NASIM

CE-13

The differential equations $M(x,y)dx + N(x,y)dy = 0$ is called homogenous differential equation if $M(x,y)$ and $N(x,y)$ are homogenous and of same degree.

01. Solve the equation. $\{x\sqrt{x^2+y^2} - y^2\}dx + xydy = 0$

Soln.

We have, $\{x\sqrt{x^2+y^2} - y^2\}dx + xydy = 0$

$$\therefore \frac{dy}{dx} + \frac{x\sqrt{x^2+y^2} - y^2}{xy} = 0 \quad \text{--- (1)}$$

Let, $y = vx$

$$\frac{dy}{dx} = v + x.\frac{dv}{dx}$$

putting this value in (1) we get.

$$v + x\frac{dv}{dx} + \frac{x\sqrt{x^2+y^2} - y^2}{xy} = 0$$

$$\text{or, } v + x\frac{dv}{dx} + \frac{x\sqrt{x^2+v^2x^2} - v^2x^2}{x^2v} = 0$$

$$\text{or, } v + x\frac{dv}{dx} + \frac{x^2\{\sqrt{1+v^2} - v^2\}}{x^2v} = 0$$

$$\text{or, } v + x\frac{dv}{dx} + \frac{\sqrt{1+v^2} - v^2}{v} = 0$$

$$\text{or, } x\frac{dv}{dx} + \frac{\sqrt{1+v^2} - v^2 + v^2}{v} = 0$$

$$\text{or, } x\frac{dv}{dx} + \frac{\sqrt{1+v^2}}{v} = 0$$

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$$\Rightarrow \frac{dv}{dx}x = -\frac{\sqrt{1+v^2}}{v}$$

$$\Rightarrow \frac{v\,dv}{dx} = -\frac{\sqrt{1+v^2}}{x}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{\sqrt{1+v^2}}{v}$$

$$\Rightarrow \frac{v\,dv}{\sqrt{1+v^2}} = -\frac{dx}{x}$$

$$\Rightarrow \frac{\frac{1}{2}2v\,dv}{\sqrt{1+v^2}} = -\frac{dx}{x}$$

$\Rightarrow$ Integrating we get

$$\frac{1}{2}\int\frac{2v\,dv}{\sqrt{1+v^2}} + \int\frac{dx}{x} = c$$

$$\Rightarrow \frac{1}{2}2\sqrt{1+v^2} + \ln x = c$$

$$\Rightarrow \sqrt{1+v^2} + \ln x = c$$

$$\Rightarrow \sqrt{1+\frac{y^2}{x^2}} + \ln x = \ln k \quad [\text{Put } c = \ln k]$$

$$\Rightarrow \sqrt{x^2+y^2} = x\ln k/x$$

Answer:

2. Solve the equation $\{y+\sqrt{x^2+y^2}\}dx - x\,dy = 0$. Find the value of $y(1)=0$!!

Solⁿ. we have $\{y+\sqrt{x^2+y^2}\}dx - x\,dy = 0$

$$\Rightarrow \frac{y+\sqrt{x^2+y^2}}{x} - \frac{dy}{dx} = 0 \quad \text{———} (1)$$

Put $y = vx$,

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From (1) we get,

$$\frac{vx+\sqrt{x^2+v^2x^2}}{x} - v - x\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{x(v+\sqrt{1+v^2})}{x} - v - x\frac{dv}{dx} = 0$$

$$\Rightarrow \cancel{v}+\sqrt{1+v^2} - \cancel{v} - x\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{dx}{x} - \frac{dv}{\sqrt{1+v^2}} = 0$$

Integrating we get,

$$\int\frac{dx}{x} - \int\frac{dv}{\sqrt{1+v^2}} = 0$$

$$\ln x - \ln\{v+\sqrt{1+v^2}\} = \ln c$$

$$\Rightarrow \cancel{\log}\ln\frac{x}{v+\sqrt{1+v^2}} = \ln c$$

$$\Rightarrow \frac{x}{y/x+\sqrt{1+y^2/x^2}} = c$$

$$\therefore \frac{x^2}{y+\sqrt{x^2+y^2}} = c. \quad \text{Answer;}$$

$y(1)=0$

when, $x=1$ and $y=0$ then we get from ①

$$\frac{1}{0+\sqrt{1^2+0}}=c$$

$\therefore c=1$

The particular solution is $\frac{x^2}{y+\sqrt{x^2+y^2}}=1.$ Answer:

3. Solve the equation $(1+e^{x/y})dx+e^{x/y}(1-x/y)dy=0.$

Sol^n: we have,

$$(1+e^{x/y})dx+e^{x/y}(1-x/y)dy=0 \text{ ——— ①}$$

~~put~~, $$\frac{dx}{dy}+\frac{e^{x/y}(1-x/y)}{1+e^{x/y}}=0$$

Put, $x=yv$

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

from ① we get,

$$\frac{dx}{dy}+\frac{e^{x/y}(1-x/y)}{1+e^{x/y}}=0$$

$$\Rightarrow v+y\frac{dv}{dy}+\frac{e^{v}(1-v)}{1+e^{v}}=0$$

$$\Rightarrow v + y.\frac{dv}{dy} + \frac{(1+e^v-1)(1-v)}{1+e^v} = 0$$

$$\Rightarrow \cancel{v} + y\frac{dv}{dy} + 1-\cancel{v} - \frac{1-v}{1+e^v} = 0$$

$$\Rightarrow y\frac{dv}{dy} + \frac{1+e^v-1+v}{1+e^v} = 0$$

$$\Rightarrow y\frac{dv}{dy} + \frac{v+e^v}{1+e^v} = 0$$

$$\Rightarrow \left(\frac{1+e^v}{v+e^v}\right).dv + \frac{dy}{y} = 0$$

Integrating we get,

$$\int\left(\frac{1+e^v}{v+e^v}\right)dv + \int\frac{dy}{y} = 0$$

$$\Rightarrow \ln(v+e^v) + \ln y = \ln c$$

$$\Rightarrow y(v+e^v) = c$$

$$\Rightarrow y\left(x/y + e^{x/y}\right) = c$$

where c is an arbitary constant.

Answer.

4. Solve the equation, $y\,dx - x\,dy = \sqrt{x^2+y^2}\,dx$

Solⁿ. we have, $y\,dx - x\,dy = \sqrt{x^2+y^2}\,dx$

$$y - x\frac{dy}{dx} = \sqrt{x^2+y^2}$$

$$\Rightarrow x\frac{dy}{dx} = y - \sqrt{x^2+y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-\sqrt{x^2+y^2}}{x} \quad \text{——— } (1)$$

Putting, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

from (1) we get,

$$v + x.\frac{dv}{dx} = \frac{vx - \sqrt{x^2+v^2x^2}}{x}$$

$$\Rightarrow v + x.\frac{dv}{dx} = v - \sqrt{1+v^2}$$

$$\Rightarrow x\frac{dv}{dx} + \sqrt{1+v^2} = 0$$

$$\Rightarrow \frac{dv}{\sqrt{1+v^2}} + \frac{dx}{x} = 0$$

Integrating,

$$\int \frac{dv}{\sqrt{1+v^2}} + \int \frac{dx}{x} = 0$$

$\Rightarrow \ln |v+\sqrt{1+v^2}| + \ln x = \ln c$

$\Rightarrow \ln x(v+\sqrt{1+v^2}) = \ln c$

$\Rightarrow x(v+\sqrt{1+v^2}) = c$

$\Rightarrow x\left(\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right) = c$

$\therefore (y+\sqrt{x^2+y^2}) = c.$ Answer:

5. Solve the equation $\frac{y}{x}\cos\frac{y}{x}\,dx = \left(\frac{x}{y}\sin\frac{y}{x}+\cos\frac{y}{x}\right)dy$

Soln. we have,

$$\frac{y}{x}\cos\frac{y}{x}\,dx = \left(\frac{x}{y}\sin\frac{y}{x}+\cos\frac{y}{x}\right)dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{y}{x}\cos\frac{y}{x}}{\frac{x}{y}\left\{\sin\frac{y}{x}+\cos\frac{y}{x}\right.} \quad \text{------ (1)}$$

Put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

from (1) we get,

$$v + x\frac{dv}{dx} = \frac{v\cos v}{\frac{1}{v}\sin v+\cos v} = \frac{v^2\cos v}{\sin v+v\cos v}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v(\sin v+v\cos v-\sin v)}{\sin v+v\cos v}$$

$$v + x\frac{dv}{dx} = v - \frac{v\sin v}{\sin v + v\cos v}.$$

$$\Rightarrow x.\frac{dv}{dx} + \frac{v\sin v}{\sin v + v\cos v} = 0$$

$$\Rightarrow \left(\frac{\sin v + v\cos v}{v\sin v}\right) dv + \frac{dx}{x} = 0$$

Integrating we get.

$$\int \frac{1}{v} dv + \int \frac{\cos v}{\sin v} dv + \int \frac{dx}{x} = \cancel{\ln c}\ 0$$

$$\Rightarrow \ln v + \ln \sin v + \ln x = \ln c$$

$$\Rightarrow \ln xv\sin v = \ln c$$

$$\Rightarrow xv\sin v = c$$

$$\Rightarrow y\sin y/x = c.$$ Answer:

6. Solve the equation $x^2 dy + (x^2 - xy + y^2) dx = 0$

Soln. we have $x^2 dy + (x^2 - xy + y^2) dx = 0$

$$\frac{dx}{dy} + \frac{x^2}{(x^2 - xy + y^2)} = 0 \quad \text{——— (1)}$$

Put, $x = vy$

$$\frac{dx}{dy} = v + y.\frac{dv}{dy}$$

from ① we get,

$$v + y\frac{dv}{dy} + \frac{v^2y^2}{v^2y^2 - vy^2 + y^2} = 0$$

$$\Rightarrow v + y\cdot\frac{dv}{dy} + \frac{\cancel{y^2}\, v^2}{v^2 - v + 1} = 0$$

$$\Rightarrow y\frac{dv}{dy} + \frac{\cancel{v^2} + v^3 - \cancel{v^2} + v}{v^2 - v + 1} = 0$$

$$\Rightarrow y\frac{dv}{dy} + \frac{v^3 + v}{v^2 - v + 1} = 0$$

$$\Rightarrow \left(\frac{v^2 - v + 1}{v^3 + v}\right) dv + \frac{dy}{y} = 0$$

$$\Rightarrow \frac{v^2 + 1 - v}{v(v^2 + 1)}\, dv + \frac{dy}{y} = 0$$

$$\Rightarrow \left(\frac{1}{v} - \frac{1}{v^2 + 1}\right) dv + \frac{dy}{y} = 0$$

Integrating we get,

$$\int \frac{dv}{v} - \int \frac{dv}{v^2 + 1} + \int \frac{dy}{y} = \ln c$$

$$\Rightarrow \ln v - \tan^{-1} v + \ln y = \ln c$$

$$\Rightarrow \ln \frac{vy}{\cancel{\tan^{-1} v}\; c} = \cancel{\ln c}\; \tan^{-1} v$$

$$\Rightarrow \frac{vy}{c} = e^{\tan^{-1} v}$$

$$\Rightarrow \frac{vy}{c} = e^{\tan^{-1} v}$$

$$\Rightarrow vy = ce^{\tan^{-1} v}$$

$$\Rightarrow x = ce^{\tan^{-1} x/y}$$

Answer:

Solve the equation $\{x^3 + y^2\sqrt{x^2+y^2}\}dx - xy\sqrt{x^2+y^2}\,dy = 0$

Solⁿ, we have,

$$\{x^3 + y^2\sqrt{x^2+y^2}\}\,dx - xy\sqrt{x^2+y^2}\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} - \frac{\{x^3 + y^2\sqrt{x^2+y^2}\}}{xy\sqrt{x^2+y^2}} = 0 \quad \text{—— (1)}$$

Puts, $y = vx$

$$\frac{dy}{dx} = v + x\,dv/dx$$

from (1) we get,

$$v + x\,dv/dx - \frac{x^3 + y^2\sqrt{x^2+y^2}}{xy\sqrt{x^2+y^2}} = 0$$

$$\Rightarrow v + x\,dv/dx - \frac{1 + v^2\sqrt{1+v^2}}{x^3 v\sqrt{1+v^2}} = 0$$

$$\Rightarrow v + x.\frac{dv}{dx} - \frac{1+v^2\sqrt{1+v^2}}{v\sqrt{1+v^2}} = 0$$

$$\Rightarrow x\frac{dv}{dx} + \frac{v^2\sqrt{1+v^2} - 1 - v^2\sqrt{1+v^2}}{v\sqrt{1+v^2}} = 0$$

$$\Rightarrow x\, {dv}/{dx} - \frac{1}{v\sqrt{1+v^2}} = 0$$

$$\Rightarrow \left(v\sqrt{1+v^2}\right) dv - \frac{dx}{x} = 0$$

Integrating we get,

$$\int v\sqrt{1+v^2}\,dv - \int {dx}/{x} = \cancel{\ln c}\; 0$$

$$\Rightarrow \frac{1}{2}\int 2v\sqrt{1+v^2}\,dv - \ln x = \ln c$$

$$\Rightarrow \frac{1}{3}(1+v^2)^{3/2} - \ln x = \ln c$$

$$\Rightarrow \frac{1}{3}(1+{y^2}/{x^2})^{3/2} = \ln(cx)$$

$$\Rightarrow {1}/{3}\,(x^2+y^2)^{3/2} = x^3 \ln(cx)$$

Answer:

8. $(\sqrt{x+y}-\sqrt{x-y})\,dx+(\sqrt{x-y}-\sqrt{x+y})\,dy=0$

Soln. we have,

$$(\sqrt{x+y}-\sqrt{x-y})\,dx+(\sqrt{x-y}-\sqrt{x+y})\,dy=0$$

$$\Rightarrow \frac{dy}{dx}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x-y}-\sqrt{x+y}}=0 \quad \text{---} \quad (1)$$

Put. $y=vx$

$$\frac{dy}{dx}=v+x.\,dv/dx$$

$$v+x\,dv/dx+\frac{\sqrt{1+v}-\sqrt{1-v}}{\sqrt{1-v}-\sqrt{1+v}}=0$$

or, $v+x.\,dv/dx-1=0$

or, $x.\dfrac{dv}{dx}+v-1=0$

or, $\dfrac{dv}{v-1}+\dfrac{dx}{x}=0$

or, $\displaystyle\int\frac{dv}{v-1}+\int\frac{dx}{x}=\int d\ln(c)$

or, $\ln(v-1)+\ln x=\ln c$

or, $\ln(v-1)x=\ln c$

or, $(v-1)x=c$

or, $(y/x-1)\,x=c$

$\therefore$ $y-x=c$. Answer:

Solve the equation $x\frac{dy}{dx} - y = 2\sqrt{x^2+y^2}$

Soln. we have,

$$x\cdot\frac{dy}{dx} - y = 2\sqrt{x^2+y^2} \qquad \frac{dy}{dx} = \frac{y+2\sqrt{x^2+y^2}}{x} \quad \text{——— (1)}$$

Put,

$$y = xv$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

from (1) we get,

$$v + x\frac{dv}{dx} = \frac{vx + 2\sqrt{x^2+v^2x^2}}{x}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + 2\sqrt{1+v^2}$$

$$\Rightarrow \frac{dv}{2\sqrt{1+v^2}} = \frac{dx}{x}$$

Integrating we get,

$$\int \frac{dv}{2\sqrt{1+v^2}} = \int \frac{dx}{x}$$

Put, $v = \tan\theta$

$$\frac{dv}{d\theta} = \sec^2\theta.$$

$$dv = \sec^2\theta\, d\theta$$

$$\theta = \tan^{-1} v$$

$$\Rightarrow \int \frac{\sec^2\theta\, d\theta}{2\sqrt{1+\tan^2\theta}} = \ln x + c$$

$$\Rightarrow \frac{1}{2}\int \frac{\sec^2\theta\, d\theta}{\sec\theta} = \ln x + c$$

$$\Rightarrow \frac{1}{2}\int \sec\theta\, d\theta = \ln x + c.$$

$\Rightarrow \frac{1}{2}\sec\theta\tan\theta = \ln x + c$

$\Rightarrow \frac{1}{2} v \sec\tan^{-1}(v) = \ln x + c$

$\Rightarrow \frac{1}{2}(y/x)\sec\{\tan^{-1}(y/x)\} = \ln x + c$

$\Rightarrow y\sec\{\tan^{-1}(y/x)\} = 2x\ln x + 2xc$

Answer:

10. Solve the equation: $y\sqrt{x^2+y^2}\,dx - x(x+\sqrt{x^2+y^2})\,dy = 0$

Soln, we have,

$$y\sqrt{x^2+y^2}\,dx - x(x+\sqrt{x^2+y^2})\,dy = 0$$

$$\Rightarrow \frac{y\sqrt{x^2+y^2}}{x(x+\sqrt{x^2+y^2})} - \frac{dy}{dx} = 0 \quad (1)$$

Let, $y = vx$

$$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$$

from (1) we get,

$$\frac{v\sqrt{1+v^2}}{1+\sqrt{1+v^2}} - v - x\cdot\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{v\cancel{\sqrt{1+v^2}} - v - v\cancel{\sqrt{1+v^2}}}{\cancel{x}\,1+\sqrt{1+v^2}} - x\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{v}{1+\sqrt{1+v^2}} + x\frac{dv}{dx} = 0$$

$$\Rightarrow \frac{dx}{x} + \left(\frac{1+\sqrt{1+v^2}}{v}\right) dv = 0$$

Integrating we get,

$$\int \frac{dx}{x} + \int \frac{dv}{v} + \int \frac{\sqrt{1+v^2}}{v}\, dv = \cancel{0}\ \ln c$$

$$\Rightarrow \ln x + \ln v + \frac{\frac{1}{v}\sqrt{1+\frac{1}{v^2}}}{2} + \frac{1}{2}\ln\left\{\frac{1}{v} + \sqrt{1+\frac{1}{v^2}}\right\} = \ln$$

where $v = y/x$

Answer.

11 $(x^2+y^2)\,dx + 2xy\,dy = 0$

We have, $\frac{dy}{dx} = -\frac{x^2+y^2}{2xy}$ ——— ①

Put, $y = vx$

$\frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$, from ① we get,

$$v + x\cdot\frac{dv}{dx} = -\frac{x^2+x^2v^2}{2x\cdot vx} = -\frac{1+v^2}{2v}$$

or, $x\frac{dv}{dx} = -\frac{1+v^2}{2v} - v = -\frac{1+3v^2}{2v}$

$\therefore \frac{dx}{x} = -\frac{2v}{1+3v^2}dv$

Integrating,

$\log x + \frac{1}{3}\log(1+3v^2) = \log c$

or, $x(1+3v^2)^{1/3} = c$

or, $x(1+3\,y^2/x^2)^{1/3} = c$. Answer:

12. Solve $x^2y\,dx - (x^3+y^3)\,dy = 0$

$$\frac{dy}{dx} = \frac{x^2y}{x^3+y^3} \text{ —— (1)}$$

Put, $y = vx$,

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

from (1) we get,

$v + x\frac{dv}{dx} = \frac{v}{1+v^3}$

$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$

or, $\frac{dx}{x} = -\left(\frac{1+v^3}{v^4}\right)dv$

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or, $\frac{dx}{x} = -\left[\frac{1}{v^4} + \frac{1}{v}\right] dv$

Integrating,

$\log x = \frac{1}{3v^3} - \log x + c$

$\Rightarrow \log vx = \frac{1}{3v^3} + c$

$\Rightarrow \log y = \frac{x^3}{3y^3} + c.$ Answer.

13. $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ —— (1)

Put, $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

from (1) we get,

$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$

or, $x\frac{dv}{dx} = \frac{v}{v-1}$

or, $\frac{dx}{x} = \left(\frac{v-1}{v}\right) dv$

or, $\frac{dx}{x} = (1-\frac{1}{v})\,dv$

Integrating,

$\log x = v - \log v + \log c$

$\log xv = v + \log c$

or, $xv = c.e^{v}$

$\therefore y = c.e^{y/x}$

Answer:

14. $\frac{dy}{dx} = \frac{y^3+3x^2y}{x^3+3xy^2}$ (1)

Put, $y = vx$, $\frac{dy}{dx} = v + x.\frac{dv}{dx}$

from (1), $x\frac{dv}{dx} = \frac{v^3+3v}{1+3v^2} - v = \frac{2v(1-v^2)}{1+3v^2}$

or, $\frac{2dx}{x} = \frac{1+3v^2}{2v(1-v^2)}\,dv$

or, $2\frac{dx}{x} = \left(\frac{1}{v} - \frac{2}{1+v} + \frac{2}{1-v}\right)dv$

Integrating

$2\log x = \log v - 2\log(1+v) + 2\log(1-v) + \log c$

$\Rightarrow x^2(1-v)^2(1+v)^2 = cv$

Answer

GENERAL DIFFERENTIAL SOLUTION

ACTIVE

Date : 7-8-15

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A differential equation is an equation which involves derivatives. for example:-

$$\frac{dy}{dx} = x+5$$

Ordinary differential equation:

Differential equations are those which involves only one independent variable and the derivatives are ordinary derivatives. Then the diff. equ. is called ordinary diff. equ.

Partial diff. equ:

Diff. equ. are those which involves two or more than two independent variables and derivatives are partial derivatives. Then the diff. equ. is called partial diff. equ.

Order of a diff. equ:

The order of a differential equ. is defined to be the order of the highest derivatives in differential equation

For example -

$(y'')^2 + (y')^3 + 3y = x^2$ has order 2.

Degree of a differential equ:

Deg. of a diff. equ. is the power of the highest differential coefficient when the equ. has been made rational (equation free from radicals and fraction).

For example –

$xy + y = 3$ has degree one.

1. Show, $y = c_1 \cos x + c_2 \sin x$ satisfies the D.E.

$$\frac{d^2y}{dx^2} + y = 0$$

$$y = c_1 \cos x + c_2 \sin x \quad \text{— (1)}$$

diff. w. r. to. x,

$$\frac{dy}{dx} = -c_1 \sin x + c_2 \cos x \quad \text{— (2)}$$

Again diff. w. r. to. x

$$\frac{d^2y}{dx^2} = -c_1 \cos x - c_2 \sin x \quad \text{— (3)}$$

from (3),

$$\frac{d^2y}{dx^2} = -(c_1 \cos x + c_2 \sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y$$

$$\therefore \frac{d^2y}{dx^2} + y = 0 \quad \text{[showed]}$$

Show, $y = 3\cos x + 2\sin x$ satisfies D.E. $\frac{d^2y}{dx^2} + y = 0$.

$y = 3\cos x + 2\sin x$ —— (1)

Diff. w.r.to. x

$$\frac{dy}{dx} = -3\sin x + 2\cos x$$

Again. diff. w.r.to.x

$$\frac{d^2y}{dx^2} = -3\cos x - 2\sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -(3\cos x + 2\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$ [Showed].

Find the D.E of a straight line passing through the origin

If Any diff. equ. of straight line passing through the origin Then we get the equ -

$$y = mx \text{ —— (1)}$$

Diff. w.r.to. x.

$$\frac{dy}{dx} = m \therefore m = \frac{dy}{dx} \text{ —— (2)}$$

putting the value of m in (1)

$$y = \frac{dy}{dx} \cdot x$$

$$y\,dx - x\,dy = 0$$

which is the required general equation.

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4. Find the D.E. $y = Ax^2 + Bx + C$.

$$y = Ax^2 + Bx + C \text{ ———— } (1)$$

Diff. w.r.to. x

$$\frac{dy}{dx} = 2Ax + B \text{ ———— } (2)$$

Again. diff. w.r.to. x.

$$\frac{d^2y}{dx^2} = 2A \text{ ———— } (3)$$

also diff. w.r.to. x.

$$\frac{d^3y}{dx^3} = 0$$

which is the required general equation.

5. Obtain the D.E. associated with the given primitive $y = c_1e^{2x} + c_2e^{x} + c_3$.

$$y = c_1e^{2x} + c_2e^{x} + c_3 \text{ ———— } (1)$$

Diff. w.r.to. x

$$\frac{dy}{dx} = 2c_1e^{2x} + c_2e^{x} \text{ ———— } (2)$$

Again. diff. w.r.to x.

$$\frac{d^2y}{dx^2} = 4c_1e^{2x} + c_2e^{x} \text{ ———— } (3)$$

also diff. w.r.to. x

$$\frac{d^3y}{dx^3} = 8c_1e^{2x} + c_2e^{x} \text{ ———— } (4)$$

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(IV) − (II) $\Rightarrow$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 4c_1e^{2x} \quad \text{—— (5)}$$

(III) − (II) $\Rightarrow$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 2c_1e^{2x} \quad \text{—— (6)}$$

(5) − 2×(6) $\Rightarrow$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 2\left(\frac{d^2y}{dx^2} - \frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0. \quad \text{Answer:}$$

6. Find the D.E. of the family of parabola with foci at the origin and axis along the x-axis.

Any equ. of the family of parabola with foci at the origin and axis along the x-axis then we get,

$$y^2 = 4A(x+A) \quad \text{—— (1)}$$

Diff. w.r.to. x

$$2y\frac{dy}{dx} = 4A.$$

$$A = \frac{y\,dy}{2dx}$$

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Putting the value of A in ① we get,

$$y^2 = 4\left(\frac{y}{2}\frac{dy}{dx}\right)\left[x + \left(\frac{y}{2}\frac{dy}{dx}\right)\right]$$

$$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$y^2 - 2xy\frac{dy}{dx} - y^2\left(\frac{dy}{dx}\right)^2 = 0.$$

Which is the required general solution.

7. Find the D.Efferential term $y = c_1e^{3x} + c_2e^{2x} + c_3e^{x}$

$$y = c_1e^{3x} + c_2e^{2x} + c_3e^{x} \quad \text{———} ①$$

Diff. w.r.to. x..

$$\frac{dy}{dx} = 3c_1e^{3x} + 2c_2e^{2x} + c_3e^{x} \quad \text{———} ②$$

Again diff. w.r. to. x

$$\frac{d^2y}{dx^2} = 9c_1e^{3x} + 4c_2e^{2x} + c_3e^{x} \quad \text{———} ③$$

Also. diff. w.r. to. x

$$\frac{d^3y}{dx^3} = 27c_1e^{3x} + 8c_2e^{2x} + c_3e^{x} \quad \text{———} ④$$

② − ① ⇒

$$\frac{dy}{dx} - y = 2c_1e^{3x} + c_2e^{2x} \quad \text{———} ⑤$$

③ − ② ⇒

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 6c_1e^{3x} + 2c_2e^{2x} \quad \text{———} ⑥$$

④ − ③ ⇒

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 18c_1e^{3x} + 4c_2e^{2x} \quad \text{———} ⑦$$

⑥ − 2×⑤ ⇒

$$\left(\frac{d^2y}{dx^2} - \frac{dy}{dx}\right) - 2\left(\frac{dy}{dx} - y\right) = 2c_1e^{3x} \quad \text{———} ⑧$$

~~⑧~~ ⑦ − 2×⑥ ⇒

$$\left(\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2}\right) - 2\left(\frac{d^2y}{dx^2} - \frac{dy}{dx}\right) = 6c_1e^{3x} \quad \text{———} ⑨$$

⑨ − 3×⑧ ⇒

$$\left(\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2}\right) - 2\left(\frac{d^2y}{dx^2} - \frac{dy}{dx}\right) - 3\left\{\left(\frac{d^2y}{dx^2} - \frac{dy}{dx}\right) - 2\left(\frac{dy}{dx} - y\right)\right\} = 0$$

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

which is the required general solution.

8. Find the primitive $y = cx^2 + c^2$

$$y = cx^2 + c^2 \quad \text{——— (1)}$$

Diff. w.r. to x,

$$\frac{dy}{dx} = 2cx$$

$$c = \frac{1}{2x} \cdot \frac{dy}{dx}$$

Putting the values of c in (1) we get.

$$y = \frac{1}{2x} \cdot \frac{dy}{dx} x^2 + \left(\frac{1}{2x}\right)^2 \left(\frac{dy}{dx}\right)^2$$

$$= \frac{x}{2} \cdot \frac{dy}{dx} + \frac{1}{4x^2} \cdot \left(\frac{dy}{dx}\right)^2$$

$$4x^2 y = 2x^3 \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2.$$ Answer:

9. Solve the D.E $x = A \sin(y + B)$.

$$x = A \sin(y+B) \quad \text{——— (1)}$$

Diff. w.r.to. y we get

$$\frac{dx}{dy} = + A \cancel{\sin} \cos(y+B) \quad \text{——— (2)}$$

Again diff. w.r.to. y

$$\frac{d^2x}{dy^2} = -A \sin(y+B) \quad \text{——— (3)}$$

$$\frac{d^2x}{dy^2} = -x$$

$$\frac{d^2x}{dy^2} + x = 0$$ Answer:

10. Find the general solution of circle:

General ~~soln~~ equation of circle,

$$x^2+y^2+2gx+2fy+c=0 \quad \text{---} (1)$$

Diff. w. r. to. x.

$$2x+2y\cdot\frac{dy}{dx}+2g+2f\cdot\frac{dy}{dx}=0$$

Again. diff. w. r. to. x

$$2+2\left\{\left(\frac{dy}{dx}\right)^2+y\cdot\frac{d^2y}{dx^2}\right\}=0$$

Also. diff. w. r. to. x.

$$\left\{2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+y\cdot\frac{d^3y}{dx^3}\right\}=0$$

$$\therefore\ y\cdot\frac{d^3y}{dx^3}+3\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}=0$$

Answer:

11. Find the differential term $(x-\mu)^2+(y-\lambda)^2=r^2$, where λ and μ are arbitary constant.

We have, $(x-\mu)^2+(y-\lambda)^2=r^2$ --- (1)

Diff. w. r. to. x

$$2(x-\mu)+2(y-\lambda)y_1=0$$

$$y_1=-\frac{(x-\mu)}{(y-\lambda)}$$ --- A

Again diff. w.r. to. x,

$$y_2 = -\left[\frac{(y-\lambda) - (x-\mu)\cdot y_1}{(y-\lambda)^2}\right]$$

$$= -\left[\frac{(y-\lambda) + \frac{(x-\mu)^2}{y-\lambda}}{(y-\lambda)^2}\right]$$

$$= -\left[\frac{(y-\lambda)^2 + (x-\mu)^2}{(y-\lambda)^3}\right]$$

$$= -\frac{r^2}{(y-\lambda)^3} \text{ ——— } (3)$$

Now we have to eliminate λ

$$1 + y_1^2 = 1 + \frac{(x-\mu)^2}{(y-\lambda)^2}$$

$$= \frac{(y-\lambda)^2 + (x-\mu)^2}{(y-\lambda)^2}$$

$$1 + y_1^2 = \frac{r^2}{(y-\lambda)^2}$$

Squaring both sides from (3) we get,

$$(y_2)^2 = \frac{r^4}{(y-\lambda)^6} = \frac{r^4}{\frac{r^6}{(1+y_1^2)^3}}$$

$$(1+y_1^2)^3 = \frac{r^2}{(y_2)^2}$$

Answer:

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12. Find the D.E. $Ax^2+By^2=1$

$$Ax^2+By^2=1 \quad \text{——— (1)}$$

Diff. w.r. to. x we get,

$$2Ax + 2By\cdot\frac{dy}{dx} = 0$$

$$\therefore \quad Ax + By\frac{dy}{dx} = 0 \quad \text{——— (2)}$$

Again diff. w.r. to. x.

$$A + B.y.\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0 \quad \text{——— (3)}$$

from (2) we get,

$$By\frac{dy}{dx} = -Ax$$

$$\frac{y}{x}\frac{dy}{dx} = -\frac{A}{B}$$

$$\therefore \quad -\frac{A}{B} = \frac{y}{x}\cdot\frac{dy}{dx}$$

from (3) we get,

$$\frac{A}{B} + y\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$-\frac{y}{x}\cdot\frac{dy}{dx} + y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y\frac{dy}{dx}$$

Which is the required differential equation.

13. Find the D.E. $r = a(1+\cos\theta)$.

$$r = a(1+\cos\theta). \quad \text{——} (1)$$

Diff. w. r. to. θ.

$$\frac{dr}{d\theta} = -a\sin\theta \quad \text{——} (2)$$

from (1). $a = \frac{r}{1+\cos\theta}$

putting the value of a in (2) we get,

$$\frac{dr}{d\theta} = -\frac{r\sin\theta}{1+\cos\theta}$$

$$\frac{dr}{d\theta}(1+\cos\theta) + r\sin\theta = 0$$

Which is the required differential equ.

14. Find the D.E. of circle which passes through the origin and lies on the x-axis.

$$x^2+y^2+2gx+2fy+c=0 \quad \text{——} (1)$$

Since equ. (1) passes through the origin also Lies on the x-axis,

Then we get, $x^2+y^2+2gx=0$ —— (2)

Diff. w. r. to. x.

$$2x+2y\frac{dy}{dx}+2g=0$$

$$x+y\cdot\frac{dy}{dx}=-g$$

putting the value of g in (2) we get,

$$x^2+y^2+2x\left(-x-y\frac{dy}{dx}\right)=0$$

$$\Rightarrow x^2+y^2-2x^2-2xy\frac{dy}{dx}=0$$

$$\Rightarrow y^2-x^2-2xy\frac{dy}{dx}=0$$

which is the required diff. equation.

Form the d.E. that represents all parabolas each of which has a latus rectum $4a$ and whose axes are parallel to axis.

Equ. of the family of such parabola is

$(y-k)^2 = 4a(x-h)$, diff. $(y-k)\frac{dy}{dx} = 2a$ ——(1)

Diff. Again $(y-k)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ ——(2)

Putting value of K from (1) in (2) we get.

$$2a\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0.$$

Which is the required differential equation.

16. Form the d.E. of all parabolas whose axes are parallel to the axis of y.

Such parabola are given by

$$(x-h)^2 = 4a(y-k)$$

Diff. $(x-h) = 2a\frac{dy}{dx}$

Diff. again, $1 = 2a\cdot\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = \frac{1}{2a}$$

Diff. once again $\frac{d^3y}{dx^3} = 0.$

This is the required differential equation.

17. Form d.E. of all conics whose axis coincide with the axes of coordinates.

such conics are given by $ax^2+by^2=1$

Diff. $\frac{dy}{dx} = -\frac{ax}{by}$

Again diff. $\frac{d^2y}{dx^2} = -\frac{a}{b}\cdot\left(\frac{1}{y} - \frac{x}{y^2}\cdot\frac{dy}{dx}\right)$

i.e. $\frac{d^2y}{dx^2} = \frac{y}{x}\frac{dy}{dx}\cdot\left(\frac{1}{y} - \frac{x}{y^2}\cdot\frac{dy}{dx}\right)$

$$xy\cdot\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx}$$

Which is the required differential equation

SEPERABLE EQUATION.

ACTIVE
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1. Solve the equn. $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$ & verify if $x = \pi/2$, $y = 0$.

We have, $(e^y+1)\cos x\,dx + e^y \sin x\,dy = 0$ ——— (1)

Dividing both sides $(e^y+1)\sin x$, we get from (1)

$$\frac{\cos x}{\sin x}dx + \left(\frac{e^y}{e^y+1}\right)dy = 0$$

$$\Rightarrow \frac{\cos x\,dx}{\sin x} + \frac{(e^y+1-1)}{e^y+1}dy = 0$$

$$\Rightarrow \frac{\cos x\,dx}{\sin x} + dy - \frac{dy}{e^y+1} = 0$$

Integrating both sides we get,

$$\int \frac{\cos x\,dx}{\sin x} + \int dy - \int \frac{e^{-y}dy}{1+e^{-y}} = \int d(c)$$

$$\log \sin x + y + \log(1+e^{-y}) = c$$

$$\Rightarrow \log(1+e^{-y})\sin x + y = c$$

Answer:

2. Solve the equ. $\sin^{-1}\frac{dy}{dx} = x+y$.

We have,

$$\sin^{-1}\frac{dy}{dx} = x+y$$

$$\frac{dy}{dx} = \sin(x+y) \quad \text{——— (1)}$$

Put $x+y = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

Putting this value in (1) we get

$$\left(\frac{dv}{dx}-1\right) = \sin v$$

$$\frac{dv}{dx} = 1+\sin v$$

$$\frac{dv}{1+\sin v} = dx$$

Integrating we get;

$$\int \frac{dv}{1+\sin v} = \int dx$$

$$\Rightarrow \int \frac{(1-\sin v)\,dv}{1-\sin^2 v} = x+c$$

$$\Rightarrow \int \frac{(1-\sin v)\,dv}{\cos^2 v} = x+c$$

$\Rightarrow \int \sec^2 v\,dv - \int \tan v \sec v\,dv = x + c$

$\Rightarrow \tan v - \sec v = x + c$

$\Rightarrow \dfrac{\sin v - 1}{\cos v} = x + c$

$\Rightarrow \dfrac{\sin(x+y) - 1}{\cos(x+y)} = x + c$. Answer:

$(x+4)(y^2+1)\,dx + y(x^2+3x+2)\,dy = 0$

We have, $(x+4)(y^2+1)\,dx + y(x^2+3x+2)\,dy = 0$ — ①

dividing ① by $(y^2+1)(x^2+3x+2)$ we get

$\dfrac{x+4}{x^2+3x+2}\,dx + \dfrac{y\,dy}{y^2+1} = 0$

$\Rightarrow \dfrac{\frac{1}{2}(2x+3+5)}{x^2+3x+2}\,dx + \frac{1}{2}\dfrac{2y\,dy}{y^2+1} = 0$

Integrating we get,

$\frac{1}{2}\int \dfrac{(2x+3)\,dx}{x^2+3x+2} + \frac{5}{2}\int \dfrac{dx}{x^2+3x+2} + \frac{1}{2}\int \dfrac{2y\,dy}{y^2+1} = \frac{1}{2}\int d(c$

$\Rightarrow \log(x^2+3x+2) + 5\int \dfrac{dx}{(x+\frac{3}{2})^2 + 2 - \frac{9}{4}} + \log(y^2+1) = c$

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$$\Rightarrow \log(x^2+3x+2) + 5\int \frac{dx}{(x+3/2)^2-\left(\frac{1}{2}\right)^2} + \log(y^2+1) = c$$

$$\Rightarrow \log(x^2+3x+2) + \frac{5}{\frac{1}{2}\cdot 2}\log\left|\frac{x+3/2-1/2}{x+3/2+1/2}\right| + \log(y^2+1) = c$$

$$\Rightarrow \log(x^2+3x+2) + 5\log\left(\frac{x+1}{x+2}\right) + \log(y^2+1) = c$$

$$\Rightarrow \log\frac{(x^2+3x+2)(x+1)(y^2+1)}{(x+2)} = c$$

$$\Rightarrow (x^2+3x+2)(x+1)(y^2+1) = (x+2)e^{c}.$$

Answer:

4. $\sqrt{(x^2-1)(y^2-1)}\,dx + xy\,dy = 0$

We have.

$$\sqrt{(x^2-1)(y^2-1)}\,dx + xy\,dy = 0$$

$$\Rightarrow \frac{\sqrt{x^2-1}}{x}\,dx + \frac{y}{\sqrt{y^2-1}}\,dy = 0$$

Integrating we get,

$$\int \frac{\sqrt{x^2-1}}{x}\,dx + \frac{1}{2}\int \frac{2y}{\sqrt{y^2-1}}\,dy = \int d(c)$$

$$\Rightarrow \int \frac{(x^2-1)}{x\sqrt{x^2-1}}\,dx + \frac{1}{2}\,2\sqrt{y^2-1} = c$$

$$\Rightarrow \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{dx}{x\sqrt{x^2-1}} + \sqrt{y^2-1} = c$$

$$\Rightarrow \frac{1}{2}\int \frac{2x\,dx}{\sqrt{x^2-1}} - \sec^{-1}x + \sqrt{y^2-1} = c$$

$$\Rightarrow \sqrt{x^2-1} + \sqrt{y^2-1} - \sec^{-1}x = c.$$ Answer:

5. Solve the equ. $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

We have, $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$$= e^x . e^{-y} + x^2 . e^{-y}$$

$$\Rightarrow \frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow e^y\,dy = (e^x + x^2)\,dx$$

$$\Rightarrow (e^x + x^2)\,dx - e^y\,dy = 0$$

Q1.

Integrating we get,

$$\int (e^x + x^2)\,dx - \int e^y\,dy = \int d(c)$$

$$e^x + \frac{x^3}{3} - e^y = c.$$ Answer:

Solve the eqn. $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$.

We have, $y - x\frac{dy}{dx} = ay^2 \cancel{dx} + a\,dy/dx$

$$\Rightarrow\ y\,dx - x\,dy = ay^2\,dx + a\,dy$$

$$\Rightarrow\ (y - ay^2)\,dx = (x+a)\,dy$$

$$\Rightarrow\ \frac{dx}{x+a} = \frac{dy}{y - ay^2}$$

Integrating we get,

$$\int \frac{dx}{x+a} = \int \frac{dy}{y - ay^2} \quad \text{--- (1)}$$

$$\int \frac{dx}{x+a} = \int \frac{dy}{y(1-ay)}$$

Now, $\frac{1}{y(1-ay)} = \frac{A}{y} + \frac{B}{(1-ay)}$

$$1 = y\,A(1-ay) + By$$

if $y=0$ Then $A=1$

also. if $y=1/a$ then $B=a$.

$$\therefore \frac{1}{y(1-ay)} = \frac{1}{y} + \frac{a}{1-ay}$$

from ① we get,

$$\int \frac{dx}{x+a} = \int \frac{dy}{y} + \int \frac{a\,dy}{1-ay}$$

$$\log(x+a) = \log y - \log(1-ay) + c$$ Answer:

Solve the equ. $\sqrt{x+y+1}\,\frac{dy}{dx} = 1$

we have, $\sqrt{x+y+1}\; dy/dx = 1$ —— ①

put, $x+y+1 = v^2$

$$1+\frac{dy}{dx} = 2v\frac{dv}{dx}$$

$$\frac{dy}{dx} = 2v\frac{dv}{dx} - 1$$

from ① we get

$$v\left(2v\frac{dv}{dx} - 1\right) = 1$$

$$\Rightarrow 2v^2\frac{dv}{dx} - v = 1$$

$$\Rightarrow 2v^2dv - v\,dx = dx$$

$\Rightarrow 2v^2 dv = (v+1) dx$

$\Rightarrow \frac{2v^2}{v+1} dv = dx$

Integrating we get,

$$\int \frac{2v^2 dv}{v+1} = \int dx$$

$$\Rightarrow 2 \int \frac{(v^2-1) dv}{v+1} + 2\int \frac{dv}{v+1} = \int dx$$

$$\Rightarrow 2 \int \frac{(v+1)(v-1) dv}{v+1} + 2\int \frac{dv}{v+1} = \int dx$$

$$\Rightarrow 2. \int (v-1) dv + 2 \log(v+1) = x+c$$

$$\Rightarrow 2 \left(\frac{v^2}{2} - v\right) + 2\log(v+1) = x+c$$

$$\Rightarrow v^2 - 2v + 2\log(v+1) = x+c$$

putting the value of v

$$\Rightarrow (x+y+1) - 2\sqrt{x+y+1} + 2\log\left(\sqrt{x+y+1}+1\right) = x+c.$$

Answer

8. Solve the e.q.u. $\ln \frac{dy}{dx} = ax+by$

Sol^n: we have, $\ln \frac{dy}{dx} = ax+by$

$$\frac{dy}{dx} = e^{ax+by} \quad \text{——} \ (1)$$

Put; $ax+by = v$

$$a + b\frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right)$$

Putting this value in (1) we get

$$\frac{1}{b}\left(\frac{dv}{dx} - a\right) = e^{v}$$

$$\frac{dv}{dx} - a = be^{v}$$

$$\Rightarrow \frac{dv}{dx} = be^{v} + a$$

$$\Rightarrow \frac{dv}{dx} = be^{v} + a$$

$$\Rightarrow \frac{dv}{be^{v}+a} = dx, \text{ Integrating we get}$$

$$\Rightarrow \frac{1}{b} \int \frac{dv}{e^{v}+a/b} = \int dx$$

$$\Rightarrow \frac{1}{b} \int \frac{e^{-v}\,dv}{1+a/b\, e^{-v}} = \int dx$$

$$\frac{1}{b}\int \frac{e^{-v}dv}{1+\frac{a}{b}e^{-v}} = \int dx$$

Put; $1+\frac{a}{b}e^{-v}$

$-\frac{a}{b}e^{-v}\,dv = dt$

$$\Rightarrow -\frac{a}{b^2}\log\left(1+\frac{a}{b}e^{-v}\right) = x+c$$

$$\Rightarrow -\frac{a}{b^2}\log\left\{1+\frac{a}{b}e^{-(ax+by)}\right\} = x+c.$$

Answer:

9. Solve the equ. $x\sin y\,dx + (x^2+1)\cos y\,dy = 0$

$$x\sin y\,dx + (x^2+1)\cos y\,dy = 0$$

$$\frac{x}{x^2+1}dx + \frac{\cos y}{\sin y}dy = 0$$

Integrating we get,

$$\frac{1}{2}\int \frac{2x\,dx}{x^2+1} + \int \frac{\cos y\,dy}{\sin y} = \int d(c)$$

$$\Rightarrow \frac{1}{2}\log(x^2+1) + \log\sin y = c$$

$$\Rightarrow \frac{1}{2}\log(x^2+1) + \log\sin y = c$$

$$\Rightarrow \log\sqrt{1+x^2} + \log\sin y = c$$

$$\Rightarrow \log \sin y\sqrt{1+x^2} = c \quad \therefore \sin y\sqrt{1+x^2} = e^c = K$$

10. Solve the equ. $(1-x^2)(1-y)dx = xy(1+y)dy$.

Sol^n; we have

$$(1-x^2)(1-y)dx = xy(1+y)dy \quad \text{———} (1)$$

Dividing both sides by $(1-y)x$ we get;

$$\frac{(1-x^2)\,dx}{x} = y\left(\frac{1+y}{1-y}\right)dy$$

$$\Rightarrow \frac{1}{x}dx - x\,dx = \frac{y}{1-y}dy + \frac{y^2 dy}{1-y}$$

$$\Rightarrow \frac{1}{x}dx - x\,dx = \frac{-(1-y-1)}{1-y}dy - \frac{(1-y^2-1)}{1-y}dy$$

$$= -dy + \frac{dy}{1-y} - (1+y)dy + \frac{dy}{1-y}$$

$$= -2dy - y\,dy + \frac{2dy}{1-y}$$

Integrating we get;

$$\int \frac{1}{x}dx - \int x\,dx = -2\int dy - \int y\,dy + 2\int \frac{dy}{1-y}$$

$$\Rightarrow \log x - \frac{x^2}{2} = -2y - \frac{y^2}{2} - 2\log(1-y) + c.$$

Answer:)

NON HOMOGENEOUS DIFFERENTIAL EQUATION

1. Solve the equ. $(3x-4y-3)\frac{dy}{dx} = (3x-4y-2)$

We have, $(3x-4y-3)\frac{dy}{dx} = (3x-4y-2)$ ①

$$\Rightarrow \frac{dy}{dx} = \frac{3x-4y-2}{3x-4y-3} \quad \text{①}$$

Let $3x-4y = v$

$$3 - 4\frac{dy}{dx} = \frac{dv}{dx}$$

$$4\, dy/dx = 3 - dv/dx$$

$$dy/dx = \frac{1}{4}(3 - dv/dx).$$

Putting this value in ① we get,

$$\frac{1}{4}(3 - dv/dx) = \frac{v-2}{v-3}$$

$$\Rightarrow 3 - dv/dx = \frac{4(v-2)}{v-3}$$

$$\Rightarrow \frac{dv}{dx} = 3 - \frac{4(v-2)}{v-3}$$

$$\Rightarrow \frac{dv}{dx} = \frac{3v-9-4v+8}{v-3}$$

$$\Rightarrow \frac{dv}{dx} = \frac{-(v+1)}{v-3}$$

$$\Rightarrow \left(\frac{v-3}{v+1}\right) dv = -dx$$

Integrating we get,

$$\int \frac{3}{v+1}\,dv - \int \frac{v}{v+1}\,dv = \int dx$$

$$\Rightarrow 3\ln(v+1) - \int\left(\frac{v+1-1}{v+1}\right)dv = x+c$$

$$\Rightarrow 3\ln(v+1) - v + \ln(v+1) = x+c$$

$$\Rightarrow 4\ln(v+1) - v = x+c.$$

$$\Rightarrow 4\ln(3x-4y+1) - 3x+4y-x = c$$

$$\therefore 4\ln(3x-4y+1) - 4x+4y = c.$$ Answer:

2. Solve the equ. $\frac{dy}{dx} = \frac{x+y-1}{x+4y+2}$

We have, $\frac{dy}{dx} = \frac{x+y-1}{x+4y+2}$ —— (1)

Let,

$x = X+h, \quad y = Y+k$

$$\frac{dy}{dx} = \frac{dY}{dx} = \frac{dY/dx}{dX/dx} = \frac{dY}{dX}$$

from (1) we get

$$\frac{dY}{dX} = \frac{X+h+Y+k-1}{X+h+4Y+4k+2}$$ —— (2)

choose h and k such that,

$$h+k-1=0$$

and $h+4k+2=0$

$$\frac{h}{2+4} = \frac{k}{-1-2} = \frac{1}{4-1}$$

$$\frac{h}{6} = \frac{k}{-3} = \frac{1}{3}$$

$h = 2$ $\quad k = -1$

From (4) we get,

$$\frac{dy}{dx} = \frac{x+y}{x+4y}$$

Let, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

putting this value in (2) we get,

$$v + x\frac{dv}{dx} = \frac{x+vx}{x+4vx} = \frac{1+v}{1+4v}$$

$$x\frac{dv}{dx} = \frac{1+v}{1+4v} - v$$

$$= \frac{1+\cancel{v}-\cancel{v}-4v^2}{1+4v}$$

$$= \frac{1-4v^2}{1+4v}$$

$$\Rightarrow \left(\frac{1+4v}{1-4v^2}\right) dv = \frac{dx}{x}$$

$$\Rightarrow \frac{dv}{1-4v^2} + \frac{1}{2}\,\frac{8v}{1-4v^2}\,dv = \frac{dx}{x}$$

$$\Rightarrow \frac{dv}{4v^2-1} - \frac{1}{2}\,\frac{8v\,dv}{1-4v^2} = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{4}\int \frac{dv}{v^2 - 1/4} + \frac{1}{2}\int \frac{8v\,dv}{4v^2-1} = -\int \frac{dx}{x}$$

$$\Rightarrow \frac{1}{4}\,\frac{1}{2\cdot\frac{1}{2}}\,\ln\left|\frac{v-\frac{1}{2}}{v+\frac{1}{2}}\right| + \frac{1}{2}\ln(4v^2-1) = -\ln x + \frac{1}{4}\ln c$$

$$\Rightarrow \ln\left(\frac{2v-1}{2v+1}\right) + 2\ln(4v^2-1) = -4\ln x + \ln c.$$

Answer:

where $v = y/x = \frac{y-k}{x-h} = \frac{y+1}{x-2}$ where c is an arbitary constant

$$\ln\left(\frac{\frac{y+1}{x-2} - 1}{\frac{y+1}{x-2} + 1}\right) + 2\ln\left\{1 - 4\left(\frac{y+1}{x-2}\right)^2\right\} = -4\ln(x-2) +$$

$$\ln\left(\frac{y+1-x+2}{y+1+x-2}\right) + 2\ln\left\{1 - 4\left(\frac{y+1}{x-2}\right)^2\right\} = -4\ln(x-2) -$$

$$\ln\left(\frac{y-x+3}{y+x-1}\right) + 2\ln\left\{1 - 4\left(\frac{y+1}{x-2}\right)^2\right\} = -4\ln(x-2) +$$

Answer:

3 Solve $\frac{dy}{dx} = \frac{2y-x-4}{y-3x+3}$

we have, $\frac{dy}{dx} = \frac{2y-x-4}{y-3x+3}$ —— (1)

Put, $x = X+h$

$y = Y+k$ $\quad \frac{dy}{dx} = \frac{dY}{dX} = \frac{dY/dx}{dX/dx} = \frac{dY}{dX}$

$$\therefore \frac{dy}{dx} = \frac{dY}{dX}$$

from (1) we get,

$$\frac{dY}{dX} = \frac{2Y+2k-X-h-4}{Y+k-3X-3h+3}$$

$$= \frac{2Y-X+2k-h-4}{Y-3X+k-3h+3}$$

choose h and k such that,

$$2k-h-4=0$$
$$k-3h+3=0$$

Solving we get,

$$\frac{k}{-3-12} = \frac{h}{-4-6} = \frac{1}{-6+1}$$

$$\frac{k}{-15} = \frac{h}{-10} = \frac{1}{-5}$$

$$k=3, \quad h=2$$

$X = x-h = x-2$

$Y = y-k = y-3$

$$\frac{dY}{dX} = \frac{2Y-X}{Y-3X}$$

Put, $Y = vX$

$$\frac{dY}{dX} = v + X\frac{dv}{dX}$$

$$v + X\frac{dv}{dX} = \frac{2vX - X}{vX - 3X}$$

or, $X\frac{dv}{dX} = \frac{2v-1}{v-3} - v = \frac{2v-1-v^2+3v}{v-3}$

or, $X\frac{dv}{dX} = \frac{5v-v^2-1}{v-3}$

or, $\frac{(v-3)dv}{v^2-5v+1} = -\frac{dX}{X}$

or, $\frac{1}{2}\frac{(2v-5)dv}{v^2-5v+1} - \frac{1}{2}\frac{dv}{v^2-5v+1} = -\frac{dX}{X}$

or, $\frac{1}{2}\frac{(2v-5)dv}{v^2-5v+1} - \frac{1}{2}\frac{dv}{(v-5/2)^2-(\sqrt{21}/2)^2} = -\frac{dX}{X}$

Integrating we get,

$$\frac{1}{2}\int\frac{(2v-5)\,dv}{v^2-5v+1}-\frac{1}{2}\int\frac{dv}{\left(v-\frac{5}{2}\right)^2-\left(\frac{\sqrt{21}}{2}\right)^2}=-\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2}\log(v^2-5v+1)-\frac{1}{2}\cdot\frac{1}{2\cdot\frac{\sqrt{21}}{2}}\log\left\{\frac{v-\frac{5}{2}-\frac{\sqrt{21}}{2}}{v-\frac{5}{2}+\frac{\sqrt{21}}{2}}\right\}$$

$$=-\log x+c$$

$$\Rightarrow \log(v^2-5v+1)^{\frac{1}{2}}-\frac{1}{2\sqrt{21}}\log\left\{\frac{2v-5-\sqrt{21}}{2v-5+\sqrt{21}}\right\}=-\log x+c$$

where $y=vx$

$$v=y/x=\frac{y-3}{x-2}$$; Answer.

LINEAR DIFFERENTIAL EQUATION

ACTIVE
Date: 7-8-18
Page: 10'pm 24

1. Solve the equation $(1+x^2)\frac{dy}{dx}+y = e^{\tan^{-1}x}$ ——— ①

$$\Rightarrow \frac{dy}{dx}+\frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

This is the linear equ. in y.

Here $P(x) = \frac{1}{1+x^2}$, $Q(x) = \frac{e^{\tan^{-1}x}}{1+x^2}$

$$I.F. = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$$

Multiplying ① by I.F. we get,

$$\frac{dy}{dx}e^{\tan^{-1}x} + \frac{e^{\tan^{-1}x}}{1+x^2}y = \frac{\left(e^{\tan^{-1}x}\right)^2}{1+x^2}$$

Integrating we get,

$$y.e^{\tan^{-1}x} = \int \frac{\left(e^{\tan^{-1}x}\right)^2}{1+x^2}dx$$

Put, $\tan^{-1}x = z$

$\frac{dx}{1+x^2} = dz$

$$= \int (e^z)^2 dz$$

$$= \int e^{2z}dz$$

$$y.e^{\tan^{-1}x} = \frac{e^{2z}}{2} + c/2 = \frac{1}{2}e^{2\tan^{-1}x} + c/2$$

$$2ye^{\tan^{-1}x} = e^{2\tan^{-1}x} + c.$$

where c is an arbitrary constant.

2. Solve the equ. $(1+x^2)\frac{dy}{dx} + 4xy = x$

$$\frac{dy}{dx} + \left(\frac{4x}{1+x^2}\right) y = \frac{x}{1+x^2} \quad \text{--- (1)}$$

This is the linear equation of y

Here, $P(x) = \frac{4x}{1+x^2}$ $Q(x) = \frac{x}{1+x^2}$

$$I.F = e^{\int \frac{4x}{1+x^2} dx} = e^{2\int \frac{2x}{1+x^2} dx} = e^{2\ln(1+x^2)}$$

$$= (1+x^2)^2$$

multiplying (1) by integrating factor we get,

$$\frac{dy}{dx}(x^2+1)^2 + \frac{4x}{x^2+1}(x^2+1)^2 y = \frac{x(x^2+1)^2}{x^2+1}$$

$$\Rightarrow \frac{dy}{dx}(x^2+1)^2 + 4x(x^2+1)y = x(x^2+1)$$

Integrating we get,

$$y(x^2+1)^2 = \int (x^3+x)\,dx + c$$

$$y(x^2+1)^2 = \frac{x^4}{4} + \frac{x^2}{2} + c$$

$$y(x^2+1)^2 - \frac{x^4}{4} - \frac{x^2}{2} = c.$$ Answer:

Solve the equation $y^2 dx + (3xy - 1)\, dy = 0$

$$y^2 dx = (1 - 3xy)\, dy$$

$$\Rightarrow \quad y^2 \frac{dx}{dy} + 3xy = 1$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{3xy}{y^2}\right) = \frac{1}{y^2}$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{3y}{y^2}\right)x = \frac{1}{y^2} \quad \text{———} \ (1)$$

which is linear equation of x.

Here, $P(y) = \frac{3y}{y^2} = \frac{3}{y}$

$Q(y) = \frac{1}{y^2}$

$$\therefore \text{I.F} = e^{\int \frac{3}{y} dy} = e^{3 \ln y} = e^{\ln y^3} = y^3$$

multiplying (1) by I.F we get

$$y^3 \frac{dx}{dy} + \frac{3}{y} x y^3 = y$$

$$\Rightarrow y^3 \frac{dx}{dy} + 3xy^2 = y$$

Integrating we gets

$xy^3 = \int y\,dy + c_1$

$xy^3 = \frac{y^2}{2} + c_1$

$2xy^3 - y^2 = 2c_1 = c$

where, c is an arbitary constant.

Answer:

4. Solve the equation $y\ln y\,dx + (x - \ln y)\,dy = 0$

$y\ln y \frac{dx}{dy} + x - \ln y = 0$

$\Rightarrow y\ln y \frac{dx}{dy} + x = \ln y$

$\Rightarrow \frac{dx}{dy} + \frac{x}{y\ln y} = \frac{1}{y}$ —— (1)

which is a linear equ. of x.

Here, $P(y) = \frac{1}{y\ln y}$

$Q(y) = \frac{1}{y}$

$$I.F = e^{\int \frac{dy}{y \ln y}}$$

Put,
$\ln y = z$
$\frac{1}{y} \cdot dy = dz.$

$$= e^{\int \frac{1}{z} dy\, dz}$$

$$= e^{\ln z}$$

$$= e^{\ln(\ln y)}$$

$$= \ln y$$

Multiplying ① by integrating factor we get,

$$\ln y \frac{dx}{dy} + \frac{x}{y} = \frac{\ln y}{y}$$

Integrating we get,

$$x \ln y = \int \frac{\ln y}{y} dy + c.$$

where c is an arbitary constant.

Answer:-

5. Solve the equation $(1+y^2)dx = (\tan^{-1}y - x)\,dy$

$$(1+y^2)\frac{dx}{dy} + x = \tan^{-1}y$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2} \quad \text{——— (1)}$$

This is the linear equation in x,

Here, $P(y) = \frac{1}{1+y^2}$

$Q(y) = \frac{\tan^{-1}y}{1+y^2}$

$$I.F = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}y}$$

Multiplying (1) both sides I.F we get,

$$e^{\tan^{-1}y}\frac{dy}{dx} + \left(\frac{e^{\tan^{-1}y}}{1+y^2}\right)x = \frac{\tan^{-1}y\, e^{\tan^{-1}y}}{1+y^2}$$

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Integrating we get,

$$xe^{\tan^{-1}y} = \int \frac{\tan^{-1}y\, e^{\tan^{-1}y}\, dy}{1+y^2}$$

Put,
$\tan^{-1}y = z$
$\frac{1}{1+y^2}\, dy = dz.$

$$= \int z.e^{z} dz.$$

$$= z.e^{z} - e^{z} + c$$

$$= e^{z(z-1)} + c$$

$$xe^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c.$$

Answer:

EXACT DIFFERENTIAL EQUATION

Find the condition that $M(x,y)dx + N(x,y)dy = 0$ may be exact D.E.

The necessary condition:

If $M(x,y)dx + N(x,y)dy = 0$ is to be exact then

$$M(x,y)dx + N(x,y)dy = d\mu(x,y)$$

$$= \frac{\partial \mu}{\partial x}dx + \frac{\partial \mu}{\partial y}dy$$

$$\therefore M = \frac{\partial \mu}{\partial x} \quad \text{--- (1)} \qquad N = \frac{\partial \mu}{\partial y} \quad \text{--- (2)}$$

from (1) differentiating partially w.r.to. y

$$\frac{\partial M}{\partial y} = \frac{\partial^2 \mu}{\partial y \partial x}$$

also from (ii) differentiating partially w.r.to x

$$\frac{\partial N}{\partial x} = \frac{\partial^2 \mu}{\partial x \partial y}$$

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y} \quad \text{--- (3)}$$

which is the necessary condition.

The sufficient condition:

Let $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ we have to show that $Mdx + Ndy = 0$ is exac differential equation.

Let $\int M dx = P$ $\quad \frac{\partial P}{\partial x} = M$

$$\frac{\partial^2 P}{\partial y \partial x} = \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad [\text{using (3)}]$$

$$\frac{\delta N}{\delta x} = \frac{\delta^2 p}{\delta y \delta x} = \frac{\delta}{\delta x}\left(\frac{\delta p}{\delta y}\right)$$

Integrating we get

$$\int \frac{\delta N}{\delta x} = \int \frac{\delta}{\delta x}\left(\frac{\delta p}{\delta y}\right)$$

$$N = \frac{\delta p}{\delta y} + f(y)$$

Now $Mdx + Ndy$

$$= Mdx + \left[\frac{\delta p}{\delta y} + f(y)\right] dy$$

$$= \frac{\delta p}{\delta x} dx + \frac{\delta p}{\delta y} dy + f(y) dy$$

$$= dp + f(y) dy$$

$$= dp + dF(y)$$

$$= d[P + F(y)]$$

Hence $Mdx + Ndy = 0$ is an exact D.E.

1. Solve the eqn. $(2x\cos y + 3x^2y)dx + (x^3 - x^2\sin y)dy = 0$
$y(0) = 2$

Soln. we have

$$(2x\cos y + 3x^2y)dx + (x^3 - x^2\sin y)dy = 0 \quad \text{--- (1)}$$

Let $M = 2x\cos y + 3x^2y$ $\quad \frac{\delta M}{\delta y} = -2x\sin y + 3x^2$

$N = x^3 - x^2\sin y$ $\quad \frac{\delta N}{\delta x} = -2x\sin y + 3x^2$

$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$

$\therefore$ equ. ① is exact diff. equ. Then we must find μ such that

$\frac{\delta \mu}{\delta x} = M = 2x\cos y + 3x^2 y$ ——— ②

and $\frac{\delta \mu}{\delta y} = N = x^3 - x^2 \sin y$ ——— ③

from ② we get,

$\mu = x^2 \cos y + x^3 y + f(y)$ ——— ④

Diff. w.r. to. y

$\frac{\delta \mu}{\delta y} = -x^2 \sin y + x^3 + \frac{\delta F(y)}{\delta y}$

$\Rightarrow N = -x^2 \sin y + x^3 + \frac{\delta F(y)}{\delta y}$

$\Rightarrow x^3 - x^2 \sin y = -x^2 \sin y + x^3 + \frac{\delta F(y)}{\delta y}$

$\Rightarrow \frac{\delta F(y)}{\delta y} = 0$

$\Rightarrow \delta F(y) = 0$

Integrating we get,

$\int \delta f(y) = c_1 \Rightarrow f(y) = c_1$

from ④ we get,

$\mu = x^2 \cos y + x^3 y + c_1$

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Hence the general solution

$$\mu(x,y) = c_2$$

$$x^2\cos y + x^3y + c_1 = c_2$$

$$\Rightarrow x^2\cos y + x^3y = c_2 - c_1 = c \quad \text{Answer}$$

when, $x=0$ and $y=2$ for the initial condition

$$0 = c.$$

$$\therefore x^2\cos y + \cancel{x}\, x^3 y = 0$$

$$\cos y + \cancel{x}\, xy = 0 \quad \text{Answer:}$$

2. Solve the equ. $2x(ye^{x^2}-1)dx + e^{x^2}dy = 0$

Sol^n. we have,

$$2x(ye^{x^2}-1)\,dx + e^{x^2}dy = 0 \quad \text{———— (1)}$$

Let, $M = 2x(ye^{x^2}-1)$, $N = e^{x^2}$

$$\frac{\delta M}{\delta y} = 2xe^{x^2}; \quad \frac{\delta N}{\delta x} = 2xe^{x^2}$$

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

equ. (1) is exact, then we must find μ such that, $\frac{\delta \mu}{\delta x} = M = 2x(ye^{x^2}-1)$ ———— (2)

$$\frac{\delta \mu}{\delta y} = N = e^{x^2} \quad \text{———— (3)}$$

from ② we get,

$$\mu = ye^{x^2} - x^2 + f(y).$$

$$\frac{\delta \mu}{\delta y} = e^{x^2} + \frac{\delta f(y)}{\delta(y)}$$

$$N = e^{x^2} + \frac{\delta f(y)}{\delta y}$$

$$e^{x^2} = e^{x^2} + \frac{\delta f(y)}{\delta y}$$

$$\frac{\delta f(y)}{\delta y} = 0$$

Integrating, $f(y) = c_1$

$$\mu = ye^{x^2} - x^2 + c_1$$

Hence the general solution

$$\mu(x,y) = c_2$$

$$ye^{x^2} - x^2 + c_1 = c_2$$

$$ye^{x^2} - x^2 = c.$$ Answer:

3. Solve the equ. $(2x^3 + 3y)dx + (3x + y - 1)dy = 0$

Sol^n. we have $(2x^3 + 3y)dx + (3x + y - 1)dy = 0$ ——— ①

Here, $M = 2x^3 + 3y$

$N = 3x + y - 1$

$$\frac{\delta M}{\delta y} = 3 = \frac{\delta N}{\delta x}$$

Hence equ. ① is exact. Then we must find μ such that

$$\frac{\delta\mu}{\delta x} = M = 2x^3 + 3y \text{ ——— } (2)$$

$$\frac{\delta\mu}{\delta y} = N = 3x + y - 1 \text{ ——— } (3)$$

from (3) we get:

$$\mu = \frac{1}{2}x^4 + 3xy + f(y)$$

Diff. partially w. r. to. y

$$\frac{\delta\mu}{\delta y} = 3x + \frac{\delta f(y)}{\delta(y)}$$

$$N = 3x + \frac{\delta f(y)}{\delta y}$$

$$3x + y - 1 = 3x + \frac{\delta f(y)}{\delta y}$$

$$y - 1 = \frac{\delta f(y)}{\delta y}$$

$$\delta f(y) = (y-1)\, dy$$

Integrating we get:

$$\int \delta f(y) = \int (y-1) dy$$

$$f(y) = \frac{y^2}{2} - y + c$$

$$\mu = \frac{1}{2}x^4 + 3xy + \frac{y^2}{2} - y + c_1$$

Hence the general solution

$$\mu(x,y) = c_2$$

$$\frac{1}{2}x^4 + 3xy - y + \frac{y^2}{2} + c_1 = c_2$$

$$\frac{1}{2}x^4 + 3xy + \frac{y^2}{2} - y = c$$ Answer:

Solve the equ. $(y^2 e^{xy^2} + 4x^3)dx + (2xye^{xy^2} - 3y^2)dy = 0$

Sol^n we have.

$$(y^2 e^{xy^2} + 4x^3)\,dx + (2xye^{xy^2} - 3y^2)\,dy = 0 \quad \text{—— (1)}$$

Here, $M = (y^2 e^{xy^2} + 4x^3)$, $\frac{\delta M}{\delta y} = 2ye^{xy^2} + y^2 e^{xy^2} \cdot 2xy$

$N = (2xye^{xy^2} - 3y^2)$ $\quad \frac{\delta N}{\delta x} = 2ye^{xy^2} + 2xye^{xy^2} y^2$

$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$ — equ. (1) exact differential equation

Then we must find μ such that

$$\frac{\delta \mu}{\delta y} = N = 2xye^{xy^2} - 3y^2 \quad \text{—— (2)}$$

$$\frac{\delta \mu}{\delta x} = M = y^2 e^{xy^2} + 4x^3 \quad \text{—— (3)}$$

From (3) we get,

$$\mu = \frac{y^2}{y^2} e^{xy^2} + x^4 + f(y)$$

$$= e^{xy^2} + x^4 + f(y) \quad \text{—— (4)}$$

diff. partially w. r. to. y

$$\frac{\delta \mu}{\delta y} = 2xye^{xy^2} + \frac{\delta f(y)}{\delta y}$$

$$N = 2xye^{xy^2} + \frac{\delta f(y)}{\delta y}$$

$$2xye^{xy^2} - 3y^2 = \frac{\delta f(y)}{\delta y} + \cancel{2xye^{xy^2}}$$

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$\delta f(y) = -3y^2 \delta y$

Integrating we get,

$\int \delta f(y) = -3\int y^2 dy$

$f(y) = -3 \cdot \frac{y^3}{3} + c_1$

$f(y) = -y^3 + c_1$

$\mu = e^{xy^2} + x^4 - y^3 + c_1$

Hence the general solution $\mu(xy) = c_2$

$\therefore e^{xy^2} + x^4 - y^3 + c_1 = c_2$

$e^{xy^2} + x^4 - y^3 = c$ Answer

where $c = c_2 - c_1$ are arbitary constant

৫. Solve the diff. equ.

$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$

Soln. we have

$(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$ ——— (1)

Here, $M = x^3 + 3xy^2$ $\qquad \frac{\delta M}{\delta y} = 6xy$

$N = 3x^2y + y^3$ $\qquad \frac{\delta N}{\delta x} = 6xy$

$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$

Hence equ (1) is exact diff. equ.

Now we must find μ such that

$$\frac{\delta \mu}{\delta x} = M = x^3 + 3xy^2 \text{ ——— } (2)$$

$$\frac{\delta \mu}{\delta y} = N = 3x^2y + y^3 \text{ ——— } (3)$$

Now from (2) we get,

$$\mu = \frac{x^4}{4} + \frac{3}{2}x^2y^2 + f(y)$$

diff. partially w.r.to. y we get

$$\frac{\delta \mu}{\delta y} = 3x^2y + \frac{\delta f(y)}{\delta y}$$

$$N = 3x^2y + \frac{\delta f(y)}{\delta y}$$

$$3x^2y + y^3 = 3x^2y + \frac{\delta f(y)}{\delta y}$$

$$\Rightarrow y^3 = \frac{\delta f(y)}{\delta y}$$

$$\Rightarrow \delta f(y) = y^3 \delta y$$

Integrating we get,

$$f(y) = \frac{y^4}{4} + c_1$$

$$\mu = \frac{x^4}{4} + \frac{3}{2}x^2y^2 + \frac{y^4}{4} + c_1$$

Hence the general solution

$$\mu(x,y) = c_2$$

$$\frac{x^4}{4} + \frac{3}{2}x^2y^2 + \frac{y^4}{4} + c_1 = c_2$$

$$x^4/4 + 3/2\, x^2y^2 + y^4/4 = c.$$ where $c = c_2 - c_1$ are arbitary constant

Answer.

μ

6. 8 Solve the equ. $(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$

Sol^n; we have

$(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy=0$ —— (1)

Here, $M=x^2-4xy-2y^2$ $\quad \frac{\delta M}{\delta y}=-4x-4y$

$N=y^2-4xy-2x^2$ $\quad \frac{\delta N}{\delta x}=-4y-4x$

$\therefore \frac{\delta M}{\delta y}=\frac{\delta N}{\delta x}$

Hence equ. (1) is exact ~~differential~~ equ.

Then we must find μ...

$\frac{\delta \mu}{\delta x}=M=x^2-4xy-2y^2$ $\quad \cancel{\frac{\delta M}{\delta y}=-4x-4y}$ —— (2)

$\frac{\delta \mu}{\delta y}=N=y^2-4xy-2x^2$ —— (3)

$\mu=\frac{x^3}{3}-2x^2y-2y^2x+f(y)$ —— (4)

Again from (4) diff. partially with respect to y we get,

$\frac{\delta \mu}{\delta y}=-2x^2-4xy+\frac{\delta f(y)}{\delta y}$

$y^2-4xy-2x^2=-2x^2-4xy+\frac{\delta f(y)}{\delta y}$

$y^2=\frac{\delta f(y)}{\delta y}$

$\delta f(y) = y^2 dy$

Integrating we get,

$$\int \delta f(y) = \int y^2 dy$$

$$f(y) = y^3/3 + c_1$$

Hence from ④

$$\mu = \frac{x^3}{3} - 2x^2y - 2y^2x + y^3/3 + c_1$$

∴ General solution

$$\mu(x,y) = c_2$$

$$\frac{x^3}{3} - 2x^2y - 2y^2x + y^3/3 + c_1 = c_2$$

$$\frac{x^3}{3} - 2x^2y - 2y^2x + y^3/3 = c_2 - c_1 = c$$

where c is an arbitary constant. Answer:

INTEGRATING FACTOR

Suppose $M(x,y)dx + N(x,y)\,dy = 0$ is not exact but
$\phi(x,y)\,[M(x,y)dx + N(x,y)dy] = 0$ is exact.
$\phi(x,y)$ is called integrating factor $\mu(x,y) = 0$

1. Solve the eqn. $(x^4+y^4)dx - xy^3dy = 0$ ——— (1)

Here, $M = x^4+y^4$, $N = -xy^3$

$$\frac{\delta M}{\delta y} = 4y^3, \quad \frac{\delta N}{\delta x} = -y^3$$

$$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$$, which is not exact

Again $Mx + N.y = (x^4+y^4)x + (-xy^3)y$

$$= x^5 + \cancel{xy^4} - \cancel{xy^4}$$

$$= x^5 \neq 0$$

since (1) is homogenious and $Mx + Ny \neq 0$

$$I.F = \frac{1}{Mx+Ny} = \frac{1}{x^5}$$

Multiplying (1) by I.F. we get

$$\frac{1}{x^5}(x^4+y^4)dx - \frac{xy^3}{x^5}dy = 0$$

$$\left(\frac{1}{x} + \frac{y^4}{x^5}\right)dx - \frac{y^3}{x^4}dy = 0$$

Now, we get, $\left(\frac{\delta M}{\delta y} = \frac{4y^3}{x^5}\right)$ and $\frac{\delta N}{\delta x} = \frac{4y^3}{x^5}$

$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$ Hence equ. is exact

also its solution $\int \frac{1}{x}dx - \frac{1}{4}\frac{y^4}{x^4} + c = 0$

$\ln x - \frac{1}{4}\frac{y^4}{x^4} + c = 0$

where c is an arbitary constant. Answer:

Solve the equ. $(3y^3 - xy)dx - (x^2 + 6x^2y^2)dy = 0$ —— ①

Soln. Here, $M = 3y^3 - xy$ $\quad \frac{\delta M}{\delta y} = 9y^2 - x$

$N = x^2 + 6x^2y^2$ $\quad \frac{\delta N}{\delta x} = 2x + 12y^2x$

$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$

Hence the equ. ① is not exact

$Mx + Ny = x(3y^3 - xy) + y(x^2 + 6x^2y^2)$

$= 3xy^3 - x^2y + x^2y + 6x^2y^3$

$= 3xy^3(1+2x)$

$I.F = \frac{1}{3xy^3(1+2x)}$ dividing both sides of equ. ①

$\frac{(3y^3 - xy)\,dx}{3xy^3(1+2x)} - \frac{(x^2+6x^2y^2)\,dy}{3xy^3(1+2x)} = 0$

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$$\left\{\frac{1}{x(1+2x)} - \frac{1}{3y^2(1+2x)}\right\}dx - \frac{x(1+6y^2)}{3y^3(1+2x)}dy = 0$$

Hence its solution

$$\left\{\frac{1}{x} - \frac{2}{1+2x} - \frac{1}{3y^2(1+2x)}\right\}dx - \frac{x(1+6y^2)}{3y^3(1+2x)}dy$$

$$\log x - \frac{2}{2}\log(1+2x) - \frac{1}{6y^2}\log(1+2x) = \log c$$

$$\log\frac{x}{1+2x} - \log(1+2x)^{1/6y^2} = \log c$$

$$\frac{x}{(1+2x)\cdot(1+2x)^{1/6y^2}} = c.$$

where c is an arbitary constant. Answer

Solve the equ. $y(1+xy)dx + x(1-xy)dy = 0$

Here, $M = y(1+xy)$, $\frac{\delta M}{\delta y} = 1+2xy$

$N = x(1-xy)$, $\frac{\delta N}{\delta x} = 1-2xy$

$$\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$$

The equ. (1) is not exact.

Now, $Mx + Ny = yx(1+xy) + xy(1-xy)$

$= 2xy$

Here, $Mx + Ny \neq 0$

Integrating factor $= \frac{1}{2xy}$

Multiplying both sides ① by I.F.

$$\frac{y(1+xy)dx}{2xy} + \frac{x(1-xy)dy}{2xy} = 0$$

$$\left(\frac{1}{2x} + \frac{y}{2}\right) dx + \left(\frac{1}{2y} - \frac{x}{2}\right) dy = 0$$

which is exact and its solution

$$\frac{1}{2}\log x + \frac{1}{2}xy + \frac{1}{2}\log y = \frac{1}{2}c.$$

$$\log x + xy + \log y = c$$

$$\log xy + xy = c.$$

where c is an arbitary constant.

Answer:

4. Solve the equ. $(3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$

$N = 2x^2 + 2y$, $\frac{\delta N}{\delta x} = 4x$

$M = 2x^2 + 4xy$ $\frac{\delta M}{\delta y} = 4x$

$$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

Hence equ. ① is exact

also its solution $x^3 + 2x^2y + y^2 = c.$

Answer:

5. $(x^2-y)\,dx + (y^2-x)\,dy = 0$

Here, $M = x^2-y$, $\frac{\delta M}{\delta y} = -1$

$N = y^2-x$ $\frac{\delta N}{\delta x} = -1$

$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$

Equation ① is exact diff. equ.
also its solution,

$$(x^2-y)\,dx + (y^2-x)\,dy = 0$$

$$\frac{x^3}{3} - xy + \frac{y^3}{3} = c.$$ Answer:

6. Show that the following equ. is exact and solve it

$$[(y-x)\,dx - 2x\,dy] / (x+y)^3 = 0$$

Here, $M = \frac{y-x}{(x+y)^3}$ $N = \frac{-2x}{(x+y)^3}$

$$\frac{\delta M}{\delta y} = \frac{(x+y)^3 - 3(x+y)^2(y-x)}{(x+y)^{6\,4}}$$

$$= \frac{x+y - 3y + 3x}{(x+y)^4}$$

$$= \frac{4x-2y}{(x+y)^4}$$

also, $\frac{\delta N}{\delta x} = \frac{-2(x+y)^3 + 2x \cdot 3(x+y)^2}{(x+y)^6}$

$$= \frac{-2x - 2y + 6x}{(x+y)^4}$$

$$= \frac{4x - 2y}{(x+y)^4}$$

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

Hence equation ① is exact diff. equation.

Also its solution

$$\frac{y-x}{(x+y)^3}\,dx - \frac{2x\,dy}{(x+y)^3} = 0$$

$$\left\{\frac{y}{(x+y)^3} - \frac{x}{(x+y)^3}\right\} dx - \frac{2x\,dy}{(x+y)^3} = 0$$

$$-\frac{y}{4(x+y)^4} - \int \frac{(x+y-y)}{(x+y)^3}\,dx = c$$

$$-\frac{y}{4(x+y)^4} - \int \frac{1}{(x+y)^2}\,dx + \int \frac{y\,dx}{(x+y)^3} = c$$

$$-\cancel{\frac{y}{4(x+y)^4}} + \frac{1}{3(x+y)^3} + \cancel{\frac{y}{4(x+y)^4}} = c$$

$$\frac{1}{3(x+y)^3} = c. \quad \text{Answer:}$$

Date:
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7. Solve the equ. $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$

$M = x^2y - 2xy^2$ $\qquad \frac{\partial M}{\partial y} = x^2 - 4xy$

$N = -(x^3 - 3x^2y)$ $\qquad \frac{\partial N}{\partial x} = 6xy - 3x^2$

$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, equ. ① is not exact.

Now $Mx + Ny = x(x^2y - 2xy^2) - y(x^3 - 3x^2y)$

$= \cancel{x^3y} - 2x^2y^2 - \cancel{x^3y} + 3x^2y^2$

$= x^2y^2$

and $Mx + Ny \neq 0$

∴ Integrating factor $= \frac{1}{x^2y^2}$

multiplying ① by I.F.

$\frac{1}{x^2y^2}(x^2y - 2xy^2)\,dx - \frac{1}{x^2y^2}(x^3 - 3x^2y) = 0$

$\Rightarrow \left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\frac{x}{y^2} - \frac{3}{y}\right)dy = 0$

Integrating for its solution.

$\frac{x}{y} - 2\ln x + 3\ln y = c$

where c is an arbitary constant.

Answer:

when $\dfrac{\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}}{N} = f(x)$; then $e^{\int f(x)dx}$ is integrating factor of $Mdx + Ndy = 0$.

8. $(12y + 4y^3 + 6x^2)\,dx + 3(x + xy^2)\,dy = 0$

Here, $M = 12y + 4y^3 + 6x^2$ $\quad \dfrac{\delta M}{\delta y} = 12 + 12y^2$

$N = 3(x + xy^2)$ $\quad \dfrac{\delta N}{\delta x} = 3(1 + y^2)$

$\therefore \dfrac{\delta M}{\delta y} \neq \dfrac{\delta N}{\delta x}$

Hence equ. ① is not exact

$$\frac{\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}}{N} = \frac{12 + 12y^2 - 3 - 3y^2}{3(x + xy^2)}$$

$$= \frac{9(1+y^2)}{3x(1+y^2)}$$

$= 3/x$, which is a function of x only

$I.F = e^{\int 3/x\, dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

Multiplying ① both sides x^3 we get,

$$(12x^3y + 4x^3y^3 + 6x^5)\,dx + (3x^4 + 3x^4y^2)\,dy = 0$$

Which is exact and it's solution

$3x^4y + x^4y^3 + x^6 = c$. Answer.

Rule 1: If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$ a function of x only, Then $e^{\int f(x)dx}$ is an integrating factor.

Rule 2: If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = g(y)$ a function of y only, then $e^{\int -g(y)dy}$ is an integrating factor.

9. $(x^2+y^2+2x)\,dx + 2y\,dy = 0$.

Here, $M = x^2+y^2+2x$, $\quad \dfrac{\partial M}{\partial y} = 2y$

$N = 2y$ $\quad \dfrac{\partial N}{\partial x} = 0$

$\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$, equ. ① is not exact.

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y-0}{2y} = 1$$

which is a function of x only. I.F $= e^{\int dx} = e^x$

multiplying ① both sides by e^x we get,

$e^x(x^2+y^2+2x)\,dx + e^x\,2y\,dy = 0$

which is exact and also its solution,

$\int e^x x^2 dx + y^2 \int e^x dx + \int 2x e^x dx = \int dc$

$e^x x^2 - \cancel{\int 2x e^x dx} + y^2 e^x + \cancel{\int 2x e^x dx} = c$

$e^x x^2 + e^x y^2 = c$. where c is an arbitary consta

Answer:

BERNOULI DIFFERENTIAL EQUATION

ACTIVE
Date: 8-8-15

The equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called a Bernouli Differential Equation. $n \neq 0, 1$

1. Solve the equation $\frac{dy}{dx} + xy = xy^2$ —— (i)

$$\frac{dy}{dx} y^{-2} + xy^{-1} = x$$

$$\Rightarrow \frac{1}{y^2} \cdot \frac{dy}{dx} + \frac{x}{y} = x \quad \text{—— (ii)}$$

Let, $v = \frac{1}{y}$

$$\frac{dv}{dx} = -\frac{1}{y^2} \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx} = -y^2 \frac{dv}{dx}$$

putting this value in (i) we get,

$$= \cancel{\frac{y^2}{y^2}} \cdot \frac{dv}{dx} + vx = x$$

$$\frac{dv}{dx} - vx = -x \quad \text{—— (iii)}$$

which is a linear equation in v.

$$I.F = e^{-\int x dx} = e^{-x^2/2}$$

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multiplying (ii) by integrating factor we get:

$$e^{-x^2/2}\frac{dv}{dx} - xve^{-x^2/2} = e^{-x^2/2}(-x)$$

Integrating we get,

$$ve^{-x^2/2} = -\int xe^{-x^2/2} + c$$

Put,

$-x^2/2 = z$

$x\,dx = -dz$

$$= \int e^{z}\,dz + c$$

$$= e^{z} + c$$

$$= e^{-x^2/2} + c.$$

$$\Rightarrow ve^{-x^2/2} = e^{-x^2/2} + c$$

$$\Rightarrow \frac{1}{y}.e^{-x^2/2} = e^{-x^2/2} + c.$$ Answer:

Solve the equation. $\frac{dy}{dx} + y = xy^3$

$$\frac{1}{y^3} \cdot \frac{dy}{dx} + \frac{1}{y^2} = x \quad \text{--- (1)}$$

Let, $v = 1/y^2$

$$\frac{dv}{dx} = -\frac{2}{y^3} \frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{2} y^3 \frac{dv}{dx}$$

putting this value in (1) we get,

$$-\frac{1}{2} \cdot \frac{y^3}{y^3} \frac{dv}{dx} + v = x$$

$$\frac{dv}{dx} - 2v = -2x \quad \text{--- (11)}$$

which is a linear equation in v.

$$I.F = e^{-2x}$$

multiplying (1) by I.F. we get,

$$e^{-2x} \frac{dv}{dx} - 2v \cdot e^{-2x} = -2xe^{-2x}$$

Integrating we get,

$$ve^{-2x} = -\int x e^{-2x}\,dx$$

$$= -\left[x\int e^{-2x} - \int\left\{\frac{d}{dx}(x)\int e^{-2x}dx\right\}dx\right]$$

$$= -\frac{x}{2}e^{-2x} + \frac{1}{4}e^{-2x} + c/4$$

$$ve^{-2x} = \left[\frac{1}{4}(-2x+1)\right]e^{-2x} + c/4$$

$$4ve^{-2x} = (-2x+1)e^{-2x} + c$$

$$\Rightarrow e^{-2x}(4v+2x-1) = c$$

$$\Rightarrow e^{-2x}\left(\frac{4}{y^r}+2x-1\right) = c$$

Answer:

Friday - Saturday
8-8-15, 3.20 am