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* Define sine series and expand it

Half range sine series: The part which contains only the sine term in fourier sereis is called half range sine series.

If range is $(0, \pi)$ which is half of $(-\pi, \pi)$ of Fourier series. In odd

Expansion:

The sine series in range $0 \leq x \leq \pi$ is

$$f(x) = \sum_{n=1}^{\alpha} b_n \sin nx \quad \ldots \text{(1)}$$

Now multiplying eq^n (1) by sin and intergrating with respect to x in $(0, \pi)$

$$\int_0^{\pi} f(x) \sin nx\, dx = \sum_{n=1}^{\alpha} b_n \int_0^{\pi} \sin^2 nx\, dx$$

$$= \sum_{n=1}^{\alpha} \frac{b_n}{2} \int_0^{\pi} 2\sin^2 nx\, dx]$$

$$= \sum_{n=1}^{\alpha} \frac{b_n}{2} \left[x - \frac{1}{2n} \sin^2 nx\right]_0^{\pi}$$

$$= \sum_{n=1}^{\alpha} \frac{b_n}{2} [(\pi - 0) = (0 - 0)]$$

$$= \sum_{n=1}^{\alpha} \frac{b_n}{2} \cdot \pi$$

$$\therefore b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx\, dx$$

Hence, in $(0, \pi)$ sine series is,

$$f(x) = \frac{2}{\pi} \int_0^{\pi} f(x) \sin^2 nx\, dx$$

This is the expansion of half range sine series

Home Work:

* Find the fourier series in the expansion of

$$f(x) = \begin{cases} 0 & \text{if } -\pi < x < 0 \\ 2x^3 & \text{if } 0 < x < \pi \end{cases}$$

Sol^n: We know, the fourier series is,

$$f(x) = \frac{a_0}{2} + \sum a_n \cos nx + b_n \sin nx) \ldots\ldots (1)$$

Here
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$
$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0} 0\,dx + \int_0^{\pi} 2x^3 dx\right\}$$
$$= \frac{1}{\pi}\left[\frac{2x^4}{4}\right]_0^{\pi}$$
$$= \frac{\pi^3}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cos nx\,dx + \int_0^{\pi} 2x^3 \cos nx\,dx\right]$$
$$= \frac{1}{\pi}\left[\int_0^{\pi} 2x^3 \cos nx\,dx\right.$$
$$= \frac{2}{\pi}\int_0^{\pi} \underset{u}{x^3}\ \underset{v}{\cos nx}\,dx$$

$\left[\begin{array}{l} uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots \\ \sin n\pi = 0 \\ \cos n\pi = (-1)^n \end{array}\right.$ Hence $u' = \frac{du}{dx}$, $v' = \int v\,dx$

$$= \frac{2}{\pi}\left[\frac{x^3}{n}\sin nx + \frac{3x^2}{n^2}\cos nx - \frac{6x}{n^3}\sin nx - \frac{6}{n^4}\cos x\right]_0^{\pi}$$
$$= \frac{2}{\pi}\left[\frac{\pi^3}{n^3}\sin nx + \frac{3\pi^2}{n^2}\cos n\pi - \frac{6\pi}{n^3}\sin nx - \frac{6}{n^4}\cos n\pi + \frac{6}{n^4}\cos n\right]$$
$$= \frac{6x}{n^2}\cos n\pi - \frac{12}{n^4\pi}\cos n\pi + \frac{12}{n^4\pi}$$
$$= \frac{6\pi(-1)^n}{n^2} - \frac{12(-1)^n}{n^4\pi} + \frac{12}{n^4\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\ \sin nx dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0 \cdot \sin n dx + \int_{0}^{\pi} 2x^3 \sin nx dx\right]$$

$$= \frac{1}{\pi}\int_{0}^{\pi} 2x^3 \sin nx dx$$

$$= \frac{2}{\pi}\left[\left(\frac{x^3}{v} x \frac{-\frac{1}{n}\cos nx}{u_1}\right) - \left(\frac{3x^2}{v'} x \frac{-\frac{1}{n^2}\sin nx}{u_2}\right) + \left(\frac{6x}{v''} + \frac{\frac{1}{n^3}\cos nx}{u_3}\right) - \left(\frac{6}{v^4} + \frac{\frac{1}{n^4}\sin nx}{u^4}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{x^3}{n}\cos nx + \frac{3x^2}{n^2}\sin nx + \frac{6x}{n^3}\cos nx - \frac{6}{n^4}\right]$$

$$= \frac{2}{\pi}\left[-\frac{\pi^3}{n}\cos n\pi + \frac{3\pi^2}{n^2}\sin n\pi + \frac{6\pi}{n^3}\cos n\pi - \frac{6}{n^4}\sin n\pi + \frac{6}{n}\right.$$

$$= \frac{2\pi^2}{n}\cos n\pi + \frac{12}{n^3}\cos n\pi$$

$$= -\frac{2\pi^2}{n}(-1)^n + \frac{12}{n^3}(-1)^n$$

$$= \frac{2\pi^2}{n}(-1)(-1)^n + \frac{12}{n^3}(-1)^n$$

$$= \frac{2\pi^2}{n}(-1)^{n+1} + \frac{12}{n^3}(-1)^n$$

Putting the values of a_0, a_n, b_n in eqn (1)

$$f(x) = \frac{\pi^3}{4} + \Sigma\left[\left\{\frac{6\pi - (-1)^n}{n^2} - \frac{12(-1)^n}{n^4\pi} + \frac{12}{n^4\pi}\right\}\cos nx + \left\{\frac{2\pi^2}{n}(-1)^{n+1} + \frac{12}{n^3}(-1)^n\right\}\sin x\right]$$

Ans!

Home Work

Q1 Given that,

$$y^2zp - x^2zq = x^2y$$

We know the lagranges auxiliary equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{y^2z} = \frac{dy}{-x^2z} = \frac{dz}{x^2y}$$

From 1st and 2nd ratio, we get,

$$\frac{dx}{y^2z} = \frac{dy}{-x^2z}$$

$$\Rightarrow \frac{dx}{y^2} = \frac{dy}{-x^2}$$

$$\Rightarrow -x^2dx = y^2dy$$

$$\Rightarrow -\frac{x^3}{3} = \frac{y^3}{3} - \frac{c_1}{3} \text{ [By integrating]}$$

$$\Rightarrow -x^3 - y^3 = -c_1$$

$$\Rightarrow x^3 + y^3 = c_1$$

From 2nd and 3rd ratio, we get,

$$\frac{dy}{-x^2z} = \frac{dz}{x^2y}$$

$$\Rightarrow \frac{dy}{-z} = \frac{dz}{y}$$

$$\Rightarrow ydy = -zdz$$

$$\Rightarrow \frac{y^2}{2} = -\frac{z^2}{2} + \frac{c_2}{2} \text{ [By integrating]}$$

$$\Rightarrow y^2 + z^2 = c_2$$

Hence the general solution is $\phi(x^3+y^3, y^2+z^2) = 0$
Where ϕ is an arbitrary function.

ii) Given that,

$$(x^2-yz)p + (y^2-zx)q = z^2-xy$$

We know the lagrange auxiliary equitations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\text{or, } \frac{dx}{x^2 y^2} = \frac{dy}{y^2 zx} = \frac{dz}{z^2 xy}$$

choosing 1,-1,0 ; 0,1,-1 and -1,0,1 as multipliannegd

$$\frac{dx}{x^2yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2xy} = \frac{dx-dy}{x^2-yz-y^2+zx} = \frac{dy-dz}{y^2-zx+z^2+xy} = \frac{-dx+dz}{-x^2+yz+z^2xy}$$

$$\Rightarrow \frac{dx}{x^2yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2xy} = \frac{dx-dy}{x^2-y^2+z(x-y)} = \frac{dy-dz}{y^2-z^2+x(y-z)} = \frac{dz-dx}{(z-x)(x+y+z)}$$

For 4th and 5th ratio, we get,

$$\frac{dx-dy}{(x-y)(x+y+z)} = \frac{dy-dz}{(y-z)(x+y+z)}$$

$$\Rightarrow \log(x-y) = \log(y-z) + \log c.$$

$$\Rightarrow \log \frac{x-y}{y-z} = \log c,$$

$$\Rightarrow \frac{x-y}{y-z} = c_1$$

From 5th and 6th ratio, we get.

$$\frac{dy-dz}{(y-z)(x+y+z)} = \frac{dz-dx}{(z-x)(x+y+z)}$$

$$\Rightarrow \log(y-z) = \log(z-x) + \log c_2$$

$$\Rightarrow \log \frac{y-z}{z-x} = \log c_2$$

$$\Rightarrow \frac{y-z}{z-x} = c_2$$

Hence the general solution is $\phi\left(\frac{x+y}{y-z}, \frac{y-z}{z-x}\right) = 0$

Where ϕ is an arbitrary function.

(iii) Given that, $(y-z)p + (x-y)q = z-x$

We know the lagrange's auxiliary equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Or $$\frac{dx}{y-z} = \frac{dy}{y-z} = \frac{dz}{z-x}$$

Choosing 1.1.1 and x,y,z as multiplier we get,

$$\frac{dx}{y-z} = \frac{dy}{x-y} = \frac{dz}{z-x} = \frac{dx+dy+dz}{y-z+x-y+z-x} = \frac{xdx+zdy+ydz}{xy-zx+zx-yz+yz-xy}$$

$$\Rightarrow \frac{dx}{y-z} = \frac{dy}{x-y} = \frac{dz}{z-x} = \frac{dx+dy+dz}{0} = \frac{xdx+zdy+ydz}{0}$$

From 4th ratio, we get,

$$dx+dy+dz = 0$$

$$\Rightarrow x+y+z = c_1 \text{ (by Integration)}$$

From 5th ratio, we get,

$$xdx+zdy+ydz = 0$$

$$\Rightarrow xdx + d(yz) = 0$$

$$\Rightarrow \frac{x^2}{2} + yz = \frac{c_2}{2} \text{ [By Integration]}$$

$$\Rightarrow \frac{x^2+2yz}{2} = \frac{c_2}{2}$$

$$\Rightarrow x^2+2yz = c_2$$

Hence the general solution is $\phi\left((x+y+z), x^2+2yz\right) = 0$

Where ϕ is an arbitrary function.

(iv)

Given that,

$$x(2y^4 - z^4)p + y(z^4 - 2x^4)q = z(x^4 - y^4)$$

We know the lagrange's auxiliary equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

or, $$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)}$$

choosing x^3, y^3, z^3 as multiplier and $x, y, z/2$ as divider we get.

$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)} = \frac{x^3dx + y^3dy + z^3dx}{x^4(2y^4 - z^4) + y^4(z^4 - 2y + z^4(x^4 - y^4)}$$

$$= \frac{\frac{dx}{x} + \frac{dy}{y} + 2\frac{dz}{dz}}{2y^4 - z^4 + z^4 - 2x^4 + 2(x^4 - y^4)}$$

or,

$$\frac{dx}{x(2y^4 - z^4)} = \frac{dy}{y(z^4 - 2x^4)} = \frac{dz}{z(x^4 - y^4)} = \frac{x^3dx + y^3dy + z^3dz}{0}$$

$$= \frac{\frac{dx}{x} + \frac{dy}{y} + 2\cdot\frac{dz}{z}}{0}$$

From 4th ratio we get.

$$x^3dx + y^3dy + z^3dz = 0$$

$$\Rightarrow \frac{x^4}{4} + \frac{y^4}{4} + \frac{z^4}{4} = \frac{c_1}{4} \text{ (By integration)}$$

$$\therefore x^4 + y^4 + z^4 = c_1$$

From 5th ratio, we get,

$$\frac{dx}{x} + \frac{dy}{y} + 2\frac{dz}{z} = 0$$

$\Rightarrow \log x + \log y + 2\log z = \log c_2$ (By integrating)

$\Rightarrow \log x + \log y + \log z^2 = \log c_2$

$\Rightarrow \log (xyz^2) = \log c_2$

$\therefore xyz^2 = c_2$

Hence the general solution is $\phi \cdot (x^4+y^4+z^4 \cdot xyz^2) = 0$,

Where ϕ is an arbitrary function.

Home Work

Problem-5: Find the integral surface of the linear partial differential equation

(i) $4yzp + q + 2y = 0$ which passes through $y^2+z^2=1$, $x+z=2$

[Hints : Taking 1st and 3rd ratio : Taking 2nd and 3rd ratio then adding and $y^2+z^2+x+z=3$ (Ans)]

(ii) $(x-y)p+(y-x-z)q=z$ which contain the circle $z=1$, $x^2+y^2=1$

[Hints : 1,1,1 and 1,-1,1 as multiplier ;

Taking 4th ration ; Taking 3rd and 5th ratio and

$z^4(x+y+z)^2-2z^4(x+y+z)-2z^2(x-y+z)+(x-y+z)^2$

$=0$ (ans)]

Problem - 5

(i)

Given that,

$$4yzp + q + 2y = 0$$

We know the lagrange's auxiliary equations are

$$\frac{dx}{P} \pm \frac{dy}{Q} = \frac{dz}{R}$$

Or, $\frac{dx}{4yz} = \frac{dy}{1} = \frac{dz}{-2y}$

Taking 1st and 3rd ratio we get,

$$\frac{dx}{4yz} = \frac{dz}{-2y}$$

Or, $\frac{dx}{2z} = \frac{dz}{-1}$

Or, $-dx = 2zdz$

Or, $dx + 2zdz = 0$

Or, $x + z^2 = c_1$ ...... ① [By integrating]

Again from 2nd and 3rd ratio we get,

$$\frac{dy}{1} = \frac{dz}{-2y}$$

Or, $-2ydy = dz$

Or, $2ydy + dz = 0$

Or, $y^2 + z = c_2$ .... ⑪ [By integrating]

Adding (i) and (ii) we get,

$$x+z^2+y^2+z = c_1+c_2$$

Or, $y^2+z^2+x+z = c_1+c_2$

Or, $1+2 = c_1+c_2$

Or, $c_1+c_2 = 3$

Or, $x+z^2+y^2+z = 3$

$\therefore y^2+z^2+x+z = 3$ Which is the required integral surface.

(ii)

Given that

$$(x-y)p + (y-x-z)q = z$$

We know the lagrange's auxiliary equations are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

Or, $\dfrac{dx}{x-y} = \dfrac{dy}{y-x-z} = \dfrac{dz}{z}$

choosing 1,1,1 and 1,-1,1 as multiplier we get,

$$\frac{dx}{x-y} = \frac{dy}{y-x-z} = \frac{dz}{z} = \frac{dx+dy+dz}{x-y+y-x-z+z} = \frac{dx-dy+dz}{x-y-y+x+z+z}$$

Or, $\dfrac{dx}{x-y} = \dfrac{dy}{y-x-z} = \dfrac{dz}{z} = \dfrac{dx+dy+dz}{0} = \dfrac{dx-dy+dz}{2x-2y+2z}$

From 4th ratio we get,

$$dx + dy + dz = 0$$

or, $x+y+z = c_1$ .... (i) [By integrating]

Again from 3rd and 5th ratio, we get,

$$\frac{dz}{z} = \frac{dx - dy + dz}{2x - 2y + 2z}$$

or, $\frac{dz}{z} = \frac{1}{2} \cdot \frac{dx - dy + dz}{x - y + z}$

or, $2\frac{dz}{z} = \frac{dx - dy + dz}{x - y + z}$

or, $2 \log z = \log(x-y+z) + \log c_2$ [By integration]

or, $\log z^2 = \log(x-y+z) + \log c_2$

or, $\frac{z^2}{x-y+z} = c_2$ .... (ii)

Since (i) and (ii) Passes through $z=1, x^2+y^2=1$

Putting $z=1$ in eq^n (i) and (ii) we get,

From (i) $x+y+1 = c_1$

$\therefore x+y = c_1 - 1$

From (ii) $\frac{1}{x-y+1} = c_2$

or, $\frac{1}{c_2} = x-y+1$

$\therefore x-y = \frac{1}{c_2} - 1$

Now,

$$x^2+y^2=1$$

or, $2(x^2+y^2)=2$ (multiplying by 2 on both side)

or, $(x+y)^2+(x-y)^2=2$

or, $(c_1-1)^2+(1/c_2-1)^2=2$

or, $c_1^2-2c_1+1+(1/c_2)^2-2.1/c_2+1=2$

or, $c_1^2+1/c_2^2-2c_1-2/c_2=0$

or, $(x+y+z)^2+\frac{(x-y+z)^2}{z^4}-2(x+y+z)-\frac{2(x-y+z)}{z^2}=0$

or, $\frac{z^4(x+y+z)^2+(x-y+z)^2-2z^4(x+y+z)-2z^2(x+y+z)}{z^4}=0$

or, $z^4(x+y+z)^2+(x-y+z)^2-2z^4(x+y+z)-2z^2(x-y+z)^2=0$

$\therefore z^4(x+y+z)^2-2z^4(x+y+z)-2z^2(x-y+z)+(x-y+z)^2=0$

Which is the required integral surface.

Home Work

problem :06

Find the complete integral of the given partial differential equation by charpit's method $2z+p^2+qy+2y^2=0$

Soln: Given that,

$$2z+p^2+qy+2y^2=0$$

Let $F(x,y,z,p,q) = 2z+p^2+qy+2y^2 = 0 \quad \cdots\cdots (i)$

We know the charpit's auxiliary equations are,

$$\frac{dp}{F_x+pF_z} = \frac{dq}{F_y+qF_z} = \frac{dz}{-pF_p-qF_q} = \frac{dx}{-F_p} = \frac{dy}{-F_q}$$

$$\text{or, } \frac{dp}{2p} = \frac{dq}{3q+4y} = \frac{dz}{-2p^2-qy} = \frac{dx}{-2p} = \frac{dy}{-y}$$

From 1st and 4th ratio, we get,

$$\frac{dp}{2p} = \frac{dx}{-2p}$$

$$\text{or, } \frac{dp}{1} = \frac{dx}{-1}$$

or, $dx+dp=0$

or, $x+p = a$ [By integrating]

$\therefore p = a-x \quad \cdots\cdots (ii)$

Now Solving (i) and (ii) we get,

$$2z+(a-x)^2+qy+2y^2=0$$

or, $qy = -2z-(a-x)^2-2y^2$

or, $q = -\frac{2z}{y} - \frac{(a-x)^2}{y} - 2y$

We know,

$$dz = pdx + qdy$$

or, $dz = (a-x)dx + \left\{-\frac{2z}{y} - \frac{(a-x)^2}{y} - 2y\right\}dy$

or, $dz = (a-x)\,dx - \frac{2z}{y}\,dy - \frac{(a-x)^2}{y}\,dy - 2ydy$

or, $dz + \frac{2z}{y}\,dy = (a-x)\,dx - \frac{(a-x)^2}{y}\,dy - 2ydy$

or, $2y^2dz + 4yzdy = 2y^2(a-x)dx - 2y\,(a-x)^2dy - 4y^3\,dy$

[multiplying by $2y^2$ on both sides]

or, $2y^2dz + 4yzdy = -2y^2(x-a)dx - 2y\,(x-a)^2\,dy - 4y^3dy$

or, $2(y^2dz + z.2ydy) = -\{2y^2\,(x-a)\,dx + 2y\,(x-a)^2\,dy\} - 4y^3dy$

or, $2d\,(y^2z) = -d\,[y^2\,(x-a)^2] - 4y^3dy$

or, $2\int d\,(y^2z) = -\int d\,[y^2\,(x-a)^2] - \int 4y^3\,dy$

or, $2y^2z = -y^2\,(x-a)^2 - 4\,y^4/4 + c$ [By integrating]

or, $2y^2z = -y^2\,(x-a)^2 - y^4 + c$

or, $2y^2z + y^2\,(x-a)^2 + y^4 = c$

Which is the required complete integral of (1)

Home Work

⑥ Expand the Fourier series in $-\pi < x < \pi$ for $f(x) = x$

⑦ Expand the Fourier series in $-\pi < x < \pi$ for

$$f(x) = \begin{cases} -\frac{\pi}{4} ; & -\pi < x < 0 \\ \frac{\pi}{4} ; & 0 < x < \pi \end{cases}$$

⑧ Expand the Fourier series in $-\pi < x < \pi$ for

$$f(x) = \begin{cases} -1 ; & -\pi < x < 0 \\ 0 ; & x = 0 \\ 1 ; & 0 < x < \pi \end{cases}$$

⑨ Expand the Fourier series in $-\pi < x < \pi$ for

$$f(x) = \begin{cases} -x ; & -\pi < x < 0 \\ x ; & 0 < x < \pi \end{cases}$$

⑩ Expand the Fourier series for

$$f(x) = \begin{cases} 0 ; & -2 < x < 0 \\ 1 ; & 0 < x < 2 \end{cases}$$

Home Work :

Problem - 6 : Expand the Fourier series in $-\pi < x < \pi$ for $f(x) = x$

Sol^n: We know the Fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \;\dots\; (1)$$

Since $f(x) = x$

$\Rightarrow f(-x) = -x = -f(x)$

So, it is odd function

and $a_0 = 0$,

$a_n = 0$

Here,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{2}{\pi}\int_{0}^{\pi} x \sin nx \, dx$$

$$= \frac{2}{\pi}\left[x \cdot \left(-\frac{1}{n}\cos nx\right) - 1 \cdot \left(-\frac{1}{n^2}\sin nx\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{1}{n^2}\sin nx - \frac{x}{n}\cos nx\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{1}{n^2}\cancel{\sin n\pi} - \frac{\pi}{n}\cos n\pi - \frac{1}{n^2}\cancel{\sin 0} - 0\right]$$

$$= \frac{2}{\pi}\left(-\frac{\pi}{n}\cos n\pi\right)$$

$$= -\frac{2}{\pi} \cdot \frac{\pi}{n} (-1)^n$$

$$= -\frac{2}{n}(-1)^n$$

$$= \frac{2}{n}(-1)^{n+1}$$

Putting these values in (i) we get,

$$f(x) = \sum_{n=1}^{\alpha} \frac{2}{n}(-1)^{n+1} \sin nx$$

$$= \frac{2}{1}\sin x + \frac{2}{2}(-1)\sin 2x + \frac{2}{3}\sin 3x + \frac{2}{4}(-1)\sin 4x + \frac{2}{5}\sin 5x + \frac{2}{6}(-1)\sin 6x + \frac{2}{7}\sin 7x + \cdots\cdots$$

$$= 2\sin x - 2\frac{\sin 2x}{2} + 2\frac{\sin 3x}{3} - 2\frac{\sin 4x}{4} + 2\frac{\sin 5x}{5} - 2\frac{\sin 6x}{6} + 2\frac{\sin 7x}{7}\cdots\cdots$$

$$= 2\left[\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \frac{\sin 5x}{5} - \frac{\sin 6x}{6} + \frac{\sin 7x}{7}\cdots\right]$$

(Ans)

Problem-7: Expand the Fourier Series in $-\pi < x < \pi$ for

$$f(x) = \begin{cases} \pi/4 \,; & -\pi < x < 0 \\ \pi/4 \,; & 0 < x < \pi \end{cases}$$

Soln: We know the Fourier series is;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} (a_n \cos nx + b_n \sin nx) \cdots\cdots (1)$$

Here,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{4}\,dx + \frac{1}{\pi}\int_{0}^{\pi} \frac{\pi}{4}\,dx$$

$$= \frac{1}{\pi}\cdot\frac{\pi}{4}\int_{0}^{\pi} dx - \frac{1}{\pi}\cdot\frac{\pi}{4}\int_{-\pi}^{0} dx$$

$$= \frac{1}{4}\int_0^{\pi} dx - \frac{1}{4}\int_{-\pi}^{0} dx$$

$$= \frac{1}{4}[x]_0^{\pi} - \frac{1}{4}[x]_{-\pi}^{0}$$

$$= \frac{1}{4}(\pi - 0) - \frac{1}{4}(0+\pi)$$

$$= \frac{1}{4}\pi - \frac{1}{4}\pi$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{4}\cos nx \, dx + \frac{1}{\pi}\int_0^{\pi} \frac{\pi}{4}\cos nx \, dx$$

$$= -\frac{1}{\pi}\int_{-\pi}^{0} \cos nx \, dx + \frac{1}{\pi}\int_0^{\pi} \cos nx \, dx$$

$$= -\frac{1}{\pi}\left[\frac{1}{n}\sin nx\right]_{-\pi}^{0} + \frac{1}{4}\left[\frac{1}{n}\sin nx\right]_0^{\pi}$$

$$= -\frac{1}{4}\left[\frac{1}{n}\sin \overset{0}{0} - \frac{1}{n}\sin\overset{0}{(-n\pi)}\right] + \frac{1}{4}\left[\frac{1}{n}\sin \overset{0}{n\pi} - \frac{1}{n}\sin \overset{0}{0}\right]$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{4}\sin nx \, dx + \frac{1}{\pi}\int_0^{\pi} \frac{\pi}{4}\sin nx \, dx$$

$= -\frac{1}{4}\int_{-\pi}^{0} \sin nx\, dx + \frac{1}{4}\int_{0}^{\pi} \sin nx\, dx$

$= -\frac{1}{4}\left[-\frac{1}{n}\cos nx\right]_{-\pi}^{0} + \frac{1}{4}\left[-\frac{1}{n}\cos nx\right]_{0}^{\pi}$

$= -\frac{1}{4}\left[-\frac{1}{n}\cos 0 + \frac{1}{n}\cos(-n\pi)\right] + \frac{1}{4}\left[-\frac{1}{n}\cos n\pi + \frac{1}{n}\cos 0\right]$

$= -\frac{1}{4}\left[-\frac{1}{n} + \frac{1}{n}(-1)^n\right] + \frac{1}{4}\left[-\frac{1}{n}(-1)^n + \frac{1}{n}\right]$

$= -\frac{1}{4}\cdot\frac{1}{n}\left[-1 + (-1)^n\right] + \frac{1}{4}\cdot\frac{1}{n}\left[-(-1)^n + 1\right]$

$= \frac{1}{4n}\left[1 - (-1)^n + 1 - (-1)^n\right]$

$= \frac{1}{4n}\left[2 - 2(-1)^n\right]$

$= \frac{2}{4n}\left[1 - (-1)^n\right]$

$= \frac{1}{2n}\left[1 - (-1)^n\right]$

Putting these values in (1) we get,

$f(x) = \sum_{n=1}^{\alpha} \frac{1}{2n}\left[1 - (-1)^n\right]$

$= 2\cdot\frac{\sin x}{2\cdot1} + 0\cdot\frac{\sin 2x}{2\cdot2} + 2\cdot\frac{\sin 3x}{2\cdot3} + 0\cdot\frac{\sin 4x}{2\cdot4} + 2\cdot\frac{\sin 5x}{2\cdot5} + 0\cdot\frac{\sin 6x}{2\cdot6} + 2\cdot\frac{\sin 7x}{2\cdot7} + \cdots$

$= \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \cdots$ (Ans)

Problem - 8: Expand the Fourier series in $-\pi < x < \pi$ for

$$f(x) = \begin{cases} -1 & : -\pi < x < 0 \\ 0 & : x = 0 \\ 1 & : 0 < x < \pi \end{cases}$$

Sol^n: We know the Fourier series is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} (a_n \cos nx + b_n \sin nx) \quad \dots \text{①}$$

Here $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -1\, dx + \frac{1}{\pi} \int_{0}^{\pi} 1\, dx$$

$$= -\frac{1}{\pi} \int_{-\pi}^{0} dx + \frac{1}{\pi} \int_{0}^{\pi} dx$$

$$= -\frac{1}{\pi} [x]_{-\pi}^{0} + \frac{1}{\pi} [x]_{0}^{\pi}$$

$$= -\frac{1}{\pi} (0+\pi) + \frac{1}{\pi} (\pi - 0)$$

$$= -\frac{\pi}{\pi} + \frac{\pi}{\pi}$$

$$= 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, . dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} -1 \cos nx\, dx + \frac{1}{\pi} \int_{0}^{\pi} 1 \cdot \cos nx\, dx$$

$= -\frac{1}{\pi}\int_{-\pi}^{0}\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}\cos nx\,dx$

$= -\frac{1}{\pi}\left[\frac{1}{n}\sin nx\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{1}{n}\sin nx\right]_{0}^{\pi}$

$= -\frac{1}{\pi}\left[\frac{1}{n}\cancel{\sin 0}^{0} - \frac{1}{n}\sin(-\cancel{n\pi})^{0}\right) + \frac{1}{\pi}\left[\frac{1}{n}\cancel{\sin n\pi}^{0} - \frac{1}{n}\cancel{\sin 0}^{0}\right]$

$= 0$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} -1.\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} 1.\sin nx\,dx$

$= -\frac{1}{\pi}\int_{-\pi}^{0}\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}\sin nx\,dx$

$= -\frac{1}{\pi}\left[-\frac{1}{n}\cos nx\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{1}{n}\cos nx\right]_{0}^{\pi}$

$= -\frac{1}{\pi}\left[-\frac{1}{n}\cos 0 + \frac{1}{n}\cos(-n\pi) + (-\frac{1}{n}\cos n\pi + \frac{1}{n}\cos 0\right]$

$= -\frac{1}{\pi}\left[-\frac{1}{n} + \frac{1}{n}(-1)^n\right] + \frac{1}{\pi}\left[-\frac{1}{n}(-1)^n + \frac{1}{n}\right]$

$= -\frac{1}{\pi n}\{(-1)^n - 1\} + \frac{1}{\pi n}\{-(-1)^n + 1\}$

$= \frac{1}{\pi n}\{-(-1)^n + 1 - (-1)^n + 1\}$

$= \frac{1}{\pi n}\{2 - 2(-1)^n\}$

$= \frac{2}{n\pi}\{1 - (-1)^n\}$

Putting these values in (1), we get,

$$f(x) = \sum_{n=1}^{\alpha} \frac{2}{n\pi}\{1-(-1)^n\} \sin nx$$

$$= \frac{2.2}{1.\pi}\sin x + \frac{2.0}{2.\pi}\sin 2x + \frac{2.2}{3.\pi}\sin 3x + \frac{2.0}{4.\pi}\sin 4x + \frac{2.2}{5.4}\sin 5x$$

$$+ \frac{2.0}{6.\pi}\sin 6x + \frac{2.2}{7.\pi}\sin 7x + \dots$$

$$= \frac{4}{\pi}\sin x + \frac{4}{3\pi}\sin 3x + \frac{4}{5\pi}\sin 5x + \frac{4}{7\pi}\sin 7x + \dots$$

$$= \frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \frac{\sin 7x}{7} + \dots\right) \text{ Ans:}$$

Problem - 9 Expand the Fourier series in $-\pi < x < \pi$ for

$$f(x) = \begin{cases} -x; & -\pi < x < 0 \\ x; & 0 < x < \pi \end{cases}$$

Soln: We know the Fourier series is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} (a_n \cos nx + b_n \sin nx) \dots (1)$$

Here,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -x\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,dx$$

$$= -\frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,dx$$

$$= -\frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= -\frac{1}{\pi}\left[0 - \frac{(-\pi)^2}{2}\right] + \frac{1}{\pi}\cdot\left[\frac{\pi^2}{2} - 0\right]$$

$$= -\frac{1}{\pi}\left(-\frac{\pi^2}{2}\right) + \frac{1}{\pi}\cdot\frac{\pi^2}{2}$$

$$= \frac{\pi}{2} + \frac{\pi}{2}$$

$$= \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -x\cdot\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cdot\cos nx\,dx$$

$$= -\frac{1}{\pi}\int_{-\pi}^{0} -x\cdot\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$$= -\frac{1}{\pi}\left[x\cdot\frac{1}{n}\sin nx - 1\cdot\left(-\frac{1}{n^2}\cos nx\right]_{-\pi}^{0} + \frac{1}{\pi}\left[x\cdot\frac{1}{n}\sin nx - 1\cdot\left(-\frac{1}{n^2}\cos nx\right)\right]_{0}^{\pi}$$

$$= -\frac{1}{\pi}\left[\frac{x}{n}\sin nx + \frac{1}{n^2}\cos nx\right]_{-\pi}^{0} + \frac{1}{\pi}\ \frac{x}{n}\sin nx + \frac{1}{n^2}\cos nx\Big]_{0}^{\pi}$$

$$= -\frac{1}{\pi}\left[0 + \frac{1}{n^2}\cos 0 + \frac{\pi}{n}\cdot\sin(-\cancel{n}\pi) - \frac{1}{n^2}\cos(-n\pi)\right] + \frac{1}{\pi}\left[\frac{1}{\pi}\left[\frac{\pi}{n}\sin\right.\right.$$

$$+ \frac{1}{n^2}\cos n\pi - 0 - \frac{1}{n^2}\cos 0\Big]$$

$$= -\frac{1}{\pi}\left[\frac{1}{n^2} - \frac{1}{n^2}(-1)^n\right] + \frac{1}{\pi}\left[\frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right]$$

$$= -\frac{1}{\pi n^2}\left\{1 - (-1)^n\right\} + \frac{1}{\pi n^2}\left\{(-1)^n - 1\right\}$$

$= \frac{1}{\pi n^2}\{(-1)^n - 1 - 1 + (-1)^n\}$

$= \frac{1}{\pi n^2}\{2(-1)^n - 2\}$

$= \frac{2}{\pi n^2}\{(-1)^n - 1\}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx \, dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} -x \cdot \sin nx + \frac{1}{\pi}\int_{0}^{\pi} x \cdot \sin nx \, dx$

$= -\frac{1}{\pi}\int_{-\pi}^{0} x \cdot \sin nx + \frac{1}{\pi}\int_{0}^{\pi} x \cdot \sin nx \, dx$

$= -\frac{1}{\pi}\left[x \cdot \left(-\frac{1}{n}\cos nx\right) - 1 \cdot \left(-\frac{1}{n^2}\sin nx\right)\right]_{-\pi}^{0} + \frac{1}{\pi}\left[x \cdot \left(-\frac{1}{n}\cos nx\right) - 1 \cdot \left(\frac{1}{n^2}\sin nx\right)\right]_{0}^{\pi}$

$= -\frac{1}{\pi}\left[-\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_{-\pi}^{0} + \frac{1}{\pi}\left[-\frac{x}{n}\cos nx + \frac{1}{n^2}\sin nx\right]_{0}^{\pi}$

$= -\frac{1}{\pi}\left[-0 + \frac{1}{n^2}\cancel{\sin 0}^{0} - \frac{\pi}{n}\cos(-n\pi) - \frac{1}{n^2}\cancel{\sin(-n\pi)}^{0}\right] + \frac{1}{\pi}\left[-\frac{\pi}{n}\cos n\pi + \frac{1}{n^2}\cancel{\sin n\pi}^{0} + 0 - \frac{1}{n^2}\cancel{\sin 0}^{0}\right]$

$= -\frac{1}{\pi}\left\{-\frac{\pi}{n}(-1)^n\right\} + \frac{1}{\pi}\left\{-\frac{\pi}{n}(-1)^n\right\}$

$= \frac{1}{n}(-1)^n - \frac{1}{n}(-1)^n$

$= 0$

Putting these values in (1), we get,

$$f(x) = \pi/2 + \sum_{n=1}^{\alpha} \frac{2}{\pi n^2}\{(-1)^n - 1\}\cos nx$$

$$= \pi/2 + \frac{2}{\pi \cdot 1^2}(-2)\cos x + \frac{2}{\pi \cdot 2^2} \cdot 0 \cdot \cos 2x + \frac{2}{\pi \cdot 3^2} \cdot (-2)\cos 3x +$$

$$+ \frac{2}{\pi \cdot 4^2} \cdot 0 \cdot \cos 4x + \frac{2}{\pi \cdot 5^2}(-2)\cos 5x + \frac{2}{\pi \cdot 6^2} \cdot 0 \cdot \cos 6x + \frac{2}{\pi \cdot 7^2} \cdot (-2)\cos 7x + \cdots$$

$$= \pi/2 - 4/\pi \cos x - \frac{4}{\pi \cdot 3^2}\cos 3x - \frac{4}{\pi \cdot 5^2}\cos 5x - \frac{4}{\pi \cdot 7^2}\cos 7x \cdots$$

$$= \pi/2 - \frac{4}{\pi}\left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \frac{\cos 7x}{7^2} + \cdots\right]$$

Ans!

Problem : 10 Expand the Fourier series for

$$f(x) = \begin{cases} 0 \; ; & -2 < x < 0 \\ 1 \; ; & 0 < x < 2 \end{cases}$$

Sol^n: We know the Fourier series is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\alpha} \left(a_n \cos \frac{n\pi x}{2} + b_n \sin \frac{n\pi x}{2}\right) \;\dots\; (1)$$

Here,

$$a_0 = \frac{1}{2} \int_{-2}^{2} f(x)\, dx$$

$$= \frac{1}{2} \int_{-2}^{0} 0 \cdot dx + \frac{1}{2} \int_{0}^{2} 1 \cdot dx$$

$$= \frac{1}{2} \int_{0}^{2} dx$$

$$= \frac{1}{2} [x]_0^2$$

$$= \frac{1}{2} (2-0)$$

$$= 1$$

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\int_{-2}^{0} 0.\frac{\cos n\pi x}{2}\,dx + \frac{1}{2}\int_{0}^{2} 1.\cos\frac{n\pi x}{2}\,dx\}$$

$$= \frac{1}{2}\int_{0}^{2} 1.\cos\frac{n\pi}{2}.x\,dx$$

$$= \frac{1}{2}\left[\frac{1}{\frac{n\pi}{2}}\sin\frac{n\pi x}{2}\right]_0^2$$

$$= \frac{1}{2}\left[\frac{2}{n\pi}\sin\frac{n\pi x}{2}\right]_0^2$$

$$= \frac{1}{2}\left[\frac{2}{n\pi}\sin n\pi^{\nearrow 0} - \frac{2}{n\pi}\sin 0^{\nearrow 0}\right]$$

$$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\sin\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\int_{-2}^{0} 0.\sin\frac{n\pi x}{2} + 2/1\int_{0}^{2} 1.\sin\frac{n\pi x}{2}\,dx$$

$$= \frac{1}{2}\int_{0}^{2}\sin\frac{n\pi}{2}x\,dx$$

$$= \frac{1}{2}\left[-\frac{1}{\frac{n\pi}{2}}\cos\frac{n\pi x}{2}\right]_0^2$$

$$= \frac{1}{2}\left[-\frac{2}{n\pi}\cos\frac{n\pi x}{2}\right]_0^2$$

$$= \frac{1}{2}\left[-\frac{2}{n\pi}\cos n\pi + \frac{2}{n\pi}\cos 0\right]$$

$$= \frac{1}{2}\cdot\frac{2}{n\pi}(1-\cos n\pi)$$

$$= \frac{1}{n\pi}(1-\cos n\pi)$$

$$= \frac{1}{n\pi}\{1-(-1)^n\}$$

Putting these values in (1) we get.

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \{1 - (-1)^n\} \sin \frac{n\pi x}{2}$$

$$= \frac{1}{2} + \frac{1}{1.\pi} \cdot 2 \cdot \sin \frac{1.\pi x}{2} + \frac{1}{2.\pi} \cdot 0 \cdot \sin \frac{2.\pi x}{2} + \frac{1}{3\pi} \cdot 2 \cdot \sin \frac{3.\pi x}{2} + \frac{1}{4\pi} \cdot 0 \cdot \sin \frac{4.\pi x}{2} + \frac{1}{5\pi} \cdot 2 \cdot \sin \frac{5.\pi x}{2} + \frac{1}{6\pi} \cdot 0 \cdot \sin \frac{6.\pi x}{2} + \frac{1}{7\pi} \cdot 2 \cdot \sin \frac{7.\pi x}{2} + \cdots$$

$$= \frac{1}{2} + \frac{2}{\pi} \sin \frac{\pi x}{2} + \frac{2}{3\pi} \sin \frac{3\pi x}{2} + \frac{2}{5\pi} \sin \frac{5\pi x}{2} + \frac{2}{7\pi} \sin \frac{7\pi x}{2} + \cdots$$

$$= \frac{1}{2} + \frac{2}{\pi} \left[\sin \frac{\pi}{2} . x + \frac{1}{3} \sin \frac{3\pi}{2} . x + \frac{1}{5} \sin \frac{5\pi}{2} . x + \frac{1}{7} \sin \frac{7\pi}{2} . x + \cdots \right]$$

Ans!