

Heaven's Light is Our Guide  
**DEPARTMENT OF CIVIL ENGINEERING**  
**RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY**  
 B.Sc. Engineering Second Year Odd Semester Examination, 2019  
**Math- 2101**  
**Mathematics III**

Full marks: 72

Time: 3 Hours

- N.B:-**
- (i) Answer any SIX questions, taking THREE from each section.
  - (ii) Figures in the margin indicate full marks.
  - (iii) Use separate answer script for each section.
  - (iv) Assume reasonable value for any data missing.

**SECTION-A**

- Q.1(a) Define order and degree of a differential equation (DE) with example. Obtain the DE of the family of curves  $y = e^x(A\cos x + B\sin x)$ . Mention the order of the obtained DE. 6.00
- (b) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 290k and the substance cools from 370k to 330k in 10 minutes, find when the temperature will be 295k. 6.00
- Q.2(a) Define homogeneous DE with two examples; solve the homogeneous DE  $(x^3 - 2y^3)dx + 3xy^2dy = 0$  using proper transformation. 6.00
- (b) What is integrating factor? Find integrating factor of  $x \frac{dy}{dx} - 2y = x^2 + \sin\left(\frac{1}{x^2}\right)$ , and solve it. 6.00
- Q.3 (a) Solve the DE  $(D^2 - 4D + 4)y = 3x^2e^{2x}\sin 2x$ , where  $D \equiv \frac{d}{dx}$ . 4.00
- (b) Find the solution of  $x \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} = 0$  when the equation does not contain y directly. 4.00
- (c) Prove that  $e^{\frac{x}{2}} \left(x - \frac{1}{2}\right) = \sum_{n=-\infty}^{\infty} J_n(x) z^n$  where  $J_n(x)$  is Bessel's function. 4.00
- Q.4 (a) State and prove Rodrigue's formula for Legendre polynomial  $P_n(x)$ . 7.00
- (b) Show that  $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$  where  $\alpha$  and  $\beta$  are the roots of  $J_n(x)$ . 5.00

**SECTION-B**

- Q.5(a) Define Fourier series. Expand the Fourier series for  $f(x) = x + x^2$  when  $-\pi < x < \pi$ . 6.00
- (b) Find a Fourier series expression for  $f(x) = x \sin x$  in the interval  $-\pi < x < \pi$ . 6.00
- Q.6(a) Find the Fourier sine integral for  $f(x) = e^{-\beta x}$  and hence show that  $\frac{\pi}{2} e^{-\beta x} = \int_0^{\infty} \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2} d\lambda$ . 6.00
- (b) Solve the one dimensional heat equation by using Fourier transformation:  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ ,  $x > 0$ ,  $t > 0$  with the condition  $u(0, t) = 0$   
 $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$  6.00
- Q.7(a) Solve:  $(y + xz)p - (x + yz)q = x^2 - y^2$ . 6.00
- (b) Find the integral surface of  $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$  which passes through  $y = 0$  and  $xz = a^3$ . 6.00
- Q.8(a) Find the complete and singular integral of  $(p^2 + q^2)y = qz$ . 6.00
- (b) Solve the Laplace equation in three dimensional Cartesian coordinate  $(x, y, z)$ . 6.00

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 (iv) Assume reasonable value for any data missing.

**SECTION-A**

Q.1(a) Define integrating factor. Find the integrating factor of  $\frac{dy}{dx} + 2xy = x^3$  and hence solve it. 4.00

(b) Find the general solution of  $\frac{d^2x}{dt^2} + n^2x = 0$ . 4.00

(c) Solve the initial value problem:  
 $\frac{dy}{dx} + 2y \tan x = \sin x; y\left(\frac{\pi}{3}\right) = 0$  4.00

Q.2 Define singular point and ordinary point of a differential equation. Find the series solution of  $4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 0$  12.00

Q.3(a) Show that  $\int_{-1}^1 \{p_n(x)\}^2 dx = \frac{2}{2n+1}, n = 0, 1, 2, \dots$  Where  $p_n(x)$  is the legendre polynomial of order  $n$ . 6.00

(b) Prove that  $p_n(x)$  is the co-efficient of  $z^n$  in the expansion of  $(1 - 2xz + z^2)^{-\frac{1}{2}}$  in ascending powers of  $z$ . 6.00  
 Also find the value of  $p_n(1)$

Q.4(a) Prove that  $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[ \frac{3}{x} \sin x + \frac{3-x^2}{x^2} \cos x \right]$  6.00

(b) Prove that,  $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0$  where  $\alpha$  and  $\beta$  are the roots of  $J_n(x) = 0$ . 6.00

**SECTION-B**

Q.5(a) Define even and odd function with example. Show that an even function can have no sine terms in its Fourier expansion. 6.00

(b) Find the Fourier series to represent the function  $f(x) = e^x$  for  $-\pi < x < \pi$  and hence derive a series for  $\frac{\pi}{\sinh \pi}$  6.00

Q.6(a) Define Fourier sine and cosine transform. Find the Fourier transform of  $f(x) = e^{-|x|}$ , where  $-\alpha \leq x \leq \alpha$  6.00

(b) For the Fourier integral to prove that  $f(x) = \frac{1}{\pi} \int_0^{\infty} \int_{-\infty}^{\infty} f(t) \cos \lambda(t-x) dt d\lambda$  6.00

Q.7(a) Define partial DE with example of 2<sup>nd</sup> order & 3<sup>rd</sup> degree. Form a partial DE from  $x + y + z = f(x^2 + y^2 + z^2)$  by eliminating arbitrary function  $f$ . 6.00

(b) Find the general solution of the partial DE  $(2xy - 1)p + (z - 2x^2)q = 2(x - y)z$  and also the particular integral which passes through the line  $x = 1, y = 0$ . 6.00

Q.8(a) Find a complete and singular integral of  $2xz - px^2 - 2qxy + pq = 0$  6.00

(b) Solve the Laplace's equation in two dimensional polar coordinate  $(r, \theta)$  6.00

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$\sqrt{\frac{2}{\pi x}}$

Math-201  
Mathematics -III

Full Marks: 70

Time: 3 Hours

- N.B.: (i) Answer SIX questions, taking THREE from each section.  
(ii) Figure in the margin indicates full marks.  
(iii) Use separate answer script for each section.

SECTION-A

- Q.1(a) Show that if  $I \frac{d^2 \theta}{dt^2} + g\theta = 0$  and if  $\theta = \alpha$  and  $\frac{d\theta}{dt} = 0$  when  $t = 0$  then  $\theta = \alpha \cos\left(t \sqrt{\frac{g}{I}}\right)$  6.00
- (b) Find the foci of the curve which satisfies the differential equation  $(1 + y^2)dx - xydy = 0$  and passes through the point (1,0). 5.67
- Q.2 Define ordinary point and singular point. Solve  $2x^2y'' - xy' + (x-5)y = 0$  by the Frobenius method. 11.67
- Q.3(a) Prove that  $P_n(x)$  is the coefficient of  $h^n$  in the expansion of  $(1-2xh + h^2)^{-1/2}$  in ascending powers of  $h$ . Also show that  $P_n(1) = 1$ . 4.00
- (b) Prove that  $\int_1^x x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$  4.00
- (c) Prove that  $xP_n'(x) - P_{n-1}'(x) = nP_n'(x)$ . 3.67
- Q.4(a) Using the recurrence relations, show that  $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ . 4.00
- (b) Prove that  $J_{3/2}(x) = \sqrt{\left(\frac{2}{\pi}\right)} \left(\frac{\sin x}{x} - \cos x\right)$ . 4.00
- (c) Prove that  $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$  3.67

SECTION-B

- Q.5(a) Define periodic function. If  $f(x) = \left(\frac{\pi-x}{2}\right)^2$  in the range 0 to  $2\pi$ , show that 6.00
- $$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$$
- (b) Expand the Fourier series in the interval  $-2 < x < 2$  when  $f(x) = \begin{cases} 2; & -2 < x < 0 \\ x; & 0 < x < 2 \end{cases}$  5.67
- Q.6(a) Find the Fourier sine integral for  $f(x) = e^{-\beta x}$  hence show that  $\frac{\pi}{2} e^{-\beta x} = \int_0^\infty \frac{\lambda \sin \lambda x}{\beta^2 + \lambda^2} d\lambda$  6.00
- (b) Prove that for the Fourier integral  $f(x) = \frac{1}{\pi} \int_0^\infty \int_{-\infty}^\infty f(t) \cos u(t-x) dt du$  5.67
- Q.7(a) Find the solution of  $\frac{\partial^2 u}{\partial x^2} = h^2 \frac{\partial u}{\partial t}$  for which  $u(0,t) = u(l,t) = 0$  and  $u(x,0) = \sin \frac{\pi x}{l}$  by the method of variable separable. 5.67
- (b) Find the complete and singular integral of  $(p^2 + q^2)y = qz$ . 6.00
- Q.8(a) Solve  $pxy + pq + qy = yz$  by Charpit's method. 5.67
- (b) Solve the three dimensional Laplace's equation in cylindrical coordinates  $(r, \theta, z)$ . 6.00

**Math 2101**

**Mathematics - III**

**Full Marks: 72**

**Time: 3 Hours**

- N.B.:-**
- (i) Answer SIX questions, taking THREE from each section.
  - (ii) Figure in the margin indicates full marks.
  - (iii) Use separate answer script for each section.
  - (iv) Assume reasonable value for any data missing.

**SECTION-A**

- Q. 1(a) Find the foci of the curve which satisfies the differential equation  $(1 + y^2)dx - xydy = 0$  and passes through the point (1,0). 4.00
- Q. 1(b) Find the particular solution of  $\cos y dx + (1 + 2e^{-x}) \sin y dy = 0$  when  $x = 0, y = \frac{\pi}{4}$ . 4.00
- Q. 1(c) Show that the general solution of the differential equation  $\frac{d^2x}{dt^2} + n^2x = 0$  is  $x = A \cos(nt + \xi)$ . 4.00
- Q. 2 Define ordinary point and singular point. Solve  $2x^2y'' - xy' + (x-5)y = 0$  by the Frobenius method. 12.00
- Q. 3(a) Prove that  $J_n(x)$  is the co-efficient of  $z^n$  in the expansion of  $e^{x(z-\frac{1}{z})}$ . Also prove that  $\cos(x \cos \alpha) = J_0(x) - 2J_2(x) \cos 2\alpha + 2J_4(x) \cos 4\alpha \dots$   
 $\sin(x \cos \alpha) = 2J_1(x) \cos \alpha - 2J_3(x) \cos 3\alpha + 2J_5(x) \cos 5\alpha \dots$  6.00
- Q. 3(b) Prove that  $\int_0^\pi x J_n(\alpha x) J_n(\beta x) dx = 0$  where  $\alpha$  and  $\beta$  are the roots of  $J_n(x) = 0$ . 6.00
- Q. 4(a) If  $z$  is a solution of Legendre equation  $(1-x^2)y'' - 2xy' + n(n+1)y = 0$  then show that  $(1-x^2)^{m/2} \frac{d^m z}{dx^m}$  is a solution of the equation  $(1-x^2)y'' - 2xy' + \left\{ n(n+1) - \frac{m^2}{1-x^2} \right\} y = 0$ . 6.00
- Q. 4(b) Derive Rodrigues formula for Legendre polynomial. 6.00

**SECTION-B**

- Q. 5(a) Define periodic function. Expand the half range cosine series. 6.00
- Q. 5(b) Expand the Fourier series for the function  $f(x) = \left(\frac{\pi-x}{2}\right)^2$  in the range 0 to  $2\pi$ . 6.00
- Q. 6(a) Find the Fourier transform of  $f(x)$  if  $f(x) = \begin{cases} x & |x| \leq a \\ 0 & |x| > a \end{cases}$ . 6.00
- Q. 6(b) Solve the boundary value problem  $\frac{\partial u}{\partial t} = 3 \frac{\partial^2 u}{\partial x^2}$  with the condition  $u(0,t) = u(2,t) = 0, t > 0$ , and  $u(x,0) = x, 0 < x < 2$  using Fourier transformation. 6.00
- Q. 7(a) Find a partial differential equation by the elimination of arbitrary function  $\phi$  from the equation  $\phi(u,v) = 0$  where  $u$  and  $v$  are the function of  $x, y$  and  $z$ . 6.00
- Q. 7(b) Solve the nonlinear partial differential equation  $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$  by Charpit's method. 6.00
- Q. 8(a) Solve:  $\frac{1}{r^2} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) + \frac{1}{\sin \phi} \frac{\partial}{\partial \phi} \left( \sin \phi \frac{\partial u}{\partial \phi} \right) + \frac{1}{\sin^2 \phi} \frac{\partial^2 u}{\partial \theta^2} \right]$ . 7.00
- Q. 8(b) The diameter of a semi-circular plate of radius  $a$  is kept at  $0^\circ C$  and the temperature at the semi-circular boundary is  $T^\circ C$ . Find the steady state temperature in the plate. 5.00

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