrac{v^2}{2g} = \dfrac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Next Class….

- Statement of Bernoulli’s Equation
- Derivation of Bernoulli’s Equation
- Problems related to Bernoulli’s Equation

Bernoulli's Equation: Statement

In an ideal incompressible fluid when the flow is steady and continuous, sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line"

Mathematically, $\frac{p}{\gamma} + \frac{V^2}{2g} + Z = Constant$

$\frac{p}{\gamma}$ = Pressure Energy

$\frac{V^2}{2g}$ = Kinetic Energy

Z = Datum or Elevation or Potential Energy

Derivation of Bernoulli's Equation

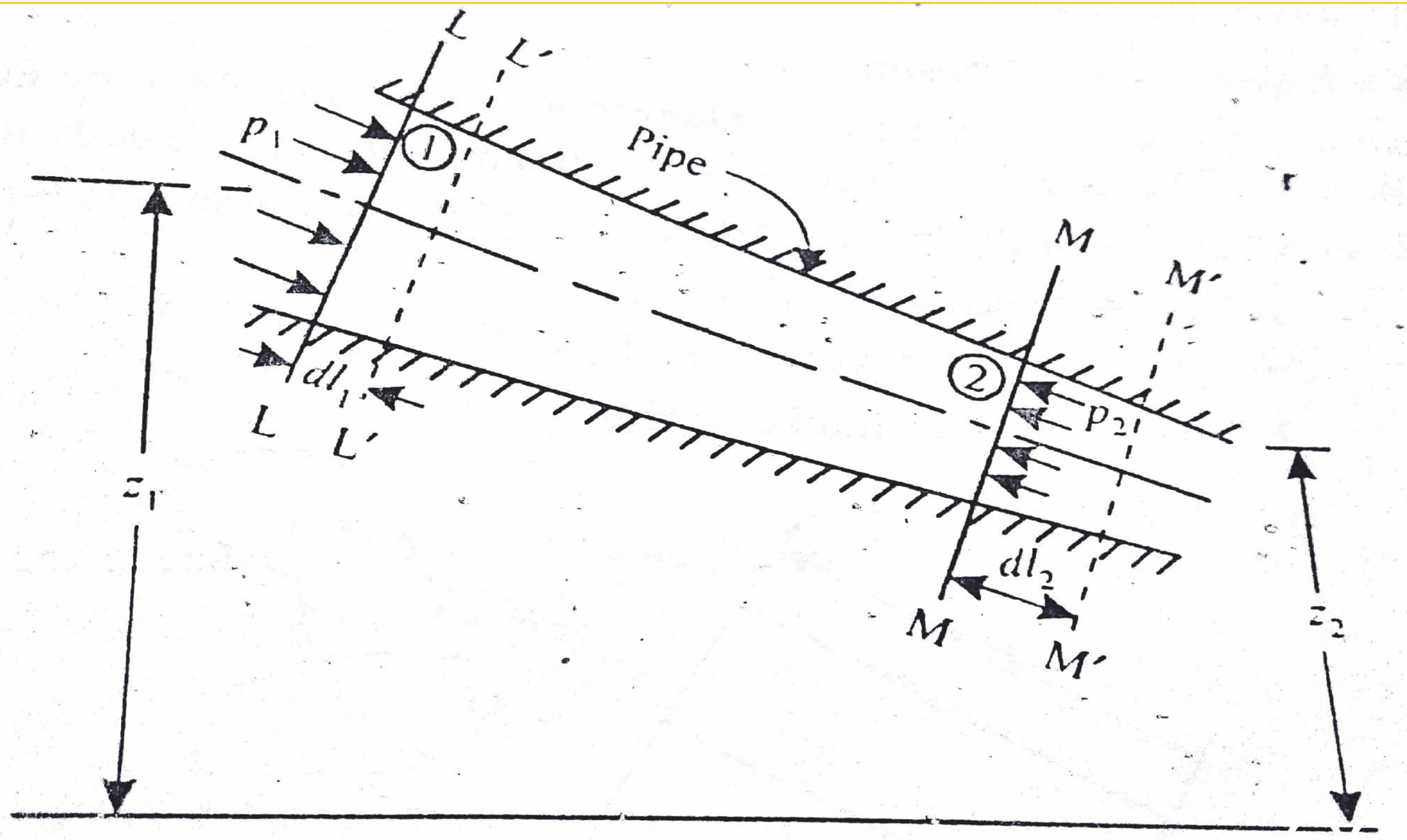


Fig. 6.1. Bernoulli's equation.

Let us consider two sections: LL and MM	
Pressure at LL= P_1	Pressure at MM= P_2
Velocity at LL= V_1	Velocity at MM= V_2
Height of LL above datum = Z_1	Height of MM above datum = Z_2
Area of Pipe at LL= A_1	Area of Pipe at MM= A_2

Weight of Liquid = Unit Weight × Volume (Area*Length)
Work Done = Force × Distance
Force = Pressure × Area

Let the liquid between the two sections LL and MM move to $L'L'$ and $M'M'$ through very small lengths dl_1 and dl_2 as shown in Fig. 6.1. This movement of liquid between LL and MM is equivalent to the movement of the liquid between LL and $L'L'$ and MM and $M'M'$, the remaining liquid between $L'L'$ and MM being unaffected.

Let, W = Weight of liquid between LL and $L'L'$.

As the flow is continuous,

$$\therefore \quad W = wA_1 \,.\, dl_1 = wA_2 \,.\, dl_2$$

or $$A_1 \,.\, dl_1 = \frac{W}{w} \qquad \text{...(i)}$$

Similarly, $$A_2 \,.\, dl_2 = \frac{W}{w} \qquad \text{...(ii)}$$

$$\therefore \quad A_1 \,.\, dl_1 = A_2 . dl_2$$

Work done by pressure at LL, in moving the liquid to $L'L'$

$$= \text{force} \times \text{distance} = p_1 \,.\, A_1 \,.\, dl_1$$

Similarly, work done by the pressure at MM in moving the liquid to $M'M' = -p_2 . A_2 . dl_2$

(- ve sign indicates that direction of p_2 is *opposite* to that of p_1)

$\therefore$ Total work done by the pressure

$$= p_1 . A_1 dl_1 - p_2 A_2 dl_2$$

$$= p_1 . A_1 dl_1 - p_2 A_1 dl_1 \qquad (\because A_1 dl_1 = A_2 dl_2)$$

$$= A_1 . dl_1 (p_1 - p_2)$$

$$= \frac{W}{w}(p_1 - p_2) \qquad \left(\because A_1 . dl_1 = \frac{W}{w}\right)$$

Loss of potential energy $= W(z_1 - z_2)$

Gain in kinetic energy $= W\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) = \frac{W}{2g}(V_2^2 - V_1^2)$

Also, loss of potential energy + work done by pressure = gain in kinetic energy

$\therefore$

$$W(z_1 - z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(V_2^2 - V_1^2)$$

Bernoulli's Equation: Assumptions

- The Liquid is ideal and incompressible
- The flow is steady and continuous.
- The flow is along the streamline i.e. it is one dimensional
- The velocity is uniform over the section and is equal to mean velocity
- The only forces acting on the fluid are gravity forces and pressure forces.

Important Terms

- Incompressible = Incompressible means that the effect of pressure on the fluid density are zero or negligible.
- Ideal Fluid = An ideal fluid is one which is incompressible and has zero viscosity (In reality, no such fluid exists)
- Steady Flow = The type of flow in which the fluid characteristics like velocity, pressure, density etc. at a particular point do not change with time.
- Stream Line = Defined as an imaginary line within the flow so that the tangent at any point on it indicates velocity at that point.

Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

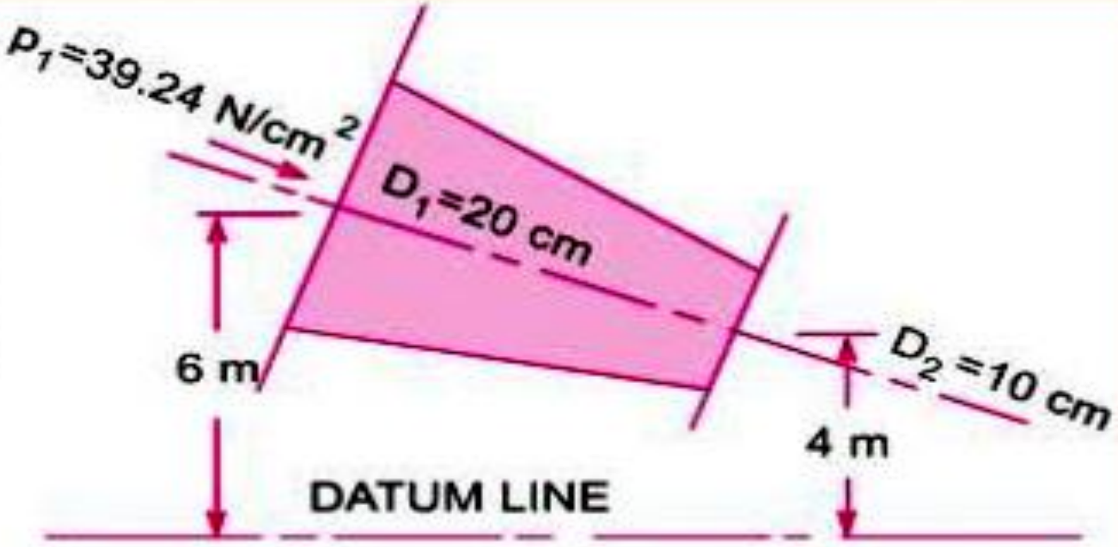


Fig. 6.3

Solution. Given :

At section 1,

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2$$
$$= 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2,

$$D_2 = 0.10 \text{ m}$$

$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Practice Problem#2

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$

and $V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or $$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}.$$ **Ans.**

Bernoulli's Equation for Real Fluid

- Bernoulli's equation based on the assumption that fluid is non-viscous and therefore friction-less.
- Practically, all fluids are viscous and as such there are always some losses in fluid flows.
- These losses have to be taken into consideration in the application of Bernoulli's equation which gets modified for real fluid.

Bernoulli's Equation for Real Fluid

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + h_f$$

$$h_f = Loss\ of\ Head\ due\ to\ friction$$

Practice Problem#3

A pipeline carrying oil of specific gravity 0.87 changes in diameter from 200 mm diameter at position A to 500 mm diameter at position B which is 4 meters at a higher level. If the pressure at A and B are 9.81 N/cm^2 and 5.886 N/cm^2 respectively, and the discharge is 200 Liter/sec. Determine the loss of head and direction of flow.

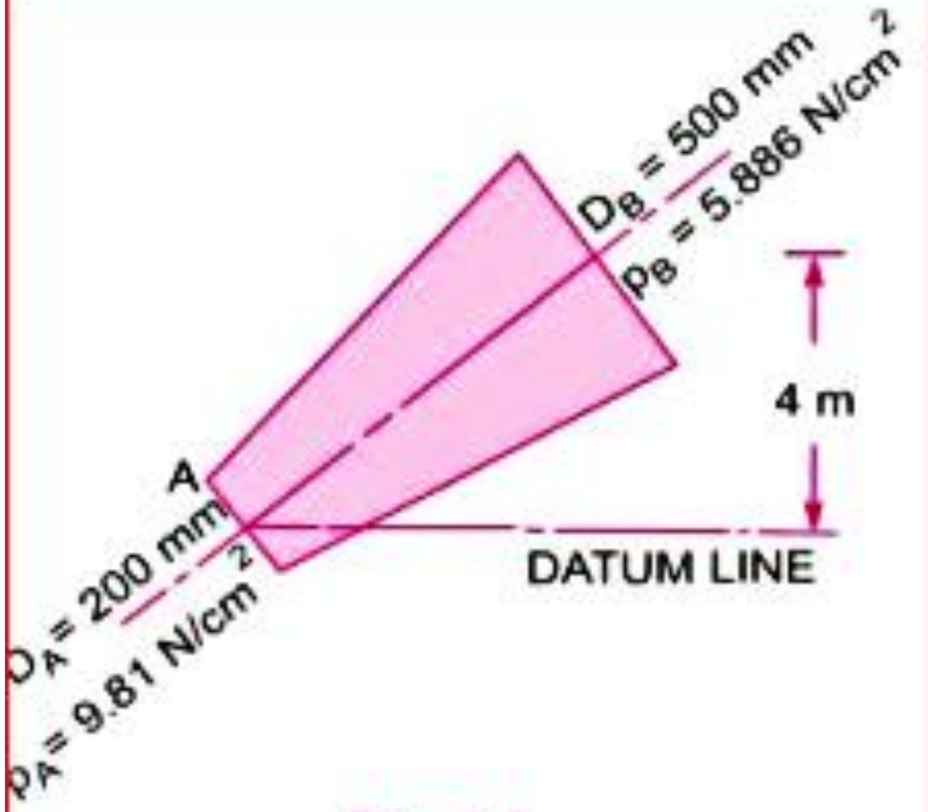


Fig. 6.8

Solution

Solution. Discharge, $Q = 200 \text{ lit/s} = 0.2 \text{ m}^3\text{/s}$

Sp. gr. of oil $= 0.87$

$\therefore$ ρ for oil $= .87 \times 1000 = 870 \dfrac{\text{kg}}{\text{m}^3}$

Given : **At section *A*,** $D_A = 200 \text{ mm} = 0.2 \text{ m}$

Area, $A_A = \dfrac{\pi}{4}(D_A)^2 = \dfrac{\pi}{4}(.2)^2$

$= 0.0314 \text{ m}^2$

$p_A = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

Solution

If datum line is passing through A, then

$$Z_A = 0$$

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369 \text{ m/s}$$

At section B, $D_B = 500 \text{ mm} = 0.50 \text{ m}$

Area, $A_B = \frac{\pi}{4} D_B^2 = \frac{\pi}{4} (.5)^2 = 0.1963 \text{ m}^2$

$$p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$$

Solution

$$Z_B = 4.0 \text{ m}$$

$$V_B = \frac{Q}{\text{Area}} = \frac{0.2}{.1963} = 1.018 \text{ m/s}$$

Total energy at A $= E_A = \dfrac{p_A}{\rho g} + \dfrac{V_A^2}{2g} + Z_A$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0 = 11.49 + 2.067 = 13.557 \text{ m}$$

Total energy at B $= E_B = \dfrac{p_B}{\rho g} + \dfrac{V_B^2}{2g} + Z_B$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4.0 = 6.896 + 0.052 + 4.0 = 10.948 \text{ m}$$

(*i*) **Direction of flow.** As E_A is more than E_B and hence flow is taking place from A to B. **Ans.**

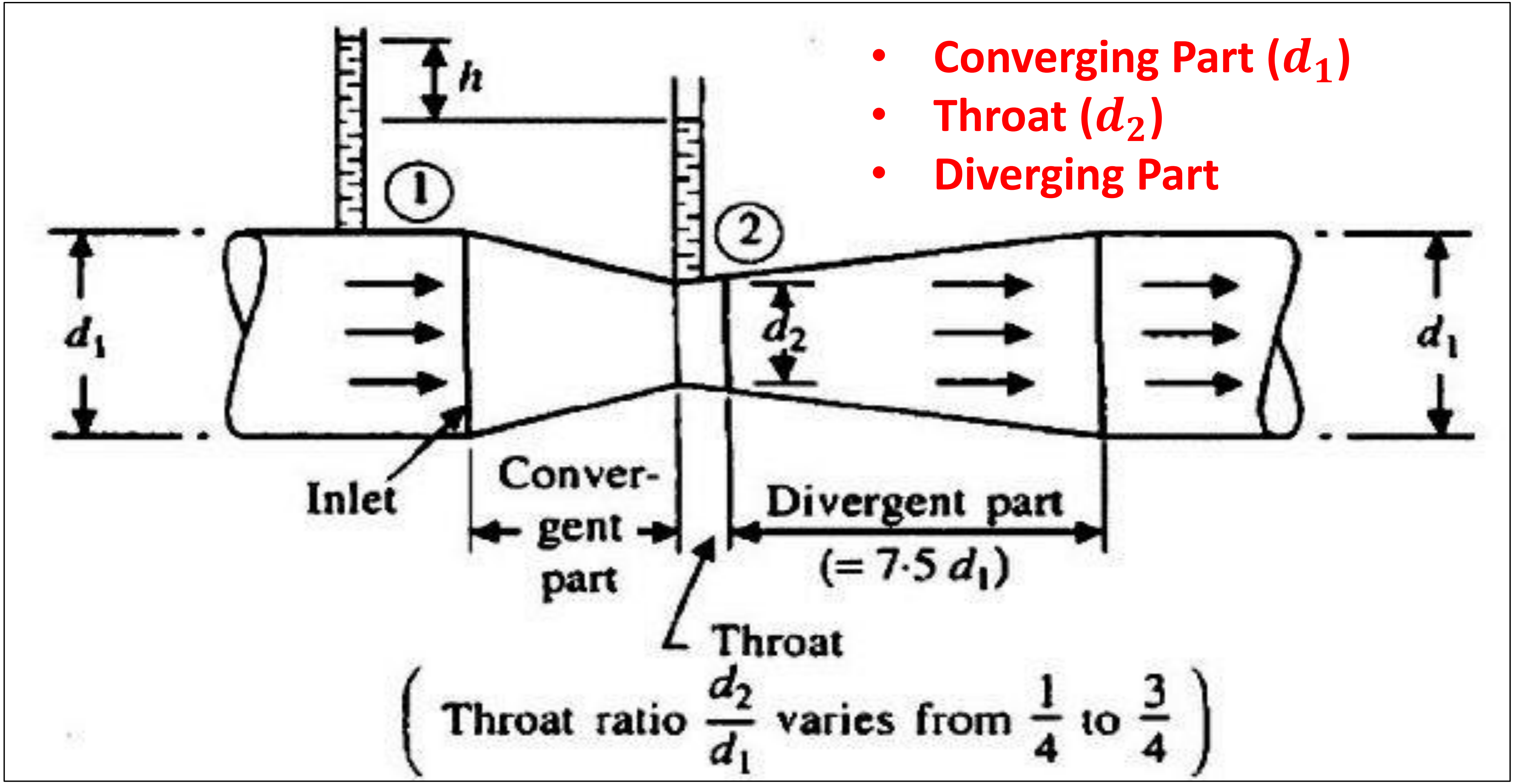
(*ii*) Loss of head $= h_L = E_A - E_B = 13.557 - 10.948 =$ **2.609 m. Ans.**

Practical Application of Bernoulli’s Equation

1. Venturimeter
2. Orificemeter
3. Rotameter and Elbow Meter
4. Pitot Tube

Venturimeter

- **Converging Part (d_1)**
- **Throat (d_2)**
- **Diverging Part**



Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) = $\frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

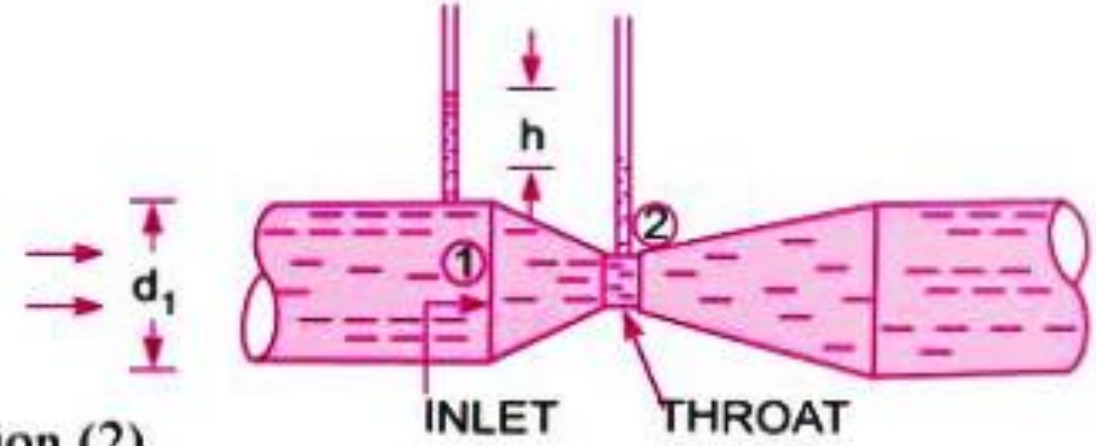


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or
$$v_2{}^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$\therefore$
$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

∴ Discharge, $Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \quad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid

S_o = Sp. gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Then

$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.9)}$$

Value of h given by Differential U-tube Manometer : Case II

If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by:

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where S_l = Sp. gr. of lighter liquid in U-tube
S_o = Sp. gr. of fluid flowing through pipe
x = Difference of the lighter liquid columns in U-tube.

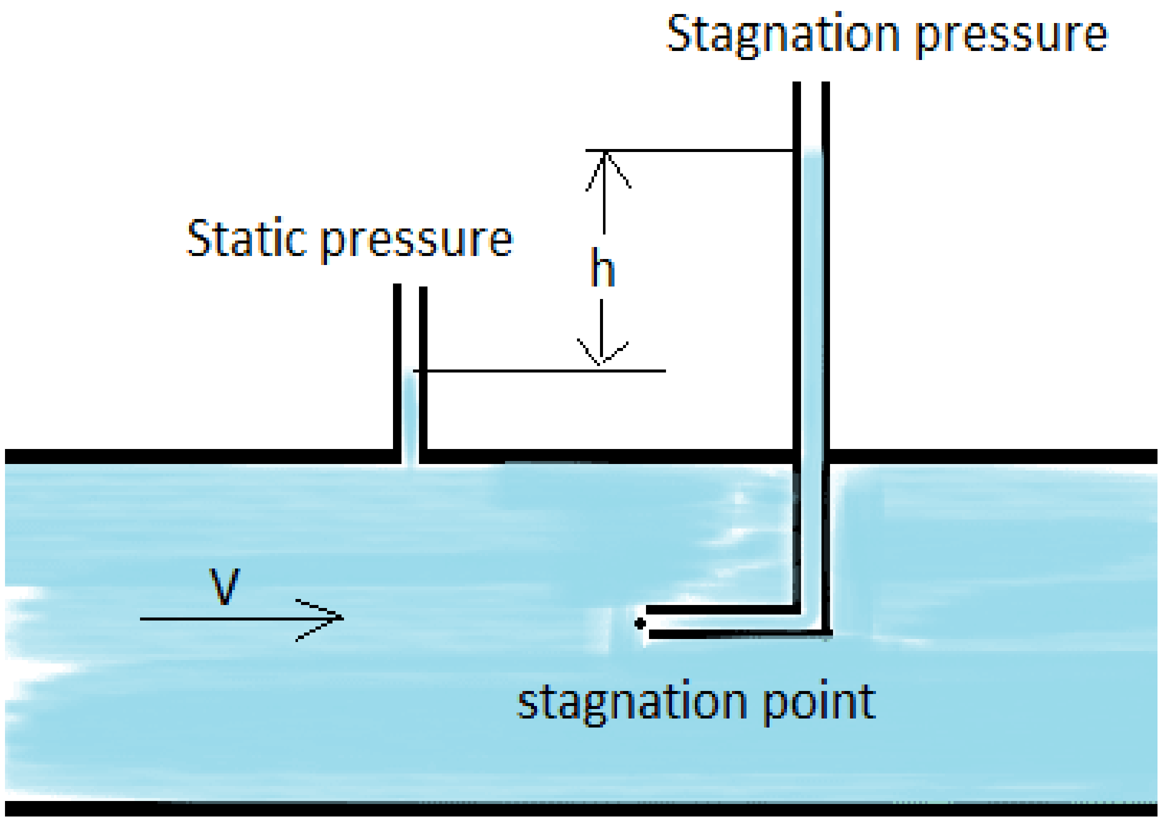
Practice Problem#4

- A Venturimeter with 150 mm diameter at inlet and 100 mm diameter at throat is laid with its axis horizontal and is used for measuring the flow of oil of specific gravity 0.90.
- The oil mercury differential manometer shows a gauge difference of 200 mm.
- Calculate the discharge.
- Assume, coefficient of discharge as 0.98.

Answer: 0.06393 m^3/sec.

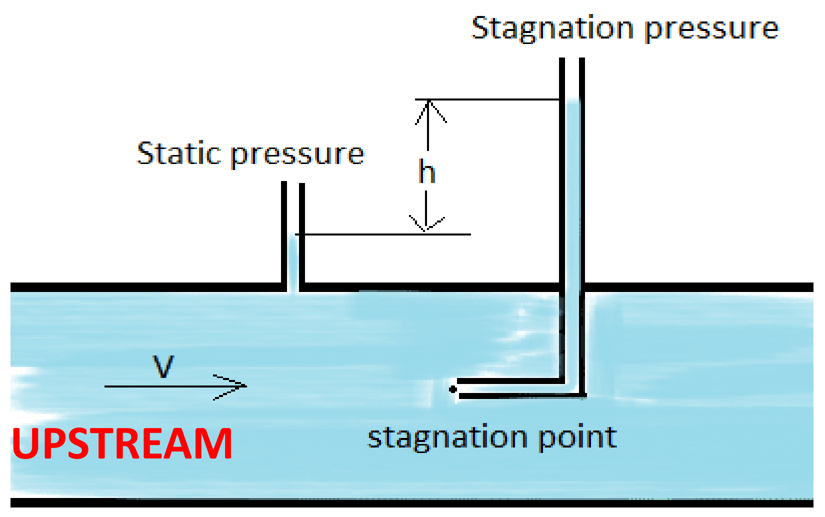
Pitot Tube

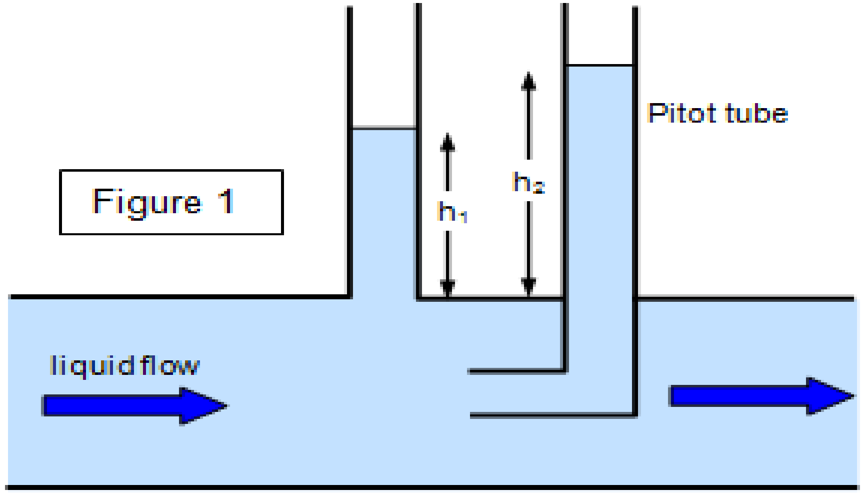
- Pitot Tube is one of the most accurate devices for velocity measurement.
- It works on the principle that, if the velocity of flow becomes zero, the pressure there is increased due to conversion of kinetic energy into pressure



Working of Pitot Tube

- It consists of a glass tube in the form of a 90 degree bend of short length open at both its ends.
- It is placed in the flow with its bent leg directed upstream so that a stagnation point is created immediately in front of the opening.
- Kinetic energy at this point gets converted into pressure energy causing the liquid to rise in the vertical limb, to a height equal to the stagnation pressure.





Theory of Pitot Tube

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equation at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

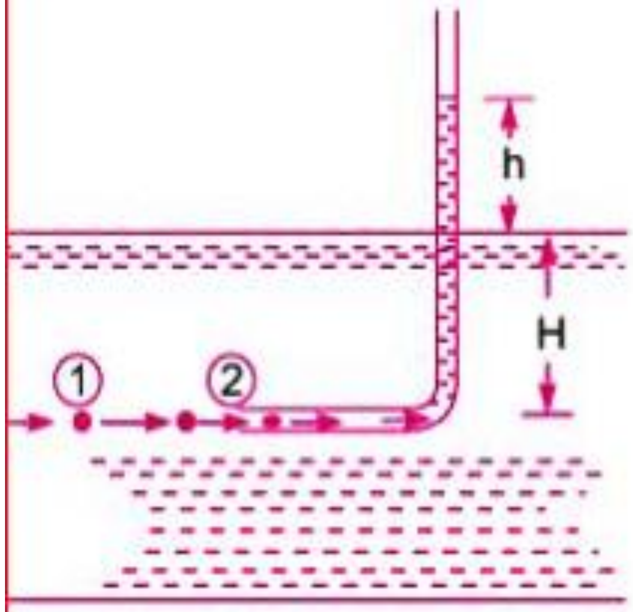
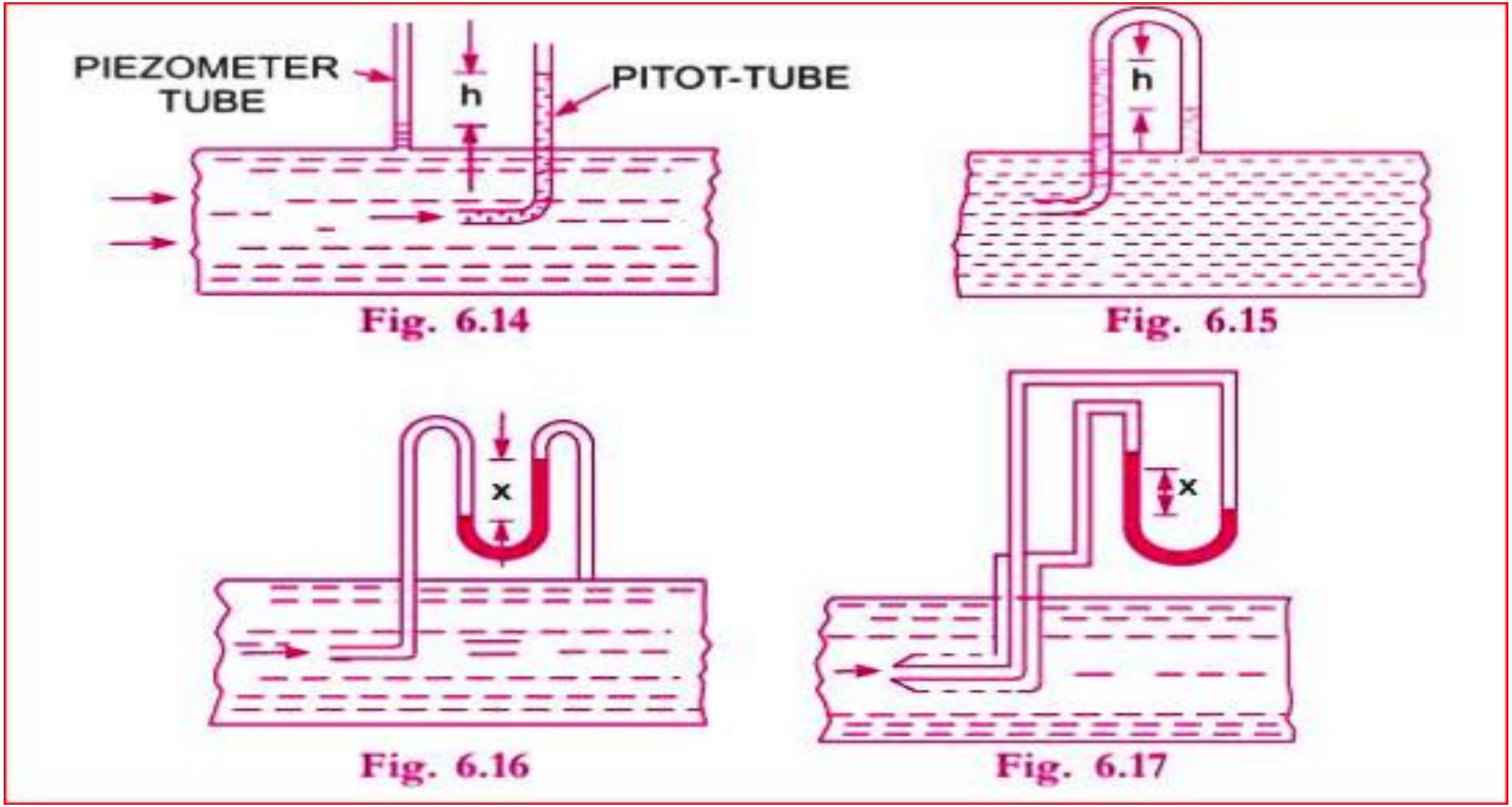


Fig. 6.13 *Pitot-tube.*

Velocity at any point, V = $C_v * V_1$

Arrangements of Pitot tube



Problem 6.27 *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea-water is 1.026 with respect of fresh water.*

Practice Problem#5

Solution. Given :

Diff. of mercury level, $x = 170 \text{ mm} = 0.17 \text{ m}$

Sp. gr. of mercury, $S_g = 13.6$

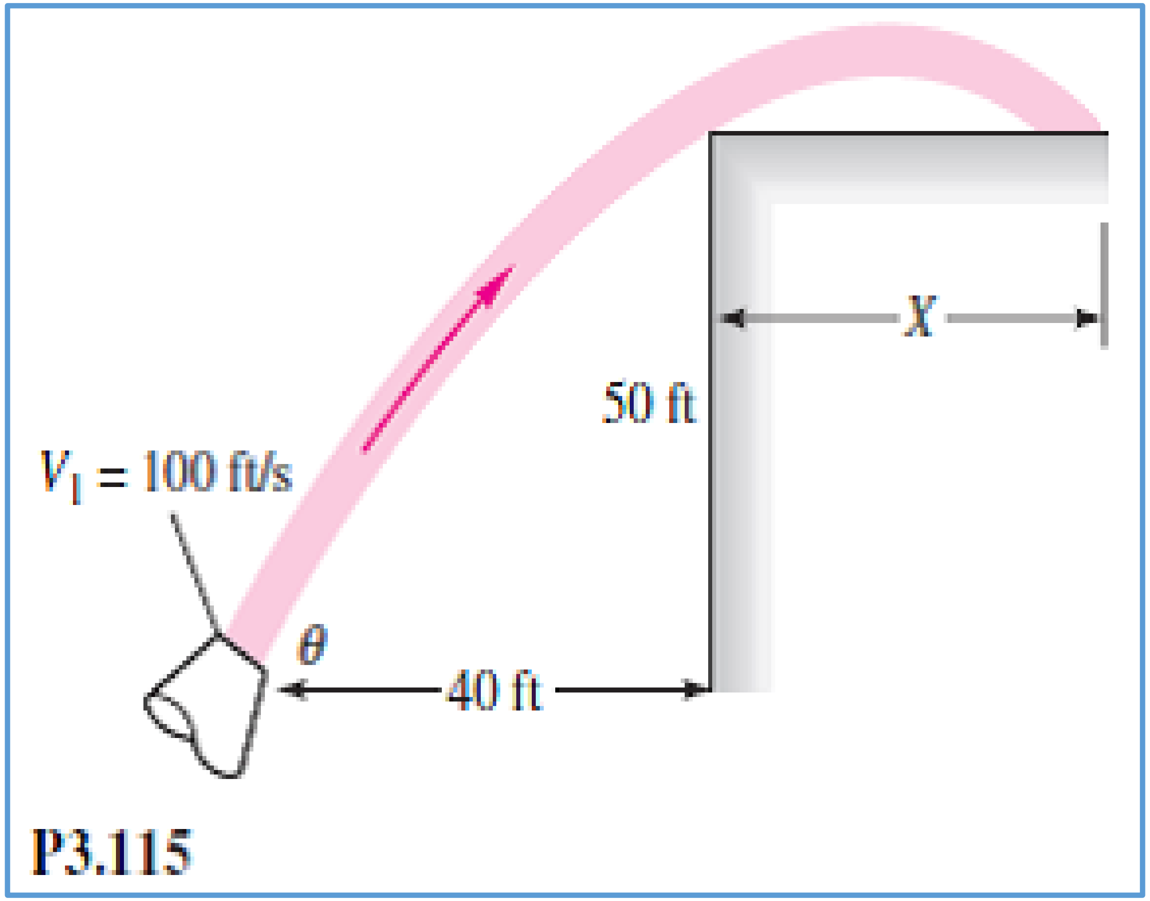
Sp. gr. of sea-water, $S_o = 1.026$

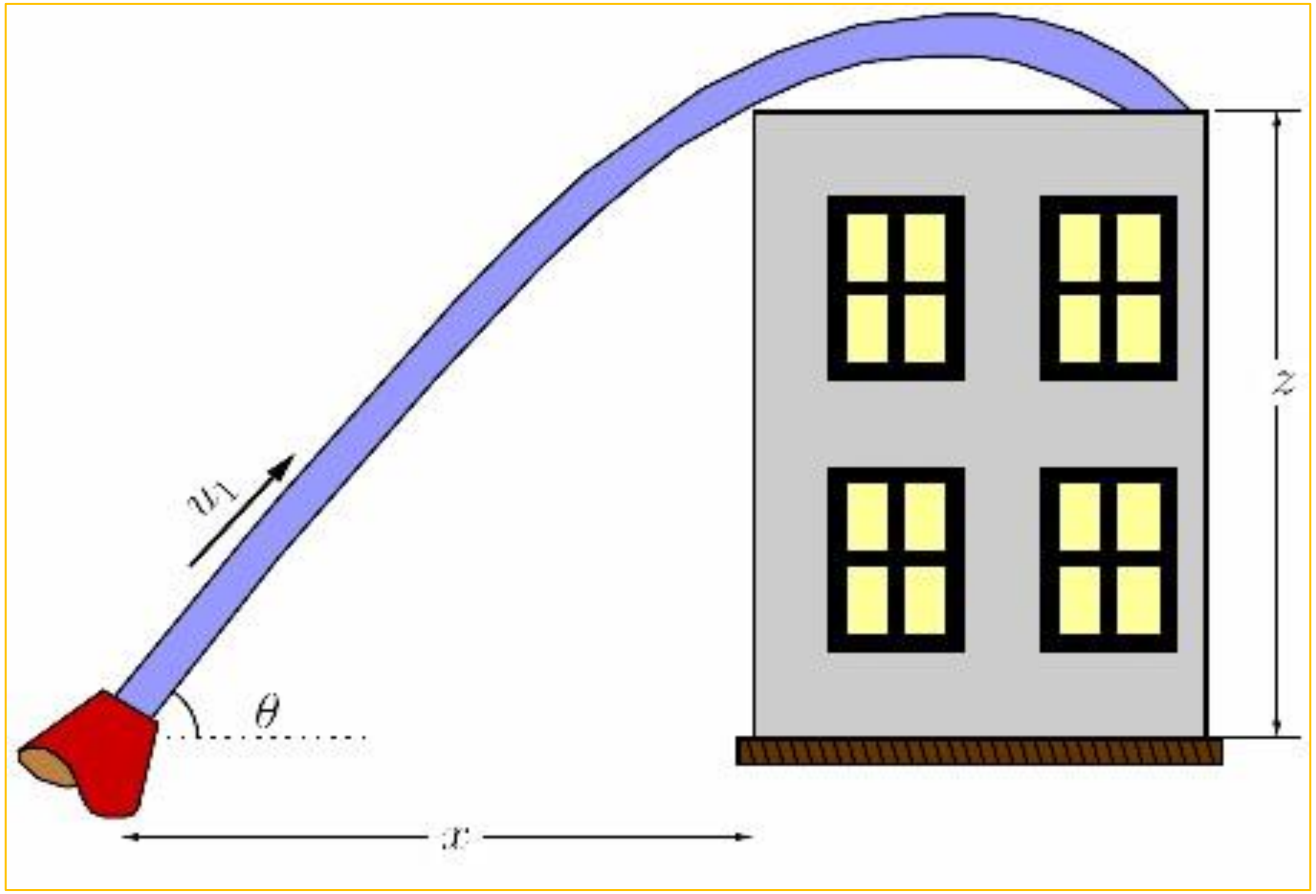
$$\therefore \quad h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834 \text{ m}$$

$$\therefore \quad V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393 \text{ m/s}$$

$$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01 \text{ km/hr.}}$$ **Ans.**

Free Liquid Jet





Free Liquid Jet

- A jet of liquid issuing from the nozzle in atmosphere is called a free liquid jet.
- The parabolic path traversed by the liquid jet under the action of gravity is known as trajectory.
- A nozzle is a device designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it exits (or enters) an enclosed chamber or pipe.

Application of Free Liquid Jet

Banani fire: Death toll rises to 25 (Source: The Daily Star)

Equation of a Free Liquid Jet

Let, the jet A make an angle with the horizontal direction. If U is the velocity of the water jet, then,

Horizontal Component

= U Cos θ

Vertical Component

= U Sin θ

Consider another point P(X,Y) on the center of the line of the jet.

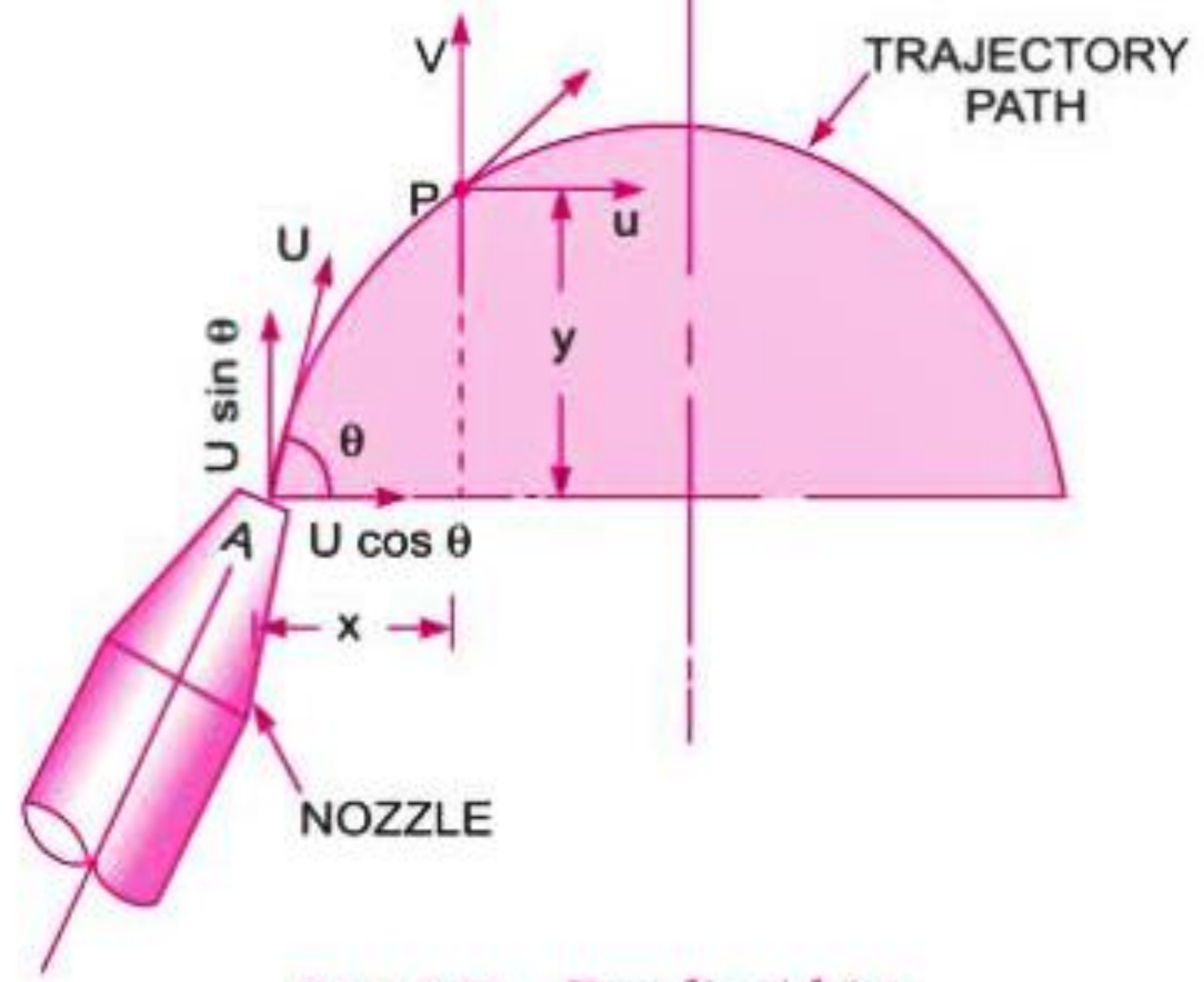


Fig. 6.30 *Free liquid jet.*

$$x = \text{velocity component in } x\text{-direction} \times t$$
$$= U \cos\theta \times t \qquad \ldots(i)$$

and

$$y = (\text{vertical component in } y\text{-direction} \times \text{time} - \frac{1}{2} g t^2)$$
$$= U \sin\theta \times t - \frac{1}{2} g t^2 \qquad \ldots(ii)$$

{$\because$ Horizontal component of velocity is constant while the vertical distance is affected by gravity}

From equation (*i*), the value of t is given as $t = \dfrac{x}{U \cos\theta}$

Substituting this value in equation (*ii*)

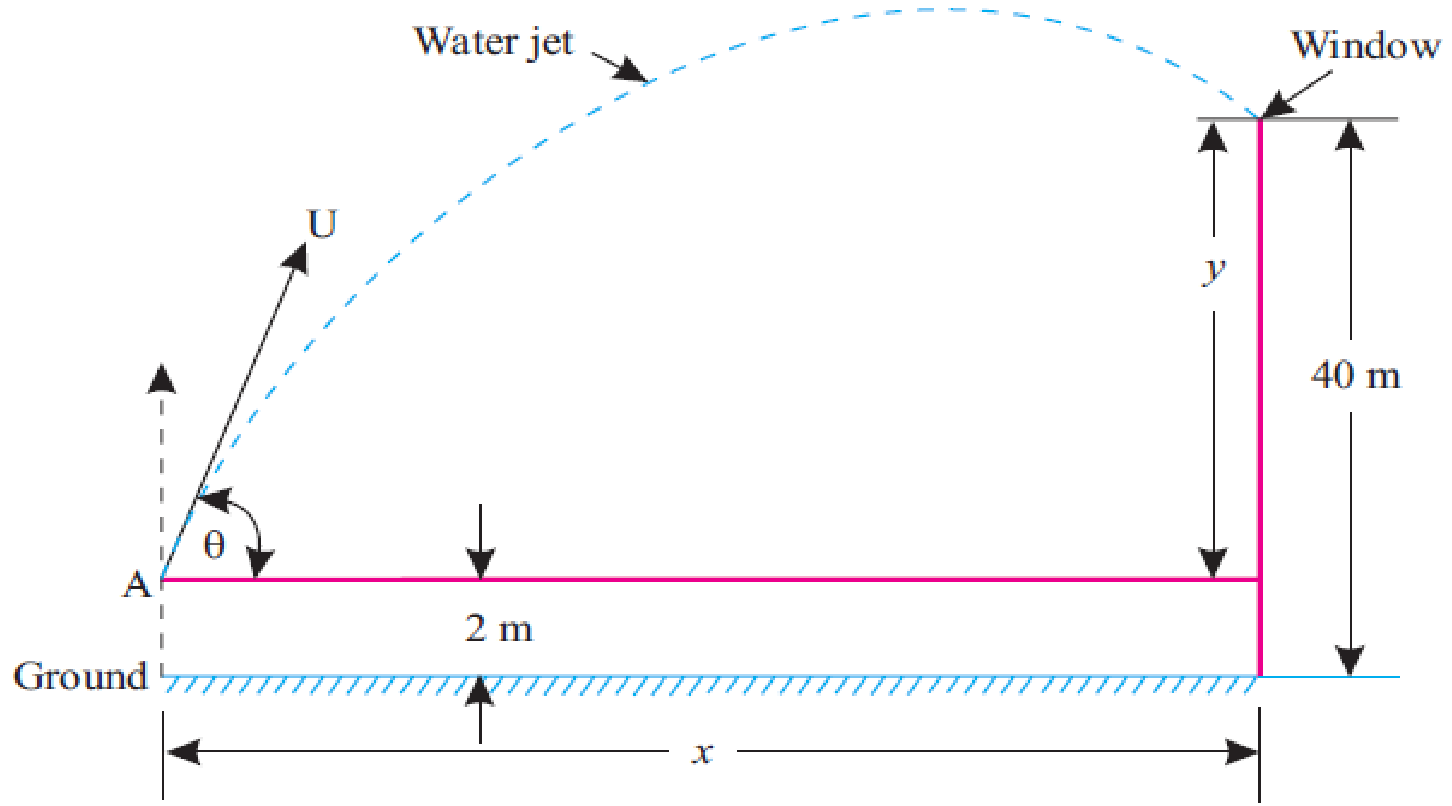
$$y = U \sin\theta \times \frac{x}{U \cos\theta} - \frac{1}{2} \times g \times \left(\frac{x}{U \cos\theta}\right)^2 = x \frac{\sin\theta}{\cos\theta} - \frac{g x^2}{2U^2 \cos^2\theta}$$

$$= x \tan\theta - \frac{g x^2}{2U^2} \sec^2\theta \qquad \left\{\because \frac{1}{\cos^2\theta} = \sec^2\theta\right\} \qquad \ldots(6.24)$$

Equation (6.24) gives the variation of y with the square of x. Hence this is the equation of a parabola. Thus the path travelled by the free jet in atmosphere is parabolic.

A fireman must reach a window 40 m above the ground of Banani tower with a water jet, issued from a nozzle 30 mm in diameter and discharging 30 kg/s. Assuming the nozzle height to be 2 m above the ground, determine the greatest horizontal distance from the building where the fireman can stand and still reach the jet into the window.

Refer to Fig. 6.45.



Problem : Rajput
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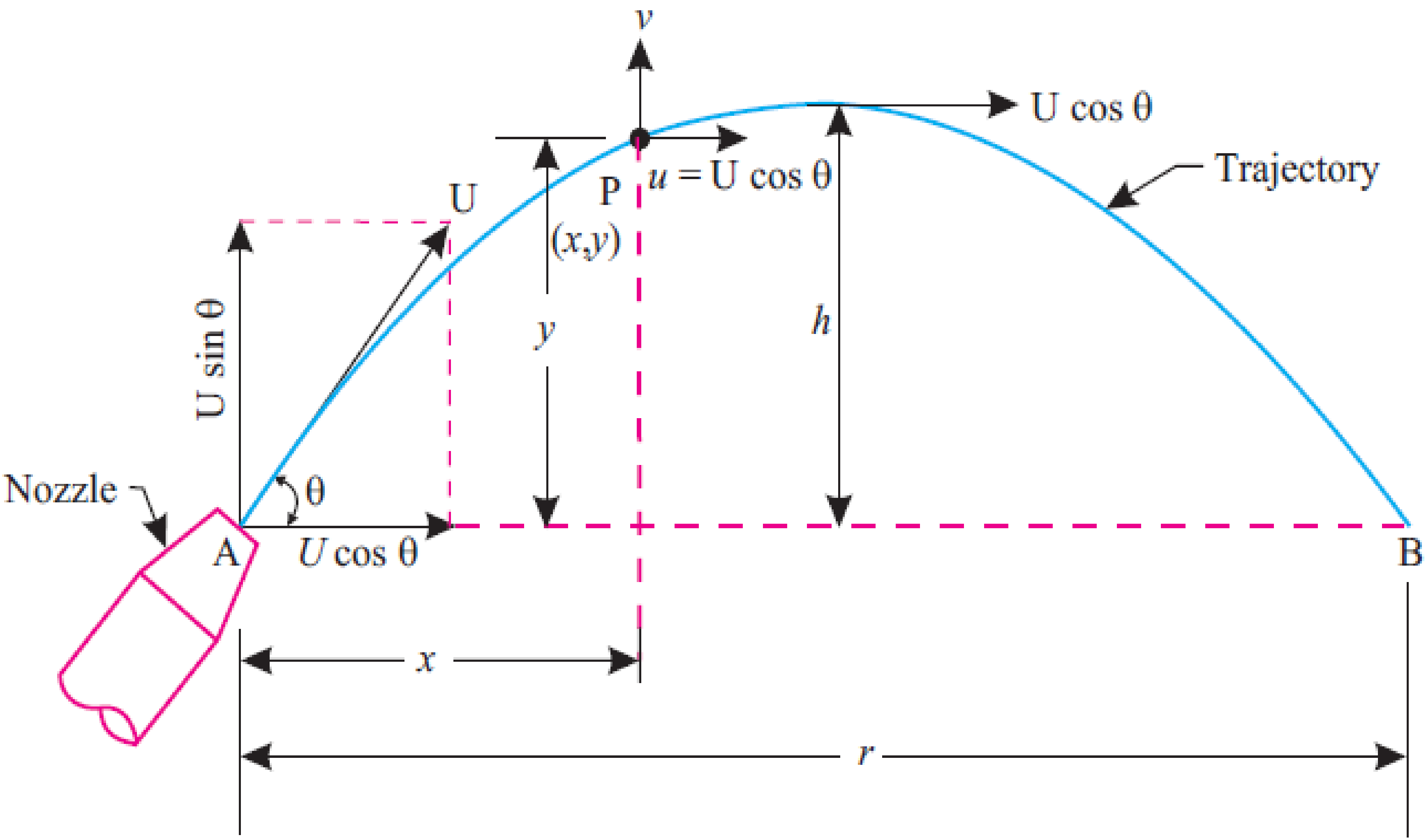
Maximum Height attained by Jet



Maximum Height attained by Jet

Using the relation, $V_2^2 = V_1^2 - 2gS$; (g is acting in the downward direction, but particles is moving up)

Here, $V_2 = 0$ at highest point ,

V_1= initial vertical component = Usinθ,

$$\therefore \quad 0 - (U \sin \theta)^2 = -2g \times S$$

where S is the maximum vertical height attained by the particle.

$$\text{or} \quad -U^2 \sin^2\theta = -2gS$$

$$\therefore \quad S = \frac{U^2 \sin^2 \theta}{2g}$$

Time of Flight

It is the time taken by the fluid particle in reaching from A to B as shown in figure. Let, T is the time of flight.

Using equation (*ii*), we have $y = U \sin\theta \times t - \frac{1}{2} g t^2$

when the particle reaches at B, $y = 0$ and $t = T$

$\therefore$ Above equation becomes as $0 = U \sin\theta \times T - \frac{1}{2} g \times T^2$

or $$0 = U \sin\theta - \frac{1}{2} g T$$

or $$T = \frac{2U \sin\theta}{g}$$

(*iii*) **Time to reach highest point.** The time to reach highest point is half the time of flight. Let T^* is the time to reach highest point, then

$$T^* = \frac{T}{2} = \frac{2U \sin\theta}{g \times 2} = \frac{U \sin\theta}{g} \quad ...(6.27)$$

(*iv*) **Horizontal range of the jet.** The total horizontal distance travelled by the fluid particle is called horizontal range of the jet, *i.e.*, the horizontal distance *AB* in Fig. 6.30 is called horizontal range of the jet. Let this range is denoted by x^*.

Then x^* = velocity component in x-direction

× time taken by the particle to reach from *A* to *B*

$= U \cos\theta \times$ Time of flight

$$= U \cos\theta \times \frac{2U \sin\theta}{g} \qquad \left\{\because T = \frac{2U \sin\theta}{g}\right\}$$

$$= \frac{U^2}{g} 2 \cos\theta \sin\theta = \frac{U^2}{g} \sin 2\theta \quad ...(6.28)$$

(*v*) **Value of θ for maximum range.** The range x^* will be maximum for a given velocity of projection (U), when $\sin 2\theta$ is maximum

or when $\sin 2\theta = 1$ or $\sin 2\theta = \sin 90° = 1$

∴ $2\theta = 90°$ or $\theta = 45°$

Then maximum range, $$x^*_{max} = \frac{U^2}{g} \sin^2\theta = \frac{U^2}{g} \qquad \{\because \sin 90° = 1\} \quad ...(6.29)$$

Practice Problem #6

A vertical wall is of 8m height. A jet of water is coming out from a nozzle with a velocity of 20 m/s. The nozzle is situated at a distance of 20m from the vertical wall. Find the angle of projection of the nozzle to the horizontal so that the jet of water just clears the top of wall.

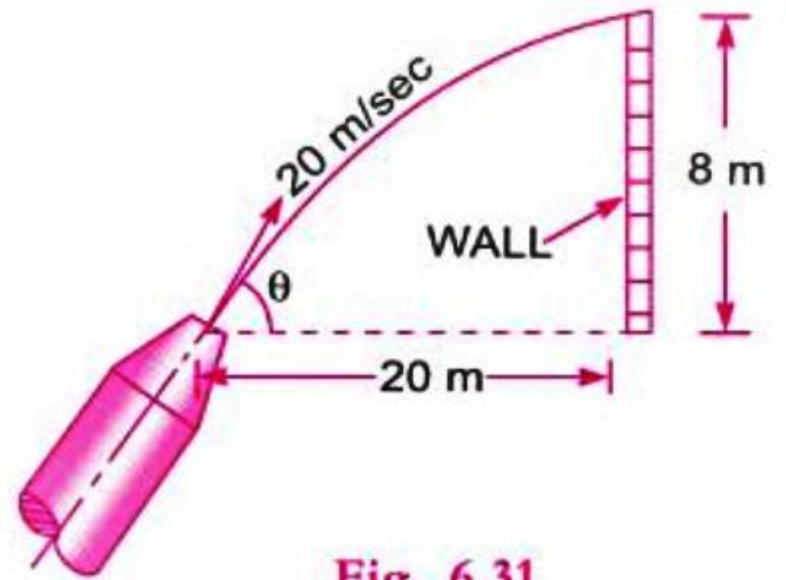


Fig. 6.31

Solution. Given :

Height of wall $= 8$ m

Velocity of jet, $U = 20$ m/s

Distance of jet from wall, $x = 20$ m

Let the required angle $= \theta$

Using equation (6.24), we have

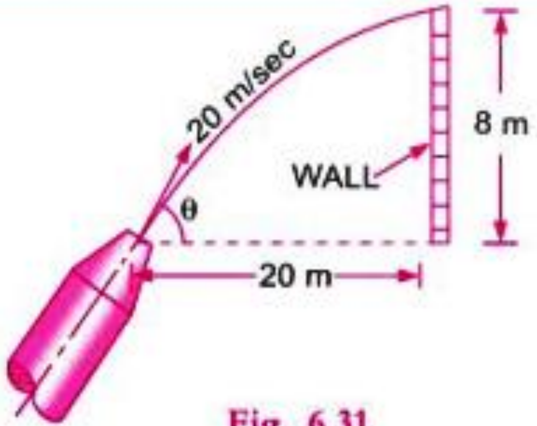


Fig. 6.31

$$y = x \tan\theta - \frac{gx^2}{2U^2}\sec^2\theta$$

where $y = 8$ m, $x = 20$ m, $U = 20$ m/s

$$8 = 20\tan\theta - \frac{9.81\times 20^2}{2\times 20^2}\sec^2\theta$$

$$= 20\tan\theta - 4.905\sec^2\theta$$

$$= 20\tan\theta - 4.905\,[1+\tan^2\theta] \qquad \{\because \ \sec^2\theta = 1+\tan^2\theta\}$$

$$= 20\tan\theta - 4.905 - 4.905\tan^2\theta$$

or $\quad 4.905\tan^2\theta - 20\tan\theta + 8 + 4.905 = 0$

or $\quad 4.905\tan^2\theta - 20\tan\theta + 12.905 = 0$

$\therefore$
$$\tan\theta = \frac{20\pm\sqrt{20^2 - 4\times 12.905\times 4.905}}{2\times 4.905} = \frac{20\pm\sqrt{400-253.19}}{9.81}$$

$$= \frac{20\pm\sqrt{146.81}}{9.81} = \frac{20\pm 12.116}{9.81} = \frac{32.116}{9.81} \text{ or } \frac{7.889}{9.81}$$

$\therefore \quad = 3.273$ or 0.8036

$\therefore \quad$ **$\theta = 73°\ 0.8'$ or $38°\ 37'$. Ans.**

Practice Problem#7 (Rajput, 376 page)

A jet issuing from a 30 mm nozzle held at 0.6 m above the ground level at an angle of 30 degree to the horizontal strikes the ground 4m away. Determine:

1. The maximum height reached.
2. The range of the jet.
3. The discharge

Answer:

0.458 m above the nozzle, 3.18 m, 0.00425 m^3/s

Practice Problem#8

It is required to place an orifice in the side of a tank at such an elevation that the jet will attain a maximum horizontal distance from the tank at the level of its base.

What is the proper distance from the orifice to the free surface when the depth of liquid in the tank is maintained at 1.2 m?

Solution. Depth of liquid in the tank = 1.2 m

$$x = \sqrt{2gh} \times t \quad \text{...(i)}$$

and, $$y = -\frac{1}{2} g t^2 \quad \text{...(ii)}$$

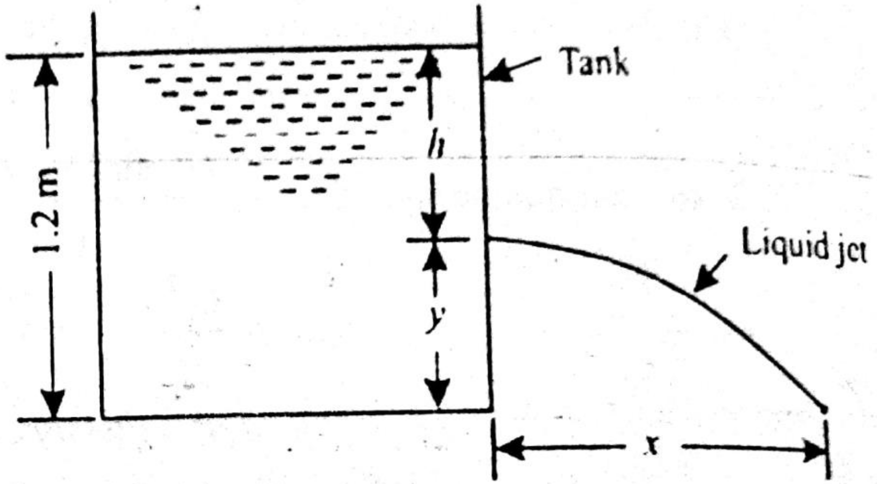


Fig. 6.43

Eliminating t, we get

$$y = -\frac{1}{2} g \times \left(\frac{x}{\sqrt{2gh}}\right)^2$$

$$= -\frac{1}{2} \times g \times \frac{x^2}{2gh}$$

or, $$y = -\frac{x^2}{4h}$$

Also, $1.2 = h + y$ or $y = 1.2 - h$

$$\therefore \quad (1.2 - h) = -\frac{x^2}{4h}$$

or, $$x^2 = -4h\,(1.2 - h) = -4.8\,h + 4h^2$$

For horizontal distance x to be maximum $\frac{dx}{dh} = 0$

$$\therefore \quad 2x\frac{dx}{dh} = -4.8 + 8h = 0 \quad \text{or} \quad h = 0.6 \text{ m}$$

Thus, the orifice should be located at a distance of **0.6 m below the free surface. (Ans.)**

Overview of Fluid Dynamics

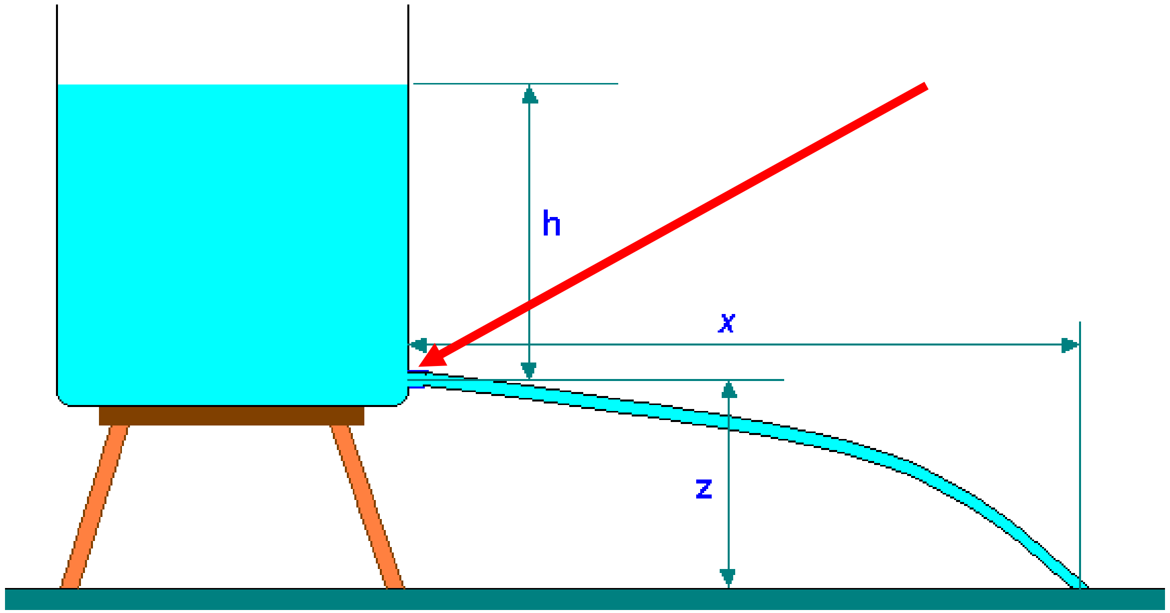
1. Bernoulli's Equation: Statement and Derivation
2. Bernoulli's Equation for Real Fluid
3. Practical Application of Bernoulli's Equation (Venturimeter, Pitot Tube)
4. Free Liquid Jet

*1st Class test on Fluid Dynamics

Flow through Orifice and Mouthpieces

- Classification of Orifice
- Flow through an orifice
- Hydraulic Coefficients (C_c , C_v , C_d , C_r)
- Experimental Determination of Coefficients
- Discharge through a large rectangular orifice
- Classification of Mouthpiece
- Discharge through an external mouthpiece

Orifice

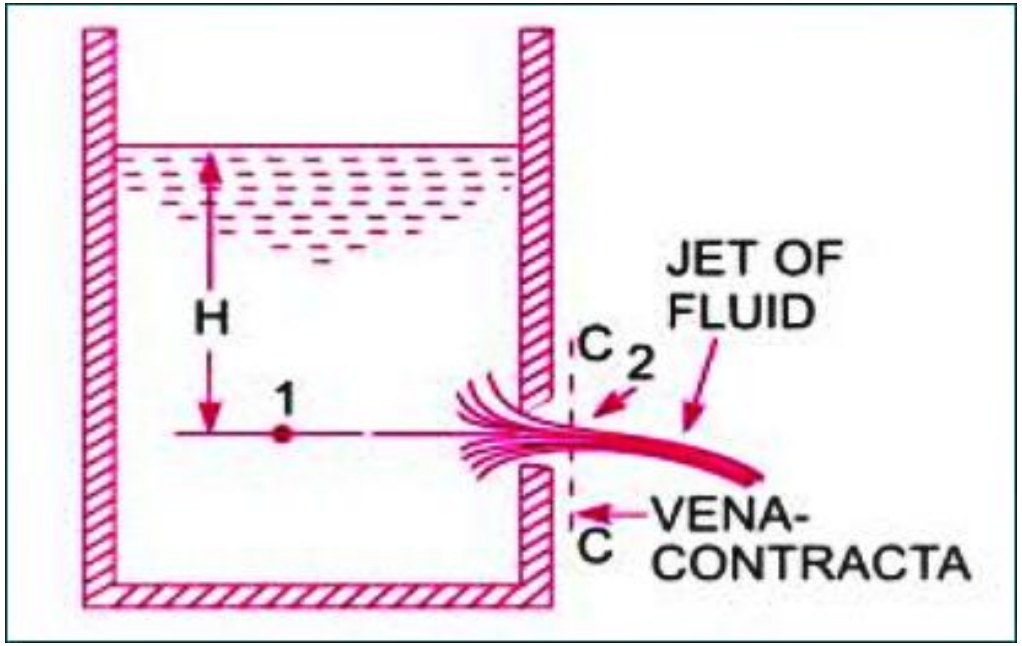


- An orifice is an opening in the wall or base of a vessel through which fluid flows.
- The top edge of orifice is always below the free surface.
- Orifices are used to measure the discharge of fluid flow.

Classification of Orifice

Parameter	**Type of Orifice**
According to Size	Small orifice
	Large orifice
According to Shape	Circular Orifice
	Rectangular Orifice
	Square Orifice
	Triangular Orifice
Shape of Upstream edge	Sharp Edged Orifice
	Bell Mouthed Orifice
According to Discharge Condition	Free Discharge Orifice
	Submerged Orifice
	Fully Submerged Orifice
	Partially Submerged Orifice

Torricelli's Theorem (flow through an orifice)



Consider two points 1 and 2 as shown in figure.

Point 1 is inside the tank and point 2 at the vena contracta.

Let the flow is steady and at a constant head H. Applying Bernoulli's equation at point 1 and point 2 gives:

Torricelli's Theorem

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Fig. 7.1 ***Tank with an orifice.***

But $$z_1 = z_2$$

$\therefore$ $$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $$\frac{p_1}{\rho g} = H$$

$$\frac{p_2}{\rho g} = 0 \text{ (atmospheric pressure)}$$

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$\therefore$ $$H + 0 = 0 + \frac{v_2^2}{2g}$$

$\therefore$ $$v_2 = \sqrt{2gH} \qquad \text{...(7.1)}$$

This is theoretical velocity. Actual velocity will be less than this value.

Hydraulic Coefficients (C_c , C_v , C_d , C_r)

Co-efficient of Contraction:

The ratio of area of jet at vena contracta to the area of the orifice is known as co-efficient of contraction. It is denoted by C_c.

$$C_C = \frac{Area\ of\ Jet\ at\ vena\ contracta}{Area\ of\ orifice} = \frac{a_c}{a}$$

Hydraulic Coefficients (C_c , C_v , C_d , C_r)

Co-efficient of resistance:

The ratio of loss of head in the orifice to the head of water available at the exit of the orifice is known as co-efficient of resistance. It is denoted by C_r.

$$C_r = \frac{Loss\ of\ head\ in\ the\ orifice}{Head\ of\ water}$$

Relationship: $C_d = C_c \times C_v$

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual area}}{\text{Theoretical velocity} \times \text{Theoretical area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual area}}{\text{Theoretical area}}$$

$$C_d = C_v \times C_c$$

Standard Value for Hydraulic Coefficients

Hydraulic Coefficients	Range of Values
Co-efficient of Velocity	0.95-0.99
Co-efficient of Contraction	0.61-0.69
Co-efficient of Discharge	0.61-0.65

Practice Problem#9 (Rajput 495 page)

An orifice 60 mm in diameter is discharging water under a head of 9 meters. If C_d= 0.60 and C_v= 0.90 find:

i. Actual Discharge

ii. Actual Velocity of Jet at Vena- Contracta

Answer:

0.02254 m^3/s

11.26 m/s

Experimental Determination of Hydraulic Coefficients

1. Determination of Co-efficient of Velocity by Co-ordinate method

$$C_v = \frac{x}{\sqrt{4yH}}$$

2. Determination of Co-efficient of Discharge

$$C_d = \frac{Q}{a \times \sqrt{2gH}}$$

Value of Actual Discharge is determined with the help of a rectangular tank.

Practice Problem#10 (Bansal)

A tank has two identical orifices on one of its vertical sides. The upper orifice is 3 meter below the water surface and lower one is 5 meter below the water surface. If the value of C_v for each orifice is 0.96, find the point of intersection of two jets.

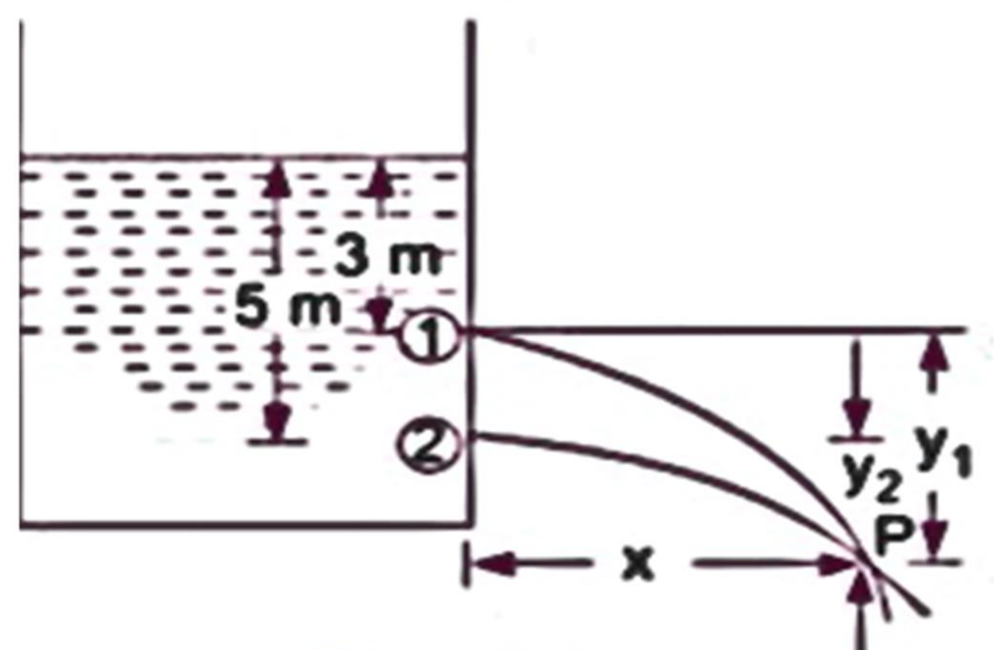


Fig. 7.5

Solution

Height of water from orifice (1), $H_1 = 3$ m

From orifice (2), $H_2 = 5$ m

C_v for both = 0.96

Let P is the point of intersection of the two jets coming from orifices (1) and (2), such that

x = horizontal distance of P

y_1 = vertical distance of P from orifice (1)

y_2 = vertical distance of P from orifice (2)

Then $y_1 = y_2 + (5 - 3) = y_2 + 2$ m

The value of C_v is given by equation (7.6) as

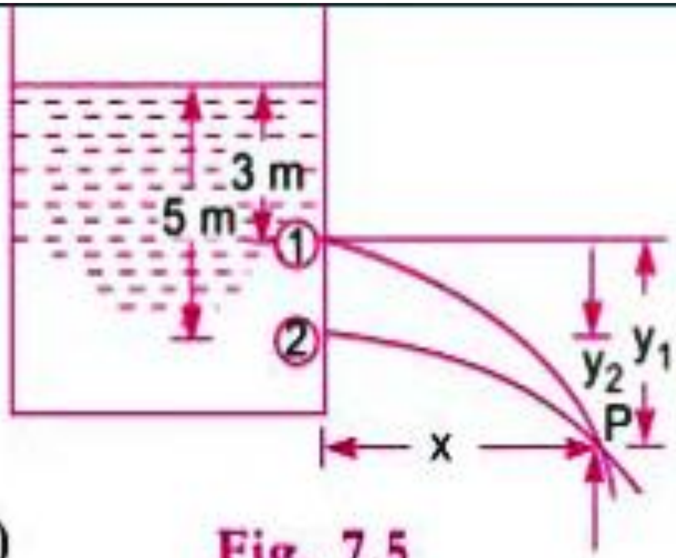


Fig. 7.5

For orifice (1), $$C_{v_1} = \frac{x}{\sqrt{4y_1 H_1}} = \frac{x}{\sqrt{4y_1 \times 3.0}} \quad \text{...}(i)$$

For orifice (2), $$C_{v_2} = \frac{x}{\sqrt{4y_2 H_2}} = \frac{x}{\sqrt{4 \times y_2 \times 5.0}} \quad \text{...}(ii)$$

As both the orifices are identical

$\therefore$ $$C_{v_1} = C_{v_2}$$

or $$\frac{x}{\sqrt{4y_1 \times 3.0}} = \frac{x}{\sqrt{4y_2 \times 5.0}} \quad \text{or} \quad 3y_1 = 5y_2$$

But $$y_1 = y_2 + 2.0$$

$\therefore$ $$3(y_2 + 2.0) = 5y_2$$

$\therefore$ $$2y_2 = 6.0$$

$\therefore$ $$y_2 = 3.0$$

From (*ii*), $$C_{v_2} = \frac{x}{\sqrt{4y_2 \times H_2}}$$

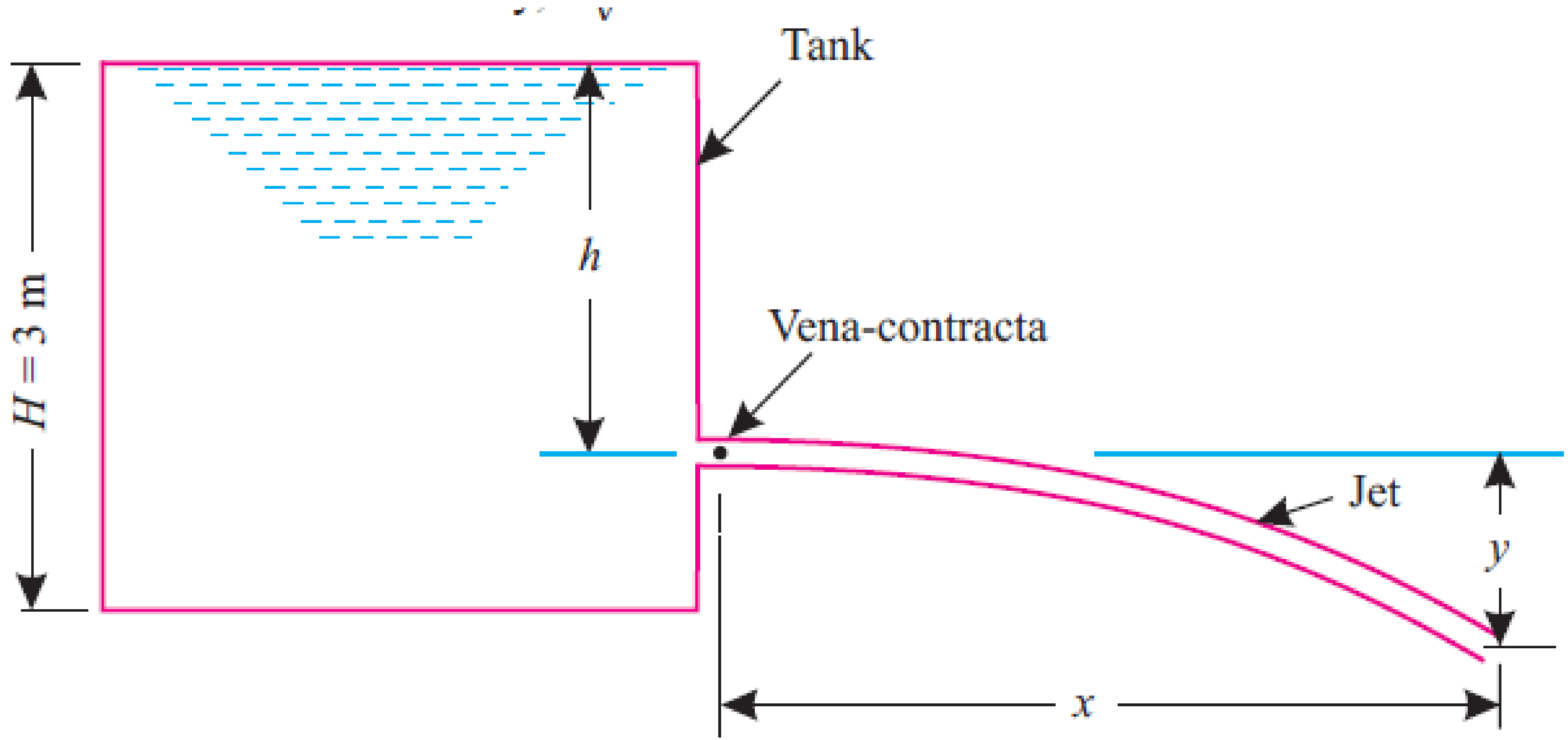
or $$0.96 = \frac{x}{\sqrt{4 \times 3.0 \times 5.0}}$$

$\therefore$ $x = 0.96 \times \sqrt{4 \times 3.0 \times 5.0}$ = **7.436 m. Ans.**

Practice Problem#11 (Rajput, Page= 457)

A 3 m tank standing on the ground is kept full of water. There is a small orifice in its vertical side with its center at depth h meters below the free surface of liquid in the tank. Find the value of h so that the liquid strikes the ground at the maximum distance from the tank. Assuming Cv= 0.97, calculate the maximum value of horizontal distance.

Practice Problem#11 (Rajput, Page= 457)



Flow through Large Orifice

- When available head of a liquid is less than 5 times the height of orifice, the orifice is called large orifice.
- In case of small orifice, the velocity is considered to be constant in the entire cross section and the discharge is calculated by the formula:

  Q= C_d × a × $\sqrt{2gH}$.
- But in case of large orifice, the velocity of a liquid, flowing through the orifice varies with the available head of the liquid and hence Q can not be calculated as mentioned above.

Flow through Large Orifice

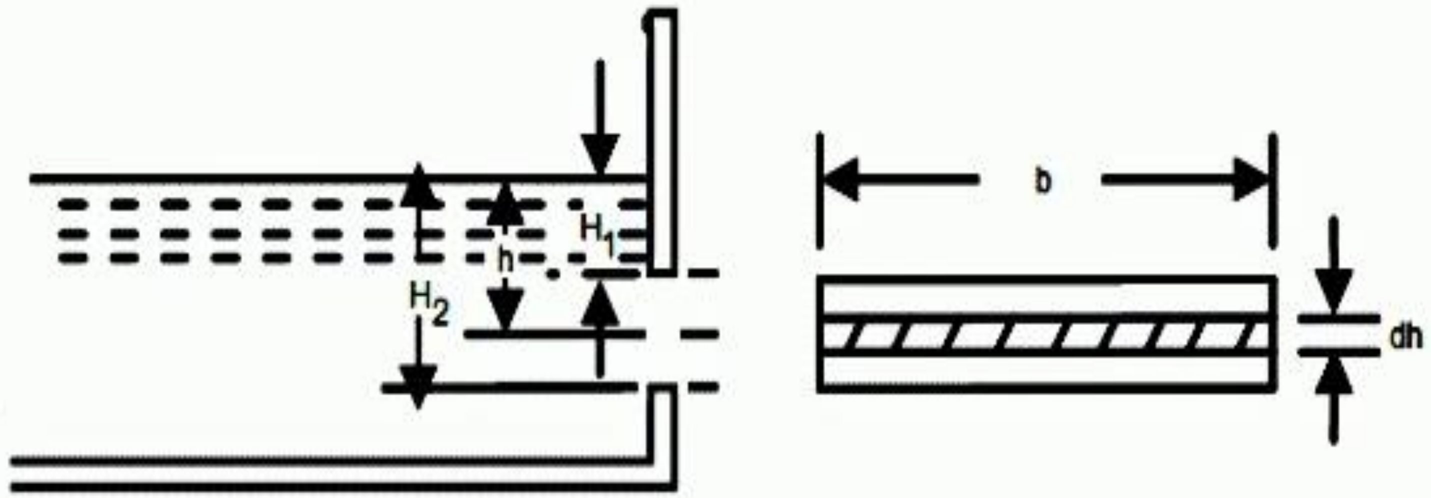


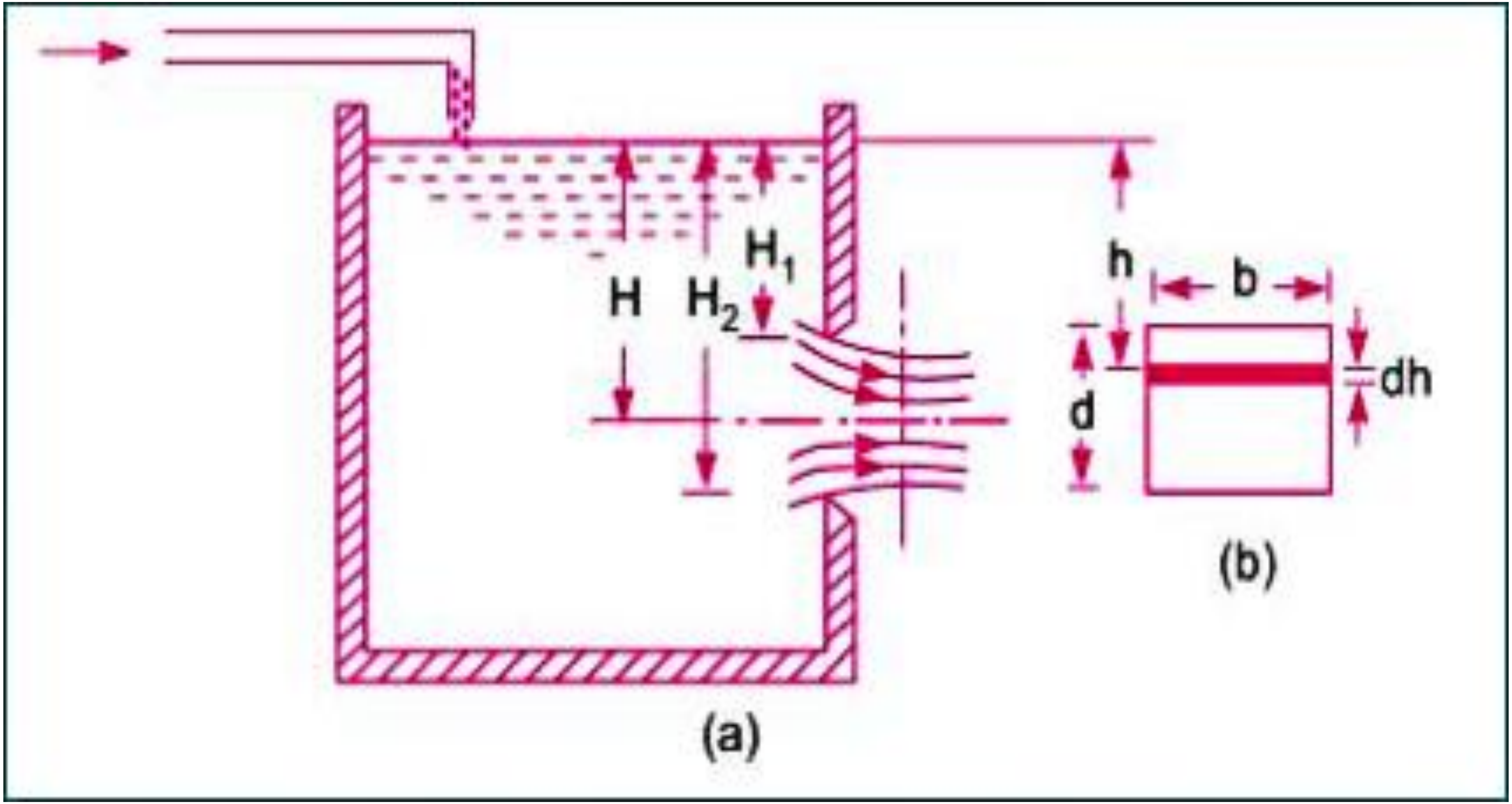
Fig: Large rectangular orifice

Discharge Q can be calculated as : $Q = C_d \times (b \times d) \times \sqrt{2gH}$

b= width of orifice

d = depth of orifice

Discharge through a large Rectangular Orifice



Discharge through a large Rectangular Orifice

Consider a large rectangular orifice in one side of the tank discharging freely into atmosphere under a constant head, H as shown in figure.

Consider an elementary horizontal strip ‘dh’ at a depth of h below the free surface of the liquid in the tank as shown in figure. Let,

H_1= Height of liquid above top edge of orifice

H_2= Height of liquid above bottom edge of orifice

C_d= Co efficient of discharge

b = Width of orifice

d = Depth of orifice = $H_2 - H_1$

$\therefore$ Area of strip = $b \times dh$

and theoretical velocity of water through strip = $\sqrt{2gh}$.

$\therefore$ Discharge through elementary strip is given

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity}$$

$$= C_d \times b \times dh \times \sqrt{2gh} = C_d b \times \sqrt{2gh}\ dh$$

By integrating the above equation between the limits H_1 and H_2, the total discharge through the whole orifice is obtained

$$\therefore \quad Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh}\ dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\ dh = C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b\ \sqrt{2g} \left[H_2^{3/2} - H_1^{3/2}\right]. \qquad \text{...(7.8)}$$

Practice Problem #12

A rectangular orifice 0.9m wide and 1.2m deep is discharging water from a vessel. The top edge of the orifice is 0.6 m below the water surface in the vessel. Calculate the discharge through the orifice if C_d= 0.60 and percentage of error if the orifice is treated as a small orifice.

Information given,

b= 0.9 m

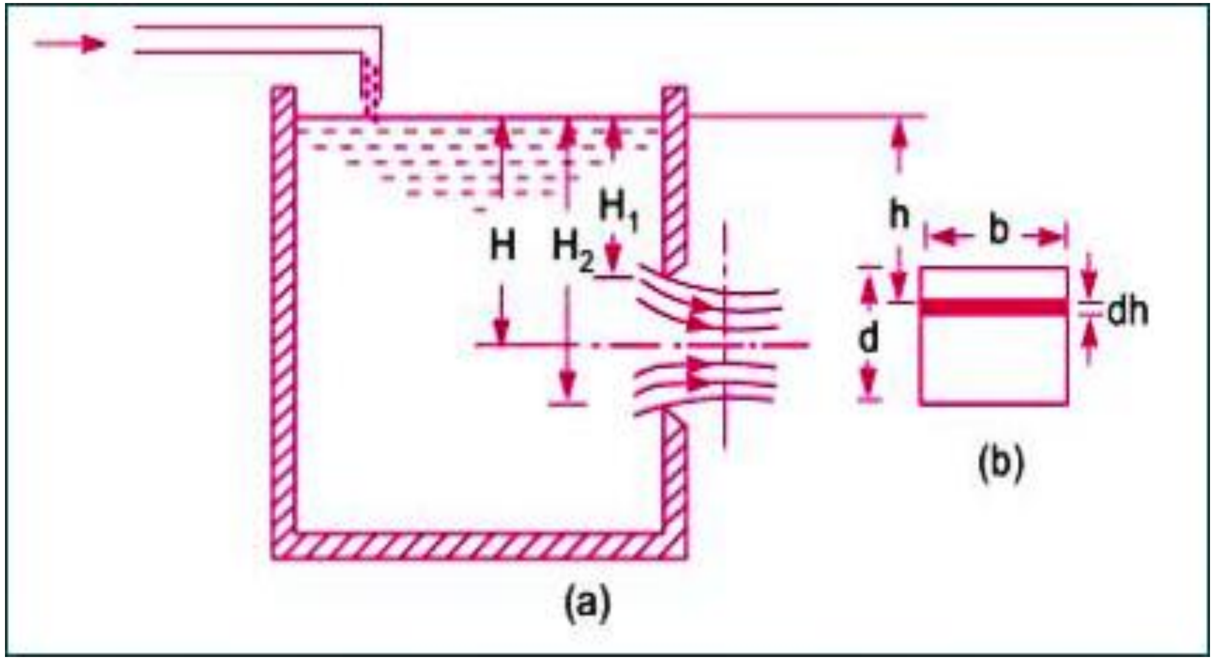
d= 1.2 m

H_1= 0.6

d= $H_2 - H_1$

So, H_2= 1.8 m

C_d= 0.60



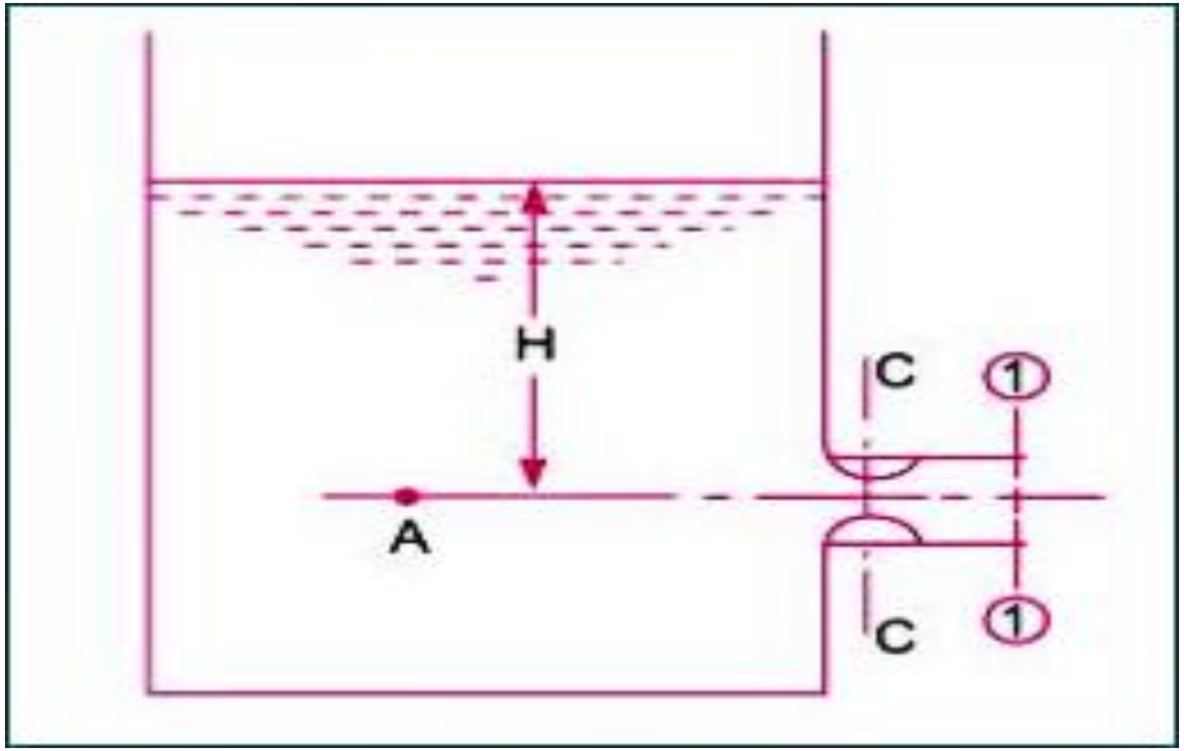
Solution

Discharge Q is given as
$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g} \times [H_2^{3/2} - H_1^{3/2}]$$
$$= \frac{2}{3} \times 0.6 \times 2.9 \times \sqrt{2 \times 9.81}\ [1.8^{3/2} - 0.6^{3/2}]\ \text{m}^3/\text{s}$$
$$= 1.5946\ [2.4149 - .4647] = \mathbf{3.1097\ m^3/s.}$$ **Ans.**

Discharging for a small orifice
$$Q_1 = C_d \times a \times \sqrt{2gh}$$
where $h = H_1 + \frac{d}{2} = 0.6 + \frac{1.2}{2} = 1.2$ m and $a = b \times d = 0.9 \times 1.2$

$$Q_1 = 0.6 \times .9 \times 1.2 \times \sqrt{2 \times 9.81 \times 1.2} = 3.1442\ \text{m}^3/\text{s}$$
$$\%\ \text{error} = \frac{Q_1 - Q}{Q} = \frac{3.1442 - 3.1097}{3.1097} = \mathbf{0.01109}\ \text{or}\ \mathbf{1.109\%}.$$ **Ans.**

Mouthpiece

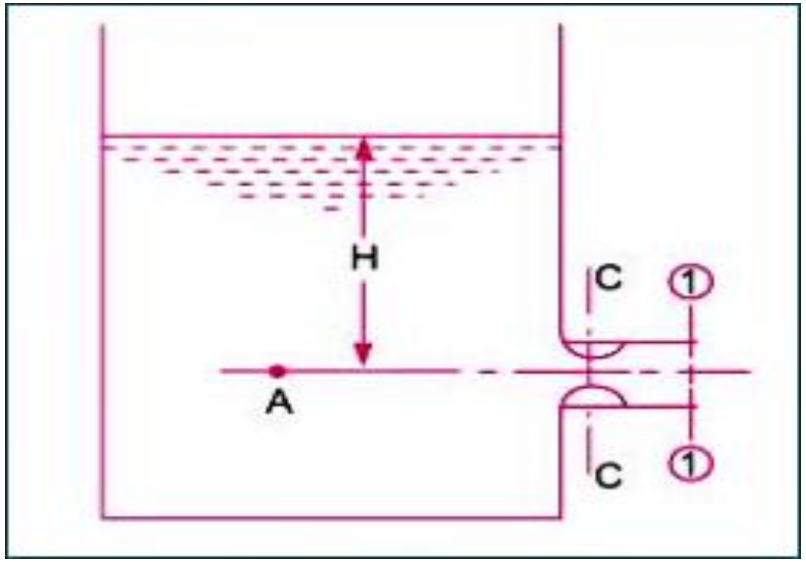


- A mouthpiece is an attachment in the form of a small tube or pipe fixed to the orifice.
- The length of pipe extension is usually 2 to 3 times the orifice diameter and is used to increase the amount of discharge.

Classification of Mouthpiece

Parameter	Type
According to Position of Mouthpiece	1. Internal Mouthpiece
	2. External Mouthpiece
According to shape of Mouthpiece	1. Cylindrical Mouthpiece
	2. Convergent Mouthpiece
	3. Convergent- Divergent Mouthpiece
According to nature of discharge	1. Mouthpiece running full
	2. Mouthpiece running free

Flow through an External Cylindrical Mouthpiece



- Consider a tank having an external cylindrical mouthpiece of cross-sectional area a_1 attached to one of its sides as shown in figure.
- The jet of liquid entering the mouthpiece contracts to form vena contracta at a section C-C.
- Beyond this section, the jet again expands and fill the mouthpiece completely.

Fig. 7.13 *External cylindrical mouthpieces.*

Let H = Height of liquid above the centre of mouthpiece

v_c = Velocity of liquid at C-C section

a_c = Area of flow at vena-contracta

v_1 = Velocity of liquid at outlet

a_1 = Area of mouthpiece at outlet

C_c = Co-efficient of contraction.

Applying continuity equation at C-C and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$\therefore \quad v_c = \frac{a_1 v_1}{a_c} = \frac{v_1}{a_c / a_1}$$

But $$\frac{a_c}{a_1} = C_c = \text{Co-efficient of contraction}$$

Taking $C_c = 0.62$, we get $\frac{a_c}{a_1} = 0.62$

$$\therefore \quad v_c = \frac{v_1}{0.62}$$

The jet of liquid from section C-C suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L* which is given as $h_L = \frac{(v_c - v_1)^2}{2g}$

But $v_c = \dfrac{v_1}{0.62}$ hence $h_L = \dfrac{\left(\dfrac{v_1}{0.62} - v_1\right)^2}{2g} = \dfrac{v_1^2}{2g}\left[\dfrac{1}{0.62} - 1\right]^2 = \dfrac{0.375\, v_1^2}{2g}$

Applying Bernoulli's equation to point A and (1)-(1)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, v_A is negligible,

$\dfrac{p_1}{\rho g}$ = atmospheric pressure = 0

$$\therefore \quad H + 0 = 0 + \frac{v_1^2}{2g} + .375\,\frac{v_1^2}{2g}$$

$$\therefore \quad H = 1.375\,\frac{v_1^2}{2g}$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\,\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is $v_{th} = \sqrt{2gH}$

Discharge through Mouthpiece is greater than Orifice.

∴ Co-efficient of velocity for mouthpiece

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

C_c for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus $$C_d = C_c \times C_v = 1.0 \times .855 = 0.855$$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice, and so discharge through mouthpiece will be more.

Practice Problem#13 (Bansal = 343 page)

An external cylindrical mouthpiece of diameter 150 mm is discharging water under a constant head of 6 m. Determine the discharge and absolute pressure head of water at vena contracta .

Take, C_d= 0.855 and C_c for vena-contracta = 0.62. Atmospheric pressure head = 10.3 m of water.

Time Required for Emptying a Hemispherical Tank

A Hemispherical tank of diameter 4 m contains water upto a height of 1.5 m. An orifice of diameter 50 mm is provided at the bottom. Find the time required by water

1. to fall from 1.5 m to 1 m
2. for completely emptying the tank.

Take $C_d = 0.6$

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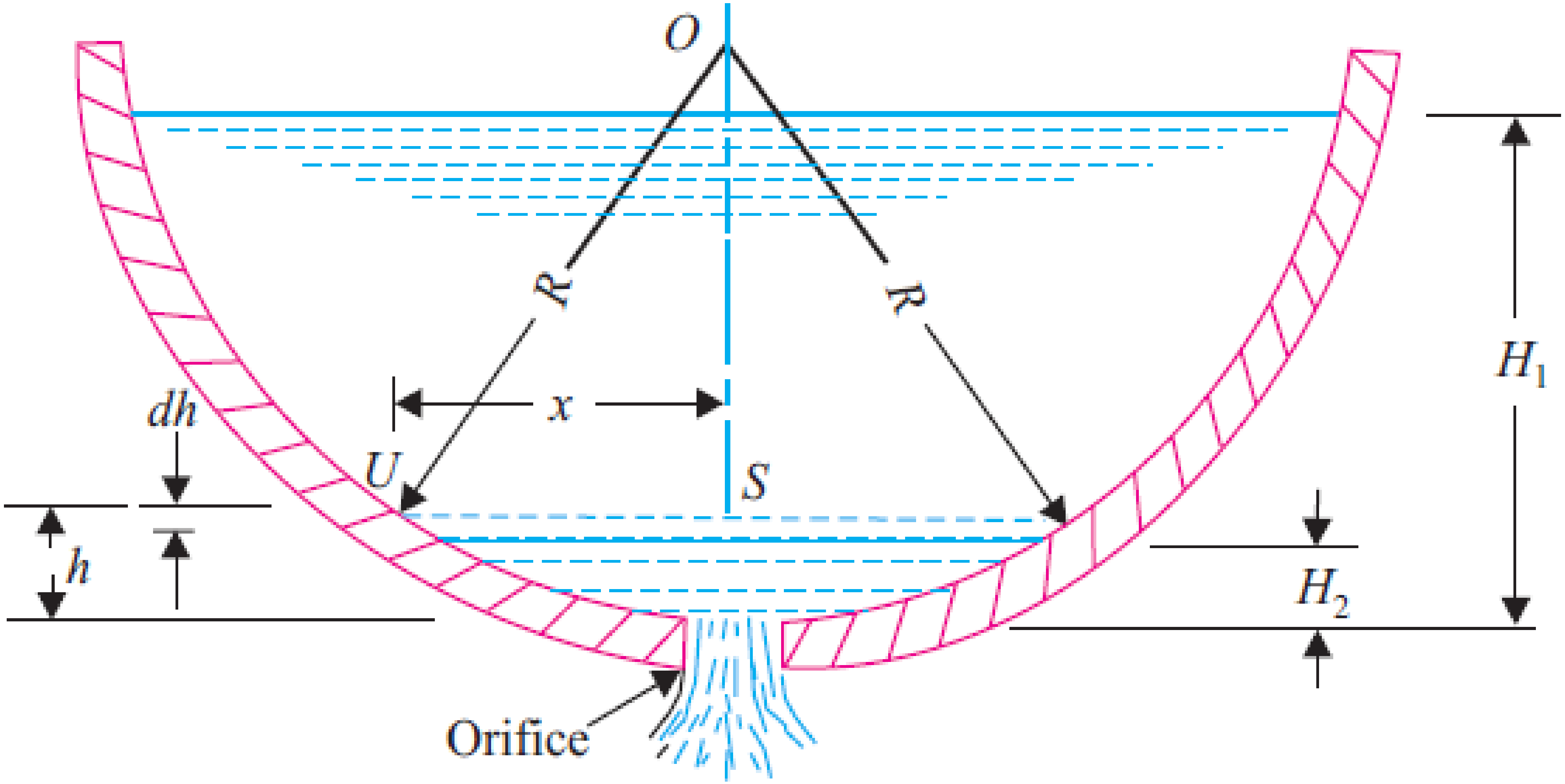
Bansal: Problem No. 7.19 (Page 336)

Time Required for Emptying a Hemispherical Tank



Time Required for Emptying a Hemispherical Tank

Let us Consider

R = Radius of the tank

a = Area of the orifice

H_1= Initial height of the liquid

H_2= Final height of the liquid

T = Time in seconds for the liquid to fall from H_1 to H_2

Step: Determine small time interval dT

Let at any instant of time, the height of liquid over the orifice is h and x be the radius of the liquid surface.

Then, area of liquid surface, $A = \pi x^2$

Theoretical velocity of liquid $= \sqrt{2gh}$

Let the height of liquid decrease by dh in a small interval of time dT. Then,

Volume of liquid leaving the tank in time dT

$$= A.dh = \pi x^2 \times dh \qquad \ldots(i)$$

Also, volume of liquid flowing through the orifice in time dT

$$= C_d \times \text{area of orifice} \times \text{velocity} \times dT$$

$$= C_d \cdot a \cdot \sqrt{2gh} \times dT \qquad \ldots(ii)$$

Equating (i) and (ii), we get:

$$\pi x^2 (-dh) = C_d \cdot a\sqrt{2gh} \times dT$$

The negative sign accounts for the *decrease in head* on the orifice with *increase in time interval*.

$$dT = \frac{-\pi x^2 \cdot dh}{C_d \cdot a \cdot \sqrt{2gh}} \qquad ..(iii)$$

Step: Determine small time interval dT

From Fig. 8.19, we have:

$$OU = R \text{ and } OS = (R-h)$$

$$\therefore \quad x = US = \sqrt{OU^2 - OS^2} = \sqrt{R^2 - (R-h)^2}$$

$$= \sqrt{R^2 - R^2 - h^2 + 2Rh} = \sqrt{2Rh - h^2}$$

or

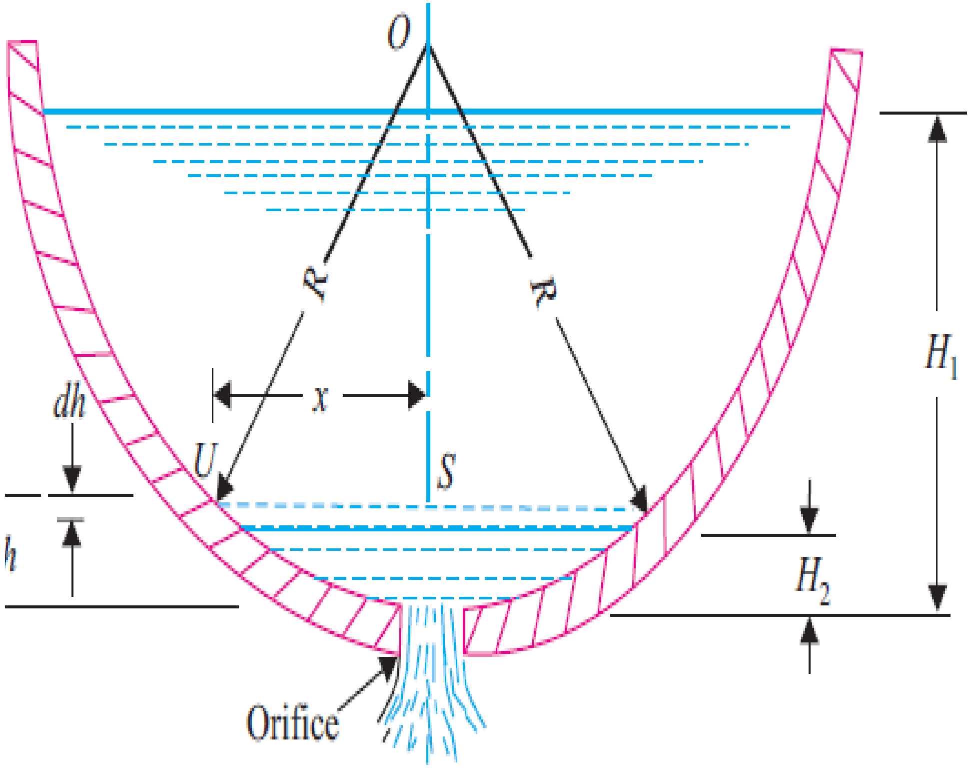
$$x^2 = (2RH - h^2)$$

Substituting this value of x^2 in eqn. (*iii*), we get:

$$dT = \frac{-\pi\,(2Rh - h^2)\,dh}{C_d . a . \sqrt{2gh}}$$

$$= \frac{-\pi}{C_d . a . \sqrt{2g}} = (2Rh - h^2)\, h^{-1/2}\, dh$$

$$= \frac{-\pi}{C_d . a . \sqrt{2g}} (2Rh^{1/2} - h^{3/2})\, dh$$



The total time T required to bring the liquid level from H_1 to H_2 is obtained by integrating the above equation between the limits H_1 to H_2.

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-\pi}{C_d . a\sqrt{2g}} (2Rh^{1/2} - h^{3/2})\, dh$$

Integrate dT to get total time T

$$T = \frac{-\pi}{C_d . a\sqrt{2g}} \int_{H_1}^{H_2} (2Rh^{1/2} - h^{3/2})\, dh$$

$$= \frac{-\pi}{C_d . a\sqrt{2g}} \left[2R \times \frac{h^{1/2+1}}{\frac{1}{2}+1} - \frac{h^{3/2+1}}{\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d . a\sqrt{2g}} \left[\frac{2}{3} \times 2R\, h^{3/2} - \frac{2}{5} h^{5/2} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d . a\sqrt{2g}} \left[\frac{4}{3} R\left(H_2^{3/2} - H_1^{3/2}\right) - \frac{2}{5}\left(H_2^{5/2} - H_1^{5/2}\right) \right]$$

or

$$T = \frac{\pi}{C_d . a . \sqrt{2g}} \left[\frac{4}{3} R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right) \right] \quad ...(8.15)$$

For emptying the tank completely, $H_2 = 0$ and hence,

$$T = \frac{\pi}{C_d . a . \sqrt{2g}} \left[\frac{4}{3} RH_1^{3/2} - \frac{2}{5} H_1^{5/2} \right] \quad ...(8.16)$$

Flow over Notches and Weirs

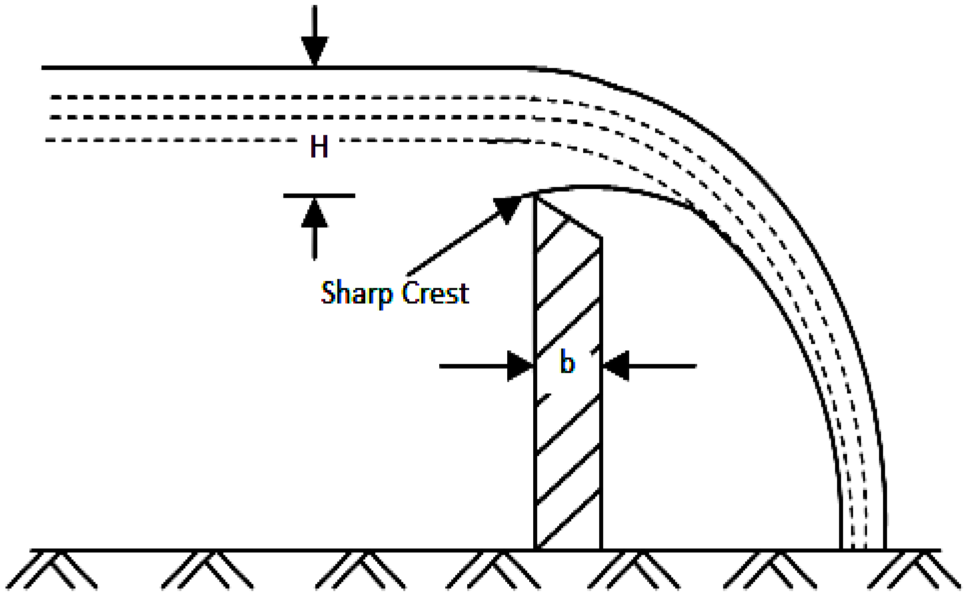


Fig : Sharp-crested weir

Notch and Weir

- Notch may be defined as an opening provided in the side of a tank or vessel such that the liquid surface in the tank is below the top edge of the opening. It is generally made of metallic plate.
- Weir may be defined as any regular obstruction in an open stream over which the flow takes place. It is made of masonry or concrete.

*Notch is sometimes called as a weir and vice versa.

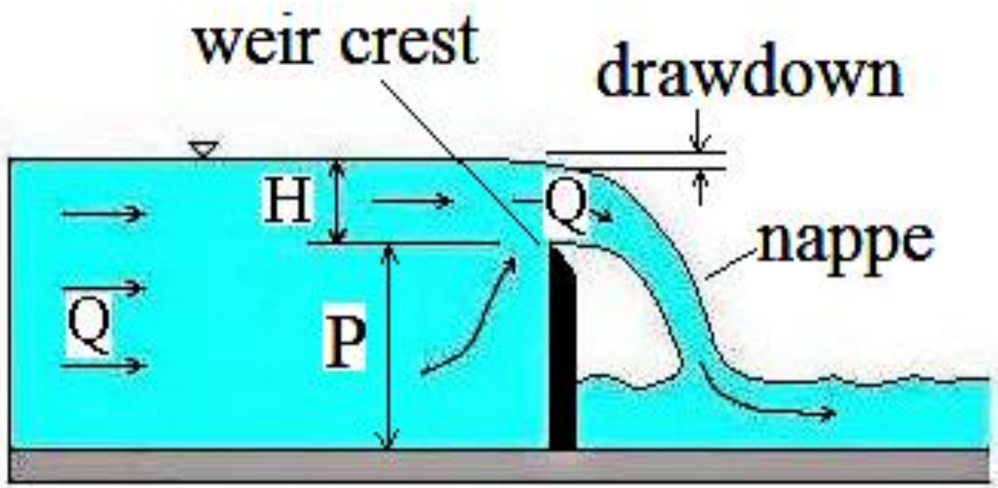
Nappe and Crest

Nappe or Vein:

The sheet of water flowing through a notch or over a weir is known as Nappe or Vein.

Sill or Crest :

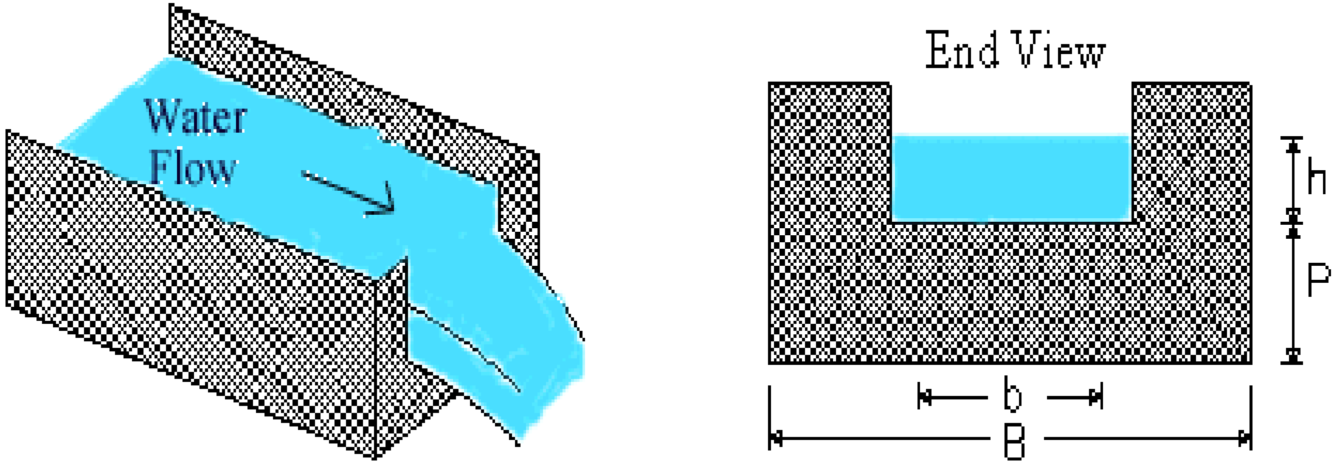
The top of the weir over which the water flows is known as sill or crest.

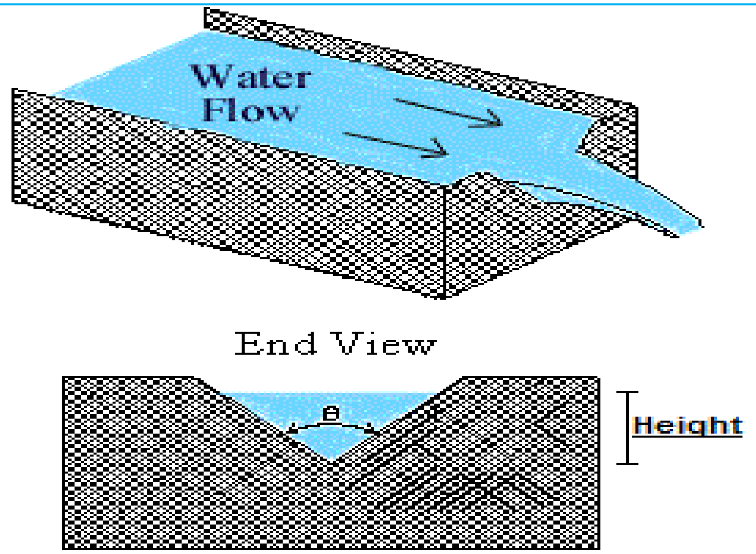


Flow Over a Sharp Crested Weir

Types of Notches

Sl. No.	Type of Notch
1.	Rectangular Notch
2.	Triangular Notch
3.	Trapezoidal Notch
4.	Stepped Notch





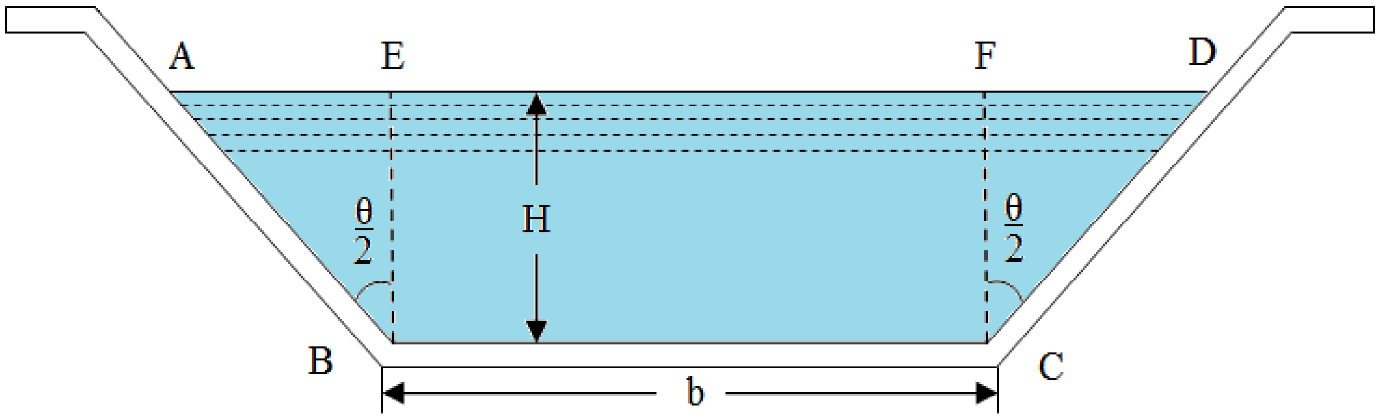


Fig : Trapezoidal Notch

Stepped Notch

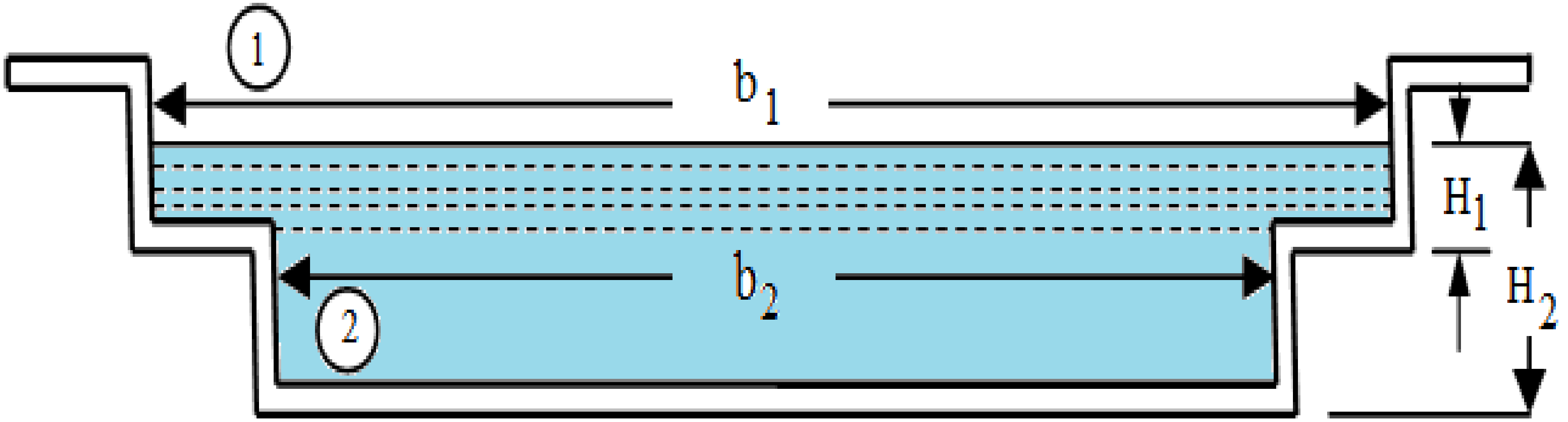


Fig : Stepped Notch

Types of Weir

Parameter	Type	
Shape	1. Rectangular Weir	2. Cippoletti weir
Nature of Discharge	1. Ordinary Weir	2. Submerged Weir
Width of Crest	1. Narrow Crested Weir	2. Broad Crested Weir
Nature of Crest	1. Sharp Crested Weir	2. Ogee Weir

Discharge over a Rectangular Notch or Weir

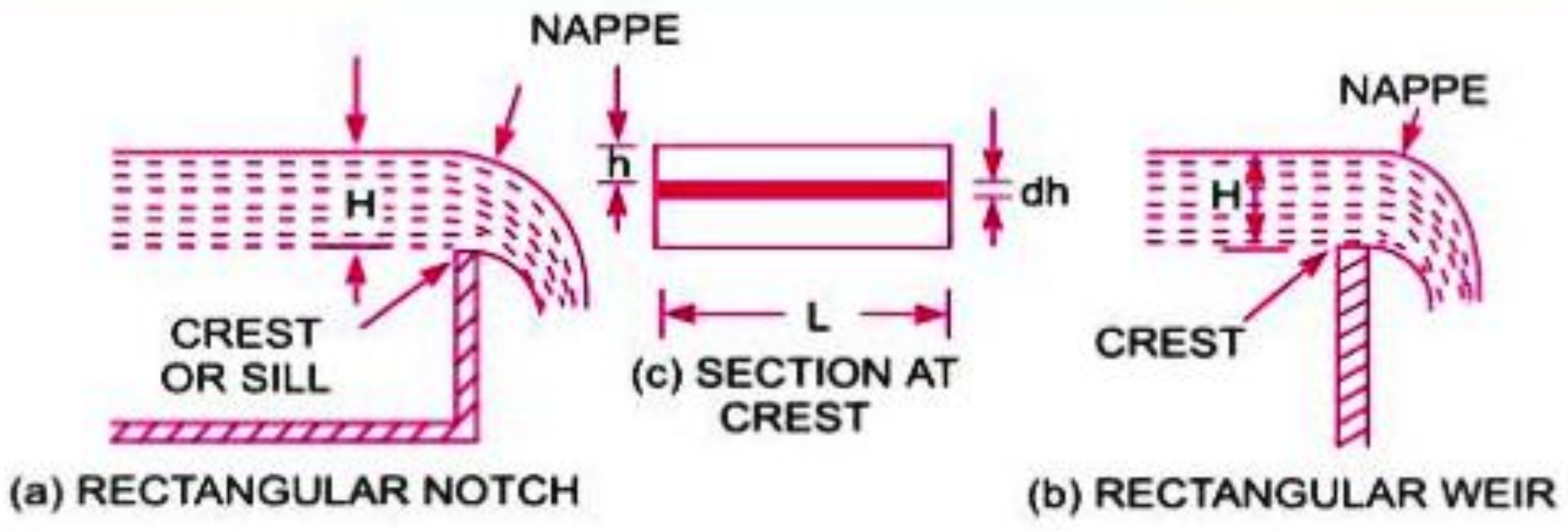


Fig. 8.1 *Rectangular notch and weir.*

Head of Water over the crest= H

Length of the Notch or Weir =L

Consider an elemental horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in figure.

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip = $\sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \qquad \text{...}(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H.

$$\therefore \qquad Q = \int_0^H C_d \,.\, L \,.\, \sqrt{2gh} \,.\, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2}\, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g}\ [H]^{3/2}. \qquad \text{...}(8.1)$$

Practice Problem#14

Determine the height of a rectangular weir of Length 6m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 liters/second. Take, C_d= 0.6.

Given Information,

Length of Weir, L = 6 m

Depth of Water, H_1= 1.8 m

Discharge, Q = 2000 Liters/second = 2 m^3/s

Co-efficient of Discharge, C_d= 0.6

Let, H = Height of water above the crest of weir

H_2= Height of weir

H_1=H_2+H

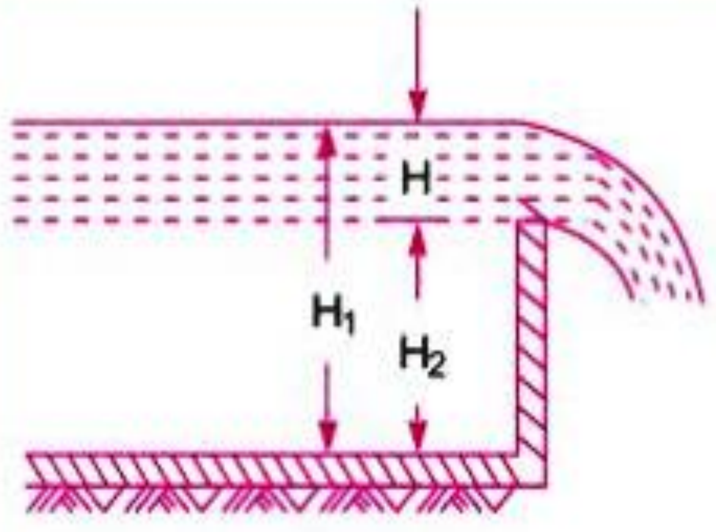


Fig. 8.2

Solution

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$

$$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623\ H^{3/2}$$

$$H^{3/2} = \frac{2.0}{10.623}$$

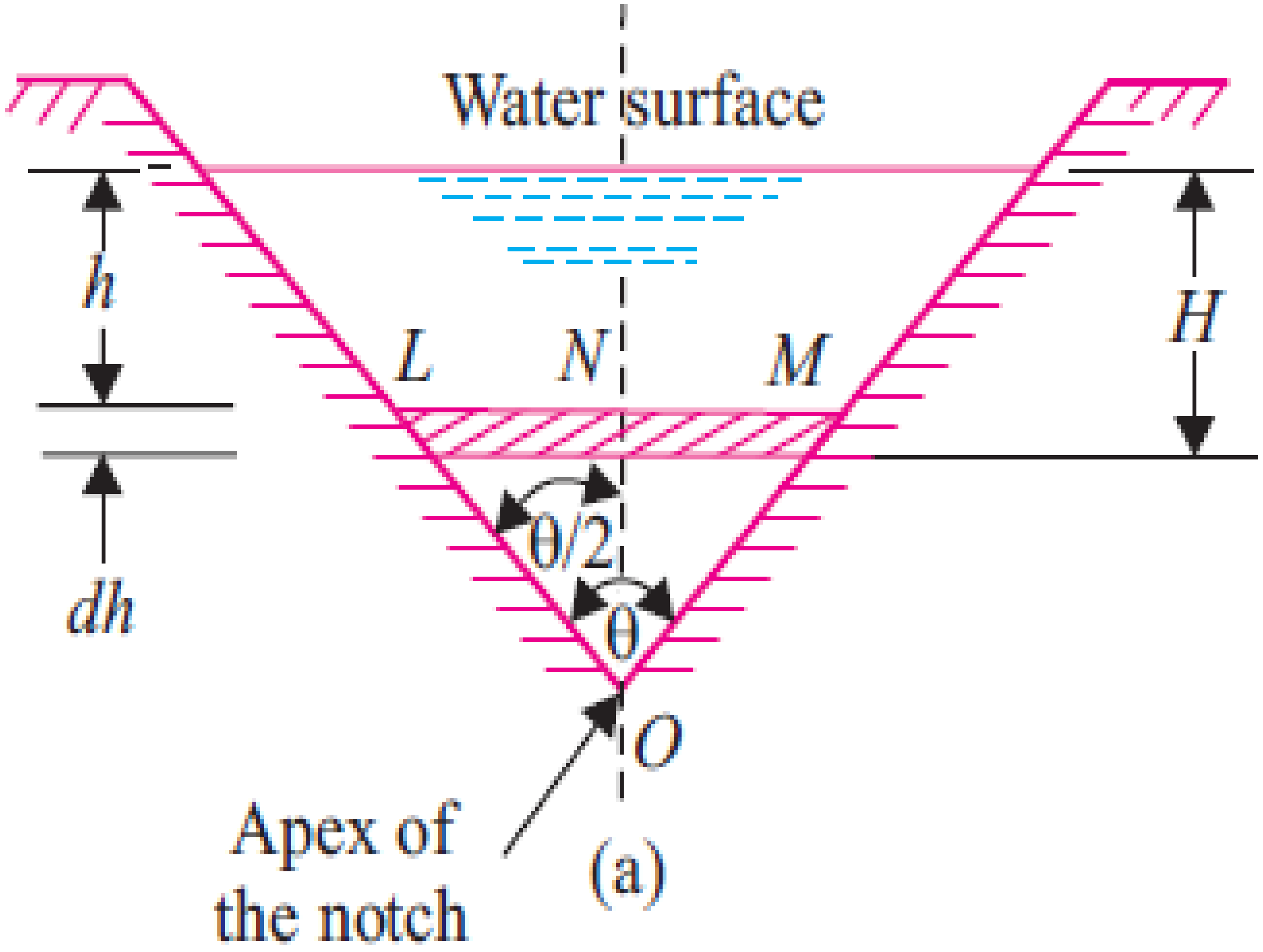
$$H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

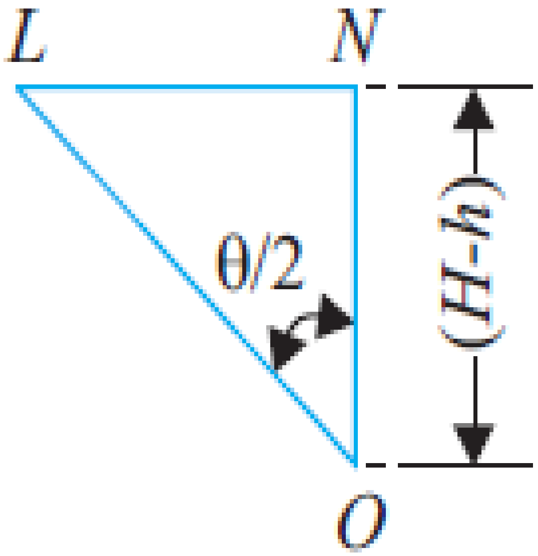
Height of weir, $H_2 = H_1 - H$

$= \text{Depth of water on upstream side} - H$

$= 1.8 - .328 = $ **1.472 m. Ans.**

Discharge over a Triangular Notch or Weir





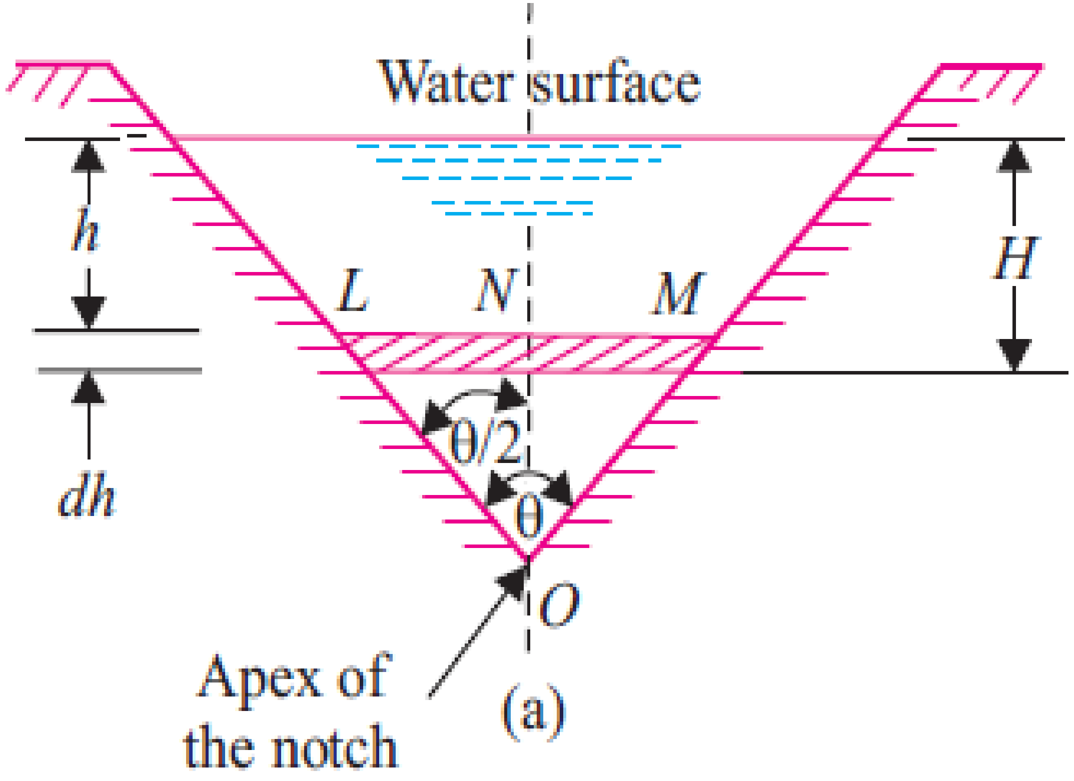
Discharge over a Triangular Notch or Weir

Let,

H = Head of water above V- notch

θ = Angle of Notch

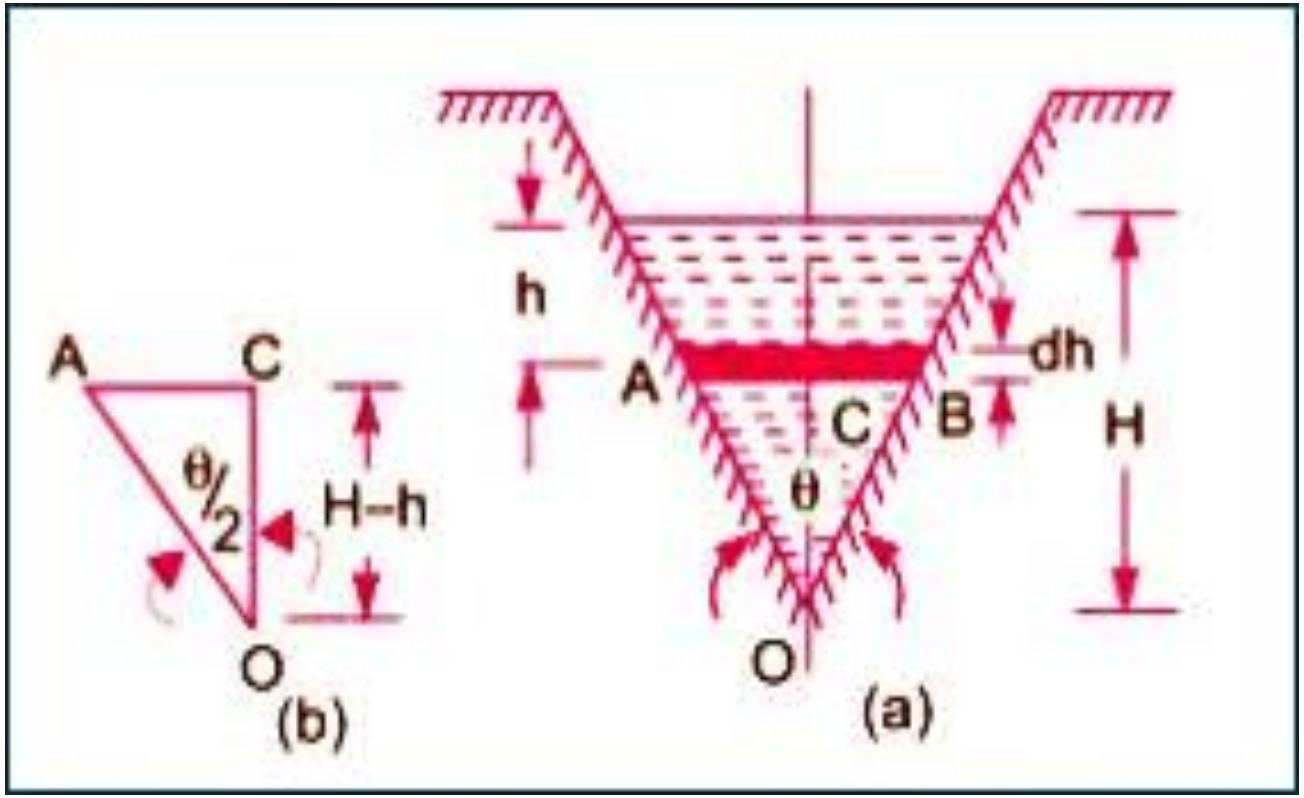
Consider a horizontal strip of water of thickness “dh” at a depth of h from the free surface of water.



From figure b, we get,

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{H-h} \quad ;$$

So, AC = $\tan\frac{\theta}{2} \times (H\text{-}h)$;



Width of strip, AB= 2×AC = $2\times \tan\frac{\theta}{2} \times (H\text{-}h)$

Area of Strip = $2\tan\frac{\theta}{2} \times (H\text{-}h) \times dh$

The theoretical velocity of water through strip = $\sqrt{2gh}$

$\therefore$ Discharge, through the strip,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H - h)\tan\frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H - h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$\therefore$ Total discharge,

$$Q = \int_0^H 2C_d\,(H - h)\tan\frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$= 2C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (H - h)h^{1/2}\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})\,dh$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H.H^{3/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$= 2 \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15} C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

For a right-angled *V*-notch, if $C_d = 0.6$

$$\theta = 90°, \quad \therefore \quad \tan\frac{\theta}{2} = 1$$

Discharge,

$$Q = \frac{8}{15} \times 0.6 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2}$$

$$= 1.417\ H^{5/2}.$$

Practice Problem#15 (Bansal 359 page)

Water flows over a rectangular weir 1m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.

Practice Problem#15 (Bansal 359 page)

Hints:

Discharge over rectangular weir = Discharge over triangular weir

Discharge over rectangular weir, Q = $\frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{\frac{3}{2}}$

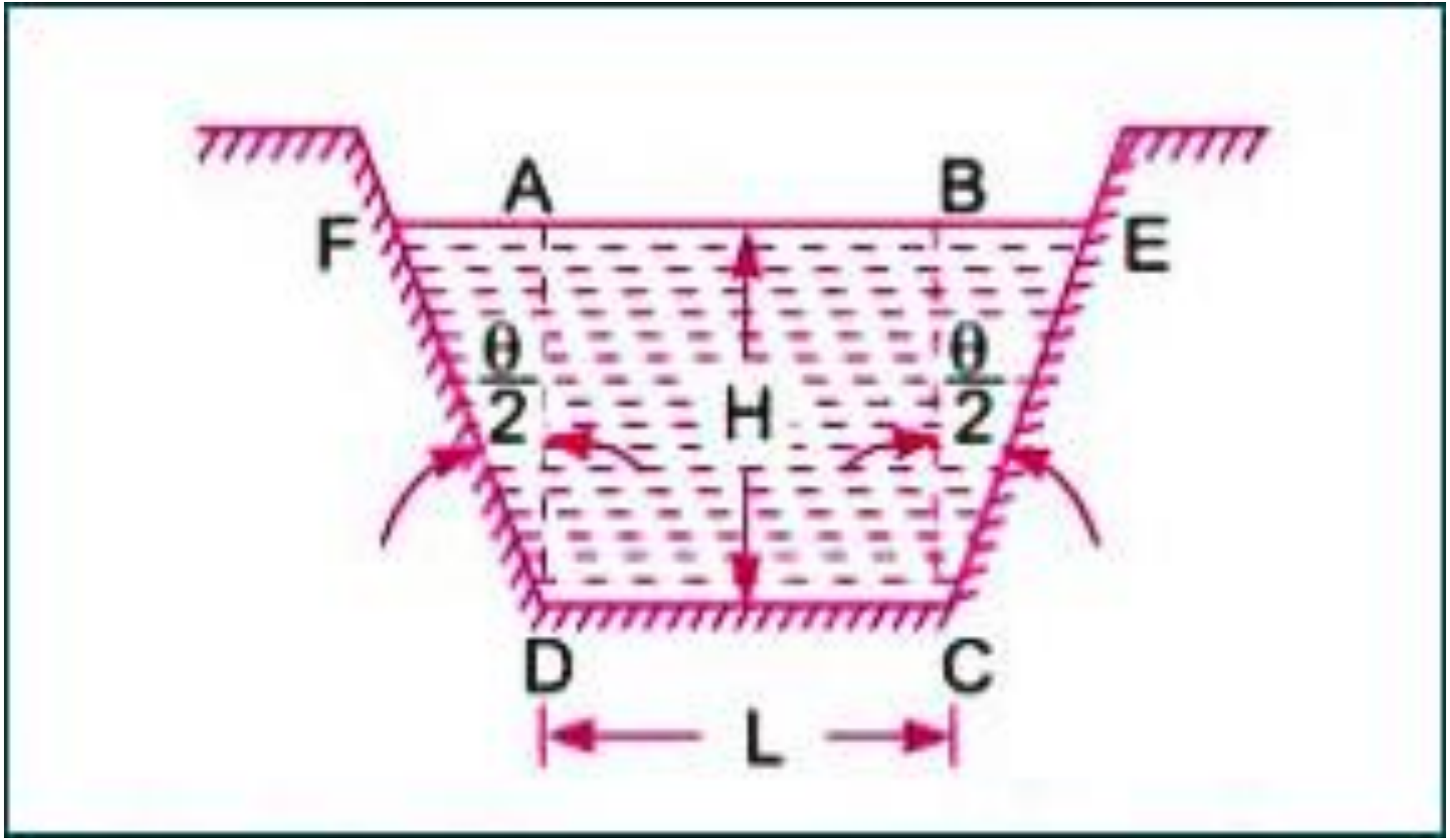
Discharge over triangular weir, Q = $\frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{\frac{5}{2}}$

Answer: 357.2 mm

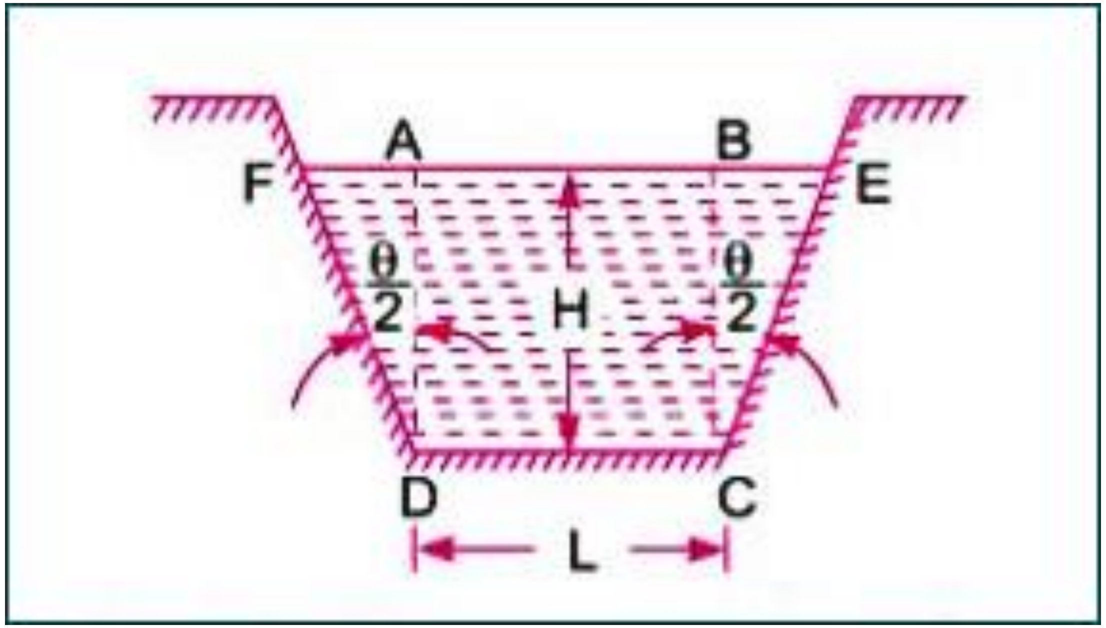
Advantages of Triangular notch over a rectangular weir

1. For a right-angled V notch or weir the expression for the computation of discharge is very simple ($Q = 1.417\, H^{2.5}$)
2. In a given triangular notch, only one reading that is head (H) is required to be taken for the measurement of discharge.
3. For low discharge a triangular notch gives more accurate results than a rectangular notch.

Discharge over a Trapezoidal Notch or Weir

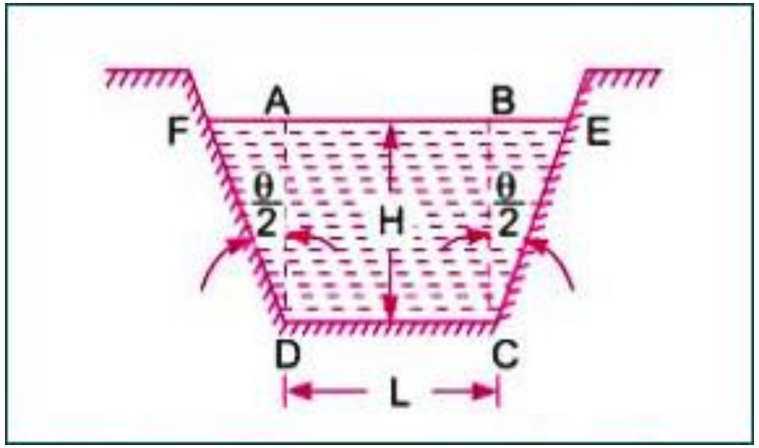


Discharge over a Trapezoidal Notch or Weir



- A trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir.
- Thus, total discharge will be equal to the sum of discharge through rectangular weir+ discharge through triangular weir.

Discharge over a Trapezoidal Notch or Weir



Total Discharge through trapezoidal weir = Discharge through triangular portion (ADF+BCF) + Discharge through rectangular portion (ABCD)

Let us consider,

H = Height of water over the notch.

L = Length of rectangular portion of the notch.

C_{d1}= Co-efficient of discharge for the rectangular portion

C_{d2}= Co-efficient of discharge for the triangular portion

Discharge over a Trapezoidal Notch or Weir

Discharge through rectangular portion ABCD is given by,

$Q_1 = \frac{2}{3} \times \ C_d \times \mathrm{L} \times \sqrt{2g} \times H^{\frac{3}{2}}$

Discharge through two triangular notches ADF and BCE is equal to the discharge through a single triangular notch of angle θ and is given by,

$Q_2 = \frac{8}{15} \times \ C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{\frac{5}{2}}$

Discharge through trapezoidal notch or weir,

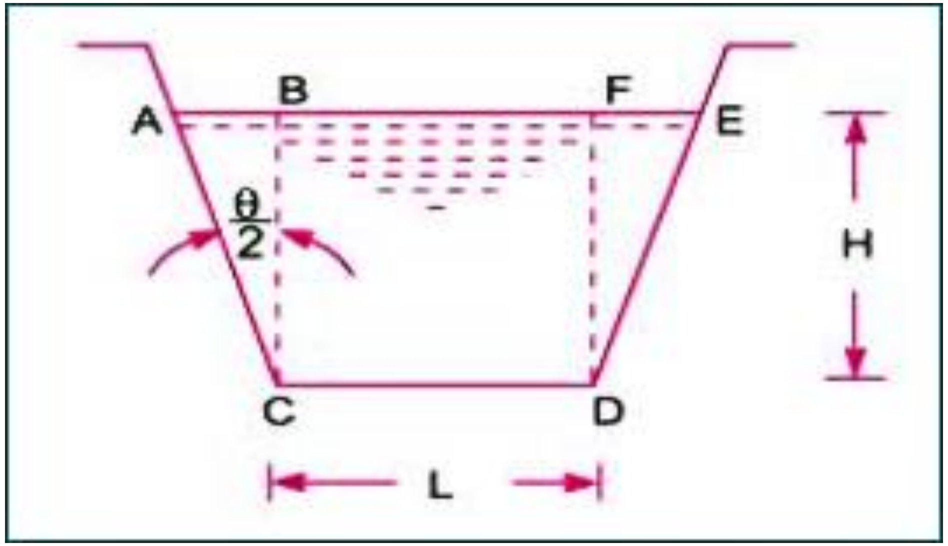
Q

$= Q_1 + Q_2$

$= \frac{2}{3} \times \ C_d \times \mathrm{L} \times \sqrt{2g} \times H^{\frac{3}{2}} + \frac{8}{15} \times \ C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{\frac{5}{2}}$

Practice Problem#16

Find the discharge through a trapezoidal notch which is 1 m wide at the top 0.40 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while triangular portion = 0.60.



Solution:

Given information,

Top width, AE = 1m

Base width, CD = L = 0.40 m

Head of water , H = 0.20 m

For rectangular portion, C_{d1} = 0.62

For triangular portion, C_{d2} = 0.60

Solution

From ΔABC, we have

Fig. 8.5

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{H}$$

$$= \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = \frac{0.3}{0.3} = 1$$

Discharge through trapezoidal notch is given by equation (8.4)

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} + \frac{8}{15} \times .60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$

$= 0.06549 + 0.02535 = 0.09084\ m^3/s =$ **90.84 litres/s. Ans.**

Discharge over Stepped Notch

A stepped notch is a combination of rectangular notches. The discharge through stepped notch is equal to the sum of discharges through different rectangular notches.

Consider a stepped notch as shown in Fig. 8.6.

Let H_1 = Height of water above the crest of notch 1,

L_1 = Length of notch 1,

H_2, L_2 and H_3, L_3 are corresponding values for notches 2 and 3 respectively.

C_d = Co-efficient of discharge for all notches

$\therefore$ Total discharge $Q = Q_1 + Q_2 + Q_3$

or

$$Q = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$+ \frac{2}{3} C_d \times L_2 \times \sqrt{2g}\ [H_2^{3/2} - H_3^{3/2}] + \frac{2}{3} C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}. \qquad \text{...(8.5)}$$

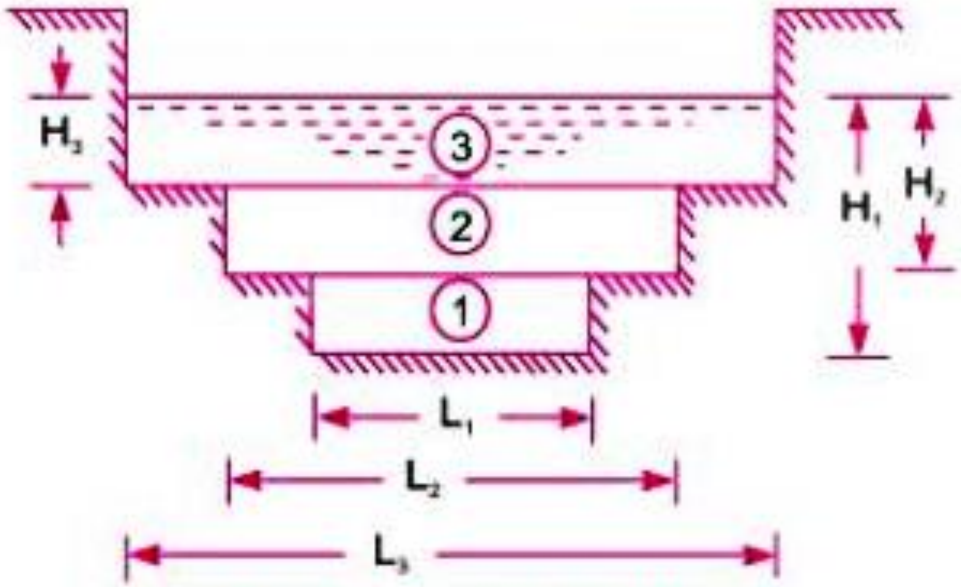


Fig. 8.6 *The stepped notch.*

Practice Problem#17

The following figure shows a stepped notch. Find the discharge through the notch if C_d for all section = 0.62

Solution:

Given information,

C_d=0.62

L_1= 40 cm

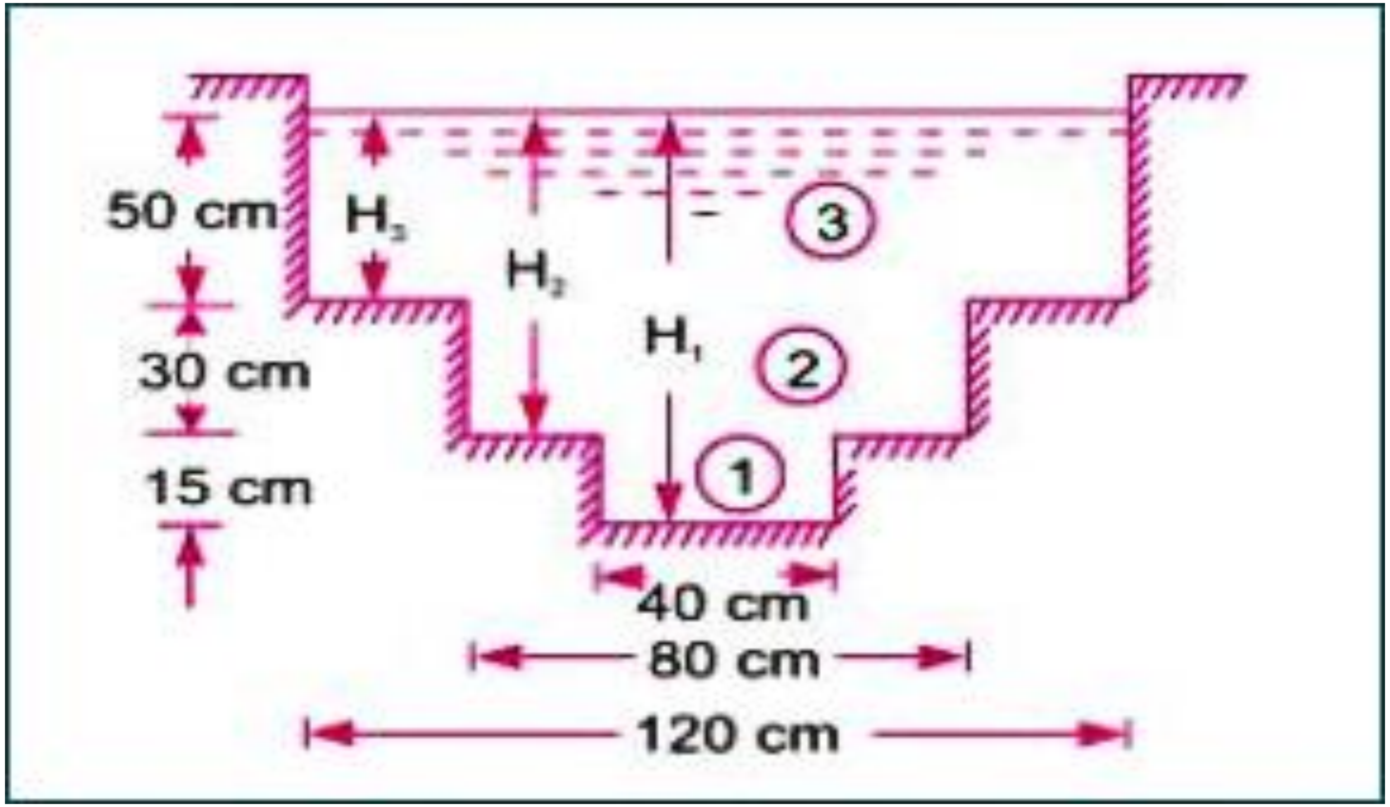
L_2= 80 cm

L_3= 120 cm

H_1= 50+30+15 = 95 cm

H_2= 50+30 = 80 cm

H_3= 50 cm



Solution: Total Discharge = $Q_1 + Q_2 + Q_3$

where $$Q_1 = \frac{2}{3} \times C_d \times L_1 \times \sqrt{2g}\ [H_1^{3/2} - H_2^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 40 \times \sqrt{2 \times 981} \times [95^{3/2} - 80^{3/2}]$$

$$= 732.26[925.94 - 715.54] = 154067 \text{ cm}^3/\text{s} = 154.067 \text{ lit/s}$$

$$Q_2 = \frac{2}{3} \times C_d \times L_2 \times \sqrt{2g} \times [H_2^{3/2} - H_3^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 80 \times \sqrt{2 \times 981} \times [80^{3/2} - 50^{3/2}]$$

$$= 1464.52[715.54 - 353.55] \text{ cm}^3/\text{s} = 530141 \text{ cm}^3/\text{s} = 530.144 \text{ lit/s}$$

and $$Q_3 = \frac{2}{3} \times C_d \times L_3 \times \sqrt{2g} \times H_3^{3/2}$$

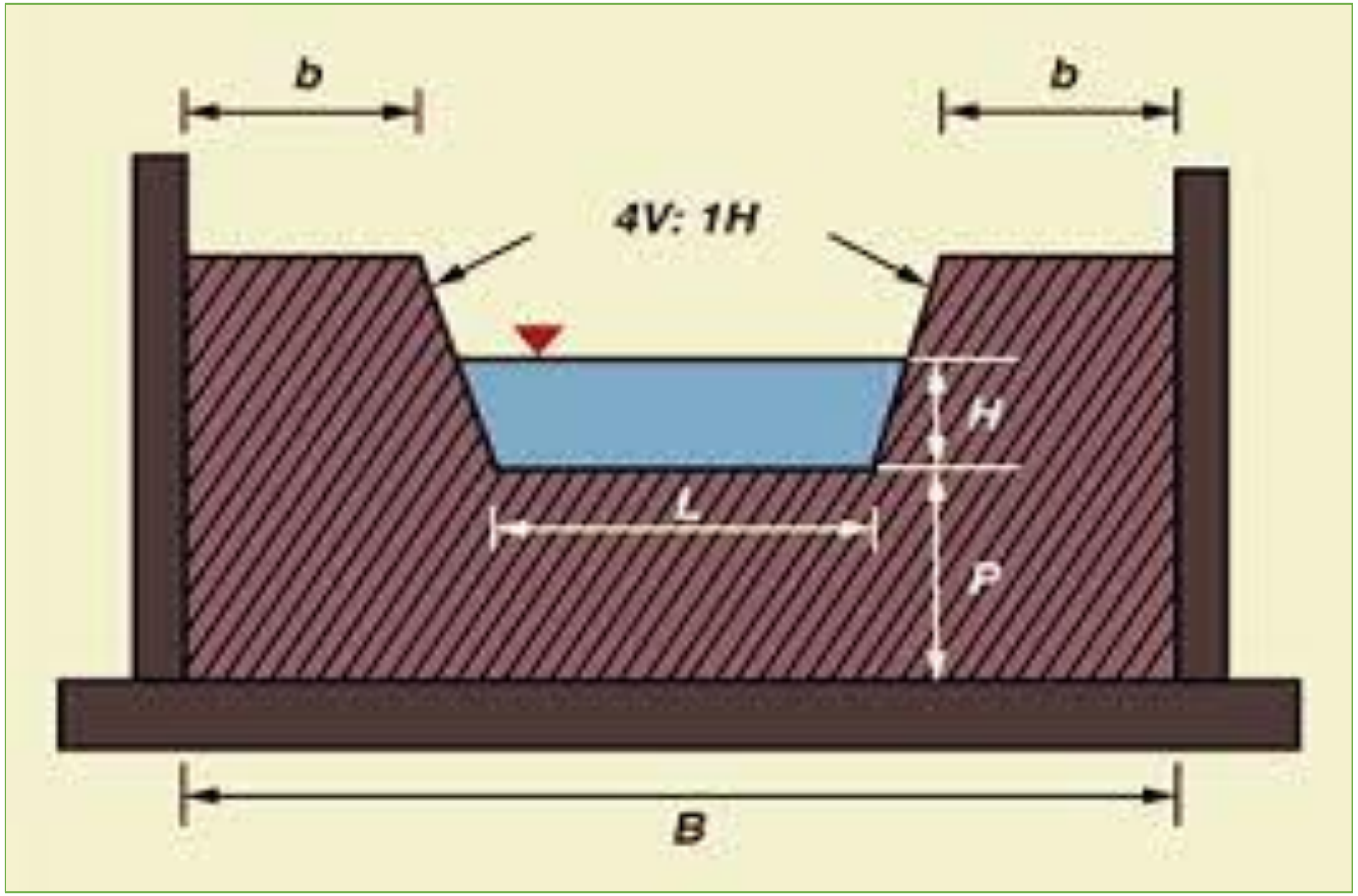
$$= \frac{2}{3} \times 0.62 \times 120 \times \sqrt{2 \times 981} \times 50^{3/2} = 776771 \text{ cm}^3/\text{s} = 776.771 \text{ lit/s}$$

$\therefore$ $$Q = Q_1 + Q_2 + Q_3 = 154.067 + 530.144 + 776.771$$

$$= 1460.98 \text{ lit/s.}$$ **Ans.**

Cipolletti Weir or Notch

Cipolletti weir is a trapezoidal weir which has side slopes of 1 horizontal to 4 vertical.



Broad Crested Weir, Narrow crested Weir

H = Height of water above the crest

L = Length of the crest

2L > H (Broad Crested Weir)

2L < H (Narrow Crested Weir)

Narrow crested weir is similar to rectangular weir and its discharge is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{\frac{3}{2}}$$

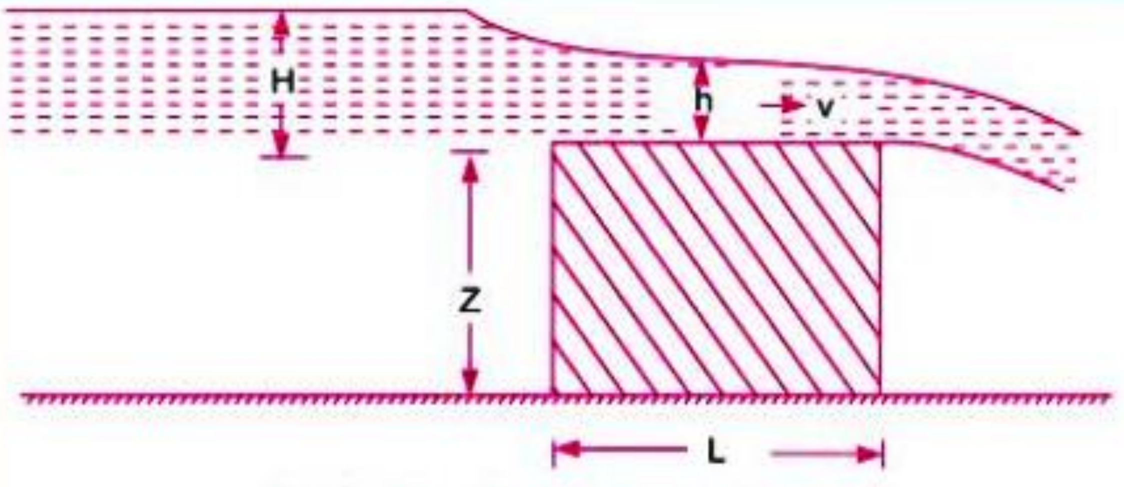


Fig. 8.10 *Broad-crested weir.*

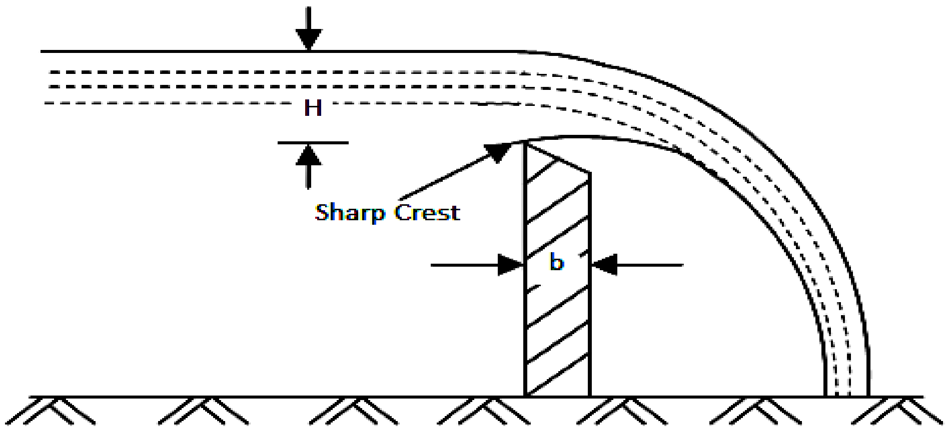


Fig : Sharp-crested weir

Ogee Weir

- The following figure shows an ogee weir in which crest of weir rises up to maximum height of 0.115*H (Height of water above crest of weir.
- Ogee weir is similar to rectangular weir and its discharge is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{\frac{3}{2}}$$

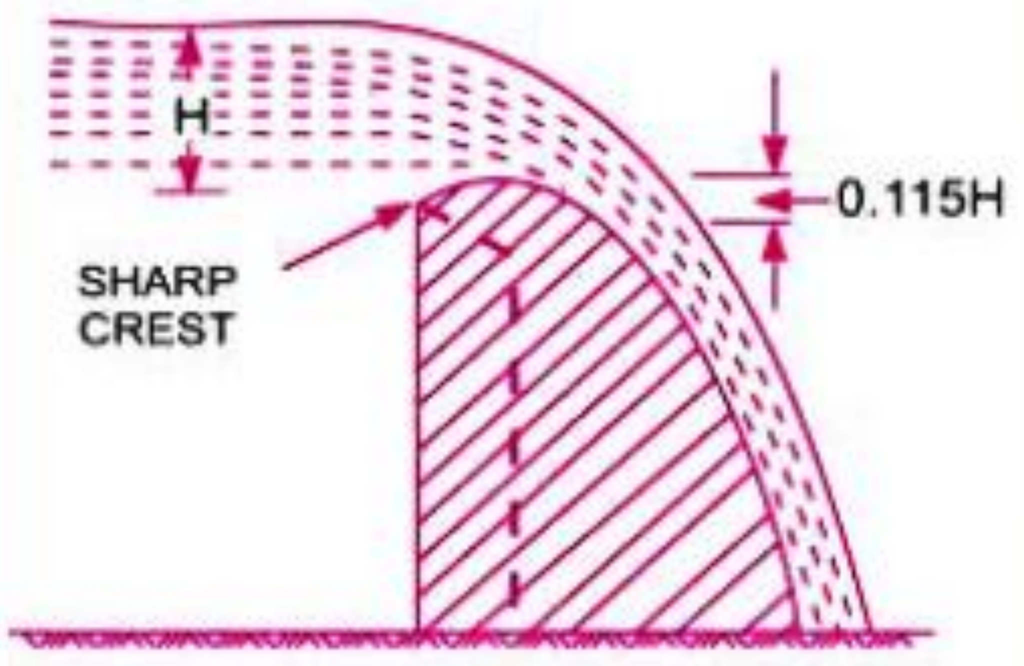


Fig. 8.11 An Ogee weir.

Laminar Flow

জীবনে চলার পথে লেমিনার (Laminar) হতে হয়। তাতে জীবনে চলার পথটা মসৃণ হয় । মানুষের সাথে বিবাদ কম হয়। আর টারবুলেন্ট (Turbulent) হলে আসলে সমস্যা, দিকবিদিক ঠিক থাকেনা, ঝামেলা লেগেই থাকবে।

Laminar Flow

- Laminar means arranged.
- Paths taken by individual fluid particles do not cross one another.

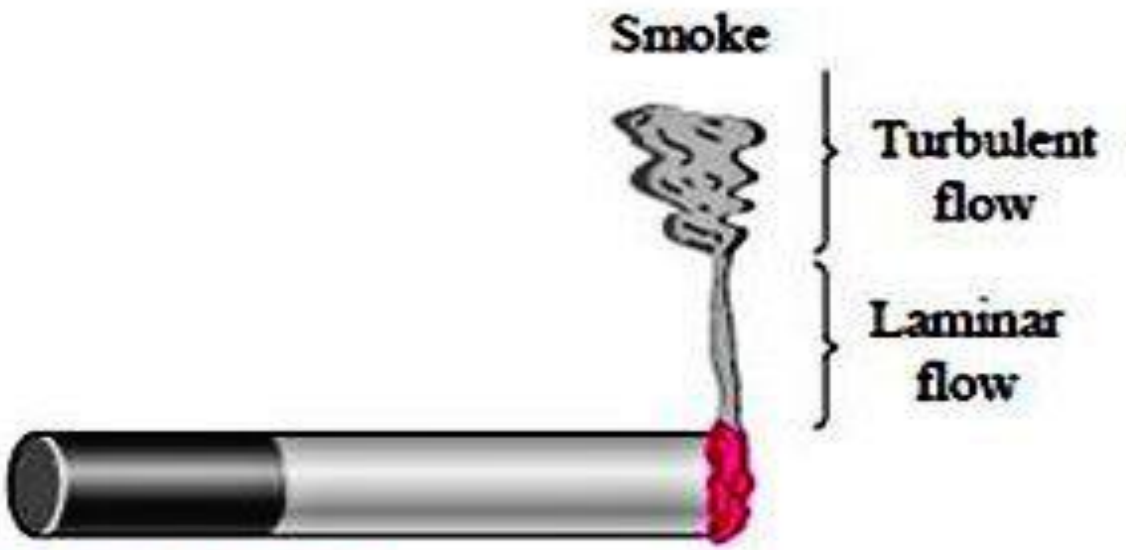
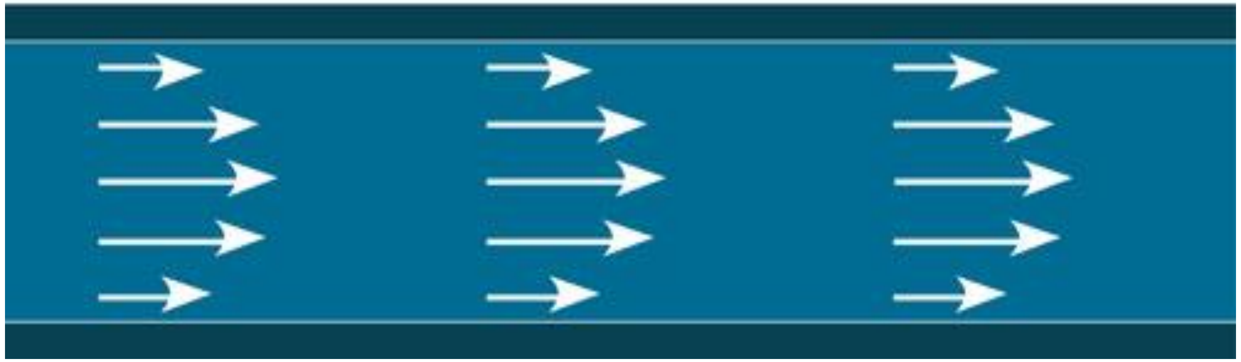


FIGURE 6–14
Laminar and turbulent flow regimes of cigarette smoke.



Newtonian Fluid

These fluids have a linear relationship between viscosity and shear stress.

It can be classified as:

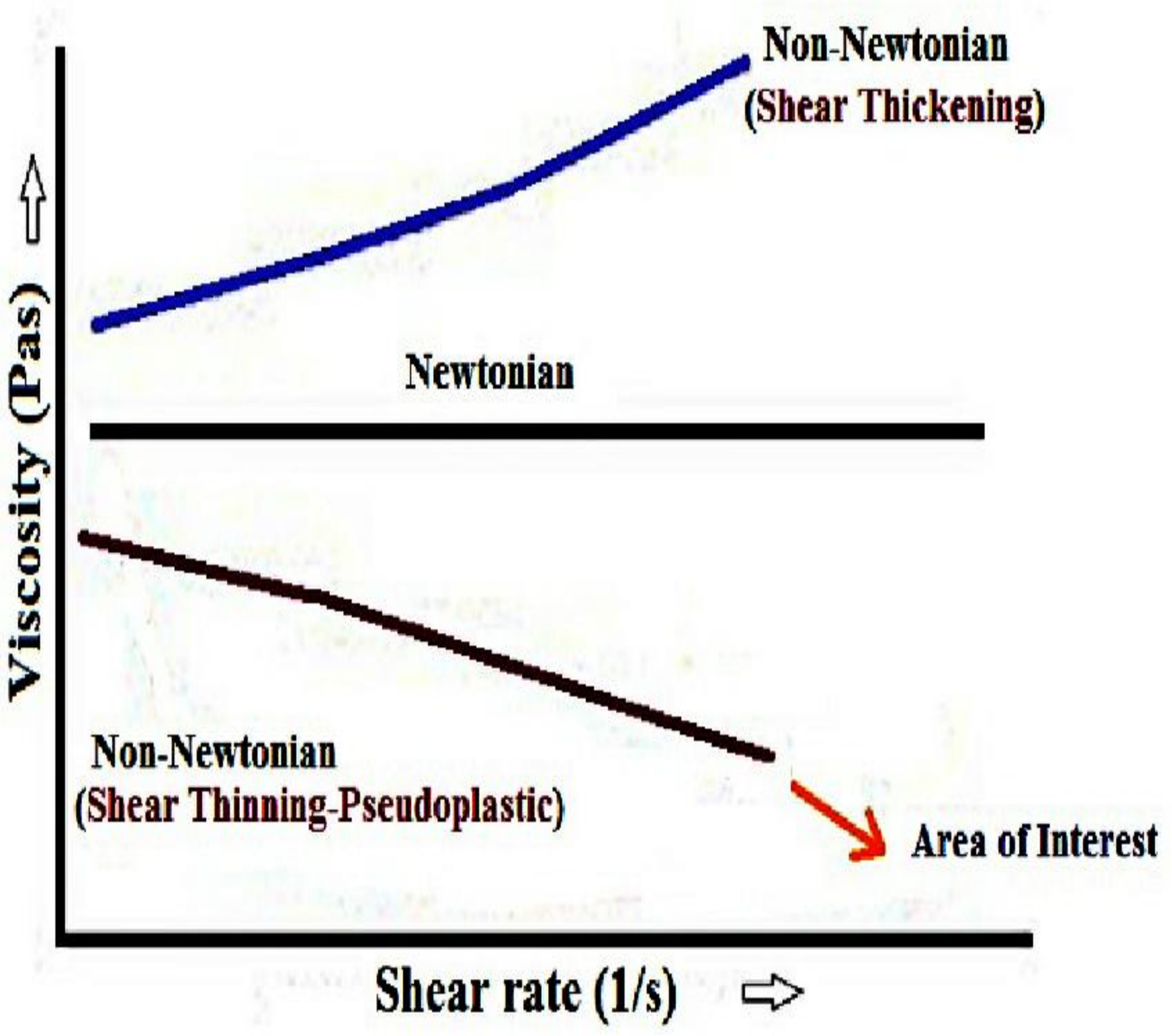
1. Laminar (or viscous)
2. Turbulent

Reynolds number, $R_e = \frac{\rho V d}{\mu}$

For Laminar flow, $R_e < 2000$

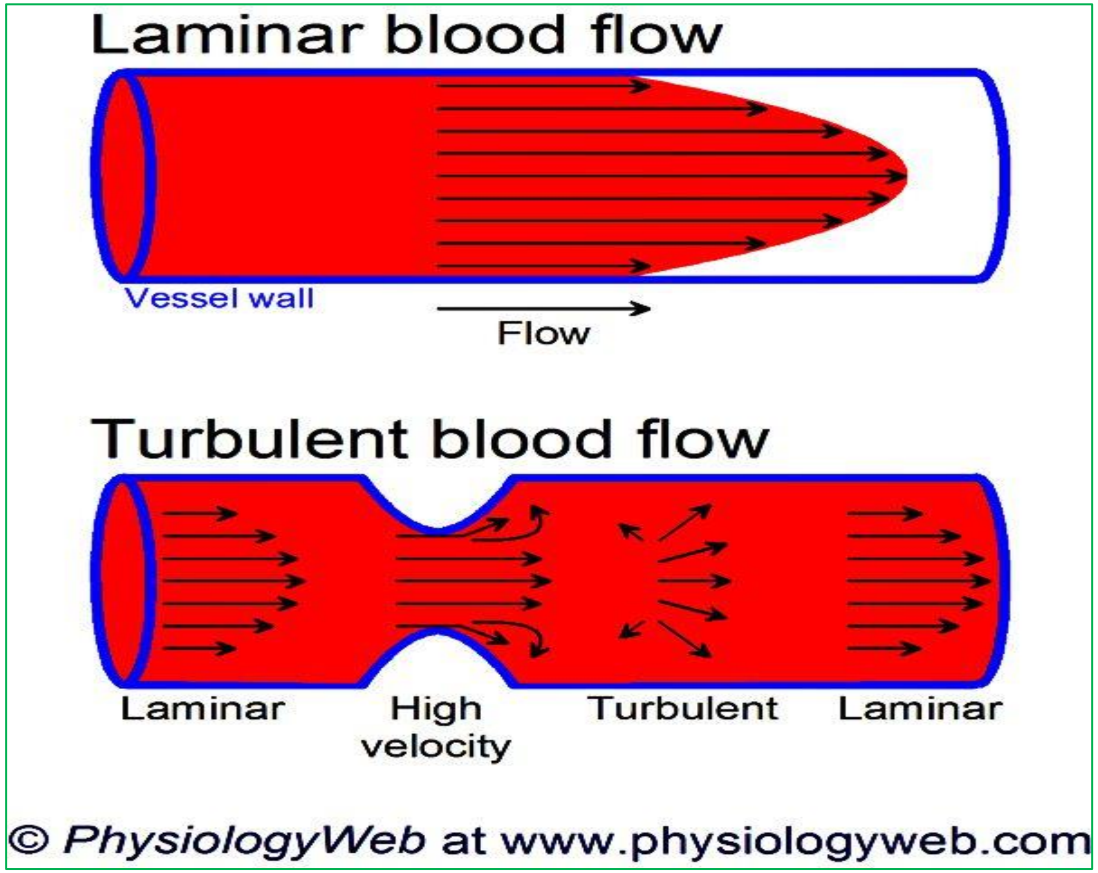
For Turbulent flow, $R_e > 4000$

R_e between 2000 and 4000 indicates transition from laminar to turbulent.

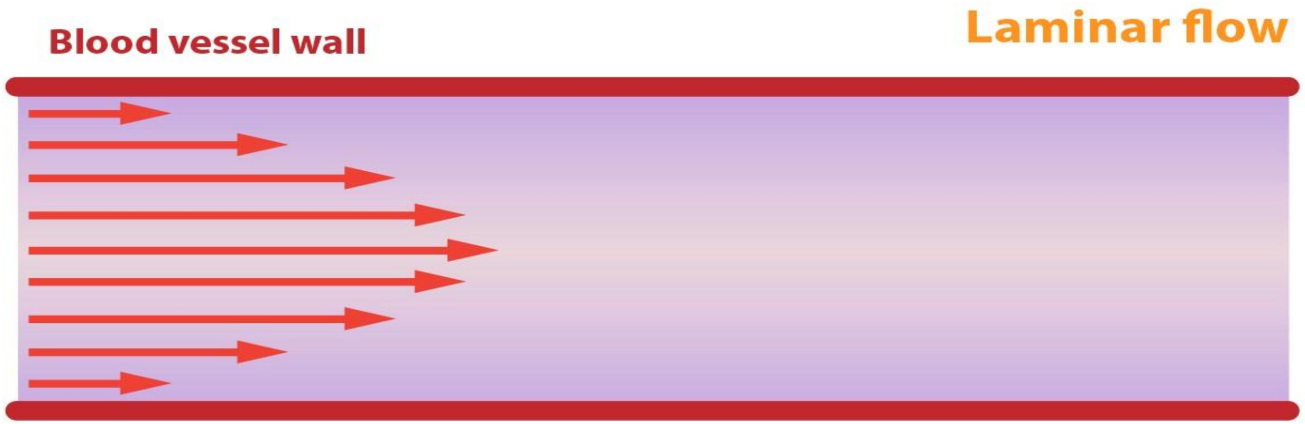


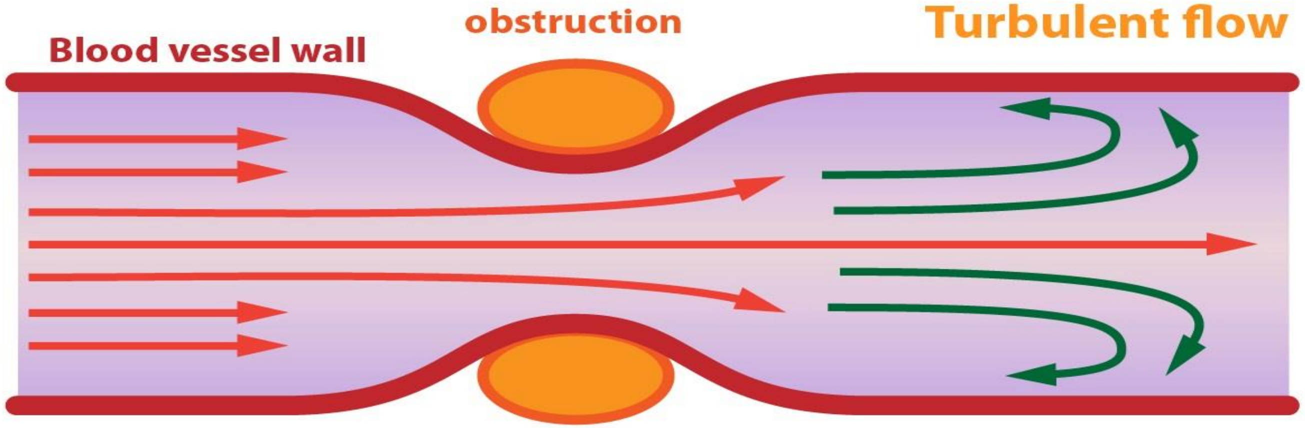
Examples of Laminar/ Viscous flow

1. Flow past tiny bodies.
2. Underground flow.
3. Movement of blood in the arteries of a human body.
4. Flow of oil in measuring instruments.
5. Rise of water in plants through their roots.



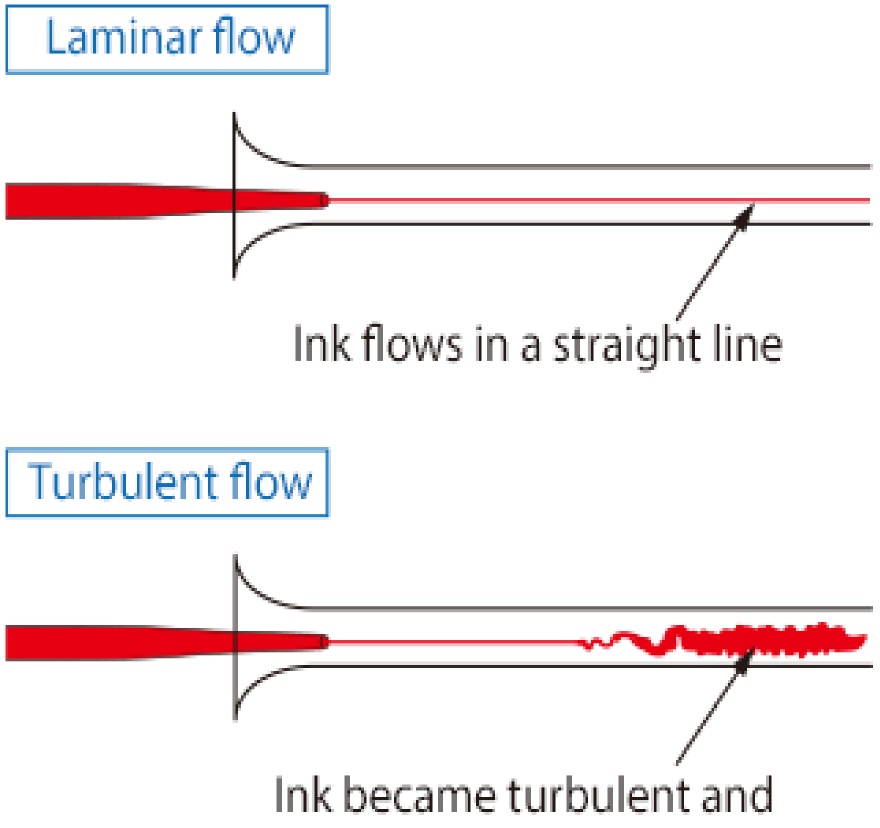
Laminar and Turbulent flow in Blood Vessel



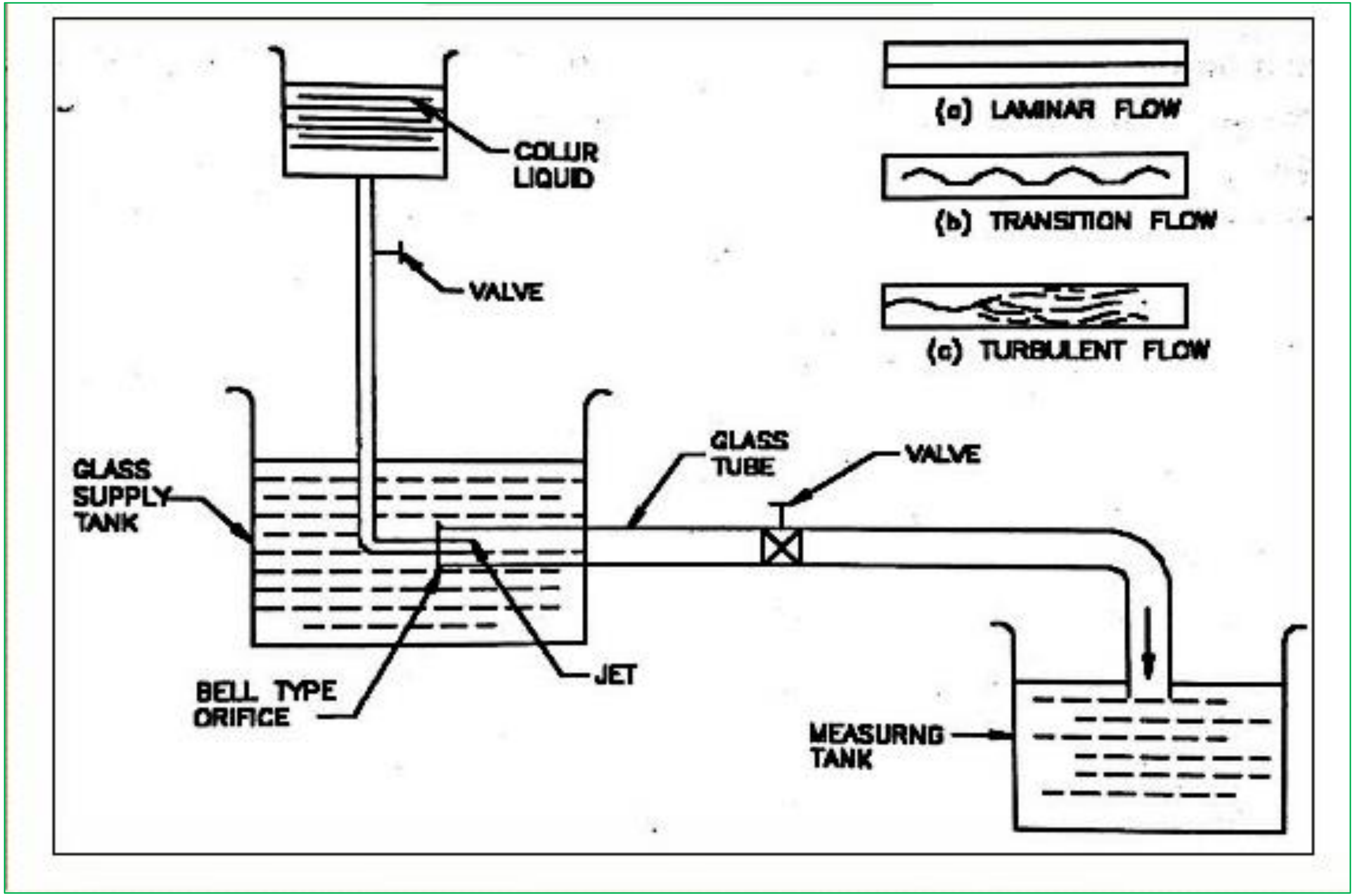


Reynolds Experiment

Osborne Reynolds in 1883, with the help of a simple experiment demonstrated the existence of the following two types of flows.



Reynolds Experiment



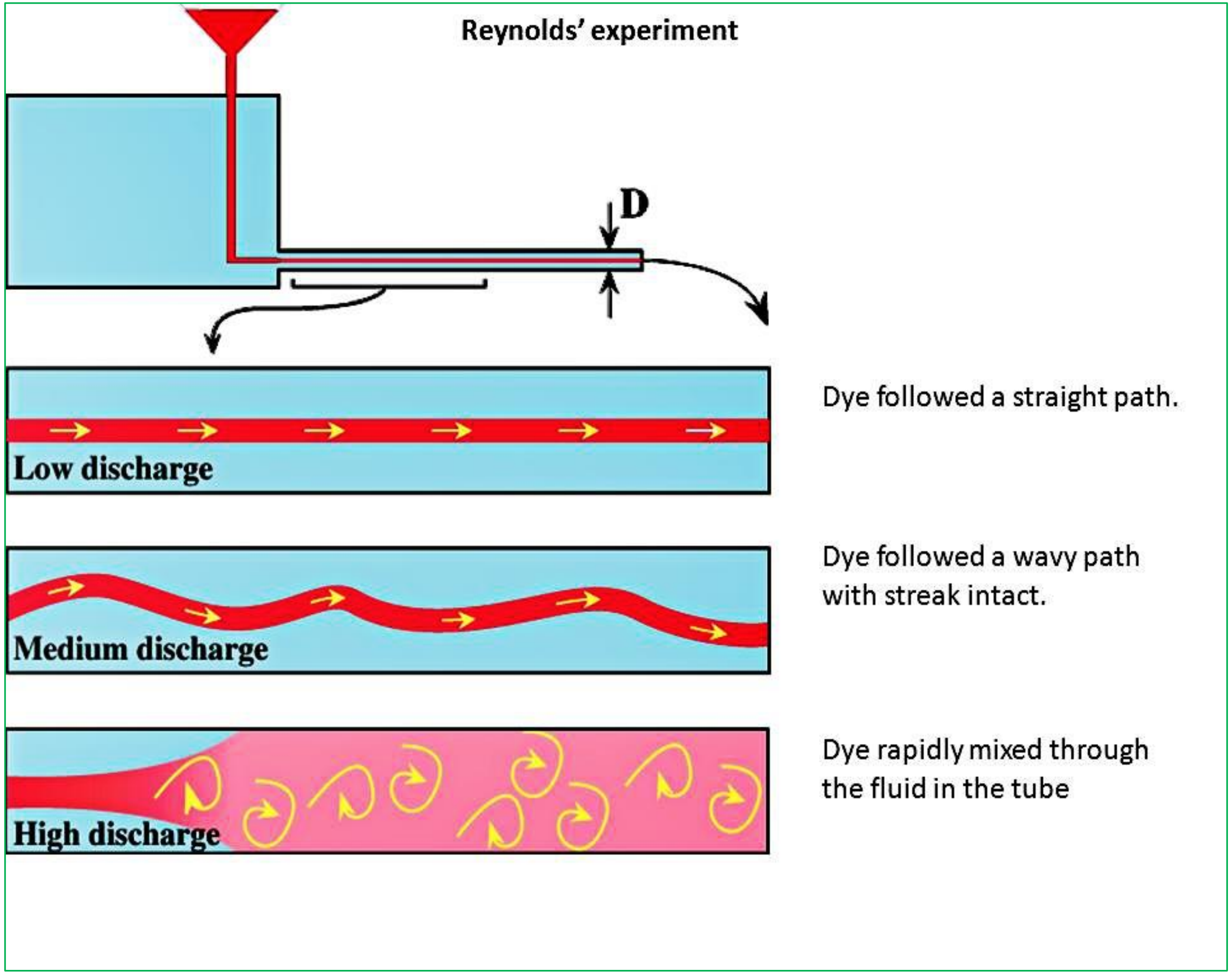
Reynolds Experiment: Apparatus

1. A constant head tank filled with water.
2. A small tank containing dye.
3. A horizontal glass tube provided with a bell mouthed entrance.
4. A regulating valve.

Reynolds Experiment: Procedure

1. The water was made to flow from the tank through the glass tube into the atmosphere.
2. The velocity of flow was varied by adjusting valve.
3. The liquid dye was introduced into the flow at the bell mouth through a small tube.

Observations made



Observations made

1. When velocity of flow is low, the dye remained in the form of a straight and stable filament passing through the glass tube. (laminar flow)
2. With the increase in velocity, a critical state was reached at which the dye filament showed irregularity and began to weaver. This is transitional state.
3. With further increase in velocity of flow in the filament of dye become more intense and ultimately the dye diffused over the entire cross-section (turbulent flow)

Loss of Head and Velocity of flow relationship

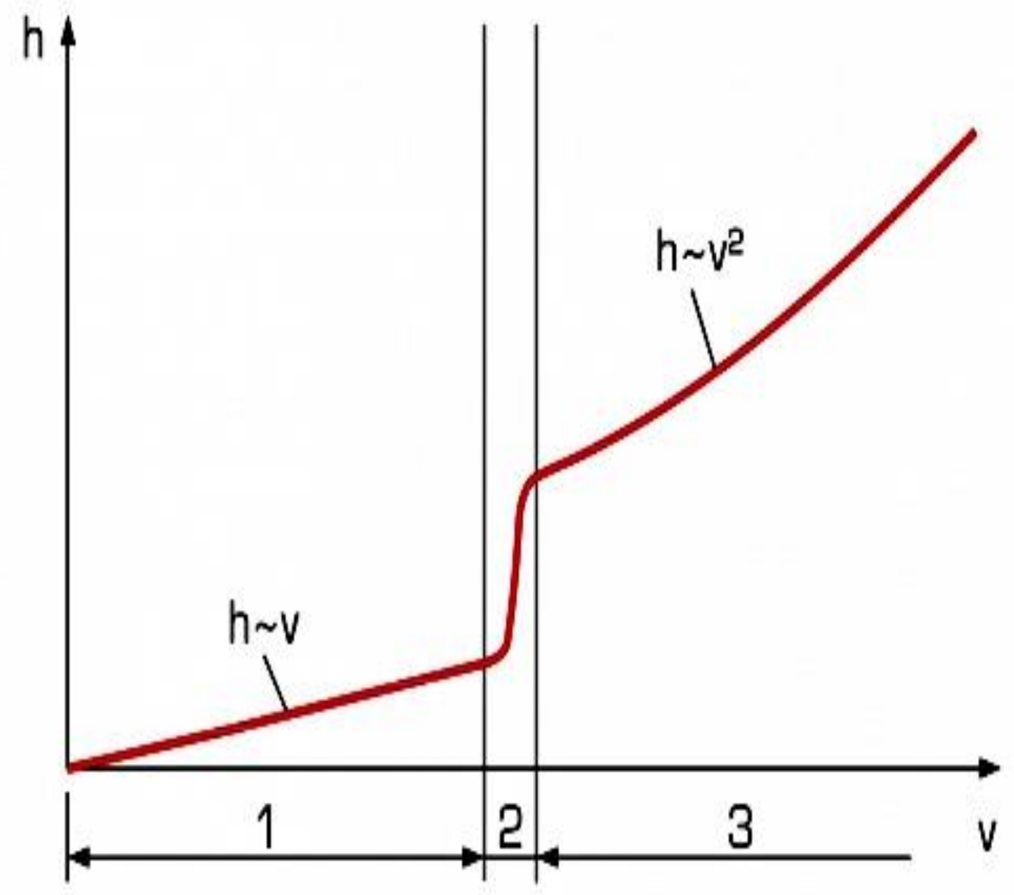
A graph is plotted between V (velocity of flow) and h_f (loss of head). Such a graph is shown in figure. It may be seen from the graph that:

Zone 1: Laminar flow ($h_f \propto v$)

Zone 2: Transition zone

Zone 3: Turbulent flow ($h_f \propto v^n$)

Value of n varies from 1.75 to 2



Reynolds Number, R_e

Reynolds from his experiment, found that the nature of flow in a closed conduit depends upon the following factors:

1. Diameter of the pipe (D)
2. Density of the liquid (ρ)
3. Viscosity of the liquid (μ)
4. Velocity of flow (V)

By combining the above variables, Reynolds determined a non-dimensional quantity equal to $\frac{\rho V D}{\mu}$ which is known as Reynolds number.

Reynolds Number, R_e

It is defined as the ratio of inertia force to the viscous force.

Inertia force (F_i)

$= mass * acceleration$

= density $*$ volume $* \frac{Velocity}{time}$

= density $* \frac{Volume}{time} *$ Velocity

= density $*$ Area $*$ Velocity $*$ Velocity

= ρAV^2

Reynolds Number, R_e

Viscous force (F_v)

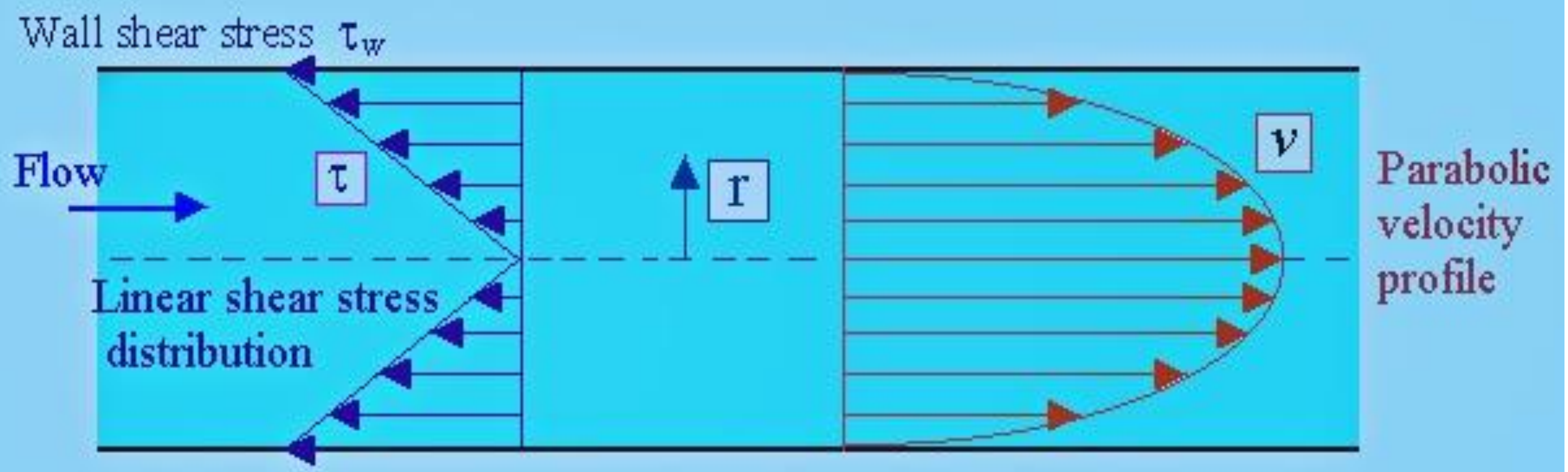
$= shear\ stress * area$

$= \mu \frac{du}{dy} * area$

$= \mu \frac{V}{L} * A$

Reynolds Number , $R_e = \frac{F_i}{F_v} = \frac{\rho VL}{\mu}$

Shear Stress and Velocity Profile



Shear stress and Velocity distributions in pipe, fully developed flow of Newtonian fluid, for Laminar flow

Why Laminar Flow is AWESOME??

https://www.youtube.com/watch?v=y7Hyc3MRKno

Flow of Viscous Fluid through Circular Pipe

- At low velocity the fluid moves in layers.
- Each layer of fluid slides over the adjacent layer.
- For Newtonian fluids the shear stress, τ, is directly proportional to the velocity gradient, du/dy, developed in the fluid.
- The proportionality constant, μ, is called the "dynamic" or "absolute", viscosity. So shear stress, $\tau = \mu \frac{du}{dy}$
- Expression for Reynolds number is given by: $R_e = \frac{\rho V D}{\mu}$
- Here, ρ = Density of fluid flowing through the pipe ; V = Average velocity of fluid; D = diameter of pipe; μ = Viscosity of fluid

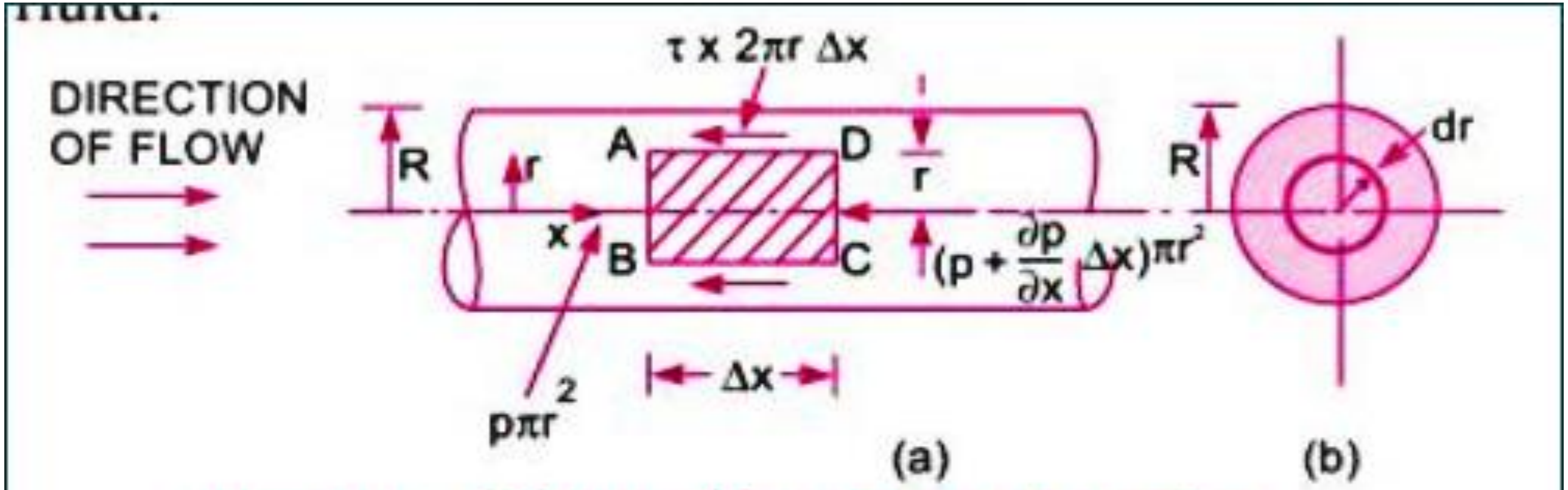


Fig. 9.1 *Viscous flow through a pipe.*

- Consider a horizontal pipe of radius R.
- The viscous fluid is flowing from left to right in the pipe as shown in figure.
- Consider a fluid element of radius r sliding in a cylindrical fluid element of radius (r+ dr)
- Let, length of fluid element = Δx

Shear Stress τ across a section = $-\frac{\partial p}{\partial x}\frac{r}{2}$

If p is the intensity of pressure on the face AB,

then the intensity of pressure on the face CD will be p+ $\frac{\partial p}{\partial x}\,\Delta x$

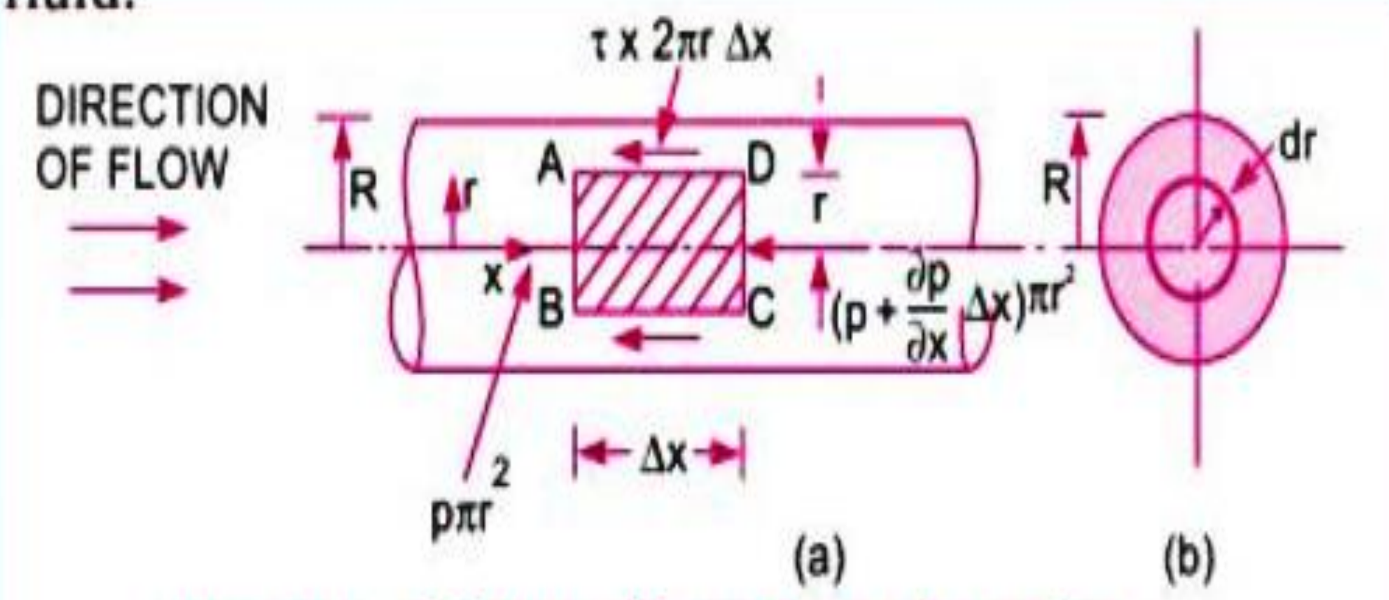


Fig. 9.1 *Viscous flow through a pipe.*

Forces acting on the fluid Element

Location of Force	Type and Magnitude
Face AB	Pressure force $= p * \pi r^2$
Face CD	Pressure force $= (p + \frac{\partial p}{\partial x} \Delta x) * \pi r^2$
Surface of fluid element ABCD	Shear force $= \tau * 2\pi r \Delta x$

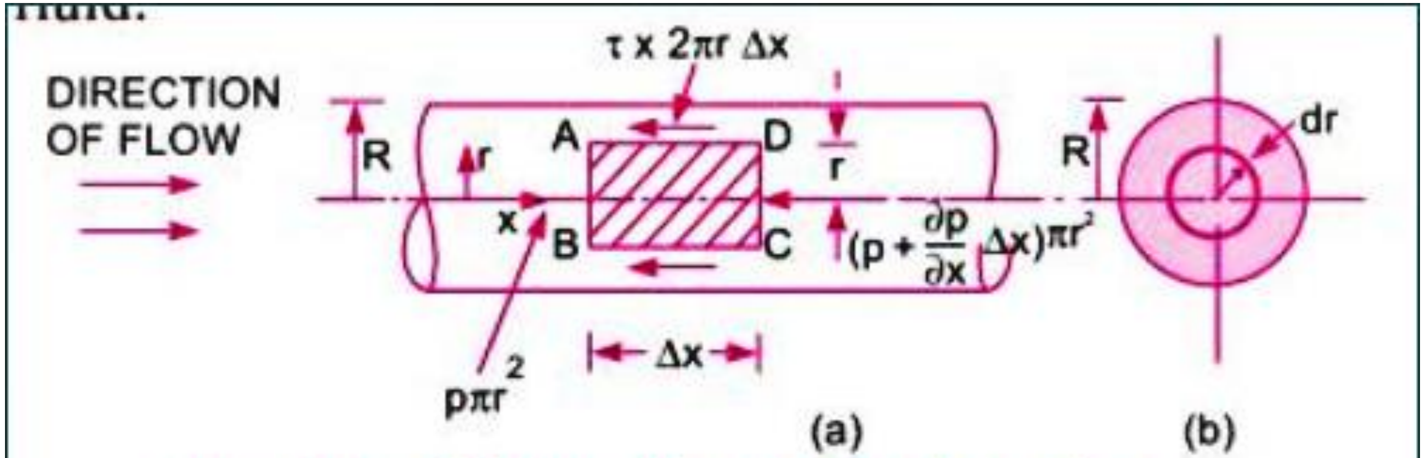


Fig. 9.1 *Viscous flow through a pipe.*

Sum of all forces in the direction of flow must be zero

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x} . r - 2\tau = 0$$

$\therefore$
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

Shear stress and Velocity distribution

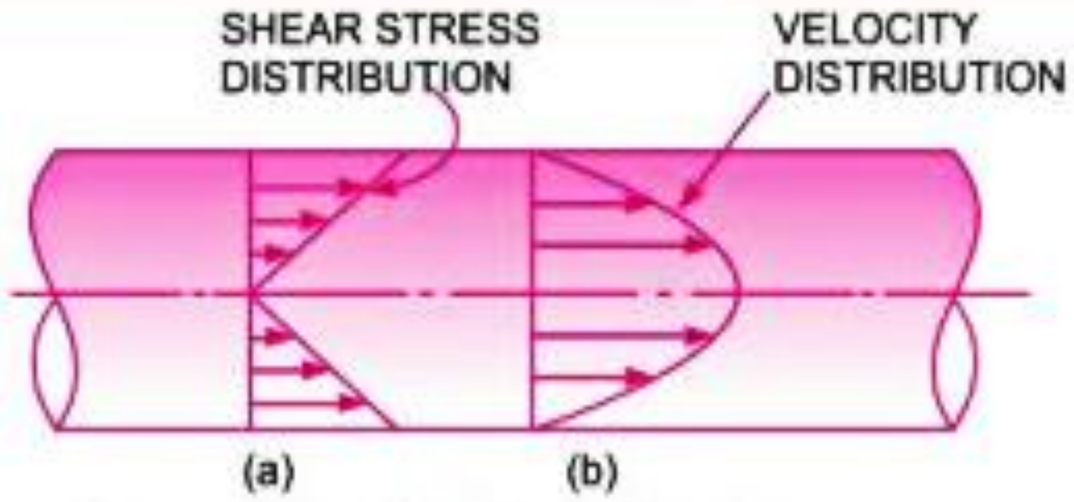


Fig. 9.2 Shear stress and velocity distribution across a section.

To obtain velocity distribution across a section,

the value of shear stress $\tau = \mu \frac{du}{dy}$ ....(1)

is substituted in equation $\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$ ....(2)

Velocity Distribution: $u = \frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$

But in the relation, $\tau = \mu \frac{du}{dy}$, y is measured from pipe wall.

Hence, $y = R - r$ and $dy = -dr$

So, shear stress, $\tau = \mu \frac{du}{dy} = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$

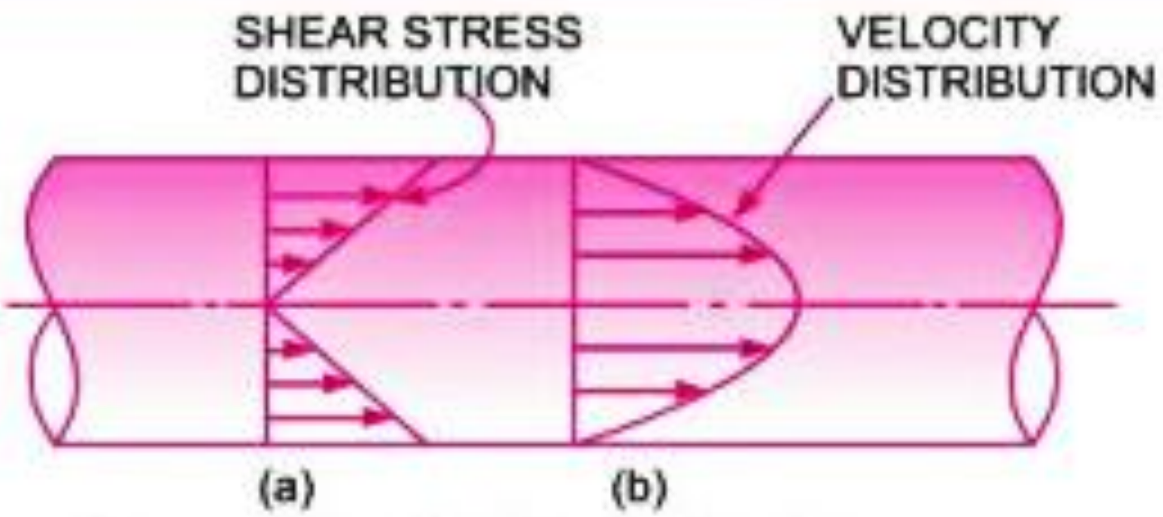


Fig. 9.2 *Shear stress and velocity distribution across a section.*

Velocity Distribution: $u = \frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{or} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

Integrating this above equation w.r.t. 'r', we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C \qquad \text{...(9.2)}$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$, $u = 0$.

$$\therefore \qquad 0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

$$\therefore \qquad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Substituting this value of C in equation (9.2), we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \qquad \text{...(9.3)}$$

Ratio of maximum velocity to average velocity

Condition:

When r = 0, velocity becomes maximum;

So, maximum velocity, U_{max}

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - 0^2]$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x} * R^2$$

The average velocity $\bar{u}$ is obtained by,

$$\bar{u} = \frac{Discharge\ of\ fluid\ across\ the\ section}{Area\ of\ the\ pipe\ (\pi R^2)}$$

Discharge Q across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr.

Discharge Q:

dQ = velocity at a radius r × area of ring element

$$= u \times 2\pi r\,dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\,dr$$

$$Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}\,(R^2 - r^2) \times 2\pi r\,dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\,r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\,dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

∴ Average velocity, $$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$$

or $$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \quad \text{...(9.5)}$$

Dividing equation (9.4) by equation (9.5),

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2} = 2.0$$

∴ Ratio of maximum velocity to average velocity = 2.0.

Drop of Pressure for a given length L of the pipe (Hagen Poiseuille Formula)

From equation (9.5), we have

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

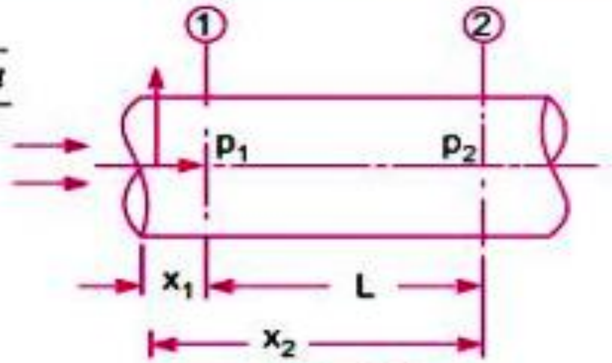


Fig. 9.3

Integrating the above equation w.r.t. x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$\therefore \quad -[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or } (p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2}L \qquad \{\because \; x_2 - x_1 = L \text{ from Fig. 9.3}\}$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2} \qquad \left\{\because \; R = \frac{D}{2}\right\}$$

or $$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2},$$ where $p_1 - p_2$ is the drop of pressure.

$$\therefore \quad \text{Loss of pressure head} = \frac{p_1 - p_2}{\rho g}$$

$$\therefore \quad \frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2} \qquad \text{...(9.6)}$$

Equation (9.6) is called **Hagen Poiseuille Formula.**

Practice Problem#18 (Rajput= Page 534)

A crude oil of viscosity 9 poise and specific gravity 0.90 is flowing through a horizontal circular pipe of 60 mm diameter. If the pressure drop in 100 m length of the pipe is 1800 kN/m^2, determine:

1. Rate of flow of oil (Q = Area* Velocity; Pressure drop, $\Delta p = \frac{32\mu\bar{u}L}{D^2}$)
2. Center-line velocity ($U_{max} = 2\bar{u}$)
3. Total frictional drag over 100 m length ($F = \tau_0 * \pi D L$; $\tau_0 = -\frac{\partial p}{\partial x}\frac{R}{2}$)
4. Velocity gradient at the pipe wall ($\tau_0 = \mu\frac{du}{dy}$)
5. Velocity and shear stress at 8 mm from the wall ($u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$)

*For similar triangle relationship, $\frac{\tau_0}{R} = \frac{\tau}{r}$

Practice Problem#19

A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.

Solution:

Given information,

D = 200 mm = 0.2 m ; R = 0.1 m = 10 cm

$U_{max} = 1.5\ m/s$

y = 4 cm.

We know, $\frac{U_{max}}{\bar{u}} = 2$;

So, mean velocity = 0.75 m/s (answer)

Solution

Velocity at any distance r from the center is given by,

$u = U_{max}[1 - (\frac{r}{R})^2]$

$0.75 = 1.5 \times [1-(\frac{r}{0.1})^2]$

r = 70.7 mm (answer)

Velocity at y = 4 cm from the pipe wall;

y = R – r

0.04 = 0.1 – r

r = 0.06 m

Velocity at y = 4 cm or r = 6 cm from the pipe wall

$$u = U_{max}[1 - (\frac{r}{R})^2]$$

$$u = 1.5 * [1 - (\frac{6}{10})^2]$$

u = 0.96 m/s (answer)

Loss of Head due to friction in Viscous flow

Hagen Poiseuille formula:

Loss of pressure head, $h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$

Darcy Weisbach formula:

Loss of head due to friction, $h_f = \frac{4fL}{D}\frac{V^2}{2g}$

Velocity in pipe is always average velocity, that is, V = $\bar{u}$

Equating, we get,

$$\frac{32\mu\bar{u}L}{\rho g D^2} = \frac{4fL}{D}\frac{\bar{u}^2}{2g}$$

Loss of Head due to friction in Viscous flow

Friction factor, f

$$= \frac{32\mu\bar{u}L*D*2g}{\rho g D^2*4L*\bar{u}^2}$$

$$= \frac{16\mu}{\bar{u}\ \rho\ D}$$

$$= \frac{16}{\frac{\bar{u}\ \rho\ D}{\mu}} = \frac{16}{R_e}$$ [By definition, Reynolds Number, $R_e = \frac{\bar{u}\ \rho\ D}{\mu}$]

Practice Problem#20

Water is flowing through a 200 mm diameter pipe with co-efficient of friction f = 0.04 ; The shear stress at a point 40 mm from the pipe axis is 0.00981 N/cm^2. Calculate shear stress at the pipe wall.

Solution:

First, find whether the flow is viscous or not.

Friction factor, f = $\frac{16}{R_e}$

Reynolds number , $R_e = \frac{16}{f} = \frac{16}{0.04} = 400\ (Laminar\ flow)$

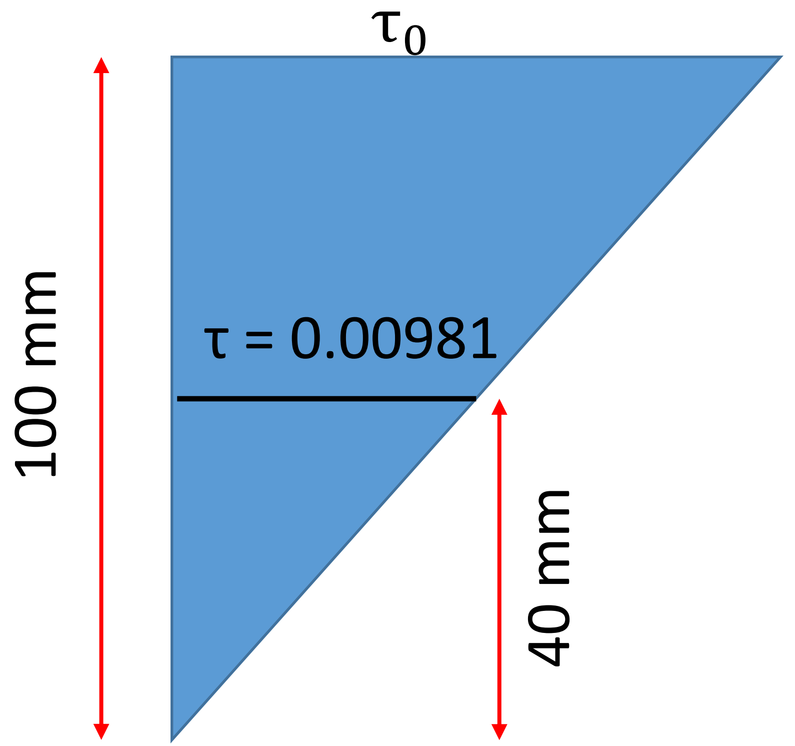
Solution

Let, shear stress at pipe wall = τ_0

From similar triangle relationship,

$$\frac{0.00981}{40} = \frac{\tau_0}{100}$$

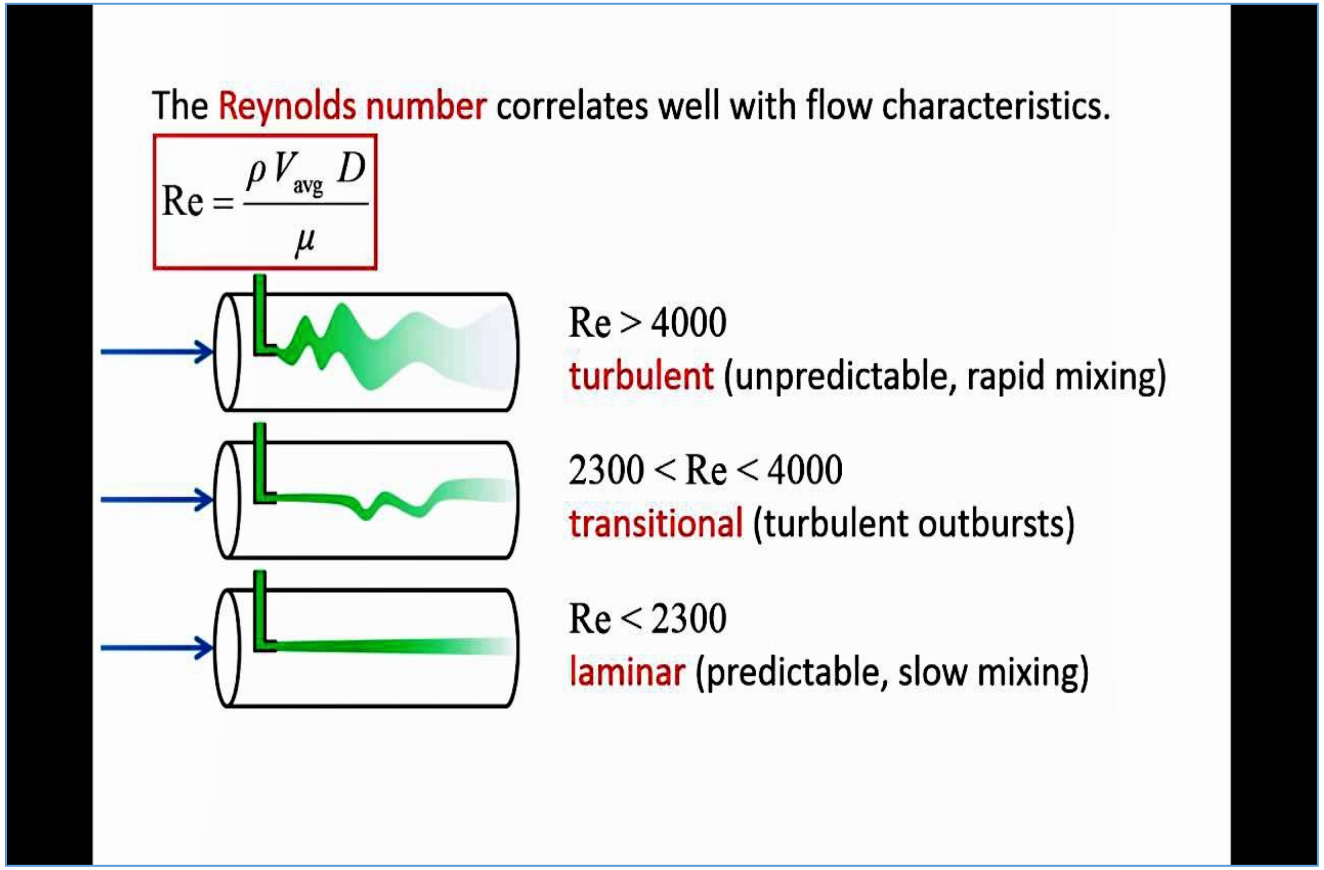
τ_0= 0.0245 N/cm^2

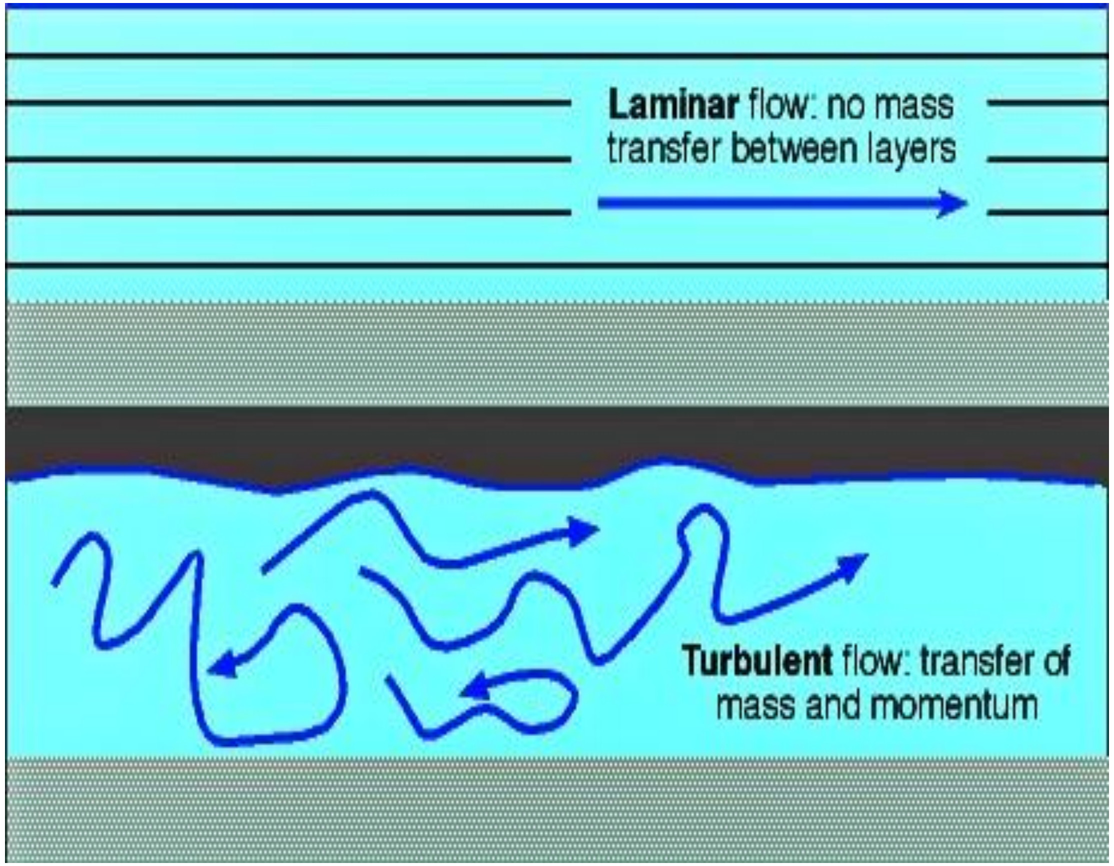


Turbulent Flow

- In a pipe, turbulent flow occurs when $R_e > 4000$.
- In a turbulent flow, fluid motion is irregular and chaotic and there is complete mixing of fluid due to collision of fluid masses with one another.
- The fluid masses are interchanged between adjacent layers.
- As the fluid masses in adjacent layers have different velocities, interchange of fluid masses between the adjacent layers is accompanied by a transfer of momentum.
- The shear in turbulent flow is mainly due to momentum transfer.

Transfer of Momentum in Turbulent flow

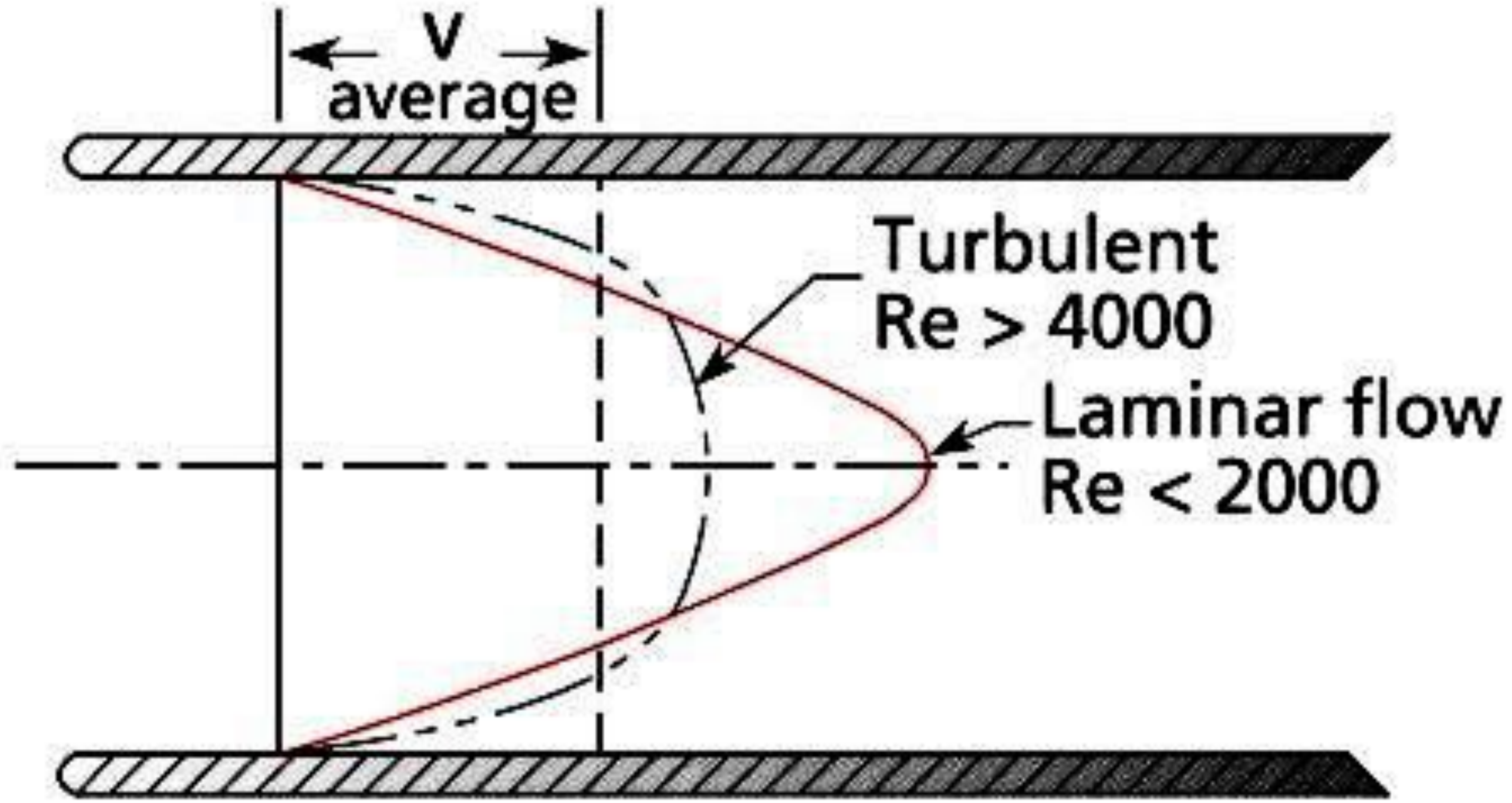




Velocity distribution curve for Laminar and Turbulent flow

- The velocity distribution in turbulent flow is more uniform than in laminar flow.
- In turbulent flow, the velocity gradients near the boundary wall shall be quite large resulting in more shear.
- In turbulent flow, the flatness of velocity distribution curve in the core region away from the wall is because of mixing of fluid layers and exchange of momentum between them.
- The velocity distribution which is paraboloid in laminar flow tends to follow logarithmic variation in turbulent flow.
- Random orientation of fluid particles in a turbulent flow gives rise to additional stresses, called Reynolds stresses.

Velocity Distribution Curve



Laminar-Turbulent

A laminar flow changes to turbulent flow when-

1. Velocity is increased.
2. Diameter of pipe is increased.
3. Viscosity of fluid is decreased

Frictional resistance for turbulent flow is:

1. Proportional to V^n ; where n varies from 1.5 to 2.0.
2. Proportional to the density of fluid.
3. Proportional to the area of surface in contact.
4. Independent of pressure.
5. Dependent on the nature of surface in contact.

Loss of Head due to Friction in Pipe flow (Darcy Equation)

Notation	Parameter
p_1	Intensity of Pressure at section 1–1
V_1	Velocity of flow at section 1–1
p_2, V_2	Intensity of pressure and Velocity of flow at section 2–2 respectively.
L	Length of the pipe between section 1–1 and 2–2
d	Diameter of the pipe
f'	Frictional resistance per unit wetted area per unit velocity
h_f	Loss of head due to friction

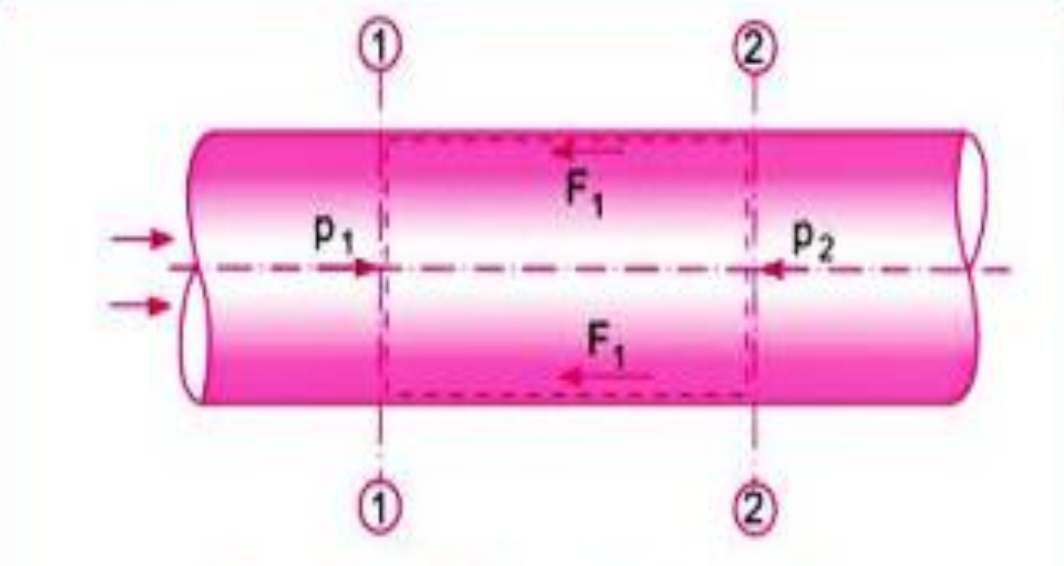


Fig. 10.3 *Uniform horizontal pipe.*

Loss of Head due to Friction in Pipe flow (Darcy Equation)

Forces acting on the fluid between section 1−1 and 2−2;

1. Pressure force at section 1 −1 = $p_1 * A$
2. Pressure force at section 2 −2 = $p_2 * A$
3. Frictional resistance F_1

Resolving all the forces in horizontal direction:

$$p_1 A - p_2 A - F_1 = 0$$

$$A\,(p_1 - p_2) - F_1 = 0$$

$$(p_1 - p_2) = \frac{F_1}{A} \text{ .... (1)}$$

Applying Bernoulli's equation at section 1—1 and 2—2;

Let, h_f = loss of head due to friction.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$z_1 = z_2 = 0 \; ; V_1 = V_2;$$

$$h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}$$

$$p_1 - p_2 = \rho g h_f \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Frictional Resistance, F_1

Frictional resistance

= frictional resistance per unit wetted area per unit velocity $*$ wetted area $*$ V^2

= $f' * \pi d L * V^2$

= $f' *$ P$*$ $L * V^2$ (Perimeter , P = πd)

From equation 1 and 2, we get,

$$\rho g h_f = \frac{F_1}{A}$$

$$\rho g h_f = \frac{f' P L V^2}{A}$$

$$h_f = \frac{f' P L V^2}{A \rho g}$$

Darcy Weisbach Equation

Hydraulic mean depth,

R = $\frac{P}{A} = \frac{Wetted\ Perimeter}{Area} = \frac{4}{d}$

Putting $\frac{f'}{\rho} = \frac{f}{2}$; $where\ f\ is\ known\ as\ friction\ factor\ we\ get,$

Loss of head due to friction in pipes flow,

$$h_f = \frac{4fL}{d}\frac{V^2}{2g}$$

Expression for Co-efficient of Friction in terms of shear stress

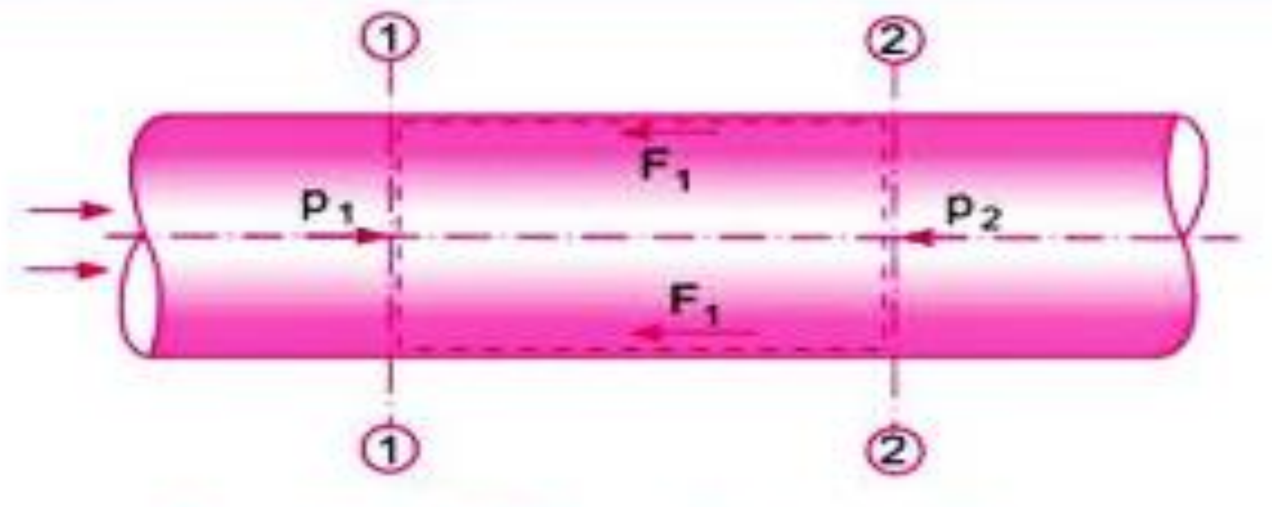


Fig. 10.3 *Uniform horizontal pipe.*

Forces acting on the fluid between section 1—1 and 2—2;

1. Pressure force at section 1 —1 = $p_1 * A$
2. Pressure force at section 2 —2 = $p_2 * A$
3. Frictional resistance F_1

Expression for Co-efficient of Friction in terms of shear stress

Forces acting on fluid between sections 1-1- & 2-2 is given by,

$$p_1 A - p_2 A - F_1 = 0$$

$$(p_1 - p_2) * \frac{1}{4}\pi d^2 = shear\ force\ due\ to\ shear\ stress\ \tau_0$$

$$(p_1 - p_2) * \frac{1}{4}\pi d^2 = \text{shear stress} * \text{surface area}$$

$$(p_1 - p_2) * \frac{1}{4}\pi d^2 = \tau_0 * \pi \mathrm{dL}$$

$$(p_1 - p_2) = \frac{4\tau_0 * \pi \mathrm{dL}}{\pi d^2} = \frac{4\tau_0 * \mathrm{L}}{d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

Expression for Co-efficient of Friction in terms of shear stress

From Darcy-Weisbach equation,

$$h_f = \frac{4fL}{d}\frac{V^2}{2g}$$

$$\frac{p_1 - p_2}{\rho g} = \frac{4fL}{d}\frac{V^2}{2g}$$

$$p_1 - p_2 = \frac{4fL}{d}\frac{V^2}{2g} * \rho g \text{ ....(2)}$$

Equating, we get,

Friction factor , f = $\frac{2\tau_0}{\rho V^2}$

Velocity distribution in turbulent flow:

Prandtl's Universal Distribution Equation,

$$\frac{u_{max} - u}{u_f} = 2.5 \ln[\frac{R}{y}]$$

Here,

u_{max} = Center line velocity

u = local velocity at distance y

u_f= Shear friction velocity = $\sqrt{\frac{\tau_0}{\rho}}$

Practice Problem#21 (Bansal= 445)

Determine the wall shearing stress in a pipe of diameter 100 mm which carries water. The velocities at the pipe center and 30 mm from pipe center are 2 m/s and 1.5 m/s respectively. The flow in pipe is given as turbulent.

Solution:

Formula: $\frac{u_{max}-u}{u_f} = 2.5 \ln[\frac{R}{y}]$

R = 50 mm.

r = 30 mm.

y = R$-$r

y = 20 mm

Velocity at pipe center, $u_{max} = 2\ m/s$

Velocity at distance y = 20 mm from pipe wall, u = 1.5 m/s.

Solution

Applying the formula:

Shear friction velocity,

$u_f = 0.2185\, m/s$

If τ_0 *is the shear stress at pipe wall,* $u_f = \sqrt{\frac{\tau_0}{\rho}}$

$\tau_0 = u_f^2 * \rho = 0.2185 * 0.2185 * 1000 = 47.676\, N/m^2$

Download Class Lecture Slides from here:

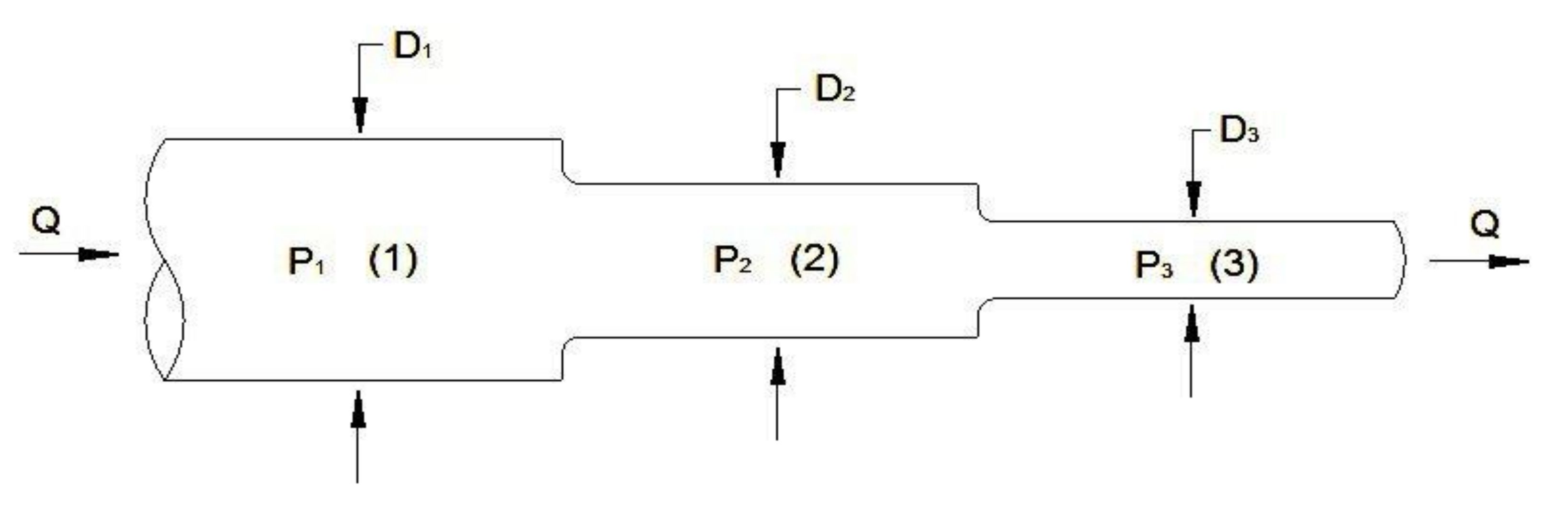
1. https://plus.google.com/+AhmedHossain090001
2. http://www.ruet.ac.bd/teacher/CE/Ahmed%20Hossain

Flow Through Pipes

Definition of Pipe:

A pipe is a closed conduit (generally of circular section) which is used for carrying fluid under pressure.

The flow in a pipe is termed as pipe flow only when the fluid completely fills the cross-section and there is no free surface of fluid.



Loss of Energy (or Head) in Pipes

When water flows in a pipe, it experiences some resistance to its motion, due to which its velocity and ultimately the head of water available is reduced.

Loss of energy (or head) is classified as follows:

A. Major Energy Losses: Loss due to friction.

B. Minor Energy Losses:

1. Sudden enlargement of pipe.
2. Sudden contraction of pipe.
3. Bend of Pipe.
4. An obstruction in pipe.
5. Pipe fittings, etc.

Major Energy Losses

These losses which are due to friction are calculated by:

1. Darcy Weisbach formula.

$$Loss\ of\ head\ due\ to\ friction, h_f = \frac{4fL}{D} * \frac{V^2}{2g}$$

f = Co-efficient of friction (function of Reynolds number, R_e)

For R_e < 2000, f = $\frac{16}{R_e}$ [Laminar flow]

For R_e = 4000 to 10^6 , f = $\frac{0.0791}{R_e^{\frac{1}{4}}}$ [Turbulent flow]

Major Energy Losses

2. Chezy's formula:

Mean velocity, V = C $\sqrt{iR}$

Here,

C = Chezy's constant

R = Hydraulic mean depth or Hydraulic radius

i = Loss of head per unit length of the pipe = $\frac{h_f}{L}$

Chezy's formula (for loss of head) is generally used for flow through open channels.

Practice Problem#21

Water is to be applied to the students of Zia Hall of RUET through a supply main. The following data is given:

- Distance of the reservoir from the campus = 3000 meter.
- Number of inhabitants = 4000
- Consumption of water per day of each inhabitant = 180 Liters.
- Loss of head due to friction = 18 meter
- Co-efficient of friction for the pipe, f = 0.007
- If half of the daily supply is pumped in 8 hours, determine size of the supply main.

Solution

Loss of head due to friction, $h_f = 18\, meter$

Co-efficient of friction, f = 0.007

Number of students = 4000

Consumption per day per student = 180 liters = 0.18 m^3

Total supply for day = 4000 $*$ 0.18 = 720 m^3 (Volume)

Since half of the supply is to be pumped in 8 hours, maximum flow for which the pipe is to be designed,

Q = $\frac{Volume}{Time} = \frac{0.50*720}{8*3600} = 0.0125\, m^3$

Velocity, V = $\frac{Q}{A} = \frac{0.0125}{0.25*\pi*D^2} = \frac{0.0159}{D^2}$

Solution

Using the relation,

Loss of head due to friction, $h_f = \frac{4fL}{D} * \frac{V^2}{2g}$

Diameter, D = 0.143 meter or 143 mm.

Minor Energy Losses

Loss of Head due to sudden enlargement:

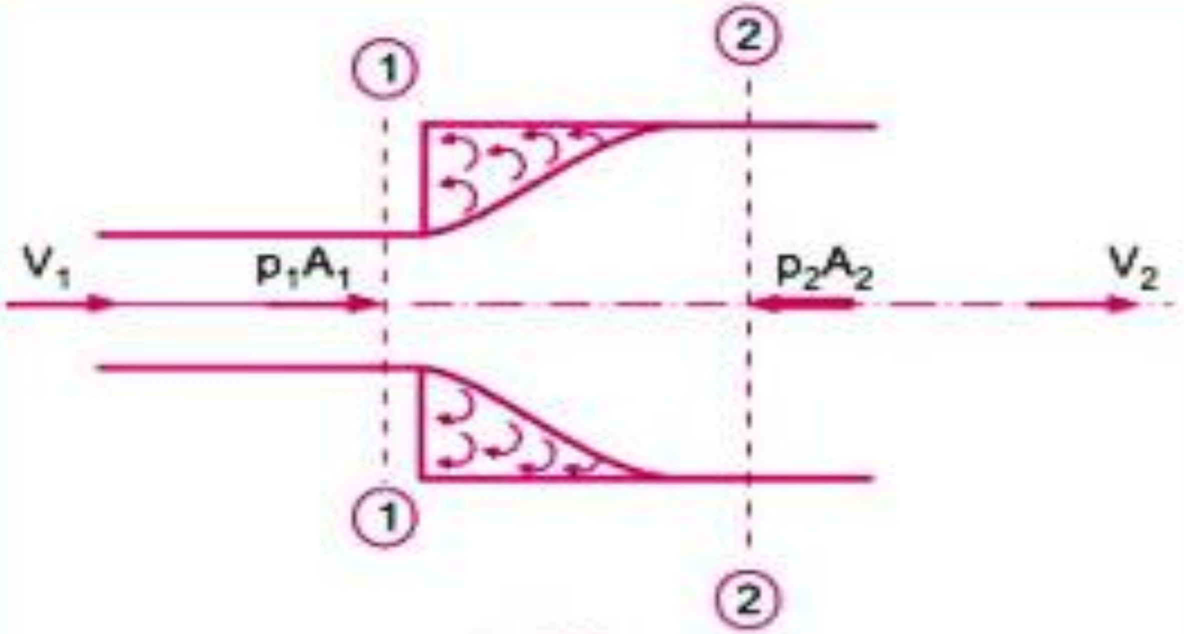


Fig. 11.1 *Sudden enlargement.*

Video link: https://vimeo.com/193012607

Step 1: Apply Bernoulli's equation

Due to sudden change of diameter of the pipe from D_1 to D_2 , the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed as shown in figure. The loss of head or energy takes place due to the formation of these eddies.

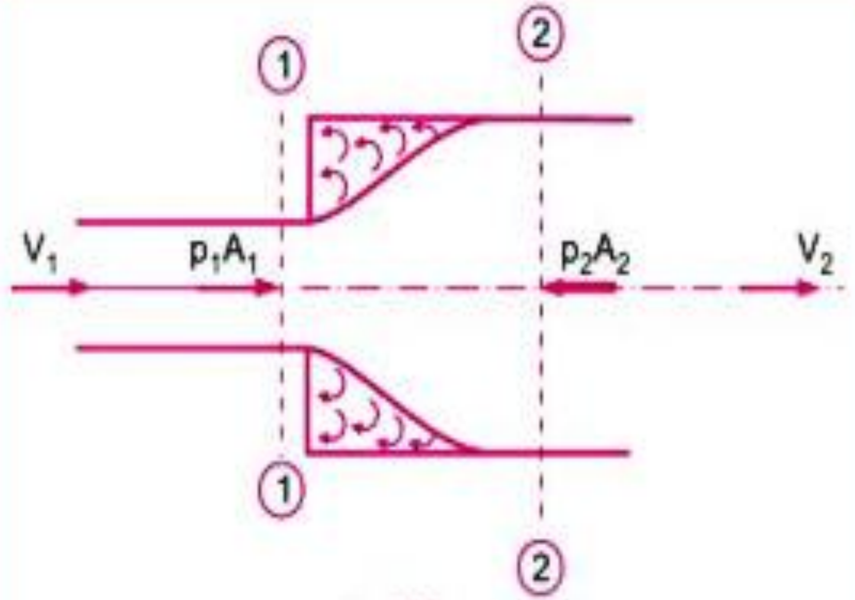


Fig. 11.1 Sudden enlargement.

Applying Bernoulli's Equation to section 1-1 and 2-2,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_e$$

$$h_e = \left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

Step 2: Change of Momentum

Momentum of liquid at section 1-1,

= mass $*$ velocity

= density $*$ volume $*$ velocity

= density $*$ area $*$ velocity $*$ velocity

= ρ $A_1 V_1^2$

Similarly, Momentum of liquid at section 2-2 = ρ $A_2 V_2^2$

Change of momentum

= ρ $A_2 V_2^2$ −ρ $A_1 V_1^2$

= ρ $A_2 V_2^2 -$ ρ $V_1^2 * \frac{A_2 V_2}{V_1}$ [continuity equation, $A_1 = \frac{A_2 V_2}{V_1}$]

= ρ $A_2(V_2^2 - V_1 V_2)$

Step 3: Net force acting on the liquid

$Net\ force, F_x$

$= p_1 A_1 + p_0(A_2 - A_1) - p_2 A_2$

= $p_1 A_1 + p_1(A_2 - A_1) - p_2 A_2$ [assuming $p_0 = p_1$]

= $(p_1 - p_2) * A_2$

According to Newton's Second Law,

Net force = Change of momentum

$(p_1 - p_2) * A_2$= ρ $A_2(V_2^2 - V_1 V_2)$

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1 V_2$$

$\frac{p_1}{\gamma} - \frac{p_2}{\gamma}$ = $\frac{V_2^2 - V_1 V_2}{g}$ [dividing both sides by g]

Loss of Head due to sudden Enlargement

$$h_e = \left(\frac{p_1}{\gamma} - \frac{p_2}{\gamma}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

Loss of head due to enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

V_1 = Velocity at section 1 = Contracted section

V_2 = Velocity at section 2 = Enlarged section

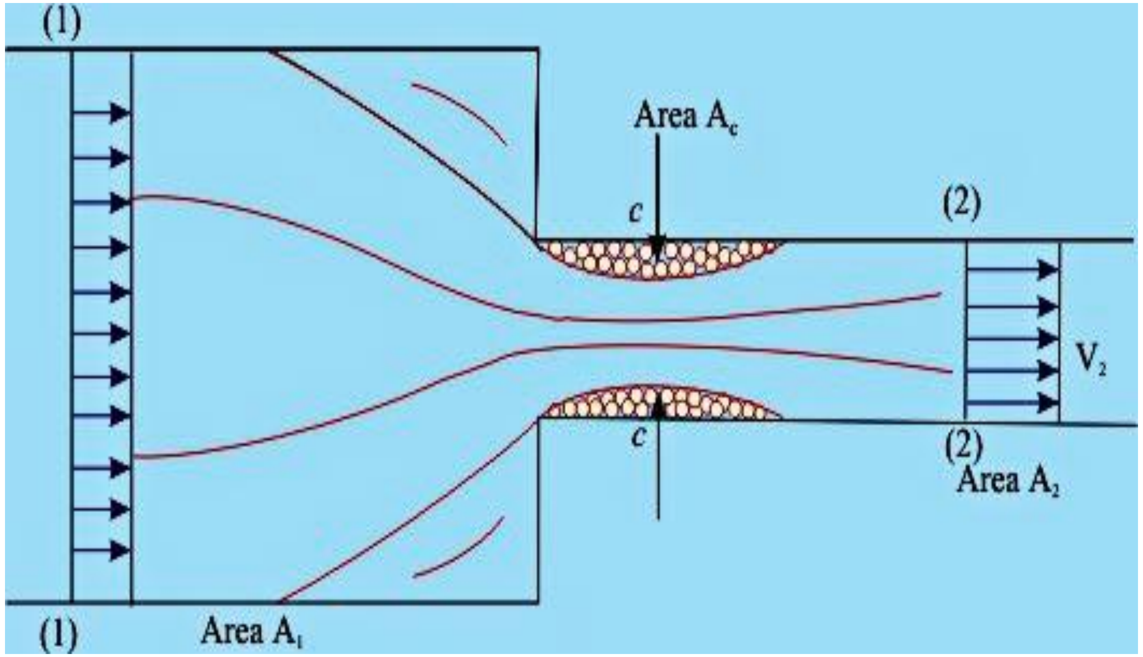
Practice Problem#22 [Rajput= 639]

A horizontal pipe 150 mm in diameter is joined by sudden enlargement to a 225 mm diameter pipe. Water is flowing through it at the rate of 0.05 m^3/s. Find

- Loss of head due to abrupt expansion.
- Pressure difference in the two pipes.
- Change in pressure if the change of section is gradual without any loss.

Loss of Head due to sudden contraction

Symbol	Notation
A_c	Area of flow at section C-C
V_c	Velocity of flow at section C-C
A_2	Area of flow at section 2-2
V_2	Velocity of flow at section 2-2
h_c	Loss of head due to sudden contraction



Loss of head due to sudden contraction

= Loss up to vena contracta + Loss due to sudden enlargement beyond vena contracta

Loss of Head due to sudden contraction

$h_c = negligibly\ small + \frac{(V_c - V_2)^2}{2g}$ ……………………………(1)

From continuity equation, we have,

$A_c V_c = A_2 V_2$

$V_c = \frac{V_2}{C_c}$ [Co-efficient of contraction, $C_c = \frac{A_c}{A_2}$]

Substituting the value of V_c in equation 1 we get,

$h_c = \frac{V_2^2}{2g} \left(\frac{1}{C_c} - 1\right)^2$

In general, $h_c = \mathrm{k} * \frac{V_2^2}{2g}$

Constant k = $\left(\frac{1}{C_c} - 1\right)^2$

Practice problem#23 (Bansal = 476)

A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. Find the loss of head due to contraction if $C_c = 0.62$; Also determine the rate of flow of water.

Solution:

Information given,

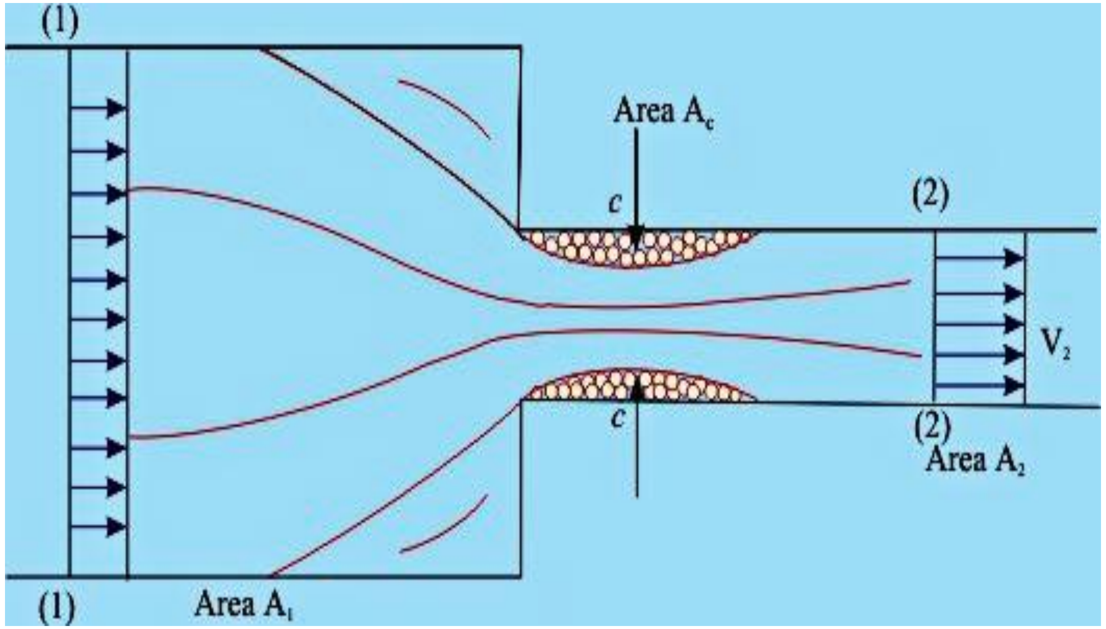
$D_1 = 500\ mm$ = 0.5 m

$D_2 = 250\ mm$ = 0.25 m

$p_1 = 13.734 * 10^4$ N/m^2

$p_2 = 11.772 * 10^4$ N/m^2

$C_c = 0.62$



Solution

Head lost due to contraction $= \dfrac{V_2^2}{2g}\left[\dfrac{1}{C_c} - 1.0\right]^2 = \dfrac{V_2^2}{2g}\left[\dfrac{1}{0.62} - 1.0\right]^2 = 0.375\,\dfrac{V_2^2}{2g}$

From continuity equation, we have $A_1V_1 = A_2V_2$

or
$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ (pipe is horizontal)

$\therefore$
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

Solution

But $$h_c = 0.375\frac{V_2^2}{2g} \text{ and } V_1 = \frac{V_2}{4}$$

Substituting these values in the above equation, we get

$$\frac{13.734\times10^4}{9.81\times1000}+\frac{(V_2/4)^2}{2g}=\frac{11.772\times10^4}{1000\times9.81}+\frac{V_2^2}{2g}+0.375\frac{V_2^2}{2g}$$

or $$14.0+\frac{V_2^2}{16\times2g}=12.0+1.375\frac{V_2^2}{2g}$$

or $$14-12=1.375\frac{V_2^2}{2g}-\frac{1}{16}\frac{V_2^2}{2g}=1.3125\frac{V_2^2}{2g}$$

or $$2.0=1.3125\times\frac{V_2^2}{2g} \text{ or } V_2=\sqrt{\frac{2.0\times2\times9.81}{1.3125}}=5.467 \text{ m/s.}$$

(*i*) Loss of head due to contraction, $h_c = 0.375\frac{V_2^2}{2g}=\frac{0.375\times(5.467)^2}{2\times9.81}=$ **0.571 m. Ans.**

(*ii*) Rate of flow of water, $Q = A_2V_2 = 0.04908\times5.467 = 0.2683 \text{ m}^3\text{/s}$ = **268.3 lit/s. Ans.**

Practice Problem#24

A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm^2 and 11.772 N/cm^2 respectively. If the rate of flow of water is 300 liter/second, Find the value of co-efficient of contraction C_c.

Solution:

Formula :

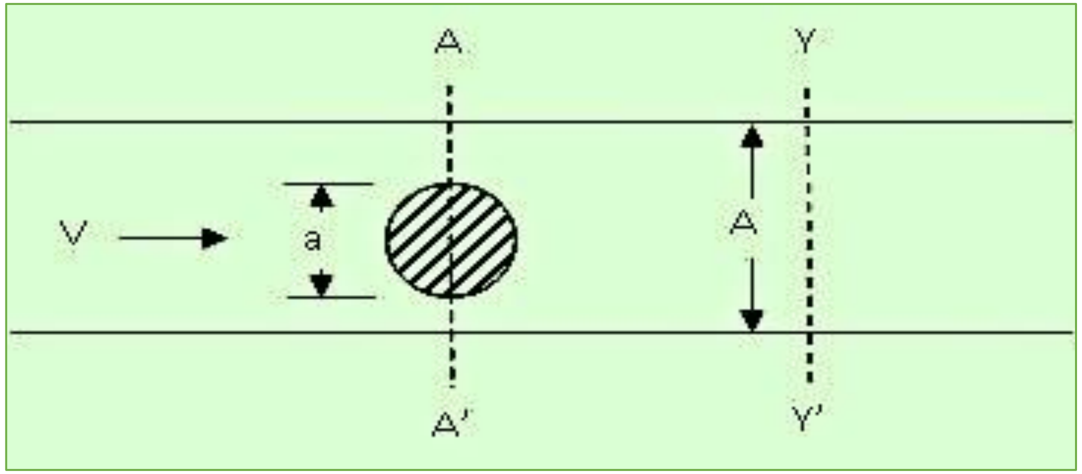
1) $h_c = \frac{V_2^2}{2g} \left(\frac{1}{C_c} - 1\right)^2$

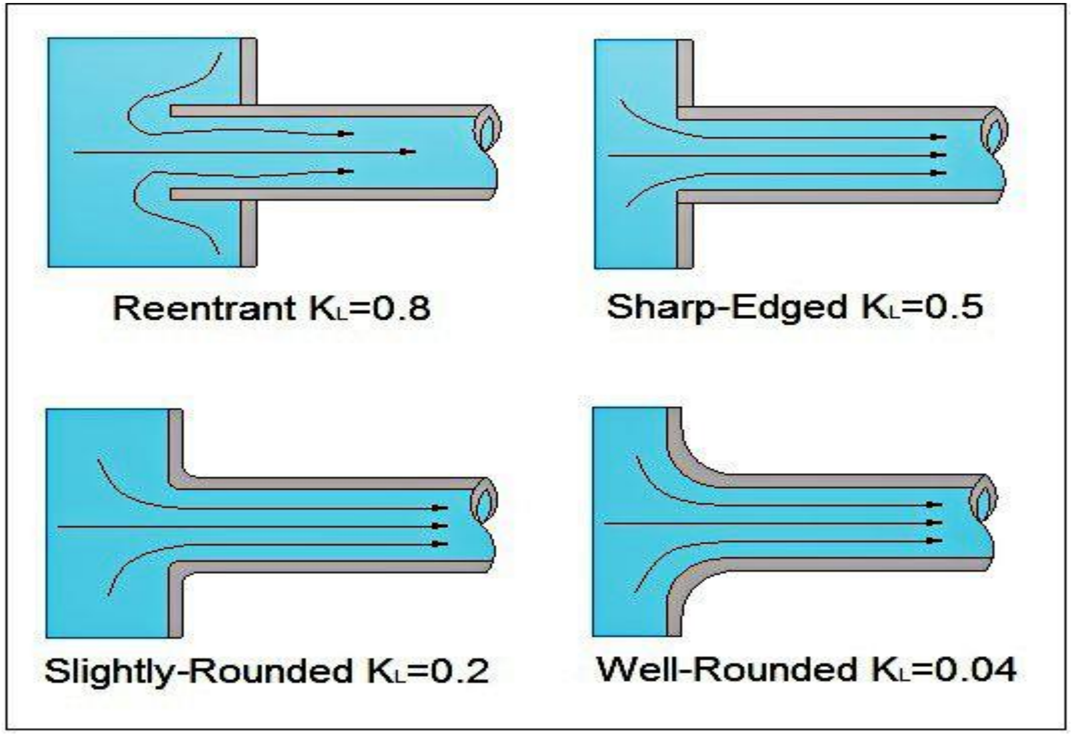
2) $\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c$

3) $A_1 V_1 = A_2 V_2$

Other Minor Losses in Pipe Flow

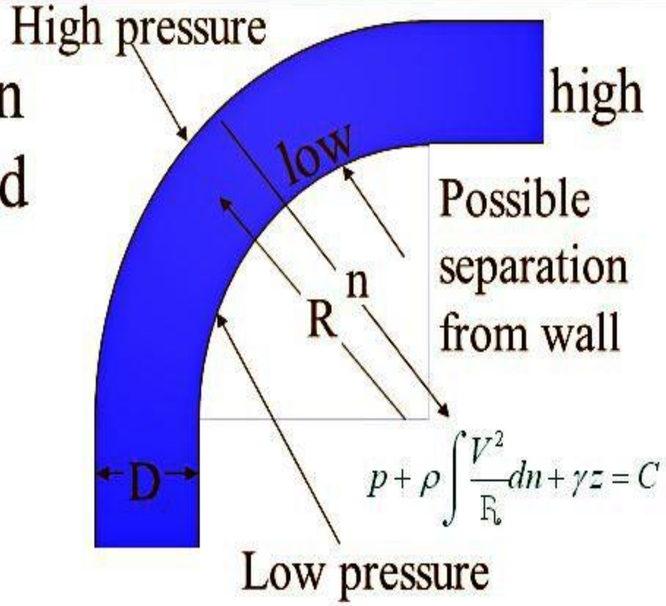
Type of Loss	Formula
Loss of head due to Obstruction, h_{obs}	$[\frac{A}{C_c(A-a)}]^2 * \frac{V_2^2}{2g}$
Loss of head at the Entrance , h_i	$0.5 * \frac{V^2}{2g}$
Loss of head at the exit, h_o	$\frac{V^2}{2g}$
Loss of head due to Bend, h_b	$k * \frac{V^2}{2g}$
Loss of head in Various Pipe fittings, $h_{fittings}$	$k * \frac{V^2}{2g}$





Head Loss in Bends

- Head loss is a function of the ratio of the bend radius to the pipe diameter (R/D)
- Velocity distribution returns to normal several pipe diameters downstream



$$h_b = K_b \frac{V^2}{2g}$$

K_b varies from 0.6 - 0.9

Total Energy Line/ Energy Gradient Line

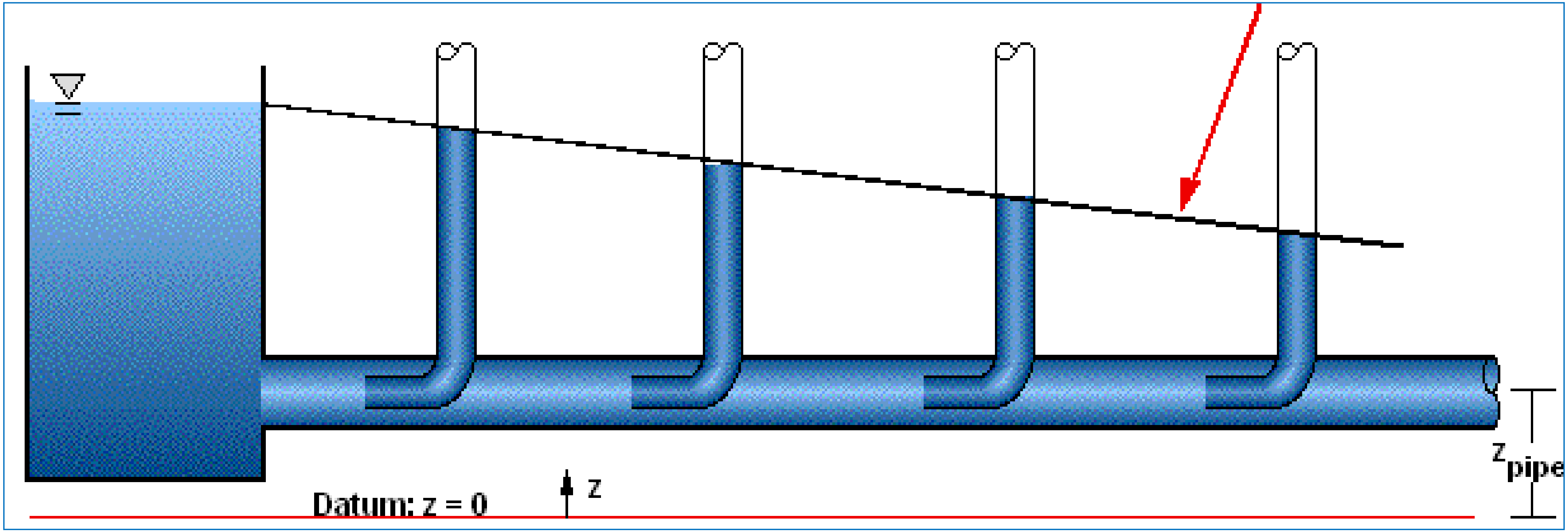
When fluid flows along the pipe, there is loss of head (energy) and the total energy decreases in the direction of flow.

If the total energy at various points along the axis of pipe is plotted and joined by a line, the line so obtained is called Energy Gradient Line (E.G.L)

Total Head (total energy per unit weight) with respect to any arbitrary datum, is the sum of pressure head, potential head and velocity head, that is

Total Head (energy) = $\frac{p}{\gamma} + z + \frac{v^2}{2g}$

EGL (Energy Gradient Line)

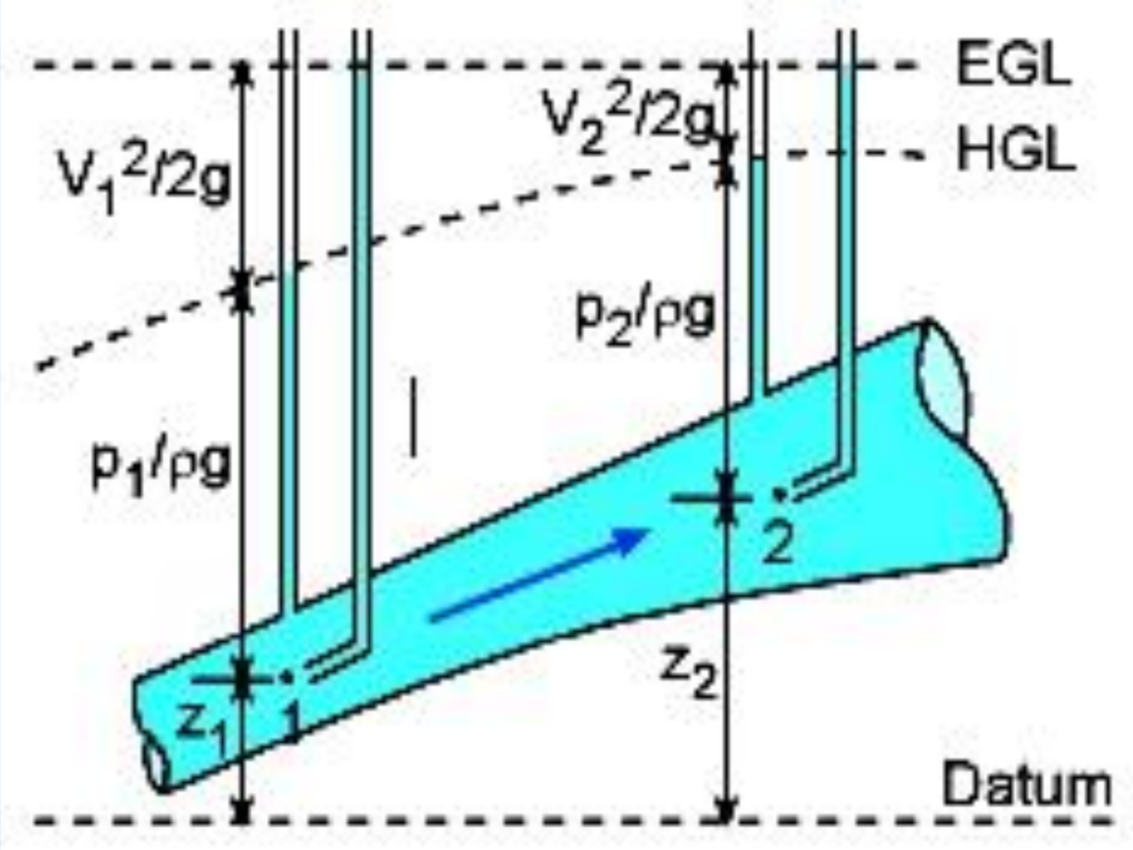


Hydraulic Gradient Line (HGL)

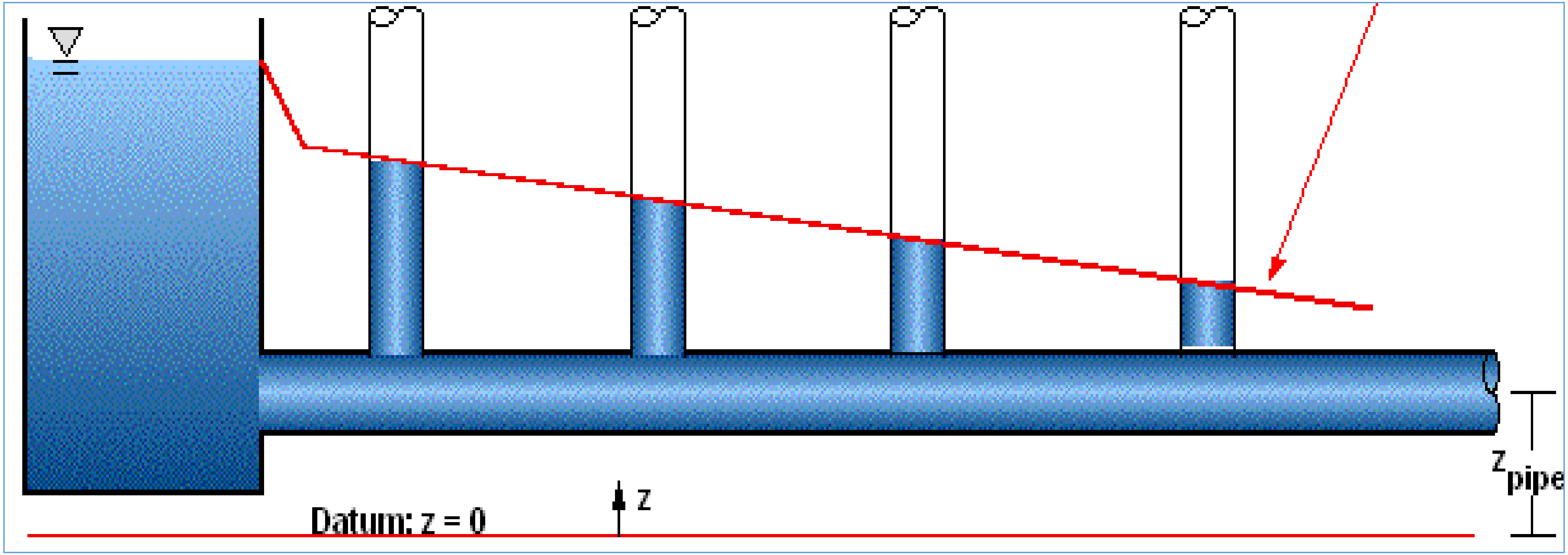
The sum of pressure head and potential head $(\frac{p}{\gamma} + z)$ at any point is called piezometric head.

If a line a is drawn joining the piezometric levels at various points, the line so obtained is called Hydraulic Gradient Line (HGL)

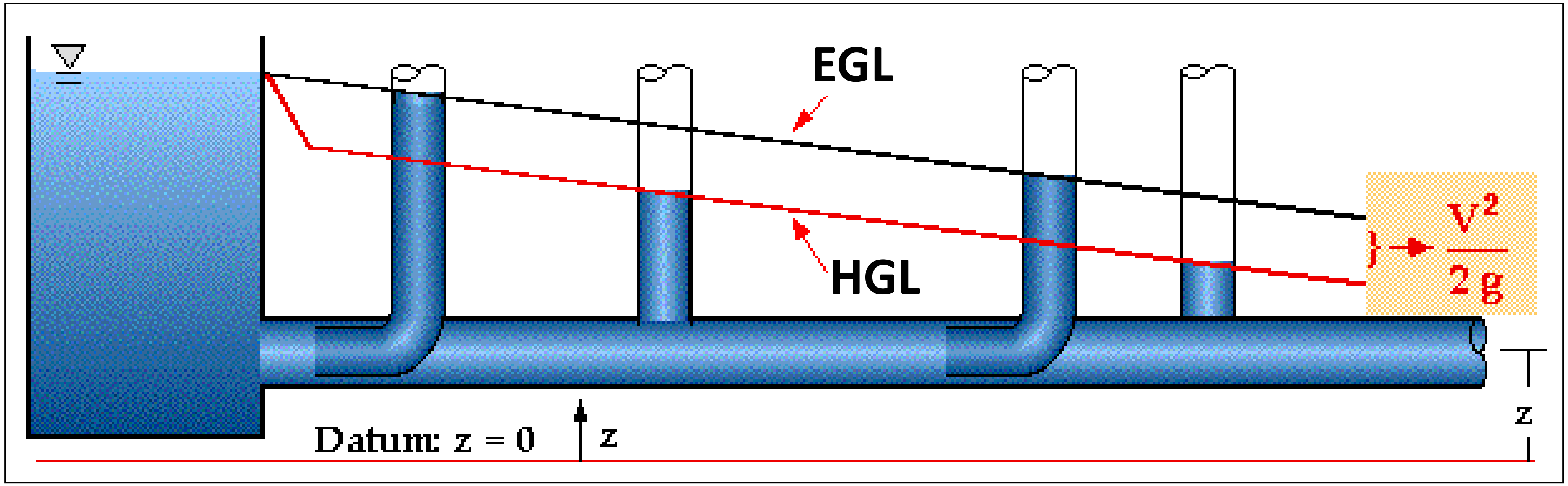
$$\text{HGL} = \frac{p}{\gamma} + z$$



Hydraulic Gradient Line (HGL)



EGL and HGL



The following points are worth noting

1. EGL always drops in the direction of flow because of loss of head.
2. HGL may rise or fall depending on the pressure changes.
3. HGL is always below the EGL and the vertical intercept between the two is equal to the velocity head $\frac{V^2}{2g}$
4. For a pipe of uniform cross-section, the slope of HGL is equal to the slope of EGL.
5. There is no relation between the slope of EGL and slope of the axis of pipe.

Practice Problem

A horizontal pipe 40 meter long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 meter length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height off water level in the tank is 8 meter above the center of the pipe. Considering all the losses of head which occur, determine the rate of flow. Take f = 0.01 for the both sections. Also draw EGL and HGL.

Solutions:

Total Length, L = 40 meter

Height of Water, H = 8 meter

Co-efficient of friction, f = 0.01

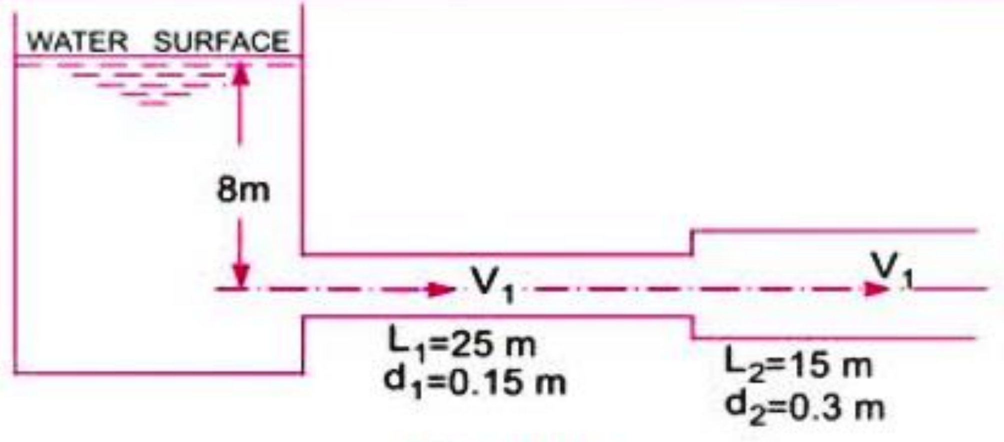


Fig. 11.5

Bernoulli's Equation

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in figure and taking reference line passing through the axis of pipe.

$$0 + 0 + 8 = 0 + \frac{V_2^2}{2g} + 0 + all\ losses$$

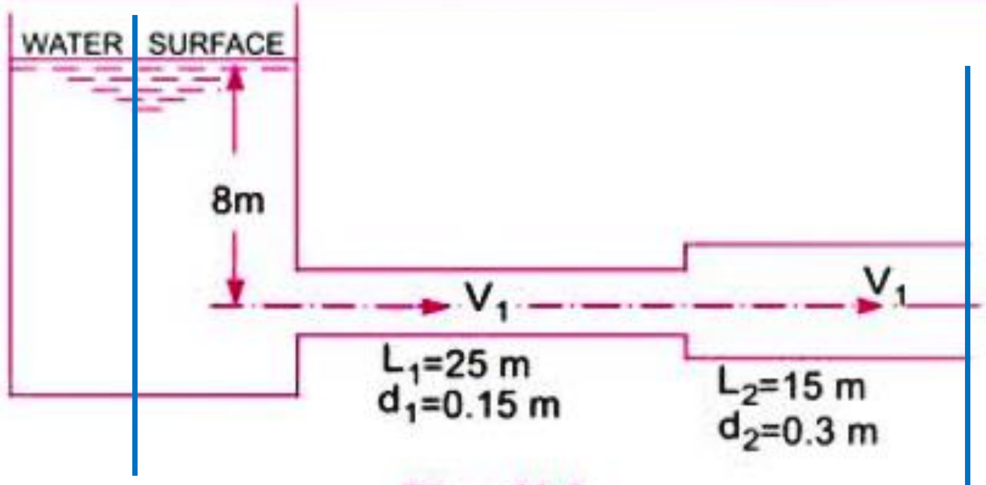


Fig. 11.5

Here, all losses included are:

1. Loss of head at entrance
2. Loss of head due to friction in pipe 1
3. Loss of head due to sudden enlargement
4. Loss of head due to friction in pipe 2
5. Loss of head at outlet

Head Losses in the direction of flow

where h_i = loss of head at entrance = $0.5\,\dfrac{V_1^2}{2g}$

h_{f_1} = head lost due to friction in pipe 1 = $\dfrac{4\times f\times L_1\times V_1^2}{d_1\times 2g}$

h_e = loss head due to sudden enlargement = $\dfrac{(V_1-V_2)^2}{2g}$

h_{f_2} = Head lost due to friction in pipe 2 = $\dfrac{4\times f\times L_2\times V_2^2}{d_2\times 2g}$

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}d_2^2\times V_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2\times V_2 = \left(\frac{0.3}{.15}\right)^2\times V_2 = 4V_2$$

Discharge, Q = $A_2 V_2$

Substituting the values of these losses in equation (*i*), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\,\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}\,[1 + 8 + 106.67 + 9 + 2] = 126.67\,\frac{V_2^2}{2g}$$

$$\therefore \quad V_2 = \sqrt{\frac{8.0 \times 2 \times g}{126.67}} = \sqrt{\frac{8.0 \times 2 \times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_2 \times V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113 = 0.07867 \text{ m}^3\text{/s}$ = **78.67 litres/s. Ans.**

Plotting of EGL, HGL

Given : $L_1 = 25$ m, $d_1 = 0.15$ m

$L_2 = 15$ m, $d_2 = 0.3$ m, $f = .01$, $H = 8$ m

The velocity V_2 as calculated in problem 11.17 is

$$V_2 = 1.113 \text{ m/s}$$

$$V_1 = 4V_2 = 4 \times 1.113 = 4.452 \text{ m/s}$$

The various head losses are $h_i = 0.5 \times \dfrac{V_1^2}{2g} = \dfrac{0.5 \times 4.452^2}{2 \times 9.81} = 0.50$ m

$$h_{f_1} = \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times .01 \times 25 \times (4.452)^2}{0.15 \times 2 \times 9.81} = 6.73 \text{ m}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4.452 - 1.11)^2}{2 \times 9.81} = 0.568 \text{ m}$$

Plotting of EGL, HGL

$$h_{f_2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g} = \frac{4 \times .01 \times 15 \times (1.113)^2}{0.3 \times 2 \times 9.81} = 0.126 \text{ m}$$

$$h_o = \frac{V_2^2}{2g} = \frac{1.113^2}{2 \times 9.81} = 0.063 \text{ m}$$

$$V_1^2/2g = \frac{4.452^2}{2 \times 9.81} = 1.0 \text{ m.}$$

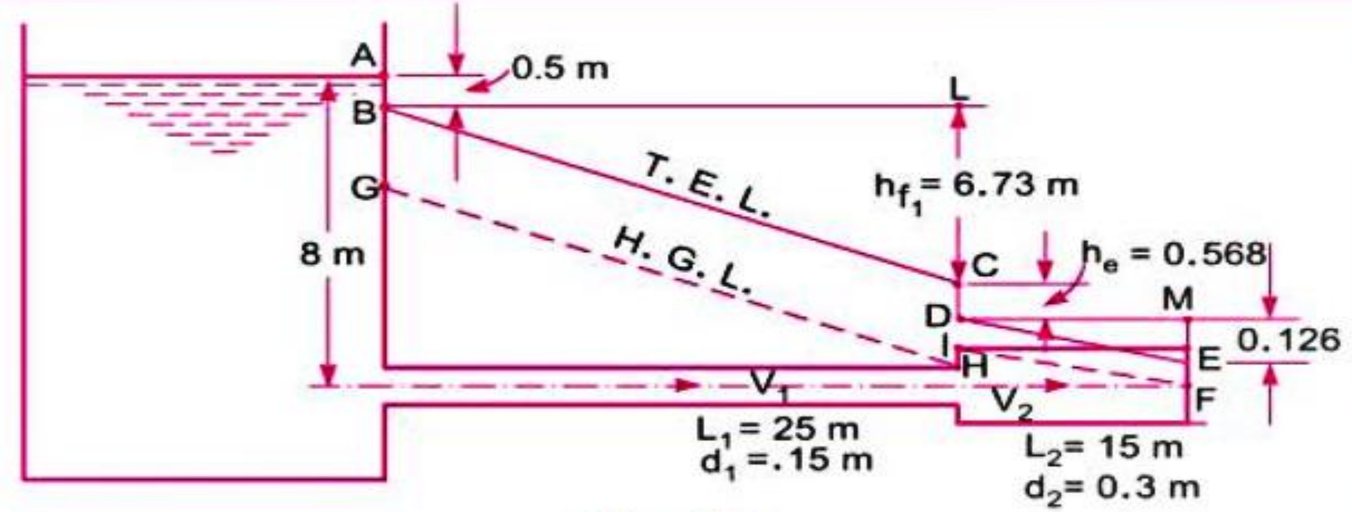
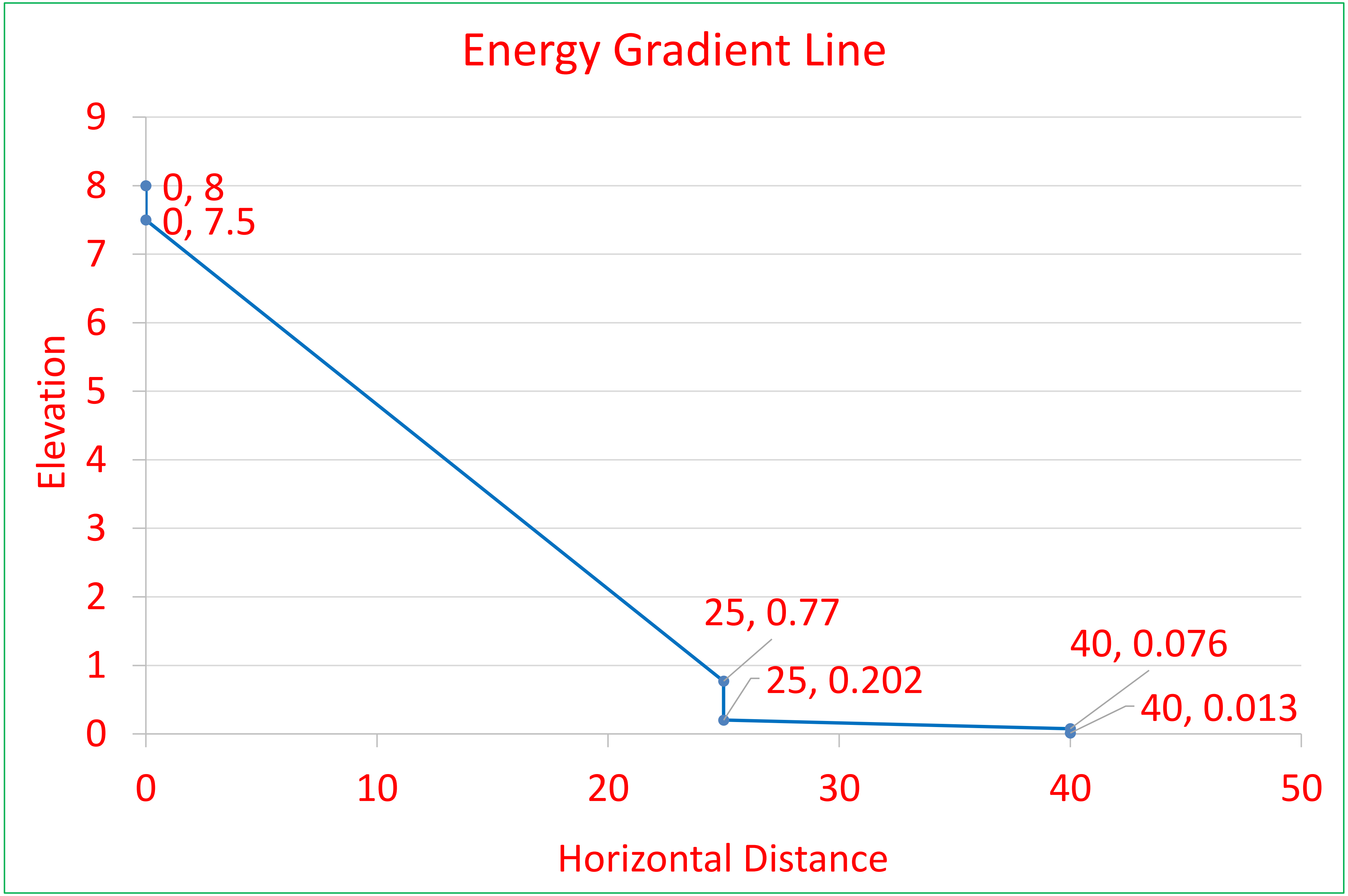


Fig. 11.9

EGL

Head Losses	Horizontal Ordinate, X	Vertical Ordinate, Y	Point
N/A	0	8	A
$h_i = 0.5$	0	7.5	B
$h_{f1} = 6.73$	25	0.77	C
$h_e = 0.568$	25	0.202	D
$h_{f2} = 0.126$	40	0.076	E
$h_o = 0.063$	40	0.013	F


Energy Gradient Line
Elevation
Horizontal Distance
0, 8
0, 7.5
25, 0.77
25, 0.202
40, 0.076
40, 0.013
9
8
7
6
5
4
3
2
1
0
0
10
20
30
40
50

HGL

1. From B, take BG = $\frac{V_1^2}{2g} = 1\ m$
2. Draw the line GH parallel to the line BC
3. From F, draw a line FI parallel to the line ED
4. Join the point H and I
5. The line GHIF represents the Hydraulic Gradient Line (HGL)

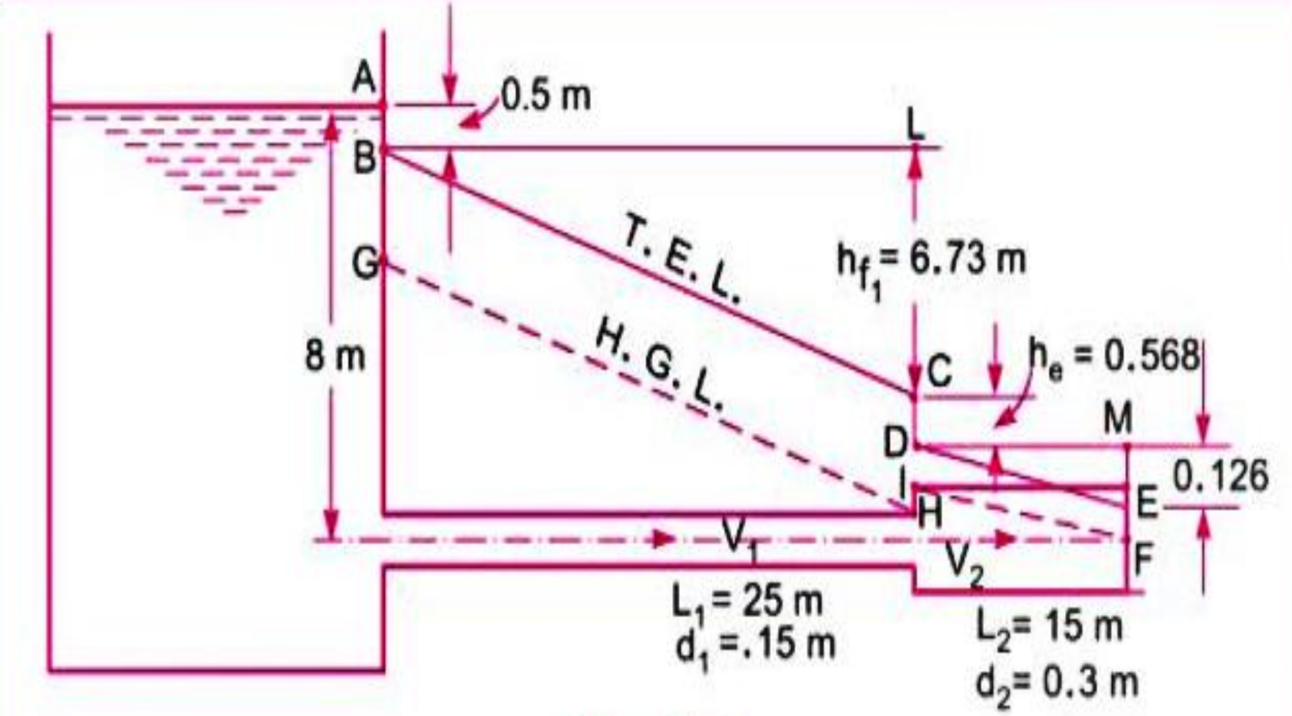
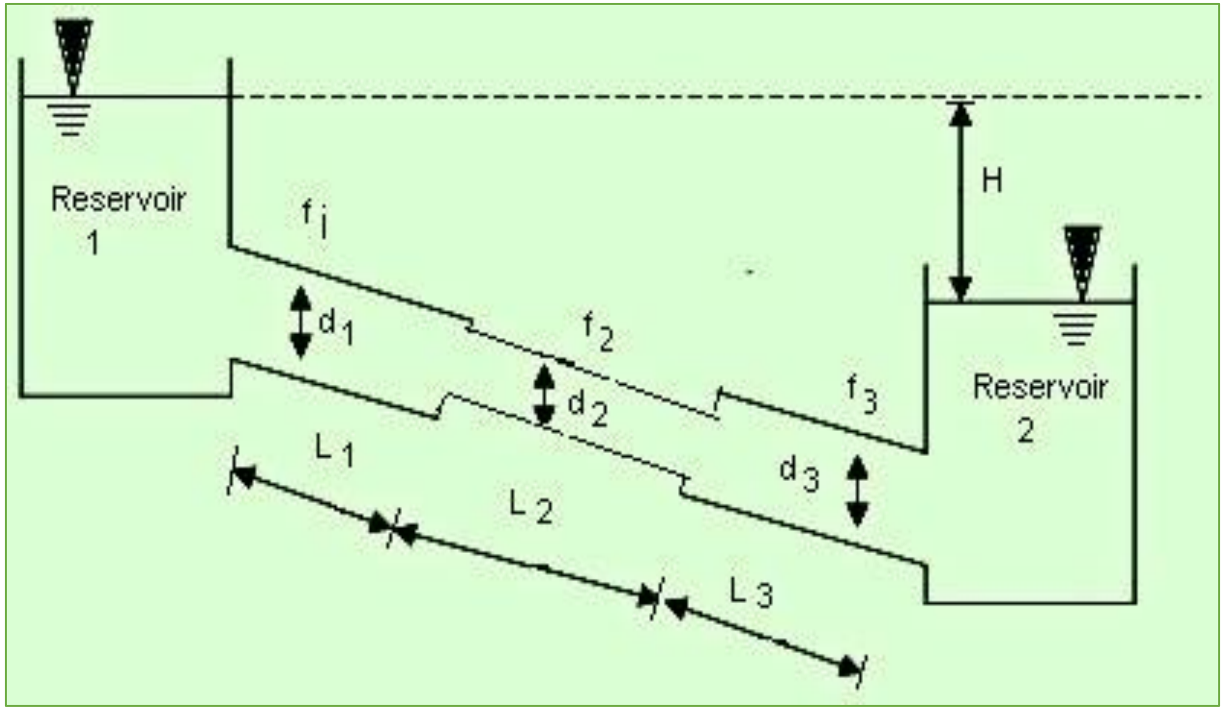


Fig. 11.9

Pipes in Series or Compound Pipes

Pipes in series or compound pipes is defined as the pipes of different length and diameters connected end to end (series) to form a pipe line as shown in figure.



$L_1, L_2, L_3 =$ Length of pipes 1,2,3 respectively
$d_1, d_2, d_3 =$ Diameter of pipes 1,2,3 respectively
$V_1, V_2, V_3 =$ Velocity of pipes 1,2,3 respectively
$f_1, f_2, f_3 =$ Coefficient of friction of pipes 1,2,3 respectively
H = Difference of water level in the two tanks.

Pipes in Series or Compound Pipes

The discharge passing through each pipe is same.

$$\therefore \quad Q = A_1V_1 = A_2V_2 = A_3V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$$\therefore \quad H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g} \quad \text{...(11.12)}$$

If minor losses are neglected, then above equation becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} \quad \text{...(11.13)}$$

If the co-efficient of friction is same for all pipes

i.e., $f_1 = f_2 = f_3 = f$, then equation (11.13) becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right] \quad \text{...(11.14)}$$

Practice Problem

Three pipes of 400 mm, 200 mm, 300 mm diameters have lengths of 400 m , 200 m and 300 m respectively. They are connected in series to make a compound pipe. The ends of this compound pipe are connected with two tanks whose difference in water levels is 16 meter. If coefficient of friction for the three pipes are same and equal to 0.005, determine the discharge through the compound pipe considering:

a) Neglecting minor losses

b) Including minor losses

Solution

Solution. Given :

Difference of water levels, $H = 16$ m

Length and dia. of pipe 1, $L_1 = 400$ m and $d_1 = 400$ mm $= 0.4$ m

Length and dia. of pipe 2, $L_2 = 200$ m and $d_2 = 200$ mm $= 0.2$ m

Length and dia. of pipe 3, $L_3 = 300$ m and $d_3 = 300$ mm $= 0.3$ m

Also $f_1 = f_2 = f_3 = 0.005$

(i) Discharge through the compound pipe first neglecting minor losses.

Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2} \times V_1 = \frac{d_1^2}{d_2^2}V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

and

$$V_3 = \frac{A_1V_1}{A_3} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_3^2} \times V_1 = \frac{d_1^2}{d_3^2}V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 1.77V_1$$

Solution

Now using equation (11.13), we have

$$H = \frac{4f_1 L_1 V_1^2}{d_1 \times 2g} + \frac{4f_2 L_2 V_2^2}{d_2 \times 2g} + \frac{4f_3 L_3 V_3^2}{d_3 \times 2g}$$

or

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300}{0.3 \times 2 \times 9.81} \times (1.77\ V_1)^2$$

$$= \frac{V_1^2}{2 \times 9.81}\left(\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2 \times 9.81}(20 + 320 + 63.14) = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$\therefore$

$$V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

Solution

$\therefore$ Discharge, $Q = A_1 \times V_1 = \frac{\pi}{4}(0.4)^2 \times 0.882 = \mathbf{0.1108\ m^3/s.}$ **Ans.**

(*ii*) **Discharge through the compound pipe considering minor losses also.**

Minor losses are :

(*a*) At inlet, $h_i = \frac{0.5\,V_1^2}{2g}$

(*b*) Between 1st pipe and 2nd pipe, due to contraction,

$$h_c = \frac{0.5\,V_2^2}{2g} = \frac{0.5\left(4V_1^2\right)}{2g} \qquad (\because V_2 = 4V_1)$$

$$= \frac{0.5 \times 16 \times V_1^2}{2g} = 8 \times \frac{V_1^2}{2g}$$

(*c*) Between 2nd pipe and 3rd pipe, due to sudden enlargement,

$$h_e = \frac{\left(V_2 - V_3\right)^2}{2g} = \frac{(4V_1 - 1.77V_1)^2}{2g} \qquad (\because V_3 = 1.77\ V_1)$$

$$= (2.23)^2 \times \frac{V_1^2}{2g} = 4.973\ \frac{V_1^2}{2g}$$

(*d*) At the outlet of 3rd pipe, $h_o = \frac{V_3^2}{2g} = \frac{(1.77V_1)^2}{2g} = 1.77^2 \times \frac{V_1^2}{2g} = 3.1329\ \frac{V_1^2}{2g}$

Solution

The major losses are $= \dfrac{4f_1 \times L_1 \times V_1^2}{d_1 \times 2g} + \dfrac{4f_2 \times L_2 \times V_2^2}{d_2 \times 2g} + \dfrac{4f_3 \times L_3 \times V_3^2}{d_3 \times 2g}$

$$= \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300 \times (1.77V_1)^2}{0.3 \times 2 \times 9.81}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

∴ Sum of minor losses and major losses

$$= \left[\frac{0.5\,V_1^2}{2g} + 8 \times \frac{V_1^2}{2g} + 4.973\,\frac{V_1^2}{2g} + 3.1329\,\frac{V_1^2}{2g}\right] + 403.14\,\frac{V_1^2}{2g}$$

$$= 419.746\,\frac{V_1^2}{2g}$$

But total loss must be equal to H (or 16 m)

∴ $419.746 \times \dfrac{V_1^2}{2g} = 16 \quad \therefore \quad V_1 = \sqrt{\dfrac{16 \times 2 \times 9.81}{419.746}} = 0.864$ m/s

∴ Discharge, $Q = A_1 V_1 = \dfrac{\pi}{4}(0.4)^2 \times 0.864 =$ **$0.1085\ m^3/s$. Ans.**

Equivalent Pipe (Pipes in Series)

Equivalent pipe is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

The uniform diameter of the equivalent pipe is known as the equivalent diameter of the series or compound pipe.

Let, L_1, L_2, L_3 etc. = Length of pipes 1,2,3 etc.

D_1, D_2, D_3 etc. = Diameter of pipes 1,2,3 etc.

H = Total head loss

L = Length of the equivalent pipe

D = Diameter of the equivalent pipe.

Total Head Loss in Compound Pipe

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} \quad \text{...(11.14A)}$$

Assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1V_1 = A_2V_2 = A_3V_3 = \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2 = \frac{\pi}{4}d_3^2V_3$

$\therefore$ $V_1 = \frac{4Q}{\pi d_1^2}$, $V_2 = \frac{4Q}{\pi d_2^2}$ and $V_3 = \frac{4Q}{\pi d_3^2}$

Substituting these values in equation (11.14A), we have

$$H = \frac{4fL_1 \times \left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] \quad \text{...(11.15)}$$

Head Loss in Equivalent Pipe

Taking same value of f in compound pipe,

Head loss in equivalent pipe, H = $\frac{4fL}{D} * \frac{V^2}{2g}$

Where, velocity, V $= \frac{Q}{A} = \frac{4Q}{\pi d^2}$

So, substituting the value of velocity, we get,

H = $\boldsymbol{\frac{4*16fQ^2}{\pi^2*2g}\left[\frac{L}{d^5}\right]}$

Head loss in compound pipe and equivalent pipe is same, therefore,

$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} +$ ……………………………. [Dupit's Equation]

If the length of the equivalent pipe is equal to the length of the compound pipe; L = $L_1 + L_2 + L_3 +$…….,diameter D of the equivalent pipe may be determined by using this equation.

Problem# Rajput (663 page)

A piping system consists of three pipes arranged in series: the length of the pipes are 1200 m, 750 m and 600 m and diameters 750 mm, 600 mm and 450 mm respectively.

1. Transform the system to an equivalent 450 mm diameter pipe and
2. Determine an equivalent diameter for the pipe, 2550 m long.

Formula: $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

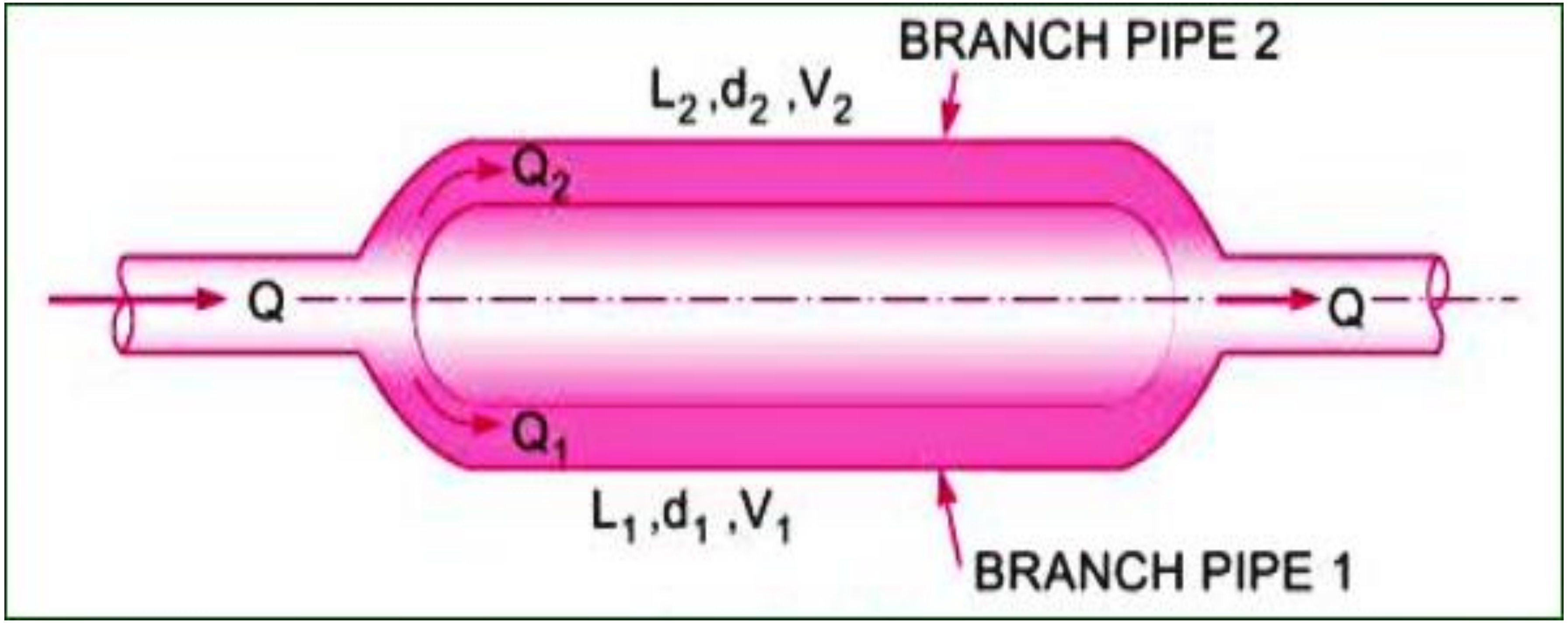
Answer:

Length, L = 871.3 m

Diameter, D = 557.8 mm

Pipes in Parallel

The pipes are said to be in parallel when a main line divides into two or more parallel pipes which again join together downstream and continues as a main line.



Pipes in Parallel

The rate of flow in the main pipe is equal to the sum of flow through branch pipes. Hence, we have, Q = $Q_1 + Q_2$

In this arrangement, loss of head for each branch pipe is same.

Loss of head for branch pipe 1 = Loss of head for branch pipe 2

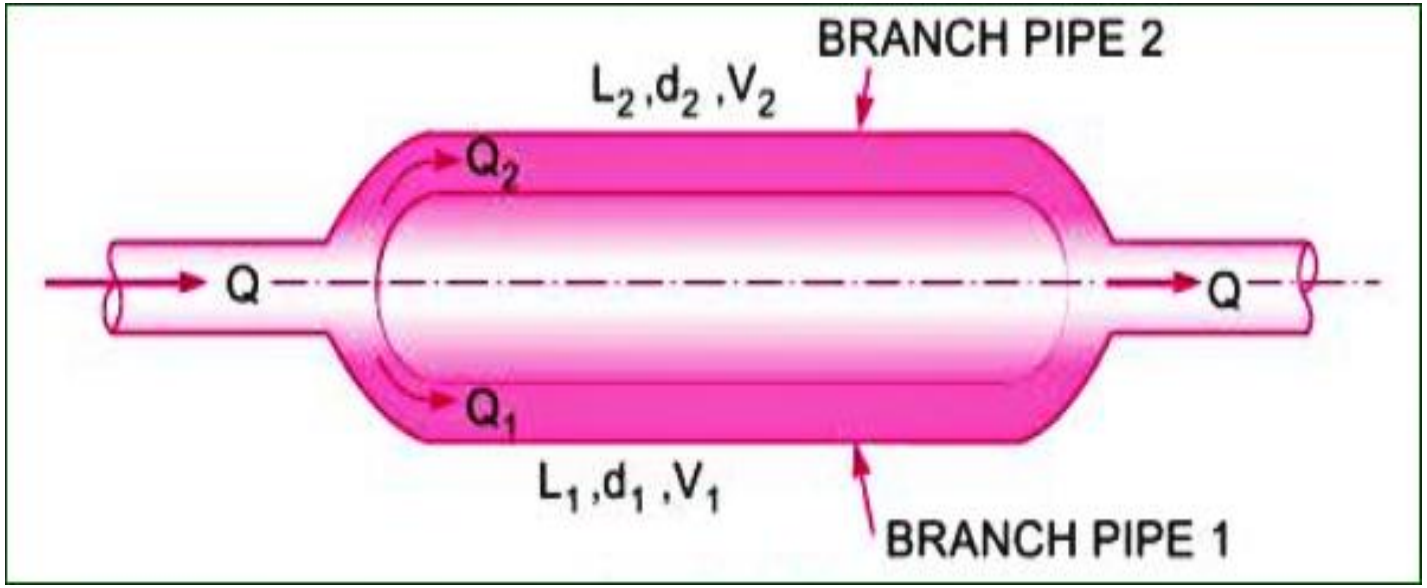
$$\frac{4f_1L_1V_1^2}{d_1 * 2g} = \frac{4f_2L_2V_2^2}{d_2 * 2g}$$

When, $f_1 = f_2$,

$$\frac{L_1V_1^2}{d_1 * 2g} = \frac{L_2V_2^2}{d_2 * 2g}$$

Problem # Bansal (Page=509)

A main pipe divides into two parallel pipes which again forms one pipe as shown in figure. The length and diameter for the first parallel pipe are 2000 m and 1 m respectively, while length and diameter for the second parallel pipe are 2000 m and 0.8 m respectively. Find the rate of flow in each parallel pipe if the total flow in the main pipe is 3 m^3/s. Co-efficient of friction for each parallel pipe is equal to 0.005.



Solution

Parameter	Value
Length of Pipe 1	L_1 = 2000 m
Diameter of Pipe 1	d_1= 1 m
Length of Pipe 2	L_2 = 2000 m
Diameter of Pipe 2	d_2 = 0.8 m
Co-efficient of Friction	$f_1 = f_2 = f = 0.005$
Total discharge	Q = 3 m^3/s = $Q_1 + Q_2$
Discharge in Pipe 1	Q_1
Discharge in Pipe 2	Q_2

Solution

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\frac{4 \times .005 \times 2000 \times V_1}{1.0 \times 2 \times 9.81} = \frac{4 \times .005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

or $$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} \text{ or } V_1^2 = \frac{V_2^2}{0.8}$$

$\therefore$ $$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894}$$

Now $$Q_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} (1)^2 \times \frac{V_2}{.894}$$

and $$Q_2 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4} (.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$$

Substituting the value of Q_1 and Q_2 in equation (i), we get

$$\frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times .64\ V_2 = 3.0 \text{ or } 0.8785\ V_2 + 0.5026\ V_2 = 3.0$$

or $$V_2[.8785 + .5026] = 3.0 \text{ or } V = \frac{3.0}{1.3811} = 2.17 \text{ m/s.}$$

Solution

Velocity in Pipe 2 , $V_2 = 2.17$ m/s

Velocity in Pipe 1, V_1= 2.427 m/s

Discharge in pipe 1, $Q_1 = 0.25 * \pi * 1^2 * 2.427$= 1.906 m^3/s

Discharge in pipe 2, $Q_2 = 3 - 1.906$ = 1.094 m^3/s

Pipe Network Analysis

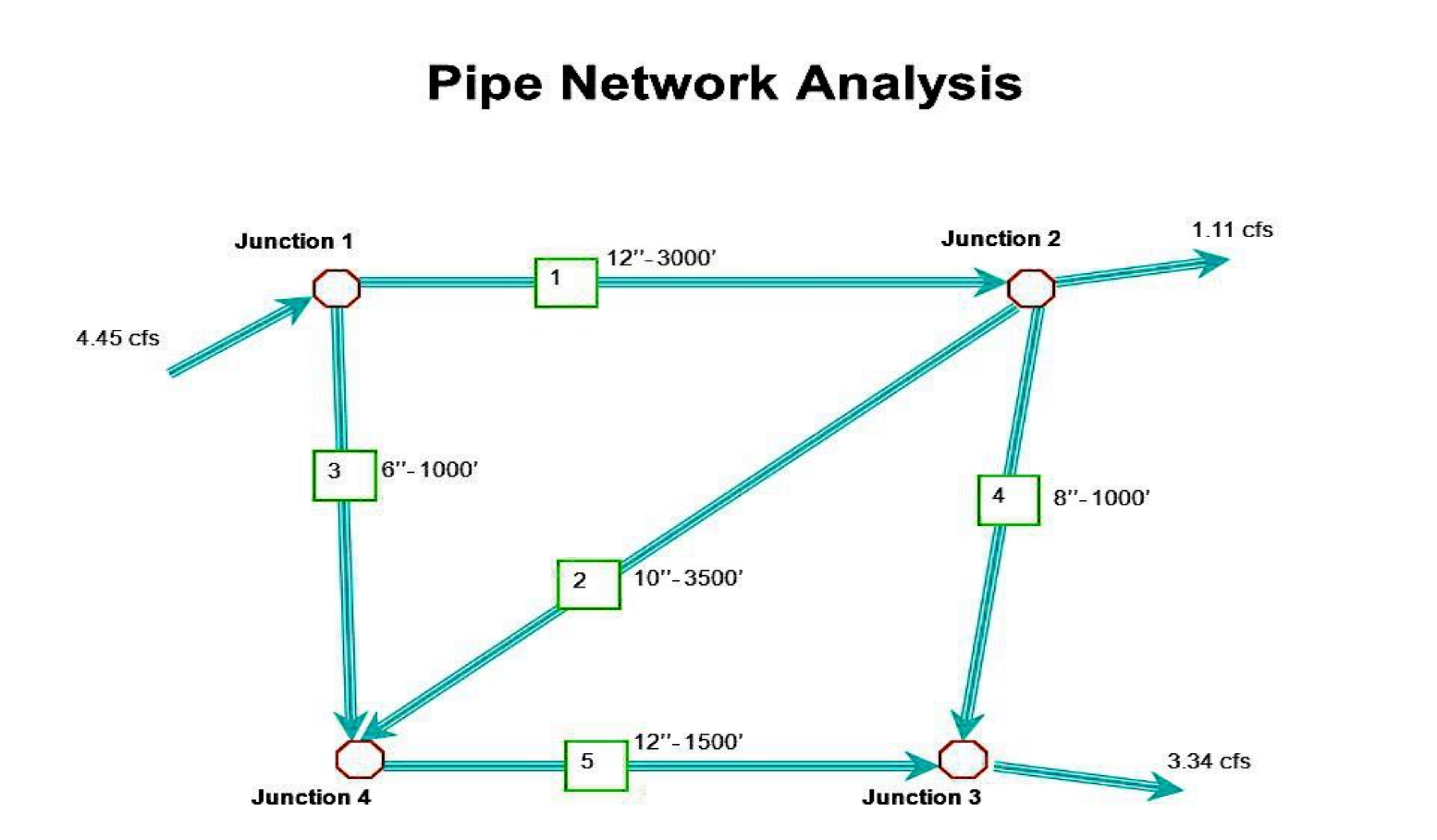


Figure 1: A Small Pipe Network

A pipe network is an interconnected system of pipes forming several loops and circuits.

Pipe Network

The following are the necessary conditions for any network of pipes.

1. The flow into each junctions must be equal to the flow out of the junctions. This is due to continuity equation.
2. The algebraic sum of head losses round each loop must be zero. This means that in each loop, the loss of head due to flow in clockwise direction must be equal to the head due to flow in anticlockwise direction.
3. The head loss in each pipe is expressed as $h_f = r\,Q^n$. The value of r depends on three factors:

a) Length of pipe

b) Diameter of pipe

c) Co-efficient of friction of pipe.

Pipe Network

$$h_f = \frac{4 \times f \times L \times V^2}{D \times 2g} = \frac{4fL \times \left(\frac{Q}{A}\right)^2}{D \times 2g} \qquad \left(\because\ V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2}\right)$$

$$= \frac{4fL \times Q^2}{D \times 2g \times \left(\frac{\pi}{4}D^2\right)^2} = \frac{4fL \times Q^2}{D \times 2g \times \left(\frac{\pi}{4}\right)^2 \times D^4}$$

$$= \frac{4f \times L \times Q^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times D^5}$$

$$h_f = r\,Q^2; n = 2\ for\ turbulent\ flow$$

Hardy Cross Method

1. In this method, a trial distribution of discharges is made arbitrarily but in such a way that continuity equation is satisfied at each node.
2. With the assumed values of Q, the head loss in each pipe is calculated according to the equation $h_f = r\, Q^n$
3. Now consider any loop. The algebraic sum of head losses round each loop must be zero.
4. Now calculate net head loss around each loop considering the head loss to be positive in clockwise flow and to be negative in anti-clockwise flow.
5. If the net head loss due to assumed values of Q round the loop is zero, then the assumed values of Q in that loop is correct.
6. But if the net head loss due to assumed values of Q is not zero, then the assumed values of Q are corrected by a correction factor ΔQ for the flows till the circuit is balanced.

Hardy Cross Method

* Let for any pipe Q_0 = assumed discharge and Q = correct discharge, then

$$Q = Q_0 + \Delta Q$$

$\therefore$ Head loss for the pipe, $h_f = rQ^2 = r(Q_0 + \Delta Q)^2$.

For complete circuit, the net head loss, $\Sigma h_f = \Sigma (rQ^2) = \Sigma r (Q_0 + \Delta Q)^2 = \Sigma r(Q_0^2 + 2Q_0 \Delta Q + \Delta Q^2)$

$= \Sigma r (Q_0{}^2 + 2Q_0 \Delta Q)$ As ΔQ is small compared with Q_0 and hence ΔQ^2 can be neglected.

$\therefore$
$$\Sigma rQ^2 = \Sigma rQ_0{}^2 + \Sigma r \times 2Q_0 \Delta Q$$

For the correct distribution, the net head loss for a circuit should be zero (*i.e.*, $\Sigma h_f = \Sigma (rQ^2) = 0$)

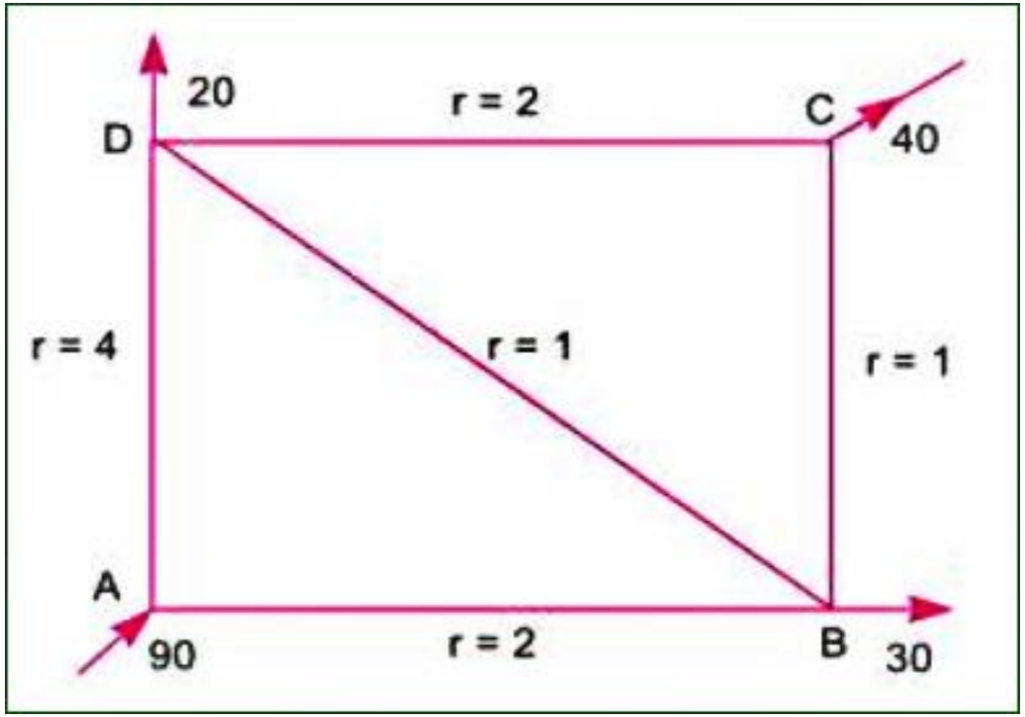
$\therefore$
$$\Sigma rQ_0^2 + \Sigma r \times 2Q_0 \Delta Q = 0$$

$\Sigma rQ_0^2 + \Delta Q \, \Sigma r \times 2Q_0 = 0$ [As ΔQ is same for one circuit, hence it can be taken out of the summation]

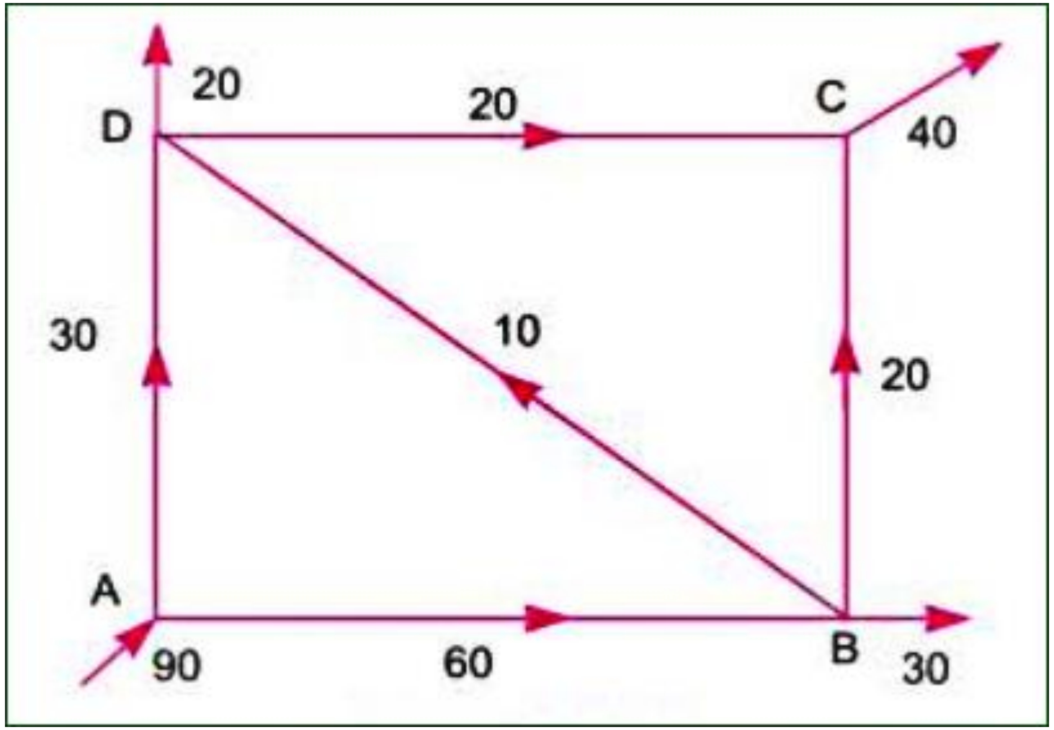
$\therefore$
$$\Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2r Q_0}.$$

Problem: Bansal#549

Calculate the discharge in each pipe of the network shown in figure. The pipe network consists of 5 pipes. The head loss h_f in a pipe is given by $h_f = rQ^2$. The values of r for various pipes and also the inflow or outflows at nodes are shown in figure.



First trial



For the first trial, the discharges are assumed as shown in figure so that continuity equation is satisfied at each node (flow into node = flow out of the node). For this distribution of discharge, the corrections ΔQ for the loops ABD and BCD are calculated.

First trial

Loop ADB				
Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	30	$4 \times 30^2 = 3600$	$2 \times 4 \times 30 = 240$
DB	1	10	$-1 \times 10^2 = -100$	$2 \times 1 \times 10 = 20$
AB	2	60	$-2 \times 60^2 = -7200$	$2 \times 2 \times 60 = 240$
			$\Sigma rQ_0^2 = -3700.$	$\Sigma 2rQ_0 = 500.$

$$\therefore \quad \Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2r Q_0} = \frac{-(-3700)}{500} = 7.4$$

Loop DCB				
Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
DC	2	20	$2 \times 20^2 = 800$	$2 \times 2 \times 20 = 80$
CB	1	20	$-1 \times 20^2 = -400$	$2 \times 1 \times 20 = 40$
BD	1	10	$1 \times 10^2 = 100$	$2 \times 1 \times 10 = 20.$
			$\Sigma 2\ rQ_0^2 = 500$	$\Sigma 2rQ_0 = 140$

$$\therefore \quad \Delta Q = \frac{-\Sigma r Q_0^2}{\Sigma 2r Q_0} \frac{-500}{140} = -3.57 \quad \simeq -3.6.$$

*Head loss h_f is positive for clockwise direction and vice versa.

*If ΔQ is positive for a loop, then it is to be added to the flow in clockwise direction and vice versa.

First trial

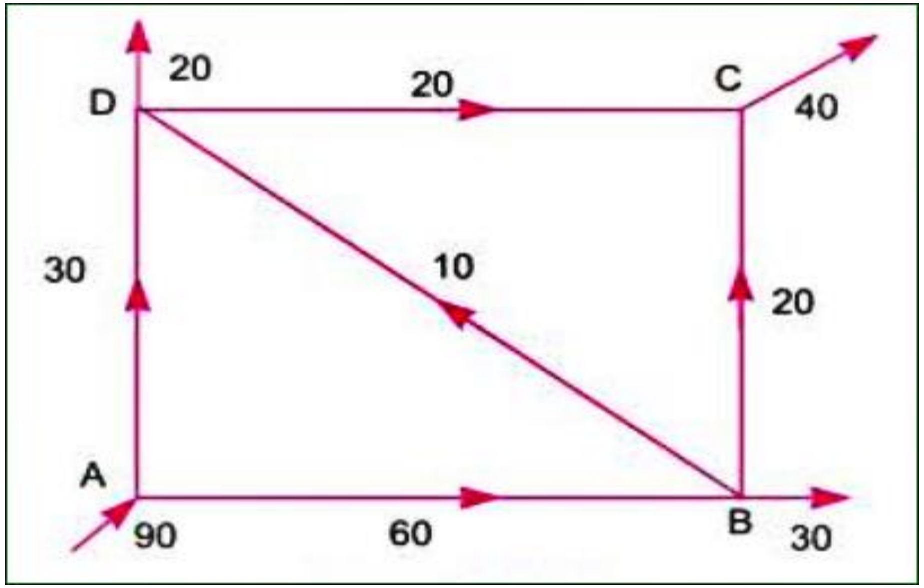
Corrected flow for each pipe is given as:

Loop ADB (ΔQ= +7.4)

Pipe AD = 30 + 7.4 = 37.4 (flow is clockwise)

Pipe AB = 60 – 7.4 = 52.6 (flow is anti-clockwise)

Pipe BD = 10 – 7.4 = 2.6 (flow is anti-clockwise)



Corrected flow for each pipe is given as:

Loop DCB (ΔQ= – 3.6)

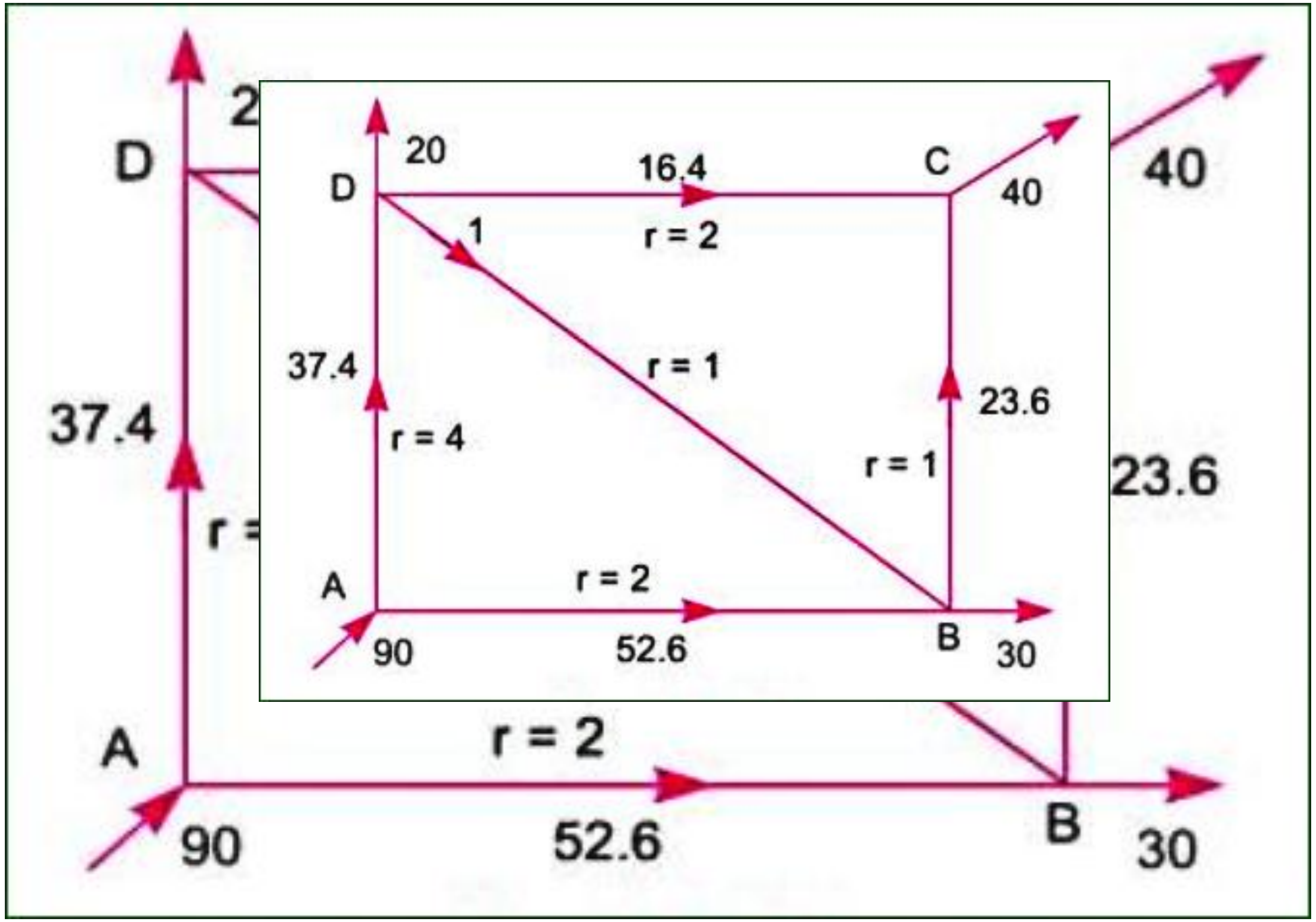
Pipe DC = 20 – 3.6 = 16.4 (flow is clockwise)

Pipe BC = 20 + 3.6 = 23.6 (flow is anti-clockwise)

Pipe BD = 2.6 – 3.6 = – 1 (flow is clockwise)

*The pipe BD is common in two loops, Hence this pipe will get two corrections.

First trial: Results



Second trial

Loop ADB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
AD	4	37.4	$4\times 37.4^2 = 5595$	$2\times 4\times 37.4 = 299.2$
DB	1	1	$1\times 1^2 = 1$	$2\times 1\times 1 = 2$
AB	2	52.6	$-2\times 52.6^2 = -5533.5$	$2\times 2\times 52.6 = 210.4$

$$\Sigma rQ_0^2 = 62.54,\qquad \Sigma 2rQ_0 = 511.6$$

$$\therefore \Delta Q = \frac{-\sum r Q_0^2}{\sum 2r Q_0} = \frac{62.54}{-511.6}$$

$$= -0.122 \simeq -0.1$$

Loop DCB

Pipe	r	Q_0	$h_f = rQ_0^2$	$2rQ_0$
DC	2	16.4	$2\times 16.4^2 = 537.9$	$2\times 2\times 16.4 = 65.6$
CB	1	23.6	$-1\times 23.6^2 - 556.9$	$2\times 1\times 23.6 = 47.2$
BD	1	1	$-1\times 1^2 = -1$	$2\times 1\times 1 = 2$

$$\Sigma rQ_0^2 = -20,\qquad \Sigma 2rQ_0 = 114.8$$

$$\therefore \Delta Q = \frac{-\sum r Q_0^2}{\sum 2r Q_0} = \frac{-(-20)}{114.8}$$

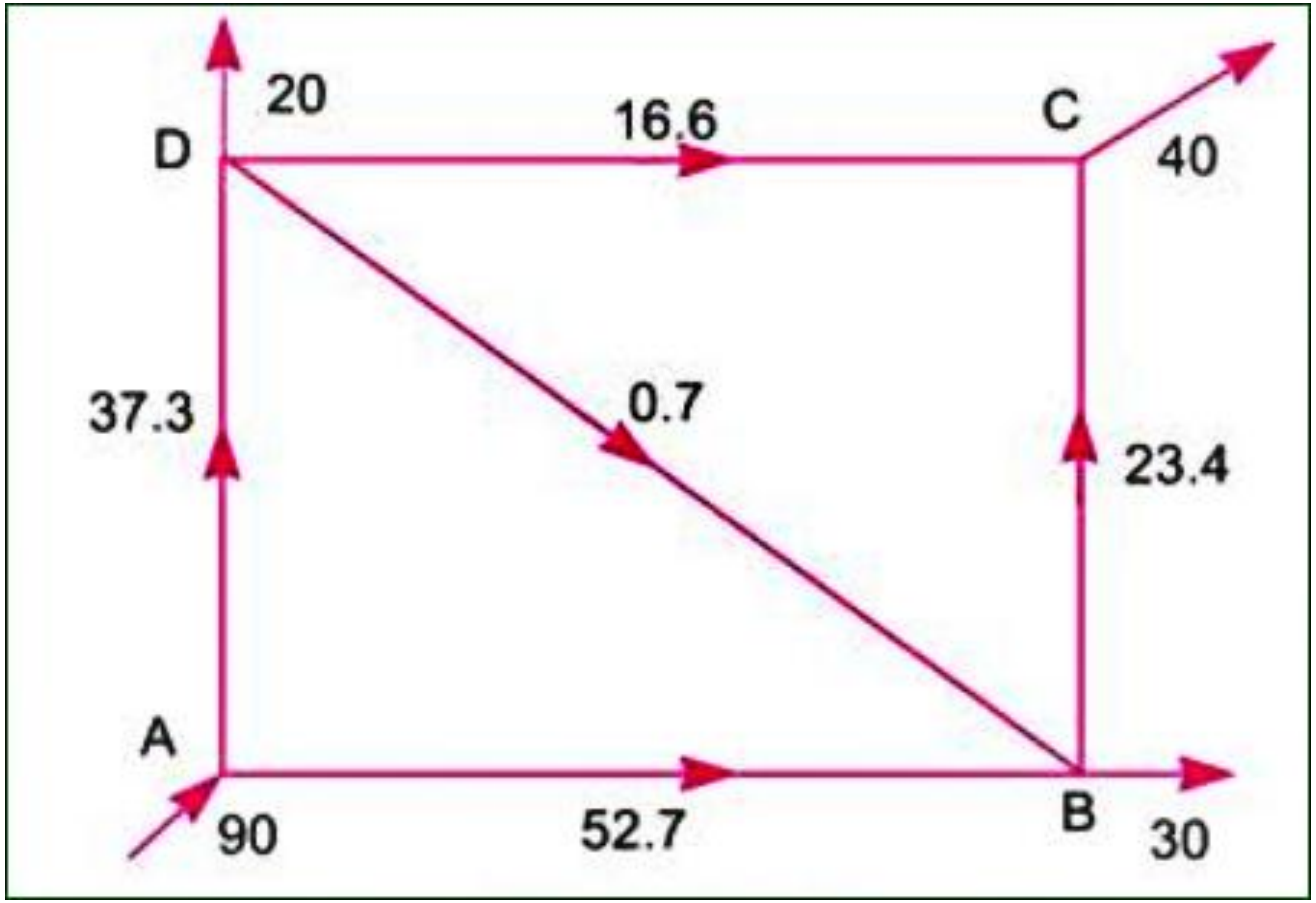
$$= \frac{20}{114.8} = 0.174$$

$$\simeq 0.2$$

Second trial: Results

Loop ADB (ΔQ = -0.1)	Loop DCB (ΔQ = + 0.2)
Pipe AD = 37.4 – 0.1 = 37.3 (Clockwise)	Pipe DC = 16.4 + 0.2 = 16.6 (Clockwise)
Pipe DB = 1– 0.1 = 0.90 (Clockwise)	Pipe CB = 23.6 - 0.2 = 23.4 (Anti-clockwise)
Pipe AB = 52.6 + 0.10 = 52.7 (Anticlockwise)	Pipe BD = 0.9 – 0.2 = 0.7 (Clockwise)
Final distribution of Discharges	
Pipe AD = 37.3 (from A to D)	
Pipe AB = 52.7 (from A to B)	
Pipe DB = 0.70 (from D to B)	
Pipe DC = 16.6 (from D to C)	
Pipe BC = 23.4 (from B to C)	

Second trial: Results



Dimensional Analysis

Dimensional analysis is a mathematical technique which makes the use of the study of the dimensions for solving several engineering problems.

Dimensional analysis helps in determining a systematic arrangement of the variables in the physical relationship, combining dimensional variables to form non-dimensional parameters.

Uses of Dimensional Analysis:

1. To test the dimensional homogeneity of any equation of fluid motion.
2. To derive rational formulae for a flow phenomenon.
3. To derive equations expressed in terms of non-dimensional parameters to show the relative significance of each parameter.
4. To plan model tests and present experimental results in a systematic manner; thus making it possible to analyze the complex fluid flow problems.

Dimensional Analysis

Advantages of Dimensional Analysis:

1. It expresses the functional relationship between the variables in dimensionless terms.
2. In hydraulic model studies , it reduces the number of variables involved in a physical phenomenon.
3. By proper selection of variables, the dimensionless parameters can be used to make certain logical deductions about the problem.
4. Design curves, by the use of dimensional analysis can be developed from experimental data or direct solution of the problem.
5. It enables getting up a theoretical equation in a simplified dimensional form.

Dimensions: Rajput#380

Quantity	Symbol	Dimensions	
		$MLT\Theta$	$FLT\Theta$
Length	L	L	L
Area	A	L^2	L^2
Volume	$\mathcal{V}$	L^3	L^3
Velocity	V	LT^{-1}	LT^{-1}
Acceleration	dV/dt	LT^{-2}	LT^{-2}
Speed of sound	a	LT^{-1}	LT^{-1}
Volume flow	Q	L^3T^{-1}	L^3T^{-1}
Mass flow	$\dot{m}$	MT^{-1}	FTL^{-1}
Pressure, stress	p, σ, τ	$ML^{-1}T^{-2}$	FL^{-2}
Strain rate	$\dot{\epsilon}$	T^{-1}	T^{-1}
Angle	θ	None	None
Angular velocity	ω, Ω	T^{-1}	T^{-1}
Viscosity	μ	$ML^{-1}T^{-1}$	FTL^{-2}
Kinematic viscosity	ν	L^2T^{-1}	L^2T^{-1}
Surface tension	Υ	MT^{-2}	FL^{-1}
Force	F	MLT^{-2}	F
Moment, torque	M	ML^2T^{-2}	FL
Power	P	ML^2T^{-3}	FLT^{-1}
Work, energy	W, E	ML^2T^{-2}	FL
Density	ρ	ML^{-3}	FT^2L^{-4}
Temperature	T	Θ	Θ
Specific heat	c_p, c_v	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	γ	$ML^{-2}T^{-2}$	FL^{-3}
Thermal conductivity	k	$MLT^{-3}\Theta^{-1}$	$FT^{-1}\Theta^{-1}$
Thermal expansion coefficient	β	Θ^{-1}	Θ^{-1}

Dimensional Homogeneity

Dimensional homogeneity states that, every term in an equation when reduced to fundamental dimensions must contain identical powers of each dimension.

Let us consider the equation,

Pressure, p = γh

Dimension of L.H.S = $\mathrm{M}L^{-1}T^{-2}$

Dimension of R.H.S = $\mathrm{M}L^{-2}T^{-2} * L$= $\mathrm{M}L^{-1}T^{-2}$

Dimension of L.H.S = Dimension of R.H.S

So, equation, p = γh is dimensionally homogeneous; so it can be used in any system of units.

Methods of Dimensional Analysis

The methods of dimensional analysis are:

1. Rayleigh's method
2. Buckingham's π method
3. Bridgman's method
4. Matrix-tensor method
5. By visual inspection of the variables involved
6. Rearrangement of differential equations.

Here, only first two methods will be dealt with.

Rayleigh's Method

In this method, a functional relationship of some variables is expressed in the form of an exponential equation which must be dimensionally homogeneous. Thus, if X is a variable which depends on X_1, X_2, X_3,....... X_n; the functional equation can be written as:

$X = f\,(X_1, X_2, X_3, \ldots\ldots\ X_n)$ .... *(1)*

In the above equation, X is a dependent variable, while X_1, X_2, X_3,....... X_n are independent variable.

A dependent variable is the one about which information is required while independent variables are those which govern the variation of dependent variable.

Thus equation (1) can be written as:

$X = C\,(X_1^a, X_2^b, X_3^c, \ldots\ldots X_n^n)$; *Where, C is a constant and a,b,c are the arbitrary powers.*

Rayleigh's Method (Modi#842)

Let Q be the discharge passing through a small orifice of diameter d under a constant head H. Also let, ρ be the mass density and μ be dynamic viscosity of the fluid flowing through the orifice. The discharge Q may be assumed to depend on these variables, d, H, ρ, μ and the gravitational acceleration g.

Find an expression for discharge Q.

Solution:

Functional relationship for Q may be written as,

$Q = f\,(\mu, \rho, d, H, g)$

By exponential form, it can be written as,

$Q = C\,(\mu^{a}\ \rho^{b}\ d^{c}\ H^{d}\ g^{e})$; where C is a dimensionless constant.

Substituting the proper dimensions for each variable in this exponential equation in M-L-T system,

Rayleigh's Method (Modi#842)

$$\frac{L^3}{T} = \left(\frac{M}{LT}\right)^a \left(\frac{M}{L^3}\right)^b L^c L^d \left(\frac{L}{T^2}\right)^e$$

For dimensional homogeneity, the exponents of each dimension on both sides of equation must be identical,

For M : 0 = a+b

For L : 3 = − a−3b+c+d+e

For T : − 1 = −a −2e

Since, there are five unknowns in three equations, three of the unknowns must be expressed in terms of other two,

b = −a

e = $\frac{1}{2} - \frac{a}{2}$

c = $\frac{5}{2} - \frac{3a}{2}$ −d

Rayleigh's Method (Modi#842)

So, according to, $Q = C\,(\mu^{a}\,\rho^{b}\,d^{c}\,H^{d}\,g^{e})$

$$Q = C[\mu^{a}\rho^{-a}\,d^{\frac{5}{2}-\frac{3a}{2}-d}\,H^{d}g^{\frac{1}{2}-\frac{a}{2}}]$$

$$Q = C[(d^{\frac{5}{2}}g^{\frac{1}{2}})\,(\mu^{a}\rho^{-a}d^{-\frac{3a}{2}}g^{-\frac{a}{2}})\,(H^{d}d^{-d})]$$

$$Q = C[(d^{2}d^{\frac{1}{2}}g^{\frac{1}{2}})\,(\frac{\mu}{\rho d^{\frac{3}{2}}g^{\frac{1}{2}}})^{a}\,(\frac{H}{d})^{d}]$$

$$Q = C[(\frac{\mu}{\rho d^{\frac{3}{2}}g^{\frac{1}{2}}})^{a}\,(\frac{H}{d})^{d-\frac{1}{2}}\,(d^{2}H^{\frac{1}{2}}g^{\frac{1}{2}})]$$

$$Q = \frac{C}{\frac{\pi}{4}\sqrt{2}}[(\frac{\mu}{\rho d^{\frac{3}{2}}g^{\frac{1}{2}}})^{a}\,(\frac{H}{d})^{d-\frac{1}{2}}\,\frac{\pi}{4}d^{2}\sqrt{2gH}]$$

$$Q = a\sqrt{2gH}\,f_{1}[\left(\frac{\mu}{\rho d^{\frac{3}{2}}g^{\frac{1}{2}}}\right),\left(\frac{H}{d}\right)]$$

Rayleigh's Method (Modi#842)

This expression may be written in usual form:

Q = $C_d * a\sqrt{2gH}$

Where, C_d is the co-efficient of discharge of the orifice which can be expressed as:

$C_d = f_1\left[\left(\frac{\mu}{\rho d^{\frac{3}{2}} g^{\frac{1}{2}}}\right), \left(\frac{H}{d}\right)\right]$

It may be pointed out that both the terms in the bracket are dimensionless and C_d is also a non-dimensional factor.

Practice Problem: Rajput#383

7.3 Find an expression for the drag force on smooth sphere of diameter D, moving with a uniform velocity V in a fluid density ρ and dynamic viscosity μ.

7.4 The efficiency η of a fan depends on the density ρ, the dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor and discharge Q. Express efficiency η in terms of dimensionless parameter.

7.6 A partially submerged body is towed in water. The resistance R to its motion depends on the density ρ, the viscosity μ of water, length L of the body, velocity V of the body and acceleration due to gravity. Show that resistance to motion can be expressed in the form:

$$R = \rho L^2 V^2 \phi \left[\left(\frac{\mu}{\rho L V}\right), \left(\frac{Lg}{V^2}\right)\right]$$

Bansal: Dimensional and Model Analysis

https://books.google.co.in/books?id=nCnifcUdNp4C&pg=PA554&source=gbs_toc_r&hl=en#v=onepage&q&f=true

Buckingham's π Method

When a large number of physical variables are involved, Rayleigh's method of dimensional analysis becomes increasingly laborious and cumbersome. Buckingham's method is an improvement over the Rayleigh's method. Buckingham designated the dimensionless group by the Greek capital letter π (Pi). It is therefore often called Buckingham's π method.

The advantage of this method over Rayleigh's method is that it let us know, in advance of the analysis, as to how many dimensionless groups are to be expected.

Buckingham's π Method (Statement)

"If there are n variables (dependent and independent variables) in a dimensionally homogeneous equation and if these variables contain m fundamental dimensions (such as M,L,T etc.), then the variables are arranged into (n−m) dimensionless terms. These dimensionless terms are called π terms"

Mathematically, if any variable X_1 depends on independent variables X_2, X_3, X_4,……. X_n; the functional equation may be written as

$X_1 = f\,(X_2, X_3, X_4, \ldots\ldots. X_n)$

It can also be written as, $f_1(X_1, X_2, X_3, \ldots\ldots. X_n) = 0$

It is a dimensionally homogeneous equation and contains n variables.

Buckingham’s π Method (Statement)

If there are m fundamental dimensions, then according to Buckingham’s π method, it can be written in terms of number of π terms (dimensionless group), in which number of π terms is equal to (n−m). Hence equation becomes as:

$f_1(\pi_1, \pi_2, \pi_3, \ldots\ldots \pi_{n-m}) = 0$

Each dimensionless π term is formed by combining m variables out of the total n variables with one of the remaining (n−m) variables that is: each π term contains (m+1) variables.

These m variables which appear repeatedly in each of π terms are consequently called repeating variables and are chosen from among the variables such that they together involve all the fundamental dimensions and they themselves don not form a dimensionless parameter.

Selection of repeating Variables

1. Repeating variables must contain jointly all the fundamental dimensions involved in the phenomenon. Usually the fundamental dimensions are M,L and T.
2. The repeating variables must not form the non-dimensional parameters among themselves.
3. As far as possible, dependent variables should not be selected as repeating variables.
4. No two repeating variables should have the same dimensions.
5. Repeating variables should be chosen in such a way that one variable contains geometric property (length, diameter, height), other variable contains flow property (velocity, acceleration) and third variable contains fluid property (mass density, weight density, dynamic viscosity).

Problem: Bansal#568

The pressure difference Δp in a pipe of diameter D and length l due to viscous flow depends on velocity V, viscosity μ and density ρ. Using Buckingham's π theorem, obtain an expression for Δp.

Step 1: Selection of repeating variables

Now Δp is a function of D, l, V, μ, ρ or $\Delta p = f(D, l, V, \mu, \rho)$

or $$f_1(\Delta p, D, l, V, \mu, \rho) = 0 \quad \text{...(i)}$$

Total number of variables, $n = 6$

Number of fundamental dimension, $m = 3$

Number of π-terms $= n - 3 = 6 - 3 = 3$

Hence equation (*i*) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0$ ...(*ii*)

Each π-term contains $m + 1$ variables, *i.e.*, $3 + 1 = 4$ variable. Out of four variables, three are repeating variables.

Choosing D, V, μ as repeating variables, we have π-terms as

$$\pi_1 = D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p$$

$$\pi_2 = D^{a_2} . V^{b_2} . \mu^{c_2} . l$$

$$\pi_3 = D^{a_3} . V^{b_3} . \mu^{c_3} . \rho$$

Step 2: Calculate π terms

First π-term $\pi_1 = D^{a_1} . V^{b_1} . \mu^{c_1} . \Delta p$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-1}T^{-1})^{c_1} . ML^{-1}T^{-2}.$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_1 + 1$, $\therefore$ $c_1 = -1$

Power of L, $0 = a_1 + b_1 - c_1 - 1$, $\therefore$ $a_1 = -b_1 + c_1 + 1 = 1 - 1 + 1 = 1$

Power of T, $0 = -b_1 - c_1 - 2$, $\therefore$ $b_1 = -c_1 - 2 = 1 - 2 = -1$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^1 . V^{-1} . \mu^{-1} . \Delta p = \frac{D\Delta p}{\mu V}.$$

Second π-term $\pi_2 = D^{a_2} . V^{b_2} . \mu^{c_2} . l$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-1}T^{-1})^{c_2} . L.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_2$, $\therefore$ $c_2 = 0$

Power of L, $0 = a_2 + b_2 - c_2 + 1$, $\therefore$ $a_2 = -b_2 + c_2 - 1 = -1$

Power of T, $0 = -b_2 - c_2$, $\therefore$ $b_2 = -c_2 = 0$

Substituting the values of a_2, b_2 and c_2 in π_2,

Step 2: Calculate π terms

$$\pi_2 = D^{-1} . V^0 . \mu^0 . l = \frac{l}{D}.$$

Third π-term $\pi_3 = D^{a_3} . V^{b_3} . \mu^{c_3} . \rho$

Substituting the dimension on both sides,

$$M^0L^0T^0 = L^{a_3} . (LT^{-1})^{b_3} . (ML^{-1}T^{-1})^{c_3} . ML^{-3}.$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_3 + 1,$ $\therefore$ $c_3 = -1$

Power of L, $0 = a_3 + b_3 - c_3 - 3,$ $\therefore$ $a_3 = -b_3 + c_3 + 3 = -1 - 1 + 3 = 1$

Power of T, $0 = -b_3 - c_3,$ $\therefore$ $b_3 = -c_3 = -(-1) = 1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^1 . V^1 . \mu^{-1} . \rho = \frac{\rho DV}{\mu}.$$

Step 3: Final expression

Substituting the values of π_1, π_2 and π_3 in equation (*ii*),

$$f_1\left(\frac{D\Delta p}{\mu V}, \frac{l}{D}, \frac{\rho DV}{\mu}\right) = 0 \quad \text{or} \quad \frac{D\Delta p}{\mu V} = \phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right] \quad \text{or} \quad \Delta p = \frac{\mu V}{D}\phi\left[\frac{l}{D}, \frac{\rho DV}{\mu}\right]$$

Experiments show that the pressure difference Δp is a linear function $\frac{l}{D}$. Hence $\frac{l}{D}$ can be taken out of the functional as

$$\Delta p = \frac{\mu V}{D} \times \frac{L}{D}\phi\left[\frac{\rho DV}{\mu}\right]. \text{ Ans.}$$

Expression for difference of pressure head for viscous flow

$$h_f = \frac{\Delta p}{\rho g} = \frac{\mu V}{D} \times \frac{l}{D} \times \frac{1}{\rho g}\phi\,[R_e] \qquad \left\{\because \frac{\rho DV}{\mu} = R_e\right\}$$

$$= \frac{\mu VL}{wD^2}\phi\,[R_e]. \text{ Ans.}$$

Model Analysis

Google Search: Model

- a three-dimensional representation of a person or thing or of a proposed structure, typically on a smaller scale than the original.
- a thing used as an example to follow or imitate.

Leonardo DiCaprio Wins Oscar For Best Actor For 'The Revenant'

Model Analysis

- In order to know about the performance of the hydraulic structures (dams, spillways etc.) or hydraulic machines (turbines, pumps etc.) before actually constructing or manufacturing them, their models are made and tested to get the required information.
- The model is the small scale replica of the actual structure or machine.
- The actual structure or machine is called Prototype.
- The models are not always smaller than the prototype.
- In some cases, a model may be even larger or of the same size as prototype depending upon the need and purpose.

Advantages of model testing:

- The model tests are quite economical and convenient.
- With the use of models, the performance of hydraulic structures/hydraulic machines can be predicted in advance.
- While designing a particular portion of the structure, if clear cut analytical and reliable method is not available then in such cases it becomes absolutely necessary to know about the safety and reliability of such parts which is possible by means of model testing.
- Model testing can be used to detect and rectify the defects of an existing structure which is not functioning properly.

Applications of the model testing:

1. Civil engineering structures such as dams, spillways, weirs, canals etc.
2. Flood control, investigation of silting and scour in rivers, irrigation channels.
3. Turbines, pumps and compressors.
4. Design of harbours, ships and submarines.
5. Aeroplanes, rockets and missiles.
6. Tall buildings (to predict the wind loads on buildings, the stability characteristics of the buildings and airflow patterns in their vicinity.

Similitude

Google search results:

- the quality or state of being similar to something.
- a comparison between two things.
- a person or thing resembling someone or something else.

The following three similarities must be ensured between the model and the prototype:

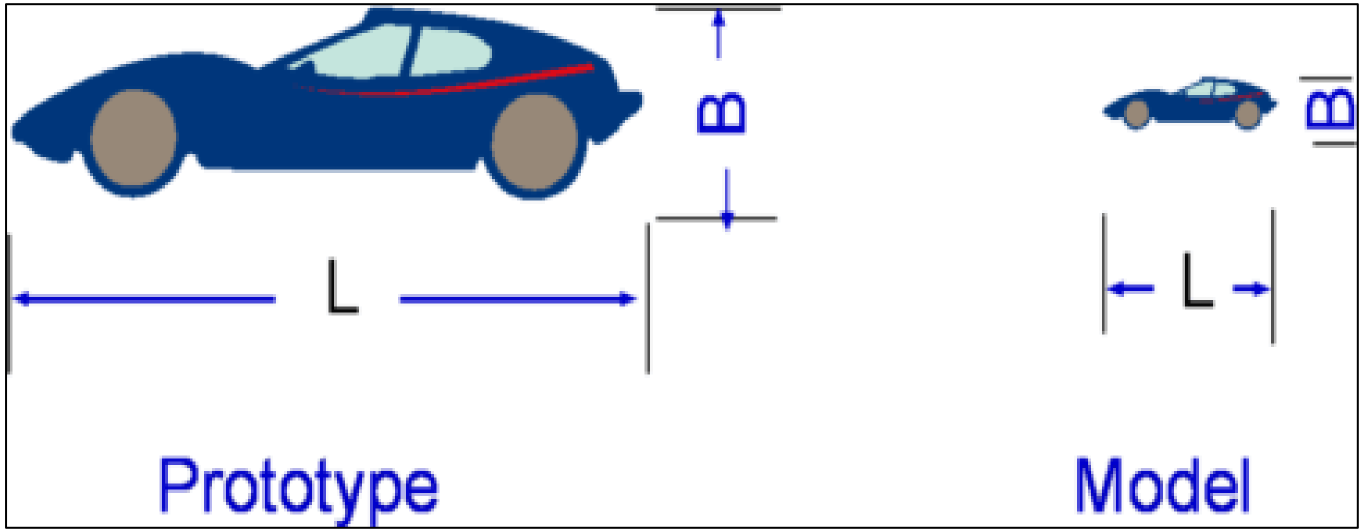
1. Geometric similarity
2. Kinematic similarity
3. Dynamic similarity

Geometric Similarity

For geometric similarity to exist between the model and the prototype, the ratios of corresponding lengths in the model and in the prototype must be same and the included angles between two corresponding sides must be same.

Models which are not geometrically similar are known as geometrically distorted model.

Let,



L_m= Length of model

H_m= Height of model

D_m= Diameter of model

A_m= Area of model

V_m= Volume of model

and L_p, H_p, D_p, A_p, V_p = Corresponding values of the prototype.

Geometric Similarity

Scale factor, $L_r = \frac{L_p}{L_m} = \frac{H_p}{H_m} = \frac{D_p}{D_m}$

Area ratio, $A_r = \frac{A_p}{A_m} = L_r^2$

Volume ratio, $V_r = \frac{V_p}{V_m} = L_r^3$

Kinematic Similarity

Kinematic similarity is the similarity of motion. If at the corresponding points in the model and in the prototype, the velocity or acceleration ratios are same and velocity or acceleration vectors point in the same direction, the two flows are said to be kinematically similar.

So, for kinematic similarity,

Velocity ratio, $V_r = \frac{V_{1p}}{V_{1m}} = \frac{V_{2p}}{V_{2m}}$

Acceleration ratio, $a_r = \frac{a_{1p}}{a_{1m}} = \frac{a_{2p}}{a_{2m}}$

*Directions of the velocities in the model and prototype should be same.

*Geometric similarity is a pre-requisite for kinematic similarity.

Dynamic Similarity

Dynamic similarity is the similarity of forces. The flows in the model and in prototype are dynamically similar if at all the corresponding points, identical types of forces are parallel and bear the same ratio.

Force ratio, $F_r = \frac{F_{ip}}{F_{im}} = \frac{F_{vp}}{F_{vm}} = \frac{F_{gp}}{F_{gm}}$

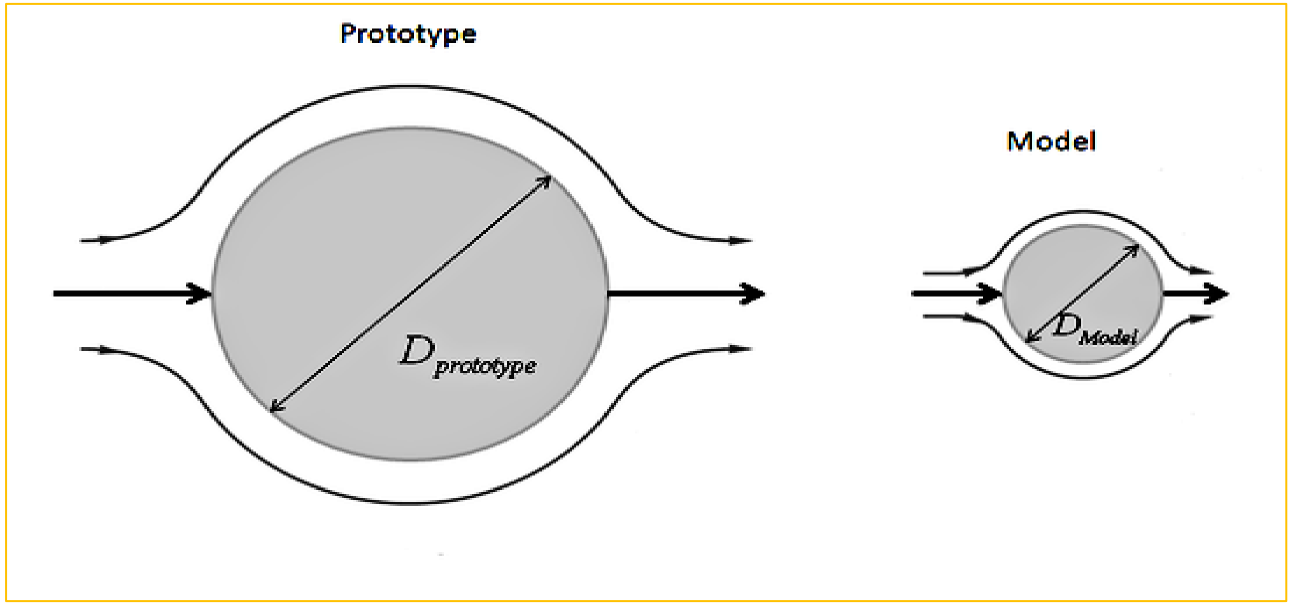
F_i = Inertia force

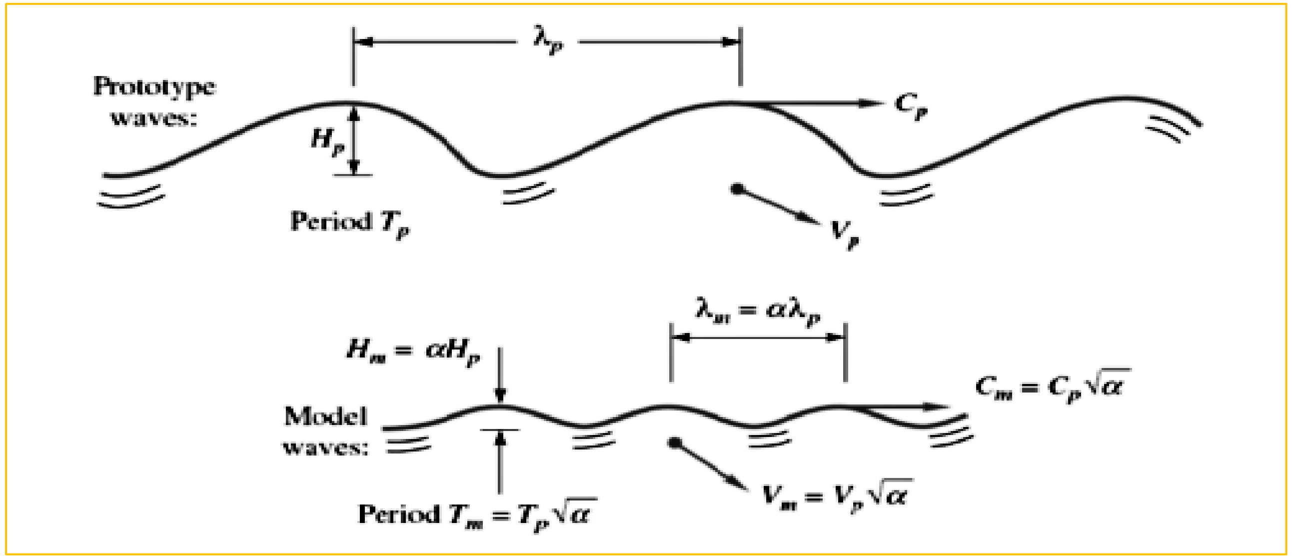
F_v = Viscous force

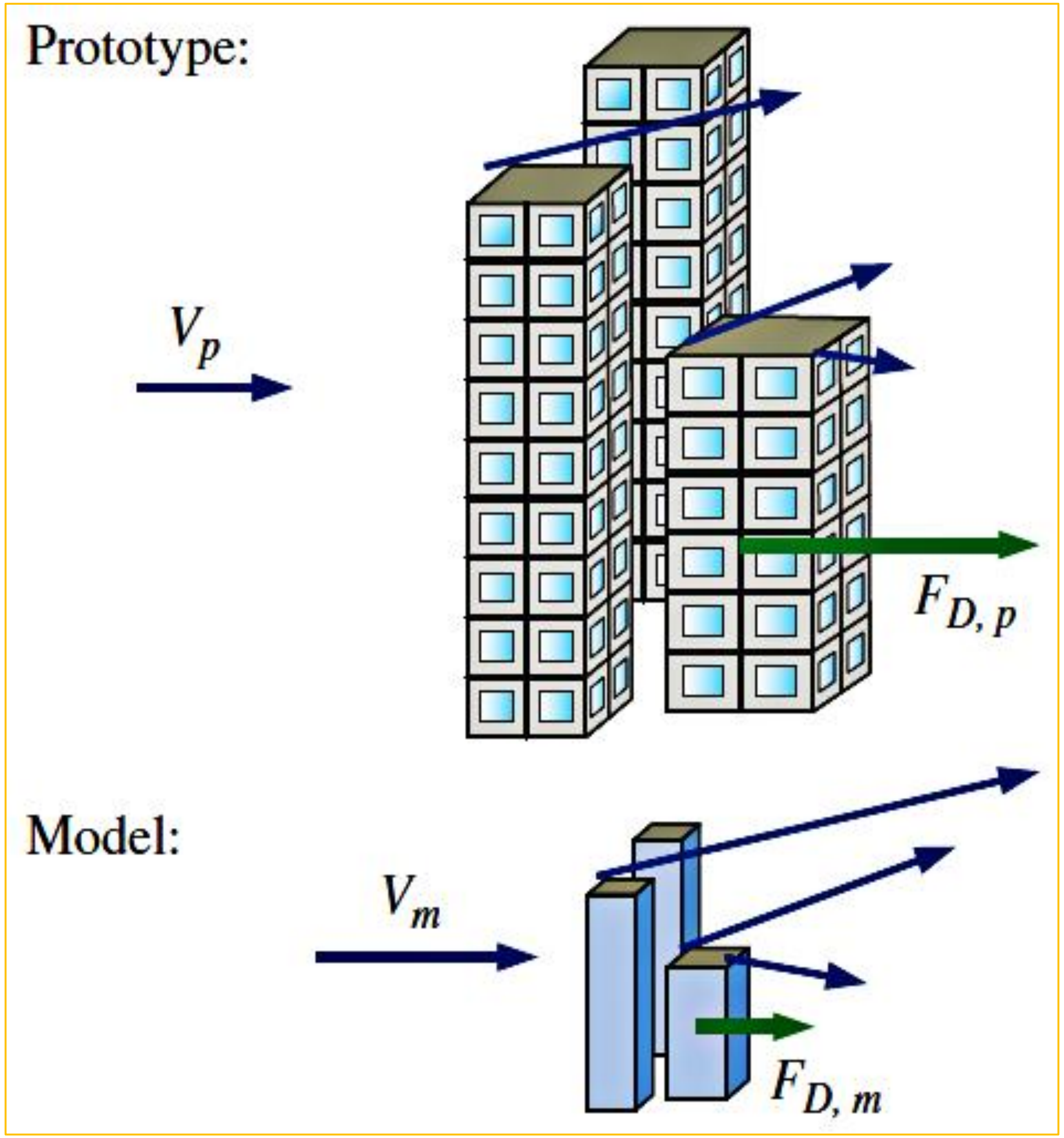
F_g = Gravity force

*directions of the corresponding forces at the corresponding points in prototype should also be same.

Similitude


Prototype
Model
$D_{prototype}$
D_{Model}


λ_p
Prototype waves:
C_p
H_p
Period T_p
V_p
$\lambda_m = \alpha\lambda_p$
$H_m = \alpha H_p$
$C_m = C_p\sqrt{\alpha}$
Model waves:
$V_m = V_p\sqrt{\alpha}$
Period $T_m = T_p\sqrt{\alpha}$


Prototype:
V_p
$F_{D,p}$
Model:
V_m
$F_{D,m}$

Factors influencing Hydraulic Phenomena

1. Inertia force (F_i):

- It always exists in the fluid flow problem.
- It is equal to the product of mass and acceleration of the flowing fluid and acts in the direction opposite to the direction of acceleration.

Inertia force, $\boldsymbol{F_i}$ = mass $*$ acceleration

2. Viscous force (F_v):

- It is present in fluid flow problems where viscosity is to play an important role.
- It is equal to the product of shear stress (τ) due to viscosity and surface area of flow.

Viscous force, $\boldsymbol{F_v}$ = shear stress $*$ surface area

Factors influencing Hydraulic Phenomena

3. Gravity force (F_i):

- It is present in case of open surface flow.
- It is equal to the product of mass and acceleration due to gravity.

Gravity force, $\boldsymbol{F_g}$ = mass $*$ acceleration due to gravity

4. Pressure force (F_p):

This type of force is present in case of pipe flow.

It is equal to the product of pressure intensity and cross-sectional area of the flowing fluid.

Pressure force, $\boldsymbol{F_p}$ = pressure intensity $*$ cross-sectional area

Factors influencing Hydraulic Phenomena

5. Surface tension force ($\boldsymbol{F_s}$)

It is equal to the product of surface tension and length of surface of the flowing fluid.

F_s = surface tension $*$ length of surface of flowing fluid

6. Elastic Force ($\boldsymbol{F_e}$):

It is equal to the product of elastic stress and area of the flowing fluid.

F_e = elastic stress $*$ area of flowing fluid

DIMENSIONLESS
Numbers

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Dimensionless numbers and their significance

1. Reynolds number
2. Froude’s number
3. Euler’s number
4. Weber’s number
5. Mach’s number

Reynolds Number, R_e

Reynolds from his experiment, found that the nature of flow in a closed conduit depends upon the following factors:

1. Diameter of the pipe (D)
2. Density of the liquid (ρ)
3. Viscosity of the liquid (μ)
4. Velocity of flow (V)

By combining the above variables, Reynolds determined a non-dimensional quantity equal to $\frac{\rho V D}{\mu}$ which is known as Reynolds number.

Reynolds Number, R_e

It is defined as the ratio of inertia force to the viscous force.

Inertia force (F_i)

$= mass * acceleration$

= density $*$ volume $* \frac{Velocity}{time}$

= density $* \frac{Volume}{time} *$ Velocity

= density $*$ Area $*$ Velocity $*$ Velocity

= ρAV^2

Reynolds Number, R_e

Viscous force (F_v)

$= shear\ stress * area$

$= \mu \frac{du}{dy} * area$

$= \mu \frac{V}{L} * A$

Reynolds Number , $R_e = \frac{F_i}{F_v} = \frac{\rho VL}{\mu} = \frac{\rho VD}{\mu}$

Froude Number

It is defined as the square root of the ratio of inertia force and the gravity force.

Mathematically, $F_r = \sqrt{\frac{F_i}{F_g}}$

Here, inertia force, F_i = ρAV^2

Gravity force, F_g

= Mass ∗ Acceleration due to gravity

= density ∗ volume ∗ g

= ρ ∗ AL ∗ g

$F_r = \frac{V}{\sqrt{Lg}}$

Euler's Number

It is defined as the square root of the ratio of the inertia force to the pressure force.

Mathematically, $E_u = \sqrt{\frac{F_i}{F_p}}$

Here, inertia force, F_i = ρAV^2

Pressure force, F_p

= intensity of pressure ∗ area

= P ∗ A

$E_u = \frac{V}{\sqrt{P/\rho}}$

Weber Number

It is defined as the square root of the ratio of the inertia force to the surface tension force.

Mathematically, $W_e = \sqrt{\frac{F_i}{F_s}}$

Here, inertia force, F_i = $\rho A V^2$

Surface tension force, F_s

= surface tension $*$ length

= $\sigma * L$

$$W_e = \frac{V}{\sqrt{\sigma/\rho_L}}$$

Mach Number

It is defined as the square root of the ratio of the inertia force to the elastic force.

Mathematically, M = $\sqrt{\frac{F_i}{F_e}}$

Here, inertia force, F_i = ρAV^2

Elastic force, F_e

= elastic stress $*$ area

= K $*$ A = K $*$ L^2

M = $\frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$; C = Velocity of sound in the fluid

Summary

Sl. No.	Dimensionless number	Aspects			
		Symbol	Group of variables	Significance	Field of application
1.	*Reynolds number*	*Re*	$\frac{\rho VL}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$	Laminar viscous flow in confined passages (where *viscous effects are significant*)
2.	*Froude's number*	*Fr*	$\frac{V}{\sqrt{Lg}}$	$\frac{\text{Inertia force}}{\text{Gravity force}}$	Free surface flows (where *gravity effects are important*)
3.	*Euler's number*	*Eu*	$\frac{V}{\sqrt{p/\rho}}$	$\frac{\text{Inertia force}}{\text{Pressure force}}$	Conduit flow (where *pressure variations are significant*)
4.	*Weber's number*	*We*	$\frac{V}{\sqrt{\sigma/\rho L}}$	$\frac{\text{Inertia force}}{\text{Surface tension}}$	Small surface waves, capillary and sheet flow (where *surface tension is important*)
5.	*Mach's number*	*M*	$\sqrt{\frac{V}{K/\rho}}$	$\frac{\text{Inertia force}}{\text{Elastic Force}}$	High speed flow (where *compressibility effects* are significant).

Model (or Similarity Laws)

Dimensionless number should be same for the model as well as the prototype; this condition is difficult to be satisfied for all dimensionless numbers.

Hence, models are designed on the basis of the force which is dominating in the flow situation. The laws on which the models are designed for dynamic similarity are called model or similarity laws.

1. Reynold's model law.
2. Froude Model law.
3. Euler model law.
4. Weber model law
5. Mach model law.

Reynolds Model Law

In flow situations where in addition to inertia, viscous force is the other predominant force, the similarity of flow in the model and its prototype can be established if Reynolds number is same for both the systems. This is known as Reynolds law.

According to this law:

$$(R_e)_{model} = (R_e)_{prototype}$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\frac{\rho_p}{\rho_m} * \frac{V_p}{V_m} * \frac{L_p}{L_m} * \frac{1}{\mu_p/\mu_m} = 1$$

$$\frac{\rho_r V_r L_r}{\mu_r} = 1$$

Here, subscript r represent the corresponding scale ratios

Reynolds Model Law: Scale ratio

Time scale ratio, $T_r = \frac{L_r}{V_r}$

Acceleration scale ratio, $a_r = \frac{V_r}{T_r}$

Force scale ratio, $F_r = \rho_r L_r^2 V_r * a_r$

Discharge scale ratio, $Q_r = \rho_r L_r^2 V_r$

Application of Reynolds model law:

1. Motion of air planes
2. Flow of incompressible fluid in closed pipes
3. Motion of submarines completely under water
4. Flow around structures and bodies immersed completely under moving fluids.

Reynolds Model Law

Water at 15^0 flows at 4 m/s in a 150 mm pipe. At what velocity must oil at 30^0 flows in a 75 mm pipe for the two flows to be dynamically similar? Take ν for water at 15^0 as 1.145$\times 10^{-6}\ m^2/s$ and that for oil at 30^0 as 3.0$\times 10^{-6}\ m^2/s$.

Solution:

$(R_e)_{model}=(R_e)_{prototype}$

$$\frac{V_m d_m}{\nu_m} = \frac{V_p d_p}{\nu_p}$$

V = 20.96 m/s

Reynolds Model Law: Problem

A pipe of diameter 1.5 m is required to transport an oil of specific gravity 0.90 and viscosity 3x10^{-2} poise at the rate of 3000 liter/s. Tests were conducted on a 15 cm diameter pipe using water at 20^0C. If viscosity of water at 20^0 is 1x10^{-2} poise, Find the velocity and rate of flow in the model.

Solution

Solution:

Prototype Data:

- Diameter, D_p= 1.5m
- Viscosity of fluid, μ_p= $3x10^{-2}$ poise
- Discharge, Q_p =3000litre/sec
- Sp. Gr., S_p=0.9
- Density of oil=ρ_p=0.9x1000
  =900kg/m^3

Model Data:

- Diameter, Dm=15cm =0.15 m
- Viscosity of water, μ_m =$1x10^{-2}$ poise
- Density of water, ρ_m=1000kg/m^3
- Velocity of flow V_m=?
- Discharge Q_m=?

For pipe flow,

According to Reynolds' Model Law

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p} \Rightarrow \frac{V_m}{V_p} = \frac{\rho_p D_p}{\rho_m D_m} \frac{\mu_m}{\mu_p}$$

$$\frac{V_m}{V_p} = \frac{900 \times 1.5}{1000 \times 0.15} \frac{1 \times 10^{-2}}{3 \times 10^{-2}} = 3.0$$

$$\text{Since } V_p = \frac{Q_p}{A_p} = \frac{3.0}{\pi / 4(1.5)^2}$$

$$= 1.697 m/s$$

$$\therefore V_m = 3.0 V_p = 5.091 m/s$$

$$and\ Q_m = V_m A_m = 5.091 \times \pi / 4(0.15)^2$$

$$= 0.0899 m^3/s$$

Froude Model Law

When the gravitational force can be considered to be only predominant force which controls the motion in addition to the inertia force, the similarity of the flow in any two such systems can be established if the Froude number for the both system is same. This is known as Froude Model law.

Application of Froude model law:

1. Free surface flows such as flow over spillways, sluices etc.
2. Flow of jet from an orifice or nozzle.
3. Where waves are likely to be formed on the surface.
4. Where fluids of the different mass densities flow over one another.

Froude Model Law

$$(Fe)_P = (Fe)_m \quad or \quad \frac{V_P}{\sqrt{g_P L_P}} = \frac{V_m}{\sqrt{g_m L_m}} \quad or \quad \frac{V_P}{\sqrt{L_P}} = \frac{V_m}{\sqrt{L_m}}$$

$$\frac{V_P}{V_m\left(\sqrt{\dfrac{L_P}{L_m}}\right)} = V_r / \sqrt{L_r} = 1; \quad where: V_r = \frac{V_P}{V_m}, L_r = \frac{L_P}{L_m}$$

Froude Model Law

$$\text{since } \frac{V_P}{\sqrt{L_P}} = \frac{V_m}{\sqrt{L_m}}$$

$$\text{Velocity Ratio: } V_r = \frac{V_P}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

$$\text{Time Ratio: } Tr = \frac{T_P}{T_m} = \frac{L_P/V_P}{L_m/V_m} = \frac{L_r}{\sqrt{L_r}} = \sqrt{L_r}$$

$$\text{Acceleration Ratio: } a_r = \frac{a_P}{a_m} = \frac{V_P/T_P}{V_m/T_m} = \frac{Vr}{Tr} = \frac{\sqrt{L_r}}{\sqrt{L_r}} = 1$$

$$\text{Discharge Ratio: } Q_r = \frac{A_P V_P}{A_m V_m} = L_r^2 V_r = L_r^2 \sqrt{L_r} = L_r^{5/2}$$

$$\text{Force Ratio: } Fr = m_r a_r = \rho_r Q_r V_r = \rho_r L_r^2 V_r V_r = \rho_r L_r^2 V_r^2 = \rho_r L_r^2 L_r = \rho_r L_r^3$$

$$\text{Power Ratio: } Pr = Fr.Vr = \rho_r L_r^2 V_r^2 V_r = \rho_r L_r^2 V_r^3 = \rho_r L_r^2 \left(\sqrt{L_r}\right)^3 = \rho_r L_r^{7/2}$$

In the model test of a spillway the discharge and velocity of flow over the model were 2 m^3/s and 1.5 m/s respectively. Calculate the velocity and discharge over the prototype which is 36 times the model size.

Solution: Given that

For Model

- Discharge over model, $Q_m = 2\ m^3/sec$
- Velocity over model, $V_m = 1.5\ m/sec$
- Linear Scale ratio, $L_r = 36$

For Prototype

- Discharge over prototype, $Q_p = ?$
- Velocity over prototype $V_p = ?$

For Discharge

$$\frac{Q_p}{Q_m} = (L_r)^{2.5} = (36)^{2.5}$$

$$Q_p = (36)^{2.5} \times 2 = 15552\ m^3/sec$$

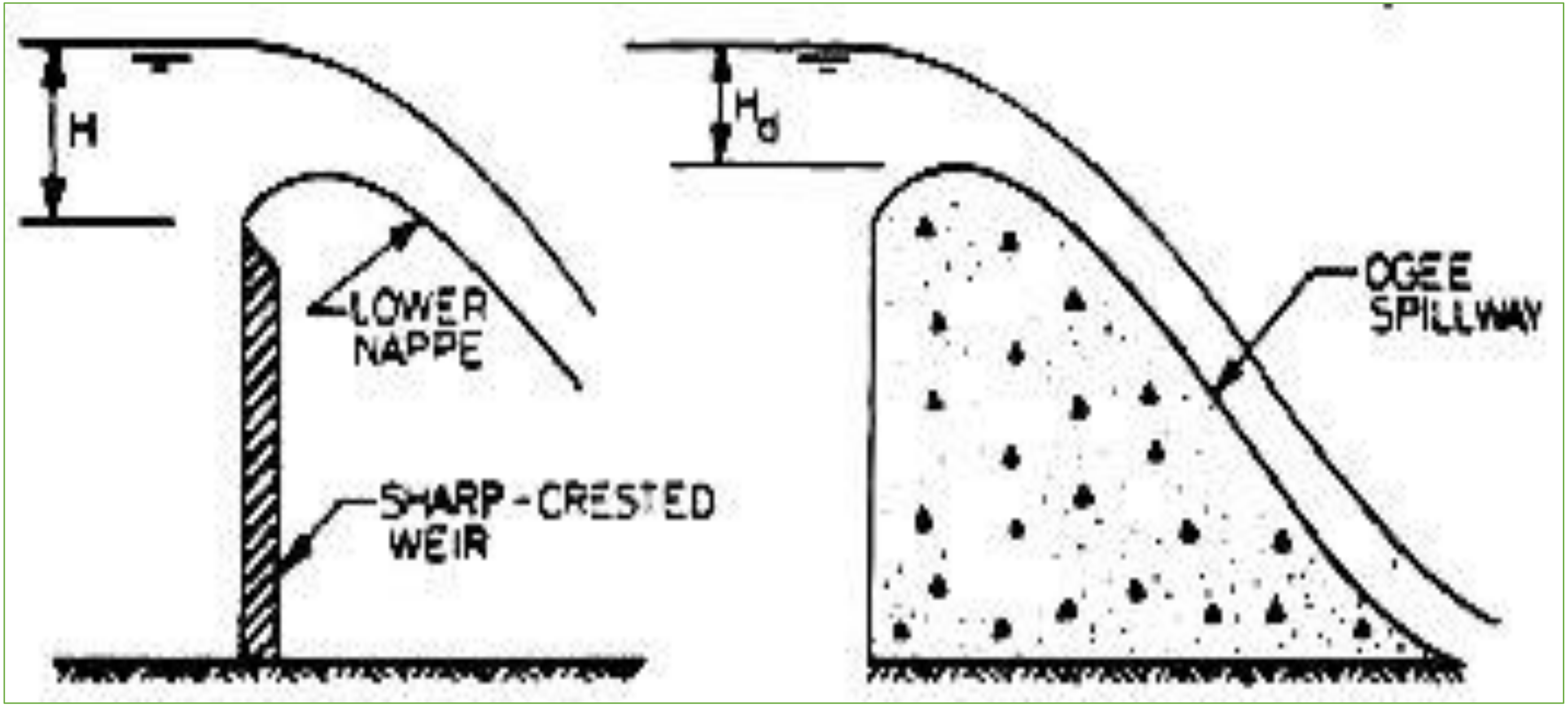
For Dynamic Similarity,

Froude Model Law is used

$$\frac{V_p}{V_m} = \sqrt{L_r} = \sqrt{36} = 6$$

$$V_p = 6 \times 1.5 = 9\ m/sec$$

Spillway



Problem: Rajput#432 [Semester Final=2017]

A 1:64 model is constructed of an open channel in concrete which has Manning's N = 0.014. Find the value of N for the Model.

Solution:

Manning's Formula:

Velocity in an open channel flow

$$V = \frac{1}{N} * R^{\frac{2}{3}} * S^{\frac{1}{2}}$$