Heaven's Light is Our Guide

DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering Second Year 2nd Semester Examination, 2016

Math-2181

Mathematics-III

Full marks: 72 | Time: 3 Hours

N.B:-
(i) Answer any SIX questions, taking THREE from each section.
(ii) Figures in the margin indicate full marks.
(iii) Use separate answer script for each section.
(iv) Assume reasonable value for any data missing.

SECTION-A

1st order

Q.1(a) Find the foci of the curve which satisfies the differential equation $(1+y^2)dx-xydy=0$ and passes through the point (1,0). 6

(b) If $M(xy)dx+N(xy)dy=0$ is a homogenous differential equation, then prove that the change of variable $y=vx$ transform this differential equation into a separable differential equation. 6

2nd order

Q.2 Define ordinary point and singular point. Solve the differential equation $2x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+(x-5)y=0$ by Forbenious method. 12

C-6

Q.3(a) Show that $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ is a solution of the Legendre's equation $(1-x2)y''-2xy'+n(x+1)y=0$ 6

(b) Show that $\int_{-1}^{1}\{Pn(x)\}^2dx=\frac{2}{2n+1}, n=0,1,2...$ where $P_n(x)$ represents the Legendre polynomial of order. 6

Bessel

Q.4(a) Prove that $Jn(x)=\frac{1}{\pi}\int_0^{\pi}\cos(n\theta-x\sin\theta)d\theta$ where $J_n(x)$ is the Bessels function. 6

(b) Prove that $J_{-5/2}(x)=\sqrt{\frac{2}{\pi x}}[\frac{3}{x}\sin x+\frac{3-x^2}{x^2}\cos x]$ 6

SECTION-B

Fourier series

Q.5(a) Find the Fourier series to represent the function $f(x)=e^x$ for $-\pi<x<\pi$ and hence derive a series for $\frac{\pi}{\sinh\pi}$. A → 854E 6

(b) Obtain Fourier series of the function $f(x)=1+\frac{2x}{\pi}, \quad -\pi<x<0$
$=1-\frac{2x}{\pi}, \quad 0<x<\pi$ 6

Q.6(a) Find the solution of the equation $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$ which satisfies the conditions $u(0,t)=u(1,t)=0$ and $u(x,0)=\varphi(x)$ A – 964 → MHU-(28, 32) 6

(b) Define Fourier sine and cosine transform. Find the Fourier transformation of $F(x)=e^{-|x|}$, where $-a\le x\le a$ 6

PDE

Q.7(a) Solve $(y+zx)p-(x+yz)q=x^2-y^2$ where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. (x, y, -z) $\frac{2}{1+\omega^2}$ 6

(b) Find integral surface of $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ which passes through $xz=a^3, y=0$ 6

Q.8(a) Find a complete and singular integral of $2xz-px^2-2qxy+pq=0$.

(b) Solve the Laplace's equation in two dimensional polar coordinate (r,θ) MHO (90) 6

****

PDE (Laplace eqn)

(x, y, -z)

$y^2+xyz-x^2-xy^2$

x^2-y^2

(r,θ)

$\frac{d}{dx}y_n = nxy$

Heaven's Light is Our Guide

DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

B.Sc. Engineering **Second Year Odd Semester Examination**, 2017

Math- 2101

Mathematics III

Full marks: 72 **Time: 3 Hours**

N.B:-
(i) Answer any **SIX** questions, taking **THREE** from each section.
(ii) Figures in the margin indicate full marks.
(iii) Use separate answer script for each section.
(iv) Assume reasonable value for any data missing.

SECTION-A

Q.1(a) A particle is projected with a velocity u at an angle $\propto$ to the horizon. Find the path. 4.00

(b) Prove that $\frac{1}{(x+y+1)^4}$ is an integrating factor of $(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$ and hence integrate the equation. 4.00

(c) Solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{p}(l - x)$, where a, R, p and l are constants subject to the conditions $y = 0, \frac{dy}{dx} = 0$ at $x = 0$ 4.00

Q.2 Find the power series solution of the differential equation $(2x^2 - 3)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + y = 0$ where $y(0) = 0$, $y'(0) = 5$ 12.00

→10

Q.3(a) Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\theta - xsin\theta)d\theta$. →6 6.00

→6

(b) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi n}}\left\{\frac{3-x^2}{x^2}sinx - \frac{3}{x}cosx\right\}$ 6.00

30

Q.4(a) Prove that $\int_{-1}^{+1} x^2 P_{n+1}(x)P_{n-1}(x)dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$ →4 4.00

(b) Prove that $(1 - 2xz + z^2)^{-\frac{1}{2}}$ is a solution of the equation $z\frac{\partial^2(zv)}{\partial z^2} + \frac{\partial}{\partial x}\left\{(1 - x^2)\frac{\partial v}{\partial x}\right\} = 0$ →4 4.00

(c) Using the recurrence relations, show that $(2n + 1)(x^2 - 1)P_n'(x) = n(n + 1)(P_{n+1}(x) - P_{n-1}(x))$ 4.00

SECTION-B

Q.5(a) Define Fourier series of a function. Obtain the Fourier co-efficient a_0, a_n and b_n. 4.00

(b) Show that an even function contains cosine terms and odd function contains sine term in its Fourier expansion. 4.00

(c) Find the Fourier series to represent the function $f(x) = e^x$ for $-\pi < x < \pi$ and hence derive a series for $\frac{\pi}{sinh\pi}$ 4.00

Q.6(a) Solve boundary value problem using Fourier transformation: 6.00

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}; x > 0, t > 0 \text{ and } u(0, t) = 0$$

→6

$$u(x, 0) = \begin{cases} 1, 0 < x < 1 \\ 0, \ x > 1 \end{cases}$$

(b) Find the solution of $\frac{\partial^2 u}{\partial x^2} = h^2\frac{\partial u}{\partial t}$ for which $u(0, t) = u(l, t) = 0$ and $u(x, 0) = sin\frac{\pi x}{l}$, by the method of variables separable. 6.00

→ 3

Q.7(a) Obtain a partial differential equation from $\phi\left(\frac{z}{x^2}, x^2 - y^2 + z^2\right) = 0$ →6 6.00

(b) Find the equation of the integral surface of $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which passes through the circle $z = 0, \ x^2 + y^2 = 2x$. 6.00

→3

Q.8(a) Solve $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$ by Charpit's method. →4 6.00

(b) Solve $\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{sin\varphi}\frac{\partial}{\partial\varphi}\left(sin\varphi\frac{\partial u}{\partial\varphi}\right) + \frac{1}{sin^2\varphi}\frac{\partial^2 u}{\partial\theta^2}\right] = 0$ →5 6.00

****