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CHAPTER – 1

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PROPERTIES OF FLUIDS

The matter can be classified on the basis of the *spacing between the molecules* of the matter as follows

1. Solid state, and
2. Fluid state,
    (i) Liquid state, and (ii) Gaseous state

Solids

Solids may be defined as substances which deform on the application of shearing stress until the internal shear resistance is equal to the externally applied stress.

Fluids

A *fluid* may be defined as a substance which is capable of flowing. It has no definite shape of its own, but conforms to the shape of the containing vessel. Further even a small amount of shear force exerted on a fluid will cause it to undergo as deformation which continues as long as the force continuous to be applied.

A fluid has the following characteristics:

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a liquid/fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.

Classification of Fluids

A fluid may be classified as follows:

1. (a) Liquid, (b) Gas, and (c) Vapour.
2. (a) Ideal fluids and (b) Real fluids

Liquids

A *liquid* is a fluid, which possesses a definite volume, which varies only slightly with temperature and pressure. Since under ordinary conditions liquids are difficult to compress, they may be for all practical purposes regarded as incompressible. It forms a free surface or an interface separating it from the atmosphere as any other gas present.

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Liquids have bulk elastic modulus when under compression and will store up energy in the same manner as a solid. As the contraction of volume of a liquid under compression is extremely small, it is usually ignored and the *liquid is assumed to be incompressible*. A liquid will withstand a slight amount of tension due to molecular attraction between the particles which will cause an apparent shear resistance, between two adjacent layers. This phenomenon is known as *viscosity*.

All known liquids vaporise at narrow pressures above zero, depending on the temperature.

Gases

A gas is a fluid, which is compressible and possesses no definite volume but it always expands until its volume is equal to that of the container. Even a slight change in the temperature of a gas has a significant effect on its volume and pressure. However, if the conditions are such that a gas undergoes a negligible change in its volume, it may be regarded as incompressible. But if the change in volume is not negligible the compressibility of the gas will have to be taken into account in the analysis.

Vapour

A *vapour* is a gas whose temperature and pressure are such that it is very near the liquid state. Thus steam may be considered a vapour because its state is normally not far from that of water.

Therefore it may be concluded that in *solids*, the molecules are very closely spaced whereas in *liquids* the spacing between the different molecules is relatively large and in *gases* the spacing between the molecules is still large. It means that inter-molecular cohesive forces are *large* in solids, *smaller* in liquids and *extremely small* in gases, and on account of this fact, solids possess compact and rigid form, liquid molecules can move freely within the liquid mass and the molecules of gases have greater freedom of movement so that the gases fill the container completely in which they are placed.

A *solid* can resist tensile, compressive and shear stresses up to a certain limit whereas a fluid has no tensile strength or very little of it and it can resist the compressive forces only when it is kept in a container. When a fluid is subjected to a shearing force it deforms continuously as long as the force is applied. The amount of shear stress in a fluid depends on the magnitude of the rate of deformation of the fluid element.

Liquids and *gases* exhibit different characteristics. The liquids under ordinary conditions are quite difficult to compress (and therefore they may for most purposes be regarded as incompressible) whereas gases can be compressed much readily under the action of external pressure and when the external pressure is removed the gases tend to expand indefinitely.

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Ideal Fluid and Real Fluid

Ideal Fluids

Ideal fluids are those fluids which have no viscosity and surface tension and they are incompressible. As such for ideal fluids no resistance is encountered as the fluid moves. However, in nature the ideal fluids do not exist and therefore, these are only imaginary fluids. The existence of these imaginary fluids was conceived by the mathematicians in order to simplify the mathematical analysis of the fluids in motion. The fluids, which have low viscosity such as air, water etc., may however be treated as ideal fluids without much errors.

Practical or Real Fluids

Practical or real fluid, are those fluids which are actually available in nature. These fluids possess the properties such as viscosity, surface tension and compressibility and therefore a certain amount of resistance is always offered by these fluids when they are not in motion.

Fluid Mechanics

Fluid mechanics is that branch of science which deals with the behaviour of the fluids at rest as well as in motion.

Hydraulics

Hydraulics may be defined as the branch of science, which deals with water at rest as well as in motion.

Sub-Divisions of Fluid Mechanics

Fluid mechanics may be divided into three divisions:

1. **Hydrostatics**

Hydrostatics that studies the mechanics of fluids at absolute and relative rest; the fluid elements are free from shearing stresses,

2. **Kinematics**

Kinematics that deals with translation, rotation and deformation of fluid elements without considering the forces and energy causing such a motion.

3. **Dynamics**

Dynamics that prescribes the relations between velocities and acceleration and the forces which are exerted by or upon the moving fluids.

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Units and Dimensions

A *dimension* is a name which describes the measurable qualities or characteristics of an object such as mass, length, time and temperature etc.

A *unit* is an accepted standard for measuring the dimensions or quality. (ω)

Physical quantities used in fluid mechanics are expressed in five fundamental dimensions, viz. *length, mass, force, time and temperature.*

Fundamental Units

The measurement of physical quantities is one of the most important operations in engineering. Every quantity is measured in terms of some arbitrary, but internationally accepted units, called fundamental units. All physical quantities, met with in this subject are expressed in terms of the following four fundamental quantities:

1. Length (L)
2. Mass (M)
3. Time (T)
4. Force (F)

Derived Units

Some units are expressed in terms of other units which are derived from fundamental units are known as derived units. e.g., the unit of volume, acceleration, velocity, pressure etc.

System of Units

There are four systems of units, which are commonly used and universally recognized. These are known as:

(1) C.G.S. units (2) F.P.S. units (3) M.K.S. units and (4) S.I. units.

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Specific Weight, Mass Density, Specific Gravity and Specific Volume

(a) Specific Weight

Specific weight (w) of a liquid is its weight per unit volume.

$$w = \frac{W}{V}$$

γ

where W is the weight of the liquid having volume V.

Since weight is expressed in kgf or Newton, the unit of measurement of specific weight is kgf/m^3 or N/m^3. In terms of fundamental units, the dimensional formula of specific weight is

$$\left[\frac{F}{L^3}\right] \text{ or } \left[\frac{M}{L^2 T^2}\right]$$

(b) Mass Density

Density (ρ) is a measure of the amount of fluid contained in a given volume and is defined as the mass per unit volume

$$\rho = \frac{m}{V}$$

where m is the mass of fluid having volume V. The units of density are kg/m^3 or kgfs^2/m^4 or Ns^2/m^4.

The dimensional formula of density in fundamental units is

$$\left[\frac{M}{L^3}\right] \text{ or } \left[\frac{FT^2}{L^4}\right]$$

The density of a fluid diminishes with rise of temperature except for water which has a maximum value at 4^{0}C.

(c) Specific Gravity

Specific gravity (s) refers to the ratio of specific weight (or mass density) of a fluid to the specific weight (or mass density) of a standard fluid. For liquids the standard fluid is water at 4^{0}C, and for gases the standard fluid is taken either air at 0^{0}C or hydrogen at the same temperature. Specific gravity is dimensionless and has no units.

lb → 62.4 lb/ft^3

সমুদ্রের পানি, 64 lb/ft^3

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(d) Specific Volume

Specific volume (v) represents the volume per unit mass of fluid. It is the inverse of the mass density

$$v = \frac{V}{m}$$

$$\text{or } v = \frac{1}{\rho}$$

The concept of specific volume is found to be practically more useful in the study of flow of compressible fluids, i.e., gases.

Compressibility and Bulk Modulus of Elasticity

Fluid mechanics deals with both incompressible and compressible fluids, i.e., with fluids of either constant or variable density. When pressure is applied to a fluid, it contracts and when pressure is released it expands. Compressibility of a fluid then characterises its ability to change its volume under pressure. The relative change of volume per unit pressure is given by the ***coefficient of compressibility***

$$\beta_c = \frac{-1}{V}\left(\frac{dV}{dp}\right)$$

where dp is the small change in pressure applied to the fluid and dV is the incremental volume change in the original volume V. The negative sign implies that a positive pressure increment results in a negative volume increment, i.e., an increase in pressure causes a decrease in volume. Quite often, the compressibility of fluid is expressed by its bulk modulus of elasticity (K), which is the inverse of the coefficient of compressibility:

$$K = \frac{1}{\beta_c} = -\frac{dp}{dV/V}$$

The bulk modulus of elasticity increases somewhat with temperature and pressure. At ordinary temperatures and pressure k = 20,000 kgf/cm^2 for water and k = 1.05 kgf/cm^2 for air. That indicates that air is approximately 20,000 times more compressible than water. Water is 80 times more compressible than mild steel.

To gain some idea about the compressibility of water, imagine that a unit cubic meter of water is subjected to a pressure of 10 kgf/cm^2. Then a change in the volume of water amounts to:

$$-dV = \frac{Vdp}{K} = \frac{1 \times 10}{20000} = \frac{1}{2000}\ m^3$$

[illegible]

Thus the application of 10 kgf/cm² to water under ordinary conditions causes the volume to decrease by only 1 part in 2000. Such a volume change is insignificant and as such water is regarded as incompressible fluid for most of the hydraulic problems. But in case of water flowing through pipes when the sudden or larger changes in pressure (e.g., water hammer) take place, the compressibility cannot be neglected.

Though the gases in general are compressible, this compressibility becomes important only when the gas velocity becomes more then 20% of the velocity of sound waves in that gas.

Viscosity (Imp by

In plain English viscosity is derived from the word '*viscous*' which means '*sticky*', '*adhesive*', or '*tenacious*'.

Thus viscosity can be defined as that property of a fluid, which resist relative motion of its adjacent layers. It is measure of internal fluid friction due to which there is resistance to flows.

A perfect fluid would have no viscosity. There is no perfect fluid, but gases show less variation in viscosity than liquids. Water is one of the least viscous of all liquids, whereas glycerine, heavy oil and molasses are liquids having comparatively high viscosities.

Newton's Law of Viscosity

(Imp. by I.M).

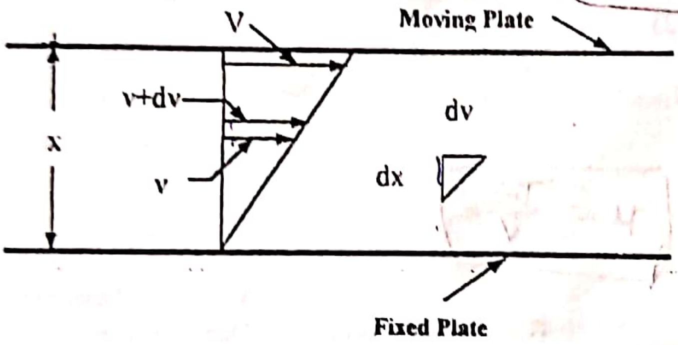

Fig. 1.1 Velocity Variation Near a Solid Boundary

The mathematical basis of viscosity may be derived from the Fig. 1.1. Consider two parallel plates of indefinite extent at distance x apart, the space between them being filled with a fluid. Consider further that one of these plates moves at velocity V parallel to the other plate.

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Three assumptions are made:

(1) That the fluid particles in contact with a moving surface have the velocity of that surface.

(2) That the rate of change of velocity is uniform in the direction perpendicular to the direction of motion.

(3) That the shearing stress in the fluid is proportional to the rate of change of velocity.

By assumption 2, from similar triangles

$$\frac{V}{x} = \frac{dv}{dx} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (1.1)$$

But by assumption 3, the unit shearing stress

$$\tau \propto \frac{dv}{dx}$$

$$\tau = \mu \frac{dv}{dx} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (1.2)$$

where μ is a proportionality factor called the *coefficient of viscosity*.

From equation (1.1) and (1.2)

$$\frac{V}{x} = \frac{\tau}{\mu} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (1.3)$$

or

$$\mu = \frac{\tau x}{V} \qquad \boxed{\mu = \frac{\tau x}{V}} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (1.4)$$

If the plates are unit distance apart and moving with unit relative velocity, then

$$\mu = \tau$$

The coefficient (μ) is known as the ***coefficient of viscosity***, or ***absolute viscosity*** or ***dynamic viscosity*** or simply ***viscosity***.

Equation (1.4) was first suggested by Sir Isaac Newton, So it is referred to as Newton's Viscosity Equation.

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The F.P.S. units in which dynamic viscosity is expressed can be evaluated from equation (1.4). Unit shear τ is in pounds per square foot, distance x is in feet, and velocity V is in feet per second. Hence the units of μ are

$$\frac{lb/ft^2 \times ft}{ft/sec} = \frac{lb\ sec}{ft^2} = \frac{slug}{ft\ sec}$$

In the metric system, the unit of viscosity is called the *poise*, 1 poise being 1 dyne sec per cm^2. A centipoise is 0.01 poise. It has been found experimentally that the dynamic viscosity of water at 68^0F (20^0C) is 1 centipoise. The ratio of the dynamic viscosity of any fluid to the dynamic viscosity of water at 68^0 F is termed as relative viscosity. Therefore, when expressed in centipoises, the dynamic viscosity and relative viscosity of any fluid are numerically equal.

Kinematic Viscosity

Kinematic Viscosity (v) is the ratio of the dynamic viscosity of a fluid to its mass density. Thus

$$\nu = \mu/\rho$$

and the units are

$$\frac{lbsec / ft^2}{lbsec^2 / ft^2} = \frac{ft^2}{sec}$$

In the metric system the unit of kinematic viscosity is called the Stoke. 1 stoke being 1 sq cm per sec.

It is to be noted that numerical values of poise and stoke in C.G.S. units are same.

Effect of Temperature on Viscosity

"The viscosity of liquids decreases but that of gases increases with the increase in temperature."

The reason is that viscosity of liquids is mainly due to intermolecular cohesion and this cohesion decreases with the increase in temperature; whereas in gases the molecules are more free to move to the adjacent layers hence intermolecular forces are nearly negligible. Thus there is a transfer of momentum between different layers of gases. That is why, the viscosity of gases is mainly due to transfer of momentum of molecules. So when temperature

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is raised the molecular activity increases, consequently, more transfer of momentum takes place, as a result the viscosity of gas is increased.

Heimholtz suggested the following empirical relationship for viscosity of water:

$$\mu = \frac{0.1776}{1 + 0.03368\ T + 0.000221\ T^2} \text{ poise} \qquad \dots\ \dots\ \dots\ \dots\ \dots (1.5)$$

as viscosity depends upon temperature only and it is practically independent of pressure.

where T = Temperature in ^{0}C between the range of 0^0C and 100^0C.

Effect of Pressure on Viscosity

The change of pressure does not affect viscosity appreciably under ordinary conditions. But viscosity of some oils has been found to increase with increase in pressure.

Surface Tension and Capillarity

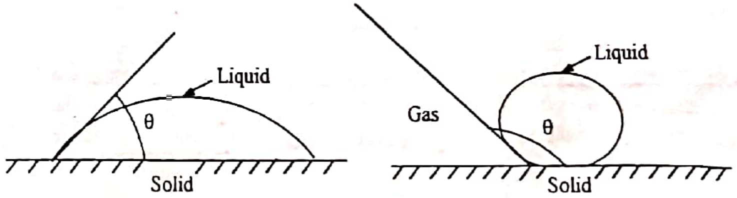
Cohesion

Cohesion means intermolecular attraction between molecules of the same liquid. It enables a liquid to resist small amount of tensile stresses. Cohesion is the tendency of the liquid to remain as one assemblage of particles. Surface tension is due to cohesion between particles at the free surface.

Adhesion

Adhesion means attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick to another body.

When a liquid like, mercury, is spilled on a smooth horizontal surface, it tends to gather into droplets because the cohesive molecular forces are greater than the adhesive forces between the mercury molecules and the material of the surface. Mercury tends to stay away from the surface and is said to be a non-wetting liquid. In case of water, adhesive forces are greater than cohesive forces. Naturally when water is poured on the same smooth horizontal surface, it would spread out and wet the horizontal surface. The wetting and non-wetting of the surface is dictated by the angle of contact between the liquid and the surface material.



$\theta < \frac{\pi}{2}$

(a) Wetting liquid (water)

$\theta > \frac{\pi}{2}$

(b) Non-wetting liquid (mercury)

Fig. 1.2 Liquid Gas Interface with a Solid Surface

Refer to Fig. 1.2 which illustrates the liquid-gas interface with a solid surface. Liquid would wet the surface when $\theta < \pi/2$ and the degree of wetting increases as θ decreases to zero. For a non-wetting liquid $\theta > \pi/2$. The contact angle is dependent on the nature and type of liquid, the solid surface and its cleanliness. For pure water in contact with a clean glass surface θ is essentially 0^0. Even when water is slightly contaminated, θ becomes as high as 25^0. Mercury, a non-wetting liquid has θ between 130^0 to 150^0.

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Surface Tension

Surface tension is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by other molecules all around and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of the molecules below it. Consequently, as shown in Fig. 1.3, there is a net inward force on the molecule. The force is normal to the liquid surface. At the free surface a thin layer of molecules is formed. This is because of this film that a thin small needle can float on the free surface (the layer acts as a membrane).

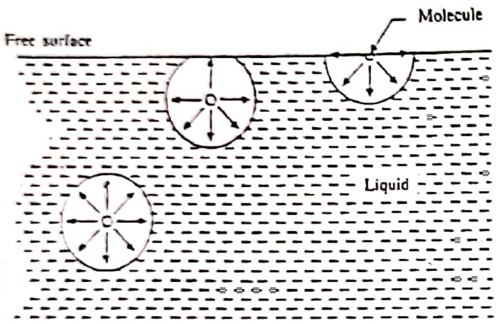

Fig. 1.3 Surface Tension

Surface tension forces are generally negligible in comparison with the pressure and gravitational forces but become quite significant when there is a free surface and the boundary dimensions are small, e.g., in the small-scale models of hydraulic engineering structures

Some important examples of phenomenon of surface tension are as follows:

(i) Raindrops (A falling raindrop becomes spherical due to cohesion and surface tension)

(ii) Rise of sap in a tree.

(iii) Birds can drink water from ponds.

(iv) Capillary rise and capillary siphoning

(v) Collection of dust particles on water surface.

(vi) Break up of liquid jets.

Dimensional Formula for surface tension

The dimensional formula for surface tension is given by:

$$\left[\frac{F}{L}\right] \text{ or } \left[\frac{M}{T^2}\right]$$

It is usually expressed in kgf/m or N/m. The value of surface tension depends upon

(i) Nature of the liquid

(ii) Nature of the surrounding matters which may be a solid, liquid or a gas.

(iii) Kinetic energy and hence the temperature of liquid molecules.

Growth in temperature results in a reduction of the intermolecular cohesive forces and hence in a reduction of the surface tension force. Surface tension values for liquids are generally quoted when it contacts with air as the surrounding medium.

For *water and air*, σ ranges from

0.0074 kgf/m (0.0727 N/m) at 20°C to

0.0058 kgf/m (0.0569 N/m) at 100°C; a drop with temperature rise.

For *mercury and air*, σ is about

0.0524 kgf/m (0.514 N/m) at 20°C.

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Capillarity or Meniscus Effect

When a small diameter glass tube, called the capillary tube is dipped into a water container, water rises in the tube to a level that stands higher than the level of water in the container. Conversely, the surface of mercury is depressed down in the capillary tubing when it is dipped in mercury. The phenomenon of liquid rise or fall in a capillary tube is called the *capillarity or meniscus effect* (Fig.1.4). Capillarity is a surface tension effect that depends upon the relative intermolecular attraction between different substances; it is due to both cohesion and adhesion.

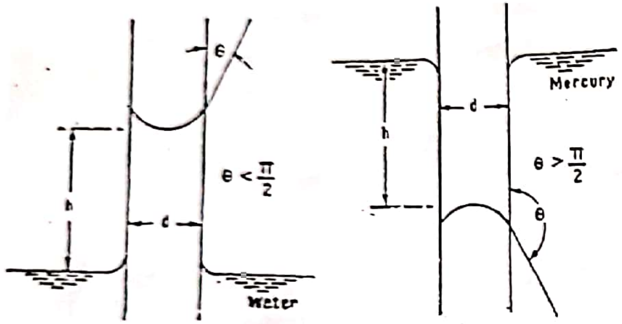

Fig. 1.4 Capillary Rise and Depression

Adhesion between glass and water molecules is greater than cohesion between water molecules. Consequently the water molecules spread over the glass surface and form a concave meniscus with small angle of contact. Opposite conditions hold good for mercury, i.e., cohesion between mercury molecules is greater than adhesion of mercury to glass. Mercury then displays a convex meniscus with angle of contact greater than 90^0.

Knowing the surface tension σ, angle of contact θ, tube diameter d and specific weight of liquid w, the rise (for water) or depression (for mercury) of the liquid in the capillary tube can be worked out by the following analysis:

Weight of liquid raised or lowered in the capillary tube

$$= (\text{Area of tube} \times \text{rise or fall}) \times \text{specific weight}$$

$$= \frac{\pi}{4} d^2 hw$$

Vertical component of surface tension force

$$= (\sigma \times \text{circumference}) \times \cos\theta$$

$$= \sigma \pi d \cos\theta$$

When in equilibrium, the downward weight of the liquid column h is balanced by the vertical component of the force of tension.

Hence

$$\frac{\pi}{4} d^2 hw = \pi d\sigma \cos\theta$$

$$\text{or } h = \frac{4\sigma \cos\theta}{wd}$$

It is to be noticed that for $0 \leq \theta \leq 90°$, h is positive (concave meniscus and capillary rise) and that for $90 \geq \theta \leq 180°$, h is negative (convex meniscus and capillary depression).

Evidently the capillary action is inversely proportional to the tube diameters. For precise work the small diameter tubes are to be avoided; recommended minimum tube diameter for water and mercury is 6 mm. Further since the presence of dirt affects the surface tension and hence the capillary rise or depression, the interior surface of the tube is to be kept clean.

Following points are worth noting:

(i) Smaller the diameter of the capillary tube, greater is the capillary rise or depression.

(ii) The measurement of liquid level in laboratory capillary (glass) tubes should not be smaller than 6 mm.

(iii) Capillary effects are negligible for tubes longer than 12 mm.

(iv) For *wetting liquid* (water): θ < π/2. For water: θ = 0 when pure water is in contact with clean glass. But θ becomes as high as 25^0 when water is slightly contaminated.

For *not-wetting liquid* (mercury): θ > π/2. For mercury θ varies between 130^0 to 150^0.

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(v) The effects of surface tension are negligible in many flow problems except those involving

- capillary rise,
- formation of drops and bubbles,
- the break up of liquid jets, and
- hydraulic model studies where the model or flow depth is small.

CHAPTER - 2

FLUID STATICS

Fluid Statics is the study of a fluid at rest; the concept includes situations where fluids are either actually at rest or undergo uniform acceleration in a container or rotate as a solid mass. No shear force is then present as the fluid particles do not move with respect to one another.

Fluid Pressure

When a fluid such as water or oil is contained in a vessel, it exerts force at all points on the surface area of the sides and bottom of the container. The fluid is at rest i.e., there is no relative velocity, hence, there would be no tangential or shear stress. It means there is no force component tangential to the walls of container. Hence the force exerted on walls by fluid, when at rest, is normal to the surface area.

The normal force exerted on a unit area of surface is called ***intensity of pressure, specific pressure, unit pressure or simply pressure.***

$$\text{Intensity of pressure} = \frac{\text{Normal force}}{\text{Area}}$$

or

$$p = \frac{F}{A}$$

From the above equation, the dimensions of pressure are FL^{-2}, hence in MKS system it may be represented in

$$\frac{\text{kgf}}{\text{m}^2} \quad \text{or} \quad \frac{\text{kgf}}{\text{cm}^2}$$

In SI units pressure is measured in N/m^2 or kN/m^2. Latest units (SI) for pressure are Pa (Pascals) or kPa and 1 Pa = 1 N/m^2.

Bar is also used as pressure and 1 bar = 100 kN/m^2.

Sometimes the normal force F is represented by symbol P, then

$$P = pa.$$

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Pressure Head

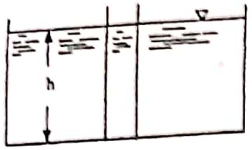

Fig. 2.1 Measurement of Pressure Head

Consider a vessel containing some liquid as shown in the Fig. 2.1. Let a cylinder be made to stand in the liquid as shown.

Let,

γ = Sp. wt. of the liquid
h = height of liquid in the cylinder
A = Area of the cylinder base

There will be some pressure on the cylinder base due to the weight of the liquid in the cylinder.

∴ Intensity of pressure or pressure

$$= \frac{\text{Weight of liquid in the cylinder}}{\text{Area of the cylinder base}}$$

$$= \frac{\gamma h A}{A} = \gamma h$$

This equation shows that the intensity of pressure at any point, in a liquid is proportional to its depth, from the surface (since γ is constant for the given liquid)

$$\therefore \quad p = \gamma h \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (2.1)$$

$$h = \frac{p}{\gamma} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (2.2)$$

Since γ for any fluid is constant, then pressure p may be expressed in terms of the height of a column of any fluid.

It is thus obvious, that the pressure can be expressed in either of the following two ways:

i) As a force per unit area (i.e. kg/cm^2, kg/m^2, or N/m^2 etc)

ii) As a height of the equivalent liquid column.

When the pressure is expressed in units of length, it is commonly referred to as *pressure head* and can be measured in meters or cm of liquid column.

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Pascal's Law

An important and unique property of hydrostatic pressure is reflected in Pascal's Law which states that

"Intensity of pressure at a point in a fluid at rest is same in all directions."

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Expression of pressure intensity at a point

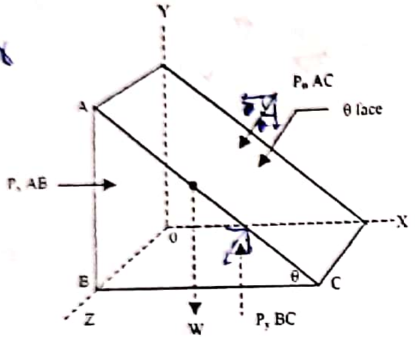

Fig. 2.2 Forces on a Fluid Element

Consider a small wedge shaped element of stationary fluid and assume that the element has a unit depth perpendicular to the plane of the paper. The element is acted upon by the normal pressure forces and the vertical force due to weight. Let p_x, p_y and p_θ be the pressure intensities on the faces AB, BC and AC respectively. Then

force on face AB = p_x AB

force on face BC = p_y BC

force on face AC = p_θ AC

The weight of the fluid element is

$$= \text{area of the triangular elements} \times \text{sp. wt.}$$

$$= \frac{1}{2}(AB) \times (BC) \times \gamma$$

and it acts through the centre of gravity. Since the fluid elements is in equilibrium, the forces in the horizontal and vertical direction must balance.

Resolving horizontally,

$$p_x\, AB = p_\theta\, AC \sin\theta$$

Recognising that AB = AC sin θ

$$\therefore p_x\, AC \sin\theta = p_\theta\, AC \sin\theta\,; \qquad p_x = p_\theta$$

Resolving vertically

$$p_y\, BC = \frac{1}{2}(AB) \times (BC) \times \gamma + p_\theta\, AC \cos\theta$$

Let the size of the elemental system approach smaller and smaller dimensions; then the gravitational force which diminishes as the product of two dimension can be neglected in comparison with the pressure forces from which the diminishing effect in proportional to the reduction on size. Thus

$$p_y\, BC = p_\theta\, AC \cos\theta$$

$$p_y\, AC \cos\theta = p_\theta\, AC \cos\theta\,; \qquad p_y = p_\theta$$

$$\therefore \quad p_x = p_y = p_\theta \qquad \text{… … … … … (2.3)}$$

This result is independent of the angle θ and therefore it follows that pressure acts equally in all directions in a stationary fluid. Pressure at point has only one value regardless of the orientation of the area upon which it is determined. *Independence of direction implies that pressure is a scalar quantity.*

Atmospheric Pressure

Air, like all gases, possesses weight and therefore it must exert some intensity of pressure. The total weight of air in the whole atmosphere and the consequent force it would exert on the earth is difficult to calculate as it is estimated that the maximum height of atmosphere above the earth surface varies from 150 to 1,000 kilometers.

It has however, been determined that the force exerted by air on an air column of one square cm in cross sectional area and having height equal to height of atmosphere and its value is approximately 1 kgf, the exact value being 1.033 kgf at N.T.P. This is termed as atmospheric pressure and is also expressed in meters of water's column and in millimeters of mercury column. The atmospheric pressure is also known as *Barometric Pressure*.

The values of one atmosphere are:

1.033 kgf/cm^2 or approximately taken as 1 kgf/cm^2 (MKS)

101,300 N/m^2 or pascal (Pa) or approximately taken as 101 kN/m^2 (SI units)

10.3 meters of water

760 mm of mercury

14.7 psi

2116 lb/ft^2

29.92 inch mercury

33.91 ft water

The above values of atmospheric pressure are based on calculations at mean sea level and mean temperature.

Variation of Atmospheric Pressure with Altitude

The following table gives the calculated values of atmospheric pressure (or barometric pressure) with the change of altitude, temperature remaining constant.

Altitude	metre	0	500	1000	1500	2000	3000	4000	5000
Barometric Pressure	mm of mercury	760.0	720.0	677.0	637.0	596.0	530.0	464.0	413.0
	m of water	10.3	9.76	8.92	8.6	8.1	7.2	6.3	5.6
	kN/m^2 or kPa	101.3	95.5	89.9	84.3	79.5	70.6	61.8	55.0

Atmospheric, Absolute, Gauge and Negative Pressure

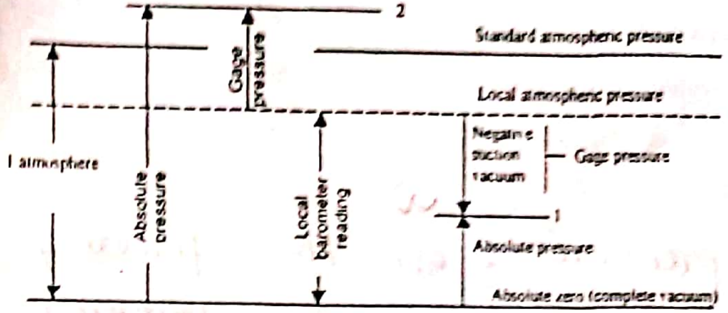

Fig. 2.3 Scales for Pressure Measurement

For pressure measurement, pressure at some reference point must be known. So a zero pressure or datum pressure must be established.

In hydraulic engineering local atmospheric pressure is taken as datum or zero pressure. The pressure measured on this scale in referred to as gauge pressure. It is so called because gauges, instruments for pressure measurement, are commonly calibrated with atmospheric pressure as zero. In case of sub-atmospheric pressure i.e., less than atmosphere, the gauge shall read less than zero. Therefore sub-atmospheric pressure is referred as ***negative gauge pressure or vacuum or suction pressure***. In complete vacuum where there are no fluid molecules at all, absolute zero pressure exist. A pressure measured with this absolute zero as a datum (or zero pressure) is called ***absolute pressure***. Hence absolute pressure has always positive value.

Unless specially mentioned pressure is understood to be gauge pressure.

Therefore,

Absolute Pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

Gauge Pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Vacuum Pressure is defined as the pressure below the atmospheric pressure.

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The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.3.

Mathematically,

P (abs) = P (atm) + P (gauge), where P (gauge) is the positive pressure.

P (abs) = P (atm) - P (gauge), where P (gauge) is the negative pressure.

(absolute pressure = atmospheric pressure + gauge pressure)

Static (P_s) and Total pressure (P_t)

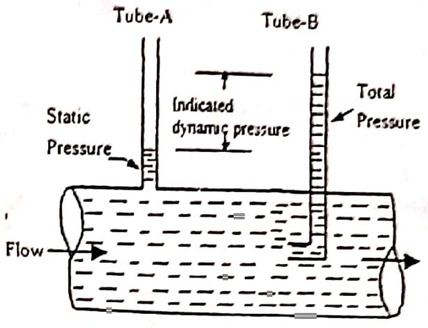


Fig. 2.4 Static and Total pressure

Static pressure is defined as the force per unit area acting on the wall by a fluid at rest or flowing parallel to the wall in a pipeline. Static pressure of a moving fluid is measured with an instrument which is at rest relative to the fluid. The instrument should theoretically move with the same speed as that of the fluid particle itself. As it is not possible to move a pressure transducer along in a flowing fluid. Static pressure is measured by inserting a tube into the pipeline at right angles to the flow path (Fig. 2.4). Care in taken to ensure that tube does not protrude into the pipeline and cause errors due to impact and eddy formation. When the tube protrudes into the stream there would be local speeding up to the flow due to its deflection around the tube; hence an erroneous reading of the static pressure would be observed.

Total or stagnation pressure is defined as the pressure that would be obtained if the fluid stream were brought to rest isentropically. In the above figure probe B sensed the total pressure while tap A senses only the static component of pressure.

The difference between the total and the static pressure gives the pressure due to fluid velocity; referred to as dynamic pressure. The dynamic pressure is due to flow speed and is also known as the velocity or impact pressure. For an incompressible fluid or for a gas flowing at low velocities, the dynamic pressure equals $V^2/2g$, where V is the velocity of fluid flow.

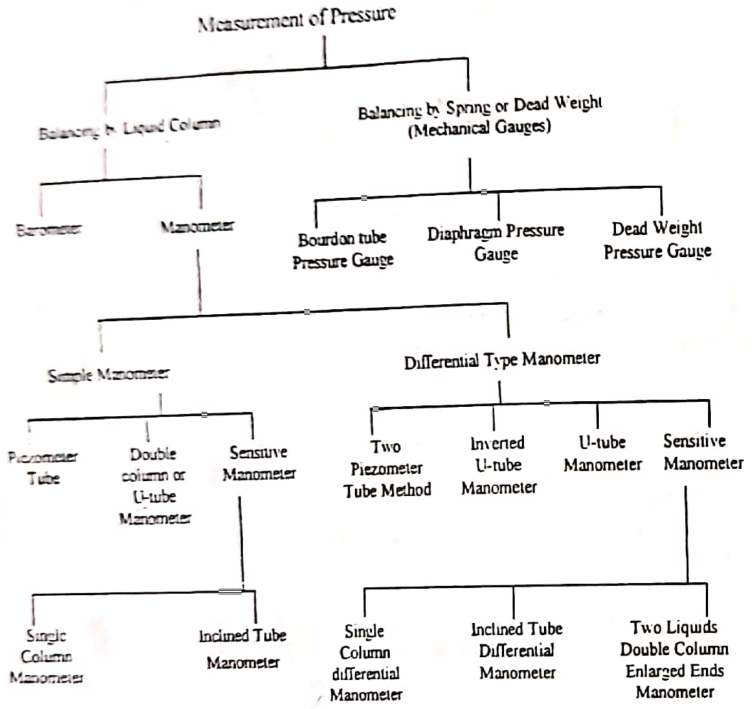
Velocity pressure = Total pressure - Static pressure

$$\frac{V^2}{2g} = (P_t - P_s)$$

$\frac{V^2}{2g}$

Pressure Measurement

All the devices designed for measurement of the intensity of hydrostatic pressure are based on either of two fundamental principles of measurement of pressure. Firstly by balancing the column of liquid (whose pressure is to be found) by the same or another known column of liquid and secondly of balancing the column of liquid by spring or dead weight.



(a) Balancing by Liquid Column

The intensity of liquid pressure p is measured by balancing against a hydrostatic column of liquid of known specific weight γ. Then the height h of the balancing column is a measure of the unknown pressure.

$$p = \gamma h \text{ kgf/m}^2 \text{ (or kN/m}^2\text{)}$$

$$h = \frac{p}{\gamma} \text{ metre}$$

$$P = \gamma h$$

$$h = \frac{P}{\gamma}$$

Manometers

Manometer are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.

Manometer liquids – The choice of balancing liquid depends upon the range of pressure to be measured and sensitivity required. For low pressure range and sensitivity, liquids of low specific weights such as carbon tetrachloride (CCl_4) – sp gr = 1.595; Acetylene-tetrabromide ($CH\,Br_2\,CH\,Br_2$) – sp gr = 2.95 or Antimony pentafluoirde ($Sb\,F_5$) – sp gr = 2.99 are used and for high pressure range mercury (Hg) – sp gr = 13.57 is employed.

For measuring small vacuums and small pressure difference of gases, water is used with high sensitivity.

(b) Balancing by Spring or Dead Weight

Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. Such instruments are used for the measurement of very high pressure where if the first method were used, the height of the liquid column would become inordinate, thus impracticable.

In mechanical gauges, the liquid (whose pressure is to be measured) exerts its force on a movable diaphragm or piston, which is resisted by a spring of known stiffness or dead weight of known value. The intensity of pressure then would be equal to the force P divided by the area a of diaphragm or piston.

i.e. $$p = \frac{P}{a}$$

Mechanical gauges have the advantages of giving direct pressure reading, portability and wider operating range. These can yield fairly accurate readings if properly calibrated. A combination of *piston type gauge* and *manometer* using jointly the principles of manometer and dead weight gauge are also in use now.

The commonly used mechanical pressure gauges are:

(a) Diaphragm pressure gauge
(b) Bourdon tube pressure gauge
(c) Dead-weight pressure gauge
(d) Bellows pressure gauge.

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Classification of Manometers

Manometers are classified under two main heads

(a) *Simple Manometer* measures in a pipe or vessel full of liquid, by a glass tube having one of the ends inserted in the pipe or vessel and the other end open to atmosphere.

(b) *Differential Manometer* measures the difference of pressure between any two points on a pipeline running full of liquid by inserting both ends of glass tube to the two points

For precise pressure measurement, various manometers are available in the market, many of them are named after their inventor's e.g.: Rosenmueller, Prandtl, Chattock, etc.

Simple Manometers

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are:

(a) Piezometer

(b) U-tube Manometer

(c) Single Column Manometer.

(a) Piezometer

It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.5. and 2.6

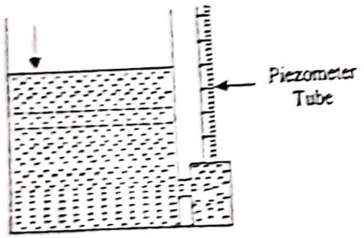

Fig 2.5 Piezometer Tube fitted to Open Vessel

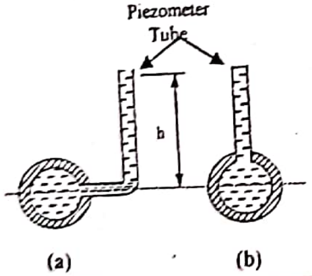

Fig. 2.6 Piezometer Tubes Fitted to Closed pipe. (a) and (b) show Different Connections.

Height of liquid in the tube is proportional to the pressure at the gauge point. Pressure head may be directly read from the graduations on the tube or on the scale attached to it. Care should be taken to ensure that the end of the tube connected to the vessel under pressure is flushing with the inside surface of the vessel. The diameter of the tube should not be less than 10 mm to avoid error due to capillary action. Height of column should be read at the centre of the meniscus i.e., lowest point of curve for water and the highest point of curve for mercury. The pressure head at any section of a pipe running full of liquid is also read directly by piezometer tube as shown in Fig. 2.6.

If h is the height of liquid column in piezometer tube in m of liquid, then intensity of pressure

$p = \gamma h$ kgf/m² in MKS or kN/m² in SI units.

where γ = specific weight of liquid, in kgf/m³ or kN/m³.

Double Column or U-Tube Manometer

If the pressure to be measured lies in a range beyond the scope of a simple piezometer, a double column or U-tube manometer (refer Fig 2.7) is used.

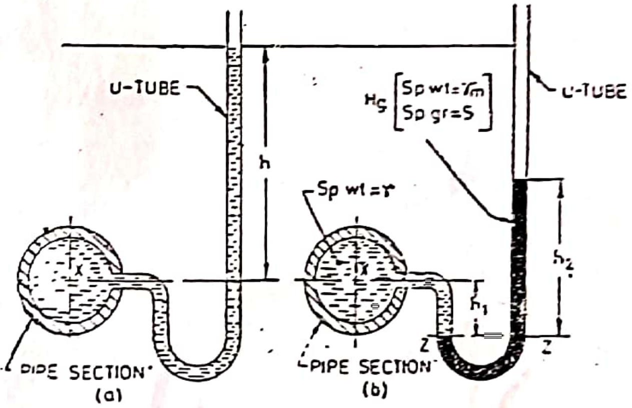

Fig. 2.7 Double Column or U-tube Manometer

(a) Using same liquid of pipe as measuring liquid

(b) Using mercury as measuring liquid

A single tube is bent in U shape, one end of which is connected to the gauge point and the other open to atmosphere. The diameter of tube should be about 10mm. The liquid commonly used in such a manometer is mercury but the information regarding liquid used for measuring various magnitudes of pressure should be a proper guide.

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If the vertical glass tube Fig. 2.6a is substituted by U-tube as shown in Fig. 2.7 even then the water from pipe will rise to the same height as in case of piezometer tube. To measure high pressures the U-tube may be filled with a liquid heavier than water (refer Fig 2.7 b). Mercury having 13.6 as specific gravity is as many times heavier than water and is commonly used for the purpose and thereby the height of U-tube can be decreased as for the same pressure, mercury will rise to a height of $\frac{1}{13.6}$ that of water.

Sensitive Manometers- Single Column Manometer

Single column manometer is a modified form of a U-tube manometer in which a reservoir having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.8 and Fig. 2.9. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

1. Vertical Single Column Manometer
2. Inclined Single Column Manometer

Vertical Single Column Manometer

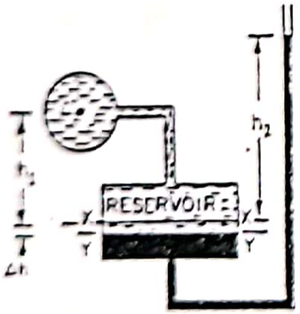

Fig. 2.8 Vertical Single Column Manometer

Fig. 2.8 shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let

Δh = Fall of heavy liquid in reservoir
h_2 = Rise of heavy liquid in right limb
h_1 = Height of center of pipe above X-X
h = Pressure at A, which is to be measured
A = Cross-sectional area of the reservoir
a = Cross-sectional area of right limb
S_1 = Sp. gr. of liquid in pipe
S_2 = Sp. gr. of heavy liquid in reservoir and right limb

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore A \times \Delta h = a \times h_2$$

$$\therefore \Delta h = \frac{a \times h_2}{A} \qquad \text{------ (2.4)}$$

Considering the datum line Y-Y as shown in Fig. 2.8. Then pressure head in the right limb above Y-Y

$$= (\Delta h + h_2)S_2$$

Pressure head in the left limb above Y-Y

$$= (\Delta h + h_1)S_1 + h$$

Equating these pressures, we have

$$(\Delta h + h_2)S_2 = (\Delta h + h_1)S_1 + h$$

$$\text{or} \quad h = (\Delta h + h_2)S_2 - (\Delta h + h_1)S_1$$

$$= \Delta h[S_2 - S_1] + h_2S_2 - h_1S_1$$

But from equation (2.4), $\Delta h = \frac{a \times h_2}{A}$

$$\therefore h = \frac{a \times h_2}{A}[S_2 - S_1] + h_2S_2 - h_1S_1 \qquad \text{............... (2.5)}$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

$$\text{Then} \quad h = h_2S_2 - h_1S_1 \qquad \text{............... (2.6)}$$

From equation (2.6), it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

Inclined Single Column Manometer

Fig. 2.9 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

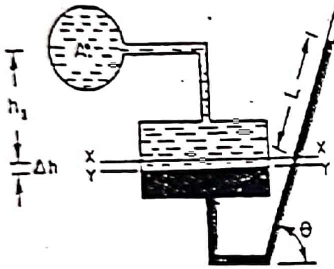

Fig. 2.9 Inclined Single Column Manometer

Let L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-X

$= L \times \sin\theta$

From equation (2.6), the pressure head at A is

$$h = h_2 S_2 - h_1 S_1$$

Substituting the value of h_2, we get

$$h = L \sin\theta \times S_2 - h_1 S_1 . \qquad \text{... ... ... (2.7)}$$

Differential manometer

When the difference of pressure between any two points in a pipe line (Fig 2.10) or in two pipes (Fig 2.11) is to be measured, they may be connected to the two ends of a manometer. This is called differential manometer.

A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are:

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

U-tube Differential Manometer

If the pressure at each of the two points is high compared to atmospheric pressure, the above two methods will be unsuitable, as the tubes required will be inordinately long. In such a case U-tube or double column manometer has to be used. The measuring liquid which fills a part of U-tube will be heavier than water flowing in the pipe or pipes. Fig. 2.10 and Fig. 2.11. shows the differential manometers of U-tube type.

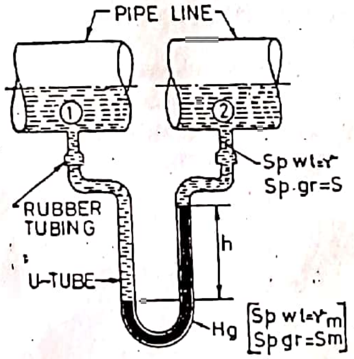

Fig. 2.10 U-tube Differential Manometer for One Pipe Line

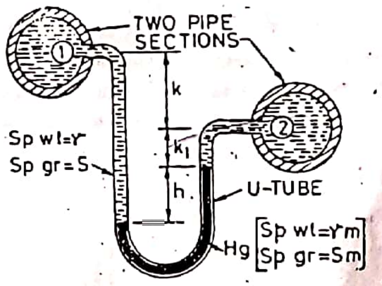

Fig. 2.11 U-tube Differential Manometer for Two Pipes

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Inverted U-tube Differential Manometer

It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.12 shows an inverted U-tube differential manometer connected to the two points A and B.

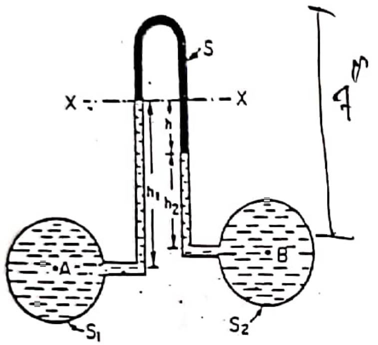

Fig. 2.12 Inverted U-tube Differential Manometer

Two Liquid Double Column Enlarged Ends Manometer (Micromanometer)

For a very high precision on measurement of pressure difference, the U-tube differential manometer is equipped with two basins (or widened ends). In order to magnify the readings two liquids of different specific gravities, immiscible in each other and in the fluid to be measured, are used in the two basins and U-tube as shown in Fig. 2.13.

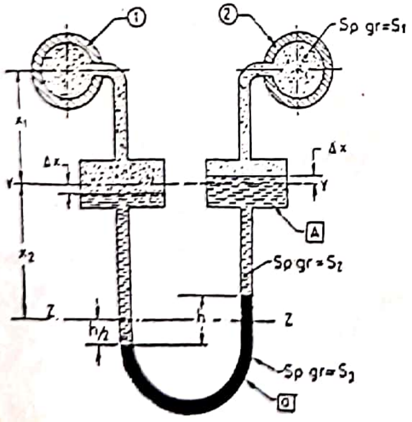

Fig.2.13 Two Liquid Double Column Enlarged Ends Manometer (Micromanometer)

The heavier liquid of sp. gr. S_3 fills the lower U-tube portion upto Z-Z, then the higher liquid of sp. gr. S_2 is added to both sides, filling the upper portion of U-tube as well as parts of basins up to Y-Y. The liquid or gas of sp. gr. S_1, running in the pipes fill the space above Y-Y.

Assuming that pipes are at the same height and fluid in both the pipes is same, the difference in pressure between points 1 and 2 can be determined as follows:

First both ends of U-tube are exposed to the same pressure with which both ends of the meniscus of heavier liquid occupy the level Z-Z. Now, let the pressure exerted at point 1 be more than that at point 2. This will cause the level of light liquid to fall in the left basin and rise in the right hand basin by an amount equal to Δx. The volume of liquid displaced in each basin is equal to the volume displaced in U-tube –

or $$\Delta x.A = \frac{h}{2}a \qquad \text{... ... ... (2.8)}$$

where A = the cross-sectional area of basin;
a = the cross-sectional area of U-tube.

Forming the manometer equation starting from point 1,

$$\left.\begin{aligned} &S_1h_1 + (x_1 + \Delta x)S_1 + \left(x_2 - \Delta x + \frac{h}{2}\right)S_2 - hS_3 \\ &-\left(x_2 - \frac{h}{2} + \Delta x\right)S_2 - (x_1 - \Delta x)S_1 = h_2S_1 \end{aligned}\right\} \qquad \text{... ... ... (2.9)}$$

Simplifying the equation,

$$S_1(h_1 - h_2) = hS_3 + \left(x_2 - \frac{h}{2} + \Delta x\right)S_2 + (x_1 - \Delta x)S_1 - (x_1 + \Delta x)S_1 - (x_2 - \Delta x + \frac{h}{2})S_2$$

Substituting the value of Δx from equation 2.8

$$\begin{aligned} S_1(h_1 - h_2) = hS_3 &+ \left(x_2 - \frac{h}{2} + \frac{h}{2}\cdot\frac{a}{A}\right)S_2 + \left(x_1 - \frac{h}{2}\cdot\frac{a}{A}\right)S_1 \\ &-\left(x_1 + \frac{h}{2}\cdot\frac{a}{A}\right)S_1 - \left(x_2 - \frac{h}{2}\cdot\frac{a}{A} + \frac{h}{2}\right)\cdot S_3 \end{aligned}$$

$$= hS_3 + x_2S_2 - \frac{h}{2}S_2 + \frac{h}{2}\cdot\frac{a}{A}S_2 + x_1S_1 - \frac{h}{2}\cdot\frac{a}{A}S_1$$

$$-x_1S_1 - \frac{h}{2}\cdot\frac{a}{A}S_1 - x_2S_2 + \frac{h}{2}\cdot\frac{a}{A}S_2 - \frac{h}{2}\cdot S_2$$

$$= h\left[S_3 - S_2\left(1-\frac{a}{A}\right) - S_1\frac{a}{A}\right]$$

$$S_1(h_1 - h_2) = h\left[S_3 - S_2\left\{1-\frac{d}{D}\right\}^2 - S_1\left(\frac{d}{D}\right)^2\right] \qquad \text{... ... ... (2.10)}$$

$$\text{or, } (h_1 - h_2) = h\left[\frac{S_3}{S_1} - \frac{S_2}{S_1}\left\{1-\left(\frac{d}{D}\right)^2\right\} - \left(\frac{d}{D}\right)^2\right] \text{ in m of liquid of sp. gr. } S_1 \qquad \text{... ... (2.10a)}$$

The whole of the quantity in the bracket of Eqn 2.10 is a constant for a particular manometer and its measuring liquids. It is therefore required to measure only one reading of the manometer, i.e. *h*. This type of sensitive manometer is useful for measuring the small pressure differences of gases, for which the water may be taken as heavier liquid as measuring fluid and oil as lighter liquid. Since the manometer is used for very high precision measurement, it is known as *micromanometer*.

Manometer Equation Procedure

(a) Start from one end of the manometer or the end at which the pressure is given. Write the intensity of pressure, at this end in meter of water. In case of simple manometer start from open end and take the intensity of pressure at this end as zero. Equation thus formed shall show gauge pressure.

(b) Add to the pressure found above, the change in intensity of pressure from one meniscus is lower than the first and negative if it is higher. This is calculated by multiplying the difference in level in the tube by the specific weight of the liquid.

(c) Continue the process as in (b) until the other end of the U-tube is reached.

(d) The algebraic sum of the pressures found at (a), (b) and (c) shall be equal to the pressure at the other end.

Note: The pressure difference may also be expressed in meters of water by forming the manometric equation as given above with specific gravity of the corresponding fluid in place of its specific weight.

Hydrostatic Forces on Submerged Surfaces

When a static mass of fluid comes in contact with a surface, either plane or curved, a force is exerted by the fluid on the surface. This force is known as total pressure. It has been observed that:

(i) The intensity of pressure p due to the weight of fluid at any point is directly proportional to its depth below the free liquid surface. This depth is known as pressure head.

(ii) The fluid static pressure cause force which acts normally at every point of the surface of container or any submerged object.

Force on a Horizontal Submerged Plane Surface

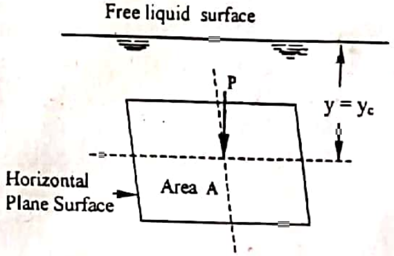

Fig. 2.14. Force on a Horizontal Submerged Plane Surface

Fig.2.14 shows as plane surface submerged and held in a horizontal position at depth Y below the free surface of the liquid. Since every point on the surface is at the same depth, the pressure intensity is constant over the entire plane surface.

From hydrostatic equation $p = wy$, and if A is the total area of the surface then total pressure force on the horizontal surface is:

$\boxed{w = \rho g}$

$$F = pA = wyA = A\,(wy)$$

For the given configuration, the depth $y = y_c$ depth of the centre of gravity (centroid) of the submerged surface below the free surface of the liquid:

$$\therefore \boxed{F = wAy_c} \qquad \text{... ... ... (2.11)}$$

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Force on a Vertical Plane Submerged Surface

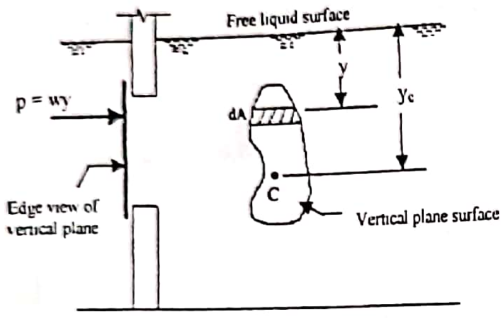

Fig. 2.15. Force on a Vertical Plane Submerged Surface

Consider a plane surface of arbitrary shape immersed vertically in a static mass of fluid (Fig. 2.15). Depth of the liquid varies from point to point; pressure intensity is thus not constant over the entire surface. Analysis for the total pressure force is then made by considering on the vertical plane surface an elementary horizontal strip lying at a depth y below the free surface of the liquid. The pressure intensity for this strip can be assumed to be constant and equal to

$$p = wy$$

Hence differential force on the strip is:

$$dF = pdA = wy\,dA$$

and the total force on the entire area is given by:

$$F = \int wy dA = w\int y dA$$

The integral $\int y dA$ represents the sum of first moments of the areas of the strip about free liquid surface and equals Ay_c where y_c is the depth to the centroid of the immersed surface. Thus

$$F = wAy_c = A\,(wy_c) \qquad \text{... ... ... (2.12)}$$

Force on an Inclined Submerged Plane Surface

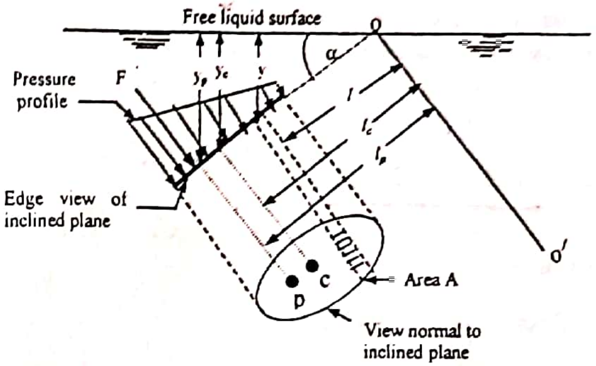

Fig. 2.16 Force on an Inclined Submerged Plane Surface

Let a plane surface of arbitrary shape be entirely submerged in a static mass of liquid. The plane of the surface intersects the horizontal liquid surface at axis $0 - 0'$ making an arbitrary angle α. The differential force dF on an elementary strip of area dA, where the pressure intensity is p, is given by:

$$dF = pdA = wydA = w \times l \sin\alpha \times dA$$

Where y is the vertical depth of the elementary area from the free liquid surface, and l is its distance from the axis $0 - 0'$. Force on the entire immersed surface can be computed by integrating the differential force dF over the entire area A.

$$F = \int pdA = w\sin\alpha\int l dA$$

$$l_c = \frac{\int l\,dA}{A}$$

Integral $\int l dA$ is the first moment of area A about $0 - 0'$ and is equal to Al_c

$$F = w\sin\alpha\, Al_c$$
$$= wA\,(l_c \sin\alpha) = wAy_c$$
$$= A\,(wy_c) \qquad \text{... ... ... (2.13)}$$

Evidently through equations (2.11), (2.12) and (2.13) we conclude that ***whatever may be the inclination of the submerged plane surface to the free liquid surface, the magnitude of the resultant hydrostatic force equals the product of the area and the pressure at the centroid of the area***

Centre of Pressure

We know that the intensity of pressure is not uniform, but increases with depth of a point on the immersed surface. As the pressure is greater over the lower portion of the surface, therefore the resultant pressure, on an immersed surface will act at some point, below the centre of gravity of the immersed surface and towards the lower edge of the figure. The point of the area at which the resultant pressure acts, is known as centre of pressure and is always expressed in terms of depth from the liquid surface.

Considering the above figure location of centre of pressure is determined by taking moments about the axis $0-0'$.

$$Fl_p = \int (dF)l = \int w \times l\sin\alpha \times dA \times l$$

$$= w\sin\alpha \int l^2 dA$$

$$\therefore l_p = \frac{w\sin\alpha \int l^2 dA}{F}$$

Substituting the value of $F = w\sin\alpha \int l dA$, we obtain

$$l_p = \frac{w\sin\alpha \int l^2 dA}{w\sin\alpha \int l dA}$$

$$= \frac{\int l^2 dA}{\int l\, dA}$$

Recognizing that $\int l^2\, dA = I_0$ = second moment of area, or the moment of inertia of the area conserved about axis $0-0'$, the above equation can be rewritten as:

$$lp = \frac{I_0}{Al_c}$$

Moment of inertia is generally quoted about an axis through the centriod; shifting the axis from the surface to parallel axis passing through the centroid gives:

$I_0 = I_c - Al_c^2$ (parallel axis theorem)

where I_c is the moment of inertia of the area about an axis through the centroid.

or $I_0 = Ak_c^2 + Al_c^2$

where k_c is the radius of gyration.

$$l_p = \frac{I_c + Al_c^2}{Alc}$$

$$= \frac{Ic}{Alc} + l_c \qquad \text{... ... ... (2.14)}$$

$$\text{or, } l_p - l_c = \frac{I_c}{Al_c} = \frac{k_c^2}{l_c} \qquad \text{... ... ... (2.14a)}$$

In terms of vertical distance from the free surface,

$$y_p - y_c = \frac{I_c\sin^2\alpha}{Ay_c} = \frac{k_c^2\sin^2\alpha}{y_c} \qquad \text{... ... ... (2.15)}$$

The following facts can be gleaned from the above equation:

1. Centre of pressure lies below the centroid, because for any plane surface the factor $(k_c^2 \sin^2\alpha/y_c)$ is always positive.
2. Deeper the surface is lowered into the liquid i.e., greater is the value of y_c, closer comes the centre of pressure to the centroid of the area.
3. Depth of centre of pressure is independent of the specific weight of the liquid and is consequently same for all liquids.
4. For a horizontal surface $\alpha = 0$ and so $y_p = y_c$, i.e., the centre of pressure coincides with the centroid.
5. For a vertical surface $\alpha = 90°$ and so
   $$y_p - y_c = \frac{I_c}{Ay_c}$$
6. If the vertical rectangular surface has breadth b and depth d, then
   $$y_p = \frac{d}{2} + \frac{bd^3/12}{(bd)d/2} = \frac{2}{3}d$$
   i.e., the centre of pressure is at a depth equal to $\frac{2}{3}$rd of the submerged height of the surface below the liquid level.

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Centre of Pressure of a Composite Section

The centre of pressure of a composite section (i.e., a section with cut out hole or other composite section) is obtained as discussed below:

1. Split up the composite section into convenient sections (i.e., rectangles, triangles or circles)
2. Calculate the pressures. P_1, P_2 ... on all the sections.
3. Then calculated the total pressure P on the whole section by the algebraic sum of the pressures
4. Then calculated the depths of centres of pressure h_1, h_2 .... for all the sections from the water surface.
5. Then equate $Ph = P_1h_1 + P_2h_2 +$ .... where h = Depth of centre of pressure of the section from the water.level.

Force on Curved Submerged Surface

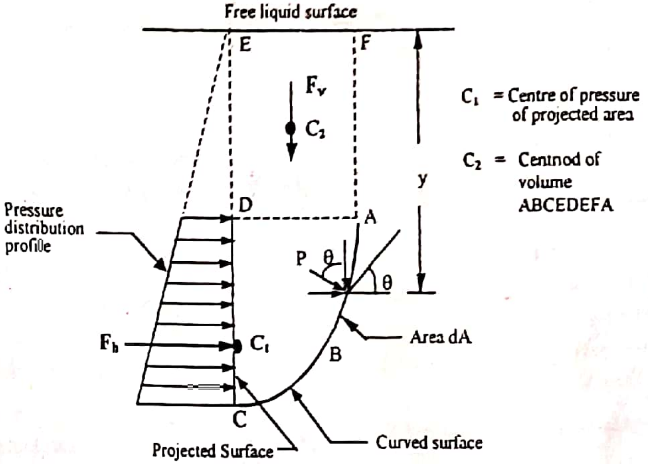

Fig. 2.17 Force on Curved Submerged Surface

Let the Fig. 2.17 represent the trace of curved surface submerged wholly in a static mass of liquid. Consider on the curved surface an elementary area dA lying at a vertical depth y below the free surface of the liquid. If p is the normal pressure intensity at the elementary area, then the differential force acting in direction normal to the surface is

$$dF = pdA = wydA$$

and the total force on the entire curved surface is

$$F = \int wydA.$$

Since for a curved surface direction of the pressure force varies from point to point, straightforward integration procedure cannot longer be applied. Computation of total pressure on a curved surface in then made possible by assessing the pressure forces acting on projected horizontal and vertical planes. For the elementary area:

$$dF_h = dF \sin\theta = pdA \sin\theta$$

$$dF_v = dF \cos\theta = pdA \cos\theta$$

Where θ is the inclination of the elementary area dA with the horizontal. Substituting p = wy in the above expressions and subsequent integration yields:

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$$F_x = \int dF_x = w \int y\,dA \sin\theta$$

$$F_y = \int dF_y = w \int y\,dA \cos\theta$$

In these expression, $dA \sin\theta$ and $dA \cos\theta$ represent respectively the vertical and horizontal projections of the elementary area dA

Consequently,

- $\int y dA \sin\theta$ represents the total pressure force on projected area of curved surface on the vertical plane. The point of application of the horizontal component F_x is at the centre of pressure of the projected area

- $\int y dA \cos\theta$ represents the total pressure on projected area of the curved surface on the horizontal plane and it equals the weight of liquid lying in the portion ABCDEFA; weight of liquid extending from the curved surface to the free surface of the liquid. The point of application of the component F_y acting vertically downward is at the centroid of the liquid volume above the curved surface

The resultant pressure force F in then equal to $\sqrt{(F_x^2 + F_y^2)}$; acting at angle $\tan^{-1}\left(\frac{F_y}{F_x}\right)$ with the horizontal

In some engineering applications, the liquid acts from below the curved surface. In that case, the vertical component of pressure force acts upwards, and equals the weight of an imaginary column of liquid above the curved surface up to the free surface.

Pressure Diagrams

A pressure diagram may be defined as a graphical representation of the variation in the intensity of pressure over a surface. Such diagrams are very useful for finding out the vertical surface (i.e., wall or dam). A vertical surface may be subjected to the following types of pressure

1. Pressure due to one kind of liquid on one side,
2. Pressure due to one kind of liquid, over another, on one side, and
3. Pressure due to liquids on both the sides

Pressure Due to One Kind of Liquid on One Side

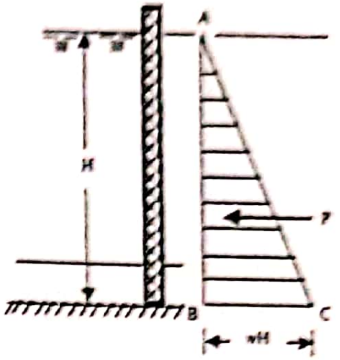

Fig. 2.18 Pressure Due to One Kind of Liquid on One Side

Consider a vertical wall subjected to pressure due to one kind of liquid, on one of its sides as shown in Fig. 2.18

Let H = Height of liquid

w = Specific weight of the liquid

P = Total pressure on the wall, per unit length.

We know that the pressure on the wall is zero at the liquid surface, and will increase by a straight line law to wH at the bottom. Therefore the pressure diagram will be a triangle ABC as shown in the figure. The total pressure on the wall per unit length

$$P = \text{Area of triangle ABC} = \frac{1}{2} \times H \times wH = \frac{wH^2}{2}$$

This pressure will be act at the centre of gravity of the triangle, i.e., at a depth of $\frac{2}{3}H$ from the liquid surface, or at a height of $\frac{1}{3}H$ from the bottom of the liqui

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Pressure Due to One Kind of Liquid over Another on One Side

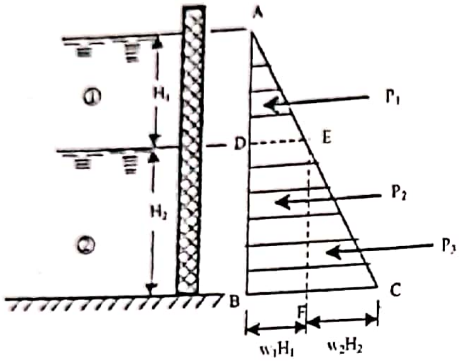

Fig. 2.19 Pressure Due to One Kind of Liquid over Another on One Side

Consider a vertical wall, subjected to pressure due to one kind of liquid, over another, on one side as shown in the figure. This will happen, when one liquid is insoluble into the other.

Let H_1 = Height of liquid 1,
w_1 = Specific weight of liquid 1,
H_2 = Height of liquid 2
w_2 = Specific weight of liquid 2
P = Total pressure on the wall per unit length

We know that the pressure in such a case in zero at the liquid surface, and will increase by a straight line law to w_1H_1 up to a depth of H_1. It will further increase, by a straight line law, to $w_1H_1 + w_2H_2$ as shown in the figure.

The pressure P_1 on the surface AD, due to liquid 1, may be found out, as usual, from the area of triangle ADE (i.e., $P_1 = \frac{w_1H_1^2}{2}$). The pressure on the surface DB will consist of pressure P_2 due to superimposed liquid 1, as well as pressure P_3 due to liquid 2. This pressure will be given by the area of the trapezium BCED (i.e., area of rectangle DBFE due to superimposed liquid i.e., $P_2 = w_1H_1 \times H_2$ and the area of triangle FCE due to liquid 2 (i.e., $P_3 = \frac{w_2H_2^2}{2}$)

The total pressure P will be sum of three pressures (i.e., $P = P_1 + P_2 + P_3$). The line of action of the total pressure may be found out by equating the moment of P, P_1, P_2 and P_3 about A.

top of the wall

Pressure Due to Liquids on Both Sides

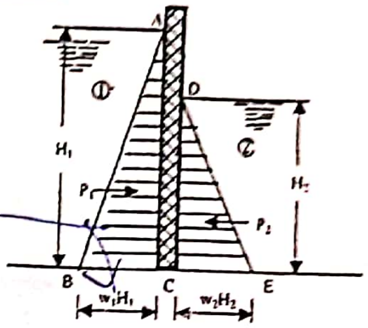

Fig. 2.20 Pressure Due to Liquids on Both Sides

Consider a vertical wall subjected to pressures due to liquids on both sides as shown in Fig.2.20.

Let H_1 = Height of liquid 1,
w_1 = Specific weight of liquid 1,
H_2 = Height of liquid 2
w_2 = Specific weight of liquid 2
P = Resultant pressure on the wall per unit length.

We know that the pressure of liquid 1 is zero at the liquid surface and will increase by a straight line law, to w_1H_1 at the bottom as shown in the above figure.

The total pressure on the wall per unit length due to liquid 1.

$$P_1 = \frac{1}{2} \times H_1 \times w_1H_1 = \frac{w_1H_1^2}{2}$$

Similarly, total pressure on the wall per unit length due to liquid 2,

$$P_2 = \frac{1}{2} \times H_2 \times w_2H_2 = \frac{w_2H_2^2}{2}$$

As the two pressures are acting in the opposite directions, therefore the resultant pressure will be given by the difference of the two pressures (i.e., $P = P_1 - P_2$). The line of action of the resultant pressure may be found out by equating the moments of P, P_1 and P_2 about the bottom of the wall.

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Hoop Tension in a Pressure Pipe

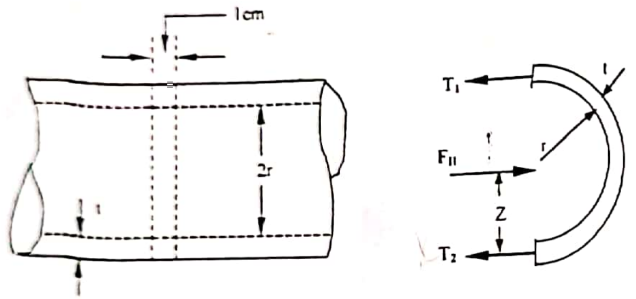


Fig. 2.21 Hoop Tension in a Pressure Pipe

A circular pipe under the action of an internal pressure is in tension around its circumference. This hoop tension causes stretching of the pipe wall and the pipe material is subjected to tensile stresses. Consider the above figure showing longitudinal section and one-half cross-section of a circular pipe. Assuming that no longitudinal stress occurs, the walls are in tension as shown. Considering 1 cm length of pipe and one-half of its circular section as a free body, let the tensile force per cm of pipe length at top and bottom be T_1 and T_2, respectively as shown in Fig.2.21

The horizontal hydrostatic force F_H acts through the centre of pressure of the projected area and has a magnitude 2rp, in which p is the pressure at the pipe centre in kg/cm^2 and r is the pipe radius in cm. For penstock pipes which are subjected to high pressures, the centre of pressure may be taken at the pipe centre, and then,

$$T_1 + T_2 = F_H$$

$$= 2pr$$

or $$2T = 2pr$$

Therefore the hoop tension force

$$T = pr \qquad \text{... ... ... (2.21)}$$

For the pipe wall thickness of t cm. the tensile stress σ in the pipe-wall

$$\sigma = \frac{T_1}{1 \times t} = \frac{pr}{t}$$

If the pressures at the top and bottom of pipe differ appreciably, the centre of pressure is computed and Z determined. To determine T_1 and T_2, the following equation may be written.

$$T_1 + T_2 = 2pr$$

taking moments of forces about the pipe bottom

$$T_1 . 2r = F_H \times Z$$

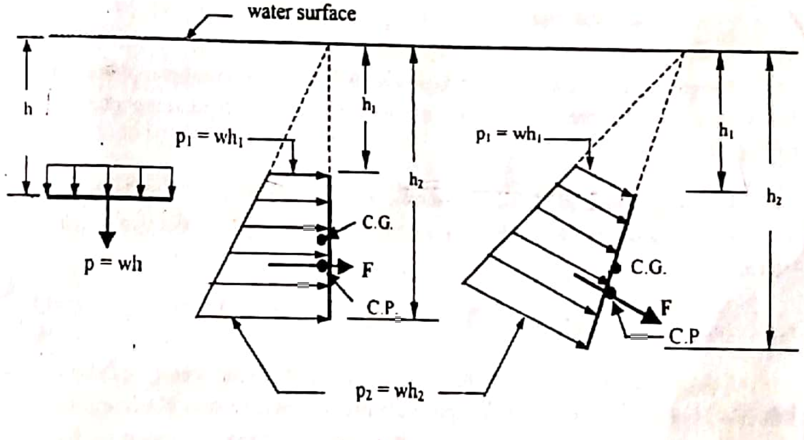
$$= 2\,pr \times Z$$

Solving the above two equation

$$T_1 = pZ \qquad \text{... ... ... (2.22)}$$

$$T_2 = p(2r - Z) \qquad \text{... ... ... (2.23)}$$

Pressure Diagrams for Horizontal, Vertical and Inclined surfaces



(a) Horizontal (b) Vertical (c) Inclined

Fig. 2.22 Pressure Diagrams for Horizontal, Vertical and Inclined surfaces

Practical Applications of Hydrostatics

There are numerous engineering applications on the subject of hydrostatics. number of hydraulic Transmission of hydrostatics pressure is applied in the design of a number of hydraulic machines such as hydraulic press, hydraulic jack, hydraulic crane etc. The engineering structures which are constructed to withstand the forces in the static liquids are:

1. Overhead water tanks
2. Sluice gates
3. Lock gates.
4. Masonry walls
5. Dams

1. Overhead Water Tanks

Overhead water tanks are used to hold water at a certain height to be supplied under gravity.

2. Sluice Gates

Gates are used in the rivers, dams, pipes and canal locks. The hydrostatic pressure or forces may be acting on one side of the gate e.g., simple gates, sluice gates, tainter gates and roller gates.

In some cases the liquid lies on both sides of gates of sluice type. A *sluice gate* is used to control the flow of water by moving it in a vertical plane through the grooves made in the outlets.

Static Pressure on Sluice Gates

A sluice gate consists of two vertical plates, known as skin plates, having a number of horizontal I-beams in between them. The pitch of the beams on the bottom side is less than that on the top side of the plates, since pressure increases with the depth. To control the flow of water the sluice gate moves up and down with the help of rollers fixed on the skin plates. The rollers travel on the vertical rails, called guides, which form a part of piers or vertical walls of outlets.

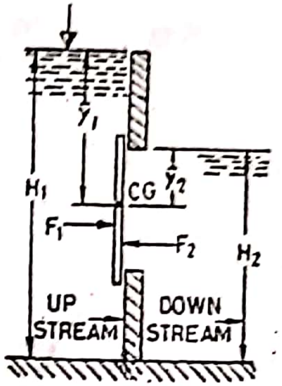


Fig. 2.23 Resultant Pressure on Sluice Gate

For the gate is acted upon by the pressure on both of its sides, the resultant static pressure action on the gate and the position of centre of pressure, is determined as follows -

Calculate the total pressure F_1 and F_2 on the gate on upstream and downstream sides respectively, and as the two pressure are acting in the opposite directions, the resultant pressure F will be the difference of F_1 and F_2 –

Total pressure on upstream side of gate $F_1 = \gamma \bar{y}_1 . A_1$

Total pressure on downstream side of gate, $F_2 = \gamma \bar{y}_2 . A_2$

∴ Resultant static pressure on gate $F = F_1 - F_2$ ... ... (2.24)

For determining the position of centre of pressure, h is calculated for both sides. The final position of c.p. is determined by taking moments about any point say top or bottom of the gate.

3. Lock Gates

A *lock gate* is used to change the water level in the canal by construction a lock for the purpose of navigation.

If the dam is constructed in a canal or river, the water levels on the two sides of the dam will be different. For the purpose of navigation or boating, a chamber known as *lock* is constructed between the two different water levels by providing two sets of lock gates. In order to transfer a both from the upper water level to the lower one, it is first admitted into the lock by opening the upstream gates with which the water enters the lock rising the level of water equal to that of the canal upstream are opened with which the water level in the lock

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falls to that of the canal downstream side. The boat is then able to proceed further downwards. For a boat to be transferred from the lower to the higher side, the above procedure has to be reversed.

Determination of Resultant Pressure on Lock Gate

The elevation and plan of lock gates are shown in Fig 2.24. AB and BC are the two gates each carried on two hinges fixed on their top and bottom as both A and C. The gates are tightly closed to one another at B by the action of water pressure. It is required to determine the magnitude of forces on hinges due to water pressure. Consider the gate AB.

Let F = the resultant water pressure, action on AB, normal to it;

N = reaction force supplied by gate BC to gate AB and acts normal to the contact surface of the two gates;

R = resultant reaction of the top and bottom hinges; which is assumed to lie in the same horizontal plane in which F and N lie;

θ = angle of inclination which the gate AB makes with the normal to the side of lock.

As N is normal to the contact surface, it follows that it will make the same angle θ with the gate AB at B, being alternate angles.

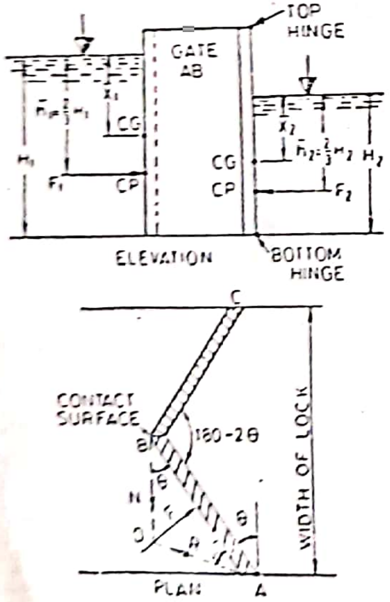

Fig. 2.24 Resultant Pressure on Lock Gates

Since F acts normal to AB at the centre of width AB, so in order that the gate to be in equilibrium, these forces N, F and R should pass through a point D. Therefore R meets at D and triangle BAD becomes an isosceles triangle having angles ADB and BAD equal to angle θ

Resolving forces along AB -

$$N.\cos\theta \quad R.\cos\theta$$

$$\therefore \quad N = R \qquad \text{------ (2.25)}$$

Resolving forces normal to AB

$$F = R.\sin\theta - N.\sin\theta \qquad \text{------ (2.26)}$$

$$\therefore \quad F = (R-N)\sin\theta = 2R\sin\theta \qquad \text{------ (2.27)}$$

The resultant pressure F is the difference of F_1 and F_2 the pressure acting in two opposite directions.

i.e., $F = F_1 - F_2$, where

$F_1 = \gamma\bar{y}_1 A_1 = \gamma \cdot \frac{H_1}{2} \cdot A$ acting at c.p. at $\frac{2H_1}{2}$ from free water surface on upstream side.

$F_2 = \gamma\bar{y}_2 A_2 = \gamma\frac{H_2}{3}A$ acting at c.p. at $\frac{2H_2}{2}$ from free water surface on downstream side.

and A = the wetted area of the gate AB.

Reactions at top and bottom hinges

The total pressure F_1 and F_2 act in the middle of gate AB. Thus only half of the pressures of F_1 an F_2 will be acting on both the hinges of gate AB and the remaining half on the edge of gate BC (i.e. on the contact surface of gates AB and BC).

Taking moments about bottom hinge,

$$(R_T \sin\theta).H = \left(\frac{F_1}{2} \times \frac{H_1}{3}\right) - \left(\frac{F_2}{2} \times \frac{H_2}{3}\right)$$

where H = height of lock gate;

Also resolving horizontally-

$$R_T \sin\theta - R_B \sin\theta = \frac{F_1}{2} - \frac{F_2}{2}$$

Find the reactions on top and bottom hinges R_T and R_B, from the above equations.

Also $R \quad R_T \quad R_B$.

Masonry Dam

Masonry Dam is a structure across a river constructed for impounding water by making a reservoir. The water thus impounded by dam is used for the purposes of irrigation and power generation. There are various types of dams, the two principle types being gravity dam and arched dam.

Water Pressure on Masonry Dams

Masonry dam is a structure across a river constructed for impounding water by making a reservoir. The water thus impounded by dam is used for the purpose of irrigation and power generation. The main forces acting on a dam are static water pressure against upstream face and the weight of masonry. For the actual design of a dam, the other forces considered are wind forces, ice thrust, uplift of water beneath or within the dam. A dam may be of any cross-section, but the following are important from the subject point of view:

1. *Rectangular dams* ✓
2. *Trapezoidal dams* ✓

Water Pressure on Rectangular Dams

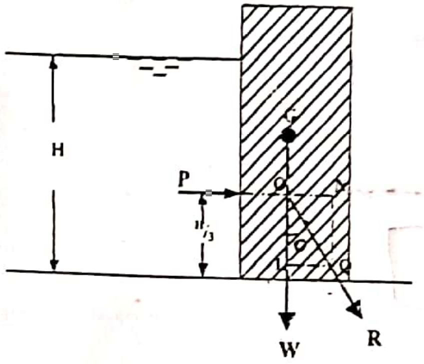

Fig. 2.25. Resultant Pressure on Rectangular Dam

Consider a rectangular dam having water on one of its sides as shown in Fig. 2.25. Now consider a unit length of the dam.

Let H = Height of water stored by dam

P = Total pressure of water

$$\frac{1}{2} wh \times h = \frac{wH^2}{2}$$

We know that this pressure P will act at a point of height $\frac{H}{3}$ above the base of the dam. ✓

Let W be the weight of the dam masonry per unit length of ~~the dam which~~ will act down wards through the centre of gravity of the dam section.

Now the resultant pressure of force P and weight W will be given by the relation

$$R = \sqrt{(P^2 + W^2)} \qquad \text{... ... ... (2.28)}$$

and the inclination of the resultant with the vertical (θ) will be given by the relation,

$$\tan\theta = \frac{P}{W} \qquad \text{... ... ... (2.28a)}$$

Now with OL and ON or LQ (equal to W and P to some scale) complete the rectangle OLQN. Then the diagonal OQ will give the resultant (R) to scale. Now extend OL and OQ to meet the base line at M and R as shown in the figure.

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Let x be the horizontal distance between the centre of gravity of the dam, and the point through which the resultant cuts the base (i.e., MR) The distance x may be found out from the similar triangles OLQ and OMR.

$$\frac{MR}{OM} = \frac{LQ}{OL}$$

or,
$$\frac{x}{H/3} = \frac{P}{W}$$

or.
$$\boxed{x = \frac{P}{W} \times \frac{H}{3}} \qquad \text{... ... ... (2.29)}$$

Water pressure on Trapezoidal Dams

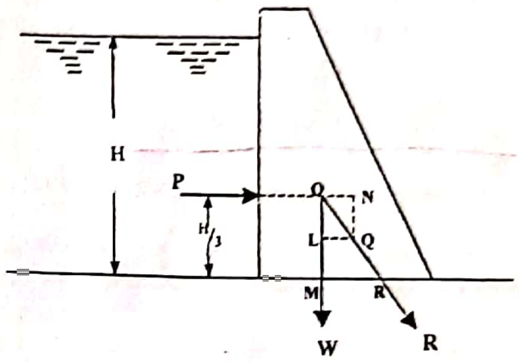

Fig. 2.26 Resultant Pressure on Trapezoidal Dam

A trapezoidal dam is more economical and also easier to construct than a rectangular dam. That is why, these days trapezoidal dams are preferred to the rectangular ones.

Consider a trapezoidal dam retaining water on one of its sides (say vertical side) as shown in Fig. 2.26.

Like the rectangular dam, the total pressure on a trapezoidal dam per metre length will be given by the relation

$$P = \frac{wH^2}{2}$$

and the horizontal distance between the center of gravity of the dam and the point, at which the resultant cents the base will also be given by the relation:

$$x = \frac{P}{W} \times \frac{H}{3}$$

If the water is retained on the inclined face, the water pressure will act normally to the face. In such a case the horizontal and vertical components of the pressure are to be used for all calculations. This may be simplified by assuming the surface to be vertical and the weight of triangular wedge of water over the inclined surface, is included in the weight of dam.

Conditions of Stability of a Dam

In the previous two articles, we have derived a relation, which gives us the point, in the base of the dam, through which the resultant cuts the base. The point in the base, through which the resultant cuts gives us very important information and helps us in finding the stability of the dam.

For a dam to be stable, the following conditions are to be satisfied

1. To safeguard against overturning, the resultant R must lie within the base.
2. The maximum and minimum stresses developed at bottom should be safe. To avoid tension at the base, the resultant R should be within central third of the base.
3. To prevent sliding, maximum frictional force that can develop should be more than horizontal force i.e., $P < \mu w$ or geometrically the angle of inclination of the resultant R with vertical must be less than the angle of fiction for masonry.
4. The maximum stress developed at the bottom of the dam should be within the permissible stress of the site.

CHAPTER-3

BUOYANCY AND FLOTATION

A body immersed partially or fully in a fluid experiences a vertical upward force. This vertical upward force on a floating or submerged body is known as buoyant force and its magnitude can be determined by Archimedes principle of buoyancy. The tendency of a submerged body to rise in a fluid because of the upward fluid pressure which opposes the downward force of gravity is known as ***buoyancy***.

Archimedes Principle of Buoyancy states that ***when a body is totally or partially immersed in a fluid, it is buoyed up (or lifted up) by a force which equals the weight of the fluid displaced by the body.***

Proof:

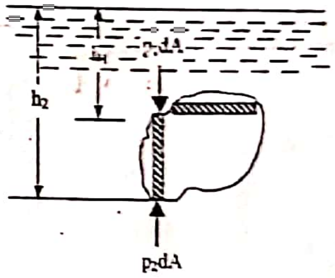

Fig. 3.1 Archimedes Principle

Let a body be immersed in a fluid of constant specific weight γ as shown in the figure. It is required to find the resultant vertical lift force acting on the body.

Divide the body into a large number of vertical prisms. Consider an elementary vertical prism of cross sectional area dA.

Then force acting upward on bottom of prism

$= p_2 \, dA - \boxed{\gamma} h_2 \, dA$

$$\boxed{p = \gamma h}$$

and force acting downwards on top of prism

$$\gamma h_2\, dA - \gamma h_1\, dA$$

Net upward force on prism

$$dF_B = \gamma\, dA\, (h_2 - h_1)$$

$$= \gamma\, dv$$

or total upward or buoyant force on lift ... ... (3.1)

$$F_B = \gamma \int dv = \gamma V$$

Thus upward or buoyant force = weight of fluid displaced

As buoyant force is vertical and is equal to the weight of fluid displaced, therefore it acts through the centre of gravity of the displaced fluid. The point of application of the buoyant force is known as *centre of buoyancy*.

Considering any horizontal prism as shown with dotted lines in the figure, the depth of fluid at both end areas is same, therefore the pressure force acting on its left end area and right end area shall be equal in magnitude but opposite is direction. Thus no resultant horizontal force acts on the submerged body. Thus follows that no force other than the buoyant force in vertical direction acts on the submerged body.

Body Immersed in Fluids of Different Specific Weights

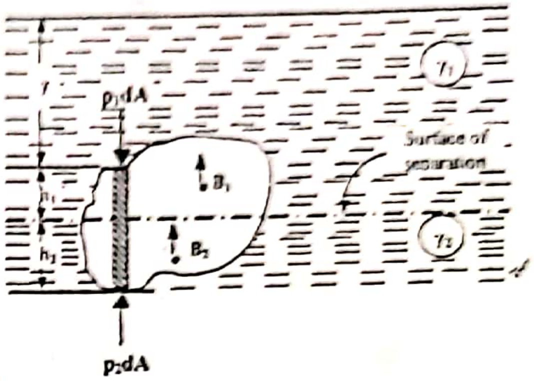

Fig. 3.2 Body Immersed in Fluids of Different Specific Weights

Suppose the body floats in between the surfaces of separation of two fluids of specific weight γ_1 and γ_2 as shown in the figure.

On an elementary vertical prism of cross-sectional area dA, the buoyant force.

$$dF_B = p_2\, dA - p_1\, dA$$

$$= dA\, [(y + h_1)\, \gamma_1 + h_2\, \gamma_2] - y\, \gamma_1\, dA$$

$$= (h_1\, \gamma_1 + h_2\, \gamma_2)\, dA$$

$$= \gamma_1\, dv_1 + \gamma_2\, dv_2$$

Total buoyant force

$$F_B = \int dF_B = \int \gamma_1\, dv_1 + \int \gamma_2\, dv_2$$

$$= \gamma_1 v_1 + \gamma_2 v_2 \qquad \text{... ... ... (3.2)}$$

$$= \text{total weight of fluid displaced}$$

Here v_1 and v_2 are the volumes of the body submerged in fluids of specific weights γ_1 and γ_2 respectively.

Principle of Flotation

Principle of floatation states that *the weight of a floating body is equal to the weight of the fluid displaced by the body.*

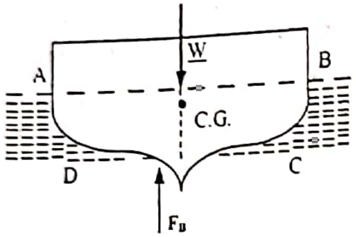

Fig. 3.3 Principle of Flotation

Proof

Consider a floating body at the free surface of a liquid.

From equation 3.2, buoyant force

$$F_B = \gamma_{air} \cdot V_{air} + \gamma_{liquid} \cdot V_{ABCD}$$

As $\gamma_{air} \quad \gamma_{liquid}$

$$F_B = \gamma_{liquid} \cdot V_{ABCD}$$

= weight of liquid displaced.

Since the body is in static equilibrium.

$$\Sigma F_{vertical} = 0$$

or $\quad F_B - W_{body} = 0$

or $\quad F_B = W_{body}$

i.e., Weight of liquid displaced = Weight of floating body.

Depending on the ratio of the weight W of a body and the buoyant force F_b, three cases are possible:

(i) $W > F_b$; the body tends to move downward, and eventually sink.

(ii) $W = F_b$; the body floats and is only partially submerged.

(iii) $W < F_b$; the body is lifted upward and rises to the surface.

From the above condition, we can say that a body can be made to float by:

(i) decreasing the weight of the body, the volume remaining the same.

(ii) increasing the volume of the body, the weight remaining the same.

Some Practical Applications

(1) Volume of an irregular solid can be determined by knowing the apparent loss of weight which the body will experience when totally submerged in fluid of known specific gravity.

(2) Specific gravity of liquid, can be determined – Principle of Hydrometer.

(3) A submarine is brought on the surface of ocean by decreasing its weight, the volume remaining the same. It is done by pumping the water out of the ballast tanks until the weight of submarine is less than the upward buoyant forces exerted by water on the outside. The submarine sinks by taking water in it until its weight is greater than the buoyant forces.

(4) Life-saving suits for ships and rubber rafts for airplanes which fly over the ocean.

(5) Modern bathing suits which can be inflated with air making possible even for non-swimmers to bathe in safety.

Principle of Hydrometer

- A floating object of weight W will sink more in a lighter fluid than in a heavier one.

- $W = \gamma V$, where V is the volume of fluid displaced and γ the specific weight of the fluid. Thus, W remaining constant the volume V displaced will vary inversely with γ.

- For two liquids of weight densities γ_1 and γ_2 and volumes V_1 and V_2,

$$W = \gamma_1 V_1 = \gamma_2 V_2$$

or $$\frac{V_1}{V_2} = \frac{\gamma_2}{\gamma_1} \qquad \text{... ... ...} \quad (3.3)$$

Statical Stability of Floating Bodies

Any floating body is subjected to two systems of parallel forces:

(i) the downward force of gravity acting on each of the particles that goes to make up the body, and

(ii) the buoyant force of the liquid acting upward on the various elements of the submerged surface.

In order that the body may be in equilibrium the resultants of these two systems of forces must be collinear, equal and opposite. Hence the centre of buoyancy and the centre of gravity of the floating body must lie in the same vertical line.

Types of Equilibrium of Floating Bodies

The equilibrium of floating bodies is of the following types:

1. Stable equilibrium
2. Unstable equilibrium
3. Neutral equilibrium

1. Stable equilibrium

When a body is given a small angular displacement (i.e. tilted slightly), by some external force, and then it returns back to its original position due to the internal forces (the weight and the upthrust), such an equilibrium is called *stable equilibrium*.

2. Unstable equilibrium

If the body does not return to its original position from the slightly displaced angular position and heels further away, when given a small angular displacement, such an equilibrium is called an *unstable equilibrium*.

3. Neutral equilibrium

If a body, when given a small angular displacement, occupies a new position and remains at rest in this new position, it is said to possess a *neutral equilibrium*.

Metacentre and Metacentric Height

Metacentre

Metacentre is defined as the point about which a body starts oscillating when the body is tilted by a small angle. It may also be defined as a point of intersection of the axis of body passing through c.g.(G) and , original centre of buoyancy (B_0), and a vertical line passing through the centre of buoyancy (B) of the tilted position of the body.

The position of metacentre, M remains practically constant for the small angle of tilt θ.

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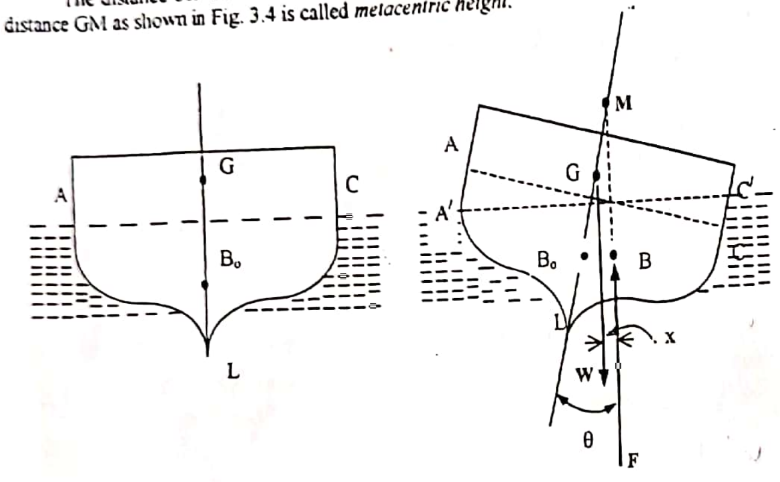
Metacentric Height

The distance between the centre of gravity of a floating body and the metacentre (i.e. distance GM as shown in Fig. 3.4 is called *metacentric height*.



(a) (b)

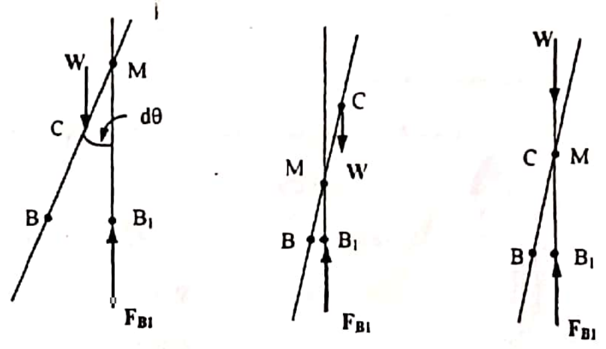
Fig. 3.4 Cross-section of a Ship

Figure 3.4 shows the cross section of a ship floating in an upright position, the axis of symmetry being vertical. For this position the centre of buoyancy lies on the axis of symmetry at B_0, which is the centre of gravity of the area ACL. The centre of gravity of the ship is assumed to be at G. If from any case, such as wind or wave action, the ship is made to heel through an angle θ, as shown in the figure, the centre of gravity of the ship and cargo remaining unchanged, the centre of buoyancy shifts to a new position, B, which is the centre of gravity of the area A'C'L'. The buoyant force F, acting downward through G, constitute a couple $W \times x$ which resists further overturning and tends to restore the ship to its original upright position.

If the vertical line through the centre of buoyancy intersects the inclined axis of symmetry at point M above the centre of gravity, the two forces F and W produce a righting moment. If, however, M lies below G an overturning moment is produced. The point M is known as the metacentre, and its distance GM form the centre of gravity of the ship is termed the metacentric height. The metacentric height is a measure of the statical stability of the ship. For small angles of inclination, the position of M does not change materially and the metacentric height is approximately constant.

Metacentric Height as Direct Measure of Stability of Floating Body

From the previous article we know that the position of metacentric is a direct measure of the stability of floating body. The condition of equilibrium of a floating body is determined by the external couple in relation to the sense of the infinitesimal tilt given to the body. Therefore, more the metacentric height, the more stable the body will be, A zero value of metacentric height means neutral equilibrium and a negative value, unstable equilibrium.



(a) (b) (c)

Configuration	**(a)**	**(b)**	**(c)**
Sense of Tilt	Clockwise	Clockwise	Clockwise
Sense of External Couple Due to W and F_{B1}	Anti clockwise	Clockwise	Nil
Action of External Couple	Restoring	Further Tilting	Nil
Behaviour of the Body	Returns to Status-Quo	Further Tilt-	Stays as it is
Condition of Equilibrium	Stable	Unstable	Neutral
Sign of GM, the Metacentric Height.	+ ve	- ve	zero

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It can be observed from the description of the above figure that stable, unstable and neutral equilibrium correspond to M above C, M below C and M coinciding with C respectively. This criterion has long been used by the hydraulic engineers. Distance CM measured from C to M is called the metacentric height should be positive. It also follows that larger the metacentric positive height more stable is the body.

It is instructive to comment that the position of equilibrium of a body is the position of minimum potential energy corresponding to the equilibrium state. This is a general conclusion arrived at in the study of mechanics. The centre of gravity C should therefore, be as low as possible. For a floating body, the condition of 'CM' to be positive is in conformity with this conclusion.

The normal values of metacentric height for different types of ships are:

- merchant ships	0.30 to 1.00 m
- sailing ships	0.45 to 1.25 m
- battle ships	1.00 to 1.50 m
- river crafts	up to 3.50 m.

Determination of Metacentric Height

For certain floating bodies like ships, pontoons, barges etc., it is very important to know the metacentric height, in order that they might not capsize. Therefore naval architects and engineers take care in designing a ship to keep the metacentre at a safe distance above centre of gravity. The position of centre of gravity of ship may be lowered by carrying ballast also. The ballast is a heavy material placed in the ship's hold for stability.

There are two methods by which the metacentric height can be determined – theoretical and experimental. The theoretical method to find the metacentric height is used at the time of designing the ship and the experimental method is employed after the ship has been afloat and ballasted.

Theoretical or Analytical Method for the Determination of Metacentric Height

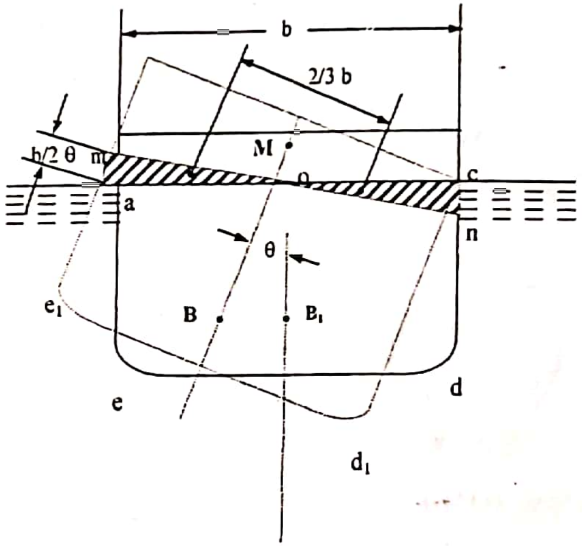

Fig. 3.5 Theoretical Method for Determination of Metacentric Height

Consider a ship floating freely in water. Let the ship be given a clockwise rotation through a small angle θ (in radians) as shown in the figure. The immersed section has now changed from *acde* to acd_1e_1.

The original centre of buoyancy B has now changed to a new position B_1. It may be noted that the triangular wedge *aom* has come out of water, whereas the triangular wedge *aom* has gone under water. Since the volume of water displaced remains the same, therefore the two triangular wedges must have equal areas.

As the triangular wedge *aom* has come out to water, thus decreasing the force of buoyancy on the left, therefore it tends to rotate the vessel in an anti-clockwise direction. Similarly as the triangular wedge *ocn* has gone under water, thus increasing the force of buoyancy on the right, therefore it again tends to rotate the vessel in an anti-clockwise direction. It is thus obvious, that these forces of buoyancy will form a couple, which will tend to rotate vessel in anticlockwise direction about O. If the angle (θ) through which the body is given rotation, is extremely small, then the ship may be assumed to rotate about M (i.e., metacenter).

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Let

l = Length of the ship

b = Breadth of the ship

θ = Very small angle (in radians) through which the ship is rotated.

V = Volume of water displaced by the ship.

From the geometry of the figure we find that

$$am = cn = \frac{b\theta}{2}$$

Volume of wedge of water aom

$$= \frac{1}{2}\left(\frac{b}{2} \times am\right) l$$

$$= \frac{1}{2}\left(\frac{b}{2} \times \frac{b\theta}{2}\right) l \qquad \left(\because am = \frac{b}{2}\right)$$

$$= \frac{b^2 \theta l}{8} \qquad \dots \dots \dots \dots \dots (3.4)$$

$\therefore$ Weight of this wedge of water

$$= \frac{wb^2\theta l}{8} \quad (w = \text{specific weight of water}) \qquad \dots \dots \dots \dots \dots (3.5)$$

and arm of the couple

$$= \frac{2}{3} b$$

$\therefore$ Moment of the restoring couple

$$= \frac{wb^2\theta l}{8} \times \frac{2}{3} b$$

$$= \frac{wb^3\theta l}{12} \qquad \dots \dots \dots \dots \dots (3.6)$$

and moment of the disturbing force

$$= wV \times BB_1 \qquad \dots \dots \dots \dots \dots (3.7)$$

Equating these two moments,

$$= \frac{wb^3\theta l}{12} = w \times v \times BB_1$$

Substituting values of $\frac{lb^3}{12} = I$ (i.e. moment of inertia of the plan of the ship) and

$BB_1 = BM \times \theta$ in the above equation.

$$w . I\theta = w \times V (BM \times \theta)$$

$$\therefore BM = \frac{I}{V} = \frac{\text{Moment of inertia of the plan}}{\text{Volume of water displaced}} \qquad - - - - - (3.8)$$

Now metacentric height,

$$GM = BM \pm BG$$

– ve sign is to be used if G is lower than B, and

- ve sign is to be used if G is higher than B.

Experimental Method for the Determination of Metacentric Height

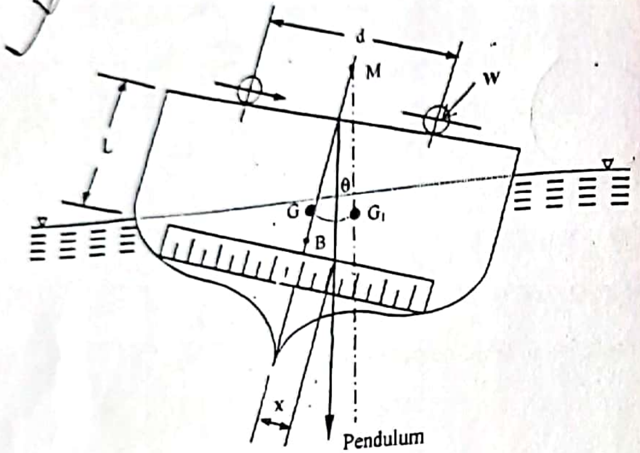

Fig. 3.6 Experimental Method for the Determination Metacentric Height

The metacentric height of ship, floating object, may also be determined experimentally: A rolling known weight is moved across the deck by which the ship heels through an angle θ. The angle is measured by noting the horizontal distance moved by a long pendulum or a plumb bob, hanging inside the ship from the centre point of the clock along a horizontal graduated scale as shown in the above figure.

Let d = the distance moved along the deck by the rolling weight;

l = the distance along GM from the centre of deck to the point from where the distance x is to be measured;

x = the distance along the graduated scale parallel to the deck;

w = the movable weight (known);

W = the weight of the ship (known);

G = the c.g. of the ship (known);

M = the metacentre of the ship

By moving the rolling weight w on the deck of ship through a distance d its moment is equal to wd. This moment should be equal to the moment of W about metacentre M.

$$\text{or} \quad W.\,GG_1 = w\,d \qquad \text{... ... ...} \quad (3.9)$$

$$\text{where} \quad GG_1 = GM \sin\theta$$

$$\text{and} \quad GM \sin\theta = GM \tan\theta \qquad (\text{as } \sin\theta \approx \tan\theta \text{ for small angle of heel})$$

$$\therefore \; W.\,GM \tan\theta = w.\,d$$

$$\text{or,} \quad GM = \frac{w.\,d}{W.\tan\theta} \qquad \text{... ... ...} \quad (3.10)$$

$$\text{where} \quad \tan\theta = \frac{x}{l}$$

$$\therefore \; \text{Metacentric height } GM = \frac{wd}{W} \cdot \frac{l}{x} \qquad \text{... ... ...} \quad (3.11)$$

By noting the position of the water-line and calculating the displaced water, the value of W can be found. Thus all the terms on right hand side of the above equation being known, the metacentric height GM can be determined.

The position of centre of gravity G being known, the above method will find the position of metacentre M.

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Maximum Length of a vertically floating Body

We know that a cube of wood (having specific gravity less than 1) can float in water, in any position. If we maintain any two sides (say breadth and thickness) of the cube, constant and go on increasing gradually the third side (say length) and try to float the block vertically in water, we can see that the block can float vertically in water upto some length. If we increase the length of the block beyond this length, we find that it cannot float vertically in water though it can float longitudinally.

The maximum permissible length of the block, floating vertically in water, may be found out by keeping the body in stable equilibrium. Or in other words, this can also be found out by avoiding the unstable equilibrium of the floating body. For doing so, the metacentre (M) should be above centre of gravity (G) of the floating body (a condition of stable equilibrium) or the metacentre (M) may coincide with the centre of gravity (G) of the floating body (a condition of neutral equilibrium i.e., by avoiding the unstable equilibrium).

Floating Bodies Anchored at the Base

We have seen in the above article that there is always a certain limit of length, up to which a body can float vertically. If the length of the body exceeds this limit, it cannot float vertically, though it can float horizontally. But, sometimes, due to certain reasons, the body is required to float vertically. For doing so, the body is anchored, by means of a chain from the centre of its base. The tension in the anchor chain, puts on additional downward force on the body, which will cause a larger volume of water to be displaced.

Let W = Weight of the body

T = Tension in the anchor chain.

Then the total downward force, which will displace the water

$$= W + T \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (3.12)$$

The remaining procedure for solving the problem is the same.

Conical Buoys Floating in Liquid

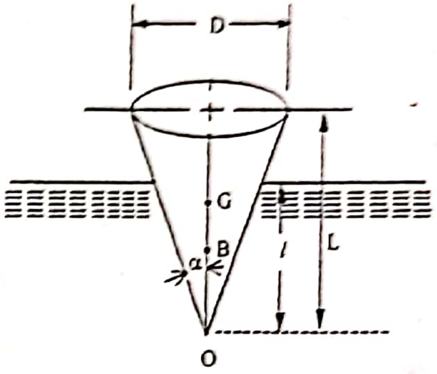

Fig. 3.6 Conical Buoy

Consider a conical buoy floating in a liquid as shown in Fig. 3.6

Let D = Diameter of the cone

d = Diameter of the cone at the liquid level

2α = Apex angle of the cone

L = Length of the cone

l = Length of the cone immersed in the liquid

From the geometry of the figure we find that the distance of centre of buoyancy from the apex O.

$$OB = \frac{3}{4} l$$

Distance of centre of gravity from the apex O.

$$OG = \frac{3}{4} L$$

Volume of liquid displaced

$$V = \frac{1}{3} \pi l^3 \tan^2 \alpha \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots \quad (3.13)$$

Moment of inertia of the circular section at the liquid level about the centroidal axis.

$$I = \frac{\pi}{64} \times d^4$$

$$= \frac{\pi}{64}(2l\tan\alpha)^4$$

$$= \frac{\pi}{4} l^4 \tan^4\alpha \qquad \text{------ (3.14)}$$

The metacentric height of the conical buoy may be found out by the relation (as before)

$$BM = \frac{I}{V}$$

$$= \frac{\frac{\pi}{4}(l^4 \tan^4\alpha)}{\frac{1}{3}\pi(l^3 \tan^2\alpha)}$$

$$BM = \frac{3}{4} l \tan^2\alpha \qquad \text{------ (3.15)}$$

Time Period of Rolling (Oscillation) of a Floating Body

Where a floating body is given a tilt, it is set in a state of oscillation as if suspended at the metacentre in a manner similar of that a simple pendulum

Let θ = angle of tilt at an instant of time t

k = radius of gyration of the body about a longitudinal axis passing through the centre of gravity

h = metacentric height, GM

w = weight of the floating body

Then angular acceleration of the body = $\frac{d^2\theta}{dt^2}$

Moment of inertia of the body, about a line through the centre of gravity

$$I = \frac{W}{g}k^2 \qquad \text{------ (3.16)}$$

Inertia torque = Moment of inertia × Angular acceleration

$$= -\frac{W}{g}k^2\left(\frac{d^2\theta}{dt^2}\right) \qquad \text{------ (3.17)}$$

(- ve sign accounts for the fact that inertia torque acts so as to decrease the angular tilt θ)

Righting or restoring moment = W × GM sin θ

= W. hθ (for small angle θ measured in radians)

Equating the inertia torque and righting moment

$$W h\theta = -\frac{w}{g}k^2\frac{d^2\theta}{dt^2}$$

$$\text{or} \quad \frac{k^2}{g}\frac{d^2\theta}{dt^2} + h\theta = 0 \qquad \text{(rearranging and dividing both sides by W)}$$

$$\text{or} \quad \frac{d^2\theta}{dt^2} + \frac{gh}{k^2}\theta = 0 \qquad \left(\text{dividing both sides by } \frac{k^2}{g}\right) \qquad \text{------ (3.18)}$$

Solution of this second order linear differential equation is:

$$\theta = A\sin\left(t\sqrt{\frac{gh}{k^2}}\right) + B\cos\left(\sqrt{\frac{gh}{k^2}}\right) \qquad \text{... ... ... (3.19)}$$

where A and B are the constants of integration.

If T is the time period of oscillation in seconds, then the boundary conditions are:

$\theta = 0$ at, $t = 0$, $\theta = 0$ at $t = \frac{T}{2}$

These boundary condition give:

$$B = 0;\ A\left(\sin\frac{T}{2}\sqrt{\frac{gh}{k^2}}\right) = 0$$

Since $A \neq 0$;

$$\sin\left(\frac{T}{2}\sqrt{\frac{gh}{k^2}}\right) = 0$$

$$\therefore \quad \frac{T}{2}\sqrt{\frac{gh}{k^2}} = \pi$$

$$\text{or} \quad T = 2\pi\sqrt{\frac{k^2}{gh}} \qquad \text{... ... ... (3.20)}$$

Evidently the time period of oscillation decreases with an increase in metacentric height h ; number of oscillations increase in h. Apparently it is not desirable to keep the metacentric height very large to avoid large number of oscillations. Recalling that a large value of metacentric height corresponds to improve stable equilibrium, we find that the two requirements are contrary to each other. In actual practice, an optimum value of metacentric height is selected and specified for the ship.

Bilge Water in Ships

All ships require some amount of water for everyday use and also to run the prime movers and boilers etc. This water in some cases is stored in the lower portion of the ship and is known as *bilge water* or *ballast water*. If this water has a free surface, its c.g. shifts in the direction of the shift to centre of buoyancy of the whole ship. The movement of the centre of gravity of the bilge water decreases the stability of the ship. The metacentric height of the ship also changes with the movement of bilge water. In order to reduce the instability due to bilge water, compartments are made in the ship to store water or the tanks are provided for the purpose.

CHAPTER – 4

FUNDAMENTALS OF FLUID FLOW

Introduction

When the fluid is at rest, the only fluid property of significance in the specific weight of the fluid. On the other hand when a fluid is in motion various other fluid properties become significant. As such the nature of flow of a real fluid is complex and not always subject to exact mathematical analysis, so often recourse to experimentation is required. However, in some cases the mathematical analysis of problems of fluid flow is possible if some simplifying assumptions are made.

The science which deals with the geometry of motion of fluids without reference to the forces causing the motion is known as ***hydrokinematics*** or simply ***kinematics***. Thus kinematics involves merely the description of the motion of fluids is terms of space-time relationship. On the other hand the science which deals with the action of forces in producing or changing motion of fluids is known as ***hydrokinetics*** or simply ***kinetics***. Obviously the study of fluids in motion involves the consideration of both the kinematics and the kinetics.

A fluid unlike solid, in composed of different particles, which move at different velocities and may be subject to different accelerations. Moreover, the velocity and acceleration of a fluid particle may change both with respect to time and space. Therefore in the study of fluid flow it in necessary to observe the motion of fluid particles at various points in space and at successive instants of time.

Description of Fluid Flow

There are in general two methods by which the motion of a fluid may be described. These are the ***Lugrangian*** method and the ***Eulerian*** method.

Lagrangian Method:

This approach refers to description of the behavior of individual fluid particles during their course of motion through space. The observer travels with the particle being studied as demonstrated in Fig. 4.1(a). The fluid velocity and acceleration are then determined as functions of position and time. Let originally the co-ordinates of a fluid particle be a, b, c and let these co-ordinates change to x, y, z after time interval t. The kinematic flow pattern is then fully described if the following equations of motion are known

$$\left.\begin{aligned} x &= f_1(a,b,c,t) \\ y &= f_2(a,b,c,t) \\ z &= f_3(a,b,c,t) \end{aligned}\right\} \qquad \text{... ... ... (4.1)}$$

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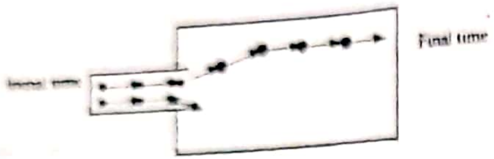
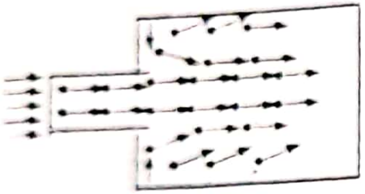


(a) Lagrangian Approach
(Study of each particle with time)



(b) Eulerian Approach
(Study at fixed stations in space)

Fig. 4.1 Lagrangian and Eulerian Method of Approach

These equations can be stated as '*position x of a fluid particle in some function of initial space co-ordinates a, b, c and the time t*'. The initial space coordinates, a, b, c and time t are called the Lagrangian variables.

The velocity and acceleration components of the fluid particle are obtained by taking derivatives with respect to time

The velocity compounds are

$$\left.\begin{aligned} u &= \frac{dx}{dt} \\ v &= \frac{dy}{dt} \\ w &= \frac{dz}{dt} \end{aligned}\right\} \qquad \ldots\ldots\ldots (4.2)$$

The acceleration components are

$$\left.\begin{aligned} a_x &= \frac{du}{dt} = \frac{d^2x}{dt^2} \\ a_y &= \frac{dv}{dt} = \frac{d^2y}{dt^2} \\ a_z &= \frac{dw}{dt} = \frac{d^2z}{dt^2} \end{aligned}\right\} \qquad \ldots\ldots\ldots (4.3)$$

Since the motion of one individual fluid particle in inadequate to describe the entire flow field, motion of all the fluid particles has to be considered simultaneously

Eulerian Method

In this method attention is focussed on the motion and properties of different fluid particles as they pass fixed points in space. The observer remains stationary and observes what happens at some particular point as demonstrated in Fig. 4.1(b). Let x, y and z be the space co-ordinates at time t. Then the components of velocity vector are functions of these space co-ordinates and time.

Symbolically

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

The velocity components for the flow field can also be represented as the rate of change of displacement:

$$u = \frac{dx}{dt}$$
$$v = \frac{dy}{dt}$$
$$w = \frac{dz}{dt}$$

Since the velocity of a fluid element or particle is a function of both position and time, the acceleration is given by the total rate of change of velocity. Thus for the x-component of acceleration a_x, we can write:

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

Applying these operations to the preceding expression for velocity components gives:

$$a_x = \frac{\partial u}{\partial t} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right)$$

Similarly the acceleration components along y and z axes are:

$$a_y = \frac{\partial v}{\partial t} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right) \qquad \ldots\ldots\ldots (4.4)$$

$$a_z = \frac{\partial w}{\partial t} + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right)$$

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The equation indicate that acceleration can result from two things:

- The terms $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$

represent *local acceleration*; the fluid particles are accelerated locally because of a change in flow with time at each point.

- The terms:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} \quad \text{and}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}$$

represent ***convective acceleration***; the fluid particles are accelerated by the convective act of moving from one position to another where velocity in different.

The total acceleration of the fluid particle is called the ***material*** or ***substantial acceleration***.

In general, the velocity is a function of space (s) and time (t), i.e.

$$v = f(x, y, z, t)$$

or $$v = f(s, t)$$

and the acceleration,

$$a = \frac{dv}{dt} = \frac{\partial v}{\partial s}\cdot\frac{ds}{dt} + \frac{\partial v}{\partial t}$$

$$a = v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \text{…………… (4.5)}$$

Here convective acceleration

$$= v\frac{\partial v}{\partial s} \qquad \text{…………… (4.6)}$$

and local acceleration

$$= \frac{\partial v}{\partial t} \qquad \text{…………… (4.7)}$$

In curvilinear motion equation (4.5) gives the '***tangential acceleration***'. A particle moving in a curved path will always have a normal acceleration (radial acceleration)

$$a_n = \frac{V^2}{r} \qquad \text{…………… (4.8)}$$

towards the center of the curved path, where r is the radius of the path, though its tangential acceleration (a_t) may be zero (as in the case of uniform circular motion).

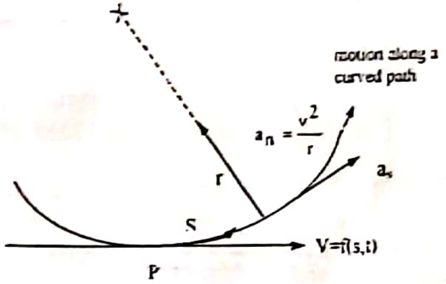

Fig. 4.2 Tangential and Normal Acceleration

In general, for motion along a curved path (Fig. 4.2)

$$a = a_t + a_n$$

or $$a = \left(v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}\right) + \frac{v^2}{r} \qquad \text{…………… (4.9)}$$

Comparison between Lagrangian and Eulerian Method

The distinction between the two approaches can be well understood with reference to the following analogy cited by *Prof. Salamo Eskinazi*:

"A study of the complete survey of automobile traffic in a large city can be accomplished by two methods: First, a complete description of the traffic can be obtained by placing an observer at every intersection and recording the speed and direction of the traffic, license numbers etc. as they cross each intersection. This information gained by concentrating at various points in the city constitutes the Eulerian method of analysis. The same traffic information can also be obtained if a motorist observer in assigned to each vehicle in the city. The observer records the speed and direction as a function of time, together with the license number of the car he is assigned to observe. This is the Lagrangian methods"

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The Lagrangian is rather cumbersome and complex: it begin an enormous task to keep track of the positions of all particles in a flow field because their relative position continuously change with time. The equations of motion are very difficult to solve and the motion is hard to understand. Further, the fluid analyst needs the flow properties at a particular point and is not interested in establishing the history or predicting the future of the fluid particle. The Eulerian approach in more practical and consequently the overwhelming majority of the fluid dynamic analysis are made on the Eulerian approach.

Lines of Flow

The following lines are considered suitable to describe the motion of the fluid =

(a) Stream Line

Stream line is said to be an imaginary line drawn through a flow field such that the tangent at each point on the line indicates the direction of the velocity of the fluid particle at that point (Fig. 4.3).

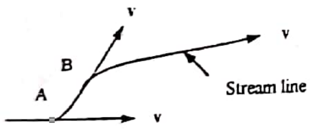

Fig. 4.3 Stream line and velocity of flow

Since at any point on a stream line the velocity in tangential to the streamline, so the component of velocity at right angles to the stream line in always zero. Thus there can be no flow occurring across any streamline. No two stream line can ever cross one another. If they cross, the fluid particle located at that point of intersection would have two velocities and this is inconsistent with the definition.

For drawing any stream line it is difficult to select just any group of fluid particles and consider a line tangent to the velocity vector, because in that case a number of particles may be left out for which no stream line can be traced which would not cross the first stream line or cross a solid boundary. The flow stream lines are interdependent and must be determined such that the entire field is satisfied. Due to this reason, the stream line in the vicinity of the solid surface conform to the outline of the boundary surface as shown in Fig. 4.3 and 4.5.

The series of stream lines will represent the flow pattern at that instant.

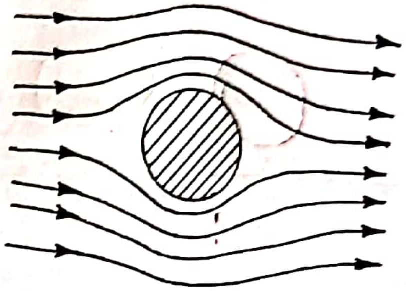

Fig. 4.4 Stream lines of Flow Past Round a Solid Object

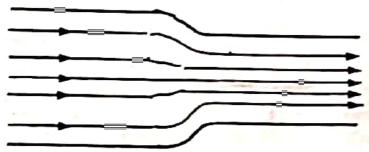

Fig. 4.5 Streamlines of Flow Within a Closed Conduit

Let during the time interval dt, a fluid particle travel a distance ds along the streamline. Further, let dx, dy, and dz be the components of the displacement ds into the co-ordinate axes. This implies that the fluid particle has traversed distance:

dx along the x-axis with velocity u
dy along the y-axis with velocity v
dz along the z-axis with velocity w

during the same time interval dt. The velocity components are then obviously given by:

$$u = \frac{dx}{dt}; \quad v = \frac{dy}{dt}; \quad w = \frac{dz}{dt}$$

Hence the equation of a general streamline is given by:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \qquad \dots \dots \dots \dots \dots \quad (4.10)$$

When cylindrical co-ordinates are used, then for the same streamline:

$$dr : r d\theta : dz \quad u : v : w \qquad \dots \dots \dots \dots \dots \quad (4.11)$$

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For a two-dimensional flow in the x-y plane,

$$\frac{dx}{u} = \frac{dy}{v}, \qquad \frac{dy}{dx} = \frac{v}{u} \qquad \text{(4.12)}$$

i.e., the slope of a plane stream line equals the ratio of velocity components

The important characteristics of streamlines are:

(i) Streamlines do not cross, otherwise the fluid particle will have two velocities at the point of intersection and that is physically impossible. Streamlines may, however, intersect at isolated points of zero velocity or infinite velocity.

(ii) There cannot any movement of fluid mass across the streamlines, i.e., the flow is only along the streamline and not across it. Evidently, a streamline is equivalent to a rigid boundary and the fluid lying between any two streamlines can be considered to be in isolation.

(iii) Streamline spacing varies inversely as the velocity; converging of streamlines in any particular direction shows accelerated flows in that direction. This point can be clearly understood by considering a portion of fluid flow between two streamlines AC and BD (Fig.4.6). Since no fluid can penetrate the streamlines, the flow passing through each of the sections AB and CD would be same. Taking depth of stream to be constant in a direction at right angles to the plane of the paper, the cross section area at section CD is greater than that at section AB. Then in accordance with continuity equation (area × velocity = const.), the flow velocity at section AB is greater than that at section CD.

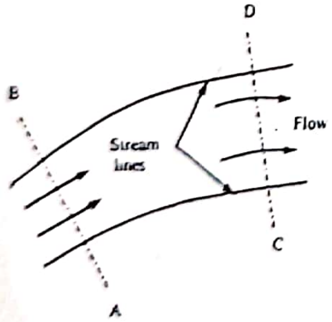


Fig. 4.6 Stream Line Spacing and the Velocity

(b) Stream surface

A stream surface is generated by a large number of closely spaced stream lines which pass through an arbitrary curve. A stream surface can be plane, asymmetric or general spatial

(c) Stream tube

In a steady field of flow the stream tube may be defined a tubular space formed by the collection of stream lines passing through the perimeter of a closed curve (Fig. 4.7).

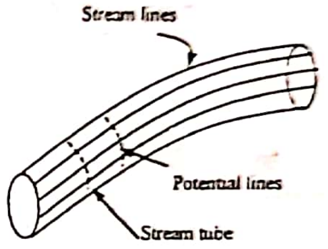


Fig. 4.7. Stream Tube Showing Streamlines and Potential lines

As it is bounded on all sides by stream lines and the velocity normal a stream line in zero, hence no fluid can enter or leave the stream tube except at the ends. Therefore stream tube behaves as if it were a solid tube.

From the Law of conservation of matter, the quality of fluid entering at one end of the stream tube and leaving at the other must be same, provided there is no sink or source in the tube. The general equation of continuity will then apply to the case of stream tube even though it has no solid boundaries.

The above-mentioned behavior of stream tube has practical significance. In complicated flow problems the whole field of flow can be divided up into a large number of stream tubes and thus a clear picture of actual pattern of flow is obtained. Such a technique is referred to as *flow net*.

Moreover in special case of unsteady flow in which the velocity direction at a point remain constant but the magnitude changes with time, the stream tube remains fixed.

In simple flow problems such as fluid flow in conduits, the solid boundaries may serve as the periphery of a stream tube since they satisfy the conditions of having no flow crossing the wall of the tube.

In general, the cross-sectional area may vary along a stream tube. Only is the steady flow field with uniform velocity will all stream lines be straight and parallel.

The contents of a stream tube are known as *'current filament.'*

(d) Path Lines

A path line is the line traced by a single particle during successive instants of time and shows the directions of velocity (of the same fluid particle) at successive instants of time. A path line can intersect itself, and the path lines may be zigzag. The path traced by a single particle of smoke issuing from a cigarette is a path line.

(e) Streak Line or Filament Line

It is an instantaneous picture of the positions of all particles in the flow which have passed through a given point. For example the line formed by smoke particles ejected into the flow from a fixed nozzle is a streamline. In an experimental work (to trace the motion of fluid particles) a coloured dye may be injected into the flowing fluid and the resulting coloured filament lines at a given location give the streak lines.

Potential Lines

The lines of equal velocity potential are known as potential lines. These are orthogonal to the stream lines.

Different Types of Displacement of Fluid Particles

Any fluid element can be translated, rotated or distorted during its course of motion. Correspondingly, the displacements of the elements are called (i) translation (ii) rotation and (iii) distortion or deformation.

Translation

In this case the elements moves bodily in some direction. For example, in flow through pipes of constant diameter or through open channel of constant width and depth, any element of fluid is simply moved from its original position (Fig. 4.8). Therefore a pure translation does not cause any stress in the element as the element can be brought to rest by change of co-ordinate system.

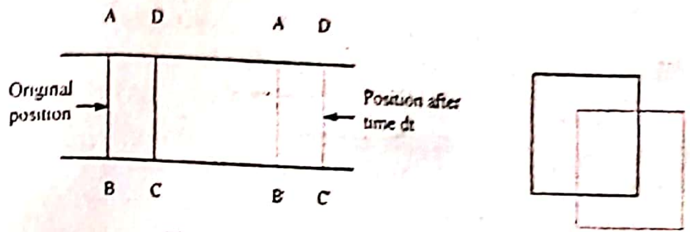


Fig. 4.8 Pure Translation of Fluid Element

Rotation

Figure 4.9 shows the case of pure rotation. Here rotation of AB and AD both are in the same direction i.e. anticlockwise. In pure rotation also, no stress in caused in the fluid element, because there is no relative motion with respect to the rotating system.

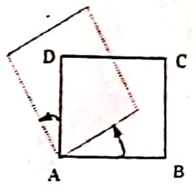


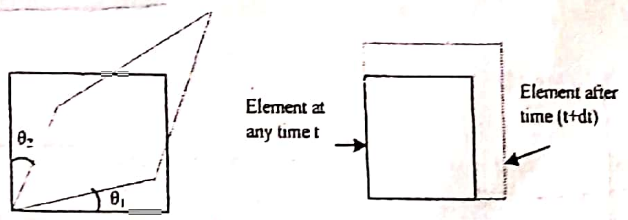
Fig. 4.9 Pure Rotation of Fluid Element

Distortion and Deformation

The distortion of a fluid element is of two types -

(a) Angular distortion (refer Fig. 4.10a)

(b) Volume or linear distortion (refer Fig. 4.10b)



(a) Angular Distortion (b) Linear Deformation (Dilatation)

Fig. 4.10 Distortion and Deformation of Fluid Element

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In the angular distortion of a fluid element, the volume remaining the same as in the case of an incompressible fluid. Shear stress is induced causing shear strain resulting in the change of angle between two adjacent sides of the element (Fig. 4.10a).

A fluid particle has net zero rotation in a plane if the average of angular velocities of two mutually perpendicular linear elements of the particle in that plane is zero, i.e. if one line rotates in an anticlockwise direction at the same rate as the other rotates in a clockwise direction, the particle is distorting but not rotating.

In linear distortion, Fig 4.10(b), the volume changes, there is no shear strain, there is only linear strain (dilatation).

In an actual flow problem, the fluid element may undergo all the three types of displacement simultaneously i.e. translation, rotation and deformation.

General Types of Fluid Flow

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The fluid in motion may be of following types

1) Steady or unsteady
2) Uniform or non-uniform
3) Laminar or turbulent
4) Compressible or incompressible
5) Pressure flow or Pressure less flow
6) Ideal or real flow
7) Rotational or irrotational
8) One, two or three dimensional
9) Sub-critical, critical or super-critical
10) Sub-sonic, sonic, supersonic or hypersonic flow

All above types of flow can exist independently of each other, making it possible to have various combinations, e.g., a flow may be unsteady, non-uniform and turbulent etc.

1. Steady and Unsteady Flow

Steady Flow

Motion of a fluid is said to be steady when the particle which succeed each other through a specific point, during a period of time, have the same flow parameters such as velocity, density, viscosity, pressure, surface tension and temperature.

If all these parameters are denoted by 'P' then mathematically it must be expressed as

$$\frac{\partial P}{\partial t} = 0 \qquad \text{for steady flow} \qquad \text{(4.13)}$$

where P is any one of the above-mentioned parameters. This equation denotes that these parameters are independent of time at any particular position.

In many problems, when velocity remains constant with time, the other parameters are also found to remain constant with time. Therefore steady flow is also mathematically expressed in terms of velocity only, i.e.

$$\frac{\partial V}{\partial t} = 0 \qquad \text{for steady flow} \qquad \ldots\ldots \text{(4.14)}$$

Examples of a steady flow are

- Liquid efflux from a vessel in which constant level is maintained.
- Flow of water in a pipeline due to a centrifugal pump being run at uniform rotational speed.

Unsteady Flow

If the fluid or flow parameters such as velocity and pressure are dependent not only on the position in the coordinate system used to describe the field of flow, but also on time, the flow is termed as *unsteady*. The path lines of successive particles will be different and thus the stream line pattern of flow will be changing every instant.

Mathematically –

$$\frac{\partial P}{\partial t} \neq 0 \qquad \text{for unsteady flow} \qquad \ldots\ldots \text{(4.15)}$$

or in terms of velocity

$$\frac{\partial V}{\partial t} \neq 0 \qquad \text{for unsteady flow} \qquad \ldots\ldots \text{(4.16)}$$

Some familiar examples of unsteady flow are:

- Liquid falling under gravity out of an opening in the bottom of a vessel.
- Liquid flow in the suction and pressure pipes of a reciprocating pump.
- Wave motion and the cyclic movement of large bodies of water in tidal flow.

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2. Uniform and Non-Uniform Flow

A uniform flow is one in which the fluid or flow parameters at any given instant remain same at every point in space.

Conversely if the fluid or flow parameter at any instant change with distance, the flow is called non-uniform flow.

Symbolically

$$\frac{\partial P}{\partial S} = 0 \qquad \text{for uniform flow} \qquad \dots\dots\dots\dots\dots (4.17)$$

$$\frac{\partial P}{\partial S} \neq 0 \qquad \text{for non-uniform flow} \qquad \dots\dots\dots\dots\dots (4.18)$$

In terms of velocity, these are expressed as

$$\frac{\partial V}{\partial S} = 0 \qquad \text{for uniform flow} \qquad \dots\dots\dots\dots\dots (4.19)$$

$$\frac{\partial V}{\partial S} \neq 0 \qquad \text{for non-uniform flow} \qquad \dots\dots\dots\dots\dots (4.20)$$

where ∂S is the displacement in any direction.

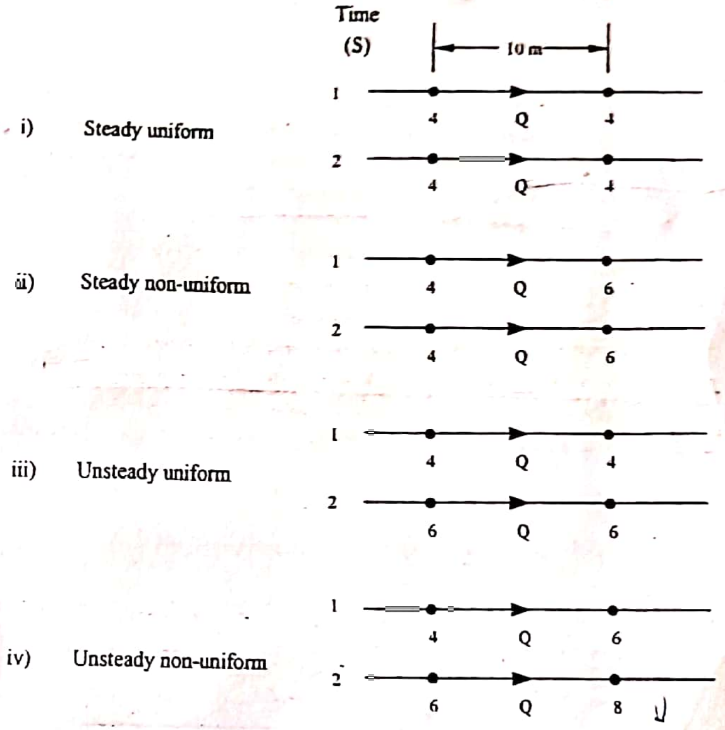
The terms uniform and non-uniform flow are more used is open channels. In case of uniform flow the size and shape of the cross section in a particular length remain constant.

Steadiness refers to no change with time and uniformity refers to no change in space. Consequently the steadiness and uniformity of flow can coexist independently and the following possible combinations can be though of:

- Steady uniform, e.g., the flow at constant rate in a pipeline of constant cross-sectional area
- Unsteady uniform flow, e.g. the flow at an increasing or decreasing rate through a pipe lien of constant cross-sectional area
- Steady non-uniform flow, e.g. flow at constant rate through a converging or diverging pipe.

- Unsteady and non-uniform flow, e.g., flow at an increasing or decreasing rate through a converging or diverging pipe.

The above four flow classifications may be shown by the following sketches:



Thus in steady uniform flow there is no acceleration. In steady non-uniform flow due to convergence or divergence of stream tube, there is only convective acceleration. In unsteady non-uniform flows both convective and local accelerations are caused.

For steady flow, V = f (x,y,z) and local acceleration in zero. i.e. the term $\partial v/\partial t$ in equations (4.5) and (4.9) and the terms

$$\frac{\partial u}{\partial t}, \frac{\partial u}{\partial t}, \frac{\partial w}{\partial t}$$

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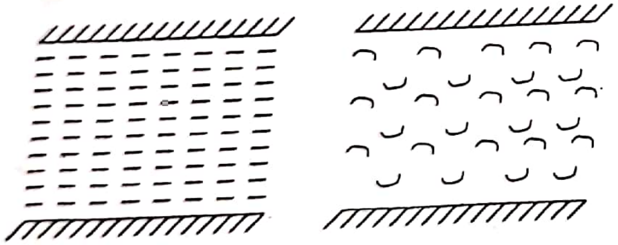
in equation (4.4) are zero. For uniform flow, the convective acceleration is zero, i.e. the terms in parentheses in equation (4.4) and the term $v\frac{\partial v}{\partial s}$ in equations (4.5) and (4.9) are zero.

3. Laminar and Turbulent Flow

A laminar flow in characterized by a smooth flow of one lamina of fluid over another. Fluid elements move in well-defined paths and they retain the same relative position at successive cross-sections of the flow passage (Fig. 4.11a). As the particles move in definite and observable paths or streamlines, this flow is sometimes known as *streamline flow*. It is also called the *viscous flow*. This type of flow occurs generally in smooth pipes when the velocity of flow is low, and also in liquids having a high viscosity.

In turbulent flow, the fluid elements move in erratic and unpredictable paths. Individual fluid particles are subject to fluctuating transverse velocities so that the motion is eddying and sinuous rather than rectilinear. The random edding motion is called turbulence. Fig 4.11(b) shows the erratic path followed by a single particle during an interval of time. It represents the irregular motion of a large number of fluid particles at any instant. Turbulent flow generally prevails in rivers, canals, and in the atmosphere. The discharge of smoke into the atmosphere from a large stack is another commonly observed example of turbulent flow.



(a) Laminar **(b) Turbulent**

Fig. 4.11 Laminar and Turbulent Flows

4. Compressible and Incompressible Flow

Flow in incompressible if the density changes, due to pressure and temperature variations, are insignificant in the flow field. For all practical purposes, liquids can be regarded as *incompressible*. This means that pressure and temperature changes have little effect on their volume. Exceptions occur only when the liquid water is subjected to severe accelerations such as in water hammer that causes compression waves.

When the density changes are appreciable, the flow is called *compressible*. The gases are readily compressible fluids. They expand infinitely in the absence of pressure and contract easily under pressure. Nevertheless when the density variation is small, e.g., flow is a ventilating system even the gas flow can be treated incompressible.

5. Pressure Flow and Pressure-less Flow

In *pressure-flows* the fluid motion is bounded by solid walls on all the sides and free surface of the liquid does not exist, i.e., the liquid surface is not exposed to atmospheric or any other constant pressure. A pipe flow is a pressure flow; the fluid flow is mainly due to pressure gradient existing along the pipe length.

In pressure-less flows, also called the *gravity flows*, the fluid motion is bounded on three sides, and the fourth side is exposed to the atmosphere. Here the fluid motion is due to its own weight. The flow in open channels, rivers and partially filled pipelines is a *pressure-less* flow.

6. Ideal and Real Flow

In the ideal friction-less flows, no shear stress in presumed to exist between adjacent fluid layers, and between the fluid layers and the boundary. This assumption is valid only when the fluid is non-viscous (inviscid μ=0) or when the velocity gradient normal to the direction of flow is zero. Only normal stresses can exist in ideal flows.

Fluids are generally viscous, and as such the shear stresses came into existence when the fluid particles are in motion. These stresses oppose the sliding of one layer over another. Real flow situations are characterized by the frictional resistance to fluid motion; this resistance is due to the viscosity of real fluids.

7. Irrotational and Rotational Flow

For irrolational flow there is no net rotation of a fluid element as it moves from point to point along a streamline, and hence each fluid element has zero angular velocity about its mass center; only distortion (rather than rotation) of the fluid element occurs.

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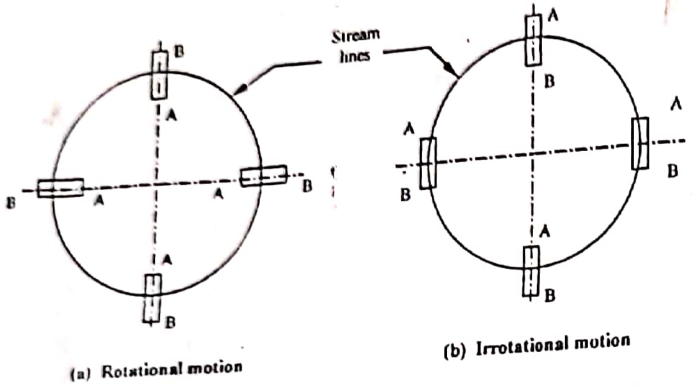

(a) Rotational motion

(b) Irrotational motion

Fig. 4.12 Rotational and Irrotational Fluid Flow

The fluid particle AB (Fig. 4.12b) rotates about its own axis when moving along a circular streamline, and constitutes the rotational flow. In Fig. 12b, the same fluid particle does not rotate about its own axis as it moves along the circular stream line and evidently the flow is irrotational.

A free vortex or whirlpool which develops above a drain in the bottom of a stationary tank is an example of irrotational flow; the velocity varies inversely with the distance from the centre; a small object floating on the surface can be seen to move in a circular path but not to rotate relative to the tank. Thus a free vertex is an irrotational vortex in which the velocity varies inversely as the radial distance from the center of the vortex, i.e. for a

Free vortex: $v \propto \frac{1}{r}$

or $V_r = \text{constant}$

A liquid in a rotating tank is an example of rotational flow; here the velocity varies directly with the distance from the center, and an object floating on the surface can-be seen to rotate in the same manner as the tank itself. Thus a force vortex is a rotational vertex in which the velocity increases directly as the radial distance from the center of vortex, i.e. for a

force vortex: $V \propto r$

or, $\frac{V}{r} = \text{constant}$

A steady irrotational flow is called *potential flow.*

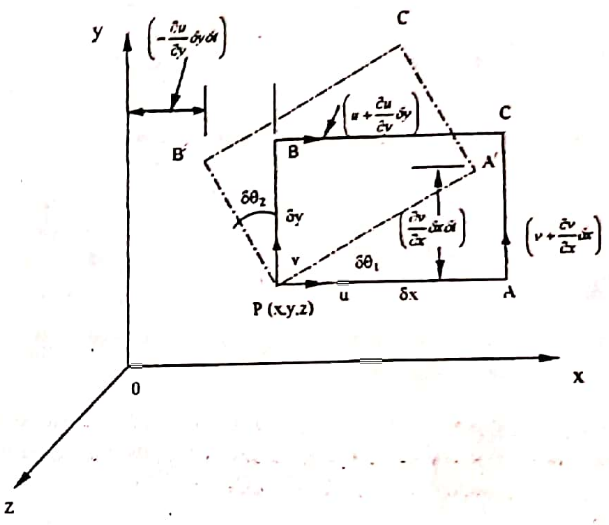

Fig. 4.13 Rotation of Rectangular Fluid Element about z Axis

The rotation of fluid element may be defined in terms of the components of rotation about three mutually perpendicular axes. Referring to Fig. 4.13 the mathematical expression for the rotation components about an axis parallel to z-axis is developed.

Let a fluid element at any point P (x,y,z) has the velocity components u and v in the x and y directions respectively. Consider any two line segments PA and PB of lengths δx and δy, taken parallel to the x and y axes respectively for the sake of convenience. The velocity at A in the y direction will be $\left(v+\frac{\partial v}{\partial x}\delta x\right)$ and the velocity at B in the x direction will be $\left(u+\frac{\partial u}{\partial u}\delta y\right)$. since the velocities at P and A in the y direction are different, there will be an angular velocity developed for the linear element PA. Similarly the velocities at P and B in the x direction are different and hence these will be an angular velocity developed for the linear element PB.

Now if during a time interval of δt the elements PA and PB which were initially perpendicular to each other, have rotated in x-y plane, by small angles $\delta\theta_1$ and $\delta\theta_2$

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respectively, have moved, relative to P to new position PA′ and PB′ as indicated by the dotted lines, then the angular velocity (ω_{PA}) of element PA about Z axis is

$$\omega_{PA} = \lim_{\delta t \to 0} \frac{\delta\theta_1}{\delta t} = \lim_{\delta t \to 0} \frac{\left[\left(v + \frac{\partial v}{\partial x}\delta x\right) - v\right]\delta t}{\delta x \delta t} = \frac{\partial v}{\partial x} \text{rad / sec.}$$

Similarly, the angular velocity (ω_{PB}) of element PB about Z axis is

$$\omega_{PB} = \lim_{\delta t \to 0} \frac{\delta\theta_2}{\delta t} = \lim_{\delta t \to 0} \frac{-\left[\left(u + \frac{\partial u}{\partial y}\delta y\right) - u\right]\delta t}{\delta y \delta t} = -\frac{\partial u}{\partial y} \text{rad/sec.}$$

The negative sign has been introduced because the motion in the anticlockwise direction has been considered as positive.

The rotation component about any axis may be defined as the average angular velocity of any two infinitesimal linear elements in the particle that are perpendicular to each other and to the axis of rotation (in this case it is Z axis).

Thus by the above definition the rotation component ω_Z of a particle situated at point P is

$$\omega_z = \frac{1}{2}(\omega_{PA} + \omega_{PB}) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \text{— — — — — (4.21a)}$$

By adopting the same procedure the rotation, components about the axes parallel to the x and y axes will be obtained as

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \text{... ... ... (4.21b)}$$

and $$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \text{... ... ... (4.21c)}$$

If at every point in the flowing fluid the rotation components ω_x, ω_y and ω_z are equal to zero, then the flow is know as irrotational flow. Thus for a flow to be irrotational the following conditions must be satisfied throughout the flow field:

For $\omega_x = 0$; $$\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \qquad \text{— — — — — (4.22a)}$$

For $\omega_y = 0$. $$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \qquad \text{— — — — — (4.22b)}$$

For $\omega_z = 0$; $$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad \text{— — — — — (4.21c)}$$

The rotation of a fluid particle is always associated with shear stress, because the rotation can be caused only by a torque exerted on the fluid particle and this will be produced by the shear forces. As such in the case of flow of fluids having larger viscosity or in the regions of flow field where the viscosity of the fluid has greater predominance the flow is invariably rotational flow. However, in the case of fluids such as air or water having small viscosity the flow in the region away from the boundary may for all practical purposes be treated as irrotational. Moreover, in the case of rapidly converging or accelerating flows the flow may be treated as irrotational. The consideration of an irrotational flow in general leads to a simplified analysis of fluid flow problems.

Circulation, Vorticity and Rotation

The flow along a closed curve is called circulation (i.e. the flow in eddies and vortices). The mathematical concept of circulation is the line integral, taken completely around a closed curve, of the tangential component of the velocity vector. Consider a closed curve C as shown in Fig. 4.14a, and let at any point on the curve the velocity of flow of fluid be V. If θ in the angle between a small element ds along the curve in the tangential direction and the velocity V, then the component of the velocity in the direction tangential to the curve in V cosθ. By the definition the circulation Γ (Greek, capital, 'gamma') around a closed curve C is

$$\Gamma = \int_c V \cos\theta \, ds \qquad \text{... ... ... (4.23)}$$

Further if u, v and w are the components of velocity v and dx, dy and dz are the components of the displacement ds, then the circulation can also be written as

$$\Gamma = \int_c (u dx + v dy + w dz) \qquad \text{... ... ... (4.23a)}$$

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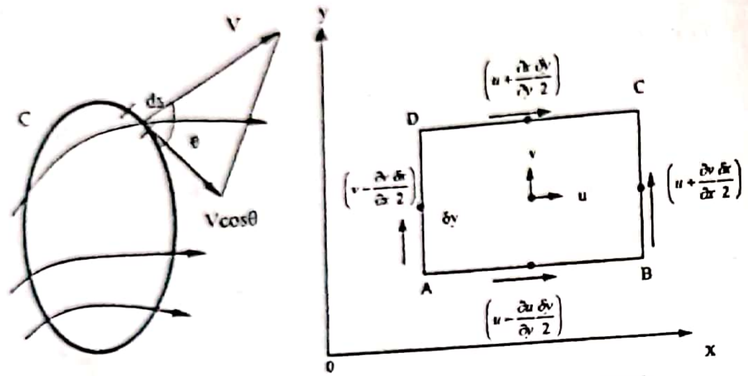

(a) Around a closed curve

b) Around an elementary rectangle

Fig. 4.14 Circulation in the Plane of a Two-dimensional Steady Flow Field

Thus the circulation around a circular cylinder is equal to the peripheral velocity of fluid around the cylinder multiplied by cylinder circumference i.e.

$$P = 2\pi r V_\theta$$

where r in the radius of the cylinder.

The circulation around an elementary rectangle with sides parallel to the axes x and y as shown in Fig 14(b) may be written as follows:

$$\text{Circulation along AB} = \left(u-\frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x$$

$$\text{Circulation along BC} = \left(v+\frac{\partial v}{\partial x}\frac{\delta x}{2}\right)\delta y$$

$$\text{Circulation along CD} = -\left(u+\frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x$$

$$\text{Circulation along DA} = -\left(v-\frac{\partial v}{\partial x}\frac{\delta x}{2}\right)\delta y$$

The positive sense of integration is such that the enclosed surface is on left when d from the side of the outward normal

Further whatever be the shape of the curve, the circulation around the periphery of the curve, must equal the sum of the circulation around the elementary surfaces of which it consists, provided the boundary of the curve in wholly in the fluid. Thus in this case the total circulation will be given by

$$\Gamma = \Gamma_{AB} + \Gamma_{BC} + \Gamma_{CD} + \Gamma_{DA}$$

$$= \left(u-\frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x + \left(v+\frac{\partial v}{\partial x}\frac{\delta x}{2}\right)\delta y - \left(u+\frac{\partial u}{\partial y}\frac{\delta y}{2}\right)\delta x - \left(v-\frac{\partial v}{\partial x}\frac{\delta x}{2}\right)\delta y = \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\delta x\delta y \quad \text{--- --- --- --- ---} \ (4.24)$$

The vorticity at any point is defined as the ratio of the circulation around an infinitesimal closed curve at that point to the area of the curve, i.e., it is defined as circulation per unit area. Thus from equation (4.24), the vorticity Ω (Greek, capital, "omega") may be expressed as

$$\Omega = \frac{\text{Circualtion}}{\text{Area}}$$

$$= \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \quad \text{--- --- --- --- ---} \ (4.25)$$

By comparing equation (4.25) and (4.21a), we get

$$\Omega = 2\omega_z \quad \text{--- --- --- --- ---} \ (4.26)$$

that is vorticity (or circulation per unit area) is equal to twice the rotation component about an axis perpendicular to the plane in which the area is lying. Vorticity is a vector quantity whose direction is perpendicular to the plane of the small curve round which the circulation in measured. Thus in a general case of three dimensional flow, equation (4.26) represents only a component of vorticity in the Z direction, i.e., $\Omega_z = 2\omega_z$. Similarly the other two components of vorticity may also be obtained as $\Omega_x = 2\omega_x$ and $\Omega_y = 2\omega_y$.

The three components of vorticity, therefore are

$$\Omega x = 2\omega_x = \left(\frac{\partial w}{\partial y}-\frac{\partial v}{\partial z}\right) \quad \text{... ... ...} \ (4.27a)$$

$$\Omega_y = 2\omega_y = \left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) \quad \text{... ... ...} \ (4.27b)$$

$$\Omega_z = 2\omega_z = \left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right) \quad \text{... ... ...} \ (4.27c)$$

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For irrotational flow the net rotation in zero i.e. $\omega = 0$. This implies that,

$$\omega_x = \omega_y = \omega_z = 0 \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ (4.28)$$

$$\text{or } \Omega_x = \Omega_y = \Omega_z = 0 \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ (4.29)$$

which gives the conditions of irrotationality of flow as

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ (4.30a)$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ (4.30b)$$

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \qquad \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ (4.30c)$$

If the vorticity is zero at all points in a region then the flow in that region is said to be irrotational. On the other hand flow in regions where the vorticity is other than zero in said to be rotational.

The above results have been derived by considering a rectangular curve. But it may be stated that the results obtained in equations (4.24) and (4.26) are independent of the shape of the closed curve considered, and the rectangular curve has been chosen only for the sake of simplicity.

Free vortex and Forced Vortex

The vortex motion is of two types: Free vortex and forced vortex.

Free Vortex

Free Vortex is that in which the fluid mass rotates without any external impressed contact force. The whole fluid mass rotates either due to fluid pressure itself or gravity or due to rotation previously impaired. energy is not expended to any outside source.

The free vortex motion is also called potential vortex or irrotational vortex.

Examples: Common examples of free vortex are:

- Whirlpool in a river
- Drainage of liquid through an opening at the bottom of a container.
- Flow of liquid in a centrifugal pump casing after it has left the impeller.
- Flow of water in a turbine casing before it enters the guide vanes.
- Flow around a circular bend.

(b) **Forced Vortex** is one in which the fluid mass in made to rotate by means of some external agency. The external agency is generally the mechanical power which imparts a constant torque on the fluid mass. The torque rotates the fluid mass either by stirring the liquid contained in the vessel or by spinning the vessel containing the liquid, about a vertical axis; consequently the whole fluid mass revolves with constant angular velocity ω. Thus in forced vortex, there is always expenditure of energy.

The forced vortex motion is also called *flywheel vortex* or *rotational vortex.*

Examples: common examples of forced vortex are:

- Rotation of liquid inside the impeller of a centrifugal pump and inside the runner of a hydraulic turbine.
- Flow through central core of a mixer.

Free or forced vortex in further characterized as cylindrical free or forced vortex, or spiral free or forced vortex.

Cylindrical Vortex

In cylindrical free or forced vortex, the fluid mass rotates in concentric circles. Hence it in also known as circulatory flow.

Spiral Vortex

when cylindrical vortex is superimposed over the radial flow described above, the resulting vortex is known as spiral vortex.

Example: Motion of liquid in spiral casing (i.e. volute) of a centrifugal pump.

8. One, Two and Three-Dimensional Flows

In a one dimensional flow, the fluid parameters (velocity, pressure, temperature and thus density and viscosity) remain constant throughout any cross-section normal to flow direction. Reference to velocity as the flow parameter, its variation across any section is insignificant and its variation occurs only along the flow direction. The flow field is represented by streamlines which are essentially straight and parallel.

A change in the flow parameters may, however, occur from section to section. A change in the cross-section of the flow passage, is however, required to be gradual. Examples of one-dimensional flow have been depicted in Fig 4.15.

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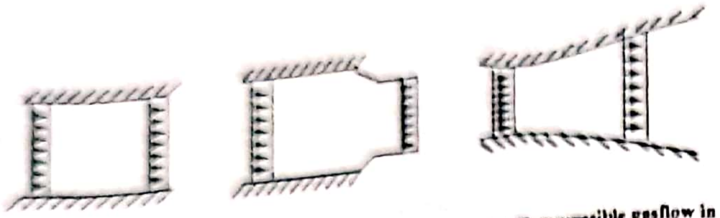

(a) Flow between parallel plates

(b) Incompressible flow in a duct

(c) Compressible gasflow in a nozzle.

Fig. 4.15 Illustrations of One-dimensional Flow

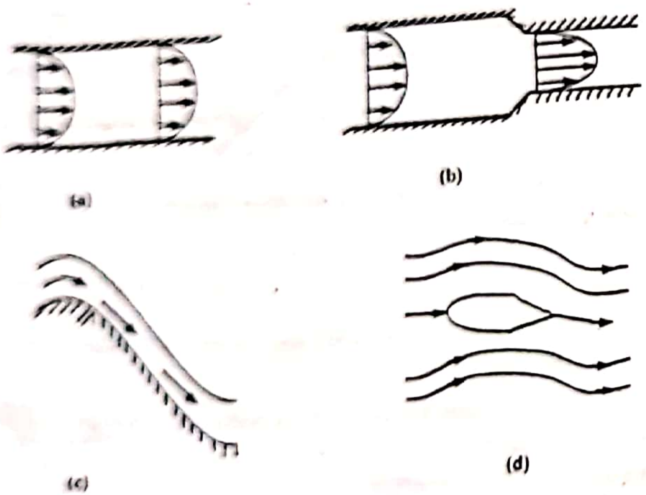

Fig. 4.16 Illustrations of Two-dimensional Flow

(a) viscous flow between parallel plates

(b) viscous flow between converging plates

(c) flow over the central part of a spillway

(d) flow at the middle portion of an airplane wing.

Fig. 4.17 Illustration of a Three-dimensional Flow

In a two-dimensional flow, the flow velocity and other fluid parameters vary along two-directions. This is because of the actual real flow conditions wherein viscous effects creep in which make the velocity change from zero at the boundary to a maximum value in the flow field. Streamlines are plane curves, are identical in parallel planes, and there is no variation of flow properties on a surface. Examples of two-dimensional flow have been illustrated in Fig. 4.16.

A three-dimensional flow stipulates that the flow properties vary in all the three directions, the streamlines are space curves (Fig. 4.17). Examples of three-dimensional flows are: flow in a river, flow within fluid machines, flow at a inlet to a nozzle.

A fluid flow with symmetrical velocity profile: about the axis is called an asymmetric flow. Such a flow in essentially a two-dimensional flow because in cylindrical co-ordinates the velocity gradients exist only in the axial and radial directions; variables do not change in the circumferential direction.

Dimensionality of a flow field for steady/unsteady flows can be mathematically prescribed as follows:

	Steady	Unsteady
One dimensional flow	$v = f(x)$	$v = f(x,t)$
Two-dimensional flow	$v = f(x,y)$	$v = f(x,y,t)$
Three-dimensional flow	$v = f(x,y,z)$	$v = f(x,y,z,t)$

For simplicity, much of the science of fluid mechanics is dependent upon the one-dimensional method of analysis.

9. Sub critical, Critical and Supercritical Flow

The ratio of the inertial forces to the gravitational forces (per unit volume) is known as the Froude Number and this may be written as

$$F = \frac{U}{\sqrt{g D}}$$

where,

U = characteristic velocity (generally taken as the average velocity over the cross-section of an open channel).

g = gravitational acceleration

D = hydraulic depth, defined as the ratio of the area of cross-section of flow to the width of the water surface.

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When $F = 1$, then the flow is said to be critical.

$F < 1.0$, then the flow is said to be subcritical or tranquil flow.

$F > 1.0$, then the flow is said to be supercritical flow, rapid flow or shooting flow

10. Subsonic, Sonic, Supersonic and Hypersonic Flow

Mach Number

Mach Number is defined as the ratio of the velocity at a point in the fluid to the velocity of sound at the point at a given instant of line. The Mach number is an important parameter in dealing with the flow of compressible fluids when elastic force becomes important and predominant. Mach number prescribes the following flow regimes:

Subsonic flow : $M < 1$
Sonic flow : $M = 1$
Supersonic flow : $M > 1$
Hypersonic flow : $M > 5$

where M is the Mach Number.

Continuity Equation

The continuity equation is actually mathematical statement of the principle of conservation of mass. A most general expression on the basis of the principle may be obtained by considering a fixed region within a flowing fluid as shown in Fig. 4.18. Since fluid is neither created nor destroyed within this region it may be stated that the rate of increase of the fluid mass contained within the region must be equal to the difference between the rate at which the fluid mass enters the region and the rate at which the fluid mass leaves the region. However if the flow is steady, the rate of increase of the fluid mass within the region is equal to zero, then the rate at which the fluid mass enters the region is equal to the rate at which the fluid mass leaves the region. This relation is used to derive a general equation of continuity for a three-dimensional steady or unsteady flow.

Fig. 4.18 **Diagrammetric Representation of the Principle of Conservation of Mass**

Continuity Equation For One-Dimensional Steady Flow

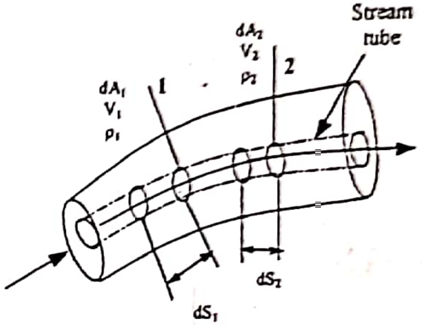

Fig. 4.19 Flow Through Closed Conduit - Derivation of Equation of Continuity

Consider the flow of fluid between sections (1) and (2) of a stream tube (Fig. 4.19). Let the cross-sectional areas across flow be A_1 and A_2 respectively. Let the velocities be V_1 and V_2 at these sections. Further let dS_1 and dS_2 be the distance traversed by fluid patients at (1) and (2) respectively in a differential time dt.

Now there are possibilities that during the differential time dt either the fluid mass flowing into will stay between sections (1) and (2) or it may flow out . By the principle of conservation of mass -

$$\left(\begin{array}{l}\text{Rate of inflow of}\\\text{fluid mass}\\\text{between section}\\\text{(1) and (2)}\end{array}\right) = \left(\begin{array}{l}\text{Rate of outflow of}\\\text{fluid mass through}\\\text{section (2)}\end{array}\right) + \left(\begin{array}{l}\text{Rate at which}\\\text{fluid mass is}\\\text{stored within the}\\\text{section (1) and (2)}\end{array}\right)$$

But in steady flow, the rate of fluid mass flowing into any control volume and that flowing out must be the same at all times so the rate at which the fluid mass is stored in zero. If does not matter whether the flow is uniform or non-uniform. Therefore in equation of continuity for steady flow becomes -

$$\left(\begin{array}{l}\text{Rate of inflow of}\\\text{fluid mass}\\\text{between sections}\\\text{(1) and (2)}\end{array}\right) = \left(\begin{array}{l}\text{Rate of outflow of}\\\text{fluid mass through}\\\text{section (2)}\end{array}\right)$$

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atically it can be written as

$$\frac{dM_1}{dt} = \frac{dM_2}{dt} \qquad \text{... ... ... (4.31)}$$

where $\frac{dM_1}{dt}$ and $\frac{dM_2}{dt}$ are the rates of fluid mass flowing into and out of sections (1) and (2) respectively

and dM = Mass = Density × volume

$$= \rho . A . dS$$

$$\therefore \quad \frac{dM}{dt} = \rho . A . \frac{dS}{dt} = \rho AV$$

where ρ, A and V denote the density of flowing fluid at that particular section, cross-sectional area and local velocity of flow across dA in the stream tube.

Using subscripts -

$$\frac{dM_1}{dt} = \rho_1 . A_1 V_1$$

$$\frac{dM_2}{dt} = \rho_2 . A_2 V_2$$

and

From Law of conservation of mass -

$$\frac{dM_1}{dt} = \frac{dM_2}{dt}$$

Hence $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ = mass rate of flow ... ... (4.32)

Continuity Equation ~~from~~ for One Dimensional Unsteady Flow:

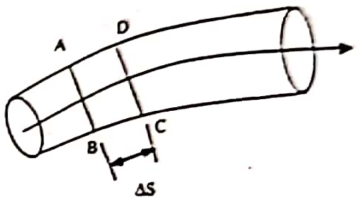

Fig. 4. 20 One-Dimensional Unsteady Flow Through Stream Tube

Consider the flow of fluid in a small length ΔS of a stream tube (refer Fig. 4.20). Let ρ, A and V be the mass density, cross-sectional area and the velocity at the center of fluid element. It is assumed that the flow does not occur inwards or outwards through its sides but only across its ends which are ΔS apart. ρ, A, and V are all function of ΔS.

Now mass of fluid entering the control volume through the face AB in small time Δt is given by

$$m_{is} = \left[\rho AV - \frac{\partial}{\partial S}(\rho AV)\frac{\Delta S}{2}\right]\Delta t$$

Mass of fluid leaving the control volume through the face DC in time Δt,

$$m_{os} = \left[\rho AV + \frac{\partial}{\partial S}(\rho AV)\frac{\Delta S}{2}\right]\Delta t$$

∴ Increase in mass of control volume in time Δt,

$$\Delta m = m_{is} - m_{os}$$

or $$\Delta m = \left[\rho AV - \frac{\partial}{\partial S}(\rho AV)\frac{\Delta S}{2}\right]\Delta t - \left[\rho AV + \frac{\partial}{\partial S}(\rho AV)\frac{\Delta S}{2}\right]\Delta t = -\frac{\partial}{\partial S}(\rho AV)\Delta S . \Delta t$$

But increase in mass of control volume in time Δt is equal to $\frac{\partial}{\partial t}(\rho \, A \, \Delta S)\,\Delta t$

or $$\frac{\partial}{\partial t}(\rho A \Delta S)\Delta t = -\frac{\partial}{\partial S}(\rho AV)\Delta S . \Delta t$$

$$\therefore \quad \frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial S}(\rho AV) = 0 \qquad \text{... ... ... (4.33)}$$

This is the *equation of continuity for one dimensional unsteady flow in differential form.*

If the flow is steady, $\frac{\partial}{\partial t}(\rho A) = 0$

Hence the equation of continuity for steady flow becomes -

$$\frac{\partial}{\partial s}(\rho A V) = 0 \qquad \dots \dots \dots \dots \dots (4.34)$$

or $$\rho A V = \text{constant} \qquad \dots \dots \dots \dots \dots (4.35)$$

i.e. $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ = mass rate of flow

where subscripts 1 and 2 denote sections 1 and 2 along the stream tube. Equation (4.35) is the same as derived in the previous article.

If the fluid is incompressible (e.g. water is taken as incompressible), then $\rho_1 = \rho_2$ and the equation of continuity reduces to

$$A_1 V_1 = A_2 V_2 = Q = \text{constant} \qquad \dots \dots \dots \dots \dots (4.36)$$

where Q is the volume rate of flow.

Equation (4.36) implies that for steady incompressible flow the volume rate of flow (or discharge) will remain the same at all sections of a tube provided fluid in neither injected to nor taken out of it.

Applicability of Equation of Continuity

It is important to note that in cases where the velocity varies over the entire cross-sectional area, the velocity in the continuity equation represents average velocity of flow across the section. The average velocity is derived in the following manner.

Let a pipe consist of a number of infinitely small stream tubes each having a cross-sectional area dA. Let the velocity of flow through such a stream tube be constant and equal to V. Now if A is the cross sectional area of the pipe and V_{mean} the mean velocity of fluid passing over a section, then the mean velocity is given by

$$V_{mean} = \frac{\int_A V dA}{A} \qquad \dots \dots \dots \dots \dots (4.37)$$

The equation of continuity ($Q = A_1 V_1 = A_2 V_2$) is applicable only when

(a) The flow is steady which is usually the case for most of the problems of fluid mechanics. If it is not mentioned whether the flow is steady or unsteady, steady flow is assumed.

(b) The density along the flow is not changing i.e. the flow is incompressible which is the case for most problems of hydraulics as compressibility in negligible. Unless specifically mentioned, flow is assumed to be incompressible.

(c) If the velocity is not uniform over the cross-section, the average velocity is assumed for each section of flow.

(d) The flow is one-dimensional. All pipe and channel flow problems are solved by this assumption, because of simplicity and practical purposes.

(e) There is no branching of stream tubes.

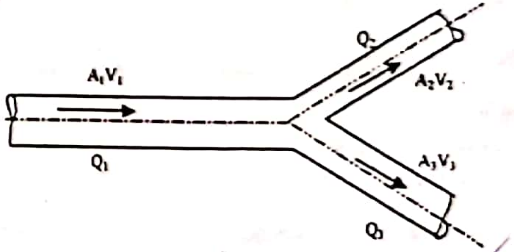

Fig. 4.21 Branching of Pipe

Referring to Fig. 4.21

$$Q_1 = A_1 V_1 \neq A_2 V_2$$

also $A_1 V_1 \neq A_3 V_3$

But $Q_1 = A_1 V_1 = A_2 V_2 + A_3 V_3$

or, $$Q_1 = Q_2 + Q_3 \qquad \dots \dots \dots \dots \dots (4.38)$$

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Continuity Equation for Two and Three-dimensional Flow

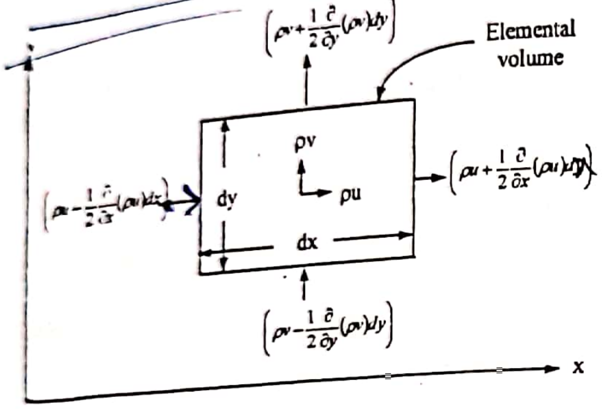

Fig. 4.22 Elemental Volume of Fluid

i) For Two Dimensional Flow

Let us apply the principle of conservation of mass to a small volume of space through which a flow takes place. This volume is an imaginary volume fixed in position and offering no resistance of any kind to flow.

In two dimensional flow, there is no component of flow along the z-axis. Sections normal to the z-axis therefore, have an identical flow pattern, so that it is sufficient to consider a unit width in the z-direction.

The fluid velocity in the x-direction will be designated by u, and that in the y-direction by v, while the density will be designated by ρ. Both the Velocity components and density are functions of position and time.

The principle of conservation of mass requires that the net outflow of mass from the volume be equal to the decrease of mass within the volume. The flow of mass per unit time and area through a surface in the product of the velocity normal to the surface and the density. Thus the x-component of the mass flux per unit area at the center of the volume in ρu. This flux, however, changes from point to point as indicated in Fig. 4.22.

The net outflow of mass per unit time, therefore is

$$\left\{\rho u+\frac{1}{2}\frac{\partial}{\partial x}(\rho u)dx\right\}dy+\left\{\rho v+\frac{1}{2}\frac{\partial}{\partial y}(\rho v)dy\right\}dx-\left\{\rho u-\frac{1}{2}\frac{\partial}{\partial x}(\rho u)dx\right\}dy-\left\{\rho v-\frac{1}{2}\frac{\partial}{\partial y}(\rho v)dy\right\}dx$$

$$=\frac{\partial}{\partial x}(\rho u)dxdy+\frac{\partial}{\partial y}(\rho v)dxdy \qquad \text{-- -- -- -- --} \quad (4.39)$$

and this must be equal to the rate of mass decrease within the element.

$$-\frac{\partial\rho}{\partial t}dx\,dy \qquad \text{-- -- -- -- --} \quad (4.40)$$

Equating (4.39) and (4.40) and simplifying we get,

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \qquad \text{-- -- -- -- --} \quad (4.41)$$

ii) For Three-dimensional Flow

If we had considered the Z-direction also, then we would get

main equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad =0 \qquad \text{-- -- -- -- --} \quad (4.42)$$

The equations (4.41) and (4.42) are known as continuity equations for two-dimensional and three dimensional flows respectively.

If the density is a constant or the fluid is incompressible, the continuity equations become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad \dots\dots\dots\dots\dots \quad (4.43)$$

and

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \dots\dots\dots\dots\dots \quad (4.44)$$

for two-dimensional and three-dimensional flow respectively.

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Continuity Equation in Two-Dimensional Cylindrical Polar Co-ordinates

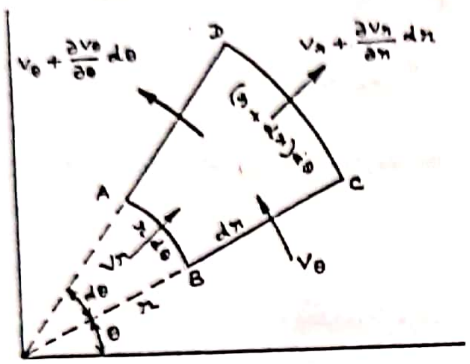

Fig. 4.23 Control Volume For The Continuity Equation in Cylindrical Polar Coordinates

Quite often a two-dimensional flow field for incompressible flow in described with reference to a system of polar cylindrical coordinates (r, θ). The flow velocity has a component V_r in the radial direction and a component V_θ in the tangential direction and the flow is assumed to take place through a differential element rdθ and dr as show in Fig.4.23.

Sides of the fluid element ABCD have the following dimensions :

AD = BC = dr

AB = rdθ and CD = (r + dr) dθ

Thickness of the element perpendicular to the plane of the paper shall be presumed to be unity. There will occur changes both in the velocity and the density as the fluid flows through in the element.

Flow in Radial Direction

Mass of fluid entering the face AB during time dt is given by:

Fluid influx = density (Velocity × Area) time as

Fluid influx = $\rho V_r\, r d\theta\, dt$ ... ... (4.45)

Mass of fluid leaving the face CD during the same time dt is

$$\text{fluid efflux} = \left[\rho V_r + \frac{\partial}{\partial r}(\rho V_r) dr\right](r + dr) d\theta\, dt \qquad \text{... ... ... (4.46)}$$

Mass accumulated in the element because of flow in radial direction in given by the difference between fluid influx and fluid efflux.

∴ mass accumulated due to flow in radial direction.

$$= \rho V_r\, r d\theta\, dt - \left[\rho V_r + \frac{\partial}{\partial r}(\rho V_r) dr\right](r + dr) d\theta\, dt$$

$$= -\left[\rho V_r\, dr\, d\theta + \frac{\partial}{\partial r}(\rho V_r) dr\, r d\theta\right] dt \qquad \text{– – – – – (4.47)}$$

(The terms containing $(dr)^2$ have been neglected)

Flow in Tangential Direction

By a similar treatment, the mass accumulation due to flow in the tangential direction would be

$$= \left[\rho V_\theta\, dr - \left\{\rho V_\theta + \frac{\partial}{\partial \theta}(\rho V_\theta) d\theta\right\} dr\right] dt$$

$$= -\frac{\partial}{\partial \theta}(\rho V_\theta) dr\, d\theta\, dt \qquad \text{– – – – – (4.48)}$$

Total gain in fluid mass

$$= -\left[\rho V_r\, dr\, d\theta + \frac{\partial}{\partial r}(\rho V_r) dr\, r d\theta + \frac{\partial}{\partial \theta}(\rho V_\theta) dr\, d\theta\right] dt \qquad \text{... ... ... (4.49)}$$

According to the law of mass conservation, the total gain in mass must equal the rate of change of fluid mass in the element ABCD which is

$$= \frac{\partial}{\partial t}(density \times volume) dt$$

$$= \frac{\partial}{\partial t}\left[\rho \frac{r d\theta + (r + dr)\, d\theta}{2} dr\right] dt$$

$$\frac{\partial}{\partial t}(\rho r\, d\theta\, dr)dt \qquad \text{(4.50)}$$

$$-\frac{\partial}{\partial t}(\rho\, d\theta\, dr)dt = -\left[\rho V_r + \frac{\partial}{\partial r}(\rho V_r)r + \frac{\partial}{\partial \theta}(\rho V_\theta)\right] dr\, d\theta\, dt$$

or $$\frac{\partial}{\partial t}(\rho r\, d\theta\, dr) + \left[\rho V_r + \frac{\partial}{\partial r}(\rho V_r)r + \frac{\partial}{\partial \theta}(\rho V_\theta)\right] dr\, d\theta = 0 \qquad \text{(4.51)}$$

For steady flow $\frac{\partial}{\partial t}(\rho r d\theta\, dr) = 0$

$$\left[\rho V_r + \frac{\partial}{\partial r}(\rho V_r)r + \frac{\partial}{\partial \theta}(\rho V_\theta) dr\, d\theta\right] = 0 \qquad \text{(4.52)}$$

Further for incompressible flow, ρ = constant

$$V_r + \frac{\partial}{\partial r}(V_r)r + \frac{\partial}{\partial \theta}(V_\theta) = 0 \qquad \text{(4.53)}$$

or $$\frac{\partial}{\partial r}(V_r r) + \frac{\partial}{\partial \theta}(V_\theta) = 0 \qquad \text{(4.54)}$$

which is the equation of continuity in cylindrical polar coordinates for two dimensional steady incompressible flow.

Continuity Equation in Three-Dimensional Cylindrical Polar Co-ordinates

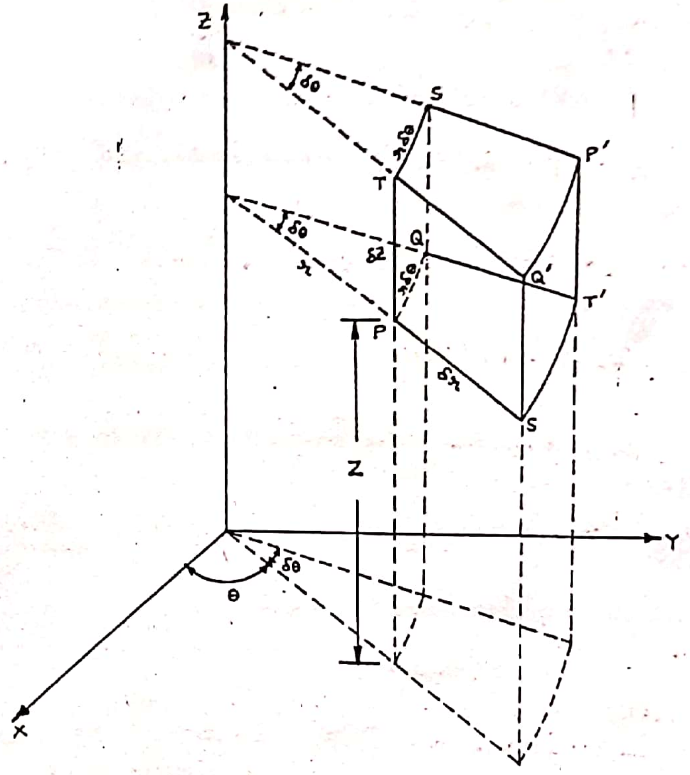

Fig. 4.24 Elementary Cylindrical Parallelopiped

Often a continuity equation is required to be used in terms of the cylindrical polar coordinates which may also be derived by adopting the procedure as indicated below.

Consider any point P (r, θ, z) in space. Let dr, dθ and dz be the small increments in the direction r, θ and z, respectively, so that PS = dr, PQ = rdθ and PT = dz, construct an elementary parallelepiped as shows in Fig. 4.24. Let v_r and v_θ and v_z be the components of the velocity v in the directions of r, θ and z at point P. Further let ρ represent the mass density of fluid at point P.

Considering the pair of faces PST'Q and P'S'TQ' the mass of fluid entering the parallelopiped per unit time through the face PST'Q

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$$= \rho v_z (dr \times rd\theta)$$

Mass of fluid leaving the parallelopiped per unit time through the face P'S'TQ'

$$= \rho v_z (dr \times rd\theta) + \frac{\partial}{\partial z}(\rho v_z dr \times rd\theta)dz$$

Therefore the net mass of fluid that has remained in the parallelopiped per unit time through this pair of faces

$$= \rho V_z (dr \times rd\theta) - \rho V_z (dr \times rd\theta) - \frac{\partial}{\partial z}(\rho V_z dr \times rd\theta)dz$$

$$= -\frac{\partial}{\partial z}(\rho V_z dr \times rd\theta)dz \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.55)$$

Similarly the net mass of fluid that remains in the paralleloped per unit time through the pair of faces PTQ'S and P'S'TQS'

$$= -\frac{\partial}{\partial \theta}(\rho V_\theta dr dz)dz \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.56)$$

and than through the pair of faces PQS'T' and P'Q'ST

$$= -\frac{\partial}{\partial r}(\rho V_r rd\theta dz)dr \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.57)$$

By adding all these expressions in Equations (4.55), (4.56) and (4.57), the net total mass of fluid that has remained in the parallelepiped per unit time through all the three pair of faces.

$$= -\left[\frac{\partial(\rho V_r r)}{rdr} + \frac{\partial(\rho V\theta)}{rd\theta} + \frac{\partial(\rho V_z)}{dz}\right](dr\, dz\, rd\theta) \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.58)$$

The mass of fluid in the parallelepiped

$$= \rho(dr\, dz\, r\partial\theta)$$

and its rate of increase with time

$$= \frac{\partial}{\partial t}(\rho\, dr\, dz\, d\theta)$$

$$= \frac{\partial \rho}{\partial t}(dr\, dz\, d\theta) \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.59)$$

Equating (4.58) and (4.59) we get

$$-\left[\frac{\partial(\rho V_r r)}{r\partial r} + \frac{\partial(\rho V_\theta)}{r\partial\theta} + \frac{\partial(\rho V_z)}{\partial z}\right](dr\, dz\, rd\theta) = \frac{\partial \rho}{\partial t}(dr\, dz\, rd\theta)$$

Dividing both sides of the above expression by the volume of the paralleopiped $(dr dz rd\theta)$ and taking the limit so as to reduced the paralleopiped to point P, the continuity equation is obtained as

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V_r r)}{r\partial r} + \frac{\partial(\rho V_\theta)}{rd\theta} + \frac{\partial(\rho V_z)}{\partial z} = 0 \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.60)$$

Equation (4.60) represents the continuity equation in cylindrical polar coordinates in its most general form which is applicable for steady or unsteady flow, uniform or non-uniform flow and compressible and incompressible fluids.

Again for steady flow since $\frac{\partial \rho}{\partial t} = 0$, equation (4.60) reduces to

$$\frac{\partial(\rho V_r r)}{rdr} + \frac{\partial(\rho V_\theta)}{rd\theta} + \frac{\partial(\rho V_z)}{dz} = 0 \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.61)$$

Further for an incompressible fluid the mass density ρ does not change with r, θ, z and t hence equation (4.60) simplifies to

$$\frac{\partial(V_r r)}{rdr} + \frac{\partial(V_\theta)}{rd\theta} + \frac{\partial(V_z)}{dz} = 0 \qquad \dots\ \dots\ \dots\ \dots\ \dots\ (4.62)$$

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Stream Function

The flow rate or mass per unit time across any line between the two stream lines and per unit thickness is known as ***stream function***. It is denoted by Greek letter ψ (psi). The concept of stream function is based on the continuity principle and the properties of stream lines. The definition of stream function provides a mathematical means of plotting and interpreting flow fields.

Physical Concept of Stream Function

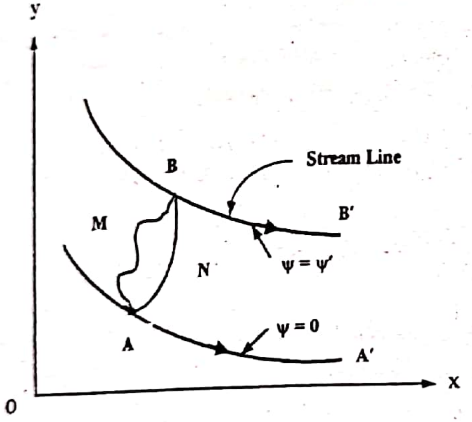
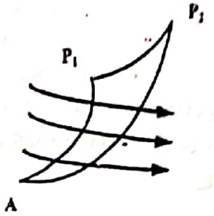


Fig. 4.25 Physical Concept of Stream Function

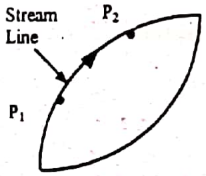
Let the flow be two-dimensional, steady and incompressible. Let AA' and BB' be the stream lines (Fig. 4.25) representing the flow. The flow per unit time across point A and B of the two steam lines is called *Stream function ψ*. The lines joining AB may be AMB or ANB, but. the flow per unit time ψ will remain the same. If point A is fixed, the flow per unit time ψ will depend upon the position B of the fluid and not on the line joining A and B. Let ψ at point A be zero, then ψ at point B will be flow per unit time, i,e, ψ.

Since the stream function measures the discharge between two stream lines, each stream line will have a constant value of ψ.

The stream function ψ has the dimensions of volume per unit length per unit time. It must be borne in mind that the existence of this function is a consequence only of the conditions for the continuity and incompressibility of the fluid, so a stream function is valid for a viscous fluid.

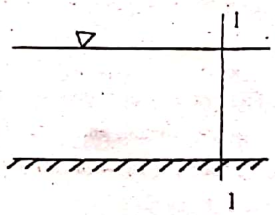


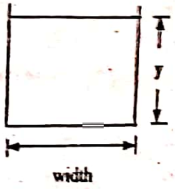
Let us now consider two points P_1 and P_2 and two curves drawn from them to a fixed point A. Let ψ_1 and ψ_2 represent the value of the stream function at points P_1 and P_2 respectively. Then the flux across the curve AP_2 is equal to the flux across the curve AP_1 plus that across the curve P_1P_2. Hence, the flux across the curve P_1P_2 is $\psi_2 - \psi_1$. It can be easily seen that if the reference point A is replaced by another point A_1, the value of the stream function $\psi_2 - \psi_1$ changes by a constant, namely the flux across A_1A.



When the point P_1 and P_2 are points on the same streamline (not necessarily coincident), the flux across P_1P_2 is $\Delta\psi = \psi_2 - \psi_1 = 0$. Since by definition there is no flow across a streamline, the stream function is constant along a steam line.

To have an important physical interpretation of stream function, let us consider a two dimensional, incompressible flow in a channel.





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The conservation of mass principle states that the amount of flow between the fixed walls of a channel is the same for all cross-section.

$$\therefore Q = \int_A u\,dA$$

In two-dimensional flow, if lines of constant x are the cross-section and y_1 and y_2 are the coordinates of the channel top and bottom, then the integration can be carried out at constant x, giving for flow per unit width,

$$q = \int_{y_1}^{y_2} u\,dy \qquad \text{... ... ... (4.63)}$$

and this is the same for all section.

The difference on stream function between the same points y_1 and y_2 at a constant value of x can be obtained in the following way:

From the definition of stream function we have

$$u = \frac{\partial \psi}{\partial y} = \frac{d\psi}{dy} \quad \text{as x is constant}$$

$$d\psi = u\,dy$$

$$\int_{y_1}^{y_2} d\psi = \int_{y_1}^{y_2} u\,dy$$

$$\text{or} \quad \psi_2 - \psi_1 = \int_{y_1}^{y_2} u\,dy \qquad \text{... ... ... (4.64)}$$

Comparing equations (4.63) and (4.64) we find that $\psi_1 - \psi_2$ is precisely the same as the flow rate per unit width, we therefore conclude that lines of constant stream function are everywhere parallel to the flow and the numerical difference in ψ between two stream lines in equal to the flow rate per unit width passing between the stream lines.

Mathematical Concept of Stream Function

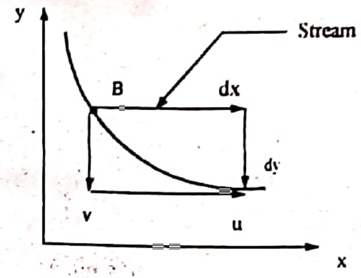

Fig. 4.26 Mathematical Concept of Stream Function

$\psi = f_r(x, y)$ for two dimensional flow,

Let point B on the streamline move by a small distance ds along the flow. The distance ds may be represented by components dy in y-direction and dx in x-direction (Fig. 4.26). Let the velocity components at point B (x, y) be u and v in x and y-directions respectively. Then equation of streamline in

$$\frac{dy}{dx} = \frac{v}{u}$$

or, udy - vdx=0

Further the discharge across dy will be

$$d\psi = u\,dy$$

$$u = \frac{\partial \psi}{\partial y} \qquad \text{... ... ... (4.65)}$$

Similarly discharge across dx is given by

$$d\psi = -v\,dx$$

$$v = -\frac{\partial \psi}{\partial x} \qquad \text{... ... ... (4.66)}$$

(Minus sign indicates the velocity v acting in downward direction in Fig.26)

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Substituting the values of u and v from equations (4.65) and (4.66) in the left hand side of the continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

we get

$$\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)+\frac{\partial}{\partial y}\left(-\frac{\partial\psi}{\partial x}\right)$$

$$\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^2\psi}{\partial x\partial y}=0 \qquad \text{... ... ... (4.67)}$$

Thus stream function satisfies the equation of continuity.

Further putting the values of u and v in the expression for vorticity in a two-dimensional flow, it gives

$$\Omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \qquad \text{(Equation of Vorticity)}$$

$$=\frac{\partial}{\partial x}\left(-\frac{\partial\psi}{\partial y}\right)-\frac{\partial}{\partial y}\left(-\frac{\partial\psi}{\partial y}\right)$$

$$=-\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}$$

$$\text{or} \qquad \frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\Omega \qquad \text{... ... ... (4.68)}$$

If $\Omega=0$, the flows is irrotational, and

$$=\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}=0 \qquad \text{... ... ... (4.69)}$$

This is Laplace Equation for ψ.

Properties of Stream function

The following are the useful properties of stream function-

(i) ψ is constant everywhere on any one stream line;
or, along a stream line ψ = constant
or, ψ = constant represents the family of stream lines
or, ψ = constant, is a steam line equation.

(ii) The flow around any path in the fluid is zero if the flow is continuous.

(iii) The rate of change of ψ with distance in an arbitrary direction proportional to the component of velocity normal to that direction.

(iv) The velocity vectors may be found be differentiating the stream function (refer equations 4.65 and 4.66)

(v) The algebraic sum of the stream function for two incompressible flow patterns is the stream function for the flow resulting from the super-imposition of these patterns,

$$\text{i.e.,} \quad \frac{\partial\psi_1}{\partial s}+\frac{\partial\psi_2}{\partial s}=\frac{\partial(\psi_1+\psi_2)}{\partial s} \qquad \text{... ... ... (4.70)}$$

Lines of Constant ψ Represents Streamlines

We know

$$\psi=f(x,y,t) \qquad \text{... ... ... (4.71)}$$

For unit time t = 1, we have from equation (4.69)

$$\psi=\psi(x,y)$$

$$\therefore d\psi=\frac{d\psi}{\partial x}dx+\frac{\partial\psi}{\partial y}dy \qquad \text{... ... ... (4.72)}$$

But $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ (from equations 4.65 and 4.66)

Substituting these values of u and v is equation (4.72) we have

$$d\psi=-v\,dx+u\,dy \qquad \text{... ... ... (4.73)}$$

For a lines constant ψ, dψ = 0 hence from equation (4.73), we get

$$\frac{dx}{u} = \frac{dy}{v}$$

which is the equation of stream line. Hence a line of constant ψ represents a stream line.

Velocity Potential

The flow takes place in a pipe line if there is a difference of pressure. The direction of flow will take place from higher to the lower pressure. Thus the velocity of flow in a certain direction will depend upon the potential difference which is known as ***velocity potential*** and denoted by ϕ (phi). The velocity potential is scalar function of position and time.

Mathematically

$$u = \frac{\partial\phi}{\partial x}, \quad v = \frac{\partial\phi}{\partial y}, \quad w = \frac{\partial\phi}{\partial z} \qquad \text{... ... ...} \quad (4.74)$$

where u, v and w are the velocity components in x, y and z direction respectively. Velocity potential function is a function of x and y such that its partial derivative with respect to displacement in any direction is equal to the velocity component.

Potentional Line or equipotential line is a line along which the velocity potential is constant. When a velocity potential function is equated to a series of constants, a family of curves is obtained. These curves are at right angles to streamlines at every point i.e. they are orthogonal to streamlines.

Some of the salient characteristics of a potential function are:

- Since ϕ is a function of x and y alone its total different is :

$$d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy \qquad \text{... ... ...} \quad (4.75)$$

Substituting the values of u and v from equation (4.74), we get

$$d\phi = u\,dx + v\,dy \qquad \text{... ... ...} \quad (4.76)$$

For an equipotential line the potential function ϕ is constant i.e., $d\phi = 0$. Therefore an equipotential line

$$u\,dx + v\,dy = 0$$

or,
$$\frac{dy}{dx} = -\frac{u}{v} \qquad \text{... ... ...} \quad (4.77)$$

which prescribes the slope of equipotential line at any point.

Substitute for u and v in terms of potential function in the expression for vorticity

$$\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial x}\right) = 0 \qquad \text{... ... ...} \quad (4.78)$$

Hence the velocity potential satisfies irrotational flow. The existence of velocity potential function means that a possible flow must be irrotational. Consequently the two dimensional irrotational flow in often called the ***potential flow.***

Combining equation (4.78) with continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \qquad \text{we get,}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial z}\right) = 0$$

or,
$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \qquad \text{... ... ...} \quad (4.79)$$

which is the Laplace equation. The velocity potential function thus satisfies the Laplace equation.

- The velocity potential function can also be defined as the integral of the tangential velocity component along a curve joining any two points. In that case, the velocity potential of point B relative to point A is :

$$\phi_B = \int_{AOB} V\cos\alpha\, ds \qquad \text{... ... ...} \quad (4.80)$$

Cauchy Riemann Equations

The above discussion with reference to stream function and velocity potential function leads us to conclude that:

(i) Stream function ψ applies to both rotational and irrotational flows. The flow has only to be steady and incompressible.

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(xi) For irrotational flow, both the stream function and the velocity potential function satisfy Laplace equation; consequently they are interchangeable. We have the following important relationships between the stream function ψ and potential function ϕ for a steady irrotational incompressible flow

$$\left.\begin{aligned} u &= \frac{\partial\psi}{\partial y} = \frac{\partial\phi}{\partial x} \\ v &= -\frac{\partial\psi}{\partial x} = \frac{\partial\phi}{\partial y} \end{aligned}\right\} \qquad \text{... ... ...} \quad (4.81)$$

In hydrodynamics, these equations are sometimes called the *Cauchy-Riemann equations.*

The corresponding relations in cylindrical polar co-ordinates are

$$\left.\begin{aligned} u_\theta &= -\frac{\partial\psi}{\partial r} = \frac{1}{r}\frac{\partial\phi}{\partial\theta} \\ u_r &= \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{\partial\phi}{\partial r} \end{aligned}\right\} \qquad \text{... ... ...} \quad (4.82)$$

Vortex Line

If a line in drawn in the fluid so that the tangent to it at each point is in the direction of vortex vector $\underline{\Omega}$ at that point, then the line is called a vortex line.

If $d\underline{l}$ is in the direction of vorticity vector then

$$\underline{\Omega}\times d\underline{l} = \begin{vmatrix} i & i & k \\ \Omega_x & \Omega_y & \Omega_z \\ dx & dy & dz \end{vmatrix} \qquad \text{... ... ...} \quad (4.83)$$

$$\underline{i}(\Omega_y dz - \Omega_z) + \underline{j}(\Omega_z - \Omega_x dz) + \underline{k}(\Omega_x dy - \Omega_y dx) = 0$$

From the above relation, the equation of vortex lines are obtained as

$$\frac{dx}{\Omega x} = \frac{dy}{\Omega y} = \frac{dz}{\Omega z} \qquad \text{... ... ...} \quad (4.84)$$

Orthogonality of Streamlines and Equipotential Lines

From equations (4.73) and (4.76) for stream function and velocity potential we have

$$d\psi = -vdx + udy \quad \text{and}$$

$$d\phi = udx + vdy$$

However, along any streamline ψ is constant, therefore

$$d\psi = 0 = -vdx + udy$$

$$\text{or,} \quad \frac{dy}{dx} = \frac{v}{u} \qquad \text{--- --- --- --- ---} \quad \text{(i)}$$

also along a potential line ϕ is constant and hence

$$d\phi = 0 = udx + vdy$$

$$\text{or,} \quad \frac{dy}{dx} = -\frac{u}{v} \qquad \text{--- --- --- --- ---} \quad \text{(ii)}$$

Combining expressions (i) and (ii)

slope of streamline × slope of equipotential line

$$= \frac{v}{u}\times\left(-\frac{u}{v}\right)$$

$$= -1 \qquad \text{... ... ...} \quad (4.85)$$

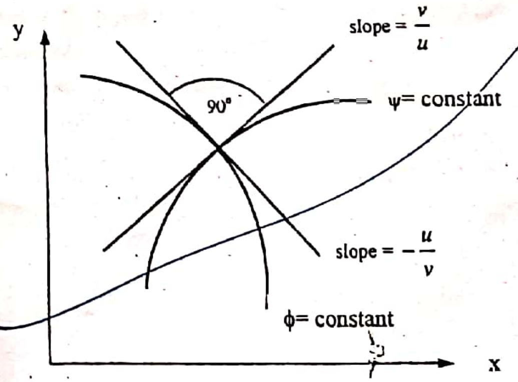

Fig. 4.27 Orthogonality of Streamlines and Equipotential Lines

Equation (4.85) is a mathematical statement of the fact that equipotential lines are normal to the stream lines. This aspect is depicted in Fig 4.27. The orthogonality between the streamlines and equipotential lines serves to draw a *flownet*, a graphical representation of these lines for a potential flow. Streamlines and equipotential lines are drawn between the flow boundaries with the requirements that they form small squares. The bounding surfaces form streamlines and the equipotential lines intersect the boundaries at right angles.

Flow Net

A grid obtained by drawing a series of streamlines and equipotential lines is known as a flow net. The flow net provides a simple graphical technique for studying two-dimensional irrotational flows especially in the cases where mathematical relations for stream function and velocity function are either not available or are rather difficult and cumbersome to solve.

With means now at hand of obtaining the velocity distribution from a given streamline pattern, then next step is to seek a method of determining the form of stream lines for any boundary geometry. The only absolute method of determination, to be sure, entails observation of the flow itself, by means of smokes, dye, or other visible agent. In many instances, however effective use may be made of a simple graphical process based upon mathematical principles of classical hydrodynamics. Since these principles embody, as a matter of fact, the only general means of even approximating a flow pattern without recourse to actual field or laboratory measurement, their graphical representation warrants careful attention at this point.

For two-dimensional motion, such graphical representation of the mathematical analysis is known as a ***flow net***. This consist in brief, of a system of stream lines so spaced that the incremental rate of flow Δq is the same between each successive pair and a system of normal lines so spaced that at any point the distance Δs between normal lines equal the distance Δn between stream lines. The velocity under such circumstances would then be inversely proportional to the distance between either the stream lines or the normal lines throughout the flow.

The flow net is a graphical representation (Fig. 4.28) of stream lines and equipotential lines which are perpendicular to each of other. The steamlines ψ_1, ψ_2, ψ_3 etc. show the direction of flow. The equipotential lines ϕ_1, ϕ_2, ϕ_3 etc., shows the lines joining the points of equal velocity potential. When the flow is irrotational, then only, the flow net can be drawn. For a two-dimensional steady irrotational and incompressible flow ψ (x, y) is constant along a stream lines and for potential flow ϕ (x, y) is constant along equipotential lines.

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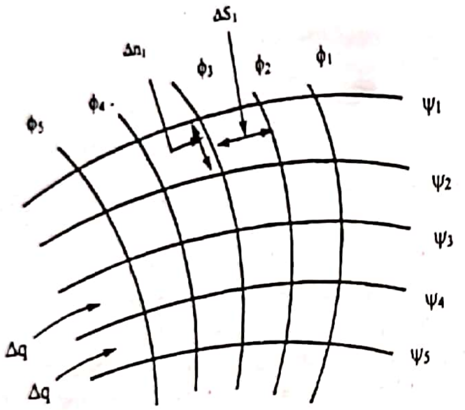

Fig. 28. Flow Net

Let Δq be the flow rate between any two stream lines say ψ_1 and ψ_2, then for a unit length (perpendicular to the plane of paper),

$$\Delta q = V_1 \Delta n_1 = V_2 \Delta n_2 = \text{constant} \qquad \ldots \ldots \ldots \ldots \ldots \quad (4.86)$$

where Δn_1, Δn_2 and V_1, V_2 are the distances between two stream lines and velocities at the two sections 1 and 2 respectively. Hence closeness of streamlines in a flow net indicates higher velocity.

From equation (86)

$$\frac{V_1}{V_2} = \frac{\Delta n_2}{\Delta n_1} \qquad \ldots \ldots \ldots \ldots \ldots \quad \text{(iii)}$$

Further $\Delta\phi = V_1 \Delta s_1 = V_2 \Delta s_2 = \text{constant}$ ... ... (4.87)

where Δs_1 and Δs_2 are the distances between two equipotential lines.

or, $$\frac{V_1}{V_2} = \frac{\Delta s_2}{\Delta s_1} \qquad \ldots \ldots \ldots \ldots \ldots \quad \text{(iv)}$$

From equations (iii) and (iv)

$$\frac{\Delta n_2}{\Delta n_1} = \frac{\Delta s_2}{\Delta s_1}$$

or, $$\frac{\Delta n_1}{\Delta} = \frac{\Delta n_2}{\Delta} = \text{constant} \qquad \ldots \ldots \ldots \ldots \ldots \quad (4.88)$$

or, $$\frac{\Delta n}{\Delta s} = \text{constant}$$

If this ratio $\frac{\Delta n}{\Delta s} = 1$, the flow net is a set of squares.

For example the net of flow between parallel boundaries would consist, as shown in Fig. 4.29 of a series of square meshes of constant size, indicating the same velocity at every point. Were the same boundaries to converge as in Fig 4.30, the meshes would no longer be exactly square but the medium lines of any mesh would still have essentially the same length; the velocity would evidently be constant along any normal line but would vary as the streamlines converged. If on the other hand, the boundaries were coaxial cylinders, the stream lines and normal lines of Fig 4.30 would simply be interchanged, as indicated in Fig. 4.31 the velocity would then be constant along the circular stream lines but would vary inversely with the radius in the normal direction.

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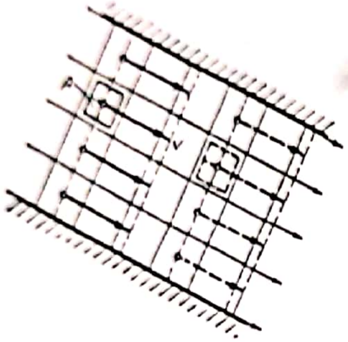

Fig. 4.29 Flow Net for Parallel Boundaries

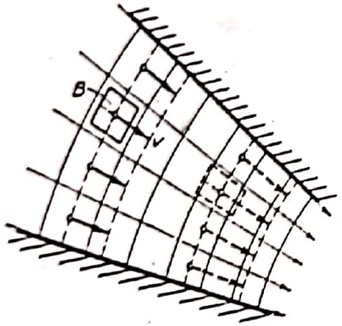

Fig. 4.30 Flow Net for Convergent Boundaries

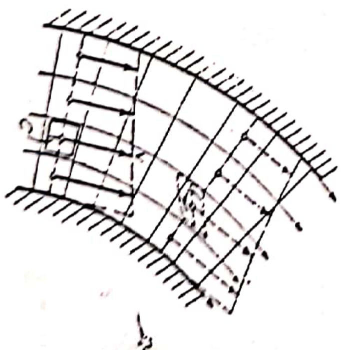

Fig. 4.31 Flow Net for Coaxial Boundaries

Methods of Drawing Flow Nets

The plane ideal fluid flow pattern through a passage or over a body of any given geometry can be plotted in the form of constant Ψ and ϕ lines by one of the following procedures :

1. Analytical method
2. Graphical Method
3. Electrical Analogy Method
4. Relaxation Method
5. Helle Shaw or Viscous Flow Analogy method

Out of these five methods, the first three methods are briefly described below.

1. Analytical Method

In this method the equations corresponding to the curves ϕ and Ψ are first obtained and the same are plotted to give the flow net pattern for the flow of fluid between the given boundary shapes. In other words, this method involves a solution of Laplace equation for ϕ and Ψ, which gives the corresponding equations for ϕ and Ψ. But this method cannot be applied in various cases on account of the boundary shapes being such that it may not be possible to obtain the solution of the Laplace equation for ϕ and Ψ. In such cases other methods may be adopted to obtain the flow net pattern for the corresponding flow.

Analytical method is used for simple cases of parallel flows, flow around a corner or flow around cylinders etc. where it is possible to obtain expression for stream function and velocity potential. There are many other practical flow cases where analytical method cannot be applied. In such cases graphical and electrical analogy methods are employed.

2. Graphical Method

The graphical method of plotting a flow net is the simplest and gives good results if carefully drawn. The streamlines are drawn equally dividing the space between the boundaries into a few equal number of flow channels. Then the equipotential lines are drawn intersecting the boundaries and the streamlines orthogonally, such that the flow fields are roughly square. If the equipotential lines are spaced at the same distance as the streamlines, the flow net results in *perfect squares* in the region of uniform flow, and *near squares* in the region of non-uniform flow. The accuracy of final flow net can be checked by drawing diagonals.

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Graphical method consumes lot of time and requires lot of erasing to get the proper shape of a flow net.

3. Electrical Analogy Method

This method is a practical method of drawing a flow net for a particular set of boundaries. It is based on the fact that the flow of fluids and flow of electricity through a conductor are analogous. These two systems are similar in this respect that electric potential is analogous to the velocity potential, the electric current is analogous to the velocity of flow, and the homogeneous conductor is analogous to the homogenous fluid.

Applicability of Flow Net

The flow net is applicable in the conditions mentioned below:

(i) The flow must be steady.

(ii) The fluid should be ideal or fluid have negligible viscosity. However, for rapidly accelerating flows, even for fluids of low viscosity the flow nets give good results.

(iii) Fluid weight should not govern the flow phenomenon. This means that fluids should be light and the flow is two-dimensional.

(iv) Flow nets can be drawn for confined converging flows, for flows with free surface but not for diverging flows of real fluids in boundaries such as flow in a sudden enlargement of a pipe.

Utility of Flow Net Analysis

Following are the utility of flow net analysis:

1. Once a flow net is constructed for a given model at a given rate of discharge, the same flow net will hold good for different rate of discharge for different geometrical similar fixed boundaries. With the application of continuity equation, velocity at any point in terms of reference velocity can be obtained.

2. With flow net known efficient boundary profile can be designed. The flow nets give a good idea about the smooth boundary profile that can be substituted for boundaries with sharp corners.

3. Flow net can be used to determine the quantity of seepage and uplift pressure below hydraulic structures.

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