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Math - 2101

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Differential Equation

$\frac{d}{dx}$ ⟶ ordinary differentiation [Independent variable সবসময় same এবং single হলে]

$\frac{\delta}{\delta x}$ ⟶ partial differentiation [more than one independent variable থাকলে]

└→ একটি ind. var. থাকলে এটাও ordinary diff এর operator হিসেবে use করা যায়।

Ex: $\frac{d(y)}{dx} + x = y$ ⟶ ordinary

$\frac{\delta(y)}{\delta x} + x = y$ ⟶ ordinary or partial হতে পারে।

Differential Equation

$\frac{d}{dx}$ ⟶ ordinary differentiation [Independent variable [illegible] same [illegible]]

$\frac{\delta}{\delta x}$ ⟶ partial differentiation [more than one independent variable থাকলে]

⟶ [illegible] ordinary diff [illegible] operator [illegible]

Ex: $\frac{d}{dx}(y) + x = y$ ⟶ ordinary

$\frac{\delta}{\delta x}(y) + x = y$ ⟶ ordinary or partial হতে পারে।

Differential Equation: (DE)

An equation involving derivatives of one or more dependent variable (s) w.r.t. to one or more independent variable (s) is called Differential Equation (DE).

Two types:

Single variable [illegible] diff.

(i) Ordinary DE: An eq^n involving ordinary derivatives of single independent variable is called ODE.

Ex: $$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x$$

more than one [illegible] diff.

(ii) Partial DE:

Ex: $$\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} + \frac{\delta^2 u}{\delta z^2} = 0$$ [lables eq^n / hit eq^n]

Order and Degree:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x$$

$\downarrow$

সর্বোচ্চ derivation → order

সর্বোচ্চ derivation যেখানে থাকবে তার power → degree

Ex:

$$\left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2} \longrightarrow \begin{array}{l} \text{degree} = 1 \\ \text{order} = 2 \end{array}$$

power 1

2য় derivation

$$\left(\frac{d^3y}{dx^3}\right)^1 + x\left(\frac{dy}{dx}\right)^5 = y \longrightarrow \begin{array}{l} \text{order} = 3 \\ \text{degree} = 1 \end{array}$$

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}} + x\left(\frac{dy}{dx}\right) = y \longrightarrow \begin{array}{l} \text{order} = 2 \\ \text{degree} = 1 \end{array}$$

[$\frac{1}{2}$ কে simplify করতে হবে]

* order or degree fraction হবে না।

Linearity and Non-Linearity:

Non-linear:

(i) $\left(\frac{d^2y}{dx^2}\right)^2 + y = x$

(ii) $y\left(\frac{d^2y}{dx^2}\right) + x = 1$

(iii) $\frac{d^2y}{dx^2} + y^2 = x$

dependent variable product হিসেবে থাকলে

(infinite series আকারে express করা যায়)

(iv) $\frac{d^2y}{dx^2} + e^y = x$

dependent variable এর transcendental function এর product হিসেবে থাকলে;

এগুলো বাদে অন্য সব linear হবে;

$$\frac{d^2y}{dx^2} + e^x = x$$

$$\frac{d^2y}{dx^2} + x^2 = x$$

$$\frac{d^2y}{dx^2} + xy = 1$$

linear

but,

$$\left(\frac{dy}{dx}\right)^2 + \sin x = e^x \longrightarrow \text{non-linear}$$

Form a ODE by eliminating arbitrary constant from $y = ax + bx^2$ — (i)

Soln:

[convert [illegible] arbitrary constant [illegible] ।]

Now,

differentiation w.r. to x two times,

$$\frac{dy}{dx} = a + 2bx \quad \text{— (ii)}$$

[constant ২টি আছে ২ বার diff করব।]

$$\Rightarrow \frac{d^2y}{dx^2} = 2b$$

$$\Rightarrow b = \frac{1}{2} \cdot \frac{d^2y}{dx^2}$$

putting the value of b in eqn (ii) →

$$\frac{dy}{dx} = a + 2 \cdot \frac{1}{2} \cdot \frac{d^2y}{dx^2} \cdot x$$

$$\Rightarrow a = \frac{dy}{dx} - x\frac{d^2y}{dx^2}$$

Putting the value of a and b in eqn (i) →

$$y = x\left(\frac{dy}{dx} - x \cdot \frac{d^2y}{dx^2}\right) + \frac{1}{2}x^2\frac{d^2y}{dx^2}$$

$$\therefore y = x\frac{dy}{dx} - \frac{1}{2}x^2\frac{d^2y}{dx^2}$$

Ans.

$c(y+c)^2 = x^3 \longrightarrow (1)$

$\frac{d}{dx}(1) \longrightarrow$

$$2c(y+c)\frac{dy}{dx} = 3x^2 \longrightarrow (ii)$$

$(ii) \div (1) \longrightarrow$

$$\frac{2c(y+c)\frac{dy}{dx}}{c(y+c)^2} = \frac{3x^2}{x^3}$$

$$\Rightarrow \frac{y+c}{2 \cdot \frac{dy}{dx}} = \frac{x}{3}$$

$$\Rightarrow y+c = \frac{2}{3}x \cdot \frac{dy}{dx}$$

$$\Rightarrow c = -y + \frac{2}{3}x \cdot \frac{dy}{dx}$$

c এর স্থান (1) নং এ বসাই $\longrightarrow$

$$\left(\frac{2x}{3} \cdot \frac{dy}{dx} - y\right)\left(\frac{2x}{3} \cdot \frac{dy}{dx}\right)^2 = x^3$$

Find the DE of all circles passing through the origin and having their centres on x-axis.

Soln: eqn of circle : $x^2+y^2+2gx+2fy+c=0$

here, for condition ① : $x^2+y^2+2gx+2fy+0=0$

for condition ⑪ : $x^2+y^2+2gx=0$ ——— ①

centre : $(-g, 0)$.

From eqn ① ⟶

$$x^2+y^2+2gx=0$$

$$\Rightarrow 2x+2y\cdot\frac{dy}{dx}+2g=0$$

$$\Rightarrow g=-\left(x+y\cdot\frac{dy}{dx}\right)$$

g এর মান ① নং এ বসিয়ে,

$$x^2+y^2+2x\left(-x-y\frac{dy}{dx}\right)=0$$

$$\Rightarrow x^2-y^2+2xy\frac{dy}{dx}=0$$

* 1st order থেকে শুরু করে ⟶ Highest order differential eqn

* 1^{st} order, 1^{st} degree হলে linear হতেও পারে নাও হতে পারে।

Ex: $\frac{dy}{dx} = -xy^3$

First order First degree DE এর solution:

$\frac{dy}{dx} \longrightarrow (x,y)dx$

Derivative form: $\frac{dy}{dx} = f(x,y)$

Differential Form: $M(x,y)\,dx + N\,dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{M}{N}$

2 রকমের format এ থাকতে পারে।

Method-1

① Separation of variables method:

$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

$\Rightarrow \tan^{-1}y = \tan^{-1}x + \tan^{-1}c$

একটা integrating cons add করতে হবে।

$\Rightarrow \tan^{-1}y - \tan^{-1}x = \tan^{-1}c$

$\Rightarrow \tan^{-1}\frac{y-x}{1+xy} = \tan^{-1}c$

* constant একটা, কারণ একবার diff. করা হয়েছে।

$\Rightarrow \frac{y-x}{1+xy} = c$

$\Rightarrow y-x = c(1+xy)$

$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

$\Rightarrow \frac{dy}{dx} = e^{x} e^{-y} + x^2 e^{-y}$

Ans: $e^{y} = e^{x} + \frac{1}{3}x^3 + c$

$\Rightarrow e^{y} dy = (e^{x} + x^2) dx$

$\Rightarrow e^{y} = e^{x} + \frac{x^3}{3} + c$

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* particular solⁿ $\longrightarrow$ c [illegible] specific value [illegible]

Find the solution of $\frac{dy}{dx} = e^{x+y}$. It is given that $y = 1$ when $x = 1$.

Find y when $x = -1$.

$\rightarrow e^{-y} dy = e^{x} dx$

$\Rightarrow -e^{-y} = e^{x} + c$

$\Rightarrow e^{-y} = -e^{x} - c$ ——— (1) [general solⁿ $\rightarrow$ c [illegible] specific [illegible]]

Putting $y = 1$ when $x = 1$, in eqⁿ (1) $\rightarrow$

$e^{-1} = -e^{1} - c$

$\Rightarrow c = -e - e^{-1}$

$\therefore e^{-y} = -e^{x} + e^{-1} + e$ ——— (2) [particular solⁿ]

When $x = -1$, then $\longrightarrow$

$$e^{-y} = -e^{-1} + e^{-1} + c$$

$$\Rightarrow e^{-y} = c$$

$$\Rightarrow y = -1$$

Variable separation difficult হলে $\longrightarrow$

* function এর একটি variable এ transform করব।

General Form

$\frac{dy}{dx} = f(ax+by+c)$

let,

$ax+by+c = v$

$$\Rightarrow a + b \cdot \frac{dy}{dx} + 0 = \frac{dv}{dx}$$

$$\Rightarrow b \frac{dy}{dx} = \frac{dv}{dx} - a$$

$$\therefore \frac{dy}{dx} = \frac{1}{b}\left(\frac{dv}{dx} - a\right)$$

$$\Rightarrow \frac{1}{b}\left(\frac{dv}{dx} - a\right) = f(v)$$

$$\Rightarrow \frac{dv}{dx} - a = b f(v)$$

$$\Rightarrow \frac{dv}{dx} = b f(v) + a$$

$$\Rightarrow \int \frac{dv}{b \cdot f(v) + a} = \int dx$$

$$\Rightarrow \int \frac{dv}{b f(v) + a} = x + c$$

$$\Rightarrow \frac{dy}{dx} = (4x+y+1)^2$$

let

$$4x+y+1 = v$$

$$\Rightarrow 4 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 4$$

$$\Rightarrow \frac{dv}{dx} - 4 = v^2$$

$$\Rightarrow \frac{dv}{dx} = v^2 + 4$$

$$\Rightarrow \frac{dv}{v^2+4} = dx$$

$$\Rightarrow \frac{1}{2}\tan^{-1}\frac{v}{2} = x + c$$

[illegible]

$$\Rightarrow \tan^{-1}\frac{v}{2} = 2x + 2c$$

$$\Rightarrow \frac{v}{2} = \tan(2x+2c)$$

$$\Rightarrow v = 2\tan(2x+2c)$$

$$\Rightarrow 4x+y+1 = 2\tan(2x+2c)$$ ⟶ [illegible]

$(x-y)^2 \frac{dy}{dx} = a^2$ ⟶ let $x - y = v$

$$\Rightarrow -\frac{dy}{dx} + 1 = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 1 - \frac{dv}{dx}$$

Homogenious Differential eqn (1st order, 1st degree) :

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right) \quad \text{or,} \quad \frac{dy}{dx} = f\left(\frac{x}{y}\right)$$

$$x\,dx + y\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y} = f(x/y)$$

$$\text{or,} \quad \frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$$

x, y এর ঘাত সমান ভাগ আকারে থাকবে।

$$(x^2+y^2)\,dx + 2xy\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^2+y^2}{2xy}$$

$$= -\frac{x^2\left(1+\left(\frac{y}{x}\right)^2\right)}{2 \cdot x^2\left(\frac{y}{x}\right)}$$

$$= f\left(\frac{y}{x}\right)^2$$

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

$\frac{dy}{dx} = f(y/x)$

let, $y/x = v$

$\Rightarrow y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = f(v)$

$\Rightarrow x\frac{dv}{dx} = f(v) - v$

$\Rightarrow \int \frac{dv}{f(v) - v} = \int \frac{dx}{x}$

$\Rightarrow \int \frac{dv}{f(v) - v} = \ln x + c$

[Homogeneous eqn কে separation of variable এ convert করে reduce করা যায়।]

$(x^2 + y^2)dx + 2xy\,dy = 0$ ——— (1)

Since it is a 1st order, 1st degree homogenus eqn, we can write

let, $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

from (1) →

$$\frac{dy}{dx} = -\frac{x^2 + y^2}{2xy}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{x^2 + v^2x^2}{2x \cdot vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{x^2(1+v^2)}{2x^2 v} = -\frac{(1+v^2)}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1+v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1+v^2+2v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1+3v^2}{2v}$$

$$\Rightarrow \left(\frac{2v}{1+3v^2}\right)dv = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{3}\ln(1+3v^2) = -\ln x + \ln c$$

$$\Rightarrow \ln(1+3v^2) = -3\ln x + 3\ln c$$

$$\Rightarrow \ln(1+3v^2) = \ln x^{-3} + \ln c^3$$

$$\Rightarrow 1+3v^2 = x^{-3}\cdot c^3$$

$$\Rightarrow 1+3\cdot\frac{y^2}{x^2} = x^{-3}\cdot c^3$$

$$\Rightarrow \frac{x^2+3y^2}{x^2} = x^{-3}\cdot c^3$$

$$\Rightarrow x^2+3y^2 = c^3/x$$

4) $x^2 y\,dx - (x^3+y^3)dy = 0$ ——— (i)

let, $y = vx$, since 1st order, 1st degree differential eqn (Homogeneous):

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——— (ii)}$$

From (i) →

$$x^2 y\,dx - (x^3+y^3)dy = 0$$

$$\Rightarrow x^2 y\,dx = (x^3+y^3)\,dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 y}{x^3+y^3}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 \times vx}{x^3+v^3x^3}$$

$$\Rightarrow v + x.\frac{dv}{dx} = \frac{x^2 \times vx}{x^3+v^3x^3}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v}{1+v^3}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v^3} - v = \frac{v-v-v^4}{1+v^3} = \frac{-v^4}{1+v^3}$$

$$\Rightarrow x\frac{dv}{dx} = -\left(v^4 + \frac{v^4}{v^3}\right) = -(v^4+v)$$

$$\Rightarrow \frac{x}{dx} = -\frac{v^4+v}{dv}$$

$$\Rightarrow \frac{dx}{x} = -dv\left(\frac{1}{v^4} + \frac{1}{v}\right)$$

$$\Rightarrow \ln x = -\frac{1}{v^4}\,dv - \ln v + c$$

$$\Rightarrow \ln x = -\frac{v^{-4+1}}{-3} - \ln v + c$$

$$\Rightarrow \ln x = -\frac{v^{-3}}{-3} - \ln v + c$$

$$\Rightarrow \ln x + \ln v = -\frac{1}{3v^3} + c$$

$$\Rightarrow \ln vx = -\frac{1}{3v^3} + c$$

1st order, 1st degree linear diff. equation solution:

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

↳ y এর কোন পাওয়ার থাকবে না

$P(x) = P$

$Q(x) = Q$

Integrative factor: I.F. $= e^{\int P dx}$

$$e^{\int P dx} \frac{dy}{dx} + P y e^{\int P dx} = Q e^{\int P dx}$$

$$\Rightarrow \frac{d}{dx}\left(y e^{\int P dx}\right) = Q e^{\int P dx}$$

$$\Rightarrow y e^{\int P dx} = \int Q e^{\int P dx} dx + c$$

$(1-x^2)\frac{dy}{dx} - xy = 1$

here,

$P(x) = -\frac{x}{1-x^2}$

$Q(x) = \frac{1}{1-x^2}$

$$\Rightarrow \frac{dy}{dx} - \frac{x}{1-x^2} y = \frac{1}{1-x^2} \quad \text{——— (1)}$$

$$\text{I.F} \Rightarrow e^{\int -\frac{x}{1-x^2} dx} = e^{\frac{1}{2}\ln(1-x^2)}$$

$$= e^{\ln(1-x^2)^{1/2}}$$

$$= \sqrt{1-x^2}$$

multiplying $\sqrt{1-x^2}$ to ① $\longrightarrow$

$$\sqrt{1-x^2}\,\frac{dy}{dx} - \frac{x}{1-x^2}\sqrt{1-x^2}\cdot y = \frac{\sqrt{1-x^2}}{1-x^2}$$

$$\Rightarrow \frac{d}{dx}\left(y\sqrt{1-x^2}\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow y\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow y\sqrt{1-x^2} = \sin^{-1}x + c$$

$$\Rightarrow y = \frac{\sin^{-1}x}{\sqrt{1-x^2}} + \frac{c}{\sqrt{1-x^2}}$$

* Integrating factor definition
(বই)

$x\frac{dy}{dx} + 2y = x^2 \log x$

$$\Rightarrow \frac{dy}{dx} + 2\cdot\frac{y}{x} = x\log x$$

$P(x) = \frac{2}{x}$

$Q(x) = x\log x$

$$I.F = e^{\int P(x)\,dx}$$

$$= e^{\int 2/x\,dx}$$

$$= e^{2\ln x}$$

$$= x^2$$

$$x^2\frac{dy}{dx} + 2xy = x^3 \log x$$

$$\Rightarrow \frac{d}{dx}(yx^2) = x^3 \log x$$

$$\Rightarrow yx^2 = \int x^3 \log x \, dx$$

$$= \log x \int x^3 dx - \int \left[\frac{d}{dx}\log x \int x^3 dx\right] dx$$

$$= \frac{x^4}{4}\log x - \int \frac{1}{x}\cdot\frac{x^4}{4} dx$$

$$= \frac{1}{4}x^4 \log x - \frac{1}{4}\int x^3 dx$$

$$= \frac{1}{4}x^4 \log x - \frac{1}{4}\frac{x^4}{4} + c$$

$$= \frac{x^4}{4}\log x - \frac{x^4}{16} + c$$

1st order, 1st degree solve → 32

Bernoulli DE (1st order, 1st degree non linear eq^n):

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \quad , \; n \neq 0, 1$$

(non-linear কে linear করতে হবে।)

$$\Rightarrow y^{-n}\frac{dy}{dx} + p(x)y^{1-n} = Q(x) \longrightarrow (1)$$

let,

$$y^{1-n} = v$$

$$\Rightarrow (1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\cdot\frac{dv}{dx}$$

(1) নং এ বসিয়ে,

$$\frac{1}{1-n}\cdot\frac{dv}{dx} + P(x)v = Q(x)$$

$$\Rightarrow \frac{dv}{dx} + (1-n).P(x)\,v = \frac{1}{1-n}Q(x) \longrightarrow \text{linear eq}^n$$

#: $x\frac{dy}{dx} - 2y = xy^4$

$\Rightarrow y^{-4}.\frac{dy}{dx} - \frac{2}{x}y^{-3} = 1$ ——— ①

let,

$y^{-3} = v$

$\Rightarrow -3y^{\cancel{-4}}\cancel{\frac{dy}{dx}}\frac{dv}{dx}$

$\Rightarrow -3y^{-4}\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow y^{-4}\frac{dy}{dx} = -\frac{1}{3}\frac{dv}{dx}$

① নং এ মান বসিয়ে,

$-\frac{1}{3}\frac{dv}{dx} - \frac{2}{x}v = 1$

$\Rightarrow \frac{dv}{dx} + \frac{6}{x}v = (-3)$

I.F

$\Rightarrow \frac{dv}{dx} + \frac{6}{x}y^{-3} = -3$ ——— ⑪

I.F $\longrightarrow e^{\int \frac{6}{x}dx}$

$= e^{6\ln x}$

$= x^6$

⑪ $\times\, x^6 \longrightarrow$

$x^6\frac{dv}{dx} + \frac{6}{x}x^6 y^{-3} = x^6(-3)$

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$$\Rightarrow x^6 \frac{d}{dx}(y^{-3}) + \frac{6}{x} y^{-3} x^6 = x^6(-3)$$

$$\Rightarrow \frac{d}{dx}(y^{-3}x^6) = -3x^6$$

$$\Rightarrow y^{-3}x^6 = -3\int x^6$$

$$\Rightarrow y^{-3}x^6 = -3 \cdot \frac{x^7}{7} + c$$

$$\Rightarrow \frac{1}{y^3} = -3 \cdot \frac{x}{7} + \frac{c}{x^6}$$

$$\Rightarrow \frac{1}{y^3} = -3 \cdot \frac{x}{7} + \frac{c}{x^6}$$

$$\Rightarrow y^{-3} = -\frac{3x}{7} + \frac{c}{x^6}$$

06/02/2020

Exact differential eqn:

1st order, 1st degree: $M\,dx + N\,dy = 0$ is said to be exact if

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}} \longrightarrow \text{step 1}$$

$y\,dx + x\,dy = 0$ ——— ①

here,

$M = y, \quad N = x$

$\Rightarrow \dfrac{\partial M}{\partial y} = 1 \quad \Rightarrow \dfrac{\partial N}{\partial x} = 1$

From ① ⟶

$y\,dx + x\,dy = 0$

$\Rightarrow d(xy) = 0$

$\Rightarrow xy = c$

$M = y$

$\Rightarrow \dfrac{\partial M}{\partial x} = xy$

$N = x$ ⟶ free করলে D

$\therefore xy + 0 = c$

$\Rightarrow xy = c$

⟶ **step : 2**

① Integrating M w.r. to x treating y as a constant. (partial diff.)

(ii) in N, terms free from x then integrate w.r. to y. (x এর কোন term থাকবে না)

(iii) then add both the terms.

#: $(y^4 + 4x^3y + 3x)dx + (x^4 + 4xy^3 + y + 1)dy = 0$ ——— (1)

Here,

$M = y^4 + 4x^3y + 3x$

$\Rightarrow \frac{\partial M}{\partial y} = 4y^3 + 4x^3$

$N = x^4 + 4xy^3 + y + 1$

$\Rightarrow \frac{\partial N}{\partial x} = 4x^3 + 4y^3$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, therefore (1) is exact DE.

Now, integrating M w.r. to x treating y as a constant.

i.e. $= xy^4 + x^4y + \frac{3}{2}x^2$

In N, terms free from x is $y+1$. Now integrating w.r. to y and

is equal to $\frac{y^2}{2} + y$

Then adding, we having,

$xy^4 + x^4y + \frac{3}{2}x^2 + \frac{y^2}{2} + y = c$

$x(x^2+y^2-a^2)\,dx + y(x^2-y^2-b^2)\,dy = 0$ ——— (1)

here,

$M = x(x^2+y^2-a^2)$

$\Rightarrow \frac{\partial M}{\partial y} = 2xy$

$N = y(x^2-y^2-b^2)$

$\Rightarrow \frac{\partial N}{\partial x} = 2xy$

here. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, therefore (1) is exact DE.

Now, integrating M w.r.t to x. treating y as a constant.

i.e. $\frac{x^4}{4} + y^2\frac{x^2}{2} - a^2\frac{x^2}{2}$

In N, terms free from x is $y^3 - yb^2$.

Integrating →

$\frac{y^4}{4} - \frac{y^2}{2}b^2$

Adding

$\frac{x^4}{4} + y^2.\frac{x^2}{2} - a^2\frac{x^2}{2} + -\frac{y^4}{4} - \frac{y^2}{2}b^2 = c$

$\Rightarrow x^4 + 2x^2y^2 - 2a^2x^2 - y^4 - 2b^2y^2 = c$

$Mdx + Ndy = 0$ but, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

<u>Rule 1:</u> If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x)$, a fn of x only, then $e^{\int f(x)dx}$ is I.F

<u>Rule 2:</u> If $\dfrac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = g(y)$, a fn of y only, then $e^{\int g(y)dy}$ is an I.F

$(x^2 + y^2 + x)dx + xydy = 0$ ——— (i)

$M = x^2 + y^2 + x$

$\Rightarrow \frac{\partial M}{\partial y} = 2y$

$N = xy$

$\Rightarrow \frac{\partial N}{\partial x} = y$

Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, then (i) is not exact.

Now,

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y - y}{xy} = \frac{y}{xy} = \frac{1}{x} = f(x)$$

$I.F = e^{\int f(x)dx}$

$= e^{\int \frac{1}{x} dx}$

$= e^{\ln x}$

$= x$

Mulliplying ① by I.F. $\longrightarrow$

$$(x^3 + xy^2 + x^2)\,dx + x^2y\,dy = 0 \quad \text{——— (ii)}$$

$$M = x^3 + xy^2 + x^2$$

$$\Rightarrow \frac{\partial M}{\partial y} = 2xy$$

$$N = x^2y$$

$$\Rightarrow \frac{\partial N}{\partial x} = 2xy$$

$$\frac{1}{4}x^4 + \frac{x^2y^2}{2} + \frac{x^3}{3} = c$$

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Solution of Higher order DE with constant co-efficient (Linear):

↳ সবসময় একটি trial solⁿ থাকবে

Sample:

$$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + P_n y = X \quad \text{---} (1)$$

$= 0$ [Homogeneous eqⁿ]

where $P_1, P_2, P_3, \ldots P_n$ are constant (not function of x)

and X is the function of x

Trial solⁿ: $y = e^{mx}$

$$\Rightarrow \frac{dy}{dx} = me^{mx}$$

$$\vdots$$

$$\frac{d^n y}{dx^n} = m^n e^{mx}$$

যদি এর right এ 0 থাকে (Homogeneous)

$$(m^n + P_1 m^{n-1} + \cdots + P_n) e^{mx} = 0$$

$e^{mx} \neq 0$ [exponential function]

$\therefore m^n + P_1 m^{n-1} + \cdots + P_n = 0$ [m এর n সংখ্যক মান বের হবে]

↳ m এর মান পাব

$\therefore m = m_1, m_2, \ldots, m_n$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \cdots + c_n e^{m_n x}$$ [General solⁿ]

$$\# \quad \frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0 \quad \text{------------} \quad (1)$$

let,

$y = e^{mx}$ be a trial solⁿ of (1).

$$\therefore \frac{dy}{dx} = me^{mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2 e^{mx}$$

From (1) $\longrightarrow$

$$(m^2 + 5m + 6)\, e^{mx} = 0$$

$$\Rightarrow m^2 + 5x + 6 = 0 \quad [\because e^{mx} \neq 0]$$

$$\therefore m = -2, -3$$

$$y = c_1 e^{-2x} + c_2 e^{-3x} \quad [\because \text{Homogeneous eq}^n]$$

$$\# \quad \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

$\hookrightarrow m = -2, -2$

$$\therefore y \neq c_1 e^{-2x} + c_2 e^{-2x}$$

$\hookrightarrow = (c_1 + c_2 x)\, e^{-2x}$

৩টি cons. থাকলে; $\longrightarrow y = (c_1 + c_2 x + c_3 x^2 + \ldots)\, e^{-2x}$

$m = -2, -2, 3$ হলে $\longrightarrow$

$y = (c_1 + c_2 x) e^{-2x} + c_3 e^{3x}$

\# $\frac{d^2y}{dx^2} + y = 0 \longrightarrow$ Auxiliary eqn is $m^2 + 1 = 0$

$\Rightarrow m^2 = -1$

$\Rightarrow m = \pm\sqrt{-1}$

$\Rightarrow m = \pm i \quad [a = 0, b = 1]$

$y = c_1 \cos x + c_2 \sin x$

$m = a \pm ib$ হলে

$\hookrightarrow y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

$(D^3 - 4D^2 + 5D - 2)y = 0$ where, $D = \frac{d}{dx}$

Trial eq^n $\longrightarrow$

$$y = e^{mx}$$

$$\Rightarrow \frac{dy}{dx} = me^{mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2 e^{mx}$$

$$\Rightarrow \frac{d^3y}{dx^3} = m^3 e^{mx}$$

From ① $\longrightarrow$

$$(m^3 - 4m^2 + 5m - 2) e^{mx} = 0$$

$$\Rightarrow m^3 - 4m^2 + 5m - 2 = 0$$

$$\Rightarrow m = 2, 1, 1$$

$y =$

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Right side G x এর function আকারে

$f(D) = (D^3 - 4D^2 + 5D - 2)y_p = e^{3x}$ where $D = \frac{d}{dx}$ ... ①

let, $y = e^{mx}$ be the trial sol^n of $(D^3 - 4D^2 + 5D - 2)y = 0$

$$y_c = c_1 e^{2x} + (c_2 + c_3 x)e^x$$

$$y_p = \frac{1}{D^3 - 4D^2 + 5D - 2} e^{3x}$$ [particular function]

$$f(D)y = e^{ax}$$

$$\Rightarrow y_p = \frac{1}{f(D)} e^{ax}$$

$$= \frac{1}{f(a)} e^{ax} \quad \text{if } f(a) \neq 0$$

here $a = 3$

$$y_p = \frac{1}{3^3 - 4.3^2 + 5\cdot 3 - 2} e^{3x}$$

$$= \frac{1}{4} e^{3x}$$

General sol^n of ① is

$$y = y_c + y_p$$

$$= c_1 e^{2x} + (c_2 + c_3 x)e^x + \frac{1}{4} e^{3x}$$

1[illegible]/02/2020

$(D^2 - 3D + 2)y = e^x$

Auxiliary,

$m = 1, 2$

$y_c = c_1 e^x + c_2 e^{2x}$

$\therefore y_p = \frac{1}{D^2 - 3D + 2} e^x$ ———— (1)

$y_p = \frac{1}{f(D)} e^{ax}$

$= \frac{1}{(D-a)^n} e^{ax}$

$= \frac{x^n}{n!} e^{ax}$ [when $f(a) = 0$]

[e^{ax} here $a = 1$]
এই মান বসিয়ে করলে-বসিয়ে $f(D)$
যদি 0 হয়, তাহলে এই process

① নং থেকে,

$y_p = \frac{1}{(D-2)(D-1)} e^x$

$= \frac{1}{D-1} \left[\frac{1}{D-2} e^x \right]$

$= -\frac{1}{D-1} e^x$

$= -\frac{x^1}{1!} e^x$

$= -xe^x$

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$(D^2-1)y = e^{2x}$

$$\Rightarrow y_p = \frac{1}{(D^2-1)} e^{2x}$$

$$= \frac{1}{3} e^{2x} \quad$$ [D এর স্থানে 2 বসিয়ে]

(Ans)

If.

$$y_p = \frac{1}{(D^2-1)} e^{-x}$$

$$= \frac{1}{(D+1)(D-1)} e^{-x}$$

$$= \frac{1}{D+1}\left[\frac{1}{D-1} e^{-x}\right]$$

$$= -\frac{1}{2}\left[\frac{1}{D+1} e^{-x}\right]$$

$$= -\frac{1}{2} \frac{x^1}{1!} e^{-x}$$

$$= -\frac{1}{2} x e^{-x}.$$

$(D^2-1)y = x^2$

$$\Rightarrow y_p = \frac{1}{D^2-1} x^2$$

$$= \frac{1}{-(1-D^2)} x^2$$

$$= -(1-D^2)^{-1} x^2$$

$$= -\left[1 \;\; D^2 + D^4 \;\ldots\ldots\right] x^2$$

$$= -x^2 - 2$$

$\Rightarrow D^2 x^2$ [2 বার diff]

$\Rightarrow 2$

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$* \ (D^2 - 2D + 1)y = x+1$

$\Rightarrow y_p = \dfrac{1}{D^2 - 2D + 1}(x+1)$

$= \dfrac{1}{(D-1)^2}(x+1)$

$= (D-1)^{-2}(x+1)$

$= [1 + (-2)$

Or,

$y_p = \dfrac{1}{1 + D^2 - 2D}(x+1)$

$= [1 + (D^2 - 2D)]^{-1}(x+1)$

$= [1 - (D^2 - 2D) + (D^2 - 2D)^2 - \cdots](x+1)$

$= x + 1 + 2$

$= x + 3$

$* \ y_p = \dfrac{1}{f(D)} \sin ax$

যেখানে D থাকবে সেটি— বাদে D^2 থাকলে $f(-a^2)$ বসাবে

when $f(-a^2) \neq 0$

$* \ (D^2 - 1)y = \sin 2x$

$\Rightarrow y_p = \dfrac{1}{D^2 - 1} \sin 2x$

$= \dfrac{1}{-2^2 - 1} \sin 2x$

$= \dfrac{1}{-5} \sin 2x$

\# $(D^2-2D+1)y = \sin 2x$

$$\Rightarrow y_p = \frac{1}{D^2-2D+1}\sin 2x$$

$$= \frac{1}{-2^2-2D+1}\sin 2x$$

$$= \frac{1}{-3-2D}\sin 2x$$

$$= \frac{(-3+2D)}{(-3-2D)(-3+2D)}\sin 2x$$

$$= \frac{-3+2D}{9-4D^2}\sin 2x$$

$$= \frac{-3+2D}{9+16}\sin 2x$$

$$= \frac{1}{25}(-3+2D)\sin 2x$$

$$= \frac{1}{25}(-3\sin 2x + 4\cos 2x)$$

\# $(D^2+1)y = \sin x$ ⟶ [illegible] D দিয়ে ভেঙেছি

[illegible] D ভেঙেছি

1 বার derivative, 1 বার x গুণ করেছি

$$\Rightarrow y_p = \frac{1}{D^2+1}\sin x$$

$$= \frac{x}{2D}\sin x$$

$$= \frac{x}{2}\frac{1}{D}\sin x$$

$$= \frac{x}{2}(-\cos x)$$

$$= -\frac{x\cos x}{2}$$

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Ⓐ $y_p = \frac{1}{f(D)} (v e^{ax})$ where v is any function

$\Rightarrow y_p = e^{ax} \frac{1}{f(D+a)} v$

Ⓔ $(D^2 - 9) y = x e^{3x}$

$\Rightarrow y_c = c_1 e^{3x} + c_2 e^{-3x}$

$y_p = \frac{1}{D^2 - 9} (x e^{3x})$

$= e^{3x} \frac{1}{(D+3)^2 - 9} x$

$= e^{3x} \frac{1}{D^2 + 6D + 9 - 9} x$

$= e^{3x} \frac{1}{D^2 + 6D} x$

$= e^{3x} \frac{1}{D(D+6)} x$

$= e^{3x} \frac{1}{D} \left[\frac{1}{D+6} x \right]$

$= e^{3x} \frac{1}{D} \cdot \frac{1}{6} \left[1 + \frac{D}{6} \right]^{-1} x$

$= \frac{e^{3x}}{6} \frac{1}{D} \left[1 - \frac{D}{6} + \cdots \right] x$

$= \frac{1}{6} e^{3x} \frac{1}{D} \left(x - \frac{1}{6} \right)$

$= \frac{1}{6} e^{3x} \left(\frac{x^2}{2} - \frac{x}{6} \right) \quad = \frac{1}{12} x^2 e^{3x} - \frac{1}{36} x e^{3x}$

$(D^2-2D+4)y = e^x \cos x$

$\Rightarrow$

$$y_p = \frac{1}{(D^2-2D+4)}(e^x \cos x)$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1)+4} \cos x$$

$$= e^x \frac{1}{D^2+2D+1-2D-2+4} \cos x$$

$$= e^x \frac{1}{D^2+3} \cos x$$

$$= e^x \frac{\cos x}{-1^2+3}$$

$$= \frac{1}{2} e^x \cos x$$

$(D^2+4)y = x \sin x$

$$x e^{ix}$$

$$= x(\cos x + i \sin x)$$

$$= x\cos x + \underbrace{i x \sin x}_{\text{imaginary part of } x\sin x}$$

$\Rightarrow$

$$y_c =$$

$$y_p = \frac{1}{D^2+4} x \sin x$$

$$= \text{Imaginary part of } (x e^{ix}) \frac{1}{D^2+4}$$

{cos দেওয়া থাকলে real part হবে

$$= \text{I.P of } e^{ix} \frac{1}{(D+i)^2+4} x$$

$$= \text{I.P of } e^{ix} \frac{1}{D^2+2iD+3} x$$

$$= \text{I.P of } e^{ix} \frac{1}{3\left(1+\frac{D^2+2iD}{3}\right)} x$$

$$= \text{I.P. of } e^{ix} \frac{1}{3}\left[1+\frac{D^2+2iD}{3}\right]^{-1} x$$

$= \text{Im. part of } \frac{1}{2} e^{ix} \left[1 - \frac{D^2+2iD}{2} + \dots \right] x$

$= \text{Im. part of } \frac{1}{2} e^{ix} \left(x - \frac{2}{2} i\right)$

$= \text{I.P. of } \frac{1}{2} (\cos x + i \sin x) \left(x - \frac{2}{2} i\right)$

$= -\frac{2}{2} \cos x + \frac{1}{2} x \sin x$

* $(D^2+1)y = x^2 \cos x$

$\Rightarrow$ $y_p = \text{Real part of } \frac{1}{D^2+1} (x^2 e^{ix})$

$= \text{Real part of } e^{ix} \frac{1}{(D+i)^2-1} x^2$

$= \text{Real part of } e^{ix} \frac{1}{D^2+2iD-2} x^2$

$= \text{Real part of } e^{ix} \frac{1}{-2\left(1 - \frac{D^2+2iD}{2}\right)} x^2$

$= \text{Real part of } \frac{e^{ix}}{-2} \left(1 - \frac{D^2+2iD}{2}\right)^{-1} x^2$

$= \text{R.P of } \frac{e^{ix}}{-2} \left[1 + \frac{D^2+2iD}{2} + \frac{(D^2+2iD)^2}{4} + \dots \right] x^2$

$= \text{R.P of } \frac{e^{ix}}{-2} (x^2 + 1 + 2xi)$

$= \text{R.P of } -\frac{1}{2} (\cos x + i \sin x)(x^2 + 1 + 2xi)$

$$= -\frac{1}{2}\left[(x^2-1)\cos x - 2x\sin x\right]$$

$$(D^2-4D+4)y = 3x^2 e^{2x}\sin 2x$$

(ii) Soln of Higher Order DE with variable co-efficient:

$$x^{(n)} \underbrace{\frac{d^{n}y}{dx^{n}}} \rightarrow \text{cossy oylar homogeneous eqn}$$

তৈরি করবো

(order same হবে and degree)

$$x^{n}\frac{d^{n}y}{dx^{n}} + P_1 x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + P_2 x^{n-2}\frac{d^{n-2}}{dx^{n-2}} + \dots + P_n y = X$$

→ constant co-efficient এ আনতে হবে

let,

Independent variable, $x = e^{z}$

$$\Rightarrow \log x = z$$

and $\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \frac{1}{x}\cdot\frac{dy}{dz}$

$$\therefore x\cdot\frac{dy}{dx} = \frac{dy}{dz}$$

again, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1}{x}\cdot\frac{dy}{dz}\right)$

$$= \frac{1}{x}\cdot\frac{d}{dz}\left(\frac{dy}{dz}\right)\cdot\frac{dz}{dx} - \frac{1}{x^2}\cdot\frac{dy}{dz}$$

$$\left[\because \frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}\right]$$

$$= \frac{1}{x}\cdot\frac{d^2y}{dz^2}\cdot\frac{1}{x} - \frac{1}{x^2}\cdot\frac{dy}{dz}$$

$$= \frac{1}{x^2}\cdot\frac{d^2y}{dz^2} - \frac{1}{x^2}\cdot\frac{dy}{dz}$$

$$\therefore \quad x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$$

[4] Homogeneous linear equations:

[3] $$x^2 \frac{d^2y}{dx^2} + y = 3x^2 \quad \text{—— (i)}$$

Soln: let,

$$x = e^z$$

$$\Rightarrow \log x = z \quad \text{—— (ii)}$$

$$x \frac{dy}{dx} = \frac{dy}{dz} \quad \text{—— (iii)}$$

$$x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz} \quad \text{—— (iv)}$$

Using (iii) and (iv) in eqn (i) $\longrightarrow$

$$\frac{d^2y}{dz^2} - \frac{dy}{dz} + y = 3e^{2z} \quad \text{—— (v)}$$

let,

$y = e^{mz}$ be the trial soln of

$$\frac{d^2y}{dz^2} - \frac{dy}{dz} + y = 0$$

A. E $\quad m^2 - m + 1 = 0$

$$\Rightarrow m = \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{3}\, i}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

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$\therefore y_c = e^{\frac{1}{2}z}\left(c_1 \cos\frac{\sqrt{3}z}{2} + c_2 \sin\frac{\sqrt{3}z}{2}\right)$ [z এর মান x এর মাধ্যমে আসবে

$$\therefore y_p = \frac{1}{D^2 - D + 1} 3e^{2z}$$

$$= 3 \cdot \frac{e^{2z}}{2^2 - 2 + 1}$$

$$= \cancel{3}e^{2z}$$

$$y = y_c + y_p$$

(ii) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$ —— (1)

Soln: let,

$$x = e^z$$

$$\Rightarrow \log x = z \text{ —— (ii)}$$

$$x\frac{dy}{dx} = \frac{dy}{dz} \text{ —— (iii)}$$

$$x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz} \text{ —— (iv)}$$

Putting (iii) and (iv) in eqn (1) ⟶

$$\frac{d^2y}{dz^2} - \frac{dy}{dz} - 2\cdot\frac{dy}{dz} - 4\cdot y = e^{4z}$$

let,

$y = e^{mz}$ be the trial soln of $\frac{d^2y}{dz^2} - 3\frac{dy}{dz} - 4y = 0$

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Ans: $y = c_1 x^4 + c_2/x + \frac{1}{5} x^4 \log x$

$m^2 - 3m - 4 = 0$

$\Rightarrow m = 4, -1$

$y_c = c_1 e^4 \quad - c_2 e$

$y_p = \frac{1}{D^2 - 3D - 4}$

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26/02/2020

(iii) DE when dependent variable is absent: if —,

$$f\left(\frac{d^n y}{dx^n}, \frac{d^{n-1} y}{dx^{n-1}}, \frac{d^{n-2} y}{dx^{n-2}}, \ldots\ldots, \frac{d^k y}{dx^k}, x\right) = 0 \quad \text{(i)}$$

$q = \frac{d^k y}{dx^k}$ [$k \neq 1$]

let, us put,

$$p = \frac{dy}{dx}, \quad \frac{d^2 y}{dx^2} = \frac{dp}{dx}, \ldots\ldots \text{ etc.}$$

1st order [illegible] p হবে,

Then eq^n (i) becomes,

$$f\left(\frac{d^{n-1} p}{dx^{n-1}}, \frac{d^{n-2} p}{dx^{n-2}}, \ldots\ldots, p, x\right) = 0 \quad \text{(ii)}$$

We will have,

$$p = r(x)$$

$$\Rightarrow \frac{dy}{dx} = r(x)$$

$$\Rightarrow y = \int r(x)\, dx + c$$

which is the required sol^n.

[illegible]

\# $2\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 4 = 0$ ——— (1)

Sol^n: Since the eqn is free from y, then put, $\frac{dy}{dx} = p$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dp}{dx}$ in eqn (1),

$$2 \cdot \frac{dp}{dx} - p^2 + 4 = 0$$

$$\Rightarrow 2\frac{dp}{dx} = p^2 - 4$$

$$\Rightarrow \frac{2\,dp}{p^2-4} = dx$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{p-2} - \frac{1}{p+2}\right) dp = dx$$

$$\Rightarrow \frac{1}{2}\ln(p-2) - \frac{1}{2}\int \frac{p}{p+2}\,dp = dx$$

$$\Rightarrow \frac{1}{2}\ln(p-2) - \frac{1}{2}\left[\frac{p+2}{p+2} - \frac{2}{p+2}\right] dp = dx$$

$$\Rightarrow \frac{1}{2}\ln(p-2) - \frac{1}{2}p$$

$$\Rightarrow \frac{1}{2}\ln(p-2) - \frac{1}{2}\ln(p+2) = x + \ln c_1$$

$$\Rightarrow \ln\frac{p-2}{p+2} = 2x + \ln c_1^2$$

$$\Rightarrow \ln\frac{p-2}{p+2} = \ln e^{2x} + \ln c_1^2$$

$$\Rightarrow \ln\frac{p-2}{p+2} = \ln e^{2x} c_1^2$$

$$\Rightarrow \frac{p-2}{p+2} = e^{2x} c_1^2$$

$$\Rightarrow p - 2 = (p+2)e^{2x} c_1^2$$

$\Rightarrow p - p e^{2x} c_1^2 = 2 + 2e^{2x} c_1^2$

$\Rightarrow p(1 - e^{2x} c_1^2) = 2(1 + e^{2x} c_1^2)$

$\Rightarrow p = \dfrac{dy}{dx} = \dfrac{2(1 + e^{2x} c_1^2)}{(1 - e^{2x} c_1^2)}$

$\Rightarrow \dfrac{dy}{dx} = 2\left(1 + \dfrac{2c_1^2 e^{2x}}{1 - e^{2x} c_1^2}\right)$

$\Rightarrow y = 2x + 2\displaystyle\int \dfrac{2c_1^2 e^{2x}}{1 - e^{2x} c_1^2}\, dx$

$\Rightarrow y = 2x + 2\displaystyle\int \dfrac{2c_1^2 e^{2x}}{1 - e^{2x} c_1^2}\, dx$

$\Rightarrow y = 2x - 2\ln(1 - e^{2x} c_1^2) + c_2$

put,

$1 - c_1^2 e^{2x} = z$

$\Rightarrow -2c_1^2 e^{2x}\, dx = dz$

* $\frac{d^4y}{dx^4} - \cot x \frac{d^3y}{dx^3} = 0$

Soln: The equation does not contain y directly and the lowest differenti co-efficient is $\frac{d^3y}{dx^3}$.

Then put $\frac{d^3y}{dx^3} = q$

$\Rightarrow \frac{d^4y}{dx^4} = \frac{dq}{dx}$ in eqn ①,

$\frac{dq}{dx} - \cot x . q = 0$

$\Rightarrow \frac{dq}{q} = \cot x \, dx$

$\Rightarrow \ln q = \log \sin x + \log c$

$\Rightarrow q = c \sin x$

$\Rightarrow \frac{d^3y}{dx^3} = c \sin x$

$\Rightarrow \frac{d^2y}{dx^2} = -c \cos x + c_1$

$\Rightarrow \frac{dy}{dx} = -c \sin x + c_1 x + c_2$

$\Rightarrow y = c \cos x + c_1 x^2/2 + c_2 x + c_3$

Independent variable absent:

The DE $f\left(\frac{d^n y}{dx^n}, \frac{d^{n-1}y}{dx^{n-1}}, \ldots, \frac{dy}{dx}, y\right) = 0$ ——— ①

The eqⁿ does not contain x directly, then—

put, $\frac{dy}{dx} = p$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dp}{dx}$

$\Rightarrow \frac{dp}{dy} \cdot \frac{dy}{dx} = p \cdot \frac{dp}{dy}$ .... etc.

putting this in eqⁿ ①,

$$f\left(\frac{d^{n-1}p}{dy^{n-1}}, \frac{d^{n-2}p}{dy^{n-2}}, \ldots p, y\right) = 0$$

Solving this, we have,

$p = f(y)$

$\Rightarrow \frac{dy}{dx} = F(y)$

$\Rightarrow y = \int F(y)\, dy + c.$

$\#\ \frac{d^2y}{dx^2} + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3 = 0$ ——— ①

Since eqn ① does not contain x directly, then,

put, $\frac{dy}{dx} = p$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{dp}{dx} = \frac{dp}{dy} \cdot \frac{dy}{dx} = p\frac{dp}{dy}$ in eqn ①,

$p\frac{dp}{dy} + p + p^3 = 0$

$\Rightarrow \frac{dp}{dy} + 1 + p^2 = 0$

$\Rightarrow \frac{dp}{dy} = -(1+p^2)$

$\Rightarrow \frac{dp}{1+p^2} = -dy$

$\Rightarrow \tan^{-1}p = -y + c_1$

$\Rightarrow p = \tan(c_1 - y)$

$\Rightarrow \frac{dy}{dx} = \tan(c_1 - y)$

$\Rightarrow \frac{dy}{\tan(c_1-y)} = dx$

$\Rightarrow \cot(c_1-y)\,dy = dx$

$\Rightarrow -\log\sin(c_1-y) = x + \log c_2$

$\Rightarrow -\log \sin(c_1-y)c_2 = x$

$\Rightarrow \log\sin(c_1-y)c_2 = \log e^{-x}$

$\Rightarrow \sin(c_1-y)c_2 = e^{-x}$

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(iii) Soln of 1st order DE but not 1st degree:

$$p^n + P_1 p^{n-1} + P_2 p^{n-2} + \dots + P_n = 0 \quad \text{---} (1) \quad \text{where, } p = \frac{dy}{dx}$$

$P_1, P_2 \rightarrow$ constant

I solvable for p :

Eqn (1) can be written as

$$[p - F_1(x,y)]\,[p - F_2(x,y)] \dots [p - F_n(x,y)] = 0 \quad \text{---} (ii)$$

Now,

$p - F_1(x,y) = 0$

$\Rightarrow \frac{dy}{dx} = F_1(x,y)$

$\Rightarrow y = \int F_1 dx + C_1$

$\Rightarrow f_1(x, y, C_1) = 0$

$p - F_2(x,y) = 0$

$\Rightarrow \frac{dy}{dx} = F_2(x,y)$

$\Rightarrow y = \int F_2 dx + C_2$

$\Rightarrow f_2(x, y, C_2) = 0$

...

$p - F_n(x,y) = 0$

$\Rightarrow \frac{dy}{dx} = F_n(x,y)$

$\Rightarrow y = \int F_n . dx + C_n$

$\Rightarrow f_n(x, y, C_n) = 0$

But $C_1 = C_2 = C_3 = \dots = C_n = C$

Hence, the general soln of (1) is

$$f_1(x,y,c)\, f_2(x,y,c) \dots f_n(x,y,c) = 0$$

(Ans)

Ⓠ Solve $p^4 - (x+2y+1)p^3 + (x+2y+2xy)p^2 - 2xyp = 0$ ——(i)

Soln: Factorize eqn (i), we have,

$p(p-1)(p-x)(p-2y) = 0$ ——(ii)

here,

$p = 0$

$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow y = c$

$p - 1 = 0$

$\Rightarrow \frac{dy}{dx} = 1$

$\Rightarrow dy = dx$

$\Rightarrow y = x + c$

$p - x = 0$

$\Rightarrow \frac{dy}{dx} = x$

$\Rightarrow y = \frac{x^2}{2} + c$

$\Rightarrow 2y - x^2 = c$

$p - 2y = 0$

$\Rightarrow \frac{dy}{dx} = 2y$

$\Rightarrow \frac{dy}{y} = 2dx$

$\Rightarrow \ln y = 2x + \ln c$

$\Rightarrow \ln y = \ln e^{2x} + \ln c$

$\Rightarrow \frac{y}{e^{2x}} = c$

$\Rightarrow \ln y = \ln ce^{2x}$

$\Rightarrow y = ce^{2x}$

hence, g.s is.

$$[(y-c)(y-x-c)(2y-x^2-c)(y-ce^{2x})] = 0$$

(Ans)

(iii) Eq^n solveable for y: (যেখানে y কে seperate করা যায়)

$$y = f(x, p) \quad \text{---} \; (i) \quad \text{where} \quad p = \frac{dy}{dx}$$

Now, diff (i) w.r.to x,

$$\frac{dy}{dx} = p = F\left(x, p, \frac{dp}{dx}\right) \quad \text{---} \; (ii)$$

Solving (ii), we have,

$$\phi(x, p, c) = 0$$

$$\Rightarrow p = \phi_1(x, c)$$

(p কে seperate করা না গেলে x কে আলাদা করে eq^n (i) এর সাথে elemination করব.)

Solve $y = 2px + p^4 x^2$ ——— ①

Soln: Diff ① w.r.t. to ① x,

$$\frac{dy}{dx} = p = 2p + 2x \cdot \frac{dp}{dx} + 2xp^4 + 4p^3 x^2 \frac{dp}{dx}$$

$$\Rightarrow -p - 2xp^4 = 2x(1 + 2p^3 x) \frac{dp}{dx}$$

$$\Rightarrow -p(1 + 2xp^3) = 2x(1 + 2p^3 x) \frac{dp}{dx}$$

$$\Rightarrow -p = 2x \frac{dp}{dx}$$

$$\Rightarrow \frac{dp}{p} + \frac{1}{2} \frac{dx}{x} = 0$$

$$\Rightarrow \ln p + \frac{1}{2} \ln x = \ln c$$

$$\Rightarrow p^2 x = c$$

$$\Rightarrow p^2 = c/x$$

Putting the value of p in ①

$$y = 2px + c^2/x^2 \cdot x^2$$

$$\Rightarrow y = 2px + c^2$$

$$\Rightarrow y - c^2 = 2px$$

$$\Rightarrow (y - c^2)^2 = 4p^2 x^2 - 4\, c/x \cdot x^2$$

[Solvable for x → [illegible]]

$$\Rightarrow (y - c^2)^2 = 4cx$$

(Ans)

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